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SPIN AXIS DIRECTION DETERMINATION FOR THE GALACTIC-JUPITER PROBE

BY
V. R. SIMAS

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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

The antenna for the envisioned spin stabilized Galactic-Jupiter Probe is an 8 foot body fixed paraboloid which at the 2.3GHz down link frequency transmits a 4° pencil beam toward the earth. The spacecraft spin axis which is concentric to the antenna beam must be precessed periodically to enable the antenna to follow the earth as it travels in its orbit about the Sun. The described (spacecraft) Spin Axis Direction Determination is a 2 wavelength single baseline radio interferometer with its two antennas mounted symmetrically with respect to the spin axis. The system's acquisition beamwidth is 24°. Operating on a received 2.1 GHz beacon signal radiated by earth stations the system employs phase monopulse techniques to derive a sinusoidally varying voltage synchronous to the spin rate. The amplitude of this voltage is proportional to the angular boresight error, ϵ; the phase, ψ, denotes the direction of the error and supplies timing signals for activating the series of propulsion pulses necessary to precess the spin axis in the proper direction to correct the error. When ϵ exceeds the prescribed threshold value of 1°, a programmed attitude correction sequence is initiated by command from earth. At a range of 5 AU and a power level of 10 kw radiated by an 85 foot dish the values of $\sigma\epsilon$ and $\sigma\psi$ at ϵ = 1° are calculated to be 0.018° and 1.02° respectively which are satisfactory for the mission.

Locating the interferometer antennas near the focal point of the parabola compounds the multipath problem due to received energy being ~~secured~~ focused onto the back lobes of the horns. Careful design will be necessary to sufficiently reduce reception in the back direction.

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INTRODUCTION

The Galactic-Jupiter Probe, now under Phase I study, is considered to be a feasible program; however, costs associated with the type of launch vehicle and spacecraft complexity, must be held to a reasonable level. To avoid excessive launch vehicle costs, the presently envisioned spacecraft weight is limited to between 300 and 500 pounds. The principal problem confronting the study group is this weight limitation; it will be necessary to exercise considerable ingenuity in providing the implementation necessary to satisfy the mission requirements for this pioneering program.

A significant function of the spacecraft is the communication link between the spacecraft and earth terminals. A relatively high gain, thus narrow beam, antenna array on the Galactic-Jupiter Probe is certainly desirable from the viewpoint of communication link sensitivity. In fact, without substantial spacecraft antenna gain, it will be impossible to maintain communications, regardless of the reduction in information transmittal rate. At the extended ranges involved with this project the space loss at 2.11 GHz is about 280 db. If information is to be telemetered over these vast distances even at rates as low as a few bits per second substantial directivity is required at the spacecraft terminal in conjunction with the ground terminal.

Probably the simplest and lightest spacecraft, ignoring communication sensitivity, results when the attitude control is derived from spin stabilization with the spin axis Sun directed. With judicious selection of the launch date and trajectory to minimize the Earth-Probe-Sun (EPS) angle as a function of the trip time, a fairly directive antenna can be utilized which will provide moderate gain at encounter and use the increased signal level at small ranges to compensate for the reduction in antenna gain resulting from the unavoidably large Earth-Probe-Sun angles. Such antenna systems result in a below margin communication system even though a 210 foot dish is employed at the ground terminal.1/ Since 85 foot dishes are 8 to 10 db less sensitive, it is clear that for a spin stabilized spacecraft the antenna, thus the spin axis, must be directed toward the earth.

Sun pointing or earth pointing for equivalent communication performance has a decided impact on the spacecraft weight. Although earth pointing requires a relatively complex and power consuming receiving system, it appears to be a smaller price to pay in comparison to the additional weight of a high powered transmitter and the required RTG power source necessary to compensate for the reduction in antenna gain. In either case, the spacecraft attitude must be altered periodically with a spin axis precession propulsion system. In view of the relief in power output level afforded the spacecraft transmitting system by the increased antenna gain allowed by earth pointing, this type of spin axis orientation appears mandatory. The problem then is costing out in terms of spacecraft weight for equal communication performance, the optical system used for sun directing in comparison to a radio system suitable for earth pointing. This report describes the performance and approximate physical dimensions of a candidate radio boresight error determination system to provide electrical signals adequate for controlling the propulsion system as required throughout the flight.

SYSTEM REQUIREMENTS

The spacecraft antenna is assumed here to be a body fixed parabola having the largest diameter that can be fitted within the launch vehicle shroud. The clearance for the envisioned launch vehicle is sufficient for approximately an 8 foot dish. The gain and beamwidth of such an antenna at the selected down link carrier frequency of 2.25 GHz is approximately 32 db and 4 degrees respectively. This dish in conjunction with the 85 foot ground antenna, and a reasonable spacecraft transmitter power of 10 watts will permit communications at ranges of 5 AU exceeding 10 bits per second which is considered satisfactory for the mission. As will be shown, the proposed attitude error determination system can achieve an accuracy commensurate with the 4° beamwidth without undue complexity and still maintain a satisfactory performance margin.

For a variety of reasons the down link frequency for the Galactic-Jupiter Probe communication link has been at least initially constrained to be within the 2.2 GHz to 2.3 GHz band. Although higher frequencies are attractive with respect to space loss considerations, (directive antennas being employed on both ends of the communication link), 2.25 GHz will be utilized for the purpose of this report. Up link transmitters operating in the frequency band from 2090 GHz to 2120 GHz and exhibiting output power levels of 10 kw exist in the GSFC Tracking and Data Networks. These units in conjunction with the 85 foot dishes will supply a radio beacon to the on-board radio attitude error determination system.

The greater the accuracy of the boresight error determination system the smaller the off-boresight loss in gain that must be endured in the 8 foot antenna with its 4° beam. A good compromise between these factors is considered to be a pointing accuracy that enables the boresight error to be maintained below plus or minus 1/4 the beamwidth = 1°. This results in an antenna gain reduction of less than 1 db. For reliable operation the system uncertainty must be a reasonable small part of this factor, or about BW/20 = 0.2°. This accuracy budget includes bias errors as well as fluctuations due to system noise. Thus, it is considered reasonable to attempt to hold the bias errors to less than 0.1° and the noise fluctuations to 0.1° rms. In order to correct boresight errors by precessing the spin axis, it is necessary to determine the boresight error orientation to an accuracy of about 2°. This is the most demanding requirement on the attitude error determination system.

The acquisition beamwidth of the on-board attitude Spin Axis Direction system is another compromise, this time between the beamwidth and the desire for much needed antenna gain at the receiving terminal of the link. A reasonable compromise is considered to be an acquisition beamwidth of 24° which corresponds to a gain of about 15 db.

Table I summarizes the communication link parameters for the system:

Table I

P_t, up link transmitter power	10 kw (carrier only)
G_t, ground transmitter antenna gain	52 db, (85 ft. dish and losses)
G_r, spacecraft interferometer antenna gain	15 db
λ, propagation wavelength at f = 2.11 GHz	14.1 cm
85 ft. ground antenna beamwidth	0.4°
Interferometer antenna beamwidth	24°
ϵ_b, bias error	<0.1 degrees
ϵ_n, noise error	<0.1 degrees rms
σ_ψ, timing errors	<2.0 degrees
R_m, maximum range	5 AU
Space loss	-277 db (5 AU)
ω_s, spin rate	6 rpm = 0.1 Hz ± .03 Hz

SYSTEM CONCEPT

The proposed boresight error determination system consists of a single channel RF interferometer with its baseline oriented orthogonal to the spin axis of the spacecraft. Figure 1 shows the interferometer antenna pair mounted on the feed box of the 8 foot parabola. The choice of the horn type antenna elements is representative only; other types will be considered. The figure gives a somewhat misleading impression of the size of these units and the resulting aperture blockage which for the system portrayed is calculated to cause only about 0.5 db reduction in antenna gain.2/

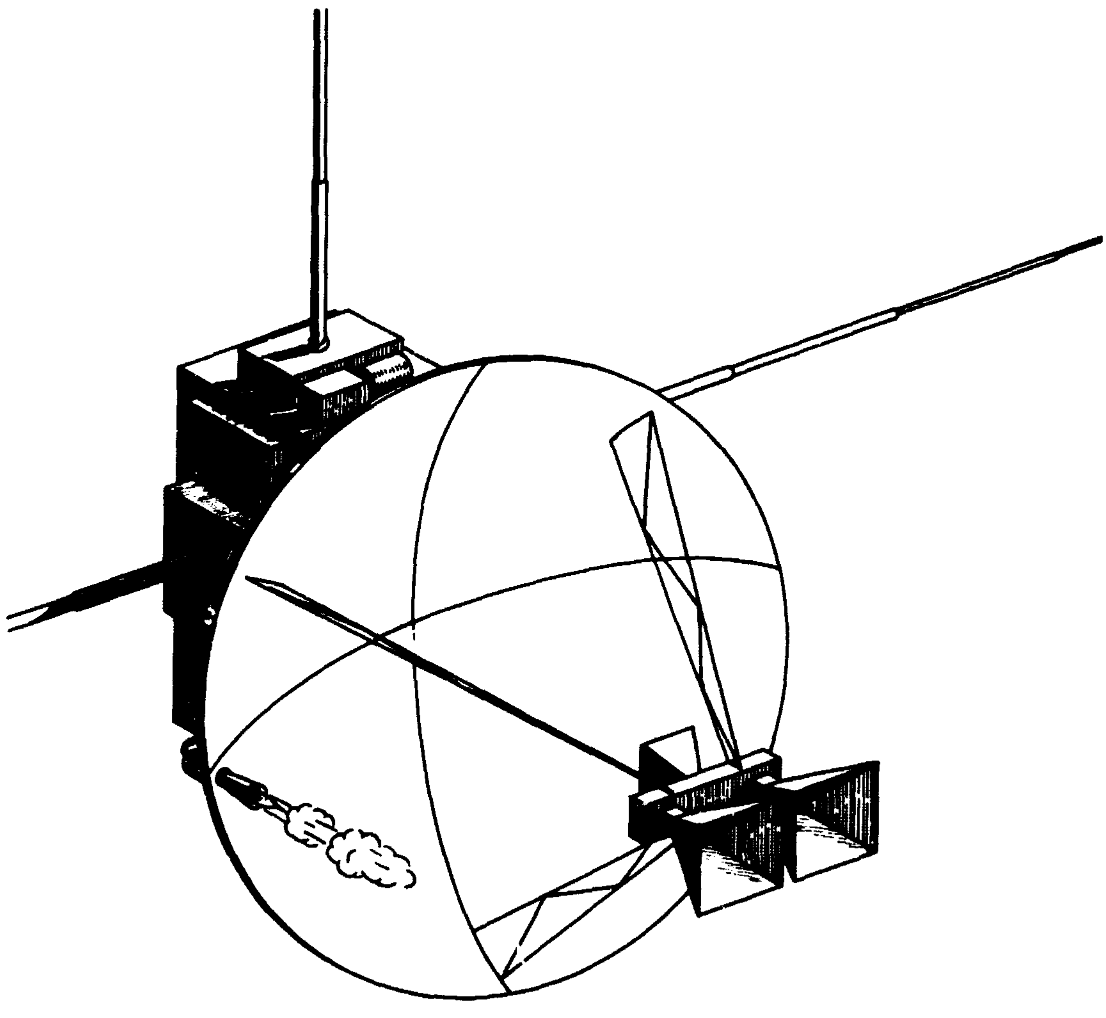

Figure 1. Interferometer Antenna Elements Mounted at the Feed Point of the Spacecraft Parabola

The basic interferometer operation follows from Figure 2. The antennas, A_1 and A_2 are separated a distance β. The angle, a, is formed by the baseline and the direction of the distant emitter, in this case the earth. The line A_1-P is the equal-phase front of the received signal. From geometry:

$$\cos a = \frac{\phi}{\beta} \tag{1}$$

where ϕ and β are measured in the same units — usually wavelengths. The interferometer determines "a" by measuring the phase difference between the received signals at A_1 and A_2. The output of the system is a voltage proportional to the value of ϕ which is equal to $\beta \cos a$.

As the spacecraft, thus the interferometer, rotates about its spin axis the output voltage fluctuates according to the relationship shown in Figure 3. The interferometer baseline is oriented along the X axis; thus the system provides a voltage at its output proportional to $\beta \cos a$ as above. The X/Y coordinate system, which contains the interferometer, is considered to rotate about the Z axis with the earth vector remaining fixed. Thus the angle ψ changes linearly as $\omega_s t$ where ω_s is the angular rotation rate of the spacecraft. The direction cosine, cos a, in terms of ϵ and ψ is determined from Figure 3 as:

$$\sin \epsilon = \frac{A \cos a}{A \cos \psi} \tag{2}$$

then

$$\cos a = \sin \epsilon \cos \psi \; . \tag{2a}$$

Since V_o is made nearly equal to $K \cos a$, where K is a constant, and $\psi = \cos \omega_s t$

$$V_o(t) = K \sin \epsilon \cos \omega_s t \; . \tag{3}$$

For small values of ϵ:

$$V_o(t) = K \epsilon \cos \omega_s t \tag{3a}$$

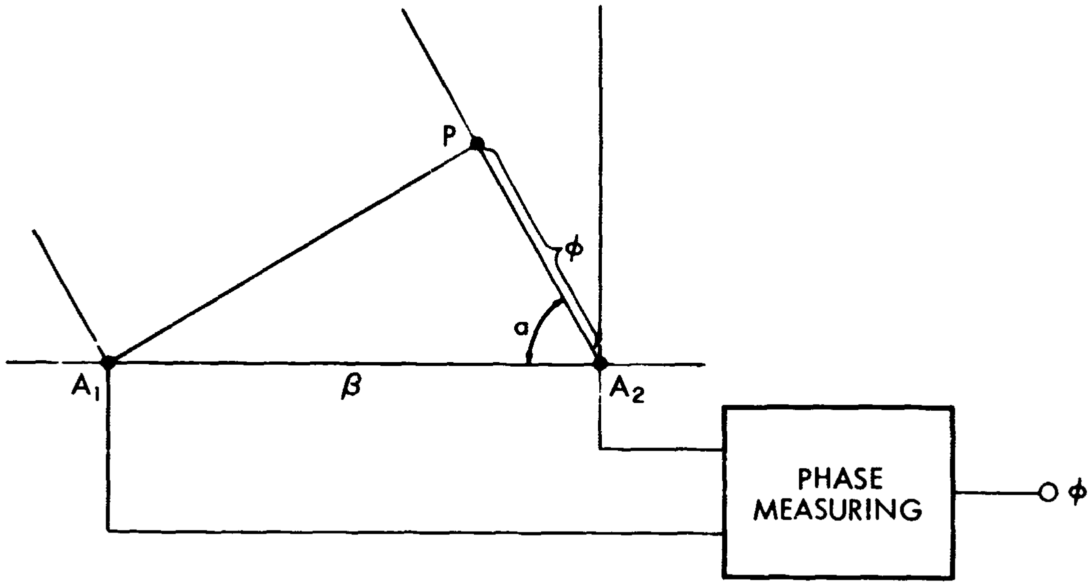

Figure 2. Interferometer Geometrical Relationship

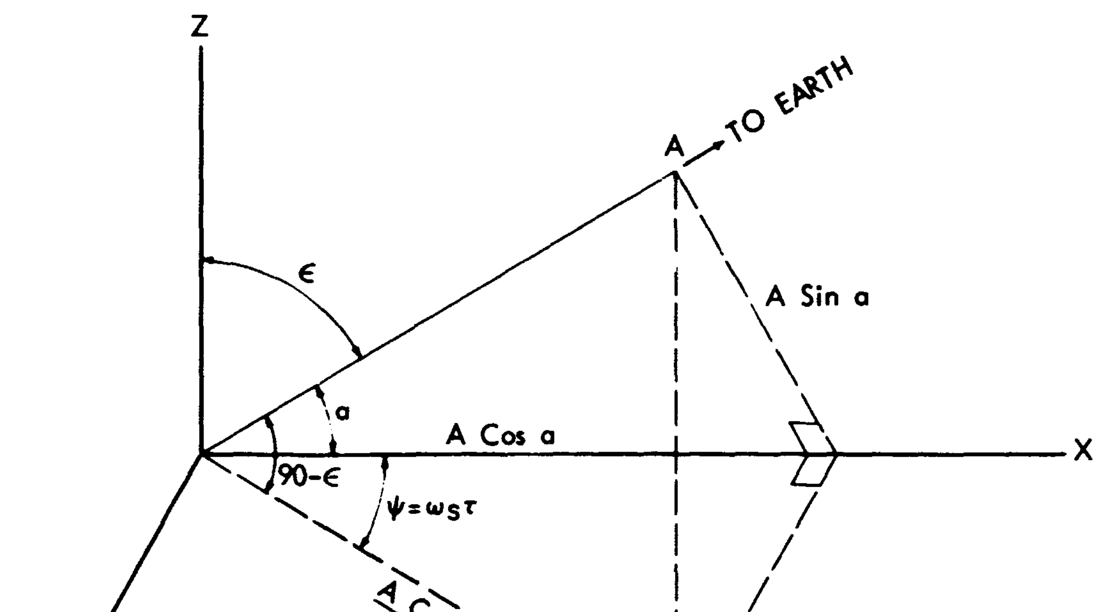

Figure 3. Interferometer Operation Resulting from Spacecraft Spin

showing that V_o varies very nearly sinusoidally, synchronous to the spin rate, with an amplitude proportional to the boresight error as depicted in Figure 4. The means are thus available for setting up threshold criteria (deadzone) which allows a boresight error correction only when a designated threshold is exceeded. Because the interferometer determines the orientation of the boresight error it can also provide timing signals for activation of the attitude correction propulsion jets at the correct value of ψ.

The design engineer has a number of methods he can employ to perform the phase measurement; these differ substantially in principles of operation. The concept shown in Figure 5 was selected for the Galactic-Jupiter Probe because it is considered to excel in weight and power consuming factors yet maintains a high sensitivity and appears to be inherently reliable because of its simplicity. This system employs techniques similar to conventional phase monopulse systems.

Referring to the block diagram, Figure 5, the outputs of the two antennas are fed to a hybrid junction which provides sum and different output signals. The hybrid functions to transform the variations in ϕ, due to variations in the boresight error angle, to amplitude variations. A 180 degree phase shift occurs

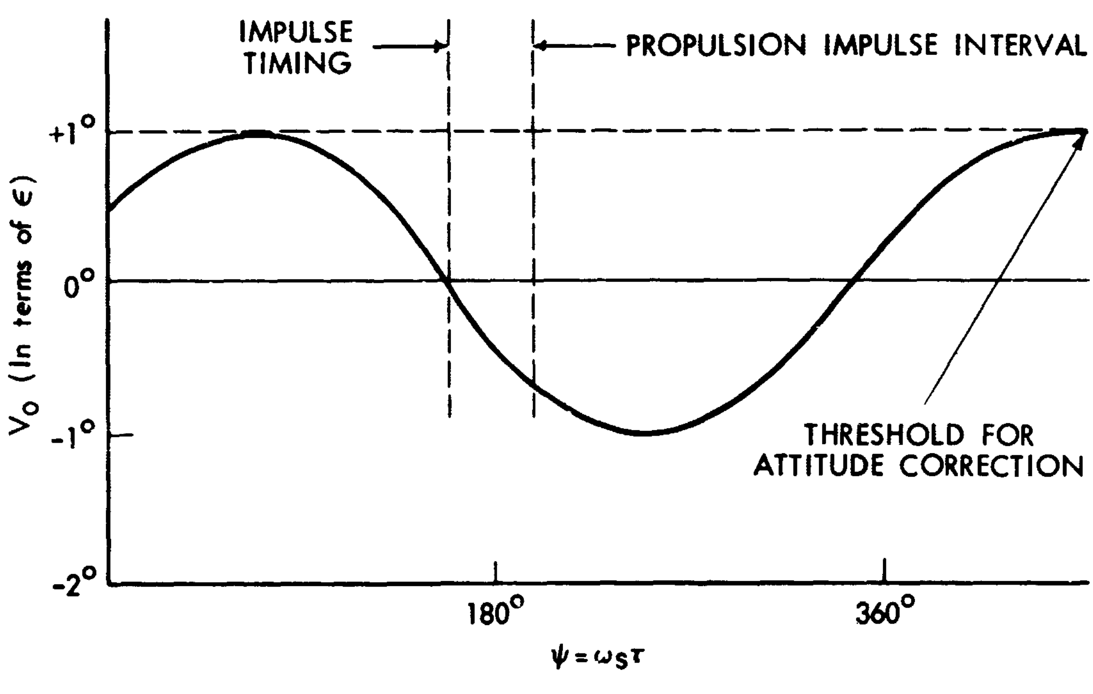

Figure 4. Product Detector Output Voltage

in the difference channel, when the sign of ϕ changes from positive to negative. The circuitry utilizes this phase shift to transform the transfer characteristic of the system from double valued to single valued thereby providing error voltage outputs which are positive for positive error angles and vice-versa. These relationships are discussed further in the performance analysis section.

Although the requirement for receiver stability is primarily one of amplitude, some phase stability is required since excessive phase variations between sum and difference channels have an effect on the amplitude of the output voltage, V_o.

Non-correlated additive noise is applied to the sum and difference signals as designated by the receiver noise figure. Compared to the envisioned receiver noise figure of 5 db, the background sky noise is negligible and will be ignored here. The signals are amplified using conventional receiver techniques except that both channels have common local oscillators represented by the VCO in the figure.

This is one of a number of systems that are characterized by having a very low post detection passband, many orders of magnitude smaller than the variations in received signal frequency due to doppler. To avoid severe deterioration of the SNR in the IF signal detector, it is necessary to narrow-band the predetection signal to raise the SNR at the detector input above unity. Although doppler will be programmed out at the ground transmitter to within an uncertainty of 20 Hz or so, it is still considered appropriate to employ a tracking filter (phase lock loop) in the sum channel instead of a passive filter to avoid gross differential phase shift errors between channels.

The AGC product detector in the sum channel provides AGC voltage to the voltage controlled attenuators, VCA, in both channels in order to prevent received signal level variations from contributing to the level of the output voltage. V_o is required to be a function of cos a only. Being driven by the properly phased reference oscillator the product detector in the difference channel coherently detects the difference signals producing V_o. To alleviate dynamic range problems this voltage, which varies as cos ω_s t as shown above, is then "cleaned up" in a low-pass filter which has a 5 Hz cut-off frequency, sufficiently high to prevent any significant phase delay at ω_s. Further filtering of this signal, free from phase shifts, is desirable from a SNR viewpoint. The tracking filter formed by the phase detector, the low pass filter and the reference VCO operating at ω_s is ideally suited for this function. The output of this tracking filter drives circuitry which decides when a voltage level corresponding to a particular threshold boresight, ϵ, results. This information is telemetered to the ground terminal where it is carefully evaluated using a-priori knowledge of the spacecraft dynamics. When the attitude correction sequence is begun by command from the ground station the switch, S_1 (frequency Δ) is opened so that the VCO, operating at, ω_s, continues to oscillate at the spin rate with its phasing set as per the start of the sequence. Thus, the timing signals remain fixed in frequency and phase during the correction sequence independent of the decay in the error voltage, V_o, as the error is reduced to or through its zero value. The above error correction operation is illustrative only; relationships with other spacecraft elements will probably require variations in techniques to attain the best overall operation.

PERFORMANCE ANALYSIS

The performance of this system depends upon the "cleanliness" of the signal presented to the threshold decider and more importantly to the reduction in uncertainty of the timing signals as derived from the axis crossings of the reference VCO signal oscillating at ω_s. Both bias and fluctuating errors must be taken into consideration. Bias errors depend upon system amplitude and phase drifts, antenna mis-alignment, etc. Fluctuating errors are due to the additive noise in the system only since sky noise is negligible.

The antenna separation, β, in the interferometer has a pronounced effect on both the sensitivity and transfer characteristics of the system. From Equation (1) it follows that:

$$\frac{d\phi}{da} = \beta \sin a \tag{4}$$

where $d\phi/da$ can be interpreted as the system sensitivity, which is proportional to the length of the baseline. To avoid ambiguity, which results when ϕ is allowed to change more than 1 cycle over the beamwidth of the antenna, the value of β is limited to a certain maximum. In this phase monopulse system, ambiguity manifests itself through the production of signal level nulls in the sum channel. Under these conditions the system becomes inoperative. The limitation on β is such that:

$$\beta \sin(\text{beamwidth}/2) \leq \lambda/2 \tag{5}$$

thus the maximum baseline is:

$$\beta_{max} = 2.4\lambda \tag{5a}$$

For this maximum baseline the sum channel at the half beamwidth of the antenna is close to a null condition. In view of the noise and other errors in the system a somewhat smaller β is desired. For practical reasons a value of β equal to 2 wavelengths is considered to be an appropriate compromise between sensitivity and functional problems. Under these conditions, from Equation (2a):

$$\phi_{max} = \beta \cos a_{max} = \beta \sin \epsilon_{max} \tag{6}$$

where ϵ_{max} is equal to the beamwidth/2 = 12° then

$$\phi_{max} = 2\lambda \times .207 = .414\lambda = 149° . \tag{7}$$

The characteristics of the signal and noise at the output of the product detector are derived in Appendix A-II. The only significant signal applied to the antenna system is the earth radiated beacon. The sun and sky noise is negligible in comparison to the receiver noise, even at the relatively good noise figure of 5 db. The Sun, which is the strongest cosmic noise radiator, emits a maximum flux density in the 2 GHz area of 7×10^{-19} watts/m^2/Hz.[3/] The effective area of the antenna, being approximately -14 db-m^2, results in a sun noise spectral density of -152 - 14 = -166 dbm/Hz at 1 AU and -180 dbm/Hz at 5 AU. The effective receiver thermal noise level, n, is $kt_0 B(F-1)$ where the noise figure, F, is assumed to be 5 db. Thus n = -171 dbm (see Appendix A-I) which is substantially greater than even the very maximum received Sun noise level. Considerations regarding the coherence between the Sun noise present in both the sum and difference channels are not pertinent here since the predetection bandwidth is narrow and the limiting action on the sum signal virtually eliminates this coherence at the input to the product detector. Thus, Sun noise appears only as a negligible amount of additive noise and can be ignored.

Additive noise as prescribed by the receiver noise figure is applied as shown on the block diagram, Figure 5. The analysis describes the operations on the signal as denoted by the circuitry. These operations include the signal combining by the hybrid junction, narrow-banding of the sum signal, the signal limiting effects of the tracking filter, the AGC action to enable the output signal level to be a function only of the interferometric relationships and finally the product detector which multiplies the difference signal by the sum signal producing V_o.

From Equation (12) in Appendix A-II, the output of the product detector, V_o, after normalizing and removing high frequency terms by filtering is:

$$V_n = \tan\frac{\phi}{2} + \frac{y_d(t)}{2E\cos\phi/2}$$

where the signal term is $\tan\phi/2$, a function only of ϕ as required. The variance of V_n is:

$$\sigma_{V_n}^2 = \frac{\sigma_n^2}{(2E\cos\phi/2)^2} \tag{8}$$

$$\sigma_{V_n}^2 = \frac{n}{P_r(2\cos\phi/2)^2} \tag{8a}$$

where

$$n = Kt_e B \;. \qquad \text{(Appendix A-1)}$$

Thus the

$$SNR = \frac{P_r(2\sin\phi/2)^2}{n} \tag{9}$$

for the small values of ϕ to be encountered during normal operation of this system the expression for V_n simplifies to:

$$V_n = \frac{\phi}{2} + \frac{y_d(t)}{2E} \tag{10}$$

$$V_m = \phi + \frac{y_d(t)}{E} \tag{11}$$

where

$$V_m = 2V_n \ .$$

That the operationally useful values of ϕ are small can be shown as follows: From Equation (2a), $\cos a = \sin\epsilon\cos\psi$. The maximum value of $\cos a$ during a spacecraft spin revolution is $\sin\epsilon$. This system is designed to operate at a threshold of about $\epsilon = 1°$; thus:

$$(\cos a)_{max} = \sin\epsilon \approx \epsilon = \frac{2\pi}{360} \ .$$

From the interferometer relationship,

$$\phi = \beta(\cos a)_{max} = 2 \times 360 \times \frac{2\pi}{360}$$

$$= 12.6° \ .$$

Thus:

$$\tan\frac{\phi}{2} \approx \sin\frac{\phi}{2} \approx \frac{\phi}{2}$$

since

$$\cos\frac{\phi}{2} \approx 1 .$$

The expression for V_m can be modified to include the spacecraft dynamics. That is, substituting for ϕ its equivalent in terms of ϵ and ψ. As derived earlier:

$$\cos a = \sin\epsilon\cos\psi \approx \epsilon\cos\omega_s t .$$

Therefore:

$$\phi = 2\pi\beta\epsilon\cos\omega_s t \text{ (radians)}$$

so that

$$V_m \approx 2\pi\beta\epsilon\cos\omega_s t + \frac{y_d(t)}{E} . \tag{12}$$

It is necessary to attain a very high signal to noise ratio if excessive uncertainties in the timing signals are to be avoided. This can be accomplished by narrow-band filtering the signal at f_s. A tracking filter is ideally suited to this task since it can provide very narrow passbands with essentially zero phase shift that would otherwise cause bias errors in the timing signals.

For simple reliable operation it is advisable to implement the tracking filter phase-lock-loop to be self-locking without search sweeping. Again this is attained if the uncertainties in the applied frequency, $f_s = \omega_s/2\pi$, are not greater than a factor of three times the tracking bandwidth, B_{L2}. Assuming that the spin rate can be held to within plus and minus 30% of its nominal value of 0.1 Hz,

$$3B_{L2} = 30\% f_s ; \qquad B_{L2} = 0.01 \text{ Hz} .$$

Filtering $y_d(t)$ in such a filter, which has a passband narrow compared to f_s, the random noise process can be expanded as follows:

$$Y_d(t) = X(t)\cos\omega_s t + Y(t)\sin\omega_s t \qquad (13)$$

where

$$\overline{y_d(t)^2} = \overline{\frac{1}{2}X^2(t)} + \overline{\frac{1}{2}Y^2(t)} = \overline{X^2(t)} = \overline{Y^2(t)} \qquad (14)$$

$$\overline{y_d(t)^2} = \text{KTeB} \qquad (14a)$$

where: $B = 2B_{L2}$, (for noise level calculation, the bandwidth must be referred to the predetection side.) Then:

$$V_m \approx 2\pi\beta\epsilon\cos\omega_s t + \frac{X(t)\cos\omega_s t + Y(t)\sin\omega_s t}{E} . \qquad (15)$$

This is a vector having an amplitude approximately equal to $(2\pi B\epsilon + X(t)/E)$ at an angle:

$$\psi = \tan^{-1}\frac{Y(t)/E}{X(t)/E + 2\pi\beta\epsilon}$$

which for sufficiently high signal to noise ratios reduces to:

$$\tan\psi \approx \psi \approx \frac{Y(t)}{2\pi E\beta\epsilon} .$$

The variance of ψ is:

$$\sigma_\psi^2 = \frac{\overline{Y^2(t)}}{(2\pi\beta\epsilon E)^2} \tag{16}$$

$$= \frac{2KT_e B_{L2}}{(2\pi\beta\epsilon)^2 P_r} \tag{16a}$$

where

$$P_r = E^2$$

$$= -140 \text{ dbm (see Appendix A-1)}$$

taking $\epsilon = 1° = -17.5$ db -radian, for the threshold condition:

$$\sigma_\psi^2 = +3 - 199 + 28 - 20 - 2(3 + 5 + 3 - 17.5) + 140$$

$$= -35 \text{ db -radian}^2$$

The standard deviation is:

$$\sigma_\psi = -17.5 \text{ db -radian}$$

$= 1.02°$ which is within the allowable 2° rms with a margin of approximately 6 db.

As stated previously it is also necessary to maintain the amplitude fluctuations on this signal below a nominal value. The fluctuations in the output voltage can be expressed as a function of the fluctuations in ϵ. The magnitude only of the output voltage is:

$$\left|V_m\right| \approx 2\pi B \epsilon + \frac{X(t)}{E} \tag{17}$$

where the quadrature term, $Y(t)$, is negligible. The variance of $\left|V_m\right|$,

$$\overline{\left|V_m\right|^2} = \sigma_m^2 = \frac{n}{E^2} \, . \tag{18}$$

Now let the noise fluctuations in the signal be replaced by equivalent fluctuations in ϵ. Then,

$$\sigma_m^2 = (2\pi\beta)^2 \sigma_\epsilon^2 \tag{19}$$

$$= \frac{n}{E^2} \tag{19a}$$

then,

$$\sigma_\epsilon = \frac{n^{1/2}}{E \cdot 2\pi\beta} \tag{20}$$

$$= \frac{\left(2\,KT_e\,B_{L\,2}\right)^{1/2}}{2\pi\beta P_r^{\,1/2}} \tag{20a}$$

$$= \frac{3 - 199 + 28 - 20}{2} - 3 - 5 - 3 + \frac{140}{2}$$

$$= -35 \text{ db -radian}$$

$$= \frac{57.3}{3.2 \times 10^3} = 0.018^\circ \text{ rms} \, .$$

This small fluctuation about the 1° threshold value will not disturb the operation of the threshold decider.

ADDITIONAL FACTORS AFFECTING PERFORMANCE

The foregoing results, namely the standard deviation of ψ and ϵ indicate a substantial margin on the performance of this system. These uncertainties are due to the additive noise in the system only. Bias errors are another matter. The important contributors to bias errors are:

1. Antenna mis-alignment.
2. Phase drifts of transmission lines and the hybrid junction.
3. Differential gain and phase shift between channels.
4. dc amplifier drift.
5. Oscillator drift.
6. Product detector balance drifts and changes with signal and noise levels.

With careful design these bias errors can be brought within tolerance. Fortunately, most of the bias errors contribute only to variations in the measurement of ϵ and not the timing signals, ψ. Proper structuring the antennas and hybrid as an integral unit without thermal stresses produces a signal null in the difference channel at boresight regardless of the receiver drift. Passing through this null produces the timing signals which are relatively free of bias error. Errors in ϵ, although designed to be less than 0.1°, could rise to several times this value without catastrophic effects. By monitoring these signals at the ground terminal, compensation can be applied to permit operation to continue in spite of some types of system malfunction.

Because of the critical nature of the attitude control operation it is considered advisable to put the ground operator in the control loop. Substantial redundancy in the electronics should be supplied which can be substituted upon command for malfunctioning circuitry. In addition, at the cost of a few information bits telemetered periodically, the performance of the system at several key points can be monitored in order to insure proper attitude correction when commanded via the up link.

Although the start of the attitude correction sequence is operator controlled, the series of timed propulsion pulses to precess the spin axis appropriately should be controlled by an on-board program with the timing supplied by the error determination system as described above. Once having substantiated proper system performance via telemetry, the on-board timing control appears most reliable for the system as described above.

To illustrate that the timing signals continue at the proper frequency and phase during the attitude correction sequency after the phase lock loop is broken by Switch #1, Figure 4, consider the VCO stability to be 1 part in 10^3 per hour. Such an oscillator will accrue phase errors at a rate given as follows:

$$\phi_{degrees} = \frac{360(\Delta F/F)\, f_o\, t^2}{\tau}$$

where

$\frac{\Delta F/F}{\tau}$ describes the oscillator stability $= 10^{-3}/\mathrm{hr}$

f_o is the operating frequency $= 10^{-1}$ Hz

t is elapsed time

thus

$$t = \left(\frac{\tau\phi}{360\, \Delta F/F\, f_o}\right)^{1/2}$$

let

$$\phi = 1^\circ ,$$

then

$$t = \left(\frac{3600}{360 \times 10^{-3} \times 10^{-1}}\right)^{1/2} = \sqrt{10^5}$$

$$= 316 \text{ sec for a 1 degree phase shift to occur.}$$

At the spacecraft spin rate of 0.1 Hz this time interval corresponds to about 31 revolutions which exceeds the number of impulses required to correct a 1° boresight error.

The lock up time for narrow band loops of this type is very substantial. When the VCO differs in frequency by less than 1 loop bandwidth with respect to the applied signal frequency the expression for lock up time is:

$$\tau_{lock} = \frac{0.53}{B_L}$$

where B_L is the loop bandwidth then

$$\tau_{lock} = \frac{0.53}{0.01} = 53 \text{ seconds.}$$

When the applied signal frequency differs by an appreciable amount with respect to the VCO, the relationship between pull-in or lock up time and frequency offset, ΔF, is:

$$\tau = \frac{4.2(\Delta f)^2}{B_L^3}$$

letting $\Delta f = 0.015$ Hz, an offset of 1.5 loop bandwidths,

$$\tau = \frac{4.2 \times 2.25 \times 10^{-4}}{10^{-6}}$$

$$= 950 \text{ sec} \approx 15 \text{ minutes.}$$

When $\Delta f = 0.03$ Hz, the maximum specified for the system:

$$\tau = \frac{4.2 \times 9 \times 10^{-4}}{10^{-6}}$$

$$= 3780 \text{ sec}$$

$$\approx 1 \text{ hour} .$$

Although these circuit operations are very lengthy, the interval between attitude correction is a matter of several days so no difficulty should be experienced from this effect. Implementation of circuitry that will perform accurately over these long periods will require digital accumulators and other relatively sophisticated techniques.

As can be realized from the substantial margins that have been calculated in the standard deviations of ϵ and ψ, the limitation in the sensitivity of this system is not the accuracy but the lock up performance of the I.F. coherent detector phase lock loop. The optimum loop bandwidth for a coherent detector is a compromise between the additive thermal noise in the system and the 1/f noise of the system oscillators, usually the VCO. For large signal to noise ratios, 10 db or more, a linear approximation of the phase error due to thermal noise is valid, i.e.: 10/

$$\sigma_{\phi} = \left(\frac{1}{2(\text{SNR})}\right)^{1/2} .$$

The component of tracking error due to oscillator noise is given by:

$$\sigma_\phi = \left(\frac{C}{B_n}\right)^{1/2}$$

where C is a constant dependent on noise level and B_n is the noise bandwidth. This relationship has been verified for a large number of oscillators.[11/] For this analysis, an oscillator was chosen having a noise spectrum representative of oscillators rated between the best available and the average of the high Q types.

For narrow bandwidths the phase error in the loop is due primarily to oscillator noise. However, for large loop bandwidths, thermal noise predominates: hence an optimum selection of bandwidth can be made. For the parameters described in the text, Figure 6 shows the standard deviation of the phase fluctuations as a function of bandwidth both for the oscillator and the thermal nose. The minimum points on the curves indicate the optimum bandwidth for that particular range, thus SNR. Since the phase noise is fairly significant at 5 AU, about 4° rms, even for the optimum bandwidth, it is advisable to supply the capability of varying the loop bandwidth via command at smaller ranges where the SNR is greater.

CONCLUSION

This report has shown that even at ranges extending out to some billions of kilometers a radio angle measuring system will supply measurements data to an accuracy of a few parts in a thousand. This is reflected in the analytically achieved values for σ_ϵ and σ_ψ.

Because this report is in the nature of a feasibility study only, a number of important factors were not considered. These will be touched upon briefly here. The accuracy of the interferometer is dependent upon the differential phase characteristics of the antennas and their variations with aspect angle, temperature and possible aging effects when exposed to the rigors of deep space. As a start, these elements should be electroformed with precision then plated with the most space endurable material. Careful design will be necessary to properly reduce antenna coupling effects since these alter the phase characteristics, thus the accuracy of the system.

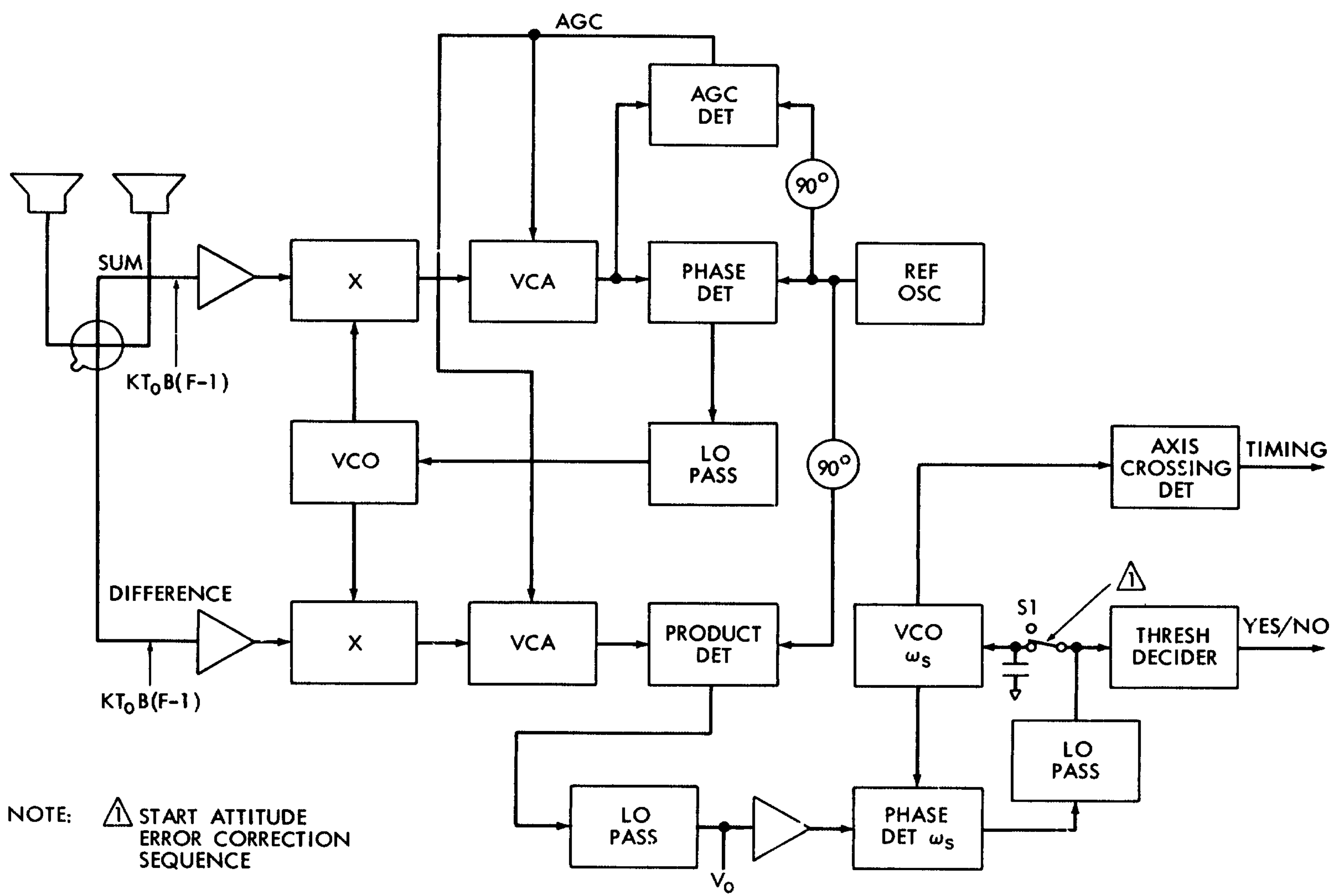

Figure 5. Block Diagram of Phase Measuring System

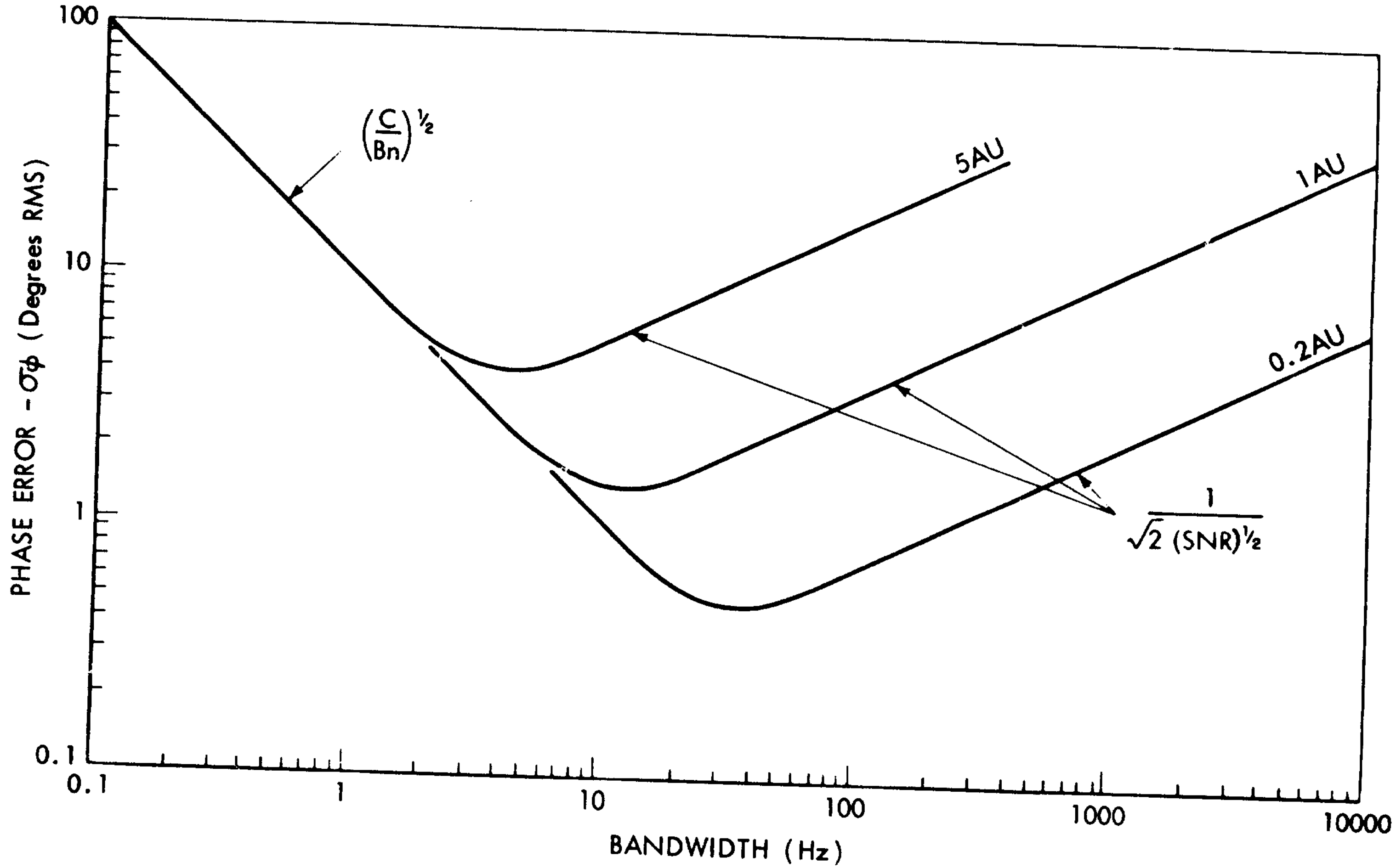

Figure 6. Optimum Loop Bandwidths for Coherent Detector

The accuracy and sensitivity performance of interferometers is very sensitive to received signals reflected from surrounding objects. As a primary consideration, therefore, it is conventional to place the interferometer antennas at a location such that surrounding objects are well out of the antenna beam. This dictated the configuration shown in Figure 1. Although this location is probably the best available for this spacecraft, it must be recognized that considerable extraneous RF energy may be focused onto the back of the horns. Careful design will be necessary to sufficiently reduce reception in the back direction. Results of experimentation by the Antenna Systems Branch at GSFC show that the back radiation of such antennas can be markedly reduced by placing them in a tunnel lined with absorbing material.

In addition to the determination of the 1° threshold boresight error condition it may be desirable to also use this system as a spacecraft attitude sensor. With some modifications and in conjunction with a Sun sensor this could be accomplished.

The signals applied to the coherent detector are not pure carrier components but contain a varying phase shift proportional to ϕ, (see Equation 8 — Appendix A-II). The question arises whether this phase variation will put undo stress on the phase lock loop thus reducing its sensitivity. The fact is, this phase varies at the spin rate of the spacecraft and is negligible for any reasonable coherent detector loop bandwidth being considered.

For the purpose of this report, none of the usual communication losses such as polarization, transmission line losses, reduction in power due to side bands, etc. were considered. Any unavoidable losses not considered can be taken up by the performance margin.

At ranges exceeding 5 AU, it will probably be necessary to supply additional beacon signal level beyond the capabilities of the 85 foot dish and the 10 kw transmitters. Incidentally, although 10 kw was used throughout this report, the two existing transmitters at the appropriate command sites will supply about 36 kw when connected in a combined output mode. This command capability may also be extended to 100 kw by 1970. At the ranges beyond 5 AU the beacon can by supplied by 210 foot dishes and 100 kw transmitters which will increase the beacon level by 18 lb. This will provide a margin of 12 db at 10 AU beyond that which exists at 5 AU for the system as described. There is no reason to expect that with this added capability the system will not operate beyond 40 AU.

ACKNOWLEDGMENTS

My thanks to Mr. Thomas E. McGunigal for the information and concepts he provided concerning phase-lock-loops and frequency stability relationships and to Mr. Dominick Santarpia for his assistance with the analytical procedures.

REFERENCES

1/ Jupiter Flyby Application, JPL EPD 358, p. 5-54.

2/ Merrell I. Skolnick, Introduction to Radar Systems, McGraw-Hill Co., Inc. 1962, p. 285.

3/ Solar System Radio Radiation, J. D. Kraus, AFCRC-TN-59-148, ASTIA Document No. AD 211944, Dec. 1958, p. 13.

4/ W. K. Bennett, Methods of Solving Noise Problems, Proc. IRE, Vol. 44, No. 5, pp. 609-638, May 1956.

5/ A comparison of the Angular Accuracy of An Amplitude Monopulse Radar and a Phase Interferometer Radar in the Presence of Noise, L. H. Kosowsky and L. Chanzit, International Conference on Military Electronics, 8th Conference Proceedings, Sept. 14-16, 1964, p. 297-301.

6/ Angle Estimation Accuracy with a Monopulse Radar in the Search Mode, S. Sharenson, IRE Transactions on Aerospace Navigational Electronics, Sept. 1962, p. 175-179.

7/ W. B. Davenport, Jr., and W. L. Root, Introduction to Random Signals and Noise, McGraw-Hill Book Co., Inc., New York, 1958.

8/ Mischa Schwartz, Information Transmission Modulation and Noise, McGraw-Hill Book Co., Inc., New York, 1959.

9/ Preliminary Trajectory Data to be Used in Communication Design in the Galactic Probe, R. E. Coady, GSFC Mission Analysis Office, Oct. 20, 1965.

10/ Floyd M. Gardner, Phaselock Techniques, John Wiley & Sons, Inc., New York, 1966.

11/ R. Sydnor, J. J. Caldwell, B. E. Rose, Frequency Stability Requirements for Space Communications and Tracking Systems, Proceedings of IEEE, Vol. 54, No. 2, Feb. 1966, p. 231-236.

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APPENDIX A-I

SIGNAL AND NOISE LEVELS

Signal Level at a Maximum Range of 5 AU

$$P_r = P_t G_r G_t \left(\frac{\lambda}{4\pi R}\right)^2$$

where:

P_t is transmitter power level = 10 kw = +70 dbm

G_r is interferometer antenna gain = 15 db

G_t is transmitter antenna gain, (85 foot dish) = 52 db

$\left(\frac{\lambda}{4\pi R}\right)^2$ is space loss at 5 AU and 2 GHz = 277 db

then:

$$P_r = +70 +15 +52 -277$$

$$= -140 \text{ dbm} .$$

NOISE LEVEL

$$n = \sigma_n^2 = K T_0 B(F-1)$$

where

K is Boltzmann's Constant = -199 db

T is Reference Temp = 290°K = 25 db

B is Bandwidth, for spectral levels = 1 Hz = 0 db

F is Receiver Noise Figure = 5 db

then

$$N = -199 + 25 + 3 = -171 \text{ dbm/Hz} .$$

EQUIVALENT NOISE LEVEL IN SUM CHANNEL DUE TO THE I.F. PHASE LOCK LOOP

To obtain maximum lock up sensitivity in the presence of additive noise it is necessary to make the tracking bandwidth as small as possible. This is accomplished by making the cut-off frequency of the low pass filter small. However, the acquisition frequency range of the system increases by increasing the cutoff frequency. To insure rapid and reliable phase lock up without the necessity of frequency searching, B_L must be no smaller than a factor of three times the uncertainties of the I.F. frequency presented to the phase detector. Because the ground transmitter will be hydrogen maser controlled the instability from this source is negligible. It is estimated that the frequency shift of the signal arriving at the spacecraft due to doppler can be programmed out at the ground site to a part in 10^8 or about 20 Hz. By carefully monitoring the two-way range-rate observations the frequency uncertainty of the spacecraft oscillators can be limited to a few parts in 10^9 or about 10 Hz. Thus $3\,B_L = 30$ Hz; and B_L then could be limited to 10 Hz. To provide a margin for this crucial lock up operation, a value of $B_L = 25$ Hz is considered reasonable.

The predetection bandwidth is equal to $2\,B_L$; then $B = 50$ Hz and the effective noise level is:

$$KT_eB = -171 + 17$$

$$= -154 \text{ dbm} .$$

APPENDIX A-II

SIGNAL AND NOISE ANALYSIS

The following analysis describes the operation of the RF interferometer signal processing equipment, on signals applied to the antenna system. As shown in Figure A-1, the signals received by each antenna are fed to a hybrid junction. The signal at A2 is shifted by an amount ϕ due to the difference in propagation path length. The sum and difference output signals from the hybrid are.

$$S = \frac{\sqrt{2}\,E}{\sqrt{2}} \cos(\omega t + \phi) + \frac{\sqrt{2}\,E}{\sqrt{2}} \cos \omega t + N_s(t) \qquad \text{(A-1)}$$

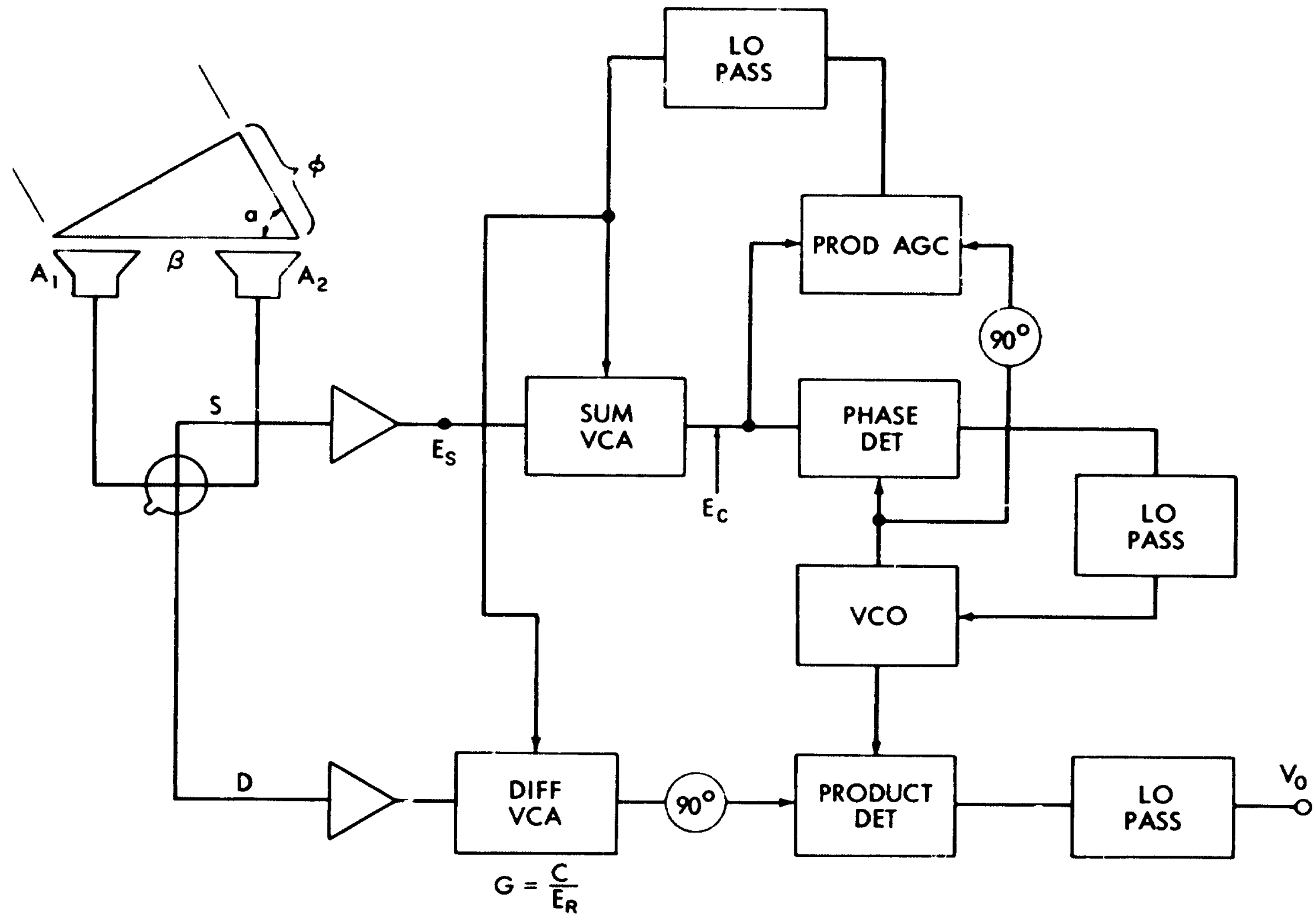
$$D = \frac{\sqrt{2}\,E}{\sqrt{2}} \cos(\omega t + \phi) - \frac{\sqrt{2}\,E}{\sqrt{2}} \cos \omega t + N_D(t) \qquad \text{(A-}$$

where $\sqrt{2}\,E$ is the peak amplitude of the received signal, and $N(t)$ is the narrow band white additive noise generated in each channel. The $\sqrt{2}$ in the denominators results from the impedance transformations in the hybrid. The additive noise power density spectrum can be approximated by the expression 4.

$$N(t) = x(t) \cos\left(\omega_t + \frac{\phi}{2}\right) + y(t) \cos\left(\omega_t + \frac{\phi}{2}\right) \qquad \text{(A-3)}$$

where $x(t)$ and $y(t)$ are slowly varying random variables whose mean is zero $\left(\overline{x(t)} = \overline{y(t)} = 0\right)$ and whose mean square value is given by: $\overline{x(t)^2} = \overline{y(t)^2} = n = N^2$, the total noise power. As shown by Bennett,[4] $x(t)$ and $y(t)$ are independent random variables and normally distributed; hence:

$$\sigma_x^2 = \sigma_y^2 = N^2 = KT_e B$$



FigureA-1. Equivalent System Block Diagram for Signal and Noise Analysis where $T_e = T_0 \, (F-1)$ and F is the receiver noise figure. The sum and difference equations may be expressed as:

$$S = E\cos\left(\omega t + \phi\right) + E\cos\omega t + x_s(t)\cos\left(\omega t + \frac{\phi}{2}\right) + y_s(t)\sin\left(\omega t + \frac{\phi}{2}\right) \quad (A-4)$$

$$D = E\cos\left(\omega t + \phi\right) - E\cos\omega t + x_d(t)\cos\left(\omega t + \frac{\phi}{2}\right) + y_d(t)\sin\left(\omega t + \frac{\phi}{2}\right). \quad (A-5)$$

Applying trigonometric identities and shifting the phase of D by 90° as required for circuit operation:

$$D_s = 2E\cos\left(\omega t + \frac{\phi}{2}\right)\sin\frac{\phi}{2} + x_d(t)\sin\left(\omega t + \frac{\phi}{2}\right) - y_d(t)\cos\left(\omega t + \frac{\phi}{2}\right) \quad (A-6)$$

and

$$S = 2E\cos\left(\omega t + \frac{\phi}{2}\right)\cos\frac{\phi}{2} + x_s(t)\cos\left(\omega t + \frac{\phi}{2}\right) + y_s(t)\sin\left(\omega t + \frac{\phi}{2}\right). \quad (A-7)$$

The analysis is much simplified if the noise in the sum channel is negligible compared to the sum channel signal level. At the far ranges, 5 AU, operation over the full 12° bandwidth, of the system will not be required. Therefore, let $\phi_{max} = 75°$, which reduces the operating beamwidth by a factor of 2 or ±6°. Then the signal level in the sum channel is:

$$2E\cos\frac{\phi}{2} = 2E\cos\frac{75°}{2} \text{ volt rms.}$$

The equivalent sum signal power level $P_s = P_r\,(2 \times .8)^2$ where P_r at 5 AU is shown in Appendix A-I to be equal to -140 db, hence:

$$P_s = -140 \text{ dbm} + 4 \text{ db}$$

$$= -136 \text{ dbm}.$$

The noise level (see Appendix A-I) is -154 dbm. Since the noise level is 18 db below the signal level, it is considered reasonable to ignore the noise terms in the sum channel. The important consideration is that reliable phase lock up in the tracking filter channel is obtained at this signal to noise ratio. This procedure cannot be followed in the difference channel because of the $\sin \phi/2$ term which can reduce the signal term to zero for $\phi = 0$. The sum signal thus reduces to:

$$S = 2E\cos\left(\omega t + \frac{\phi}{2}\right)\cos\frac{\phi}{2} \quad . \tag{A-8}$$

The information content of the received signals consists solely of the value of ϕ. Unless a particular form of AGC is applied to the signals the output will also be a function of the received signal level. The AGC requirements are satisfied by the arrangement of voltage controlled attenuators, VCA's, shown in the block diagram, Figure A-1. Both VCA's have identical gain control characteristics. The control voltage is derived coherently by the AGC product detector such that the output of the sum VCA is held to a constant level using conventional AGC servo techniques. Since the amplitude input to this VCA is $2E\cos\phi/2$, it follows that:

$$G \times 2E\cos\frac{\phi}{2} = C_1$$

then

$$G = \frac{C_1}{2}E\cos\frac{\phi}{2}$$

where G is the gain law of the VCA and C_1 represents the constant sum-VCA output level.

The difference channel signal level is also subject to this gain variation resulting in:

$$D_G = D_s \times G = C_1\tan\frac{\phi}{2}\cos\left(\omega t+\frac{\phi}{2}\right) + \frac{C_1 x_d(t)}{2E\cos\phi/2}\sin\left(\omega t+\frac{\phi}{2}\right) + \frac{C_1 y_d(t)}{2E\cos\phi/2}\cdot\cos\left(\omega t+\frac{\phi}{2}\right) \quad . \tag{A-9}$$

Through the action of the phase lock loop the VCO is locked in phase with the sum signal. Since its output is constant is has the characteristics of the sum signal such that:

$$S_{vco} = C_2 \cos\left(\omega t + \frac{\phi}{2}\right) . \tag{A-10}$$

The product detector multiplies D_G by S_{vco} which after filtering results in:

$$V_o = \frac{C_1 C_2}{2} \tan\frac{\phi}{2} + \frac{C_1 C_2 y_d(t)}{4 E \cos \phi/2} . \tag{A-11}$$

After normalizing:

$$V_n = \tan\frac{\phi}{2} + \frac{y_d(t)}{2 E \cos \phi/2} \tag{A-12}$$

where

$$V_n = \frac{2 V_o}{C_1 C_2}$$

and E is the rms value of the received signal.