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ABSTRACT

The intensity, spectrum and polarization of gyro-synchrotron radiation from an arbitrary distribution of electrons in a magnetoactive plasma are calculated. These calculations are based on detailed theories of gyro-synchrotron emission and absorption as well as on the equations of radiation transfer in the limit of large Faraday rotation. By performing extensive numerical calculations, for both isotropic and anisotropic electron distributions, we investigate the influence of reabsorption and the Razin effect on the spectrum and polarization of the radiation. Using the same formalism and calculations, we also investigate the possibility of negative absorption in an anisotropic gyro-synchrotron source.

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GYRO-SYNCHROTRON EMISSION AND

ABSORPTION IN A MAGNETOACTIVE PLASMA

I. INTRODUCTION

Gyro-synchrotron radiation is the electromagnetic emission generated by mildly relativistic electrons moving in a magnetic field, while the term synchrotron radiation is used to describe the emission from ultrarelativistic electrons. The theory of synchrotron emission and absorption, in vacuum as well as in a plasma, has been treated by a number of authors (e.g. Ginzburg and Syrovatskii, 1969), and it is widely believed that it can account for the observed radio emission from a variety of cosmic sources. Synchrotron theory, however, is not directly applicable to radio emission produced by mildly relativistic electrons in regions of strong magnetic fields and dense plasmas. In such regions, the ambient plasma and gyro frequencies are large and comparable, within an order of magnitude, to the observed radio frequencies; the propagation of electromagnetic waves is anisotropic; there is more than one distinct mode of wave propagation; and there may be a large Faraday rotation. This type of radiation is probably responsible for radio emission from large solar flares as well as for emission generated in the atmospheres of certain stars and planets. For the detailed understanding of its spectrum and polarization, it is necessary to consider in detail the theory of gyro-synchrotron emission and absorption in a magnetoactive plasma.

The theory of gyro-synchrotron emission, for electrons in circular orbits in vacuum, was first treated by Schott (1912). Subsequently, Schwinger (1949), Takakura (1960a) and Landau and Lifshitz (1962) have further developed this theory, and Takakura (1960b) has extended it to electrons in helical orbits. The general problem of electromagnetic radiation by charged particles in a magnetoplasma has been treated by a number of authors (Twiss, 1958; Eidman, 1958; Liemohn, 1965 and Mansfield, 1967). These treatments provide expressions for the spectral and angular distribution of the radiation, in a given mode of wave propagation, for single electrons of given energies and pitch-angles. By considering the special case of an isotropic ambient medium, Ramaty (1968) obtained an expression for the total radiation spectrum, integrated over all emission angles, for electrons in circular orbits. A numerical analysis of this spectrum revealed that if the Lorentz factor of the radiating electron is less than the plasma-to-gyro-frequency ratio of the ambient medium, the radiation at low frequencies is strongly suppressed. This effect is well known for synchrotron radiation, and is generally referred to as the Razin effect.

The radiation at low frequencies, however, is influenced not only by the Razin effect, but also by a variety of propagation phenomena, such as absorption below the plasma frequency, gyro-resonance and free-free absorption by the ambient electrons, and gyro-synchrotron reabsorption by the radiating electrons themselves.

The theories of gyro-resonance and free-free absorption were discussed by several authors (e.g. Kundu, 1965), and we shall not treat them, therefore, in the present paper. The theory of gyro-synchrotron reabsorption was treated by Twiss (1958), for radiation in vacuum, and was extended by Kawabata (1964) to radiation in a dilute plasma with Faraday rotation. An attempt has been made by Kai (1965) to apply Kawabata's (1964) results to solar radio emission, but because of the complexity of the equations, no detailed numerical results were obtained. In a complete treatment of gyro-synchrotron emission and absorption, the Razin effect and reabsorption must be treated in a consistent fashion since these two processes are highly interdependent. As the influence of the ionized medium becomes more pronounced, the Razin effect not only suppresses the emissivity of individual electrons but also reduces the corresponding absorption coefficient. Such a simultaneous treatment of these two effects is not available in the published literature.

Closely connected with the theory of gyro-synchrotron reabsorption is the problem of negative absorption. The possibility of the amplification of synchrotron radiation was first considered by Twiss (1958), but Bekefi, Hirshfield and Brown (1961) showed that under the circumstances described by Twiss (1958) (isotropic, ultrarelativistic electrons in a vacuum) the absorption coefficient will always be positive. Negative absorption, however, may occur if the radio source consists of mildly relativistic electrons. Bekefi, Hirshfield and Brown (1961) have investigated this possibility and showed that if the electron spectrum increases with increasing energy but cuts off before the electrons become ultrarelativistic, maser action will occur just below the gyro-frequency and its second harmonic. Even though Twiss' original suggestion for maser action was proven to be untenable, the possibility of amplification of synchrotron radiation from ultrarelativistic electrons was reconsidered recently by a number of authors. McCray (1966) and Zheleznyakov (1967) have shown that maser action may occur if the radiation is produced in a plasma and the electrons have an inverted energy spectrum. Heyvaerts (1968) has considered the possibility of maser action due to pitch-angle anisotropies and showed that amplification of synchrotron radiation is possible if the angular spread of the particle distribution is of the order of the aperture angle of the radiation pattern. Obviously, for ultrarelativistic electrons, extremely well collimated particle beams are necessary. For mildly relativistic electrons, however, maser action would require much smaller anisotropies. The possibility of the amplification of gyro-synchrotron radiation owing to an anistropic pitch-angle distribution, has not been yet considered in the published literature.

In the present paper we shall provide a detailed treatment of the theory of gyro-synchrotron emission and absorption by an arbitrary distribution of radiating electrons in a cold, collisionless magnetoactive plasma. We shall evaluate Stokes' parameters by using a general expression for gyro-synchrotron emission (Liemohn, 1965), a detailed formula for the absorption coefficient (Bekefi, 1966), and the theory of gyro-synchrotron radiation transfer in the limit of large Faraday rotation. By performing detailed numerical calculations we investigate the simultaneous effects of the ambient medium and reabsorption. We find that both these effects tend to suppress the radio flux at low frequencies but lead to different polarization of the resultant radiation. We also consider the effects of an anisotropic electron distribution. Since the radiation pattern of ultrarelativistic electrons is much narrower than that of mildly relativistic particles, the emission spectrum and polarization from an anistropic distribution will strongly depend on the direction of observation. We find that if this direction differs from that of maximum anistropy, the radiation at high frequencies is strongly suppressed while the emission at the longer wavelengths remains essentially unchanged. This suppression effect coupled with the Razin effect at low frequencies may lead to the total suppression of the entire radiation spectrum.

Finally, we consider the possibility of maser action in an anistropic gyro-synchrotron source. We find that maser action is indeed possible, even if the electron spectrum decreases with increasing energy, and, for moderate anistropies it occurs at essentially the same frequencies as found by Bekefi, Hirshfield and Brown (1961) for the isotropic distribution with an inverted energy spectrum.

II. GYRO-SYNCHROTRON EMISSION AND ABSORPTION

We consider a homogeneous system of energetic electrons moving in a homogeneous, cold and collisionless electron plasma, permeated by a static and uniform magnetic field. If the density of the plasma electrons is much higher than that of the energetic particles, wave propagation in this system is governed solely by the ambient medium. Waves of two modes can propagate in such a medium, the ordinary and extraordinary modes of magnetoionic theory (Ratcliffe, 1959; Ginzburg, 1964). The indices of refraction, $n_{\pm}$, and polarization coefficients, $a_{\theta\pm}$ and $a_{k\pm}$, as functions of the frequency ν, and of the angle θ between the wave normal $\hat{k}$ and the static field $\vec{B}$, are given by (Ginzburg, 1964):

$$n_{\pm}^2(\theta) = 1 + \frac{2\nu_p^2(\nu_p^2-\nu^2)}{\pm\left\{\nu^4\nu_B^4\sin^4\theta+4\nu^2\nu_B^2(\nu_p^2-\nu^2)^2\cos^2\theta\right\}^{1/2} - 2\nu^2(\nu_p^2-\nu^2) - \nu^2\nu_B^2\sin^2\theta} \tag{1}$$

$$a_{\theta\pm}(\nu,\theta) = -\frac{2\nu(\nu_p^2-\nu^2)\cos\theta}{-\nu^2\nu_B\sin^2\theta \pm \left\{\nu^4\nu_B^2\sin^4\theta+4\nu^2(\nu_p^2-\nu^2)^2\cos^2\theta\right\}^{1/2}} \tag{2}$$

$$a_{k\pm}(\nu,\theta) = -\frac{\nu_p^2\nu_B\nu\sin\theta - a_{\theta\pm}\nu_p^2\nu_B^2\cos\theta\sin\theta}{\nu_p^2(\nu_B^2\cos^2\theta-\nu^2)-\nu^2(\nu_B^2-\nu^2)}\ . \tag{3}$$

The subscripts (+) or (−) correspond to the ordinary or extraordinary modes, respectively, and the plasma and gyro frequencies, ν_p and ν_B, are given by

$$\nu_p = e\left(n_e/\pi m\right)^{1/2}\ ; \qquad \nu_B = \frac{1}{2\pi}\frac{eB}{mc}$$

where n_e is the electron density and B is the magnitude of the static magnetic field. The polarization coefficients are defined in terms of the components of the electric vector of the radiation field

$$i\alpha_\theta = \frac{E_\theta}{r} ; \qquad i\alpha_k = \frac{E_k}{E_x} \tag{4}$$

where E_x and E_θ are the transverse components of $\vec{E}$, with E_θ lying in the plane defined by $\vec{B}$ and $\hat{k}$, and E_k is the component of $\vec{E}$ along the wave normal direction $\hat{k}$.

The indices of refraction, $n_\pm$, have several cutoffs ($n = 0$) and resonances ($n \to \infty$), and for a non-vanishing B there are frequency bands below the plasma frequency in which either or both the ordinary or extraordinary mode indices are real and greater than unity. It can be shown, however, (Pawsey and Bracewell, 1955), that for $\theta \neq 0$, radiation at a frequency ν, in the ordinary or extraordinary modes, will not escape from a radio source for which both n_e and B fall off toward the observer, unless ν is greater than ν_p or ν_x, respectively, where $\nu_x = \left(\nu_p^2 + \nu_B^2/4\right)^{1/2} + \nu_B/2$. Furthermore, for $\nu > \nu_p$, n_+ is real and less than unity, and for $\nu > \nu_x$, n_- is also real and less than unity. We shall limit our discussion, therefore, to these frequency ranges.

As can be seen from Equations (3) and (4), the electric vector of the radiation field in a plasma has in general a non-vanishing longitudinal component, E_k. Unless we invoke coupling between longitudinal oscillations and transverse electromagnetic waves, the energy associated with this component will not escape from the radio source, and therefore, in the subsequent calculations we have set

$\alpha_k = 0$. We wish to point out, however, that this cannot introduce any serious error in our calculations, since from the numerical evaluations which will be described below, we find that for the frequency range of interest ($\nu > \nu_p$ and $\nu > \nu_x$), the emissivity associated with E_k is negligible in comparison with the total radiated energy.

The polarization coefficient, $\alpha_{\theta^\pm}$, for an arbitrary direction of propagation, is finite and greater than zero. The radiation is therefore elliptically polarized. For the special cases of longitudinal ($\theta = 0$) or transverse ($\theta = \pi/2$) propagation, the radiation becomes circularly or linearly polarized, respectively. In particular, for $\theta = \pi/2$

$$\left.\begin{aligned} n_+^2 &= 1 - \nu_p^2/\nu^2 \\ n_- &= 1 + \nu_p^2\left(\nu_p^2 - \nu^2\right)/\left(\nu^4 - \nu^2\,\nu_B^2 - \nu^2\,\nu_p^2\right) \end{aligned}\right\} \qquad (5)$$

and

$$\left.\begin{aligned} \alpha_{\theta^+} &\rightarrow -i\infty \\ \alpha_{\theta^-} &= 0\,. \end{aligned}\right\} \qquad (6)$$

We now consider the radiation produced by the energetic electrons. If we set $\alpha_k = 0$, the frequency and angular distribution of the emission from a single electron, into the ordinary or extraordinary modes, is given by (Liemohn, 1965):

$$\eta_{\pm}(\nu, \theta, \gamma, \phi) = \frac{2\pi e^2}{c} \nu^2 \sum_{s=-\infty}^{\infty} \frac{n_{\pm}}{1 + a_{\theta^{\pm}}^2}$$

$$\cdot \left[-\beta \sin\phi \, J_s'(x_s) + \left\{ \alpha_{\theta^{\pm}} \left(\frac{\cot\theta}{n_{\pm}} - \beta \frac{\cos\phi}{\sin\theta} \right) J_s(x_s) \right\} \right]^2$$

$$\cdot \left(1 + \frac{\partial \ln n_{\pm}}{\partial \ln \nu} \right) \delta\left(\nu - \frac{s\nu_B}{\gamma} - n_{\pm} \nu \beta \cos\phi \cos\theta \right) \frac{\text{ergs}}{\text{sec. sr. Hz}} \quad (7)$$

where

$$x_s = \frac{s n_{\pm} \beta \sin\phi \sin\theta}{1 - n_{\pm} \beta \cos\phi \cos\theta} . \quad (8)$$

In these expressions, β is the electron velocity in units of c; $\gamma = (1 - \beta^2)^{-1/2}$; ϕ is the electron pitch-angle with respect to $\vec{B}$ and J_s is a Bessel function or order s. Equation (7) can be used for all frequencies for which $n^2 > 0$. Positive values of s correspond to the normal Doppler effect, negative values of s, to the anomalous Doppler effect and s = 0, to Cerenkov radiation. For values of ν greater than ν_p and ν_x, however, $0 < n^2 < 1$, and, therefore, the index s ranges only over values greater than zero.

The emissivity η, given in Equation (7) is equal to the energy lost by the particle due to radiation into unit frequency interval and unit solid angle and not to the observed radiation by a distant observer. It was shown by a number of authors (Epstein and Feldman, 1967; Scheuer, 1968; Ginzburg and Syrovatskii, 1969) that if the radiating electron moves in a helical orbit, these two expressions are not identical, the total energy loss being smaller by approximately a factor of $\sin^2 \phi$ than the integrated radiation crossing a large but fixed surface surrounding the moving charge. This difference is the result of the nonstationary nature of the radiation field which leads to a change in field energy within a fixed volume of space. It was shown by Scheuer (1968) and by Ginzburg and Syrovatskii (1968), however, that in order to compute the radiation from a given number of electrons in a fixed volume in space, one still has to use Equation (7) for the power emitted by an electron and not the modified expressions that would correspond to the observed radiation by distant observers. Equation (7) should be treated, therefore, as the volume emissivity from a monoenergetic distribution of electrons with unique pitch-angles, normalized to 1 electron of Lorentz factor γ and pitch-angle ϕ.

The emissivities, $j_{\pm}$, and absorption coefficients, $K_{\pm}$, for an arbitrary energy and pitch-angle distribution can be obtained from the following general expressions (Bekefi, 1966; Melrose, 1968):

$$j_{\pm}(\nu, \theta) = \int \eta_{\pm}(\nu, \theta, \gamma, \phi)\, f(\vec{p}\,')\, d^3 p' \text{ ergs/(sec sr Hz cm}^3) \quad (9)$$

$$K_{\pm}(\nu, \theta) = \frac{c^2}{n\nu^2} \int \eta_{\pm}(\nu, \theta, \gamma, \phi) \frac{1}{h\nu} \left[f(\vec{p}) - f(\vec{p}\,')\right] d^3 p' \ \mathrm{cm}^{-1} \tag{10}$$

where $f(\vec{p})\, d^3 p$ is the number of electrons per unit volume, with momenta in $d^3\, p$ around $\vec{p}$, and the sign of K is defined such that by emitting a photon of energy $h\nu$ an electron changes its momentum from $\vec{p}\,'$ to $\vec{p}$. Since the photon energy $h\nu$ is much smaller than the electron energy γmc^2, $f(\vec{p}) - f(\vec{p}\,')$ can be reduced to

$$f(\vec{p}) - f(\vec{p}\,') = -\frac{h\nu}{pc} \beta\gamma \frac{\partial f}{\partial \gamma} + \mathrm{tg}\, \Delta\phi \frac{\partial f}{\partial \phi} \tag{11}$$

where $\Delta\phi$ is the change in the pitch-angle of the radiating electron resulting from the emission of a photon of energy $h\nu$ into the direction θ, and is given by

$$\mathrm{tg}\, \Delta\phi = \frac{h\nu}{pc} \frac{n\beta \cos\theta - \cos\phi}{\beta \sin\gamma} . \tag{12}$$

As can be seen from Equations (10) and (11), for an isotropic distribution ($\partial f/\partial\phi = 0$), a necessary condition for negative absorption is $\partial f/\partial\gamma > 0$. For an anisotropic distribution, however, negative absorption can occur even if $\partial f/\partial\gamma < 0$. A necessary condition for this is

$$\Delta\phi \frac{\partial f}{\partial \phi} < 0$$

which simply means that owing to the emission of a photon, the electron pitch-angle is changed from a more densely populated to a less densely populated pitch-angle state. We shall further investigate this type of instability in the numerical calculations to be described below. We wish to point out, however, that maser action from a moderately anisotropic distribution will be confined mainly to the low harmonics of gyro-synchrotron emission, since for ultra-relativistic electrons the emission is strongly peaked in the direction of the instantaneous velocity, and amplification will occur only if the angular spread of f is of the order of the aperture angle of the radiation pattern. This point was made by Heyvaerts (1968).

We now consider a radio source which has a total volume V and contains N electrons with energies greater than some given minimal value. We assume that the distribution function $f(\vec{p})$ can be separated into an energy dependent part, $u(\gamma)$, and a pitch-angle dependent part, $g(\phi)$ such that

$$f(\vec{p}) = \frac{N}{V} \frac{1}{p^2} \frac{d\gamma}{dp} u(\gamma)\, g(\phi) \,. \tag{13}$$

In terms of these functions, j and K can be written as:

$$j_{\pm}(\nu, \theta) = 2\pi \int_1^{\infty} u(\gamma)\, d\gamma \int_{-1}^{1} d\cos\phi\, g(\phi)\, \eta_{\pm}(\nu, \theta, \gamma, \phi) \; \frac{\text{ergs}}{\text{cm}^3\ \text{sec sr Hz}} \tag{14}$$

$$K_{\pm}(\nu, \theta) = \frac{2\pi}{m\nu^2 n} \int_1^{\infty} u(\gamma)\, d\gamma \int_{-1}^{1} d\cos\phi\, g(\phi)\, \eta_{\pm}(\nu, \theta, \gamma, \phi)$$

$$\cdot \left[-\frac{\beta\gamma^2}{u(\gamma)} \frac{d}{d\gamma} \left(\frac{u(\gamma)}{\beta\gamma^2} \right) + \frac{n\beta\cos\theta - \cos\phi}{\gamma\beta^2 \sin\phi} \frac{1}{g(\phi)} \frac{dg(\phi)}{d\phi} \right] \text{cm}^{-1} . \quad (15)$$

By substituting Equation (7) for $\eta_{\pm}$ we can directly evaluate j and K for both the ordinary and extraordinary modes. The double integrals, $d\gamma\, d\cos\phi$, in Equations (14) and (15) can be reduced to single integrals by virtue of the delta function in Equation (7). If $\theta \neq \pi/2$, this eliminates the integral over $d\cos\phi$, while if $\theta = \pi/2$ the delta function is used to eliminate the integral over $d\gamma$. Carrying out these integrations, we obtain:

$$j_{\pm}(\nu, \theta) = \frac{BN}{V} \frac{e^3}{mc^2} G_{\pm}\left(\frac{\nu}{\nu_B}, \theta\right) \quad (16)$$

$$K_{\pm}(\nu, \theta) = \frac{N}{BV} (2\pi)^2 e H_{\pm}\left(\frac{\nu}{\nu_B}, \theta\right) . \quad (17)$$

For $\theta \neq \pi/2$, the functions G and H are given by

$$\begin{bmatrix} G_{\pm} \\ H_{\pm} \end{bmatrix} = \frac{2\pi}{\cos\theta} \int_1^{\infty} d\gamma \frac{u(\gamma)}{\beta\gamma} \sum_{s=s_1}^{s_2} \frac{s}{1 - n_{\pm}\beta\cos\theta\cos\phi_s} \frac{g(\phi_s)}{1 + \alpha_{\theta\pm}^2} \left(1 + \frac{\partial \ln n_{\pm}}{\partial \ln \nu}\right)$$

$$\cdot \left\{ -\beta\sin\phi_s J_s'(x_s) + \alpha_{\theta\pm} \left(\frac{\cot\theta}{n_{\pm}} - \beta\frac{\cos\phi_s}{\sin\theta} \right) J_s(x_s) \right\}^2$$

$$\cdot \begin{bmatrix} 1 \\ \frac{1}{n_{\pm}^2} \left(\frac{\nu_B}{\nu}\right)^2 \left(-\frac{\beta\gamma^2}{u(\gamma)} \frac{d}{d\gamma} \left(\frac{u(\gamma)}{\beta\gamma^2} \right) + \frac{n\beta\cos\theta - \cos\phi}{\gamma\beta^2\sin\phi} \frac{1}{g(\phi)} \frac{dg}{d\phi} \right) \end{bmatrix} \quad (18)$$

where

$$x_s = \frac{s n_\pm \beta \sin\theta \sin\phi_s}{1 - n_\pm \beta \cos\theta \cos\phi_s} \tag{19}$$

$$s_{1,2} = \frac{\nu}{\nu_B} \gamma \left(1 \pm n_\pm \beta \cos\theta\right) \tag{20}$$

$$\cos\phi_s = \frac{1 - s\nu_B/\gamma\nu}{n_\pm \beta \cos\theta} . \tag{21}$$

For $\theta = \pi/2$ we obtain:

$$\begin{bmatrix} G_\pm \\ H_\pm \end{bmatrix} = 2\pi n_\pm \sum_{s=1}^{\infty} s\beta_s^2 u(\gamma_s) \left(1 + \frac{\partial \ln n_\pm}{\partial \ln \nu}\right) \int_{-1}^{1} d\cos\gamma \, Y_\pm \, g(\phi_s)$$

$$\cdot \left[\frac{1}{n_\pm^2}\left(\frac{\nu_B}{\nu}\right)^2 \left(-\frac{\beta_s \gamma_s^2}{u(\gamma_s)} \frac{d}{d\gamma}\left(\frac{u}{\beta\gamma^2}\right)_{\gamma_s} - \frac{\cot\phi}{\gamma_s \beta_s^2} \frac{1}{g(\phi)} \frac{dg}{d\phi}\right)\right] \tag{22}$$

where

$$\left.\begin{aligned} Y_+ &= \cos^2\phi \, J_s(x_s) \\ Y_- &= \sin^2\phi \, J_s'(x_s) \end{aligned}\right\} \tag{23}$$

$$\gamma_s = s\nu_B/\nu ; \qquad \beta_s = \left(1 - \frac{1}{\gamma_s^2}\right)^{1/2} \tag{24}$$

$$x_s = s\beta_s m_\pm \sin\phi . \tag{25}$$

In previous studies of the effect of the ionized medium on solar radio emission (Ramaty and Lingenfelter 1967, 1968), it was convenient to introduce a parameter α, defined as

$$\alpha = \frac{3}{2}\frac{\nu_B}{\nu_p} . \tag{26}$$

Then, in terms of this parameter, gyro-synchrotron emission from an electron of Lorentz factor γ is strongly suppressed at the low frequencies if $\alpha\gamma < 1$, but remains unaffected by the ionized medium if $\alpha\gamma \gtrsim 1$ (Ramaty, 1968). Moreover, if $\alpha\gamma << 1$, the total emission will also be strongly suppressed. Since the indices of refraction and polarization coefficients, as functions of ν/ν_B, depend only on the "intrinsic" parameters α and θ, the functions G and H also depend on these parameters only and on the normalized distribution functions $u(\gamma)$ and $g(\phi)$, but are independent of such "extrinsic" parameters as the intensity of the magnetic field, the size and shape of the radio source and the densities of the energetic and ambient electrons.

III. POLARIZATION AND RADIATION TRANSFER

Partially polarized radiation at a given point in space is completely determined by 4 independent parameters such as the Stokes parameters I, Q, U, and V. These functions in turn, can be obtained from any other set of independent parameters that specify the radiation. In a magnetoactive plasma, the natural set of parameters consists of the intensities of the ordinary and extraordinary modes, I_+ and I_-, and the phase relations, I_s and I_c, which are defined in terms of $I_\pm$ and the phase difference, δ, between the modes:

$$I_s = \left(I_+ I_-\right)^{1/2} \sin\delta$$

$$I_c = \left(I_+ I_-\right)^{1/2} \cos\delta \ .$$

Since, as mentioned above, the longitudinal component of $\vec{E}$ is set to zero ($\alpha_k = 0$), the radiation field is completely determined by its transverse components. Using Equation (4), E_x and E_θ, as functions of $I_\pm$ and δ, can be written as:

$$\begin{pmatrix} E_x \\ E_\theta \end{pmatrix} = \left(\frac{2\pi}{c}\right)^{1/2} \left[I_+^{1/2}\,\hat{e}_+ + e^{-i\delta}\, I_-^{1/2}\,\hat{e}_-\right] \tag{27}$$

where the unit vectors $\hat{e}_\pm$ are defined by

$$\hat{e}_\pm = \left(1 + \alpha_{\theta\pm}^{2}\right)^{-1/2} \begin{pmatrix} 1 \\ i\alpha_{\theta\pm} \end{pmatrix}$$

and where the x and θ axes are chosen such that the θ direction is along the perpendicular component, $\vec{B}_\perp$, of the static field $\vec{B}$. In terms of the transverse components of the electric field, Stokes' parameters are given by (Born and

Wolf, 1964)

$$\left.\begin{aligned} I &= \frac{c}{2\pi}\left(E_x E_x^* + E_\theta E_\theta^*\right) \\ Q &= \frac{c}{2\pi}\left(E_x E_x^* - E_\theta E_\theta^*\right) \\ U &= \frac{c}{2\pi}\left(E_x E_\theta^* + E_\theta E_x^*\right) \\ V &= \frac{c}{2\pi}\, i\left(E_x E_\theta^* - E_\theta E_x^*\right) \ . \end{aligned}\right\} \tag{28}$$

The degree of polarization, π, the ratio of the axes of the polarization ellipse, p, and the angle χ between the major axis and the x direction can then be written as

$$\left.\begin{aligned} \pi &= \left(Q^2 + U^2 + V^2\right)^{1/2}\Big/ I \\ p = tg^{-1}\beta; \qquad \sin 2\beta &= V\Big/\left(Q^2 + U^2 + V^2\right)^{1/2} \\ tg\, 2\chi &= U/Q \ . \end{aligned}\right\} \tag{29}$$

These quantities are defined such that, if seen by an observer, the electric vector rotates clockwise or counterclockwise if V is negative or positive, respectively. $V = 0$ corresponds to linear polarization and $Q = U = 0$ corresponds to circular polarization. The angle $\chi (0 \leq \chi < \pi)$, as seen by an observer, is measured counterclockwise from the x direction; if $U > 0$, $0 < \chi < \pi/2$ and if $U < 0$, $\pi/2 < \chi < \pi$; if

$U = 0$, $\chi = 0$ or $\chi = \pi/2$ for $Q > 0$ or $Q < 0$, respectively. These definitions are consistent with Figure (1.8) in Born and Wolf (1964).

Substituting Equation (27) into Equations (28), we get

$$\left.\begin{aligned}
I &= I_+ + I_- \\
Q &= I_+ \frac{1-\alpha_{\theta^+}^2}{1+\alpha_{\theta^+}^2} + I_- \frac{1-\alpha_{\theta^-}^2}{1+\alpha_{\theta^-}^2} + \frac{4I_c}{\left(1+\alpha_{\theta^+}^2\right)^{1/2}\left(1+\alpha_{\theta^-}^2\right)^{1/2}} \\
U &= \frac{2I_s\left(\alpha_{\theta^+} - \alpha_{\theta^-}\right)}{\left(1+\alpha_{\theta^+}^2\right)^{1/2}\left(1+\alpha_{\theta^-}^2\right)^{1/2}} \\
V &= 2\left\{\frac{I_+\,\alpha_{\theta^+}}{1+\alpha_{\theta^+}^2} + \frac{I_-\,\alpha_{\theta^-}}{1+\alpha_{\theta^-}^2} + \frac{I_c\left(\alpha_{\theta^+} + \alpha_{\theta^-}\right)}{\left(1+\alpha_{\theta^+}^2\right)^{1/2}\left(1+\alpha_{\theta^-}^2\right)^{1/2}}\right\}
\end{aligned}\right\} \quad (30)$$

For radiation in vacuum, $\alpha_{\theta^\pm} = \pm 1$, and Equations (30) reduce to

$$\left.\begin{aligned}
I &= I_+ + I_- \\
Q &= 2I_c \\
U &= 2I_s \\
V &= I_+ - I_- \,.
\end{aligned}\right\} \quad (31)$$

Equations (30) or (31) provide the transformation between the two independent set of parameters, I, Q, U, V and $I_\pm$, I_s, I_c. Similar equations were obtained by Kai (1965). They contained, however, some numerical errors, and therefore could not be used in the present study.

If the radiation is completely polarized, the phase difference δ has a well defined value, and

$$I_s^2 + I_c^2 = I_+ I_- \ .$$

Using Equations (29) and (30) or (31), we verify that in this case the degree of polarization, π, indeed equals unity.

For a single particle in vacuum, $\delta = 0$, and, therefore, the radiation specified by Equations (31) is elliptically polarized with the major axis perpendicular to $\vec{B}_\perp$. For ultrarelativistic particles at high frequencies $I_+ = I_-$, and the radiation becomes linearly polarized with the plane of polarization perpendicular to $\vec{B}_\perp$. For a single particle in a plasma, δ is not equal to zero and its value depends in a complicated way on the energy and pitch-angle of the particle, as well as on the direction of propagation and the properties of the plasma. We shall circumvent these complications, however by limiting our treatment to the case of large Faraday rotation in the source region, for which the phase relations are randomized and $I_s = I_c = 0$. The assumption of large Faraday rotation in the source is almost always well satisfied for gyro-synchrotron radiation. The rotation angle, $\Delta\psi$, is approximately given by

$$\Delta\psi \approx \frac{L}{\lambda} \frac{\nu_p^2 \nu_B}{\nu^3}$$

where L is of the order of the linear dimensions of the system and λ is the wavelength of the radiation. Since gyro-synchrotron radiation generally occurs at frequencies ν, comparable, within an order of magnitude, to both ν_p and ν_B, whereas $L >> \lambda$, $\Delta\psi$ will in general be much larger than 2π and the amount of Faraday rotation will be large.

In order to obtain the intensity and polarization of the radiation from a system of particles, it is necessary to solve the transfer equations for either I, Q, U and V or $I_{\pm}$, I_s and I_c. General treatments of such equations, without specific solutions, were given by Kawabata (1964), Kai (1965), and Ginzburg and Syrovatskii (1969). The phase relations, I_s and I_c, provide coupling between the two modes, and this considerably complicates the solution of these equations. If the Faraday rotation is large, however, $I_s = I_c = 0$, the transfer equations for I_+ and I_- decouple and take on the simple form

$$\frac{dI_{\pm}(\nu, \theta)}{dz} + K_{\pm}(\nu, \theta)\, I_{\pm}(\nu, \theta) = j_{\pm}(\nu, \theta) \tag{32}$$

where $j_{\pm}$ and $K_{\pm}$ are given by Equations (9) and (10).

For a homogeneous source region, the solution of Equation (32) is given by

$$I_{\pm}(\nu, \theta) = \frac{j_{\pm}}{K_{\pm}}\left[1 - \exp\left(-K_{\pm} L\right)\right] \tag{33}$$

where L is the depth of the source. If the radio source has an effective area A such that its volume is $V \approx AL$, and if in addition to the suppression of the longitudinal component of the radiation field, discussed above, no other boundary conditions are required, then by using Equations (16), (17), and (33), the intensity of the escaping radiation can be written as

$$I_\pm(\nu, \theta) = \frac{e^2}{(2\pi)^2 mc^2} \frac{(BN/A)\, G_\pm\left(\nu/\nu_B\right)}{(N/BA)\, H_\pm\left(\nu/\nu_B\right)} \left[1 - \exp\left(-(2\pi)^2 e \frac{N}{BA} H_\pm\right)\right] . \qquad (34)$$

For $I_s = I_c = 0$ Equations (30) also reduce to

$$\begin{aligned} I &= I_+ + I_- \\ Q &= I_+ \frac{1-\alpha_{\theta+}^2}{1+\alpha_{\theta+}^2} + I_- \frac{1-\alpha_{\theta-}^2}{1+\alpha_{\theta-}^2} \\ U &= 0 \\ V &= 2\left(\frac{I_+\,\alpha_{\theta+}}{1+\alpha_{\theta+}^2} + \frac{I_-\,\alpha_{\theta-}}{1+\alpha_{\theta-}^2}\right). \end{aligned} \qquad (35)$$

Equations (34) and (35), with $G_\pm$ and $H_\pm$ given by Equations (16) through (25), and $\alpha_{\theta\pm}$ given by Equation (2) completely determine the intensity and polarization of the radiation. Since $G(\nu/\nu_B)$ and $H(\nu/\nu_B)$ depend only on the "intrinsic" parameters α and θ and the normalized distributions $u(\gamma)$ and $g(\phi)$, the "extrinsic" properties of the source depend only on N/BA and BN. In particular, the reabsorption depends only on the parameter N/BA (i.e. on the total number of energetic electrons along the line of sight per unit area and per unit B).

We consider now the Stokes parameters given by Equation (35). Using Equations (29), we find that the degree of polarization is given by

$$\pi = \left| I_+ - I_- \right| \Big/ \left(I_+ + I_- \right) .$$

Thus, when both modes escape from the source region, because of Faraday rotation, π is always less than unity. Since U = 0 (which is also a result of Faraday rotation) one of the axes of the polarization ellipse will be parallel to $\vec{B}_\perp$. From the numerical calculations which are described below, we find that the values of Q are such that the major axis of the ellipse is parallel or perpendicular to $\vec{B}_\perp$ depending on whether I_+ is greater or smaller than I_-, respectively. Linear polarization will occur if $\theta = \pi/2$, since then, by Equations (6) and (35) $V = 0$ and $Q = I_- - I_+$. Thus, depending on whether I_- is greater or smaller than I_+, the plane of polarization is perpendicular or parallel to $\vec{B}_\perp$, respectively.

In the next section we shall use the formalism developed above to study the spectral and polarization properties of typical gyro-synchrotron sources.

IV. NUMERICAL RESULTS

We first consider monoenergetic electrons with an isotropic pitch-angle distribution, $g(\phi) = 1/4\pi$. Using Equation (16), we have evaluated the emissivity j, equal to $j_+ + j_-$, for a variety of values of α, θ and γ. The resultant spectra, as functions of ν/ν_c, where

$$\nu_c = \frac{3}{2} \nu_B \gamma^2 \tag{36}$$

are shown as solid lines in Figure 1 for: $\alpha = 0.25$; $\theta = 70°$ and $45°$; and $\gamma = 2$, $\gamma = 5$ and $\gamma \to \infty$. The dashed lines in Figure 1 represent the emissivity j_{UR}, evaluated from the formula

$$j_{UR}(\nu, \theta) = \frac{\sqrt{3}}{4\pi} \frac{e^3 B}{mc^2} \sin\theta \left(\frac{\gamma_1}{\gamma}\right) \left(\frac{\nu_c}{\nu_{c1}}\right) \int_{\nu/\nu_{c1}}^{\infty} K_{5/3}(y)\, dy \tag{37}$$

where

$$\nu_{c1} = \nu_c \left(\frac{\gamma_1}{\gamma}\right)^3 \tag{38}$$

and

$$\gamma_1 = \frac{\gamma}{\left[1 + \frac{9}{4} \frac{\gamma^2 - 1}{\alpha^2 \left(\nu/\nu_B\right)^2}\right]^{1/2}} . \tag{39}$$

Equation (37) appears to be the generalization of the well known emissivity formulas for ultrarelativistic electrons in helical orbits in vacuum (Ginzburg and Syrovatskii, 1964, 1965) and for ultrarelativistic electrons in circular orbits in a medium (Ginzburg and Syrovatskii, 1964, 1965; Ramaty and Lingenfelter, 1967). As can be seen from Figure 1, for both $\theta = 70°$ and $\theta = 45°$, there is a significant discrepancy between j_{UR} and the exact spectra, j, obtained from Equation (16). The ratios, j_{UR}/j, of the spectra shown in Figure 1, can be compared with the ratios F_{UR}/F that were evaluated by Ramaty (1968) for the integrated emissivity (over all angles θ) from mildly relativistic electrons in circular orbits in an ionized medium. From this comparison (see Figure 3, Ramaty (1968)) we find that the ratios j_{UR}/j, for $\theta = 70°$, are almost the same as the corresponding ratios F_{UR}/F. For $\theta = 45°$, however, the discrepancy between the ultrarelativistic and mildly relativistic results, shown in Figure 1, are significantly larger than those obtained for the circular orbits. For example, for $\theta = 45°$, if $\nu/\nu_c = 2$, $F_{UR}/F \approx 8$ (Figure 3, Ramaty, 1968), whereas, according to Figure 1, at the same value of ν/ν_c, $j_{UR}/j \approx 60$. The larger discrepancy for small values of θ must be attributed to the combined effects of the non-relativistic nature of the particles, the helical orbits and the ionized medium. The subsequent calculations, therefore, will entirely be based on the exact formalism developed in the previous sections.

We next consider an ensemble of electrons with an isotropic pitch-angle distribution, $g(\phi) = 1/4\pi$, and an energy spectrum of the form

$$u(\gamma) \sim (\gamma - 1)^{-\Gamma} \tag{40}$$

with a low-energy cutoff at $\gamma = 1.2$. It was found (e.g. Holt and Ramaty, 1969) that such a distribution, with $\Gamma \approx 3$, can reproduce the x-ray spectrum between 100 to 500 keV that was observed for the solar flare of 7 July 1966. At higher energies, direct observations of solar flare electrons (Cline and McDonald, 1968) also indicate that the spectrum is a power law with an index which may vary from flare to flare, but in general has a value close to that deduced from the x-ray data. In the subsequent calculations, we shall use Equation (40) with $\Gamma = 3$.

The emissivities, $j_{\pm}$, and absorption coefficients $K_{\pm}$, as functions of ν/ν_B, are shown in Figure 2, for $\theta = 45°$, and for $\alpha = 3$ and 0.5. As can be seen, for the small value of α, the emissivity and absorption coefficients, in both modes, are strongly suppressed at low frequencies. For $\alpha = 3$, this suppression effect dissappears, and, at low frequencies, one can clearly see the harmonic structure of both j and K. From Figure 2 we also see that for both $\alpha = 3$ and $\alpha = 0.25$, $j_- > j_+$, $K_- > K_+$, but $j_+/K_+ > j_-/K_-$. In an optically thin source, therefore, the extraordinary mode will dominate, whereas if the source is optically thick, the ordinary mode radiation will be larger than that in the extraordinary mode. This is a special result for non-thermal sources, since in an optically thick thermal source, because of Kirchhoff's law, j/K will be the same in both modes. The dominance of the ordinary mode in a optically thick non-thermal source is caused by the greater relative contribution of higher energy electrons to radiation in this mode. This effect is most pronounced at typical gyro-synchrotron frequencies, since in the limiting case of radiation from ultrarelativistic electrons, the contributions from both modes become equal.

As can be seen from Figure 2, the conditions for a given source to be optically thin or thick depend critically on the values of the parameters N/BA and α. In order to investigate these conditions and the simultaneous dependence of the spectrum and polarization on N/BA and α we have evaluated Equations (34) and (35), for various values of these parameters, and for $\theta = 45°$, $\Gamma = 3$, and $g(\phi) = 1/4\pi$. The total intensity, I, as a function of ν/ν_B, is shown in Figures (3), (4), (5) and (6) for N/BA = 10^9, 10^{11}, 10^{13} and 10^{15} respectively. The asterisks on the spectra denote the frequency, ν^*, at which $I_+ = I_-$. Since for the frequency range of interest ($\nu > \nu_p$ and $\nu > \nu_x$) the absolute values of both a_{θ^+} and a_{θ^-} are close to unity ($a_{\theta^+} \gtrsim 1$; $a_{\theta^-} \gtrsim -1$), the frequency ν^* also corresponds to complete depolarization (Q = U = V = 0). For $\nu \ll \nu^*$, the source is optically thick and $I_+ > I_-$. For $\nu \gg \nu^*$ the source becomes optically thin, and $I_- > I_+$. Holt and Ramaty (1969) made a study of microwave emission from solar flares based on the theory developed in the present paper. They showed that the transition from the optically thick to the optically thin regime, discussed above could account for the observed reversal of polarization of solar microwave bursts.

From Figures (3), (4), (5) and (6), we can also see that for a given value of N/BA one can define a critical value of α, α^*, such that: for $\alpha < \alpha^*$ the source is optically thin, the radiation at low frequencies is dominated by the Razin effect and it is polarized in a sense corresponding to the extraordinary mode; for $\alpha > \alpha^*$, the radiation at low frequencies is dominated by reabsorption and, at $\nu = \nu^*$, the polarization changes from the ordinary to the extraordinary mode.

In Figure (7) we plot N/BA as a function of α^*. This curve was obtained from Figures (3), (4), (5) and (6) as well as from similar calculations for other values of N/BA. As indicated, the area to the left of the curve corresponds to an optically thin source, while for values of N/BA and α corresponding to the area to the right of the curve, the polarization reverses at $\nu = \nu^*$.

In order to illustrate the polarization properties of the source, we evaluated the Stokes parameters I, Q and V, using Equations (35). (As discussed above, because of Faraday rotation, U = 0.) These are shown in Figure 8 as functions of ν/ν_B for N/BA = 10^{13} and for α = 1 and 0.25. As can be seen, for α = 1, the polarization reverses at $\nu/\nu_B \approx 10$. For α = 0.25, this reversal does not occur. Since $Q \ll V$, the degree of polarization is given approximately by $\pi \approx V/I$. The value of π is of the order of 10% to 20% in the optically thick regime and may become as large as 40% at frequencies where the source is optically thin. Since Q is very small, the polarization is almost circular. Nevertheless, the measurement of the sign of Q is possible in principle (see Bekefi (1966) pp. 23). If one adopts the model of Holt and Ramaty (1969) for the polarization reversal of microwave bursts, the measurement of the sign of Q and of the direction in which its value is maximal would determine the orientation of the polarization ellipse and, therefore, also of $\vec{B}_\perp$ at the source of the microwave emission.

We proceed now to consider the effects of an anisotropic pitch-angle distribution. For an isotropic distribution, a variation in the angle of observation θ has the net effect of reducing the total emission and the emission frequencies, both approximately by a factor of $\sin\theta$. As long as θ is not equal to or very close to zero, these effects are small. On the other hand, for a highly anisotropic distribution, the radiation pattern will depend strongly on the angle between the direction of observation and that of maximum anisotropy. In order to illustrate this effect we have considered a pitch-angle distribution of the form

$$g(\phi) \sim \sin^m \phi \tag{41}$$

and have chosen m = 45 for which $g(80°)/g(90°) = 0.5$. Using Equations (40) and (41), we have evaluated the total intensity, I, for a variety of values of the parameters α, θ and N/BA. From these calculations we find that, as for an isotropic distribution, the spectrum at low frequencies will be determined either by the Razin effect or by reabsorption, but will essentially be independent of the value of θ. At the high frequencies, however, the intensity will depend critically on the direction of observation. This is illustrated in Figures (9) and (10), for $\alpha = 1$ and 0.25, respectively.

As in the previous figures, the asterisks denote the frequency at which $I_+ = I_-$. By comparing the $\theta = 90°$ curve in Figure (9) with that corresponding to $\alpha = 3$ in Figure 5, we see that if the direction of observation coincides with that of maximum anisotropy, the total intensity, spectral shape and frequency of polarization reversal for the anisotropic distribution are all comparable to the corresponding quantities in the isotropic case (since for the latter $\theta \neq 90°$, both the intensity and frequency scale are reduced by approximately a factor of

$\sin\theta$). From Figures (9) and (10) we also see that if $\theta < \pi/2$, the emission intensity at high frequencies is strongly suppressed. As discussed above, this high-energy cutoff is the direct result of the different emission patterns of mildly relativistic and ultrarelativistic electrons. In Figure (10) we see the combined effects of the anisotropy and the Razin effect. Since now the radiation is suppressed at both the high and low frequencies, a situation may develop in which the total emission is also strongly suppressed.

Finally, we consider the polarization of the spectra shown in Figures (9) and (10). By comparing Figure (9) with Figures (3), (4), (5) and (6), we see that, for comparable values of α, the frequency of polarization reversal is lower for the anisotropic distribution and it decreases with decreasing θ. This is a direct consequence of the suppression of the emission of the relativistic electrons, since as discussed above, the dominance of the ordinary mode in an optically thick source results from the enhanced contribution of the high energy electrons to the emission at a given frequency. Because of the strong Razin effect, the spectra shown in Figure (10) are all due to an optically thin source and their polarization corresponds to that of the extraordinary mode.

Finally, we consider the question of negative absorption. As mentioned above, Bekefi, Hirshfield and Brown (1961) have investigated the possibility of maser action in a gyro-synchrotron source for which the distribution function f, at the appropriate energies, satisfies $\partial f/\partial\gamma > 0$. These authors have considered an isotropic electron distribution of the form $f \sim \beta^p \exp\left(-b\beta^2\right)$, where both p and b are non-negative constants, and showed that for $p > 0$ maser action will occur just below ν_B and its second harmonic. In the present paper we shall investigate negative absorption that results from an anisotropic electron distribution in a magnetoactive plasma. We shall only consider electron distributions for which $\partial f/\partial\gamma < 0$, so that maser action will result only from the anisotropy.

We have chosen two specific examples:

$$u(\gamma) \sim \exp(-\gamma) \tag{42}$$

and

$$u(\gamma) \sim \gamma^{-2.5} \tag{43}$$

and a pitch-angle distribution of the form given by Equation (41). Using Equations (22) through (25) (for $\theta = \pi/2$), we have evaluated K_+ and K_- for $m = 5$ and for various values of α. We find that for such a moderate anisotropy there is no maser action for $\alpha \to \infty$ (emission in vacuum). The reason for this is that the high-energy electrons contribute large positive quantities to both K_+ and K_-, and these cancel the negative contribution that results from the anisotropy. As α is decreased, however, K_+ becomes negative, in essentially the same frequency range ($\lesssim \nu_B$) as found by Bekefi, Hirshfield and Brown (1961) for the isotropic distribution with an inverted energy spectrum. This is illustrated in Figures (11) and (12) for the spectra given by Equations (42) and (43), respectively. The occurrance of an instability in a given range of frequencies in the presence of a plasma, is the result of the suppression of the positive contribution of the relativistic electrons to radiation at that frequency. Qualitatively, this result is the same as that obtained by McCray (1966) and Zheleznyakov (1967), who showed that even though synchrotron radiation in a vacuum can never be amplified, in the presence of a plasma maser action is possible. Furthermore, by comparing Figures (11) and (12) we also see that the range of frequencies in which K_+ is negative is larger for the exponential spectrum than for the power law. This also is the result of the different contributions of the high-energy electrons, since the spectrum given by Equation (42), obviously consists of less energetic particles than the one given by Equation (43).

For a moderate anisotropy, such as that given by $m = 5$, maser action is confined to $\nu \lesssim \nu_B$ and unless these frequencies can escape from the source, amplified radiation will not be observed. As α is further decreased, therefore, ν_p will eventually become larger than ν_B, and, as can be seen from Figures (11) and (12), K_+ will be positive for all frequencies of interest. By the same token, since the characteristic frequency ν_x, defined above is larger than ν_B, for the distributions used in Figures (11) and (12), K_- is positive for all values of ν that escape from the source. For much larger anisotropies or for inverted energy spectra, maser action is not limited to $\nu \lesssim \nu_B$, and, therefore, negative values of K_- may occur as well. In the present paper, however, we shall not treat maser action resulting from such highly anisotropic distribution.

Limiting ourself to the spectrum given by Equation (43) and to small values of m, we consider the effects of the anisotropy. In Figure (13), K_+ is plotted for $\alpha = 2$ and for various values of the parameter m. As can be seen, there is no maser action for the isotropic distribution but as m is increased, an instability sets in and the range of frequencies over which K_+ is negative increases with increasing m.

Another important aspect of amplified gyro-synchrotron radiation is its dependence on the direction of observation. In order to investigate this effect, we have evaluated the functions G and H for various values of the observation angle θ. We have used Equations (18) through (21) for $\theta \neq \pi/2$ and Equations (22) through (25) for $\theta = \pi/2$. The total intensity I, obtained from Equations (34) and (35) is plotted in Figure (14) for several values of θ. As can be seen, the radiation is highly amplified at $\nu \lesssim \nu_B$, but falls off rapidly as the angle between the direction of observation and that of maximum anisotropy is increased. Since the amplified radiation is entirely due to the ordinary mode and since it is confined to $\theta \approx \pi/2$, even though Faraday rotation is large, the emission is 100% linearly polarized. Because of its highly collimated nature and large degree of polarization, this type of radiation may be responsible for radio emission from the recently observed pulsars.

V. SUMMARY

We have developed a detailed formalism for the calculation of the emissivity and absorption coefficient, for a given direction of observation, from an ensemble of electrons with arbitrary energy and pitch-angle distributions in a cold collisionless electron plasma and a static uniform magnetic field. By assuming that all spatial distributions are homogeneous, we have solved the transfer equations in the limit of large Faraday rotation, i.e. when the phase difference between the ordinary and extraordinary modes is completely random so that the two modes propagate independently. This condition is generally well satisfied for gyro-synchrotron sources.

Using this formalism we have studied the influence of reabsorption and the Razin effect on the spectrum and polarization of the radiation. Both effects tend to lower the flux at low frequencies, but since the Razin effect suppresses not only the emissivity but also the absorption coefficient, a source which is optically thick in the absence of an ambient plasma, may become optically thin if the influence of the medium is taken into account. Gyro-synchrotron radiation from an optically thin source is polarized in the extraordinary mode, but because of the non-thermal nature of the radiating electrons, in an optically thick source the ordinary mode dominates. This transition is characteristic of gyro-synchrotron radiation and it may account for the observed polarization reversal of solar microwave bursts.

We have also investigated the effects of an anisotropic electron distribution. For such distributions, if the direction of observation differs significantly from that of maximum anisotropy, the radiation at high frequencies is strongly suppressed. This cutoff, together with the low frequency Razin suppression can effectively suppress the entire radiation spectrum. Moreover, the pitch-angle anisotropy, by suppressing the radiation from the high-energy electrons, has the additional effect of lowering the frequency of polarization reversal, mentioned above.

Finally, we have studied the possibility of maser action. We find that an anisotropic distribution can cause negative absorption, even if the electron spectrum, at all energies, decreases with increasing energy. For moderate anisotropies maser action will be limited to the ordinary mode and to frequencies just below the gyro frequency. The radiation will be 100% polarized and if the pitch-angle distribution is peaked around circular orbits, the amplified radiation will be linearly polarized and confined to a narrow cone centered around the direction of maximum anisotropy.

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FIGURE CAPTIONS

Figure 1. Gyro-synchrotron emission spectra from monoenergetic electrons. The dashed lines were obtained from the ultrarelativistic approximation (Equation 37) and the solid lines from the exact formalism.

Figure 2. The emissivity (solid line) and absorption coefficient (dashed line) from an ensemble of electrons with isotropic pitch-angle distribution and power law energy spectrum.

Figure 3. The radiation intensity from an isotropic, power law electron distribution for various values of the Razin parameter α and the reabsorption parameter N/BA equal to 10^9.

Figure 4. The radiation intensity from an isotropic, power law electron distribution for various values of the Razin parameter α and the reabsorption parameter N/BA equal to 10^{11}.

Figure 5. The radiation intensity from an isotropic, power law electron distribution for various values of the Razin parameter α and the reabsorption parameter N/BA equal to 10^{13}.

Figure 6. The radiation intensity from an isotropic, power law electron distribution for various values of the Razin parameter α and the reabsorption parameter N/BA equal to 10^{15}.

Figure 7. Relationship between the Razin and reabsorption parameters, α and N/BA, respectively. The area to the left corresponds to optically

thin sources, whereas in the area to the right, the source is optically thick below ν^* but becomes optically thin above ν^*.

Figure 8. Variation of the Stokes parameters I, Q and V with frequency for two choices of α and $N/BA = 10^{13}$.

Figure 9. The radiation intensity from a highly anisotropic electron distribution for various values of the emission angle θ and for $\alpha = 1$.

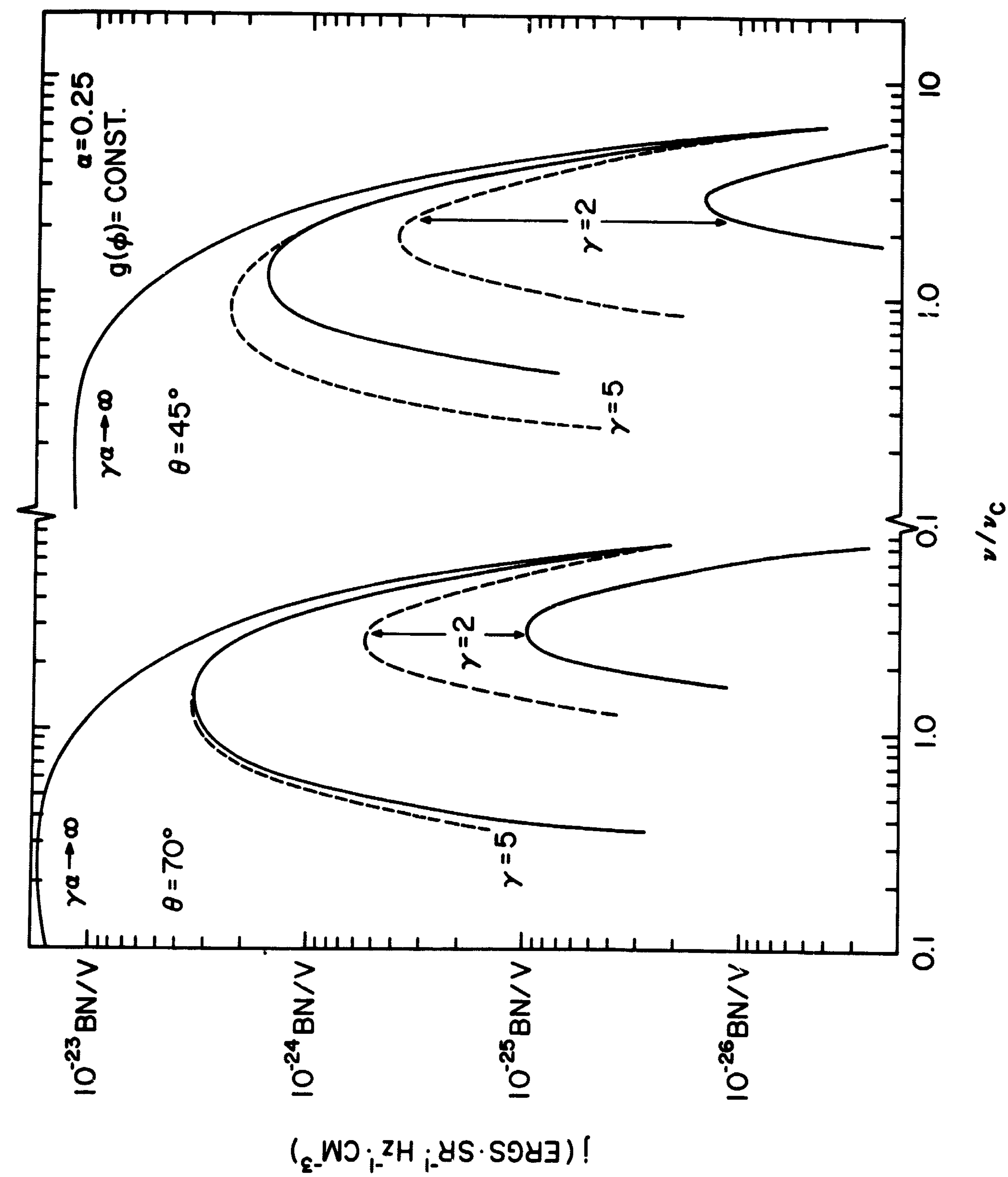
Figure 10. The radiation intensity from a highly anisotropic electron distribution for various values of the emission angle θ and for $\alpha = 0.25$.

Figure 11. Absorption coefficients in the ordinary mode for various values of the Razin parameter α and a power law electron spectrum.

Figure 12. Absorption coefficients in the ordinary mode for various values of the Razin parameter α and an exponential electron spectrum.

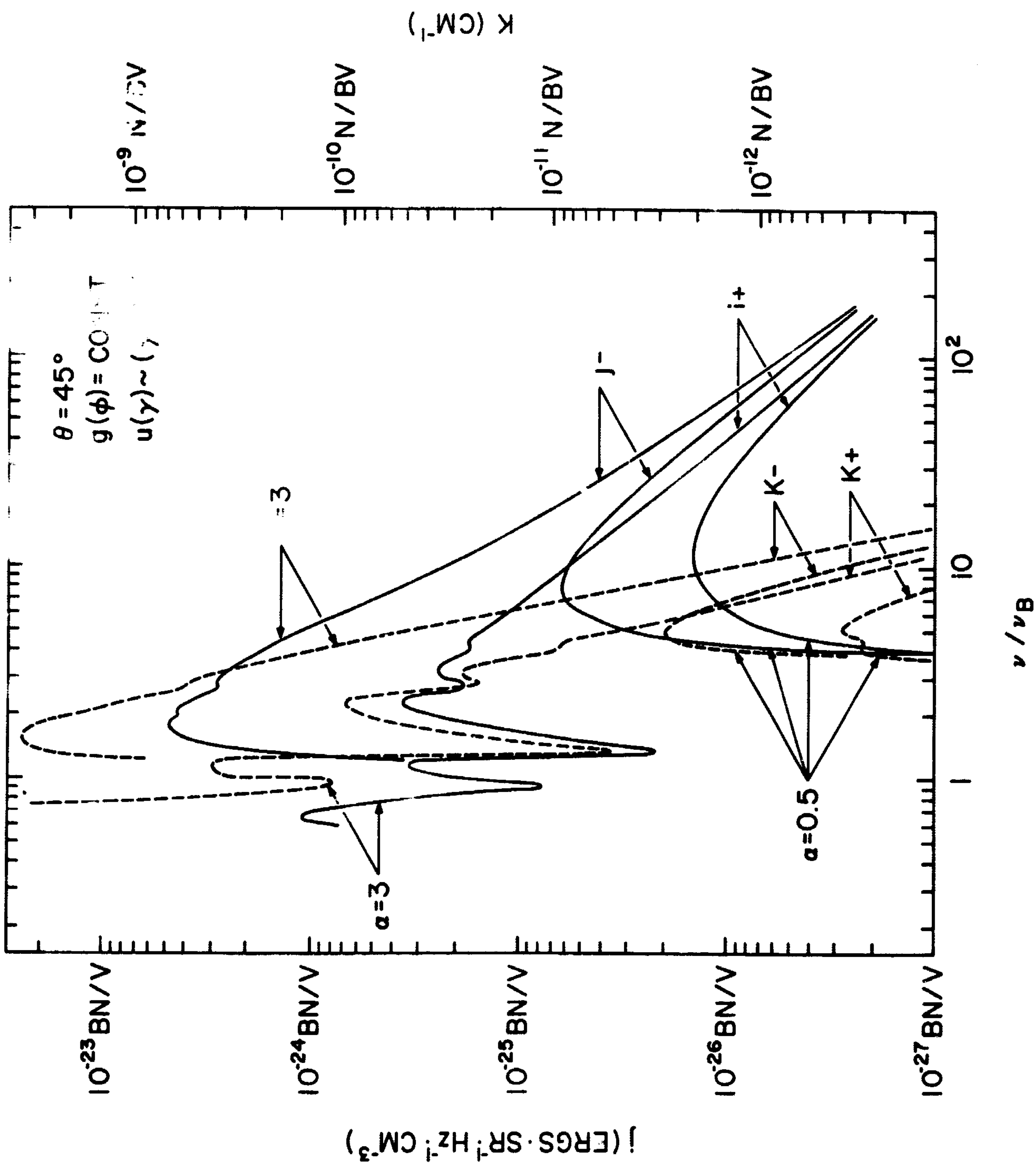
Figure 13. Absorption coefficients in the ordinary mode for various values of the anisotropy parameter m.

Figure 14. Total radiation intensity which exhibits a strong enhancement due to maser action at $\nu \lesssim \nu_B$.



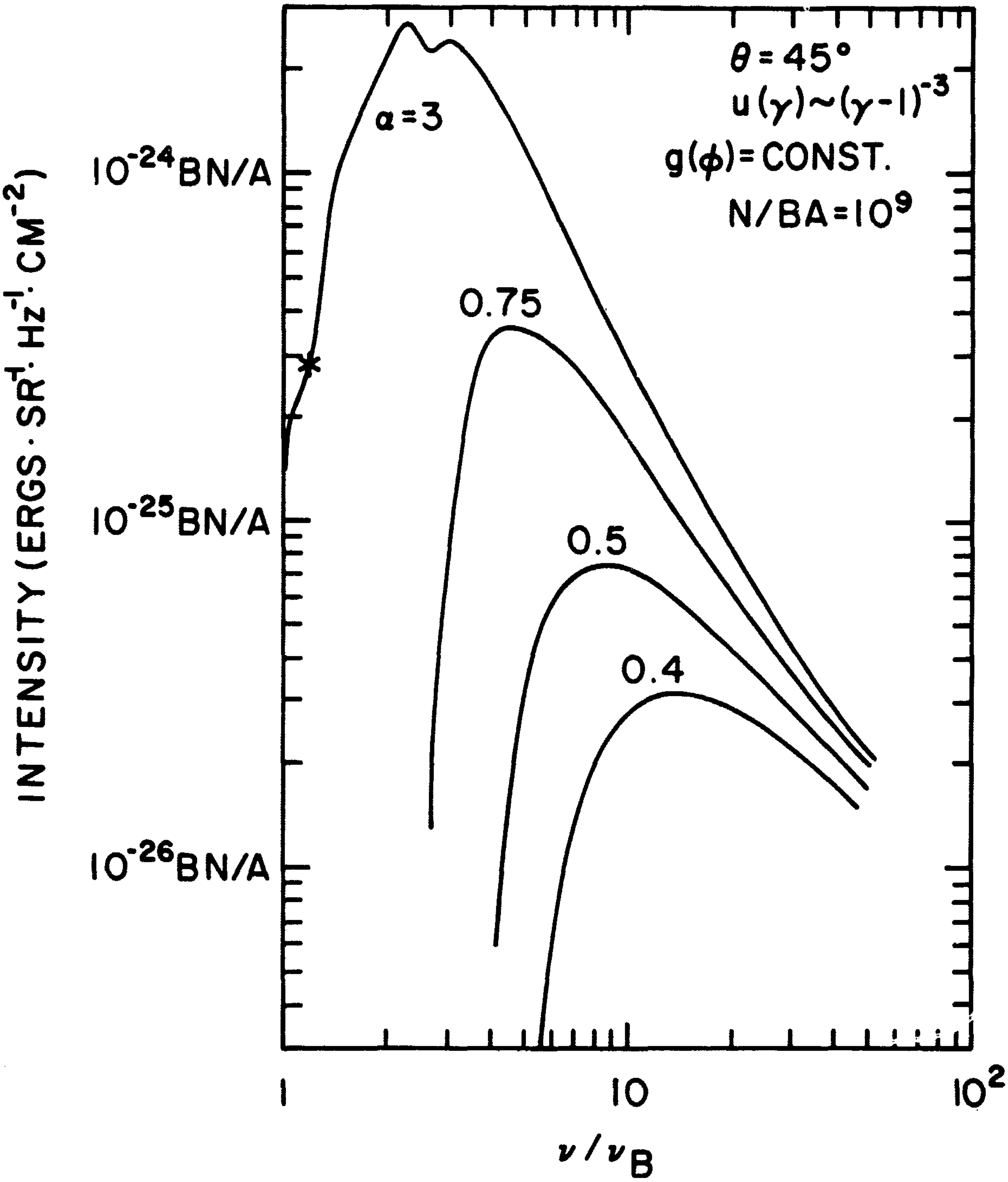
α=0.25
$g(\phi)$= CONST.
$\gamma\alpha \rightarrow \infty$
θ=45°
γ=2
γ=5
θ=70°
$\gamma\alpha \rightarrow \infty$
γ=2
γ=5
10^{-23}BN/V
10^{-24}BN/V
10^{-25}BN/V
10^{-26}BN/V
j (ERGS · SR^{-1} Hz^{-1} CM^{-3})
0.1
1.0
0.1
1.0
10
ν/ν_C

Figure 1



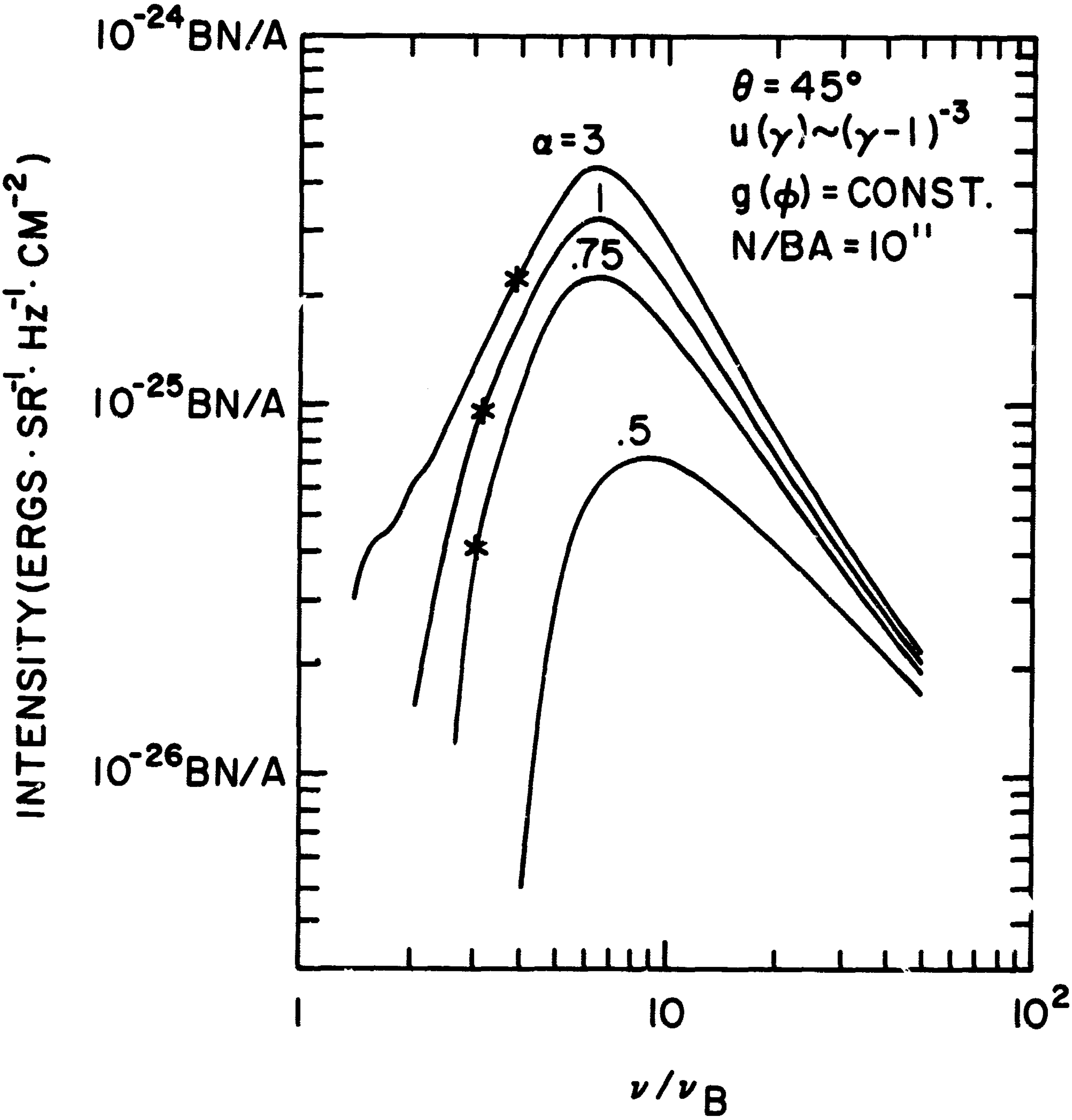
K (CM⁻¹)
10⁻¹⁰ N/BV
10⁻¹¹ N/BV
10⁻¹² N/BV
θ = 45°
j−
i+
K−
K+
a=0.5
a=3
ν / ν_B
10
10²
1
10⁻²³ BN/V
10⁻²⁴ BN/V
10⁻²⁵ BN/V
10⁻²⁶ BN/V
10⁻²⁷ BN/V
j (ERGS·SR⁻¹ Hz⁻¹ CM⁻³)

Figure 2



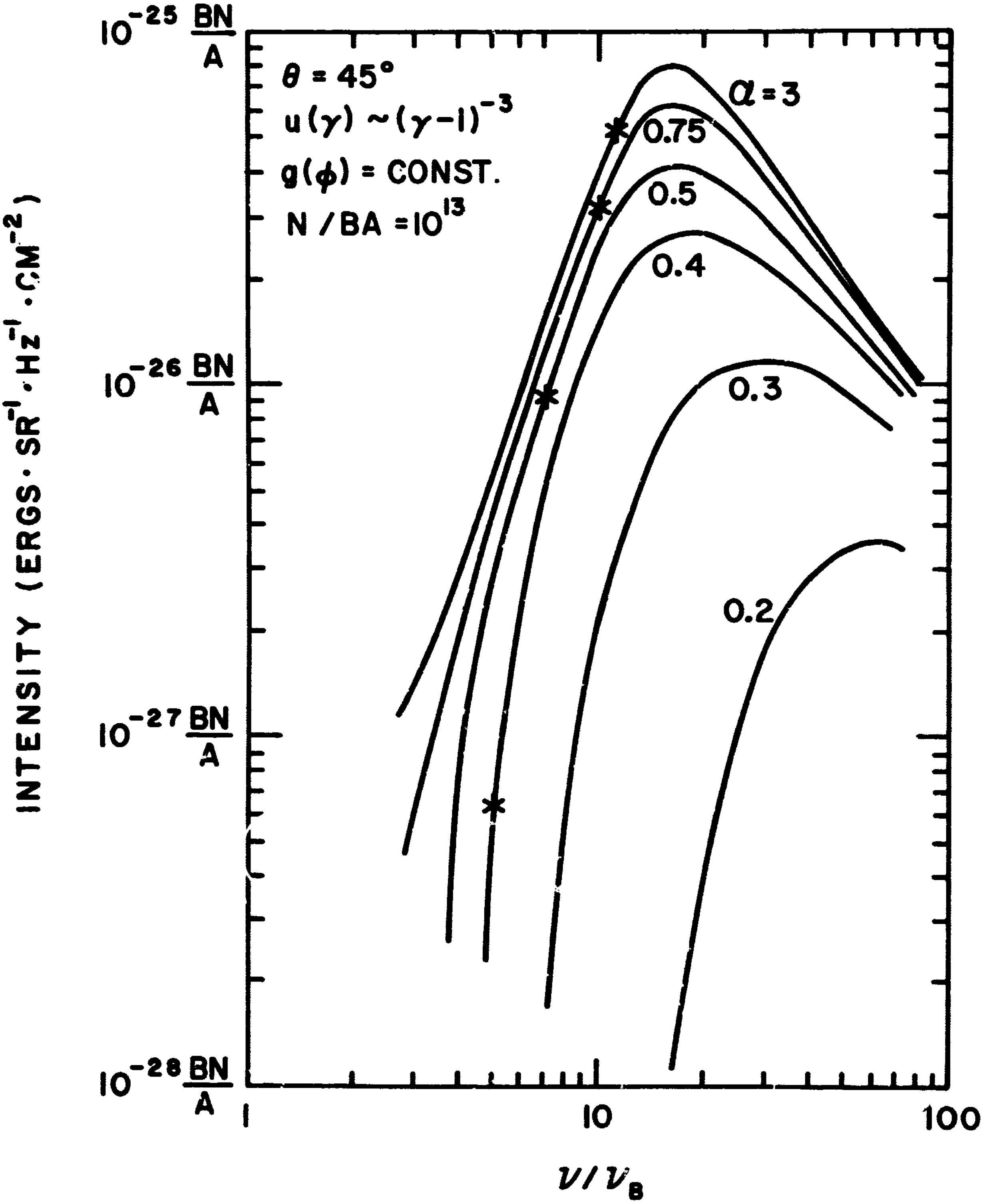
θ = 45°
u(γ)~(γ-1)^-3
g(φ) = CONST.
N/BA = 10^9
α = 3
0.75
0.5
0.4
10^-24 BN/A
10^-25 BN/A
10^-26 BN/A
INTENSITY (ERGS · SR^-1 · Hz^-1 · CM^-2)
1
10
10^2
ν / ν_B

Figure 3



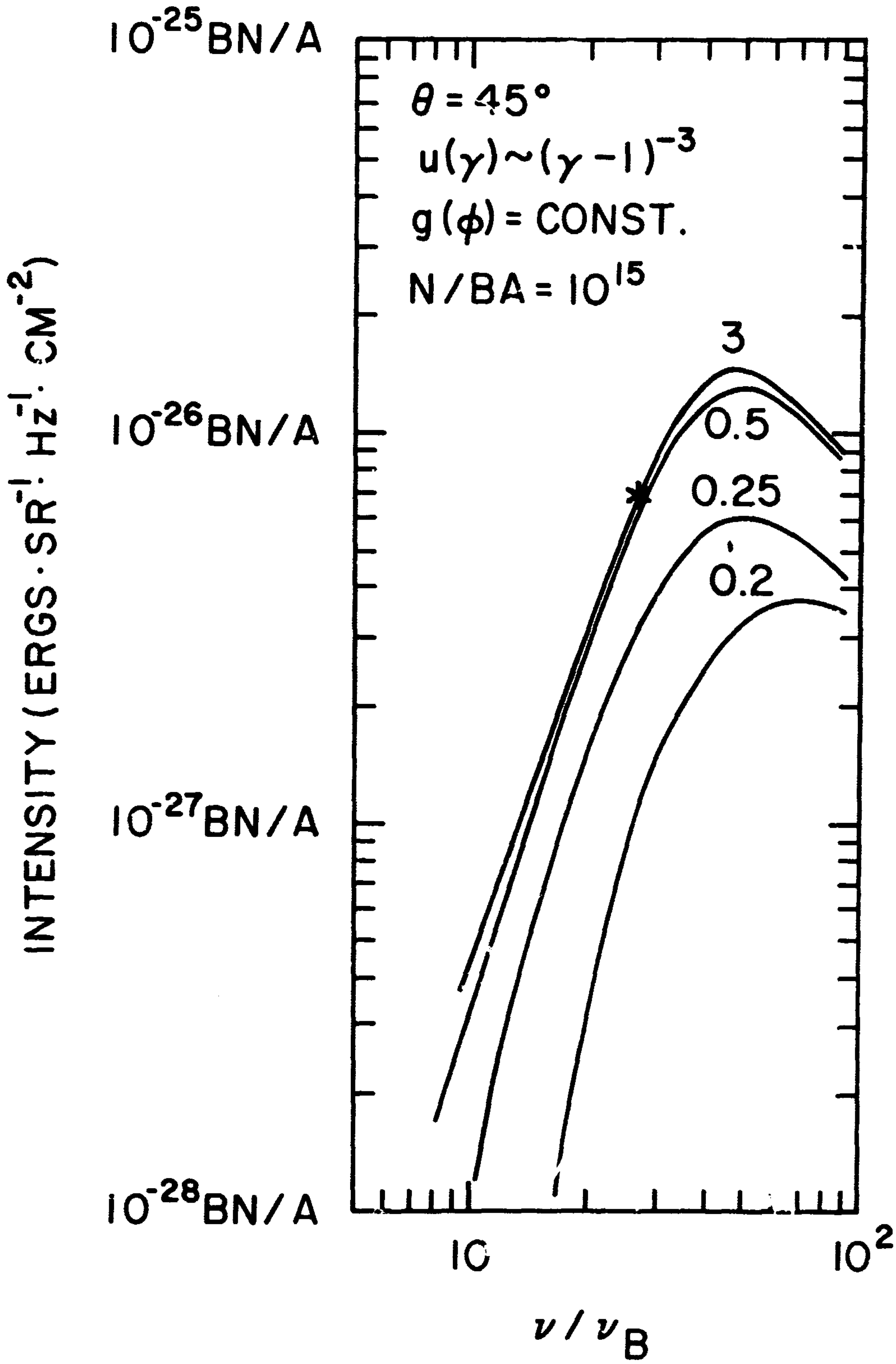
10^{-24}BN/A
10^{-25}BN/A
10^{-26}BN/A
INTENSITY(ERGS · SR^{-1} · Hz^{-1} · CM^{-2})
$\theta = 45°$
$u(\gamma) \sim (\gamma-1)^{-3}$
$g(\phi)$ = CONST.
N/BA = 10^{11}
$\alpha = 3$
1
.75
.5
1
10
10^2
ν/ν_B

Figure 4



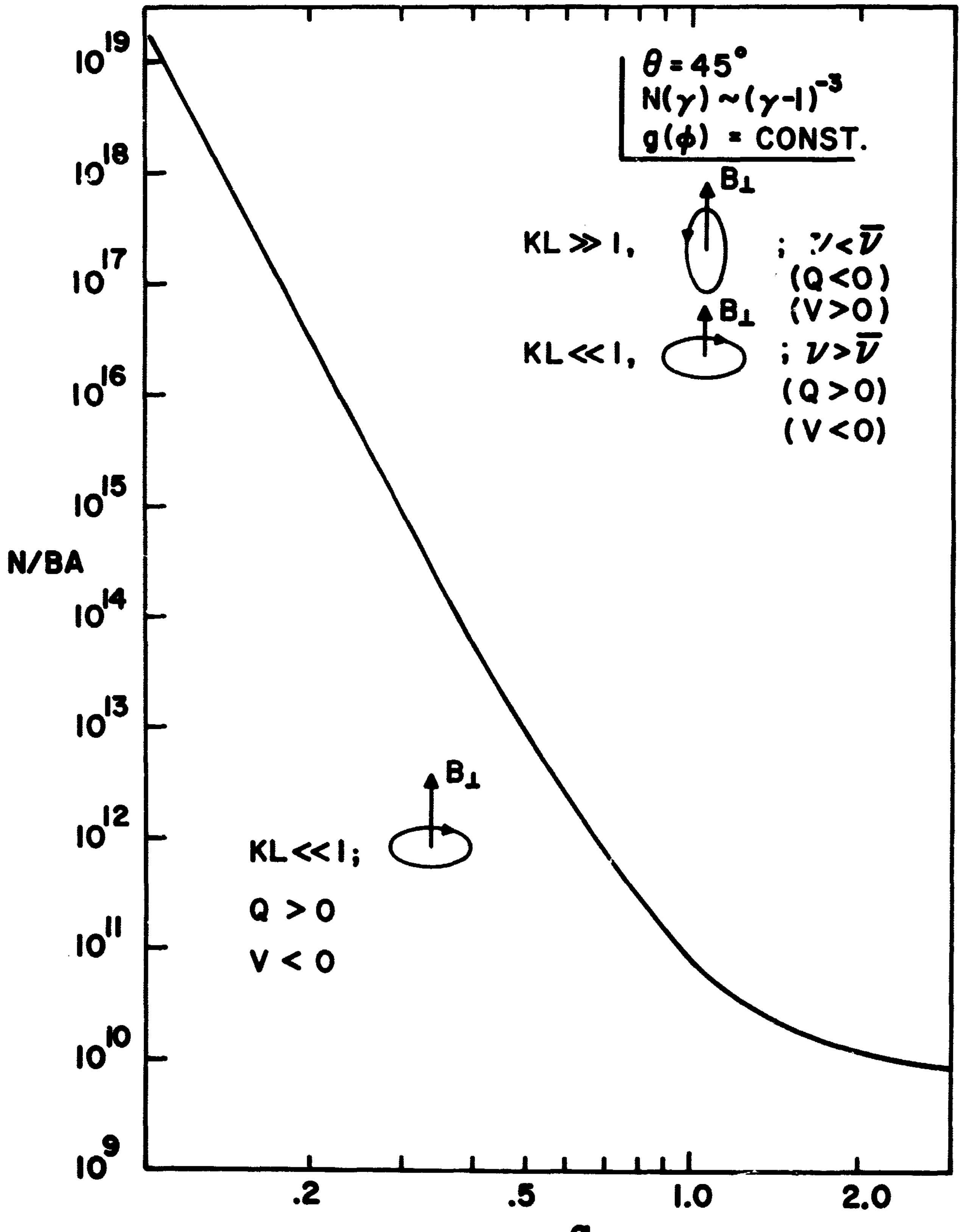
$10^{-25}\frac{BN}{A}$
$10^{-26}\frac{BN}{A}$
$10^{-27}\frac{BN}{A}$
$10^{-28}\frac{BN}{A}$
INTENSITY (ERGS · SR^{-1} · Hz^{-1} · CM^{-2})
$\theta = 45°$
$u(\gamma) \sim (\gamma-1)^{-3}$
$g(\phi)$ = CONST.
$N/BA = 10^{13}$
$\alpha = 3$
0.75
0.5
0.4
0.3
0.2
1
10
100
ν/ν_B

Figure 5



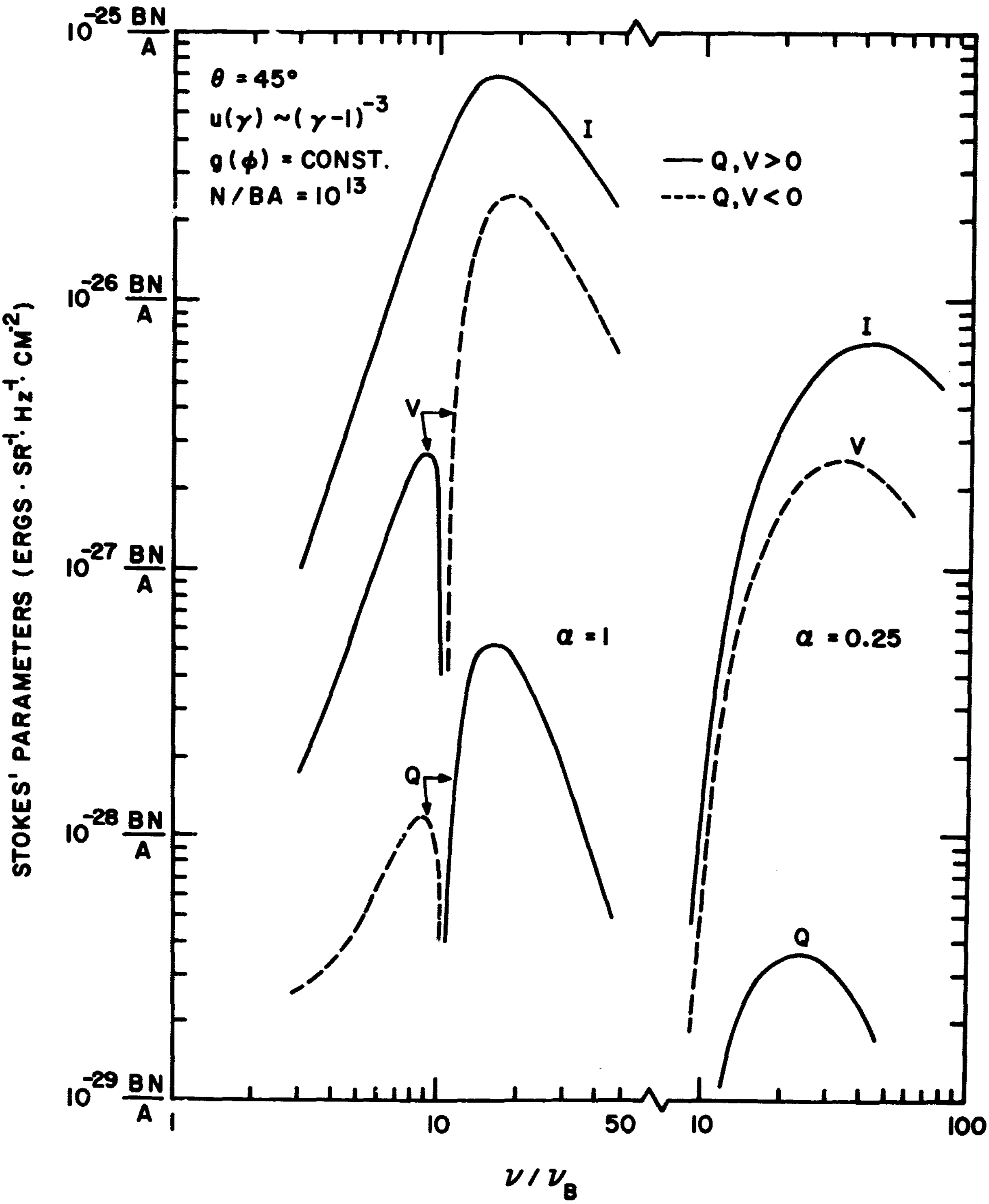
10-25BN/A
10-26BN/A
10-27BN/A
10-28BN/A
INTENSITY (ERGS · SR-1 · Hz-1 · CM-2)
θ = 45°
u(γ) ~ (γ − 1)-3
g(φ) = CONST.
N/BA = 1015
3
0.5
0.25
0.2
10
102
ν / νB

Figure 6



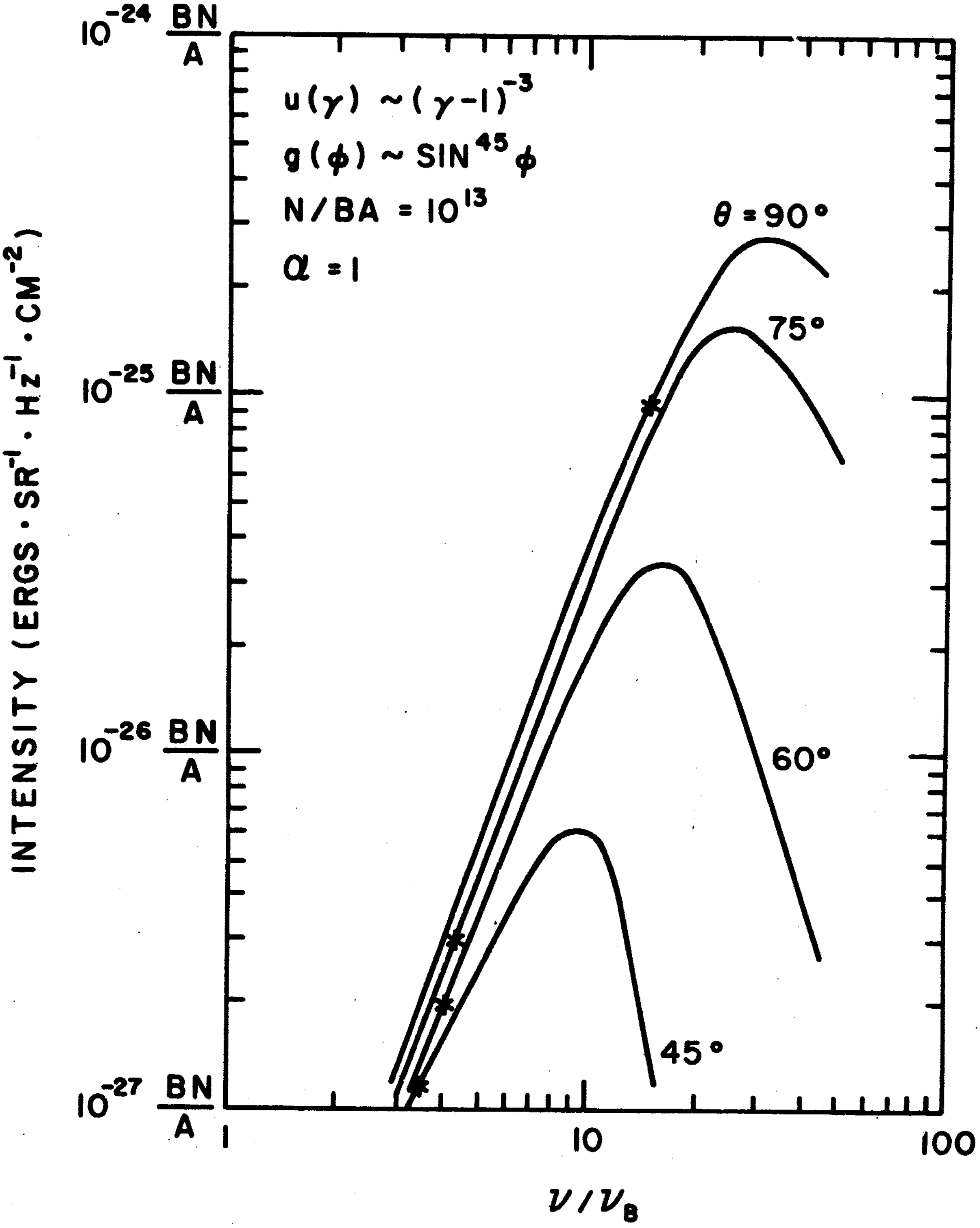
θ = 45°
N(γ) ~ (γ-1)^-3
g(φ) = CONST.
B⊥
KL ≫ 1,
; ν < ν̄
(Q < 0)
(V > 0)
B⊥
KL ≪ 1,
; ν > ν̄
(Q > 0)
(V < 0)
B⊥
KL << 1;
Q > 0
V < 0
N/BA
10^19
10^18
10^17
10^16
10^15
10^14
10^13
10^12
10^11
10^10
10^9
.2
.5
1.0
2.0
α

Figure 7



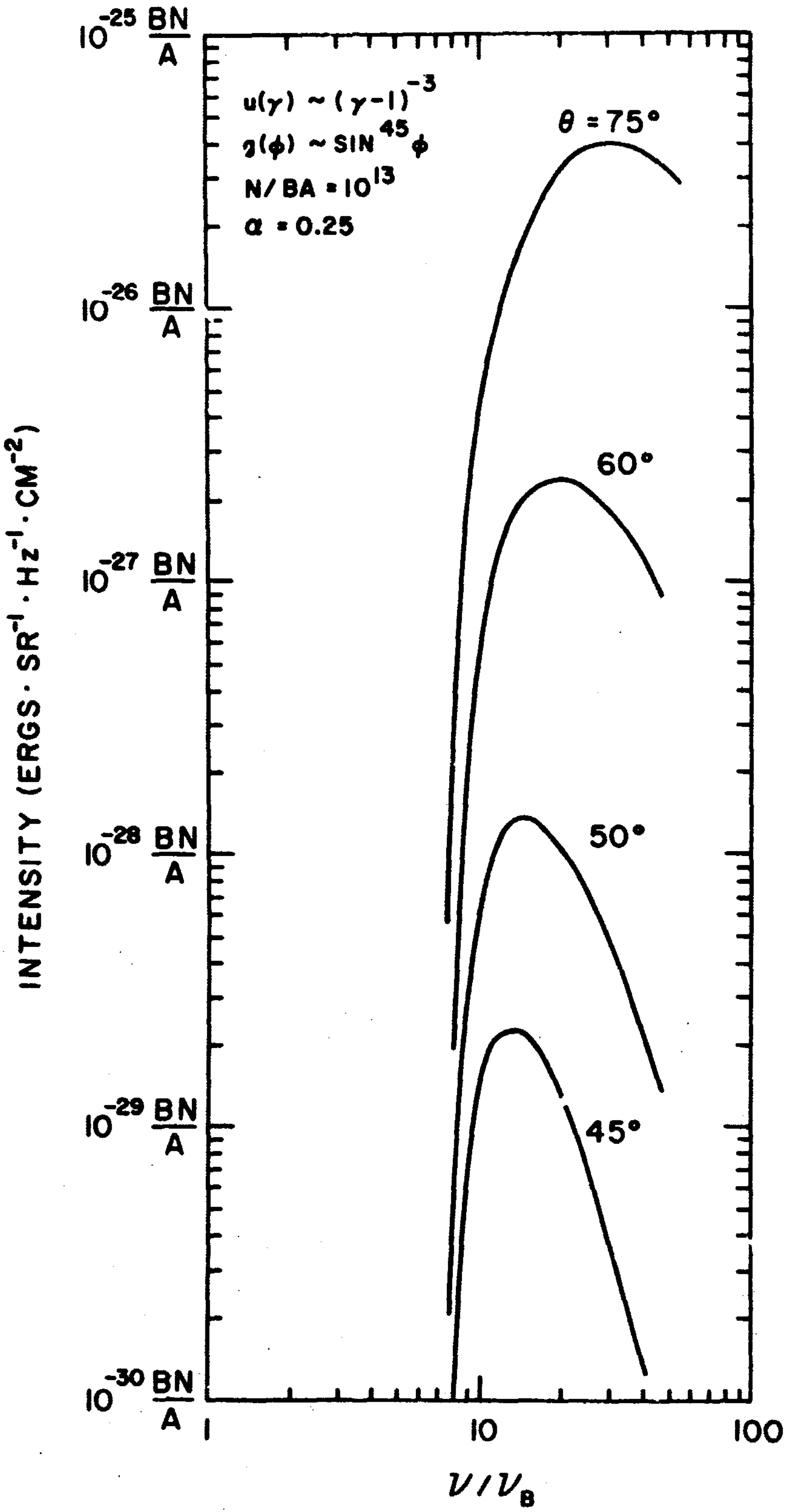
$10^{-25}\frac{BN}{A}$
$10^{-26}\frac{BN}{A}$
$10^{-27}\frac{BN}{A}$
$10^{-28}\frac{BN}{A}$
$10^{-29}\frac{BN}{A}$
STOKES' PARAMETERS (ERGS · SR^{-1} · Hz^{-1} · CM^{-2})
$\theta = 45°$
$u(\gamma) \sim (\gamma - 1)^{-3}$
$g(\phi)$ = CONST.
$N/BA = 10^{13}$
— Q, V > 0
---- Q, V < 0
I
V
Q
$\alpha = 1$
$\alpha = 0.25$
1
10
50
10
100
ν / ν_B

Figure 8



10^{-24} BN/A
10^{-25} BN/A
10^{-26} BN/A
10^{-27} BN/A
INTENSITY (ERGS · SR^{-1} · Hz^{-1} · CM^{-2})
$u(\gamma) \sim (\gamma-1)^{-3}$
$g(\phi) \sim \mathrm{SIN}^{45}\phi$
N/BA = 10^{13}
α = 1
θ = 90°
75°
60°
45°
1
10
100
ν/ν_B

Figure 9



$10^{-25}\,\frac{BN}{A}$
$10^{-26}\,\frac{BN}{A}$
$10^{-27}\,\frac{BN}{A}$
$10^{-28}\,\frac{BN}{A}$
$10^{-29}\,\frac{BN}{A}$
$10^{-30}\,\frac{BN}{A}$
INTENSITY (ERGS · SR^{-1} · Hz^{-1} · CM^{-2})
$u(\gamma) \sim (\gamma-1)^{-3}$
$g(\phi) \sim \mathrm{SIN}^{45}\phi$
$N/BA = 10^{13}$
$\alpha = 0.25$
$\theta = 75°$
60°
50°
45°
1
10
100
ν/ν_B

Figure 10

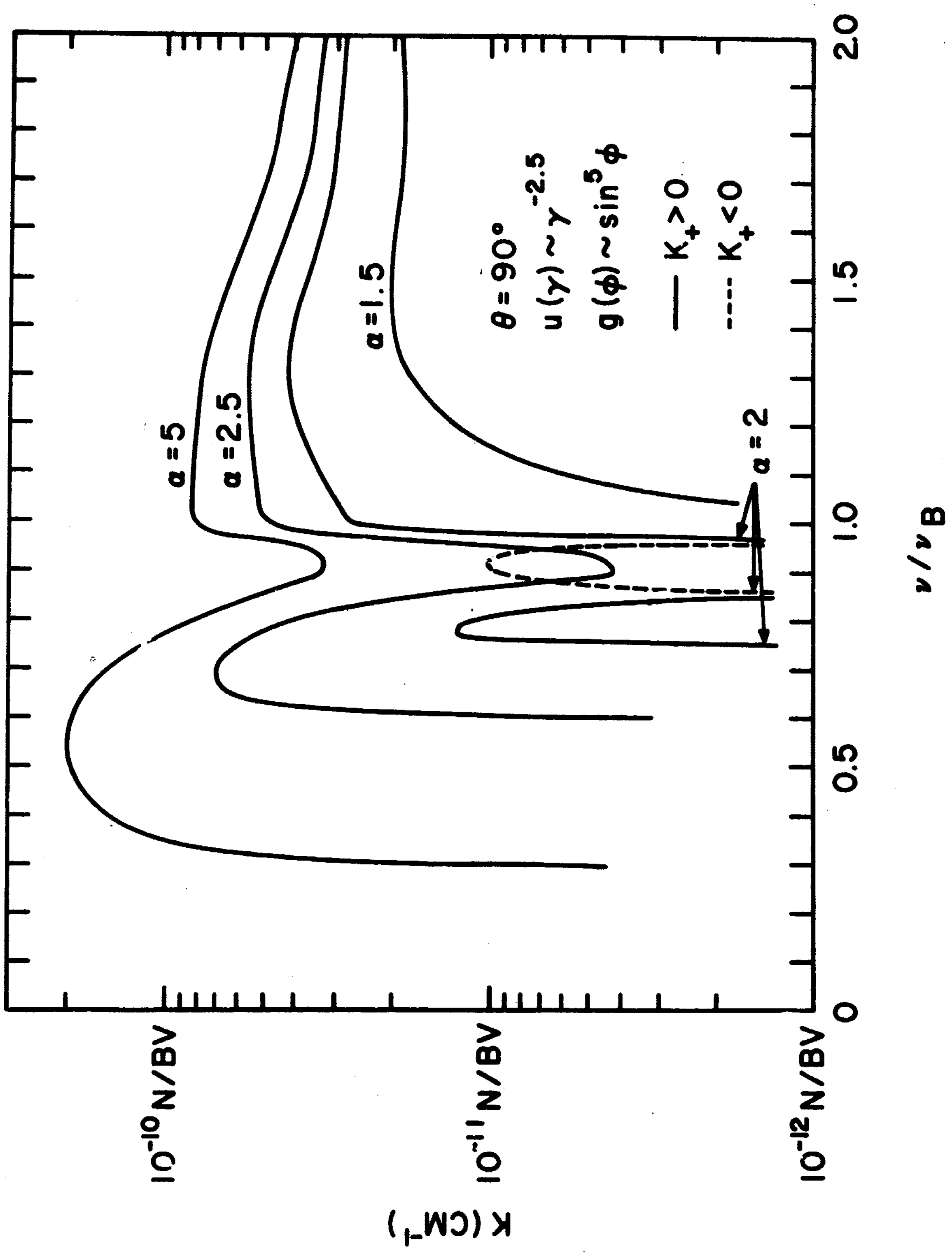
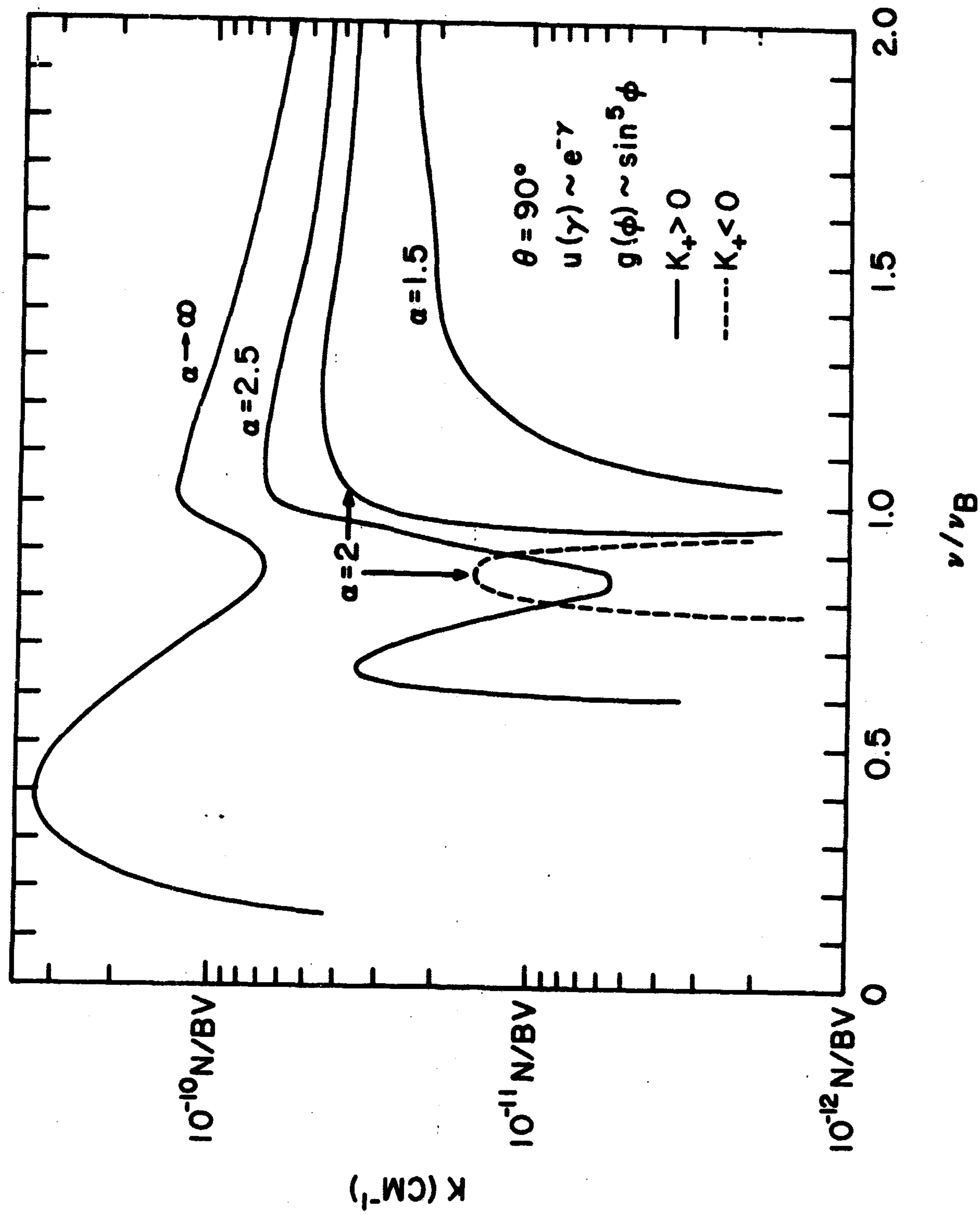
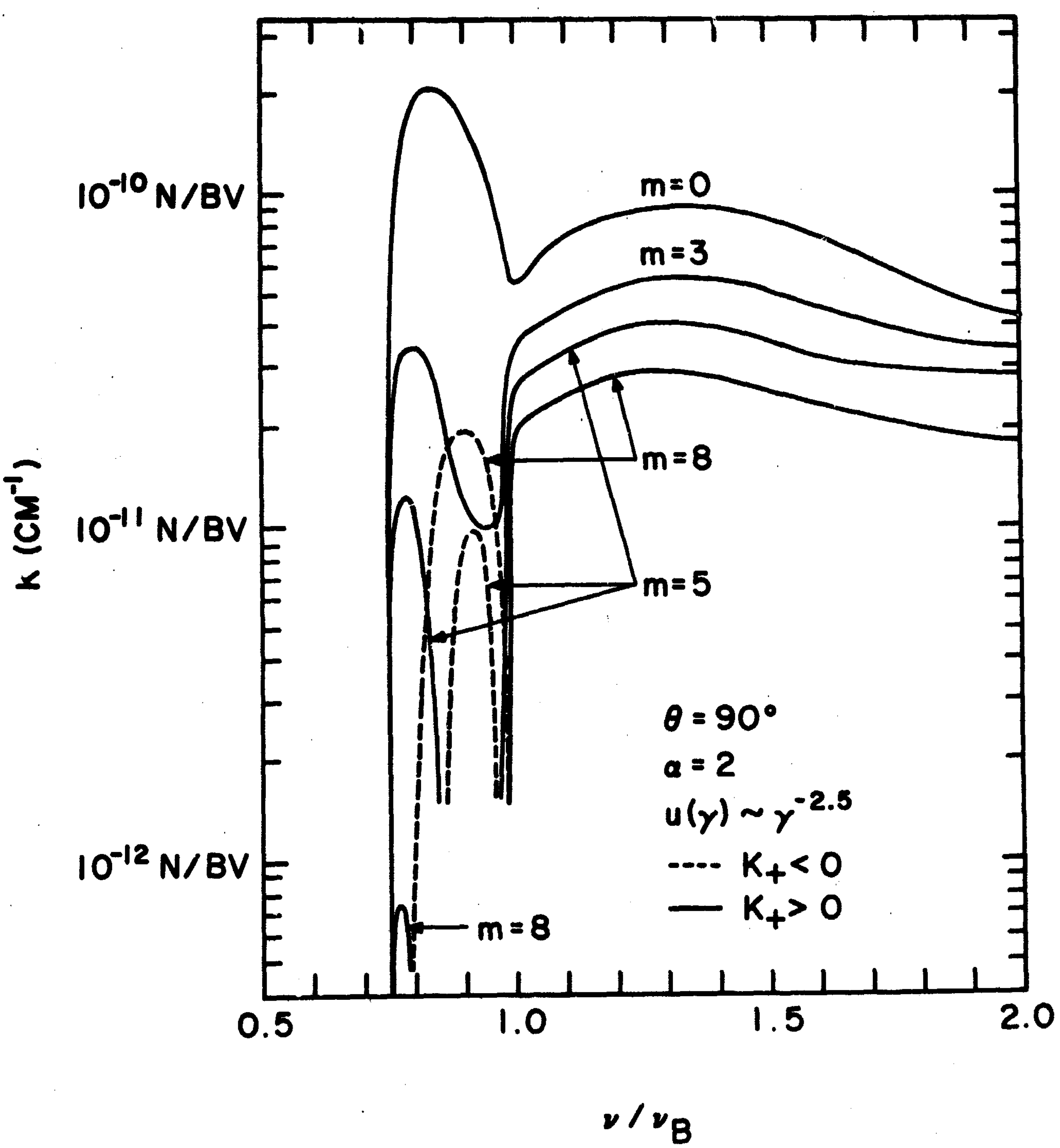

α = 5
α = 2.5
α = 1.5
α = 2
θ = 90°
u(γ) ~ γ^-2.5
g(φ) ~ sin^5 φ
— K+ > 0
---- K+ < 0
0
0.5
1.0
1.5
2.0
ν/ν_B
10^-10 N/BV
10^-11 N/BV
10^-12 N/BV
K (CM^-1)

Figure 11



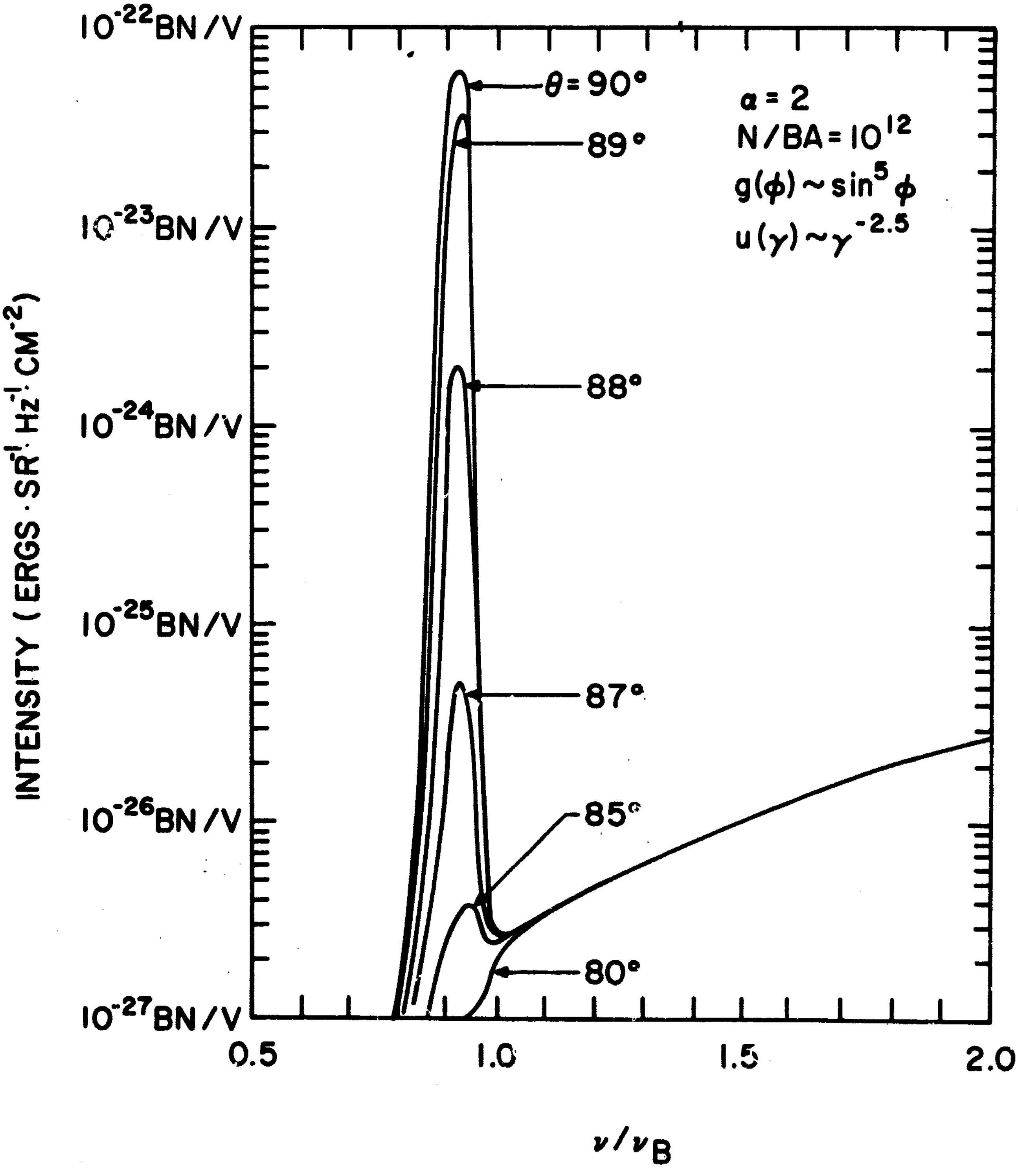
$\alpha \rightarrow \infty$
$\alpha = 2.5$
$\alpha = 2$
$\alpha = 1.5$
$\theta = 90°$
$u(\gamma) \sim e^{-\gamma}$
$g(\phi) \sim \sin^5 \phi$
$K_+ > 0$
$K_+ < 0$
10^{-10} N/BV
10^{-11} N/BV
10^{-12} N/BV
K (CM^{-1})
0
0.5
1.0
1.5
2.0
ν / ν_B

Figure 12



10^{-10} N/BV
10^{-11} N/BV
10^{-12} N/BV
κ (CM^{-1})
m=0
m=3
m=8
m=5
m=8
θ = 90°
α = 2
$u(\gamma) \sim \gamma^{-2.5}$
---- $K_+ < 0$
— $K_+ > 0$
0.5
1.0
1.5
2.0
ν / ν_B

Figure 13



10^{-22}BN/V
10^{-23}BN/V
10^{-24}BN/V
10^{-25}BN/V
10^{-26}BN/V
10^{-27}BN/V
INTENSITY (ERGS·SR^{-1}·Hz^{-1}·CM^{-2})
θ=90°
89°
88°
87°
85°
80°
α = 2
N/BA = 10^{12}
$g(\phi) \sim \sin^5\phi$
$u(\gamma) \sim \gamma^{-2.5}$
0.5
1.0
1.5
2.0
ν/ν_B

Figure 14