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The Conceptual Design of a Small Solar Probe
(Sunblazer)
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## THE CONCEPTUAL DESIGN OF A SMALL SOLAR PROBE (SUNBLAZER)

## A3STRACT

A scientific experiment to measure the electron density of the solar corona will use apparatus consisting of a small, solar-pressure-oriented spacecraft containing a multiple-frequency, pulsed, 2-kilowatt, solid-state transmitter, and a terrestrial $50-\mathrm{dB}$ dipole phased array receiving antenna.

Included are analyses of the communication system, the $0.63-\mathrm{AU}$ perihelion solar orbit, the semi-passive attitude control system, the spacecraft power and thermal control systems.

The conceptual electrical and mechanical design of the spacecraft hardware covers the basic configuration, RF and digital electronics design and packaging; and includes fundamental stabilization, separation, despin and deployment schemes.

4


Plate 1 Sunblazer Spacecraft


Plate 2 Launch configuration -- front view.


Plate 3 Launch configuration -- rear view.

## TABLA: OF CONTL:NTS

Chapter Page
! INTRODUCTORY REVIEW OF THF SUNBIAZER IROGRAM
1.1 Mission Considerations ..... 1
1.1.1 Scientific Mission Objectives ..... 1
1.1.2 Other Flight I'rogram Responsibilities. ..... 1
1.1.3 Future On-Board Experiments ..... 2
1.1.4 Missions and Experiments for a Ten-Year !rogram. ..... 3
1.2 Propagation and Communication Considerations. ..... 3
1.2.1 The Choice of 75 MHz Frequency ..... 3
1.2.2 The Propagation Experiment. ..... 4
1.2.3 Communication Bit-Kates ..... 5
1.2.4 Tracking Kequirements ..... 6
1.3 The Ground Receiving Terminal ..... 6
1.3.1 Initial Proposal. ..... 6
1.3.2 High-Gain Elements ..... 7
1.3.3 Change to Closely-Packed "Single" Frequency Experiment ..... 8
1.3.4 Additional Advantages of the $50-\mathrm{dB}$ Broadband Array ..... 10
2 CHANNEL CHARACTERIZATION
2.1 Time Delay ..... 13
2.2 Time Dispersion ..... 14
2.3 Frequency Dispersion ..... 18
2.4 Signal Selection ..... 19
3 THE SUNBLAZER ORBIT
3.1 Basic Orbital Objective ..... 21
3.2 Relative Motions of Spacecraft a:rd Earth ..... 21
3.3 Evaluation of Possible Orbits ..... 22
3.4 The Generation of Three Superior Conjunctions ..... 23
3.4.1 Influence of Launch Time on the Retrograde Motion ..... 25
Chapter TABLF: OF CONTH:NTS (Cont.) linge
3.4.2 In-Plane firrors. ..... 25
3.4.3 Eiscape Ve'ocity Firror ..... 26
3. © Computer-Generated Numerical and Graphical Orbit Information. ..... 27
COMMUNICATION SYSTE:M
4. 1 System liequirements and Constraints. ..... 8!
4.2 Functional Description of the Communication System ..... 90
4. 3 Signal Mesign ..... 92
4.3.1 Main Transmitter ..... 93
4.3.1.1 Pulse Einvelope Duration ..... 93
4. 3. 1.2 Peak Power ..... 95
4.3.1.3 Int rapulse Coding ..... 96
4.3.1.4 Telemetry Modulation ..... 98
4,3.2 Beacon Transmitter ..... 100
4.3.2.1 Pulse Finvelope ..... 100
4.3.2.2 Telemetry Modulation ..... 102
4. 4 Signal-to-Noise Hatio ..... 103
4. 4. 1 Main Communication Link. ..... 103
4.4.2 Beacon Communication Link ..... 104
4. 5 System Performance and Measurement Accuracy ..... 107
4.5.1 The Correlation Receiver ..... 107
4.5.2 Time of Arrival Measurement Accuracy ..... 110
4.5.3 Main Telemetry ..... 115
4:5.4 Beacon Telemetry. ..... 118
4.6 Spacecraft Programer. ..... 121
4.6.1 Functional Description ..... 121
4.6.2 Logic Design. ..... 121
4.6.3 Pulse Format ..... 123
4.6.4 Hardware Implementation ..... 125
5 GROUND AKRAY
5.1 Function of the Ground Array ..... 127
5.1.1 Relationship to the Sunblazer Spacecrait ..... 127
5.1.2 Relationship to the Correlation Receiver ..... 128
5. 1. 3 Other Uses for the Ground Array ..... 128
5. 2 System Considerations ..... 129
5.2.1 Antenna Gain and Number of Elements ..... 129
5.2.2 Frequency Sensitivity ..... 131
5.2.3 Noise Considerations ..... 133
Chapter TABI.t: OF CONTENTS (Cont.) Page
5.3 Wencription of the Vroponed Arraya ..... 136
5.3.1 The Indi.idual Dipole tilement ..... 136
3.3.2 The itx-1)tpole, 14-dB Antenna filement ..... 136
5.3.3 The Pilot Array ..... 139
5.3.3.1 General Description ..... 139
5. 3.3.2 Fitectronics for Pilot Array ..... 141
5.3.3.3 Testing Col, iderations ..... 143
5. 3.4 The tixpanded Array ..... 146
5.3.4.1 General Commente ..... 146
5.3.4.2 Fxpanded 40-dB Array Electronics ..... 146
5.3.4.3 Testing. ..... 149
5.3.5 The 50-di3 Array ..... 149
6 SPACECRAFT CONSTRAINTS
6.1 Distribution of the Payload Weight ..... 15!
6. 2 Dimensional Limitations. ..... 153
6. 3 Spacecraft - Adapter Interface ..... 153
6, 3.1 Attachment Method ..... 153
6.3.2 Despin - Deployment Actuator ..... 156
6. 4 Environmental Testing Prior to June 1968 ..... 156
6.4.1 Vibration ..... 156
6. 5 Structural Analysis ..... 158
6.5.1 Introduction ..... 158
6.5.2 Calculations ..... 159
6.5.3 Discussion ..... 163
7 DESCRIPTION OF THE SPACECRAFT
7.1 Platform-Radiator Sub-Assembly ..... 165
7.1.1 Fabrication ..... 165
7. 1.2 Area Avallable for Solar Cells ..... 168
7.1.3 Platform-Radiator Drawings. ..... 168
7. 2 The Electronics Sub-Assembly ..... 168
7.2.1 The Hub ..... 171
7.2.2 Hub and Module Drawings ..... 171
7.2.3 The Compartment Shield ..... 173
7.2.4 The Vertical Member ..... 173
7.2.5 Cover ..... 173
7.2.6 Plating Specifications and Process ..... 174
7.2.7 Electronics Sub-Assembly Critical $P_{i}$ is Tests ..... 174

## TABlA: OF CONT:NTS (Cont.)

'rage
7. 3 Salla ..... 177
7.3.1 Gall Irlve Mechanimm ..... 178
7,4 The Sparecrafte Mechanical Sequence at Injection ..... 179
7.4.1 Dosalble: Separation Dynamics Compensation Hequirement: ..... 181
7.5 Weighe Distribution of the Spacecraft ..... 183
\%. $\%$ Moments of Inerita ..... 184
ATTITLDE: CONTHOL SYSTEM
H. 1 Introduction ..... 187
H. 1.1 Oflentation of Spacecraft vithin $10^{\circ}$ of Sun ..... 188
8. 1. 2 Spin Rate of Spacecrath Between Two Limits. ..... 188
8. 1.3 Simplicity of Control System. ..... 189
8. 2 Solar Pressure Stabilization ..... 90
8.3 Basic Stabilization Syatems ..... 192
8. 4 Specici Sunblazer Conditions ..... 193
8.4.1 Initial Conditions ..... 193
8.4.2 Tracking Requirements. ..... 194
8,4,3 Orbit Condition: ..... 194
8.4.4 Weight and Volume Requirements ..... 194
8. 5 Basic Description of Vanes System. ..... 195
8, 6 Filementary Dynamics. ..... 198
8.7 Vector Dynamics ..... 201
8. 8 Vane Torques ..... 207
8.9 Dynamical Motion of Spacecraft ..... 215
8.9.1 Actual Results ..... 215
8.9.2 Optimization of Vanes' Cant Angle. ..... 220
8.9.3 Sensitivity to Initial Conditions ..... 229
8. 1.4 Parametric Investigation of Long-Term Motion. ..... 229
8.9.5 Nominal Conditions ..... 229
8.9.6 Possible Pitfalls in Motion ..... 237
8. 10 Ancillary Problems. ..... 239
8. 11 Nutations. ..... 240
8. 1. 1 Convention Used to Distinguish Nutational from Precessional Modes ..... 240
8.11.2 General Dynamical Equations for an Axisymmetric Body and Conversion into the Standard Notation. ..... 241
8. 11.3 Spacecraft Motion in Terms of Energy ..... 244
8. 11.4 Nutational Amplitude .....  247
8.11. 5 Reradiative Damping ..... 250

## TAlBIA: (OF CONTH:NTS (Cont.)

l'age
8. 12 Heading and Cone Angle Change (Launch through Depluy- ment) ..... 252
8.12.1 Int roduction ..... 252
8.12.2 Analysis ..... 252
8.12.3 Results ..... 264
8.12.4 Discussion ..... 264
$!$
THF:RMAI. BALANCE:
3.1 Discussion ..... 267
9. 2 Requirements of Spacecraft Thermal Design ..... 267
!. 3 Problems ..... 268
9. 4 Solutions ..... 270
!3.4.1 Thermal Analysis ..... 270
9.4.2 Sail Temperature ..... 273
9.4.3 Materials ..... 273
9.4.3.1 Surfaces. ..... 273
9.4.4 Other Considerations ..... 277
3.4.5 Results ..... 278
9.5 Summary ..... 278
9.6 Spacecraft Thermal Test ..... 278
10 ASPECT SENSOR AND CONTROL
10.1 The Detector Mechanical Configuration ..... 281
10.1.1 The Detector Design ..... 283
SUNBLAZFR SPACECRAFT POWER SYSTEM
11.1 Introduction ..... 287
11.2 System Description ..... 287
11.2.1 Solar-Cell Panels ..... 289
11.2.2 Power Converters ..... 290
11.2.2.1 "L" Converter ..... 290
11.2.2.2 "X" Converter ..... 291
11.3 Energy Storage Tests ..... 296
11.3.1 Interpretation of a Typical Pulsed Power Test ..... 298
11.3.2 Capacitor Vacuum Test ..... 303
11.3.3 Internal Pressure Relief Test. ..... 303
11.4 Power System Summary ..... 303

## TABLE OF CONTENTS (Cont.)

RF SURSYSTEM
12.1 RF Subsyatem Functional Description . . . . . . . . . 305
12.2 Generation of the Three Discrete Frequencies . . . . . . 311
12.3 RF and de Power Irofile of the 2 kW Transmitter. . . . . 313
12.4 Circuit Intormation . . . . . . . . . . . . . . . . 313
12.4.1 5 MHz Crystal Oscillator . . . . . . . . . . 315
i2.4.2 215 Frequency Multiplier . . . . . . . . . . 317
12.4.3 10V Bias Keying Network . . . . . . . . . . 318
12.4.4 N-Way Power Divider or Combinei Networks . . 318
12.4.5 Basic Amplifiers . . . . . . . . . . . . . . 320
12. 4. 6 3-dB Quadrature Hybric Circuits . . . . . . . . 323
12.5 Construction Techniques . . . . . . . . . . . . . . 324
12.6 The 50-Watt Beacon 'Transmitter . . . . . . . . . . . 327
12.7 The Spacecraft Integrated Antenna System . . . . . . . 329
12.7.1 Introduction. . . . . . . . . . . . . . . . 329
12.7.2 Main Transmitter Antenna . . . . . . . . . . 331
12.7.3 Beacon Transmitter Antenna . . . . . . . . . 348
12.7.4 Mutual Coupling . . . . . . . . . . . . . . 350

REFERENCES . . . . . . . . . . . . . . . . . . . . . . . . . 355

## APPENDICES

1 Analysis of Optimal Energy Storages
2 Power Requirements and Self-Optimizing Electronic Power Supply of a Small Solar Probe
3 Thermal Transfer and Radiation from a Thin Circular Plate Source--Thin Cylindrical Shell Radiator

## IIS'T OF゙ ILILCSTRATIONS

Figur" Pagi"
2-1 Additional delay $\mathrm{T}_{\mathrm{d}}(75)$, relative delay $\mathrm{T}_{\mathrm{d}}\left(f_{1}\right)-\mathrm{T}_{\mathrm{d}}\left(f_{2}\right)$, and standard deviation of delay $\sigma_{d}(75)$, vs palh offset ${ }^{d} 2^{2}$. . ..... 15
2-2 Dispersion time $\tau$, and reciprocal frequency spreading $1 / B$, vs path offset. Frequency: 75 MHz ..... 17
3-1 Motion of Sunblazer relative to the earth ..... 24
3-2 Solar encounter profile, January launch ..... 28
3-3 Solar encounter profile. Fehruary launch ..... 29
3-4 Solar encounter profile, March launch ..... 30
3-5 Solar encounter profile, April launch ..... 31
3-6 Solar encounter profile, May launch ..... 32
3-7 Solar encounter poufile, June launch ..... 33
3-8 Solar encounter profile, July launch ..... 34
3-9 Solar encounter profile, August launch ..... 35
3-10 Solar encounter profile, September launch ..... 36
3-11 Solar encounter profile. Octobes launch ..... 37
3-12 Solar encounter profile, November launch ..... 38
3-13 Solar encounter profile, December launch ..... 39
4-1 Energy ratio at .5 MHz (neglecting solar noise) ..... 105
4-2 Signal/noise (power) ratio "or beacon link. ..... 108
4-3 Maximum likelihood correlation receiver ..... 111
4-4 Form of sampled correlation function ..... 112
4-5 Central peak of autocorrelation function ..... 114
4-6 RMS measurement error as percentage of measured quantity ..... 116
4-7 Error in measurement of pulse duration ..... 119
5-1 Beam steering by progressive phase shifts ..... 132
5-2 Noise model. ..... 134
5-3 Individual dipole element ..... 136
5-4 Six-dipole configuration ..... 137
5-5 Mercury-switched phased loops (schematic). ..... 138

## 1.IST OF ILIISTRATI(ANS (cont)

5-6 192-dipole array ..... 146
5-7 Electronics system block diagram ..... 142
5-8 Time delay circuit ..... 144
5-9 Expanded 40-dB array ..... 147
5-10 Block diagram of phase-shifting and signal-summing electronics ..... 148
5-11 Propesed plan for 50-dB array ..... 150
6-1 Solar probe performance using Scout fifth stage - Wallops Station ..... 152
6-2 Location of spacecraft in $\mathbf{- 4 0}$ heat shield ..... 154
6-3 Axial alignment of spacecraft ..... 155
6-4 Spacecraft mounting. ..... 160
6-5 Primary structure model (box beam). ..... 161
6-6 Bnx beam section ..... 162
7-1 Platform-radiator sub-assembly segment ..... 166
7-2 Assembly jig. ..... 167
7~3 Electronics sub-assembly ..... 169
7-4 Section view intraconnect ..... 172
7-5 Sail drive mechanism ..... 180
7-6 Schematic separation sensor ..... 182
7-7 Weight distribution ..... 185
8-1 Radiation pressure geometry ..... 191
8-2 Configuratior of stabilizing vanes ..... 196
8-3 Simple precession ..... 199
8-4 Force on vares ..... 200
8-5 Angular momentum ..... 202
8-6 Euler angles ..... 204
8-7 Vanes system ..... 208
8-8 Torques calculated for Sunblazer vanes ..... 216
8-9 Universal angle vs spin rate ..... 217
8-10 Initial spiral $-60^{\circ}, 20 \mathrm{r} / \mathrm{min}$ ..... 218
8-11 Initial $\theta$ and $\omega$ vs time ..... 219
8-12 Long-term $\theta$ and $\omega$ vs time. ..... 221
8-13a Long-term spirals (1) ..... 223
8-13b Long-term spirals (2) ..... 224
8-13c Longnterm spirals (3) ..... 225
8-13d Long-term spirals (4) ..... 226

## LIST OF II.I.ISTRATIONS (cont)

8-13e l.ong-term spirals (5) ..... 227
8-13f long-term mpirals (6) ..... 228
8-14 Optimization of cant angle - stabilization time ..... 230
8-15 Optimization of cant angle long-term cycles ..... 231
8-16 Stabilization time vs initial 0 ..... 232
8-17 Initial stabilization spirals - different initial conditions ..... 233
8-18 Stabilization time vs initial $\omega$ ..... 234
8-19 Iong-term number of cycles vs pitch error ..... 235
8-20 long-term cycles vs reradiation damping constant ..... 236
8-21 Long-term cycles vs initial spin rate ..... 238
8-22 Halfman's coordinates ..... 242
8-23 Nutation coordinates ..... 245
8-24 Sunblazer deployment ..... $25 ?$
8-25 Angular deviation during coasting period ..... ,7
8-26 Angular variations during despin ..... 60
8-27 Effects of sail deployment on spacecraft dynamics ..... 263
8-28 Resulting headings from separation, despin and deployme ..... 265
9-1 Sunblazer surface properties. ..... 271
9-2 Sunblazer equivalent thermal circuit ..... 272
9-3 Flectronic compartment temperature ..... 274
9-4 Thermal isolation ..... 275
9-5 Sail ..... 276
10-1 Aspect sensor detector ..... 282
10-2 Detector functional output ..... 284
10-3 Aspect sensor detector output format ..... 285
11-1 Power system description ..... 288
11-2 "L" converter ..... 292
11-3 "X" converter ..... 294
11-4 Capacitor descharge ..... 297
11-5 "L" converter pulsed power tests ..... 298
11-6 Capacitor power output vs duty cycle ..... 302
12-1 Sunblazer electronics system ..... 306
12-2 Spacecraft electronics system diagram ..... 307
12-3 Different intrapulse coding schemes ..... 310
12-4 Block diagram for generation of the ( $75 \pm 5$ ) MHz bandwidth ..... 312
12-5 . 5 MHz crystal oscillator ..... 316
12-6 5 MHz to 75 MHz frequency multiplier ..... 317

## TATMOH ILTHSTRATIONS (cont)

12-7 (omparison of the oatput waveforms of regalar and step recovery limes ..... 317
$12-8$ heying network ..... 318
12-6 Noway power-divider or combiner networks ..... $31!$
12-10 Transistor power vs frequenry curves ..... 321
12-11 Basic amplifies ..... 322
12-12 3-13 quadrature hybrid ..... 323
12-13 NF circuit packuging ..... 326
12-14a Block diagram of the beacon transmitter ..... 328
12-14b Beacon antenna switeh ..... 328
12-15 Spacecraft projection ..... 330
12-16 Determination of $\phi$ and $A$ for any point in space ..... 333
$12-17 \mathrm{am}$ C Normalized $|\vec{F}|$, $|\mathrm{H}|$, and $|\vec{S}|$ patterns ..... 338for I .100 cm and $\mathrm{f}=70 \mathrm{MHz}$ ..... 339
$f=76 \mathrm{MHz}$
$f=76 \mathrm{MHz}$ $\mathrm{f}=80 \mathrm{MHz}$ ..... 340
12-18 Normalized $|\vec{S}|$ patterns for $L_{1}=100 \mathrm{~cm}$ and $f=75 \mathrm{MHz}$ ..... 341
12-19 Interface between transmitter and its antenna. ..... 343
12-20a Radiation resistance as a function of rod length, rod spacing and frequency. ..... 344
12-20b Antenna reactance as a function of rod length, rod spacing and frequency ..... 345
12-21 Geometry and current distribution of a sail ..... 347
12-22 Evaluation of the vector potential due to a current strip of width w and height $\mathrm{dz} \mathbf{z}^{\prime}$. ..... 347
12-23 Normalized $|\vec{H}|$ and $|\vec{E}|$ patterns for a dipole constituted by a pair of tilted rods ..... 351

## LIST OF TA:3LES

Table Page
3-1 First-order computations ..... 22
5-I Signal frequency bands ..... 127
5-II Gain per element. ..... 130
5-III Design summary 29-dB pilot array ..... $13!$
5-IV Pilot array electronics ..... 145
$5-\mathrm{V}$ Design summary of $40-\mathrm{dB}$ array ..... 146
5- VI Design summary of $50-\mathrm{dB}$ array ..... 149
6-I Distribution of payload weight ..... 151
7-I Results of RFI shiclding tests ..... 174
7-II Test results on filtercons ..... 176
7-III Thread strength test results (averaged) ..... 176
7-IV Results: moments of inertia. ..... 186
9-I Sunblazer spacecraft thermal constants ..... 269
9-II Expected spacecraft temperatures for 3/4-year orbit ..... 278
11-I Nominal current and voltage output of Sunblazer array ..... 289
11-II "L" Converter parts list ..... 293
11-III " X " Converter parts list ..... 295
11-IV Capacitor test results ..... 301
12-I $\quad \mathrm{DC}$ to RF conversion table for the different sections of the transmitter ..... 314
12-II Basic parameters of beacon transmitter ..... 329

## CHADPER 1

### 1.0 INTRODHCTOHY HEVIEW OF THE SUNBLAZER HROZARAM

### 1.1 Mission Considerations

### 1.1.1 Scientific Mission Objectives

The primary ubjective is to measure the electron density !rofile accurately and unambiguously over the 5 -to- 100 solar radii distance from the sun, where it has been inferred only with considerable ambiguity from radio star occultation measurements.

A secondary ohjective is to measure, clearly, the scale of turbulence in the inner corona and the outward-moving velocity of the inhomogeneities in the inner corona by measuring the scintillations in angle and time of arrival of coherent transmissions as occultation progresses.

Other ubjectives are: to infer some qualitative information about the existence of a general solar magnetic field; and, through observaticn of Faraday rotation or pulse splitting, to carry a variety of particle and field experiments to the 0.52 AU region of interplanetary space.

### 1.1.2 Other Flight Program Responsibilities

There appear to be no other NASA flight programs which could, for either economical or technical reasons, satisfy all of the mission objectives of the Sunblazer program. These objectives are: primarily, to make electron density profiles in the region from 5 or so solar radii out to 100 solar radii; and, through the measurement of fluctuation phenomena observed on the ray paths near occultation, to determine the scale of turbulence in the inner corona; and perhaps, through Doppler-broadening measurements, to measure the velocity of the inhomogeneities in a general radial direction.

To accomplish these objectives, one should have spacecraft reaching perihelia of 0.52 to 0.65 AU so that occultations will occur in a reasonable time such as 1 to 1.5 years after launch. To the best of our knowledge,
nome of the Mafmery spacecraft will dothis. Similarly, the l'onereg mission, which might have gome inte this region, is no lomger planned. llowevart, even if it wetre, the loms-tern froquifenemt to make met omly a simply isolated orcoltation meastiroment but perhaps a sucoession of these. so that the highly variable coronal densities could be more accurately characteri:od, would underscore the importance of a relatively ine xpensive latuch vehicle and spareoraft for making such a serites of turastrenwonts.

## 1.1. is finture in-lBoard fixperiments


 Lnad ot 55 pounds into a 0.65 Al orbit. "The word "payload" in this vast" applies wall weight appended to the fifthestage rocket casing. 'Gere are at least 10 to 15 puunds of excessive telemetry which have been put aboard the 5-stage Scont vehicle as performance-testing telemetry which presumably could be removed on later launches. We would expect that this additional weight could he used for adoitional science on Sunblazer. A large mumber of sugkestions for quite meritorious experiments have beren proHused, relating mustly to particle and field experiments in the 0.52 to 0.65 Al region that Sunblazer will traverse. Dr. Van Allen has proposed a Geiger counter experiment to count energetic particle fluxes in this region. Dr. Simpson has proposed a cosmic ray experiment to determine the cosmic ray gradients in the region closer to the sun; this is also a simple counting experiment. MIT experimenters have propused placing an $x$-ray detector on Sunblazer, looking in an antisolar direction, to measure the $x$-ray sky background over a wide range of angles as Sunblazer orbits the sun, which would be a very rewarding experiment. A simplified plasma probe for measuring the solar wind density and velocity at 0.52 AU is also a possibility, and Dr. Shapiro has discussed with us the possibility of placing aboard Sunblazer a coherent $x$-band beacon so that his fourth test of general relativity could be undertaken. This is a paiticularly practical and attractive way to conduct Shapiro's experiment. Weight limitations clearly will not permit all of these being done on any one launch, but combinations, perhaps two at a time, could be done over a succession of launches over several years. It may also be possible to employ a flux gate magnetometer to measure magnetic field strength at 0.52 AU.

All of the experiments lisied are typically those that count events over a one-day per' ud and require the contents of a counter to be transmitted back once a day or so for the experiment to be accomplished. These are
quite suitable for the low-data-rate transmiseston system that Sunblazer will have.
1.1.t Missions and Experiments for a Ten-Year Program

It has always been felt that the great advantage of a light-weight, low-cost spacecraft for interplanetary missions is that it would offer the possibillty of repeated measurements of coronal densities and scale sizes through a major portion of a solar cycle. It secms feasible to plan tolaurh two spacecraft per year over a 10 -year period, with the basic propagation experiment being done on all of them, since this is essentially built moto the communications system on the spacecraft. Orbital choices and onboard experiments can vary over the time period, depending on the total weight available in each launch. It is believed that the first three mis sions should probably be 0.65 Al ones with conjunction occurring in 1.5 years. One might regard these as providing exploratory measurements of the corona and as proving out the design of the spacecraft itself. They would aiso provide, if all three were successful, the opportunity for making spatial and temporal correlations of the coronal densities at different azimuthal positions in the ecliptic. The 0.65 AC orbit provides a very slow passage through conjunction. It would be desirable to return to ou' original orbit having a perihelion at 0.52 AU and a rapid conjunction passage at one year after launch to make electronic density profi'e measurements which are more nearly fixed in time, and it is suggested that three 0.52 Al launches be made.

It is also possible, by causing the hyperbolic excess velocity after escape to be almost orthogonal to the ecliptic, to achieve an inclined orbit where the inclination might be as much as $10^{\circ}$. In this case an out-of-plane distance of perhaps 30 million kilometers could be obtained. A number of these launches could be considered partly to make propagation measurements on ray paths out of the ecliptic, but principally to carry on-board experiments significantly far from the ecliptic plane.

On all of these missions, the $0.52 \mathrm{AU}, 0.65 \mathrm{AU}$ and the out-of-the-ecliptic inclusion of two out of the five or six possible on-buard experiments could also be made, to provide the opportunity for a number of particle and field experiments.

### 1.2 Propagation and Communication Considerations

### 1.2.1 The Choice of 75 MHz Frequency

The arguments on chnice of frequency, and the belief that a frequency in
the 75 Mllf is bext to use, are complleated but these are the high pointe of the argument. If it is assumed that the two trequencies used in the experiment are a probing frequency and an infinite reference frequency. then the amount of relative delav betwern identical aignals tranmitted on these twocarriers varies inversely as the socond power of the fiequency. At 75 Mliz it is predicted the average corona will have a relative delay of $535 \mu \mathrm{~s}$ at 100 solar radit and about 10 ms at 5 solar radis

If, for the sake of reducing the complexity and cost of the pround equapment. and also th the process simplify.ng the spacecraft. one uses a reference frequency some $10^{\circ}$, greater than the probing frequency, then these delays are divided by a factor of 5 and berome 2 ms and $107 \mu \mathrm{~s}$. They are still readily mosurable by the pulse techniques which we propose for Sublazer where the effective pulse definition can be made of the order of $5 \mu \mathrm{~s}$. If a 150-M11z carrier is used, these delays become reduced by a factor of $t$ and become $500 \mu$ s and $25 \mu s$ which, while still measurable by the same methods, would suggest greatly reduced accuracy. Similarly, if the frequencies used were as high as 2200 MHz , the relative delays become ridiculously small and can no longer be measured by group delay methods.

A similar arpument applies to the important scintillations and Dopplerbroadening measurements. The angular scintillations at the 4 -meter wavelength are iairly readily measurable in the 5 to 20 solar radii range, whereas, with the higher frequencies they would become too small to measure accurately.

### 1.2.2 The Propagation Experiment

By now, many studies nave been made of the extent to which the turbulent coronal plasma would be expected to distort a pulse transmitted through either the frequency broadening which would arise from the motion of the turbulent refractive inhomugeneities or, correspondingly, from the delay . Iistortion which would occur in the time domain through the dispersive nature of the merium or through multipath. It has been concluded, on the basis of both theoretical studies and the examination of the data provided by radio star occiltation and by the Mariner 4 frequency-broadening measurements made during its occultation by the sun, that a 3 -ms pulse at 75 MHz has an excellent probability of being detected in as close as 5 solar radii, and further that this pulse cen be composed of $5-\mu s$ elementary pulses distinguished by phase-reversa! modulation. In effect, there would be a $5 \mu s$ resolution, and any multipath contributions would be identifiable, as well as those contributions arriving on different
 \&roupedelay method hws over the kind of sinusotdal noodulation enyployed tu the f'iomeer propmgetion experiment in thet the imerisdicity of that mudu* lation precludes the possithility of identifying maltipath and other perhaps unanticipated propagation modes, and makes cifficult. If not risky. the interpretation of the delay data.

The othe $r$, in our view, qreat deficiency of the Pionore experiment that the delay masurements are made on the u, link, for engineering conventence, but the interpretation must be done then by a relatively inept machine, and not by the scientific observer who is avaliable if downink transmissions are empoyed. We recognize, as dies anyone with space communication experience, that uplink sensitivities catalways be made supertor, but this is hardly important in vie $w$ of the grat advantage of gromm-liased interpretation if downlink transmissions can be made aderguate, as we belfeve they are in Sunblacer. In Pioneer, for example, all possibility of scintillation measurements, friquency-broadening measurements and Faraday-rotation measurements are excluded unless an extremely complicated receiver on the spacecraft is postulated. We are convinced from an engineering viewpoint that the MIT propagation measurement has been proved feasible.

### 1.2.3 Communication Bit-Raies

If the closely-packed $75-\mathrm{MHz}$ frequency-hoping experiment is the one we design for, then with the $45-$ to $50-\mathrm{dB}$ ground antenna, a bit-rate of approximately 1 bit per second can be expected with low-error probability when Sunblazer is 2 AU from the earth. Assuming that the spacecraft is visible at least two hours per day, this is a capability of 7200 bits, which is more than enough to communicate all of the engineering data such as temperature, battery voltage, orientation angle, etc., as well as the results of any simple on-board experiments. During the summer periods when the visibility is more likely to be six hours per day, a correapondingly greater amount of information can be telemetered. As a matter of interest, the Sunblazer bit-rate of 1 bit per second is a remarkably high bit-rate for such a simple spacecraft, when compared to the Mariner bit-rate of 8 bits per second using directive antennas on the spacecraft, 210 -foot antennas, the Goldstone maser receivers, and a much shorter transmission range of only 50 rillion miles.

### 1.2.4 Tracking Hequirements

Tracking and communications are not a problem if the normal NASA tracking nets are not used. The angle and Doppler information we qet on Sunhatar fam the phased array will be sufficiently accurate to establish the orbit with enough precision for the propagation experiment. For the propagation experiment, the only important parameter is the displacement of the say path from the solar center. The actual locations of the terminals, assuming that both are well outside of the region of greatest electron density, are unimportant. Evan so, from the angle and Doppler measurements we expect to be able to locate the spacecraft within $10,000 \mathrm{~km}^{3}$ with high reliability.

There is no uplink contemplated in the Sunblazer mission, and the modest amount of telemetry indicated will be multiplexed on the same $75-\mathrm{MHz}$ carrier emanating from the spacecraft.

The Ground Receiving Terminal

### 1.3.1 Initial Proposal

When the Sunblazer experiment was first proposed, it was hoped that the relatively high energy to be transmitted per pulse, consistent with the comparatively long ( $25-\mathrm{ms}$ ) pulse width, ${ }^{(1,2)}$ would allow one of the ex'iting metric-wave-length radio astronomical telescopes to be employed as the ground-receiving antenna. Several unsuccessful attempts were made to get a commitment of more than a few hours a week of observing time on a number of suitable facilitins, such as the Arecibo, P. R. telescope and a local 150-foot paraboloid. It was concluded that the only way in which observing time, consistent with the investment in spacecraft and launch costs, could be guaranteed would be for Sunblazer to have its own receiving antenna. The least expensive means of obtaining a suitable aperture was believed to be through an array of dipoles patterned after the $38-\mathrm{MHz}$ solar radar telescope at El Campo, Texas which had been operated for some time, although design differences were require* by the higher 75-and $225-\mathrm{MHz}$ frequencies for the initially proposed two-frequency Sunblazer experiment. It was further proposed to take advantage of new developments in scid-- tate circuitry to make the array automatically phaseable, as opposed to the time-consuming cable-plugging method then in use at El Campo, and an increase was planned from 1,000 to 4, 000 dipole elements to give an improved receiving gain of some $36 \mathrm{~dB} .{ }^{(3)}$ This corresponds approximately to the gain in the Arecibo telescope at 75 MHz ,

After some considerable amont of taboratory development and test work on the electromic amplifier and phasing package to be assoctated with each dipole element, the design of these was sufficiently firm so that bids from sevetal elentenic manufacturers could be obtained to give realistic cost data on the array's electronic components. These outside estimates indicated two things: 1) That mure than $90^{\circ}$, of the cust of the two-frequency array was in the electronics, and 2) that the cost of a dual-polarization, two-frequency, toou-dipule array was likely to he considerably in excess of the $\$ 3$ million estimate which had earlier been pustulated. These sumewhat discouraging but realistic conclusions were reported to a group from NASA lleadquarters and langley at a meeting of the Sunblazer coordinating committee held in Cambridge on 22 May 1967.

### 1.3.2 High-Gain Elements

As a result of this meeting a fundamental redirection of the array program was made. Since the maior cost items of the array were in the electronic system, any attempt at significant cost reduction required a less complex electronic system. For a given over-all system gain this implies replacement of the individual $5-\mathrm{dB}$ dipole elements with higher-gain types. A cosi a lysis, based upon various cost-versus-gain functions indicated that minimum overall system cost results when the olement (mechanical) cost and the electronic system cost are approximately equal. It was further indicated that an array of $14-\mathrm{dB}$ to $20-\mathrm{dB}$ elements would yield a reduction of an order of magnitude in electronic system complexity over the !revious individual dipole system. Several typical high-gain elements were considered, but no decision was taken as to the final element design.

The approach was discussed at a Sunblazer coordinating committee meeting in Washington on 13 July 1967, and again in Cambridge on 23 August 1967. At the August meeting it was agreed that the 128 -element pilot array would continue to be installed at El Campo using dipoles as the radiating element since that work was well along. However, the $225-\mathrm{MHz}$ array would be redesigned to achieve $25-\mathrm{dB}$ gain using an array of high-gain elements. Subsequently the backfire configuration was selected for this array. The $225-\mathrm{MHz}$ array was completely redesigned and typical anterna patterns taken on the element. A complete set of electronic and mechanical hardware was prepared. However, the $225-\mathrm{MHz}$ system was not installed due to a change, dictated in part by budgetary cunsiderations, in the format of the pulse transmitted from the spacecraft. This format is fully described in Section 1.3.3, and was described at the Sunblazer Review meeting on

15-14 March 1968 and 23 July 1968. The closely-packed, simgle-frequency format eliminates the need for the $225-\mathrm{MHz}$ array and has: additional advantage of higher permissible gain at 75 MHz for constant facility cost.

There remained, however, the question of the element for the $75-\mathrm{Mliz}$ aיray. Although the backfire configuration was satisfactory for the 225-MHz system, it possessed mechanical disadvantages at 75 MHz due to the relatively large ( 4 -meter) wavelength. Based upon cost figures obtained from the construction of an in-house model and estimates obtained from oatside vendors, it was determined that the cost-per-unit of gain of this antenna type was inordinately high. The approach to this problem was to keep the element gain constant at 14 dB while investigating other elements of equivalent gain. Details of this evolution are given in the document "History ard Dusign Summaxy, Sunblazer Phased Array". (4) The fundamental conclusion was: "The best method, in terms of satisfying the Sunblazer tracking requirements (both engineering and science shots) at minimum overall cost (initial installation, operation and maintenance) is to construct a cross-potarized wideband dipole array at El Campo, Texas." In this array the element is realized by the interconnection of six dipoles. Basically, it has been determined that a $14-\mathrm{dB}$ antenna element is less expensive and has better performance when constructed by connecting six dipole elements together as opposed to employing single or multiple backfire or other slow. wave type elements. A detailed description of the $50-\mathrm{dB}$ array using this six dipole (double-tee) interconnection is given in the Sunblazer Ground Antenna Report (February. 1969).

### 1.3.3 Change to Closely-Packed "Single" Frequency Experiment

 The foregoing changes would have been final ones except that information recently gleaned from the Mariner 4 solar occultation experiment ${ }^{(5)}$ suggested some changes in the transmitted.pulse format, which further interacted with the ground antenna design. This experiment measured for the first time the frequenfy broadening which a coherent signal undergoes when transmitted on ray paths within four to six solar radii of the solar corona. When the $2200-\mathrm{MHz}$ Mariner 4 measurements are scaled to 75 MHz by theoretical methods justified by some recent results in our laboratory ${ }^{(6,7)}$, they strongly suggest that the $25-\mathrm{ms}$ pulse for Sunblazer earlier proposed should be reduced to a pulse no longer than 3 ms , if coherent integration and coding within the pulse are t: be employed. Pulse-coding is desirable to achieve a 5- to $10-\mu$ s resolution and accuracy in our time-delay measurements. Without a substantial increase in peak power, the reduced pulsewidth means a correspondingly reduced energy per pulse which can only be made up by an increase in antenna gain on the apacecraft or on the ground. For reasons assuciated with the simplicity and the reliability of the spacecraft, we preferred not to change the radiation pattern of its antenna from an essentially omnidirectional one, which meant that to obtain an acceptable signal-to-noise ratio on a single pulse, a ground antenna gain in excess of $36 \mathrm{~d} l 3$ would be required. This, in turn, implied the addition of many more elements and returned the design problem again to one of overcoming a cost constraint.

The solution this time, in an attempt to stay within the $\$ 3$ million budget, was to simplify the experiment in what was believed to he an acceptable way, by using a separation between the probing and reference frequencies of sume $10^{\circ t}$, such that buth frequencies could be received within the bandwidth achievable in a single array. This reduces somewhat the accuracy of the delay experiment but does not compromise the experimental results. On the other hand the elimination of the $225-\mathrm{MHz}$ array represented a substantial saving in cost which could then be applied to increase the gain of the $75-\mathrm{MHz}$ array to a $50-\mathrm{d} \beta$ level. This provides an excellent ( $16-\mathrm{dB}$ ) signal-to-noise ratio on the narrow ( $3-\mathrm{ms}$ ) pulse now to be transmitted from the Sunblazer spacecraft, and results, it is ielieved, in a better and a simpler all-around experiment.

A $50-\mathrm{AB}$ antenna at 75 MHz is a rather remarkable instrument in its own right and is some 14 dB greater than the gain of the telescope at Arecibo at this wavelength. It is also some 14 dB greater than the circular interferometer composed of 9645 -foot-diameter paraboloids operated by CSIRO in Australia. Yet it is believed that the present development and test experience with both the $75-\mathrm{MIlz}$ dipole pilot array, and measurement design efforts to date on the high-gain backfire elements at 225 MHz , allow one to say with some degree of confidence that the electronically phased wideband array of $102420-\mathrm{dB}, 75-\mathrm{MHz}$ elements with dual polarization is a realistic design and can be built for approximately the $\$ 3$ million figure.

While, indeed, there have been a number of major changes in the overall Sunblazer propagation experiment and in the receiving array, they have been made for good reasons and are precisely the kind of changes in concept or design that one should expect for work done on supporting research and technology funds. In making these changes, the primary motivation has been to select carrier frequencies and modulation formats which are most likely to lead to the pruduction of useful data in early Sunblazer
propagation experiments and, at the same time, are conducive to spaceroft and ground antenna designs which must satisfy serious cost constraints.

### 1.3.4 Additional Advantages of the 50-d! Broatband Array

A further benefit of the use of a $50-\mathrm{dli}$ ground termina! is the rather high bit-rate (possibly in excess of several bits per second) which ean he made available for the telemetry of data from on-l) ard experiments over distances as great as 2 AU. This is a matter of sume considerable importance for on-board experimenters, and to the future use of Sunblazer as an interplanetary ubservatury.

As a radio telescope the array also has important ancillary uses, the most recent and exciting of which concerns the observation of the newly discovered pulsars at a frequency somewhat below that on which they have thus far been observed. The high gain and directivity of this array at 75 MHz may well provide important new information and improved resolution of the signals from the sources observed thus far, and its greater sensitivity may lead to the discovery of still more sources.
The modular nature of the array also makes it practicable to provide small solid-state power amplifiers in the 250 - to 1000 watt range on each elew ment, which makes possible conversion of the array into a radar telescope or a future uplink station to Sunblazer of impressive performance. The power amplifiers required are essentially indentical to those developed for the Sunblazer spacecraft and, while some uesign work is involved in adapting and packaging them for the ground array, their addition to the array is largely a matter of cost. That is, the overall transmitter power could be somewhere between one and four MW, depending on the funds available and the interest in adding the transmitter facility. This was an item in the original proposal, but was eliminated at an early stage because of cost considerations. It is, however, a growth itnm which should not be forgotten in future planning.

Solar radar echoes have never been observed at 75 MHz , and the reason for this is believed to be that sufficient radar telescope sensitivity has never been available to counteract the additional absorption losses in the corona, which occur along a ray path prior to the reflection point. The great advantage in making solar radar observations with this instrument would be improved sensitivity and a greatly improved angular resolution, which would make it possible to resolve quadrants of the solar disk, and to improve the capability for observing average Doppler and Dopplerbroadening of the signals which contain important, and otherwise
umohtamable, infomation on the mention of the solar conona at histances conse to the photosphere

As a phan*tary radar it wouk aphear to be the first instrument that could detect reflections from the planet dupiter and could wake important ohservations on Venus and Mars at a moch lowne frequency than has heretom fore been used, The rather surprisimg oseillation in ratio cross set' on of the phanet Vents whict dames ${ }^{(8)}$ has ohserved at 38 mhe combthe better studied with the additional sensitivity and the greater dange of this new inswament.

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## CHAPTER 2

## 2. 0 CHANNEL. CHAKACTERIZATION

In order to measure the integrated electron density along the communication path and the fluctuation of this density, it is necessary to deturmine how these two quantities will affect signals propagating through such an ionized medium.

The integrated electron density will cause a transmitted pulse to have an additional delay over the free-space delay and cause it to be dispersed in time. The fluctuations of this integrated electron density will cause the arrival time to be random and fluctuate from pulse to pulse. In addition, it will also cause the received signal to be dispersed in frequency. The bandwidth and duration of the transmitted pulse must be chosen so as to minimize the time and frequency dispersion effects while at the same time allowing the measurement of the pulse delay and delay fluctuation to a reasonable degree of accuracy.

The main reason for the minimization of the dispersion effects is so that a simply near-optimum detection-estimation scheme may be used to make the desired measuremerts. At present only the frequency and time dispersion of the channel are known. In general complete statistical description of the channel is necessary to design the optimum receiver. However, if the sagnal duration and bandwidth can be chosen so as to make the channel appear stationary during the pulse duration, a simple detection scheme results. In effect, a transmitted pulse will be received with a random delay, frequency shift, amplitude and phase which are constant over the pulse duration. The optimum detection scheme is then a bank of matched filters, covering the expected frequency shifts; each filter being matched to a frequency shifted version of the transmitted signal and followed by a square-law envelope detector.

### 2.1 Time Delay

Due to the integrated electron density over the communication path between Sunblazer and the earth, a narrow-band pulse will suffer an additional delay
ovet the space free delay of:

$$
\begin{equation*}
T_{x 1}(f)=\frac{1.35 \times 10^{-14} N}{r^{2}} \text { sex } \tag{2.1}
\end{equation*}
$$



 and hetween 75 and 80 MIF as a function of the offict distance from the sum
 shawn fur comparison.

The varnation in the integrated electron flensity will cathse a fluctuation in $T_{4}(f)$. This has heoth studied hy Hollwege ${ }^{(1)}$ and the stamdard ibevathon of $\mathrm{T}_{4}(\mathrm{f})$. nuted by $\mathrm{o}_{4}(\mathrm{f})$ is shown in Fig. 2-1. It is assumed that the "hbols " are 200 km in size and that the rms electron density fluctuation is $0.0 r$, times the average dectron density.

The measurement of the relative delay will give the value of N , the integrated alectron density, while a measurement of the standard deviation of this delay will give a value for $\sqrt{b(\Delta n)^{2}}$ where $b$ is the "blob" size and $\sqrt{\overline{(\Delta n)^{2}}}$ the rms electron density fluctuation.

### 2.2 Time Dispersion

The integrated clectron density along the communication path also causes the time dispersion of a narrow-band pulse; this effect has heen studiod by lindyck ${ }^{(2)}$. The Sunblazer signal will be spread in time by two phenomena. The first is dispersion of the various frequency components of the signal due to the frequency-dependent phase delay in the medium. The second is multipath spreading, due to the random bending of various rays from the transmitter by the coronal refractive index fluctuations, and the resultant varying path lengths of these rays from the transmitter to the receiver.
The behavior of the dispersive time spreading was studied some timr ago ${ }^{(2)}$. Assuming that the medium is constant over the pulse duration, the channel can be modeled by a linear dispersive network with a transfer function $H(\omega)=\exp \{-j \phi(\omega)\}$, where:

$$
\begin{equation*}
\phi(\omega)=\mathrm{T}_{\mathrm{o}}\left(\omega-\frac{\overline{\omega_{\mathrm{p}}^{2}}}{\omega}\right) \tag{2.2}
\end{equation*}
$$

in which $T_{o}$ is the free-space propagation time and $\overline{\omega_{p}^{2}}$ is the average plasma frequency along the path. A study was made of the output signal as a function of the proposed orbit, assuming that the receiver contains a filter matched


Fis. 2-1 Additional delay $\mathrm{T}_{\mathrm{d}}(75)$, relative delay $T_{d}\left(f_{1}\right)-T_{d}\left(f_{2}\right)$, and standard deviation of delay $0_{d}(75)$, vs path offset.
to the transmitted signal. followed by a square-law envelope detector. If the pulse is a binary-phase modulated carrere at $w_{o}$. with a bit time of $t_{1}$ the nutput of the detertor is not greatly degraded as lonk as $t_{1}>{ }^{7}{ }^{\circ}=$ $\sqrt{\pi \mid 0^{\circ}\left(w_{0} \mid\right.}$. In the absence of dispersion (and noise) the output is the square of the envelope of the autocorrelation function of the transmitted pulse (assuming the Doppler has been removed). The correlation peak has a width of $2 / t_{1}$ and reaches a maximum at the arrival time plus the signal duration. With dispersion present and for $t_{1}=\tau_{0}$ the peak width is doubled. and its maximum value is down 3 " $R$. The "dispersion time", $\tau_{0}$. is shown in Fig. 2-2 as a function of the offset distance for the proposed orbit.

Althouph $T_{o}$ pives a good measure for the dispersion of one pulse. it does not incluck the effects of multipath spreading. This is causeci by the arrival of a continuum of delays of the transmitted pulse, over a time comparable to the pulse duration. In all regions where dispersion is noticeable, the multipath spreading is significantly greater than the dispersion. The nature of this time spreading can best be understond from the angular power spertrum of the received electromagnetic signal. It is well known that radiation from a point source will be scattered or randomly bent by the corona. and this radiation will arrive at a receiver with a random angle (or angles) of incidence. If we represent the corona by a thin refracting or phase changing screen at the center of the propagation path, then energy arriving from an angle $\theta$ with respect to the line-of-sight path will suffer an additional propagation delay $T_{0} \theta^{2}$ with respect to the direct path. where $T_{o}$ is the line-of-sight propagation time delay from the transmitter to the receiver. Thus, if the mean square angle of arrival fluctuation is $\theta_{0}^{2}$, we might expect an average multipath spreading of the signal on the order of $T_{0} \theta_{0}^{2}$. A rigorous analysis of the received field from thin sereen has shown that this intuitive result is very nearly correct ${ }^{(3)}$.

A meaningful description of the multipath spreading in the channel is found in the delay scattering function. This function tells us on the average how the power from a very short narrowband pulse is spread out in time by the channel. If a very large number of these short pulses were transmitted through the channel and the received power from each pulses recorded and averaged over the ensemble of pulses, then the result would be the delay scattering funciion.

For an anisotropic thin screen model of the corona, the delay scattering function has been shown to be:

$$
\begin{equation*}
\sigma(\tau)=\frac{4}{T_{0} \theta_{0} \theta_{i}} e^{-\frac{2 t^{\prime}}{T_{0}}}\left[\frac{1}{\theta_{0}^{2}}+\frac{1}{\theta_{i}^{2}}\right] I_{0}\left(\frac{2 t^{\prime}}{T_{0}}\left[-\frac{1}{2}+\frac{1}{\theta_{0}^{2}}\right]\right. \tag{2.3}
\end{equation*}
$$



Fig. 2-2 Dispersion time $\tau_{0}$, an reciprocal frequency spreading $1 / \mathrm{B}$, vs path offset. Frequency: 75 MHz .
where $\theta_{i}$ and $0_{0}$ are the rms scattering angles in and out of the plane of the ecliptic. To is the propagation time delay from the recesver to the transmitter along the line-of-sight path. $\boldsymbol{x}=0$ is the arrival time of the pulse along this path. and $I_{o}$ is the zero-order modified Bessel function (Bussel function with an imaginary argument). If the medium is isotropic ( $\theta_{i}=\theta_{0}$ ) this exprission reduces to an exponential. whereas if $\theta_{0} \Rightarrow n_{1}$ or vice versa the scattering function approaches the chi-squared density. In any case the mean multipath delay is found to be

regardless of the anisotropy ratio.
Since the mean square scattering angles are highly variable. we would expeet the multipath spreading to vary widely from day to day. Thus when the Sunblazer path offset decreases to the point where the average multipath spread is comparable to the pulse length, operation will probably berome intermittent. In this case a measurement of the multipath spreading characteristics of the channel will also yield valuable information.

### 2.3 Frequency Dispersion

If a long unmodulated carrier is transmitted through the solar coiona, the received signal will be spread in frequency. The only experimental data on such phenomena is given by Goldstein ${ }^{(4)}$. Mariner IV transmitted an unmodulated carrier throust the corona and the spectrum of the received signal was estimated. If it is assumed that the received signal is a sample function from a stationary random process. then the power apectrum estimated is the Fourier traneform of the statistical correlation function of the process. Analysis of the corona validates this assumption. Hollweg ${ }^{(1)}$ has shown that the spectral broadening is proportional to $f^{-1}$ and Goldstein ${ }^{(4)}$ concluded that for small antenna beamwidths the broadening is proportional to the beamwidth. Scaling the Mariner IV data taken at 2295 MHz with a beamwidth of 0.14 degrees to Sunblazer's 75 MHz and 0.3 degrees, the equivalent noise bandwidth of the received signal is

$$
\begin{equation*}
B=\frac{2.2 \times 10^{4}}{p^{3}} \mathrm{~Hz} \quad 3 \leq \rho \leq 6 \tag{2.5}
\end{equation*}
$$

The data art valid only between 3 and 6 solar radii. However, Hollweg's ${ }^{(1)}$ results indicate that for $\rho>10$ solar radii $B$ is proportional to $\rho^{-1.5}$, 1/B is plotted in Fig. 2-2 with the correct extrapolation made.

### 2.4 Siynal Selection

The correct signal handwidth and duration can now be deduced from the above data. Since it is desired to make accurate time-of-arrival measurements, the transmitted pulse must possess a good ambiquity function. This can be accomplished by phase modulating a currier pulse of duration $T$ with a Maximum Lerip, th Linear Recurring Sequence of M bits, the bit time being $t_{1}=\mathrm{T} / \mathrm{M}$. The nandwidth of the signal is then $W=1 / t_{1}$ and the time-bandwitth product $T W=M$.

Consilering first the dispersion time $\tau_{0}$, it is seen that for $t_{1}>\tau_{0}$ the received signal will not be greatly dispersed $:$ distorted in time. By letting $t_{1}=25 \mu$ sec. dispersion will not be noticed until 6 solar radii. At this point the detection loss is 1.5 dB and the correlation peak width is about 1.5 times the undispersed case.

Having fixed the bit time it is now necessary to see if the bandwidth of the signal is reasonably larger than the spread in frequency caused by the medium. As seen in Fig. 2-2 the bandwidth $W$ is at least one hundred times $B$ for $\rho>4$. Hence, the received signal will not be noticeably dispersed in frequency.

The signai now is not dispersed in time or frequency. However. the pulse duration $T$ has to be adjusted so that the received signal will be coherent. As noted in Section 2.3, B is a measure of the bandwidth of a frequency disperscd carrier. Hence, the signal will tend to be correlated over a time of $1 / B$ sec. By choosing $T=3 \mathrm{msec}$, it is seen from Fig. 2-2 that the received signal will be coherent up to about 6 solar radii. After this point the signal starts to become distorted and the matched filter square-law envelope detector is no longer optimum.

From the above analysis it appears that based on the present knowledge of the solar corona that a binary phase-modulated pulse of duration 3 msec and a bit time of $25 \mu \mathrm{sec}$, in conjunction with a matched-filter, square-law envelope detector receiver, should produce reliable measurements of tie electron density and its fluctuations up to about 6 solar radii, or until the multipath spreading hegins to dominate.

## CTOM - PACE ELANK MOT REMO.

## CHAPTER 3

## ©. 0 THE SUNBLAZER ORBIT

### 3.1 Basic Orbital Objective

In order that the main experiment in the Sunblazer program be conducted properly, it is necessary for the Sunblazer vehicle to be placed in a solar orbit which allows it to pass behind the sun's dise (as seen from the earth) within a reasonably short time after launch. In other words, the orbit must eventually cause the vehicle to line up approximately with the sun and the earth.

## 3. 2 Relative Motions of Spacecraft and Earth

This result can be most reliably accomplished if we first note that, in general, the vehicle's orbit and the ecliptic (i.e., the earth's orbital plane) will be mutually inclined by some small angle, due to errors in injection velocity. Irrespective of these errors, the earth will certainly be in Sunblazer's orbital plane not only at launch, but also after each six-month interval after launch (i.e., after the earth has traversed $n \pi$ radians from the launch point where $n$ is an integer). This means that, if superior conjunction with the sun should be required to occur after the earth has traversed $n \pi$ radians in central angle about the sun, the vehicle must have correspondingly traversed $n \pi \pm \pi$ radians during the same time; thus, the required period of the vehicle's orbit about the sun must be $\frac{n}{n \pm 1}$ times the earth's orbital period (under some additional assumptions given below). If this condition is achieved, then regardless of the inclination of Sunblazer's orbit, after $n$ six-month intervals or $\frac{n}{2}$ years, 1) the earth will be in the vehicle's orbital plane, and 2) the vehicle will have traversed exactly $\pi$ radians more or less than the earth in that plane.

This condition on the vehicle's orbital period comes out so neatly for all $n$ only because the line of apsides of the vehicle's orbit (the line running through perihelion and aphel:on) will nearly coincide with its line of nodes
(i. e. . the line of intersection of its orbital plane with the ecliptic) for any "reasonable" injuction scheme, But this latter restriction applies only if $n+1$ is old. for if $n+1$ is even, it is easily shown that any single-impulse infection will bring about the desired result. The important point to be malle here is that, if a nominal orbit is selected for Sunblazer with a sideral period of $\frac{n}{n \pm I}$ years, then comparatively large launch- vehicle injection errous can be tolerated by the main experiment and the reliability of the mission consequently increased.

## 3. 3 Evaluation of Possible Orbits

: Table :3-T

|  | Firstmorder computations. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| n | aidereal period (yrs) | Semi-major axis (AU) | perihelion (AU) | aphe ${ }^{\text {m }}$ lion (AU) | time to sup. conj. (yrs) | $\begin{gathered} \text { inj. } \\ \text { vel. } \\ (\mathrm{km} / \mathrm{sec}) \end{gathered}$ |
| 1 | $1 / 2$ <br> degenerate | $0.63$ | $0.26$ | $1.00$ | $1 / 2$ - | $10.7$ |
| 2 | $\begin{gathered} 2 / 3 \\ 2 \end{gathered}$ | $\begin{aligned} & 0.76 \\ & 1.59 \end{aligned}$ | $\begin{aligned} & 0.52 \\ & 1.00 \end{aligned}$ | $\begin{aligned} & 1.00 \\ & 2.17 \end{aligned}$ | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | $\begin{aligned} & 5.1 \\ & 5.1 \end{aligned}$ |
| 3 | $\begin{aligned} & 3 / 4 \\ & 3 / 2 \end{aligned}$ | $\begin{aligned} & 0.83 \\ & 1.31 \end{aligned}$ | $\begin{aligned} & 0.65 \\ & 1.00 \end{aligned}$ | $\begin{aligned} & 1.00 \\ & 1.62 \end{aligned}$ | $\begin{aligned} & 11 / 2 \\ & 11 / 2 \end{aligned}$ | $\begin{aligned} & 3.3 \\ & 3.3 \end{aligned}$ |
| 4 | $\begin{aligned} & 4 / 5 \\ & 4 / 3 \end{aligned}$ | $\begin{aligned} & 0.86 \\ & 1.21 \end{aligned}$ | $\begin{aligned} & 0.72 \\ & 1.00 \end{aligned}$ | $\begin{aligned} & 1.00 \\ & 1.42 \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 2.5 \\ & 2.4 \end{aligned}$ |
| 5 | $\begin{aligned} & 5 / 6 \\ & 5 / 4 \end{aligned}$ | $\begin{aligned} & 0.89 \\ & 1.16 \end{aligned}$ | $\begin{aligned} & 0.77 \\ & 1.00 \end{aligned}$ | $\begin{aligned} & 1.00 \\ & 1.32 \end{aligned}$ | $\begin{array}{ll} 2 & 1 / 2 \\ 2 & 1 / 2 \end{array}$ | $\begin{aligned} & 2.0 \\ & 1.9 \end{aligned}$ |

These figures assume the earth to be a massless point in a circular orbit about the sun. The actual values depend upon the time of year of launch since the earth is in a slightly eccentric crbit.

Other considerations besicles sensitivity to out-of-plane velocity error are displayed in Table 3-I. In calculating the required injection velocity, it is assumed that the most efficient use is made of the launch vehicle by launching it either parallel or anti-parallel to the earth's orbit velocity. It is seen that for each $n$ there is both an orbit which remains entirely outside the earth's orbit (and corresponds to injection parallel to the earth's orbital velocity), and another which remains entirely inside (and corresponds to launching anti-parallel).

However. orhits which remain outside 1 astronomical unit have been repected for the initial missions because. 1) the avalable solar mower would be decreasel, and 2) the corresponding communication distances would be longer.

Of the remaining candidates (which extend no farther than 1 astronomical unit from the sun), the 1/2-year orbit must be rejected. For, despite its apnealing property of achieving superior conjunction in only half a year, it not only requires a rather large injection velocity, but also subjects the vehicle us a solar readiation-flux at perihelion 15 times greater than that existing at 1 astronomical unit. On the other hand, although the 5/6-year orbit requires a low injection velocjty and subjects the spacecraft to only a small increase in solar intensity. it has the disadvantage of requiring $21 / 2$ years for superior conjunction. It therefore, must be rejected from considerations of spacecraft deterioration.

Of the remaining candidates (2/3. $3 / 4$ and $4 / 5$ ) the $3 / 4$-year orbit possesses a special advantage which is not apparent from the acompanying table: it carries the spacecraft through not one, but three superior conjunctions with the sun (Fig, 3-1). Furthermore, this orbit permits the vehicle to remain within a subtended angle of $3^{\circ}$ from the sun for more than six months. compared with less than one month for the 2/3- and 4/5-year orbits.

Sunblazer design work has determined that the 2/3-year orbit both reduces the permitted payload weight to marginal levels for a first mission, and creates cooling problems for this lightened vehicle at perihelion. Therefore. in order to achieve a balance between the conflicting requirements of low injection-velocity and a minimum time to superior conjuncetion. as well as to take advantage of its other special properties, the $3 / 4$-year orbit has been selected.
3. 4 The Generation of Three Superior Conjunctions

In order to understand why the selected orbit yields three superior conjunctions with the sun instead of just one, we need only call attention to three simple facts. First, any orbit which remains wholly inside the earth's orbit must have, on the average, an angular velocity greater than that of the earth. Second, because the vehicle is launched from the earth, that launch point is at the aphelion of the 3/4-year orbit where the angular velocity is actually less than that of the earth (recall that the injection direction is opposite to that of the earth's orbital velocity). Third, at superior conjunction $11 / 2$ years after launch, the vehicle will be back at exactly the same point in its orbit from which it was launched (i.e., after having made exactly two revolutions about the sun, it is again at aphelion and has a sma?ler angular velocity than the earth).


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Fig. 3-1 Motion of Sunblazer relative to the earth.

We see from the first fact that the vehicl. must. on the average, gain central angle on the earth; but from the second fact it appears that the earth will gain on the vehicle whenever the latter is near aphelion (including the launch point). Therefore. we see that the vehicle will make steady gains on the earth. interm rupted periodically by small segments of apparent retrograde motion. Furthermore. from the third fact we note that one of these retrograde segments will occur "behind the sun" after 1 1/2 years. thus giving rise to a triple superior conjunction.

### 3.4.1 Influence of Launch Time on the Retrograde Motion Amplitude

IMring the first mission-planning studies of the 3/4-year orbit done at MIT, it was discovered that the amount of this apparent retrograde motion of the spacecraft about the triple superior conjunction was markedly influenced by the earth's orbital position at launch 1 1/2 years earlier (or equivalently, by the calendar date of the launch). It was subsequently concluded that the seemingly small eccentricity of the earth's orbit ( 0.016726 ) was responsible for this effect (Figs, 3-2 to 3-13) by means of the following mechanism.

If the vehicle is launched around July first, when the earth is near its own aphelion. then the aphelion of the 3/4-year orbit must be at least as great as that of the earth, or greater than 1 AU . Correspondingly, the perihelion must decrease in order to keep the major axis (and the orbital period) constant. This increase in aphelion and decrease in perihelion for a July first launch results in a slightly larger eccentricity for the vehicle's orbit. thus increasing the extremes of angular velocity at these points. In particular, the spacecraft's angular velocity at aphelion is even smaller than it would be if launched tangentially at exactly 1 astronomical unit. But $11 / 2$ years from launch it will be January first, and the earth will be at perihelion where its angular velocity is greatest. Thus, the small change in conditions caused by launching on July first adds two independent inputs to the magnitude of the vehicle's apparent retrograde motion $1 / 2$ years later, so that the subtended amplitude of this retrograde motion is $6^{\circ}$ as seen from the earth. Similarly, the decrease in eccentricity of the venicle's orbit for a January third launch has been found to decrease the amplitude of retrograde motion to only about $1^{\circ}$.

### 3.4.2 In-Plane Errors

Although the sensitivity of the 3/4-year orbit to out-of-plane launch error has been minimized, it is still necessary to consider the effect of the inplane component of error. To do this, it was first determined that the sensitivity of the orbital period to a change from the nomina? heliocentric
velocity of the spacecraft was about 20 days $/ \mathrm{km} / \mathrm{sec}$ (e. R. . if the nominal launch velocity were changed in magnitude only from 3.3 to $3.3 \pm 0.2 \mathrm{~km} / \mathrm{se} \cdot \mathrm{C}$. then the orbital period would he changed from $3 / 4$ of a year to about $274 t$ [0. 2] [20]. or $274 \pm 4$ days).

If we assume that the vehicle's launch velocity is 200 meters per second ton high. then its orbital period will be shortened by four days so that $11 / 2$ years later. after two revolutions, it will arrive at its aphelion point right days sooner than nominal. Since the earth moves ahout $1^{0}$ per day, it will then be about $8^{\circ}$ from the spacecraft-sun line and since the spacecraft will be about 2 AU from the earth, the subtended angle between the vehicle and the sun will be $4^{\circ}$ as seen from the earth. This means that, if the launch velocity is 200 meters per second too high, every point in the solur- 20 nounter profile will be moved about $4^{\circ}$ to the left. This would be acceptable. at least for the first missions, since the spacecraft would still remain within a trgion of interest for more than half a year.

It should also be noticed here that all of the solar-encounter profiles could be centered by changing the initial velocity by less than 100 meters per second. The reason some of the profiles are not centered is that the period of the orbits has been fixed at 3/4-year, and the earth is in an orbit of finite eccentricity. Also, the nominal inclination to the ecliptic is zero, and the selection of $1 / 2^{\circ}$ inclination in the solar-encounter profiles is only to demonstrate the effect of a finite inclination.

### 3.4.3 Escape Velocity Error

It should be cinphasized here that all of the previous figures ignore the hyperbolic escape trajectory from the earth or even the need for achieving escape velocity. They apply only at the earth's sphere of influence, which is at a radius of about $920,000 \mathrm{~km}$. Unfortunately, the errors present at launch-vehicle burnout (without 300 km of the earth's surface) are magnified considerably at the sphere of influence. To see why, define $v_{b}$ as the burnout velocity. $v_{\infty}$ as the velocity at the sphere of influence, and $v_{e}$ as the earth's escape velocity as the burnout altitude. Conversation of energy yields the equation $v_{\infty}=\sqrt{v_{b}^{2}-v_{e}^{2}}$. If we take the derivative of $v_{\infty}$ with respect to $\mathrm{v}_{\mathrm{b}}$, we will get:

$$
\begin{equation*}
\frac{d v_{\infty}}{d v_{b}}=\frac{v_{b}}{\sqrt{v_{b}^{2}-v_{e}^{2}}}=\frac{v_{b}}{v_{\infty}} \simeq \frac{12}{3.3} \simeq 31 / 2 \tag{3,1}
\end{equation*}
$$

for the 3/4-year orbit. This means that. if the burnout velocity is in error by 1 meter per second. the error at the earth's sphere of influence will be about $31 / 2$ meters per second. Consequently, the 200-meter-per-second error discussed previously reduces to alout 60 meters per serond, or about 180 feet per second for the launch vehicle at burnout.

## 3. 5 Computer-Generated Numerical and Graphical Orbit Information

The position of the spacecraft during the $3 / 4$-year orbit is graphically shown in 10-day increments in Fig. 3-2 through 3-13, and the angular displacement effect on the triple conjunction characteristics due to launch time is also shown in the same figures entitled "Solar lincounter Profile".

The numerical data which were utilized to generate these graphs are also presented.

## SOLAF ENCGUNTER PROFILE

## 



SUNELAZER ORBIT
Fig. 3-2.
solar encounter profile


SUNBLAZER OREIT
Fig. 3-3.

SOLAR ENCOUNTEG PRCFILE


SUNBLAZER ORBIT
Fig. $3-4$.

SOLAK ENCOUNTER PROFILE


Fig. 3-5.

## SOLAR ENCOUNTER PR.IFILE


sunglazer orbit
Fig. 3-6,

## golar encounter prifile




Fig. 3-7.

## golar encu'jnter frofile



Fig. 3-3.

## SOLAR ENCOUNTER RROFILE



SUNBLAZER ORBIT
Fig. 3-9.

## SOLAR ENCIUNTER PROFILE



SUNBLAZER ORBIT
Fig. 3-10.

SOLAR ENCOUNTER PROFILE



SUNBLAZER GRBIT
Fig. 2-11.

## SOLAn ENCOUNTER PROF ILE



SUNBLAZER ORBIT
Fig. 3-12.

SOLAR ENCOUNTER PROFILE


SUNBLAZER ORBIT
Fig. 3-13.

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TINITE O.U IUFLI: 5.0 IFINLE 700.0 THFG= U.U SFMI MAJOR AXIS=U.H2B4EIT7



| 120.0 | 1.00859 | 0.68530 | 0.50334 | 123.214 | 150.043 | 37.917 | 0.340 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 125.0 | 1.00488 | 0.67061 | 0.55867 | 128.056 | 199.228 | 39.221 | 0.212 |  |
| 130.11 | 1.01108 | 0.67075 | 0.61605 | 132.887 | 169.220 | 40.171 | 0.102 |  |
| 135.0 | 1.01221 | 0.66794 | 0.67477 | 137.708 | 119.037 | 40.821 | 0.008 |  |
| 140.0 | 1.01325 | 0.66827 | 0.73407 | 142.519 | 188.891 | 41.220 | -0.070 |  |
| 145.0 | 1.01419 | 0.67175 | 0.73320 | 147.321 | 198.689 | 41.419 | -0.136 |  |
|  |  |  |  |  |  |  |  |  |
| 150.0 | 1.01503 | 0.67824 | 0.85141 | 152.114 | 208.342 | 41.466 | -0.189 |  |
| 155.0 | 1.01576 | 0.68747 | 0.90807 | 156.901 | 217.774 | 41.402 | -0.232 |  |
| 160.0 | 1.01638 | 0.69919 | 0.96262 | 161.682 | 226.923 | 41.267 | -0.265 |  |
| 165.0 | 1.01688 | 0.71298 | 1.01462 | 166.458 | 235.743 | 41.091 | -0.240 |  |
| 170.0 | 1.01126 | 0.72844 | 1.06376 | 171.230 | 244.208 | 40.902 | -0.308 |  |
| 175.0 | 1.01751 | 0.74519 | 1.10980 | 175.999 | 252.307 | 40.720 | -0.320 |  |


| 180.9 | 1.01764 | 0.76283 | 1.15262 | 180.766 | 200.039 | 40.560 | -0.326 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 185.0 | 1.01764 | 0.78102 | 1.29213 | 185.33 | 267.417 | 40.434 | -0.327 |
| 190.0 | 1.01752 | 0.79944 | 1.22833 | 190.300 | 274.456 | 40.349 | -0.344 |



| 420.11 | 10．0rums， | C．R174． | 1.27535 | 41 n ．hue | － $21: 108 \mathrm{~cm}$ | 13．201 | －1）．1／16， |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 425.6 | U．94113 | $11.6749 \%$ | 1．6U2＊4 | 423.424 | 911.310 | 13.321 | －11．lui |
| 4510.1 |  | ＂－timene | 1．A，sola | $42 \% .024$ | 314.7184 | 11.0 .47 | －4．13s |
| 413．0． | U－testel | d．lulll | Leb3620 | $415 \cdot 1 / 21$ | 54.40 es | 16．3u＇ | －11．132 |
| 440.1 | 2）．94534 | 11.11 l | 1．A．N St．1 | $43 \mathrm{H}, \mathrm{NuO}$ | 547.5 Hz | 9.534 | －0．1 HU |
| 445．0＇ | U．4）傃 | 0．1s17： | 1．10842 | 443.164 | tububery | 7.324 | －0．1\％ |
| 450.11 | 1．4．4． 50 | 1．140r4 | 1．713144 | 44R．117 | （1） 18.9 HC | h．Sue． | －1．0．un |
| 455．11 | Ualcides | deluble | 1．15644 | 4330651 | 0．1．445 | 2．206 | －6．216 |
| $40.10 \cdot 1$ | 1．0014\％ | $11.144^{\prime \prime}$ | 1．PIMNU | $45 \mathrm{H.5/5}$ |  | $4.18{ }^{\text {c }}$ | －0．2ic |
| 40\％．！ | 1．06：${ }^{\text {a }}$ | 1084 19 | 1．80こ32 | 403.481 | ASt．4te | 3．30．4 | －ubayd |
| 470.1 | 1.0045 | wotiler | 1.48360 | 4，RH．Sts |  | ？．610 | $-11.8<u^{\text {c }}$ |
| 413.11 | 1.00519 | 1．0estars． | 1.34412 | 413.246 | （14\％．44， | 1.301 | －11．21\％ |
| 4 nr .1 | 1．1011M | $1.8670 r$. | 1．46sin | 478.184 | 625．0．0？ | 1.411 | －11．run |
| 4ど5．4 | 1．lycios | U．t！73！${ }^{\text {a }}$ | 1.88196 | $4 \mathrm{HP.97} \mathrm{\%}$ | 640.480 | 11.360 | －0．1．9 |
| 430.1 | 1.00947 |  | 1.89476 | 4H7．R2？ | Hut．573 | 0． $0^{4} 7$ | －0．1 1 KN |
| 44b．l1 | 1．01103 | 0.410441 | 1．91\％${ }^{\circ}$ | 498.324 | 612．0．0） | 0． 2 －3th | －11．1／\％ |
| 500.10 | 1．01210 | $0.4181 \%$ | 1.93624 | 441.415 | 617.327 | n．ors？ | －11．141 |
| \＄05．0 | 1.01320 | 0－13017 | 1.4434 | 2102.266 | 062.401 | $-0.107$ | －0．140 |
| 510.1 | 1.01415 | 11．142014 | 1.45617 | hul．0nn | 6n7．41811 | －0．144 | －11．121 |
| 315.1 | 1.01490 | 0．y 207 | 1.96704 | 311.882 | 64\％．314 | －0． 1.845 | －0．112 |
| 320.11 | 1．01573 | C．4t076 | 1.97641 | 310．664 | hu7．1k3 | －0．${ }^{24} 50$ | －0．11）4 |
| －25．010 | 1.01635 | C． 76801 | 1.98440 | 221.460 | 10．4．401 | －0．433 | －0．1016 |
| 530.11 535.0 | 1.01176 1.01724 | 0.41841 0.41844 | 1.97082 | ＇， $26.27 \%$ | 7ut－htik | －0．112 | －u．11） 1 |
| 335.0 | 1．C1724 | U．41844 | 1.99564 | 330.494 | 711.178 | －0．0．71 | －1）．1） 88 |
| 240．：1 | 1.917511 | 13．314， 1 | $1.798+7$ | 535.768 | 115．734 | 10.0 | －10．11\％ |
| 545.0 | 1.011704 | Veye362 | 2.04067 | 240.315 | 120．308 | 0.112 | 0.1161 |
| 550．11 | 1.01764 | 0.94314 | d．00077 | 545．302． | 7．4．t．） 5 | 0.216 | U．11／1 |
| 55b．0 | 1.01753 | 0．t小17\％ | 1．7392\％ | 354.007 | 724．407 | 0.324 | $0.11 / 40$ |
| 540．1 | $1.617{ }^{13}$ | U．478．4 | 1.94670 | 544.838 | 735．474 | 0.425 | 4.1124 |
| 563．9 | 1.016 .2 | 0.41471 | 1．4＇） | 5h．t．610 | 75 SH | 11.510 | 0．：18 |
| 570.9 | 1.016 .43 | 0．96．80？ | 1.98535 | 564.385 | 743.735 | 0.760 | U．100 |
| 575.9 |  | 10．40．19！ | 1．9176？ | 569.165 | 147．949 | 11.593 | 0.114 |
| 38U．0 | 1.01411 | $0.4+343$ | 1．9684？ | 375.951 | 752．140 | 11.58 .5 | U．1st |
| 54．3．1 | 1．01428 | 0.16350 | 1.45170 | 578．745 | 711．6．624 | 0.543 | 0.147 |
| 540．1） | 1.01335 | $0.4374{ }^{5}$ | 1.44577 | 585．645 | 707．0ild | 0.448 | U．162 |
| 295.1 | 1.01232 | 0.23005 | 1.43233 | 588．35＇\％ | 701．140 | 0.237 | 0.176 |
| 600.11 | 1．011）0 | 0． 010148 | 1.91750 | 573.114 | 713．1004 | 0.082 | 0.164 |
| 605．11 | 1.01000 | 0．${ }^{\text {（1）180 }}$ | i．90176 | 598.004 | 118．444 | －0． 214 | 0.200 |
| 610.11 | 1.00877 | 0.47 lll | 1.88471 | 602.846 | 784．004 | －0．570 | 0.204 |
| 615．0） | 1.00138 | 0.8 .2754 | 1．8629\％ | 607.694 | 784.405 | －1．011 | 0.216 |
| 620.11 | 1．0059\％ | 0.84222 | 1.84738 | 612.565 | 745.974 | －1．354 | 0.221 |
| 625．0 | 1.00453 | 0.82433 | 1.82723 | 617.444 | 802.304 | －2．141 | 0.224 |
| 630．11 | 1.00307 | 0.80605 | 1.80616 | 622.336 | 808.918 | －2．933 | 0.223 |
| 635．01 | 1.00157 | 0.78763 | 1.78422 | 627.242 | 815.841 | $-3.785$ | 0.220 |
| 640．．1 | 1.00006 | 0.7693 ？ | 1.76145 | 632.162 | 823.045 | －4．752 | 0.213 |
| 645．1） | 0.94854 | 0.75143 | 1.73791 | 637.096 | 830.698 | －5．836 | 0.202 |
| 650．1 | 0.94704 | 0.73433 | 1.71360 | 642.045 | 83 H .665 | －7．041 | 0.188 |
| － |  | －－ | － |  |  |  | －－ |


| 655.01 | *.0.0nors | U.718in | 1.tanst | 647.1101 | H4t.030 | -H.3ns | 0.110 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AhOUll |  | charer | Le.6.264 | 6hleyti | Rusutila | -4.8u1 | 1 ClaH |
| 6n5.0 | 11.0420. | 0.61145 | 1.6503\% | 656. 714 | H04.174 | -11.347 | 0.122 |
| 670.0 | U.9ylis | 0.04126 | 1.60411 | col. 411 | H/4.Ves | -12.4ys | 0.045 |
| 075.1 | 0.9700: | 11.67574 | 1.58731 | 666. 414 | HE S.f.0. 7 | -14.124 | 0.0020 |
| 6AO.U | 0.98881 | U.60417 | 1.55514 | 672.021 | 4.3.403 | -16.526 | 0.025 |
| 683.0 | 0.9 H/61 | 0.61767 | 1.327 ml | 677.061 | 713.24n | -1R.Anl | -0.012 |
| 690.0 | 0.98603 | 0.00498 | Labulut | 642.111 | 11s.vou | -20.210 | -U.0.0 |
| 694.0 | O. 0 Hstas | 0.0.1416 | 1.473114 | 617.112 | 4/1.nss | -28.118 | -0.10ny |
| 100.0 | U. 78484 | U. 68.186 | 1.45022 | 042.240 | + 32.400 | -24.06) | -0.120 |

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SUNBLATER HREIT


PFRITIU SFNS TG INJECT VIL $=20.54$ LAYS PFR/KM/SFG VSHSIII=U.IIU7A IPAE U.U

| TIMF | KFR | RSi: | UISt | FHR | 1 1p | AzIMUTII | elevation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.10 | 0.98417 | 0.94557 | 0.00610 | 3.241 | ?. H9O | -46.924 | 4.017 |
| 5.0 | U. 98560 - | 0.48478 | 0.01550 | 8.311 | 1.414 | -86.540 | 4.102 |
| 10.0 | 11.98651 | 0.9H2'5 7 | 0.02418 | 13.312 | 11.951 | -80.1771 | 4.103 |
| 15.0 | U. 98.857 | $0.978{ }^{0} 9$ | 11.03341 | 14.423 | 16.519 | -14.180 | 4.107 |
| 20.0 | 0.48870 | 0.97376 | U.04285 | 23.463 | 21.176 | -68. 112 | 4.073 |
| 250 | 0.98941 | 0.90 .112 | 0 | 28.443 | 25.769 | $-62.461$ | 4.079 |



| 90.0 | 1.00859 | 0.77684 | 0.25550 | 92.657 | 94.606 | 21.593 | 1.497 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 95.0 | 1.00488 | 0.75845 | 0.29079 | 91.500 | 107.063 | 25.685 | 1.247 |
| 100.0 | 1.01108 | 0.74064 | 0.3301 | 107.331 | 114.884 | 29.192 | 1.018 |
| 105.0 | 1.01221 | 0.12380 | 0.37341 | 101.151 | 183.078 | 32.144 | 0.812 |
| 110.0 | 1.01325 | 0.70447 | 0.42051 | 111.967 | 131.647 | 34.580 | 0.629 |
| 115.1 | 1.01419 | 0.64484 | 0.4714 | 116.764 | 140.574 | 36.543 | 10.408 |


| 120.0 | 1.01503 | 0.68341 | 0.5248 H | 121.553 | 149.830 | 38.079 | 0.327 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 125.0) | 1.01576 | 0.67455 | 0.58119 | 126.344 | 159.365 | 39.233 | 0.205 |  |
| 130.0 | 1.01638 | 0.668356 | 0.63945 | 131.12' | 109.112 | 40.055 | 0.099 |  |
| -35.0 | 1.01688 | 0.66569 | 0.69891 | 135.901 | 118.990 | 40.542 | 0.008 |  |
| 140.0 | 1.01726 | 0.60604 | 0.75880 | 140.618 | 188.905 | 40.894 | -0.008 |  |
| 145.1 | 1.01751 | 0.669 .59 | $0 . \mathrm{HLR36}$ | 145.442 | 148.763 | 41.010 | -0.132 |  |
| 150.0 | 1.01764 | 0.61621 | (1. 27685 | 151.210 | 208.471 | 40.984 | -0.1 04 |  |
| 155.0 | 1.01764 | 0.683665 | 0.93360 | 154.976 | 217.0122 | 40.860 | -0.226 |  |
| 160.0 | 1.01752 | 0.69758 | 0.98910 | 153.744 | 227.141 | 40.673 | -0.25\% |  |
| 165.0) | 1.01727 | 0.71162 | 1.03949 | 164.513 | 235.993 | 40.457 | -0.284 |  |
| 170.0 | 1.01690 | 0.72731 | 1.08867 | 169.284 | 244.483 | 40.236 | -0.301 |  |
| 175.0 | 1.01641 | 0.74440 | 1.13423 | $17+.0611$ | 25?.597 | 40.031 | -0.313 |  |


| 180.0 | 1.01579 | 0.76 .234 | 1.17645 | 178.841 | 260.339 | 39.856 | -0.319 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 185.0 | 1.01507 | 0.78081 | 1.21527 | 183.627 | 261.719 | 39.723 | -0.321 |  |
| 190.0 | 1.01423 | 0.74950 | 1.25061 | 188.421 | 274.756 | 39.637 | -0.319 |  |





| 540.0 | 1.01643 | 0.98370 | 1.49984 | 513.828 | 715.767 | -10.953 | -0.014 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 545.0 | 1.01583 | 0.98528 | 2.00089 | 538.609 | 720.249 | -0.834 | 0.001 |  |
| 550.0 | 1.01511 | 0.98539 | 2.00034 | 543.395 | 724.822 | -0.704 | 0.021 |  |
| 555.0 | 1.01428 | 0.98402 | 1.99819. | 548.188 | 729.351 | -0.575 | 0.040 |  |
| 560.0 | 1.01335 | 0.48118 | 1.99446 | 552.484 | 733.844 | -0.451 | 0.059 |  |
| 565.0 | 1.01232 | 0.97688 | 1.98916 | 557.799 | 738.480 | -0.333 | 0.018 |  |


| 570.1 | 1.01120 | 0.97113 | 1.98230 | 562.618 | 743.109 | -0.244 | 0.046 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 575.0 | 1.01000 | 0.96395 | 1.97394 | 567.448 | 747.801 | -0.174 | 0.114 |
| 580.0 | 1.00872 | 0.95538 | 1.96409 | 572.290 | 752.569 | -0.149 | 0.131 |
| 585.0 | 1.00738 | 0.94544 | 1.95281 | 577.143 | 757.432 | -0.144 | 0.141 |
| 590.0 | 1.00599 | 0.93418 | 1.94014 | 582.009 | 762.404 | -0.201 | 0.162 |
| 595.0 | 1.00455 | 0.92165 | 1.92616 | 586.887 | 767.504 | -0.301 | 0.176 |


| 600.0 | 1.00307 | 0.90792 | 1.91091 | 541.780 | 7772.752 | -0.464 | 0.189 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 605.0 | 1.00151 | 0.87301 | 1.89447 | 576.686 | 778.168 | -0.701 | 0.200 |  |
| 610.0 | 1.00006 | 0.87720 | 1.87691 | 601.606 | 783.715 | -1.013 | 0.210 |  |
| 615.0 | 0.99854 | 0.86042 | 1.85830 | 606.540 | 789.594 | -1.412 | 0.217 |  |
| 620.0 | 0.99704 | 0.84288 | 1.83870 | 611.489 | 745.650 | -1.906 | 0.222 |  |
| 625.0 | 0.99555 | 0.82475 | 1.81819 | 616.451 | 801.968 | -2.500 | 0.225 |  |


| 630.0 | 0.99410 | 0.80621 | 1.79683 | 621.427 | 808.574 | -3.200 | 0.224 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 635.0 | 0.99269 | 0.78752 | 1.77468 | 626.418 | 815.492 | -4.013 | 0.221 |
| 640.0 | 0.99133 | 0.76893 | 1.75179 | 631.421 | 822.747 | -4.945 | 0.214 |
| 645.0 | 0.99003 | 0.75076 | 1.72819 | 636.437 | 830.357 | -5.999 | 0.204 |
| 650.0 | 0.98881 | 0.73336 | 1.70394 | 041.465 | 838.337 | -7.176 | 0.184 |


| 655．0） | 0.48767 | 11.71711 | 1.57915 | n4n．tus | A46．0．172 | －8．416 | 0.171 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6600 0 | Canatis | 0．10241 | 1－643641 |  | A．15．417 | －1．846 | 1014 |
| 665.0 | 1）．9650t | 11.00407 | 1.627511 | 6＇56．615 | H64．4H7 | －11．426 | 0.123 |
| 670.0 | 0.98484 | 0.67929 | 1.60115 | 601.684 | 813．811 | －13．062 | 0.0 .85 |
| 675.0 | 0.98417 | $0.6116 \%$ | 1.5744 ？ | 6.66 .760 | H83．607 | －14．1110 | 0．02 1 |
| 680．1 | －0．94352 | 0.66695 | 1.54754 | 611．845 | 843.320 | －16．585 | 0.025 |
| 685.0 | 0.911304 | 0.66544 | 1．920：2 | 676.431 | 903．325 | －1H．4 SH | －0．01？ |
| 690.0 | 0.98270 | 0.66717 | 1．41411 | ．682．023 | 913.127 | －20．328 |  |
| 645.0 | 0.48244 | 0.612015 | $1.46,434$ | 687.114 | 92.2911 | －27．231 | －0．064 |
| ．．． 700.1 | 0.93241 | 0.61 リッ1 | 1．44016 | 692.214 | 932．551 | －24．130 | －0．1く1 |

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## SUNHLA/FR TIRBIT

TINITE $0 . \overline{0}$ TDFLTE S.O TFINL $=700.0$ TAETIE 61.0 SFMI MAJOR AXIS=0̈.62548I7T

## FCCFNTRICITY=(0.20157915 INCIINATION= O.JONU SA INJECTIIN VELQGITY=U.89719

PFRIOD SFNS TO INJECT VILE 20.42 UAYS PFR/KMIAFC VSBSIIIEU. 11213 TPAE U.O


| 120.0 | 1.01765 | 0.67810 | 0.54124 | 119.832 | 147.225 | 37.743 | 0.320 |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 125.0 | 1.01763 | 0.66872 | 0.59875 | 124.549 | 1.28 .904 | 39.012 | 0.201 |
| 130.0 | 1.01748 | 0.66239 | 0.65812 | 129.367 | 108.813 | 39.753 | 0.048 |
| 135.0 | 1.01721 | 0.65935 | 0.71864 | 134.136 | 178.863 | 40.216 | 0.009 |
| 140.0 | 1.01681 | 0.65971 | 0.77946 | 138.909 | 188.454 | 40.450 | -0.006 |
| 145.0 | 1.01629 | 0.66347 | 0.83980 | 143.685 | 198.982 | 40.503 | -0.128 |


| 150.0 | 1.01566 | 0.67048 | 0.89888 | 148.467 | 208.849 | 40.424 | -0.180 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 155.0 | 1.01491 | 0.68046 | 0.95603 | 153.255 | 218.468 | 40.254 | -0.221 |
| 160.0 | 1.01406 | 0.64305 | 1.01070 | 158.049 | 227.773 | 40.032 | -0.254 |
| 165.0 | 1.01310 | 0.70784 | 1.06246 | 162.853 | 236.718 | 39.790 | -0.278 |
| 170.0 | 1.01205 | 0.72438 | 1.11101 | 167.665 | 245.274 | 39.554 | -0.246 |
| 175.0 | 1.01091 | 0.74224 | 1.15615 | 172.487 | 253.433 | 39.345 | -0.308 |

$\left.\begin{array}{lllllllll}180.0 & 1.00969 & 0.76101 & 1.19780 & 177.320 & 261.198 & 39.116 & -0.314 \\ 185.0 & 1.00839 & 0.78029 & 1.23591 & 182.164 & 268.584 & 39.058 & -0.316 \\ 190.0 & 1.00704 & 0.74976 & 1.27050 & 187.020 & 275.612 & 38.997 & -0.313\end{array}\right)$



| 655.0 | 0.4829 | 0.1818 .1 | 1.67394 | 64t.0.1N | 0498836 | - $\mathrm{H} .000 \%$ | 0.113 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -660.0. |  | chathather | 1-6atic3 | -64.111 | sumatiz | -9,644 | 01015 |
| 665.0 | 0.914246 | U. bram 11 | 1.62756 | 6, 6.0 M(16 | Pt3.H12 | -11.047 | 0.11: |
| 670.0 | 0.98241 | U.67374, | 1.59641 | 601.903 | 8\%3.326 | -12.711 | U. $1: 6$ |
| 615.1 | 11.914244 | 0.60369 | 1.57007 | 666.999 | H63.112 | -14.410 | 0.160 |
| 680. 1 | $0.93<70$ | 0.06068 | 1.54544 | 672.094 | $8 \times 3.094$ | -16.310 | 0.026 |
| 685.0) | 0.74305 | $0.6{ }^{\circ} \mathrm{P} 410$ | 1.51695 | 677.18 He | 903.174 | -14.20H | -0.01? |
| 690.14 |  | $0.6 .60 y 1 ~$ | 1.49087 | 082.274 | 913.25c | -20.143 | -0.03) |
| 695.0 | U.9+443 | 0.60 hun | 1.46557 | 687.351 | 42.1.214 | -27.047 | -0.014 |
| . 200.0 | U.98485 | 0.6744 L | 1.44143 | 6)2.433 | n32.94y | -24.043 | -0.121 |

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SUNALALER TRPII
TINITE 0.0 TDELTE 5.0 TIINLE 700.0 TBEGE YI.O SEMI MAJUR AXISEO.H2S48IT7
ECCFNTRICIIYE0.2116432H IHCLINATINN: 0.5000 SA INJECTION VELOCITYEO.BATMI
PERIIIN SENS TO INJECT VILE 2C. 21 UAYS PLR/KM/SEC VSUSOIEO.11340 IPAE J.O

| IIME | RER | HSO | UIST | FER | -90 | ALIMUIII | Elevarion |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | $1.001: 21$ | 1.00019 | 0.00618 | 3.120 | 2.771 | -H9. 5 HH | 3.917 |
| 5.0 | $1.00173=$ | 0.94941 | 0.01574 | 8.038 | 7.148 | -81.015 | 3.744 |
| 10.0 | 1.00323 | $0.9470 n$ | 0.02532 | 12.943 | 11.518 | -75.172 | 3.442 |
| $-15.0$ | 1.00470 | 0.99316 | 0.01419 | 17.834 | 13.956 | -69.645 | 3.947 |
| 20.0 | 1.00613 | 0.44771 | 0.04408 | <2.711 | 211.416 | -64.087 | 3.912 |
| 25.0 | 1.00752 | 0.98074 | 0.05324 | 27576 | 26.431 | -58, 3116 | 18846 |


| 30. 0 | $1.008 \mathrm{H6}$ | 0.97225 | 0.06231 | 32.426 | 29.521 | -52.4H3 | 3.847 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35.0 | 1.01012 | 0.96229 | 0.01144 | 31.268 | 34.147 | -46.325 | 3.788 |  |
| 40.0 | 1.01132 | $0.950 \mathrm{H9}$ | 0.08080 | 42.096 | 3 4.978 | -39.911 | 3.704 |  |
| 4500 | 1.01243 | 0.91802 | 0.09067 | 46.915 | 43 1983 | -33.260 | 3.588 |  |
| 50.0 | 1.01345 | 0.92395 | 0.10137 | 51.724 | 4A.931 | -26.42\% | 3.438 |  |
| 55.0 | 1.01431 | 0.90854 | 0.11330 | . 26.524 | 54.143 | -19.497 | 3.231 |  |
| 60.0 | 1.01519 | 0.84145 | 0.12692 | 61.316 | 59.542 | -12.600 | 3.030 |  |
| 65.0 | 1.01584 | 0.87428 | 0.1426 h | 66. 102 | 65.163 | -6.833 | 2.182 |  |
| 70.0 | 1.01644 | 0.85567 | 0.16101 | 10.882 | 71.001 | n.658 | 2.513 |  |
| 75.0 | 1.01646 | 0.81626 | 0.18238 | 75.657 | 11.116 | 6.107 | 2.235 |  |
| 80.0 | 1.01732 | 0.81626 | C. 20716 | 80.424 | 83.526 | 12.301 | 1.958 |  |
| 85.0 | 1.01755 | 0.79588 | 0.23564 | 85.147 | 40.262 | 17.359 | 1.684 | , |


| 90.0 | 1.01765 | 0.77534 | 0.26806 | 89.964 | 97.354 | 21.850 | 1.435 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 95.0 | 1.01763 | 0.15512 | 0.30458 | 94.731 | 104.828 | 25.770 | 1.198 |
| 100.0 | 1.01748 | 11.75543 | 0.34528 | 99.498 | 117.710 | 29.136 | 0.482 |
| 105.0 | 1.01721 | 0.71672 | 0.39010 | 104.268 | 121.015 | 11.969 | 0.787 |
| 110.0 | 1.01681 | 0.64946 | 0.43891 | 104.040 | 129.747 | 34.302 | 0.613 |
| 1150 | 1.01624 | 0.64413 | 0.49140 | 113.817 | 138.897 | 36.169 | 0.428 |



| 180.1 | 1.00120 | 0.75942 | 1.20970 | 176.708 | 262.343 | 38.751 | -0.311 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 185.0 | 0.94968 | 0.77976 | 1.24730 | 181.631 | 269.714 | 38.668 | -0.313 |  |
| 190.0 | 0.99817 | 0.80025 | 1.28129 | 186.569 | 276.747 | 38.649 | -0.310 |  |



| 420.0 | 1.01494 | 0.610641 | 1.59814 | 416.292 | 561.463 | 15.575 | -0.015 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| +25.0 | 1.01515 | -0.66507 - | 1.62695 | 421.084 | 511.441 | 11.641 | -0.141 |  |
| 430.0 | 1.015 \% | 0.67840 | 1.63486 | 425.470 | 501.226 | 4. HI 13 | -0.135 | \% |
| 43500 | 1.01646 | N-6itus | 1.68117 | 410.650 | 540.6113 | 8.020 | -10.124 |  |
| $\begin{aligned} & 440.0 \\ & \$ 45.0 \end{aligned}$ | $\begin{aligned} & 1.01644 \\ & 1.017 .30 \end{aligned}$ | $\begin{aligned} & 0.70661 \\ & 0.72430 \end{aligned}$ | $\begin{aligned} & 1.70764 \\ & 1.73247 \end{aligned}$ | $\begin{aligned} & 435.421 \\ & 440.197 \end{aligned}$ | $\begin{aligned} & 3.94 .517 \\ & 608.127 \end{aligned}$ | $\begin{aligned} & 6.484 \\ & 5.018 \end{aligned}$ | $\begin{aligned} & -0.118 \\ & -0.114 \end{aligned}$ |  |
|  |  |  |  |  |  |  |  |  |
| 450.0 | 1.01754 | 0.74374 | 1.75630 | 444.965 | 616.250 | 3.680 | -0.206 |  |
| 45500 | 1.01765 | 0.76174 | 1.77015 | $4 \times 4.131$ | 623.225 | 2.471 | -0.214 |  |
| $460.1)$ | 1.01763 | 11.78414 | 1.80105 | 454.500 | 611.262 | 1.408 | -0.21H |  |
| 465.0 | $1.01 / 49$ - | 0.80461 | 1.82201 | 459.267 | 634.148 | 0.412 | -0.219 |  |
| 470.0 | 1.01727 | 11.82487 | i. 84203 | 464.036 | 644.742 | -0.342 | -0.216 |  |
| 415.0 | 1.01643 | U.8446\% | 1.86110 | 468.809 | 6)1.072 | -1.028 | -0.212 |  |
| 480.0 | 1.01632 | 0.86373 | 1.87917 | 473.585 | 657.070 | -1.601 | -0.205 |  |
| -485.0 | 1.01569 | U. 86196 | 1.89621 | 47 H. 366 | 662.813 | -2.067 | $-0.145$ |  |
| 490.1 | 1.014 .95 | 0.89918 | 1.91217 | 483.154 | 6OH. 350 | -2.433 | -0.104 |  |
| -495.11 | 1.01410 | 0.9132 H | 1.42698 | 487.948 | .673.646. | -2.704 | -0.112 |  |
| 5000 | 1.01315 | 0.93015 | 1.94060 | 492.751 | 67A.786 | -2.469 | -0.154 |  |
| 50500 | 1.01210 | 10.94373 | 1.95295 | 497.563 | 683.770 | -2.996 | $-0.141$ |  |
| 510.0 | 1.01046 | 0.95548 | 1.96398 | 502.385 | 688.614 | -3.031 | -0.127 |  |
| 515.0 | 1.00975 | 0.46673 | 1.97364 | 507.217 | 643.354 | $-3.003$ | -0.110 |  |
| 520.0 | 1.00846 | 0.97606 | 1.98186 | 512.060 | 647.970 | $-2.917$ | -0.0.02 |  |
| 525.0 | 1.00711 | 0.98190 | 1.98860 | 516.916 | 702.546 | $-2.783$ | -0.014 |  |
| $\begin{array}{r} 530.0 \\ 535.0 \end{array}$ | 1.00570 1.00425 | 0.94023 0.94502 | 1.99383 1.99751 | 521.784 526.666 | 707.056 711.477 | -2.606 -2.345 | -0.036 -0.037 |  |


| 540.0 | 1.00277 | 0.94825 | 1.99960 | 531.561 | 715.881 | -2.150 | -0.018 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 545.0 | 1.00127 | 0.99993 | 2.00010 | 536.464 | 720.265 | -1.847 | 0.001 |  |
| 550.0 | 0.94976 | 1.00005 | 1.99900 | 541.393 | 724.639 | -1.674 | 0.020 |  |
| 555.0 | 0.99824 | -0.94860 | 1.99629 | 546.329 | 729.021 | -1.347 | 0.094 |  |
| 560.0 | 0.99674 | 0.94558 | 1.99197 | 551.281 | 733.421 | -1.069 | 0.058 |  |
| 565.0 | 0.99526 | 0.94102 | 1.98608 | 536.246 | 737.855 | -0.402 | 0.076 |  |


| 570.0 | 0.99381 | 0.98492 | 1.97863 | 561.225 | 742.337 | -0.555 | 0.045 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 575.0 | 0.94241 | 0.97730 | 1.96967 | 566.211 | 746.086 | -0.330 | 0.112 |
| 580.0 | 0.99106 | 0.96418 | 1.95923 | 571.224 | 751.566 | -0.140 | 0.124 |
| 585.0 | 0.98978 | 0.95760 | 1.94737 | 576.243 | 756.225 | 0.059 | 0.145 |
| 590.0 | 0.98858 | 0.94559 | 1.93416 | 581.273 | 761.056 | 0.108 | 0.161 |
| 595.0 | 0.98746 | 0.93221 | 1.91965 | 586.315 | 766.020 | 0.153 | 0.175 |


| 600.0 | 0.98643 | 0.91752 | 1.90394 | 591.367 | 771.135 | 0.124 | 0.184 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 605.0 | 0.96551 | 0.90160 | 1.88709 | 596.429 | 776.424 | 0.071 | 0.0149 |  |
| 610.0 | 0.98469 | 0.84453 | 1.86919 | 601.500 | 781.911 | -0.190 | 0.209 |  |
| 615.0 | 0.98399 | 0.86644 | 1.85034 | 606.577 | 787.620 | -0.487 | 0.216 |  |
| 620.0 | 0.78342 | 0.84747 | 1.83061 | 611.661 | 793.579 | -0.887 | 0.222 |  |
| 625.0 | 0.98247 | 0.82778 | 1.81009 | 616.750 | 799.816 | -1.401 | 0.225 |  |


| 630.0 | 0.98265 | 0.80759 | 1.78884 | 621.843 | 806.361 | -2.0338 | 0.225 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 635.0 | 0.98246 | 0.78713 | 1.76692 | 626.938 | 813.246 | -2.805 | 0.222 |
| 640.0 | 0.98241 | 0.76669 | 1.74434 | 632.035 | 820.447 | -3.709 | 0.216 |
| -665.0 | 0.98249 | 0.74662 | 1.72126 | 637.131 | 828.143 | -4.753 | 0.206 |
| 650.0 | 0.98270 | 0.72729 | 1.69756 | 642.220 | 836.204 | -5.940 | 0.142 |


| 655.0 | (1).04303 | $0.71+17$ | 1.6732H | 647.314 | 844.642 | -7.211 | 0.174 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6ache 0 | 0.48363 | 10.01264 | 1-64841 | 652.406 | He_Lebus | $-1.740$ | 0.152 |
| 665.0 | 0.98413 | 0.67824 | 1.62302 | 657.489 | 867.925 | -10.341 | 0.126 |
| 620.0 | 0.984E5 | 0.66653 | 1.59706 | 662.565 | 412.012 | -12.062 | 0.046 |
| 675.0 | 0.7856 | U.6.1 Hz | 1.57 Ue 7 | 647.633 | HEP.tivi | $-13.808$ | 0.043 |
| .680.0 | 0.9866s | 0.65249 | 1.54198 | 472.613 | 8.82 .805 | -15.800 | 0.026 |
| 685.0) | 0.987 CP | 0.63018 | 1.51184 | 677.743 | 913.1/1 | -17.775 | -0.012 |
| 690.0 |  | 0.65214 | --1.4.3016 | 682.743 | 913.412 | -19.7.91 | -0.0.21 |
| 695.0 | 0.99014 | 0.65811 | 1.46472 | 647.411 | 923.623 | -21.422 | -0.0.40 |
| 700.0 | 0.94131 | U. Cu 124 | 1.44015 | 692.821 | 933.594 | -23.847 | -0.128 |

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## SUNBLAZFK NKBII



## ECCENTKICITVEU.22IAGYち3 INCI.INATINNE O.5000 SB INJECIIDN VELIEITYEO.87833

PERIOD SENS IN INJFLI VEL $=19.94$ UAYS PEK/KM/SFL VSHSOI=0.11415 TPA= 0.0


| 90.0 | 1.01552 | 0.77481 | 0.26305 | 89.232 | 96.070 | 20.538 | 1.465 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 95.0 | 1.01475 | 0.75345 | 0.29905 | 99.021 | 103.550 | 29.665 | 1.225 |
| 100.0 | 1.01387 | 0.73263 | 0.33936 | 98.818 | 111.460 | 28.202 | 1.005 |
| 105.0 | 1.01290 | 0.71280 | 0.38398 | 103.623 | 119.823 | 31.197 | 0.805 |
| 110.0 | 1.01183 | 0.64445 | 0.43217 | 108.437 | 128.647 | 33.669 | 0.627 |
| 115.0 | 1.01067 | 0.67809 | 0.48546 | 113.261 | 137.924 | 35.654 | 0.408 |


| 120.0 | 1.00943 | 0.66427 | 0.54161 | 118.096 | 147.624 | 37.191 | 0.328 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 125.0 | 1.00813 | 0.65348 | 0.60062 | 122.943 | 157.690 | 38.325 | 0.207 |  |
| 130.0 | 1.00676 | 0.64516 | 0.66175 | 127.801 | 168.039 | 39.106 | 0.101 |  |
| 135.0 | 1.00535 | 0.64264 | 0.72411 | 132.673 | 178.564 | 39.586 | 0.011 |  |
| 140.0 | 1.00389 | 0.64306 | 0.78677 | 137.558 | 189.139 | 39.820 | -0.065 |  |
| 145.0 | 1.00240 | 0.64742 | 0.84880 | 142.456 | 199.636 | 39.865 | -0.128 |  |
|  |  |  |  |  |  |  |  |  |
| 150.0 | 1.00049 | 0.65550 | 0.90933 | 141.368 | 209.933 | 39.776 | -0.180 |  |
| 155.0 | 0.99938 | 0.66698 | 0.96762 | 156.295 | 219.927 | 39.600 | -0.221 |  |
| 160.0 | 0.99787 | 0.68139 | 1.02308 | 157.235 | 229.541 | 39.383 | -0.253 |  |
| 165.0 | 0.99637 | 0.69822 | 1.07527 | 122.190 | 238.723 | 39.159 | -0.277 |  |
| 170.0 | 0.99490 | 0.71693 | 1.12391 | 167.159 | 247.448 | 38.957 | -0.295 |  |
| 175.0 | 0.99346 | 0.73701 | 1.16883 | 172.142 | 255.713 | 38.798 | -0.306 |  |

$\left.\begin{array}{lllllllll}\hline 180.0 & 0.99207 & 0.75798 & 1.20999 & 177.138 & 263.529 & 38.695 & -0.311 \\ 185.0 & 0.99074 & 0.77941 & 1624739 & 182.147 & 270.920 & 38.658 & -0.312 \\ \hline 190.0 & 0.98947 & 0.80092 & 1.28108 & 187.168 & 277.914 & 38.691 & -0.310\end{array}\right)$

| $\begin{aligned} & 195.0 \\ & 200.0 \\ & 205.0 \end{aligned}$ | $\begin{aligned} & U .9 H 8<9 \\ & 0.98119 \\ & 0.9 H 219 \end{aligned}$ | $\begin{aligned} & 0.82222 \\ & 0.84301 \\ & 0.36309 \end{aligned}$ | $\begin{aligned} & 1.31111 \\ & 1.31776 \\ & 1.30098 \end{aligned}$ | $\begin{aligned} & 147.202 \\ & 1+1.246 \\ & 707.301 \end{aligned}$ | $\begin{aligned} & 2114.543 \\ & 290.81,2 \\ & 2.16 . H 41 \end{aligned}$ | $\begin{aligned} & 3 H .7 \% \\ & 38.410 \\ & 33.210 \end{aligned}$ | $\begin{aligned} & -v .311 \\ & -v .2 y 4 \\ & -0.213 s \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.10 .0 | 0.98524 | U.88228 | 1.38097 | 207.36 | 302.57s | 34.512 | -0.204 |
| 215.0 | 0. ${ }^{\text {OHA }}$ | 0.9104 ? | 1.34785 | 212.431 | 104.06n | 19.80H | -0.254 |
| 220.0. | 0.98384 | 0.91738 | 1.41178 | 217.511 | 313.353 | 40.275 | -0.230 |
| 225.0 | 0.983? | (1) प131) | $1.4 \% 767$ | 222.60\% | 31A.45? | 40.714 | -0. 217 |
| 23011 | 10.4828 | 10.14141 | 104.1124 | $22106+2$ | 3216.364 | 41.196 | -0.121 |
| 235.0 | 0.9132 づ | $0.4(1) 31$ | 1.43717 | 232.786 | 32A.1H6 | 41.709 | -0.116 |
| 240.0 | 0.98241 | 0.91184 | 1.44006 | 211.881 | 312.803 | $42.22 \%$ | -0.154 |
| 245.0 | ().98241 | 0.93181 | 1.44142 | 242.973 | 317.448 | 47.733 | -0.131 |
| 2500 | 1 | 18.4029 | 1044101 | 24801674 | 1416421 | 4.282 | -0.147 |
| 255.0 | 1).48278 | 0.47710 | 1.43833 | 253.1ns | 346.14 H | 43.80 ) | -0.002 |
| 260.0. | 11.98316 | 1.00238 | 1.43381 | 253.2'9 | 350.113 | 44.31) 3 | -0.0.06 |
| 265.0 | 10.48366 | 1.00606 | 1.42771 | 203.346 | 355.043 | 44.771 | -0.0.30 |
| 27000 | 11.98430 | 13 | 1.42022 | 208.421 | 129.341 | 45.216 | -0.0us |
| 275.11 | 0.98503 | 1.00854 | 1.41151 | 273.502 | 363.640 | 45.6116 | $0.0<3$ |
| 280.0 | 0.98591 | 1.00744 | .1.40179 | 278.508 | 307.956 | 45.942 | 0.020 |
| 245.0) | 0.9868 H | 1.00464 | 1.39127 | 213.620 | 312.248 | 46.214 | 0.011 |
| 290.0 | - U. 98785 | 1.00052 | 1.38010. | 248.673 | 376.511 | 46.411 | 0.103 |
| 295.0 | U.98911 | 0.99436 | 1.36870 | 243.710 | 350.974 | 46.525 | 0.130 |
| 300.0 | 0.79035 | 0.98683 | 1.35713 | 298.736 | 385.427 |  | 0.156 |
| 305.0 | 0.94167 | 0.97714 | 1.34571 | 303.744 | 389.951 | 46.460 | 0.161 |
| -310.0 | 0.99304 | 0.90113 | 1.31468 | 108.149 | 314.507 | 46.276 | 0.246 |
| 315.0 | 0.99446 | $0.9550 ?$ | 1.32434 | 313.735 | 399.273 | 45.9179 | 0.828 |
| 32000 | 0.94593 | 0.94147 | 1.31495 | 318.708 | 404.148 | 45.537 | 0.244 |
| 325.0 | 0.94742 | 0.92654 | $1.306 \mathrm{H2}$ | 323.667 | 409.152 | 44.975 | (1.20t |
| 330.0 | 0.99893 | 0.91429 | 1.30021 | 328.612 | 414.321 | 44.278 | 0.284 |
| 335.0 | 1.00044 | 0.89280 | 1.29544 | 333.543 | $419.6+8$ | 43.442 | 0.298 |
| 3400 | 1.010146 | 0.817420 | 1.29270 | 338.454 | 425.211 | 42.467 | 0.301 |
| 345.0 | 1.00345 | 0.85462 | 1.29251 | 343.367 | 431.134 | 41.353 | 0.313 |
| 350.0 | 1.00492 | 0.81420 | 1.29486 | 148.250 | 43.7 .2 .7 | -40.101 | 0.314 |
| 355.0 | 1.00635 | 0.81317 | 1.30000 | 353.126 | 443.692 | 38.714 | 0.311 |
| 3600 | 1.0077 | 0.79174 | 1.30827 | 357.984 | 4120.413 | 37.200 | 0.303 |
| 365.0 | 1.00905 | 0.71022 | 1.31960 | 362.838 | 457.633 | 35.504 | 0.289 |
| 370.0 | 1.01031 | 0.74844 | 1.33411 | 367.677 | . 465.202 | 33.816 | 0.271 |
| 375.0 | 1.01144 | 0.72810 | 1.35174 | 372.504 | 413.208 | 31.965 | 0.244 |
| . 380.0 | 1.01259 | 0.70814 | 1.37235 | 317.321 | 4111.66? | 30.021 | 0.220 |
| 385.0) | 1.01359 | 0.61077 | 1.39571 | 382.128 | 490.591 | 27.998 | 0.188 |
| 390.0 | 1.01450 | 0.61491 | 1.42146 | 386.927 | 499.962 | 25.909 | 0.153 |
| 395.0 | 1.01530 | 0.66169 | 1.44915 | 391.719 | 509.746 | 23.769 | 0.115 |
| 400.0 | 1.01599 | 0.65161 | 1.47827 | - 396.503 | 514.880 | 21.548 | 0.016 |
| 405.0 | 1.01657 | 0.64509 | 1.50825 | 401.282 | 530.277 | 19.415 | 0.036 |
| 410 | 1.01702 | 0.64240 | 1.53857 | 406.057 | 540.823 | 17.244 | -0.003 |
| 415.0 | 1.01736 | 0.64367 | 1.56871 | 410.827 | 551.392 | 15.108 | -0.041 |



| 055.0 | 0.4A741) | (1).71475 | 1.67836 | 648.429 | 843.536 | -6. $2: 11$ | 0.175 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -660.010 |  | 068:114 | 1-6.1724 | 661.401 | H42.564 | -7.816 |  |
| 665.1) | 0.99074 | U.6/1H5 | 1.623'37 | 6511.4.t2 | $802.01 / 8$ | -9.47M | 0.17 .1 |
| 610.0 | 0.99100 | 0.65926 | 1.60257 | 003.506 | B11. Hod | -11.213 | 0.097 |
| $675.1)$ | 0.99217 | $0.61 .9 \% 1$ | 1.51544 | 668.507 | H42.00l | -13.1n3 | 0.063 |
| 680.0 | 0.94434 | 0.64418 | 1.54889 | 073.4 | 8.82 .514 | -1\%.102 | 0.027 |
| 685.0 | 0.7 (1) | 0.64234 | 1.52152 | 678.4611 | 903.1102. | -17.247 | -0.011 |
| 690.11 | U.94115 | 0.64445 | 1.44421 | 683.427 | 913.639 | -19.354 | -0.0.01 |
| 695.0 | O.9y88n | 0.67 (1) | 1.46738 | 6B4.373 | 9く4.001 | -21.471 | -0.0.00 |
| -100.0 | 1.cods 7 | 0.64001 | 1.44146 | 641.304 | 9 34.242 | $-23.5+2$ | -0.129 |

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$\qquad$ ....

SUNAIAIRR IIRBIT
TINIT $=$ U.O TDFI.TE 5.0 IFINLE 700.0 IBEGEISP.I SEMI MAJOR AXISEU.A254甘I77



| TIME. | NFR | RSA | IJST | FER | - So | AzImutil | flfVAIION |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.11 | 1.01514 | 1.01431 | 0.00625 | 3.044 | 2.647 | -92.374 | 3.A21 |
| 5.10 | 1.01602 . | 1.01110 | _0.01614 | 1.834 | 6.914. | -81.210 | 9.145 |
| 10.11 | 1.01659 | 1.01124 | 0.02587 | $12.61 \%$ | 11.185 | -17.312 | 3.743 |
| 15.0 | 1.01704 | 1.00713 | 0.03542 | 17.386 | 15.464 | -72.771 | 3.748 |
| 20.11 | 1.01717 | 1.00140 | 0.04487 | 22.151 | 1\%. 784 | -67.908 | 9.180 |
| 25.0 | 1017758 | 0.42405 | 0.05404 | 26.423 | 24014 | -62.744 | 3.101 |



| 90.0 | 1.00943 | 0.77451 | 0.25252 | 89.194 | 45.178 | 18.654 | 1.527 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 95.0 | 1.00813 | 0.75238 | 0.28738 | 94.041 | 102.659 | 23.104 | 1.277 |
| 100.0 | 1.00676 | 0.73078 | 0.32669 | 98.900 | 110.587 | 26.951 | 1.047 |
| 105.0 | 1.00535 | 0.71015 | 0.37047 | 103.771 | 118.988 | 30.212 | 0.838 |
| 110.0 | 1.00389 | 0.69102 | 0.41862 | 108.656 | 127.874 | 32.416 | 0.652 |
| 115.0 | 1.00240 | 0.67393 | 0.47087 | 113.555 | 117.234 | 15.047 | 0.406 |


| 120.0 | 1.00089 | 0.65946 | 0.52681 | 118.467 | 147.053 | 36.798 | 0.340 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 125.0 | 0.99938 | 0.64815 | 0.58581 | 123.393 | 157.259 | 38.066 | 0.214 |
| 130.0 | 0.99787 | 0.64046 | 0.64710 | 128.334 | 107.769 | 38.954 | 0.105 |
| 135.0 | 0.29637 | 0.61676 | 0.70978 | 133.288 | 178.468 | 39.520 | 0.012 |
| 140.0 | 0.99490 | 0.63721 | 0.71286 | 138.257 | 189.722 | 39.823 | -0.066 |
| 145.0 | 0.99346 | 0.64178 | 0.83535 | 143.240 | 199.890 | 39.923 | -0.131 |


| 150.0 | 0.99207 | 0.65027 | 0.89637 | 148.236 | 210.345 | 39.879 | -0.183 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 155.0 | 0.99074 | 0.66230 | 0.95511 | 153.245 | 220.473 | 39.745 | -0.265 |
| 160.0 | 0.98947 | 0.67738 | 1.01097 | 158.267 | 230.196 | 39.566 | -0.257 |
| 165.0 | 0.98829 | 0.64495 | 1.06351 | 163.300 | 239.460 | 39.380 | -0.281 |
| 170.0 | 0.98719 | 0.71445 | 1.11244 | 168.345 | 243.242 | 39.217 | -0.243 |
| 175.0 | 0.98619 | 0.73533 | 1.15760 | 173.400 | 256.540 | 39.098 | -0.309 |

$\left.\begin{array}{lllllllll}\hline 180.0 & 0.98529 & 0.75708 & 1.19895 & 178.464 & 264.370 & 39.037 & -0.314 \\ 185.0 & 0.98451 & 0.77927 & 1.23650 & 183.536 & 271.759 & 39.043 & -0.315 \\ 190.0 & 0.98384 & 0.80150 & 1.27032 & 188.615 & 278.737 & 39.119 & -0.312\end{array}\right)$

| $\begin{aligned} & 195.0 \\ & 200.0 \\ & 205.0 \end{aligned}$ | $\begin{aligned} & 0.98324 \\ & 0.94288 \\ & 0.48254 \end{aligned}$ | $\begin{aligned} & 1.8 .1341 \\ & 0.84486 \\ & 0.801,54 \end{aligned}$ | $\begin{aligned} & 1.30050 \\ & 1.32714 \\ & 1.35049 \end{aligned}$ | $\begin{aligned} & 104.701 \\ & 198.791 \\ & 209.484 \end{aligned}$ | $\begin{aligned} & 206.1411 \\ & 201.604 \\ & 2.97 .505 \end{aligned}$ | $\begin{aligned} & 19.2661 \\ & 39.461 \\ & 34.760 \end{aligned}$ | $\begin{aligned} & -0.3115 \\ & -0.2 .76 \\ & -0.2 H 4 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 210.0 | 0.98243 | 0.84525 | 1.310ヶ\% | 208.980 | 303.2ら1 | 40.098 | -0.210 |
| 215.0 | 0.94241 | 0.90 Sts | 1.30751 | 214.076 | SUR.6.1 | 40.48 H | -0.254 |
| 220.0 | 0.98233 | 0.92125 | 1.40152 | 214.172 | 313.930 | 40.423 | -0.237 |
| 225.0 | 0.9H2 ${ }^{\text {PH }}$ | 0.95737 | 1.41770 | 224.261 | 314.976 | 41.345 | -0.218 |
| 23000 | - |  | $18421 / 2$ | 229.308 | whallll | 41.895 | - 0.148 |
| 235.0 | 0.96306 | 0.46324 | 1.42772 | 234.44) | 12H. h(1) | 4?.417 | -6.110 |
| -240.0 | 0.98430 | 0.976\%1 | 1.43045 | 2.31.126 | 333.222 | 42.90 | -6.154 |
| 245.0 | 0.98505 | 0.9R71\% | 1.41225 | 244.600 | 357.741 | 43.4 Hl | -0.1s1 |
| 2500 | C0.34541 | 0.40276 | 1.613154 | 244.461 | 142.114 | 44.04 | -0.106 |
| 255.0 | 0.986 H | 1.01276 | 1.42902 | 254.724 | 34R.5.3H | 44.5 .96 | -0.082 |
| 260.0 | 0.98795 | 1.00415 | 1.42472 | 259.112 | 3501.848 | 45.031 | -0.026 |
| 265.0 | 0.98911 | 1.01190 | 1.41845 | 264.409 | 3 ¢5.119 | 45.414 | -0.030 |
| 27000 | 0.99045 | 1.01402 | 1.41160 | 269.834 | 154,366 | 45.216 | -0.004 |
| 275.0 | 0.94167 | 1.01449 | 1.41,310 | 274.847 | 307.602 | 46.254 | 0.043 |
| 280.0 | 0.99304 | 1.01331 | 1.39312 | 279.847 | 367.841 | 46.603 | 0.020 |
| 285.0 | 0.99446 | 1.01050 | 1.38349 | 284.834 | 317.046 | 46.430 | 0.017 |
| 290.0. | 0.99593 | 1.00604 | 1.37269 | 289.807 | 376.342 | 47.620 | 0.103 |
| 295.0 | 0.99742 | 0.99096 | 1.36154 | 2\%4.76 | 380.714 | 47.1114 | U.lso |
| 300.0 | 0.99893 | 0.44227 | 1.35028 | 299.710 | 305.105 | 42.045 | 0.156 |
| 305.0 | 1.00044 | 0.94249 | 1.33916 | 304.641 | 349.57 .9 | 46.984 | 0.181 |
| 310.0 | 1.00196 | 0.97214 | 1.32845 | 109.557 | 344.133 | 46.761 | 0.2015 |
| 315.0 | 1.00345 | 0.95977 | 1.31839 | 314.460 | 308.803 | 46.422 | 0.228 |
| 320.0 | 1.000692 | 0.94591 | 1.30929 | 114.349 | $463061{ }^{4}$ | 45.926 | 0.240 |
| 325.0 | 1.00635 | $0.93116 \%$ | 1.30140 | 324.724 | 40R.553 | 45.364 | 0.208 |
|  | - .- |  |  |  |  |  |  |
| -330.0 | 1.00773 | 0.91397 | 1.29504 |  | 413.670 |  | 0.284 |
| 335.0 | 1.00905 | 0.84605 | 1.29048 | 333.937 | 418.447 | 43.771 | 0.248 |
| 34000 | 1.01031 | 0.87695 | 1.28800 | 318.772 | 424.542 | 42.765 | 0.301 |
| 345.0 | 1.01149 | 0.816612 | 1.28789 | 345.603 | 430.342 | 41.621 | 0.313 |
| 350.0 | 1.01259 | 0.83582 | 1.29037 | 348.419 | 436.427 | 40.340 | 0.315 |
| 355.0 | 1.01359 | 0.81413 | 1.29568 | 353.227 | 442.832 | 38.926 | 0.112 |
| 36000 | 1.01450 | 0.74202 | 1.30399 | 358.026 | 444.591 | 37.381 | 0.304 |
| 365.0 | 1.01530 | 0.76976 | 1.31541 | 362.817 | 456.740 | 35.71 K | 0.241 |
| . 370.0 | 1.01599 | 0.74171 | 1.33000 | 367.602 | 404.314 | 33.940 | 0.272 |
| 375.0 | 1.01657 | 0.72627 | 1.34770 | 372.381 | 472.341 | 32.057 | 0.249 |
| . 380.0 | 1.01702 | 0.70592 | 1.36840 | 371.155 | 480.843 | 10.080 | 0.221 |
| 385.0 | 1.01736 | 0.68718 | 1.34184 | 381.926 | 489.835 | 28.020 | 0.190 |
| 390.0 | 1.01757 | 0.67060 | 1.41165 | 386.694, | 499.299 | 25.893 | 0.134 |
| 395.0 | 1.01766 | 0.65676 | 1.44540 | 391.461 | 509.202 | 23.712 | 0.116 |
| 400.0 | 1.01761 | 0.64619 | 1.47454 | 396. 2.28 | 519.482 | 21.448 | 0.077 |
| 405.0 | 1.01745 | 0.61933 | 1.50449 | 400.996 | 510.043 | 19.270 | 0.037 |
| 410.0 | 1.01715 | 0.61650 | 1.53471 | 405.766 | 540.765 | 17.054 | -0.003 |
| 415.0 | 1.01674 | 0.63784 | 1.56467 | 410.539 | 531.512 | 14.874 | -0.041 |


| 420.0 | 1.01620 | 0.64328 | 1.54994 | 415.316 | 562.146 | 12.756 | -0.076 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 425.0 | 1.01555 | 0.65296 | 1.622411 | 420.098 | 512.534 | 10.725 | -0.1U8 |  |
| 430.0 | 1.01474 | 0.6652? | 1.64973 | 424.887 | 587.587 | 8.hus | -0.156 | 1 |
| 435010 | 1.01302 | Catales | 1667596 | $424 \times 684$ | 542.214 | 7 l 106 | -0.01al | , |
| 440.11 | 1.01795 | 1. 600891 | 1.70109 | 434.48 H | 601.176 | 5.344 | -0.100 |  |
| 445.0 | 1.01194 | 0.11841 | 1.12521 | $49 \%$ 302 | 6.10 .054 | 3.040 | -0.116 |  |
| 450.1 | 1.01075 | 0.719 .97 | 1.7483 H | 444.125 | 61H.251 | 2.483 | -0.207 |  |
| 4550 | $1 \times 00949$ | C.76180 | 1677069 | 448.9611 | 6250985 | 1.284 | -14.214 |  |
| 460.0 | 1.00819 | 0.18407 | 1.79218 | 453.806 | 637.243 | 0.296 | -0.211 |  |
| 465.0 | 1.00085 | U.80623 | 1.81288 | - 458.664 | 6410.110 | -1).614 | -0.211 |  |
| 470.0 | 1.00547 | 0.87819 | 1.43280 | 461.535 | 646.710n | -1.439 | - 0.210 |  |
| 475.0 | 1.00346 | 0.84451 | 1.85190 | 464.419 | 632.902 | $-2.1105$ | -0.211 |  |
| 480.1) | 1.04247 | O.HAyPs | 1.87016 | 473.117 | 6, H.ROU $^{\text {a }}$ | -7.548 | -0.204 |  |
| 485.0 | 1.00047 | 0.88932 | 1.84751 | 478.228 | 664.433 | $-2.420$ | -0.1 44 |  |
| 490.1 | 0.94945 | 0.91760 | 1.90389 | 433.154 | 669.832 | -3.174 | -0.103 |  |
| 495.0 | 0.99794 | 0.41479 | 1.91922 | 48H.044 | 675.023 | -3.339 | -0.110 |  |
| $500.1)$ | 0.99644 | 0.9405H | 1.9334? | 443.04 H | 6 H0.032 | -3.3.12 | -0.150 |  |
| 505.1 | 0.99497 | 0.45496 | 1.94642 | 498.016 | 684488 | $-3.304$ | -0.141 |  |
|  |  |  |  |  |  |  |  |  |
| 510.0 | 0.94353 | 0.96787 | 1.95814 | 202.998 | 684.598 | -3.251 | -0.1c |  |
| 515.0 | 0.99213 | 0.91927 | 1.96852 | 507.994 | 694.146 | -3.047 | -0.108 |  |
| 520.0 | 0.99080 | (1).4891? | 1.97747 | 513.002 | 698.694 | -2.844 | -0.041 |  |
| 52500 | 0.94953 | 0.92118 | 1.94496 | 218.021 | 701.111 | -2.255 | -0.013 |  |
| 530.0 | 0.98834 | 1.0040115 | 1.94042 | 523.056 | 707.463 | -2.221 | -0.0.0) |  |
| 535.0 | 0.98724 | 1. 2090 H | 1.94531 | 52H.100 | . 2111.163 | $-1.893$ | -0.056 |  |
|  |  |  |  |  |  |  |  | 1 |
| 5\%0.0 | 0.98624 | 1.01248 | 1.99810 | 533.154 | 116.028 | -1.455 | -0.01H |  |
| 545.0 | 0.98533 | 1.01426 | 1 | 538.218 | 121.271 | -1.044 | velus |  |
| 550.0 | 0.98454 | 1.01418 | 1.99880 | 549.290 | 724.501 | -0.615 | 0.020 |  |
| 555.0 | 0.98387 | 1.01285 | 1.99671 | 543.364 | 728.14日 | -0.146 | 0.034 |  |
| 560.0 | 0.98342 | 1.00968 | 1.99298 | 553.454 | 733.004 | 0.219 | 0.0 .7 |  |
| 565.0 | 0.98289 | 1.0048 | 1.98765 | 550.544 | 737.303 | 0.626 | U.013 |  |
| 570.0 | 0.98260 | $0.9+845$ | 1.94075 | 563.637 | 741.646 | 1.006 | 0.0 .13 |  |
| 575.0 | 0.98244 | 0.99042 | 1.97232 | 568.732 | 146.052 | 1.347 | 0.110 |  |
| 580.0 | 0.98241 | 0.94081 | 1.96240 | 573.829 | 750.537 | 1.646 | $0.1<7$ |  |
| 585.0 | 0.98252 | 0.46963 | 1.95101 | 578.925 | 155.119 | 1.841 | 0.143 |  |
| 590.0 | 0.98216 | 0.95644 | 1.93838 | 584.020 | 759.815 | 2.076 | 0.158 |  |
| 595.0 | 0.98113 | 0.94276 | 1.92443 | 289.111 | 764.646 | 2.186 | 0.112 |  |
| 600.0 | 0.98364 | 0.92718 | 1.90924 | 594.198 | 769.631 | 2.217 | 0.185 |  |
| 605.0 | 0.98426 | 0.91025 | 1.89305 | 599.280 | 714.794 | 2.156 | 0.196 |  |
| 610.0 | 0.98501 | 0.89206 | 1.87580 | 604.354 | 780.161 | 1.944 | 0.206 |  |
| 615.0 | 0.98587 | 0.87274 | 1.85765 | 604.421 | 785.727 | 1.721 | 0.214 |  |
| 620.0 | 0.98683 | 0.85241 | 1.83865 | 614.479 | 741.615 | 1.328 | $0 . ? 20$ |  |
| -625.0 | 0.98790 | 0.83124 | 1.81891. | -619.528 | 197.766 | 0.805 | 0.223 |  |
| 630.0 | 0.98905 | 0.80444 | 1.79847 | 624.565 | 804.741 | 0.145 | 0.224 |  |
| 635.0 | 0.99029 | 0.18127 | 1.71739 | 629.591 | 811.083 | -0.660 | 0.221 |  |
| 640.0 | 0.99160 | 0.7650 ? | 1.75570 | 634.604 | 818.320 | -1.618 | 0.216 |  |
| -645.0 | 0. 29297 | 0.74306 | 1.73338 | 639.605 | 825.989 | -2.733 | 0.206 | ) |
| 650.0 | 0.99439 | 0.72182 | 1.71043 | 644.592 | 834.116 | -4.004 | 0.143 |  |


| 655.0 | 0.945H6 | 0.71177 | 1.64671 | (14\%).506 | H47.123 | -5.434 | 0.175 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -660.0 | 0.40212 | crathata | 10.60234 | -6xabla | Anlolla | -106le | 0141 |
| 665.0 | $0.9 y+36$ | 0.60140 | 1.63710 | (15).47! | Het. 1717 | - $\mathrm{H} .14 \%$ | U.127 |
| 610.0 | 1.OUCS 7 | 0.65421 | 1.61100 | 664.402 | H/1.g04 | -10.611 | $0.0 \times 1$ |
| 615.0 | $1.0014 \%$ | 0.64440 | 1.511006 | 664.3211 | 141.117 | -12.514 | 0.064 |
| 680.1 | 1.003st | U. osust | 1.55641 | 674.223 | H42.323 | -16.660 | 0.027 |
| 685.11 | 1.00485 | 0.6364\% | 1.52827 | 074.11) | पut.llot | -16.1\% | -1).011 |
| 090.0 | 1.0UE28 | 0.63806 | 1.50000 | (8).'Jut | 913.741 | -1R.7AU | -u.usl |
| 695.1 | 1.00?6a | 0.64495 | 1.47203 | \%*リ.8') |  | -21.116 | -0.0.0 |
| 100.0 | 1.00899 | U.6's) 000 | 1.44481 | 6ys. 702 | 954.115 | $-23.362$ | -0.12y |

$\qquad$
$\qquad$

SUNBI ALER NRP! I


PEKIIII SFNS IN INJFCI VKLE 19.79 WAYS PEK/KM/SFC VSESSOIEV.11439 TPAE U.O


$\left.\begin{array}{cccccccc}\hline 180.0 & 0.98242 & 0.75677 & 1.17896 & 180.395 & 264.647 & 39.645 & -0.320 \\ 185.0 & 0.98243 & 0.77924 & 1.21700 & 185.492 & 272.073 & 39.728 & -0.320 \\ 190.0 & 0.98257 & 0.80174 & 1.25133 & 190.588 & 279.045 & 39.826 & -0.316\end{array}\right)$


| 420.0 | $1.01: 44$ | 0.641711 | 1.5HG24 | 415.48 H | 56, $\cdot 2 \cdot 7$ | 12.794 | -0.077 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 425.0 | 1.00924 | 0.61066 | 1.61419 | 420.324 | 572.708 | 10.704 | -0.10y |  |
| 430.0 | 1.00743 | 0.01 tell | 1.64113 | 425.173 | 387.8114 | $8.84 ?$ | -0.1) 137 | 1 |
| 43500 | 1.0005 | 0.47452 | 1066714 | 4-10.014 | 542.470 | 7 l 106 | -ublo2 | , |
| 440.0 | 1.00513 | $0.0+7+44$ | $1.69196$ | 484.407 | 601.660 | 5.423 | -0.1*2 |  |
| 445.11 | 1.00366 | $0.71797$ | $1.71595$ | $43^{\prime 4} .744$ | 410.3:7 | 3.934 | $-0.1 \times 7$ |  |
| $\begin{array}{r} 450.0 \\ 450.0 \end{array}$ | $\begin{aligned} & 1.00217 \\ & 1.00006 \end{aligned}$ | $\begin{aligned} & 0.71947 \\ & 0.76155 \end{aligned}$ | $\begin{aligned} & 1.73410 \\ & 1.76147 \end{aligned}$ | $\begin{aligned} & 444.694 \\ & 449.609 \end{aligned}$ | $61 \mathrm{H} . \mathrm{beq}$ <br> 6) 3ul | $\begin{aligned} & 2.603 \\ & 1.424 \end{aligned}$ | $\begin{aligned} & -0.218 \\ & -0.216 \end{aligned}$ |  |
| 460.11 | 0.74015 | $11.7+405$ | 1.78312 | 454.537 | 634.546 | 0.420 | -0.214 |  |
| 465.l | O.94704 | U.8ら6!2. | 1.80407 | 451.4401 | 6411.484 | -0.448 | -0.720 |  |
| 410.0 | 0.94014 | O. R280 | 1.82430 | 464.437 | 647.001 | -1.165 | -0.211 |  |
| 475.0 | 0.93467 | U.85013 | 1.84380 | 404.4108 | 6, 1.104 | -1.740 | $-0.212$ |  |
| 4HO.0 | 0.94324 | 0.81081 | 1.86249 | 474.393 | 699.066 | -2.1.14 | -0.204 |  |
| 485.0 | 0.29186 | 0.89048 | 1.88033 | 474.341 | 644.082 | $-2.505$ | -0.145 |  |
| 490.0 | 0.99054 | 0.40401 | 1.89722 | 484.402 | 610.063 | -2.710 | -0.183 |  |
| -445.0 | 0.98929 | .). 92628 | 1.91310 | 489.425 | 675.2 .14 | -2.804 | -0.170 |  |
| 500.0 | $0.9+12$ | 0.94220 | 1.92787 | 444.460 | 6811. 224 | -2.814 | -0.150 |  |
| 5050 | 0.98703 | 11.42670 | 1.44145 | 449.506 | 6850055 | -2.12 | $-0.141$ |  |




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## SUNBLALFR ORI?IT


FCCENTKICITY=0.22405754 INLLINATIDNE U.50OO SB INJECTINN VFLOEITY=0.871T1
PFRIIII SFNS TO IVJECT VFL $=14.84$ UAYS PER/KM/SEG VSHSOI=0.11437 TPAE U.O TIMF REK KSB FIST FSB AZIMUTH IILFAIIIN

| 0.11 | 1.01537 | 1.01456 | 0.00625 | 3.044 | 2.647 | -82.34H | 3.871 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5.0 | 1.01458 | 1.01374 | 0.01605 | 7.840 | 6.954 | -86.523 | 3. H16 |
| 10.0 | 1.01367 | 1.01177 | 0.02587 | 12.634 | 11.184 | -83.9122 | 3.745 |
| 15.0 | 1.01269 | 1.00716 | U. 03545 | 17.445 | 15.462 | -80.022 | 3.740 |
| 20.1) | 1.01160 | 1.00143 | 0.04482 | 22.261 | 19.783 | -75.612 | 3.784 |
| 25.0 | 1001043 | 0.39408 | 0.05348 | 27.087 | 24.160 | -70.814 | 1.718 |



| 90.0 | $0.941 H 0$ | 0.77451 | 0.22660 | 90.998 | 45.173 | 14.415 | 1.702 |  |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 95.0 | 0.99048 | 0.75238 | 0.25837 | 96.010 | 102.654 | 19.645 | 1.421 |  |
| 100.0 | 0.98923 | 0.73077 | 0.29476 | 101.034 | 110.582 | 24.290 | 1.161 |  |
| 105.0 | 0.98806 | 0.71014 | 0.33583 | 106.070 | 118.983 | 28.207 | 0.925 |  |
| 110.0 | 0.98698 | 0.69100 | 0.38149 | 111.116 | 127.869 | 31.478 | 0.715 |  |
| 115.0 | 0.28600 | 0.67341 | 0.43153 | 11.2 .173 | 137.235 | 34.144 | 0.530 |  |


| 120.0 | 0.981213 | 0.65943 | 0.48 .91 | 121.239 | 147.050 | 36.257 | 0.369 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 125.0 | 0.98436 | 0. 0.64812 | 0.54285 | 126.313 | 157.257 | 37.874 | 0.231 |
| 130.0 | 0.98372 | 0.64043 | 0.60279 | 131.39? | 167.768 | 39.058 | 0.113 |
| 135.1 | 0.98320 | 0.63673 | 0.66443 | 136.480 | 178.468 | 39.813 | 0.013 |
| 140.0 | 0.98281 | 0.63717 | 0.72678 | 141.570 | 189.222 | 40.386 | -0.0710 |
| 145.0 | 0.98255 | 0.64175 | 0.78885 | 146.664 | 149.842 | 40.665 | -0.138 |
| 150.0 | 0.98242 | 0.650124 | 0.84973 | 151.760 | 210.347 | 40.713 | -0. 143 |
| 155.0 | 0.98243 | 0.66228 | 0.90861 | 156.857 | 220.477 | 40.766 | -0.237 |
| 160.0 | 0.98257 | 0.67736 | 0.96484 | 161.953 | 230.199 | 40.695 | -0.210 |
| 165.0 | -0.98284 | 0.69494 | 1.01796 | 167.046 | 239.464 | 40.599 | -0.244 |
| 170.0 | 0.98325 | 0.71444 | 1.00760 | 172.137 | 248.246 | 40.510 | -0.311 |
| 175.0 | 0.98378 | 0.73532 | 1.11375 | 177.223 | 256.545 | 40.449 | -0.321 |


| 180.0 | 0.98444 | 0.75707 | 1.15616 | 182.303 | 264.375 | 40.432 | -0.326 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 185.0 | 0.98521 | 0.77926 | 1.19488 | 187.376 | 271.764 | 40.408 | -0.326 |
| 190.0 | 0.98610 | 0.80150 | 1.22797 | 192.440 | 278.742 | 40.562 | -0.322 |


| $\begin{aligned} & 195.0 \\ & 200.0 \\ & 205.0 \end{aligned}$ | $\begin{aligned} & 0.98709 \\ & 0.98818 \\ & 0.98916 \end{aligned}$ | $\begin{aligned} & 0.82347 \\ & 0.84490 \\ & 0.86555 \end{aligned}$ | $\begin{aligned} & 1.26153 \\ & 1.28964 \\ & 1.31441 \end{aligned}$ | $\begin{aligned} & 197.446 \\ & 202.542 \\ & 207.576 \end{aligned}$ | $\begin{aligned} & 244.549 \\ & 241.609 \\ & 247.567 \end{aligned}$ | $\begin{aligned} & 40.714 \\ & 40.923 \\ & 41.184 \end{aligned}$ | $\begin{aligned} & -0.31 b \\ & -0.3 u 5 \\ & -0.2+2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 210.0 | D.99041 | 0.84526 | 1.33408 | 212.594 | 303.255 | $41.4+3$ | -0.217 |
| 215.0 | 0.94194 | 0.90387 | 1.35406 | 217.609 | 308.701 | 41.844 | -6.260 |
| 220.u | 0. 99312 | 0.42127 | 1.31014 | 222.641 | 313.934 | 42.230 | -0.242 |
| 225.0 | 0.94475 | 0.45755 | 1.3832 h | 227.541 | 318.974 | 42.644 | -0.222 |
| 2300 | 0.94622 | 0.98304 | 1.34354 | 232.661 | 323.462 | 43.018 | -0.201 |
| 235.0 | 0.94772 | 0.96527 | 1.40134 | 237.517 | 328.6,04 | 43.526 | -0.117 |
| 240.0 | 0.99923 | 0.97700 | 1.40681 | 24.n.459 | 313.225 | 43.980 | -0.115 |
| 245.0 | 1.00075 | 0.94718 | 1.41007 | 247.307 | 337.742 | 44.433 | -0.13 |
| 25000 | 1.00226 | 0.9127 78 | 1.41130 | 252.301 | 142.176 | 44.876 | -4.148 |
| 255.0 | 1.00375 | 1.00279 | 1.41063 | 257.200 | 346.519 | 45.103 | -0.089 |
| 260.0 | 1.00521 | 1.00818 | 1.40823 | 262.086 | 350.849 | 45. 105 | -0.0.7 |
| 265.0 | 1.006 .63 | 1.01193 | 1.41425 | 266.959 | 355.120 | 46.1075 | -0.031 |
| 2700 | 1.00800 | 1001405 | 1.39888 | 271.819 | 359.306 | 46.406 | $-0.004$ |
| 275.0 | 1.00931 | 1.01452 | $1.3922 \%$ | 276.667 | 303.602 | 46.688 | 0.023 |
| 280.0 | 1.01055 | 1.01332 | 1.38463 | 281.503 | 367.840 | 46.916 | 0.020 |
| 285.0 | 1.01172 | 1.01053 | 1.37616 | 286.328 | 372.045 | 47.080 | 0.077 |
| 290.0 | 1.01240 | 1.00608 | 1.36702 | 291.143 | 376.381 | 47.114 | 0.104 |
| 295.0 | 1.01378 | 0.94999 | 1.35745 | 295.949 | 38C.712 | 47.184 | 0.150 |
| 300.0 | 1.01467 | 0.99230 | 1.34766 | 300.746 | 385.104 | 42.117 | 0.196 |
| 305.0 | 1.01545 | 0.98302 | 1.33789 | 305.536 | 389.571 | 46.951 | 0.181 |
| 310.0 | 1.01612 | 0.91217 | 1.32838 | 310.319 | 394.130 | 46.684 | 0.205 |
| 315.0 | 1.01667 | 0.95980 | 1.31937 | 315.097 | 398.800) | 46.309 | 0.228 |
| 3200 | 10.1710 | 0.444594 | 1.31113 | 319.871 | 463.600 | 45.819 | 6.24y |
| 325.0 | 1.01741 | 0.93065 | 1.30393 | 324.641 | 40A. 549 | 45.209 | 0.267 |
| 330.0 | 1.01700 | 0.91399 | 1.29804 | 32.2.409 | 413.672 | 44.475 | 0.284 |
| 335.0 | 1.01766 | 0.89607 | 1.29374 | 334.176 | 418.933 | 43.612 | 0.297 |
| 3400 | 1.01759 | 0.87697 | 1.29130 | 338.943 | 424.538 | 42.614 | 0.307 |
| 345.7 | 1.01740 | 0.85683 | 1.29099 | 343.711 | 430.337 | 41.444 | 0.312 |
| 3510.0 | 1.01704 | 0.83582 | 1.29304 | 348.481 | 436.422 | 40.239 | 0.314 |
| 355:0 | 1.01664 | 0.81414 | 1.29769 | 353.255 | 442.827 | 38.854 | 0.311 |
| 360.0 | 1.01608 | 0.14202 | 1.30511 | 358.033 | 449.586 | 37.346 | 0.303 |
| 365.0 | 1.01541 | 0.76976 | 1.31544 | 362.817 | 456.735 | 35.717 | 0.291 |
| 370.0 | 1.01462 | 0.74770 | 1.32875 | 367.607 | 464.309 | 33.976 | 0.273 |
| 375.0 | 1.01373 | 0.72626 | 1.34502 | 372.405 | 472.336 | 32.151 | 0.250 |
| 380.0 | 1.01274 | 0.70591 | 1.36414 | 377.211 | 480.841 | 30.191 | 0.222 |
| 385.0 | 1.01166 | 0.68716 | 1.38591 | 382.027 | 489.830 | 28.168 | 0.190 |
| 390.0 | 1.01049 | 0.61058 | 1.41001 | 386.853 | 499.245 | 26.076 | 0.155 |
| 395.0 | 1.00924 | 0.65673 | 1.43603 | 391.689 | 509.199 | 23.929 | 0.117 |
| 400.0 | 1.00793 | 0.64616 | 1.46348 | 396.538 | 519.480 | 21.748 | 0.011 |
| 405.0 | 1.00655 | 0.63930 | 1.49182 | 401.398 | 530.042 | 19.555 | 0.037 |
| 4100 | 1.00513 | 0.63647 | 1.52054 | 406.272 | 540.763 | 17.373 | -0.003 |
| 415.0 | 1.00366 | 0.63781 | 1.54918 | 411.159 | 531.513 | 15.230 | -0.041 |



| 655.0 | 1.01275 | 0.70115 | 1.70564 | 6)0.910 | 847.119 | -4.830 | 0.173 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6600 | 101374 | 12.68342 | 1-64106 | 655.316 | 851812 | -6,472 | 0142 |
| 665.0 | 1.01463 | 0.66757 | 1.65540 | 660.514 | H61.313 | -H.254 | 0.126 |
| 610.0 | 1.01541 | 0.664418 | 1.62863 | 665.304 | 8.11.363 | -10.164 | $0.0 \%$ |
| 675.0 | 1.01609 | 0.64437 | 1.60081 | 670.088 | 841.715 | -12.186 | O.vos |
| .680.0 | 1.01664 | -1.638315 | 1.57201 | -614.866 | 8122.321 | -14.297 | 0.027 |
| 685.0 | 1.01708 | 0.61641 | 1.54263 | 674.639 | 903.062 | -16.473 | -0.011 |
| 690.11 | 1.01740 | 0.61861 | 1.51290 | -684.410 | 913. 196 | -18.690 | -0.020 |
| 695.0 | 1.01759 | 0.64492 | 1.48331 | 689.178 | 924.340 | -20.920 | -0.090 |
| 200.0 | 1.01 .766 | 0.63447 | 1.45449 | 643.9442 | 914.711 | -23.134 | -0.128 |

区

SUNBLA7FR HKP:I

ECCEVTRICITY=0.22164446 INCLINATION: 0.5000 SA INJLCIIUN VELOCITY=0.A7E54



| 120.0 | 0.98241 | 0.66447 | 0.46946 | 122.901 | 147.641 | 36.228 | 0.379 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 125.0 | 0.98244 | 0.65365 | 0.52554 | 128.051 | 157.701 | 37.965 | 0.236 |
| 130.0 | 0.98261 | 0.64634 | 0.58435 | 133.153 | 168.048 | 39.255 | 0.115 |
| 135.1 | 0.98211 | 0.64283 | 0.64488 | 138.246 | 178.567 | 40.166 | 0.012 |
| 140.0 | 0.98 .334 | 0.64325 | 15.70625 | 143.335 | 149.137 | 40.766 | -0.012 |
| 145.0 | 0.98390 | 0.64760 | 0.76751 | 148.420 | 199.628 | 41.120 | -0.142 |


| 150.0 | 0.98458 | 0.65567 | 0.82771 | 153.499 | 209.921 | 41.293 | -0.198 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 155.0 | 0.98538 | 0.66713 | 0.84626 | 158.570 | 219.911 | 41.339 | -0.241 |
| 160.0 | 0.98629 | 0.68152 | 0.94234 | 163.633 | 229.520 | 41.317 | -0.275 |
| 165.1 | 0.98730 | 0.69832 | 0.99553 | 168.687 | 238.699 | 41.239 | -0.300 |
| 170.0 | 0.98841 | 0.71701 | 1.04549 | 173.730 | 247.423 | 41.163 | -0.317 |
| 175.0 | 0.98961 | 0.73706 | 1.09203 | 178.763 | 255.687 | 41.104 | -0.327 |

$\left.\begin{array}{lllllllll}180.0 & 0.94087 & 0.75801 & 1.13504 & 183.783 & 263.503 & 41.078 & -0.332 \\ 185.1 & 0.99221 & 0.77941 & 1.17450 & 188.791 & 270.844 & 41.094 & -0.332 \\ 190.0 & 0.99360 & 0.80091 & 1.21045 & 193.786 & 277.888 & 41.159 & -0.328\end{array}\right)$

| $\begin{aligned} & 195.0 \\ & 200.0 \\ & 205.0 \end{aligned}$ | $\begin{aligned} & 0.19505 \\ & 0.94652 \\ & 0.49 R 07 \end{aligned}$ | $\begin{aligned} & 0.82218 \\ & 0.84235 \\ & 0.86307 \end{aligned}$ | 1.24244 1.21201 1.24195 | 198.767 203.134 203.688 | $\begin{aligned} & 2114.514 \\ & 230.818 \\ & 2+6 . A 1 R \end{aligned}$ | $\begin{aligned} & 1.1 .214 \\ & 41.417 \\ & 41.547 \end{aligned}$ | $-0.3: 0$ <br> $-0.310$ <br> $-0.297$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 210.0 | 0.2935\% | 0.84219 | 1.32011 | 211.627 | 302.532 | 41.400 | -0.282 |
| 215.0 | 1.00105 | 0.90031 | 1.34041 | 218.532 | 304.04H | $42.1 \% 1$ | -0.264 |
| .220.12 | 1.00254 | 0.41726 | 1.35136 | 223.403 | 313.314 | 42.513 | -0.246 |
| 225.0 | 1.00404 | 0.94295 | 1.37150 | 228.360 | 318.434 | 42.862 | -1).220 |
| 23010 | 1.00545 | 0.94220 | 1.388141 | 213.241 | 323.375 | 43.230 | -0.204 |
| $235.1)$ | 1.00641 | 0.96022 | 1.37210 | 23H.113 | $32 \mathrm{H.179}$ | 44.611 | -0.162 |
| 240.0. | 1.04827. | 0.91164 | 1.39883 | 242.971 | 312.892 | 43.84 | - 0.158 |
| 245.0) | 1.00956 | 0.98164 | 1.40341 | 241.416 | 317.428 | 44.384 | -(). 134 |
| 2500 | 1.01079 | 0.112006 | $1{ }^{1} \times 05186$ | 2520650 | 3418 | 44.764 | -0.0.01 |
| 255.0 | 1.01174 | 0.49692 | 1.40644 | 257.473 | 346.342 | 45.128 | -0.084 |
| .260.0 | 1.01100 | 1.00220 | 1.40526 | 262.286 | 120.711 | 45.472 | -0.028 |
| 265.0 | 1.01347 | 1.005137 | 1.40244 | 267.090 | 355.041 | 45.786 | -0.031 |
| 2700 | 101483 | 1.00296 | 1639428 | 2718885 | 3298.346 | 46.064 | -0.0064 |
| 275.0 | 1.01579 | 1.00841 | 1.39281 | 276.674 | 363.611 | 46.302 | 0.023 |
| 280.0 | 1.01624 | 1.00126 | 1.38625 | 281.456 | 361.234 | 46.484 | -0.050 |
| 285.0) | 1.01676 | 1.001451 | 1.37817 | 286.233 | 312. 253 | 46.618 | 0.077 |
| 290.0 | 1.01717 | L.00014 | 1.37058 | 291.000 | 316.548 | 46.682 | 0.104 |
| 295.0 | 1.01746 | 0.4 .1418 | 1.36188 | 295.776 | 380.984 | 46.673 | 0.131 |
| 300.0 | 1.01762 | 0.98666 | 1.35288 | 300.543 | 3058.438 | $46.54 b$ | 0.122 |
| 305.0 | 1.01765 | 0.47757 | 1.34381 | 305.310 | 389.964 | 46.410 | 0.182 |
| 310.0 | 1.01756 | 0.96697 | 1.33489 | 310.077 | 344.581 | 46.141 | 0.206 |
| 315.0 | 1.01734 | 0.95487 | 1.32637 | 314.846 | 394.309 | 45.710 | 0.228 |
| 32000 | 1.01700 | 0.944133 | 1.31850 | 319.617 | 404.162 | 45.242 | $0.24 \%$ |
| 325.0 | 1.01654 | 0.92641 | 1.31155 | 324.392 | 409.171 | 44.701 | 0.26 .7 |
| 330.0 | 1.01596 | 0.91017 | 1.30577 | 329.171 | 414.348 | 43.942 | 0.243 |
| 335.0 | 1.01526 | 0.84271 | 1.30145 | 333.456 | 419.720 | 43.161 | 0.296 |
| 3400 | 1.01445 | 0.81417 | 1.29884 | 138.8.747 | 425.315 | 42.205 | 0.306 |
| 345.0 | 1.01354 | 0.83455 | 1.29420 | 343.547 | 431.159 | 41.123 | 0.311 |
| 350.0 | 1.01253 | 0.83415 | 1.29978 | 348.355 | 437.283 | 39.414 | 0.313 |
| 355.0 | 1.01143 | 0.81314 | 1.30379 | 353.172 | 443.719 | 38.581 | 0.310 |
| 36000 | 1.01024 | 0.79173 | 1.31044 | 358.000 | 450.501 | 37.127 | Qesut |
| 365.0 | 1.00898 | 0.77024 | 1.31984 | 362.839 | 437.661 | 35.556 | 0.234 |
| 370.0 | 1.00766 | 0.74898 | 1.33208 | 367.690 | 465.230 | 33.875 | 0.211 |
| 375.0 | 1.00627 | 0.72836 | 1.34715 | 372.553 | 473.236 | 32.042 | 0.248 |
| 380.0 | 1.00484 | 0.70883 | 1.36499 | 311.429 | 481.695 | 30.216 | 0.221 |
| 385.0 | 1.00337 | 0.69088 | 1.38540 | 382.319 | 490.615 | 28.260 | 0.189 |
| 390.0 | 1.00187 | 0.67504 | 1.40810 | 387.222 | 499.983 | 26.235 | 0.154 |
| 395.0 | 1.00036 | 0.66185 | 1.43274 | 392.139 | 509.763 | 24.160 | 0.110 |
| 400.0 | 0.99885 | 0.65178 | 1.45887 | 397.071 | 519.893 | 22.052 | 0.077 |
| 405.0 | 0.99734 | 0.64527 | 1.48600 | 402.016 | 530.284 | 19.933 | 0.037 |
| 410.0 | 0.99585 | 0.64258 | 1 c 51368 | 406.976 | 540.825 | 17.827 | $-0.004$ |
| 415.0 | 0.99438 | 0.64385 | 1.54147 | 411.950 | 551.389 | 15.760 | -0.041 |



| 655.0 | 1.01716 | 0.71445 | 1.71164 | 650.775 | A43.561 | -5.224 | 0.172 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 64000 | 1-201761 | 060824 | 1-68666 | 645.645 | A62.674 | -6arin | 0.1410 |
| 665.0 | 1.01761 | 0.671 .97 | 1.66060 | 68.0 .313 | 862.03H | -R.619 | $0.1<4$ |
| 670.0 | 1.01145 | 0.63442 | 1.65144 | 665.079 | 811.898 | -10.944 | 0.045 |
| 675.0 | 1.11175 .7 | 0.650011 | 1.60590 | 669.1146 | A82.043 | -12.474 | 0.002 |
| .680.0 | 1.01736 | 0.64437 | 1.57627 | 674.615 | 842.525 | -14.547 | 0.027 |
| 685.0 | 1.01702 | 0.6425? | 1.54663 | 674.365 | 903.082 | -16.616 | -0.011 |
| .690.0. | 1.01636 | 0.64463 | -1.516/6 | 684.15t | 913.635 | -18.848 | -0.050 |
| 695.0 | 1.01599 | 0.65060 | 1.48714 | 68H.937 | 924.055 | -21.011 | -0.089 |
| 200.0. | 1.01530 | 0.66017 | 1.45831 | 693.722 | 934.220 | -23.205 | -0.121 |

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## SUNBI AIEK RR.II


FCCIMTRICITY=0.21172720 INCLINATIOF: 1.500 SO INJECTIIN VFLICITY=0.6077S

TIMF KRK HSM HIST FFA FSH AZIMUTH ELEVATIUN

| 0.0 | 1.00024 | 1.00076 | 0.00610 | 3.119 | 2.770 | -84.542 | 4.tc7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5.0 | 0.94077 | 0.9 .1948 | 0.01587 | 8.1051 | 7.140 | -42.084 | $3.7 \% 1$ |
| 10.0 | 0.9 .1771 | 0.99113 | 0.02550 | 11.494 | 11.536 | -88.461 | 3.913 |
| 15.0 | 0.94518 | 0.91923 | 0.01494 | 17.97is | 15.954 | -64.807 | 3.915 |
| 20.0 | 0.91432 | 10.9n7/k | 11.104418 | 22.492 | 20.411 | -80.204 | 3.1902 |
| 25.0 | 0.99240 | 0.94060 | 0053107 | 21.921 | 24.924 | $-15.296$ | Le8y\% |
| 30.0 | 0.97153 | 0.91737 | 0.06160 | 32.42\% | 29.517 | -10.080 | 3.841 |
| 35.0 | -0.97022 | 0.96235 | 0.06974 | 37.916 | 34.1.72 | -64.535 | 3.418 |
| 40.0 | 0.98849 | 0.95095 | 0.07710 | 42.465 | 38.973 | - 58.624 | 3.851 |
| 45010 | 10.28764 | 0.93814 | 0.08544 | 48.1001 | 43,477 | -122, 312 | 1.8u6 |
| $\begin{aligned} & 50.0 \\ & 55.0 \end{aligned}$ | $\begin{aligned} & 0.9867 A \\ & 0.98542 \end{aligned}$ | $\begin{aligned} & 0.92400 \\ & 0.90154 \end{aligned}$ | $\begin{aligned} & 0.09332 \\ & 0.10157 \end{aligned}$ | $\begin{aligned} & 53.049 \\ & 58.148 \end{aligned}$ | $\begin{aligned} & 48.924 \\ & 34.1 .5 \end{aligned}$ | $\begin{aligned} & -45.566 \\ & -38.402 \end{aligned}$ | $\begin{aligned} & 3.756 \\ & 3.6<7 \end{aligned}$ |
| 60.0 | 0.98446 | 0.89199 | 0.11066 | 6.3.175 | 59.534 | -30.875 | 3.416 |
| -65.0 | 0.98423 | 0.81432 | 0.12111 | 68.250 | 65.144 | -23.089 | 3.217 |
| 70.0 | 0.98360 | U.H5570 | 0.13350 | 13.332 | $70.9+2$ | -15.116 | 3.031 |
| 75.0 | 0.98311 . | 0.83628 | 0.14850 | 78.420 | 1.7.106 | -1.435 | 2.746 |
| 80.0 | 0.98274 | 0.81627 | 0.16666 | A3.511 | 43.516 | 0.237 | 2.434 |
| 85.0 | 0.98251 | U.74588 | 0.18851 | 88.606 | -40.252 | -6.215 | 2.111 |
| 90.0 | 0.98241 | 0.77534 | 0.21443 | 43.702 | 47.343 | 13.281 | 1.741 |
| 95.0 | 0.90244 | 0.75511 | 0.24478 | 48.798 | 104.818 | 18.883 | 1.441 |
| 100.0 | 0.98761 | 0.73540 | 0.27146 | 103.894 | 112.710 | 23.748 | 1.213 |
| 105.0 | 0.98241 | 0.71669 | 0.31 .906 | 108.987 | 121.005 | -27.892 | 0.963 |
| 110.0 | 0.913334 | 0.69942 | 0.36249 | 114.077 | 129.738 | 31.353 | 0.741 |
| 1150 | 0.98340 | 0.68468 | U.41142 | 119.161 | 138.884 | 34.185 | 0.547 |

\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline 120.0 \& \& \& \& \& \& \& \& <br>
\hline 125.0 \& r.964

.98588 \& 0.67115 \& 0.46291 \& 124.240 \& 14H.427 \& 36.446 \& 0.380 \& <br>
\hline 130.0 \& ¢.98629 \& 0.65427 \& \& 129.312 \& 158.298 \& 38.200 \& 0.236 \& <br>
\hline 135.0 \& 0.98730 \& 0.65092 \& 0.63524 \& 139.428 \& 178.708 \& 40.451 \& 0.012 \& <br>
\hline 140.0 \& 0.98841 \& 0.65139 \& 0.64554 \& 144.472 \& 189.037 \& 41.081 \& -0.074 \& <br>
\hline 145.0 \& . .98960 \& 0.65544 \& 10.25594 \& 149.504 \& 199.296 \& 41.465 \& -0.143 \& <br>
\hline 150.1 \& 0.24087 \& 0.66247 \& 0.81545 \& 154.524 \& 219.3i5 \& 41.663 \& -0.144 \& <br>
\hline 155.0 \& 0.24221 \& 0.67368 \& 2.87338 \& 154.532 \& 219.181 \& 41.730 \& -0.244 \& <br>
\hline 160.0 \& 0.97360 \& 0.68717 \& 0.92911 \& 164.527 \& 228.640 \& 41.711 \& -0.278 \& <br>
\hline 165.0 \& 0.99505 \& 0.70297 \& 0.98215 \& 169.508 \& 217.703 \& 41.645 \& -0.30s \& <br>
\hline 170.0 \& 0.99652 \& 0.72058 \& 1.03218 \& 174.476 \& 246.346 \& 41.563 \& -0.320 \& <br>
\hline -175.0 \& 0.99802 \& 0.73954. \& 1.07896 \& 174.429 \& 254.559 \& 41.487 \& -0.330 \& <br>
\hline
\end{tabular}

| 180.0 | 0.99454 | 0.75940 | 1.12238 | 184.308 | 262.353 | 41.434 | -0.335 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 185.0 | 1.00105 | 0.77916 | 1.16238 | 189.293 | 269.743 | 41.416 | -0.335 |
| 190.0 | 1.00256 | 0.80025 | 1.19898 | 194.204 | 276.756 | 41.418 | -0.331 |


| $\begin{aligned} & 195.0 \\ & 200.0 \\ & 205.0 \end{aligned}$ | $\begin{aligned} & 1.00404 \\ & 1.00564 \\ & 1.100641 \end{aligned}$ | $\begin{aligned} & 0.8700,8 \\ & 0.84049 \\ & 1.85474 \end{aligned}$ | $\begin{aligned} & 1.23221 \\ & 1.20217 \\ & 1.24894 \end{aligned}$ | $\begin{aligned} & 149.101 \\ & 205.404 \\ & 208.854 \end{aligned}$ | $\begin{aligned} & 289.420 \\ & 249.164 \\ & 244.414 \end{aligned}$ | $\begin{aligned} & 41.503 \\ & 41.612 \\ & 41.704 \end{aligned}$ | $\begin{aligned} & -0.324 \\ & -0.313 \\ & -0.300 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 210.0 | 1.00827 | 0.87418 | 1.3126 .1 | 213.112 | 301.614 | 41.976 | -0.245 |
| 215.0 | 1.00936 | O. H 1563 | 1.35336 | 21A.557 | 307.177 | 42.185 | -0.268 |
| 220.11. | 1.01079 | U. 41148 | 1.35124 | 223.391 | 312.533 | 42.442 | -0.244 |
| 225.0 | 1.01194 | 0.92717 | 1.36640 | 228.214 | 317.708 | 42.726 | -0.2<8 |
| 23000 | 1801300 | 0 0.9aucs | 1-374)6 | 2341027 | 122.122 | 416010 | -0.202 |
| 235.0 | 1.111347 | 0.75350 | 1.3 H9Un | 237.831 | 367.574 | 43.348 | -0.184 |
| 24000 | 1.01483 | U. 96460 | 1.3:682 | 242.027 | 312.355 | 43.615 | -0.100 |
| 245.0 | 1.01559 | 0.47425 | 1.40 () 37 | 247.415 | 397.010 | 44.019 | -0.136 |
| 2500 | 101024 | W.98242 | 1.40556 | 252.144 | 34654 | 44.328 | -0.10 |
| 255.0 | 1.01676 | 0.9 .1917 | 1.40742 | 256.474 | $346.0 \mathrm{H4}$ | 44.6142 | -0.04) |
| 260.0 | 1.01717 | 0.94419 | 1.64720 | 261. 747 | 350.513 | 44.938 | -0.058 |
| 265.0 | 1.01746 | 0.41776 | 1.40555 | 266.511 | 354.944 | 45.211 | -0.031 |
| 27000 | 101762 | 1.94977 | 10612012 | 271.246 | 359.329 | 450461 | -0,004 |
| 275.0 | 1.01765 | 1.00022 | 1.34758 | 276.052 | 363.704 | 45.698 | $0.0<3$ |
| 280.0 | 1.01756 | 0.919111 | 1.39160 | 280.414 | 368.063 | 45.818 | 0.020 |
| 285.0 | 1.017 .34 | 0.94643 | 1.38486 | 285.587 | 372.477 | 45.927 | 0.078 |
| 290.0 | -1.01700 | . 0.99212 | 1.37714. | 290.354 | 37f.902 | 45.971 | 0.145 |
| 295.0 | 1.01654 | 0.94642 | 1.36424 | 295.133 | 3Hi.312 | $45.961)$ | 0.131 |
| 300.0 | 1.01596 | 0.91912 | 1.36021 | 249.912 | 345.202 | - 45.8 .70 | 0.151 |
| 305.0 | 1.01526 | 0.97031 | 1.35214 | 304.647 | 340.507 | 45.700 | 0.162 |
| 310.0 | 1.01445 | 0.96004 | 1. 34358 | 304.489 | 345.204 | -45.441 | 0.206 |
| 315.0 | 1.01354 | 0.94833 | 1.33534 | 314.248 | 400.0119 | 45.086 | 0.228 |
| 320.0 | 1253 | 0.921523 | 1.32765 | 319.046 | 404.942 | 44.613 | 0.249 |
| 325.0 | 1.01143 | 0.92061 | 1.32077 | 323.914 | 410.028 | 44.072 | 0.267 |
|  |  | -- | - | - |  |  |  |
| 330.0 | 1.01024 | 0.90514 | 1.31497 | 328.741 | 415.211 | 43.378 | 0.283 |
| 335.0 | 1.00898 | 0.88830 | 1.31051 | 333.586 | 420.713 | 42.607 | 0.246 |
| 3400 | 1.0 ¢76a | 0.87041 | 1.30704 | 338.414 | 426.312 | 41.647 | 0.365 |
| 345.0 | 1.00627 | 0.85161 | 1.30664 | 343.294 | 432.275 | 40.665 | 0.310 |
| 350.0 | 1.00434 | 0.83205 | 1.30772 | 348.170 | 438.450 | - 39.512 | 0.312 |
| 355.0 | 1.00337 | 0.81144 | 1.31113 | 354.000 | 444.927 | 38.238 | 0.348 |
| 360.0 | 100187 | 0.79150 | $1 \times 31763$ | 157.963 | 451.734 | 36.845 | 0.300 |
| 365.0 | 1.00036 | 0.77103 | 1.32558 | 362.880 | 45H.907 | 35.340 | 0.281 |
| 370.0 | -0.99865 | 0. 0.75084 | 1.33685 | 367.812 | 466.467 | 33.727 | 0.209 |
| 375.0 | 0.99734 | 0.73131 | 1.35086 | 372.757 | 414.438 | 32.015 | 0.246 |
| 380.0 | .U.94585 | 0.71286 | 1.36755 | 377.717 | 4H2.834 | 30.213 | 0.219 |
| 385.0 | 0.94438 | 0.69597 | 1.38615 | 382.691 | 491.654 | 28.334 | 0.141 |
| 390.0 | 0.99296 | 0.68110 | 1.40823 | 387.6.78 | 500.896 | 26.394 | 0.1 .53 |
| 395.0 | 0.99159 | 0.66874 | 1.43166 | 392.614 | 511.510 | 24.398 | 0.115 |
| 400.0 | 0.99028 | 0.65934 | 1.45666 | 397.643 | 520.442 | 22.316 | 0.016 |
| 405.0 | 0.98905 | 0.65327 | 1.4 A27A | 402.714 | 530.611 | 20.345 | 0.036 |
| 410.0 | 0.98789 | 0.65076 | 1.50961 | 401.766 | 540.915 | 18.328 | -0.003 |
| 415.0 | 0.98683 | 0.65195 | 1.53674 | 412.H04 | 531.239 | 16.348 | -0.041 |




| 540.0 | 0.99795 | 0.99832 | 1.99544 | 539.184 | 115.882 | 1.634 | $\begin{aligned} & 0.018 \\ & 0.001 \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 545.0 | 0.94946 | 1.00000 | 1.94833 | 544.124 | 720.265 | 1.912 |  |
| 550.0 | 1.00098 | 1.00012 | 1.99461 | 549.054 | 7\%4.634 | 2.207 | 0.020 |
| 555.0 | 1.00248 | 0.99866 | 1.99928 | 553.966 | 729.019 | 2.470 | 0.039 |
| 560.0 | 1.00397 | 0.94565 | 1.99736 | 558.864 | 733.414 | 2.711 | 0.0080 |
| 535.0 | 1.00542 | 0.92102 | 1.99387 | 563.748 | 7.37.1152 | 2.927 | 0.076 |


| 570.0 | 1.00684 | 0.98499 | 1.48883 | 568.619 | 742.333 | 3.108 | 0.094 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 575.0 | 1.00820 | 0.97736 | 1.98227 | 573.477 | 746.878 | 3.249 | 0.111 |  |
| 580.0 | 1.00950 | 0.96824 | 1.97423 | 578.323 | 751.501 | 3.340 | 0.128 |  |
| $58 . .00$ | 1.01073 | 0.95766 | 1.96478 | 583.157 | 756.219 | 3.376 | 0.144 |  |
| 540.0 | 1.01189 | 0.94565 | 1.95395 | 587.980 | 761.051 | 3.348 | 0.159 |  |
| 595.0 | 1.01295 | 0.93226 | 1.44180 | 592.794 | 766.014 | 3.250 | 0.173 |  |


| 600.0 | 1.01392 | 0.91757 | 1.92841 | 697.598 | 771.127 | 3.075 | 0.185 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 605.0 | 1.01479 | 0.90164 | 1.91382 | 602.395 | 776.416 | 2.814 | 0.196 |
| 610.0 | 1.01 .54 | 0.88457 | 1.89810 | 607.183 | 781.902 | 2.460 | 0.206 |
| 615.0 | 1.01621 | 0.86647 | 1.88131 | 611.966 | 787.611 | 2.005 | 0.213 |
| 620.0 | 1.01674 | 0.84749 | 1.86351 | 616.743 | 743.568 | 1.444 | 0.218 |
| 625.0 | 1.01716 | 0.82780 | 1.84473 | 621.516 | 799.806 | 0.768 | 0.221 |


| 630.0 | 1.01745 | 0.80759 | 1.82502 | 626.286 | 806.352 | -0.044 | 0.221 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 635.0 | 1.01761 | 0.78713 | 1.80440 | 631.054 | 813.235 | -0.952 | 0.218 |
| 640.0 | 1.01765 | 0.76668 | 1.78287 | 635.821 | 820.487 | -2.005 | 0.211 |
| 645.0 | 1.01757 | 0.74660 | 1.76042 | 640.587 | 828.133 | -3.193 | 0.202 |
| 650.0 | 1.01736 | 0.72727 | 1.73703 | 645.356 | 8.36 .194 | -4.516 | 0.188 |


| 655.0 | 1.01702 | 0.76911 | 1.71266 | 650.127 | A44.6H3 | -5.973 | 0.170 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 660.0 | 101656 | Onat201 | 1-68124 | Ohar Bug | 853.647 | -7.562 | 0.169 |
| 665.0 | 1.01599 | 0.61H23 | 1.66088 | 699.618 | 862.918 | -9.273 | 0.123 |
| 670.0 | 1.01530 | 0.64647 | 1.63350 | 664.463 | 812.4V6 | -11.096 | 0.044 |
| 675.0 | 1.01444 | 0.63176 | 1.6052 ? | 669.254 | 842.597 | -13.016 | 0.001 |
| 680.0 | 1.01359 | 0.65243 | 1.51 .622 | 674.054 | 482.803 | $-15.015$ | 0.026 |
| 685.0 | 1.01258 | 0.65071 | 1.54678 | 678.A61 | 413.171 | -17.014 | -0.011 |
| 690.0 | 1.01148 | 0.65267 | -1.51727 | -683.678 | 913.434 | -19.168 | -0.0 |
| 695.0 | 1.01030 | 0.61874 | 1.48814 | 688.505 | 923.687 | -21.276 | -0.0.084 |
| 200.0 | 1.00902 | 6.66718 | 1.45921 | .693.344. | 931.524 | $-23.311$ | -0.126 |

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## SUNBLALER MRRIT



## ECCENTRICITY=0.2013474T INCLINATION= 0.5000 SA IN.IEC.IION VELOCITY=1).8474L

PERICD SENS TO INJECT VII. $=20.43$ UAYS RER/KMiSFC VSASUI 0.11210 THAE U.O

| IME | RER | RSB | U15 | FER | +5B | AZIMUTH | elevailon |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.99176 | 0.99169 | 0.00618 | 3.183 | 2.833 | -43.808 | 3.911 |
| 5.0 | $0.989+1$ | 0.99095 | 0.01512 | 8.200 | 7.295 | -93.066 | 4.0 we |
| 10.0 | 0.98875 | 0.948864 | 0.02522 | 13.229 | 11.731 | -89.065 | 4.002 |
| 15.0 | 0.98762 | 0.98489 | 0.03454 | 18.269 | 10.214 | -84.455 | 3.998 |
| 20.0 | 0.98658 | 0.97962 | 0.04359 | 23.320 | 20. Fl 9 | -79.594 | 3.9 .97 |
| 25.0 | 0.98564 | 0.97288 | 0.05233 | 28.380 | 25.420 | $-74.367$ | 3.944 |


| 30.0 | 0.98481 | 0.96464 | 0.06072 | 33.444 | 30.042 | -68.915 | 3.900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35.0 | 0.98409 | 0.95506 | 0.06880 | 38.526 | 34.8 .51 | -63.147 | 3.909 |
| 40.0 | 0.98350 | 0.94406 | 0.07665 | 43.609 | 39.714 | -57.017 | 3.938 |
| 45.0 | 0.98303 | 0.931172 | 0.08442 | 48.697 | 44.649 | $-50.488$ | 3.884 |
| $\begin{array}{r} 50.0 \\ 55.0 \end{array}$ | $\begin{aligned} & 0.98269 \\ & 0.98248 \end{aligned}$ | $\begin{aligned} & 0.41811 \\ & 0.90329 \end{aligned}$ | $\begin{array}{r} 0.09236 \\ 0.10081 \end{array}$ | $\begin{aligned} & 53.789 \\ & 5 H .884 \end{aligned}$ | $\begin{aligned} & 49.825 \\ & 55.112 \end{aligned}$ | $\begin{aligned} & -43.549 \\ & -36.214 \end{aligned}$ | $\begin{aligned} & 3.800 \\ & 3.617 \end{aligned}$ |
| 60.0 | 0.98240 | 0.88736 | 0.11023 | 63.98C | 60.543 | -28.561 | 3.508 |
| 65.0 | 0.98247 | 0.88704 | 0.12116 | 64.077 | 66.201 | -20. 722 | 3.290 |
| 70.0 | 0.98266 | 0.85259 | 0.13416 | 74.172 | 12.170 | -12.851 | 3.020 |
| 75.0 | 0.98299 | 0.83405 | 0.14986 | 79.265 | 78.337 | -5.188 | 2.726 |
| 80.0 | 0.98344 | 0.81448 | 0.1687 A | 84.353 | 84.788 | 2.110 | 2.405 |
| 85.0 | 0.98403 | 0.79560 | 0.19140 | 89.437 | 91.532 | 8.834 | 2.078 |


| 90.0 | 0.98473 | 0.77616 | 0.21806 | 94.514 | 48.653 | 16.903 | 1.760 |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 95.0 | 0.98555 | 0.75699 | 0.24899 | 99.584 | 106.120 | 20.257 | 1.460 |
| 100.0 | 0.98648 | 0.73841 | 0.28432 | 104.645 | 113.768 | 24.889 | 1.187 |
| 105.0 | 0.98751 | 0.72082 | 0.32402 | 109.697 | 122.211 | 28.823 | 0.941 |
| 110.0 | 0.98864 | 0.70464 | 0.36798 | 114.738 | 130.849 | 32.102 | 0.724 |
| 115.0 | 0.98985 | 0.69030 | 0.41594 | 112.768 | 133.870 | 34.781 | 0.535 |





| 510.0 | 0.94825 | 0.94893 | 1.94743 | 509.438 | 648.044 | 0.640 | -0.124 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 515.0 | 0.99917 | 0.95934 | 1.958993 | 514.375 | 692.860 | 0.742 | -0.112 |
| 520.0 | 1.00128 | 0.96836 | 1.96941 | 519.297 | 697.580 | 0.843 | -0.0 .14 |
| 525.0 | 1.00278 | 0.97594 | 1.97842 | 524.206 | 702.220 | 0.919 | -0.075 |
| 530.0 | 1.00426 | 0.98205 | 1.98541 | 529.101 | 706.745 | 1.140 | -0.056 |
| 535.0 | 1.00571 | 0.98669 | 1.92186 | 535.982 | 711.320 | 1.319 | -0.032 |


| 540.0 | 1.00712 | 0.98982 | 1.99623 | 538.851 | 715.810 | 1.507 | -0.018 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 545.0 | 1.00847 | 0.99144 | 1.99901 | 543.706 | 120.278 | 1.700 | 0.001 |
| 550.0 | 1.00976 | 0.99155 | 2.00020 | 548.550 | 724.739 | 1.889 | 0.020 |
| 555.0 | 1.01097 | 0.94015. | 1.99974 | 553.382 | 729.205 | 2.067 | 0.040 |
| 560.0 | 1.01211 | 0.98724 | 1.99779 | 558.203 | 733.691 | 2.229 | 0.038 |
| 565.0 | 1.01315 | 0.98282 | 1.99422 | 563.015 | 738.209 | 2.367 | 0.077 |


| 670.0 | 1.01410 | 0.97692 | 1.98910 | 567.818 | 742.716 | 2.474 | 0.045 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 575.0 | 1.01495 | 0.96955 | 1.98246 | 572.612 | 747.405 | 2.544 | 0.113 |
| 580.0 | 1.01570 | 0.96075 | 1.97433 | 577.400 | 752.112 | 2.571 | 0.127 |
| 585.0 | 1.01632 | 0.95053 | 1.96477 | 582.181 | 756.914 | 2.546 | 0.145 |
| 590.0 | 1.01683 | 0.93895 | 1.95382 | 586.957 | 161.227 | 2.464 | 0.160 |
| 5950 | 1.01722 | 0.92606 | 1.94153 | 591.729 | 766.869 | 2.317 | 0.174 |


| 600.0 | 1.01749 | 0.91192 | 1.92796 | 596.499 | 712.062 | 2.097 | 0.147 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 605.0 | 1.01763 | $0.8966 ?$ | 1.91316 | 601.266 | 777.425 | 1.800 | 0.197 |  |
| 610.0 | 1.01765 | 0.88023 | 1.89720 | 606.033 | 782.982 | 1.415 | 0.207 |  |
| 615.0 | 1.01754 | 0.86290 | 1.88013 | 6100.800 | 788.754 | 0.940 | 0.214 |  |
| 620.0 | 1.01730 | 0.84475 | 1.86177 | 615.569 | 794.770 | 0.369 | 0.219 |  |
| 625.0 | 1.01694 | 0.82596 | 1.84285 | 020.340 | 801.055 | -0.324 | 0.221 |  |
|  |  |  |  |  |  |  |  |  |
| 630.0 | 1.01646 | 0.80673 | 1.82273 | 625.114 | 807.637 | -1.117 | 0.221 |  |
| 635.0 | 1.01586 | 0.78729 | 1.80167 | 629.895 | 814.542 | -2.029 | 0.218 |  |
| 640.0 | 1.01514 | 0.76792 | 1.77968 | 634.680 | 821.798 | -3.065 | 0.211 |  |
| 645.0 | 1.01432 | 0.74896 | 1.75677 | 639.472 | 829.424 | -4.226 | 0.201 |  |
| 650.0 | 1.01339 | 0.73075 | 1.73293 | 644.273 | 837.439 | -5.512 | 0.187 |  |


| 6600 | 1.01125 | 10.4R26 | Lantay | 0541002 | Mharath | -8.657 | 0.168 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 665.0 | 1.01006 | 0.64485 | 1.65591 | 658.731 | H63.843 | -10.105 | 0.122 |
| 620.0 | 1.00879 | 0.61341 | L.62854 | 661.572 | $413 n 342$ | $-11.857$ | 0.093 |
| 675.0 | 1.00745 | 0.64 .581 | 1.60038 | 668.426 | 843.123 | -13.644 | 0.060 |
| ..680.0 | 1.00606 | 0.6048 .2 | 1.51114. | . 613.291 | 893.100 | -15.617 | 0.023 |
| 685.0 | 1.00462 | 0.65928 | 1.54287 | 67H.16A | 903.176 | -17.589 | -0.012 |
| 690.0 | 1.00314 | 0.64110 | 1.51414 | -683.061 | 913.241 | $\ldots=19.595$ | $-0.020$ |
| 695.0 | 1.00165 | 0.66620 | 1.48596 | 687.965 | 423.211 | -21.616 | -0.048 |
| 200.0 | 1.00013 | 0.67456 | $-1.45880$ | 642.885 | 912.976 | -23.629 | -0.126 |

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| IIMF | KFK | RSH | UIST | Ff.R | FSt | ALIMUTH | elfvation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.98481 | U.4 555 | 0.00618 | 3.241 | 2. 188 | -96.806 | 4.025 |
| 5.0 | 0.98409 - | $0.9 H 482$ | 0.01566 | 8. 318 | 1.414 | -92.213 | 4.002 |
| 10.0 | 0.98350 | $0.9+261$ | 0.02495 | 13.401 | 11.952 | -87.223 | 4.081 |
| 15.0 | 0.98303 | 0.978 .6 | 0.03411 | 18.484 | 16.517 | -82.048 | 4.063 |
| 20.0 | 0.98267 | 0.77379 | 0.04301 | 23.581 | 21.1:4 | -76.803 | 4.083 |
| 25.0 | 0.98248 | 0.46722 | 0.05163 | 28.676 | 25.187 | -71.319 | 4.078 |



| 90.0 | 0.99113 | 0.77683 | 0.22781 | 94.578 | 99.601 | 17.382 | 1.681 |  |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 95.0 | 0.99248 | 0.75844 | 0.26004 | 99.584 | 107.057 | 22.303 | 1.394 |  |
| 100.0 | 0.99389 | 0.74066 | 0.29650 | 104.576 | 114.878 | 26.544 | 1.133 |  |
| 105.0 | 0.99534 | 0.72387 | 0.33715 | 109.554 | 123.073 | 30.133 | 0.900 |  |
| 110.0 | 0.99682 | 0.70845 | 0.38184 | 114.519 | 131.642 | 33.116 | 0.693 |  |
| 11500 | 0.99432 | 0.64482 | 0.43031 | 119.469 | 140.570 | 35.545 | 0.513 |  |


| 120.0 | 0.99984 | (). 68338 | 0.48216 | 124.405 | 149.826 | 37.477 | 0.356 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 125.0 | 1.00135 | 0.67 .451 | 0.53687 | 129.328 | 159.362 | 38.967 | 0.221 |
| 130.0 | 1.00285 | 0.66853 | 0.59380 | 134.236 | 169.111 | 40.072 | 0.106 |
| 13500 | 1.00433 | 0.66566 | 0.65224 | 139.130 | 178.989 | 40.851 | 0.009 |
| 140.0 145.0 | 1.00578 1.00718 | 0.66600 0.66955 | 0.71140 0.77052 | 144.011 148.878 | 188.905 148.764 | 41.358 41.647 | -0.072 -0.140 |


| 150.0 | 1.00853 | 0.67617 | 0.82884 | 153.733 | 208.473 | 41.769 | -0.144 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 155.0 | 1.00982 | 0.68562 | 0.88570 | 158.576 | 217.954 | 41.770 | -0.238 |
| 160.0 | 1.01103 | 0.69756 | 0.94053 | 163.408 | 227.144 | 41.689 | -0.272 |
| 165.0 | 1.01216 | 0.71160 | 0.99288 | 168.229 | 235.997 | 41.561 | -0.297 |
| 170.0 | 1.01320 | 0.72735 | 1.04242 | 173.040 | 244.487 | 41.413 | -0.315 |
| 175.0 | 1.01415 | 0.74439 | 1.08891 | 177.842 | 252.602 | 41.266 | -0.326 |


| 180.0 | 1.01499 | 0.76233 | 1.13220 | 182.637 | 260.344 | 41.137 | -0.332 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 185.0 | 1.01573 | 0.78081 | 1.17223 | 187.424 | 267.724 | 41.037 | -0.333 | -0.330 |
| 190.0 | 1.01635 | 0.79950 | 1.20898 | 192.205 | 214.760 | 40.974 |  |  |


| $\begin{array}{r} 195.0 \\ -200.0 \\ 205.0 \end{array}$ | $\begin{aligned} & 1.01686 \\ & =1.01724 \\ & 1.01750 \end{aligned}$ | $\begin{gathered} 0.81813 \\ 0.83643 \\ 0.85421 \end{gathered}$ | $\begin{gathered} 1.24247 \\ 1.22276 \\ \hline 1.29991 \end{gathered}$ | $\begin{aligned} & 196.981 \\ & 201.753 \\ & 206.522 \end{aligned}$ | $\begin{aligned} & 281.676 \\ & 242.849 \\ & 294.041 \end{aligned}$ | $\begin{aligned} & 40.951 \\ & 40.971 \\ & 41.034 \end{aligned}$ | $\begin{aligned} & -0.323 \\ & -0.313 \\ & -0.300 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21000 | 1.101764 | -0.82120 | 1.32643 | 21.240 | 299.962 | 41.117 | -0.285 |
| 215.0 | 1.01764 | 0.86750 | 1.34521 | 216.056 | 305.622 | 41.274 | -0.268 |
| -220.0 | 1.01753 | 0.90273 | 1.36356 | 220.826 | 311.104 | 41.454 | -0.242 |
| 225.0 | 1.01729 | 0.91687 | 1.37914 | 225.543 | 316.410 | 41.661 | -0.229 |
| 2380 | 1.01602 | 0.92984 | 1.30221 | 230.364 | 321.561 | 41.843 | -0.248 |
| 235.0 | 1.01643 | 0.94157 | 1.40276 | 235.140 | 326.577 | 42.145 | -0.1.45 |
| 200.0 | 1.01583 | 0.95199 | 1.41024 | 219.920 | 311.416 | 42.413 | -0.161 |
| 245.0 | 1.01511 | 0.96106 | 1.41688 | 244.706 | 316.216 | 42.69 C | -0.136 |
| 250 | 101428 | 0.961876 | 1 | 264.649 | 140.443 | 42.971 | -0.111 |
| 255.0 | 1.01335 | 0.97501 | 1.42261 | 254.300 | 345.642 | 43.250 | -0.085 |
| -260.0 | 1.01232 | 0.41283 | 1.42266 | 259.110 | 350. 239 | 43.519 | -0.058 |
| 265.0 | 1.01120 | 0.98320 | 1.42103 | 263.929 | 354.797 | 43.774 | -0.031 |
| 270.0 | 1001000 | 0.98510 | 1841788 | 268.760 | 359.332 | 44.006 | -0.004 |
| 275.0 | 1.00872 | $0.9855 ?$ | 1.41336 | 273.601 | 363.855 | 44.209 | 0.023 |
| -280.0 | 1.00738 | 0.98447 | 1.40765 | 278.454 | 368. 381 | 44.311 | 0.051 |
| 285.0 | 1.00599 | 0.98174 | 1.40092 | 283.320 | 372.924 | 44.500 | 0.078 |
| .290.0 | 1.00455 | 0.97795 | 1.30335 | 288.148 | 371.497 | 44.573 | 0.106 |
| 295.0 | 1.00307 | 0.97251 | 1.38516 | 293.091 | 3H2. 115 | 44.587 | 0.152 |
| -300.0 | 1.00157 | 0.46563 | 1.37654 | 297.997 | 386.741 | 44.534 | 0.128 |
| 305.0 | 1.00006 | 0.95735 | 1.36772 | 302.917 | 341.542 | 44.408 | 0.183 |
| 310.0 | 0.99854 | 0.94170 | 1.35892 | 307.851 | 396.382 | 44.149 | 0.207 |
| 315.0 | 0.99704 | 0.93672 | 1.35039 | 312.800 | 401.324 | 43.902 | 0.229 |
| 3200 | 0.99455 | 0.932445 | 16.34237 | 317.762 | 406.400 | 43.504 | 0.249 |
| 325.0 | 0.99410 | 0.91097 | 1.33512 | 322.739 | 411.615 | 43.014 | 0.267 |
| -330.0 | 0.99269 | 0.84635 | 1.32891 | 327.729 | 416.294 | 42.410 | 0.283 |
| 335.0 | 0.99133 | 0.88069 | 1.32400 | 332.732 | 422.557 | 41.694 | 0.295 |
| 34000 | 0.99003 | 0.86409 | 1.32065 | 3372748 | 428.328 | 40.862 | 0.304 |
| 345.0 | 0.98881 | 0.84669 | 1.31911 | 342.776 | 434.331 | 19.913 | 0.309 |
| . 350.0 | 0.98767 | 0.82867 | 1.31963 | 347.816 | 440.591 | 18.844 | 0.310 |
| 355.0 | 0.98663 | 0.81020 | 1.32242 | 352.860 | 447.134 | 37.658 | 0.306 |
| 3600 | 0.98568 | n. 29151 | 1.32766 | 357.926 | 4236881 | 16.358 | 0.297 |
| 365.0 | 0.98484 | 0.77287 | 1.33550 | 362.995 | 461.164 | 34.947 | 0.284 |
| 370.0 | 0.98412 | 0.75458 | 1.34601 | 368.071 | 468.697 | 33.434 | 0.206 |
| 375.0 | 0.98352 | 0.73698 | 1.35922 | 373.154 | 476.547 | 31.82\% | 0.242 |
| 380.0 | 0.98305 | 0.72044 | 1.37507 | 378.242 | 484.873 | 30.133 | 0.215 |
| 385.0 | 0.98270 | 0.70537 | 1.313342 | 383.334 | 493.520 | 28.367 | 0.184 |
| 390.0 | 0.98249 | 0.69217 | 1.41407 | 388.429 | 502.521 | 26.541 | 0.149 |
| 395.0 | 0.98241 | 0.68126 | 1.43672 | 393.525 | 511.842 | 24.672 | 0.112 |
| 40000 | 0.98246 | 0.67298 | 1.46104 | 348.622 | 521.430 | 22.717 | 0.073 |
| 405.0 | 0.98265 | 0.66765 | 1.48663 | 403.717 | 531.213 | 20.874 | 0.034 |
| 4100 | 0.98297 | 0.666546 | 1.51311 | 404.810 | 54.10108 | 18.983 | -0.0304 |
| 415.0 | 0.98342 | 0.66649 | 1.54010 | 413.899 | 551.019 | 17.126 | -0.041 |



| 655.0 | 1.00462 | 0.71709 | 1.69940 | 647.960 | 846.648 | -7.787 | 0.104 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 660.0 | 1-00210 | 0.7023a | 1.67260 | 0620453 | A5Sala | -9,206 | 0.102 |
| 665.0 | 1.00165 | 0.68964 | 1.64706 | 657.757 | 804.4H6 | -10.855 | 0.122 |
| 670.0 | 1.90013 | 0.62926 | 1.61986 | 662.612 | 873.847 | -12.543 | Q. 042 |
| 675.0 | 0.9486? | 0.67159 | 1.59213 | 667.611 | 883.504 | -14.314 | 0.000 |
| 680.0 | 0.99711 | 0.66692 | 1.56609 | -672.55a | 893. 314 | -16.164 | 0.025 |
| 685.0 | 0.94562 | 0.66541 | 1.53602 | 677.520 | 903.224 | -18.063 | -0.012 |
| 690. 2 | 0.94412 | 0.66713 | 1.50826 | -682.496 | Y13.121 | $-12.245$ | -0.0.0 |
| 695.0 | 0.99275 | 0.61202 | 1.48120 | 687.486 | 922.412 | -21.941 | -0.088 |
| 200.0 | 0.94139 | 0.67948 | 1.65521. | 692488 | 912.5121 | -23.882 | -0.126 |

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## CHAPTER 4

## 4. 0 COMMUNICATION SYSTEM

This chapter provides an overall description of the communication system requirements and constraints, and the functional design of each of the major elements of the system. Topics covered include the signal design, the anticipated signal-to-noise ratios, analysis of the expected system performance, and the hardware implementation of thie spacecraft equipment.

### 4.1 System Requirements and Constraints

The purpose of the communication system is to satisfy the mission objectives described in Chapter 1 of this report. Within this general framework there are two notable subdivisions.

The propagation experiment of immediate interest requires the transmission of accurately timed pulses of coherent RF energy at two different frequencies, and the precise measurement of relative transmission time delays, and frew quency and delay perturbations induced by the transmission medium. As described fully in Chapter'1, these measurements are expected to yield unique data describing the electron density surrounding the sun, and the nature of the turbulence.

Another important communication system requirement is to provide engineering data confirming the viability of the small, inexpensive, solar stabilized, spacecraft as a vehicle for the propagation experiment and for possible subsequent particle and fields experiments in the 0.5 AU region of interplanetary space. Primarily, this requires a reliable acquisition and tracking capability and the provision of a telemetry data link for evaluation of the spacecraft behavior and for the return of data from experimental packages carried on the spacecraft.

Principal communication system constraints are imposed by the expected perturbations in the channel that is to be in.estigated, and by the very low signal- to-noise ratio. The latter is limited severely by the very long range
over which communication must be maintained. Additionally, the apacecraft size and wright restrictions conspire to limit both the average power available to the apacecraft transmitter and the peak power that can be realized with avallable epace-quality components.

### 4.2 Functional Description of the Communication System

Seven major syatem elements are relevant directly to a discusaion of the total system function and performance:

1. the spacecraft programmer, which generates all the timing pulses and reference frequencies required in the spacecraft;
2. the main transmitter, concerned primarily with the short pulses required by the propagation experiment;
3. the beacon transmitter, used primarily to provide a spacecraft tracking signal and a telemetry link for spacecraft engineering evaluation data;
4. the communication channel, which also is the medium that is to be investigated;
5. the receiving antenna, a large phased array;
6. the receiver RF section, which provides power gain with the least possible additive noise; and
7. the data evaluation section of the receiver, which is concerned with the accurate measurement of time and frequency perturbation.
Functional diagram, D-106-400000, shows the relation between these various elements.

In the spacecraft a stable, crystal-controlled, oscillator, operating at a frequency close to 5 MHz , provides a basic timing reference signal. Output of this oscillator is frequency multiplied to define the required RF' carrier irequencies of 70,75 , and 80 MHz . Output of the oscillator is also counted down in the programer to define the time intervals during which the transmitters are turned on. Derivation of the timing pulses and carrier frequencies from the same source ensures that the transmitted pulses are completely coherent and, as will be discussed later, makes it possible to phase code the pulse in a manner which enhances substantially the time measurement capability of the system.

The main transmitter emits pulses with a duration of 3 ms , internally phase coded with a baud time of $25 \mu \mathrm{~s}$. Peak power output of this transmitter is 2 kW . The pulse repetition rate varies, in sympathy with the actual power available from the solar cell array, from a minimum of one pulse in 120 s to a maximum of one pulse in 0.8 s . Eighty percent of the total energy output
of this transmitter is used to emit pulses alternately at the two extreme carrier frequencies of 70 and 80 MHz . These are the pulses used in the relative delay time measurement. Twenty percent of the energy output is transmitted in the form of similar short pulses at a carrier frequency of 75 MHz . These pulses serve an important function during the sigual acquisition and receiver synchronization process, and, also, permit some of the freqency spreading effects of the channel to be measured.

The beacon transmitter emits pulses with a peak nower of 50 watts at a carrier frequency of 75 MHz . This transmitter operates in two modes. In the first few days of flight, before the spacec 'aft solar-cell array is oriented properly towards the sun, the beacon transmitter operates from a battery power supply and transmits pulses with a duration ranging from 50 to $!50 \mathrm{~ms}$ at a repetition rate of one pulse in 120 seconds. These pulses provide an immediate spacecraft tracking signal, while the pulse duration is used to convey telemetry data describing the behavior of the spacecraft during the initial solar-acquisition and spacecraft-stabilizacion phase. After the spacecraft has been oriented, and sufficient power is available from the solar cell array, the main transmitter is activated. In this mode, the beacon pulse duration is increased to range from 5 to 7 seconds. These longer pulses are interleaved infrequently with the main transmitter output. This permits some additional frequency-spreading data to be measured, and also provides a continuing, redundant telemetry link.

Pulses transmitted from the spacecraft must pass through the communication channel, which is the physical entity that is to be investigated. It is expected that the channel will introduce significant frequency dependent time delays and apparent frequency and time perturbations, These effects are described fully in Chapter 2 of this report. Here we note only that the anticipated relative time delays are expected to range from a few $\mu s$ up to the order of 1 ms , and that the probable frequency perturbations range up to a few hundred Hz at the carrier frequencies used by Sunblazer. As will be seen, these parameter ranges affect importantly the optimum signal design.

The phased array receiving antenna and associated RF amplifiers also are described fully in Chapter 12 of this report. Here we note orly that the measurement precision required when the communication line of sight passes close to the sun dictates a combination of high gain antenna and low noise amplifier with an effective signal-to-noise ratio of the order of 50 dB . Later sections of this chapter will show this in detail.

The data csaluation sections of the receiver include a $5-\mathrm{MHz}$ oscillator that is slaved to the oscillator in the spacecraft. Three separate control loops are used in different combinations. For initial acquisition, the oscillator is tuned manually by an operator referring to a display of the received aignal. The oscillator then is tuned more finely by a pulsed frequency discriminator that is used to examine the $75-\mathrm{MHz}$ pulses emanating from the main spacecraft transmitter. And, finally, the oscillator is tuned precisely by measuring the time interval between received pulses; that is, by measuring the pulse repetition rate which is known to be integrally related to the transmitted carrier frequency. In a manner analogous to that used in the spacecraft, the oscillator output is counted down to provide accurate timing pulses, and is frequency multiplied to provide coherent RF reference signals. These RF signals are used as reference inputs to the mixers in the front end of the receiver.

Prime measurements are conducted by banks of correlators in the receiver. By reference to the basic oscillator, these correlators compare the signal actually received with a multiplicity of postulated signals generated in the receiver. The actual time of signal arrival is obtained by a maximum likelihood choice based on the outputs of the correlator banks. This subject is discussed in later sections of this chapter. Here we note only that the required system precision necessitates the use of a few hundred correlators for the examination of each of the two extreme carrier frequencies of 70 and 80 MHz .

Other sections of the spacecraft and receiver complex are concerned with the conversion, coding, decoding, and display of telemetered data. This is superposed on the beacon link by pulse duration modulation, and on the main link by pulse position modulation. The rationale for this is discussed in later sections of this chapter. Here we note only that the format is designed so that the prime propagation experiments are not degraded by this piggyback data.

### 4.3 Signal Design

Two separate but related commurication links are included in the Sunblazer system. For maximum reliability, these two links are used redundantly as much as possible. Both links offer a capability for the measurement of frequency broadening induced by the transmission medjum, and both provide a capacity for the superposition of tclemetry data. Each link, however, has a difference prime function that dictates a different optimum form for the signal coding. This optimum signal design is the subject of this section of the report.

### 4.3.1 Main Transmitter

The signal coding used in the main transmitter is designed to optimize the prime propagation experiment. This experiment requires that the spacecraft emit pulses of relatively short duration with large peak power at each of two different carrier frequencies, so that the relative transmission delay imposed by the medium can be measured accurately.

Practically, there is a fairly broad range of acceptable values for the duration and peak power of the transmitted pulse. A somewhat arbitrary 'optimum' combination of these parameters has to be chosen on the basis of a tradeoff among several conflicting system constraints. This compromise leads to the selection of a pulse with an envelope duration of the order of 3 ms , a peak power of the order of 2 kW , and an internal phase coded structure with a baud time of the order of $25 \mu \mathrm{~s}$. Telemetry is best superposed on this structure by pulse-position modulation. The rationale for these choices is discussed below.

### 4.3.1.1 Pulse Envelope Duration

Consider first the most desirable duration for a single transmitted pulse. In order to enhance the time resolution capability of the system, and to minimize the effective sample aperture time for the experiment, the pulse duration should be as short as possible. But, in order to minimize both the transmitter peak power requirement and the receiving antenna gain requirement, the pulse duration should be as long as possible. A compromise between these two conflicting requirements can be effected by considering the limitations of the receiving antenna and the postulated behavior of the medium when the spacecraft is close to superior conjunction.

A practical lower limit for the beamwidth of the receiving antenna is of the ord." 0.5 degrees. This minimum beamwidth is primarily constrained by the necessity for tolerating angular scintillations which cause an apparent random motion of the signal source.

This finite antenna beamwidth will prohibit reception when the angular offset between the solar disc and the line of sight from spacecraft to earth is less than about one degree. In this condition a portion of the solar disc will intercept the antenna beam. And since the sun has an effective noise temperature of a few million degrees at the frequencies of interest, the total background noise picked up by the receiving antenna will increase prohibitively.

Now, as the spacecraft approaches superior conjunction the range over which communication has to be maintained is 2 AU , or $3 \times 10^{11}$ meters. This causes
a path loss of:

$$
\begin{equation*}
\frac{1}{4 \pi R^{2}}=\frac{1}{4 \pi\left(3 \times 10^{11}\right)^{2}} \approx-240 \mathrm{~dB} \tag{4,1}
\end{equation*}
$$

Because of this very substantial signal attenuation it is necessary to design the receiver to detect the presence of a pulse by coherent integration of the total energy in the pulse. But for this to be possible, each pulse must be individually coherent. That is, the RF carrier phase must remain essentially undisturbed for the duration of a single pulse.

Some quantitative speculation relevant to this requirement is provided by Chapter 2 of this report. There it is shown that the average channel coherence time is likely to be related to the offiset of the line of sight from the solar disc in the following manner:

| Offset <br> (Degrees) | Coherence time <br> (milliseconds) |
| :---: | :---: |
| 0.5 | 0.3 |
| 1.0 | 3.0 |
| 1.5 | 10.0 |

It is apparent from these figures that the "optimum" pulse duration is of the order of 3 ms .

A pulse significantly shorter than 3 ms would require either the transmitter peak power or the receiving antenna gain, or both, to be increased to maintain the same collected signal energy per pulse. But these rather difficult and expensive changes would not improve the experiment significantly. Since the channel coherence time reduces very rapidly for offset angles less than about one degree, and the solar noise contribution increases very rapidly at the same time, the possible decrease in minimum observable angular offset is very limited.

At the expense of a rather significant increase in the minimum observable offset angle, a pulse longer than 3 ms could be used to justify a reduction in the transmitter peak power or receiving antenna gain. However, this increase in minimum observable angular offset would materially degrade the experiment in a region of particular interest.
Further, if the increased pulse duration were used to justify a reduction in the transmitter peak power, then the total radiated energy per pulse would be unchanged and no other system constraints would be relieved. The only effect of this change, therefore, would be to reduce slightly the difficulty
involved in the design of the transmitter at the expense of a degradation in the experiment.

On the other hand, if the transmitter peak power is assumed to be a constant, a longer pulse duration could be used to justify a reduction in the required receiving antenna gain. But this is not an unalloyed blessing. Since the energy per transmitted pulse is increased by this change, while the average power collected by the spacecraft solar cell array remains constant, the pulse repetition rate would have to be reduced proportionately. Thus the sample data points obtained at the two different carrier frequencies would be more widely separated in time and the total sampling rate would be reduced. Both of these changes represent a further degradation of the experiment.

### 4.3.1.2 Peak Power

The most desirable transmitter peak power also has to be chosen as a compromise between two conflicting sets of constraints. It is desirable that this be as low as possible, both in order to permit the maximum possible pulserepetition rate compatible with the limited average power available in the spacecraft, and also in order to simplify the design of the transmitter. But the peak power should be as high as possible to provide a maximum overall signal-to-noise ratio so as to achieve a maximum precision in the time of arrival measurement for a given receiving antenna gain, or, equivalently, to minimize the required receiving antenna gain for a specified measurement accuracy. The compromise here is settled largely on the basis of the transmitter design difficulty and the achieved pulse-repetition rate.

Available transistors permit a single power amplifier stage to generate about 70 W at the frequencies of interest. As described elsewhere in this report, a larger total RF output is created by using many such stages in parallel. In principle, this procedure can be extended as far as necessary. Practically, however, the inputs and outputs of the paralleled stages must be isolated by lussy power dividers and combiners to prevent destructive interference between the parallel stages. As the number of stages is increased, the losses introduced in the combining process tend to increase progressively due to the increasing difficulty of maintaining a precise phase match between the various pareilel paths.

Extensive design and experimentation have shown that a few tens of stages may be connected in parallel with a consequent total peak-power output of the order of a few kW . At the present time, this represents a practical upper limit for this procedure. And, in turn, this sets one rough bound on the choice of peak-power level.

One of the physical parameters of interest in the propagation experiment is the scale size of the turbulence in the electron density. Currently, it is believed that the plasma will contain 'blobs' with a diameter of the order of 200 to 300 km moving radially outward from the sun, with an initial velocity of the order of 200 to 300 km per second. These 'blobs', then, may be expected to traverse the communication line of sight in a time of one to ten seconda.

It follows that the propagation experiment must achieve a sampling rate of the order of one sample per second. This will, at least, permit the observation of large inhomogeneities moving diagonally across the line of sight.

The average power output of the solar cell array is of the order of 18 W when the spacecraft approaches superior conjunction. After allowing for some inefficiencies in the energy conversion processes, and for the fixed overhead drain of the spacecraft timing and control circuits, this allows an average RF output power of the order of 6 W at the two extreme carrier frequencies involved in the propagation experiment.

The required combination of one pulse per second and an average output power of 6 W requires the energy of a single transmitted pulse to be 6 joules. And since the pulse duration has already fixed as 3 ms , this requires that the peak power be 2 kW .

As shown earlier, this requirement for $2-k W$ peak poweris compatible with the capability that can be achieved in the transmitter. It will be shown later, also, that a 6-joule pulse permits the required measurement precision to be obtained with a receiving antenna gain of the order of 50 dB

### 4.3.1.3 Intrapulse Coding

The 3 -ms-duration pulses are internally coded to improve the time of arrival measurement precision that can be attained. This internal coding (modulation) increases the effective bandwidth of the signal and, in combination with a matched filter receiver, permits a measurement precision equivalent to that which could be obtained with a much shorter duration but equal energy pulse.

Since the pulse envelope duration has been chosen so that the received pulse is coherent, in the sense that the carrier phase reinains correlated for the total duration of the pulse, the required coding can be superposed by phase modulation of the transmitted signal. That is, the total pulse is divided into a number of discrete but contiguous segments distinguished by the phase of the RF carrier.

Selection of an optimum segment duration, or baud time, requires some consideration of the effective bandwidth of the channel. It will be shown later that the error in the estimation of pulse arrival time is related directly to the baud duration. Ideally, therefore, the baud duration should be very short. Practically, the usable duration is determined by the necessarily finite bandwidth of the channel. This bandwidth imposes a minimum limit below which the signal will be severely attenuated, with a consequent serious waste of signal energy.

In the Sunblazer system the frequency selective nature of the channel imposes the most stringent limitation on the minimum usable baud duration. Note that the spectrum of the transmitted signal will include numerous sidebands with a total frequency extent that is inversely proportional to the chosen baud time. Since the channel is frequency selective, these sidebands will be delayed selectively in a manner that causes the envelope of the received pulse to be different from the envelope of the transmitted pulse. This effect can cause a significant reduction in the output of the receiver matched filter which, in the absence of prior knowledge of the channel characteristics, is optimally matched to the transmitted signal.

As noted in Chapter 2 of this report, this effect has been studied in detail. It transpires that the channel can be characterized by a 'dispersion time', and that the effective system signal-to-noise ratio is related to the channel dispersion time, $\mathrm{T}_{\mathrm{c}^{\prime}}$ and the selected baud duration, $\mathrm{T}_{\mathrm{b}}$, by the factor:

$$
\begin{equation*}
G=\frac{1}{1+\frac{T_{c}}{T_{b}}} \tag{4,2}
\end{equation*}
$$

Also shown in Chapter 2 is some quantitative speculation relating the channel 'dispersion time' to the angular offset of the communication line of sight from the solar disc. Using these data, it is postulated that the signal attenuation will be related to the angular offset and the chosen baud duration in the following manner:

| Baud duration <br> (microseconds) | Angular offset (degrees) |  |  |
| :---: | :---: | :---: | :---: |
| 0.5 | 1.0 | 1.5 |  |
| 35 | -3.3 dB | -1.5 dB | -1.2 dB |
| 25 | -4.2 | -2.0 | -1.7 |
| 15 | -5.6 | -3.0 | -2.6 |

From this tabulation it is seen that a baud duration of 25 to $30 \mu s$ will permit continued observation to an angular offset of about one degree without intolerable signal degradation. This choice is consistent with the receiving
antenna beamwidth and channel coherence time limitations discuased previously. Thus we have the desirable mituation that no individual syatem parameter is unduly restrictive.

Now, since the best pulse envelope duration if of the order of 3 ms , and the best $h$ ad duration is of the order of $25 \mu \mathrm{~s}$, it follows that the intrapulse coding ahould be chosen to yield about 120 segments. Two important additional restrictions are that the code should have a good aperiodic autocorrelatior. function with a strong central peak and minimum sidelobes, and that it should be easy to generate with a minimum amount of hardware in the apacecraft. A less important, but not negligible, restriction in that the baud duration chosen should be a binary multiple of the period of the basic $5-\mathrm{MHz}$ oscillator in the spacecraft. This, also, will tend to minimize the complexity of spacecraft hardware.

One class of codes satisfying these requirements is the set of 127-bit pseudonoise sequences that can be generated readily with a seven stage shift register using linear feedback. Several hundred possible sequences are available within this class. The particular sequence chosen, on the basis of minimum hardware complexity, is such that:

$$
\begin{aligned}
& \text { Bit }_{1}=\text { Bit }_{2}=\ldots \ldots=\text { Bit }_{7} \\
& \mathrm{Bit}_{(i+7)}=\operatorname{Bit}_{(i)} \oplus \text { Bit }_{(i+6)} \quad[1 \leq i \leq 120]
\end{aligned}
$$

The particular baud-duration chosen, again on the basis of minimuri hardware complexity, is $25.6 \mu \mathrm{~s}$. Thus the exact pulse envelope duration is:

$$
127 \times 25.6 \times 10^{-3}=3.2512 \mathrm{~m} / 3
$$

### 4.3.1.4 Telemetry Modulation

There are two ways in which telemetry data may be superposed on the main communication channel without materially affecting the prime propagation experiment: either by choosing a different intrapulse coding sequence to represent different telemetry data bits or combinations of bits, or by using pulse position modulation in which the telemetry data bit is represented by the relation between the actual pulse transmission time and an arbitrarily defined time reference. These two approaches differ mainly in the achievable telemetry data rates and the amount of hardware required in the receiver.

The telemetry data rate that can be achieved using pulse position modulation
is limited by the number of different time slots that can be recognized unambiguously. In the Sunblazer system the relative transmission delay may be as large as a few tens of milliseconds. The time separation between pulses transmitted at the two extreme carrier frequencics may, in some circumstances, be as $s$ mall as 0.4 s . In this context the choice is restricted to, say, four different time slots. Correspondingly, each transmitted pulse may represent one of four different quantized levels with a consequent data rate of two bits per pulse.

By contrast, there are several hundred distinctive, neariy orthogorial, intrapulse code sequences that may be used. With this system, therefore, each transmitted pulse may be used to represent one of several hundred different quantized levels, with a consequent data rate of the order of eight to ten data bits per pulse.

The amount of receiver hardware required to detect telemetry data superposed by pulse position modulation is much less than that required to detect $a$ multiplicity of different codes. In each case, the pulse time of arrival has to be estimated for the purpose of the propagation experiment. This requires a bank of filters matched to the expected pulse. If the data are superposed by pulse position modulation, the same bank of filters may be used repeatedly at times corresponding to the possible data time slots. But if the data are superposed by the use of unique codes, a separate bank of filters must be provided to match each possible code.

Another distinction may be made on the basis of the probable error rate. For a particular signal-to-noise ratio and data rate in a given channel, there is a corresponding theoretic bound on the bit error rate. This error rate increases rather rapidly as the data rate is pushed toward the limiting channel capacity. Physically, we may note that, since the pulse rate is fixed by the requirements of the propagation experiment, an increase in telemetry data rate requires that the receiver be presented with a larger number of distinctive signals. This, in turn, increases the probability that the receiver will incorrectly identify the received signal.

Since the telemetry data rate required in the initial Sunblazer flight is quite low, we elect to use the simplest possible system in which each transmitted pulse is used to represent one data bit, and the bit is identified by transmitting the pulse at either one of two predefined times. For subsequent flights, additional hardware can be added to the receiver to increase the system capability without in any way inhibiting continued reception from the first spacecraft.

### 4.3.2 Beacon Tranamitter

Signal coding for the beacon transmitter is designed primarily to enhance the performance of this communication link during the critical first few days of the spacecraft fight. In this time era, while the spacecraft is orienting the solar cell array toward the aun, the beacon must provide accurate apacecraft tracking data and engineering telemetry deacribing the behavior of the spacecraft.

These functions are performed by transmitting a pulse with a peak power of 50 W at a constant carrier frequency of 75 MHz . Telemetry is superponed by analog modulation of the pulse duration from 50 to 150 ms . The rationale for this choice is discussed below.

### 4.3.2.1 Pulse Envelope

Accurate tracking of the spacecraft and prediction of its orbit require measure of two important parameters: the spacecraft direction with respect to earth at the time of observation, and its relative motion. A determination of the current spacecraft direction requires mainly an ability to point the receiving antenna beam so as to maximize the received signal amplitude. A determination of relative n:otion can be made by measuring the Doppler thift induced in the known carrier frequency.

The signal coding used in the beacon is designed to ensure that these measurement can be made with adequate precision. Primarily, the signal power is chosen to yield a received signal-to-noise ratio sufficient to permit the antenna to be pointed accurately. The minimum pulse duration is chosen to permit an adequate measure of the received frequency and, correspondingly, the Doppler shift.

At any instant, the received signal-to-noise (power) ratio is:

$$
\begin{equation*}
\frac{S}{N}=\frac{P_{t} A_{e}}{4 \pi R^{2} k T \Delta f} \tag{4,3}
\end{equation*}
$$

where $P_{t}$ is the peak power of the transmitter,
$R$ is the spacecraft-to-earth range,
$k$ is Boltzmann's constant,
$T$ is the combined effective noise temperature of the observed background and the receiver front end, and
$\Delta f$ is the receiver bandwidth;
and $A_{e}$, the effective area of the receiving antenna, is given by:

$$
\begin{equation*}
A_{e}=\frac{\lambda^{2} G_{r}}{4 \pi} \tag{4,4}
\end{equation*}
$$

where $G_{r}$ is the directional gain of the receiving antenna.
Analyais of the spacecraft dynamics, described elsewhere in this report, indicates that the initial stabilization phase, during which the beacon may be the only communication link, will continue for a period of about ten days. Orbit calculations, also described elsewhere in this report, show that in this period the naximum range achieved will be $3.9 \times 10^{9}$ metars, and that the maximum Doppler frequency will be 800 Hz . The average galactic background noise temperature af 75 MHz is $1800^{\circ} \mathrm{K}$, and the receiver frontend equivalent-noise temperature is $850^{\circ} \mathrm{K}$. Assuming a receiving antenna gain of 30 dB , which present schedule and funding limitations indicate is all that may be available at the time of the first engineering test launch, and assuming a receiver bandwidth equal to the maximum expected Doppler frequency, we have:

$$
\begin{align*}
\frac{S}{N} & =\frac{p_{t} \times 4^{2} \times 10^{3}}{\left(4 \pi \times 3.9 \times 10^{9}\right)^{2} \times 1.38 \times 10^{-23} \times(1800+850) \times 800} \\
& \approx 0.23 \mathrm{P}_{t} \tag{4,5}
\end{align*}
$$

However, a proper receiving antenna beam pointing capability requires a reasonable signal-to-noise ratio of, say, 10 dB . This, then, requires the peak power of the transmitted pulse to be on the order of:

$$
P_{t} \approx \frac{10}{0.23} \approx 44 \mathrm{~W}
$$

Available transistors permit use of a single stage to generate a fairly long pulse with a peak power of 50 W . This, therefore, is selected as the best peak power to use for the beacon transmitter.

To permit adequate accuracy in the prediction of the spacecraft orbit: ine rms error in the measurement of Doppler frequency must be held to about 1 Hz . In terms of the pulse duration and the signal-to-noise (power) ratio, this measurement error is:

$$
\begin{equation*}
\sigma=\frac{1}{T_{p} \times 2 \pi \sqrt{\frac{S}{N}}} \tag{4,6}
\end{equation*}
$$

We therefore require:

$$
T_{p}=\frac{1}{\sigma \times 2 \pi \sqrt{\frac{S}{N}}}
$$

$$
\begin{equation*}
T_{p} \approx \frac{1}{1 \times 2 \pi \sqrt{0}} \approx \frac{1}{18.8} \approx 50 \mathrm{~mm} \tag{4,7}
\end{equation*}
$$

Fifty milliseconds, therefore, is selected as the beat minimum pulse duration for the beacon transmitter.

### 4.3.2.2 Telemetry Modulation

A prime requirement for the telemetry superposed on the beacon tranamitter in the early phase of the flight is that the data rate be reasonobly high in order to provide a comprehensive description of the initial behavior of the spacecraft. For maximum reliability it is desirable, also, that the hardwar'c be as simple as possible. And it is necessary that each transmitted pulse convey as much information as possible so that the number of pulses and the consequent drain on the battery power supply may be reatricted.

For these reasons the beacon telemetry is implemented as an analog syatem in which the magnitude of the data point is represented by the duration of the transmitted pulse. Since the minimum pulse duration has already been fixed for other reasons, it is only necessary now to choose the maximum duration $s 0$ that the available range will permit a tolerable data measurement accuracy.

In terms of the receiver bandwidth and the received signal-to-nose (power) ratio, the rms error associated with the measurement of a pulse duration is:

$$
\begin{equation*}
\sigma=\frac{1}{B \sqrt{2 \frac{S}{N}}} \tag{4,8}
\end{equation*}
$$

or, with the assumptions used previously, is:

$$
\begin{align*}
\sigma & =\frac{1}{800 \sqrt{20}} \\
& \approx \frac{1}{3580} \approx 280 \mu \mathrm{~s} . \tag{4,9}
\end{align*}
$$

The full-scale data range is chosen as 100 ms in order to limit the contribution of this error source to a tolerably small $0.25 \%$. That is, the beacon pulse is designed to be modulated from a minimum of 50 ms to a maximum of 150 ms .

### 4.4 Sirnal-:0-Noise Ratio

In this section we discuss and evaluate the nature and magnitude of the various signal-to-noise ratios that are of interest. It is shown that the ratio of interest in the main communication link is the energy ratio and that the minimum value of this ratio is 16 dB . For the beacon link the parameter of interest is the power ratio which has a minimum value of 13 dB .

### 4.4.1 Main Communication Link

The prime purpose of the mair: communication link is to permit the accurate measurement of pulse arrival time that is required by the propagation experiment. This measurement is best accomplished by the use of a bank of matched filters in the receiver. This matched filter reception procedure performs coherent detection such that the receiver output signal-to-noise ratio is related to the input signal-to-noise energy ratio. That is, the critical ratio determining system performance is the ratio between the total energy in each recpived pulse and the total system-noise energy or spectral density. The signal-to-noise (energy) ratio is:

$$
\begin{equation*}
\frac{E}{N_{0}}=\frac{E_{t} G_{t} A_{e}}{4 \pi R^{2} k T} \tag{4,10}
\end{equation*}
$$

or, substituting:

$$
\begin{gather*}
A_{e}=\frac{\lambda^{2} G_{r}}{4 \pi}  \tag{4.11}\\
E_{N_{0}}=\frac{E_{t} G_{t} \lambda^{2} G_{r}}{(1 \pi R)^{2} k T} \tag{4.12}
\end{gather*}
$$

where $E_{t}$ is the energy of the transmitted pulse, in joules,
$G_{t}$ is the gain of the spacecraft antenna in the direction of earth,
$\lambda$ is the wavelength of the RF carrier, in meters,
$\mathbf{G}_{r}$ is the directional gain of the receiving antenna,
$R$ is the range from spacecraft to earth, in meters,
$k$ is Boltzmann's constant, in joules per degree $K$,
$T$ is the total equivalent noise temperature of the galactic background and the receiver front end, in degrees $K$.

The transmitted pulse energy is:

$$
E_{t}=3.25 \times 10^{-3} \times 2 \times 10^{3}=6.5 \text { joules. }
$$

The main spacecraft antenna provides a directional gain of 3 dB in the galactic plane, and the receiving antenna is assumed to have a directional gain of 50 dB . The average galactic background noise temperature at 75 MHz is $1800^{\circ} \mathrm{K}$, and the equivalent noise temperature of the receiver front end is $850^{\circ} \mathrm{K}$.

Substituting these values:

$$
\begin{align*}
\frac{\mathrm{E}}{\mathrm{~N}_{0}} & =\frac{6,5 \times 2 \times 4^{2} \times 10^{5}}{(4 \pi \mathrm{R})^{2} 1.38 \times 10^{-23} \times 2.65 \times 10^{3}} \\
& \approx \frac{3.6 \times 10^{24}}{R^{2}} \tag{4,13}
\end{align*}
$$

Evaluation of this ratio yields Fig. 4-1 which shows the energy ratio as a function of the angle between the communication line of sight and the solar disc throughout the spacecraft flight. It is seen that the energy ratio remains above 16 dB up to the time at which the receiving antenna beam intercepts part of the solar disc and the direct solar noise contribution becomes prohibitive. As will be shown later, this energy ratio is sufficient to yield the required measurement accuracy.

### 4.4.2 Beacon Communication Link

One prime purpose of the beacon communication link in the early phase of the flight is to permit rapid signal acquisition using non-coherent detection to minimize the search difficulties. In the later phase of the flight, the beacon transmitter is used to emit a relatively long pulse designed to permit a direct measurement of the frequency spreading induced by the medium: the nature of the disturbance which is to be measured will make the received signal incoherent, again necessitating non-coherent detection. The critical system parameter for this link, therefore, is the ratio between the received peak signal-power and the total system noise-power within a defined bandwidth.

The signal- to noise (power) ratio is:

$$
\begin{equation*}
\frac{S}{N}=\frac{P_{t} G_{t} \lambda^{2} G_{r}}{(4 \pi R)^{2} k T \Delta f} \tag{4,14}
\end{equation*}
$$

where $P_{t}$ is the peak power of the transmitted pulse, in watts,
$\Delta f$ is the receiver bandwidth, in Hz .
and all other terms are as previously defined.
The beacon transmitter peak power is 50 W . Substituting this and other previously defined values yields:


Fig. 4-1 Energy ratio at 75 MHz (neglecting solar noise).

$$
\begin{align*}
\frac{\mathrm{S}}{\mathrm{~N}} & =\frac{50 \times 2 \times 4^{2} \times \mathrm{G}_{r}}{(4 \pi R)^{2} \times 1.38 \times 10^{-23} \times 2.65 \times 10^{3} \Delta f} \\
& \approx \frac{2.8 \times 10^{20} \mathrm{G}_{\mathrm{r}}}{\mathrm{R}^{2} \Delta f} \tag{4,15}
\end{align*}
$$

There are three different time regimes, and corresponding sets of system parameters, for which this must be evaluated.

For the first ten days of the flight. a prime task is to acquire the signal rapidly without requiring an accurate knowledge of the Doppler shift. To permit this, the receiver bandwidth must be of the order of 800 Hz , which is the maximum frequency shift expected in the first few days. Another constraint is set by current funding and schedule limitations which indicate that, for the first engineering test launch, the receiving antenna gain will be limited to 50 dB .

After these first few days it is expected that the spacecraft will orient the solar cell array toward the sun, permitting the main transmitter to be activated. In this mode the beacon pulse duration is increased to five seconds. This long pulse, coupled with the fact that prior measurements will than permit a fairly accurate prediction of spacecraft position and motion, will allow the receiver bandwidth to be reduced to, say, 100 Hz .

The third era of interest commences about six months after launch, when a measurement of the medium induced frequency spreading starts to become interesting. For this experiment the receiver bandwidth must be maintained in the order of 20 Hz to accommodate the expected perturbations, and the full receiving antenna gain of 50 dB is required, For the three different eras, then, we have:

$$
\begin{array}{ll}
\frac{S}{N}=\frac{2.8 \times 10^{20} \times 10^{3}}{R^{2} \times 8 \times 10^{2}}=\frac{3.5 \times 10^{20}}{R^{2}} & {[0 \leq \text { Days } \leq 10]} \\
\frac{S}{N}=\frac{2.8 \times 10^{20} \times 10^{3}}{R^{2} \times 1 \times 10^{2}} \approx \frac{2.8 \times 10^{21}}{R^{2}} & {[10 \leq \text { Days } \leq 40]} \\
\frac{S}{N}=\frac{2.8 \times 10^{20} \times 10^{5}}{R^{2} \times 20} \approx \frac{1.4 \times 10^{24}}{R^{2}} & {[200 \leq \text { Days }] .} \tag{4,16}
\end{array}
$$

Evaluation of these ratios yields Fig. 4-2, which shows the beacon link signal-to-noise (power) ratio throughout the three interesting segments of the spacecraft flight. It will be shown later that the achieved ratios are sufficient to permit the required measurement accuracies to be obtained.

## 4. 5 System Performance and Measurement Accuracy

This section demonstrates the validity of the communication system design by showing that it is possible to obtain detection probabilities and measurement accuracies that are consistent with the aims of the experiment. Clearly, the system performance will depend not only on the signal design described previously, but also on the efficiency with which the received signal is processed. For this reason we start with a description of some of the important characteristics of the receiver and the manner in which it operates.

### 4.5.1 The Correlation Receiver

In the ground receiver we are confronted by the classic radar problem: a noisy input must be processed to detect the presence of a signal of known form; and the time of arrival of that signal must then be estimated. In the Sunblazer system the problems are particularly challenging for two reasons. First, as a consequence of the wide bandwidth signal that must be employed, the peak power signal-to-noise ratio is very low. Second, the received signal will be subject to substantial pulse to pulse time perturbations of a pseudo-random nature due to the turbulence of the medium. These problems, however, are a matter of degree affecting the detail and complexity rather than the form of the solution.

An optimum detection procedure for this situation involves the calculation of the cross correlation between a postulated signal of the known form and the actual signal received, and the subsequent comparison of the correlator output with a predetermined threshold. A decision "signal present" or "signal absent" is made, depending on whether or not the correlator output exceeds the threshold. The arrangement may be said to test the hypothesis that a signal of the postulated form exists as one component of the noisy input.

Two kinds of error can occur. A 'false alarm' occurs if the receiver decides that a sigial is present when it is truly absent, and a 'miss' occurs if the receiver decides that a signal is absent when it is truly present. A tradeoff between these two kinds of errors may be made by adjustment of the threshold. A low threshold setting will decrease the probability of a miss, but increase the probability of a false alarm, and yice versa.


Fig. 4-2 Signal/noise (power) ratio for beacon link.

Simultaneous minimization of both kinds of error can only be accomplished by increasing the signal-to-noise (energy) ratio. We are not concerned here with a detailed examination of the receiver performance in this respect. We note only that it is possible to achieve both a miss probability and a false alarm probability of 0.002 if the energy ratio is of the order of 13 dB , and a simulizneous miss probability and false alarm probability of 0.00001 if the energy ratio is of the order of 16 dB . As shown previously, the minimum expected energy ratio in the Sunblazer system is, indeed, 16 dB . Since the pulse repetition rate is i:: the order of one pulse per second we may therefore expect a maximium combined statistical detection error rate of the order of two events in 28 hours of observation time. Evidently, this is not the most critical problem that has to be solved.

Since the energy ratio is sufficient to provide a high level of confidence in the detection procedure. we may now turn our attention to the next problem of defining the time of arrival of the signal. This is an extension of the detection problem: we now wish to know not only that a signal is present but, also, that it is present at a particular time.

By analogy, an optimum procedure for this time estimation involves the calculation of the cross correlation between the noisy input and a signal of the known form occurring at a known time. Subsequent comparison of the correlator output with a known threshold will then permit a decision "bignal present at this time" or "signal absent at this time" to be made.

But a further improvement in detection strategy can be effected by calculating simultaneously the cross correlation between the noisy input and a multiplicity of signals of the known form occurring at different times. An estimate of signal arrival time is then made by noting the correlator which provides the maximum output; or, equivalently, by noting a pair of correlators which bracket the maximum and provide equal outputs.

This is the 'maximum likelihood' detection strategy. In detection parlance, the arrangement may be said to yield the aposteriori probability that the signal arrived at time $t$, given the reception of the signal.

This arrangement is commonly implemented with a parallel group of matched filters, exemplified by the tapped delay line. This implementation, however, is less than optimum for the Sunblazer system for several reasons. First, the time-bandwidth products involved make a tapped delay line prohibitively expensive. Second, analog signal processing tends to require frequent and critical calibration. Third, it is not clear that an analog system could be adjusted readily to accommodate the expected signal pertisrbations. Fourth,
a hard wired system lacks the nexibility required to cope with the sequential observation of several spacecraft transmitting different signals.

We elect, therefore, to use an implementation that is largely digital in nature. This is shown as Fig. 4-3. In a synchronous demodulator the received signal is multiplied by a reference carrier to effect a frequency translation. The resultant video is processed in several parallel paths. In each path it is multiplied by a version of the expected modulation. These various locally generated signals are displaced successively in time by a small increment, $t$. Outputs of these multipliers are integrated separately. The integrator outputs are then compared in order to select that path in which the signal is most closely aligned with the locally generated expected signal.

Most of the hardware is concerned with the code multiplication process. This is inherently a simple, inexpensive procedure, making it practical to replicate the parallel processing paths in large numbers. Typically, we envision several hundred individual correlators making.it possible to use a small separation time increment for adequate measurement resolution with a large total time span coverage for adequate capture range.

The foregoing description has glossed over many of the practical difficulties involved in the implementation; a full discussion is provided by the reporting of the associated contract for the development of the phased array and receiver complex. One point worth noting here is that the lack of a stable phase lock between the received carrier and the local reference used in the "synchronous" demodulator makes it necessary to calculate not the cross correlation function itself, but the square of this function. This affecte the system performance analysis, but does not mifect the validity of the comparison procedure used in the maximum likelihood estimation strategy.

### 4.5.2 Time of Arrival Measurement Accuracy

One criterion in the signal design is the use of an intrapulse code having a 'good' autocorrelation function with a strong central peak and minimum sidelobes. The maximum likelihood, multiple correlation, procedure yields a set of sample points tracing out this autocorrelation function in te. ms of the time displacement between the received signal and the locally generated postulated signals. The form of this output is shown as Fig. 4-4.

Evidently, the measurement precision cannot be better than the resolution that is determined by the time displacement between successive correlators. To permit an analysis of the system accuracy, therefore, we assume that, at least in the vicinity of the central peak, there are enough, melators


Fig. 4-3 Maximum likelihood correiation receiver.


Fig. 4-4 Form of sampled correlation function.
separated by a sufficiently small increment that the output is : virtually continuous tracing of the autocorrelation function.

It is also evident that a discussion of system accuracy is meaningless if the receiver mistakenly selects a sidelobe rather than the central feak. We also assume, therefore, that the form of the autocorrelation function and the system signal-to-noise ratio are adequate to avoid this ambiguity.

With these assumptions, the interesting portion of the sampled function is as shown by Fig. 4-5. The ideal central peak is a triangle with amplitude $A$ determined by the energy of the received signal pulse, and base width $2 \mathrm{~T}_{B}$ determined by the baud time of the intrapulse code. Noise is superposed on this peak with a relative magnitude determined by the system signal-tonoise ratio.

Consider, then, a time estimate made by noting which correlator output sample first exceeds $50 \%$ of the maximum output; or, equivalently, the time at which the "leading edge" of the function crosses a threshold set at A/2. The error in this time estimate will be proportional to $n(t)$, and inversely proportional to the slope of the function. But this slope is $A / T_{B}{ }^{80}$ :

$$
\begin{align*}
\Delta T & =\frac{n(t)}{A / T_{B}} \\
& =\frac{T_{B}}{A / n(t)} \tag{4.17}
\end{align*}
$$

and for repeated measurements, the variance will be:

$$
\begin{equation*}
\sigma^{2}=\overline{\Delta T^{2}}=\frac{T_{B}^{2}}{\left[A^{2} / n^{2}\right]} \tag{4.18}
\end{equation*}
$$

Now, the IF energy ratio may be shown as:

$$
\begin{equation*}
\left(\frac{E}{N}\right)_{I F}=\frac{E_{r}}{k T} \tag{4.19}
\end{equation*}
$$

where $\mathrm{E}_{\mathrm{c}}$ is the signal energy at the output of the receiver RF preamplifier and mixer,
$k$ is Boltzmann's constant,
$T$ is the effective system noise temperature, including the contribution of the RF section of the receiver.
Synchronous demodulation doubles the signal-tomnoise ratio. This is a consequence of the fact that the frequency translated ignal sidebands add coherently, while the noise components add incoherently. So, following synchronous demodulation, the energy ratio is:


Fig. 4-5 Central peak of autocorrelation function.

$$
\begin{equation*}
\left(\frac{E}{N}\right)_{V I D E O}=2\left(\frac{E}{N}\right)_{I F}=\frac{E_{r}}{k T / 2} \tag{4,20}
\end{equation*}
$$

Then after the matched filter and integrator, we have:

$$
\begin{equation*}
A=\int_{0}^{T} \sqrt{\frac{E_{r}}{T_{p}}} \frac{1}{\sqrt{T_{p}}} d t=\sqrt{E_{r}} \tag{4.21}
\end{equation*}
$$

and

$$
\begin{equation*}
\sqrt{\overline{n^{2}}}=\sqrt{\frac{k T}{2}} \tag{4.22}
\end{equation*}
$$

Hence the measurement variance is:

$$
\begin{equation*}
\sigma^{2}=\frac{T_{B}^{2}}{\left[A^{2} / n^{2}\right]}=\frac{T_{B}^{2}}{2\left[\frac{E_{r}}{k T}\right]} \tag{4,23}
\end{equation*}
$$

But the form of the autocorrelation function is such that the output noise is uncorrelated at sample times separated by intervals of $T_{B}$. We can, therefore, make a similar, independent measurement on the "trailing edge" of the function at the same threshold level of $A / 2$. Averaging these measurements will improve the overall measurement by a factor of 2 . Hence the total rms error in the measurement of pulse arrival time will be:


Then, setting $T_{B}$, the code baud time, equal to $25 \mu \mathrm{~s}$, and using the data of Fig. 4-1 which shows the signal-to-noise (energy) ratio throughout the spacecraft flight, and the data of Fig. 2.1, which shows the predicted magnitude of the relative delay that is to be measured, we obtain Fig. 4-6 which shows the rms error as a percentage of the measured quantity. It is evident that the measurement accuracy is consistent with the aims of the experiment.

### 4.5.3 Main Telemetry

As described previcusly, data are superposed on the main communication link by pulse position modulation such that each transmitted pulse has only two possible positions in time. These times are chosen to be far enough apart that there is no possibility of a reception ambiguity caused by a sudden fluctuation of the total signal propagation time.


Fig. 4-6 RMS measurement error as percentage of measured quantity.

An urdetected data bit error, therefore, requires the receiver to "miss" a pulse by indicating that a signal is absent when it is truly present at one time; and, for the other one of the piair of possible time slots, to cause a "false alarm" by deciding that a signal is present when it is truly absent. We require to know, therefore, the proisability that these two different kinds of error will occur jointly.

The probability of a "miss" and the probability of a "false alarm" are both related to the threshold used in the detection procedure and the effective system signal-to- noise (energy) ratio. These probabilities are tabulated in the literature. A convenient description is given by Barton ${ }^{(1)}$.

Using these data, and setting the threshold so that the probability of a "miss" is equal to the probability of a "false alarm", we find that the joint probability of an undetected error is related to the energy ratio in the following manner:

| Energy Ratio <br> dB | "Miss" <br> probability | "False Alarm" <br> probability | Jount Enror <br> probability |
| :---: | :--- | :--- | :--- |
| 10 | 0.03 | 0.03 | $9 \times 10^{-4}$ |
| 12 | 0.005 | 0.005 | $2.5 \times 10^{-5}$ |
| 14 | 0.0007 | 0.0007 | $5 \times 10^{-7}$ |
| 16 | 0.00001 | 0.00001 | $1 \times 10^{-10}$ |

It is instructive to compare this with the number of telemetry data bits received during the spacecraft flight. The pulse repetition rate is of the order of one pulse per second and an average of four hours observation time is expected each day. Hence the number of data bits received per day is $1.4 \times 10^{4}$, and in 400 days the total is $5.6 \times 10^{6}$. The error rate associated with an energy ratio of 16 dB indicates a loss of one bit in about 2000 spacecraft flights. If the energy ratio decreases to about 12 dB , a statistical undetected error rate of one data bit in each spacecraft flight can be expected.

Of course, these extrapolations are not very significant because the number of data bits involved is not large enough to make the statistical error rate meaningful. Most likely, errors will occur in bursts when detection is less than optimum for some reason, such as a gross mispointing of the receiver antenna. However, the calculated data reliability shows, at least, that no serious problem is involved.

### 4.5.4 Beacon Telemetry

In the first fen days of the flight the beacon transmitter provides the only communication link. Telemetry is superposed on this link by pulse duration modulation to describe the initial behavior of the spacecraft. In this time era the power aignal-to-noise ratio is amply large to ensure reliable detection of the signal. We are principally concerned, therefore, with the precision with which the duration of a received pulse can be measured.

Consider the possible error in the measurement of a single pulse of the form shown as Fig. 4-7. An estimate of the time of occurrence of the leading edge can be made by noting the time at which the pulse amplitude first exceeds a threshold set at, say, $50 \%$ of the maximum amplitude. For reasonably large signal-to-noise ratios, the error in this measurement will be:

$$
\begin{equation*}
\Delta T_{r}=\frac{n(t)}{A / t t_{r}} \tag{4,25}
\end{equation*}
$$

A similar, independen frror will occur in the measurement of the time of occurrence of the trailing edge of the pulse, so the combined error in the estimate of pulse duration will be:

$$
\begin{equation*}
\Delta T=\sqrt{2} \Delta T_{r}=\frac{\sqrt{2} n(t)}{A / t_{r}}=\frac{\sqrt{2} t_{r}}{A / n(t)} \tag{4,26}
\end{equation*}
$$

Then the measurement variance will be:

$$
\begin{equation*}
\sigma^{2}=\frac{2 t_{r}^{2}}{\left[A^{2} / \mathrm{n}^{2}\right]}=\frac{2 t_{r}^{2}}{[S / N]} \tag{4,27}
\end{equation*}
$$

where $S / N$ is the video signal-to-noise (power) ratio. But the pulse rise time will be limited primarily by the receiver bandwidth so:

$$
\begin{equation*}
t_{r} \approx \frac{1}{2 F} \tag{4.28}
\end{equation*}
$$

where $F$ is the effective bandwidth, in hertz. Further, the signal-to-noise ratio may be written as:

$$
\begin{equation*}
\frac{S}{N}=\frac{S_{r}}{k T F} \tag{4.29}
\end{equation*}
$$

where $\mathbf{S}_{\mathbf{r}}$ is the received signal power,
$k$ is Boltzmann's constant,
$T$ is the total effective system noise temperature, and
$F$ is the receiver bandwidth.
So:


Fig. 4-7 Error in measurement of pulse duration.

$$
\begin{equation*}
\sigma=\sqrt{\frac{2 k T F}{(2 F)^{2} S_{r}}}=\sqrt{\frac{k T}{2 F S_{r}}} \tag{4.30}
\end{equation*}
$$

Note that the receiver bandwidth appears in the denominator of this expression. This is a consequence of the fact that the pulse rise time is limited primarily by the receiver handwidth. In practice, little will be gained by making this bandwidth greater than, say, 50 times the reciprocal of the pulse duration. Since, as shown previously, the bandwidth required to accommodate the Doppler shift is of the order of 800 Hz , this can be used as the receiver bandwidth.

Now, the received signal power level is:

$$
\begin{equation*}
S_{r}=\frac{P_{T} \times G_{T} \times \lambda^{2} \times G_{R}}{(4 \times \pi \times R)^{2}} \tag{4.31}
\end{equation*}
$$

where $P_{T}$ is the transmitted power, in watts,
$\mathbf{G}_{\mathbf{T}}$ is the directional gain of the spacecraft antenna,
$\lambda$ is the wavelength, in meters,
$G_{R}$ is the directional gain of the receiving antenna, and $R$ is the range, in meters.
So, for a $30-\mathrm{dB}$ receiving antenna, which may be all that is available for the first engineering test flight, we have:

$$
\begin{align*}
S_{r} & =\frac{50 \times 1 \times 4^{2} \times 1000}{\left(4 \times \pi \times 3 \times 10^{9}\right)^{2}} \\
& \approx 5 \times 10^{-16} \mathrm{watts} \tag{4,32}
\end{align*}
$$

And, typically:

$$
\begin{equation*}
k T=1.38 \times 10^{-23} \times 2.5 \times 10^{3} \approx 3.5 \times 10^{-20} \mathrm{~W} / \mathrm{Hz} \tag{4.33}
\end{equation*}
$$

Substituting these values yields:

$$
\begin{align*}
\sigma & =\sqrt{\frac{3.5 \times 10^{-20}}{2 \times 800 \times 5 \times 10^{-16}}} \\
& \approx 2 \times 10^{-4}=200 \mu \mathrm{~s} \tag{4.34}
\end{align*}
$$

Since the full scale range of the analog data representation is from 50 to 150 ms , the rms measurement error is:

$$
\begin{equation*}
\epsilon=\frac{200 \times 10^{-6} \times 10^{2}}{(150-50) \times 10^{-3}}=0.2 \% \tag{4,35}
\end{equation*}
$$

This is acceptably small, both in comparison with the probable data conversion errors in the spacecraft and in terms of the function required of this telemetry link.

### 4.6 Spacecraft Programer

This section of the report describes the hardware implementation of the spacecraft programer and telemetry subsystem. Topics covered inelude the functional operation of the subsystem, detail of the hardware implementation, and the telemetry format. This format is consistent with the signal design criteria discussed earlier.

### 4.6.1 Functional Description

Drawing No. D-106-400000 shows a functional diagram of the total Sunblazer system. The spacecraft equipment includes a programer that is responsible for timing all the activities on the spacecraft, a beacon transmitter and its associated telemetry encoding equipment, a main transmitter and its associated telemetry encoding equipment, the aspect sensor and sail controller, and a power subsystem. In this section of the report, we are concerned primarily with the programer and telemetry encoding subsystems.

A more detailed functional diagram of the spacecraft is shown by Drawing No. D-106-400001. All timing pulses and RF cerriers used in the spacecraft are developed from a basic $5 \cdot \mathrm{MHz}$ oscillator. Output of this oscillator is counted down by the pulse duration counter and the pulse spacing counter to generate "transmit" controls for both the beacon and main transmitters. Other outputs of the countdown chain are used to control the operation of the telemetry encoders so that appropriate data modulation can be added to the transmitted pulses. Output of the oscillator is also frequency multiplied to generate the required $R F$ carrier frequencies.

### 4.6.2 Logic Design

Logic diagram E-106-400017 and timing diagram E-106-400018 show the detailed implementation of the spacecraft programer and the resulitant pulse and telemetry formats. The logic symbology used is chosen to maintain a one-to-one correspondence between the logic elements and the actual hardware elements. This symbology is explained as a part of the logic diagram.

The pulse duration and pulse spacing counters together form a 29-stage binary counter. Principal outputs of this counter chain are: a square wave with a period of $25.6 \mu \mathrm{~s}$, used to define the baud duration for the phase coded main transmitter pulses; a square wave with a period of 3.2 ms , used to
define the anvelope $\dot{\text { sisfation for the main transmitter pulse; and a square }}$ wave with a period ranging from 0.3 to 107 seconds, used to define the pulse repetition rate for the transmitters. Other outputs of the counter chain are available to control the remainder of the timing and encoding hardware.

A mode control section (flip-flops 30 and 31 ) define which one of the two operating modes is in effect. In the "beacon only" mode, the apacecraft operates from a battery power supply, and the main tranmaitter and all its associated control hardware are inactive. In the "main" mode, both the main transmitter and the beacon transmitter are used, with power being derived from the spacecraft soler cell array.

The pulse duration modulator ( Iip-flops 32 through 38) controls the duration of the pulse that is emitted by the beacon transmitter. In the "beacon only' mode, the minimum pulse duration is fixed as 50 ms by the control gates associated with flip-flop 33. The maximum duration of the pulse is 150 ms . This is controlled by the element shown in the logic diagram as flip-flop 38. This element has the form of an astable multivibrator in which the pulse recovery time is controlled by an analog input.

The controlling analog input is obtained from the output of the beacon data multiplexer. This is a set of 16 gates activated sequentially to select one of 15 possible data inputs. One position is left blank to provide a beacon telemetry synchronization signal. At present, we envision seven of the data inputs to be digital in nature, and eight of them to be analog. However, the instrumentation is such that this grouping can be changed readily to accord with the final definition of the data points that are required to be handled by this telemetry data link. The multiplexer is controlled by the beacon word sounter which simply counts sequentially through all possible positions of the multiplexer in synchronization with the beacon transmitter output.

The pulse interval controller (flip-flops 55 through 57) is activated only in the "main" mode. It controls the operation of the last seven stages of the basic countdown chain (flip-flops 23 through 29) so as to determine the actual transmitted pulse rate for the main transmitter. The pulse interval controller is a reversible counter permitting eight possible output states with a consequent eight possible pulse repetition rates. Modification of the content of this counter can occur only at the end of a complete data frame.

The pulse position modulator (nip-flops 43 through 46) determines which one of two possible time slots shall be used for the main transmitter pulse. In this way the telemetry data is superposed on the main transmitter by pulse position modulation. A data bit of zero is distinguished by transmitting the main transmitter pulse in the first of two possible time slots. A data bit of one is distinguished by transmitting the main pulse in a time slot that is 200 ms later.

The requence generator (flip-flops 47 through 54) is responsible for the phase modulation of the main transmitter pulse. The kroup of eeven flip- flops, 48 through 54, form a seven-stage shift register with linear fofdback uned to generate 3 127-bit maximal length pseudo-noise code.

The bit counter and parallel-to- Berial converter, (flip-flops 50 through 64) is responsible for converting the quantized data representation to a serial form and assembling this with word synchronizing pulses into the required output telemetry data format. Flip- flops 59 through 63 form the equivalent of a ten-stage shift register which sequentially selects one of the quantized data input bits or word mynchronizing bits.

The quantized data bits may be obtained directly from an eight-bit digital input, or from the output of an eight-bit analog-to-digital converter. This quantized data is stored in flip-flops 70 through 77. The data are quantized in this register by allowing this group of flip-flops to act as a crunter for a time interval that is proportional to the magnitude of the analog data.

Selection of the main input data is controlled by the main data multiplexer, which is a group of 32 gates activated sequentially under control of the main word counter. At present, we envision two digital input channels, one of which will be used to transmit the status of the beacon word counter as an additional synchronizing signal, and one of which accepts an eight-bit digital word from an external source. The other main data inputs are analog in nature. Again, this grouping of digital and analog data sources can be changed readily to accord with the final selection of telemeti, $v$ data inputs.

### 4.6.3 Pulse Format

The pulse forrnat is shown by sheet 1 of Drawing No. E-106-400018. In the "beacon only" mode, pulses are transmitted at a fixed repetition interval of 107 seconds. A group of 16 sequential pulses forms a data frame. One pulse in each frame is deleted to provide telemetry synchronization. Each of the other 15 pulses is used to represent one data point by pulse duration modulation:. The duration is related linearly to the magnitude of tne analog data point isuch that 0 volts is represented by a $50-\mathrm{ms}$ pulse, and 5 volts is represented by a $150-\mathrm{ms}$ pulse. Digital data are represented by a pulse with a duration of either 60 or 120 ms .

In the "main" mode, the beacon pulse duration is increased to range from 5 to 7 seconds, again proportional to the magnitude of the data point that is to be transmitted. These pulses are interleaved relatively inf requently with the main transmitter pulses. The longest frame duration is controlled by the time required to cycle completely through the sixteen beacon pulses.

The sixteen wegments corresponding to the beacon words are defined as "major frames". Each major frame is divided into two "minor frames" diatinguished only by word zero. In one minor frame the beacon transe - Itter is activated in this time interval; in the other minor frame word zero t left blank. This provides telemetry frame nynchronization for the main communication charnel.

The remainder of each minor frame comprises 31 data :'ords. Each word is formed by ten bits. Two of these are transmitted at 75 MHz . The remaining eight are transmitted alternately at the two extreme carrier frequencies of 70 and 80 MHz . The uncoded $75-\mathrm{MHz}$ pulses are used in the initial acquisition of frequency synchronization in the receiver, to permit a direct measure of the frequency perturbations induced by the medium, and to serve the function of telemetry data word synchronizing pulses. The coded pulses transmitted at 70 and 80 MHz form the backbone of the experiment. These are the pulses that permit the relative delay time experiment to be conducted. Additionally the pulses are coded by pulse position modulation to form the main telemetry data link.

Ordinarily, the nulse repetition interval for the main transmitter pulses is 1.6 seconds. Thus, the word duration is 16 s , the ininor frame duration is 514 s , the major frame duration is 1024 s or 17 minutes, and the beacon frame duration is 272 minutes or $41 / 2$ hours.

These "normal" rates are chosen so as to utilize fully the average power available from the solar cell array when the spacecraft is 1 astronomical units distant from the $F \cdots$. As the spacecraft approaches closer to the sun. however, the average available power also increases. The pulse interval controller then moves to double the pulse rate so as to take full advantage of the extra power.

As a safety precaution, the pulse interval controller also can reduce the average pulse repetition rate by successive factors of two. If, for example, $50 \%$ of the solar cell array should fail, the rate would be reduced from one pulse every 1.6 s to one pulse every 3.2 s; the experiments and the reception of telemetry data could thus continue, although at a lower data rate. In this mode, the beacon frame duration would extend to nine hours. This may be longer than the observation period available in one day, so it may not be pusisible to see a complete frame of beacon telemetry data. To circumvent this problem, the content of the beacon word counter is transmitted as one word of the main telemetry frame so that beacon telemetry synchronization atill can be achieved.

The pulse interval controller is actually implemented to make possible increase of the pulse repetition interval by successive factors of two, up to as much as 107 seconds. This emergency backup mode requires very little extra hardware and permits at least some experimental data to be obtained ever if the power source degrades to about $5 \%$ of the nominal value.

## 4. 6. 4 Hardware Implementation

The spacecraft programer uses the Fairchild 9040 series of low power diode-transistor logic integrated circuits. This series provides the best combination of low power, logic flexibility, and space-qualified reliability that is available today. This integrated circuit family is being used extensively in many other NASA oriented equipments and is the subject of continuing and close scrutiny by NASA Goddard Quality Assurance.

Recent production runs have shown serious reliability problems. However, the production process responsible for this has been identified by NASA Goddard Quality Assurance and by Fairchild. Our information is that Fairchild is now modifying the production process and that qualified units will be available again in early 1969. Since this matter is of considerable moment to many NASA programs, it is attracting a great deal of attention and we are confident that the problems will be solved completely long before the Sunblazer flight hardware has to be committed.

Should this hope not be realized, however, there is a recourse available. Amelco is now starting production of a second source copy of the Fairchild units. On the basis of contacts with NASA Goddard Quality Assurance, it is understood that the Amelco units avoid the questionable production process and are, therefore, likely to be qualified in the very near future. If even this recourse should fail, the next alternative would be to redesign the programer to use the nearly equivalent Texas Instrument family of low power TTL circuits. The modifications required would not be extensive. The major problem would be a slight increase in total subsystem power consumption.















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## CHAPTER 5

### 5.0 GROUND ARRAY

### 5.1 Function of the Ground Array

From a systems point of view, the Sunblazer ground array may be considered a linear-active transducer. Regarded as such, the array provides coupling between an incident radiation field, originating from the spacecraft, and earth-station data-processing equipment. Thus the array must interface with the on-board electronics system and the correlation receiver, both of which are described in other sections of this document. The antenna itself is characterized by its effective area or gain, frequency response, polarization, noise characteristics and tracking capability. The interrelationship of these quantities with the on-board electronice and the correlation receiver is given below, followed by a brief description of addíional uses and capabilities of the array system. Section 5.2 presents a systems analysis of the array; Sec. 5.3 is a detailed description of the Sunblazer array electronic organization and hardware.

### 5.1.1 Relationship to the Sunblazer Spacecraft

To be compatible with the Sunblazer communications system, antenna characteristics such as gain and noise figure must be maintained at the frequencies radiated by the on-board transmitter, plus an allowance for modulation bandwidths and dispersion due to Doppler, attenuation and plasma effects. Three specific frequency bands are of primary interest:

## Table 5-I

Signal frequency bands.
$69.72 \mathrm{MHz} \pm 500 \mathrm{KHz}$
$74.70 \mathrm{MHz} \pm 500 \mathrm{KHz}$
$79.68 \mathrm{MHz} \pm 500 \mathrm{KHz}$

Since it is impractical to provide separate band-tuning networks for each of the frequency bands, the array system is being designed to operate over the range 69.2 MHz to 80.2 MHz .

In addition to bandwidth considerations, the polarization of the transmitted signal from the spacecraft is of some importance. The on-board antenna is linearly polarized, but the radiating elements rotate at a mean angular velocity of about 1 rpm with respect to a fixed direction in the plane of the receiving array. The resulting polarization of the received energy varies with time. In addition, expected Faraday rotation effects will cause displacement of the received electric field vector. Therefore, to insure a maximum received signal and to estimate the effects of Faraday rotation, the ground array must be capable of receiving two orthogonally polarized waves. For the remainder of this paper, a dual cross-polarized array is assumed unless explicitly stated otherwise.

### 5.1.2 Relationship to the Correlation Receiver

The output of the array must interface with the correlation receiver. This interface may be considered a low noise mixer which coherently translates the received and amplified RF signals to video. At this point the signal is again amplified and made available to the correlation receiver. In order to preserve the accuracy of the propagation experiment data, the energy-signal-to-noise ratio of the signal supplied to the correlation receiver must be maintained as high as possible. This requirement implies the following constraints upon the array for all frequencies and view-times of interest.

1. The array gain must be maintained as high as possible ( 50 dB ).
2. The overall noise figure must be as low as possible (approximately 5 dB ).
3. Intermodulation and cross-modulation products must be held to a minimum, and system dynamic range must be high.
4. All losses, regardless of origin, e.g., mutual coupling, mismatch or attenuation, must be held to a minimum.
5. Pattern grating lobes and side lobes must be held to a minirnum.

More quantitative values will be given in Sec. 5.2 and 5.3 for these system parameters. For the present, it should be noted that every design compromise which reduces the array gain or increases its system noise contribution reduces the accuracy of the data output of the correlation receiver.

### 5.1.3 Other Uses for the Ground Array

Because of the stringent requirements imposed upon the array by the propagation experiment, the antenna system is not limited to data acquisition and
tracking of the Sunblazer, but is a sensitive radio telescope in its own right. The additional advantages of the array have been detailed elsewhere ${ }^{(1)}$. The following is a summary of the additional array usea:

1. It can serve as radiometer for the observation of the newly-discovered pulsars.
2. By the addition of power amplifiers at each element the array would become a radar telescope for a future uplink station to Sunblazer.
3. As a solar radar, improved sensitivity and resolution would make it possible to resolve quadrants of the solar disk in Doppler and Doppler broading radar observations.
4. As a planetary radar, this instrument could detect the planet Jupiter, and could make observations of venus and Mars at a much lower frequency than has been used in the past (1).

### 5.2 System Considerations

### 5.2.1 Antenna Gain and Number of Elements

An overall antenna gain (G) of 50 dB (resulting from communication system considerations such as "free-space-attenuation loss" and propagation-delay measurement accuracy) is the primary design goal of the Sunblazer ground array This specification implies an effective collecting aperture, $A_{\text {eff }}$ of greater than $10^{5}$ square meters ( $A_{\text {eff }}=G \frac{\lambda_{0}^{2}}{4 \pi}, \lambda_{0}=4$ meters). The beam width, which depends upon element spacing, is less than $0.6^{\circ}$. An aperture having these characteristics obviously implies the use of an array of elements. The exact array dimensions and beam characteristics are dependent on the latitude of the site selected for the final array. The system outiined in tais paper will be suitable for either El Campo, Texas site (latitude $29^{\circ} \mathrm{N}$ ) or Saint Croix, V.I. (latitude $17.5^{\circ} \mathrm{N}$ ).
In considering the problem of synthesizing a phased array to satisfy the conflicting requirements of high gain, wide bandwidth and low noise, two concepts are of fundamental importance: 1) the superposition principle, and 2) the pattern-multiplication rule. The superposition principle requires that, in the far field of an array of elements, the resulting field at a point is the vector sum of the fields due to the individual elements. The patternmultiplication rule, as applied to an array of identical eleinents, states that the resulting antenna pattern of an array is the product of the element pattern and a polynomial characteristic of the array. The polynomial is commonly referred to as the array factor. In general terms the beampattern function $E(\theta, \phi)$ may be expressed as a product of the form:

$$
\begin{equation*}
E(\theta, \phi)=E_{e} A(\theta, \phi) \tag{5.1}
\end{equation*}
$$

where $E_{e}$ is the element factor, and $A$ is the array factor. When the array geometry is symmetric, Eq (5.1) may be used in simplified form, and permits a straightforward presentation of the salient features of the array.

However, the pattern-multiplication rule must be applied with caution ance its use includes the implicit assumption that mutual coupling between elements may be neglected. The pattern of an element, in general, will change when it is brought in close proximity to other elements. "The fact that all elements are physically identical does not insure that all elements of the array have the same pattern. "(2) When the array elements are spaced "far enough" apart and are highly directive, the superposition principle requires that the resultant array power-gain is merely the py oduct of the number of elements in the array and the gain per element. In general, however, the array gain is a function of the element factor, the number of elements and the element spacing, as has been shown by several authors. ${ }^{(3,4)}$

To a first-order approximation then, the gain of the proposed array is:

$$
\begin{equation*}
N G_{E}=10^{5} \tag{5,2}
\end{equation*}
$$

where $N$ is the number of elements in the array and $G_{E}$ is the power gain per element. If $N$ is now restricted to values of the form $2^{q}$ where $q$ is an integer, the element gain is also specified. Table 5-II gives the values of the total number of elements N as a function of the gain per element.

Table 5-II

Gain per element.

| $G_{E}(d B)$ | $N$ |
| :---: | ---: |
| 23 | 512 |
| 20 | 1024 |
| 17 | 2048 |
| 14 | 4096 |
| 11 | 8192 |

The value $N=4096$ and $G_{E}=14 \mathrm{~dB}$ have been selected as a compromise between a very large number of elements and high gain per element. (In the final array $\mathbf{N}$ is made somewhat larger than 4096.)

The selection of the $14-\mathrm{dB}$ element is of f : imary importance to the array design since it determines the general level of complexity of the electronics system. The actual description of array electronics
is given in Sec. 5.3. However, there is a fundamental question of the physical realization of the $14-\mathrm{dB}$ element: "Of the many types of element available in this frequency range, which element exhibits the desirable properties of low cost, wide bandwidth and high gain?" During the course of this program many antenna types were evaluated, and each was found to have limitations for array use. A detailed description of the evolution of the element design and selection is given in Sec. 1.3 of this report and in the document entitled "History and Design Summary, Sunblazer Phased Array" (5) In summary, a system parametric study was made for each of the element types, including a cost evaluation of mechanical and electrical array components. These cost-performance studies led to the following fundamental conclusion concerring the element:
"The best method, in terms of satisfying the Sunblazer tracking requirements (both engineering and science) at minimum overall cost (initial installation, operation and maintenance) is to construct a cross-polarized wide. band dipole array at El Campo, Texas."(5)
Details of the various costs are given in Ref. 5. Basically, it has beeen determined that a $14^{-d B}$ antenna element arrayed through electronic means is less expensive and has better performance when constructed by connecting 6 dipole elements, as opposed to employing single or multiple backfire, yagis or helices. A detailed description of the $14-\mathrm{dB}$ dipole element is given in Sec. 5.3.2. Figure 5-4 shows the physical laynt of the 6 -dipole (double tee) 14-dB element.

### 5.2.2 Frequency Sensitivity

Because of the relatively wide bandwidth of the array, system performance will, in general, exhibit frequency-dependent effects. Both the dipole elements and the overall array dimensions (as measured in wavelengths) are frequency-sensitive. However, both these frequency variations are of a second-order nature and will not be discussed in detail. There is a firstorder irequency-sensitive effect associated with the beam steering that requires additional discussion since it is of fundamental importance to the method by which the electronic-phasing system is organized.

The general method of beam scanning is obtained as shown in Fig. 5-1. This figure indicates that properly choser phase delays located at each 14-dB element will permit beam scanning to some desired angle. Consider now only the array elements along the $x$-axis with an energy incident at some

angic $\epsilon_{x}$. If it is desired to steer the beam in the direction $\epsilon_{x}$, then a phasedelay taper, $\beta$, from element to element (progressive phase), must be supplied to ach element where:

$$
\begin{equation*}
\beta=\frac{2 \pi D_{x}}{\lambda} \sin \epsilon_{x}=\frac{2 \pi D_{x} f}{c} \sin \epsilon_{x} . \tag{5,3}
\end{equation*}
$$

If the first element is the reference, the phase delay required at the second element is $\beta$, at the third element $2 \beta$, etc. Therefore, the required phase term at each element is linearly dependent upon frequency. To accomplish this frequency dependence, the device used to affect the phase change must have a linear phase-frequency characteristic over the frequency band of interest. A non-dispersive delay line exhibits this property, and its use as a phasing element will insure that the array beam will point in the same direction for all frequencies. The resultant system configuration is termed a delayed array as distinguished from a phased array in which frequency sensitivity is not important. ${ }^{(6)}$ The Sunblazer ground array is a delayed array, and details of its electronic system organization are given in Sec.5.3.

### 5.2.3 Noise Considerations

The concept of total system gain was of fundamental importance in the array aperture design since it set limits on the array electronics, dimensions, number of elements and element type. In an almost completely analogous way, the concepts of receiver noise temperature, $\mathrm{T}_{\mathrm{r}}$, and sky brightness temperature, $T_{B}$, are characteristic of the array system noise performance. There are several factors which contribute to these temperatures. Some of these, such as amplifier noise figure, are under the control of the system designer; while others, such as external man-made interference are not. This section will define some of the factors which contribute to the total system noise.

In the final array each $14-\mathrm{dB}$ element will have its own amplifier. However, for noise-analysis purposes, it is sufficient to consider the entire array as one antenna followed by a single amplifier (as shown in the array system noise model of Fig. 5-2). Noise may enter the system or be generated by each of the components, the antenna, connecting cables and receiver. The major contributions to the system noise follow:

1. Background sky noise, characterized $r y$ the brightness temperature $T_{B}$. The intensity of this quantity is primarily a function of the system operating frequency. For the Sunblazer ground array an average value of $1850^{\circ}$ is characteristic.
SIDELOBE NOISE
grating lobenoise
$T_{G}$
SKY NOSE
Fig. 5-2 Noise model.

2. Noise due to antenna side lobes and grating lobes pointing pis galactic hot spots. An estimated value of this quantity is $200^{\circ}$.
3. Noise due to antenna element losses, cable losses, impedance mismatches and mutual coupling. These have been estimated as 1 dB or $75^{\circ}$.
4. Front-end amplifier noise. For the design under consideration a value of 3 dB or $290^{\circ}$ is typical.
5. Spurious man-made noise entering the antenna at and near the desired signal frequencies. Additional noise may be added by intermodulation products due to high-level signals entering the amplifier. The exact specification of intermodulation depends upon local conditions, but as an example: for competing TV signals of the order $50 \mu \mathrm{~V}$, intermodulation products should be down by 80 dB for satisfactory system performance. It is clear that system dynamic range and low intermodulation distortion are related, and both are required for system linearity.

Neglecting the noise contribution due to intermodulation, the total receiver noise temperature is $565^{\circ}$. Therefore, the total equivalent system noise temperature is $\mathrm{T}_{\mathrm{r}}+\mathrm{T}_{\mathrm{B}} \simeq 2400^{\circ}=\mathrm{T}_{\mathrm{s}}$. Because $\mathrm{T}_{\mathrm{B}}$ is relatively large compared to $T_{r}$, improvements in $T_{r}$ will have only second-order effects upon $T_{s}$. System noise performance will not substantially be improved by a reduction of $T_{r}$, but rather by the selection of a site in which local RFI is at a minimum.

Although discrete sky-noise sources may contribute to total system noise, they are useful in a practical way for antenna-calibration purposes. In the $50-\mathrm{dB}$ array, the near field of the antenna $\left(\frac{2 \mathrm{D}^{2}}{\lambda}\right)$ extends to an altitude of approximately 100 kilometers above the surface of ine earth. This distance is so large that only a system utilizing a satellite could be expected to perform conventional far-field pattern measurements. However, a radio star such as Cassiopeia A provides a solution for this measurement problem. The meanurement is performed by directing the beam of the array toward a source of known intensity and spectral distribution. As the beam is allowed to pass through the source, the output of the array is a measure of the gain of the antenna elements, the electronics and the beamwidth of the antenna, This test is also an indication of the accuracy with which the beam may be steered. A similar method was used to test an experimental dipole re-ceiving-array constructed at El Campo.

## 5. 3 Description of the Proposed Arrays

In this section, the individual dipole design is shown. Following this description, there is a discussion of the element and how it is constructed, mounted in the ground, connected with its five neighbors to form a 14-dB antenna element, etc. Then, an outline of the electronics system necessary to form these individual $14-\mathrm{dB}$ element into a $20-\mathrm{dB}$ pilot array is presented. Section 5.3.4 indicates how the 29-dB pilot arrays are electronically joined to form an expanded $43-\mathrm{dB}$ array. Section 5.3 .5 briefly describes the final $50-\mathrm{dB}$ antenna system concept.

### 5.3.1 The Individual Dipole Element

The individual dipole element is to be constructed from 3/4-inch OD aluminum tube, mounted on a treated wooden post by wax-impregnated wooden dowels. The wooden post is suitably held in the clay ground of El Campo, without the necessity for a concrete footing, by using sand poured around the post as shown in Fig. 5-3.


Fig. 5-3 Individual dipole element.

From the experience with the 38 MHz solar-radar dipole array, as well as the Sunblazer 75 MHz (narrowband) dipole array at El Campo, it has been demonstrated that the feed cable may be connected to the dipole without the use of a balanced-to-unbalanced matching network (balun) as shown.

### 5.3.2 The Six-Dipole, 14-dB Antenna Element

Figure 5-4 shows some details of the proposed connection pattern of six dipoles to form a 14-dB antenna element. Dipoles 1, 2, and 3 are connected


Fig. 5-4 Six-dipole configuration.
by $75 \Omega$ cables at point $a$, as are dipoles 4, 5 , and 6 at point b. Summing points a and $b$, therefore, have a $25 \Omega$ impedance which is transformed to $100 \Omega$ levels by a $3 / 4$ wavelength $50 \Omega$ cables to summing point c. Point $c$, consequently, may be properly terminated with a $50 \Omega$ cable.

The six-dipole configuration of Fig. 5-4 yields a gain of approximately 14 dB with an EW beamwidth ( 3 dB ) of $40^{\circ}$ and a NS beamwidth of $28^{\circ}$. This "double $T$ ' interconnection pattern was selected over several possibilities, namely "four-dipole square", "four-dipole line element", and "four-dipole zizgag", because it results in relatively high gain, and requires that only two of the six dipoles be manually phased about four times per year in declination, yet affords about two hours of daily viewing time. Also, this configuration is easy to match, utilizing our standard cables, and the RF power-surnming is accomplished without electronic components.

The manual phasing for a six-dipole set can be accomplished by utilizing switched delay lines, physically located on posts 3 and 6 . The necessary time delay (phasing loops) can be switched with mercury switches connected as shown in Fig. 5-5.


Fig. 5-5 Mercury-switched phased loops (schematic).

Based on several years experience with manually-phased 1000 -dipole, 38 MHz El Campo solar radar, the estimated phasing time with this concept is about 0.005 hours/switch location. For a $40-\mathrm{dB}$ array ( 3072 dipoles with 1024 phasing locations) the array can be phased in 8 man hours.

The mercury switches are inexpensive, exhibit excellent RF performance (insertion loss 0.04 dB , isolation 30 dB , and are judged to be more eliable than connectors which must be plugged and unplugged. Also, these switched delay line systems may in the future be made fully automatic by replacing the manual switches with mercury relays.

### 5.3.3 The Pilot Array

### 5.3.3.1 General Description

Thirty-two of the 14 dB dipole subsets of Fig. $5-4$ combine to form a 192 dipole array as shown in Fig. 5-6. This pilot array measures $142 \times 100$ feet.

Each of the six-dipole subsets is connected to an electronic phase-shifter box located at the geometric center of the pilot. The function of the central electronic box is to shift the phase of the signals from each of the six-dipole subsets so that the RF signals may be summed to form one output per polarization. To interconnect the pilot erray requires about 8300 feet of cable (including both polarizations), of which 3000 feet are used in the subset and 5300 feet to connect the subsets to the central electronics. To insure relatively-constant operating temperature and a resulting phase stability, all cables are buried in shallow trenches.

Table 5-III summarizes the characteristics of the pilot antenna.

Table 5-III
Design summary 29-dB pilot array.

```
Operating Frequencies
Total Array Gain
Element Type
Realized Gain Per Dipole
Number of Dipoles
Dipole Grouping
Dipole Spacing
Array Area
Grating Lobes
Polarization
Beamwidth
Phasing
69.72 MHz, 74.7 MHz, 79.68 MHz
28.9 dB
\lambda/2 dipole, }\lambda/4\mathrm{ above a ground plane
dB
192
6 per group in double T interconnection
0.63\lambda echelon
10.68 }\lambda\mathrm{ by 7.65 }\lambda\mathrm{ (142!' by 100')
None in Zenith pointing array
Two independent polarizations NS and EW
60}\times\mp@subsup{8}{}{0
Hybrid system: rapid electronic scan for short-term tracking, manual phasing using mercury switches for long-term (declination) scans.
```



### 5.3.3.2 Electronics for Pilot Array

The governing design philosophy for a $50-\mathrm{dB}$ array has been to design a pilotarray module which contains all of the system compromises and trade-offa regarding performance and cost, and then to construct a large array by joining these self-contained pilots to obtain the required overall antenna aperture

There are several feasible organizations of a pilot model. For example, the RF outputs of individual dipoles could be returned to a central point at which all time-delay and control circuits are located; however, this would be prohibitively expensive. On the other extreme, the electronics could be more or less evenly distributed over the array aperture and the combining and control performed at many points in the pilot array. The solution of providing phasing by manually-switched delay lines at the $14-\mathrm{dB}$ element level, and connecting these points directly to a centrally located electronic control point as outlined by Fig. 5-4, 5-5, 5-6 yielded good performance and reduced both cable and electronic cost.

The block diagram of Fig. 5-7 shows the centrally located electronics required to combine the signals from each of the 32 six-dipole elements. To trace a signal path, consider, for example, the energy arriving from the 14-dB element designated as point 1. The signal is first amplified in a lownoise broadband amplifier, and then supplied to network $W$. The signal from point 2 , which is adjacent to point 1 in the first column, is also amplified and supplied to network $W$. In this circuit the amount of signal delay is dependent upon the desired array look-angle, and then summed. A second level of combining is performed by network $X$, which delays and sums the output from two adjacent $W$ networks. The output of network $X$ is a complete column output of the pilot array. There are eight such column outputs in the array. These outputs are delayed and combined in a way similar to the above by the operation networks $Y, Z$, and $T$. The RF output of network $T$ represents the sum of all 192 dipoles. All of the combining networks are of the same design, and there are only two types of RF circuits used in the entire system: 1) a broadband amplifier, and 2) a time-delay summation network. There is only one amplifier per $14-\mathrm{dB}$ element per polarization. The noise figure of the amplifier is an important design consideration; for an overall pilot system noise-figure of 5 dB , the front-end noise figure must be about 3 dB . This in turn requires a transistor with a noise figure of somewhat less than 3 dB to allow for some mismatch and attenuation losses in the coupling circuit between the RF input and the device. Fortunately, however, the gain required in the front end is relatively low. The

Fig. 5-7 Electronics system block diagram.
amplifier can then be designed with a minimum-noise, rather than a marimumgain criterior, and is therefore readily realizable.

The amplifier frequency response and linearity have also been carefully considered, for it is somewhat impractical in a low-cost, high-volume production to provide independent RF pass bands for each of the design irequencies. Accordingly, the amplifier has been designed with a 3-dB bandwidth covering the frequency range from 60 MHz to 90 MHz . But since this frequency band is a region of high RF interference, due to local TV and other commercial services, the linearity, dynamic range, crossmodulation and intermodulation of the amplifier are of prime importance and have been carefully considered in the design.

The time-delay circuit for shifting the RF signal phase (for example network W) shown in simplified form in Fig. 5-8, is composed of sections of RF cables which are switched in and out of the circuit via diodes to obtain the required time delay. At each level, in both the intracolumn and intercolumn combining, the time-delay function is performed in a similar way with the exception that the dealy lines are made longer as the summation progresses toward the system output.

Table 5-IV gives a summary of the total number of circuits for the pilot along with pertinent characteristics.

### 5.3.3.3 Testing Considerations

The primary reason for the construction of a pllot array is to obtain engineering data on design and performance problems such as amplifier and phaser uniformity, mutual coupling effects, precise antenna gain, losses, etc. that cannot be accurately anticipated. A search for such effects will be made as the array is constructed and tested. The theoretical performance including gain and effect of grating lobes and mutual coupling will be experimentally verified. The reliability of the field electronics will be determined under actual weather and working conditions. ficcurate cost and construction techniques for the final array will also be determined.

After the array is constructed, an overall evaluation will be made by using known celestial sources. Cassiopeia, Cygnus, and the Sun may be usei for the pilot array; Virgo, Taurus, and several others, including pulsating sources, may be used to test the $40-\mathrm{dB}$ array.
COAXIAL CAble lengTh $\lambda$

Fig. 5-8 Time delay circuit.
Table 5－IV
Pilot array electronics．

| Circuit Type | Number Required | Description and／or Major Specifications |
| :---: | :---: | :---: |
| Broadband Amplifier | 80 | Noise Figure $=3 \mathrm{~dB}$ <br> 3 dB Bandwidth $=30 \mathrm{mc}$ <br> Gain $=17.5 \mathrm{~dB}$ average |
| Time Delay and Summation Network＂W＂ | 32 | Maximum Delay $=17 / 8 \lambda_{0}$ in $\lambda_{0} / 8$ steps Insertion Loss $=6 \mathrm{~dB}$ max． <br> Amplitude Variation $=1 \mathrm{~dB}$ Coherently combines two signals from 14 dB elements |
| Time Delay and Summation Network＂X＂ | 16 | ```Maximum Delay = 3 7/8 的 in }\mp@subsup{\lambda}{0}{}/8\mathrm{ steps Insertion Loss =6 dB max. Amplitude Variation = 1 dB Coherently combines two signals from network 'W"``` |
| Time Delay and Summation Network＂ $\mathbf{Y}$＂ | 8 | ```Insertion Loss = 6 dB max} Maximum Delay = 17/8 (in in (%/8 steps Amplitude Variation = 1 dB Coherently combines two signals from network "X"``` |
| Time Delay and Summation Network＂Z＂ | 4 | ```Insertion Loss = 6 dB max. Maximum Delay = 3 7/8 的拄 }\mp@subsup{\lambda}{0}{}/8\mathrm{ steps Amplitude Variation =1 dB Coherently combines two signals from network "Y"``` |
| Time Delay and Summation Network＂T＂ | 2 | ```Maximum Delay = 7 7/8 \mp@subsup{\lambda}{0}{}}\mathrm{ in }\mp@subsup{\lambda}{0}{}/8\mathrm{ steps insertion Loss = 7 dB max. Amplitude Variation =1 dB Coherently combines two signals from Network "Z"``` |
| Fixed Delay | 4 | $2.67 \lambda_{0}$ phase－equalizing cable |
| Power Supply | 1 | 120 Vac＠ 1.1 amp 130.0 watts input <br> $3.6 \mathrm{Vdc} @$ 1.2 amp 3.6 watts  <br> $12 \mathrm{Vdc} @$ 10.0 amp 80.0 watts  <br> 15 V dc＠ 0.4 amp 6.0 watts  |
| Logic | 4 | 4－，5－，and 6－bit counters with SCR drivers for beam pointing control |

### 5.3.4 The Expanded Array

### 5.3.4.1 General Comments

Since the pilot array is completely self-contained, i.e. it is designed as a "module", an increase in array-effective aperture may be obtained by adding pilot array modules to realize the desired gain. Two pilot modules will yield a gain improvement of 3 dB , four modules will result in 6 dB improvement, etc. For the Sunblazer engineering payload, the required minimu:. receiving system gain which will provide satisfactory telemetry and tracking data is judged to be about 40 dB , which may be realized by expanding or adding the 15 pilot arrays as indicated in Fig. 5-9. A summary of the antenna system characteristics for the expanded array is given in Table 5-V.

Table 5-V

Design summary of $40-\mathrm{dB}$ array.

| Operating Frequency | $69.72 \mathrm{MHz}, 74.7 \mathrm{MHz}, 79.68 \mathrm{MHz}$ |
| :--- | :--- |
| Total Array Gain | 40.9 dB |
| Gain Per Dipole | 6 dB |
| Total Number Dipoles | 3072 |
| Dipole Grouping | Double T |
| Dipole Spacing | $0.63 \lambda$ |
| Array Area | $568^{\prime} \times 400^{\prime}$ |
| Beamwidth | $1.3^{\circ} \times 1.9^{\circ}$ |

### 5.3.4.2 Expanded 40-dB Array Electronics

Each of the pilot array outputs is returned to a central building. The electronic circuits for each of the 16 proposed pilot arrays are identical to the system described above; but, of course, electronics must be constructed to shift the phase of the RF signal from each pilot and sum the results.

Figure 5-10 shows a block diagram of the electronic system to shift the phase and sum the signals. As for the pilot, the signal energy from adjacent segments is summed in pairs, the two pairs are then summed, and finally the columns are treed together to form one output per polarization.

The time-delay network $W^{\prime}, X^{\prime}, Y^{\prime}$, and $Z^{\prime}$ is identical in design to that of the pilot array ( $W, ~ X, Y$, and $Z$ ), except that extra stages of cable loops are added to accommodate increased delay resulting from the fact that the


Fig. 5-9 Expanded 40-dB array.

# "Page missing from available version" 

signals come from phase centers which are more widely separated in physical distance.

### 5.3.4.3 Testing

Each segment of the expanded array will be tested individually as an antenna in a manner similar to the original pilot section. The resulting sum will then be charasterized for gain, noise figure, beamwidth, bandwidth, etc. The central building (control center) for the array will contain the necessary equipment to operate, i.e. steer the array, receive and record signals, etc. from the Sunblazer engineering spacecraft.

### 5.3.5 The 50-dB Array

A $50-\mathrm{dB}$ array may be realized by continuing the expansion philosophy outlined in the discussion of the pilot and $40-\mathrm{dB}$ arrays. Essentially, a $50-\mathrm{dB}$ array results from adding the signals from 140 pilot arrays. The electronics, construction techniquess, trenches, cables, etc, are all the same type.

Technical difficulties encountered in expanding the $40-\mathrm{dB}$ array into a $50-\mathrm{dB}$ array should be virtually non-existent. In fact, one may realistically view the modular and orderly growth procedure outlined here as minimizing the likelihood of any fumdamental problem remaining undiscovered prior to construction of the final antenna.

Figure 5-11 shows the proposed plan for the $50-\mathrm{dB}$ array, and Table 5-VI gives some characteristics.

Table 5-VI

Design summary of $50-\mathrm{dB}$ array.

| Operating Frequencies | $69.72 \mathrm{MHz}, 74.7 \mathrm{MHz}, 79.68 \mathrm{MHz}$ |
| :--- | :--- |
| Total Array Gain | 50.4 dB |
| Gain Per Dipole | 6 dB |
| Total Number of Dipoles | 26880 |
| Dipole Grouping | Double T |
| Dipnle Spacing | $0.63 \lambda$ |
| Array Area | $1400^{\prime} \times 1400^{\prime}$ |
| Beamwidth | $0.5^{\circ} \times 0.5^{\circ}$ (Zenith) |


Fig. 5-11 Proposed plan for 50-dB array.

## CHAPTER 6

### 6.0 SRACECRAFT CONSTRAINTS

The concept of using a relatively small, unguided launch vehicle to provide the escape velocity required for a solar-orbiting scientific experiment implies the use of a very small spacecraft as the payload. The standard fourstage Scout vehicle has been selected to launch the Sunblazer spacecraft; and to provide the escape velocity required, a BE-3 motor has been added to the vehicle as the fifth stage.
6.1 Distribution of the Payload Weight

The Scout User's Manual Fig.5-43, reproduced here as Fig. $8-1$, shows a performance curve of a five-stage Scout, which indicates the capability of placing a 57 -pound payload into 0.65 astronomical unit inferior solar orbit. The approximate distribution of this payload weight for the initial Sunblazer experiments is:

Table 6-I

| Distribution of Payload Weight |  |  |  |
| :--- | :---: | :--- | :--- |
| Spacecraft | 28.00 lbs | $58.5 \%$ |  |
| Sunblazer Adapter | 5.41 | 58 |  |
| Upper F Structure | 3.25 |  |  |
| Ignition Timer | 4.13 |  |  |
| Balance Weights | 1.00 | $41.5 \%$ |  |
| Despin System | 0.95 |  |  |
| Telesponder System | 8.37 |  |  |
| Performance T/M | 5.89 |  |  |
| Total |  |  |  |

The initial system engineering launches, which will monitor the performance of the modified Scout vehicle and the Sunblazer spacecraft, limit the engineering model spacecraft's weight to 28 pounds.

SOLAR PROBE MISSIONS

Wallops Island Launch
126 Degrees Asimuth
85 Degrees Elevation


Fig. 6-1 Solar probe performance using Scout fifth stage - Wallops Station.

### 6.2 Dimensional Limitations

The standard Scout heat shield for the payload has been increased in length by 15 inches to accommodate the additiot. of the fifth stage and the spacecraft. In Fig. 6-2 the relative positions of the payload configuration and the envelope of this extended heat shield are snown. The adapter (transtage) used to position the spacecraft derives its height from a spacecraft sail configuration option, which allows a geometric flexlbility in the design of these devices.

One of the basic requirements of the spacecraft design is to provide a solar cell-mounting area which is capable of delivering about 18 watts of power at 1 astronomical unit. This requirement establishes the initial dimension of the spacecraft at about 20 inches diameter. Since the volume in which the spacecraft is placed is a truncated cone, the upper positional limit of the 20 inch diameter has been calculated to be Station -21.94.

The separation plane of the spacecraft and adapter is located at Station $\mathbf{- 1 5 . 0 0}$, and the total height of the spacecraft is 6.501 inches, which locates the solar cell platform flanges at Station -21.50.
The cylindrical launch geometry of the spacecraft is, therefore, approximately. defined by a 20 -inch diameter and a 6.5 -inch height, or a total of 2042 cubic inches, with a maximum weight of 28 pounds.
6.3 Spacecraft - Adapter Interface

The orientation of the spacecrais to the yaw and pitch axes of the launch vehicle is generally predetermined by the interfacing bolt pattern provided by the design of the LTV Sunblazer Adapter.
The LTV drawing of this assembly, J23-003691 (View A-A) shows two of the interface bolt holes diametrically aligned in the launch vehicle yaw axis. Since these holes are displaced from the geometric centerlines of the spacecraft by 22.5 degrees, the resultant orientation of the spacecraft to the launch vehicle is shown in Fig. 6-3.

The spacecraft is mounted to the adapter so that Enere, Storage Capacitors 1 and 5 lie in the yaw axis, with Capacitor 1 located on the range side and Capacitor 5 on the tower side.
6.3.1 Attachment Method

To permit the attachment of the spacecraft to the adapter without requiring a disassembly of a portion of the spacecraft, the following procedure is proposed.

The separation springs of the adapter are compressed and secured by the mounting plate and pyrotechnic bolt.


Fig. 6-2 Location of spacecraft in -40 heat shield.


Fig. 6-3 Axial alignment of spacecraft.

The spacecraft has the sails and antennas in the stored (launch) position. The separation trigger mechanism, which initiates despin, is retained by a temporarily installed inhibitor device.

The spacecratt is axially oriented to the vehicle. The mounting serews are inserted through clearance holes in the upper ring flange, and seated on the mounting plate by screwing into the hub of the spacecraft. [Due to the close proximity of upper-ring sidewall stiffeners in this area, Allen-head screws will most likely be used in association with a specially-designed right-angle ratchet wrench

When the eight interface screws are properly torqued, the inhibiting devices on the separation trigger will be removed. The mechanical interface will then be complete.
6.3.2 Despin-Deployment Actuator

Mechanical separation sensors are mounted on the exposed portion of two vertical members of the electronic compartment, and are referenced to a small pad which is mounted on the sides of the upper ring-assembly of the adapter.

The two separation sensors used are basically push-rods which will have a radial force of five pounds or less acting upon each of them.

### 6.4 Environmental Testing Prior to June 1968

### 6.4.1 Vibration

The first vibration testing of the Sunblazer vehicle took place February 3, 1966. It was foreseen then that the vehicle would be changed, but it was desirable to test the principle oî a spacecraft made of aluminum sheet metal. The results were extremely satisfactory. Brittle shellac was used in order to have an idea of the magnitude of the safety factor. The stress level never went above $4,000 \mathrm{psi}$ on any part. Since the vehicle interface had not been specified at that time, the vehicle was hard-mounted to the vibrator, with the solar cell panel facing the vibrator. The test program was:

Thrust Axis
Run $1-4.5 \mathrm{~g}$ sinusoidal for 120 seconds, 40 to 80 cps $2-6.0 \mathrm{~g}$ sinusoidal for 144 seconds, 20 to 2000 cps $3-3.0 \mathrm{~g}$ random for 284 seconds, 20 to 2000 cps $4-9.0 \mathrm{~g}$ sinusoidal for 120 seconds, 40 to 80 cps $5-12.0 \mathrm{~g}$ sinusoidal for 120 seconds, 20 to 80 cps

## Tranaverse Axes

(along the compartmert and between the compartmente

$$
45^{\circ} \text { away) }
$$

Run $1-6.0 \mathrm{~g}$ sinusoidal for 144 seconds, 20 to 2000 cps
2-3 0 g random for 284 seconds, 20 to 2000 cps
3-12.0g sinusoidal for 120 seconds, 20 to 80 cps
On March 10, 1967, a test was run with the first transtage. Three sets of electronic sections were aboard; the solar cell panel faced away from the vibrator. There was a new input on the test levels (described below). A sinusoidal sweep in the thrust axis was made at 2 octaves per minute for four minutes. The level of vibration was:

| 20 to 50 cps | 1 g |
| :--- | :--- |
| 50 to 500 cps | 4 g |
| 500 to 2000 cps | 8 g |

The random vibration in the thrust axis lasted $21 / 2$ minutes under the following conditions:

20 to $2000 \mathrm{cps}, 7.7 \mathrm{~g} \mathrm{rms}$, and $0.03 \mathrm{~g}^{2} / \mathrm{cps}$ power spectral density, (This is a flight acceptance level.)

The sinusoidal sweep in the transverse axis (only one transverse axis was vibrated because of symmetry and tirae) was made twice at two different levels:
(Sweep speed 2 octaves per minute)

| 20 to 50 cps | 0.6 g |
| :---: | :---: |
| $5^{\prime}$ to 500 cps | 02 g |
| 500 to 2000 cps | 1.6 g |

(This is a flight acceptance level)

| 5 to 10 cps | 0.2 in. |
| :---: | :---: |
| 10 to 50 cps | 1.0 g |
| 50 to 500 cps | 1.5 g |
| 500 to 2000 cps | 2.5 g |

(This is a qualification level)
The random test in the thrust axis was also made at two levels, and in only one transverse axis.

20 to $2000 \mathrm{cps}, 4.4 \mathrm{~g} \mathrm{rms}, 0.01 \mathrm{~g}^{2} / \mathrm{cps}$ power spectral density
(This is a flight acceptance level) time 4 minutes

[^0](This is a qualificaion leval) time 4 minutes
The test with brittle shellac showed that the transtage had a safety factor of two, and that the sail, as then designed, was sound. Following is a summary of the effect of vibration on the electronics that were aboard:

1. 270-Watt Amplifier

Evaluation after shake revealed the presence of a nondestructive, low-frequency oscillation. This was traced to a repositioning of amplifier input circuit inductors during shake. Restoring these components to their original positions resulted ir normal operation. No other failures were observed. The inductors will be replaced by a more rigid type in future amplifiers.
2. 28-Volt, 5-Watt Beacon

The beacon wers inoperative after shake, Loss of output was traced to two open component leads which resulted from poor soldering practice. The beacon operated after these leads were restored, but a tendency toward low-frequency oscillation was noted. This oscillation was traced to the pre-driver stage, and the apparent cause isolated to coil deformation. An epoxy base for these coils would seem to be called for. In the future, breadboard circuits will not be expected to undergo shake.
3. Low Level Stages

Operation after shake was essentially the same as before, with slight detuning of output circuits noted.

### 6.5 Structural Analysis

### 6.5.1 Introduction

Weight saving is of prime importance in spacecraft structure design, and structures with maximum stiffness-to-weight ratio are employed. However, Sunblazer's special features (oriented during flight), and their implications, have to some extent dictated the basic configuration of the spacecraft. Maximizing the stiffness-to-weight ratio is thus restricted to the selection of optimum material dimensions and placing of required stiffeners. It is, therefore, sufficient for this analysis to verify that the selected structure design will withstand the expected loadings. This is done by computing the stresses in models of those parts of Sunblazer expected to experience critical loadings. By choosing a conservative (weaker than reality) model, acceptable computation results will assume a safe structure.

A more detailed analysis is planned, applying the stifiness or flexibility method to the entire structure along with a vibration analysis.

### 6.5.2 Calculations

The structure should withstand acceleration forces of 100 g . With the spacecraft mounted as shown in Fig. 6-4, the critical stresses are expected to appear . the compartment side walls and covers, parts of the platform, and in the vertical member supporting the electronics. A primary structure model, Fig. 6-5 is used to represent the entire structure, except for the electronics. In this model the weights of the platform-radiator, capacitors, sails, etc., are assumed is be concentrated along a ring of diameter $D=15$ inches. These weights are supported by four box beams mounted to the hub which is assumed to be rigid. The assumed weights $W$ are

$$
W=6150 \mathrm{gm}
$$

or per beam

$$
\frac{W}{4}=1537 \mathrm{gm}=3.38 \mathrm{lbs} .
$$

From beam theory the stress due to bending is

$$
\begin{equation*}
S=\frac{M I}{C} \tag{6-1}
\end{equation*}
$$

where $S=$ normal stress,
$M=$ bending moment,
$I=$ moment of inertia, and
$C=$ distance from neutral layer to outer fiber.
Thus (see Fig. 6-6 for I and C of box beam) for 100 g loading,

$$
S=\frac{338 \times 4.5 \times 1.85}{3}=940 \mathrm{psi} .
$$

The load gives rise to a shear stress of (see Fig. 6-6 for A)

$$
T=\frac{100 \frac{W}{4}}{A}=\frac{338}{0.365}=93 \mathrm{cpsi} .
$$

The other critical part of the structure is the vertical member in the elec tronics compartments. Each typically supports eight printed-circuit boards weighing 100 gm each. Under an acceleration load of 100 g the compression stress will be

$$
S=\frac{177}{0.150}=1180 \mathrm{psi}
$$



Fig. 6-4 Spacecraft mounting.


Fig. 6-5 Primary structure model (box beam).

and the shearing stress at the connection to the compartment shield will be

$$
T=\frac{177}{0.180}=985 \mathrm{psi}
$$

6.53 Discussion

Although the above computations oversimplify the real situation, they fulfill the requirements set up in the introduction (6.5.1). The model employed for the calculations of the primary structure is certainly conservative in the sense that it is much weaker than the real structure. Also, by lumping all the weights (excluding electronics) at the ends of the box beams, the situation depicted for the model is worse than the actual. With acceptable values for the model, it must be concluded that the actual structure is safe. The same arguments apply to vertical members. Buckling has not been mentioned at all since buckling, in those parts where it can be expected, can be prevented by adding stiffeners which do not change the basic structure or significantly increase the weight.

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## CHAPTER 7

### 7.0 DESCRIPTON OF THE SPACECRAET

The spacecraft consists of six principal sub-assemblies:

1. Platform-Radiator
2. Electronics and Power
3. Aspect Sensor
4. Salls
5. Antennas
6. Despin - Deployment

### 7.1 Platform-Radiator Sub-Assembly

The plarform-radiator sub-assembly is an aluminum sheet metal structure, which is thermo-mechanically designed to be a mount for solar cells, salls, and antennas, and to provide a passively-controlled thermal housing for the electronics, energy storage, aspect sensor and sail-drive mechanism.

## 7.j. 1 Fabrication

The platform-radiator sub-assembly is developed from four aymmetrical segments, (Fig. 7-1), each of which consists of the following parts: segment, radiator, left- and right-hand compartment sidewalls and gussets. These parts are cut and bent by use of guides and stops on the shear and break, with tolerances being held to 0.005 inch.

An assembly jig (Fig. 7-2) is used to position ine parts properly with respect to dimensions, parallelism and/or perpendicularity. The pre-cleaned parts are fastened by rivets, while in the assembly jif, and after post-assembly inspection, each segment is dip-brazed.

The dip-brazing unitization of the platform-radiator segments is used to provide a maximum thermal conductivity between parts by filling junction voids with metal. This process has the secondary effect of increasing the mechanical strength of the entire sub-assembly.


Fig. 7-1 Platform-radiator sub-assembly segment.


Fig. 7-2 Assembly jig.

### 7.1.2 Area Available for Solar Cells

The total area of one 90 -degree segment is $70.71 \mathrm{in}^{2}$, or a total platform area of $282.84 \mathrm{in}^{2}$
The front area of the aspect sensor is $23.76 \mathrm{in}^{2}$ Area for the main solar cell array is:

$$
282.84 \mathrm{in}^{2}-23.76 \mathrm{in}^{2}=259.08 \mathrm{in} .^{2}
$$

Converted to metric, the area equals $259.08 \mathrm{in}^{2} \times 6.452 \frac{\mathrm{~cm}^{2}}{\mathrm{in}^{2}}=1671.58 \mathrm{~cm}^{2}$
The number of solar cells are: 4 sets of 40 series-connected cells per segment: or 160 cells per segment times 4 segments: or 640 cells total. Basic area of the cells is $2 \mathrm{~cm}^{2} \times 640=1280 \mathrm{~cm}^{2}$. Area required for interconnections of the cells $=1280 \mathrm{~cm}^{2} \times 1.09=1395.2 \mathrm{~cm}^{2}$

Total mounting area $=1671.58 \mathrm{~cm}^{2}$
Total area of the
cells $=1395.20 \mathrm{~cm}^{2}$
Non-cell area $=276.38 \mathrm{~cm}^{2}$
The non-cell area of the platform will have thin second-surface mirrors mounted as an aid in passive thermal control of the spacecraft.

Area distribution is:
cells $=\frac{1395.20 \mathrm{~cm}^{2}}{1671.58 \mathrm{~cm}^{2}} \approx 84 \%$
platform
Solar cells $=84 \%$ of available area for power
Mirrors $=16 \%$ used for thermal control.
7.1.3 Platform-Radiator Drawings

MIT-CSR drawings used for the construction of the platform-radiator are:

| D-106-001 | Segment Assembly |
| :--- | :--- |
| D-106-202-B | Segment |
| D-:06-203-B | Radiator |
| D-106-217 | Compartment Wall, Left |
| D-106-218 | Compartment Wall, Right |

### 7.2 The Electronics Sub-Assembly

The electronics sub-assembly consists of five components, the central Hub and four electronic modules positioned at 90 -degree intervals around the Hub


Fig. 7-3 Electronics sub-assembly.

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## as shown in Fig. 7-3.

This packaging configuration was selected to establish thermal and RFI control and to provide for the elimination of standard connectors and epoxy foam encapsulants, while maintaining mechanical and electrical integrity.

### 7.2.1 The Hub

The Hub is machined from a section of tubular NEMA LE(N), retaining the tubular form over most of its length, with th. exception of the front and rear octagonai flanges which are used for hard-mounting the electronic components. The Hub functions as the central mechanical stiffener of the spacecraft; as the housing for the aspect sensor; as the support for intraconnecting wiring; as the initial support of the electronic components; and as the principal launch vehicle-spacecraft mechanical interface member.

The intraconnections of the electrical system are accomplished by placing eight epoxy-Fiberglas boards (G-10, 1/16 in. thick) around the periphery of the Hub; these are used as mounting plates for the receptacles and their associated flexible printed-circuit wiring. Low-level electrical power and control signals are transferred from the electronic modules to the intraconnect wiring through RF filters, which mate with the receptacles due to a compatible hard-mount of the filters in an associated Compartment Shield.

Figure 7-4 shows a sectional view of the electrical intraconnection scheme. The module (PI, PII, PIII, or PIV) may be inserted or removed from the intrawiring around the Hub during system checkout, due to the electromechanical connections provided by the receptacle and RF filter. Prior to flight, all of these connections are to be soldered to provide a parallel current path through the junction and, by sc doing, eliminate the necessity of relying on a failure-prone connector in flight.

### 7.2.2 Hub and Module Drawings <br> Parts and drawings associated with the Hub are: <br> MIT-CSR Drawing, D-106-201-B <br> Amp Receptacle, part no. 380598-1 Erie Filtercon, part no. 1250-003

The Electronic Modules (Components) PI, PII, PIII, PIV
The electronic module consists of four basic mechanical parts:

1. Compartment Shield D-106-200-C
2. Vertical Member C-106-207-B
3. Cover B-106-206
4. Printed Circuit D-106-901 Board (Master)

Fig. 7-4 Section view intraconnect.

### 7.2.3 The Compartment Shield

The Compartment Shield is machined aluminum alloy (2024) and features two parailel flanges (front and rear) which are used for mounting to the Hub; a longitudinal, off-center flange on the opposite side to mount the Vertical Member; and a groove on three sides to accept Metex RFI shielding (Part No. 10-309). The fourth side of the Compartment Shield has the cover permanently attached. Erie RF Filtercons (Part No. 1250~003) are thread-mounted into the two long edges of the Compartment Shield in Specified locations to transfer electrical functions in and out of the electronic compartment.

### 7.2.4 The Vertical Member

The Vertical Member is a $0.032-\mathrm{in}$. piece of sheet aluminum (6061). It is used to support printed-circuit boards and to function as a thermal sink for the electronics located in its particular compartment. RF circuit boards are slotted one-half of their length, and are inserted into the Vertical Member by mating with similar slots contained by that part. The aluminum parts and the printed-circuit boards of the electronic module are unitized by soldering, in order to establish an electrical ground return and a thermallyconductive path from the printed-circuit boards. To make practicable the soldered interface within the electronic module, all aluminum parts are copper-plated ( 0.0005 ), and then tin-lead plated ( 0.0005 ).

The Verticcil Member is preheated by an induction method to eliminate its heat-sinking capability. The printed-circuit boards are then fillet-soldered to the Vertical Member with a standard soldering iron.

### 7.2.5 Cover

The Cover is 0.032 in . sheet aluminum (6061) and is centrally slotted within its length to accommodate the passage of a portion of the Vertical Member through it. The Cover's functions are: to form the sixth side of the electronic compartment ( 4 fixed, 2 moveable); to heat-sink and provide thermal radiation; to develop mechanical stiffening for the compartment sidewalls and RFI shielding to the compartment. The cover compartment sidewall-closure shielding is accomplished by a strip of RF gasketing, 0.020 in . thick, Eccoshield SV-R.

### 7.2.6 Plating Specifications and Process

Platings of the Compartment Shield, Vertical Member, and Cover are performed to the following specifications:

```
Copper - MIL-C-14550 Class 2
Tin-lead (solder) - MIL-F-14072-M222
```

The total cleaning and plating of these parts is as follows:
Caustic etch
Nitric hydrofloric pickling
Rezincating
Nitric hydrofloric pickling
Rezincating
Copper flash
Copper plate ( 0.0005 inch $)$

Bake for $1 / 2$ hour at $375^{\circ} \mathrm{F}$; if no blisters, then electrochemically solder plate: (0.005 in.).
7.2.7 Electronics Sub-Assembly Critical Parts Tests

Braid Shielding (RFI)
In order to evaluate the RFI shielding capability of various braid and mounting groove configurations, a mock-up compartment shield was mounted centrally in a box which provided a variable braid compression (gap adjustment). A $75-\mathrm{MHz}$ source was located in cie side of the shield and a receiver on the opposite side. In this manner the relative attenuation of the mechanical configuration could be observed.

Table 7-I
Results of RFI shielding tests.

| Test No. | Braid Size | Groove Size | Average Gap | Attenuation |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $1 / 8^{\prime \prime}$ Diam. | $0.125^{\prime \prime} \times \prime$ <br> $0.093^{\prime \prime}$ | 0.012 | $\sim 60 \mathrm{~dB}$ |
| 2 | $1 / 8^{\prime \prime}$ Diam. | $0.125^{\prime \prime} \times$ <br> $0.093^{\prime \prime}$ | 0.016 | $\sim 60 \mathrm{~dB}$ |
| 3 | $1 / 8^{\prime \prime}$ Diam. | $0.125^{\prime \prime} \times$ <br> $0.040^{\prime \prime}$ | 0.060 | $\sim 62 \mathrm{~dB}$ |
| 5 | $1 / 16^{\prime \prime} \times 1 / 8^{\prime \prime}$ <br> flat <br> $1 / 16^{\prime \prime} \times 1 / 8^{\prime \prime}$ <br> flat | $0.125^{\prime \prime} \times$ <br> $0.055^{\prime \prime}$ <br> $1 / 16^{\prime \prime} \times 1 / 8^{\prime \prime}$ <br> flat | NONE | 0.018 |

Tests 5 and 6 indicate that a flat braid, when reasonably compressed 0.018 in. , does not provide the shi, ding capability of a round braid seated in a groove, as shown in test 3 whicn had a compression of 0.025 in .

It can be seen from Tests 1 and 2 that a similarity in the cross-sectional area of the groove and the cross-sectional area of the braid provides an attenuation capability which is not appreciably affected by the amount of compression provided.
Tests 1 and 2: Groove cross-sectional area $=0.125 \times 0.093=0.012 \mathrm{in}^{2}$

$$
1 / 8^{\prime \prime} \text { diameter braid cross sectional area }=\pi \times 0.0625^{2}
$$

$=0.0123 \mathrm{in}^{2}$.
Projection of braid from groove $=0.125-0.093=0.032 \mathrm{in}$.
Test 1: $\quad$ Compression $=0.032-0.012=0.020 \mathrm{in}$.
Test 2: $\quad$ Compression $=0.032-0.016=0.016 \mathrm{in}$.
Compartment shield drawing D-106-200 shows the braid mounting groove $=0.93 \mathrm{in} . \times 0.075 \mathrm{in}$.
Groove C.S. area $=0.093 \times 0.075=0.006975 \mathrm{in} .{ }^{2}$
Braid selected is Metex Part No. 10-309 (3/32 in. diam.)
Braid C.S. area $=\pi(0.046875)^{2}=0.006902 \mathrm{in}^{2}{ }^{2}$
Basic mounting dimension (width) of the braid $=1.830 \mathrm{in}$.
Total Width $=1.830+2(0.09375)=2.0175 \mathrm{in}$.
Compression $=\frac{0.0175}{2}=0.00875 \mathrm{in}$.
$G a p=\frac{2.00-1.98}{2}=0.010 \mathrm{in}$.

## RFI Filters

All wiring external to the electronics compartments is exposed to the $2 \mathrm{kilo}-$ watts of radiated RF. Leads entering or leaving the electronics compartments are therefore filtered to attenuate this induced signal.

This RF filtering will be provided by Erie Filtercons (Part No. 1250-003). Manufacturer's specifications include the following:

```
Weight = 1.3 grams
Minimum Insertion Loss over }}={\begin{array}{l}{2 Amp DC load 45dB}
```



```
Capacitance = 1500 pF
Working voltage = 100 WV dc@ + 125'0}\textrm{C
    200 WV dc @ +850}\textrm{C
Operating temp. = -55' C to + 125' C
```

Max dc and low freq current = 10 amperes
Samples of this part were subjected to high-current pulses to monitor possible changes in waveforms, the voltage drop across the device, and their high-current capability. The test pulse format was two 100 ms pulses separated by 100 ms repeated at $52-s e c o n d$ intervals.

Table 7-II
Test results on filtercons.

|  | Riselime | $\pm 0.04 \mu \mathrm{~s}$ | Falltim | $\pm 0.04$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pulse Current | With Filter | Without Filter | With Filter | Without Filter | Pulse Voltage | Load |
| 4.5A | $0.20 \mu \mathrm{~s}$ | $0.20 \mu \mathrm{~s}$ | $0.16 \mu \mathrm{~s}$ | $0.16 \mu \mathrm{~s}$ | 34 V | $7.6 \Omega$ |
| 8.5A | $0.20 \mu \mathrm{~s}$ | $0.20 \mu \mathrm{~s}$ | 0.20رs | $0.20 \mu \mathrm{~s}$ | 34V | $4.0 \Omega$ |
| 13.2A | $0.20 \mu \mathrm{~s}$ | $0.20 \mu \mathrm{~s}$ | $0.25 \mu \mathrm{~s}$ | $0.25 \mu \mathrm{~s}$ | 33 V | $2.5 \Omega$ |
| 20.6A | 0.20us | $0.20 \mu \mathrm{~s}$ | $0.30 \mu \mathrm{~s}$ | $0.30 \mu \mathrm{~s}$ | 31 V | $1.5 \Omega$ |
| 43.4A | $0.20 \mu \mathrm{~s}$ | $0.20 \mu \mathrm{~s}$ | $0.30 \mu \mathrm{~s}$ | $0.30 \mu \mathrm{~s}$ | 26V | $0.6 \Omega$ |
| Maximum voltage drop across the filter is 0.04 volts at 43.4 amperes. |  |  |  |  |  |  |

## Hub Material NEMA-LE

A comparison test of the tapped screw-thread strength of NEMA-LE was made with aluminum and magnesium, using standard tapped holes and Helicoil inserts.

A stainless steel 4-40 screw was inserted in the thread to be tested, leaving sufficient clearance under the screw head to mount the sensor of a dialindicator micrometer. Compression force was applied to the screw through a Dillon force gauge.

Table 7-III
Thread strength test results (averaged).

| STD. Thread | NEMA-LE | Magnesium | Aluminum |
| :--- | :---: | :---: | :---: |
| Force <br> Deflection | 725 lb | 900 lb | 1600 lb |
| 0.017 in. | 0.014 in. | 0.026 in. |  |
| Heli-coil <br> insert |  |  |  |
| Force | 600 lb | 1000 lb | 1500 lb |
| Deflection | 0.016 in. | 0.020 in. | 0.029 in. |

Part of the deflection indicated was, of course, due to compression of the test screw. Most of the tests terminated in a shearing of the top two threads of the various materials. These forces represent the thread fallure of the material, and not the maximum allowable force.
7.3 Salle

Four triangular-shaped salls are used for orientation and stabilization of the spacecraft. Each of these has its vertex located between the electronic compartments, where it is attached to its associated pitch-drive mechanism. The presently-considered sail is fabricated with aluminized mylar which is stretched between two longerons of prestressed spring-steel tape and stiffened by a cross-frame rod at the extremity (base).
Each sail has a vertex angle of $14^{\circ}$ and an area of about $0.14 \mathrm{~m}^{2}$, for a tctal sail area of $0.56 \mathrm{~m}^{2}$. With the flight position of the salls determined by a rearward (aft) canting of $\mathbf{2 6}{ }^{\circ}$, (from the perpendicular to the spacecraft's spin axis), the total projected zero-sun-angle area would be about $0.47 \mathrm{~m}^{2}$.

During the launch phase, each of the sails is stored in a tubular configuration adjacent to the rear of the radiator. The storage of the sails during launch is provided to insure against physical damage from centrifugal loading caused by spinup and despin of the launch vehicle and the spacecraft. Thermal protection from aerodynamically-generated heat is provided, since all exposed aluminized myiar is attached to the helixed longeron which will act as a heat sink.

Sail storage is accomplished by winding the sail from the base toward the vertex on a $1 / 1 / 4 \mathrm{in}$. diameter pipe in the rearward (aft) direction. This action causes the longerons to form an overlapping, bifilar helix spring, which has sufficient stored energy to deploy a test sail to the flight position against against a 1 g load.

The aspect sensor control logic will provide a negative pitch command to the sails when the spacecraft's spin axis approaches a small angle to the sun line, an action that is principally provided to prevent the spacecraft's spin rate from approaching too close to zero. The secondary benefits from the negative pitch control are: the obviation of design concern in establishing an absolute mechanical zero sail-pitch angle, and the necessity of maintaining a wrinklefree sail surface, either of which could cause residual torque errors if the sails were to be positioned in a suppysed zero-torque situation.

## 7. 3.1 Sall Drive Mechanism

The design problems associated with the sall drive mechanism are centered about its control syatem and the all-pitch drive motos. Three typen of control candidates are possible: one is a proportional system which provides sall-pitch corrections in proportion to the pointing and/or apin error. A second type is the bang-bang approach, a system which delays sall motion until a pre-set minimum or maximum angle or rate is detected, and then applies maximum pitch movement. The third system is a combination of the two, in which bang-bang methods are used when large angle or rate errors are involved, and proportional control is applied when small errors are to be corrected.

In the engineering models of the spacecraft, the design concern for controlyatem reliability places a priority emphasis on the simple approach, which would be an "on-off" concept, or the bang-bang system.

Once the spacecraft has accomplished its initial sun orientation, torques caused by minor geometric unsymmetries could still spin-up (or spin-down) the craft, with a resultant pointing error being induced. Rather than wait for the pointing error to reach some pre-set maximum angle before applying a corrective pitch to the sail, an immediate small pitch could be applied, which indeed, might have the effect of generating an exact counter-torque (or nearly so) that would have the general tendency of minimizing the total number of sail-pitch adjustments required during the mission.

The combination of a bang-bang control, which is designed to override the proportional control, seems to provide a system that is reasonably simple, a redundancy for insurance, and has a built-in capability of autogenerating a near-equaliberatory spacecraft geometry.

## Stepping Motor

Possible design candidates for the stepping motor which will be used for sallpitch motion are divided into two categories, both of which have been success fully utilized in space application.

The first is the type which is constructed with a permanent-magnet armature, and has a sequentially-coded pulse format applied to the field, which incrementally causes a rotation of the field, with the resultant rotation of the magnetic armature. These motors are avallable in a variety of physical sizes and output torques, but require an electronic system to provide the various pulse-code formats for stepping the motors in either direction.

A second type of stepping motor is the solenoid-activated device which has a
mechanical linear-to-rotary converter. Two types of convertera are used, one of which is ratchet and pawl, the other a ball and wedge-ghaped detent design. These motora have a high torque-to-weight parameter, but are limited to a monodirectional rotation. To compensate for this restriction, the ball converter can use two motors in a back-to-back configuration to provide the bi-directional capability, but the ratchet-and-pawl type requires an auxillary solenoid to provide a direct-gear transfer in the driven gear train to accomplish a rotational ditection change.

A consiferation of the advantages and disadvantages of all these motor types makes the back-to-back, solenoid-activated ball and detent converter to be most favorable for selection. Typical of these motors is the ledex Size 11, which was redesigned for space and used on Surveyor to drive the solar panel arrays and the earth-pointing antenna.

Although the gearing and bearings on the sail-drive mechanism are designed to help prevent an accumulation of friction and space-induced cold welding, the design simplicity of a high-torque output solenoid appears to have a greater, more prolonged capability of overcoming vacuum-induced stiction.

The proposed sail drive is shown in Fig. 7-5. A commercial version of the stapper motor is shown (Ledex 213227-029), which drives a 12.44-to-1 reduction gear assembly through two non-slip, low-friction belts.

### 7.4 The Spacecraft's Mechanical Sequence at Injection

Timing apparatus associated with the Scout's fifth stage provides the electrical ignition of the BE-3, which has a 10-second burn time; and after an appropriate delay (possibly several minutes) the central explosive bolt is fired and sheared, causing a spring-forced separation of about $3 \mathrm{ft} / \mathrm{s}$ velocity between the spacecraft and the adapter-motor assembly.

The spin rate of the total assembly up to this point is in the approximate range of 180 to 200 rpm . In order to help minimize spacecraft tip-off during separation and to despin known moments of inertia, this spin rate is maintained until after separation, when the mechanical sequence of despinning and deployment will take place.

Design calculation and demonstrative models of the separation sensors, despin and deployment hardware places the initial weight of these devices in the 500 to 600 gm range. Of this total weight, about $68.2 \%$ is associated with sail and antenna restrainers and release hardware, $22.7 \%$ with the despin assembly, and $9.1 \%$ with the separation detectors.


Fig. 7-5 Sall crive mechanism.

Figure 7-6 shows a schematic representation of the separation sensors (see 6.3.2.), which are spring-loaded pin-pullers whose linear motion upon separation releases the despin masses. When the despin masses have moved from a tangential to a radial position (end of despin), their release is accomplished by a combination of their outward radial force and the now-unrestrained despin anchor.

The departure of the despinner is followed $b_{j}$ the activation of the two sail pin-pullers, which unlock the sall-restraining cables and covers, allowing deployment of the sails to occur.
The antennas which are displaced from the sail axes by $45^{\circ}$ are restrained from deploying by a wedge of material on the despin wire. The wedge is held in position during despin by the tangential force on that wire, which is removed by the despinner moving radially about $45^{\circ}$ before final spacecraft despin has occurred. The time difference between the deployment of the antennas and the sails will be small, due to the fact that, although the spacecraft at this time moves through $45^{\circ}$ at a low angular rate, the despin wire moves from the tangent to the radial position ( $00^{\circ}$ ) with a significantly higher angular rate.

### 7.4.1 Possible Separation Dynamics Compensation Requirements

The mechanical sequence outlined in Sec. 7.4 is based on the premise that the booster motor-spacecraft assembly has a slow rate of cone angle buildup during the long coast period (delay time to separation).

If the cone-angle buildup is rapid and approaches $30^{\circ}$ during the coasting period after the fifth-stage burnout, the mechanical sequence of initiating despin upon separation becomes a marginal safety situation, For the danger of collision between the deploying spacecraft sails and the coning adapter (transtage) is increased, due to the tangential separation velocity (caused by the coning), on the spacecraft approaches its axial separation velocity. This action would produce a separation angle approaching $45^{\circ}$, and with despin and deployment occurring in 0.5 to 0.7 second after separation, the sail tips could be below or in the plane of the coning transtage.

Similarly, the long-coast phase from burnout of the fifth stage to spacecraft separation appears to be capable of causing a large deviation in the nominal spacecraft spin axis-sunline angle.

Tentative solutions to these dynamically-produced problems include:


## More rigid control of cone-generating parameters. Shorter coast period. <br> Increased separation-spring energy. <br> Sun-sensed activation of separation. <br> An immediate transfer of spin to the major moments axis of the boosi"s-adapter assembly after separation occurs. <br> A delay in activating despin of the spacecraft.

If engineering tolerances preclude a reasonable tightening of alignment parameters which are involved with generating an excessive coning angle of the final launch vehicle assembly, the sun-sensed activation of separation may be utilized. This would be simply the logically ANDED separation signal from the timer and the amplified output of a photoelectric device to cause separation to occur in the most favorable sector of the cone.

The transfer of spin to the major moment axis of the booster-adapter assembly may be accomplished by providing a despin system which has one-half of the despin device mounted on each side of the center of mass of the assembly.

The separation-to-despin delay can be accomplished for about the same weight ( 50 gm ) as the direct-acting mechanical system. Power would be supplied to an $E$-cell timing circuit by a separation-activated switch, with the amplified output of this circuit providing the ignition energy to a pair of pyrotechnic dimple-moter activated pin-pullers which would release the despin weights.

Initial tests of hardward built to this design concept indicate a total energy requirement of about 0.2 joule to provide a five-minute delay, plus the pyrotechnic release which de:nonstrate load-release capability in excess of twenty pounds.

### 7.5 Weight Distribution of tie Spacecraft

Many of the spacecraft parts are multi-functional, therefore could be listed in any one of several categories. But in this distribution analysis, each part was included in an area considered representative of its primary function:

| Sub-Assembly | Weight (gm) | Percentage |
| :---: | :---: | :---: |
| Main Electronics | 3995.8 gm | 31.2 |
| Power | 2636.0 gm | 20.6 |
| Thermal and/or Mechanical | 3259.8 gm | 25.5 |
| Orientation | 2904.6 gm | 22.7 |
| TOTAL | 12,796.2 gm | 100.0 |

In the derivation of these weights, the following contingencies were used:

| Sub-assemblies considered frozen | $=3 \%$ |
| :--- | :--- |
| Sub-assemblies partially completed | $=10 \%$ |
| Sub-assemblies in design | $=25 \%$ |

The breakdown of the sub-assembly weight is as follows:
Main Electronics

| RF Sub-system | $1880.0 \mathrm{gm}+10 \%$ | $=2068.0 \mathrm{gm}$ |
| :--- | ---: | :--- |
| Digital Sub-system | $383.1 \mathrm{gm}+10 \%$ | $=421.4 \mathrm{gm}$ |
| Intraconnections | $351.2 \mathrm{gm}+3 \%$ | $=361.7 \mathrm{gm}$ |
| Chassis | $897.8 \mathrm{gm}+3 \%$ | $=924.7 \mathrm{gm}$ |
| Antennas | $200.0 \mathrm{gm}+10 \%$ | $=220.0 \mathrm{gm}$ |
| $\quad$ |  |  |

## Power

| Solar Colls | (calculated) | 516.0 gm |
| :--- | ---: | ---: |
| Converters | $347.6+25 \%$ | 434.5 gm |
| Storage Capacitors | $1636.0+3 \%$ | 1685.5 gm |
| TOTAL |  | 2636.0 gm |

Thermal and/or Mechanical

| Platform-Radiator | (weighed) | 2034.8 gm |
| :--- | ---: | ---: |
| Sail Release | $373.5+3 \%$ | 384.7 gm |
| Despin | $175.0+3 \%$ | 180.3 gm |
| Balance Weights | $600.0+10 \%$ | 660.0 gm |
| TOTAL |  | 3259.8 gm |

Orientation
Aspect Sensor
Sails and Drive

| $755.4+25 \%$ | 944.3 gm |
| ---: | ---: |
| $1903.2+3 \%$ | 1960.3 gm |
|  | 2904.6 gm |

These listed weights are shown graphically in Fig. 7-7

### 7.6 Moments of Inertia

The moments of inertia of the spacecraft have been calculated for the spin and yaw-pitch axes for both the deployed and undeployed states. The numerical values obtained from the use of weights given in Sec 7.5 are listed in the table below.

Fig. 7-7 Weight distribution.

Table 7 -IV
Results: moments of inertia.

| Part | $\begin{aligned} & \text { Spin }\left(\times 10^{-6} \mathrm{gm-cm}^{2}\right) \\ & \text { Deploy. Not Deploy. } \end{aligned}$ |  | Pitch, Yaw ( $\times 10^{-6} \mathrm{gm}-\mathrm{cm}^{2}$ ) Deploy. Not Deploy. |  |
| :---: | :---: | :---: | :---: | :---: |
| Platform | 025190 | see deploy. | 0.19600 | see deploy. |
| Radiator | 0. 39100 | , | 0.32050 | " |
| Compart. Walls | 0.09398 | 1 | 0.05485 | " |
| Compart. Covers | 0.01379 | " | 0.01334 | " |
| Electronics | 0.64170 | " | 0.39146 | " |
| Hub | 0.01321 | see deploy. | 0.01579 | see deploy. |
| Contents of llub | 0.01250 | " | 0.02651 | " |
| Damper | 0.00292 | " | 0.00786 | " |
| Capacitors | 0.52036 | " | 0.30015 | " |
| Antennas | 0.23963 | 0.12696 | 0.09537 | 0.07950 |
| Sail Drive | 0.28084 | see deploy. | 0.15737 | see deploy. |
| Sail | 2.02748 | 0.32293 | 1.38889 | 0.18257 |
| Despin Mech | 0.08000 | see deploy. | 0.03920 | see deploy. |
| Sail. Depl. Mech | 0.07220 | " | 0.00320 | 1 |
| TOTAL | 4.642 | 2.824 | 3.011 | 1.789 |

## CHAI-TER 8

## ATTITUDE CONTROL SYSTEM

8.1 Introduction

The Sunblazer is a spinning, oriented spacecraft which faces the sun for the purpose of transmitting a radio wave through the corona.

Orientation of the spacecraft is desirable for several reasons:
1a. Solar cell power is maximized when all the cells are in a plane facing the sun. Solar cell impedance matching and thermal variations are also minimized when the solar input is nearly static, as it would be for an oriented spacecraft.

1b. Thermal design of spacecraft experiment and electronics packages is simplified when their thermal surroundings are not subjected to pseudo-random solar inputs.

1c. Antenna patterns can be maximized for a spacecraft with an axis oriented toward the sun. For transmission through the solar corona between two points in the ecliptic plane (spacecraft to earth), system performance can be improved by maximizing the antenna pattern in the direction of the sun. With a spinning spacecraft, the antenna pattern would ideally by symmetric about the spin axis, with gain versus $\theta$ roughly inversely proportional to distance-squared (as a function of $\theta$ ).

A spinning spacecraft is desirable for several reasons:
2a. A reliable passive mechanism which would despin the spacecraft down to a spin rate on the order of one revolution per hour is not known. However, spin rates on the on the order of $1-10 \mathrm{r} / \mathrm{min}$ are believed feasible.

2 b . Disturbances of micrometeorites, or of other momentum impulses to the spacecraft spinning at $1 \mathrm{r} / \mathrm{min}$, will be hundreds
of times smaller than to a static spacecraft. For the Sunblazer, based on Pegasus data (NASA-TM-X-1316), the probability of a disturbance greater that $2^{\circ}$ to the spinning spacecraft is on the order of $1 \%$ (in one year).
2c. Motion of spacecraft simplifies to precessional mode for spin rates on the order of $1 \mathrm{r} / \mathrm{min}$, making control simpler to achieve than for a slower spacecraft requiring description of motion by complete equations of motion. For the spacecraft ( $\omega \sim 1 \mathrm{r} / \mathrm{min}$ ), nutational motion also simplifies (to the forcefree approximation).

2d. Rotation of the spacecraft makes possible a limited scan of a sensor, or some other device, in a circle about the spin axis. This rotation, and the fact that the spin axis is not pointed exactly at the sun in the normal mode of motion, allow measurement of the angular velocity by an on-board sensor.

Considering the preceding reasons for having an oriented, spinning spacecraft, a list of tentative design objectives for the Sunblazer attitude-control system will be discussed in sections 8.1.1 and 8.1.2.
8.1.1 Orientation of Spacecraft within $10^{\circ}$ of Sun

Twenty-five degrees would be sufficient to ensure roughly $90 \%$ of maximum solar-cell power, but $10^{\circ}$ is desirable to avoid losses in antenna pattern when spacecraft-sun-earth angle is greater than, say, $30^{\circ}$ from conjunction.

### 8.1.2 Spin Rate of Spacecraft Between Two Limits

A lower limit of about $0.1 \mathrm{r} / \mathrm{min}$, which would make the spin rate at least five times larger than the libration rate (as discussed below under the equations of motion), would ensure that the motion is a reasonable approximation to the precessional mode. A high spin rate would be desirable for reasons 2a-2d above, in addition to the increased sensitivity of the motion to torque imbalances at low spin rates. An upper limit of about $19 \mathrm{r} / \mathrm{min}$ would ensure that the pseudo-equilibrim orientation of the spin axis will not exceed roughly $5^{\circ}$ from the sun. These limits assume a precession-torque coefficient of about 0.2 dyn-cm/deg, which would apply to the vane system described below ${ }^{(1)}$.
Spin rates near the lower limit of $0.1 \mathrm{r} / \mathrm{min}$ should occur after the initial phase of acquisition, as illustrated in the solutions below. Spin rates near the upper limit can occur as a result of four causes:

1. Momentum buildup as a result of solar-pressure torque imbalances on the sail atructure (described below). For the typical crude design considered, such motion is likely to build to an asymptotic maximum spin rate of roughly $10 \mathrm{r} / \mathrm{min}$ within about a year.
2. Mechanical fallure of the control system, with the vanes stuck in one position, resulting in a large spin torque. In this case, the whole spacecraft will probably fall.
3. Fallure of a despin mechanism. This could be caused by tumbling of the rocket-payload prior to separation and/or a faulty separation mechanism.
4. Impact of a particle with the spacecraft, followed by conversion of axes (assuming an internal damping mechanism), resulting in a relatively high spin rate about the spacecraft axis of maximum moment of inertia (and axis of symmetry), with the spin axis, in general, not directed at the sun.

In cases 3 and 4, the vehicle would probably stabilize (assuming that the control system were properly activated), but stabilization could take a long time, depending on the residual spin rate.

Generally speaking, spin control is not so critical as $\theta$-control. That is why spin rate is allowed to vary within such wide limits. In fact, for the system described below, it is not felt necessary to control the spin rate actively because 1)the probability of a disturbance may not be great enough to justify the complexity of an independent, active spin-control sub-system, and 2) some disturbances can be corrected for the proposed system, with the help of a passive mechanism, thermal damping ${ }^{(2)}$. Thermal damping should help stabilize the system at very low spin rates, and the vanes should work at very high spin rates. These cases have been studied on a computer to see how wide a margin of stability exists (as a function of system parameters, erior conditions, etc). (See Section 8, 9)

### 8.1.3 Simplicity of Control System

Simplicity of the control system is desirable for the reliability of a spacecraft which is to maintain its stability for a period on the order of one year.

Ideally, the control system would be completely passive (no moving parts and no expenditure of material), and would control both $\theta$ and spin rate to desired values. The control system which is described below is semi-
pasaive (no expenditure of material) and can only be aaid to control 0 within limits (it provides practically no control of apin rate)..

### 8.2 Solar Pressure Stabilization

The use of solar pressure as a means for stabilizing a solar-oriented atellite has definite advantages:

1. The solar radiation flux has an inherent reference direction, toward which the satellite is to be directed.
2. Solar radiation provides a practically unlimited source of momentum. Completely passive stabilization systems based on solar pressure have been studied before ${ }^{(1),(3)}$. (See Section 8.3 for systems based on solar momentum.)

Solar radiation is considered to be a flux of energy parallel to the line from the spacecraft to the sun. (The finite angular size of the sun, and the slight convergence of the solar rays, will not be considered here.) Solar radiation can be considered a flux of momentum, which is absorbed and reflected according to the laws of geometrical optics (4). Given a flux of energy of I watts $/ \mathrm{m}^{2}$ in a given direction, the flux of momentum is

$$
\begin{equation*}
p=\frac{I}{c} \tag{8,1}
\end{equation*}
$$

(MKS)
across a surface perpendicular to the given direction, where $c=$ velocity of light (MKS). If the flux of energy is reflected from a surface perpendicular to the given direction, then the radiation pressure on the surface is

$$
\begin{equation*}
p_{0}=\frac{2 I}{c} \quad(N \cdot K S) \tag{8,2}
\end{equation*}
$$

If the flux of energy is reflected from a surface with a normal making an angle $\Theta$ with the flux direction, then the radiation pressure on the surface is (see Fig. 8-1)

$$
\begin{equation*}
p_{r}(\theta)=\frac{2 I}{c} \cos ^{2} \theta \quad(M K S) \tag{8.3}
\end{equation*}
$$

(This expression is derived by reflecting the component of momentum perpendicular to the surface.)
Assuming a numerical value for $I=1380 \mathrm{watt} / \mathrm{m}^{2}$, (5) a numerical value for $p_{0}$ can be calculated as

$$
\begin{equation*}
p_{0}=0.92 \mathrm{dyn} / \mathrm{m}^{2} \tag{8.4}
\end{equation*}
$$



Fig. 8-1 Radiation pressure geometry.

By reflecting and absorbing this momentum at wurfaces which ure geometrically asymmetrical with reapect to a plane made by the sun line and an axia of the spacecraft, it is pomalble to produce torques which act in that plane perpendicular to that axis and along that axis. These torques can be used to act on a apacecraft which is spinning about an axim, respectively to precess that axis toward the sun and to change the angular velocity about that axis.

These torque components exist in all the systems which have been studied and are described below. Incidently, the force components have been worked out for the vane suntem described below, and the effect on the orbital parameters of the saiellite can easily be estimated.

### 8.3 Basic Stabilization Sygtems

Several stabilization systems utilizing solar pressure have been studied, and are mentioned in the following:

1. Totally pasaive, librating system with weathercock sail, using a thermal damper and/or a mechanical damper. Problems of total despin and of sensitivity to disturbances eliminated this system.
2. Semi-passive, librating system with movable sall. Problems of control-systein requirements and of sensitivity to disturbances eliminated this system.
3. Totally passive, spinning system with reradiative damper ${ }^{(2)}$ Problems of deapin to a low rate and of longoterm spin control eliminated this system.
4. Totally passive, spinning system with simple, asymmetrical sail and no spin control ${ }^{(1)}$. Longterm spin control is a problem with this system.
5. Totally pasaive, spinning system with a compound, asymmetrical sail providing angle and spin control. Difficulty of constructing this sail and instability of syatem for angles greater than $90^{\circ}$ eliminated this system ${ }^{(3)}$.
6. Totally passive, spinning system with a compound, asymmetrical providing angle and spin control, and an inaproved, ringtype, reradiative damper providing additional angle stability. Difficulty of construction, instability for angles rifeater than $90^{\circ}$, and the relatively low absolute efficiency of such a system
have eliminated this system in favor of the semi-passive vanetype system, (At this point, absolute efficiency can be defined as maximum erecting torque/total sail area, although it should be realized that a final evaluation of performance should involve total erecting tire, which is a complicated function of erecting and spin torquis, moments of inertia, and initial and final conditions).
7. Semi-passive, spinning system with movable vanes providing angle control. Spin is not controlled, but is expected to stay within reasonable limits. This system has the big advantages of being much faster than the passive systems and of providing stability out to large angles, near $180^{\circ}$. This system has the disadvantages of requiring a mechanical device to rotate the vanes, which will probably have to be activated twice (one cycle, on-off) in a period of several weeks. It also lacks a spincontrol subsystem, which would be desirable in the improbable occurrence of a momentum disturbance caused by a micrometeorite or by outgassing of a component, etc.

All of these systems have been investigated. However, only the Falcovitz sail $(5,6)$ and the vane-type sail (7) have been studied in sufficient detail to show feasibility for the Sunblazer application. The Falcovitz sai ${ }^{1}$ has been referenced, and the vane system is presented below ${ }^{(3)}$.

### 8.4 Special Sunblazer Conditions

Special conditions peculiar to the Sunblazer spacecraft place special constraints on the design of the attitude-control system. Such conditions are initial conditions, tracking requirements, orbit conditions, and weight and volume requirements.

### 8.4.1 Initial Conditions

Initial conditions are determined by the lannch trajectory and spin history of the rocket. In the absence of more accurate information, it is assumed that the initial angle and spin rate are respectively $60^{\circ}$ from the sun and 200 $r / r$ nin (referred to the major axis of the spacecraft), and that the initial aingle between the spacecraft axis and the sun line may vary between $45^{\circ}$ and $90^{\circ}$. Another initial condition which is assumed is that the spacecraft is also nutating with a half-cone angle of $10^{\circ}$. However, it is realized that, because of an attempt to coast after burnout with the last-stage rocket
motor attached to the spacecraft, this angle could increase to near $90^{\circ}$ as the spin of the motor-spacecraft converts to tumble; in this case, a despin mechanism on the spacecraft would almost certainly fail (if the spacecraft worf tumbling). Another uncertainty in the initial conditions could be caused by an attempt to despin the motor-spacecraft prior to separation; because of uncertainty in the burnout moment of inertia, a sizable error could be caused by a yo-yo type of despin mechanism. Repeating, the nominal conditions are assumed to be an angle of $60^{\circ}$ from the sun, nutations with a half-cone angle of $10^{\circ}$, a spin rate of $200 \mathrm{r} / \mathrm{min}$.

The inital conditions are important to the initial phase of stabilization because, for the vanes system: 1) stabilization time increases very rapidly as the initial angle increases; 2) stabilization time, to a first approximation, varies linearly with the initial spin rate; and 3) nutations, or more seriously, tumbling, can confuse a sun sensor or foul up a despin mechanism.

### 8.4.2 Tracking Requirements

Tracking requirements make it desirable that the Sunblazer orient toward the sun and start transmitting as soon as possible, in order to make an estimate of the orbital parameters as accurately as possible while the signal-tonoise ratio is stil relatively high. Tracking requirements should not be very critical for the experiment, because the beamwidth of the pruposed antenna should be wide enough that the angles from the orbit to the earth should not need to be known to better than about $0.1^{\circ}$. However, accurate tracking may be desirable to evaluate the performance of the launch vehicle.

### 8.4.3 Orbit Conditions

Orbit conditions do not significantly affect the dynamical motion of the spacecraft. It can be shown that the dynamical trajectories of the spacecraft's rigid-body motion are relatively invariant to motion around the orbit. Assuming that the spacecraft is launched at 1 AU , we can neglect the variation in initial solar intensity caused by the slight eccentricity of the earth's orbit. The most significant effect on orbit conditions is the change in spacecraft temperature distribution, which will not significantly affect the spacecraft rigid-body dynamics.

### 8.4.4 Weight and Volume Requirements

Weight and volume requirements place an upper limit on the size of the atti-tude-control system. For the vanes system, it is felt that the whole system could be built at less than 200 g and could be stowed within the available
nosecone space without too much loss in performance. For the Falcovitz system (mentioned above), there are critical tolerar., on the design of the sail, possibly requiring a more rigid structure weighing around 500 g , probably requiring storage of the whole sall without folding and deployment. But for the vanes system, weight and volume requ!rements are not felt to be critical.

## Basic Description of Vanes System

The vanes system is a semi-passive attitude-control systern utilizing the effect of solar pressure on the vanes to produce torques which will align the spacecraft spin axis toward the sun. The system is called semi-passive because 1) it does not expend any material, but 2) it does require the mechanical motion of the vanes to change the torques as the angle between the spacecraft spin axis and the sun exceeds certain limits.

As presently conceived, the vanes system (shown in Fig. 8.2 below) consists of several parts:

1. Four triangular vanes, attached to the spacecraft at points spaced $90^{\circ}$ around the rear of the spacecraft cylindrical radietor. The vanes are to be made of mylar, attached to a furlable structure.
2. Four stepping motors, to rotate the vanes about their longitudinal a:-s. For the following anglysis, the vanes are assumed to have three positions: $+35.26^{\circ}, 0^{\circ}\left( \pm 1^{\circ}\right),-35.26^{\circ}$. The position $35.26^{\circ}$ was found to maximize the spin and erecting torques. The $0^{\circ}$ position is considered to have some error because of imperfect design and construction of the vane and stepping mechanism.
3. A control sub-system, to tell the vanes when to switch between the three positions. The vanes are assumed to work synchronously, because,tor a fast-spinning spacecraft, the advantages of switching them asynchronoulsy would require switching each vane twice for each revolution of the spacecraft (on the order of twice per minute) instead of once every few days, which could make the mechanical system less reliable, in addition to requiring a more complex control logic than for the proposed system. Such asynchronous switching would have very definite advantages if the spacecraft were trapped in a very low-spin mode, but the probability of this occurring is not felt to be high


Fig. 8-2 Configuration of stabilizing vanes.
enough to justify the additional complesity.
4. A sensor, to measure the angle between the spacecraft axis of symmetry and the sun line, and a sensor to measure the spin rate of the spacecraft. These measurements are used as inputs to the control circuits which decide when to $s$ witch the vanes.

The vanes are controlled as a function of angle, not as a function of spin rate, with one exception: if the spin rate decreases, heading toward the danger zone near $0 \mathrm{r} / \mathrm{min}$, it would be desriable to trim the vanes. Trimming by increments on the order of a fraction of a degree to provide a positive spin torque and an increasing spin rate, will insure the normal mode of motion discussed below and in Section 8.9.

Normal operations of the attitude-control system can be divided into two modes: initial and long term. Both modes can be controlled by the same logic. The common logic, which controls the vanes of the proposed system for all modes, is very simple: If the spacecraft-sun angle is greater than an upper limit, say $10^{\circ}$, then the vanes turn to $\pm 35.26$ (depending on the spin direction). If the spacecraft-sun angle is less than a lower limit, say $2^{\circ}$, then the vanes go to $0^{\circ}$. However, if the spacecraft spin rate starts to decrease, then the vanes should be trimmed to provide a positive spin torque.

Given this logic, the typical motion of the spacecraft is summarized as follows: Initially, the spacecraft is at a large angle (assumed $\sim 60^{\circ}$ ), spinning fast (it is assumed that after despin by the MIT mechanism, the : nin rate would be about $20 \mathrm{r} / \mathrm{min}$ ) therefore, the vanes would be pitched to a full $-35.26^{\circ}$. It can be shown that the initial motion can be approximated by a functional relationship between $\theta$ and $\omega$ (angle and spin rate). This relationship neglects angular velocity of the spacecraft in its orbit. With the vanes on, $\theta$ and $\omega$ will both decrease approximately along this curve until $\theta$ is less than a lower limit $\left(2^{\circ}\right)$. Then the vanes would be nulled (pitched to $\sim 0^{\circ}$ ). For the initial conditions given, with four vanes each $25 \mathrm{~cm} \times 100 \mathrm{~cm}$ canted at $70^{\circ}$ to the spin axis, this initial phase is estimated to take about 13.7 days. After "nulling" the vanes, some slight error in the vanes' twist angle is assumed, resulting in error torques in the erecting and spin directions. To simplify matters, so that the spin rate does not go through zero, it is assumed that the "null" error is opposite in sense to that of the initial pitch of the vanes. In this case, the spin rate and angle of the spacecraft will start to increase and will continue to increase until the angle exceeds the upper limit $\left(10^{\circ}\right)$. For a pitch error of $1^{\circ}$, the time required for this process will initially be several weeks; however, as a result of long term stabilizing effects, which tend to increase the average spin rate, the spin-up period will increase to several months by the end of a year.

Another effect which has not been mentioned above is the change in the nutation cone angle. Nutations can increase as a result of radiation torques, and can decrease as a result of thermal damping and of internal dissipation, possibly by an onboard damper (assuming a nutation about the axis of maximum moment of inertia). The effect of a static torque on nutations has been investigated by Peterson (Section 8.11). The effects of reradiative torques and of mechanical damping have also been investigated (8.11.5, 8.12).

This description of the predicted performance of the attitude-control system is derived in part by analytical methods, in part by numerical methods, based on the equations of motion ior a rigid body, which are derived in the following sections.

### 8.6 Elementary Dynamics

A most elementary understanding of the precessional motion of a spinning, rigid body can be obtained from the treatment given in a basic physics book. (6) (See Fig. 8-3) A force couple acting on a given axis produces a torque which tends to precess the axis about which the body is spinning in the direction given by the torque.
For the Sunblazer vanes system, when the vanes are pitched to $\sim 0^{\circ}$, as shown in Fig. 8-4, the effect of solar radiation pressure is shown as a force acting on the vane in the plane defined by the sun line and the spin axis of the spacecraft. The force is perpendicular to the vane, which is assumed to be pitched at $0^{\circ}$, such that the force is also in the plane of the sun and the spin axis. As is well known from elementary statics, a force acting on a body can be resolved into a lorce acting at the center of mass of the body (which we can neglect at the present time, since it accelerates the body linearly) and a torque acting about the center of mass. For the picture of the vane, the torque is acting about an axis perpendicular to the plane of the sun and the spin axis, with the magnitude of the torque given by the product of an effective moment arm (not exactly the same as a moment arm which could be derived by a geometric construction on the picture, which is only schematic) times the magnitude of the force, such that the spin axis of the spacecraft would tend to precess in a direction perpendicular to the plane of the sun and the spin axis. The rate of precession is derived ${ }^{(6)}$ to be



Fig. 8-3 Simple precession.


Fig. 8-4 Force on vanes.

In the absence of any other torques, the spin axis would simply precess uniformly forever about the sun line. Because the torque is perpendicular to both the spin axis and the sun line, the angle between the spin axis and the sun line would never change, so that the spin axis would always lie in the surface of a cone centered on the sun line. Thus this motion is sometimes described as "coning".
For the Sunblazer vanes, when the vanes are pitched to $-35.26^{\circ}$, as shown in Fig. 8-4, the effective force on the vane (assumed perpendicular to the vane due to reflected solar radiation) is no longer in the plane of the spin axis and the sun line, so that there are components of force (force is considered as a vector) which produce torques acting in the plane of the spin axis and the sun line. The component of torque asting along the spin axis will tend to change the magnitude of the langular velocity, while the component of torque acting in the plane perpendicular to the spin axis will tend to precess the spin axis toward (or away from) the sun. The precession rate, or erection rate is given by Eq (8.5).

These two components of torque are very useful in controlling the attitude and the spin rate of the spacecraft. These torque components are present, to some degree, in all the control systems which have been studied and are mentioned above. The torque components for the vanes system will be derived below.

### 8.7 Vector Dynamics

A second, more satisfactory description of the precessional motion of a spinning, rigid spacecraft, is given in terms of the vector equation of motion,

$$
\begin{equation*}
\dot{\mathrm{L}}=\overrightarrow{\mathrm{N}} \tag{3.6}
\end{equation*}
$$

where $\vec{L}=$ angular momentum vector, $\vec{N}=$ torque vector, and

$$
\begin{equation*}
\vec{L}=\tilde{I} \vec{\omega} \tag{8.7}
\end{equation*}
$$

I is the inertia tensor. However, if the body is spinning about its axis of maximum moment of inertia, $\Gamma_{U}$, then $\tilde{I} \vec{\omega}$ is simply a vector of magnitude $I_{3} \omega$ in the direction of the spin axis ${ }^{(7)} \quad$ The equation of motion holds in any inertial frame; for a rotating frame, an additional term will be added later.

Keeping in mind the vector angular momentum $\overrightarrow{\mathrm{L}}$, as shown in Fig. 8-5, tise effect of the torque vector $\overrightarrow{\mathrm{N}}$ acting according to Eq (8.6) is to cause $\overrightarrow{\mathrm{L}}$ to move along a curve whose tangent is always in the direction of $\overrightarrow{\mathrm{N}}$. The effect


Fig. 8-5 Angular momentum .
of the component of $\vec{N}$ in the direction of $\vec{i}$ is to change the magnitude of $\vec{L}$, while the components of $\vec{N}$ perpendicular to $\vec{i}$, will tend to precess $\overrightarrow{\mathrm{L}}$ toward (or away from) and around the sun. The component of $\vec{N}$ in the direction of $\overrightarrow{\mathrm{L}}$ will be called the apin component; the component of $\vec{N}$ perpendicular to $\overrightarrow{\mathrm{L}}$, and in the plane of the sun and $\vec{L}$, will be called the erecting component; and the component perpenducular to the plane of the sun and $\overrightarrow{\mathrm{B}}$ will be called the precession component. The qualitative effect of these torques has been explained in Section 8.6 in reference to Fig. 8-3. One fact which will be obvious, once the average torques $\vec{N}$ have been derived as a function of $\theta$ for the vanes, is the fact that since the erecting component of $\vec{N}(\theta)$ will be shown to be positive for all 0 , the spacecraft will always erect toward the sun (showing stability for this mode of motion).

For the purpose of obtaining a more exact description of the motion, we will describe the motion in terms of Euler angles ${ }^{(8)}$ (see Fig 8-6), which are basically three angles specifying: 1) the angle, $\theta$, between the spin axis (or $\overrightarrow{\mathbf{L}}$ ) and the sun line; 2) a precession angle, $\phi$, around the sun line (for our purposes, the reference zero for $\phi$ will be defined to be in the plane of the spacecraft orbit); and 3 ) a rotation angle, $\psi$, about the spin axis (this angle we will ignore, but its derivative $\psi$ will be equated to $\omega$, the spin rate). We should like to transform the vector equations of motion for $\phi, \theta, \dot{\psi}$, so that thesequanities can be solved for as a function of time on a digital computer.

Before deriving the individual equations of motion, it will be desirable to add a term to the right hand side of Eq (8.6), representing the effect of a rotating frame of coordinates (such as the rotation of the sun line during the orbit and, more importantly, the rotation of coordinates which will be necessary to refer our equations to body-centered coordinates, since our torques $\overrightarrow{\mathrm{N}}$ will be derived in body coordinates); and to put the equations in a completely rigorous form, so that the subsequent derivation will at least have a firm reference set of equations as a basis.
Equation (8,6) referred to a coordinate system rotating at a vector velocity $\vec{\Gamma}_{0}$ is changed to ${ }^{\text {( }}$ )

$$
\begin{equation*}
\dot{\vec{L}}=\overrightarrow{\mathrm{N}}-\vec{\Gamma}_{o} \times \overrightarrow{\mathrm{L}} \tag{8.8}
\end{equation*}
$$

Now given this equation, we would like to put

$$
\begin{equation*}
\vec{\Gamma}_{0}=\vec{\Gamma}_{1}+\vec{\Omega} \tag{8.9}
\end{equation*}
$$



Fig. 5-6 Euler angles.
where $\vec{\Gamma}_{1}$ refers the equations to body coordinates, where $\theta$ and $\phi$ are not varying, and $\vec{\Omega}$ is the angular velocity of the spacecraft in its orbit. These rotations essentially refer the equations back to an inertial frame of coordinates. In termis of Euler angles, these vectors are given by ${ }^{(10)}$

$$
\begin{align*}
& \vec{L}=\overrightarrow{i I}_{1} \dot{\theta}+\vec{j} I_{1} \dot{\phi} \sin \theta+\overrightarrow{k I_{3}}(\dot{\psi}+\dot{\phi} \cos \theta)  \tag{8.10}\\
& \vec{\Gamma}_{1}=\vec{i} \dot{\theta}+\vec{j} \dot{\phi} \sin \theta+\vec{k} \dot{\phi} \cos \theta  \tag{8.11}\\
& \vec{\Omega}=\Omega \overrightarrow{(i} \cos \phi-\vec{j} \cos \theta \sin \phi+\vec{k} \sin \theta \sin \phi) \tag{8.12}
\end{align*}
$$

where 1) $\vec{i}, \vec{j}$, and $\vec{k}$ are unit vectors respectively in the precession, erecting, and spin directions. These vectors are in the body; however, they do not rotate with $\psi$, as shown by $\vec{\Gamma}_{1}$; and 2) we have also assumed that the body is inertially axi-symmetric (i.e. $I_{1}=I_{2}$ ); this will simplify our equations (and the solution for nutations). Performing the operations indicated by Eq ( 8,8 ) leads to the results given below:

$$
\begin{align*}
& \overrightarrow{\mathrm{L}}=\overrightarrow{\mathrm{iI}} \mathrm{I}_{1} \ddot{\theta}+\vec{j}_{\mathrm{j}}^{1}(\ddot{\phi} \sin \theta+\dot{\phi} \dot{\theta} \cos \theta)+\overrightarrow{\mathrm{k}} \mathrm{I}_{3} \frac{\mathrm{~d}}{\mathrm{~d}}(\dot{\psi}+\dot{\phi} \cos \theta)(8.13) \\
& \vec{\Gamma}_{1} \times \vec{L}=\vec{i}\left[I_{3} \dot{\phi} \sin \theta(\dot{\psi}+\dot{\phi} \cos \theta)-I_{1} \dot{\phi}^{2} \sin \theta \cos \theta\right] \\
& \left.+\vec{j}^{[ } I_{1} \dot{\phi} \dot{\theta} \cos \theta-I_{3} \dot{\theta}(\dot{\psi}+\dot{\phi} \cos \theta)\right]  \tag{8,14}\\
& \vec{\Omega} \times \overrightarrow{\mathrm{L}}=\Omega\left[-\vec{i}\left(\mathrm{I}_{3} \sin \phi \cos \theta(\dot{\psi}+\dot{\phi} \cos \theta)+\mathrm{I}_{1} \dot{\phi} \sin ^{2} \theta \sin \phi\right)\right. \\
& -\vec{j}\left(I_{3} \cos \phi(\dot{\psi}+\dot{\phi} \cos \theta)-I_{1} \dot{\theta} \sin \theta \sin \phi\right) \\
& \left.+\vec{k}\left(I_{1} \dot{\phi} \sin \theta \cos \phi+I_{1} \dot{\theta} \cos \theta \sin \phi\right)\right] \tag{8.15}
\end{align*}
$$

The important equations of motion, whicin we wanted to derive, are now obtained from the $\vec{i}, \vec{j}, \vec{k}$ components of Eq (8.8) after substituting Eq (8.13), (8.14), (8.15):

$$
\begin{gather*}
I_{1} \ddot{\theta}-I_{1} \dot{\phi}^{2} \sin \theta \cos \theta+I_{3} \dot{\phi} \omega \sin \theta=N_{1}(\theta)-(\vec{\Omega} \times \overrightarrow{\mathrm{L}})_{1}  \tag{8.16}\\
I_{1}(\ddot{\phi} \sin \theta+2 \dot{\phi} \dot{\theta} \cos \theta)-I_{3} \dot{\theta} \omega=N_{2}(\theta)-(\vec{\Omega} \times \overrightarrow{\mathrm{L}})_{2}  \tag{8.17}\\
I_{3} \frac{d \omega}{d t}=N_{3}(\theta)-(\vec{\Omega} \times \vec{L})_{3} \tag{8.18}
\end{gather*}
$$

where $\omega=\dot{\psi}+\dot{\phi} \cos \theta$
(8.19)
and $(\vec{\Omega} \times \vec{L})$ is not transcribed, pending the following simplifications.
There are two cases in which $\dot{\phi}, \dot{\psi}, \dot{\theta}$ are of comparable magnitude: 1) when we are studying high-frequency nutations (as studied by Peterson, see Sec. 8.11), the nutational period is so short that any coupling with $\vec{\Omega}$ can be neglected; and 2) when $\psi$ is very low. If either of these situations exists we have a nearlibrational situation, and the librational period is also short enough to decouple $\vec{\Omega}$ so that in both cases we will assume (low spin rate and nutations)

$$
\begin{equation*}
\vec{\Omega} \times \vec{L}=0 \tag{8.20}
\end{equation*}
$$

In a case when $\dot{\psi} \gg \dot{\phi}, \dot{\theta}$ (precessional $r$ :山e), we can assume $\Omega \ll \dot{\psi}, \dot{\phi}, \dot{\theta}$, resulting in the following simplification (high spin rates):

$$
\begin{equation*}
\stackrel{\rightharpoonup}{\Omega} \times \stackrel{\rightharpoonup}{L}=\Omega\left[-\mathrm{II}_{3} \dot{\psi} \cos \theta \text { sin }-\overrightarrow{\mathrm{I}}_{3} \dot{\psi} \cos \phi\right] \tag{8.21}
\end{equation*}
$$

For the case of hirh $\dot{\psi}$, further simplifications will be made, to $\mathrm{Eq}(8,16)$, (8.17), (8.18), (8.21), In order to derive the precessional equations of motion, which hold for high $\dot{\psi}$, we shall be able to neglect $\dot{\phi}$, $\dot{f}$ so that we have the following assumptions:

| $\ddot{\phi}=0$ | $(8.22)$ |
| :--- | :--- |
| $\ddot{\theta}=0$ | $(8.23)$ |
| $\dot{\phi} \ll \omega$ | $(8.24)$ |
| $\dot{\theta} \ll \omega$ | $(8.25)$ |

which, when applied to Eq (8.16), (8.17), (8.18), lead to the precessional equations of motion for high apin rate:

$$
\begin{gather*}
I_{3} \omega \dot{\phi} \sin n \dot{\omega} N_{1}(\theta)+I_{3} \Omega \omega \cos \theta \sin \phi  \tag{8.26}\\
I_{3} \omega \dot{\theta}=-N_{2}(\theta)-I_{3} \Omega \omega \cos \phi  \tag{8.27}\\
I_{3} \dot{\omega}=N_{3}(\theta) \tag{8.28}
\end{gather*}
$$

These equations have been solved (integrated) on a digital computer, and the solution which has been obtained so far will be described below, after deriving the vane torques in the next section.

### 8.8 Vane Torques

The vanes syatem has four triangular vanes, assumed to be 100 cm long $\times 25$ cm bage, attached to the rear of the spacecraft radiator at four points spaced by $90^{\circ}$. (See Fig. 8.7). The vanes are assumed to be canted $70^{\circ}$ up from the spin axis, and to be pitched $-35.26^{\circ}$ (measured about the longitudinal axia of the vane, with pitch $=0^{\circ}$ when the vane is tangent to a $70^{\circ}$ cone containing the longitudinal axes of the vanes). The vanes are further assumed to be made of mylar, aluminized on the front with absorptivity $=0.2$ and emissivity $=0.05$, black on the back with absorptivity $=$ emissivity $=0.9$. The spacecraft radiator is assumed to be black with absorptivity $=$ ernissivity $=1$. (This part is not too important because it is near the spacecraft center of mass and is relatively symmetrical).

A most elementary estimate of the torques on this model is made by assumin that 1) there will be no shadowing of vanes, 2) no rariation will fall on rear of vanes, and 3) torques on the central spacecr radiator section can be ignored. These assumptions are good for small $\theta\left(\theta<50^{\circ}\right)$. In this case, we shall first derive the torques as a function of $\psi$; and then average the torques for $0<\psi<2 \pi$ (which will give us the average torques, independent of $\psi$ ), applied to the vanes when the spacecraft is rotating uniformly and fast enough that variation in $\psi$ can be ignored.

With the assumptions above (primarily, no shadowing), the force on a vane can be represented as a vector acting at a point $2 / 3$ of the distance along the longitudinal axis of the triangular vane; this is the effective center of pressure, given by

$$
\begin{equation*}
\ell_{0}=\frac{\int_{0}^{\ell} f \lambda^{2} d \lambda}{\int_{0}^{l} \mathrm{f} \lambda \mathrm{~d} \lambda}=\frac{2}{3} \ell \tag{8.29}
\end{equation*}
$$

The force vector has three physically-distinct components: 1) an absorbed momenturn component, in the direction of the incident radiation $\vec{I}$; 2) a reflected momentum component, normal to the vane $\vec{P}$; and 3) a reradiated momentum component, also normal to the vane $\overrightarrow{\mathbf{P}}$. The following definitions will be made; and, in the following derivation, because the algebraic expressions are quite lengthy, sin and cos will be represented simply as $s$ and $c$.

$$
\begin{align*}
& \vec{P}=\text { unit normal vector to vane }  \tag{8.30}\\
& \vec{I}_{0}=\text { unit vector to sun }  \tag{8.31}\\
& I=\underset{\left(\text { watts } / \mathrm{m}^{2}\right)}{ } \text { solar radiation intensity } \tag{8.32}
\end{align*}
$$



Fig. 8-7 Vanes system.

$$
\begin{array}{ll}
c=\text { velocity of light (MKS) } & (8.33) \\
\text { abs = absorptivity of vane } & (8.34) \\
\text { emiss = emissivity of vane } & (8.35) \\
\alpha=\text { cant angle of vane } & (8.36) \\
\beta=\text { pitch angle of vane } & (8.37)
\end{array}
$$

With these definitions, and with reference to Fig. 8-7,

$$
\vec{P}=\vec{i}(c \alpha c \beta c \psi+s \beta \quad s \psi)+\vec{j}(c \alpha c \beta \quad s \psi-s \beta \quad c \psi)
$$

$$
\begin{gather*}
+\vec{k}(s \alpha c \beta)  \tag{8.38}\\
\overrightarrow{\mathbf{p}}=\overrightarrow{\mathrm{la}} \mathrm{c} \psi^{\prime}+\overrightarrow{\mathrm{ja}} \mathrm{~s} \psi^{\prime}+\overrightarrow{\mathrm{k} b} \tag{8.39}
\end{gather*}
$$

or
$a=\sqrt{1-s^{2} \alpha c^{2} \beta}=\sqrt{1-b^{2}}$
$b=s \alpha c \beta$

$$
\begin{equation*}
\psi^{\prime}=\psi-A \tag{8.41}
\end{equation*}
$$

$A=\tan ^{-1}\left(\frac{s \beta}{c \alpha c \beta}\right)$

$$
\begin{equation*}
\overrightarrow{\mathrm{r}}_{0}=\overrightarrow{\mathrm{j}} s \theta+\overrightarrow{\mathrm{k}} \mathrm{c} \theta \tag{8.43}
\end{equation*}
$$

Using these expressions, we can find the cosine of the angle between the normal to the vane and the incident radiation to be(See Eq 8.3):

$$
\begin{equation*}
c(\theta)=\vec{P} \cdot \vec{I}_{0}=a s \theta s \psi^{\prime}+b c \theta . \tag{8.45}
\end{equation*}
$$

Using this cosine, the absorbed and reflected radiation pressure components are given by:
$\vec{F}\left(\psi^{\prime}\right)=$ Area $\cdot$ abs $\cdot \frac{I}{\mathrm{C}} \cdot\left(\overrightarrow{\mathrm{P}} \cdot \overrightarrow{\mathrm{I}}_{0}\right) \cdot\left(-\vec{I}_{0}\right)$
absorbed

$$
\begin{align*}
& \vec{F}\left(\psi^{\prime}\right)=\text { Area } \cdot(1-\text { emiss }) \cdot \frac{2 I}{C} \cdot\left(\vec{P} \cdot \vec{I}_{0}\right)^{2} \cdot(-\overrightarrow{\mathrm{P}}) \\
& \text { reflected } \tag{8.47}
\end{align*}
$$

For a reradiative force, resulting from a reradiated intensity $I_{R}$, assuming isotropic radiation, according to Lambert's law, the effective normal component of the intensity and the force are given by:

$$
\begin{align*}
& I_{N}=I_{R} \frac{\int_{0}^{\frac{\pi}{2}} \sin \theta \cos ^{2} \theta d \theta}{\int_{0}^{\frac{\pi}{2}} \sin \theta \cos \theta d \theta}=\frac{2}{3} I_{R}  \tag{8.48}\\
& \vec{F}\left(\Psi^{\prime}\right)=\text { Area } \cdot \frac{I_{N}}{C} \cdot(-\vec{P}) \\
& \text { reradiated } \tag{8.49}
\end{align*}
$$

$I_{R}$ can be separated into two parts: 1) a part which varies with $\psi^{\prime}$ (this leads to an erecting torque, as first investigated by Peterson ${ }^{(2)}$ (CSR-T-66-3)); and 2) an average value which produces a spin torque. These are relatively small for the most important stabilization conditions ( $\beta=-35.26^{\circ}, \theta<50^{\circ}$ ), and therefore will be derived iater.

The major torque component for these primary conditions ( $\beta=-35.26$, $\theta<50^{\circ}$ ) is the reflective torque, obtained from the force by taking $\vec{r} \times \vec{F}$ in Eq (8.47), leading to:

$$
\vec{N}\left(\psi^{\prime}\right)=\text { Area } \cdot(1-\text { eniss }) \cdot \frac{2 I}{C} \cdot\left(\vec{P} \cdot \vec{I}_{0}\right)^{2} \cdot(\vec{P} \times \vec{r}) \quad(8.50)
$$ reflected

where we will take

$$
\begin{align*}
& \vec{r}=\text { center of pressure of vane }  \tag{8.51}\\
& \vec{r}=\overrightarrow{i r}_{1} c \psi+\overrightarrow{j r}_{1} s \psi-\vec{k} z_{1}  \tag{8.52}\\
& r_{1}=r_{0}+\ell_{0} s \alpha  \tag{8.53}\\
& z_{1}=\ell_{0} c \alpha \tag{8.54}
\end{align*}
$$

$\ell_{0}$ is given by Eq (8.29) and, for approximate results, numerical values can be assigned to: $\ell=100 \mathrm{~cm}, r_{0}=20 \mathrm{~cm}$, Area $=0.125 \mathrm{~m}^{2}$ and, as before, $a b s=0.2$, emiss $=0.05$. So we have:

$$
\begin{align*}
& \vec{P} \times \vec{r}=\vec{l}\left[-\left(a z_{1}+b r_{1} c A\right)=\psi^{\prime}-b r_{1} s A c \psi^{\prime}\right] \\
&+ \vec{j}\left[\left(a z_{1}+b r_{1} c A\right) c \psi^{\prime}-b r_{1} s A s \psi^{\prime}\right] \\
&+\overrightarrow{k r_{1}} s \beta . \tag{8.55}
\end{align*}
$$

From Eq (8.45),

$$
\begin{equation*}
\left(\vec{P} \cdot \vec{I}_{0}\right)^{2}=s^{2} \theta\left(a^{2} s^{2} \psi^{\prime}-b^{2}\right)+2 b a s \theta c \theta s \psi^{\prime}+b^{2} . \tag{8.56}
\end{equation*}
$$

Now, using Eq (8.50), (8.55), and (8.56), we are in a position to average the reflected torque for $0 \leq \psi^{\prime} \leq 2 \pi$ :

$$
\begin{equation*}
\vec{N}_{\text {reflected }}=\frac{1}{2 \pi} \int_{0}^{2 \pi} \underset{\substack{\mathrm{~N} \\ \text { reflected }}}{ } \tag{8.57}
\end{equation*}
$$

giving

$$
\begin{align*}
& \vec{N}_{\text {reflected }}=\text { Area } \cdot(1-\text { emiss }) \cdot \frac{2 I}{C} x \\
& {\left[-\vec{i}\left(s^{2} \alpha c \alpha c^{3} \beta r_{1}+s \alpha c \beta\left(1-s^{2} \alpha c^{2} \beta\right) z_{1}\right) s \theta c \theta\right.} \\
& -\vec{j}\left(s^{2} \alpha s \beta c^{2} \beta r_{1}\right) s \theta c \theta \\
& \left.+\vec{k}\left(s^{2} \alpha s \beta c^{2} \beta+s \beta\left[\frac{1}{2}-\frac{3}{2} s^{2} \alpha c^{2} \beta\right] s^{2} \theta\right) r_{1}\right] \tag{8.58}
\end{align*}
$$

This torque is the major torque acting for $\beta= \pm 35.26^{\circ}, \theta<50^{\circ}$ and could be used in $\operatorname{Eq}(8.26),(8.27),(8.28)$ to give a description of the initial stabilization. Two points should be noted in Eq (8.58): 1) the sign of $\mathrm{N}_{2}(\theta)$ always remains the same for $\theta<90^{\circ}$; and 2) for small angles, $-\mathrm{N}_{2}(\theta) / \theta$ $\% \mathrm{~N}_{3}(\theta)$. Point 1) indicates stability for $\theta<90^{\circ}$, and point 2) will be shown to result in a simplified description of the motion, based on these torque components, at small angles.

The second most important torque is the absorbed torque, obtained from the absorbed force in Eq (8.46) by taking $\vec{r} \times \vec{F}$ absorbed and averaging for $0<\psi^{\prime}<2 \pi$ as outlined by the following equations:

$$
\begin{equation*}
\vec{N}_{\text {absorbea }}=\frac{1}{2 \pi} \int_{0}^{2 \pi} \vec{N}\left(\psi^{\prime}\right) d \psi^{\prime} \tag{8.59}
\end{equation*}
$$

$$
\begin{equation*}
\underset{\text { absorbed }}{\vec{N}\left(u^{\prime}\right)}=\text { Area } \cdot \text { abs } \cdot \frac{1}{C} \cdot\left(\vec{P} \cdot \vec{I}_{0}\right) \cdot\left(\mathbf{I}_{0} \times \vec{r}\right) \tag{8,60}
\end{equation*}
$$

From Eq. (8. 38), (8.44), and (8.52):

$$
\begin{gather*}
\overrightarrow{\mathrm{I}}_{0} \times \overrightarrow{\mathrm{r}}=\overrightarrow{\mathrm{I}}\left(-\mathrm{z}_{1} s \theta-r_{1} c \theta s \psi\right) \\
+\overrightarrow{\mathrm{j}} \mathrm{r}_{1} c \theta c \psi-\vec{k} r_{1} s \theta c \psi  \tag{8.61}\\
\overrightarrow{\mathrm{p}} \cdot \overrightarrow{\mathrm{I}}_{0}=s \theta(\mathrm{c} \alpha c \beta s \psi-s \beta c \psi)+c \theta(s \alpha c \beta) \tag{8.62}
\end{gather*}
$$

so after averaging:

$$
\begin{align*}
\vec{N}_{\text {absorbed }} & =\text { Area abs. } \frac{I}{C}\left\{\vec{i}\left[-z_{1} s \alpha c \beta-\frac{r_{1}}{2} c \alpha c \beta\right] s \theta c \theta\right. \\
& \left.+\vec{j}\left[-\frac{r_{1}}{2} s \beta s \theta c \theta\right]+\vec{k}\left[\frac{r_{1}}{2} s \beta s^{2} \theta\right]\right\} . \tag{8.63}
\end{align*}
$$

The third most important torque is the reradiative torque, mentioned briefly above (see Eq (8, 49)), which can be differentiated into; 1) a small erecting component, and 2) an average spin component. These will be derived below.

The erecting component will be derived only for $\beta=0$ because only near $\beta=0$ is the erecting component large enough to be comparable to the reflecting component of torque. In the interest of obtaining an analytical solution, we shall also assume $\theta \sim 0$, minimizing the temperature variations on the vanes. Assuming that the body is spinning uniformly at an angle $\theta$ to the sun, as usual, the input radiation intensity into a vane is proportional to $\overrightarrow{\mathrm{P}} \cdot \overrightarrow{\mathrm{I}}_{0}$, which is periodic in $\psi^{\prime}$, as given by Eq (8.45). Given this periodic input intensity, the temperature on the vane (assumed constant over the vane) satisfies a differential equation similar to that studied by Peterson ${ }^{(2)}$ which is:

$$
\begin{equation*}
C_{P}^{*} \frac{d T}{d t}+\epsilon \sigma T^{4}=a b s \cdot I \cdot\left(b c \theta+a s \theta s \psi^{\prime}\right) \tag{8.64}
\end{equation*}
$$

Linearizing the equation, and assuming that:

$$
\begin{equation*}
T=T_{0}\left(1+a_{1} s \psi^{\prime}+b_{1} c \psi^{\prime}\right) \tag{8.65}
\end{equation*}
$$

leads to the solution:

$$
\begin{equation*}
a_{1}=\frac{\left(\frac{a s \theta}{48 \alpha}\right)}{1+\left(\frac{\omega}{\omega_{\mathrm{opt}}}\right)^{-}} 2^{-} \tag{8.66}
\end{equation*}
$$

$$
\begin{gather*}
b_{1}=\frac{\left(\frac{a s \theta}{4 \pi a}\right)}{\frac{\omega}{\omega_{\mathrm{opt}}}+\frac{\omega_{\mathrm{opt}}}{\omega}}  \tag{8.67}\\
\omega_{\mathrm{opt}}=\frac{4}{C_{F}}(\mathrm{abs} \cdot I \cdot s a)^{3 / 4} \cdot(\epsilon \sigma)^{1 / 4} . \tag{8.68}
\end{gather*}
$$

For aluminum with a thickness of 0.1 mm ,

$$
\begin{equation*}
C_{P}=C_{p} \Delta t=0.0246 \frac{\text { Joule }}{\mathrm{cm}^{2}} \tag{8.89}
\end{equation*}
$$

Assuming $I=1380$ watt $/ \mathrm{m}^{2}$, abs $=0.2, \alpha=70^{\circ}, c=0.95, \sigma=5.672 \times 10^{-8}$ watt $/ \mathrm{m}^{2}-\mathrm{o}_{\mathrm{K}}{ }^{4}$, we get:

$$
\begin{equation*}
\omega_{o p t}=\frac{3.95}{C_{\mathrm{P}}^{*}} \frac{\text { watt }}{\mathrm{m}^{2}-{ }^{o_{K}}}=0.016 \frac{\mathrm{rad}}{\mathrm{Bec}} \tag{8,70}
\end{equation*}
$$

and

$$
\begin{equation*}
T_{0}=\left[\frac{a b s \text { I s }}{\epsilon \cdot \sigma}\right]^{1 / 4}=263 \frac{1}{2}^{o} K \tag{8,71}
\end{equation*}
$$

This implies a reradiated intensity distribution of approximately :

$$
\begin{equation*}
I_{R}=a b s \cdot I \cdot\left[b c \theta+\frac{a s \theta s \psi^{\prime}}{1+\left(\frac{\omega}{\omega_{\mathrm{opt}}}\right)^{2}}-\frac{a s \theta c \psi^{\prime}}{\left(\frac{\omega_{\mathrm{opt}}}{\omega_{\mathrm{op}}}\right)+\left(\frac{\omega_{\mathrm{opt}}}{\omega}\right)}\right] \tag{8.72}
\end{equation*}
$$

Using this expression for the intensity of $I_{R}$ and averaging the torque given by $\vec{r} \times \vec{F}\left(\psi^{\prime}\right)$, with $\vec{F}\left(\psi^{\prime}\right)$ from $E q(8.49)$, we get:

$$
\begin{equation*}
\underset{\text { erecting }}{\mathrm{N}_{\text {reradiative }}} \approx C_{r} \frac{1}{2}\left(a z_{1}+b r_{1} c A\right) \frac{a s \theta}{\left(\frac{\omega}{\omega_{\mathrm{opt}}}+\frac{\omega_{\mathrm{opt}}}{\omega}\right)} \tag{8.73}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{N}_{\substack{\text { reradiative } \\ \text { anin }}}^{\mathrm{C}_{\mathrm{r}} \cdot \mathrm{r}_{1} \mathrm{~s} \beta \cdot \mathrm{c} \theta} \tag{8,73}
\end{equation*}
$$

with

$$
\begin{equation*}
C_{r}=\frac{2}{3} \frac{I_{0}}{C} \text { Area } \cdot \text { abs } \cdot \frac{(e m i s s ~ 1-e m i s s ~ 2)}{(e m i s s ~ 2+e m i s s ~ 1)} \tag{8.75}
\end{equation*}
$$

where pertinent quantities oppear in Eq (8.4), (8.40), (8.41), (8.43), (8.53), and (8.54).

The erecting component of the reradiative torque ( Eq ( 8,73 )) is only significant near $\beta=0$ (relative to reflective torque) and near $\omega=\omega_{\text {opt }}$, where we are studying error torques in the coast phase (see Sec 8.9). The spin component of the reradiative torque (Eq ( 8.75 ) is proportional to and a fraction of ( 0.08 for abs $=0.2$ ) the spin component of the reflected torque, for small $\theta$. For very large $\theta$, when the absorptive rear of the vanes is exposed, this component will be a major factor in causing a reversal of the spin torque. For the initial performance, when $\beta=-35.26^{\circ}$ and $\theta<60^{\circ}$, the torque can be mainly represented by the reflective torque, $\mathrm{Eq}(8.58)$. When $\theta<10^{\circ}$, this torque can be simplified by letting $\cos \theta \rightarrow 1, \sin ^{2} \theta \rightarrow 0$, leaving approximately linear terms in $\theta$ for the precession and erecting components of torque, and a constant for the spin component, the constant being equal to the coefficient of the erecting torque (see discussion of Eq (8,79)).

After the initial stabilization, the vanes are "nulled" to $\beta \simeq 0$; however, the error in $\beta$ will result in erior torques in the spin and erecting directions which will be added to the reradiative torques, causing motion of the spacecraft which, over a relatively long period, will be unstable. The reflective error torque can be linearized for $\beta \sim 0^{\circ}$, changing Eq (8.58) to:

$$
\begin{array}{ll}
\vec{N}_{\substack{\text { reflected } \\
\beta \sim 0 \\
\theta \sim 0}}=-\overrightarrow{i s} \alpha c \alpha\left(r_{o} s \alpha+\ell_{0}\right) s \theta \\
& -\vec{j}_{s^{2} \alpha s \beta r_{1} s \theta} \\
& +\vec{k} s^{2} \alpha s \beta r_{1} .
\end{array}
$$

The reradiative torque component we are primarily interested in is the erecting component, Eq (8.73), which reduces to:

$$
\begin{equation*}
\underset{\substack{\text { erecting } \\ \sim 0}}{\mathrm{~N}_{\text {reradiative }}}=\mathrm{C}_{\mathrm{r}} \frac{1}{2}\left(\ell_{\mathrm{o}}+r_{\mathrm{o}} s \alpha\right) \frac{\mathrm{C} \alpha \beta \theta}{\left(\frac{\omega}{\omega_{\text {opt }}}+\frac{\omega_{\text {opt }}}{\omega}\right)} \tag{8,77}
\end{equation*}
$$

With such torques derived for $\beta \sim 0, \theta \sim 0$, we shall be able to get some estimate of the long-term behavior of the spacecraft.

The simplified torques derived above are derived for the purpose of demonstrating some more general features. The actual torques have been cal-
culated as a part of a relatively complex computer program, which includes the effect of reflective and absorptive forces and shadowing by a geometrical structure such as is illustrated in the ligures. Such an analyais will not be presented here, but the results are shown in Fig. 8-8.
8.9 Dynamical Motion of Spacecraft

Given the vector torques as shown in the preceding section, it is possible to determine the motion of the spacecraft, both short-term and long-term, from the equations of motion derived in Sec 8.7. In order to get an idea of what should happen in the initial phase of atabilization, before discussing the exact solution as integrated on a digital computer, one can take Eq (8.27), ( 8.28 ) and, neglecting terms involving $\Omega$ obtain the result

$$
\begin{equation*}
\frac{d \omega}{\omega d \theta}=\frac{-N_{3}(\theta)}{N_{2}(\theta)} \tag{8,78}
\end{equation*}
$$

or

$$
\begin{equation*}
\ln \left(\frac{\xi \xi}{\omega_{0}}\right)=-\int_{\theta_{0}}^{\theta} \frac{N_{3}(\xi)}{N_{2}(\xi)} d \xi \tag{8.79}
\end{equation*}
$$

which gives an approximate relationship between $\omega / \omega_{0}$ and $\theta$, neglecting certain terms involving $\Omega$ and $\omega$. For small $\theta$, the integrand $\sim d \xi / \xi$, as discussed in reference to $\mathrm{Eq}(8,58)$ so that $\theta$ and $\omega$ are related by a power relationship (approximately linear), as shown in Fig. 8-9.

In addition, since the spin torque is roughly constant for small $\theta$, as evidenced by Fig. 8-8 and Eq (8.58) for the major component, one would expect that $\dot{\omega} \approx$ const, from $E q(8.28) ;$ so that $\omega \approx \omega_{0}-K_{1}{ }^{t}$ and $\theta / \theta_{0} \approx$ $\left(\omega / \omega_{0}\right)^{K_{2}}, K_{2} \approx 1$. Actual results do not fit this explanation too well becarse of deviation of the torques for large $\theta$ and because of rotation of sun linc (at rate $\Omega$ ). (Compare Fig. 8-9 and 8-11.)

### 8.9.1 Actual Results

The actual results of integrating the precessional equations of motion, Eq (8.26), (8.27), and (8.28), for the initial_stabilization period of $\sim 13.7$ days when the spacecraft is erecting and despinning from its initial conditions of $\theta=60^{\circ}$ and $\omega=20 \mathrm{r} / \mathrm{min}$ to its final orientation of $\theta=2^{\circ}$ and $\omega \approx 1.1 \mathrm{r} / \mathrm{min}$, are shown in Fig. 8-10 and 8-11. Figure 8-10 plots the initial spiral (motion of figure axis) in terms of $\theta$ and $\phi$ (Euler angles, as shown in Fig. 8-6. Fig. 8-11 plote $\theta$ and $\omega$ ve time for the initial stabilization period. As seen from

Fig. 8-8 Torques calculated for Sunblazer vanes.



Fig. 8-10 Initial spiral-60, $20 \mathrm{r} / \mathrm{min}$.


Fig. 8-11 Initial $\begin{gathered}\text { and } \\ \text { a } \\ \text { va time。 }\end{gathered}$
this plot, $\dot{\omega}$ is not constant, nor does $\theta / \theta_{0}=\left(\omega / \omega_{0}\right)^{K_{2}}$. After this period, the investigation moves to the long term stabilization.

Long-term motion of the spacecraft is characterized by cycles during which the spacecraft drifts out to $\theta=10^{\circ}$ (as a result of spin-up and negative erecting torques), at which point the vanes are to be actively switched to a pitch angle of $\beta \mathbb{\Xi}-35^{\circ}$. This will bring the spacecraft axis back to within $2^{\circ}$ of the sun in a short period. These cases have also been investigated on a digital computer, integrating the precessional equations of motion (Eq (8.26), i. (8.27), and (8.28)) as before, but switching the vanes' pitch angle between $\beta=+1^{\circ}$ and $\beta=-35.26^{\circ}$ when $\theta$ exceeds the limits $2^{\circ}$ and $10^{\circ}$ respectively. (In the computer program, $\theta$ is never allowed to exceed the limits by more than $0.001^{\circ}$ before $s w i t c h i n g$ occurs.) The long-term behavior of $\theta$ and $\omega$ vs time is plotted in Fig. 8-12, showing approximately 6 cycles of $\theta$ from $2^{\circ}$ to $10^{\circ}$ and back again. On this plot, an asymptotic increase in average spin rate is also evident. In addition, each individual cycle from $2^{\circ}$ to $10^{\circ}$ and back has been plotted to show the spiraling motion of the spacecraft axis of symmetry in $\theta-\phi$ coordinates. Typical spirals are shown in Fig. 8-13. For the later spirals, when the average spin rate is approaching its asymptote, the motion is slower, and the center of the spiral moves higher above the ecliptic plane ( $\phi=0^{\circ}$ ), consistent with Colombo's derivation (1). In addition, the spirals are unstable, of increasing amplitude. This is consistant with the torques derived in Eq ( 8,55 ) $,(8,63),(8,73)$, and $(8,74)$ above.

Furthermore, a closer inspection of Fig. 8-12 indicates that $\dot{\boldsymbol{u}}$ is not constant during the drift periods (pitch angle $\beta=+1^{\circ}$ ), as might be expected by the unsuspecting observer. This fact is simply explained by the variation in solar intensity during the 3/4-year orbit. The final phase of the investigation involves optimization of the vanes' constants, and sensitivity of the system to initial conditions and error conditions. Results of this investigation are summarized in Fig. 8-i4 to 8-21 below.

### 8.9.2 Optimization of Vanes' Cant Angle

The vanes' cant angle has been varied from $45^{\circ}$ to $75^{\circ}$, and the initial stabilization time and long-term number of limit cycles has been plotted (Fig. 8-14, 8-15). These results vary significantly with the cant angle, but $70^{\circ}$ was chosen to give nearly the minimum initial stabilization time without, hurting the long-term stabilization too much.




Foldoat Frame 2


$\begin{array}{lllllll}250.00 & 200.00 & 250.00 & \mathbf{2 0 0} 00 & 200.00 & 300.00 & 310.00\end{array}$
Fig. 8-12 Long-term $\theta$ and $\omega$ vs time.
foldout frame 3

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Fig. 8-13a Long-term spirals (1).


Fig. 8-13b Long-term spirals (2).


Fig. 8-13c Long-term spirals (3).


Fig. 8-13d Long-term spirale (4).


Fig. 8-13e Long-term spirals (5).


Fig. 8-13f Lcng-term spirals (6).
89.3 Sensitivity to Initial Conditiors

Comprehensive plots of initial stabilization time and initial spirals (motion of figure axis) have been plotted vs initial theta and vs initial omega (in r/min). (See Fig, 8-16, 8-17, 8-18). As expected, initial stabilization time is approximately linear with initial spin rate; and initial stabilization time is relatively constant for $10^{\circ}<$ inltial theta $<70^{\circ}$, but changes very rapidly below $5^{\circ}$ and above $75^{\circ}$.
8.9.4 Parametric Inves.igation of Long-Term Motion

The number of long-term limit cycles of the vanes has been taken as an index of the long-term performance. New results in this area indicate that the primary characteristics of the long-term motion are relatively independent of some of the conditions investigated here.

The vanes' cant angle affects the number of limit cycles, with the results shovin in Fig, 8-15. If the area of each vane were trimmed to give the same initial stabilization time for each cant angle, then the number of long-term limit cycles would be nearly constant for cant angles between $45^{\circ}$ and $70^{\circ}$.

The vanes' pitch error angle is roughly linear with the number of limit cycles, as expected. (See Fig 8-19).

The vanes' reradiation damping-time constant is not critical. Here a change in time constant by 3 orders of magnitude changes the number of limit cycles by a factor of 3.6 , from 2.5 to 8.8 , as shown in Fig. 8-20. In addition, analysis of torques shows that the system with damping is unstable for pitch angles greater than a fraction of a degree.

Initial conditions do not significantly affect the long-term performance, as shown in Fig. 8-21. Two extreme cases (initial omega of $5 \mathrm{r} / \mathrm{min}$ and 80 $\mathrm{r} / \mathrm{min}$ ) show the conversion of the system (over a period of one year) from its initial low spin state to a pseudo-equilibrium high spin state. Initial theta affects only the long-term performance as it affects the despin ratio from initial theta to $2^{\circ}$.
8.9 5 Nominal Conditions

All the preceding cases were run for fixed conditions, except for the condition of parameter which was varied:

Spacecraft moment of inertia
Vanes' cant angle
Vanes' pitch error angle
$4.64 \times 10^{6} \mathrm{gm}^{-\mathrm{cm}^{2}}$
$70^{\circ}$
$1^{\circ}$


Fig. 8-14 Optimization of cant angle - stabilization time.


Fig. 8-15 Optimization of cant angle - long-term cycles.


Fig. 8-16 Stabilization time vs initial $\theta$.
$00 \cdot 8$
$00 \cdot 0$

Fig. 8-17 Initial stabilization spirals - different initial conditions.


Fig. 8-18 Stabilization time vs initial $\omega$.


Fig. 8-19 Long-term number of cycles vs pitch error.


Fig. 8-20 Long-term cycles vs reradiation damping constant.

Reradiatoon damper riste
Initi.al spaceroralt-sum angle
lniti.t spin rate
$0.017 \mathrm{rad} / \mathrm{sicc}$
$30^{11}$
$20.1 / m \cdot n$

8 (1) if Pos:sible Pilfalls in Rotlon
A magor dymamices program has beon pun which atoprates the complete aquatoons of motion for a rigid boly and inclades the following variables.

1. Trmperature on all follo valles

2 Three differont moments of inertia

3 Arbitrary : 3 aligmment betwern iorque axin (whero orocting torque - 0) and premのipal axes
4. Arbitrary initial conditions
5. Arbitrary absorptivities and emissivities for vane's

The results have shown that an incredibly small nutation amplitude ' 0.00 : :0 0.01 fra- 0 tonal amplitude) can result in extremely large libration-oscillation amplitudes ( $20^{\circ}$ to $76^{\circ}$ ) if the error torque of the vanes results form a $-1^{(1)}$ error in the pitch angle, in the same sonse as the initial - $35.26^{\circ}$ pitch, so that the spin rate goes through 0 . A further integration of the equations of motion has shown that, with modified logic of the vanes control system, the motion might eventually stabilize to a small angle after a number of wild oscillations, bet it is felt that the simulation of a few special cases does not give enough infurmation to guarantee the stability of the system ingeneral. It is felt, at the present stage of development, that special precautions should be taken to prevent the spin rate from going through 0 .

A second major pitfall could result from an attempt to reverse the vanes (to $+35.26^{\circ}$ ) when the initial thota is large $\left(120^{\circ}\right)$, hoping that the spacecraft would despin and go to $180^{\circ}$, because of the reversal in spin torq af of the vanes above $80^{\circ}$. Hopefully, the spacecraft would then flip around and wind up at some small angle where the conventional control system would stabilize it. A promising fact is that it takes about as long in this mode to go from $120^{\circ}$ to $179^{\circ}$ as it does in the normal mode from $60^{\circ}$ to $2^{\circ}$. However, the problem of going from $178^{\circ}$ hopefully to a small angle through 0 spin rate is not understood in sufficient detail to design a predictable control system.

A third major pitfall would result from an attempt to null the vanes after separation (at $60^{\circ}$ ), and hopefully wait for the sun line to rotate into the axis of symmetry of the spac araft. Here, the precession of the spacecraft would have to be taken into account; and initial results indicate that, even at an

Fig. 8-21 Long term cycles vs initial spin rate.
iritial rate of $160 \mathrm{r} / \mathrm{min}$, the angle between the spacecraft axis and the sun would oscillate between $17^{\circ}$ (which it would reach at 57.1 days) and $155^{\circ}$ (at 158.2 days)

Nutation motion of the spacecraft (see Sec 8.11 ) ean be derived very simply from the complete equations of motion $\mathrm{Eq}(8.16)$ and (8.17), by assuming $\ddot{\phi}=0=\ddot{j}=\dot{\dot{\omega}}$, leading to

$$
\begin{equation*}
\dot{\phi}=\frac{I_{i j}}{I_{1}} \frac{\omega}{\cos \theta_{0}} \tag{8.80}
\end{equation*}
$$

which is basically the formula for force-free precession (1:), and:

$$
\begin{equation*}
\dot{\theta}_{0}=\frac{N_{2}\left(\prime{ }_{0}\right)}{I_{3} \omega^{\prime}} \tag{8.81}
\end{equation*}
$$

The amplitude of the nutation circle, " 0 , can increase (or decrease) as a result of direct or reradiative torques and of internal dissipation. See Sec 8.11 for a more detailed discussion of radiative iorques. Internal dissipation has not been determined; however, if sufficient dissipation is not available in the structure, it may be necessary to design an internal damper.

## Ancillary Problems

Some ancillary problems might be mentioned
Surfaces for the spacecraft and vanes have not been investigated in sufficient detail. Although literature exis's which indicated that the assumed surface characteristics are not unreasonable, the problem still remains of fabricating a structure and measuring the surfaces.

An aspect sensor, probably based on the technique successfully developed by the Goddard Space Center, of using multiple sensors and a mask to provide the maximum information with the minimum electrical hardware, does not seem to be a real problem

Control logic consisting of a fow AND and OR gates to switch the vanes, depending on the angle, is not felt to be a real problem either.

Micrometeorites are not felt to be a real threat to the spacecraft, either as momentum disturbance, or by erosion of the surfaces. (See Sec 8.1).

A practical test of the vane torques and the effect of the control system should definitely be made in vacuum.

### 3.11 Nutations

### 8.11.1 Convention Used to Distinguish Nutational From Precessional Modes

There are three main types of dynamicai instabilities which are of concern in orienting the Sunblazer vehicle:

> 1. Librational instability for very low spin rates;
> 2. Precessional instability (increasing cone angle);
> 3. Nutational instability for moderate and high spin rates.

Because the nominal precession rate is always more than two orders of magnitude below the nominal spin rate, nutations can be analyzed dynamically as a "torque free" precession superposeci upon a very slow and steady torqued precession. The convention to be followed here in distinguishing a nutational mode from a precessional mode is that the former causes the vehicle's spin axis to oscillate at an angular frequency comparable to that of the spin rate, while leaving both the direction and magnitude of the angular momentum vector essentially unchanged during a cycle. A precessional mode, on the other hand, causes the vehicle's axis to oscillate at angular frequency much lower than the spin rate (in fact, about inversely proportional to the spin rate), and actually changes the direction of the angular momentum during a cycle. In gener'al, both modes will be simultaneously present to some extent. The problem here will be to determine whether or not the amplitude of the nutational mode wi.l increase or decrease with time under the various sets of circumstances Sunblazer might be expected to encounter.

Before proceeding to a more mathematical description of this problem, it should be stated that, although the actual derivation to be given here differs from others in this report, both the basic equations and the final notation are identical to the previous treatments (see Sec 8.7). Since the unsimplified basic equations themselves cannot be handled analytically, an approximation will be made which can adequately account for at least the nominal operating conditions of Sunblazer. In a few special cases, where insufficient confidence could be expected from purely analytical techniques, an analog computer has been used, which employs no approximations whatever.

Basic to the mathematical description will be a cascading of two uniformlyrotating coordinate systems. The uniform rotations rates will comprise one of the approximations, (see Fig. 8-13). For the first, system, we will temporarily use the same notation as Halfman ${ }^{(12)}$, and then convert this to our standard notation. However, the definitions of $\theta, \phi$, and $\mu$ for the cone angle, precession angle and spin angle respectively will be the same in both notations.
8.11.2 General Dynamical Equations for an Axisymmetric Body and Conversion into the Standard Notation

Proceeding then to page 220 of Halfman, and also referring to the labeling in parentheses of Fig. 8-22, we find that the most general dynamical equations for an axisymmetric body are.

$$
\begin{array}{r}
* M_{x}=I_{X} \frac{d \omega_{x}}{d t} \\
* M_{y}=I_{T} \frac{d \Omega_{y}}{d t}-\Omega_{x} I_{\Gamma} \Omega_{z}+\Omega_{z} I_{x} \omega_{x}  \tag{8.82}\\
* M_{z}=I_{T} \frac{d \Omega_{z}}{d t}-\Omega_{y} I_{x} \omega_{x}+\Omega_{x} I_{T} \Omega_{y}
\end{array}
$$

where in this particular case

$$
\begin{align*}
& * \omega_{x}=\dot{\psi}+\dot{\phi} \cos \theta \\
& * \Omega_{x}=\dot{\phi} \cos \theta  \tag{8.83}\\
& * \Omega_{y}=\omega_{y}=\dot{\theta}  \tag{8.84}\\
& * \Omega_{z}=\omega_{z}=\dot{\phi} \sin \theta
\end{align*}
$$

Substituting these last three relations into Eq (8.82) one obtains:

$$
* M_{x}=I_{x} \frac{d \omega_{x}}{d t}
$$

$$
\begin{align*}
* \mathrm{M}_{\mathrm{y}} & =\mathrm{I}_{\mathrm{T}} \ddot{\theta}-\mathrm{I}_{\mathrm{T}} \dot{\phi}^{2} \sin \theta \cos \theta+\mathrm{I}_{\mathrm{x}} \dot{\phi} \omega_{\mathrm{x}} \sin \theta  \tag{8.85}\\
* \mathrm{M}_{\mathrm{z}} & =\mathrm{I}_{\mathrm{T}} \ddot{\phi} \sin \theta+2 \mathrm{I}_{\mathrm{T}} \dot{\phi} \dot{\theta} \cos \theta-\mathrm{I}_{\mathrm{x}} \omega_{\mathrm{x}} \dot{\theta}
\end{align*}
$$

Converting these equations into the standard notation so that

$$
M_{x} \rightarrow N_{z}, M_{y} \rightarrow N_{x}, M_{z} \rightarrow N_{y}, I_{x} \rightarrow I_{3}, I_{T} \rightarrow I_{1}, \omega_{x} \rightarrow \omega_{z}, \omega_{z} \rightarrow \dot{\psi}+\dot{\phi} \cos \theta
$$

* Not standard notation.

Fig. 8-22 Halfman's coordinates.
we wall eventually get [c.f. Fiq. (8.16), (8.17), (8.18)]

$$
\begin{array}{cc}
\left(N_{p}\right) & N_{x}-I_{1} \ddot{\theta}-I_{1} \dot{\phi}^{2} \sin \theta \cos \theta+1_{3} \dot{\phi} \omega_{z} \sin \theta \\
\left(N_{e}\right) & N_{y}=I_{1} \ddot{\phi} \sin \theta-2 I_{1} \dot{\phi} \dot{\theta} \cos \theta-I_{3} \omega_{z} \dot{\theta} \\
\left(N_{s}\right) & N_{z}=1_{3} \frac{d \omega_{2}}{d t} \tag{8.88}
\end{array}
$$

Note that now the z-axis is along the axis of symmetry (instead of the x-axis as before) and the x-axis is perpendicular to the vet' 'e-sun line (instead of the previous $y$-axis). Because torques about certein coordinate axes are closely associated with definite motions of the spacecraft, $\lambda_{x} \lambda_{y}$ and $N_{z}$ are commonly referred to as the "precession torque", "erecting torque", and "spin torque" respectively ( $\mathrm{N}_{\mathrm{p}}, \mathrm{N}_{e}$, and $\mathrm{N}_{s}$ ). These torques are usually a function of $\theta$ only. It is also important to notice that, until now, no approximations have been made, and the only assumption used has teen that of axial symmetry (about the $z$-axis) for the spacecraft.

In order to determine the steady rotation rates of each of the two cascaded systems, we first must require that $\dot{\theta}=\ddot{\theta}=\ddot{\phi}=\dot{\omega}_{z}=0$. Then if we solve Eq (8.86) for $\dot{\phi}$ using the quadratic formula, we get

$$
\begin{equation*}
\dot{\phi}=\frac{\mathrm{I}_{3} \omega_{z}}{2 \mathrm{I}_{1} \cos \theta}\left[1 \pm \sqrt{\left(1-\frac{1}{\mathrm{I}_{3}^{2} \omega_{z}^{2} \sin \theta}\right)}\right] \tag{0.89}
\end{equation*}
$$

where the assumption of steady precession has been introduced. As usual in such situations, the minus sign corresponds to the slower mode of precession in which we are interested, and as long as $\mathrm{N}_{\mathrm{x}}$, the precession torque, is sufficiently small (i.e.,

$$
\left.\mathrm{N}_{\mathrm{x}} \ll \frac{\mathrm{I}_{3}^{2} \omega_{\mathrm{z}}^{2} \sin \theta}{4 \mathrm{I}_{1} \cos \theta}\right)
$$

we may write

$$
\begin{equation*}
\dot{\phi} \cong \frac{\mathrm{N}_{\mathrm{x}}}{\mathrm{I}_{3} \omega_{\mathrm{z}} \sin \theta} \tag{8.90}
\end{equation*}
$$

This approximation corresponds to the case when the direction of angular momentum and the z-axis nearly coincide. One now requires that the first of the two cascaded systems rotate at the constant rate given by Eq (8.89),
as well as that its z-axis be aligned, the angular momentum of the vehicle (instead of the figure axis as before). This should work out all right, even relaxing the requirement that $\ddot{\theta}, \dot{\theta}$, and $\dot{\phi}$ be zero as long as the absolute value of $\mathrm{N}_{\mathrm{x}}$ does not change much with a given range of $\theta$ so that the angular momentum vector will precess at a nearly constant rate (recall that the angular momentum does not change much in a nutation cycle).

The second rotating coordinate system of $x^{\prime}, y^{\prime}$, and $z^{\prime}$ (see Fig. 9-23) will rotate about the (slowly) precessing $z$-azis of the first system, will have its $z^{\prime}$-axis always aligned with the vehicle's axis of symmetry, and its $x^{\prime}$-axis perpendicular to the $z$-axis. If $N_{X}$ is small or does not change appreciably within the range of any given $\theta$ oscillation (nutation), then for this $\theta$ the net effect of $N_{x}$ on the nutation amplitude will be negligible, at least in one cycle. Also, because the nutational angular rates are much, much faster than the precessional rates, we may solve approximately for the motion in the second system by writing down the same equations as before but with $N_{x}=0$. Solving for $\dot{\phi}^{\prime}$ (with no further approximations):

$$
\begin{equation*}
\dot{\phi}^{\prime}=\frac{I_{3}}{I_{1}} \frac{\omega_{z}}{\cos \theta^{\prime}} \tag{3.91}
\end{equation*}
$$

which corresponds to the "torque-free" or "wobble" mode. Also, $\dot{\phi}^{\prime}$ will be the constant rate at which the $x^{\prime}, y^{\prime}, z^{\prime}$ system precesses about the more slowly precessing z -axis.

### 8.11.3 Spacecraft Motion in Terms of Energy

Two very important points can now be made. First, suppose that there are absolutely no torques on the spacecraft and that it is both spinning at $\omega_{z}$ and "wobbling" at $\dot{\phi}^{\prime}$. Given the fact that some of the mechanical energy of such a system can be converted into heat by non-elastic structural damping aboard the spacecraft (such as loose mylar in the vanes), this motion will not necessarily be stable. In the absence of external torques, the angular momentum of the spacecraft must remain constant, but its kinetic energy can change subject to this constraint and in general will tend towards a minimum. If we consider the two possible extremes of the spacecraft's motion in terms of energy, we find that it might be spinning steadily about its axis of symmetry on one hand or a transverse axis on the other. In both cases the angular momentum, L , will be the same so we may write

$$
\begin{equation*}
L=I_{1} \omega_{1}=I_{3} \omega_{3} \tag{3.92}
\end{equation*}
$$


Fig. 8-23 Nutation coordinates.

However, the kinctic energy, $r$, will be different in each case so that

$$
\begin{gather*}
T_{1}=1 / 2 I_{1} \omega_{1}^{2}=1 / 2 I_{1}\left(\frac{I_{3}}{I_{1}} \omega_{3}\right)^{2}=\frac{I_{3}}{I_{1}} T_{3}  \tag{8.93}\\
T_{3}=1 / 2 I_{3} \omega_{3} \tag{8.94}
\end{gather*}
$$

if we substitute for $\omega_{1}$ from above. Thus it is readily seen from $\mathrm{Eq}(8.93)$ that the spin axis of smallest kinetic energy depends entirely upon the ratio of the moments of inertia. In particular, if $\mathrm{I}_{3}>\mathrm{I}_{1}, \mathrm{~T}_{1}>\mathrm{T}_{3}$ and a steady spin about the axis of symmetry has the smaller kinetic energy and hence is the stable motion.

To illustrate this result in a more rigorous nanner, we find that the angular momentum and kinetic energy of a body "wobbling" with a cone angle of $\theta^{\prime}$ are respectively:

$$
\begin{align*}
& L^{2}=I_{1}^{2} \phi^{\prime} \sin ^{2} \theta^{\prime}+I_{3}^{2} \omega_{z^{\prime}}^{2}  \tag{8.95}\\
& T=1 / 2\left(\mathrm{I}_{1} \phi^{\prime} 2 \sin ^{2} \theta^{\prime}+\mathrm{I}_{3} \omega_{z^{\prime}}^{2}\right) \tag{8.96}
\end{align*}
$$

Eliminating $\omega_{z}$ :

$$
\begin{equation*}
T=\frac{L^{2}}{2 I_{3}}+\frac{I_{1}}{2}\left(1-\frac{I_{1}}{I_{3}}\right) \dot{\phi}^{2} \sin ^{2} \theta^{\prime}=\frac{L^{2}}{2 I_{3}}\left[1+\left(\frac{I_{3}}{I_{1}}-1\right) \sin ^{2} \theta\right] \tag{8.97}
\end{equation*}
$$

It is clear from this last equation that if $I_{3}>I_{1}$, then $T$ monotonically decreases with decreasing $\theta^{\prime}$, reaching a minimum at $\theta^{\prime}=0$. Therefore as before, the case where $\theta^{\prime}=0$ (or a pure spin about the $z^{\prime}$-axis) will be the only stable dynamical mode. For this reason, the Sunblazer spacecraft has been designed to have $I_{3}>I_{1}$ so that rotation about the axis of symmetry will be stable. If $\mathrm{I}_{1}>\mathrm{I}_{3}$, then it would be energetically possible for the properlyspinning spacecraft to convert axes and end up tumbling instead.
The second important point to be noticed here is a possible confusion of the aspect sensor on the front of the spacecraft. Because as we have just shown, the spacecraft must be designed so the $I_{3}>I_{1}$, it can be seen from Eq (8.91) that the vehicle actually precesses faster than it spins! If nutations or "wobbling" of the spacecraft were not damped out, and if the amplitude were large enough so that the nutational mode caused the axis of symmetry to encircle the sun, then the aspect sensor would record the spacecraft as spinning opposite to its true spin sense; this could
badly affect the attitude-control system logic. In order to fix this dea a litto more rigorously, if $w_{z}{ }^{\prime}$, is the actual sidereal spin rate, then dt can
 In the nominal operating mode $\boldsymbol{w}_{2}^{\prime} \gg \dot{c}^{\circ}$ so that $\boldsymbol{y}^{\prime}{ }^{\prime} \boldsymbol{\omega}_{z}{ }^{\prime}$. However, since for the torque-free mode $\dot{\phi}^{\prime}>\omega_{2}^{\prime}$ whenever $I_{3}>I_{1}($ see Eq (is.91)). it is apparent that if the sun is ever encircled by the asial projection of the nutathonal mode, $\psi^{\prime}$ will be negative even though both $w_{2}^{\prime}$ and $\dot{\phi}^{\prime}$ are positive. Therefore, it is most fortunate that, because of its rather large induced angular accelerations, the nutational mode is readily dissipated by small, passive mechanical devices; in fact, it has been demonstrated through many hours of experience in U.S. space programs (e.g. ().S. ().) that such devices can be both lightweight and mechanically simple.

### 8.11.4 Nutational Amplitude

Even though existing and space-tested hardware can effectively eliminate the nutational mode, it is still desirable for the sake of completeness to determine how its amplitude will be affected by the known external torques on the spacecraft. In order to do this, we now return to the two cascaded coordinate systems used to derive Eq (8.91). Kecall that we had synthesized a combination of steady precession and steady torque-free modes to approximate a real nutation caused by very weak exteinal torques. In order to determine the behavior of $\theta^{\prime}$ under the influence of these torques, we now write Eq (8.86) (8.88) in the $x^{\prime}, y^{\prime}, z^{\prime}$ system, and in each substitute Eq (8.91) fol $\phi^{\prime}$ to get:

$$
\begin{gather*}
N_{x^{\prime}}=I_{1} \ddot{\theta}^{\prime}  \tag{8.98}\\
N_{y^{\prime}}=I_{1} \ddot{\phi}^{\prime} \sin \theta^{\prime}+\dot{\theta}^{\prime} I_{3} \omega_{z}^{\prime}  \tag{8.99}\\
N_{z}^{\prime}=\dot{\omega}_{z^{\prime}} \tag{8.100}
\end{gather*}
$$

where it must be kept in mind that the $\mathrm{N}_{\mathrm{x}}{ }^{\prime}, \mathrm{N}_{\mathrm{y}}{ }^{\prime}$, and $\mathrm{N}_{\mathrm{z}}{ }^{\prime}$ are the result of time averaging a combination of $N_{x}, N_{y}$, and $N_{z}$ resolved into the $x^{\prime}, y^{\prime}, z^{\prime}$ system. For a more mathematical description of this, expand $N_{x}(\theta), N_{y}(\theta)$, and $N_{z}(\theta)$ in a first-order Taylor series about $\theta_{o}$ (assuming that these torques are a function of $\theta$ only) so that:

$$
\begin{align*}
& \mathrm{N}_{\mathrm{x}}(\theta) \approx \mathrm{x}_{\mathrm{o}}+\mathrm{x}_{1}\left(\theta-\theta_{0}\right)  \tag{8.101}\\
& \mathrm{N}_{\mathrm{y}}(\theta) \approx \mathrm{y}_{\mathrm{o}}+\mathrm{y}_{1}\left(\theta-\theta_{0}\right)  \tag{8.102}\\
& \mathrm{N}_{\mathrm{z}}(\theta) \approx \mathrm{z}_{\mathrm{o}}+\mathrm{z}_{1}\left(\theta-\theta_{0}\right) \tag{8.103}
\end{align*}
$$

Then if the nutation amplitude is small (a justifiable assumption for a stability analysis) we find that, as a direct consequence of the dynamical geometry and defintions (see Fig. 8-13):

$$
\begin{equation*}
\theta\left(\phi^{\prime}\right) \approx \theta_{0}-\theta^{\prime} \sin \phi^{\prime} \tag{8.104}
\end{equation*}
$$

and so:

$$
\begin{align*}
& N_{x}\left(\phi^{\prime}\right)=x_{0}-x_{1} \theta^{\prime} \sin \phi^{\prime}  \tag{8.105}\\
& N_{y}\left(\theta^{\prime}\right)=y_{0}-y_{1} \theta^{\prime} \sin \phi^{\prime} \\
& N_{z}\left(\phi^{\prime}\right)=z_{0}-z_{1} \theta^{\prime} \sin \phi^{\prime}
\end{align*}
$$

Notice further that the resolved components of $N_{x}, N_{y}$, and $N_{z}$ onto $N_{x}$ / $N_{y}{ }^{\prime}$, and $\mathrm{N}_{\mathrm{z}}$ are such that

$$
\begin{gather*}
N_{x^{\prime}}=-\sin \phi^{\prime} N_{x}+\cos \phi^{\prime} N_{y}  \tag{8.108}\\
N_{y^{\prime}}=-\cos \phi^{\prime} N_{x}-\sin \phi^{\prime} N_{y}  \tag{8.109}\\
N_{z^{\prime}}=N_{z} \tag{8.110}
\end{gather*}
$$

replacing $\mathrm{N}_{\mathrm{x}}, \mathrm{N}_{\mathrm{y}}$, and $\mathrm{N}_{\mathrm{z}}$ through Eq (8. 105) - (8. 107):

$$
\begin{align*}
\mathrm{N}_{\mathrm{x}^{\prime}}\left(\phi^{\prime}\right)= & -\sin \phi^{\prime}\left(x_{0}-\mathrm{x}_{1} \theta^{\prime} \sin \phi^{\prime}\right)  \tag{8.111}\\
& +\cos \phi^{\prime}\left(y_{0}-y_{1} \theta^{\prime} \sin \phi^{\prime}\right)
\end{align*}
$$

$$
N_{y^{\prime}}\left(\phi^{\prime}\right)=-\cos \phi^{\prime}\left(x_{o}-x_{1} \theta^{\prime} \sin \phi^{\prime}\right)
$$

$$
\begin{equation*}
-\sin \phi^{\prime}\left(y_{0}-y_{1} \theta^{\prime} \sin \phi^{\prime}\right) \tag{8.112}
\end{equation*}
$$

$$
\begin{equation*}
N_{z} \prime\left(\phi^{\prime}\right)=z_{o}-z_{1} \theta^{\prime} \sin \phi^{\prime} \tag{8.113}
\end{equation*}
$$

Averaging these equations over one complete nutation cycle or for $0<\phi^{\prime}<2 \pi$ :

$$
\begin{gather*}
\overline{\mathrm{N}}_{\mathrm{x}^{\prime}}=1 / 2 \mathrm{x}_{1} \theta^{\prime}  \tag{8.114}\\
\overline{\mathrm{N}}_{\mathrm{y}^{\prime}}=1 / 2 \mathrm{y}_{1} \theta^{\prime}  \tag{8.115}\\
\overline{\mathrm{N}}_{z^{\prime}}=\mathrm{z}_{0} \tag{8.116}
\end{gather*}
$$

Recalling that $x_{1}=\left.\frac{d N_{x}(\theta)}{d \theta}\right|_{n_{0}}, \quad y_{1}=\left.\frac{d N_{y}(\theta)}{d \theta}\right|_{0} ^{0} \quad_{0}=N_{2}(\theta)$


$$
\begin{gathered}
1 /\left.2 \theta^{\prime} \frac{d N_{x}^{(\theta)}}{d \theta}\right|_{0}=1_{1} \ddot{\theta}^{\prime} \\
1 /\left.2 \theta^{\prime} \frac{d N_{y}(\theta)}{d \theta}\right|_{0}=I_{1} \ddot{\phi}^{\prime} \sin \left(1+\dot{\theta}^{\prime} I_{3} \omega_{z^{\prime}}\right. \\
\left.N_{z}^{\prime}(\theta)\right)=\dot{\omega}_{z^{\prime}}
\end{gathered}
$$

If $\theta^{\prime}$ is small then, consistent with the first-order analysis, $\ddot{\theta}^{\prime}$ can surely be neglected, thus eliminating liq (8,117). Likewise for a stability analysis, neglect the product, $\dddot{\phi}^{\prime} \sin \theta^{\prime}$, compared to $\theta^{\prime} \omega_{x}$, and get:

$$
\begin{gather*}
1 /\left.2 \theta^{\prime} \frac{\mathrm{d} \mathrm{~N}_{\mathrm{y}}(\theta)}{\mathrm{d} \theta}\right|_{\theta}=1_{3} \omega_{z^{\prime}} \dot{\theta}^{\prime}  \tag{8.120}\\
\mathrm{N}_{\mathrm{z}}\left(\theta_{0}\right)=\dot{\omega}_{z^{\prime}}
\end{gather*}
$$

or, solving Eq (8.120) for $A$ :

$$
\begin{equation*}
\dot{\theta}^{\prime}=\left.\frac{\theta^{\prime}}{2 \mathrm{I}_{3^{\omega} \mathrm{z}^{\prime}}} \frac{\mathrm{dN}_{y}(\theta)}{\mathrm{d} \theta}\right|_{\theta} \tag{8.121}
\end{equation*}
$$

Notice that $\ddot{\theta}^{\prime}$ has the same sign at a given $\theta$ as does $\frac{d N_{y}{ }^{(\theta)}}{d \theta}$ (if we take $\omega_{z^{\prime}}$ to be positive).

In order to estimate further the effects that $N_{y}(\theta)$ will have on an initial $\theta^{\prime}$, recall that $N_{y}(\theta) \approx \mathrm{k} \sin 2 \theta$ for $\theta<60^{\circ}$. Soive Eq (8.87) approximately for $\dot{\theta}$, using this value for $\mathrm{N}_{\mathrm{y}}(\theta)$, and including Eq (8.125):

$$
\begin{align*}
& \dot{\theta} \tilde{\underline{E}}-\frac{\mathrm{k} \sin 2 \theta}{\mathrm{I}_{3} \omega_{2}}  \tag{8.122}\\
& \dot{\theta}^{\prime}=\frac{\mathrm{k} \cos 2 \theta}{\mathrm{I}_{3} \omega_{z}{ }^{\prime}} \theta^{\prime} \tag{8.123}
\end{align*}
$$

Dividing Eq (8.123) by Eq (8.122) and integrating:

$$
\begin{align*}
& \frac{d \theta^{\prime}}{d \theta}=-(\cot 2 \theta) \cdot \theta^{\prime}  \tag{8.124}\\
& \frac{\theta^{\prime}}{\theta_{2}^{\prime}}=\sqrt{\frac{\sin 2 \theta_{2}}{\sin 2 \theta_{1}}} \tag{8.125}
\end{align*}
$$

Thus as $\theta$ decreases from $60^{\circ}$ to $10^{\circ}$, $\theta^{\prime}$ will increase by about a factor of 1.6 (this large value may invalidate the first-order analysis). However, this is actually not a serious problem to cope with, especially since this increase would occur over a period of several days, during which time existing mechanical nutation dampers, which have time-constants that are typically less than one hour, would effectively eliminate this mode. The preceding analysis has been at least qualitatively verified by an analog computer, and more exploration is currently being conducted in this particular area. It is hoped that future results can confirm the conclusions of the preceding treatment, as well as provide a more detailed discuission of mecharical dampers.

### 8.11.5 Reradiative Damping

In addition to the effect of a direct or static torque on a nutation amplitude, there is the damping effect of energy reradiated from the vanes with a lag relative to the input solar intensity. Using a method similar to the method which was originally suggested for Peterson's thesis and which was used to derive Eq (8.66) - (8.75), it is possible to show that the reradiative torques stabilize nutations of the vanes system (derivation is again for small nutational amplitude and hig!: $\omega$ ).
Starting with Eq. (8.64), linearizing the equation, and assuming that:

$$
\begin{gather*}
\theta \approx \theta_{0}+A_{1} \sin \Omega t  \tag{8.126}\\
\psi^{\prime} \approx \omega_{\psi^{\prime}}  \tag{8.127}\\
\omega_{\psi}=\omega\left(1-I_{3} / I_{1}\right) \tag{8.128}
\end{gather*}
$$

and

$$
\begin{equation*}
\Omega \approx \omega \mathrm{I}_{3} / \mathrm{I}_{1} \tag{8.129}
\end{equation*}
$$

as in Eq (8.91) one is led to the following expression:

$$
\text { abs. } 1 \cdot\left[\operatorname{loc} \theta_{0}+a \operatorname{as} \theta_{0} s \psi^{\prime}-b s n_{0} \cdot 0_{1} \sin \Omega t\right.
$$

$\left.-a \quad \frac{c \theta_{0}}{2} \cdot \theta_{1} \cos \left(\Omega t+\psi^{\prime}\right)+a \frac{c \theta_{0}}{2} \cdot \theta_{1} \cos \left(\Omega 2 t-\psi^{\prime}\right)\right]$

Expanding $T$ in the appropriate harmonics, one can solve for the term in $\sin \left(\Omega-\omega_{\psi}\right) \mathbf{t}$ which directly affects the nutation amplitude:

$$
\left.\mathrm{t}_{9}=\frac{\left[\frac{\operatorname{abs} \cdot \mathrm{I} \cdot \mathrm{ac}_{\mathrm{o}}{ }^{\theta} \theta_{1}}{8 \epsilon \sigma \mathrm{~T}_{\mathrm{o}}^{3}}\right]}{\left\{\frac{\left(\Omega-\omega_{\psi}\right) \mathrm{C}_{\mathrm{p}}^{*}}{4 \epsilon \sigma \mathrm{~T}_{\mathrm{o}}^{3}}+\frac{4 \epsilon \sigma \mathrm{~T}_{\mathrm{o}}^{3}}{\left(\Omega-\omega_{\psi}\right) \mathrm{C}_{\mathrm{p}}^{*}}\right.}\right\}
$$

giving an effective "nutational" torque, as in Eq (8.121):

$$
\begin{gather*}
\mathrm{I}_{3} \omega \dot{\theta}_{1} \approx \mathrm{~T}_{\text {nut }}  \tag{8.132}\\
\mathrm{T}_{\text {nut }}=\frac{\left[\frac{\mathrm{abs} \cdot \mathrm{I} \cdot \mathrm{ac} \theta \cdot \theta_{1}}{4}\right]}{\left\{\frac{\Omega-\omega_{\psi}}{\omega_{\mathrm{opt}}}+\frac{\omega_{\mathrm{opt}}}{\Omega-\omega_{\psi}}\right\}} \tag{8.133}
\end{gather*}
$$

where $\omega_{\rho \text { pt }}$ is given by $\mathrm{Eq}(8.68)$ or $\mathrm{Eq}(8.70)$. From this equation it appears that this "torque" is on the same order of magnitude as the direct destabilizing "torque" which could be extracted from the similar Eq (8.121), when $\Omega-\omega_{\psi} \approx \omega_{\text {opt }}$. For higher $\omega$ in the normal range, say two orders of magnitude above $\omega_{\text {opt }}$, the reradiative "torque" becomes much smaller than the direct destabilizing "torque" acting in Eq (8.125). This is another reason why a mechanical damper may be important for the spacecraft.
8.12 Heading and Cone Angle Change (Launch Through Deployment)
8.12.1 Introduction

In order to predict the time necessary to orient the spacecraft utilizing the radiation pressure, knowlenge of the initial angle betereen the spin axis of the spacecraft and the sunline is of great importance, especially if this angle should approach $90^{\circ}$. In this case, the time for orientation increases rapidly.

After burn-out of the last stage of the rocket, the assembly will be in a force-frce precession ' mode, i.e., the spin axis will precess about the angular momentum vector at an angle $\theta$. Events during injection (separation, despin, etc.) will change the angular momentum in both magnitude and direction (heading). The change in heading is proportional to the conc angle before the event, and will either increase or decrease the angle between the original heading and the sunline, according to where on the precessional cone the event occurs.

It is obvious, if the events occur successively at disadvantageous locations on large precession cones, that the angle between heading and sunline can increase to an undesirable (if not unacceptable) value.

Thus, to avoid large heading changes with each event, it is paramount to reduce the cone angle preceding that event.
8.12.2 Analysis

The launch sequence during which the heading may change is as follows (Fig. 8-24): After the successive burn-out of all five stages, it has been recommended that the assembly of the fifth stage and Sunblazer coast for five minutes, during which residual thrust is allowed to tail off. Separation of the payload, ccuring next, will initiate the yo-yo despin. In separating from the spacecraft, afte, nompleted de. pin, the yo-yos will trigger the deployment of sails and anvennas. A damper will then act to decrease the cone angle to zero.

The contribution of each event to the final heading change will now be analyzed separately.

## 1. Rocket Performance

Due to misalignment of the separation mechanism, unbalanced mass, etc, the cone angle after fourth-stage burn-out is expected to be $1.5^{\circ}$ ( $3 \sigma$ value) ${ }^{(13)}$. This has been substantiated by measurements during previous flights. No measurements exist for the new fifth stage. Separation of the fifth stage from the fourth will not appreciably change the cone angle. However, it is expected


Fig. 8-24 Sunblazer deployment.
that thrust misalignment in the fifth stage will increase the angle by $4.5^{\circ}$ for a spin rate of $180 \mathrm{r} / \mathrm{min}{ }^{(13)}$. It is not obvious from the cited reference whether the above $4.5^{\circ}$ is a $1 \sigma$ or $3 \sigma$ value, such that two $3 \sigma$ values should be considered for the cone angle at fifth-stage burn-out:

$$
\begin{align*}
& \text { a. } \theta=1.5^{\circ}+4.5^{\circ}=6^{\circ}  \tag{8.134}\\
& \text { b. } \theta=3 \times(0.5+4.5)=15^{\circ} .
\end{align*}
$$

2. Coasting Period

If a body is in a force-free precessional mode and dissipates energy it will change $t$ ) a configuration of least kinetic energy. The kinetic energy $T$ can be expressed in terms of the cone angle $\theta$, or $\theta$ as a function of $T$, as follows (for an axisymmetric body) ${ }^{(14)}$ :

$$
\begin{equation*}
\sin ^{2} \theta=\frac{I_{1}}{\left(I_{3}-I_{1}\right) L^{2}}\left(2 I_{3} T-L^{2}\right) \tag{8.135}
\end{equation*}
$$

where $I_{1}=$ moment of inertia about transverse axis
$I_{3}=$ moment of inertia about spin axis
$\mathrm{L}=$ angular momentum .
If the energy loss is so small that the rigid body motion is not affected, the rate of change in $\theta$ with T is obtained by differentiating Eq (9.135) with respect to time:

$$
\begin{equation*}
2 \sin \theta \cos \theta \dot{\theta}=\frac{2 \mathrm{I}_{1} \mathrm{I}_{3}}{\left(\mathrm{I}_{3}-\mathrm{I}_{1}\right)} \quad \frac{\dot{\mathrm{T}}}{\mathrm{~L}^{2}} \tag{8.136}
\end{equation*}
$$

or, for small $\theta$

$$
\begin{equation*}
\theta \dot{\theta}=\frac{\mathrm{I}_{1} \mathrm{I}_{3}}{\left(\mathrm{I}_{3}-\mathrm{I}_{1}\right)} \frac{\dot{\mathrm{T}}}{\mathrm{~L}^{2}} \tag{8.137}
\end{equation*}
$$

Since $\dot{T}$ is negative, $\theta$ will increase if $I_{1}>I_{3}$, which is the case for the assembly of the fifth stage and payload.

The dissipation of energy is due to the damped motion of the structural particles relative to each other. The motion can be represented by the equation for forced vibration with damping:

$$
\begin{equation*}
m \ddot{x}+\frac{K g}{\omega} \dot{x}+k x=F_{o} \cos \omega t \tag{8.138}
\end{equation*}
$$

where $m=$ mass
$\mathrm{k}=$ spring constant
$g$ = structural damping coefficient
$\left[\begin{array}{ll}\frac{\mathrm{Kg}}{\omega} & \text { is called the equivalent viscous damping coefficient for } \\ \text { structural damping }\end{array}\right]$
$\omega=$ frequency of forcing function
$F_{0}=$ amplitude of forcing function.
Solving Eq (8.138) gives:

$$
x=\frac{F_{o}}{k} n \cos (\omega t-\eta)
$$

where $\eta=\eta(\omega)$
$n=n(\omega) \cong$ constant for $\omega$ range considered.
The energy dissipated by the damping force is:

$$
\mathrm{E}=\int_{\mathrm{cycle}} \frac{\mathrm{Kg}}{\omega} \dot{x} \mathrm{dx}=\pi \mathrm{Kg}\left(\frac{\mathrm{~F}_{\mathrm{o}}}{\mathrm{k}} \mathrm{n}\right)^{2}
$$

or per unit time

$$
\begin{equation*}
\dot{\mathrm{T}}=\frac{1}{2} \mathrm{Kg} \omega\left(\frac{\mathrm{~F}_{\mathrm{O}}}{\mathrm{k}} \mathrm{n}\right)^{2} \tag{8,139}
\end{equation*}
$$

The forcing function in this case is the inertial force (mass $\times$ acceleration) acting on each point of the structure. The acceleration can be expressed as:
$\ddot{x}=\omega_{z}^{2}\left|\left(\frac{I_{3}}{I_{1}}\right)^{2} z \tan \theta \sin \left[\omega_{z}\left(1-\frac{I_{3}}{I_{1}}\right) t\right]+\left(\frac{I_{3}}{I_{1}}\right)^{2} \tan ^{2} \theta \cos ^{2}\left[\omega_{z}\left(1-\frac{I_{3}}{I_{1}}\right) t\right]-x\right|$.
This is for a point with coordinates ( $x, o, z$ ), $z$ being along the spin axis.
The angular velocity about the spin axis is $\omega_{z}$.
For small $\theta$, neglecting higher orders of $\theta$, and considering only the oscillatory part:

$$
\begin{equation*}
\ddot{x} \sim \omega_{z}^{2} \theta z\left(\frac{I_{3}}{I_{1}}\right)^{2} \sin \left[\omega_{z}\left(1-\frac{I_{3}}{I_{1}}\right) t\right] \tag{8.141}
\end{equation*}
$$

The most important feature in Eq (8.141) is the relation:

$$
\ddot{x}=\ddot{x}(\theta)
$$

consequently

$$
F_{o} \sim F_{o}(\theta)
$$

and using Eq (8.139)

$$
\dot{\mathrm{T}} \sim \dot{\mathrm{~T}}\left(\theta^{2}\right)
$$

Upon sub)stitution of Eq (8.139) into Eq (8.137) and introduction of 7 , the equation for 0 will be

$$
\theta \dot{\theta}=\frac{\theta^{2}}{T}
$$

Integrating gives

$$
\begin{equation*}
\theta=\theta_{0}{ }^{\mathrm{t}} \mathrm{e}^{\mathrm{t} / \tau} \tag{8.142}
\end{equation*}
$$

Thus, the cone angle increases exponentially with the dimensionless time $t / \tau$. $\tau$ has the following expression:

$$
\begin{equation*}
\tau=\frac{2 \mathrm{KL}^{2}}{\mathrm{I}_{3} \mathrm{~g} \omega_{2}^{5}\left(\frac{\mathrm{I}_{3}}{\mathrm{I}_{1}}\right)^{4} n^{2} \mathrm{f}^{2}(\mathrm{~m}, \mathrm{l})} \tag{8.143}
\end{equation*}
$$

$\mathrm{f}(\mathrm{m}, \mathrm{l})$ is a function of mass and size. To evaluate $r$ analytically would be impossible; however, Eq (8.143) can be used to scale $\tau$ from one configuration to another.
This was done ${ }^{(14)}$ for Sunblazer, using information transmitted by LRC. However, this information was so sparse that the deduced $\tau$ values for Sunblazer must still be regarded as guesses. This does not mean that nothing was gained by performing the analysis. It is felt that a range of likely $\tau$ values was established. The corresponding $\theta$ values after five minutes coasting are shown in Fig. 8-25.

$$
\begin{align*}
& \left(\frac{\theta}{\theta_{\mathrm{o}}}\right)_{\mathrm{I}}=2.7 \\
& \left(\frac{\theta}{\theta_{\mathrm{o}}}\right)_{\mathrm{II}}=1.055 \tag{8.144}
\end{align*}
$$

## 3. Sunblazer Separation from Fifth Stage

In separating Sunblazer from the rocket, the springs will ideally impart a linear momentum to the vehicle, leaving the angular velocities about the spin axis and the transverse axis, $\omega_{z}$ and $\omega_{t}$ respectively, unchanged. From geometric considerations:

$$
\begin{equation*}
\tan \theta=\frac{\mathrm{I}_{1} \omega_{t}}{\mathrm{I}_{3} \omega_{z}} \tag{8,145}
\end{equation*}
$$

Applying Eq (8.145) to the configuration, before separation and after, with $\omega_{z}$ and $\omega_{t}$ constant, gives


$$
\begin{equation*}
\tan \theta_{1}=\frac{\left(\frac{I_{1}}{I_{3}}\right)^{1}}{\left(\frac{1}{I_{3}}\right)_{0}} \tan \theta_{0} \tag{8.146}
\end{equation*}
$$

where $\left(\frac{1}{I_{3}}\right)_{0}$ is the moment ratio for the combined Sunblazer and rocket configuration, and $\left(\frac{l_{1}}{I_{3}}\right)_{1}$ for Sunblazer alone.

$$
\begin{aligned}
& \left(\frac{I_{1}}{I_{3}}\right)_{o}=10.8 \\
& \left(\frac{I_{1}}{I_{3}}\right)_{1}=0.635
\end{aligned}
$$

Thus for small $\theta$

$$
\begin{equation*}
\theta_{1}=\frac{1}{17} \theta_{0} \tag{8,147}
\end{equation*}
$$

As mentioned above, this would be true for the ideal release. However, design tolerances in mass unbalance, plunger cant angle, etc., will introduce torque impulses into the system, thus changing the angular velocities. These were shown to be negligible ${ }^{(13)}$. Another deviation from the ideal separation is introduced by a spring force unbalance.

In order to analyze this this effect, consider a symmetric disc spinning steadily about its figure axis at a rate, $\omega$, which has an impulse delivered to it of magnitude $J$, directed parallel to the figure axis. If the disc has a mass $m$, and undergoes a velocity change, $\Delta u$, then the magnitude of the impuise must have been $m \Delta u$. However, suppose this impulse acted on the disc at a distance $r$ from the axis of symmetry. Then if the disc has a moment of inertia $I$, the original angular momentum was $I \omega$ directed along the figure axis. Meanwhile, the impulse has instantaneously introduced angular momentum perpendicular to this axis of magnitude $\mathrm{Jr}=\mathrm{m} \Delta u r$. Therefore, the angle between the original and final angular momentum vectors, $\theta$, is given by

$$
\begin{equation*}
\tan \theta=\frac{m r \Delta u}{I \omega} \tag{8.148}
\end{equation*}
$$

If the spacecraft is not coning initially. then this angle $\theta$ will represent the heading error as well as the coning angle. In the more general case when the vehicle is already coning at separation, $\theta$ represents the maximum change

In the existing coning angle. In this connection, $r$ is to be computed by finding the location of the center of force of the springs relative to the vehicle's axis of symmetry. Suppose $m=10,000 \mathrm{gm}, \Delta!=100 \mathrm{~cm} / \mathrm{s}, \mathrm{r}=\mathrm{cm}, 1=3 \times 10^{6}$ $\mathrm{gm}-\mathrm{cm}^{2}$ and $\omega=20 \mathrm{rad} / \mathrm{s}$, then from Eq (8.148):

$$
\begin{equation*}
\tan \theta=\frac{1}{60} \text { or } \theta \sim 1^{\circ} . \tag{8.149}
\end{equation*}
$$

Since the displacement of the force center is of the order of 1 mm the contribution due to the above effect is negligible, and Eq (8.147) is assumed to be valid.

During this instantaneous change of cone angle the spacecraft remains fixed in space, thus the angular momentum (i.e., heading) must have charged by an amount

$$
\begin{equation*}
\theta_{0}-\theta_{1} \quad . \tag{8,150}
\end{equation*}
$$

## 4. Yo-Yo Despin

An extensive yo-yo despin analysis is given in ref 15. However, no mention is made of the heading change. An indication of the maximum heading change for certain despin parameters is given by idealizing the despin process. Consider an instantaneous despin, Fig. 8-26a, where only the angular velocity, $\omega_{z}$, about the spin axis is decreased. The angular velocity $\omega_{t}$ is assumed to remain constant. Thus, the angular momentum vector $\vec{L}_{o}$ before despin is decreased by an amount $\Delta \overrightarrow{\mathrm{L}}$, which is parallel to the spacecraft- spin axis, resulting in $\vec{L}_{1}$. After the despin process the spin axis then precesses about $\overrightarrow{\mathrm{L}}_{1}$. The cone angle is increased to $\theta_{1}$, where:

$$
\begin{equation*}
\tan \theta_{1}=\frac{I_{1} \omega_{t}}{I_{3} \omega_{z 1}} \tag{8.151}
\end{equation*}
$$

or in terms of the cone angle $\theta_{0}$ before despin:

$$
\tan \theta_{1}=\frac{\omega_{z o}}{\omega_{z 1}} \tan \theta_{0}
$$

With appropriate values for Sunblazer $\left(\omega_{z o}=200 \mathrm{r} / \mathrm{min}, \omega_{z 1}=33 \mathrm{r} / \mathrm{min}\right)$ this leads to

$$
\begin{equation*}
\tan \theta_{1}=6.06 \tan \theta_{0} \tag{8.152}
\end{equation*}
$$

From Fig. 8-26 it is seen that the heading has changed from the original (before despin) by an amount

$$
\begin{equation*}
\theta_{1}-\theta_{0} \tag{8.153}
\end{equation*}
$$



Fig. 8-26 Angular variations during despin.

In the plane formed by $\vec{L}_{0}$, and the instantaneous position of the spin axis at despin. Figure 8-266 is a top view of Fig. 8-26a showing the angular momenturn shift.

However, the actual despin process is not instantaneous since the precession rate is of the order of $20 \mathrm{rad} / \mathrm{s}$ proor to despin, and the de spin takes place in about one second. Thus, insteat of tracing a straight line on a plane parallel to the $x-y$ plane as in Fig. $8 \cdot 26 b \overrightarrow{\mathrm{l}}$. will be displaced along a curved lime in that plane (Fig. 8-Z6c). Since sunblazer's spin axis will precess about $\vec{l}$. fastest in the beginning of the despin process and slower towards the . when the spin is greatly reduced, the line from $l_{0}{ }^{0} l_{1}$, will have a 'imall radtus of curvature in the loggoning and approach a straight line close to $L_{1}$. Clearly, the length of the lire between $L_{\text {, }}$ and $I_{1}$ is a measure of the angular momentum reduction; therefore, if this is the same amount for both cases (instantaneous and actual) the distance between $L_{O}$, and $L_{1}$ is less in the actilal case. i.e. smaller coning angle and heading change. If. in addition the transverse angular velocity $\omega_{i}$ is reduced, this will reduce the cone angle and heading change even more.

Thus, the instantaneous analysis will give the maximum heading change and cone angle for a particular despin process. It will be used as an approximation to the actual process, bearing in mind that the increases in cone angle and heading are larger than the real changes.
Curves for 0 in ref 15 show values less than the idealized values and also indicate that $\omega_{t}$ is reduced, confirming the above.
5. Deployment of Sails and Antennas

The primary effect of the deployment on the dynamics of motion is a reduction in spin. This is due to the increased moment of inertia about that axis. Since the angular momentum is constant during this process

$$
\begin{equation*}
\left(I_{z} \omega_{z}\right)_{o}=\left(I_{z} \omega_{z}\right)_{1} \tag{8.154}
\end{equation*}
$$

With $\mathrm{I}_{\mathrm{zo}}=2.8 \times 10^{6} \mathrm{gm-cm}^{2} \mathrm{I}_{21}=4.6 \times 10^{6} \mathrm{gm}_{\mathrm{gm}}{ }^{2}$ and $\omega_{z_{0}}=33 \mathrm{r} / \mathrm{min}$.
Eq (8. 154) gives

$$
\omega_{z 1}=20 \mathrm{r} / \mathrm{min}
$$

In the ideal deployment the cone angle and heading will not change (angular momenta constant). In case the sails do not deyly at the same time, the following can happen:

Consider Fig. 8-27a. The case of one sail deploying at a time is investigated. The rolled- up sail ( $m_{2}$ ) is idealized as a mass, and the potential energy $E$ is assumed to be stored in the hent rod connecting it to the vehicle ( $m_{1}$ ). When $t$ 'ie sail is released, part of that energy will be converted, through the action of the bending moment $M$, itio rotational energy of the vehicle, and similarly for the sail. However, since no net torque is applied to the combined body, the angular momentum about the combined $\mathbf{C} . \mathrm{M}$. has to be constant. (For convenience, it is taken to be zero.) If both bodies are acted upon only by moments, the angular momentum of each is due to rotation about their own C. M.

Thus,

$$
\begin{gather*}
I_{1} \omega_{1}+I_{2} \omega_{2}=0  \tag{8.155}\\
E=\frac{1}{2} I_{1} \omega_{1}^{2}+\frac{1}{2} I_{2} \omega_{2}^{2} . \tag{8,156}
\end{gather*}
$$

Eq (8. 155) and (8, 156) give solutions for $\omega_{1}$ and $\omega_{2}$

$$
\begin{gather*}
\omega_{1}=\sqrt{\frac{2 E}{I_{1}\left(1+\frac{I_{2}}{I_{1}}\right)}} \\
\omega_{2}=\sqrt{\frac{2 E}{I_{2}\left(1+\frac{I_{2}}{I_{1}}\right)}} \tag{8.157}
\end{gather*}
$$

The angular momentum given to the vehicle (or sail) is

$$
\begin{equation*}
\mathrm{I}_{1} \omega_{1}=\sqrt{\frac{2 \mathrm{EI}_{1}}{\left(1+\frac{I_{1}}{\mathrm{I}_{2}}\right)}} \sim \sqrt{2 \mathrm{EI}_{2}}=\mathrm{I}_{2} \omega_{2} \tag{8.158}
\end{equation*}
$$

since $I_{1}=0\left(10^{6}\right)$ and $I_{2}=0\left(10^{2}\right)$. For a spinning vehicle this would result in a cone angle $\theta$

$$
\begin{equation*}
\tan \theta=\frac{\mathrm{I}_{1} \omega_{1}}{\mathrm{I}_{\mathrm{z}} \omega_{z}} \tag{8.159}
\end{equation*}
$$

ch for Sunblazer gives $\theta \sim 1^{\circ}$. However, since the sail will remain attached to the vehicle, they both cannot keep rotating indefinitely. Due to the opposite rotations of sail and vehicle, the sail will bend the other way after being stretched out, and accumulate potential energy until the motion is stopped. Thus, an oscillatory metion will result which in time will be
(a)


(c)

(d)

Fig. 8-27 Effects of seil deployment on spacecraft dynamics.
damped out by internal friction. This induced motion should not affect the force-free precession at some cone angle (angula! .oments have not changed). but can be regarded as a nutation (Fig. 8-27b).

The above analysis is true if energy is stored by rolling the sails as in Fig. 8-27c. This way, the potential energy has the same sign and the resulting moment the same direction. If the sail is rolled as in Fig. 8-2.7d, the spring constant can be adjusted so that each bend contains the same energy in magnitude, but with reversed signs. The total stored energy is zero and no motion can result after deployment.
8.123 Results

The analysis gave the quantitative effect of each step in the launch sequence. Depending on the position in which, relative to a fixed coordinate system in space, these steps happen, different headings can result. The best and worst positions are shown graphically in Fig. 8-28. Although both the worst and best cases are shown for the positions at which these steps occur only the worst case expected for each event is shown; i.e., the smallest $\tau$ value (Fig. 8-25) was applied to Eq (8.134) (3 6 values) resulting in cone angles after coasting of $16^{\circ}$ and $32^{\circ}$ respectively. In despinning the vehicle, the ideal conditions irt zhown (Eq (8.152)) rather than the lesser cone angle for the actual despin.
This way, the heading can make an angle with the sunline of $100^{\circ}$ and $80^{\circ}$ in the worst case (posi on-wise) (corresponding to $15^{\circ}$ and $6^{\circ}$ in Eq (8. 134), respectively) and $20^{\circ}$ and $40^{\circ}$ in the best case.

The most optimistic result would be to take $1 \sigma$ values after rocket performance, i.e., $2^{\circ}$ instead of $6^{\circ}$ in Eq (8.134). Then assume the largest $\tau$ expected, which gives a cone angle of $3.16^{\circ}$ after coasting. The effect of separation is fixed, i.e., reduction of cone angle by a factor of 17 , giving a cone angle before despin of $0.186^{\circ}$. Without numerically solving the problem, only an estimate can be made for the angle increase due to despin. If it is estimated that this increase will be a factor of 3 , the final heading in the best and worst positions will be $56.6^{\circ}$ and $63.4^{\circ}$, respectively.

### 8.12.4 Discussion

As stated in the introduction, the concern is about the largest possible angle between heading and sunline. The results show that this angle can be expected to lie between $63.4^{\circ}$ and $100^{\circ}$-- a wide range. From Fig. 8-28 it is seen that the largest contribution is due to the cone angle after coasting; subsequent events only increase the heading error by minor amounts (i.e., from $76^{\circ}$ after

coasting to $80^{\circ}$ after deployment; or $92^{\circ}$ to $100^{\circ}$ ). Of course, the cone angle after coasting is dependent on the angle aftrer rocket performance.

Unfortunately, the information received from LRC on both these events is not adequate to predict a narrow range of cones angles after coasting. However, it is felt that a $15^{\circ}$ cone angle after rocket performance is quite unlikely; this would eliminate the $32^{\circ}$ cone in Fig. $8-28$ and the final heading angle of $100^{\circ}$, and leave the range of heading between $63.4^{\circ}$ and $80^{\circ}$.

If it is felt that this range is too close to the $90^{\circ}$ mark, and unless future information about rocket performance and coasting effects reduce the maximum expected angle, there are two ways of avoiding the large heading angles:

1. Reduce the coasting period to a minimum.
2. Or, constantly monitor the attitude of Sunblazer relative to the sunline, so that the events can be initiated at favorable positions on the precession cones. In this case, large cone angles are desirable.

However, before these steps are employed, the heading change due to the present sequence has to be established more firmly.

## (MAPTER!

### 0.0 THERMAL BALANCE

### 9.1 Discussion

This discussion of the thermal balance for the Sunblazer sparecraft system is divided irito three sections. The first section describes the overall system requirements, the second relates the problems that must be solved in order to satisfy these requirements, while the third section describes the general solutions to these problems.

The Sunblazer spacecraft is to have a retrograde launch into a heliocentric solar orbit. Consequently, the spacecraft will experience a time-varying solar flux which is, on the average, equal to about 1.5 equivalent suns. It varies in magnitude from about 0.96 suns (July launch) to about 2.44 suns. This relatively large average solar flux, coupled with the long duration of the flight, complicates the spacecraft thermal design in that the thermalcontrol surfaces must be stable for an extended period ( 1.5 to 3 years) in a harsh solar environment (1.5 suns). Early in the program it was recognized that, oo obtain spacecraft surfaces whose parameters would remain relatively stable for 4.5 equivalent sun years (1.5 suns times 3 years), was a significant problem. Indeed, when this problem was first studied in 1964 , these requirements were considered to be beyond the state of the art. This is probably not true in 1968; there have been significant advances regarding surface materials in the past four years. Neyertheless, surface stability was/is a formidable problem which has heavily influenced many of the spacecraft design concepts such as solar orientation, disc-cylinder (platform-radiator) construction. wide temperature range electric design, etc.

### 9.2 Requirements of Spacecraft Thermal Design

General requirements of the spacecraft thermal design can be described as follows. The spacecraft electro-mechanical configuration and thermal design must be such that the utilization of currently-existing chemical and mechanical
space components and technologies will result in a thermal system that yirlds spacecraft operatonal-temperature extremes and gradients enmpatible with engineering concepts to satisfy the experimental physics goals. This general statement can in turn be translated into the following spacecraft system constants which are listed in Table $9-1$. A few comments are important in order to interpret meaningfully the data of that table.

1. The results of Table ©-I were derived from "gross or first-order" calculations, and are to be considered accurate to $\pm 5 \%$. Considerably more accurate calculations are possible because the Sunblazer vehicle is a small (low thermal inpedance) symmetrical vehicle without complicated error inducing shapes and booms.
2. Detailed thermal calculations are not deemed important (particularly at this time) as all of the expected operating temperatures are considerably within allowable operating limits of the various electrical, mechanical, and electro-mechanical components, subsystems and systems.
3. The center of the temperature (hence, the limits) range for each individual electronic item, with the possible exception of the solar-cell panels, can be scaled up or down if need be, by adjusting the size and/or the surface coating ( $\frac{\alpha}{\epsilon}$ ratio) of the radiating areas.
4. The solar-cell panel will be set to operate at a minimum temperature in order to maximize energy-conversion efficiency.
5. Inasmuch as the present spacecraft is not intended to carry onboard any sensitive (hence, usually highly temperature-dependent) scientific instruments, the actual spacecraft operating temperatures are not critical. The general desire, therefore, is to operate at a low temperature in order to maximize available power and estend component life. It is especially important to start at as low an operating temperature as feasible, in that degradation of the ther-mal-control surface will ultimately bias the thermal balance corstants toward higher values.

### 9.3 Problems

There are two fundamental problems associated with Sunblazer thermil balance. The first is a large variation in solar flux, which accounts for the rather large temperature ranges shown in Table 9-I. Second is the long operating life required of the spacecraft. This second requirement is a
Table 9-I
Sunblazer spacecraft thermal constants.

*Program and Telemetery (Low power digital and analog circuits)
problem because the relatively high average of solar flux (about 12, 700 equivalent sun hours per year) generally affects the thermal-control surfaces so as to reduce their effectiveness, i.e. . long-term drift of $\frac{\alpha}{\epsilon}$ (the absorptivity, $\alpha$, increases, and the emittance, $\epsilon$, decreases with time). (1, 2, 3, 4)

The Sunblazer design concent minimizes these long-term effects because the spacecraft is oriented toward the sun. This co:sequently allows the use of higi-performance (low absorptivity), highly-stable (low drift) secondsurface mirrors on the sunlit portion; while on the shaded (dark) side of the spacecraft the surface which is made highly emissive and is unaffected by the sun's ultraviolet, since it is in perpetual darkness. Unfortunately, the surface properties of the solar cell areas, which necessarily represent a large fraction of the sunlit side, will degrade in time (principally through ultraviolet darking of the cover shields). However, the design attempts to minimize this degradation by utilizing annealed sapphire covers instead of quartz. ${ }^{(5,6)}$

A third problem affecting thermal balance concerns control of thermal gradients by adjusting the various thermal impedances. The conceptual design calls for minimizing the thermal impedances between the solar cells and the platform, as well as between the platform and the radiator. Additionally, the design should maximize the thermal impedance (minimize thermal leakage) between the platform-radiator assembly and the electronic compartments. Low-impedance joints are to be constructed by brazing the platform-radiator assembly. The high-impedance joints (shields) can be constructed by inserting several layers of highly-reflective material between the electronics and compartment walls. ${ }^{(7,8)}$

### 9.4 Solutions

The Sunblazer thermal balance concepts, along with the solutions to the system problems, are described by the following analyses.
9.4.1 Thermal Analysis

Referring to Fig. 9-1 and 9-2, Eq (1) represents the thermal balance condition. From Eq (1) the temperature may be expressed in terms of the solar input, as shown in Eq (2). The results of Eq (2) are plotted in the upper righthand corner of Fig. 9-2, and we conclude that if the Sunblazer spacecraft were an isothermal body in thermal equilibrium, the temperature range of the spacecraft for 1 AU and 0.635 AU would be $7^{\circ} \mathrm{C}$ to $77^{\circ} \mathrm{C}$.


Fig. 9-1 Sunblazer surface properties.


A simplified calculaton for the rectronce compartment temperature is shown In Fig. :-3. The values used for the solamerell power. RF power output, and radhator surface areas are mperentative of the actual spacecratt and we calculate that the varation the ambent temperature for the electromes. neglecting leakage from the compartment walls, is from $-10^{\circ} \mathrm{C}$ t $1+38^{\circ} \mathrm{C}$.
Fhgure !- thows ${ }^{(9)}$ that the thermal leakage (worst case that is evaluated at the maximum temperature) about 12 W per compartment thes when compared to the 21 W of internal dissipation, is essentally negligible.

### 9.4.2 Sail Tomperature

The values used for the computathon of the sall equilibrium temperature shown in Fig, 9-5 are indsative of the actual sall configuration constructed with aluminum-coated mylar. The computation neglects leakage from the snacecraft, wheh conceptually may be made equal to zero (the sails operate at the same temperature as the spacecraft radiator) or larger if the salls are utilized as additional radiation surfaces. (This technique could be used th reduce the spacecraft temperature.)

Although the variation in sail temperature for a 0.635 orbit is $70^{\circ} \mathrm{C}$, a salient feature regarding this parameter is the difsign ability to set this temperature to cover almost any desired range by varying the $\frac{\alpha}{\epsilon}$ ratio of the front and back surfaces of the sail. The range from $-30^{\circ} \mathrm{C}$ to $+40^{\circ} \mathrm{C}$ seems to be a reasonable choice when utilizing aluminum-coated mylar.

### 9.4.3 Materials

### 9.4.3.1 Surfaces

The sunlit surface of the spacecraft is entirely covered (99\%) with $1400 \mathrm{~cm}^{2}$ of solar cells and $425 \mathrm{~cm}^{2}$ of second- surface mirrors (silver-fused silica). Although second-surface mirrors are extremely resistant to ultraviolet and other radiation damage ${ }^{(10)}$, the solar cell covers are subjected to "ultraviolet darkening" (browning) from prolonged exposure to space environment.

Annealed sapphire is more resistant to radiation damage than are fused quartz, synthetic fused silica, or the non-browning lead glasses, therefore will yield superior performance in the Sunblazer orbit. The thresholdintegrated flux for low-energy proton damage in fused-silica material is apparently between $5 \times 10^{16}$ and $1 \times 10^{17}$ protons $/ \mathrm{cm}^{2}$. A total integrated dosage of $1.3 \times 10^{17} 10-\mathrm{keV}$-protons $/ \mathrm{cm}^{2}$ at $298^{\circ} \mathrm{K}$ results in a measurable increase in absorptivity. The above numbers indicate that radiation damage due to the solar wind is probably not a problem. However, little useful data

## Solar Cells Output (19-33)watts

Dissipation - (19-33)w-(6-12)w(Rf) $=(12-21) w$
Radiator Curface $360 \mathrm{~cm}^{2}$

$$
\begin{gathered}
T_{\max } \cdot \sqrt[4]{\frac{21 W}{A_{r} \varepsilon 0}} \cdot\left(\frac{21}{360\left(0.915 .67 \times 10^{-12}\right.}\right)^{1 / 4} \cdot 338^{\circ} \mathrm{K}\left(65^{\circ} \mathrm{C}\right) \\
T_{\min } \cdot \sqrt[4]{\frac{13}{A_{r} \varepsilon 0}} \cdot\left(\frac{13}{360\left(0.915 .67 \times 10^{-12}\right.}\right)^{1 / 4} \cdot 290^{\circ} \mathrm{K}\left(17^{\circ} \mathrm{C}\right) \\
\triangle T=48^{\circ} \mathrm{K} \quad\left(-10^{\circ} \mathrm{C} \sim 38^{\circ} \mathrm{C}\right)
\end{gathered}
$$

Fig. 9-3 Electronic compara: ant temperature.

$$
\begin{align*}
& \left.\left.\int_{\varepsilon_{1}}\right|_{\varepsilon_{2}} a_{r} A A \quad \cdot \frac{1}{\frac{1}{\varepsilon_{1}}+\frac{1}{\varepsilon_{2}}-1} O \pi_{1}^{4}-T_{2}^{4}\right) \tag{I}
\end{align*}
$$

$$
\begin{align*}
& \mathrm{T}_{1} \cdot 350^{\circ} \mathrm{K}  \tag{2}\\
& \mathrm{~T}_{2} \cdot 325^{\circ} \mathrm{K} \\
& \left.\left.\left.\right|_{\varepsilon_{1}} \cdot\right|_{\varepsilon_{2} \cdots \varepsilon_{n}}\right|_{\left(q_{1} / A\right)_{n}} \cdot\left(q_{r} / A\right)_{0} \frac{1}{1+N}  \tag{3}\\
& \left.\right|_{\varepsilon=C .09, \varepsilon=0.5} q / A \quad \cdot 20 \frac{1}{\frac{1}{0.09}+\frac{1}{0.05}+\frac{1}{0.5}-1} \cdot \frac{20}{21} \approx 1 \mathrm{mw} / \mathrm{cm}^{2} \tag{4}
\end{align*}
$$

Where: $\quad\left(g_{r} / A\right)_{0}=$ Net Radiation Exchange Without Shield Compartment Polished AI $\varepsilon=0.09 \quad \varepsilon_{\text {Electronics Board }}=0.5$

$$
A=450 \mathrm{~cm}^{2} \quad H_{T} \approx 0.45 \mathrm{w} / \text { Compartment }
$$

Fig. 9-4 Thermal is slation.

$$
\begin{aligned}
& H_{\text {in }}=\cos \theta A a S \\
& H_{\text {in }}=0.9\left(\frac{1}{4}\right)(0.27) 1300 \approx 30 \text { watts } \\
& H_{0}=0 \varepsilon \therefore r^{4} \cdot H_{\text {in }} \\
& T=\left(\frac{H_{i n}}{\sigma \varepsilon A}\right)^{1 / 4}=276^{\circ} \mathrm{K} \quad\left(3^{\circ} \mathrm{C}\right) \quad \text { (Temperature at I AU) } \\
& \left.\Delta T=70^{\circ} \quad \text { (I AU to } 0.635 \mathrm{AU}\right)
\end{aligned} \begin{aligned}
& \text { Fig. } 9-5 \text { Sail. }
\end{aligned}
$$


are avalable regarding solat flares. Ristmates indicate that the solar flare proton (greater than 5 MeV ) flux that the spacecraft might experience is on the order of $1 \times 10^{6}$ protonshem ${ }^{2}$ socest. ${ }^{(11)}$ The characteristics of flare electron flux are unknown. In anv event, use of annealed sapphire and the low-resistance ( 102 cm material) n on p solar cells yields a desinn with the least amount of time degradat on. Beyond this, as a safety factor, the spacecraft power system and electronics will be designed to operate with a greatly reduced power level. This is done by relating the average pulse rate to the solar-cell power available.

The coating material for a dark (sun-shadowed surface) presents no problem. Several surface materials are acceptable. Tentatively, subject to further and more detailed mestigation with surface finish, experts on such matters as ease of application, susceptibility to handling damage, i.e., fingerprints, etc., cost, etc., the coatings such as S-13G-type coating based on silicatetreated zinc oxide* or IIT's 293 (emittance greater than 0.9 ) are acceptable. The sail surfaces merit special consideration; fundamentally, from the thermal balance and control viewpoint, the sail surface is straightforward. As indicated in the simplified analysis, an aluminum-coated mylar with polished front (sunlit) side has low absorptivity, and a high-emittance dark side (painted with zinc oxide) will yield an acceptable temperature range ( $7^{\circ} \mathrm{C}$ to $77^{\circ} \mathrm{C}$ ) over the entire orbit. Also, if a lower $\alpha$ material such as polished silver ( $\alpha=0.07$ ) were used on the front side, or any other material with a low $\frac{\alpha}{\epsilon}$ ratio, the sail temperature can be reduced to a very low value. However, the problem which must be solved is to maintain the integrity (performance) of the surfaces when the sail has been deployed after having been stored for launch in a compact configuration. Additionally, a surface material must be chosen with regard to the stabilization torques, as discussed in Chapter 8. Aluminum-coated mylar appears to satisfy all the requirements and is presently being used; however, we are considering other materials.

### 9.4.4 Othe $r$ Considerations

There do not appear to be any fundamental problems connected with the thermal isolation of the electronics. Indeed, as the analyses indicate even without any intention of thermally isolating the electronics, the operating temperatures are acceptable. The problem of maintaining an acceptably low the rmal impedance, as determined from a number of tests reported in the past, has been solved by dip-brazing the structure.

[^1]
### 9.4.5 Results

Table $9-11$ gives the expected temperatures of the various parts of the spacecraft at 1 AU and 0.635 AU .

Table 9-II
Expected spacecraft temperatures for 3/4-year orbit.

|  | 1 AU | $0,635 \mathrm{AU}$ |
| :--- | ---: | ---: |
| Solar Cells | $7^{\circ} \mathrm{C}$ | $77^{\circ} \mathrm{C}$ |
| Front Plate | $0^{\circ} \mathrm{C}$ | $65^{\circ} \mathrm{C}$ |
| Front Half Cylinder | $-5^{\circ} \mathrm{C}$ | $55^{\circ} \mathrm{C}$ |
| Back Half Cylinder | $-15^{\circ} \mathrm{C}$ | $38^{\circ} \mathrm{C}$ |
| Center Tube | $0^{\circ} \mathrm{C}$ | $5^{\circ} \mathrm{C}$ |
| Electronics | $0^{\circ} \mathrm{C}$ | $10^{\circ} \mathrm{C}$ |
| Sails | $-20^{\circ} \mathrm{C}$ | $50^{\circ} \mathrm{C}$ |
| Aspect Sensors | $0^{\circ} \mathrm{C}$ | $30^{\circ} \mathrm{C}$ |
| Motors | $0^{\circ} \mathrm{C}$ | $20^{\circ} \mathrm{C}$ |
| Antenna | $-10^{\circ} \mathrm{C}$ | $60^{\circ} \mathrm{C}$ |

9.5 Summary

The Sunblazer thermal-balance and control problem has been greatly simplified by designir.g a sun-oriented spacecraft with eiernally sunlit and dark surfaces, maintaining relatively short thermal paths, optimizing the thermal radiator dimensions (see Appendix 3 ), and thermally isolating the electronics and electronic radiators from the main body of the spacecraft. Additionally, the varicus electronics have been designed for wide-temperature operation; and the use of highly temperature-sensitive components has been kept to a minimum. There is much detailed work to be accomplished; but conceptually, the overall design and analysis show that contemporary spacecraft-design techniques, materials, and components are available to accomplish readily the proposed scientific and engineering goals.

## 9. 6 Spacecraft Thermal Test

A study made in June 1967 determined that, if the spacecraft did not orientate into position within a day, the total temperature balance could be below $-7^{\circ} \mathrm{C}$. This was corrected by increasing the radiator's length, which then gave the spacecraft an average temperature above $-7^{\circ} \mathrm{C}$ no matter what its orientation.

A test chamber then was built using infrared radiation as an energy source. The cold wall reached $-313^{\circ} \mathrm{F}$ and the vacuum was $10^{-5}$ torr. Although the source did not match the solar spectrum, it did match its energy flux and was a good "in-house" test chamber. There is only one chamber that will give a true picture of what the vehicle will see, and that is outerspace. Every earth-bound chamber has its shortcomings. The information obtaincd from the test chamber was correlated with that which had been calculated. The information received from the test, which would have been very difficult to calculate was:

1. The electronic side wall runs, on the average, $6^{\circ} \mathrm{C}$ higher than the radiator.
2. The electronic radiator, with a simple rubber insulator will produce a $25^{\circ} \mathrm{C}$ gradient with the electronics side wall at 0.63 AU .

Calculations were made to determine the size of the electronic radiators once it was known how much heat energy had to be dissipated from euch section.

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## CHAPTFR 10

### 10.0 ASPECT SENSOR AND CONTROL

The use of an active solar pressure orientation and stabilization system on the spacecraft requires a device which is capable of measuring the angle formed between the sun line and the spacecraft's major (spin) axis. A subsystem capable of performing this function, which is independent of the spacecraft's main electrical system, consists of a detector, signal-processing electronics, photo-electric transducers, energy storage and an RF tiansmitter. This sub-system is called the Aspect Sensor.

The Aspect Sensor has two principal functions, which are of about equal importance. The first is to process information concerning spin rate and sense with respect to sun angle, and to provide electrical pulses to the saildrive stepping motors to cause a properly sensed pitch of the sails to occur.

The second function of the Aspect Sensor is to provide basic engineering telemetry data during the time interval between launch and initial spacecraft sun aquisition.

### 10.1 The Detector Mechanical Configuration

The detector design consists of photo-electric diodes which have their light source interrupted (chopped) by means of mechanical apertures which are sequentially exposed to sunlight due to the spacecraft's rotation and sunalignment error.

The detector is mounted on the spacecraft's spin axis at the forward end of the Hub, and projects beyond the main solar_cell array by about 1.5 inches. In Fig 10-1 the detector configuration is shown. It is essentially a truncated cone which has a central tubular compartment surrounded by six radially isolated compartments. A photodiode is positioned in each of the seven compartments, and is thereby provided with an individual view angle determined by an aperture which ranges from a pin hole to a nearly full-length slot.


Fig. 10-1 Aspect sensor detector.
 and the photodiodes assoriated with therse are ddentified as wl, w2 and 3.

The central pin hole and the there shots with varied side position and length are used to obtain measurements with angle (o), and their associated photodiodes are identified as 91, 92,93 , and 94.

### 10.1.1 The luetector 1)resign

The wl, wh, and wi photodiode aperture slots are mechanically situated to provide light to their associated diode when the sun angle is greater than 0. $5^{0}$
 light to illuminate $\omega 1$, w2, and wis when the spacecraft potation is counterclockwise as vewed from the solaraell array (normal eotational sense during launch). The three electrical pulses generated by the photodiodes are digitally processed to provide a measurement of rate, since the time betweoneach pulse represents one third of a revolution; and the sense of rotation by monitoring whether the sequence is $\omega 1, \omega 2$, w3 or $\omega 3$, $\omega 2$, w1.

The apertures provided for the measurement of 9 are designed logarithmically to provide four incremental steps below $12^{\circ}$ of sun angle, and three steps between $12^{\circ}$ and $90^{\circ}$.

Figure 10-2 shows this logarithmic progression of $\theta$ diode view angles:
$\theta 1$ observes between $0^{\circ}$ and $3.5{ }^{\circ}$
$\theta 2$ observes between $1.8^{\circ}$ and $12^{\circ}$
$\theta 3$ observes between $6.5^{\circ}$ and $44^{\circ}$
$\theta 4$ observes between $24^{\circ}$ and $90^{\circ}$
The overlap of view angles provides seven distinct measurement steps, which are:

$$
\begin{aligned}
& \theta 1=0^{\circ} \pm 01.8^{\circ} \\
& \theta 1 \cdot \theta 2=1.8^{\circ} \text { to } 3.5^{\circ} \\
& \theta 2=3.5^{\circ} \text { to } 6.5^{\circ} \\
& \theta 2 \cdot \theta 3=6.5^{\circ} \text { to } 12^{\circ} \\
& \theta 3=12^{\circ} \operatorname{to} 24^{\circ} \\
& \theta 3 \cdot \theta 4=24^{\circ} \operatorname{to~} 44^{\circ} \\
& \theta 4=44^{\circ} \text { to } 90^{\circ}
\end{aligned}
$$

The pulses generated by the $\omega$ and $\theta$ photodiodes are shown in Fig. 10-3. The seven distinct pulse formats generated by the $\theta$ diodes can be processed and stored in a three-bit digital register as the numbers one through seven.

Fig. 10-2 Detector functional output.


Fig. 10-3 Aspect sensor detector output format.

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## ('IAPTVIR 11

## 

### 11.1 Introduction

The fundamental consideration regarding the Sunblazer spacereralt ponere system ls to generate at all times as moch "useful" power frer mit of mass as possible. In order to do this, the spacecraft is designed so that the solar cell panels are oriented towards the sun and operate at as low a temperature as practicable (see ('hapter 9). The solar-cell energy conversion officiency is maximized by adequately shielding the solar cells to prevent excessive degradation with time, and by operating the panels at the maximum power point throughout the orbit.
11.2 System Description

A simplified diagram of the power systen appears in Fig. 11-1. The 640 ( $1 \times 2 \mathrm{~cm}$ ) n on p blue-shifted $1-\Omega-\mathrm{cm}$ solar cells will deliver approximately 19 W at $0^{\circ} \mathrm{C}(1 \mathrm{Al})$, and 33 W at $70^{\circ} \mathrm{C}(0.635 \mathrm{AU})$, if the solar-cell panel is operated at the maximum power point (MPP).
There are two "I, converter units: ${ }^{*} \%$ one low-power, which operates the tracking beacon and circuits that must be energized during the spacecraft orientation phase; and a high-power unit, which is switched on to supply the high-power pulsed transmitter when the spacecraft is oriented towards the sun.

A set of nickel-cadmium batteries, which is continually recharged from the low power "L" converter, is used as a source of initial energy to activate the solar-sail stepping motors and to run the tracxing beacon. There are several different spacecraft operational modes which depend upon the amount of solar-cell power available (pointing angle with reference to the sun).
These modes are controlled by the sensing power available, and

[^2]
Fig. 11-1 Power system descrindion.
by selection of the proper mode.

### 11.2.1 Solar-cell Panels

Plate 2 shows how the individual solar celis are placed to form the Sunblazer solar panel. Since so much well-detailed material has been documented regarding sol.r-cell panel design, manufacture, operation, and testing for many different space applications, it is not necessary to delineate a set of solar-cell panel specifications in this report. When the Sunblazer panels are manufactured the specifications will be such that: 1) state-of-the-art cells will be used; and, 2) strict compliance with acceptable manufacturing practices (materials and workmanship), as described in various NASA documents, will be enforced.

Special aspects of the Sunblazer panels and mission are:

1. The relatively small number of solar cellis per space-craft (640) should: nable the panel manufacturer to select cells from the upper portion of the distribution curve in order to obtain higher energy efficiency than is feasible with large arrays, and still maintain adequate yields.
2. Better control over initiai cell selection and matching is possible (that is, tighter selection limits can be set on various cell characteristics), with fewer cells, thereby decreasirg the mismatch loss of this array.
3. Flat arrays, where the sun angle to all cells on the array is the same, further decreases the mismatch loss when compared to cylindrical arrays.
The Sunblazer array is made up of 16 parallel strings with 40 cells per string. Table $11-\mathrm{I}$ shows the nominal current and voltage output of the array.

Table 11-I
Nominal current and voltage output
of Sunblazer array

| 1 AU |  |  |  | 0.635 AU |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{T}_{\mathrm{AU}}$ | I | V | P | $\mathrm{T}_{\mathrm{AU}}$ | I | V | P |  |
| $5^{\circ} \mathrm{C}$ | 0.96 A | 20 V | 19.2 watts | $75^{\circ} \mathrm{C}$ | 2.3 A | 14.1 V | 33 watts |  |

### 11.2.2 Power (onverters

### 11.2.2.1 "L." Converter

The problem of utilizing the maximum available power from a power-limited source such as a solar-cell array can be solved by providing an adaptive de-to-dc converter, which automatically adjusts its input parameters so is to take the maximun power from the source.

There are several possible approaches to the problem of providing an adapt ive peak-power-seeking converter. Perhaps the most obvious approach is to sense the power drawn from the source, and to adjust the input parameters so as to maximize this power. If this power-sensing approach is used, the peak-power point can be tracked by continuously varying the converter input parameters and by reversing the sense of parameter variation whenever the sensed power begins to drop. This scheme causes the converter to dither its operation in a small range about the true peak-power point.

A more subtle approach is to use the principle of impedance matching. One notable characteristic of a solar-cell array is the high degree of convexity of its $V-I$ characteristic. Because of this high degree of convexity, it is possible to choose an impedance which will define a point that remains near the peak-power point over a wide temperature and illumination range. A theory of an adaptive converter based on this fact has been discussed in detail in Appendix 2. This conversion technique has the difficulties of a number of possible "latch-up" conditions which must be avoided, and also of doing only an approximate job of tracking the true peak-power point. In a study of techniques to eliminate the aforementioned latch-up conditions with a minimum of extra complexity, an equally simple and very similar system was developed, which would both eliminate the latch-up conditions and provide true peak-power tracking.

It can be shown that, when the static impedance of a load is equal to the incremental impedance of the source to which it is connected, the power being transferred is at a maximum. If the dynamic $V-I$ characteristic of a source is equivalent to the power-determining dc $V-I$ characteristic, then the incremental impedance of the source can be determined by allowing some ripple current to flow in the source, and observing the ratio of the $\Delta V$ and $\Delta I$ produced at the source.

The static input impedance of the converter is of course determined by the ratio $V / I$, the input voltage to the input current. If $\frac{\Delta V}{\Delta I}=\frac{V}{I}$ 'hen maximum
power is being transferred. Rearranging this equation gives the peak-power condition as met when $\frac{\Delta V}{V}=\frac{\Delta I}{I}$. This form of the equation suggests the use of a converter in which $\Delta V$ and $\Delta I$ are controlled to be a fixed fraction of $V$ and I respectively.

By using an inductive flyback 'boost" converter, in which the state of the power switch is controlled by a flip-flop that changes state whenever the input voltage or current falls a fixed percentage of the pre-existing input voltage or current, peak-point power conversion can be accomplished.

The "L." converter that has been built (Fig. 11-2) utilizes this principle. Q 7 is the power switch, and $Q 5$ and $Q 6$ act as a high-current-gain driver to minimize the load on the fip-flop.

Transistors $Q 8, Q 9, Q 10$, and $Q 12$ form the flip-flop, which is set or reset through clamped coupling networks that remove the kase drive from the flip-flop's input transistors whenever the voltage at the input to the coupling network has fallen a given percentage of its initiai value. Q1, Q2, Q3, and Q4 form a linear amplifier, which amplifies the small voltage drop across the current-sensing resistor $R 1$ to a level where it can be coupled directly to the flip-flop. A dual FET input stage is used in this amplifier to eliminate the need for a negative supply voltage to obtain the near-zero input voltage offset that is required in this application. D5 and Q11 form a series regulator to provide control-circuit power.
For this converter $\frac{\Delta V}{V}=\frac{\Delta I}{I}=0.1$. It was found that this was a nearly optimum figure, in that a smaller value would require a correspondingly larger inductor, thus increasing the size and weight of the package; and a larger vaiue would result in less efficient peak-point conversion because of the larger per-cycle variations from the true peak-power point.

It is interesting to note that this method of tracking the peak-power point of a solar-cell array or other electrical-power source does not depend on the source having a cọnvex V-I characteristic or any other special nonlinear V -I characteristic. It will seek the maximum power point and operate nicely, for example, with a source composed of a constant emf and a resistor.
11.2.2.2 " X " Converter

Basically, then, the " X " converter consists of two converters operated at 10 kHz , of which one is a standard Royer circuit. The primary voltage is supplied by a battery through a series regulator. The secondary supplies base drive to the transistors of the second converter, and a rectified dc input to a second series regulator. This provides the required $1 \%$-regulated voltage at output. (See Fig. 11-3)

Fig. 11-2 "L" converter.

Table 11-II
"L" Converter parts list.

| ITEM | COIN: | HESCMIPTION | MANUFACTURI:R | MFC PART NO. |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 181 | Hasistor 0.2.4 8 B W formed in beradmord model from two 0.478 restetors in parallel, pach being: | IHC | 'rype HWH |
| 2 | R2 | Hesistor 1965 | Ohmite | RCOOCiF |
| 3 | 123 | Resistor 300k $5^{\prime \prime}$, | Ohmite | RCOOCiF |
| 4 | R4, R14 | Resistor 100K 5 "\% | Ohmite | RCOO7CiF |
| 5 | R25, R133 <br> R15. R17 <br> R29, R30 | Resistor 10k $\mathrm{S}^{\prime \prime \prime}$ | Ohmite | RC07CiF |
| 6 | Rii, R20 | Resistor 1. $5 \mathrm{~K} 5^{\prime \prime \prime}$, | Ohmite | RCOOCF |
| 7 | 127, R 8 | Hesistor 47K 5 ", | Ohmite | RCOORCF |
| 8 | R ${ }^{1}$ | Resistor 2.7K $5^{\prime \prime \prime}$ | Ohmite | RC200F |
| 9 | R10, R19 | Resistor 0.1K $5^{\prime \prime}$ : | Ohmite | RCOOCiF |
| 10 | 1211 | Resistor 9.1K 5 \% | Ohmite | RC20GF |
| 11 | 112 | Hesistor 4.7K $5^{\prime \prime}$ | Ohmite | RCO7CF |
| 12 | R16 | Resistor 5.6 $\Omega 10 \%$ | Ohmite | RC20GF |
| 13 | R18 | Resistor 1.8K $5^{\circ} \mathrm{F}$ | Ohmite | RC07GF |
| 14 | $\begin{aligned} & \mathrm{R21}, \mathrm{H} 23 \\ & \mathrm{R} 31 \end{aligned}$ | Resistor $62 \mathrm{~K} 5^{\prime \prime}$ | Ohmite | RCO CFF |
| 15 | $\begin{aligned} & \mathrm{R} 22, \mathrm{R} 24 . \\ & \mathrm{R} 32 \end{aligned}$ | Resistor 18K $5^{\prime \prime \prime}$. | Ohmite | RC07GF |
| 16 | R25, R28 | Resistor $27 \mathrm{~K} 5^{\%}$ | Ohmite | RC07GF |
| 17 | 1226 | Resistor 3K $5^{\text {\% }}$, | Ohmite | RCOAGF |
| 18 | R27 | Resistor $36 \mathrm{~K} 5^{\prime \prime}$ | Ohmite | RCO7GF |
| 19 | Q1 | Transistor | Texas Instruments | 2N5045 |
| 20 | Q2, Q3 | Transistor | Solitron Devices | 2 N 2605 |
| 21 | Q4, Q8, Q9, Q10. Q11, Q12 | Transistor | Solitron Devices | 2N930 |
| 22 | Q5 | Transistor | Texas Instruments | 2N718A |
| 23 | Q6 | Transistor | Texas Instruments | 2N4000 |
| 24 | Q7 | Transistor | Texas Instruments | 2N3421 |
| 25 | 11 | Zener Diode | Motorola | 1N746 |
| 26 | D2, D3 | Diode | General Electric | 1 N 4444 |
| 27 | 1)4 | Rectifier | Unitrode | UTR4410 |
| 28 | 105 | Zener Diode | Motorola | 1 N 968 B |
| 29 | C1, C4 | Capacitor $0.022 \mu \mathrm{~F}$ | Cornell-Dubilier | DMF1S22 |
| 30 | C2 | Capacitor $0.01 \mu \mathrm{~F}$ | Cornell-Dubilier | DMF2S 1 |
| 31 | C3 | Capacitor 10 pF | Sprague | 10TCC-G:0 |
| 32 | C5 | Capacitor 0. $22 \mu \mathrm{~F}$ | Cornell-Dubilier | DMF1P22 |
| 33 | T1 | Tapped Inductor | Wilmore Electronics | Special |


Table 11-III
" X " Converter parts list.

| ITEM | CODE | DESCRIPTION | MANUFACTURER | MFG PART NO. |
| :---: | :---: | :---: | :---: | :---: |
| 1 | C1 | $1.0 \mu \mathrm{~F} 50 \mathrm{~V}$ capacitor | Kemet | K1J50KS |
| 2 | C2, C4 | $0.33 \mu \mathrm{~F} 35 \mathrm{~V}$ capacitor | Kemet | KR33J35KS |
| 3 | C3 | $1.5 \mu \mathrm{~F} 20 \mathrm{~V}$ capacitor | Kemet | K1R5J20KS |
| 4 | C5 | $0.5 \mu \mathrm{~F} 500 \mathrm{~V}$ |  |  |
| 5 | D1, D2, <br> D3, D4, <br> D5. D13, |  |  |  |
|  | D14 ${ }^{\text {d }}$ | Diode | General Electric | 1 N 4444 |
| 6 | $\begin{aligned} & \text { D6, D7, } \\ & \text { D10 } \end{aligned}$ | Diode | General Electric | 1N64 |
| 7 | D8, D9, D11, Di2 | Diode | Unitrode | 'TR12 |
| 8 | D15-D18 | Bridge Rectifier | Unitrode | CBRE |
| 9 | D19 | Zener Diode | Motorola | 1N071B |
| 10 | D20 | Žener Diode | Motorola | 1N4104 |
| 11 | R1, R8 | Resistor 82 K | Ohmite | RC20GF |
| 12 | R2, R3 | Resistor $2400 \Omega$ | Ohmite | RC20GF |
| 13 | R4 | Resistor $5600 \Omega$ | Ohmite | RC20GF |
| 14 | R5, R6 | Resistor $120 \Omega$ | Ohmite | RC20GF |
| 15 | R7 | Resistor 0-2K (select) | Ohmite | RC20GF |
| 16 | Q1, Q7 | Transistor | Solitron | 2N930 |
| 17 | $\begin{aligned} & \text { Q2, Q3, } \\ & \text { Q4, Q5, } \end{aligned}$ |  |  |  |
|  |  | Transistor | RCA | 2N2405 |
| 18 | T1 | Transformer | Wiimore | Special |
| 10 | T2 | Transformer | Wilmore | Special |

### 11.3 Energy Storage Tests

A transistorized 8 kll de switch was designed to enable capacitor-pulse-discharge testing. The overall test configuration simitaied the spacecraft power system, since it incorporated a Solar Array Simulator (SAS), a prototype "L," converter, and eight parallel-connected capacitors ( $9000 \mu \mathrm{H}, 40 \mathrm{~V}$ each).

The SAS is powered by a $35-\mathrm{volt}$, 0-3 A power supply, and blased by a $6-\mathrm{V}$ battery. The SAS uses solar cells, of the type to be used in the spacecraft, in the feedback path of a high-power operational amplifier, such that the amplifier's output is a scaled-up representation of the voltage-current characteristics of the solar cells.

The frequency-response characteristics of this closed-loop amplifier are so much higher than the frequencies involved in the maximum-power-sceking "L" converter that the accuracy of both the de simulation and of the dynamic simulation is excellent.

Two controls are provided to allow independent adjustment of the SAS output voltage and current, in order to simulate spacecraft attitude and/or orbital positions.

The "L" converter receives the SAS output power, and essentially transforms the array into a current source, which is capable of charging the energystorage capacitors to any desired voltage amplitude, dependent only on the duty cycle of the pulse format and the available solar-cell power.

This stored energy is then transferred into a resistive load, by the switch which is driven into conduction by a pulse train similar to the spacecraft's $\kappa F$ transmission format.

The load resistor used is four parallel lengths of nichrome resistance wire (zero thermal coefficient of resistance). Resistance is measured on General Radio bridge, and is then rechecked dynamically by monitoring its $\frac{E}{I}$ characteristics. Test-load resistances used are generally in the 0.3 to $0.5 \Omega$ range. With this load resistor of known value, the various parameters of the energystorage system can be empirically observed.

The total capacitance of the 8 paralleled capacitors can be interpreted by observing the time required to discharge to $36.8 \%$ of the initial voltage through the load resistor. Fig. 11-4 shows such a discharge curve, where $E=5 \mathrm{~V} / \mathrm{cm}$, and $\mathrm{T}=5 \mathrm{~ms} / \mathrm{cm}, \mathrm{E}_{0}=40 \mathrm{~V}, \mathrm{E}_{\mathrm{f}}=40 \times 0.368=14.72 \mathrm{~V}$. This occurs at $6.2 \mathrm{~cm} \times 0.5 \times 10^{-3} \mathrm{sec}=0.031 \mathrm{sec}$.

$$
C=\frac{T}{R}=\frac{0.031}{0.46 \Omega}=0.06739 \quad F=67,390 \mu \mathrm{~F}
$$



5 voits/cm
$5 \mathrm{msec} . / \mathrm{cm}$
$\mathrm{R}_{\mathrm{L}}=0.46 \Omega$

Fig. 11-4 Capacitor discharge.
11.3.1 Interpretation of a Typical lulsed lower Test

The " ${ }^{\prime \prime}$ " converter is deshigned to track the varying perak power of the solarcell array as the spacecraft travels nearer to the sun. For obsorve this unfute eharacteristic of the converter, the Soler Array Simulator (SA.S) was energized and loaded by a variable resistor. 'The two controls of the SAS were advanced in position increments, and the peak-power point of each control setting was determined by observation of the maximum voltage and current obtainable (peak power) at the load reststance. These data provided the $s A S$ output peak-power curve shown in lig. 11-5.

With the SAS now supplying power to the "I." converter, which in turn charges a paralleled capacitor bank that is pulsed-discharged by the de switch, an ob= servation of the converter and capacitor characteristics can be made.

By setting the $E:$ and I dial controls of the $S \Lambda S$ to positions similar to those used in the dummy-load test, the duty cycle of the pulse format is changed to allow an instantaneous amplitude of 45 V on the capacitors at the begiming of the pulse discharge. In this manner, the data shown in Table 11-IV were obtained, and the indicated power levels have been plotted in $F$ Fig. 11-5 with the dummy-load power curve.

Since the load resistor's value has been carefully determined, the amount of capacitance in the energy storage can be determined, and the joule dis tribution in the system can be examined.

$$
\begin{aligned}
\text { The capacitance } & =C=\frac{t}{\ln \frac{V_{0}}{V_{\mathrm{f}}} \times R_{\mathrm{L}}} \\
C & =\frac{0.003 \mathrm{sec}}{\ln \frac{45}{40} \times 0.39}=\frac{0.003}{0.0442}=67,876 \mu \mathrm{~F}
\end{aligned}
$$

which is in agreement with the $C$ determined previously with a different $R_{L}$ by the time-constant method.

The energy delivered by the capacitors can be determined by the difference in the joules stored in the bank before and after the discharge pulse.

$$
\begin{aligned}
& J_{o}=\frac{C v^{2}}{2}=\frac{678 \times 10^{-4} \times 45^{2}}{2}=68.65 \\
& J_{f}=\frac{678 \times 10^{-4} \times 40^{2}}{2}=54.24 \\
& J_{O_{0}}-J_{f}=68.65-54.24=1.4 .41
\end{aligned}
$$



Fig. 11-5 "L" converter pulsed power tests.

Joules in the pulse $=464 \mathrm{n} \mathrm{W} \times 0.003 \mathrm{~s}=13.93$.
Joules dissipated in capacitors $=14.41-13.93=0.48$
Watts dissipated during pulse $=\frac{0.48 \mathrm{~J}}{3 \times 10^{-3} \mathrm{sec}}=160 \mathrm{~W}$
Average watts dissipated at $12-\mathrm{W}$ input $=160 \times 183 \times 10^{-5}=0.29 \mathrm{~W}$
Average watts dissipated at 27.6-W input $=160 \times 468 \times 10^{-5}=0.75 \mathrm{~W}$
Assuming $0.5-\mathrm{W}$ average dissipation per capacitor bank, the average power dissipated per capacitor $=$

$$
\frac{0.5 \mathrm{~W}}{8 \text { capacitors }}=0.062 \mathrm{~W}
$$

The average efficiency of the capacitors is determined by

$$
\frac{\mathrm{P}_{\mathrm{av}}}{\mathrm{P}_{\mathrm{L}}} \quad \text { and is } 89.7 \%
$$

The discharge efficiency of the capacitors is determined by

$$
\frac{J_{\text {load }}}{J_{\text {del }}}=\frac{13,93}{14.41}=97 \%
$$

The recharge efficiency must then be:

$$
\frac{\text { Total Av. Eff. }}{\text { Discharge Av Eff }}=\frac{89.7 \%}{97.0 \%}=92.5 \%
$$

Table $12-\mathrm{V}$ indicates a requirement of 4370 W de input power to the RF subsystem.
The excess pulse power de livered in test $=4642 \mathrm{~W}-4370 \mathrm{~W}=272 \mathrm{~W}$.
The average excess power at $12-\mathrm{W}$ inpu: $=272 \mathrm{~W} \times 183 \times 10^{-5}=0.5 \mathrm{~W}$. The average excess power at $27.6-\mathrm{W}$ input $=272 \times 468 \times 10^{-5}=1.28 \mathrm{~W}$. The average excess power for use in operating the " X " converter and low-level power system over the test input-power range is

$$
\frac{1.28+0.5}{2}=0.89 \mathrm{~W}
$$

The average power delivered to the load in this test has been plotied versus a duty cycle which provides a peak amplitude of 45 volts (in Fig. 11-6). It can be observed that a duty cycle of $3 \times 10^{-3}$ will deliver 14 W to an equivalent load of $0.39 \Omega$.

With an RF sub-system requirement of 4370 W , the equivalent load resistor would be:
Table 11-IV
Capacitor test results.

| SASDial Settings |  | SAS Output |  |  | "L" Converter Output |  |  | Pulse Amplitude |  |  | Duty Cycle | Pulse $\mathrm{P}_{\text {av }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | E | 1 | E | P | I | $\mathrm{E}_{\text {av }}$ | P | $\mathrm{E}_{0}$ | $\mathrm{E}_{\mathrm{av}}$ | $\mathbf{E}_{\mathbf{f}}$ |  |  |
| 70 | 80 | 0.75 | 16.1 | 12.08 | 0.23 | 42.5 | 9.78 | 45 | 42.5 | 40 | 0.60183 | 8.50 W |
| 75 | 85 | 0.9 | 16.8 | 15.12 | 0.3 | 42.5 | 12.75 | 45 | 42.5 | 40 | 0.06242 | 11.24 |
| 80 | 90 | 1.13 | 18.1 | 20.45 | 0.395 | 42.5 | 16.79 | 45 | 42.5 | 40 | 0.00319 | 14.86 |
| 85 | 95 | 1.43 | 19.3 | 27.60 | 0.54 | 42.5 | 22.95 | 45 | 42.5 | 40 | 0.00468 | 21.73 |

Pulse width $=\mathbf{3} \mathrm{ms}$
$R_{L}=0.398$
Average Peak Pulse Power $=\frac{\mathrm{E}_{0}{ }^{2}+\mathrm{E}_{\mathrm{av}}{ }^{2}+\mathrm{E}_{\mathrm{f}}{ }^{2}}{3 \times 0.39 \Omega}=4642 \mathrm{~W}$


Fig. 11-6 Capacitor power output vs duty cycle.

$$
R_{L}=\frac{E_{a y}^{2}}{P}=\frac{42.5^{2}}{4370}=0.4133 \Omega
$$

rather than the $0.39 \Omega$ load used in the test.

### 11.3.2 Capacitor Vacuum Test

Twelve specially-designed capacitors were purchased from Sprague Electric Company for evaluation as energy-storage devices for a pulsed-discharge system operating in a vacuum.

Three of these capacitors ( $A, B$ and $C$ ) were operated in a pressure equal to $1 \times 10^{-5}$ torr, and were delivering about 2 kW pulses of 3 ms in length at a duty cycle of $U .004$ from Octover 25, 1967 to February 29, 1968 ( 4 months).

After the construction of the high-power switct, ten capacitors in parallel were tested at the $4-\mathrm{kW}$ pulse amplitude, $3-\mathrm{ms}$ width and a 0.003 duty cycle at $1 \times 10^{-5}$ turi from March 8, 1968 to May 27, 1968 ( $21 / 2$ months). Three f the capacitors in this group of ten were the original $A, B$ and $C$ which showed no weight loss after $61 / 2$ months exposure to low pressure.

### 11.3.3 Internal Pressure Relief Test

The Sprague capacitors were designed with the container scored, intended to perform the venting in case of high internal pressure. This design was selected over the standard vent, which is a thin diaphragm of plastic, usually positioned as a hole cover in the terminal plate of the capacitor.

To check the proper functioning of the scored can vent, a selected capacitor was overcharged by 150 percent ( 60 V ), and within one hour the scoring fractured slightly, developing a relatively small orifice which provided a controlled exhaust of the artificiall.y-generated internal pressure.

### 11.4 Power System Summary

The power system consists of $6401 \times 2 \mathrm{~cm}$ solar cells in 16 parallel strings of 40 cells each. Because the spacecraft is solar-oriented, the number of cells per spacecraft is kept to a minimum and it is economically feasible to select only the cells of highest performance, and carefully match cell performance, to maximize the energy-conversion efficiency. This design allows minimum power per unit mass. The power converters have been designed to track automatically the solar cell panel V-I cheracteristics over the entire orbit (see Appendix 2). The design of the spacecraft energy-storage system, which is based on considerable analysis (Appendix 1), utilizes eight capacitors and has a redundancy factor greater than two. That is,
only three capacitors in parallel are required to operate the system.
The prototype of the entire Sunblazer power system, utilizing a solar-cell panel simulatur, has tisen bench-tested, and the design proven.

## ChAPTER 12

### 12.0 RF SUBSYSTEM

The primary function of the Sunblazer electronics system is to generate pulse signals which will be used to probe the Solar Corona. Three subsystems, digital, RF, and power, are involved. (See Fig 12-1)

The power subsystem converts the solar energy into electrical energy, maintains the charge on the capacitor package used for energy storage, and provides and regulates the voltages which are necessary for operation of the transmitter and digital subsystems.

Signal generation and housekeeping information-processing are handled by the digital subsystem.

The RF subsystem consists of a 2 kW transmitter which generates this power at three discrete frequencies, $70 \mathrm{MHz}, 75 \mathrm{MHz}$, and 80 MHz , which are necessary for the main experiment. In order to phase-lock the RF pulses and their envelope, a common clock drives the digital and RF subsystems. The 2 kW RF power level is achieved by cascading amplifiers of progressively higher power-handling capabilities, with the ultimate amplification being produced in the final stages by the paralleling of several single-stage amplifiers of one type, called the basic-power amplifier.

### 12.1 RF Subsystem Functional Description

The Sunblazer electronics system diagram print (see drawing No. 106-615, Fig. 12-2) shows the relationship of the RF subsystem to the digital and power subsystems. The power supply delivers to the transmitter the following required dc bias voltages: $10 \mathrm{~V}, 10 \mathrm{~V}$ regulated, 30 V and 45 V .

The RF chioin starts with a 5 MHz crystal oscillator (Fig. 12-5). The output of the oscillator is fed into a two-stage feedback amplifier to provide load isolation and to bring its microwatt level up to about 1 mW .

The feedback amplifier drives a saturated $5-\mathrm{MHz}$ line driver which provides a stable 6.0 mW with respect to variations in temperature.





Fig. 12-2 Spacecraft electronics system diagram. FOLDOUT PRAME 3

## ..ECEDING PACE BLANK NOT FILMBD,

This power is divided by three, and distributed to the digital subsystem (clock signal), to the sidebande generator (ar a-m modulating signal), and to the $\times 15$ frequency multiplier (Fig. 12-6).

The 75 MHz output of the frequency multiplier is further amplified to 4.0 mW by the two-stage 2 N 918 RF amplifiers.

This power is divided by two: one part of it constitutes the high-frequency component in the sidebands generator, the other makes 75 MHz available at the diode switch input port.

The mixing of 75 MHz and 5 MHz signals is processed in the sidebands generator whose 70 MHz and 80 MHz outputs are then also fed to the diode switch input.

Thus, three discrete frequencies, $70 \mathrm{MHz}, 75 \mathrm{MHz}, 80 \mathrm{MHz}$, are uvailable in CW mode at the diode switch input port.

The diode switch, which is normally in the "off" condition (i.e., no signals reach the modulator), is activated through frequency-selecting gated sigrials generated by the digital subsystem. In this manner, transmission through the main chain is made possible at any of these frequencies.

The selected frequency is fed into a phase modulator where it is multiplied by +1 or -1 in order to generate a desired code word. The command to this $0^{\circ}$ or $180^{\circ}$ phase shifter is also generated in the logic subsystem.

In order to increase the overall de-to-RF efficiency of the system, the 2 kW transmitter is turned on only when a signal is to be transmitted. This is realized by keying (Fig. 12-8) the 10V biasing voltage of the two-stage 2 N 918 amplifier, which brings its own RF output power level to 30.0 mW .

The keyer also phase-locks any RF signal with its envelope.
Thus, the transmitter must operate under a pulsed condition. At the present time, the pulse format is fixed as a 3 ms pulse occuring every second. This corresponds to a duty cycle of

$$
\text { Duty cycle }=\frac{3 \mathrm{~ms}}{1 \mathrm{~s}} \times 100 \%=0.3 \%
$$

A fact worth noting at this point is tide capability of this new Sunblazer transmitter system to generate (if necessary) three different intrapulse coding schemes by properly activating the diode switch and the modulator. We can define these groups as follows (Fig. 12-3):

Group 1: Code words generated at one frequency by means of $0^{\circ}$ and $180^{\circ}$ phase shifter (binary code).


> Group 2: Code words generated at a constant phase (all $0^{\circ}$ or all $180^{\circ}$ phase shifted) by means of a sequence of different frequencies (ternary code).

> Group 3: Code words generated from the combination of the two previous groups.

The 30 mW peak power at the output of the two-stage 2 N 918 amplifiers is connected to a cascaded two-stage 2 N 2219 and a saturated two-stage 2 N 4128 , the output of which brings the power level up to 25 W . The use of a saturated system at this point is to preserve the output level, and consequently to provide a constant drive to the RF chain, which is thereby isolated from the fluctuations in the low-level power. To establish load isolation and to prevent amplifier oscillations, a 3 dB hybrid (Fig. 12-12) is used as a two-way power divider. This provides 25 W RF input power, the required drive to the two paralleled 3TE225 basic amplifiers (Fig. 12-11).

The outputs of both basic amplifiers are combined and padded down to give a drive level of 78 W . A second paralleling of six basic amplifiers by means of 3 dB hybrids (which have an efficiency of $97 \%$ and a minimum isolation between the two output ports of -30 dB ) and three-way power combiner-divider networks (Fig. 12-9) brings the power level up to 420 W .

In the final stage, 32 basic amplifiers are paralleled by using the same techniques to provide 2200 W RF power to the antenna.

One of the major characteristics of the transmitter chain which is to be emphasized is the requirement of a 10 MHz bandwidth with a center frequency of 75 MHz starting at the output of the modulator and continuing through all the supplementary circuitry to the antenna.

### 12.2 Generation of the Three Discrete Frequencies

Figure 12-4 is a block diagram of the frequency-generator system. Set $\mathbf{f}_{\mathrm{m}}=$ 5 MHz as the modulating frequency and $f_{c}=75 \mathrm{MHz}$ as the carrier frequency, with $f_{c}$ obtained from $f_{m}$ by the $\times 15$ frequency multiplication.

Thus, $\cos 2 \pi f_{c} t$ is made available at the output of the system. In order to generate the sideband frequencies, i.e., $\cos 2 \pi\left(f_{c}-f_{m}\right) t$ and $\cos 2 \pi\left(f_{c}+f_{m}\right) t$, we feed both $\cos 2 \pi f_{c} t$ and $\cos 2 \pi f_{m} t$ into two balanced modulators, as indicated in Fig. 12-4. The upper balance modulator has then an output

Fig. 12-4 Block diagram for generation of the $(\mathbf{7 5} \pm 5) \mathbf{M H z}$ bandwidth.

$$
\begin{align*}
\cos 2 \pi f_{c} t \cos 2 \pi f_{m} t & =1 / 2\left[\cos 2 \pi\left(f_{c}+f_{m}\right) t\right. \\
& \left.+\cos 2 \pi\left(f_{c}-f_{m}\right) t\right] \tag{12,1}
\end{align*}
$$

The lower-balance modulator gives an output

$$
\begin{align*}
\sin 2 \pi f_{c} t \sin 2 \pi f_{m}^{t} & =1 / 2\left[\cos 2-\left(f_{c}-f_{m}\right) t\right. \\
& \left.-\cos 2 \cdot\left(f_{c}+f_{m}\right) t\right] \tag{12.2}
\end{align*}
$$

The addition of (12.1) and (12.2) delivers the lewer sideband signal $\cos 2 \pi\left(f_{c}-f_{m}\right) t$. In order to obtain the upper sideband signal, (12.2) must be subtracted from (12.1). This is done by the $180^{\circ}$ phase-shifte: and combiner networks. One should note that in this mathematical analysis, for reasons of simplicity, the amplitudes of the signals have been deliberately assumed to be equal to unity. In a practical case, however, one should take into consideration the different amplitude levels and, therefore, design the mixer so that an optimum cancellation of unwanted frequency components on each of the three frequency chains can be achieved.

### 12.3 RF and dc Power Profile of the 2 kW Transmitter

Table 12-I indicates the estimated RF and dc power requirements for each section of the transmitter. It is assumed that:

1. Combiner-divider networks have $95 \%$ efficiency,
2. 3 dB hybrids have $97 \%$ efficiency,
3. Basic amplifiers have 8 dB gain with 75 W RF output $45 \mathrm{~V} \mathrm{~B}+$ level and $70 \%$ de-to-RF efficiency.

The sections are sequentially listed from the high RF output power level (i.e., required power for transmission) to the low end (i.e., oscillator).

### 12.4 Circuit Information

It is obvious that some of the circuitry designed for the previous 650 W transmitter may be useful in the present 2 kW version. With the exception of the 70 MHz and 80 MHz frequency generators, the circuitry for the low-level power remains basically unchanged (see Fig. 12-2, drawing no. 106-615). No major circuit modifications except broadbanding, which is present at the output of the 2 X 2 N 4128 saturated stages, are introduced up to the 25 W level.

Generation of broadbanded higher RF power by the use of ITT 3TE225 transistors and new broadband matching and paralleling techniques makes the high
Table 12-I
DC to RF conversion table
for the different sections of the transmitter.

| Quantity | Section Description | RF Input Power (W) |  | $\begin{gathered} \mathrm{n}=\underset{\text { DCin }}{\text { RFout }} \\ \text { (\%) } \end{gathered}$ | RF Input Power <br> (w) | $\begin{aligned} & \text { B+ } \\ & (\mathbf{V}) \end{aligned}$ | $\mathrm{Ide}$ (A) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Antenna (assuming no losses) | 2200 | 2200 |  |  |  |  |
| 1 | 4-way Power Combiner (HPN) | 2320 |  |  |  |  |  |
| 32 | Paralleled Basic Amplifiers (8 hasic quads $x$ 4 basic amplifiers/quad) |  | 2400 | 70 | 3430 | 45 | 76.5 |
| 4 | 3dB Hybrid Power Dividers | 400 | 384 |  |  |  |  |
| 1 | 4-way Power Divider (LPN) | 420 |  |  |  |  |  |
| 1 | 3dB Hybrid Power Combiner | 430 |  |  |  |  |  |
| 6 | Paralleled Basic Amplifiers | 72 | 450 | 70 | 645 | 45 | 14.3 |
| 1 | 3dB Hybrid Power Divider | 78 | 76 |  |  |  |  |
| 1 | 3dB Hybrid Power Combiner |  | 145 |  |  |  |  |
| 2 | Paralleled Basic Amplifiers | 24 | 150 | 70 | 214 | 45 | 4.76 |
| 1 | 3dB Hybrid Power Divider | 25 |  |  |  |  |  |
|  | The 45 V section of the transmitter has the following features: | 25 | 2200 | 51.4 | 4289 | 45 | 296 |
|  | The 30 V section of the transmitter consisting of $2 \times 2 \mathrm{~N} 2219$ and $2 \times 2 \mathrm{~N} 4128$ has the following features: | 0.03 | 20 | 25 | 80 | 30 | =2. 76 |
|  | The 10 V sections of the transmitter consume a very small amount of power compared to the 30 V and 45 V sections. Therefore, they don't affect the overall efficiency of the system. <br> 2KW Transmittor Data |  | er |  | Max. 1 | 10 | 0.1 |
|  |  |  | 2200 | 50.5 | 4370 |  |  |

RF power section of the 2 kW transmitter unique.

### 12.4.15 MHz Crystal Oscillator

The following block diagram briefly explains the theory of oscillators.

$\mathrm{a}(\mathrm{s})$ and $\mathrm{f}(\mathrm{s})$ are the network functions of both the active and passive networks. $a(s)$ is the forward transmission; $f(s)$ is the feedback network. $\phi(s)$ represents the Laplace transform of the sigral, which can be either a voltage or a current.

The following equations can be written:

$$
\begin{aligned}
& \phi_{\text {out }}(s)=a(s) \phi_{\text {in }}^{\prime}(s) \\
& \phi_{\text {in }}^{\prime}(s)=\phi_{\text {in }}(s) \neq \phi_{\text {out }}(s) \cdot f(s)
\end{aligned}
$$

The overall gain of the system is:

$$
\begin{equation*}
A(s)=\frac{\phi_{\text {out }}(s)}{\phi_{\text {in }}(s)}=\frac{a(s)}{1-a(s) f(s)} \tag{12.3}
\end{equation*}
$$

Oscillations take place when $A(s)=\infty$

$$
\begin{aligned}
& a(s) f(s)=1 \text { therefore, } \\
& |a(s) f(s)|=1 \text { and, }
\end{aligned}
$$

$$
\text { angle } a(s) f(s)=0^{\circ} \text { or } 360^{\circ}
$$



Fig. 12-5 5 MHz crystal oscillator.

Figure 12-5 shows the circuit of the : MHz crystal oscillator designed according to the principles previously presented. The variable capacitor MC621 in the $f(s)$ passive circuitry provides an adjustment in frequency over a range of approximately 20 ppm . Frequency variations with temperature are limited to 30 ppm by the crystal. Double regulation $\{10 \mathrm{~V}$ reg. and 5.6 V zener diode) is used to dc-bias the oscillator, thereby minimizing frequency shifts caused by dc voltage variations.

The device used to achieve frequency multiplication is a step-recovery diode (Fig. 12-6) which has in the off condition an abrupt recovery time (see the characteristic curves Fig. 12-7.).

According to the Fourier analysis of periodic functions the output of the steprecovery diode contains several harmonics cf is MHz.

The use of the 2 dB pad at the input helps to stabilize the operation of the multiplier with temperature variations. The series-resonant circuit, $L_{1} C_{1}$, provides a clean 5 MHz input to the diode which is operated far below its cut off frequency. The operating point of the diode is defined by $R_{b}$. At the output, $L_{2}, C_{2}, L_{3}, C_{3}$ and $C_{4}$ constitute filtering and matching networks for 75 MHz .


Fig. 12-6 5 MHz to 75 MHz frequency multiplier.


Fig. 12-7 Comparison of the output waveforms of regular and step recovery diodes.


Fig. 12-8 Keying network.
$R_{1}$ through $K_{4}$ are biaबing resistors determining the simultaneous switching levels of both switching transistors $Q_{1}$ and $Q_{2}$. A positive pulse fed into the input of the keying network turns on $Q_{1}$ and $Q_{2}$ and causes a 10 V bias level to appear at the keyed RF amplifiers.
12.4.4 N-Way Power Divider or Combiner Networks

Power-divider or -combiner networks differ, as their names indicate, only in their application. Both could be represented by the same black box with a one-port on one side and an N -port on the other. In the case of a divider network, for example, power is fed into the box through the single port, and an N -way power division is obtained at the output.

This type of network requires the following features:

1. For the deaigned frequency, it must introduce the mame phase whift tor all N -ways. (Equiphane mignala)
2. For the designed frequency, all the $N$-way must be equally loaded. (Equiamplitude aignala)
3. The network must be applicable for any value of $N$. ( $N$ could be odd or even)
4. In order to minimize the effect of errors (short circuit, mismatch, open circuit) fror, one channel to the other, a good isolation between the individual N -way ports is necensary.
5. It must have a low insertion loss.

Figure 12-9 shows a network designed with $95 \%$ efficiency and a minimum isolation of $\mathbf{- 3 0} \mathrm{dB}$.


Fig. 12-9 N-way power-divider or combiner networks.

The trensmiasion lines have a characteristic impedance of $Z_{o}$ and a length of $\lambda / 4$. They can be synthesized with lumped LC netvorks in highpass (HPN) or lov' pass (LIPN) configurations. The analytical relatione are:

$$
\begin{gathered}
R_{x}=R_{\text {lond }} \\
Z_{o}=\sqrt{N} R_{\text {load }} \\
R_{s}=Z_{\text {input }}=\frac{Z_{0}^{2}}{R_{\text {load }}}=R_{\text {load }}
\end{gathered}
$$

### 12.4.5 Basic Amplifiers

Some of the major factors which prescribe limitations in designing a apacecraft transmitter are the following:

1. Weight: The weight of the transmitter must be held to a minimum because of the limited payload capabilities of the launch vehicle used.
2. Volume: In order to locate all the electronic subsyatems in the volume available in the spacecraft, the circuitry has to be as miniaturized as possible.
3. Power: Regardless of these physical limitations, the transmitter is required to deliver the maximum possible highfrequency output power.

These arguments justify the importance of the curves RF power out ve frequency (Fig. 12-10) for different types of transistors, and the selection of ITT 3TE225 devices as typical for application in the basic amplifier.

Besides their high RF-output power capability, the 3TE225's assure a high de-to-RF efficiency. This feature has two advantages. First, for a given RF output power, the selected transistors reduce the high dc power requirement. Second, due to the smaller internal power dissipation of the device, an increase in its longevity is anticipated. Several teats and measurements on these devices show the possibility of achieving efficiencies of $\mathbf{8 0 \%}$ in a large signal, class $C$, pulsed operation.

Fig. 12-10 Transistor power vs frequency curves.


Fig. 12-11 Basic amplifier.

Figure 12-11 represents the circuitry of the basic broadband amplifier. The components $\mathrm{C}_{0}, \mathrm{R}_{1}, \mathrm{~L}_{1}, \mathrm{C}_{1}, \mathrm{~L}_{2}, \mathrm{C}_{2}$ constitute a low Q and, therefore, a broadband matching network. Matching is done at $50 \Omega$.

The components $R_{3}, L_{3}, C_{3}$ constitute a notching filter designed for the center frequency. Its function is to reduce the level of the RF-output power of the basic amplifier at the center frequency to the power levels at the sideband frequencies. The 75 MHz resonating output network is mainly constituted by $L_{4}, C_{4}$ and the output capacitance of tic device. $C_{5}, L_{5}, C_{6}$ represent the matching network at the output. In addition, $\mathrm{C}_{7}, \mathrm{C}_{8}, \mathrm{C}_{9}$ are bypass capacitors and $\mathrm{C}_{1}, \mathrm{C}_{5}$ also have de blocking functions.
The design goal is to obtain, with amplifier circuit under class $C$ operation, a pulsed output power of 75 W with a $\mathrm{B}+=45 \mathrm{~V}$, and dc-to-RF efficiency of 70 percent.
12.4.6 3 dB Quadraturn Hybrid Circuits ${ }^{(1)}$


Fig. 12-12 3 dB quadrature hybrid.
The lumped circuit realization of the quadrature hybrid is given in Fig. 12-12. In the figure all the inductances are equal to a value $L$, where $L$ is given by

$$
\begin{equation*}
\mathrm{L}=\frac{\mathrm{Z}_{\mathrm{o}}}{\omega} \tag{12.4}
\end{equation*}
$$

and $Z_{0}$ is the characteristic impedance of the transmission line system in which the coupler is used and $\omega$ is the design frequency. The capacitors $C_{1}$ and $C_{2}$ are equal, and related to the characteristic impedance by

$$
\begin{equation*}
Z_{o}=\frac{1}{\omega C_{1}}=\frac{1}{\omega C_{2}} \tag{12.5}
\end{equation*}
$$

The capacitors $C_{3}, C_{4}$ and $C_{5}$ are equal to $\frac{C_{1}}{2}$.
The operation of the circuit is outlined below. If points 2,3 , and 4 are terminated to ground by the characteristic impendance $Z_{0}$, and the circuit driven by a matched generator at point 1 to ground, power is divided equally and wii.sout loss between the loads connected at points 3 and 4. Point 2 is decoupled. On the 'ther hand, if puint 2 is driven, point 1 is decoupled, but again power is divided equally between the loads connected at points 3 and 4. Since the circuit is symmetric, points 3 or 4 may be driven with similar power-division characteristics. In any case, the voltages at the output points are always $90^{\circ}$ out of phase.

The design of the circuit is based on a scattering matrix representation of the coupler. The components $L_{1}, L_{2}$ and $C_{1}$ form an equivalent circuit of a quarter-wavelength transmission line of characteristic impedance $Z_{0^{\circ}}$. The components $L_{3}, L_{4}$ and $C_{2}$ form an identical circuit. If points 1 and 2 are driven in phase, an equivalent circuit of a quarter-wavelength line of characteristic impedance $Z_{0} / 2$ results. Now, if points 1 and 2 are driven $180^{\circ}$
out of phase, inductors $L_{1}, L_{3}$, and capacitors $C_{3}, C_{4}$ form the equivalent circuit ot a quarter-wavelength transmission line of characteristic impedance $2 Z_{0}$. Under this driving condition, there is a similar circuit formed by inductors $L_{2}, L_{4}$ and capncitors $C_{5}$ and the series combination of $C_{1}, C_{2}$. Since these two identical quarter-wavelength circuits are in cascade, the total electrical length under this out-of-phase excitation is $180^{\circ}$. By satisfying the above impedance and phase conditions, all of the requirements (at a single frequency) for a 3 dB quadrature hybrid have been satisfied.

An experimental circuit was constructed for a center frequency of 75 MHz , and the observed insertion loss was less than 1 dB , the phase angle between the equal outputs was approximately $90^{\circ}$, and the output at the decoupled port was 20 dB below the input. A usable bandwidth of 20 MHz was observed, and the input VSWR was 1.1.

### 12.5 Construction Techniques

In order to build a transmitter capable of withstanding the shock and vibration of the launch, to keep the individual stages or sections isolated from each other, both thermally and electrically, and to prevent oscillation and reduce spurious output signals, the approach generally used in the past has been to build the individual stages into compartments which have been milled out of a large piece of metal such as brass, aluminum, magnesium or some other suitable material (point-to-point component mounting), then to put covers on the top and bottom to shield these circuits from the next black box.

The other alternative has been to build the transmitter on one or more printed circuit boards provided with shields mounted in appropriate places, then to mount these boards into boxes, and, in some cases, to foam-encapsulate the transmitter, then readjust the many variable components to compensate for the effects of the foam and the proximity of the box. Either of these methods, while proven reliable and effective, is satisfactory for a transmitter consisting of a dozen or so stages, but rapidly becomes impractical for space application because of size and weight considerations.

Consider for a moment a 2200 W transmitter consisting of 55 stages starting with a precision oscillator producing a few $\mu \mathrm{W}$, built by milling out compartments from a block of aluminum. It is not difficult to imagine the remaining aluminum, with no RF parts included, weighing more than our entire transmitter, not to mention the volume taken up by a transmitter that approaches the total volume of our spacecruft. If,instead, printed circuit boards with strategically-located shields were used and then mounted in boxes, an improvement would be achieved; but the configuration would still be too
large and too heavy for this application. Clearly, a different approach is needed.

The solution proposed is to use the printed circuit boards themselves as shields between the various circuits. When laying out the printed circuit boards, which are $5.14^{\prime \prime} \times 1.98^{\prime \prime}$ cards that mate with the approximately $5^{\prime \prime} \times 5^{\prime \prime}$ vertical member in each plug, care is taken to keep all printed lines on one side (component side) of the board, while the paths of ground current (input and output) of the stage are isolated as much as possible, both to reduce magnetic coupling between inductors in the input and output by physical placement and/or orientation and to keep lead lengths as short as possible. Further, as much unbroken copper as possible is left on both sides of the printed board, and this area is connected to ground potential. (Enough copper is etched away around printed wires and terminals to make the necessary connections, and the rest is left.) The result is that we have left about $80 \%$ to $90 \%$ of the copper on the reverse side of the board and a large amount on the component side to act as a shield from the circuit below and provide lowinductance ground paths. The vertical member upon which the circuit boards are stacked, one above the other, forms an effective shield from the circuit on the adjacent side of the vertical member. The result of this stacking leaves us with a pair of compartments between each pair of printed circuit boards, isolated from its neighboring compartment by the ground plane on its own reverse side, the ground plane on the board above, the vertical member, the side wall of the compartment, the end wall of the compartment, and the compartment shield. The 34 such compartments within which the circuits are constructed give excellent isolation between stages. This type of structure provides a common electrical ground for all cards, serves as a heat sink, and provides mectanical strength for launch environment. (Fig. 12-13)

This construction technique, coupled with the fact that we have reduced the number of variable components (which are heavy and large) to the bare minimum (one variable capacitor in the entire system) by careful measurement and substitution methods, taking into account the stray capacitance and lead inductances of components, printed lines and terminals, therefore makes the circuit boards an integral part of the RF circuit (an accomplishment in itself) and has permitted packing of VHF circuits much in the manner in which on would package low-frequency circuits. Further, we are able to construct these circuits with components out of stock, with a minimum of selection and with minor trimming to compensate for gain and phase variations due to device variations.

Fig. 12-13 RF circuit packaging.

By the use of the above techniques we have effected a substantial reduction in the overall size and weight regitirements for the transmitter, and at the same time preserved the mechanical integrity of the spacecraft.

### 12.6 The 50-Watt Beacon Transmitter

Located in the rear section of the Hub is an auxilliary 50 W , battery-powered transmitter which provides a tracking beacon and pertinent telemetry during solar acquisition of the spacecraft.

The pulsed 75 MHz signal provided by this beacon transmitter is encoded by pulse-width modulation to provide orientation data from the aspect sensor, along with selected deployment, voltage, and temperature information.

The beacon transmitter is shown in Fig. 12-14a and consists of one class $A$ stage and three cascaded class $C$ amplifiers, with the final stage producing 50 watts. The drive signal for the beacon is provided by the same oscillator and multiplier which drives the main transmitter, and its output signal is radiated by one pair of sails.

A single-pole, single-throw, solid-state switch, operated by a digital command, is provided to prevent damage to the beacon transmitter's final amplifier due to voltage induced in the sail by the main transmitting antenna. The switch has an isolation of 45 dB , and an insertion loss of less than 0.2 dB , and is connected between the beacon final amplifier and the sails.

The switch-control circuitry is shown in Fig. 12-14b. The switch passes an RF signal when zero voltage is applied to the base of $Q_{1}$; then transistors $Q_{1}$, $Q_{2}, Q_{3}$ are cut off, leaving the control diode back-biased at -300 volts. In this condition, the beacon signal is transmitted. If, on the contrary, a positive voltage is applied to the base of $Q_{1}$, transistors $Q_{1}, Q_{2}$ and $Q_{3}$ conduct from the 10 V supply, forward biasing the control diode, and thus disconnecting the sails from the beacon transmitter.


Fig. 12-14a Block diagram of the beacon transmitter.


Fig. 12-14b Beacon antenna switch.

Table 12-1I
Basic parameters of beacon transmitter.

| Stage | de Supply | Drive Level | Output | $\eta$ | Trans ister Type | Class | Gain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Firat | 12 V | 1 mW | 50 mW | 40\% | 2N918 | A | 17.0 dB |
| 2nd | 12 V | 50 mW | $1 / 2 \mathrm{~W}$ | 60\% | 2N3924 | C | 10.0 dB |
| 3rd | 24 V | $1 / 2 \mathrm{~W}$ | 13 W | 70\% | 2N4128 | C | 14.0 dB |
| 4th | 24V | 3.6 W | 50 W | 70\% | $\begin{aligned} & \text { Pair } \\ & 3 \text { TE225 } \end{aligned}$ | C | 11.5 dB |
| Switch | Insertion Loss | Isolation | Forward Bias Current 20 ma |  | Reverse Bias Voltage |  |  |
|  | $<0.2 \mathrm{~dB}$ | 45 dB |  |  |  | 00V |  |

### 12.7 The Spacecraft Integrated Antenna System

### 12.7.1 Introduction

There are several alternatives for the spacecraft antenna system, varying in complexity from a simple monopole to an elaborate array, each with its own degree of complexity in deployment and feed system. For this initial launch, however, it is desirable to use something consistent with gain and pattern requirements of the experiment, and as simple in deployment and feed as possible.

Figure 12-15 represents the Sunblazer spacecraft with the antenna sjstems under consideration. Radiation of the pulsed 2 kW main transmitter signal at the $70 \mathrm{MHz}, 75 \mathrm{MHz}$ and 80 MHz frequencies is achieved by a pair of rods, each with a length $L$, mounted perpendicular to the spin axis on each side of the spacecraft solar-cell platform. The separation between these two links of the dipole thus formed is shown as 2 d , which is equal to the diameter of the platform. The $75 \mathrm{MHz}, 50 \mathrm{~W}$ beacon-pulsed signals are transmitted from a pair of sails which act as dipoles, but whose major function is to stabilize the dynamic motion of the spacecraft in conjunction with another set of sails mounted in an orthogonal axis. In the deployed configuration, the sails are canted forward by an angle of $70^{\circ}$ with respect to the spin axis. Furthermore, sighting along the spin axis shows that any pair of sails to be used as the beacon antenna is displaced from the main transmitter antenna by an angle of $45^{\circ}$.


### 12.7.2 Main Transmitter Antenna

There are two problems of major importance:

1. Matching each antenna to its tranamitter.
2. Antenna patterns over the 10 MHz bandwidth.

These problems require the calculation of the input impedance $Z_{s}=R_{s}+j X_{s}$ of the antenna, the $\vec{E}$-field, $\vec{H}$-field, and $\vec{S}$ pattern analysis over the 10 MHz bandwidth, for different rod lengths $L$ and the given spacing $2 d$.

Maxwell'; equations for electromagnetic waves varying sinusoldally in time are

$$
\begin{array}{ll}
\nabla X \mathbf{F}=-j \omega \mu \vec{H} & \text { (a) } \\
\nabla \mathbf{X H}=\boldsymbol{J}+j \omega \mathrm{C} \overrightarrow{\mathrm{E}} & \text { (b) } \\
\nabla \cdot \vec{H}=0 & \text { (c) } \\
\nabla \cdot \vec{E}=\frac{\rho}{6} & \text { (d) }
\end{array}
$$

Here any vector $\vec{\nabla}=\vec{\nabla}\left(\vec{r}_{o}, t\right)=\operatorname{Re}\left[\underline{V_{e}} e^{j\left(\omega t-k r_{0}\right)}\right]$

$$
\begin{equation*}
\text { and } \quad \stackrel{\rightharpoonup}{V}=\underline{V e}^{-j k r_{0}} \tag{12.7}
\end{equation*}
$$

The electric field $\vec{E}$ and the magnetic field $\mathfrak{i f}$ at each point in space are given by the solutions of Eq (12.6).
The solutions are

$$
\begin{align*}
& \vec{H}=\nabla \times \stackrel{\rightharpoonup}{A} \\
& \stackrel{\rightharpoonup}{E}=-j \omega \mu \stackrel{\rightharpoonup}{A}-\dot{\nabla} \phi \tag{12,8}
\end{align*}
$$

where the retarded vector potential is defined as

$$
\begin{equation*}
\vec{A}(x, y, z)=\frac{1}{4 \pi} \int_{V^{\prime}} \frac{\vec{J}\left(x^{\prime}, y^{\prime}, z^{\prime}\right) e^{-j k r}}{r} d V^{\prime} \tag{12,9a}
\end{equation*}
$$

and the retarded electric potential is defined as

$$
\begin{equation*}
\phi(x, y, z)=\frac{1}{\frac{1}{\pi}} \int_{V^{\prime}}^{0} \frac{\rho\left(x^{\prime}, y^{\prime}, z^{\prime}\right) e^{-j k r}}{r} d V^{\prime} \tag{12.9b}
\end{equation*}
$$

One has to remark here that $\phi$ can be obtained from $\hat{\wedge}$ by using the Lorentz condition, i.e.:

$$
\begin{equation*}
\phi=\frac{-1}{j \omega x} \nabla \cdot \vec{A} \tag{12.0c}
\end{equation*}
$$

In calculating the solution for an antenna with a given current distribution, one ass ' 3 it is composed of several very short dipoles of different con-stant-current distributions. The scalir and vector potentials at any point in space, due to a very short dipole of length $2 \ell$ and constant-current distribution $I_{o}$ located at the origin of a coordinate system along the z -axis, are given by the following equations:

$$
\begin{align*}
& \phi=\frac{2 \ell I_{o}}{j \omega 4 \pi \ell}\left[j \frac{\omega}{c\{ }+\frac{1}{R^{2}}\right] \cos \theta e^{-j k R}  \tag{12.10a}\\
& \stackrel{\rightharpoonup}{A}=\frac{2 \ell I_{o}}{4 \pi} \quad \frac{e^{-j k R}}{R} \hat{z}=A_{z} \hat{z} \tag{12.10b}
\end{align*}
$$

Where $\vec{R}$ is the radlus vector from the origin of the coordinate system to the point in space, $\theta$ is the angle between the positive $z$-axis and the vector $\overrightarrow{\mathrm{R}}$,

$$
\begin{aligned}
c & =\frac{1}{\sqrt{\mu_{0} c_{0}}}=3 \times 10^{10} \frac{\mathrm{~cm}}{\mathrm{sec}^{\prime}} \\
\text { and } \quad k & =\frac{2 \pi}{\lambda} .
\end{aligned}
$$

Consider the dipole system represented in Fig. 12-16. The current distribution is given by

$$
\begin{equation*}
I_{z}\left(z^{\prime}\right)=I_{0} \sin k\left(h-\left|z^{\prime}\right|\right) \tag{12.11}
\end{equation*}
$$

For any point in space, Eq (12.10) may now be written for an incremental dipole of length $d z^{\prime}$ :

$$
\begin{equation*}
d \phi=\left\{\frac{d z^{\prime} I_{0} \sin k\left(h-z^{\prime}\right)}{j \omega 4 \pi \epsilon}\left[\frac{j \omega}{c r_{1}}+\frac{1}{r_{1}^{2}}\right] \frac{z}{r_{1}} z^{\prime} e^{-j k r_{1}}\right. \tag{12.12a}
\end{equation*}
$$



Fig. 12-16 Determination of $\phi$ and $A$ for any point in space.

$$
\begin{array}{r}
\left.+\frac{d z^{\prime} I_{0} \sin k\left(h+z^{\prime}\right)}{j \omega 4 \pi \epsilon}\left[\frac{j \omega}{c r_{2}}+\frac{1}{r_{2}^{2}}\right] \frac{z+z^{\prime}}{r_{2}} e^{-j k r_{2}}\right\} \\
d \stackrel{(A}{A}=\frac{d z^{\prime} I_{0} \sin k\left(h-z^{\prime}\right)}{4 \pi} e^{-j k r_{1}} \frac{d z^{\prime} I_{0} \sin k\left(h+z^{\prime}\right)}{4 \pi}+\frac{e^{-j k I_{2}}}{r_{2}} \tag{12.12b}
\end{array}
$$

By making the assumption that $r_{1}, r_{2}, r_{0}:>2 L+2 d$, one obtains the following simplified expressions:

$$
\begin{align*}
& r_{1,2}=r_{0}-z^{\prime} \cos \theta  \tag{12.13}\\
& \frac{z-z^{\prime}}{r_{1}}=\frac{z+z^{\prime}}{r_{2}}=\frac{z}{r_{0}}=\cos \theta
\end{align*}
$$

Equations (12.12) and (12.13) together give the expressions of the scalar and vector potentials for a separated dipole:

$$
\begin{aligned}
\phi= & \frac{I_{0} \cos \theta}{4 \pi \epsilon c} \frac{e^{-j k r_{0}}}{r_{o}}\left[\int_{d}^{h} \sin k\left(h-z^{\prime}\right) e^{j k z^{\prime} \cos \theta} d z^{\prime}\right. \\
& \left.+\int_{-h}^{-d} \sin k\left(h+z^{\prime}\right) e^{j k z^{\prime} \cos \theta} c z^{\prime}\right] \\
& +\frac{I_{0} \cos \theta}{4 \pi \epsilon j \omega} \frac{e^{-j k r_{o}}}{r_{0}^{2}}\left[\int_{d}^{h} \sin k\left(h-z^{\prime}\right) e^{j k z^{\prime} \cos \theta_{d z^{\prime}}}\right. \\
& \left.+\int_{-h}^{-d} \sin k\left(h+z^{\prime}\right) e^{j k z^{\prime} \cos \theta} d z^{\prime}\right] \\
\stackrel{A}{A}= & \frac{I_{0}}{4 \pi} \frac{e^{-j k r_{o}}}{r_{0}}\left[\int_{d}^{h} \sin k\left(h-z^{\prime}\right) e^{j k z^{\prime}} \cos \theta_{d z^{\prime}}\right. \\
& \left.+\int_{-h}^{-d} \sin k\left(h+z^{\prime}\right) e^{j k z^{\prime} \cos \theta} d z^{\prime}\right]
\end{aligned}
$$

and finally

$$
\begin{equation*}
\phi\left(\vec{r}_{0}\right)=\frac{I_{0}}{4 \pi \epsilon} \cos \theta F(\theta, h, d, \lambda) \quad\left[\frac{1}{c r_{0}}+\frac{1}{j \omega r_{0}^{2}}\right] e^{-j k r_{0}} \tag{12.14a}
\end{equation*}
$$

$$
\begin{equation*}
\stackrel{\rightharpoonup}{A}\left(\vec{r}_{o}\right)=\frac{I_{0}}{4 \pi} \quad F(\theta, h, d, \lambda) \frac{e^{-j k r_{0}}}{r_{0}} \hat{z} \tag{i2.i4b}
\end{equation*}
$$

where

$$
\begin{align*}
F(\theta, h, d, \lambda)= & {[2\{\cos [k h \cos \theta]-\cos k L \cos [k d \cos \theta]} \\
& +\cos \theta \sin k L \sin [k d \cos \theta]\}] / k \sin ^{2} \theta \tag{12.14c}
\end{align*}
$$

Equations (12.8) and (12.14), solved for a spherical coordinate system, give the component values for the $\stackrel{\rightharpoonup}{E}$ and $\stackrel{H}{H}$ fields:
$H_{r_{0}}\left(r_{o}, \theta, \phi\right)=H_{\theta}\left(r_{0}, \theta, \phi\right)=0$,
$H_{\phi}\left(r_{0}, \theta, \phi\right)=\frac{I_{0}}{4 \pi} \sin \theta\left\{j \frac{k}{r_{o}} F(\theta, h, d, \lambda)-\frac{1}{r_{o}^{2} \sin \theta} \frac{\partial}{\partial \theta}[F(\theta, h, d, \lambda) \cos \theta]\right\} e^{-j k r_{o}}$,
and
$E_{r_{0}}\left(r_{0}, \theta, \phi\right)=\frac{I_{0}}{4 \pi} \cos \theta F(\theta, h, d, \lambda)\left\{2 \sqrt{\frac{\mu_{0}}{\epsilon_{0}}} \frac{1}{r_{0}^{2}}-\frac{2 j^{\omega}}{\omega \in r_{0}^{3}}\right\} e^{-j k r_{o}}$,
$E_{\theta}\left(r_{0}, \theta, \phi\right)=\frac{I_{0}}{4 \pi} \sin \theta\left\{\sqrt{\frac{\mu_{0}}{\epsilon_{0}}} j \frac{k}{r_{o}} F(\theta, h, d, \lambda)-\sqrt{\frac{\mu_{0}}{\epsilon_{0}}} \frac{1}{r_{o}^{2} \sin \theta} \frac{\partial}{\partial \theta}[F(\theta, h, d, \lambda) \cos \theta]\right.$

$$
\left.+\frac{j}{\omega \in r_{0}^{3}} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta}[F(\theta, h, d, \lambda) \cos \theta]\right\}
$$

and
$E_{\phi}\left(r_{0}, \theta, \phi\right)=0$.

The average power density is therefore given by

$$
\stackrel{\overleftrightarrow{S}}{ }\left(r_{0}, \theta, \phi\right)=\frac{1}{2}\left(\stackrel{\rightharpoonup}{E} \times \stackrel{\rightharpoonup}{H}^{*}\right) .
$$

Thus,

$$
\begin{aligned}
& \left.S_{r_{0}}\left(r_{0}, \theta, \phi\right)=\frac{I_{0}^{2} \sin ^{2} \theta}{32 \pi^{2}} \right\rvert\, \sqrt{\frac{\mu_{0}}{\epsilon_{0}}} k^{2} \frac{F^{2}(\theta, h, d, \lambda)}{r_{0}^{2}} \\
& +\left[\sqrt{\frac{\Psi_{0}}{\epsilon_{0}}} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta}[F(\theta, h, d, \lambda) \cos \theta]\right]\left[F(\theta, h, d, \lambda)+\frac{1}{\sin \theta} \frac{\theta}{\partial \theta}(F(\theta, h, d, \lambda) \cos \theta)\right] \frac{1}{r_{0}^{4}} \\
& \left.-\frac{j}{\omega \epsilon r_{o}^{5} \sin ^{2} \theta}\left[\frac{\partial}{\partial \theta}(F(\theta, h, d, \lambda) \cos \theta)\right]^{2}\right\} . \\
& S_{\theta}\left(r_{0}, \theta, \phi\right)=\frac{\mathrm{I}_{0}^{2} \sin \theta \cos \theta}{32 \pi^{2}} F(\theta, h, d, \lambda)\left\{j \sqrt{\frac{\mu_{0}}{\epsilon_{0}}} \frac{2 k}{r_{0}^{3}} F(\theta, h, d, \lambda)\right. \\
& +2 \sqrt{\frac{\mu_{0}}{\epsilon_{0}}}\left[F(\theta, h, d, \lambda)+\frac{1}{\sin \theta} \frac{\partial}{\partial \theta}((F(\theta, i, d, \lambda) \cos \theta)] \frac{1}{r_{0}^{4}}\right. \\
& \left.-\frac{2 j}{\omega \in r_{o}^{5} \sin \theta} \frac{\partial}{\partial \theta}(F(\theta, h, d, \lambda) \cos \theta)\right\} \text {, }
\end{aligned}
$$

and

$$
\begin{equation*}
S_{\phi}\left(r_{0}, \theta, \phi\right)=0 \tag{12.17}
\end{equation*}
$$

Considering only the far fields one obtaines from Eq (12.15), (12.16) and (12.17)

$$
\begin{equation*}
H_{\phi}\left(r_{0}, \theta, \phi\right)=\frac{I_{0}}{4 \pi} \sin \theta j \frac{k}{r_{0}} F(\theta, d, h, \lambda) \tag{12.18a}
\end{equation*}
$$

$$
\begin{equation*}
E_{\theta}\left(r_{o^{\prime}} \theta, \phi\right)=\frac{I_{o}}{4 \pi} \sin \theta \sqrt{\frac{\mu_{0}}{\epsilon_{0}}} j \frac{k}{r_{o}} F(\theta, h, d, \lambda), \tag{12.18b}
\end{equation*}
$$

and

$$
\begin{equation*}
S_{r_{0}}\left(r_{0}, \theta, 4\right)=\frac{I_{0}^{2} \sin ^{2} \theta}{32 \pi^{2}} \sqrt{\frac{\mu_{0}}{\epsilon_{0}}} \frac{k^{2}}{r_{0}^{2}} F^{2}(\theta, h, d, \lambda) . \tag{12.18c}
\end{equation*}
$$

Normalizing these to the peak value of the beam for a tuned dipole without separation, i.e. for

$$
\theta=90^{\circ}, 2 \mathrm{~d}=0, k L=\frac{\pi}{2}, h=L,
$$

one obtains the following normalized quantities:

$$
\begin{aligned}
& \mathrm{H}_{\phi \mathrm{n}}\left(\mathrm{r}_{0}, \theta, \phi\right)=\frac{\sin \theta \mathrm{F}(\theta, \mathrm{~h}, \mathrm{~d}, \lambda)}{\mathrm{F}\left(\theta=90^{\circ}, \mathrm{h}=\frac{\lambda}{4}, 2 \mathrm{~d}=0, \lambda\right)} \text { (in } \theta \text { direction): } \\
& \mathrm{E}_{\theta \mathrm{n}}\left(\mathrm{r}_{0}, \theta, \phi\right)=\frac{\sin \theta \mathrm{F}(\theta, \mathrm{~h}, \mathrm{~d}, \lambda)}{\mathrm{F}\left(\theta=90^{\circ}, \mathrm{h}=\frac{\lambda}{4}, 2 \mathrm{~d}=0, \lambda\right)}(\text { in } \theta \text { direction) ; }
\end{aligned}
$$

and

$$
\begin{equation*}
S_{r_{o} n}\left(r_{0}, \theta, \phi\right)=\frac{\sin ^{2} \theta \mathrm{~F}^{2}(\theta, h, d, \lambda)}{F^{2}\left(\theta=90^{\circ}, h=\frac{\lambda}{4}, 2 d=0, \lambda\right)} \quad \text { (in } r_{o} \text { direction) } \tag{12,19}
\end{equation*}
$$

Figures 12-17a, 12-17b, 12-17c, are plots of Eq (12.19) in polar coordinates for the special case of the antenna stem length of $L=100 \mathrm{~cm}\left(\frac{\lambda}{4}\right.$ at 75 MHz ). The cases of $70 \mathrm{MHz}, 75 \mathrm{MHz}$, and 80 MHz frequencies for the separation $2 \mathrm{~d}=0$ and $2 \mathrm{~d}=46 \mathrm{~cm}$ are considered, and the electric field, magnetic field, and power density patterns are plotted.

In addition, in Fig. 12-18 an extreme case of the spacing $2 \mathrm{~d}=\lambda=400 \mathrm{~cm}$ is considered, showing effectively the variation of the patterns as a function of 2d.

The next important quantity which varies as a function of frequency and


Fig. 12-17a Normalized $|\vec{E}|$, $|\vec{H}|$, and $|\vec{S}|$ patterns for $L=100 \mathrm{~cm}$ and $f=70 \mathrm{MHz}$.


Fig. 12-17b Normalized $|\vec{E}|,|\vec{H}|$, and $|\vec{S}|$ patterns for $L=100 \mathrm{~cm}$ and $\mathrm{f}=75 \mathrm{MHz}$.


Fig. 12-17c Normalized $|\vec{E}|$, $|\vec{H}|$, and $|S|$ patterns for $L=100 \mathrm{~cm}$ and $f=90 \mathrm{MHz}$.

Fig. 12-19 Normalized $|\vec{S}|$ patterns for $L=100 \mathrm{~cm}$ and $\mathbf{f}=75 \mathrm{MHz}$.
spacing is the input impedance at the antenna feedpoints A-B. This impedance can be defined as

$$
\begin{equation*}
z_{A-B}=R_{A-B}+j X_{A-B} \tag{12,20}
\end{equation*}
$$

and is called the radiation impedance of the antenna. In practice, the antenna will be connected to its transmitter as shown in Fig. 12-19. The $180^{\circ}$ hybrid is necessary in order to create a dipole characteristic in the pair of rods. Matching ${ }^{7} \mathrm{~A}-13$ at a specific frequency to the $180^{\circ}$ hybrid output impedance $(50+j 0) \Omega$ therefore means a notwork design which compensates the effect of $j \frac{X_{A-B}}{2}$, and impedance transforms $\frac{R_{A-B}}{2}$ to $50 \Omega$. The following equations give $R_{A-B}$ and $X_{A-B}$ as a function of frequency for a spacing $2 \mathrm{~d}=0$ :

$$
\begin{align*}
\frac{R_{A-B}}{\Omega}= & 30\{2[S i(4 k L)-2 S i(2 k L)] \cot k L \\
& +4[E+\ln (2 k L)-C i(2 k L)] \cot ^{2} k L \\
& \left.-[E+\ln (4 k L)-C i(4 k L)]\left[\cot ^{2} k L-1\right]\right\} ; \tag{12.21a}
\end{align*}
$$

and

$$
\begin{align*}
\frac{X_{A-B}}{\Omega}= & -30\left\{2\left[\ln \left(\frac{L}{k \rho}\right)-E+2 C i(2 k L)-C i(4 k L)\right] \cot k L\right. \\
& \left.+[S i(4 k L)-2 S:(2 k L)]\left[\cot ^{2} k L-1\right]-\frac{2}{\sin ^{2} k L} \operatorname{Si}(2 k L)\right\} . \tag{12.21b}
\end{align*}
$$

Figures 12-20a (Real) and 12-20b (Imaginary) contain the results of Eq (12.21) for the case of

$$
L=\frac{\lambda_{75 \mathrm{MHz}}}{4}=100 \mathrm{~cm}
$$

It is possible to calculate the radiation impedance $Z_{A-B}$ for the more general case of any spacing $2 d$ and any rod length $L$ by considering the total driving power at the feedpoints.

From the divergence law applied to the Poynting vector it follows

$$
\begin{equation*}
\oint_{A}(\vec{E} \times \vec{H}) \cdot d \vec{A}=\int_{V} \nabla \cdot(\vec{E} \times \vec{H}) d V \tag{12.22}
\end{equation*}
$$



Fig. 12-19 Interface between transmitter and its antenna.


Fig. 12-20a Radiation resistance as a function of rod length, rod spacing and frequency.


Fig. 12-20b Antenna reactance as a function of rod length, rod spacing and frequency.
setting

$$
\nabla \cdot(\vec{E} \times \vec{I})=\vec{H} \cdot \nabla \times \vec{E}-E \cdot \nabla \times \vec{H}
$$

and using the results of Eqs (12.6), Eq (12.22) becomes

$$
\begin{equation*}
\oint_{A}(\vec{E} \times \vec{H}) d \vec{A}=-\frac{\partial}{\partial t} \int_{V} \frac{1}{2}(\vec{H} \cdot \vec{B}+\vec{E} \cdot \vec{D}) d V-\int_{V} \vec{J} \cdot \vec{E} d V \tag{12.23}
\end{equation*}
$$

Ohm's law written in the form of Maxwell's equation is

$$
\begin{equation*}
\vec{J}=\sigma\left(\vec{E}+\vec{E}_{e}\right), \tag{12.24}
\end{equation*}
$$

where $\sigma$ is the conductance of the material, $\vec{E}$ is the induced field, and $\vec{E}_{e}$ is the emf field (externally applied field).

Equations (12.23) and (12.24) solved for the total driving power $P$ at the feedpoints give

$$
P=\int_{V} \vec{J} \cdot \vec{E}_{e} d V=\frac{\partial}{\partial t} \int_{V} \frac{1}{2}(\vec{H} \cdot \vec{B}+\vec{E} \cdot \vec{D}) d V+\int_{V} \frac{\mid \vec{J}}{}^{2} d V+\oint_{A}(\vec{E} \times \vec{H}) \cdot d \vec{A} .
$$

Assuming no losses in the antenna, and because the average of the energy contained in the volume, $V$, is constant in time for sinusoidally-varying quantities, $\mathrm{Eq}(12,23)$ and (12.24) break down to

$$
\begin{equation*}
P=\int_{V} \vec{J} \vec{E}_{e} d V=\oint_{A}(\vec{E} \times \vec{H}) \cdot d \vec{A} \tag{12.26a}
\end{equation*}
$$

and

$$
\begin{equation*}
P=-\int_{V} \vec{J} \cdot \vec{E} d V . \tag{12.26b}
\end{equation*}
$$

Either Eq (12.26a) or (12.26b) can be used to calculate the impedance $\mathrm{Z}_{\mathrm{A}}$ - ${ }^{\text {. }}$


Fig. 12-21 Geomstry and current distribution of a sail.


Fig. 12-22 Evaluation of the vector potential due to a current strip of width $W$ and height $\mathrm{dz}^{\prime}$.

From (12.26a) it follows for $\mathrm{R}_{\mathrm{A}-\mathrm{B}}$ that

$$
\begin{equation*}
\frac{R_{A-B}(h, d, \lambda)}{o h m s}=\frac{60 \pi^{2}}{\lambda^{2} \sin ^{2} k L} \int_{\theta=0}^{\pi} \sin ^{3} \theta F^{2}(\theta, h, d, \lambda) d \theta \tag{12,27}
\end{equation*}
$$

From the condition of continuity of the tangential component of $\vec{E}$, and with the $\mathrm{Eq}\left(12.26 \mathrm{~b}\right.$ ) and (12.11), it follows for $\mathrm{X}_{\mathrm{A}-\mathrm{B}}$ (2) that

$$
\begin{align*}
\frac{X_{A-B}(h, d, \lambda)}{o h m s}= & -\frac{15}{\sin ^{2} k L} \int_{0}^{2 k L}\left[S_{3}(x) \sin x+S_{4}(x) \cos x\right. \\
& \left.+T_{3}(x) \cos (2 k h-x)+T_{1}(x) \sin (2 k h-x)\right] d x \tag{12,28}
\end{align*}
$$

where

$$
\begin{aligned}
& S_{3}(x)=S i(x)-S i(2 k L-x), \\
& S_{4}(x)=\ln \left[\frac{x(2 k L-x)}{k^{4} \rho^{4}}\right]+C i(x)+C i(2 k L-x)-2 E, \\
& T_{3}(x)=\ln \left[\frac{2 k(L+2 d)-x}{2 k(2 L+2 d)-x}\right]+C i[2 k(L+2 d)-x]-C i[2 k(2 L+2 d)-x],
\end{aligned}
$$

and

$$
T_{1}(x)=S i[2 k(L+2 d)-x]-\operatorname{Si}[2 k(2 L+2 d)-x]
$$

$\mathrm{R}_{\mathrm{A}-\mathrm{B}}$ and $\mathrm{X}_{\mathrm{A}-\mathrm{B}}$ are plotted for various conditions in Fig. 12-20a (Real) and 12-20b (Imaginary).

### 12.7.3 Beacon Transmitter Antenna

For the study of the sail characteristics as an antenna set for the beacon transmitter, one starts again from the parameters of a very short dipole given in Eq (12.10a) and (12.10b). Figure 12-21 represents a sail in the $(y-z)$ plane, with $\alpha$ a small angle such that $W \simeq z^{\prime} \alpha$. Furthermore, the current density at $z^{\prime}$ is $\frac{I_{z}\left(z^{\prime}\right)}{W\left(z^{\prime}\right)}$. The first step is now to study the vector
potential and the electric potential of such a narrow current strip of width W and length $\mathrm{dz}^{\prime}$. This can, however, be considered as constituted of several very short, parallel dipoles, all aligned next to each other along the $y$-axis and each carrying a current $\frac{I_{z}\left(z^{\prime}\right)}{W\left(z^{\prime}\right)}$ dy'. This is again represented in Fig. 12-22.

For the small, short dipole, one may now write Eq (12.10b):

$$
\begin{equation*}
d \stackrel{\rightharpoonup}{A}=\frac{d z^{\prime}}{4 \pi} \frac{I_{z}\left(z^{\prime}\right)}{W\left(z^{\prime}\right)} d y^{\prime} \frac{e^{-j k R}}{R} \hat{z}^{\prime} \tag{12.29}
\end{equation*}
$$

With $R \cong R_{0}-y^{\prime} \cos \gamma$, it follnws for the vector potential of the narrov current strip shown in Fig. 12-22 that

$$
\stackrel{\rightharpoonup}{A}=\frac{d z^{\prime}}{4 \pi} \frac{I_{z}\left(z^{\prime}\right)}{W\left(z^{\prime}\right)} \frac{e^{-j k R_{0}}}{R_{0}} \hat{z} \int_{-\frac{W\left(z^{\prime}\right)}{2}}^{\frac{W\left(z^{\prime}\right)}{2}} e^{j k y^{\prime} \cos \gamma} d y^{\prime}
$$

and finally

$$
\begin{equation*}
\dot{\mathrm{A}}=\frac{d z^{\prime}}{4 \pi} I_{0} \frac{e^{-j k R_{0}}}{R_{0}} \frac{\sin \frac{k W}{2} \cos \gamma}{\frac{k W}{2} \cos \gamma} \hat{z} \tag{12.30}
\end{equation*}
$$

Analog considerations show that the expression for the electric potential at any point in space for a current strip of width, $W$, and length, $d z$, is

$$
\begin{equation*}
\phi=\frac{d z^{\prime}}{4 \pi j \omega \epsilon} I_{0}\left[j \frac{k}{R_{0}}+\frac{1}{R_{o}^{2}}\right] \frac{z}{R_{0}} \frac{\sin \frac{k W}{2} \cos \gamma}{\frac{k W}{2} \cos \gamma} . \tag{12.31}
\end{equation*}
$$

A comparison of the Eq (12.10a) and (12.10b) with Eq (12.30) and (12.31) shows that these two sets of equations differ only by the factor

$$
\begin{equation*}
\frac{\sin t}{t} \text { where } t=\frac{k W}{2} \cos \gamma . \tag{12,32}
\end{equation*}
$$

The smaller $W$, the closer is $t$ to 0 and, therefore, $\frac{\sin t}{t} \simeq 1$. In the worst case, where $W=20 \mathrm{~cm}$, $t$ becomes

$$
t=\frac{2 \pi}{\lambda_{75} \mathrm{MH}_{z}} \frac{20}{2}=\frac{2 \pi}{400} \quad \frac{20}{2}=9^{\circ}
$$

and

$$
\frac{\sin t}{t}=\frac{0.156}{0.157} \simeq 1
$$

Therefore, Eq (12.30) and (12.31) are similar to (12.10a) and (12.10b), hence the electromagnetic properties of a pair of sails are essentially the same as for a pair of rods. Consequently, the sails look like a pair of tilted rods, for which case the normalized patterns are given in Fig. 12-23 and $12-24^{*}$.

### 12.7.4 Mutual Coupling

The problem here is to determine how much of the 2 kW RF power of the main transmitter is received by the beacon antenna. This power in its turn will set the on-off characteristics (requirements) of the beacon antenna switch (see Section 12.6). The main function of the beacon antenna switch is to isolate the beacon transmitter from the main transmitter. Because of the relative complexity in geometry of the two antenna systems on the spacecraft, an analytical solution here would be quite cumbersome. The best way to determine the mutual effect is to measure the amount of power received by a pair of sails when the 2 kW transmitter is activated. However, in order to attain a qualitative feeling for the amount of the RF power coupled to the beacon antenna, a very simplified system was considered, in which the total power received by the beacon antenna was assumed to be composed of two separate parts.

That is:
Total RF received power $=\mathrm{RF}$ received power due to radiation ( $\int \overrightarrow{\operatorname{Sd}} \vec{A}$; physical aperture) +RF received power due to capacitive coupling.
For the geometry of Fig. 12-15 the radiation part of the RF received power was found to approximately 50 watts. As to the second term, the coupling capacity of the system was found to be about 0.52 pF . This corresponds at 80 MHz to a reactance of $4 \mathrm{k} \Omega$. Thus, the RF power received at

[^3]Fig. 12-23 Normalized $|\vec{H}|$ and $|\vec{E}|$ patterns for a dipole constituted by a pair of tilted rods.


Fig. 12-24 Normalized $|\vec{S}|$ pattern for a dipole constituted by a pair of tilted rods.
the beacon in a $50 \Omega$ system due to capacitive coupling would be:

$$
20 \log \left(\frac{4050}{50}\right)=38 \mathrm{~dB}
$$

38 dB down as the 2 kW main transmitter power. This is approximately 0.3 W , which is essentially negligible when compared to the 50 W due to direct radiation.

In conclusion, there will therefore be a mutual coupling of approximately

$$
\text { Coupling }=10 \log \left(\frac{50}{2000}\right)=-16 \mathrm{~dB}
$$

-16 dB between the main and beacon antenna systems; and the design of the beacon antenna switching system has been based on the 50 W power level.

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## analzsis of oftinal energy storages

by

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## ANALYSIS OF OPTIMAL ENERGY STORAGES

by

## DIETER VILLNER

Submitted to the Department of Electrical Engineering on May 17, 1968, in partial fulfillment of the requireitents for the Degrces of Master of Science and Electrical Engineer.


#### Abstract

The problem investigated is the beat design of energy storage systems to match a given power input - output relation. To consider a specific application a sateliite system is selcetcd and two models are investigated. These were battery and a corabined battery-capacitor atorage. They were analytically finvestigated and optimized to yield a maximum number of pulses that can be transmitted under the condition that the voltage at the transinitter does not drop under a given minimum. In order to do so, assumptions and inearizaticns arenecessary. The battery is modeled as a very large capacitor with internal resistance (as long as the voltage drop is ilmited to a certain small value). A periodic process is assumed and it is shown to have a steady state. The solutions of both models are calculated by computer and graph1cally compared.

It is found that the battery capacitor model has in general a higher pulse efficiency than the battery model. The reasons for this are discussed. The dependence of the solutions on the parameters are found and compared.


THESIS SUPERVISOR: David C. White
TIILE: Ford Profeseor of Engineering,

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## TABIE OF CORTEN'S

Page
6
cincter I
6
I. 1 Problem: Statenent
9
J. 2 . Mathematical Model of the Battery
11
I. 3 Practical Energy Storages
I. 4 Optirai Case Mode 116
CHAPTER II ..... 17
II. 1 Analyais of Model 1 and \#2 ..... 17
II. 2 Node1 \#1 ..... 17
II. 3 Model \#2 ..... 23
II.3.1 Switch S Open ..... 24
II.3.2 Switch S Closed ..... 27
II.3.3 Steady State Solution ..... 27
II. 4 Conclusions ..... 33
CHAPTER III ..... 34
III. 1 Calculation of N ..... 34
III. 2 Model \#1 ..... 35
IIJ.2.1 Calculation of $N\left(W_{t}\right)$ ..... 36
III.2.2 Calculation of $N(T)$ ..... 38
IIf.2.3 Calculation of $N(B)$ ..... 38
III. 3 Nodel \#2 ..... 39
III.3.1 Calculation of $N_{B}, N_{C}=f\left(R_{C}\right)$ ..... 40
III.3.2 Calculation of. $N(C)$ ..... 42
III.3.3 Calculation of $N\left(W_{t}\right)$ ..... 43
III.3.4 Calculation of $N(\tau)$ ..... 43
III.3.5 Calculation of N( $\beta$ ) ..... 45
III. 4 Comparison between Model $\# 1$ and Model $\# 2$ ..... 46
III.4.1 Optimi:nation over $R_{c}, N_{2}=N_{2}\left(R_{c}\right)$ ..... 46
III.4.2 Optimization over $C, N_{2}=N_{2}(C)$ ..... 48
III.4.3 Plot of $N=N\left(N_{t}\right)$ ..... 48
III.4.4 Plot $N=N(\tau)$ ..... 56
IIJ. 4.5 Plot of $N=N(B)$ ..... 56
III. 5 Conclusions ..... 59

## TABLE OF CONTENTS (Continued)

Pare
CHAPTER IV ..... 60
IV. 1 Conclusions ..... 60
IV. 2 Critique ..... 61.
CHAPTER V - APPENDIX ..... 63
V. 1 Efficiency of Capacitors ..... 63
V. 2 Battery Current for Maximun Energy Output ..... 64
V. 3 Coriparison of Energy Densities ..... 65
V. 4 Co:nputer Program for: $N_{1}, N_{2 B}, N_{2 C}$ ..... 66
V. 5 Tables ..... 68
REPERENCtS ..... 86

## CHAPTER I

## I. 1 Problem Stateinat

This investigation is concerned with optimizing an energy storage for an input - output relation, that appears quite frequently In applications where the Input is continuous at low level and the output is in the form of large energy pulses. The problem of energy storage arises when the instantaneous power generation is lower than the instantaneous power requirements $\left(P_{1}>P_{0}\right)$. Clearly over some average time interval the generated energy has to be higher than the required energy. That is

$$
\int_{T_{0}}^{T_{1}} P_{i} d t>\int_{T_{0}}^{T_{1}} P_{0} d t
$$

where

$$
\begin{aligned}
P_{i} & =\text { generated power } \\
P_{0} & =\text { required power } \\
T_{1}-T_{0} & =\text { sone time interval. }
\end{aligned}
$$

E:ar:ples of this kind of applicnifna are satellite transmitters, remote pulse transmitters ( rather stations) and stroboscopic lights. They of ten have the following input - output characteristic.


Figure 1
$P_{1}$ is the power ganerated (usually by oolar cells, batterics os very amall reactora), $P_{0}$ is the required power (or received poicer) by the load. If the process has a period $T$, then for

$$
\left.\int_{t}^{t+T} P_{i} d t>\int_{t}^{t+T} P_{0} d t \quad \text { (for all } t\right)
$$

simple energy storeges like springs, capucil:ors or inductors are sufficient for atoring energy, but in the casc of:

$$
\int_{t_{0}}^{t_{0}+T} P_{i} d t<\int_{t_{0}}^{t_{0}+T} P_{0} d t \text { for scine } t_{0}
$$

ve need storage elements with high energy capacity in wi:ich the energy is stored at $t<t_{0}$ like spinning masses, elevated masses in a gravitational field, batterics, or high magnetic fields stored using superconducting coils.

To evaluate the easential factors of such a cystem, we will desfign an cuargy atorage syaten for a exeicllite. It should hata a
high reliability (this often mars simplicity and redundancy in parts), high efficiency, and a low weight. Except: for the required leg weight, the other requirements occur in bung ocher fields of application. The following overall network will he proposed:


Figure 2
The transmitter is modeled as resistor (no inductive or capacitive reactance) with two states $\left[R=R_{d}\right.$ (discharge resistor) for $(T-\tau)<t \leq T$ and $R=\infty$ elscmare $].$


Figure 3

The energy etorage systom will be asinumid to be a network of batteries, resistors or capacituri. We will show liat inductorf are not woll suited for thia purpose because their ratio of weight to etored encrgy in too large. Thin will be shom in the appendi\%. Storing cnergy in rotating devicea is very Lempting and cortainly very effective for very large anounts of energy over short tiacn, buf for small mounts of encrgy the rntio of friction lossen to stored encrey is unfavorable. The energy torage we wlll consider conetste of high reliability dements like capacitors (with negligible small internal. reoistance), batterien (with internal resictance), or a combination of both (linked by resiators).

## I. 2 Mathemeticnl Nodol of the Batecy'y

Since we want to inveatignte the ntorage syoteme analyticall.y, we need a mathe:natical nodel for the buthery thet should be closc to the bchavior of real batterics. Batterics are asscoued to have the following charactaristics. [2], [3].


Fig:ra h.a

$31: 1 \% 4.1$.

The characteristics for Ag-2n, Ag-Cd, Ni-Cd and other types of batteries are very similar. The characteristics show that we can achieve highest efficiency when we discharge the battery at low currents. It looks as if the losses in the battery (due to polarimaticim and internal resistance) are proportional to the decrease In efficiency. Using this observation we sn that the efficiency is highest when the losses in the battery are lowest.

Energy lost in battery $-\int_{0}^{T} I^{2} R_{i} d t=$ Min.
where $R_{f}=$ internal resistance
I - battery current.

We use the following model for the battery for $U_{0} \geq U_{B} \geq B U_{0}$


Instead of minirileing the lose oncrgy ve can maxinize the output energy.

$$
E_{\text {out }}=\int_{0}^{I}\left(U_{0} I-I^{2} R_{i}\right) d t=M a x
$$

Asking that the battery delivere a charge $k$ in the tince intervel. results in the following constraint:

$$
k(A h)=\int_{0}^{T} I d t
$$

Using variational calculus (sce appendix) we find that the oulput energy is a maximum for I - const. = k/T.

## I. 3 Practical Encray Storases

since the load (in our case a transmitter) requires a high power we need an energy storage system output with very $10: 1$ resistance. A cepacitor has practically no internal resistence but stores only emall amounts of encrey per pound. The battery with its very high storage capacity has internal resistance that is not negilgible.

We will investigate two storage systems: one vith a baticery only, and the other one a combination of battery, resiftor and capacitor.


Model 1
Figure $6 . a$


Mode 12
Figure 6.b

Looking at Model ${ }^{2}$ we ask if it is possible to gain over the performance of Model 1. There is an additional stage In the storage system and as show in the appendix, we can get out only oue-half of the energy supplied to charge the capacitor. This model is at beet $50 \%$ efficient. First, consider Model \#1 with its high current over very short tina. If we take the total time ( $N$ t, $N$ number of pulses, $T$ pule width) we might get a very short total discharging time ( $0-3$ mintues as it turns out later), where the battery efficiency from Figure $4 b$, page 9 is very low.

Model 2 dissipates approximately the same snout of energy that it delivers to the load, but due to the rime discherining of the battery. ( $0.05-2$ hours) it will work in a region with much higher efficiency.

Both proposed models (battery only and battery, capacitor, resistor model) do not have constant discharge current. Model \#1

a very hish current. Model 2 (battery, capacitor, resistor) docs the sane, only that the high current in not delivered by the battexy, But by the capacitor (with negligible Internal resistance). The capacitor is then charged by the battery with a low (exponential) current (sce Figure 7b).
a) Model 1 .
battery current

b) Model $\$ 2$
load current


Figure 7

Thoush nelther Model 11 nor he have a coistant battexy current, Model \#2 has a low battery current, when we choose $R_{i}+K_{c}$ (the charge resistance between battery and capact.tor) in the appropriate way.

## I. 4 Optimal Case Model

There are possibilities to discharge the battery with
constant current $I=k / T$.


Figure 8
Assume $U_{0}=$ constr.

$$
U_{B}=U_{0}-I R_{i}
$$

I - cont.

$$
U_{B}=\text { cons. }
$$

$$
I=\frac{d Q}{d t}=\frac{d}{d t}\left[C U_{c}\right]=\frac{d U^{c}}{d t} c_{c}+U_{c} \frac{d C}{d t}
$$

$$
Q \text { = charge of capacity: }
$$

$$
u_{c}=\text { constr. } \rightarrow \frac{d U_{c}}{d t}=0
$$

$$
I=U_{B} \frac{d C}{d t}=\frac{k}{T} \text { (from optinfyation) }
$$

$$
\rightarrow d C=\frac{k}{\bar{U}_{B}^{T}} d t
$$

or

$$
c(t)=c_{0} t, c_{0}=\frac{k}{U_{B} T}
$$

for

$$
t=T-T
$$

and

$$
T \ll T \longrightarrow t \approx i
$$

$$
\begin{gathered}
C(T)=C_{0} T=\frac{k}{U_{B}} \\
U_{B}=U_{0}-I R_{1}=\frac{U_{0} T}{T+T_{C}}
\end{gathered}
$$

where $T_{c}=R_{1} C$
then

$$
\begin{aligned}
& c_{0}=\frac{k\left(T+T_{c}\right)}{U_{0}^{T}} \\
& I=\frac{U_{0} C_{0}}{T+T_{c}}=\text { cons. }
\end{aligned}
$$

This is a realizable solution. Varying the capacitor $c(t)=C_{0}(t)$ for $0<t \leq T$ produces a constant current. But since we need additional. (mechanical) networks we need additional energy sources. Therefore this solution is not practical.


## I. 5 Conclusfons

We are looking for an optimal energy storage systam for given input - output reletioin. We say the eystem is optimal, when it delivers the maximum number of pulses at a given weight. Proposing aeveral models we ruled out moving energy storages, capacitors and inductors, becalise of low storage capacity and proposed Nodel $\$ 1$ and 2 , which are investigated and compared in the follewing chapters.

## II. 1 Analysis of Model and 12

Both Nodels can be represcised in the following form:


Figure 10
We want to find the number of pulses that each model cen produce, providuri that certain ilmita within the storaga aystems will not be exceeded. These limits will be that the voltage across the battery and the voltage acrose the load stay above certain values.

For this purpose we bet $P_{1}=0$ and calculate this number N. If it turns out that the total discharge time of the batceries (the tine for $N$ pulses) is short compared with the cherging tias, then we do not only got the pulse efficiency (number of pulses par loaded battery) but a number thint is very close to the maximun number that we can send without time out for charging the batterics.

## II. 2 Madel 1

Assumptions: (a) all elements are linear in the working range:
(b) switch $S=1$ ical $\quad R=0 \quad S$ closed $R=\infty S$ open

Figure $1!$


$$
1_{1}=0 \longrightarrow p_{1}=0
$$

We model the battery as a large capacitor with internal
resistance. The battery emf is:

$$
\begin{equation*}
U_{0}(t)=U_{0}(0)[1-a q(t)] \tag{1}
\end{equation*}
$$

where: $U_{0}(0)=$ battery voltage of the charged battery, for $1=0$,
and

$$
\begin{aligned}
t & =0 \\
U_{0}(t) & =U_{B}(t) /_{i=0} \\
\alpha & =1 / Q_{B} \\
Q_{B} & =t o t a l \text { battery charge } \\
G(t .) & =\int_{0}^{t} 1 d t=\text { charge inlivi ged b; the battery }
\end{aligned}
$$

$$
Q_{B}=q(\infty)=\int_{0}^{m} 1 d t
$$

Since the puise width $r$ is constant and there in no charging, $1_{1}=0$, we can substitute

$$
\begin{equation*}
t=n t \tag{2}
\end{equation*}
$$

where $n$ is the number of pulses (ve consider only the discharge tims).

## Substituting

$$
U_{0}(t)=i\left(R_{i}+R_{d}\right)
$$

and

$$
\begin{gathered}
R=R_{1}+R_{d} \\
1\left(R_{i}+R_{d}\right)=U_{0}(0)[1-0.4(i)]
\end{gathered}
$$

The bettery current is then

$$
\begin{equation*}
I(t) \quad \omega \frac{U_{0}(0)}{R} \exp \left[-\frac{V_{0}(0)}{R} t\right] \tag{3}
\end{equation*}
$$

The cinage taken out of the battery:

$$
\begin{equation*}
q(t)=\int_{0}^{t} 1(t) d t=\frac{2}{a}\left\{1-\operatorname{cosip}\left[-\alpha \frac{u_{0}(0)}{R}-t\right]\right\} \tag{1}
\end{equation*}
$$

The emf actoss the battery 1s:

$$
\begin{equation*}
u_{g}(:) \quad \because \quad u_{i}\left((1) \cdots: \frac{u_{0}(0)}{k}=\right] \tag{5}
\end{equation*}
$$

The voltage acroen cin batcery is:

$$
\begin{gather*}
U_{B}(t)=U_{0}(1)-1(t) R_{1}  \tag{1}\\
U_{B}(t)=\frac{R_{d}}{R_{1}+R_{d}} U_{0}(1) \operatorname{cosp}\left[-\frac{U_{0}(0)}{R} \cdot t\right] \tag{7}
\end{gather*}
$$

In order to protect the buttery fron destruction ve linit the voltage drop $\beta$

$$
B=\frac{U_{0}(0)-U_{B}(t)}{U_{0}(0)}
$$

The allowable voltage drop $\beta$ depends very much on the type of battery, but seldom exceeds 0.15-0.20. Usually the battery is recharged for $\beta \geq 0.1$.

The minimurn voltage is

$$
\begin{equation*}
U_{B \min }(t)=(1-\beta) U_{0}(0) \tag{8}
\end{equation*}
$$

Inscrting (8) in (7)

$$
\begin{equation*}
1-\beta=\frac{U_{B m m_{1}}(t)}{U_{0}(0)}=\frac{R_{d}}{X_{i}+k_{d}} \exp \left[-U_{-0} \frac{U_{0}(0)}{R} t\right] \tag{9}
\end{equation*}
$$

From this equation we can determine the total discharge tive $t$ :

$$
\begin{equation*}
=-\frac{R}{C_{0}(0)} l_{1}\left[\frac{R_{d}}{R_{i}+R_{d}} \frac{1}{j-i}\right] \tag{1.0}
\end{equation*}
$$

Inserting (2). $t=$ Ni into (10) we obtain the number of pulses:

$$
\begin{equation*}
N=\frac{t}{i}-\frac{R}{d_{n}(0) i} \ln \left[\frac{R_{d}}{R_{1}+R_{d}} \frac{1}{1-B}\right] \tag{11}
\end{equation*}
$$

In the case where the alloivabla voltage drop at the load (i:) Is Joan than the allowed voltage drop at the battery ( $A$ ), we att $B$ - $x_{0}$

There are the following two caries:
(a) B<K choose B
(b) B2K set $B=K$

## Optimisaticil of N

In order to have positive $N$ it follows from (11)

$$
\begin{align*}
& \frac{R_{d}}{R_{1}+R_{d}} \frac{1}{1-\beta} \geq 1 \\
\longrightarrow & R_{1} \leq \frac{B}{1-\beta} R_{d}
\end{align*}
$$

Where is the maximum of $N$ with respect to $R_{i}$ ?

$$
\frac{d N}{d R_{1}}=\frac{E N}{d R} \cdot \frac{d R_{1}}{d R_{1}}=0
$$

since

$$
\begin{aligned}
R & =R_{1}+R_{d} \\
\longrightarrow d R & =d R_{1}
\end{aligned}
$$

and

$$
\frac{d N}{d R_{1}}=0=\frac{1}{C U_{0}(0) \tau}\left\{\ln \left[\frac{R}{R} \frac{1}{1-\beta}\right]-1\right\}=0
$$

Then the maximum $\left(\frac{d^{2} N_{2}}{d R_{i}^{2}} \leq 0\right)$ occure fol

$$
\begin{equation*}
R_{1}=R_{d}\left[\frac{1}{c(1-c)}-1\right] \tag{13}
\end{equation*}
$$

where e - basc of nntural logarithm.

```
For most applicatcons & \leq 0.2
```

$$
R_{1} \leq-0.54 R_{d}
$$

For all

$$
R_{1} \geq R_{d}\left(\frac{1}{C(1-\beta)}-1\right)
$$

$$
\rightarrow \frac{d N}{d R} \leq 0
$$



Figure 12
Therefore we conclucie that the feasth 1 . manim: occurs for $\|_{1} m$ or $R_{1}$ ay emall es possibie. In other vords the better the battery ( $R_{1}$ is omall) the higher the number of pulses.
11.3 Model 2


Figure 13
Assumptions: (a) all elements arc lincar in the working range;
(b) switch $S$ is idenl $R=0 \quad S$ closed $R=\infty \quad S$ open

We assume that the system has performed n periods, where one period is the time $t$ :

$$
m T<t \leq(m+1) T
$$

where the capacitor will be altomistely ciarged add discharged.


Figure 14

Let us açume:

$$
\begin{aligned}
& 1_{1}=0 \\
& \text { charge resistance }=R=R_{1}+R_{c} \\
& \text { battery emf } \quad-U_{0}(l)=U_{0}(0)[1-\operatorname{ciq}(t)] \\
& \text { charge } \\
& =q(t)=\int_{0}^{t} 1(t) d t \\
& \text { battery constant } \quad a=1 / q(\infty)
\end{aligned}
$$

and for

$$
n T \leq v \leq(n+1) T
$$

1et

$$
v=n T+t
$$

This inplies

$$
\begin{equation*}
t=v(\bmod T) \tag{14}
\end{equation*}
$$

Rewrite:
the battery emf $=U_{0}(v)=U_{0}(n T+t)=U_{n}(t)$
the batiory voltage $=U_{B}(v)=U_{B}(n T+t)=U_{B n}(t)$
the capacitor voltage $=U_{C}(v)=U_{C}(n T+t)=U_{C n}(t)$
$U_{C C n}=$ maximum capacitor voltage during the n'th period.

## II.3.1 Sritch S Ope:2

When the switch $S$ is open, we can write the following equation for the $n$ 'th period:

$$
\begin{align*}
U_{0}(v)=U_{n}(t) & =U_{n}(0)\left\{1-\alpha_{n} \int_{0}^{t} i(t) d t\right\} \\
& =1(t) R+\frac{1}{C} \int_{0}^{t} 1(t) d t+U_{c n}(0) \tag{16}
\end{align*}
$$

If we solve this equation for tell current $1(t)$, we get the follcing result:

$$
\begin{equation*}
i(t)=\frac{U_{n}(0)-U_{c n}(0)}{R} \exp \left\{-\left[\frac{1}{R \bar{C}}+u_{n} \frac{u_{n}(0)}{R}\right] l\right\} \tag{17}
\end{equation*}
$$

 maximum voltrge of the capacitor.

$$
\begin{align*}
U_{c e: n} & =U_{c 11}(T-t)=U_{c r i n}(T) \quad \therefore U_{c r i}(0)+\frac{1}{c} \int_{0}^{T} 1(t) d t \\
& =U_{c 11}(0)+\frac{1}{C} q_{n 1}(T) \tag{18}
\end{align*}
$$

 cycle.

$$
\begin{equation*}
q_{n}(T)=\int_{0}^{T} 1 d t=\frac{u_{n 1}(0)-u_{i n}(0)}{\frac{I}{C}+a_{n} u_{n}(0)}\left\{1 \cdots \operatorname{ces}\left[-1 \frac{1}{n \pi}+a_{n} \frac{u_{n}(0)}{-1}\right]\right\} \tag{19}
\end{equation*}
$$

If we substitute

$$
\begin{equation*}
\gamma_{n}^{x}=\exp \left[-\left(\frac{1}{R C}+e_{n} \frac{u_{n}(0)}{k}\right) r\right] \tag{20}
\end{equation*}
$$

We cia write Uzen

$$
\begin{equation*}
u_{c c n}=U_{c r_{1}}(0)+\frac{u_{r i}(0)-U_{c r 1}(0)}{1+v_{n} U_{i i}(0) c}\left(1-i_{n}^{x}\right) \tag{21}
\end{equation*}
$$

Invaricace of $\gamma_{n}^{N}, \operatorname{rin}_{n} U_{n}(0) C$
We will show that $a_{n} l_{n}(0) C, \gamma_{n}{ }^{x}$ do not cepasad on a.

We karin from the definition of an $\frac{1}{Q}$

$$
a_{n}=\frac{1}{a_{n}}
$$

where $Q_{n}$ - total battery charge remaining in the battery after n periods;

Q $=$ original charge.

$$
Q_{n}=Q-q(n)
$$

with

$$
q(n)=\sum_{n=0}^{n} q_{n}(T)
$$

then the battery voltage (assuming we extract $q(\mathcal{H})$ during on a period from the battery)

$$
\begin{aligned}
U_{n}(0) & =U_{0}(0)[1-\varepsilon q(n)] \\
a_{n} U_{n}(0) & =\frac{U_{n}(0)}{Q_{n}}=\frac{U_{0}(0)[1-a q(n)]}{Q-q(n)} \\
& =U_{0}(0) \frac{1}{Q} \cdot \frac{0-0(n)}{Q-q(n)} \\
a_{n} U_{n}(0) & =\alpha U_{0}(0)
\end{aligned}
$$

the product $\alpha_{n} U_{n}(0)$ does not depend on $n$ and similarly $\gamma_{n}^{x}=\gamma_{0}^{x}=\gamma^{x}$. Then

$$
\begin{equation*}
u_{c \in n}=U_{c n}(0)+\frac{U_{n_{1}}(0)-[]_{c n_{i}}(0)}{1+n_{0}(0)}\left(1-\gamma^{x}\right) \tag{22.}
\end{equation*}
$$

$$
\begin{equation*}
q_{n}(T)=\frac{U_{n}(0)-U_{c n}(0)}{\frac{I}{C}+c U_{0}(0)}\left(1-r^{v}\right) \tag{23}
\end{equation*}
$$

### 11.3.2 Switch Closed

Since we assumed that: $\tau$ << $T$, we conclude that the battery will not contribute to the current in $R_{d}$ for $T-T \leq t \leq T$ and we have the following equivalent network:

for $t \geq 1-\tau$

$$
\begin{gather*}
\left.U_{e n}(t)=U_{c \in n} \operatorname{exf} t \frac{t-(T-T)}{R_{d} c}\right)  \tag{24}\\
U_{c n}(T)=U_{c n+1}(0)=U_{c \in n} \exp \left(-\frac{\tau}{R_{d} c}\right) \tag{25}
\end{gather*}
$$

substituting

$$
\begin{gather*}
\delta=\operatorname{cxp}\left(-\frac{T}{R_{d} C}\right) \\
\cdot  \tag{26}\\
U_{c n+1}(0)=U_{c c n^{\prime}}
\end{gather*}
$$

## 1I.3.3 Steady State Solution

Now we have all the answers of the $n$ 'th period, given the int ital conditices of the $n$th perbri, We could concave end determine the initial conditions for the ( $n+1$ )'th period and
enleulate the $(n+1)$ 'th pariod, Bit for very large $n$ we would spend tou much tite, therefore we try to determine the conditions of the n'th perilod, given tic initial condition at tro for n=0.
let us assume

$$
\begin{equation*}
U_{c n}(0)-c_{n} U_{n}(0), \tag{27}
\end{equation*}
$$

and ou say the systen is in steady state for

$$
\begin{equation*}
\varepsilon_{n}=\varepsilon_{n+1} \text { for all } n(n=0,1,2, \ldots) \tag{28}
\end{equation*}
$$

We calculate $\varepsilon_{n+1}=\frac{U_{6 n+1}(0)}{U_{n+1}(0)}$ with (27), (26), (25), (23), (22), (1)

$$
\begin{aligned}
c_{n+1} & =\frac{\delta\left\{U_{c n}(0)+\frac{u_{n}(0)-U_{c n}(0)}{1+a U_{0}(0) c}\left(1-r^{n}\right)\right\}}{U_{n}(0)\left\{1-a_{n} \frac{u_{n}(0)-U_{c n}(0)}{\frac{1}{c}+a U_{0}(0)}\left(1-\gamma^{*}\right)\right\}} \\
& =\frac{\delta\left\{\frac{u_{c n}(0)}{U_{n}(0)}+\frac{1-\frac{U_{c n}(0)}{U_{n}(0)}}{1+a U_{0}(0) c}\left(1-\gamma^{*}\right)\right\}}{1-\frac{U_{c n}(0)}{U_{n}(0)}}
\end{aligned}
$$

Before we continue we will. make anoticer oubstitution for the product $\alpha U_{0}(0) C$.

$$
\begin{aligned}
a & =1 / Q \\
Q & =U_{0}(0) C_{B} \quad \text { battery charge } \\
C_{B} & =\text { euqdivalent capacitor } \\
a U_{0}(0) C & =\frac{U_{0}(0) C}{U_{0}(0) C_{B}}=\frac{C}{C_{B}}=\rho
\end{aligned}
$$

Substitute

$$
\begin{equation*}
a U_{0}(0) c=a_{n} U_{n}(0) c=p \tag{29}
\end{equation*}
$$

Since we assume there exists an $\varepsilon=\varepsilon_{n}$ then

$$
\begin{align*}
& \frac{U_{C n}(0)}{U_{n}(0)}=\frac{U_{c n+1}(0)}{U_{n+1}(0)}=\varepsilon \\
& \varepsilon=\frac{\delta\left[\varepsilon+\frac{1-\varepsilon}{1+p}\left(1-r^{k}\right)\right]}{1-p \frac{1-\varepsilon}{1+p}\left(1-\gamma^{k}\right)} \tag{30}
\end{align*}
$$

This gives the follwolng quadratic equation in $\varepsilon$

$$
\begin{equation*}
\varepsilon^{2}+\varepsilon \underbrace{\frac{\lambda+\rho \gamma-\delta\left(\gamma^{2}+\rho\right)}{\rho(1-\gamma)}}_{p}-\underbrace{\frac{\delta}{\rho}}_{m}=0 \tag{31}
\end{equation*}
$$

which can be solved in thin normal way

$$
\varepsilon=-\frac{p}{2} \pm \sqrt{\frac{p^{2}}{4}-m}
$$

Since all terms are independent of $n$, $c$ exists and $c c_{n}$ for all $n$. For the canc of constant battery voltage $U_{0}(0)=U_{1}(0)=\ldots=U_{n}(0)$, this inplice infinite battery change $\rightarrow \alpha=C, f=0$, then

$$
\begin{align*}
& r^{*}=\exp \left[-\left(\frac{1}{R C}+\frac{a U_{0}(0)}{R} \cdot r\right)\right] \\
& r^{\wedge}=\exp \left(-\frac{T}{R C}\right)=r \\
& c=\frac{1-r}{\frac{1}{\delta}-r} \tag{32}
\end{align*}
$$

## Apprny:imation for later computation:

For most applications $\rho \ll 1$ and we can substitute

$$
c=\frac{1-y}{\frac{1}{\delta}-\gamma}
$$

without making large errors.
Now we know there exists an initial condition
$U_{C O}(0)=\varepsilon U_{0}(0) \quad s o t h a t$
$U_{c n}(0)=c U_{n}(0)$ for all $n$, but we have not found on easy expression for
$U_{n}(0)=\left\{\left(U_{0}(0)\right\}\right.$.
We asounc there exists a E, such that:

$$
U_{1}(0)=\xi U_{0}(0)
$$

or

$$
U_{n}(0)=E^{n_{U}}(0)
$$

$0 r$

$$
\begin{gathered}
u_{n}(0)=\delta u_{n \cdot f}(0) \\
\varepsilon=\frac{u_{n}(0)}{v_{n-1}(0)} \cdot \frac{v_{n-1}(0)\left(1-a_{n} q_{n}\right)}{U_{n-1}(0)}=1-a_{n} q_{n}
\end{gathered}
$$

with

$$
\begin{aligned}
p & =a_{n} u_{n}(0) c \\
a_{n} & =\left(u_{6 c n}-v_{6 n}\right) c \\
u_{c c n} & =\frac{1}{6} v_{c n+1}(0)-\frac{1}{6} U_{n+1}(0) \\
u_{n+1}(0) & =\xi U_{n}(0)
\end{aligned}
$$

then

$$
\begin{align*}
\xi & =1-\alpha_{n} q_{n} \\
& =1-\alpha_{n}\left[\frac{1}{\delta} \varepsilon \xi U_{n}(0)-\varepsilon U_{n}(0)\right] c \\
& =1-0\left(\frac{\varepsilon}{\delta} \xi-\varepsilon\right) \\
\varepsilon & =\frac{1+\varepsilon \rho}{1+\frac{\varepsilon}{\delta} D} \tag{33}
\end{align*}
$$

With $\xi$ we can determine all values for ary given $n$, provided we start with $U_{c o}^{\circ}(0)=c U_{0}(0)$. But aven if we din't ste:it with $U_{c o}^{*}(0)$ we can use this formulag since this eysten hehaves 11t:e a dyamic system with dampina, every eolution will convorise te the steady otate solution for every initial conditina. For large n we will therefore neelect the effect of the initial co:idition and assuae that the system is in stady state for all tiacs.

$$
\begin{equation*}
u_{f:}(0)=u_{c}(0) \varepsilon^{n} \tag{3!}
\end{equation*}
$$

We can calculate the minimua battory voltage, onabining that the mindumen oseurs at the begiming of a periois.

$$
\begin{align*}
U_{B m i n} & =U_{c n}+\left(U_{n}-U_{: n}\right) \frac{K_{c}}{R_{1}+1 l_{c}} \\
& =U_{0}(0) \xi^{n}\left[(1-\varepsilon) \frac{R_{c}}{R_{1}+R_{c}^{-}}+\varepsilon\right] \tag{35}
\end{align*}
$$

We linit the voltage drop at the battery to $B U_{0}(0)$

$$
\begin{equation*}
U_{B \min }=(1-6) U_{0}(0) \tag{36}
\end{equation*}
$$

with (35) in (36)

$$
1-\beta=\xi^{n}\left[\frac{R_{c}}{R_{1}+1_{c}}-(1-c)+\varepsilon\right]
$$

or

$$
\begin{equation*}
N_{B}=\frac{1}{\ln \xi}\left\{\ln (1-\beta)-\ln \left[\frac{R_{c}}{R_{2}+R_{c}}-(1-c)+c\right]\right\} \tag{37}
\end{equation*}
$$

Limiting the voltage drop at the cepecitor to $\mathrm{KU}_{0}(0)$ leada to:

$$
\begin{equation*}
U_{c \operatorname{niln}}=U_{c n}(0)=c U_{n}(0)=c \xi^{n} U_{0}(0)=(1 \cdots) u_{0}(0) \tag{3i}
\end{equation*}
$$

or

$$
\begin{equation*}
N_{c}=\frac{1}{\ln \xi} \ln \frac{1-\kappa}{\varepsilon} \tag{0}
\end{equation*}
$$

New we heve the number of pulses when we weach rise mifulate bettex, voltage or when we reach the mininum ceiracitor voltage, cienryy, w

$$
\begin{gathered}
-33- \\
N=\operatorname{Nin}\left(N_{B}, N_{C}\right)
\end{gathered}
$$

### 11.4 Conclueions.

Assuming that we can model the bateury an a large capact tor with intex:'al impedance, we were able to calculate the number of pulser fer Model 11 and Nodel \#2. In order to do so for Moded 12 we had to introduce

$$
c=\frac{U_{c n}(0)}{U_{n}(0)}
$$

and

$$
\xi=\frac{u_{n}(0)}{u_{n-1}(0)}
$$

A further difficulty in detexminfig the number of pulses for Modal 12 In that wo obtained two resulte (36), (38) and we have to choose the emaller value of both.' The reanca for theon two resul.ts is the one degrec of frecdom wo have in chooing a combination of battery and capacitor, while in Modsl (1 everything is fixcd. The assumption $T \ll T$ is not necessary for Model \#1, but it is for $\# 2$. Therefore, for some $\tau>T_{0}$ (where $T_{0}$ is eme limit for $\tau$ ) the number of pulsen for 02 will be: in error and chould only be used for $T \leq T_{0}$.

## CIAPTLR III

In Chapter II we dorived thr necepsary foimulan to dotemine the numher of pulses for each Mode]. In thin clinpter wo will ufie the formulan and calculate an optimal cuergy btorage for the Sunblasore auteliftc. By varying differunt paranetere we wiil fínd the senoitivity of ench etorago system to these paramerern. Tien we will propose a cont function to evaluate the "8oodinens" of each modch and to male a comparison botwcen notrage syitems easicr and rore meaningful.

## III. 1 Calculation of $N$

What are the specifications for atelife applicetion? There will be a wight ilmit, which we will call ${ }_{t}$ where $t$ tands for total. The other specifications are the pulse pariod $T$, the pulue widsh $t$, the minimun and ma:inun tranmilter voltage $U$ orion , $U_{0}(0)$. Since we want to have a high nu:iber of plisen, we will use high capacity clements like $N$-Cd battarics ano electrolytic capacitore. These elenents are also very reliable wh!ch 1.8 one of the requirements for apace appilention. They have constant specific encrgy deneity (etored enerey por kg) over wide voltese rangea. Tils is not truc for capecitois, certeinly not at a law voltage level. Therafore we assume to have a de - de energy converter without losscs that shiftis the voltacc into a risce, Wine the energy density is approxinutely incependent of the voltage. We will choose the voltage

$$
U_{0}(0) \therefore \quad \therefore \quad \mathrm{V}
$$

(equale the oparating voltage in the sateluite).
the capacitor const.ant

$$
p_{c}=0.111 \mathrm{~F} / \mathrm{k}_{\delta} .
$$

the battery constant

$$
\rho_{8}=4000 \mathrm{As} / \mathrm{kg}
$$

and the internal battery resistance constant

$$
\rho_{R}=60, n_{i}-\rho_{R} \cdot \frac{U_{0}(0)}{Q_{L}}
$$

( $U_{0}(0)=$ battery emf at $t=0, Q_{\mathbb{E}}=$ total battery charge).
These constants are obtained by averagil: the avalloble enpirical data from references as [1],[2].

Necessary wires, resistors and switches (trensistors) ane considered to be negligible in weight and are not included in the weight: total. The computation han been dose on a digital compiter. The program is printed in the appendix.

## III.2 Model fil

We use the formula

$$
N=\frac{R_{i}+R_{d}}{a U_{0}((1) \tau} \ln \left[\frac{R}{R_{i}+R_{d}} \frac{1}{1-\beta}\right]
$$

Specifying

$$
\begin{gathered}
W_{t}=\text { total waight } \\
U_{0}(0)=\text { indtial basteriy anf }
\end{gathered}
$$

```
r - pulse width
B - allowable voltage drop al the battery
\(R_{\&} \times\) transmitter resistance
```

we can calculate

$$
Q_{b}=W_{t} \rho_{B}
$$

where

$$
Q_{B}=\text { total battery charge }
$$

and

$$
a=1 / Q_{B}
$$

then

$$
R_{i}=p_{k} \frac{U_{0}(0)}{Q_{B}}
$$

and hence $N$ which is in this case $N_{\text {max }}$.

III .2.1 Calculation of $N\left(W_{t}\right)$
Constant parameters are: $\mathbb{R}_{d}, \bar{T}, T, B, U_{0}(0)$.
Variables are: $W_{t}, R_{i}=f\left(W_{t}\right)$.
$R_{i}$ is small. compared to $R_{d}$, therefore a change in $R_{i}$
produces only minor changes in $N$. But $\mathbb{R}_{i}$ determines the region for which $N \geq 0$.

$$
N \geq 0 \text { for } \frac{R_{d}}{R_{i}+R_{d}} \frac{1}{1-\beta} \geq 1 \quad \text { from (12) }
$$

substituting

$$
R_{i}=\frac{U_{0}(0)}{\rho_{R}} \frac{\rho_{B} U_{0}(0)}{Q_{B} W_{t}}
$$

$$
W_{1} \geq \frac{D_{A} U_{0}(0)(1-\beta)}{D_{B}} \frac{R_{d}}{B}
$$

for

$$
\begin{aligned}
& B=0.15, \quad U_{0}(0)=30 \mathrm{~V}, \quad \mathbb{R}_{d}=1.0 \Omega, \\
& Q_{R}=60, \rho_{8}=4000
\end{aligned}
$$

then

$$
W_{t 0}=2.55 \mathrm{~kg}
$$

If we substitute in (11)

$$
a=1 / Q=1 / P_{B} \cdot W_{t}
$$

we see that $N$ (neglecting change in $R_{i}$ ) is directly proportional to W . Therefore we can write

$$
N=k_{0}+k_{1} W_{t}
$$


III.2.2 Calaulation of $N(T)$

Fixed paranoters are $W_{t}, B, U_{0}(0), R_{d}$,
then

$$
N(\tau)=N_{0} / \tau \text { is a hypeibola }
$$

with

$$
N_{0}=\frac{R_{i}+R_{d}}{U_{0}(0)} \ln \frac{R_{d}}{R_{i}+I_{d}} \frac{1}{1-\beta}
$$



Figure 17
III.2.3 Calculation of $N(B)$

$$
\text { Fixed parametcrs are: } W_{f}, U_{0}(0), R_{d}, ?
$$

then

$$
\begin{aligned}
N(\beta) & =N_{1}\left(x+1 n \frac{1}{1-\beta}\right) \\
& =N_{1} r-N_{1} 1 H[1-\beta]
\end{aligned}
$$

where

$$
N_{1}=\frac{R}{a U_{0}(0) \tau}, \quad r=\ln \frac{R_{d}}{R_{1}+R_{d}}
$$


III. 3 Mode 1. 12

We use the formulas (37) and (39)

$$
N_{B}=\frac{1}{\ln \xi}\left\{\ln (1-\beta)-\ln \left[\frac{R_{c}}{R_{i}+R_{c}}(1-\varepsilon)+\varepsilon\right]\right\}
$$

and

$$
N=\frac{1}{\ln \xi} \ln \frac{1-k}{\varepsilon}
$$

where

$$
\begin{aligned}
& \xi=\frac{1+c \rho}{1+\frac{\varepsilon}{\delta} \rho} \\
& \varepsilon=\delta \frac{1-\gamma}{1-\delta \gamma} \\
& \gamma=\exp \left\{-\left(\frac{1}{R C}+\frac{\alpha U_{0}(0)}{R}\right)_{T}\right\} \\
& \delta=\exp \left(-\frac{T}{R_{d} C}\right)
\end{aligned}
$$

We specify $W_{t}, U_{0}(0), T, E, R_{d}$ like in Model 11 and additionally

```
T - pulce pariod
    k - allowable voltoge drop at the cepacitor
    C - capacitor
R
```

From C we calculate

$$
W_{c}=c / p_{c}
$$

where $W_{G}=$ weight of the capacitor,

$$
W_{B}=W_{t}-W_{C}
$$

where $W_{g}$ " weight of battery.
III.3.1 Calculation of $N_{H_{C}}=f\left(R_{C}\right)$

We calculate and plot $N_{B}, N_{G}=f\left(R_{G}\right)$

ligure .9
$N_{B}=f\left(R_{c}\right)$ is a monotonic incre:aing function of $R_{f}, N_{f} \approx g\left(R_{f}\right)$ ic monotonically desruazing

$$
N=\operatorname{Mn}\left(N_{B}, N_{c}\right)
$$

Thin implies that

$$
N_{\max }=\operatorname{Max} N
$$

occurs for

$$
N_{\Delta}=N_{C}
$$

By choosing an arbitrary $R_{f}$, we will generally not get the maximum, therefore we developed an algorithm that leads to the maximum. We plot


Figure 20
The first circulation is for an arbitrois $R_{c}$. The nexis: $P_{c}$, we will try is $R_{c}$ ain $-R_{G}$ obit $+\left(N_{C}-N_{G}\right) P$. If we choose a $P$ of right magnitude our next approximation will bo closer to the maximum. If the iteration diverges we have to chinese another (isunaly smaller:) P, in order to malice the calculation convizegent. Ne continue until

III.3.2 Cnlculation of N(C).

Since the total weight fiplites fito the weifint fer the batherg and the weight foi the capacitor, we have an infinite pensibjilfty to combine these two componente under the condition that the total. weight remeins constant. Thercfore we ralculate $\|=N(C)$ for various $C$. $N=N(C)$ is not juat any $N(C)$ but $N_{\max }(C)$, the maximum posaible $N$ (that means we try al.i possible ( $R_{6}$ ) as in section III.3.1.).


Fipure 21
$N(C)$ goes to acro for $C \rightarrow C_{0}$ and $C=.103 \%$. For $W_{C}=100 \%$ there is no batteis left to charge the capacitor. For

$$
\begin{aligned}
& C \leq C_{0} \\
& 0-\exp \left(-\frac{T}{R C}\right) \leq 1-k
\end{aligned}
$$

then

$$
C_{0}=-\frac{T}{R_{d}} \ln (1-k)
$$

with

$$
K=\text { marimun voltace drep at the cepaciter. }
$$

This impliea that the voleage at $R_{d}$ in amalier than the minjmun voleage (1.-к) $U_{0}$ during the first pulse; therefure, $N=0$. $C_{0}$ depencis only on $T_{1}, R_{d}, R_{i}$, but is independent of $W_{t}$ and the other paramoicers. This cxplains why all $N=N_{\text {max }}\left(C_{9}\right)=0$ for all. $W_{8}$.

## III.3.3 Calculation of $N\left(H_{*}\right)$

Constant parancters ois: $R_{d}, B_{!}, K, T, T_{0}$
Variables are $C, R_{c}, W_{t}$.
Nol we find the maximum of every $N_{*}{ }^{*}$ constant from the diagram. 'that is

III.3.4 Gilculation of N(i).

Constant parametcrs ato: $W_{t}, R_{i}, B_{3} \cdot K, T$.
Variables are: $R_{6}, C, \tau_{\text {. }}$

$$
N=\operatorname{Miax}_{R_{c}, C} N(v)
$$



This curve has a hyperbolic character. Assuming that we draw constant power $P$ fro: the elipacitor during $t$, then

$$
E_{0}=\mathbf{P t}_{\boldsymbol{t}} \text { per pulse. }
$$

In $W_{t}$ we can store $E_{\text {max }}$. Then line number, of puloci, we can get from the storage systems is

$$
N \approx \frac{E_{\operatorname{mnx}}}{E_{0}}=\frac{E_{1 n y}}{P_{i}}
$$

which is a hyperbola. For ton large $N$ goes toward aero (iv * 0).
 proporidonel to r. For large 1 the capacitor will weigh be, so that the battery gets smaller and miller as i fucrones.

$$
N=0 \text { cor } W_{c}=V_{t}=\frac{c}{n_{s}}=\frac{-T}{\rho_{s}} \frac{\pi}{N_{f}} \overline{1 n}(1-k)
$$

## So the maximum $T$ is:

$$
T \leq-W_{t} O_{c} R_{d} \ln (1-1:)
$$

with

$$
D_{6}=0.11, R_{d}=1.0, k=0.7
$$

then

$$
\begin{aligned}
& \tau_{\text {max }} \approx 0.0396 \text { bec/ke } \\
& \tau_{\text {max }} \hat{i} 0.04 \text { sce:/kg }
\end{aligned}
$$

## III.3.5 Calculation of N(BL

Constant paramoters arc: $W_{t}, R_{d,} K_{0}, T, T$. Variables are: $R_{6}, C, B$.


 the differcuce between battery voltage and capactor voltage is too small to cherge the capacitor. Then the increasc in li, $\Delta N \cdot 0$ ns $B \rightarrow N$ 。

## 

- 46 ..

In the acetione $2 \pi 7.2$ and III. 3 we have aeen the dependenee of N on ceci paronctor. It would lo diden to hrive a sunction is (of all parametcist), that tells us whech ecmblration of pare, eteris and when motel. is the best. 2hin function vewte lie a coet fanction where the paranetern che welelited by oume factorn mad shenever thite function is an extrenum ve would have a bent or worbt model.

In thenc epplications uearly oild parmeters are fixed, like $\tau, T, W_{t}, R_{d}, U_{0}(0), K$. Since we use Ni-Cd batterice the voleage drop $G$ is fixed ton, and we coicuinted $N$ with $; 0.15$ Wheh is paciatically the upper limit. For Nodel 11 there is no optiminntina, because the battery is chesen na large an posbibla $W_{B} m W_{t}$. But for Model ti2 we optirilsed over $W_{A}, N_{C}, R_{C}\left(W_{t}=V_{E}+W_{c .}\right)$ and found a masimum of $N$ for a particular comilination of $W_{B}, W_{C}, K_{C}$.
 Since this fuction is nembinear and not: ceplicelty solveble the optimization is not a straight formard procedure. It is done graphically. In the following we will nosign $\mathrm{N}_{2}$ to N for: Model ${ }^{12}$ and $N_{1}$ to $N$ for Mocici \#1. $N_{2}$ aplita intor $N_{2 B}, N_{2 c}$ as in formulas (37) and (3!) $N_{B}=N_{2 B}$ and $N_{C}=N_{26}$ respectively.

Conetent parencters are: Vt, C, $k_{d}, \tau, \tau, G, k$.
For the optimal combination of $W_{c}$ and $W_{d}, N_{a c}$ end sie are



that an error in $R_{6}$ is not very critical with respect to $N_{\text {minx }}$ e. 3 Ions as $n_{c}{ }^{2} R_{\text {op: }}$ -

The diagram aloe shows the deponderiet: of $\mathrm{N}_{26}, \mathrm{~N}_{26}$ on $R_{c}$ for different $\%^{\prime}$ To compare $N_{2}$ and $N_{1}$ the ding: an also containing $N_{1}$.
III.4.2 Optimization over $C, N_{2}=N_{2}(C)$. Diagraifi 2, 4, 6

Constant parameters are: $W_{t}, R_{4}, T, T, B, K$.
For $R_{c}=R_{\text {opt }}$ diagram (2) page 49 shows the plot, of $\mathrm{N}_{2}=\mathrm{N}_{2}\left(\mathrm{~N}_{\mathrm{C}}\right)$. $\mathrm{W}_{6}$ is directly proportional to C , so this diagram also represents $N_{2}=N_{2}(C)$. The plot shown for all $C>C_{0}$ (see III.3.2) a very strong dependence of $N_{2}$ on $C$. Small deviations in $C$ from $C_{\text {opt }}$ result in large decreases of $N_{2}$, especially for $C_{0}<C<C_{\text {opt }}$. In the shaded areas $N_{1}>N_{2}$

Diagram (3) page 50 shows $W_{B}=f\left(W_{c}\right)$, with $W_{G}=W_{t}-W_{c}$ and the parameter $\mathrm{N}_{2}=$ constant. This graph makes it easier to find the optimal $W_{c}$ and $N$ for any $W_{\text {: }}$ given that $R_{d}, 6, T, T, K$ are fixed.
Di.egrant (5,7) pages 52,54 represent $N_{2}$ : $N_{2}(C)$ and $W_{a}{ }_{a} f\left(W_{G}\right)$ as described above but for different $T$. As $r$ increases the region with $N_{2}>N_{1}$ decreases, because the capacitor grows larger and larger and cuts down on the battery size (see III.3.4).

## 1II.1.3 Plot of $N=N\left(W_{f}\right)$. Diagram 8

Constant parameters are: $R_{d}, f, T, B, B:$
$N_{1}, N_{2}=f\left(N_{4}\right)$ are as from ITY. 2.1 and III. 3.3 expected. $N_{2}<N_{1}$ for all $W$ but with $r$ increasing then $N_{1} \rightarrow N_{2}$.






Clen
Diagrem 7. $\begin{aligned} & :_{z}=f\left(Y_{e}\right) \\ & z_{i}=i-n, I=\end{aligned}$

. 5.5 -


## 

## 




$$
N_{2}=\operatorname{NM}_{N_{0}, N_{6}, R_{6}} N\left(H_{6}, N_{6}, N_{6}\right)
$$

 (bece III.3.1). Sirice we made the nasumptina that $T$ <e $T$, the calculation is veld only for mall i, but fit shers the dependrase of N on 1:res \%. Fot $\operatorname{large} \mathrm{i}+\mathrm{H}_{1}>\mathrm{N}_{2}$.


Constant paramerers are: $W_{t}, H_{C}, T, T, K$.
Ir III.2.3 we found that $N_{1}(\beta)=0$ \{0i $R_{i}=\frac{\beta}{\sqrt{i-\beta}}$.

 function of $\beta$ for mand $B$, but as $p \rightarrow k, N \rightarrow$ constant.



## 1II. 5 Conclurton:

 and \#2, fown the limita to differumt panamoters and silowed in acer Son JII. 4 that the valuns ohtalned by calculntion ngreed with the expectod plots. It is abluziging that the denondenes of F on the various paranaters is in many conc: Lincer or indfrectiy proportional In sono regions, though it looke from the formuian an if we vonlal have to expect: much lesf linear dependence. The reason for: this nearly lineal depentience of $N$ on the paranetera is the linentization of the mod: 1 nad the small change of the paremeters.

## 

- 60 -


## CHAPTER IV

## IV. 1 Conclu:son:!

In Chapter III we coripsured lindes "1 and Node. \#\%. Modol 12.
has a higher number of pulces than 11 for njmost eveiy cinojec of
 we fould $N_{1}>N_{2}$; for all other cases ve foumil $N_{2}: N_{1}$. Tilis was to be expected from the ciscusslon in Chapter I. The total dincharge time ( $T_{d}$ ) for Mudal \#1, $T$ - $N_{1} r$ is small compared with the discharse tinc for Model ${ }^{\text {\#2 }}$

$$
T_{12}=\mathrm{N}_{2} \mathrm{~T}
$$

 is especially advantageous for small total weight. Due to the
 voltage drops fast under the value of the mintrum viof tree for
 can release the enorgy with only very emall losses but drate fiteclf only a smaj? current from the battery. Thourh this stage is less than $50 \%$ efitciont, Model $\# 2$ has a higher pulse efficioncy thon Model \#1. Ne allow the voleage drop at the Jood to be higiler thon the one at the battexy. Model $\$ 2$ can hancje thin situation, Model $\# 1$ cannot, this is amother reason why $\mathrm{N}_{2}>\mathrm{N}_{1}$.

Since the tetal discharge times are small copaled with
tin charging ti:ns (for $P_{i} n$ 1-5N the charge time for a 5 kg energy



## Now we see the main advantages of Model $\mathbf{2 : ~}^{\text {: }}$

1) since the battery is smaller, the charging time is maller;
2) since the pulse efficiency is larger than for Model. 伹. \#2
does not uciod to be clanged as often as $1 ; 3$ ) for $1_{i}>0$ we can directly chisege tise capaciter. In Model \#1 we charge for $0 \leq t \leq T-T$ the battioy and only for $T-T<t<T, f_{i}$ contributes to the current In $R_{A}$. When we use Model ${ }^{\prime \prime} 2$ in a satellite, the satcilite can transmit more informatic: than a satcllite equipped with Model. W1. The first satclife In serfes of the Sunblayer satellites will be: equipped with a capacitor ( $0.072 \mathrm{~F}, 1.8 \mathrm{~kg}$ ) as energy storage. This is sufficient because it can be guaranteed that

$$
\int_{t}^{t+1} P_{i} d t>\int_{t}^{t+T} P_{0} d t
$$

Further Sunidazer satellites will be equipped with more sophisticeted encrgy stornges that store energy over 22 hours and transmit sigr., for 2 hours per diy. In that casc Nodel fi vould be a geod solution.

## IV. 2 Critiuge

This is a completely mathematical investigation. Though the assumpticns made are justifiable fron the enginees:'s point of viev, the rosults should be supported by seme experimential data. For larger $t$ it would scem to be worthwhilc to investigate encrgy


the byetem has to be very efficiont; magretic fields stored in
 to sture energy have not been funcetifated, heeause of the jow waight rnd the high reliability requ: rement.

## V. 1 Effictuley of Capacitons

The crpacitor $C$ will he chareed with conitirnt voltinge
$U_{0}$ over a resintor $R_{c}$
The curyent is

$$
1=\frac{n_{0}}{R_{c}} \exp \left[-\frac{t}{R_{c} C}\right]
$$

Tie input energy

$$
\begin{aligned}
& =\frac{U_{0}^{2}}{R_{c}} \cdot x_{c} c=U_{0}^{2} c
\end{aligned}
$$

Sir:ce the cancgy btored in the capacitor

$$
E_{G}=\frac{1}{2} C U_{0}^{a}
$$

The charge efficiency 1 is
$\eta=\frac{F_{S}}{E_{i}}=\frac{1}{2}=50 \%$

## V. 2 Buttery Current: for Maximum Emery nu! mut

$$
\begin{equation*}
E=\operatorname{Na}: \int_{0}^{T}\left(U_{0}-i u_{1}\right) i d u \tag{1,0}
\end{equation*}
$$

where $U_{0} n$ battery emf assumed constant.
$R_{i}=$ internal resistance
1 - battery current

$$
\begin{equation*}
\text { Constraint: } k=\int_{0}^{T} i d t \tag{1,1}
\end{equation*}
$$

Using Variational Calculus

$$
\int F d x=N a x
$$

$$
\begin{gathered}
\text { Constraint: } \int G \mathrm{Cdx} \cdot \mathrm{co:istant} \\
H \\
=F-\lambda G
\end{gathered}
$$

then luger's equation

$$
\frac{d}{d t}\left(\frac{\delta}{o(1)} \cdot 11\right)-\frac{\delta}{\delta j} 11=0
$$

leads to

$$
U_{0}-2 i R-\lambda=0
$$

where $\lambda=$ constant:
then

$$
1=\frac{U-\lambda}{2 \pi}=\text { constant }
$$

with (42) in (41.)

$$
1=\frac{k}{T}=\text { contitsint }
$$

## V. 3 Comperison of Energy Densityes

From [1] and nunerous catnlogs the energy densiticon ru:

Capacitors
Chokes (inductors)
Ni-Cd battorics
Nerciry batteries
Spinning mesce (like syroscopes) up te $50 \times 10^{4} \mathrm{Ws} / \mathrm{ks}$;
Energy to losk ratio:
$\left.\begin{array}{ll}\text { Coaducio:s } & 2-5 \% \\ \text { Capacitors } & \text { less than } 0.1 \% \\ \text { Batterics } & \text { less than } 0.01 \% \\ \text { Gyroscopes } & \text { less than } 0.5 \%\end{array}\right\}$ of stored energy

The capedtor and buttery onerify density and dena
pazcenteges ase i.me fevorale than those of other chenent:.

#  

.. 66 ..

## V. 4 Cumptiter Proinin, for $\mathrm{K}_{1}, \mathrm{X}_{11}, \mathrm{~N}_{2 \mathrm{C}}{ }^{\circ}$

( Fortran ll)

ROA! $30, R n, C O,: 1 T, R K, E: O n$
RFA! 3?, U", CH,TA, T, !

ROC:O 0.111
RCD: A (0) 0 .
bIR:: ITT


PRII:T 42, D, RI


If(73.1.0)!r, ro, 5!

51 Punint
FURiNT (Bll:ranT=0.6/)
GO TO 56
55 TJ=1.OCF (Tj)
$T 2=(1!1-1!!) /(\therefore \cup U!+T A)$
$A \cdot\{6: T 1+T 2$


5G. COHT1:T:
$\mathrm{C}=\mathrm{CO}$
()! $D=0=0$

Fil: =0. 0
1\%0
J: i: 1
$C=C \cdot 0,10 * 50$
Wrec/RwR

If (:3) $130,930,8 ?$
$A \Gamma=1 /(1, * R O B)$


75
Ro-nsaremil:

l:=? 10
h:"?
CiO TO : $\because 1$
K: ! 1 +1
IF (1...7) 80,60, 13
continle
FRIIT: 41


FOMAT (11, 3F10.S)
IT (K-7) 80.05.6.5


$G_{i}=1 \times P F(\cdot T \cdot(1 .(0+P(i) /(R+n))$

$\Gamma=(1,0)-6)!(1,11 / 0 \cdots(1)$




1F (ra!) ! 5, 0.90
00
05
IF (12) $25,85,91$
collthays
presi:T asi

(a) 119



ir (rameseres.i?
9) Gompl!is

$311 \quad$ rim $11:=$


If (J-3: 30: 125.1\%
1.95
co.dThir
nos 0.5
CO:TIM:
EMI

## V. 5 TABLISS


B. 0.15
$R_{d}=1.0$
$w_{t}=5.0$
$\mathrm{k}: 0.3$


| $\beta=0.15$ | $\tau=0.02$ |
| :--- | :--- |
| $K_{d}=1.0$ | $W_{t}=2.0$ |
| $N_{1}=523$ | $R_{11}=0.225$ |




| $\beta=0.15$ | $\tau=0.02$ |
| :--- | :--- |
| $\Gamma_{d}=1.0$ | $H_{t}=3.0$ |
| $N_{1}=523$ | $k_{i 1}=0.15$ |

Dianram 2, 3; Paree 19, 50


$$
\begin{array}{ll}
\beta=0.15 & \tau=0.02 \\
R_{d}=1.0 & W_{t}=4.0 \\
N_{1}=1650 & R_{i 1}=0.1125
\end{array}
$$

Dlagrams 2,3; Pages 49, 50


$$
\begin{array}{ll}
\beta=0.15 \\
R_{d}=1.0 & T=0.02 \\
K_{1}=2774 & W_{t}=5.0 \\
R_{11}=0.00
\end{array}
$$

Diagrams ?, if eac: 49, 50


$$
\begin{array}{ll}
6=0.15 \\
R_{d}=1.0 & \tau=0.02 \\
N_{1}=3379 & W_{t}^{2=6.0}
\end{array}
$$

Diagrams 2, 3; Pages 49, 50


Diagrams 4, 5; Fages 51, 57.


$$
\begin{array}{ll}
\beta=0.15 & \tau=0.03 \\
R_{d}=1.0 & W_{t}=3.0 \\
N_{1}=349 & R_{i J}=0.15
\end{array}
$$

Diagrams 4, 5; Pages 51, 52


$$
\begin{array}{ll}
\beta=0.15 \\
R_{d}=1.0 & T=0.03 \\
N_{1}=1106 & N_{t}=4.0 \\
& R_{11}=0.11 .25
\end{array}
$$

Diagrams 4, 5; Pagen 51, 52


| $\rho_{1}=0.1 .5$ | $t=0.03$ |
| :--- | :--- |
| $R_{d}=1.0$ | $r_{t}+5.0$ |
| $N_{1}=1.849$ | $R_{i 1}=0.09$ |

Diagrams 4, 5; Pages 51, 52

$\beta=0.15$
$R_{t}=1.0$
$t=0.03$
$w_{t}=6.0$
$N_{1}=2586$
$R_{11}=0.075$

Diagrans 4, 5; Pancs 51, 52

|  | $\begin{aligned} & C \\ & \left\{F^{n} \mid\right. \end{aligned}$ | $\left\|\begin{array}{c} C \\ \text { p.e. of } \\ l_{i n} \end{array}\right\|$ | $\begin{aligned} & C \\ & C l: 0\rangle \end{aligned}$ | $\begin{gathered} R_{i} \\ \langle n\} \end{gathered}$ | a-mer Sic So? | $\begin{aligned} & R_{i}+R_{c} \\ & \left.1 \Omega_{0}\right] \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 106 | 0.1 .84 | 0.828 | 1.656 | 1.31 | 2.54 | 3.85 |
| 163 | 0.168 | 0.757 | 1.54 | 0.92 .5 | 2.04 | 2.97 |
| 197 | 0.152 | 0.685 | 1.37 | 0.714 | 1.50 | 2. 2.1 . |
| 171 | 0.133 | 0.601 | 1.202 | 0.564 | 0.88 | 1.42 |
| 100 | 0121. | 0.549 | 1.098 | 0.499 | 0.62 | 1.12 |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |


| $8=0.15$ | $\tau=0.04$ |
| :--- | :--- |
| $R_{d}=1.0$ | $N_{t}=2.0$ |
| $N_{1}=0$ | $R_{i 1}=0.225$ |

Diagrams 6, 7; Paģes 53, 54

$C=0.15$
$R_{d}=1.0$
$N_{1}{ }^{2} 262$
$T r 0.04$
$W .3 .0$
$R_{11}=0.15$

Dingrame 6, 7; Pages 53, 54

- 82 -


$$
\begin{array}{ll}
\beta=0.15 & T=0.04 \\
R_{d}=1.0 & N_{t}=4.0 \\
N_{j}=829 & R_{11}=0.1125
\end{array}
$$

Diagrams 6, 7; Pages 53, 54


$$
\begin{array}{ll}
6 \mathrm{me}(1.15 \\
R_{d}=1.0
\end{array} \quad \begin{aligned}
& \left(\begin{array}{l}
\prime \prime \\
\hline
\end{array} .04\right. \\
& N_{1}=1387
\end{aligned} \quad \begin{aligned}
& R_{11}=0.0
\end{aligned}
$$

Diagrams 6, 7; Paṣes 53, 54


| $\beta=0.15$ | $\tau=0.04$ |
| :--- | :--- |
| $R_{d}=1.0$ | $W_{t}=6.0$ |
| $N_{1}=1.939$ | $R_{i 1}=0.075$ |

Diagrams 6, 7; Pages 53, 54


DLagz:an 10, Pase 53

$$
\begin{aligned}
& W_{t}=4.0, R_{d}=1.0, T=1.0, \\
& 1=0.02,1:=0.3
\end{aligned}
$$

Rי: Pab:
 July 1, 1965, Conter ef Spract kencirch, M.T.'R.

 p. 79.

Power Requiremente and BelfOptimising Rlectronic pownr supply of a emall solar Probe. by

Richard H. Baker
Dr. Andrear Boehringer TR-67-3

Power Recuizemente and self-ontimising Electronic Power supply of a small Solar Probe<br>by<br>Richard H. Bakar<br>Dr. Andreas Boehringer<br>\section*{ABsTRACT}

This report datails the dasian of the basic nower conditioning unit for the Sunhlazer apacecraft. Wor lightweight spacecraft utilising eolar cells it is important to optimize the Power. System ovar the full temperature and illumination range. The design discussed utilized optimal control techniques to match the temperature - 111umination dependent array characteristics to a bateery svntem. The result is an efficient lightweight converter with a minimum number of active components.
A. Performance Requiremente

1. Input ..... 1
2. Output ..... 2
3. Operating Conditions ..... 2
4. Reliability, Waight, Efticiency ..... 2
5. Powar Requiraments of the Load ..... 2
B. Principle Design
6. Main Circuit and Switching Conditions. ..... 4
7. Cholce of $\Delta V$ and $\Delta I$. ..... 5
8. 8tability ..... 5
9. Principle Design of the Control Circuit ..... 6
10. 8tart Up From Arbitrary Initial Conditions ..... 7
11. Overvoltage Protection. ..... 7
C. Simplified Calculation
12. Introduction of Deviations and Normalized Volum ..... 8
13. Expansion of the given VI-Characteristic in a Maclaurin series. ..... 10
14. Celculation for the Lincarized VI- Characteristic. ..... 11
15. Approximations for Time History of Current and Voltage ..... 14
D. Choice of the system Paramers
16. Number of Series Connected Solar Cells. ..... 15
17. $\Delta V, \Delta I$ and Frequency ..... 18
18. Design of Coil, Power Transistor and Power Diode. ..... 19
F. Losses ..... 20
G. Summary ..... 20
ह. Appendices ..... 21
I. Ratio of UPP-Voltage at 1 AU and 0.635 AU ..... 21
II. Calculation for a Parabolic Approximation of the VI-Characteristic. ..... 21

## APPENDIX 2

ITEORTICAL STUDY OF A DC POWER CONVERTER FOR THE POWER SUPPLY OF A SMALL SOLAR PROBE (SUNBL,AZER)

## A. Performance Requirements

1. Input

The input of the de power converter will be an array of series-parallel connected solar cells. In contrary to the output conditions the input conaitions will change in a wice range (picture 1). As the aunblazer has to operate in an orbit with


Picture A: Power feneration of Rolar Cell Array a) an aphelion of 1 AU and b) a perihelion of 0.625 AU the ratio of the shortcut currents ( $I_{s c}$ ) of the solar cells is about

$$
\frac{I_{\mathrm{sca}}}{I_{\mathrm{scb}}}=\frac{0.635^{2}}{1}=\frac{1}{2.5}
$$

(Tha solar conatant is inversely proportional to the square of the sun distance). The ratio of the open circuit vol'sges (on the basis of changing temperature will be about

$$
V_{\text {oca }}=1.9 \quad \text { (see Appendix I) }
$$

## 2. cuatput

As it can ive seen today, the ue power cónverter will operate at its output on an array of eeries-paraliel connected nickelcadmium cells with a nearly constant voltage of $\mathrm{E}=35 \mathrm{~V}$.

## 3. Oparating Conditions

since these substantial changes, shown above in the voltageי" current characteristic of the solar batteries over the orbit, due to changes in temperature and solar constant, the power converter principally has to be designed in a way, which allowe that it operates always at or close to baximum Power Point (NPP), independent of the momentary power profile [1].

## 4. Reliability, Weight, Efficiency

a) The reliability of the power converter has to be az high as possible. Therefore the number of parts, especially semiconductors has to be as small as possible.
b) The weight has to be as mall as possible.
c) The efficiency has to be as high as possible, however, efficiency is not quite as important as mall weight.

Original material deleted because not applicable to present design requirements
B. Principle Design

1. Main circuit and Switching Conditions

A good and simple way to match tha conditions shown under A seems to be the following (picture 3).

$V_{m} I_{m}$. Mean Values of voltage and Current
V, I A Actual Values of voltage and Current
$V_{d}$. $I_{d}$ Difference between actual Value and Mean Value Picture 3. Princinle Design of Main Circuit

The array of solar cells and the battery system are interconnected by only three components in the main circuits.

The ewitch 8 is turned on and off by a flip-flop which is triggered by the differencen between actual value and mean value of voltage: $V_{d}=V-V_{m}$ and current: $I_{d}=I-I_{m}$ with the following trigger conaitions:

$$
\begin{aligned}
& V_{d}=-c_{1} \cdot \Delta V \rightarrow \text { switch is turned off } \\
& I_{c}=-c_{2} \cdot \Delta I \rightarrow \text { switch is turned on }
\end{aligned}
$$

where $\Delta V>0$ and $\Delta I>0$ are given values and $c_{1}=\frac{1}{2}$ and $c_{2}=\frac{1}{2}$, see C.4. For the following it shall be assumed that the solar cell array has come up to a point $\alpha$, where the switch was closed.

Then it will run down to lower voltages and higher currents. $V$ and herewith $V_{\dot{d}}$ is becoming smaller. When the circuit has reached point $B, V_{d}$ has become equal to $-c_{1} \cdot \Delta V$. Thus the switch is turned off.

When $E>V_{m}+\left(1-c_{1}\right) \cdot \Delta V$ (what must bol), the current then will decrease and the voltage increase until V and I reach the point $\alpha$, where $I_{d}$ becomes equal to $-C_{2} \cdot \Delta I$. Then the switch is closed and the described process starts again.

## 2. Choice of $\Delta V$ and $\Delta I$

The ratio $\frac{\Delta V}{\Delta I}$ must be chosen so that in that situation where the power output of the solar cell array is at its lowest (1 AI) the power converter operates exactly at the MPP.

FOE MPP: $V+I \frac{d V}{d I}=0 \rightarrow \frac{\Delta V}{\Delta I}=\left.\frac{V}{I}\right|_{M P P}$
$\Delta V$ (or $\Delta I$ ) itself will be chosen to about 10 pct. of their mean values, thus providing an operation always close to the ifPP. To those values (and for the same VI-characteristic) must be chosen the according values of $c_{1}$ and $c_{2}$, either by calculation (see C.4) or by experiment. However, this is not very important, since those values will be both very close to 1/2 and, for the practical design, can be taken to be exactiy 0.5. Thus the power convertex will operate in this point of the sunblazer orbit (1 AU) exactly around the MPP. In other points of the orbit, the system will not work exactly around the dpp, but very close to it. The power output of the converter will even though increase because the whole power output of the solar cell array there will be nigher and because the power-current characteristic has a flat maximum in the NPP.

## 3. Principle Design of the Control Circuit



Picture 4. Princiole Desion of the Control Circuit.

Picture 4 shows the principle design of the measuring circuit for $V_{d}$ and $I_{d}$. For an enough sensitive flip flop the value of the shunt $R_{s}$ can be kept so small, that this resistance will cause only an unessential influence on the efficiency of the whole system. The diode $D^{\prime}$ probably isn't necessary.

## 4. Start Up From Arbitrary Initial Conditions

If some failure happens to the system (for example, when the whole sunblazer turns around in space and $V$ and $I$ become zero for some time), the power converter must start to operate again properly, when the failure is over.

This can be guaranteed, when ever an either-or circuit the flip flop is controlled too by two absolute values $V_{u}$ and $I_{u}$ (Picture 3), so that for the switcining conditions must be written:

$$
\begin{aligned}
& V_{d}=-c_{1} \Delta V \\
& \quad \text { or } \\
& V \leqq V_{u} \\
& I_{d}=-c_{2} \Delta I \\
& \\
& \quad \text { or switch is turned off } \\
& I \leqq I_{u}
\end{aligned}
$$

The value of $I_{u}$ must be chosen higher than the arrent at that point, where the stationary characteristic E,I' of the storage battery and load intersects the voltage current profile of the solar cell array. But since there is concluded, that for a
battery voltage less than 908 of its nominal value the load will be aisconnected from the battery, $I_{u}$ must be chosen only higher than the current at that point, where the stationary charging characteristic E'I' of the storage battery along intersects the voltage cursent profile of the colar cell array (at 1 AU). Thus it is reasonable to take $I_{u}$ and $V_{u}$ to about 108 of $I_{M P P}$ and $V_{\text {MPP }}$ (at 1 AU).

## 5. Stability

With the $V_{i}, I_{d}$ control design the system will work stable only between the points $\alpha$ anc $\beta$ (as wanted), given by the VIcharacteristic, $\Delta V$ and $\Delta I\left(\operatorname{and} c_{2}\right.$ and $\left.C_{2}\right)$. Assumed, the system is out of this region and (for example) the mean values $V_{m}$ and $I_{m}$, stored in the capacitors are $V_{m}=0$ and $I_{m}=I_{s c}$ (very unfavorable conditions), the system will operate shortly between the points $\gamma\left(I=I_{u}\right)$ and $\varepsilon\left(I=I_{s c}-c_{2} \cdot \Delta I\right)$.


But operating for some time in this circle will diminish $I_{m}$ and enlarge $V_{\text {in }}$, so that both points $c$ and $r$ walk up on the VIcharacteristic until $\alpha$ ana $\beta$ are reached. Thus, in sinort time, the system will come to work properly between $\alpha$ and $\beta$.

## 6. Overvoltage Protection of Switch and Storage Battery

a) To protect the switch (transistor) against overvoltage (when turned off) there should be a capacitor parallel to the storage battery (picture 6). Since the inner inductance of the nickel-cacimium cells is very small, this capacitor can be kept mall too.


Picture 6 - Overvoltaqe Prntection of Batterv and Switch
b) To protect the battery system against overloading, the switch (by the flip flop) can be turned on all the time or turned off all the time (there is no important difference between these both possibilities), when the voltage of the battery exceeds its allowable value. This may be cione, for example, by the circuit shown in picture 6.


Dinture 7 . Monel for the simnlified Calemlatinn

## C. Simplified Calculation

1. Introduction of deviations and normalized values

The differential equations for the simplified circuit shown in picture 7 are:
switch turned on: $V=L \cdot \frac{d I}{d t}$
switch turned off: $V=L \cdot \frac{d I}{d t}+E$

The circuit shall operate around a reference point $P_{0}$ with the voltage $V_{0}$ and the current $I_{0}$. Introducing the deviations of the actual values from there reference values:

$$
\begin{align*}
& V_{1}=v-V_{0}  \tag{3}\\
& I_{1}=I-I_{0} \tag{4}
\end{align*}
$$

the equations (1a) and (2a) become:
switch turned on: $\quad V_{1}+V_{0}=L \cdot \frac{d I_{1}}{d t}$
switch turned off: $\cdot V_{1}+V_{0}=L \cdot \frac{d I_{1}}{d t}+E$
Introcucing the normalized values

$$
i_{1}=\frac{I_{1}}{I_{0}} \text { and } v_{1}=\frac{V_{1}}{V_{0}}
$$

they become
Switch turned on: $\quad \frac{d i_{1}}{d t}=v_{1}+1$
Switch turned off:

$$
\begin{equation*}
\frac{d i_{1}}{d t}=v_{1}+1-e \tag{lc}
\end{equation*}
$$

with the abbreviations:

$$
\begin{align*}
& \frac{V_{0}}{I_{0}}=R_{0} \frac{L}{V_{0} I_{0}}=\frac{L}{R_{0}}=T_{0}  \tag{5}\\
& \frac{t}{T_{0}}=t \frac{E}{V_{0}}=e(>1)
\end{align*}
$$

## 2. Expansion of the qiven VI - Characteristic Into a vaclaurin 8 orice

In the following calculation the original VI-profile of the solar cell array is expanded in a Maclaurin series in the reference point $V_{0}, I_{0}$. Since $I_{1}$ will always be mall, it is reasunable to break off the series behind the second term:
where

$$
\begin{equation*}
v_{1}=\frac{d v_{1}}{d i_{1}} I_{0} \cdot i_{1}+\left.\frac{1}{2} \frac{d^{2} v_{1}}{d i_{1}{ }^{2}}\right|_{0} \cdot i_{1}{ }^{2} \tag{6}
\end{equation*}
$$

1

$$
\begin{equation*}
v_{1}=a \cdot i_{1}+b \cdot i_{1}{ }^{2} \tag{7}
\end{equation*}
$$


and

$$
\begin{align*}
b= & \left.\frac{1}{2} \frac{d^{2} v_{1}}{d I_{1}^{2}}\right|_{0}=\frac{\frac{1}{2}}{\frac{d^{2} V_{1}}{V_{0}}} \frac{d\left(\frac{I_{1}}{I_{0}}\right)^{2}}{I_{0}} I_{I_{1}=0}=  \tag{8}\\
& \left.\frac{1}{2} \frac{d^{2} V}{d I^{2}}\right|_{I_{0}} \cdot \frac{I_{0}^{2}}{V_{0}}=\left.\frac{1}{2} \frac{d^{2} V}{d I^{2}}\right|_{I_{0}} \cdot \frac{I_{0}}{R_{0}}<0 \tag{9}
\end{align*}
$$

Thus the equations (1) and (2) become:
switch turned on: $\frac{d i_{1}}{d t}=1+a i_{1}+b i_{1}{ }^{2}$
switch turned off:

$$
\begin{align*}
& \frac{d i_{1}}{d t}+1-e+a_{1}+b i_{1}{ }^{2}  \tag{2d}\\
& a<0, b<0 \text { and } e \geqslant 1 .
\end{align*}
$$

where
Herefrom one may get the explicit equations $\tau=E\left(I_{1}\right)$ (Appendix II) and with the reaults of this calculation there can be shown, that
for the given VI prosile and $\left|i_{1}\right| \leqq 0,05$ (what means $\frac{\Delta I}{S_{0}}=0.1$ ) there is only a very mall difference between $b=0$ and $b \neq 0$. Therefore it is reasonable to linearize the VI profile in $\mathrm{P}_{0}$ (1.e. to take $\downarrow=0$ ).

## 3. Calculation for the Lincerized VI Profile



Picture 8 : Determination of point $P_{0}$

It leads to an easy and well-arranged calculation when the point $P_{0}$ for given VI characteristic and given $\Delta I$ and $\Delta V$ is determined by the way shown in picture 8. (The numbers in the small circles show the sequance in which this point is determined).

Then can be written:

$$
\begin{align*}
& V_{1}=-\frac{\Delta V}{\Delta I} \cdot I_{1} \\
& V_{1}=-\frac{\Delta V / V_{0}}{\Delta I / I_{0}} \cdot I_{1} \\
& V_{0}  \tag{10}\\
& v_{1}=a \cdot I_{1} \text { with } a=-\frac{\Delta V}{\Delta I} \cdot I_{0}=-\frac{\Delta V}{\Delta I} \cdot \frac{1}{V_{0}}
\end{align*}
$$

and

$$
\begin{aligned}
& i_{i_{0}}=-\frac{1}{2} \frac{\Delta I}{i_{0}}=-\frac{1}{2} \Delta i \\
& i_{10}=+\frac{1}{2} \frac{\Delta I}{I_{0}}=+\frac{1}{2} \Delta i
\end{aligned}
$$

Thus the equation (1) and (2) become:
ewitch turned on: $\quad \frac{d i_{1}}{d t}=a i_{1}+1$
switch turned off: $\frac{d i_{1}}{d t}=a i_{1}-(e-1)$
and their solutions:
switch turned on: $\quad=r_{0}+\frac{1}{a} \ln \left(1+a i_{1}\right)$
switch turned off: $\tau=\tau_{0}+\frac{1}{a} \ln \left(e-1-a i_{1}\right)$
The normalized value for the time, the system needs to get from a to s switch turned on) is:

$$
\Delta \tau_{a \rightarrow \beta}=\Delta T+=-\frac{1}{2} \cdot \ln \frac{1-\frac{1}{2} a \Delta i}{1+\frac{1}{2} \Delta i}
$$

With (5) and (10) we get for the real time:

$$
\Delta t t=\frac{L}{\Delta V / \Delta I} \cdot \ln \frac{1+\frac{1}{2} \frac{\Delta V}{V_{0}}}{1-\frac{1}{2} \frac{\Delta V}{V_{0}}}
$$

or with an expansion into series for the logarithm:

$$
\Delta \pm t=\frac{L}{\Delta V / \Delta I} \cdot L\left(\frac{1}{2} \frac{\Delta V}{V_{0}}+\frac{1}{3}\left(\frac{1}{2} \frac{\Delta V}{V_{0}}\right)^{3}+\frac{1}{5}\left(\frac{1}{2} \frac{\Delta V}{V_{0}}\right)^{5}+\ldots .\right)
$$

since $\frac{1}{2} \frac{\Delta V}{V_{0}} \ll 1: \quad \Delta t=L \cdot \frac{\Delta I}{V_{0}}$
Analogues we get:

$$
\begin{aligned}
\Delta \tau_{B \rightarrow a}=\Delta \varepsilon^{+} & =-\frac{1}{a} \ln \frac{e-1-\frac{1}{2} a \Delta i}{1-1+\frac{1}{2} a \Delta i} \\
\Delta t_{+} & =\frac{L}{\Delta V / \Delta I} \ln \frac{1+\frac{1}{2} \frac{\Delta V}{V_{0}} \cdot \frac{1}{1-\frac{1}{2} \frac{\Delta V}{V_{0}} \cdot \frac{1}{e-I}}}{1}
\end{aligned}
$$

and since

$$
\begin{align*}
& \frac{1}{2} \frac{\Delta V}{V_{0}} \cdot \frac{1}{e-1} \ll 1: \\
& \Delta t_{4}=L \cdot \frac{\Delta I}{V_{0}} \cdot \frac{1}{E_{0}-1} \tag{12}
\end{align*}
$$

The time $\Delta t$ for one cycle is

$$
\begin{aligned}
& \Delta t=\Delta t t+\Delta t+=L \cdot \frac{\Delta I}{V_{0}}\left(1+\frac{1}{\frac{E}{V_{0}}-1}\right) \\
& \Delta t=L \cdot \frac{\Delta I}{V_{0}} \cdot \frac{1}{1-\frac{V_{0}}{E}}
\end{aligned}
$$

and so the frequency

$$
\begin{align*}
& E=\frac{1}{\Delta t}=\frac{V_{0}}{\Delta I} \cdot \frac{1}{E}\left(1-\frac{V_{0}}{E}\right) \\
& E=\frac{E}{\Delta I} \cdot \frac{1}{E} V_{0}\left(1-\frac{V_{0}}{E}\right) \tag{13}
\end{align*}
$$

The voltage across the coil is

$$
V_{\text {coil }}=L \cdot \frac{d I}{d t}
$$

The man value of this voltage for the time when the switch is turned on therefore is:

$$
\begin{aligned}
\frac{1}{\Delta t} \cdot \int_{t_{a}}^{t_{a}} L \cdot \frac{d I}{d t} \cdot d t & =\frac{1}{\Delta t t} \cdot L\left(I_{\alpha}-I_{\beta}\right) \\
& =\frac{1}{d t t} \cdot L \cdot \Delta I
\end{aligned}
$$

Analogues the mean value of this voltage for the time when iso switch is opened is:

$$
\begin{aligned}
\frac{1}{\Delta t+} \int_{t_{B}}^{t_{a}} L \cdot \frac{d I}{d t} d t & =\frac{1}{\Delta t+} \cdot L\left(I_{B}-I_{a}\right) \\
& =-\frac{1}{\Delta t^{4}} \cdot L \cdot \Delta I
\end{aligned}
$$

Therefore the rectified mean value for the whole cycle is:

$$
\begin{align*}
& v_{\mathrm{Em}}=\frac{1}{\Delta E t+\Delta E 4}(L \cdot \Delta I+L \cdot \Delta I) \\
& v_{\mathrm{rm}}=E \cdot 2 \cdot L \cdot \Delta I \tag{14}
\end{align*}
$$

or with (13)

$$
\begin{equation*}
v_{r m}=2 E \cdot V_{0}\left(1-\frac{V_{0}}{E}\right) \tag{15}
\end{equation*}
$$

4. Approximations for Time History of Jurrent and Voltage

By taking the point $V_{0}, I_{0}$ in the way shown in picture 8 the current will change during one cycle from $I_{0}+\frac{1}{2} \Delta I$ to $I_{0}-\frac{1}{2} \Delta I$ and back. During the switch is turned on (from a to B) the slope $\frac{d I}{d t}$ changes in the ratio

$$
\frac{\left(\frac{d I}{d t}\right)_{a}}{\left(\frac{d I}{d t}\right)_{B}}=\frac{V_{0}+V_{1 a}}{V_{0}+V_{1 B}}=1+\left(\frac{V_{1 a}}{V_{0}}-\frac{V_{1 . \beta}}{V_{0}}\right)=1+\frac{\Delta V}{V_{0}}
$$

Analogues, when the switch is turned off (from $B$ to a) the sises $\frac{d I}{d t}$ changes in the ratio

$$
\begin{aligned}
\frac{\left(\frac{d I}{d t}\right)}{\left(\frac{d I}{d t}\right)}=\frac{E-v_{0}-V_{1}}{E-V_{0}-V_{1}} & =1+\left(\frac{V_{1}}{v_{0}}-\frac{v_{1}}{V_{0}}\right) \cdot \frac{1}{\frac{E}{V_{0}}-1} \\
& =1+\frac{\Delta V}{V_{0}} \cdot \frac{1}{\frac{E}{V_{0}}-1}
\end{aligned}
$$

since $\frac{\Delta V}{V_{0}}$ will be probably not bigger than about 0.1 and $\frac{E}{V_{0}}>1.5$, the current will be asw-tooth-function with nearly constant slope $\frac{d I}{d t}$ between two switching points (Picutre 9).


Picture 9: Time History of Current

The time history of the voltage belonging to this may be taken out of the original V,I profile. By integrating this voltage there san be obtained on even more exact current curve, hereto the voltage out of the original V,I-characteristic and so on.

Already the result of the firat iteration will be vary exact. Herefrom there can be obtained the mean values $V_{m}$ and $I_{m}$ and according to $V_{B}=V_{m}-c_{1} \Delta V$, the exact value for $c_{1}$ and analergues, according to $I_{\alpha}=I_{m}-C_{2} \Delta I$ the exact value for $c_{2}$ (VI-characteristic at aphelion, See B.2). But since these values will both be very close to $1 / 2$, for the practical design they may be chosen to 1/2.

## D. Choice of the System Parameters

1) Number of Serien connected solar cells.

For the choice of the number of series connected solar cells two points are important.
a) For the sunblazer are provided 8 segments of solar cells, where on each can be placed between $N=72$ and $N=82$ cells. To get a simple wiring (the same in each segment) it is reasonable to provide in each segment a series connection c

$$
N^{\prime}=N \text { or } \frac{1}{2} N \text { or } \frac{1}{3} N \ldots
$$

b) To get a small (light-weight) power coil and small power losses therein too, it is reasonable to keep the change of the switching frequency (over the orbit) as small as possible. According to equation (14) that would make it possible too, to get the power for load V (see A,5) out of a second winding on the power coil. Following equations (13):

$$
f=\frac{E}{\Delta I} \cdot \frac{1}{E} \cdot \frac{V_{O}}{E}\left(1-\frac{V_{O}}{E}\right)
$$

and (15)

$$
v_{r m}=2 E \cdot \frac{V_{0}}{E}\left(1-\frac{V_{O}}{E}\right)
$$

for given $E, \Delta I$ and $L$ the frequency and the rectified man vajus at the power coil are direct proportional to

$$
F=\frac{V_{O}}{E}\left(1-E_{E_{O}}^{V_{0}}\right)
$$

$F=F\left(\frac{V_{0}}{E}\right)$ is shown in picture 10.


$$
\text { Picure 10: } F=F\left(\frac{V_{0}}{E}\right)
$$

To get the mallest change in frequency, there should be

$$
\left.F(V)_{\text {max }}\right)=F\left(V_{0 \min }\right)
$$

i.e.

$$
\frac{V_{0 \max }}{E}\left(1-\frac{V_{0 \max }}{E}\right)=\frac{V_{0 \min }}{E}\left(1-\frac{v_{0 \min }}{E}\right)
$$

Therefrom one can get (for given $\frac{V_{0 \text { max }}}{V_{0 \text { min }}}$ :

$$
\begin{equation*}
\left.\frac{v_{0 \text { max }}}{E}\right|_{o p t}=\frac{v_{0 \text { max }}}{V_{0 \text { min }}} \frac{v_{0 \text { max }}-1}{v_{0 \text { min }}} \frac{\left.V_{0 \text { max }}\right)^{2}}{V_{0 \text { min }}} \tag{16}
\end{equation*}
$$

For the qiven ratio $\frac{V_{0 \text { max }}}{V_{0 \text { min }}}=1.9$ (See Appendix 1$)$
one gets

$$
\left.\frac{v_{0 \text { max }}}{v}\right|_{\text {ont }}=0.65
$$

and

$$
\left.\frac{V_{0} \min }{2}\right|_{\text {opt }}=0.344
$$

For these both extreme cases the frecuency rilli be the same. The highest freauency will occur at $\frac{V_{0}}{E}=0.5$.

The ratio in which the frequency changes in this case therefore is:

$$
\frac{F_{\max }}{F\left(V_{0 \max }\right)}=\frac{F_{\max }}{F\left(V_{0 \mathrm{~min}}\right)}=\frac{0.22 R}{0.225}=0.912
$$

The rectified mean value of the voltage from a second winding would change only within $\pm 4,5 \%$.

Number of series connected cells:
Assuming the lowest temperature is $-2 n^{\circ} \mathrm{C}$, one gets (Annendix I)

$$
\begin{aligned}
v_{0 \max } & =\left[0.41-\frac{0.0025}{{ }^{\circ} \mathrm{C}}\left(-.5 n^{\circ} \mathrm{C}\right)\right] \times \mathrm{N} \text { volts } \\
& =0.535 \times \mathrm{N} \text { volts. }
\end{aligned}
$$

With $V_{0 \text { max }} l_{\text {opt }}=0.65 \cdot \mathrm{E}=22.8 \mathrm{~V}$ therefrom results a number of series connected cells:

$$
N_{o p t}^{\prime}=\frac{22.8 \mathrm{~V}}{0.535 \mathrm{v}}=43
$$

This number would require a total number of $\mathrm{N}=2 \mathrm{~N}^{\prime}=86$ cells per segment (probably too big).

The following table shows the eittreme values of $F$ (which is proportional to the frequency and the rectifind mean value of the voltage of a second winding on the nower coil) for $E=35 v$, $T_{\max }=72,{ }^{\circ} \mathrm{C}, T_{\min }=-28^{\circ} \mathrm{C}$


| 36 | 72 | 0.550 | $0.22 \%$ | 0.206 | 0.250 | 0.823 | $\pm 98$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 38 | 76 | 0.581 | 0.306 | 0.212 | 0.250 | 0.850 | $\pm 7.5 \%$ |
| 40 | 80 | 0.606 | 0.322 | 0.218 | 0.2 .50 | 0.871 | $\pm 6.5 \%$ |
| 42 | 84 | 0.641 | 0.338 | 0.224 | 0.250 | 0.898 | $\pm 5 \%$ |
| 43 | 86 | 0.658 | 0.346 | 0.227 | 0.250 | 0.909 | $\pm 4.5 \%$ |

However, there must be said that with changind the number of cells from $N=72$ (as planned today), there will he some change of the temperature of the whole sunblazer and thus some change of $v_{0 \text { max }}$ and $v_{0 \text { min }}$ too.

## 2. $\Delta V, \Delta I$ and Frequency.

The short-cut current of one string of solar cells at 1 AU will be $I_{s c}=0.06 \mathrm{~A}$.
The whole short-cut will be
a) $\left.I_{8 c}\right|_{1} A U=8 \times 2 . I_{8 C}=I A$
and thus
b) $\left.I_{\mathrm{sc}}\right|_{0.635 \mathrm{AU}}=\frac{1}{0 . C 3 \tilde{E}^{2}}=2.5 \mathrm{~A}$

It seems reasonable to take $\Delta I=0.1 I_{a c} I_{1 A U}=0.1 \mathrm{~A}$ and, according to A. $3 \Delta V=\left.\Delta I \cdot \frac{V}{I}\right|_{\text {MPP, }} 1 A U^{m} 0.1 A \cdot \frac{19.25}{1} \frac{v}{A}$ $=2.0 \mathrm{~V}$.

To get a mail coil, but at the same time mall lossen in tres coil and the transistor, it seems reasonable to take a frecpuancy of about

$$
E=50 \mathrm{KH}_{1} .
$$

3. Design of Coile Power Transistor and Power Diode.
a) Following equation (13) the inductivity of the coil has to be:
with $\mathrm{F}=0.22$

$$
L=\frac{E}{\Delta I} \cdot \frac{1}{E} \cdot F
$$

$f=50 \mathrm{KHz}_{2}$
$E=35 \mathrm{~V}$
$\Delta I=0.1 \mathrm{~A}$
one gets.

$$
\begin{aligned}
& L=\frac{35 V}{0.1 \mathrm{~A}} \cdot \frac{1}{50} \cdot 10^{-3} \mathrm{sec} \cdot 0.22 \\
& L=1.54 \mathrm{mHy}
\end{aligned}
$$

If this inductivity should be ton big, $\Delta I$ must be increased to about $\Delta I=0.2$ A and/or the frequency to about 100 (200) KHz.

The copper of the coil has to be chosen for $I_{\text {max }}=2.5 \mathrm{~A}$ and the air gap so that $I_{\text {max }}$ the coil isn't yet in saturation.
b) Transistor: max. voltage: 40V
max. current: 2.5A
mean value of current. 2.5A
c) Diode:
max. voltage: 35 V
max. current: 2.5 A
mean value of current: 2.0 A

## F. Losses

The main losses in the suggested circuit will be:
a) Copper and iron losses in the coil
b) Switching and conducting losses in the powar tcansister
c) Losses in teh drive circuit of the power transistor
d) Conducting losses in the diode.

For the final design of the coil and drive circuit there should be considered that the losses in point b) ( 0.635 A.U., high current low voltage) may be higher than in point a) (1 A.U., low current, high voltage) because in perihelion ( 0.635 A.U.) the output power will be higher than in aphelion (1 A.U.).

## G. Summary

The proposed circuit for the de power converter is very simple and therefore will be very reliable. In addition, it is easy to increase the reliability for example by dividing the solar cell array in two or four parts, each with its own convertsr and adding up the power in the storage battery through the power diodes.

It seems to be very important too that this system is selfadaptive and not a programmed one. It will work well also when the circumstances change, for example, the output of the solar cells as a result of mocrometeorite erosion or radiation effects.

## H. APPENDICES

I. Ratio of the voltages in the MPP at 1 AU and 0.535 . $\%$

Given is: $V_{0 a}=\left(0.41-\frac{0.0025}{0}\left(T_{a}-22^{\circ} \mathrm{C}\right)\right]$ volts $x \mathrm{~N}$

$$
v_{o b}=\left[0.41-\frac{0.0025}{C}\left(T_{b}-22^{\circ} \mathrm{C}\right)\right] \text { volts } \times \mathrm{N}
$$

therefrom:

$$
v_{o a}-v_{o b}=\frac{0.0025}{C}\left(T_{b}-T_{a}\right) \text { volte } \times \mathrm{N}
$$

Assuming, the mean value of the temperature will be $\frac{T_{a}+T_{b}}{2}=22^{\circ} \mathrm{C}$ and the difference will be $T_{b}-T_{a}=100^{\circ} \mathrm{C}$, we get:

$$
\frac{v_{o a}+v_{o b}}{v_{o a}-v_{o b}}=\frac{0.82}{v_{.25}}
$$

or

$$
v_{o \mathrm{ob}}=1.9
$$

The ratio of the voltages for open circuit (A. 1 ) will be nearly the same:

$$
\frac{V_{\text {OCa }}}{V_{\text {OCb }}}=1.9 .
$$

II. Calculation for a parabolical Approximation of the VI-Charneteristic

From C. 2 equations (1a) and (2d) we get:
switch turned on: $d r=\frac{d i_{1}}{1+d i_{1}+b i_{1}}$
ewitch turned off: dr $=\frac{d i_{1}}{\Gamma-0+d i_{1}+b I_{1}^{2}}$
when $a, b<0$ and $0 \geq 1$.
For the solutions is important, whether the diseriminant $D: n>0$ or $D<0$.
switch turned on: $D_{o n}=4 b-a^{2}$
switch turned off: $D_{\text {off }}=4 b(1-c)-a^{2}$
Example from "Hoftman Einal report for Integral class Coatings for solar Cella"

Page 45, Curve 2, solar simulator
Maximum powar point: $V_{0}=420 \mathrm{mV}, I_{0}=61 \mathrm{~mA}$
Table for the points $a$ and $B\left(\right.$ with $\left.1_{1 a}=-1_{1 B}=-\frac{1}{2} \frac{\Delta I}{I_{0}}=-0.05\right)$

$$
\begin{array}{l|l|l|l}
i_{1_{\alpha}}=-0.05 & I_{\alpha}=57.95 \mathrm{~mA} & V_{\alpha}=436 \mathrm{mV} & V_{1_{\alpha}}=0.038 \\
\hline i_{1 \beta}=+0.05 & I_{\beta}=64.05 \mathrm{~mA} & V_{\beta}=394 \mathrm{mV} & V_{1 \beta}=-0.062
\end{array}
$$

These points must fit the equation

$$
v_{1}=a i_{1}+b i_{1}^{2}
$$

Point $a: \quad V_{1 a}=0.038=-0.05 a+0.0025 b$
Point B: $\quad V_{1 \beta}=-0.062=+0.05 a+0.0025 b$
Therefore:

$$
a=-1, b=-4.8
$$

Thus we get for the discriminant, when the awitch is turned off:

$$
D_{\text {off }}=4 b(1-e)-a^{2}=-19.2(1-e)-1
$$

and with

$$
\bullet \geq 1.5, D_{\text {off }}>0
$$

so we get for the solutions of (1f) and (2f):
switch turned on: $t=r_{0}-\frac{2}{\sqrt{-D_{o n}}}$ arc $\tan h \frac{2 b i_{1}+a}{\sqrt{-D_{o n}}}$
awiteh turned off: $t=\tau_{0}^{\prime}+\frac{2}{\sqrt{D_{0 f f}}}$ arc $\tan \frac{2 b i_{1}+a}{\sqrt{D_{0 f f}}}$
$10 \cdot$

The melated value for th time, the symem needs to get from a to a (switch turned on) is:
$\Delta \tau_{\alpha \rightarrow \beta}=\Delta T+=-\frac{2}{\sqrt{-D_{O n}}}$ [arc $\tan h \frac{2 b i_{1 \beta}+a}{\sqrt{-D_{O n}}}-\operatorname{arc} \tan h \frac{2 b i_{1 \alpha}+a}{\sqrt{-D_{O n}}}$;

and with $\quad i_{1_{B}}-i_{1_{\alpha}}=\frac{\Delta I}{I_{0}}=\Delta I$

$$
\begin{align*}
& i_{1 \beta}+i_{1 \alpha}=0 \\
& \Delta \tau+=\frac{2}{\sqrt{-D_{O n}}} \arctan \operatorname{h} \frac{2 b \Delta i}{\sqrt{-D_{O n}}-\frac{a^{2}-(b \Delta i)^{2}}{-D_{O n}}} \tag{1.7}
\end{align*}
$$

and analogues:


$$
\text { gives } \quad \Delta t+=0.1005
$$

The corresponding values from equations (11) and (12) $(b=0)$ are:

$$
\begin{aligned}
& \Delta T t=0.200 \\
& \Delta T t=0.200
\end{aligned}
$$

As one can see, the difference is malier than 1\%.
Literature:
(1) Study of a small solar Probe (Sunblazer) Part II, spacecraft and Payload Design, PR-5255, July 1, 1965.

$$
\begin{aligned}
& \begin{array}{l}
\Delta T+\frac{2}{\sqrt{D_{O E E}}} \text { arc } \tan \frac{-2 b \Delta 1}{D_{O E E}+\frac{a^{2}-(E A L)^{2}}{\sqrt{D_{O E I}}}}
\end{array} \\
& -1.5 \quad b=-4.8 \\
& \Delta t+=0.1989
\end{aligned}
$$

## Thermal Transfer and Radiation from a Thin Circular

 Plate Source--Thin Cylindrical Shell Radiator
## 1. Introduction

For the Sunblazer satellite, it is of interest to calculate the radiative properties of thin shells. In the configuration of Fig. 42, which is a simplified drawing of the Sunblazer satellite, there are two such shells. The first is the front plate, a thin plate which is covered with solar cells. The second is the cylinder, a thin shell which radiates excess heat into space.

## II Summary

Assuming no radiative heat transfer within the cylinder, the temperature on the shells and the rate of radiation of the shells is determined solely by a dimensionless variable 5 , where:

$$
\begin{aligned}
& 5=1 / 2 \frac{c 0}{K t} T_{0}{ }^{3} x^{2} \\
& \text { c Emissivity of shell ( } 0.85 \text { for aluminum) } \\
& 0=5.6710^{-12} \text { watts } / \mathrm{cm}^{2}-{ }^{\circ} \mathrm{K}^{4} \\
& \text { K - Conductivity of shell (2.05W/cm- }{ }^{\circ} \mathrm{K} \text { for aluminum) } \\
& \text { t - Thickness of shell }
\end{aligned}
$$



Figure 42

THE RADIATOR GEOMETRY

The smaller 5 is, the closer the system approximates an isothermal
system. In terms of 5 , the temperature on the cylinder is given by:

$$
\begin{equation*}
T=T_{0}\left(1+\zeta+\frac{2}{3} \zeta^{2}+\frac{26}{45} \zeta^{3}+\frac{116}{315} \zeta^{4}+\frac{251}{1050} \zeta^{5}+\ldots .\right) . \tag{2}
\end{equation*}
$$

The heat conducted down the cylinder is given by:

$$
\begin{align*}
& n=k 2 \pi r t \frac{d T}{d Z} K \frac{4 \pi r t T_{0}}{Z}\left(\zeta+\frac{4}{3} \zeta^{2}+\frac{78}{45} \zeta^{3}+\frac{464}{315} \zeta^{3}+\ldots\right)  \tag{3}\\
& H=2 \pi r Z \varepsilon \sigma T_{0}^{4} \quad\left(1+\frac{4}{3} \zeta+\frac{78}{45} \zeta^{2}+\frac{464}{315} \zeta^{3}+\frac{251}{210} \zeta^{4} \ldots\right), \tag{4}
\end{align*}
$$

where $r$ = radius of cylinder.
The temperature on the plate is given by:*

$$
\begin{align*}
& T=T_{0}\left(1+\rho+\frac{1}{2} \rho \zeta+\frac{1}{3} \rho^{2} \zeta+\frac{1}{9} \rho \zeta^{2}+\frac{5}{43} \rho^{2} \zeta^{2}+\frac{1}{72} \rho \zeta^{3}+\ldots\right)  \tag{5}\\
& Z=T_{\rho}+T_{0} \rho \Sigma_{1}, \tag{6}
\end{align*}
$$

where

$$
\begin{equation*}
\rho=\frac{1}{4}\left[\frac{\varepsilon \sigma}{K t} T_{0}{ }^{3} r^{2}-\frac{\alpha Q}{K t T_{0}} r^{2}\right] . \tag{7}
\end{equation*}
$$

$\alpha=$ Absorptivity of front plate
$Q=$ Heat flux per unit area incident on front plate
$\varepsilon_{1}=1+\frac{1}{2} \zeta+\frac{1}{3} \zeta^{2}+\frac{1}{72} \zeta^{3}+\ldots$

The heat conducted out of the plate is given by:

$$
\begin{equation*}
H=K 2 \pi r t \frac{d T}{d r}=4 \pi K t T_{0} \rho \Sigma_{2}, \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
\Sigma_{2}=1+\zeta+\frac{1}{3} \zeta^{2}+\frac{1}{18} \zeta^{3}+\ldots \tag{10}
\end{equation*}
$$

[^4]

Defining

$$
\begin{equation*}
T_{0}=\sqrt[\frac{1}{4}]{\frac{a Q}{\varepsilon 0}} \tag{11}
\end{equation*}
$$

equations [6], [7], [10] and [11] can be solved to give
$H=8 \pi K t \frac{\Sigma_{2} \zeta}{1+2 \Sigma_{1} \bar{\zeta}} \quad\left(T_{00}-T\right)$
and

$$
\begin{equation*}
\Delta T=\frac{2 \Sigma_{1 \xi}}{1+2 \Sigma_{15}} \quad\left(T_{00}-T\right) \tag{13}
\end{equation*}
$$

By plotting $H$ vs. $T$ from equations [2], [3] and [12], the common operating point of the cylinder and the plate can be found, as shown in Fig. 43.

Then the temperature rise along the cylinder and the plate can be found from equations [2] and [13]. The temperatures on the cylinder and plate have nearly parabolic dependence on distance, as shown in Fig. 44.


Figure 44. Temperature profiles on cylinder and plate

If the cylinder were at a constant temperature $T$, it would radiate heat at a certain rate $R$. The actual rate of radiation divided by $R$ gives an efficiency

$$
\begin{equation*}
n_{1}=\frac{1+\frac{4}{3} \zeta^{2}+\frac{78}{45} 5^{2}+\frac{464}{315} 5^{3}+\frac{251}{210} r^{4}+\ldots}{\left(T / T_{0}\right)^{4}} \tag{14}
\end{equation*}
$$

which is shown in Fig. 45.


If we have cylinders of given mass and wish to maximize the heat radiated from the cylinder by optimizing the $x$ to $t$ ratio, we get the following equation,

$$
\begin{equation*}
n_{2}=\frac{5^{1 / 3}\left(1+\frac{4}{3} \zeta+\frac{78}{45} 5^{2}+\frac{464}{315} 5^{3}+\frac{251}{210} \zeta^{4}+\ldots\right)}{0.422\left(T / T_{0}\right)^{3}} \tag{15}
\end{equation*}
$$

Thus a cylinder of any mass will radiate a maximum amount of heat for $5=0.20$.

## III Derivation of Equations

We have assumed no radiative heat transfer within the cylinder. This is a fairly good assumption because [l] in the inside of the cylinder contains electronic packages which obstruct heat flow, [2] the radiative heat transfer within the cylinder will be less than the conductive heat transfer and [3] given the above solution, we can easily obtain a good approximation to the complete solution by (a) assumino the given temperature distribution, (b) calculating the radiative heat inputs to the various areas of the shell, and then (c) re-calculating the temperature distribution (keeping (b) constant). This process could be iterated to obtain a closer approximation. For the cylinder with no radiative heat transfer, the equation of heat balance is

$$
\begin{equation*}
\frac{\mathrm{d}^{2} \mathrm{~T}}{\mathrm{dX}}=\frac{\varepsilon \sigma}{K t} T^{4} \tag{16}
\end{equation*}
$$

Assuming a solution of the form

$$
\begin{equation*}
T=T_{0}\left(1+a X^{2}+b X^{4}+c X^{6}+\ldots . .\right) \tag{17}
\end{equation*}
$$

Expanding [16] and equating like powers of $X$ leads directly to equation [2]. Differentiating [17], we get

$$
\begin{equation*}
\frac{d T}{d X}=\frac{2 T_{e}}{X}\left(a x^{2}+2 b X^{4}+3 c X^{6}+\ldots\right) \tag{18}
\end{equation*}
$$

which is equivalent to [3].
For the front plate with heat input, from the sun, the equation of heat balance is

$$
\begin{equation*}
\text { Kt } \frac{d}{d r}\left(r \frac{d T}{d r}\right)-\operatorname{cor} T^{4}+\alpha Q r=0 \tag{19}
\end{equation*}
$$

By similar methods, we obtain Equations [5] through [13]. Because the front plate tends to equilibrium at a temperature near the maximum possible temperature, one of the properties of the solution is that the rate of heat conduction out of the plate is nearly a linear function of temperature drop.


[^0]:    20 to $2000 \mathrm{cps}, 6.3 \mathrm{~g} \mathrm{rms}, 0.02 \mathrm{~g}^{2} / \mathrm{cps}$ power spectral density

[^1]:    * Developed under JPL Contract NASA-100, Sub- Contract $9 こ 1737$ and NASA MSFC Contract NAS8-5379.

[^2]:    *State-of-the-art with matched selected cells
    **See Appendix 2

[^3]:    * Computation of those patterns was made by M. Matsushita.

[^4]:    * NOTE: For this calculation we have assumed a plate 16 " in diameter, ending at the beginning of the cylinder. A slightly larger plate will have a similar temperature profile, depending similarly upon 5 .

