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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Launch-Man Sequencing for Extended
Mission Effectiveness Study

TM-68-1033-6

DATE- December 16, 1968

FILING CASE NO(S)- 101, 103, 710

AUTHOR(S)- L. D. Nelson

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ABSTRACT

This memorandum contains a necessary part of a compre-
hensive analysis of the psychological and physiological effects
upon astronauts as their length of exposure to in-flight space
environment varies. The possible man-mission resupply schedules
that exist subject to a set of fixed constraints and capable of
several sets of desired properties are determined and those
schedule types which maximize various measures of effectiveness
are singled out.

This is accomplished as follows. Three of the fixed
constraints aid in defining an initial segment of a man-mission
resupply schedule; the derived eleven initial segments partition
all admissible mission profiles into eleven classes; the classes,
and hence the schedules, are analyzed via their initial segments.
The analyses are performed by constructing charts exhibiting the
capabilities of the initial segments with respect to the 54 sets
of desired properties, reducing the charts by deleting every
initial segment less capable than some remaining segment, and
extracting the maximum duration entry for each set of desired
properties. Other measures of effectiveness analyzed are number
of astronauts exposed, mission duration/number of launch vehicles
and number of astronauts/number of launch vehicles.

It is felt that further measures of effectiveness and
other constraint sets could be efficiently analyzed in a manner
similar to that eluded to by the construction and proofs in the
Appendices.

BA-145A (8-68)

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BA-146 (8-64)

SUBJECT: Launch-Man Sequencing for Extended Mission Effectiveness Study - Cases 101, 103, 710

DATE: December 16, 1968

FROM: L. D. Nelson

TM-68-1033-6

TECHNICAL MEMORANDUM

I.   INTRODUCTION

In a comprehensive analysis of the effects of the space environment upon astronauts as their length of exposure in orbit varies it is necessary to determine the man-mission resupply schedules possible under various constraints. The need for such an analysis was originally suggested by D. B. Hoffman who also aided in the definition in the next section. Using some of the constraints a classification of all possible missions into 11 mission types is done and an analysis of these 11 mission types is performed subject to the remaining constraints and desired capabilities. Comparable analyses are clearly possible for other constraint sets.

II.   STATEMENT OF PROBLEM

The problem is to determine the number of mission types that exist and then to list these types given the following constraints and desired properties:

A.   Fixed Constraints

1.   There are three astronauts in orbit at all times,

2.   the minimum lag between resupply launches is 30 days,

3.   the maximum lag between resupply launches is 90 days,

4.   two launch vehicles are required for the initial launch,

5.   one launch vehicle is required for each resupply,

6.   an initial sequencing is required of a 30 day man being returned before any man has been exposed for 60 days, a 60 day man being returned before any man has been exposed for 90 days and a 90 day man being returned before any men have been exposed for greater than 90 days,

7.   the mission is terminated at the end of the longest exposure period,

8. the bounds on the number of astronaut periods of exposure
    are

Length of period in days	Number of periods		
		Minimum	Maximum
30		1	3
60		1	3
90		1	3
120			3
150		1	3
180			3
210		0	3
240		0	3
270		0	3
300		0	3
330		1	3
360			2

B. Varying Constraints or Desired Properties

1. The longest exposure period (P) required is either
    330 days or 360 days in length,

2. the limit (m) on the number of astronauts that can
    be exchanged at any resupply is either 1, 2 or 3,

3. the limit (r) on the resupply lag immediately preceding
    the termination of the mission is either 30, 60 or 90 days,

4. the number (ℓ) of launch vehicles.

Next, for each of the constraint sets determined by
fixing those constraints of B above, consider for maximization
the following measures of effectiveness

a. mission duration (D),

b. number of astronauts exposed (n),

c. $\dfrac{\text{mission duration}}{\text{number of launch vehicles}} = \dfrac{D}{\ell}$

d. $\dfrac{\text{number of astronauts exposed}}{\text{number of launch vehicles}} = \dfrac{n}{\ell}$

Other considerations were the number of men used and
the number of man-days in orbit; however, these are equal to
3 · [number of launch vehicles -1] and 3 · [duration] respec-
tively. Thus, these were only implicitly considered and can
be obtained by applying the conversion formulae.

III. CLASSIFICATION BY INITIAL SEGMENTS

For simplicity, due to the minimum lag constraint
(II.A.2), a 30-day period will be called a unit period (or a
unit) and visually, a 30-day period will be represented as a
block 1 unit in length, a 60-day period as a block 2 units in
length, a 30·k day period as a block k units in length.

To facilitate analysis, the set of all missions will
be partitioned into a sufficiently small collection of distinct
classes each characterized by its "initial segment." The
initial segments are arrived at and defined by using the fol-
lowing constraints from those of Section II.A:

   i)   there are 3 astronauts in orbit at all times (II.A.1),

   ii) no more than 3 each of length 1, length 2 and length 3
        blocks can be used (II.A.8), and

   iii) the precedence relations that at least one length 1
        block must end before any block exceeds length 1, at
        least one length 2 block must end before any block
        exceeds length 2 and at least one length 3 block must
        end before any block exceeds length 3 (II.A.6).

Indeed, condition iii) implies that no longer block
can start before the first 1 unit block, the first 2 unit block,
or the first 3 unit block starts, respectively. Hence, an initial
segment will be defined to be a segment from the beginning of a
mission to and including the first 3 unit block with all three
astronaut positions (condition i) filled by 1 or 2 unit blocks to
at least the beginning of the first 3 unit block. This precludes
any block 3 units or longer from starting too soon. Two segments
are said to be symmetric if one can be obtained from the other by
rearranging the blocks from line to line while keeping each block
in its own fixed time interval. This symmetry is illustrated by
the following example:
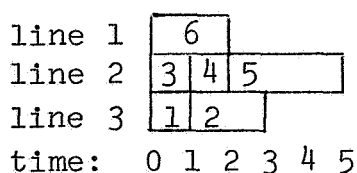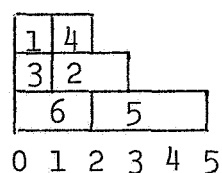
line 1	6	1 4
line 2	3 4 5	3 2
line 3	1 2	6 5
time:	0 1 2 3 4 5	0 1 2 3 4 5
	(a)	(b)

The segment (a) is symmetric to the segment (b) and vice versa.
Condition ii) implies that the maximum length of an initial
segment is 6; the minimum length is obviously 3. Since the first
1 unit block must begin at 0, by an enumeration process of begin-
ning the first 3 unit block at 0, then 1, then 2, then 3 while
beginning the first 2 unit block at 0, then 1 and observing
i) - iii), it is found that there are eleven and only eleven
initial segments within symmetry (see Appendix I). Therefore,
any mission satisfying constraints II.A.1, A.6 and A.8 must have
as its beginning one and only one of these 11 initial segments;
i.e., all such missions are classified by these 11 initial seg-
ment equivalence classes. The missions will thus be analyzed by
analyzing the initial segments listed in Figure 1. Figure 1 has
sub-categorized the initial segments due to the variable constraint
II.B.2 and the number of launches required to accomplish the initial
segments.

IV.   ANALYSIS ON INITIAL SEGMENT CLASSES

Using the constraints of Section II a capability chart
(defined later) for the initial segments will be constructed;
this chart will then be reduced by observing that some initial
segments are more "capable" than others; the resultant chart will
further be reduced and consolidated by extracting all entries
which maximize the mission duration for some fixed set of con-
straints; finally, analysis of the initial segments with the use
of the charts is explained.

A.   Capability Chart Construction

A launch is needed to start a block or blocks at a
specific point along the mission; hence, each initial segment
determines a set of launches needed to accomplish it. In turn,
using constraints II.A.4 and II.A.5 which require two launch
vehicles to start the mission and one at each launch thereafter,
the number of launch vehicles for each initial segment is deter-
mined and is one more than the number of launches. Figure 1 shows
at which points (denoted by L_v) launches are required (e.g.,

initial segment 7 requires launches at $L_1 = 0$, $L_2 = 1$, $L_3 = 3$ and
$L_4 = 4$).

Consider an initial segment I with launches at points
L_1, L_2, ..., L_s. Let the maximum resupply interval (constraint
II.A.3) be p=3 units long, a mandatory resupply launch be
r($1 \le r \le 3$) units before termination (constraint II.A.7 and II.B.3),
a longest exposure period P (constraint II.B.1) be k(k=11,12) units
long and a mission duration be D, (max[L_s+p, 1+k] $\le$ D $\le$ L_s + p + k)

units long. Since P ends the mission (constraint II.A.7) it
must start at point j=D-k. If this is a valid point for P to
start using initial segment I, the minimum number of launch
vehicles needed for a mission thus constrained is given by

$$(1) \qquad N = s + 1 + \alpha + \left\lceil \frac{D-\beta-r}{p} \right\rceil * \qquad ,$$

where

$$\alpha = \begin{cases} 0 \text{ if } j \epsilon \{L_1, L_2, \ldots, L_s\} \\ \\ 1 \text{ if } j \notin \{L_1, L_2, \ldots, L_s\} \end{cases}$$

and

$$\beta = \max[L_s, j] \quad .$$

The proof of (1) is contained in Appendix II for general $p \geq 3$.
The minimum duration for the specific constraints of section II
is 12 units or 360 days and the minimum number of launch vehicles
is 7.

An initial segment is $\underline{capable}$ of a desired property or
properties (Section II.B) if it $\underline{admits}$ the construction of a mis-
sion having the desired property or properties. Thus a $\underline{capability}$
$\underline{chart}$ expressing the capabilities with regard to the desired pro-
perties of Section II.B will be constructed for the 11 initial
segments. This chart includes only 7, 8 and 9 launch vehicles
because 7 is the minimum number and the chart becomes "regular"
at 9, i.e., it shifts the reduced 9 launch vehicle chart 3 dura-
tion units down for each additional vehicle. By setting p=3 and
varying the parameters I, k, D, and N most of the capability
chart for initial segments, Figure 2, can be constructed keeping
in mind constraints II.A.6. The remaining portion can be filled
in immediately by realizing that N is a minimum and thus the
terminal launch lag of 30 or 60 days is attained when an extra
launch is added for a given duration mission. (See Appendix III.)

B. $\underline{Capability \ Chart \ Reduction}$

The capability chart (Figure 2) will be reduced by ob-
serving that some initial segments are more capable than others.
A minimum terminal lag of 30 days includes or "covers" a minimum

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$*\lceil x \rceil$ signifies the smallest integer $\geq$ x.

terminal lag of 60 days (i.e., if it is possible to launch 30 days prior to termination, it is possible to launch instead 60 days or 90 days prior to termination without violating the other pertinent constraints and without adding another vehicle). This says $\oplus$ covers $\ominus$ and $\bigcirc$; also, $\ominus$ covers $\bigcirc$ and any one covers a blank (e.g., Figure 2). Therefore, a column or initial segment within a table can be said to "cover" another column or initial segment within the same table if every entry of the first column "covers" the corresponding entry of the second column within that table. Thus for a given longest exposure period P and for a given number of launch vehicles ℓ, any mission with the first initial segment has at least the mission duration and terminal launch lag capabilities of the second. For example, in the table for P of 330 days, $\ell = 8$, initial segment 2 has more mission duration and terminal launch lag capability than initial segment 3.

Furthermore, if an initial segment covers another initial segment for every table of Figure 2 the former has at least the longest exposure period and number of launch vehicle capability of the latter (see again initial segments 2 and 3). If the resupply astronaut exchange limit, m, is to also be considered, only those initial segments within a fixed m group should be compared.

Thus from the capability chart, Figure 2, considering the facets of a) 330 or 360 day longest exposure periods, b) 7, 8 or 9 launch vehicles, c) mission duration, d) 30, 60 or 90 day terminal launch lag and e) 1, 2 or 3 resupply astronaut exchange limit, initial segments 1, 2, 7, 8 and 9 are uniquely capable. Figure 3 is the reduced chart.

If it is desired to maximize mission duration, a further reduction can be accomplished and the tables consolidated by extracting all entries which maximize the mission duration for a fixed set of constraint variables (k, m, r, ℓ). For example, say P is a 330 day period (k=11) (constraint II.B.1), the astronaut exchange limit is 2(m=2)(II.B.2), the terminal launch lag limit is 60 days (r=2) (II.B.4) then the maximum duration D with 8 launch vehicles is 15 units or 450 days accomplished by some mission having initial segment 2 (From Figure 2, initial segment 3 could also be used; however, only one will be chosen.). The resulting chart is shown in Figure 4.

C. Capability Chart Use for Analysis of Effectiveness Measures

The missions will be analyzed in terms of the measures of effectiveness given in Section II.B: mission duration, mission duration/number of launch vehicles, number of men exposed and number of men exposed/number of launch vehicles.

C.1 Maximum Mission Duration

The maximum mission duration for any fixed set of II.B constraints (k, m, r, ℓ) is accomplished by some mission having the initial segment corresponding to the entry in Figure 4 satisfying those constraints, if any mission can satisfy those constraints (Note that $\oplus$ "covers" $\ominus$ or $\bigcirc$ and that $\ominus$ "covers" $\bigcirc$.)

The maximum duration can also be determined by Figure 4 within given bounds for the variable constraints of Section II.B and keeping the number of launch vehicles at or below 9 since there are so few entries to consider. This is done by considering

$$\max\{D(k,m,r,\ell)\,|\,k_1 \leq k \leq k_2,\ m_1 \leq m \leq m_2,\ r_1 \leq r \leq r_2,\ \ell_1 \leq \ell \leq \ell_2\}$$

e.g.,

$$\mathrm{Max}\{D(k,m,r,\ell)\,|\,11 \leq k \leq 12,\ m=2,\ 2 \leq r \leq 3,\ \ell=8\}$$

$$= \mathrm{Max}\{D(11,2,2,8),\ D(11,2,3,8),\ D(12,2,2,8),\ D(12,2,3,8)\}$$

$$= \mathrm{Max}\{15,15,14,16\} = 16 \text{ units or } 480 \text{ days.}$$

C.2 Maximum Mission Duration/Number of Launch Vehicles

The maximum mission duration divided by the number of launch vehicles, D/ℓ, can be determined in a similar manner from Figure 4. Table 1 gives the maximum duration for each fixed set of m,k,r; an initial segment used to accomplish the max D/ℓ is also shown.

C.3 Other Analyses Related to the Duration

Figures 2 and 3 can be used for other analyses such as the following. Fix the duration and consider the ranges of the other constraints or capabilities in Section II.B; e.g., setting D=16 units (480 days) from Figure 3, it is seen that at least 8 vehicles must be used and if only 8 are used, to accomplish a resupply at 2 units (60 days) from termination requires P having 330 days exposure, at least one 3 astronaut exchange and a mission with initial segment 8; the effectiveness measure $D/\ell = 16/8$ as a maximum for D=16.

Set Constraints			Obtain	
m	k	r	max D/ℓ	by using I
1	11	1	13/8 = 1.625	1
1	11	2	17/9 = 1.900	1
1	11	3	17/9 = 1.900	1
1	12	1	15/9 = 1.667	1
1	12	2	14/8 = 1.750	1
1	12	3	18/9 = 2.000	1
2	11	1	15/9 = 1.667	2
2	11	2	18/9 = 2.000	2
2	11	3	18/9 = 2.000	2
2	12	1	16/9 = 1.778	2
2	12	2	16/9 = 1.778	2
2	12	3	19/9 = 2.111	2
3	11	1	17/9 = 1.889	9
3	11	2	20/9 = 2.222	9
3	11	3	20/9 = 2.222	9
3	12	1	18/9 = 2.000	9
3	12	2	18/9 = 2.000	9
3	12	3	21/9 = 2.333	9

Table 1   MAXIMIZE D/ℓ FOR FIXED m,r,k

This is obtained from Figure 3 by varying D and ℓ for a fixed set: astronaut exchange limit m, longest exposure period k (k=11 is 330 days; k=12 is 360 days) and minimum terminal lag r.

Here are some general conclusions drawn from Figure 4 and the constraints.

1) If one has the two man exchange capability and desires a longer duration, there is a trade-off between
   a) adding an additional man exchange capability and
   b) adding an extra launch vehicle and cutting the exchange capability by one man,

2) there is generally a trade-off between a) requiring a longest exposure period of 360 days with a 90 day terminal launch lag and b) a longest exposure period of 330 days with a 60 day terminal lag and 30 days less duration but a fresher crew member to pilot reentry,

3) for x(x>9) launch vehicles, the maximum duration capability chart would be similar to that for 9 launch vehicles in Figure 4 in that it would be shifted 3 units in duration down for each extra launch vehicle, e.g., ⊕ under initial segment 2 would have the duration of 9 launch vehicles plus $3(x-9)$ or $D=18 + 3(x-9)$. Hence, it is seen that D/ℓ becomes "regular" after 9 launch vehicles, i.e., in units

$$D(k,m,r,\ell) = D(k,m,r,9) + 3(\ell-9) , \quad \ell \geqslant 9 .$$

This is due to the launch point j being greater than or equal to the maximum resupply lag past the end point of any initial segment;

4) therefore, the change in maximum D/ℓ is predictable as one launch vehicle is added (for $\ell > 9$).

$$\frac{D(k,m,r,\ell+1)}{\ell+1} = \frac{D(k,m,r,9) + 3[(\ell+1)-9]}{\ell+1} = \frac{D(k,m,r,\ell) + 3}{\ell+1}$$

implies

$$\Delta\left(\frac{D}{\ell}\right) = \frac{D(k,m,r,\ell+1)}{\ell+1} - \frac{D(k,m,r,\ell)}{\ell} = \frac{3\ell-D(k,m,r,\ell)}{\ell(\ell+1)}$$

which becomes almost insignificant as ℓ becomes large;

5) due to constraint II.A.8 the limit on mission duration is 75 units which requires at least 27 launch vehicles; thus, the maximum D/ℓ for the problem is $\frac{75}{27} = 2.778$ since $\Delta(\frac{D}{\ell}) > 0$, for $\ell > 9$; i.e., D/ℓ increases as ℓ increases (this is proved using 3), maximum $D(k,m,r,9) = 21$, and the definition of $\Delta(D/\ell)$.)

C.4 Number of Astronauts Exposed

The initial segment and fixed constraints determined by an optimal entry fixes most of a mission profile, the remaining portion to be filled in by observing all constraints and perhaps optimizing, subject to these fixed constraints, the number of astronauts exposed divided by the number of launch vehicles. Some exemplary mission profiles are shown in Figure 5 with that portion shaded which has been fixed within symmetry by a Figure 3 entry.

Now consider the number of men exposed while satisfying the constraints of Section II. An upper bound $n(D,k)$ on this number can be computed as a function of the duration D and the longest exposure period P with length k units. This is done by solving the following problem:

$$n(D,k) = \max \left[\sum_{i=1}^{k} n_i \right]$$

such that (constraint II.A.8)

$$n_i \leq 3 \, , \, i = 1, 2, \ldots, k-1$$

$$n_k \leq 2 \, ,$$

$$n_i \geq 1 \, , \, i = 1, 2, 3, k \quad ,$$

$$n_4 + n_5 + n_6 \geq 1 \, ,$$

$$n_{k-1} + n_k \geq 1 \quad ;$$

n_i being non-negative integers representing the number of exposure blocks of length i, and

$$\sum_{i=1}^{k} i \cdot n_i \leq 3 \cdot D \quad .$$

Table 2 gives bounds for k = 11, 12; D = 12, 13, ..., 21.

k \ D	12	13	14	15	16	17	18	19	20	21
11	11	12	13	13	14	15	15	16	16	17
12	--	12	13	13	14	14	15	16	16	17

Table 2.   Upper Bound, n(D,k)

An Upper Bound On The Number Of Men Exposed As A Function Of Mission Duration D And Longest Exposure Period Length k.

An additional upper bound on the number of astronauts exposed is found which depends upon the number of launch vehicles, ℓ, and the maximum astronaut exchange number, m, imposed upon each launch except the first,

$$b\,(\ell,m) \;=\; m \cdot (\ell-2) \;+\; 3 \quad .$$

Table 3 gives b(ℓ,m) for the ranges of m and ℓ considered.

ℓ \ m	1	2	3
7	8	13	18
8	9	15	21
9	10	21	24

Table 3.   b(ℓ,m)

An Upper Bound On The Number Of Men Exposed As A Function Of The

Number Of Launch Vehicles ℓ And The Astronaut Exchange Number m.

A compound upper bound on n as a function of (D,k,ℓ,m) is

$$n'(D,k,\ell,m) = \min[n(D,k),b(\ell,m)] \quad .$$

With this upper bound on the number of astronauts exposed, one can tell when a mission profile is near optimum. There may be so few remaining possibilities in some instances that the maximum can be determined, e.g., if a mission profile is obtained which attains the upper bound, then a maximized mission profile is obtained. Figure 5 shows some example mission profiles which have been selected for their salient characteristics. In particular, those shown for the one-astronaut exchange limit (i.e., m=1) have all reached the bound imposed by Table 3; they are also maximum duration missions for k=11, m=1 with $(\ell,r) = (7,3)$, $(8,1)$ and $(8,2)$ respectively, reading top to bottom. Those in the initial segment number 2 class have the maximum number of astronauts exposed and maximum duration for k=11, m=2 and appropriate choices of (ℓ,r) except for the mission with duration D=12 and having $\ell=8$, which has the maximum n for $(D,k) = (12,11)$. Note that using initial segment number 9, the last launch could be moved to a time one unit later, given a terminal launch lag of 2 units (60 days) and still satisfy all constraints except the bound on the number of 2 unit blocks; in these cases it would seem reasonable to relax this bound to obtain a relatively fresh man for the return.

C.5 Number of Astronauts Exposed/Number of Launch Vehicles

Using the bounds of Tables 2 and 3 and the Capability Chart, Figure 4, upper bounds can be obtained for the $\frac{\text{number of astronauts exposed}}{\text{number of launch vehicles}}$, n/ℓ, for a fixed constraint set of Section II.B; the maximum duration for that constraint set is determined from Figure 4 and the bounds are read from the Tables 2 and/or 3. Within the gross bounds of the Section II.B constraints and $\ell=7,8,9$ an upper bound on n/ℓ is 17/9 = 1.889 obtained by comparing with each other the longest duration's $n(k,D)$ for each ℓ divided by that ℓ.

Table 4 gives these upper bounds for n' and n'/ℓ as functions of ℓ,m between their maximums and minimums. This can be done since ℓ determines an upper bound on D which in turn determines an upper bound on $n(D,k)$. n'/ℓ is thus seen to decrease as ℓ increases.

		Upper bound on number of men exposed, n*, for		Upper bound on number of men exposed/no. of launch vehicles	
		m = 3, 2	m = 1	m = 3, 2	m = 1
	7	14	8	2.000	1.143
	8	15	9	1.875	1.125
	9	17	10	1.888	1.111
	10	18	11	1.800	1.100
	11	20	12	1.818	1.091
	12	21	13	1.750	1.083
	13	22	14	1.692	1.077
	14	23	15	1.643	1.071
	15	24	16	1.600	1.067
Number of launch vehicles ℓ	16	25	17	1.563	1.063
	17	26	18	1.529	1.059
	18	27	19	1.500	1.056
	19	28	20	1.474	1.053
	20	29	21	1.450	1.050
	21	30	22	1.429	1.048
	22	31	23	1.409	1.045
	23	32	24	1.391	1.043
	24	33	25	1.375	1.042
	25	33	26	1.320	1.040
	26	34	27	1.308	1.038
	27	35	28	1.296	1.037

TABLE 4   UPPER BOUNDS $n'(D,-,\ell,m)$ and $n'(D,-,\ell,m)/\ell$

on the number of men exposed and the number of men exposed divided by the number of launch vehicles, respectively for ℓ between its minimum and maximum.

In conclusion, any one measure of effectiveness can
be maximized for a fixed set of constraints. The mission dura-
tion, number of astronauts exposed and mission duration divided
by the number of launch vehicles can simultaneously be maximized
as ℓ increases; however, the number of astronauts exposed
divided by the number of launch vehicles generally decreases as
the number of launch vehicles increases, thus cannot be maximized
with the others. All can be maximized for a fixed ℓ.

V. SUMMARY

The problem given was to determine the possible man-
mission resupply schedules that exist subject to a set of fixed
constraints and capable of various desired properties; then to
choose those schedule types which maximized various measures of
effectiveness.

The fixed constraints pertained to the number, size
and sequencing of periods of exposure, the lag between and num-
ber of vehicles for resupply launches, and the mission termina-
tion. The desired properties were: one of two sizes for the
longest exposure period, one of 3 limits on the number of astro-
nauts exchanged at any resupply, one of 3 terminal resupply lags,
and, effectively, one of 3 numbers of launch vehicles. The
measures of effectiveness to be maximized were mission duration,
number of astronauts exposed, mission duration/number of launch
vehicles and number of astronauts exposed/number of launch vehicles.
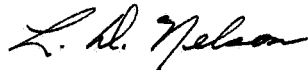
The problem was attacked by using 3 of the fixed con-
straints to aid in defining an initial segment. The total of
eleven initial segments (Figure 1) partitioned all mission pro-
files which satisfied the constraints into eleven classes. The
classes were analyzed via their initial segments. The analyses
were performed by constructing charts (Figure 2) showing the
initial segment capabilities with regard to the variable pro-
perties, reducing the charts to the more capable segments
(Figure 3) and extracting the maximum duration entry for each
of the 54 sets of fixed constraints (shown as Figure 4).

The maximum mission duration/number of launch vehicles
(Table 1) was easily computed using the reduced chart; some
general comments on duration related analyses were given. Next,
since the number of astronauts exposed is not a direct computa-
tion, upper bounds (Table 4) on this number as well as on the
number of men exposed/number of launch vehicles were computed
via a linear program. The initial segment and schedule proper-
ties exhibited by an entry in a chart determine much of a mission
profile; the remaining portion can be completed with perhaps addi-
tional salient features while continuing to observe the given
constraints. Some such example mission profiles were given
(Figure 5).

        In conclusion, it was noted that any of the measures
of effectiveness given could be maximized for a fixed number
of launch vehicles, or likewise for a fixed set of constraints;
also, that all measures increased as the number of launch
vehicles increased except the number of men exposed/number of
launch vehicles which in general (but not always) decreased.

        It is felt that other measures of effectiveness and
other comparable constraint sets could be efficiently analyzed
in a similar manner as eluded to by the construction and proofs
in the Appendices.

1033-LDN-jr   L. D. Nelson

Attachments
    Appendices I, II, III
    Figures 1 – 6

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APPENDIX I

CONSTRUCTION OF INITIAL SEGMENTS TO CLASSIFY MISSIONS

An initial segment is defined to be a segment from the beginning of a mission to and including the first 3 unit block with all three astronaut positions filled by 1 or 2 unit blocks to at least the beginning of the first 3 unit block and satisfying the following conditions:

i)   no more than 3 each of length 1, length 2 and length 3 blocks can be used, and

ii) the precedence relations that at least one length 1 block must end before any block exceeds length 1, at least one length 2 block must end before any block exceeds length 2 and at least one length 3 block must end before any block exceeds length 3.

Two mission segments are said to be symmetric if one can be obtained from the other by rearranging the blocks from line to line while keeping each block in its own fixed time interval. It will be shown that there are only eleven initial segments within symmetry.

Condition ii) implies that no longer block can start before the first 1 unit block, the first 2 unit block and the first 3 unit block respectively. With condition i), this implies that the first 1 unit block must start at time 0, the first 2 unit block must start at time 0 or 1 and the first 3 unit block must start at time 0, 1, 2 or 3. This follows since the total of block units available for blocks shorter than the first block of length ℓ is

$$\sum_{s=0}^{\ell-1} 3 \cdot s = \frac{3 \cdot \ell(\ell-1)}{2}$$

which will fill the 3 astronaut positions to a maximum time of $t_\ell = \frac{\ell(\ell-1)}{2}$; $t_\ell = 0$ for $\ell=1$, 1 for $\ell=2$ and 3 for $\ell=3$. In Figure 6 the possible locations for the first block of length 2 and the first block of length 3 are shown along the vertical and horizontal axes, respectively. Two combinations are not possible: one because the first 3 unit block ends where the first 2 unit block ends failing to satisfy condition ii); the other because all three 1 unit blocks must be used between

times 0 and 1 while another would be necessary to fill the gap
between time 1 and the first 3 unit block failing to satisfy
condition i). The remaining combinations are possible within
symmetry; these are listed, each with all valid combinations
within symmetry of 1 and 2 unit blocks filling the astronaut
positions to at least the beginning of the first 3 unit block.
All missions which satisfy the constraints of Section II.A must
satisfy the constraints defining an initial segment; hence, each
mission's beginning segment must be symmetric to one of the
initial segments. But symmetry is an equivalence relation and
since no two initial segments are symmetric, each mission falls
into one and only one of the eleven initial segment equivalence
classes.

APPENDIX II

PROOF FOR THE MINIMUM NUMBER OF LAUNCH VEHICLES N(I,k,D,r)

Consider a mission with an initial segment I, I having required launches at the points L_1, L_2, ..., L_s. Let the maximum resupply interval be $p \geq 3$ units long, a mandatory resupply launch be r ($1 \leq r \leq p$) units before termination, a longest exposure period P be k($k \geq p$) units long and a mission duration be D ($\max[L_s + p, 1+k] \leq D \leq L_s + p + k$) units long. Let the mission be terminated at the end of P; thus P must start at point j = D - k. If this is a valid point to start P using initial segment I, the minimum number of launch vehicles needed for a mission thus constrained is given by

(1)
$$N = s + 1 + \alpha + \left\lceil \frac{D-\beta-r}{p} \right\rceil \quad *$$

where

$$\alpha = \begin{cases} 0 \text{ if } j \; \epsilon \{L_1, L_2, \ldots, L_s\} \\ \\ 1 \text{ if } j \; \notin \{L_1, L_2, \ldots, L_s\} \end{cases}$$

and

(2)
$$\beta = \max[L_s, j]$$

The proof of (1) follows. Since j = D-k, $j \leq L_s + p$ and $L_s \leq \beta = \max[L_s, j] \leq L_s + p$; hence, the launch (or no launch)

---

* $\lceil x \rceil$ signifies the smallest integer $\geq$ x.

provided by α is the only possible launch other than the s launches (s+1 launch vehicles since 2 are required at the start) at $\{L_1, L_2, \ldots, L_s\}$ required by the mission up to and including a launch at the point β. From the following diagram
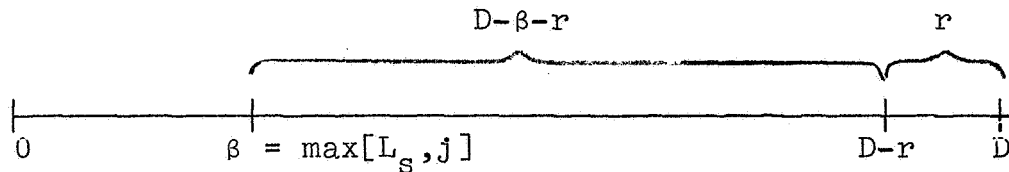
since a launch is required at point $D-r$, if $D-\beta-r \geq 0$ then the minimum number of additional launches to accomplish the mission is

$$\left\lceil \frac{D-\beta-r}{p} \right\rceil \quad , \quad 0 < r \leq p \ .$$

But $D-\beta-r \geq 0$ follows since both

$$D - j - r = D - (D-k) - r = k - r \geq p - r \ ,$$

$$D - L_s - r \geq (L_s + p) - L_s - r = p - r \ ,$$

and

$$p - r \geq r - r = 0 \quad .$$

End of proof.

APPENDIX III

CONSTRUCTION OF CAPABILITY CHART FIGURE 2

With the assumptions and constraints of Appendix I, the capability chart is constructed as follows:

1. Fix p.

2. Fix I. This determines s and $\{L_1, L_2, \ldots, L_s\}$

3. Fix k. With 1 and 2 above, this fixes the bounds on D,

$$\max[L_s+p, \; 1+k] \leqq D \leqq L_s + p + k$$

4. Fix D. This fixes j = D-k, which in turn fixes α and β. From (1) Appendix I and the definition of χ

$$\frac{D-\beta-r}{p} \leq N - s - \alpha - 1 < \frac{D-\beta-r}{p} + 1$$

which implies

$$K - pN \leqq r < K - pN + p$$

where $K = D - \beta + p(s+\alpha+1)$.

5. Fix N. This fixes the above constraint on r; with the additional constraint on r

$$1 \leqq r \leqq p \quad,$$

the minimum r is determined or non-existent for the entry with fixed p, I, k, D and N by also satisfying the constraints II.A.8.

The above suffices to construct the portion of the chart using the minimum number of launch vehicles; to complete the chart drop the constraint that r < K - pN + p.
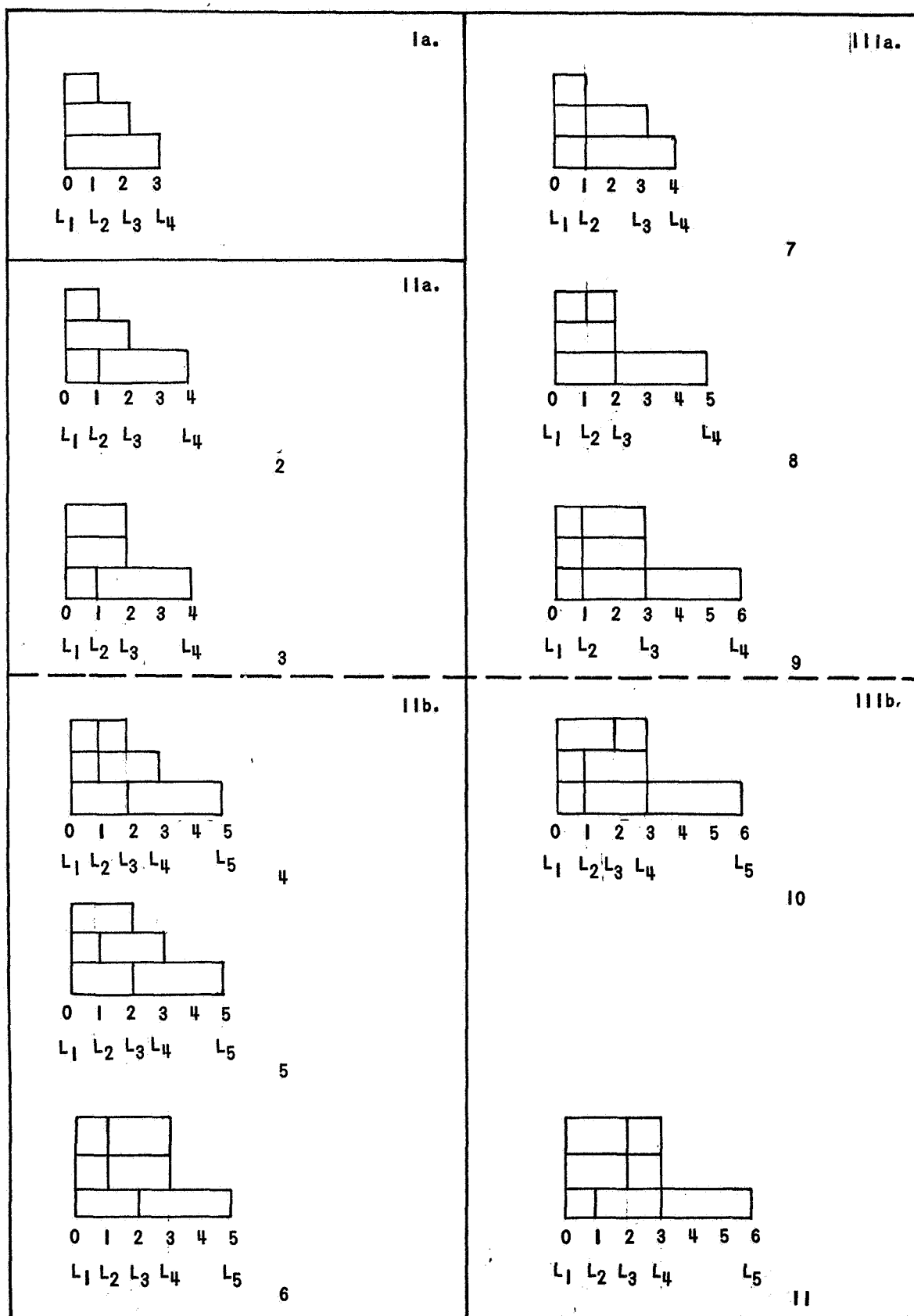
FIGURE I. - THE SET OF POSSIBLE INITIAL SEGMENTS FOR 3-MAN, MAXIMUM OF 3 EXPOSURE
PERIODS OF THE SAME LENGTH ONE-TWO-THREE LONGER PERIOD REQUIRED
SEQUENCING.   THEY ARE CATEGORIZED INTO:

      I. - INITIAL SEGMENTS REQUIRING A MAXIMUM OF I-MAN EXCHANGE,
      II. - INITIAL SEGMENTS REQUIRING A MAXIMUM OF 2-MAN EXCHANGE,
      III. - INITIAL SEGMENTS REQUIRING A MAXIMUM OF 3-MAN EXCHANGE.

FURTHER, THE REQUIRED LAUNCHES ARE NOTED BELOW EACH SEGMENT BY L_i AND
THE SEGMENTS ARE SUBDIVIDED INTO THOSE REQUIRING a) 4 AND b) 5 LAUNCHES.
EVERY UNIT PERIOD IS A 30 DAY PERIOD.
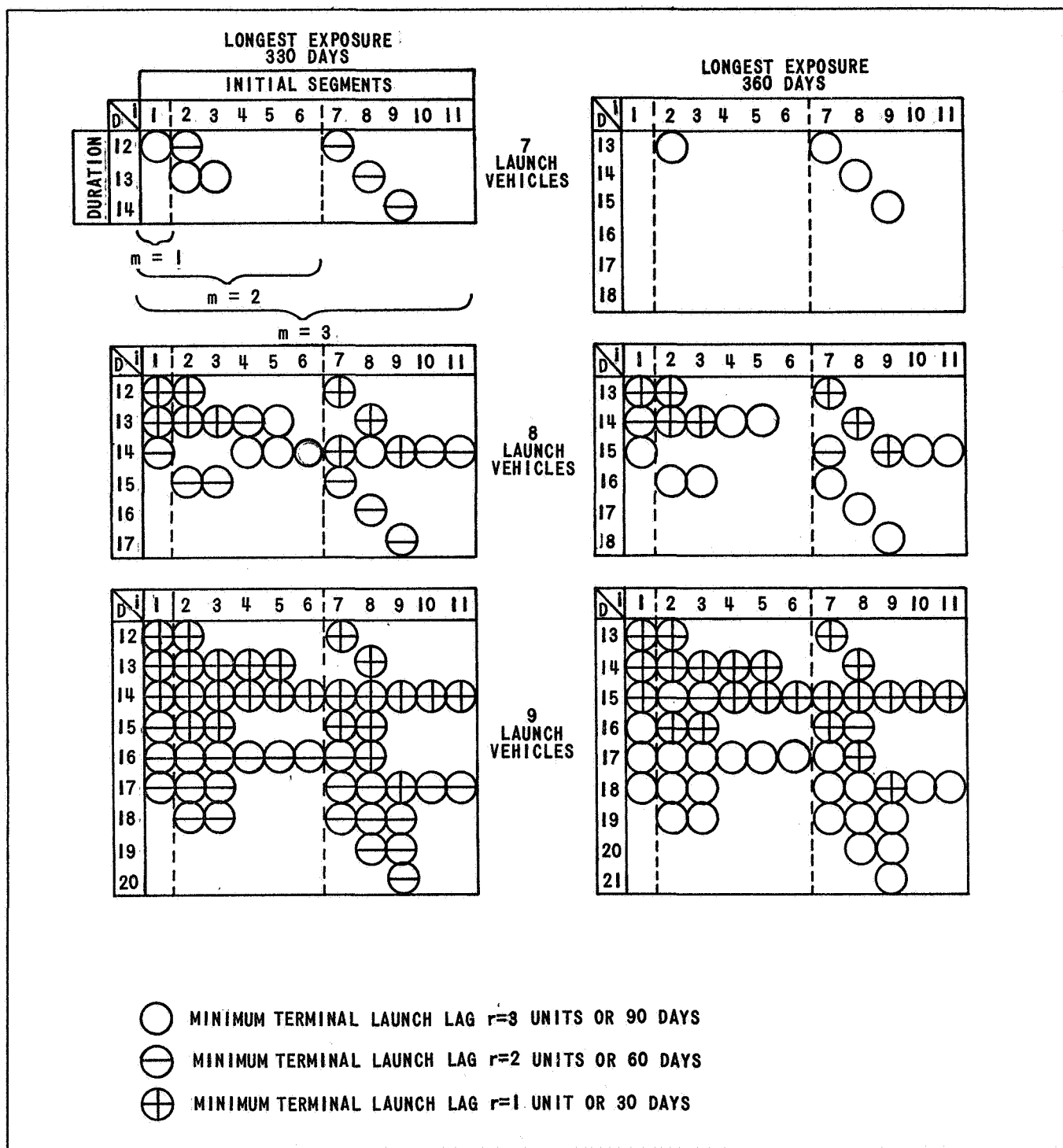
FIGURE 2. - CAPABILITY CHART FOR INITIAL SEGMENTS. THE INITIAL SEGMENTS ARE IN-
DEXED BY i; THE MISSION DURATION IN UNITS IS GIVEN BY D; THE RESUPPLY
ASTRONAUT EXCHANGE LIMIT IS GIVEN BY m.
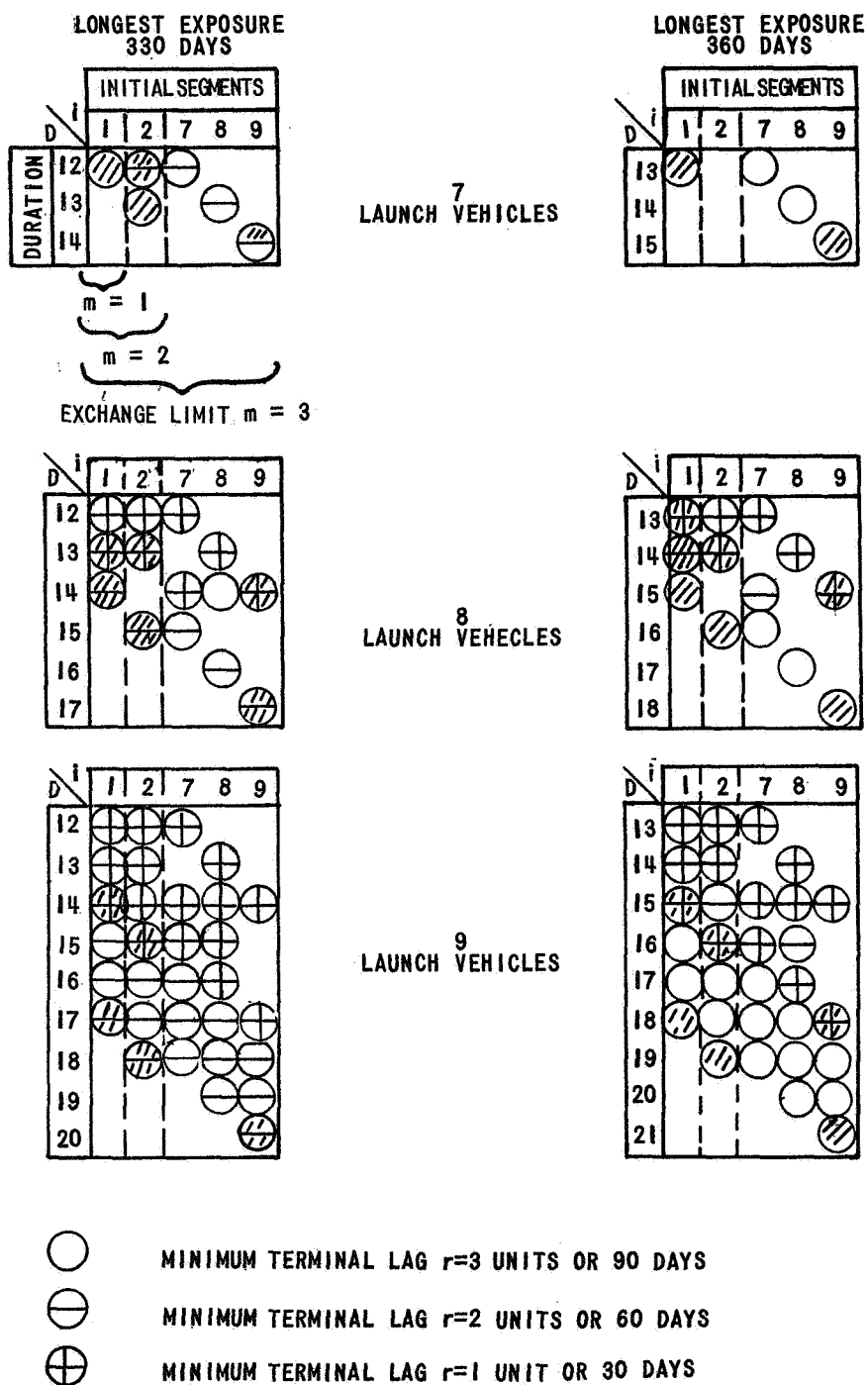
FIGURE 3. - REDUCED CAPABILITY CHART FOR INITIAL SEGMENTS. IF TRYING TO MAXIMIZE DURATION D AND RETAIN SAME CAPABILITIES OF a) LONGEST EXPOSURE PERIODS b) NUMBER OF LAUNCH VEHICLES c) TERMINAL LAUNCH LAG AND d) RESUPPLY ASTRONAUT EXCHANGE LIMIT, CONSIDER ONLY THOSE ENTRIES WHICH ARE SHADED. THE SAME IS TRUE IF TRYING TO MAXIMIZE THE DURATION DIVIDED BY THE NUMBER OF LAUNCH VEHICLES, D/ℓ.
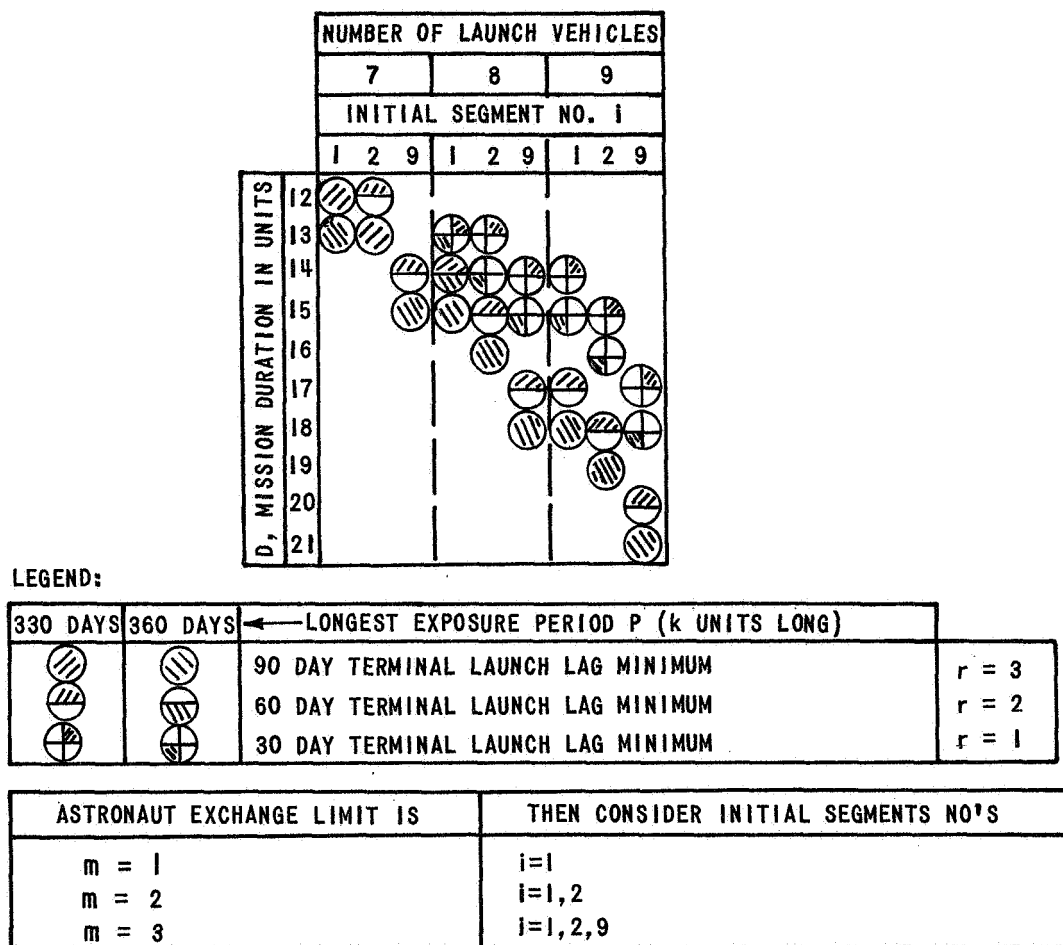
FIGURE 4. - MAXIMUM DURATION CAPABILITY CHART FOR INITIAL SEGMENTS. THIS CHART EXPRESSES THE CAPABILITIES OF THOSE INITIAL SEGMENTS HAVING THE MAXIMUM DURATION PROPERTY FOR EACH VALID FIXED SET OF PROPERTIES a) 7,8, OR 9 LAUNCH VEHICLES, b) 330 OR 360 DAY LONGEST EXPOSURE PERIOD, c) 30,60, OR 90 DAY TERMINAL LAUNCH LAG AND d) 1,2, OR 3 ASTRONAUT EXCHANGE LIMIT.

TIME PERIODS
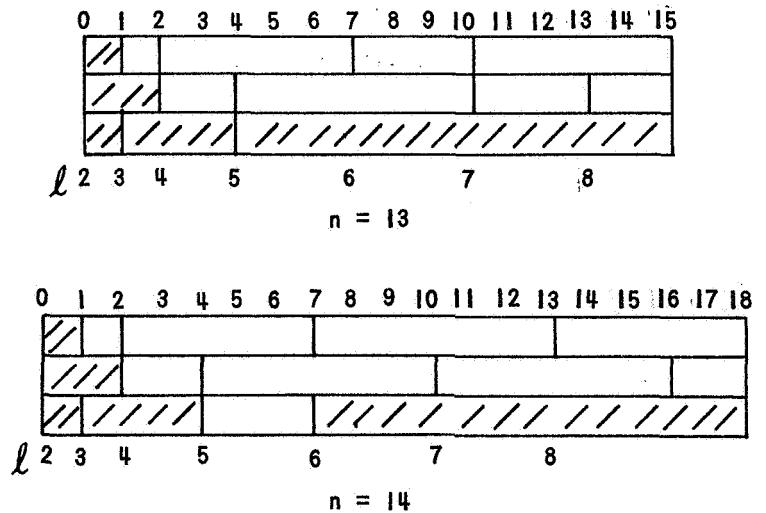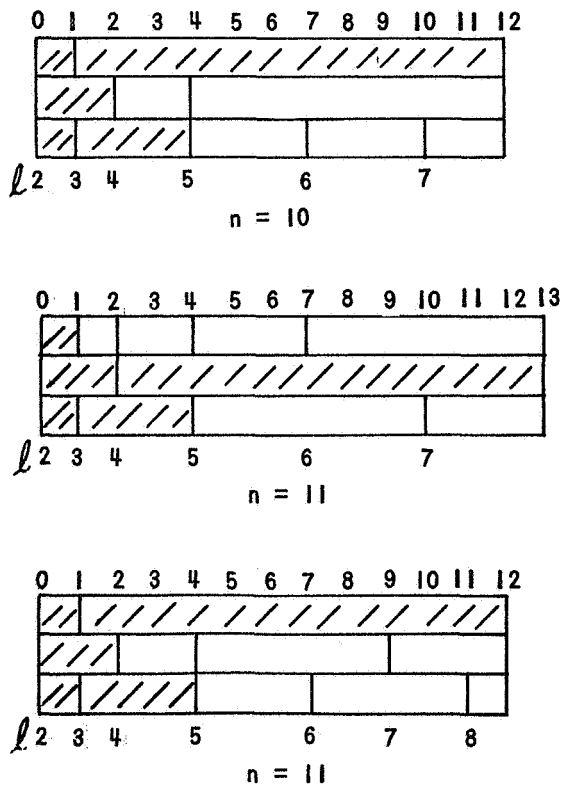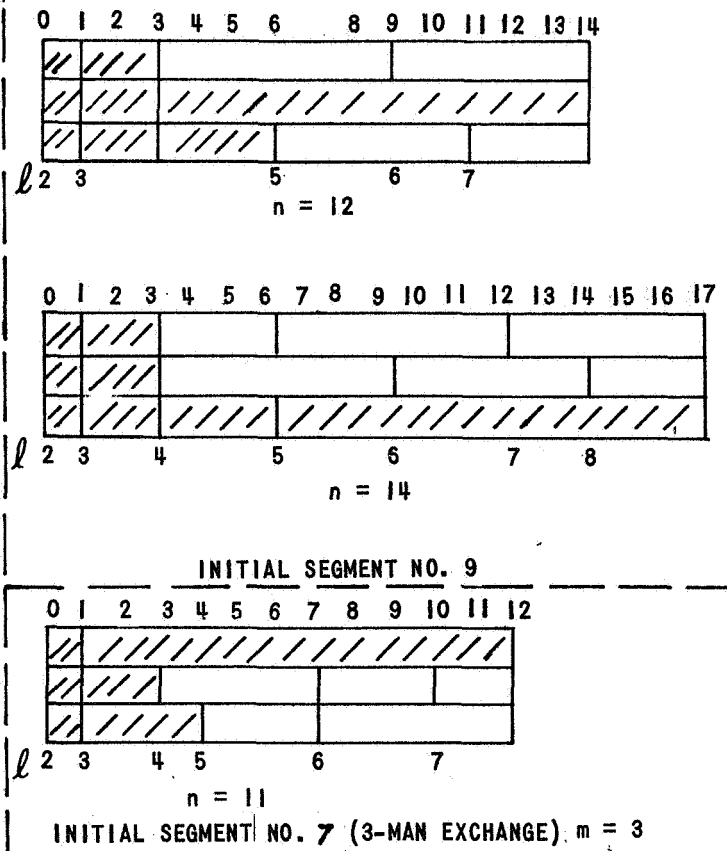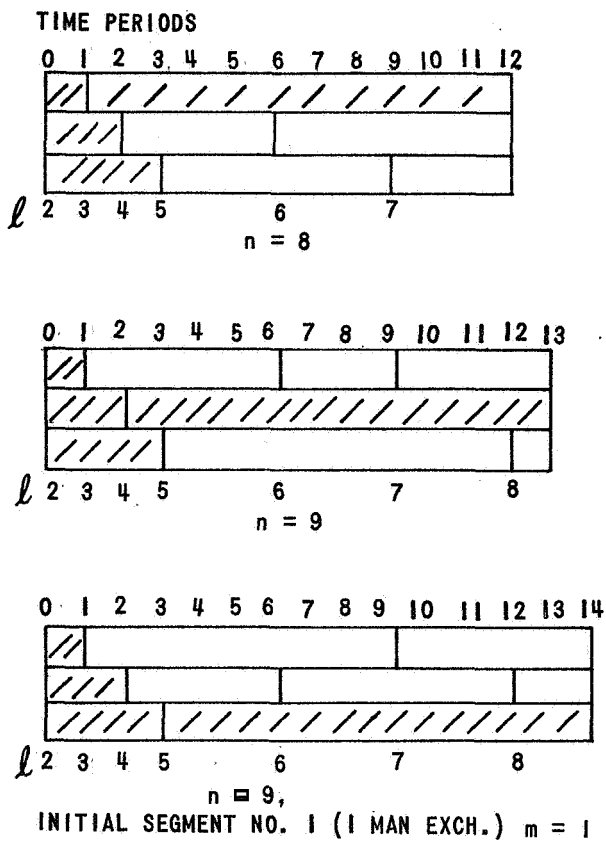
ℓ: ACCUMULATED NUMBER OF LAUNCH VEHICLES
n: NUMBER OF MEN EXPOSED
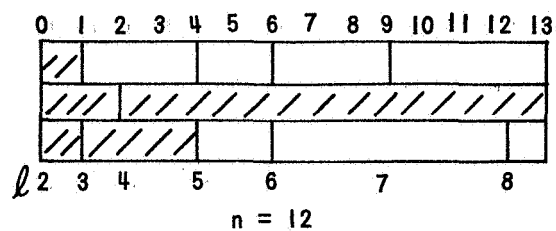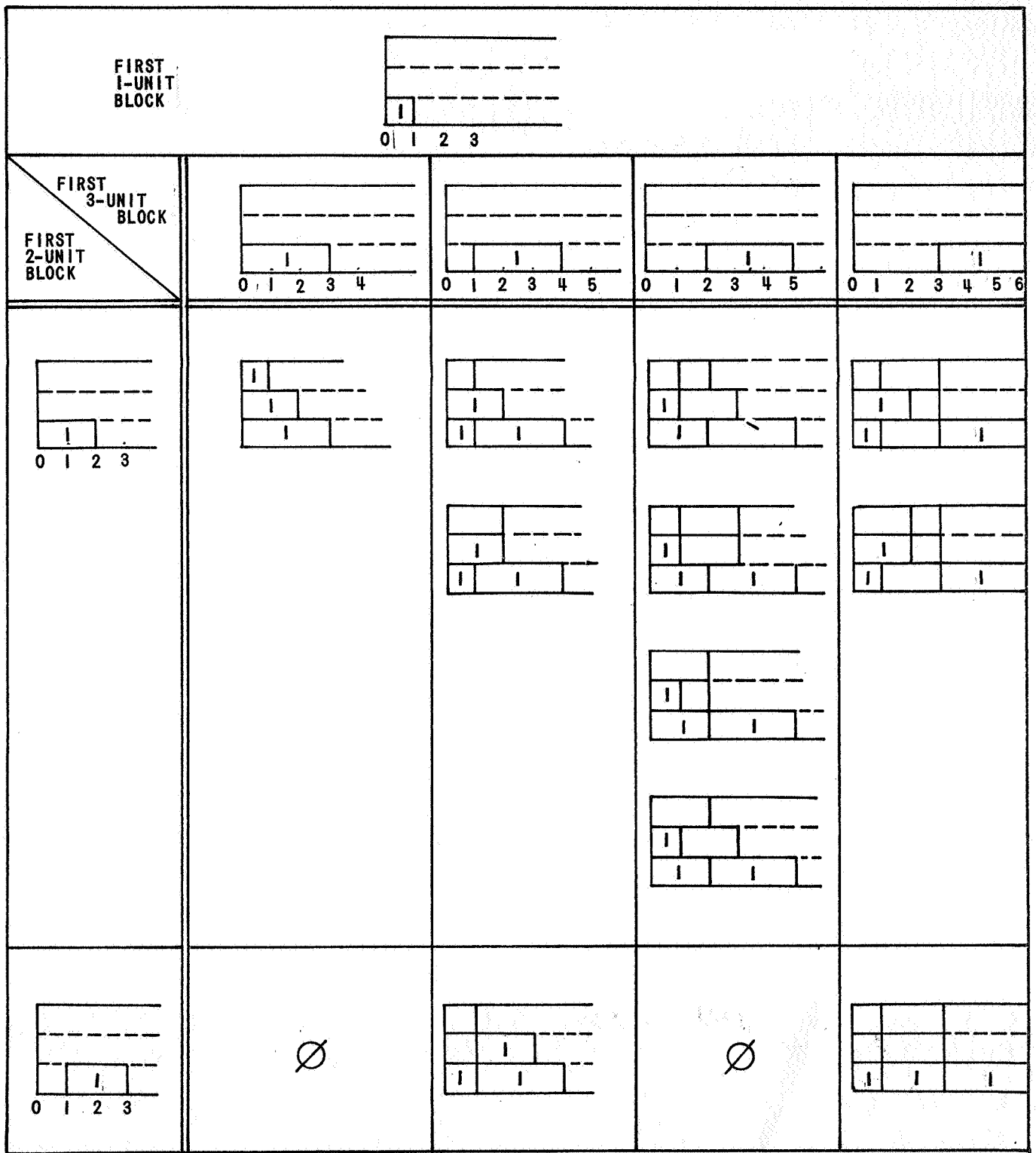
FIGURE 5. — EXTENDED MISSION EXAMPLES

FIGURE 6. - ENUMERATION OF THE ELEVEN INITIAL SEGMENTS BY THE START TIME OF THE FIRST 1 UNIT, 2 UNIT, AND 3 UNIT BLOCKS.