NASA TECHNICAL
MEMORANDUM

NASA TM X-53816

February 20, 1969

NASA TM X-53816

A PROBABILITY DISTRIBUTION FOR THE NUMBER OF THUNDERSTORM EVENTS AT CAPE KENNEDY, FLORIDA

By Lee W. Falls
Aero-Astrodynamics Laboratory

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

MSFC - Form 3190 (September 1968)

NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER

Technical Memorandum X-53816

February 20, 1969

A PROBABILITY DISTRIBUTION FOR THE NUMBER OF THUNDERSTORM EVENTS AT CAPE KENNEDY, FLORIDA

By

Lee W. Falls

TERRESTRIAL ENVIRONMENT BRANCH
AEROSPACE ENVIRONMENT DIVISION
AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

TECHNICAL MEMORANDUM X-53816

A PROBABILITY DISTRIBUTION FOR THE NUMBER OF THUNDERSTORM EVENTS AT CAPE KENNEDY, FLORIDA

By

Lee W. Falls

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

ABSTRACT

The object of this report is to determine an underlying or basic theoretical statistical model for making probability inferences in regard to thunderstorm activity at Cape Kennedy, Florida. The negative binomial distribution with probability density function

$$P(x) = \frac{\Gamma(x + k)}{x!\ \Gamma(k)}\ p^k (1 - p)^x;\ x = 0, 1, 2, \ldots \qquad k > 0,\quad 0 \leqq p \leqq 1$$

is presented to represent the variation in thunderstorm events per day at Cape Kennedy. Statistical theory and methods are developed using the latest and most comprehensive thunderstorm data available. The conclusion is reached that the negative binomial distribution is the logical choice for an underlying model to represent thunderstorm events at Cape Kennedy, Florida.

NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER

TECHNICAL MEMORANDUM X-53816

A PROBABILITY DISTRIBUTION FOR THE NUMBER OF THUNDERSTORM EVENTS AT CAPE KENNEDY, FLORIDA

SUMMARY

This investigation was made to determine an underlying, or basic, theoretical statistical model for making probability inferences in regard to thunderstorm activity at Cape Kennedy, Florida. The negative binomial distribution with probability density function

$$P(x) = \frac{\Gamma(x + k)}{x!\ \Gamma(k)} p^k (1 - p)^x; \quad x = 0, 1, 2, \ldots$$
$$k > 0, \quad 0 \leqq p \leqq 1$$

is presented to represent the variation in thunderstorm events per day at Cape Kennedy. The statistical properties necessary for the application of the negative binomial are presented, and it is shown that these attributes are present in the distributions of thunderstorm events at Cape Kennedy. The latest and most comprehensive thunderstorm data available are analyzed, and the conclusion is reached that the negative binomial distribution is the logical choice for an underlying model to represent thunderstorm events at Cape Kennedy, Florida.

I. INTRODUCTION

Statistical methods of analysis may be divided into two general categories, descriptive and analytical, both of which depend on the basic laws of probability. Descriptive methods reduce large amounts of data to a few meaningful "statistics" such as means and standard deviations. A theoretical statistical model (distribution function) is assumed for the observations, and analytical methods are used to determine how well the empirical data fit this model; i.e., the analytical procedures determine the "goodness of fit" between theory and observation.

The purpose of this report is to determine an underlying, or basic, theoretical distribution for making probability inferences in regard to thunderstorm activity at Cape Kennedy, Florida.

Thunderstorms are of primary concern in the design of launch vehicles, in the planning of space missions, and in launch operations at Cape Kennedy because of high winds, lightning hazard, and extreme turbulence associated with this atmospheric phenomenon. The combinations of environmental conditions, including unstable air with a relatively high moisture content, and some type of lifting action present during the summer months make Florida one of the major thunderstorm genesis areas over the entire earth. The negative binomial distribution is presented to represent the variation in thunderstorm events per day at Cape Kennedy.

The author wishes to acknowledge the assistance of David Riggenbach, Terrestrial Environment Branch (R-AERO-YT), for compiling the frequency distributions of thunderstorm events for Cape Kennedy from the National Weather Records Center data.

II. STATISTICAL MODEL

In practical statistics, a discrete probability law is required to describe events which seem to occur at random; for example, the arrivals of customers at a service point or the number of accidents and breakdowns in a factory. It is common practice to assume that the frequencies of such events fit a Poisson distribution. However, the Poisson series requires the assumption that the probability of the event remains constant. This assumption implies that the variance of the distribution equals its mean. In reality, it is rarely true that the probability of the event remains constant. Any variation in the probability of the event, in particular, the tendency for one event to increase the probability of another, will increase the variance of the distribution in relation to the mean -- which means a negative binomial distribution will better describe the data. A report by the weather observer of a thunderstorm is proof that the atmosphere is in a state of instability and conditions are present for the formation of further thunderstorm cells; i.e., the probability of the event is increasing.

Let us consider the first application of the negative binomial probability distribution by Yule in 1910 [1]. We will make an analogy between this application and the distribution of thunderstorms at Cape Kennedy. Suppose we have a population of people subjected to recurring exposures to a disease and that during an exposure each member of the population has an equal probability p of contracting the disease. After x exposures, the proportions who have contracted the disease 0, 1, 2, ... times will be given by

$$q^x, \ xpq^{x-1}, \ \frac{x(x-1)}{2!} p^2 q^{x-2}, \ \ldots \qquad (1)$$

where $q = 1 - p$. The terms given by (1) are terms of the binomial series $(q + p)^x$. If k unfavorable exposures to the disease are fatal to the individual, the proportion surviving after x exposures will be given by the first k terms of the binomial $(q + p)^x$. The proportion dying during the xth exposure will be those who contracted the disease $(k - 1)$ times in the first $(x - 1)$ exposures and who contract it again during the xth exposure; i.e., it will be

$$\binom{x-1}{k-1} p^{k-1} q^{x-k} \cdot p = \binom{x-1}{k-1} p^k q^{x-k} \tag{2}$$

and since deaths do not begin until the kth exposure, the proportion of deaths at the kth, $(k + 1)$th ... exposure will be

$$p^k \left[1,\ kq,\ k(k+1)\frac{q^2}{2!},\ \ldots\right] \tag{3}$$

which are successive terms in the expansion of $p^k(1 - q)^{-k}$, a binomial with a negative index. Thus, the proportions of the original population dying during successive exposures are given by successive terms of the negative binomial distribution with the first deaths occurring at the kth exposure.

Now, the probability of exactly x events (density function) is given by

$$P(x) = \binom{x+k-1}{k-1} p^k q^x. \tag{4}$$

Suppose in Yule's classic example we let the people exposed to the disease be analogous to the days in some month, say, June, being exposed to the synoptic condition favorable for the formation of thunderstorms at Cape Kennedy. Now, the number of deaths that result from exposure to the disease will be analogous to the number of thunderstorms that actually develop in June. Now, we have all the days in June subjected to recurrent exposures of synoptic conditions favorable for the formation of thunderstorms. We must assume that each day in June that is exposed to the favorable synoptic conditions has an equal probability p of having a thunderstorm develop. This is a reasonable assumption. Continuing our analogy, the proportion of thunderstorms that develop at the kth, $(k + 1)$th ... exposure will be given by (3), successive terms in the expansion of $p^k(1 - q)^{-k}$, a negative binomial whose density function is given by (4).

Thus, statistical theory indicates the negative binomial distribution as the appropriate model for the distribution of thunderstorm activity at Cape Kennedy, Florida.

III. ESTIMATION

Numerous estimators for the parameters of the negative binomial distribution have been proposed. We have chosen to use the first two-moment method proposed by Cohen [2]. The negative binomial density function given by (4) may be written in terms of the gamma function as

$$P(x) = \frac{\Gamma(x + k)}{x!\ \Gamma(k)}\, p^k q^x, \quad x = 0, 1, 2, \ldots; \quad k > 0, \quad 0 \leq p \leq 1. \tag{5}$$

The distribution function is given by

$$F(x) = \sum_{x=0}^{n} \frac{\Gamma(x + k)}{x!\ \Gamma(k)}\, p^k q^x, \tag{6}$$

which gives the probability of obtaining a value of x less than or equal to some particular value of x, say x_o.

Now, after some algebraic manipulation of Cohen's estimators, we have for the moment estimators of the parameters k and p

$$k^* = \frac{\bar{x}^2}{s^2 - \bar{x}}, \qquad p^* = \frac{k}{k + \bar{x}}, \tag{7}$$

where $\bar{x}$ is the sample mean and s^2 is the sample variance.

The mean M of the negative binomial distribution is given by

$$M = \frac{kq}{p},$$

and the variance V is

$$V = \frac{kq}{p^2}.$$

The efficiency of estimating p and k by the method of moments is derived by Fisher [3]. In terms of the parameters used here, the reciprocal of the efficiency is given by

$$\frac{1}{E} = 1 + 2\left[\frac{1}{3} q \frac{2}{(k+2)} + \frac{1}{4} q^2 \frac{2\cdot 3}{(k+2)(k+3)}\right.$$

$$\left. + \frac{1}{5} q^3 \frac{2\cdot 3\cdot 4}{(k+2)(k+3)(k+4)} + \ldots\right] .$$

IV. DATA SAMPLE

According to standard United States weather observing procedure, a thunderstorm is reported whenever thunder is heard at the station. It is reported along with other atmospheric phenomena on the standard weather observer's form WBAN-10 when thunder is heard and ends 15 minutes after thunder is last heard. Notice that the standard definition of a thunderstorm may include multiple occurrences of thunderstorms. For this reason, we have chosen to use the term "thunderstorm event" as a more appropriate definition for our statistical analysis.

Since reliable representative data concerning the number of thunderstorms actually passing over Cape Kennedy (or the launch site) is not available, it should be pointed out that the statistics presented in this paper are applicable only to an area surrounding Cape Kennedy defined by the distance at which thunder can be heard by the weather observer. An observer can hear thunder up to a radius of approximately 25 kilometers. The statistics are not appropriate for making probability inferences in regard to the number of thunderstorms that actually strike the launch pad.

Also, the type of statistical analysis presented is useful primarily for the planning of missions rather than for application to operations. Statistics may be useful up to a few days before a mission. However, at this time the weather forecaster's predictions should be more accurate, and the transition is made from statistical inference to weather forecasting dependent upon the synoptic situation prevailing a few days before the mission.

The data sample used was produced by ESSA, National Weather Records Center, Asheville, North Carolina, under government order number H-76789 for the Terrestrial Environment Branch, Aerospace Environment Division, and is the latest and most comprehensive thunderstorm data available for Cape Kennedy, Florida. The period of record is January 1957 through December 1967.

V. ANALYSIS

The negative binomial and Poisson distributions were tried as prospective models for thunderstorm events at Cape Kennedy. Table 1 summarizes observed frequencies of days that experienced x thunderstorm events for all months, and for the spring, summer, and fall seasons at Cape Kennedy. Table 1a gives the relative frequency of occurrence of days that experienced at least one thunderstorm event at Cape Kennedy for the same reference periods.

Theoretical summaries of the months and seasons that experience significant thunderstorm activity at Cape Kennedy are given in tables 2 through 12. In all cases, the sample variance was significantly greater than the sample mean, indicating the negative binomial distribution as the appropriate model. The Kolmogorov-Smirnov statistic was used for a "goodness of fit" test.

Notations used in tables 1 through 12 are as follows:

- x = the number of thunderstorm events per day
- f_o = the observed number of days during the 11-year period of record that experienced x thunderstorm events
- r.f. = the relative frequency of occurrence of x thunderstorm events
- F_o = the observed distribution function
- f_e = the expected frequencies using the negative binomial distribution
- $F(x)$ = the negative binomial distribution function
- $\bar{x}$ = the sample mean
- s^2 = the sample variance
- k^*, p^* = parameter estimators of the negative binomial distribution
- n = sample size

D_{α}^{n} = the tabulated Kolmogorov-Smirnov statistic for sample size n and rejection level $\alpha = 0.05$

$|F_o - F(x)|$ = the maximum absolute difference between the observed distribution function and the negative binomial distribution function.

Conditional probabilities are also included in the tables.

Consider the month of June (table 5) as an example. There were 40 days out of 330 days (11 years of Junes) that had exactly two thunderstorm events. This gives a relative frequency (probability) of occurrence of 0.121 of having exactly two thunderstorm events during any day in June. The observed distribution function (F_o) gives a probability of 0.921 of having two or less thunderstorm events during any day in June, or a probability of (1 - 0.921 = 0.079) of having more than two thunderstorm events during any day in June. The negative binomial distribution predicts 36.9 days in June that will experience exactly two thunderstorm events and the probability (F(x)) is 0.928 of having two or less thunderstorm events during any day in June, or a probability of (1 - 0.928 = 0.072) of having more than two thunderstorm events during any day in June. The agreement between theory and observation is very good. Comparing F_o with F(x) shows a maximum absolute difference in the distribution functions of 0.017 occurring at x = 0. Since the Kolmogorov-Smirnov statistic

$$D_{\alpha}^{n} = D_{0.05}^{330}$$

is equal to 0.075, the value of 0.017 is not sufficiently large to reject the hypothesis at the 5 percent rejection level that this sample can be fitted by a negative binomial distribution.

The conditional probabilities are computed from the theoretical frequencies (f_e) by using a double summation technique due to O. E. Smith.*

The tabulation on page 8 is an example of this technique using the month of June (see table 5). Each element in the second summation

---

*Chief, Terrestrial Environment Branch, Aerospace Environment Division, Aero-Astrodynamics Laboratory, Marshall Space Flight Center, Alabama.

x	f_e	$\sum$	$\sum\sum$	CONDITIONAL PROBABILITIES					
				i					
0	181.5			1	2	3	4	5	6
1	87.7	148.0	244.2	$\frac{244.2}{244.2} = 1$					
2	36.9	60.3	96.2	$\frac{96.2}{244.2} = .394$	$\frac{96.2}{96.2} = 1$				
3	14.7	23.4	35.9	$\frac{35.9}{244.2} = .147$	$\frac{35.9}{96.2} = .373$	$\frac{35.9}{35.9} = 1$			
4	5.7	8.7	12.5	$\frac{12.5}{244.2} = .051$	$\frac{12.5}{96.2} = .130$	$\frac{12.5}{35.9} = .348$	$\frac{12.5}{12.5} = 1$		
5	2.2	3.0	3.8	$\frac{3.8}{244.2} = .016$	$\frac{3.8}{96.2} = .040$	$\frac{3.8}{35.9} = .106$	$\frac{3.8}{12.5} = .304$	$\frac{3.8}{3.8} = 1$	
6	.8	.8	.8	$\frac{.8}{244.2} = .003$	$\frac{.8}{96.2} = .008$	$\frac{.8}{35.9} = .022$	$\frac{.8}{12.5} = .064$	$\frac{.8}{3.8} = .211$	$\frac{.8}{.8} = 1$

($\Sigma\Sigma$) is divided by the appropriate top element in each column as indicated in order to obtain the conditional probabilities; i.e., in each column under conditional probabilities, given i thunderstorm events (i = 1, 2, 3, ...), the probability of having k additional thunderstorm events (k = 0, 1, 2, ...) is given by

$$P(i + k \mid i) = \frac{(i + k)\text{th element}}{(i)\text{th element}} . \tag{8}$$

For example, for i = 2; given two thunderstorm events on any day in June, what is the probability of having two additional thunderstorm events (k = 2) on that day in June? From equation (8),

$$P(4 \mid 2) = \frac{4\text{th element}}{2\text{nd element}} = \frac{12.5}{96.2} = 0.130.$$

Also, given four thunderstorm events on any day in June (i = 4), the probability of having one additional thunderstorm event (k = 1) on that same day in June is 0.304.

VI. CONCLUSIONS

There are many advantages in the use of a theoretical statistical model for predicting a variable such as thunderstorm events at Cape Kennedy. Once sufficient representative samples have been collected and analyzed and the validity of the theory is established by an appropriate statistical test, the theoretical model becomes "deterministic" and may be applied universally to the variable under consideration. Another advantage of theory over empirical statistics is the use of the acceptable theoretical function for making probability inferences concerning values of the variable outside of the range of observation. It is often desired to make predictions relating to these "never observed" values, and the theoretical approach permits one to do so. It should be pointed out that no theoretical function can explain all observations for which it is the proposed model. Some areas of non-agreement must occur between theory and observation. These areas should be considered as expected deviations of the observations from the "fitted" theoretical curve.

The physical properties necessary for the application of the negative binomial distribution have been shown to be present in our experiment concerning the number of thunderstorm events at Cape Kennedy. In all the samples considered, the sample variance exceeded the sample mean, indicating the negative binomial as the appropriate model. Our comparison with Yule's classic application of the negative binomial substantiates its validity to represent the number of thunderstorm events at Cape Kennedy.

In all 11 samples considered, the conclusion was reached that the negative binomial distribution could not be rejected at the 5 percent rejection level as being an acceptable model for the number of thunderstorm events per day at Cape Kennedy. Furthermore, the negative binomial gave a "better" fit in all cases than the Poisson distribution.

Using statistical theory and methods, we have demonstrated that the negative binomial distribution is the logical choice for an underlying model to represent thunderstorm events at Cape Kennedy, Florida.

TABLE 1. Frequencies of the Observed Number of Days that Experienced x Thunderstorm Events at Cape Kennedy, Florida for the 11-year Period of Record January 1957 through December 1967.

x	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Spring	Summer	Fall
0	335	295	308	299	266	187	177	185	228	311	321	334	873	549	860
1	4	9	20	18	43	77	80	89	54	17	6	3	81	246	77
2	2	4	9	10	25	40	47	30	33	9	3	2	44	117	45
3		2	3	3	3	17	26	24	12	4		2	9	67	16
4			1		3	6	9	10	3				4	25	3
5					0	2	2	3					0	7	
6					1	1							1	1	
n	341	310	341	330	341	330	341	341	330	341	330	341	1012	1012	1001

TABLE 1a: Relative Frequency of Days that Experienced at Least One Thunderstorm Event at Cape Kennedy, Florida.

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Spring	Summer	Fall
.018	.048	.097	.094	.220	.433	.481	.457	.309	.088	.027	.021	.137	.458	.141

TABLE 2. March-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	$F(x)$	Conditional Probability			
						i			
						1	2	3	4
0	308	.902	.902	305.4	.896				
1	20	.059	.961	25.5	.970	1			
2	9	.026	.987	6.7	.990	.277	1		
3	3	.009	.996	2.2	.996	.078	.281	1	
4	1	.003	1.000	.8	.999	.016	.059	.211	1

$\bar{x} = 0.150$ $s^2 = 0.268$ $k^* = 0.189$ $p^* = 0.558$ $n = 341$

Goodness of fit: $D_\alpha^n = 0.074$

$|F_o - F(x)| = 0.009$

TABLE 3. April-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	$F(x)$	Conditional Probability		
						i		
						1	2	3
0	299	.906	.906	295.9	.897			
1	18	.055	.961	25.3	.973	1		
2	10	.030	.991	6.1	.992	.226	1	
3	3	.009	1.000	1.8	.997	.042	.186	1

$\bar{x} = 0.142$ $s^2 = 0.237$ $k^* = 0.214$ $p^* = 0.600$ $n = 330$

Goodness of fit: $D_\alpha^n = 0.075$

$|F_o - F(x)| = 0.013.$

TABLE 4: May-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	F(x)	Conditional Probability					
						i					
0	266	.779	.780	262.6	.770	1	2	3	4	5	6
1	43	.126	.906	52.4	.924	1					
2	25	.073	.979	16.6	.972	.339	1				
3	3	.009	.988	5.9	.989	.120	.354	1			
4	3	.009	.997	2.2	.996	.041	.122	.345	1		
5	0	.000	.997	.9	.998	.013	.037	.106	.306	1	
6	1	.003	1.000	.3	.999	.003	.007	.021	.061	.200	1

$\bar{x} = 0.352$ $s^2 = 0.621$ $k^* = 0.460$ $p^* = 0.567$ $n = 341$

Goodness of fit: $D_\alpha^n = 0.074$

$|F_o - F(x)| = 0.017$

TABLE 5: June-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	F(x)	Conditional Probability					
						i					
0	187	.567	.567	181.5	.550	1	2	3	4	5	6
1	77	.233	.800	87.7	.816	1					
2	40	.121	.921	36.9	.928	.394	1				
3	17	.052	.973	14.7	.972	.147	.373	1			
4	6	.018	.991	5.7	.989	.051	.130	.348	1		
5	2	.006	.997	2.2	.996	.016	.040	.106	.304	1	
6	1	.003	1.000	.8	.999	.003	.008	.022	.064	.211	1

$\bar{x} = 0.752$ $s^2 = 1.169$ $k^* = 1.354$ $p^* = 0.643$ $n = 330$

Goodness of fit: $D_\alpha^n = 0.075$

$|F_o - F(x)| = 0.017.$

TABLE 6: July-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	$F(x)$	Conditional Probability i				
						1	2	3	4	5
0	177	.519	.519	166.2	.487	1				
1	80	.234	.753	99.4	.779					
2	47	.138	.891	45.4	.912	.399	1			
3	26	.076	.967	18.6	.967	.143	.357	1		
4	9	.026	.993	7.2	.988	.044	.110	.307	1	
5	2	.006	1.000	2.7	.996	.009	.023	.066	.214	1

$\bar{x} = 0.874$ $s^2 = 1.277$ $k^* = 1.893$ $p^* = 0.684$ $n = 341$

Goodness of fit: $D^n_\alpha = 0.074$

$|F_o - F(x)| = 0.032$

TABLE 7: August-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	$F(x)$	Conditional Probability i				
						1	2	3	4	5
0	185	.542	.542	180.2	.528	1				
1	89	.261	.803	92.2	.799					
2	30	.088	.891	40.5	.918	.399	1			
3	24	.070	.961	16.9	.967	.146	.366	1		
4	10	.029	.990	6.8	.987	.046	.116	.316	1	
5	3	.009	1.000	2.7	.995	.010	.026	.070	.221	1

$\bar{x} = 0.809$ $s^2 = 1.280$ $k^* = 1.391$ $p^* = 0.632$ $n = 341$

Goodness of fit: $D^n_\alpha = 0.074$

$|F_o - F(x)| = 0.026$

TABLE 8: September-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	F(x)	Conditional Probability i			
						1	2	3	4
0	228	.691	.691	219.2	.664				
1	54	.164	.855	73.1	.886	1			
2	33	.100	.955	24.8	.961	.316	1		
3	12	.036	.991	8.5	.987	.089	.283	1	
4	3	.009	1.000	2.9	.995	.018	.057	.203	1

$\bar{x} = 0.509$ $s^2 = 0.777$ $k^* = 0.967$ $p^* = 0.655$ $n = 330$

Goodness of fit: $D_\alpha^n = 0.075$

$|F_o - F(x)| = 0.031$

TABLE 9: October-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	F(x)	Conditional Probability i		
						1	2	3
0	311	.911	.911	307.7	.902			
1	17	.050	.961	24.2	.973	1		
2	9	.026	.987	6.1	.991	.235	1	
3	4	.012	1.000	1.9	.997	.045	.192	1

$\bar{x} = 0.138$ $s^2 = 0.242$ $k^* = 0.182$ $p^* = 0.570$ $n = 341$

Goodness of fit: $D_\alpha^n = 0.074$

$|F_o - F(x)| = 0.011$

TABLE 10: Spring (March, April, May)-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	F(x)	Conditional Probability					
						i					
						1	2	3	4	5	6
0	873	.863	.863	863.6	.853						
1	81	.080	.943	103.7	.956	1					
2	44	.043	.986	29.2	.985	.312	1				
3	9	.009	.995	9.8	.994	.106	.339	1			
4	4	.004	.999	3.5	.998	.035	.113	.335	1		
5	0	.000	.999	1.3	.999	.011	.034	.101	.303	1	
6	1	.001	1.000	.5	1.000	.002	.007	.022	.066	.217	1

$\bar{x} = 0.215$ $s^2 = 0.386$ $k^* = 0.271$ $p^* = 0.557$ $n = 1012$

Goodness of fit: $D^n_\alpha = 0.043$

$|F_o - F(x)| = 0.013$

TABLE 11: Summer (June, July, August)-Negative Binomial Distribution for Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	F(x)	Conditional Probability					
						i					
						1	2	3	4	5	6
0	549	.542	.542	527.8	.522						
1	246	.243	.785	279.6	.798	1					
2	117	.116	.901	122.7	.919	.404	1				
3	67	.066	.967	50.1	.969	.153	.379	1			
4	25	.025	.992	19.7	.988	.054	.133	.352	1		
5	7	.007	.999	7.6	.995	.017	.041	.108	.307	1	
6	1	.001	1.000	2.9	.998	.004	.009	.023	.067	.216	1

$\bar{x} = 0.812$ $s^2 = 1.245$ $k^* = 1.523$ $p^* = 0.652$ $n = 1012$

Goodness of fit: $D^n_\alpha = 0.043$

$|F_o - F(x)| = 0.021$

TABLE 12: Fall (September, October, November)-Negative Binomial Distribution For Thunderstorm Events at Cape Kennedy, Florida

x	f_o	r.f.	F_o	f_e	F(x)	Conditional Probability			
						i			
						1	2	3	4
0	860	.859	.859	845.2	.844				
1	77	.077	.936	109.5	.954	1			
2	45	.045	.981	30.6	.984	.286	1		
3	16	.016	.997	10.1	.994	.080	.281	1	
4	3	.003	1.000	3.6	.998	.017	.058	.208	1

$\bar{x} = 0.227$ $s^2 = 0.397$ $k^* = 0.302$ $p^* = 0.571$ $n = 1001$

Goodness of fit: $D_\alpha^n = 0.043$

$|F_o - F(x)| = 0.018$

REFERENCES

1. Kendall, M. G. and A. Stuart: The Advanced Theory of Statistics, Vol. 1, Hafner Publishing Co., New York, N. Y., 1958.

2. Cohen, A. Clifford, Jr.: "Estimation in the Negative Binomial Distribution," NASA TM X-53372, Marshall Space Flight Center, Alabama, December 21, 1965.

3. Fisher, R. A.: Contributions to Mathematical Statistics, John Wiley and Sons, Inc., New York, N. Y., 1950.

APPROVAL NASA TM X-53816

A PROBABILITY DISTRIBUTION FOR THE NUMBER OF THUNDERSTORM EVENTS AT CAPE KENNEDY, FLORIDA

by Lee W. Falls

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

O. E. Smith
Chief, Terrestrial Environment Branch

W. W. Vaughan
Chief, Aerospace Environment Division

E. D. Geissler
Director, Aero-Astrodynamics Laboratory