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RESONANT MODE ANALYSIS OF THE FITZGERALD APPARATUS

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ABSTRACT

The machine developed by E. R. Fitzgerald to investigate dynamic mechanical properties of solids is analyzed for extraneous resonant modes. Mechanical models are presented to represent several nonideal machine modes; the effect of these modes upon drive tube motion is calculated. Fitzgerald's procedure for data analysis is used to calculate the effect of nonideal modes upon the dynamic compliance. The calculations show resonances in the calculated compliance which are indistinguishable from those reported by Fitzgerald.

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SUMMARY

The machine developed by E. R. Fitzgerald to investigate dynamic mechanical properties of solids is analyzed for extraneous resonant modes. Mechanical models are presented to represent several nonideal machine modes; the effect of these modes upon drive tube motion is calculated. Fitzgerald's procedure for data analysis is used to calculate the effect of nonideal modes upon the dynamic compliance. The calculations show resonances in the calculated compliance which are indistinguishable from those reported by Fitzgerald.

INTRODUCTION

Fitzgerald (refs. 1 and 2) describes an apparatus which he uses to investigate the mechanical properties of solids by measurement of the dynamic shear compliance. The apparatus shown in figure 1, subjects a pair of samples to two mechanical forces: (1) a sinusoidal shearing force of preselected frequency which is applied by means of an aluminum drive tube, and (2) a static compressive force which clamps the samples to both the drive tube and the inertial floating mass. The static clamping force is developed by a screw-driven wedge in the floating mass assembly (see fig. 1). The alternating force is developed by an alternating current flow in coils 1A and 2A; the coils being situated in a radial magnetic field.

Fitzgerald determines the mechanical compliance of the sample by measurement of the electrical impedance of a coil 2A; coil 2A forms on an element of an impedance bridge circuit (refs. 1 and 2). The measured electrical impedance of the coil of wire is altered by the induced voltages that arise from the velocity of the drive tube. In references 1 and 2, Fitzgerald shows how this electrical impedance is measured and how it is related to the mechanical impedance of the moving system.

The mechanical impedance thus obtained by Fitzgerald combines the mechanical impedances of the drive tube, the floating mass, and the samples. In order to determine

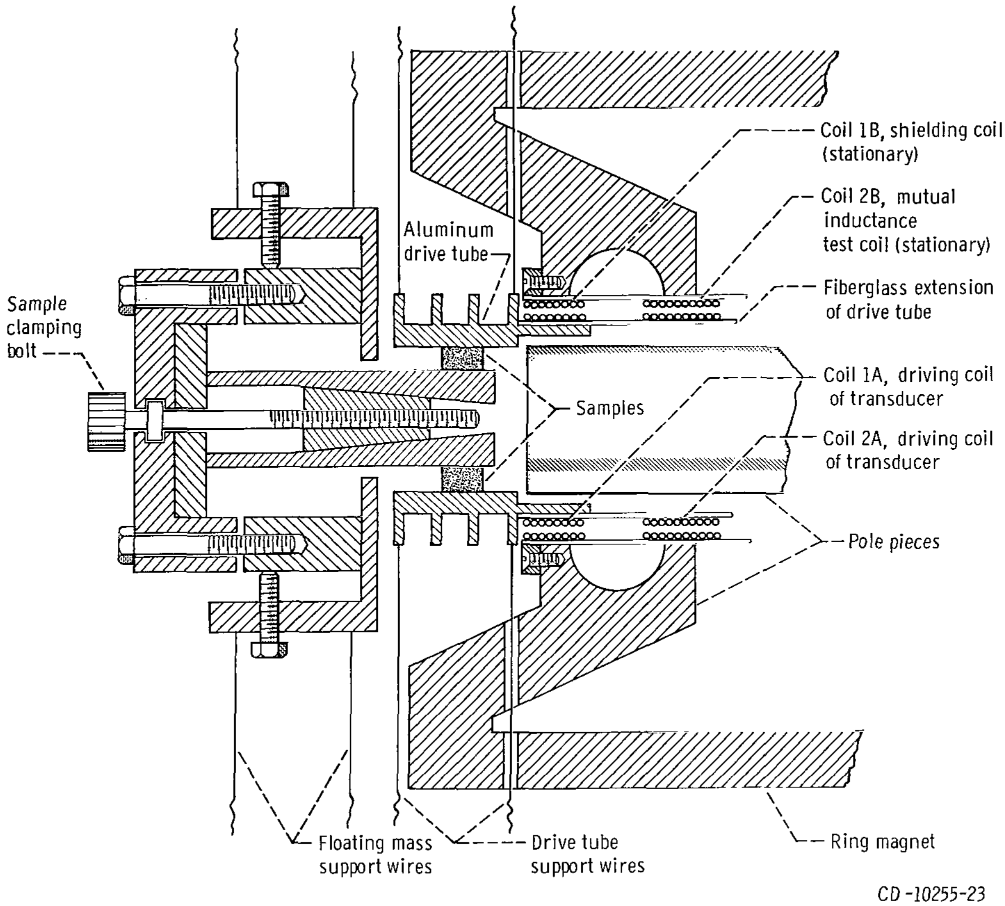


Figure 1. - Simplified view of Fitzgerald apparatus. Note portion of aluminum drive tube that extends into magnetic field.

the mechanical impedances of the samples alone, Fitzgerald resorts to a vector subtraction of mechanical impedances based upon his own mechanical model of the machine. This method requires knowledge of the mechanical impedance of the machine in three basic configurations: (1) the free tube impedance obtained with nothing attached to the drive tube; (2) the clamped tube impedance obtained with the floating mass and drive tube clamped directly together without samples, and (3) the clamped sample impedance (i.e., the mechanical impedance of the system with the drive tube, samples and floating mass all clamped together). These mechanical impedances are determined experimentally; the first two comprise the so-called calibration data for the machine.

The sample mechanical impedance deduced by Fitzgerald is reported in the form of a dynamic shear compliance spectrum in which the vector compliance is plotted against frequency and is resolved into two components: the first component, designated J', is proportional to the energy stored and recovered in one cycle of the shearing motion; the second component, designated J'', is proportional to the energy dissipated in a cycle.

The frequency range of the machine is nominally 50 to 5000 Hz.

The shear compliance spectra reported by Fitzgerald (refs. 3 to 7) and Gotsky and Stearns (ref. 8) for hard crystalline samples show marked differences from compliance values deduced from elastic theory. The three principal differences are as follows:

(1) A low frequency (50 to 500 Hz) in-phase compliance value, which is much larger than elastic theory prediction, suggests that the sample is softer than elastic predictions.

(2) Several resonances appear in the compliance spectra which are both unexpected and unexplained.

(3) The quadrature component of the compliance near large high-frequency resonances (about 3000 Hz) is asymmetric about the resonance frequency and is sometimes negative immediately above the resonance frequency. Fitzgerald states that the negative sample damping implied by a negative loss compliance is the result of a process internal to the sample.

In references 1 and 2, Fitzgerald presents his analysis of the machine. The assumptions contained in that analysis which are pertinent to this report are as follows:

(1) The four sample surfaces which contact the machine (i.e., contact the drive tube and the floating mass) connect with the machine in such a manner as to provide atom-to-atom contact between sample and machine.

(2) The floating mass assembly behaves as rigid mass at all frequencies of measurement.

(3) The samples, drive tube, and floating mass are always in perfect alinement.

(4) The motion of the drive tube is uniaxial and only two ideal machine modes exist; namely, the axial modes produced by the tube mass resonating with the restoring force constant provided by the suspension system and the sample.

Fitzgerald's analysis makes no provision for nonideal machine performance. As a consequence the subtraction method for determining sample properties allows a nonideal machine mode to be interpreted as a sample effect. The purpose of this report is to show the effect of nonideal machine modes on the dynamic shear compliance. The procedure is as follows:

(1) A mechanical model of the machine is presented and several nonideal modes which must exist are defined.

(2) The mechanical model is used to compute the drive tube velocity as a function of frequency.

(3) Using Fitzgerald's analytical method and the calculated drive tube velocity, the dynamic shear compliance is computed as a function of frequency for each type of machine mode.

MECHANICAL MODELS

The Ideal Machine

The mobility analog (ref. 9) is used to construct a mechanical or stick model of the Fitzgerald machine. In the stick model, lines represent rigid massless rods, blocks represent rigid masses, coils represent compliances, and dashpots represent dampers. The mechanical model of the Fitzgerald machine is shown in figure 2 where a switch is used to indicate the three configurations of the machine. With the switch in position 1, the free drive tube configuration is represented. In positions 2 and 3, the clamped tube and clamped sample configurations, respectively, are represented.

The stick model of the machine consists of a sinusoidal constant force generator connected between earth and the drive tube mass. The force generator represents the constant amplitude shearing force produced by the magnetic field and the alternating electric current which passes through the coils wound on the drive tube. The drive tube support system provides the restoring force for axial displacement and is represented by a compliance and damper connected between earth and the drive tube mass. This portion of figure 2 represents the free tube configuration of the machine.

The ideal clamped tube configuration is achieved by providing a rigid attachment of the floating mass to the drive tube. Position 2 of figure 2 represents this idealized condition; the floating mass suspension system is represented by a damper and a compliance connected between the floating mass and earth.

The mechanical model for the ideal clamped sample configuration is shown as position 3 of figure 2. The two samples are represented by a single compliance and a damper connected in parallel; one end of the parallel combination is attached to the drive tube mass, and the other end is attached to the floating mass. The mass of the sample

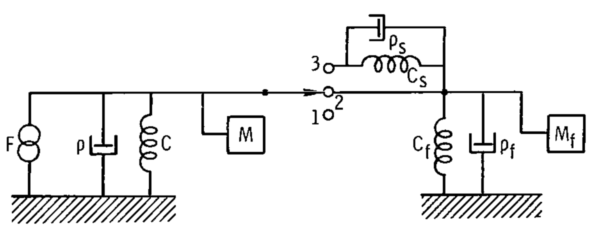


Figure 2. - Mechanical model of ideal Fitzgerald machine where F is constant amplitude sinusoidal force generator; ρ, C, and M are damping, compliance, and mass of drive tube; ρ_s and C_s are damping and compliance of sample; ρ_f, C_f, and M_f are damping, compliance, and mass of floating mass assembly. Three basic machine configurations are available with the switch shown: position 1 is model for free drive tube; position 2 is model for clamped drive tube; position 3 is model for clamped-sample configuration.

is neglected in this model. In this model we assume that the two samples actually used in the machine can be represented by a single sample; in a subsequent section this restriction shall be removed.

The Nonideal Machine

The model presented in figure 2 represents the sample as a compliance with damping. Fitzgerald's analysis assumes that the value of the sample compliance is determined from sample dimensions only and is independent of the static clamping force. A correct analysis of the Fitzgerald machine must recognize that, for hard crystalline samples, the static clamping force greatly alters the value of the effective sample compliance. The effective sample compliance is defined as the reciprocal of the restoring force constant that the sample provides for the drive tube; thus the effective sample compliance is the only compliance measured by the Fitzgerald machine, and it is only indirectly related to the dimensions and elastic properties of the samples.

The effective sample compliance is dependent on static clamping because of surface roughness which exists on the clamping surface of the machine and the loading faces of the samples. It has been shown (ref. 10), that the presence of a small surface roughness on loading surfaces will prevent complete contact of sample and machine so that the effective sample compliance is many times larger than the value calculated assuming perfect contact. If the sample is a metal or other hard crystalline solid, the area of contact between sample and machine will be dependent on static clamping, and the effective sample compliance will decrease with increasing static clamping.

A second assumption by Fitzgerald asserts that, at all frequencies, the floating mass behaves as a rigid body. Figure 1 presents a sketch of the floating mass assembly, from which it is clear that the floating mass is composed of many masses which are bolted together; both the contact region between the different pieces and the bolts themselves act as compliances; the assembly behaves as a collection of masses coupled together with springs. From this it follows that at some frequency a portion of the floating mass will produce an extraneous resonant mode and the assembly will not move as a rigid body. In figure 3 a mechanical model of a floating mass is presented which represents one possible method of nonrigid behavior. In this model the sum of all masses equals the total mass of the floating mass and the two samples are represented by a single compliance and damper. This model assumes a single degree of freedom for the two parts of the floating mass; while this greatly restricts the possible configurations which can be represented, the effect of all such nonrigid modes on the drive tube is expected to be similar.

Another type of mode can result from a nonrigid floating mass. A mechanical model

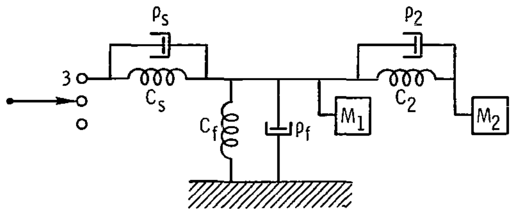


Figure 3. - Mechanical model of nonrigid floating mass mode where ρ_s and C_s are sample damping and compliance, respectively; ρ_f and C_f are damping and compliance properties of floating mass assembly; $M_1 + M_2 = M_f$; ρ_2 and C_2 are damping and compliance for decoupled mass M_2. This model attaches to switch position 3 of figure 1 as an alternative for nonrigid floating mass modes.

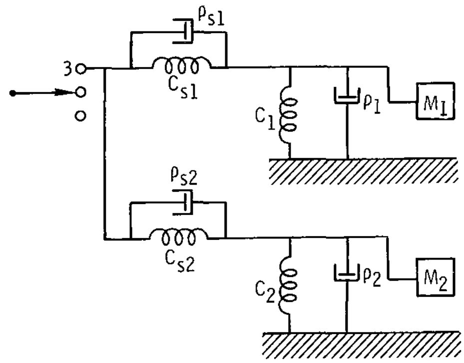


Figure 4. - Mechanical model of nonrigid floating mass mode; two samples represented; ρ_{s1} and C_{s1}, and ρ_{s2} and C_{s2} are damping and compliances for each of two samples explicitly represented by this model; ρ_1, C_1 and M_1 represent floating mass portion connected to sample 1, and ρ_2, C_2 and M_2 represent portion of floating mass connected to sample 2. Physical consistancy requires that $M_1 + M_2 = M_f$ and that $1/C_{s1} + 1/C_{s2} = 1/C_s$ and that $\rho_{s1} + \rho_{s2} = \rho_s$.

presented in figure 4, explicitly represents each of the samples used in the machine with a compliance and damper. Each compliance is attached to the drive tube on one end, and to part of the floating mass on the other. Clearly, if the compliances C_{s1} and C_{s2} and dampers ρ_{s1} and ρ_{s2} are equal (implying equal sample stiffness, damping, and surface roughness) and the floating mass is divided such that $M_1 = M_2$, $C_1 = C_2$, and $\rho_1 = \rho_2$, then the model of figure 4 will reduce identically to the ideal model of figure 2. If, however, the compliances and dampers, which represent the two samples are not equal, or if the floating mass does not decouple into two equal masses, then for some frequencies each sample will execute a motion independent of the other; the drive tube will not move in the ideal manner and a new mode will exist.

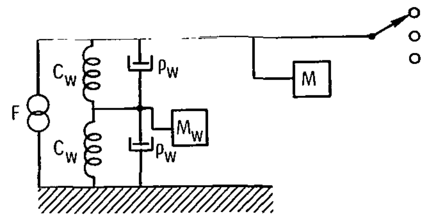


Figure 5. - Mechanical model of drive tube support wire mode where M_W is effective mass of wire and ρ_W and C_W are damping and compliance of one-half the wire length such that $2C_W = C$ and $\rho_W = 2\rho$. Model approximately represents fundamental mode of a support wire.

Another one-dimensional mode present in the Fitzgerald machine is produced by the drive-tube support wires. A mechanical model for a fundamental resonance of a support wire is shown in figure 5; the fundamental mode of the wire is represented by a mass attached by springs, to both the earth and the drive tube. The mass is approximately equal to the mass of the wire (i.e., 0.02 g). Since the drive tube mass is 31.4 grams; the effect of the wire mode is negligible at all frequencies except its resonance frequency. The low damping associated with the wire mode results in large vibration amplitude at resonance and drive tube motion is affected. The band width of these wire modes is about 1 Hz; their principal deleterious effect on the operation of the machine is that they cause nonaxial tube motion which in turn, couples energy into nonaxial machine modes. The higher frequency harmonics of the eight support wires for the drive tube, place nonaxial excitations throughout the frequency range of the machine.

Many nonaxial modes exist in the Fitzgerald machine; these modes arise from bouncing, rocking and/or twisting motions of the drive tube, samples and floating mass collectively and individually. An analysis of drive tube motion in the presence of modes with two or more degrees of freedom is beyond the scope of this work. It is clear, however, that any nonaxial mode which derives its energy of excitation from the moving drive tube, will perturb the motion of the drive tube; since Fitzgerald's analysis interprets any abnormal drive tube motions as arising from sample effects, the nonaxial mode will appear in the reported data in much the same way that other machine modes appear.

CALCULATIONS

Drive Tube Velocity Calculations

The velocity of the drive tube of the Fitzgerald machine is calculated from the mechanical models presented in the preceeding section and from the equation

$$V = FY_m \tag{1}$$

where V, F, and Y_m are complex quantities designating the drive tube velocity, the sinusoidal force applied to the drive tube, and the mechanical admittance of the moving system, respectively. The mechanical admittance Y for the ideal free tube model of figure 1 is given by:

$$Y = \left[\rho + i\left(\omega M - \frac{1}{\omega C}\right)\right]^{-1} \tag{2}$$

where M, C, and ρ are the mass, compliance, and damping of the free drive tube, respectively; ω is 2π times the frequency. Knowing the resonant frequency of the free drive tube $\omega_0/2\pi$, the compliance of the support system can be calculated from the relation

$$C = \left(\omega_0^2 M\right)^{-1} \tag{3}$$

In an analogous manner, the admittance of the ideal floating mass Y_f can be written as

$$Y_f = \left[\rho_f + i\left(\omega M_f - \frac{1}{\omega C_f}\right)\right]^{-1} \tag{4}$$

where M_f, ρ_f, and C_f are the mass, damping, and compliance, respectively, of the floating mass assembly. The mechanical admittance of the entire moving system for the clamped tube configuration is Y_{ct}; thus,

$$Y_{ct} = \frac{YY_f}{Y + Y_f} \tag{5}$$

The mechanical admittance of the sample depicted in the model of figure 1 is Y_s, and is given by

$$Y_s = \left(\rho_s - i\frac{1}{\omega C_s}\right)^{-1} \tag{6}$$

where ρ_s and C_s denote sample damping and compliance, respectively. The mechanical admittance for the entire moving system for the clamped sample configuration is Y_{cs}, and is given by

$$Y_{cs} = \frac{Y(Y_s + Y_f)}{Y + Y_s + Y_f} \tag{7}$$

The mechanical admittances for the other mechanical models are derived in like manner. By equation (1), the drive tube velocity is proportional to the mechanical admittance of the moving system as viewed from the drive tube. In order that the calculated drive tube velocity appear similar to the actual drive tube velocity measured experimentally, it is necessary to add a small constant "noise" term to the calculated drive tube velocity. This noise component reflects the limit of resolution of the Fitzgerald bridge measurement method. Because of the large change in magnitude of drive tube velocity with frequency, the results are displayed as a plot of the logarithm of drive tube velocity against frequency; this display is referred to as a velocity plot. The values of machine parameters used in these calculations are values obtained on the NASA Fitzgerald machine; these values are summarized as follows:

Drive tube mass (with coils), g	31.4
Floating mass, g	2500
Free drive tube resonance frequency, Hz	16.8
Shorted-turn constant, L_c/R_c, sec	1.49×10^{-5}
Transducer magnetic-force constant, ohms-dynes-sec/cm	1.73×10^{5}
Geometric constant, K, cm	3.49

Compliance Calculations

The calculated drive tube velocity for a clamped sample can be used to calculate a mechanical shear compliance; the procedure followed is that of Fitzgerald (refs. 1 and 2). From equation (11) of reference 1, the dynamic shear compliance J^*, is given by

$$J^* = J' - iJ'' = -i \cdot K \cdot \frac{Y_m^*}{\omega} \tag{8}$$

where K is a geometric constant of the sample, J' is the component of compliance that is in-phase with the applied shearing force, and J'' is the quadrature component of compliance; Y_m^* is the mechanical admittance of the sample as deduced by Fitzgerald's analysis and is given as (ref. 1)

$$Y_m^* = \frac{Y_m Y}{Y - Y_m} - Y_f \tag{9}$$

where Y_m is the mechanical admittance of the entire moving system (which from our point of view may include effects due to nonideal machine modes), and Y and Y_f are the mechanical admittances of the ideal free tube and ideal floating mass, respectively, as defined in equations (2) and (4). It is worthy of note that if the mechanical admittance of the moving system Y_m is exactly equal to the mechanical admittance of the clamped sample model Y_{cs} (eq. (7)), then the deduced sample mechanical admittance Y_m^* will exactly equal the mechanical admittance of the ideal sample Y_s (eq. (4)). Equation (9) shows how nonideal machine modes enter into what Fitzgerald calls the dynamic shear compliance of the sample.

The calculated compliance data are presented in terms of the resolved components J' and J'' given in equation (8); the components are plotted against frequency to give what is called a compliance spectrum.

The Shorted Turn

A principal defect in the design of the Fitzgerald machine is the presence of a thin aluminum strip directly beneath the current carrying coil of the drive tube. This forms a shorted turn on the drive tube; the shorted turn moves in the radial magnetic field of the machine. The eddy currents which flow in this shorted turn produce an additional component of force on the drive tube; the magnitude and phase of the force component are frequency dependent. This problem has been analyzed in detail in reference 10, wherein it is shown that the effect of the shorted turn is to produce a mixing of the storage and loss compliances according to the relations

$$J_F' = \frac{J' - J''\left(\dfrac{\omega L_c}{R_c}\right)}{1 + \left(\dfrac{\omega L_c}{R_c}\right)^2} \tag{10}$$

$$J''_F = \frac{J'' + J'\left(\frac{\omega L_c}{R_c}\right)}{1 + \left(\frac{\omega L_c}{R_c}\right)^2} \tag{11}$$

where J'_F and J''_F are the compliances measured by Fitzgerald in the presence of a shorted turn, of inductance L_c and resistance R_c; J' and J'' are the correct compliances that would be measured with a machine that did not have a shorted turn on the drive tube; and ω is the angular frequency. From this relation it is clear that the effect of the shorted turn will be most obvious near a high frequency resonance, since J'' is small elsewhere and J' is large both positive and negative near a resonance. These equations will be used to calculate the effect of the shorted turn on a high frequency resonance.

RESULTS

The Ideal Machine Model

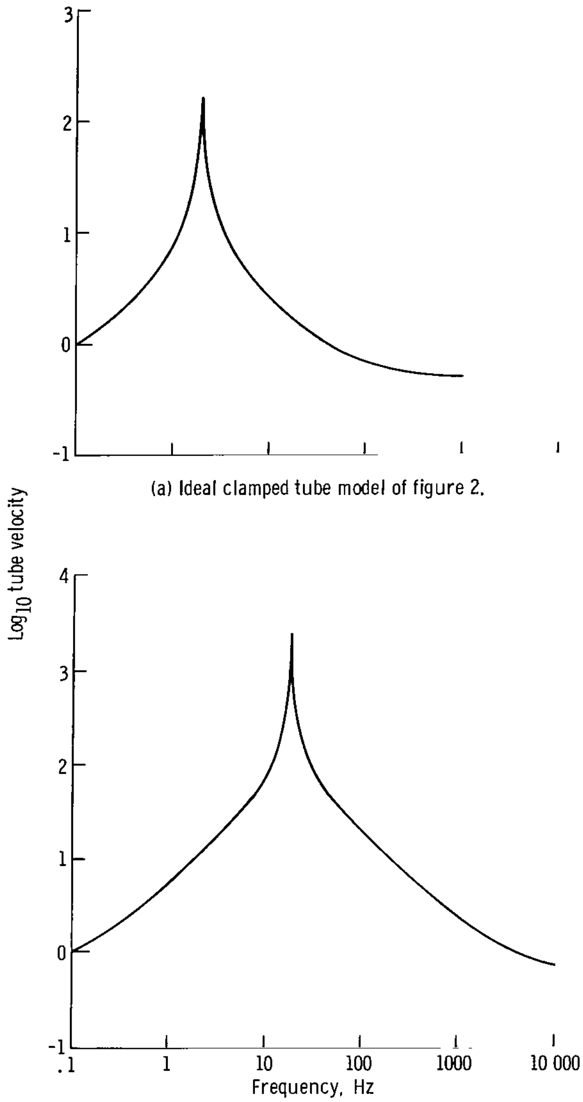
The calculated drive tube velocity for the ideal free tube is presented in figure 6 along with the velocity plot for the clamped tube model of figure 1. A single mode exists for each of these ideal models; resonant frequencies are 16.8 and 2.1 Hz for the free tube and clamped tube models respectively (these are the experimentally observed values).

The ideal clamped-sample model of figure 2 (switch position 3) has two resonant frequencies; one is the same as the clamped tube resonance, the other is determined approximately by the relation

$$\omega_s = (C_s \cdot M)^{-1/2} \tag{12}$$

where $\omega_s/2\pi$ is the system resonance frequency and C_s and M are defined in equations (6) and (2), respectively. The value of ω_s is usually more than 100 times larger than ω_0 defined by equation (3).

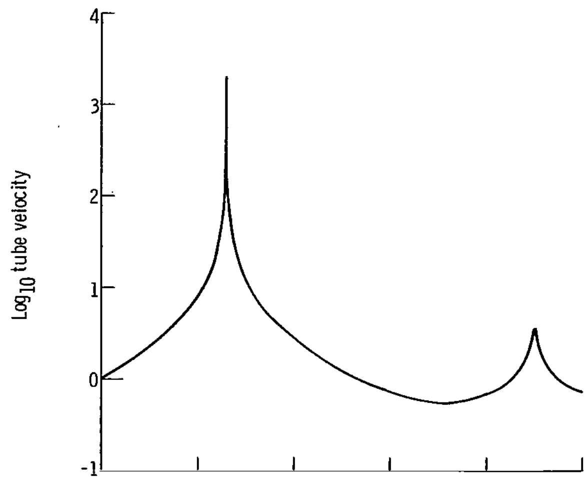
The velocity plot for the ideal clamped-sample model is shown in figure 7, along with the compliance spectrum for this mode. The real component of compliance J', is equal to the sample compliance deduced from equation (12); in this figure the system



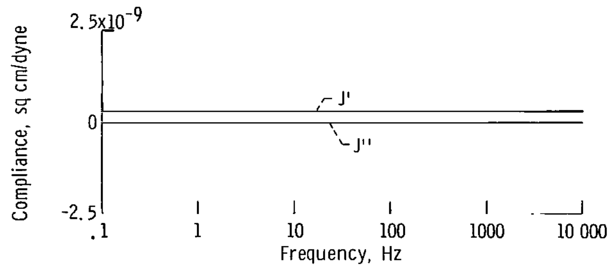
(a) Ideal clamped tube model of figure 2.

(b) Ideal free drive tube model of figure 2.

Figure 6. - Log drive tube velocity as function of frequency.



(a) Log drive tube velocity as function of frequency.



(b) Storage compliance J' and loss compliance J''.

Figure 7. - Ideal clamped-sample model of figure 2.

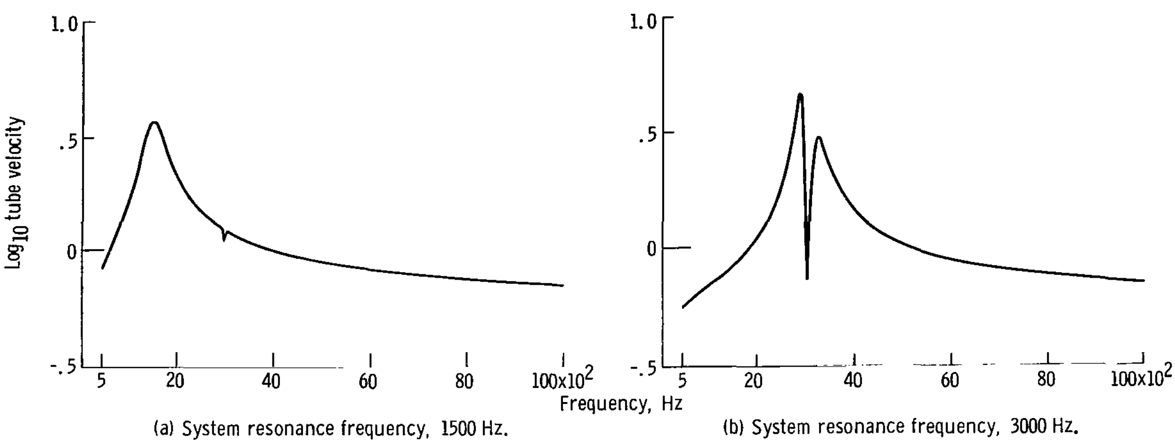
resonance frequency is 3000 Hz. The velocity plot shows the two machine modes which Fitzgerald's analysis assumes; one located at 2.1 Hz, the other at about 3000 Hz.

The Nonideal Machine Models

The calculated drive tube velocity for the nonrigid floating mass model of figure 3 is presented in figure 8; three values of system resonance frequency $\omega_s/2\pi$ are used to display the interaction between mode frequency (here 3000 Hz) and drive tube velocity. Figure 9 presents the compliance spectrum of this nonideal model.

The calculated drive tube velocity for the two-sample nonrigid mass model of figure 4 is given in figure 10, along with the compliance calculation for the same model.

The drive tube velocity calculated for the fundamental resonance of a support wire (represented by the model of fig. 5) is presented in figure 11 for two different system



(a) System resonance frequency, 1500 Hz.

(b) System resonance frequency, 3000 Hz.

(c) System resonance frequency, 6000 Hz.

Figure 8. - Velocity plots for floating mass model of figure 3. Floating mass mode occurs at 3000 Hz in each figure; system resonance frequency $w_s/2\pi$ is varied from 1500 to 6000 Hz.

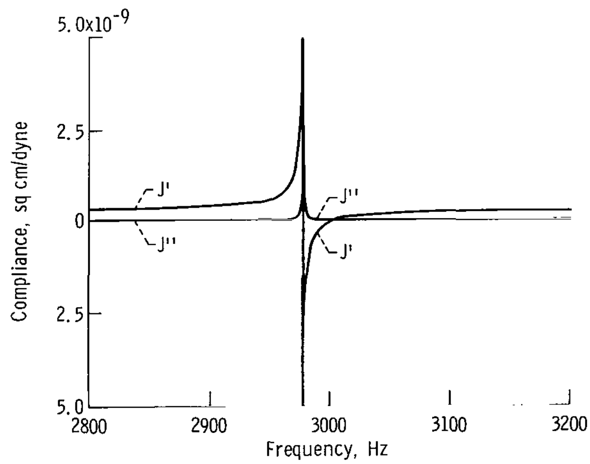
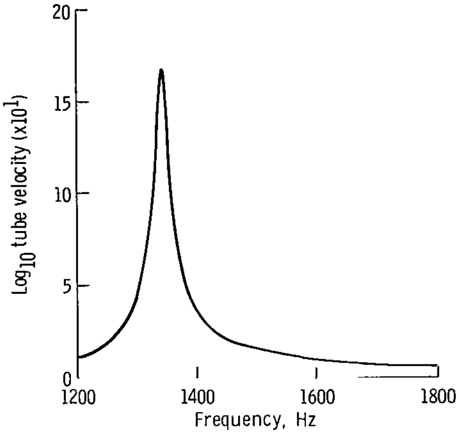
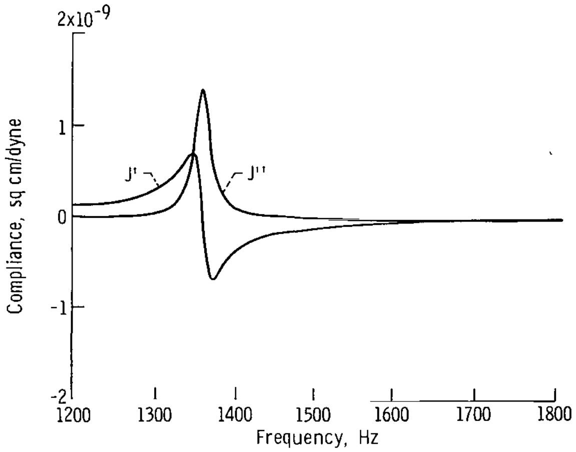


Figure 9. - Calculated compliance for floating mass mode of figure 3. Storage compliance J' and loss compliance J'' are plotted against frequency. System resonance frequency at 3000 Hz.

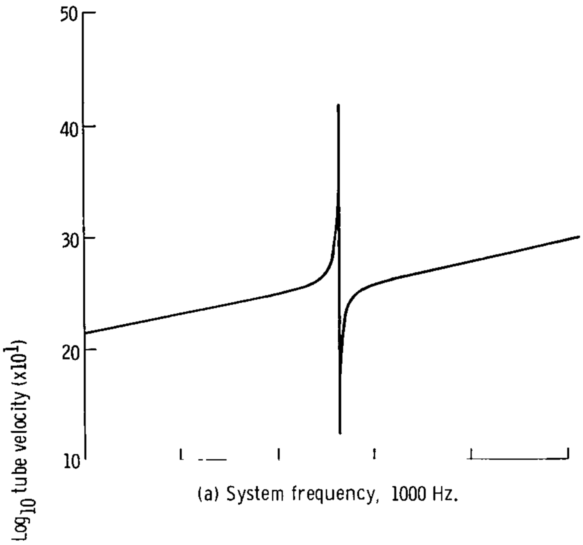


(a) Velocity plot; M_1 is 1500 grams and M_2 is 1000 grams.

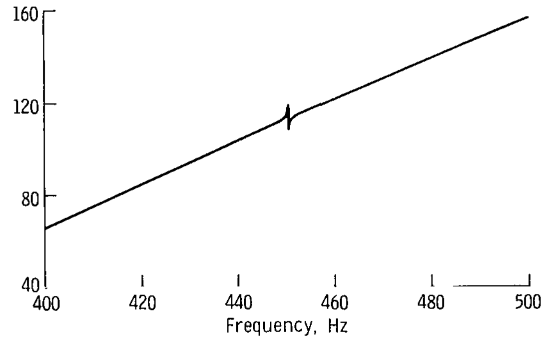


(b) Compliance calculation; storage compliance is J' and loss compliance is J''.

Figure 10. - Two-sample model of figure 4.



(a) System frequency, 1000 Hz.



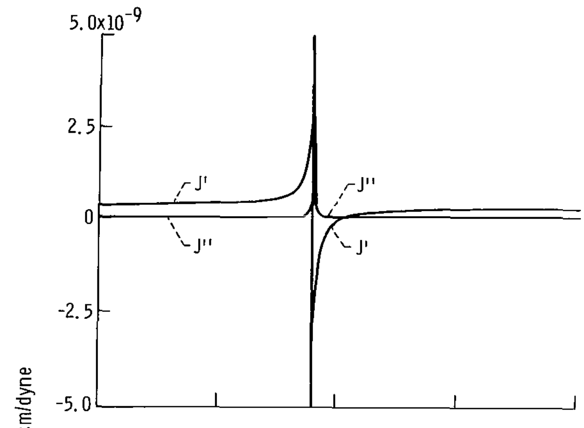
(b) System frequency, 3000 Hz.

Figure 11. - Support wire model. Wire mode resonant frequency, 450 Hz; system resonance frequency, varied from 1000 to 3000 Hz to show effect of drive tube velocity on size of mode in velocity plot; M_W is taken to be 0.1 gram.

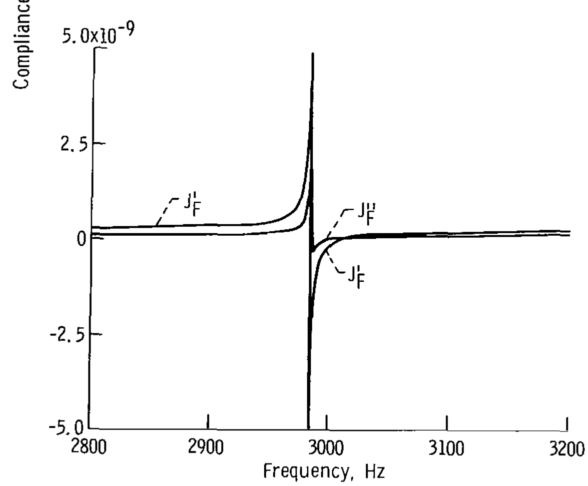
resonance frequencies. The frequency scale is expanded to show the resonance in greater detail.

The Shorted Turn

The floating mass mode shown in figure 3 is used to obtain a normal compliance spectrum for this mode. The result is shown in figure 12 along with a spectrum of the mode as seen with a machine which has a shorted turn on the driving tube. The mixing is as prescribed by equations (10) and (11).



(a) Compliance resonance without shorted turn on drive tube.



(b) Compliance resonance with shorted turn on drive tube.

Figure 12. - Comparison of compliance resonance with and without mixing effects of shorted turn on drive tube.

DISCUSSION

In order to understand the significance of Fitzgerald's data it is necessary to have a clear understanding of the role of the sample in the machine. In the models presented above, the sample is represented by a massless spring with damping, which connects the drive tube to the floating mass. The mass of the sample can be ignored in comparison to the drive tube mass since all sample information is deduced from velocity changes of the drive tube. As is shown in the models for a support wire (see fig. 11) the perturbation of the drive tube velocity is confined to a very narrow frequency range if the mass associated with the mode is small. Thus the sample mass would have to be an appreciable fraction of the drive tube mass before any significant modes could result involving the mass of the sample.

The effective sample compliance for hard sample materials is determined primarily from considerations of surface roughness and applied static load; increased static loading increases the area of contact between sample and machine by deforming surface roughness and the effective sample compliance decreases. Fitzgerald (refs. 3 and 5 to 7) and Gotsky and Stearns (ref. 8) observe that the measured compliance decreases with increased static clamping and with the passage of time; both observations would be expected from a surface roughness model. It would appear that the large value of low frequency compliance measured by Fitzgerald is simply the compliance of the surface roughness region.

Other aspects of Fitzgerald's observations can be understood in the light of the results presented herein. The effect of the shorted turn of the drive tube on the observed compliance has been shown. The origin of the negative values for the loss compliance J''_F, is the mixing of the correct compliances J' and J'' which is brought about by the eddy currents in the shorted turn.

Fitzgerald (refs. 3 and 5 to 7) and Gotsky and Stearns (ref. 8) observe that some compliance resonances increase in frequency if static clamping is increased. Resonant modes in the floating mass have compliances which are produced by the surface roughness of mating pieces of the floating mass assembly. Increasing the static clamping of the sample will most certainly increase the force on these mating surfaces and thus change the area of contact between two pieces. Increased area of contact results in decreased compliance for that particular mode and the mode resonance frequency increases. The highly nonlinear character of stiffness associated with the contact region surface roughness can cause certain modes to increase in resonance frequency beyond the range of measurement for sufficiently large applied force.

The mechanical models presented are deduced from an inspection of the mechanical constitution of the Fitzgerald machine. Consequently, it is certain that these modes exist in the Fitzgerald machine; the only uncertainty being the resonant frequency of a particular mode. The resonant frequencies of some machine modes are known; fundamental modes of support wires 300 to 600 Hz; free drive tube 16.8 Hz; clamped tube 2.1 Hz. In other cases, primarily where the compliance of the mode is determined from a surface roughness condition, we choose the mode compliance so as to place the resonance at some convenient position in the operating range of the machine. Damping values are determined by a comparison of calculated results and experimental data; the experimental velocity plot for the free tube configuration of the NASA machine is used to obtain drive tube suspension damping; loss compliance data of Fitzgerald is used to obtain damping values for the sample and nonideal modes.

While only one-dimensional models have been employed in this study, it is expected that more complex models will produce similar effects on the drive tube velocity plot and therefore produce similar effects in the compliance calculation.

CONCLUSIONS

The origin of the anomalous high compliance at low frequencies, characteristic of Fitzgerald's results, is shown to arise from the high compliance of the surface roughness of the contact region of sample and machine. The effect of the shorted turn on a compliance resonance of sufficient size and frequency is shown to produce the negative values for the loss compliance characteristic of Fitzgerald's data.

The mechanical models presented above together with supporting calculations show that the many resonances, both large and small, reported by Fitzgerald can be produced by the machine itself; this can be the case even if the sample behaves in an ideal manner (i.e., as a damped spring). Thus it appears that if the machine does indeed execute nonideal motion over its range of operation, then the machine itself must be considered the principal source of the unusual effects reported by Fitzgerald.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 27, 1968,
129-03-15-01-22.

APPENDIX - SYMBOLS

C	compliance of drive tube suspension
C_f	compliance of floating mass suspension
C_s	compliance of sample
C_w	compliance of support wire in model
C_1	compliance of part of floating mass suspension
C_2	compliance of part of floating mass suspension
F	sinusoidal force applied to drive tube
J'	storage compliance without shorted-turn effects
J''	loss compliance without shorted-turn effects
J'_F	storage compliance with shorted-turn effects
J''_F	loss compliance with shorted-turn effects
J^*	complex compliance
K	geometric constant of sample
L_c	inductance of shorted turn on drive tube
M	mass of drive tube
M_f	mass of floating mass
M_w	mass of wire in model
M_1	mass of part of floating mass
M_2	mass of part of floating mass
R_c	electrical resistance of shorted turn on drive tube
V	complex drive tube velocity
Y	mechanical admittance of free drive tube
Y_{cs}	mechanical admittance of clamped-sample configuration
Y_{ct}	mechanical admittance of clamped-tube configuration
Y_f	mechanical admittance of ideal floating mass
Y_m	generalized complex mechanical admittance
Y_m^*	mechanical admittance of sample as deduced by Fitzgerald's analysis
Y_s	mechanical admittance of ideal sample
ρ	mechanical damping of ideal free tube
ρ_f	mechanical damping of floating mass
ρ_s	mechanical damping of sample
ρ_w	mechanical damping of part of support wire
ρ_1, ρ_2	mechanical damping of portion of floating mass
ω	2π times frequency
ω_s	2π times frequency of system resonance
ω_0	2π times resonant frequency of free tube

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