NASA TECHNICAL MEMORANDUM

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OPEX-II, A RADIATION SHIELD OPTIMIZATION CODE

by Gerald P. Lahti

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Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1969

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ABSTRACT

A radiation shield optimization procedure based on the computer code, OPEX-II, is described. The OPEX-II code, based on an earlier, steepest-descent-method code OPEX, has been recoded to improve coding, simplify data input, use spherical geometry, and alter the dose-thickness relation when a layer has been removed. A complete description of how to obtain the necessary input data for OPEX-II from other transport calculations is given. Data input instructions, FORTRAN IV code listing, and a sample problem optimizing a seven-layer shield of tungsten and lithium hydride for a space power reactor are given.

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SUMMARY

A radiation shield optimization procedure based on the computer code, OPEX-II, is described. The OPEX-II code, based on an earlier, steepest-descent-method code OPEN, has been recoded to improve coding, simplify data input, use spherical geometry, and alter the dose-thickness relation when a layer has been removed. A complete description of how to obtain the necessary input data for OPEX-II from other transport calculations is given. Data input instructions, FORTRAN IV code listing, and a sample problem optimizing a seven layer shield of tungsten and lithium hydride for a space power reactor are given.

INTRODUCTION

The radiation shield designer is faced with the task of selecting shield materials and material arrangements which will not only provide adequate protection against radiation but will also minimize shield weight, cost, or space. If components of the total dose are independent of one another (i.e., primary gammas, and fission neutrons), and the geometry is simple, analytic expressions for the radiation dose may be defined and a closed form solution for minimum weight may be obtained. Reference 1 reviews such cases.

When radiation groups are not independent, which is generally the case for secondary gammas generated by neutron absorptions and inelastic scatters throughout the shield, closed form solutions are no longer attainable so numerical iterative methods must be employed. Two of these numerical, iterative optimization techniques are (1) the method of Lagrange multipliers (as applied to the shield weight optimization problem in ref. 2) and (2) the method of steepest descent (as applied to the shield weight optimization problem in ref. 3).

In both methods, an empirical analytical expression (hereinafter called the dose-thickness relation) is assumed which relates the radiation dose at some reference detector point to all thicknesses of material present. The parameters in this empirical expression are obtained by fitting them to some accurate detailed radiation transport calculations of dose for a given base configuration and perturbations of that configuration. The geometry and thicknesses of material determine weight and derivative of weight with respect to thickness. With first derivative of weight and dose with respect to thickness as determined from the dose-thickness relation, the optimization procedure alters the base configuration to obtain a set of thicknesses corresponding to a minimum weight configuration (or at least a local minimum) for some dose constraint. Optimization codes do not select materials but merely alter initial configurations. Optimization codes may eliminate layers but they never add layers. The optimum weight estimate is only as good as the parameters which describe changes of dose with thickness. A final, detailed proof calculation is necessary to confirm the predictions of the optimization code.

The steepest-descent method of reference 3 was incorporated in a rudimentary computer program called OPEX (ref. 4) but was limited to slab geometry. This report describes a revision of OPEX, called OPEX-II, and how it is applied to a radiation shield optimization. The basic steepest-descent method of reference 3 has been maintained, but the code OPEX has been completely rewritten to improve coding, simplify data input, use spherical geometry, expand the output, and alter the dose-thickness relation when a layer has been removed by the optimization code. In this report, a complete description of how to obtain the necessary input data for OPEX-II from other transport calculations is given. Data input instructions, FORTRAN IV code listing, and a sample problem optimizing a seven-layer shield of tungsten and lithium hydride for a space power reactor are given.

DOSE-THICKNESS RELATION

The total radiation dose rate D at some reference point in space is defined, for purposes of the optimization procedure, to be

$$D = \sum_{i}^{IMAX} D_i$$

where

D total dose rate

D_i i^{th} component of total dose rate (e.g., dose due to capture gammas from first shield layer or dose due to inelastic gammas from last shield layer, etc.)

IMAX number of components of dose

Each dose component D_i is further assumed to be of the form

$$D_i = C_i \exp\left(-\sum_{j=1}^{NREG} \mu_{ij}\, t_j\right) \qquad (2)$$

where

C_i fitted parameter

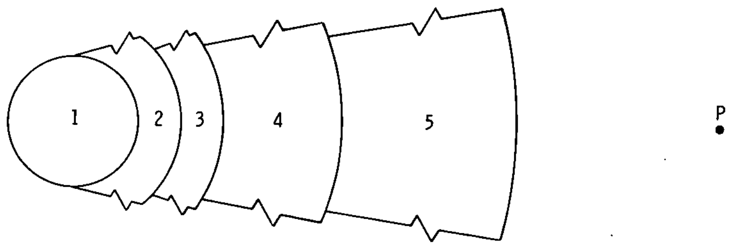
t_j thickness of j^{th} region

μ_{ij} "attenuation coefficient" which describes effect of change in thickness t_j on D_i

NREG number of regions present

The i^{th} dose component need not be associated with the j^{th} region. There may be more than one contribution to the total dose from a given layer. Other layers may contribute negligible secondary gamma dose and their dose contribution is omitted. Generally because of differences in formation rate and gamma attenuation, a given high-Z shield region will have one dose component due to capture gammas and a second due to inelastic gammas. The core will have primary neutrons and primary gammas. A hydrogeneous layer may have negligible dose.

For example, consider the spherical reactor-shield described in the following sketch and table with a dose constraint at point P:



(a)

Region j	Description	Radiation dose component i	Description
1	Reactor core	1 2	Core neutron Core gamma
2	Reflector	3 4	Capture gamma Inelastic gamma
3	Lithium hydride	---	Negligible dose
4	Tungsten shield	5 6	Capture gamma Inelastic gamma
5	Lithium hydride	---	Negligible dose

In this example, $\mu_{2,4}$, $\mu_{3,4}$, and $\mu_{4,4}$ represent the attenuation of gamma rays by the tungsten layer for the various gamma sources in the core and layers between the core and the tungsten. The coefficient $\mu_{1,4}$ represents the attenuation by the tungsten of neutron dose due to neutrons born in the core. In contrast, $\mu_{5,3}$ represents the attenuation by lithium hydride (layer 3) of neutrons which give rise to secondary gamma sources in the tungsten (layer 4).

For coefficients such as $\mu_{5,4}$, describing the effect of a thickness on its own source strength, equation (2) becomes inadequate particularly when the thickness of the layer goes to zero. OPEX-II does, however, set $C_i = 0$ if a particular layer is eliminated by the code. At this point, however, the attenuation coefficients μ_{ij} should be recalculated for the new configuration. Regions of origin of the various dose components are required data input to the code to facilitate this operation.

FITTING PARAMETERS TO DOSE-THICKNESS RELATION

The coefficients of equation (2) are obtained as precisely as possible by performing a series of transport calculations. A starting, base configuration is selected, preferably as close to an optimum configuration as possible based on past experience. With the transport calculation, such as a discrete ordinates calculation, the individual dose components D_i are evaluated. Each layer of the base configuration to be altered by the optimization code is then systematically increased by a nominal amount (say, 1 cm) and the dose components are reevaluated with the transport code. From equation (2), then,

$$\left.\begin{aligned} D_i(t_1, t_2, \ldots t_j, \ldots, t_{NREG}) &= C_i \exp\left(-\sum_{j=1}^{NREG} \mu_{ij} t_j\right) \\ D_i(t_1, t_2, \ldots t_j + \Delta t_j, \ldots, t_{NREG}) &= C_i \exp\left(-\sum_{j=1}^{NREG} \mu_{ij} t_j\right) \exp(-\mu_{ij}\Delta t_j) \end{aligned}\right\}$$

Solving the above pair of equations for μ_{ij} results in

$$\mu_{ij} = \frac{1}{\Delta t_j} \ln\left[\frac{D_i(t_1, t_2, \ldots, t_j, \ldots t_{NREG})}{D_i(t_1, t_2, \ldots, t_j + \Delta t_j, \ldots t_{NREG})}\right] \tag{3}$$

The coefficients μ_{ij} are determined for all i and j in this manner.

The basic configuration data, that is, the set of thicknesses t_j, dose components D_i, and "attenuation coefficients" μ_{ij} constitute the required input data. For the set of base configuration, then, the coefficient C_i is calculated by the code OPEX-II from equation (2); that is,

$$C_i = D_i \exp\left(\sum_{j=1}^{NREG} \mu_{ij} t_j\right)$$

WEIGHT-MINIMIZATION PROCEDURE

The procedure for obtaining the minimum weight configuration by the steepest-descent method is presented in this section. The equations are basically from references 3 and 4. The narrative is expanded to illustrate the flow of computation and to comment on code output.

The mathematical problem to be solved is that of minimizing the weight w, a function of thicknesses t_j, while constraining the total dose D to some particular value; that is,

$$\text{Minimize } w(t_1, t_2, \ldots, t_{NREG})$$

$$\text{with constraints (a) } D = \sum_i D_i = \sum_i C_i \exp\left(-\sum_j \mu_{ij} t_j\right) = \text{constant}$$

$$\text{(b) } t_j \geq 0$$

$$\text{(c) } t_\ell = \text{constant for any desired values of } \ell$$

Constraint (b) ensures a physical solution. The optional constraint (c) is useful if it is desired that some thicknesses be kept from changing during the computation (e.g., the reactor core and reflector thicknesses). Constraint (c) is necessary for the spherical geometry programmed presently to prevent the trivial case of reducing reactor core size to zero.

An n-dimensional Euclidian vector space with Cartesian coordinates $t_1, t_2, \ldots t_{NREG}$ is defined. The following vectors are defined on this space:

$$\bar{t} \equiv (t_1, t_2, \ldots t_{NREG})$$

$$\bar{g} \equiv \left(\frac{\partial w}{\partial t_1}, \frac{\partial w}{\partial t_2}, \ldots, \frac{\partial w}{\partial t_{NREG}}\right)$$

$$\bar{a} \equiv \left(\frac{\partial D}{\partial t_1}, \frac{\partial D}{\partial t_2}, \ldots, \frac{\partial D}{\partial t_{NREG}}\right)$$

The notation $\bar{t} = (t_1, t_2, \ldots t_{NREG})$ means $t_1\hat{X}_1 + t_2\hat{X}_2 + \ldots$ where $\hat{X}_i$ are unit vectors in the i^{th} direction. Vectors $\bar{g}$ and $\bar{a}$ represent the gradient of weight and dose. The components of $\bar{g}$ are evaluated from analytic expressions of weight as a function of thickness, and depend on geometry. The components of $\bar{a}$ are evaluated from the partial derivatives of (1) and (2), namely,

$$\frac{\partial D}{\partial t_k} = \sum_{i=1}^{IMAX} \frac{\partial D_i}{\partial t_k} = \sum_{i=1}^{IMAX} \frac{\partial}{\partial t_k}\left[C_i \exp\left(-\sum_{j=1}^{NREG} \mu_{ij}t_j\right)\right]$$

$$= \sum_{i=1}^{IMAX} (-\mu_{ik})C_i \exp\left(-\sum_{j=1}^{NREG} \mu_{ij}t_j\right)$$

$$= \sum_{i=1}^{IMAX} \mu_{ik}D_i$$

The unit vector $\hat{u}$ (see ref. 3 for derivation)

$$\hat{u} = \frac{-\bar{g} - \left(\frac{\bar{a} \cdot \bar{g}}{\bar{a} \cdot \bar{a}}\right)\bar{a}}{\left[\bar{g} \cdot \bar{a} - \frac{(\bar{a} \cdot \bar{g})^2}{\bar{a} \cdot \bar{a}}\right]^{1/2}}$$

points in the direction of greatest weight decrease (steepest descent) along a hyperplane tangent to the hypersurface described by the equation

$$D(\bar{X}) = D(t_1, t_2, \ldots t_{NREG}) = \text{Constant}$$

Components of $\hat{u}$, namely u_j, represent increments of thickness to be added to each t_j to approach the minimum weight criterion.

The optimization code proceeds as follows:

(1) A fraction f of each component u_j of $\hat{u}$ is added to each thickness t_j. The fraction f is an input parameter. (A value of $f = 1.0$ has given satisfactory results.)

(2) The new set of thicknesses generally does not return the correct dose constraint so a first-order correction is applied to each t_j to return the dose constraint. This correction is

$$\bar{t}_{new} = \bar{t}_{old} + \left[D(\text{constraint}) - D(\text{calc})\right] \times \frac{\bar{a}}{(\bar{a} \cdot \bar{a})}$$

Steps 1 and 2 are repeated until the relative change in weight from one iteration to the next is less than some prescribed value.

The code output includes final thicknesses and individual dose components as calculated from the dose-thickness relation for each iteration. It is incumbent on the user to make a final detailed proof calculation to verify the results of the prediction of the optimization code. Experiences have indicated that if input coefficients are determined accurately, final predictions of the optimization code are quite good provided the configuration is not radically changed If the configuration is changed severely, a recalculation of coefficients is in order.

SOME OTHER USES OF OPTIMIZATION CODE

Once an optimized base case is obtained, effects of nominal changes in reactor size, power level, and dose constraint on shield weight may be estimated using the optimization code. This is done by altering reactor radius, by scaling dose components proportionately, or by specifying a different dose constraint, respectively, and allowing OPEX-II to seek a new minimum weight configuration.

Cost optimization (minimization) may also be performed by specifying cost per unit volume rather than density (weight per unit volume) for each region.

Input information to the code requires knowledge of each component of the dose. Output information also includes contribution of each component to the total dose. Thus one learns which regions are important and most sensitive to the calculation.

OPEX-II CODE

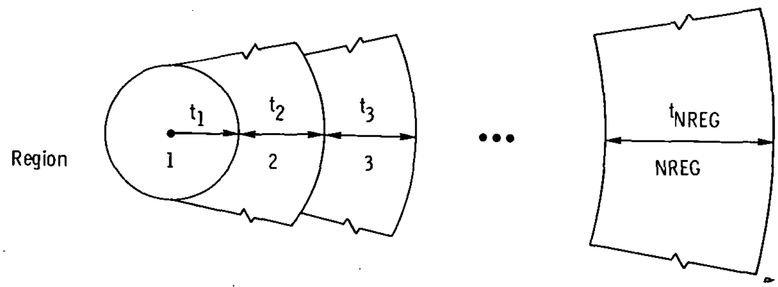
In this section the code details are presented, including

(1) Flow chart for data input

(2) FORTRAN IV listing

(3) Sample problem and sample problem output

The geometry presently coded is spherical and is illustrated in sketch (b)



(b)

The data input consists of a set of thicknesses for each region. Radii are calculated internally. A thickness must be specified for each region.

Flow Chart for Data Input

1 Control card

FORMAT (3I5, 5F10.4)

Card column	Variable	
1-5	NREG	Number of regions (thicknesses) in this problem (≤ 25)
6-10	MAX	Maximum number of iterations allowed
11-15	IMAX	Number of items in dose equation (≤ 25)
16-25	DES	Desired dose rate (used only if > 0.0)
26-35	EPS	Convergence criterion for weight
36-45	EPSD	Convergence criterion for initial dose (EPS, EPSD typically 0.001)
46-55	CØN	Fractional step size on $\hat{u}$ ($0.5 \leq CON \leq 1.0$)
56-65	CA	Fractional step size for initial dose ($0.5 \leq CA \leq 1.0$)

2 T(J) Thickness of j^{th} region; NREG values required. FORMAT (7E10.4)

3 RHØ(J) Density of j^{th} region; NREG values required. FORMAT (7E10.4)

4 NB(J) Thickness constraint flag FORMAT (25I1)
NB(J) = 0 constrain j^{th} region to constant thickness
NB(J) = 1 allow j^{th} region to change; NREG values required.

5 — μ_{ij} — FORMAT (7E10.4)

For each region j read a new card (or set of) cards with IMAX values of μ_{ij}. NREG cards (or sets of cards) required.

6 — D(i) — FORMAT (7E10.4)

i^{th} dose component corresponding to initial geometry read in above; IMAX values required.

7 — NIJ(I) — FORMAT (14I5)

NIJ(I) is the region number physically associated with the i^{th} dose term. This cross referencing is necessary to zero the correct C(I) in the event the j^{th} region is diminished to zero thickness by the optimization process. A region may be identified more than once. IMAX values required.

Perform calculation

Go to 1

FORTRAN IV Listing

Presented in this section is the IBM-7094-II FORTRAN IV listing for the OPEX-II code. SUBROUTINE WEIGHT is coded for spherical geometry. Indication of how to alter this subroutine for the geometry of slabs bounded by a conical surface is given in that subroutine. The code can handle up to 25 regions and dose components as presently dimensioned. Running times are typically less than 0.1 minute on the IBM 7094-II for 12-region, 16-dose component problems.

Sample Problem

A sample problem consisting of a reactor, molybdenum reflector, and a shield consisting of seven alternating layers of lithium hydride and tungsten is illustrated in figure 1. Region descriptions, densities, and initial thicknesses (guessed) are listed in table I. The configuration is to satisfy a dose constraint of 2 mrem per hour at a point 20 meters distant. A series of discrete ordinates calculations for both neutrons and gammas was made to calculate doses from each source; perturbations were made to determine the attenuation coefficients μ_{ij}. The results of these calculations are listed in tables II and III. Because the core radius, plenum, pressure vessel, and reflector thicknesses are to be constrained in this calculation, a value of $\mu_{ij} = 0.0$ is arbitrarily assigned to these regions.

The complete computer output for this OPEX-II calculation to seek a minimum weight for this seven-layer configuration is given in table IV. The output consists, first, of a listing of all input information, followed by the value of the dose constraint (DES), the value of the calculated dose for the initial configuration (DOS), and the weight (WT) in grams of the initial configuration. If DES and DOS do not agree to within the parameter EPSD, a new set of thicknesses is calculated and printed; this new set of thicknesses satisfies the dose constraint. The listing of DES, DOS, and WT is followed by the values of the dose components D(I) and the thickness of each region T(I).

The results of each OPEX-II iteration for the present problem are shown in figure 2. Shown, for each iteration, is the size and relative position of each of the layers as adjusted in the course of the calculation. The final thicknesses and values of the dose components are listed in tables I and II for comparison with initial values. The initial, guessed configuration weighed 3.594×10^7 grams (79 000 lb); the final, 2.999×10^7 grams (66 000 lb).

TABLE I. - REGION DESCRIPTION

Region j	Description	Density, g/cm^3	Thickness, cm (initial guess)	Final prediction of shield thicknesses, cm
1	Reactor core	9.957	26.0 radius	-----
2	Plenum	8.647	2.50	-----
3	Pressure vessel	16.763	.60	-----
4	Molybdenum reflector	9.234	11.00	-----
5	Lithium hydride	.75	(17.90)	20.52
6	Tungsten	19.3	(7.00)	9.71
7	Lithium hydride	.75	(14.00)	12.32
8	Tungsten	19.3	(5.00)	2.82
9	Lithium hydride	.75	(10.00)	10.32
10	Tungsten	19.3	(3.50)	2.33
11	Lithium hydride	.75	(59.50)	39.29

TABLE II. - DOSE COMPONENTS

Region j	Dose component, D_i	Value of dose component, D_i, mrem/hr	
		With initial shield thicknesses	With final shield thicknesses
1	Neutron	0.02430	0.2527
2	Core gamma	.00303	.0079
3	Plenum, pressure vessel capture gamma	.00196	.0049
4	Plenum, pressure vessel inelastic gamma	.00220	.0055
5	Reflector capture gamma	.204	.4782
6	Reflector inelastic gamma	.00504	.0129
7	Region 6 tungsten capture gamma	.0921	.4007
8	Region 6 tungsten inelastic gamma	.00974	.0949
9	Region 8 tungsten capture gamma	.0988	.2696
10	Region 8 tungsten inelastic gamma	.0278	.0759
11	Region 10 tungsten capture gamma	.201	.2650
12	Region 10 tungsten inelastic gamma	.0947	.1320
	Total	0.7647	2.000

TABLE III. - COEFFICIENTS

Region j	Description	Dose component i											
		1	2	3	4	5	6	7	8	9	10	11	12
		Neutron	Core gamma	Plenum n, γ	Plenum n, n'γ	Reflector n, γ	Reflector n, n'γ	W(6) n, γ	W(6) n, n'γ	W(8) n, γ	W(8) n, n'γ	W(10) n, γ	W(10) n, n'γ
		Coefficient, μ_{ij}, cm^{-1}											
1	Core	0	0	0	0	0	0	0	0	0	0	0	0
2	Plenum	0	0	0	0	0	0	0	0	0	0	0	0
3	Pressure vessel	0	0	0	0	0	0	0	0	0	0	0	0
4	Reflector	0	0	0	0	0	0	0	0	0	0	0	0
5	Lithium hydride	.1347	.0189	.0147	.0147	.0212	.024	.3629	.2297	.2018	.1885	.1837	.1759
6	Tungsten	.2480	.774	.772	.772	.805	.795	.2719	.1812	.2386	.2536	.2543	.2577
7	Lithium hydride	.1369	.0228	.0204	.0204	.0167	.0206	.0210	.0250	.2257	.1983	.1910	.1795
8	Tungsten	.2316	.770	.766	.766	.790	.781	.798	.827	.1779	.1255	.2052	.2368
9	Lithium hydride	.1218	.024	.022	.022	.0201	.0239	.0187	.0245	.0230	.0250	.2231	.1808
10	Tungsten	.2306	.768	.763	.763	.805	.776	.782	.812	.810	.851	.2407	.1107
11	Lithium hydride	.1187	.0244	.0226	.0226	.0196	.0252	.0232	.0286	.0232	.0291	.0231	.0297

TABLE IV. - OPEX-II LISTING AND OUTPUT FOR SAMPLE PROBLEM

```
$IBFTC OPT      DECK

C       MAIN PROGRAM---ADOPTED FROM OPEX (AI)  DIST. BY RSIC/ORNL
C       OPEX BASIC EQUATIONS COMPLETELY REPROGRAMMED BY---
C                     G.P.LAHTI  **NASA-LEWIS RESEARCH CENTER
C                                  CLEVELAND, OHIO
C
      COMMON       NREG, MAX,  IMAX, DES,  CON,  CA,   EPS,  EPSD,
     1 DOS, AA,    WT,   GG,   AG,   IT,   NB(25),     NIJ(25),
     2 E(25),      A(25),      C(25),      EMU(25,25),
     3 T(25),      RHO(25),    G(25),      U(25)
 7777 CALL INPUT
      CALL WEIGHT
      CALL DOSE
      IF( DES .GT. 0.0 )  CALL INIT
      DS = DOS
      DO 120 IT =1, MAX
      W = WT
      SQ = SQRT(GG -AG*AG/AA )
      AGA= AG/AA
      DO 60 I =1, NREG
      IF( NB(I) .EQ. 0)  GO TO 60
      U(I) =(-G(I) + AGA * A(I) )/ SQ
      T(I) = T(I)  + CON * U(I)
      IF( T(I).LE. 0.0)  CALL CLEAR(I)
   60 CONTINUE
      CALL DOSE
      CONST = (DS-DOS)/AA
      DO 80 I=1,NREG
      IF( NB(I) .EQ. 0)  GO TO 80
      T(I) = T(I) + CONST * A(I)
      IF( T(I).LE. 0.0)  CALL CLEAR(I)
   80 CONTINUE
      CALL WEIGHT
      CALL DOSE
      WRITE (6,600) IT, WT, DOS, (D(I), I=1,IMAX)
      WRITE (6,601) (T(I), I=1,NREG)
      IF( ABS(WT-W)/ WT  -EPS) 110, 110, 120
  120 CONTINUE
  110 GO TO 7777
  600 FORMAT(1H0/7H0 IT = I3,7X,5HWT = 1PE12.4,7X,6HDOS = 1PE12.4/
     1  7HOD(I)= 1P9E12.4/(7X,1P9E12.4))
* 601 FORMAT(7HOT(I)= 1P9E12.4/(7X,1P9E12.4))
      END
```

```
$IBFTC INPUTT  DECK

      SUBROUTINE INPUT
      COMMON       NREG, MAX,  IMAX, DES,  CON,  CA,   EPS,  EPSD,
     1 DOS, AA,    WT,   GG,   AG,   IT,   NB(25),     NIJ(25),
     2 E(25),      A(25),      C(25),      EMU(25,25),
     3 T(25),      RHO(25),    G(25),      U(25)
      READ(5,400)  NREG, MAX, IMAX, DES, EPS, EPSD, CON, CA
      WRITE(6,500) NREG, MAX, IMAX, DES, EPS, EPSD, CON, CA
      READ (5,401) (T(I),   I=1,NREG)
      READ (5,401) (RHO(I), I=1,NREG)
      READ (5,403) (NB(I),  I=1,NREG)
      WRITE(6,510) ( I,  T(I), RHO(I),NB(I), I = 1, NREG)
      WRITE(6,550)
      DO 7 J=1,NREG
      READ (5,401) (EMU(I,J),  I=1,IMAX)
    7 WRITE (6,570) J,(EMU(I,J), I=1,IMAX)
      READ (5,401) (D(I),    I=1,IMAX)
      READ(5,402) (NIJ(I), I=1,IMAX)
C     NIJ(I) IS THE REGION NUMBER ASSOCIATED WITH THE ITH DOSE TERM
      DO 9 I=1,IMAX
      BB = 0.0
      DO 8 J=1,NREG
    8 BB = BB + EMU(I,J)*T(J)
    9 C(I) = D(I)*EXP(BB)
      WRITE(6,530) (I, C(I), D(I), NIJ(I), I=1,IMAX)
  400 FORMAT(3I5, 5F10.4)
  401 FORMAT(7F10.4)
  402 FORMAT(14I5)
  403 FORMAT(25I1)
  500 FORMAT(9H1 NREG = I3/ 9H  MAX  = I3/ 9H  IMAX = I3/ 9H  DES  =
     1  1PE12.4/ 9H  EPS  = 1PE12.4/ 9H  EPSD = 1PE12.4/ 9H  CON  =
     2  1PE12.4/ 9H  CA   = 1PE12.4)
  510 FORMAT(1H0/34HOREGION      T(I)     RHO(I)  NB(I)/(I7,2F10.3,I7))
  530 FORMAT(1H0/34H0   I          C(I)          D(I)  NIJ/(I5,1P2E12.4,I5))
  550 FORMAT( 30H0REGION-J            MU(I,J)      )
  570 FORMAT(I6, 3X, 1P9E12.4/(9X,1P9E12.4))
      RETURN
      END
```

TABLE IV. - Continued. OPEX-II LISTING AND OUTPUT FOR SAMPLE PROBLEM

```
$IBFTC INITXX  DECK

      SUBROUTINE  INIT
      COMMON        NREG, MAX,  IMAX, DES,  CON,  CA,   EPS,  EPSD,
     1 DOS, AA,   WT,   GG,   AG,   II,   NB(25),     NIJ(25),
     2 D(25),     A(25),      C(25),      EMU(25,25),
     3 T(25),     RHO(25),    G(25),      U(25)
      WRITE(6,170) DES, DOS, WT, (D(I), I=1,IMAX)
      Q= 1.0
      DO 55 K =1,MAX
      DOSA = DOS
      AAA = SQRT(AA)
      CONST = CA*Q  /AAA
      IF (DOS-DES) 62, 10, 60
   60 CONST = -CONST
   62 DO 70 I=1,NREG
      IF( NB(I) .EQ. 0)  GO TO 70
      T(I) = T(I) + CONST * A(I)
      IF( T(I).LE. 0.0)  CALL CLEAR(I)
   70 CONTINUE
      CALL DOSE
      IF( ABS(DOS-DES)/DES .LE. EPSD)  GO TO 10
      IF(((DOS-DES)*(DOSA-DES)) .LE. 0.0 )  Q = 0.5 * Q
   55 CONTINUE
      K = 777
   10 CALL WEIGHT
      CALL DOSE
      WRITE(6,100) K
  100 FORMAT(  32H0INITIAL THICKNESSES FOUND AFTER I3,11H ITERATIONS)
      WRITE(6,160)     (T(I), I=1,NREG )
  160 FORMAT( 31H0CALCULATED INITIAL THICKNESSES/  (1P9E12.4))
      WRITE(6,170) DES, DOS, WT, (D(I), I=1,IMAX)
  170 FORMAT(7H0DES = 1PE12.4, 6X, 6HDOS = 1PE12.4, 6X, 5HWT = 1PE12.4/
    1  7H0D(I)= 1P9E12.4/ (7X,1P9E12.4))
      RETURN
      END
```

```
$IBFTC CLEANX  DECK

      SUBROUTINE CLEAR(J)
      COMMON        NREG, MAX,  IMAX, DES,  CON,  CA,   EPS,  EPSD,
     1 DOS, AA,   WT,   GG,   AG,   II,   NB(25),     NIJ(25),
     2 D(25),     A(25),      C(25),      EMU(25,25),
     3 T(25),     RHO(25),    G(25),      U(25)
C     THE JTH REGION HAS JUST BEEN WIPED OUT
      T(J) = 0.0
      NB(J) = 0
      DO 5 I=1,IMAX
      IF( J.EQ. NIJ(I) )  C(I)= 0.0
    5 CONTINUE
      RETURN
      END
```

TABLE IV. - Continued. OPEX-II LISTING AND OUTPUT FOR SAMPLE PROBLEM

```
$IBFTC DOSEXX  DECK

      SUBROUTINE DOSE
      COMMON        NREG, MAX,  IMAX, DES,  CON,  CA,   EPS,  EPSD,
     1 DOS, AA,    WT.   GG,   AG,   IT,   NB(25),      NIJ(25),
     2 E(25),      A(25),      C(25),      EMU(25,25),
     3 T(25),      RHO(25),    G(25),      U(25)
      DOS  = 0.0
      DO 40 I=1,IMAX
      BB = 0.0
      DO 30 J=1,NREG
   30 BB = BB + EMU(I,J)* T(J)
      D(I) = C(I) * EXP(-BB)
   40 DOS = DOS + D(I)
C     DOS  IS TOTAL DOSE
C     A(K) = (C-DOSE)/(D-XK)
C     AA = A.A
C     AG = A.G
      AA = 0.0
      AG = 0.0
      DO 60 K = 1,NREG
      A(K) = 0.0
      IF( NB(K) .EQ. 0)  GO TO 60
      DO 50 I = 1,IMAX
   50 A(K) = A(K) -  EMU(I,K) * D(I)
      AA = AA + A(K)**2
      AG = AG + A(K)*G(K)
   60 CONTINUE
      RETURN
      END
```

```
$IBFTC WTSPHR  DECK

      SUBROUTINE WEIGHT
C     CODING FOR OPEX SHIELDING OPTIMIZATION CODE
C     SPHERICAL SHELL GEOMETRY      -G.P.LAHTI  -NASA-LEWIS
C     FOR PLANE SLABS BOUNDED BY CONE OF HALF ANGLE THETA,
C     THIS SUBROUTINE MAY BE USED SIMPLY BY REPLACING
C     THE TERM  FORPI BY 4*(TAN(THETA))**2
      DATA  FORPI / 12.56636/
      DIMENSION ROUT(25)
      COMMON        NREG, MAX,  IMAX, DES,  CON,  CA,   EPS,  EPSD,
     1 DOS, AA,    WT,   GG,   AG,   IT,   NB(25),      NIJ(25),
     2 D(25),      A(25),      C(25),      EMU(25,25),
     3 T(25),      RHO(25),    G(25),      U(25)
C     CALCULATE OUTER RADIUS OF EACH REGION, AND WEIGHT
      ROUT(1)= T(1)
      RRR= ROUT(1)**3
      WEIGHT= RHO(1)*RRR
      DO 5 I=2,NREG
      R3= RRR
      ROUT(I)= ROUT(I-1)+T(I)
      RRR= ROUT(I)**3
    5 WEIGHT= WEIGHT+RHO(I)*(RRR-R3)
      WT= WEIGHT*FORPI/3.
C     CALCULATE PARTIAL WT DERIVATIVES  (DW/DXI) = G(I)
C     GG = G.G
      GG=0.0
      DO 9 I=1,NREG
      RR= 0.0
      G(I) = 0.0
      IF( NB(I) .EQ. 0)  GO TO 9
      DO 8 J=1,NREG
      R2= RR
      RR= ROUT(J)**2
    8 G(I) = G(I) + RHO(J)*(RR-R2)
      G(I) = G(I) * FORPI
      GG = GG + G(I)**2
    9 CONTINUE
      RETURN
      END
```

TABLE IV. - Continued. OPEX-II LISTING AND OUTPUT FOR SAMPLE PROBLEM

```
NREG =  11
MAX  =  50
IMAX =  12
DES  =   2.0000E 00
EPS  =   1.0000E-03
EPSD =   1.0000E-03
CON  =   7.0000E-01
CA   =   7.0000E-01


REGION      T(I)      RHO(I)   NB(I)
     1    26.000      9.957      0
     2     2.500      8.647      0
     3     0.600     16.763      0
     4    11.000      9.234      0
     5    17.900      0.750      1
     6     7.000     19.300      1
     7    14.000      0.750      1
     8     5.000     19.300      1
     9    10.000      0.750      1
    10     3.500     19.300      1
    11    59.500      0.750      1

REGION-J               MU(I,J)
     1    0.          0.          0.          0.          0.          0.          0.          0.          0.
          0.          0.          0.
     2    0.          0.          0.          0.          0.          0.          0.          0.          0.
          0.          0.          0.
     3    0.          0.          0.          0.          0.          0.          0.          0.          0.
          0.          0.          0.
     4    0.          0.          0.          0.          0.          0.          0.          0.          0.
          0.          0.          0.
     5    1.3470E-01  1.8900E-02  1.4700E-02  1.4700E-02  2.1200E-02  2.4000E-02  3.6290E-01  2.2970E-01  2.0180E-01
          1.8850E-01  1.8370E-01  1.7590E-01
     6    2.4800E-01  7.7400E-01  7.7200E-01  7.7200E-01  8.0500E-01  7.9500E-01  2.7190E-01  1.8120E-01  2.3860E-01
          2.5360E-01  2.5430E-01  2.5770E-01
     7    1.3690E-01  2.2800E-02  2.0400E-02  2.0400E-02  1.6700E-02  2.0600E-02  2.1000E-02  2.5000E-02  2.2570E-01
          1.9830E-01  1.9100E-01  1.7950E-01
     8    2.3160E-01  7.7000E-01  7.6600E-01  7.6600E-01  7.9000E-01  7.8100E-01  7.9800E-01  8.2700E-01  1.7790E-01
          1.2550E-01  2.0520E-01  2.3680E-01
     9    1.2180E-01  2.4000E-02  2.2000E-02  2.2000E-02  2.0100E-02  2.3900E-02  1.8700E-02  2.4500E-02  2.3000E-02
          2.5000E-02  2.2310E-01  1.8080E-01
    10    2.3060E-01  7.6800E-01  7.6300E-01  7.6300E-01  8.0500E-01  7.7600E-01  7.8200E-01  8.1200E-01  8.1000E-01
          8.5100E-01  2.4070E-01  1.1070E-01
    11    1.1870E-01  2.4400E-02  2.2600E-02  2.2600E-02  1.9600E-02  2.5200E-02  2.3200E-02  2.8600E-02  2.3200E-02
          2.9100E-02  2.3100E-02  2.9700E-02


  I         C(I)          D(I)   NIJ
  1  2.9425E 05  2.4300E-02    1
  2  4.9449E 03  3.0300E-03    1
  3  2.3997E 03  1.9600E-03    2
  4  2.6936E 03  2.2000E-03    3
  5  3.5981E 05  2.0400E-01    4
  6  1.1522E 04  5.0400E-03    4
  7  2.1973E 06  9.2100E-02    6
  8  2.2521E 04  9.7400E-03    6
  9  9.5079E 04  9.8800E-02    8
 10  2.0543E 04  2.7800E-02    8
 11  1.1039E 05  2.0100E-01   10
 12  2.8426E 04  9.4700E-02   10

DES =   2.0000E 00      DOS =   7.6467E-01      WT =   3.5939E 07

C(I)=   2.4300E-02  3.0300E-03  1.9600E-03  2.2000E-03  2.0400E-01  5.0400E-03  9.2100E-02  9.7400E-03  9.8800E-02
        2.7800E-02  2.0100E-01  9.4700E-02

INITIAL THICKNESSES FOUND AFTER 11 ITERATIONS

CALCULATED INITIAL THICKNESSES
 2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7701E 01  6.5022E 00  1.3850E 01  4.4536E 00  9.8800E 00
 2.8226E 00  5.9468E 01

DES =   2.0000E 00      DOS =   2.0030E 00      WT =   3.2192E 07

C(I)=   3.8960E-02  1.1539E-02  7.4045E-03  8.3112E-03  8.1703E-01  1.9633E-02  2.9957E-01  3.0622E-02  2.2937E-01
        6.4559E-02  3.2958E-01  1.4374E-01
```

TABLE IV. - Continued. OPEX-II LISTING AND OUTPUT FOR SAMPLE PROBLEM

```
 IT =    1        WT =   3.1942E 07       DOS =   2.0005E 00

D(I)=    4.1991E-02  9.6449E-03  6.1709E-03  6.9265E-03  6.7567E-01  1.6311E-02  3.4357E-01  3.5056E-02  2.6938E-01
         7.4807E-02  3.6601E-01  1.5493E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7360E 01  6.8982E 00  1.3516E 01  4.4422E 00  9.6920E 00
         2.6959E 00  5.9234E 01


 IT =    2        WT =   3.1772E 07       DOS =   2.0004E 00

D(I)=    4.5363E-02  7.9752E-03  5.0914E-03  5.7148E-03  5.5197E-01  1.3414E-02  3.8134E-01  3.8937E-02  2.9864E-01
         8.1425E-02  4.0285E-01  1.6771E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7056E 01  7.2748E 00  1.3174E 01  4.3829E 00  9.5066E 00
         2.6602E 00  5.8920E 01


 IT =    3        WT =   3.1652E 07       DOS =   2.0004E 00

D(I)=    4.9506E-02  6.8541E-03  4.3631E-03  4.8974E-03  4.6772E-01  1.1471E-02  4.0820E-01  4.1901E-02  3.0352E-01
         8.0718E-02  4.3785E-01  1.8335E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.6848E 01  7.5684E 00  1.2842E 01  4.2309E 00  9.3440E 00
         2.7466E 00  5.8481E 01


 IT =    4        WT =   3.1553E 07       DOS =   2.0003E 00

D(I)=    5.4284E-02  6.4299E-03  4.0862E-03  4.5665E-03  4.3372E-01  1.0738E-02  4.1939E-01  4.3600E-02  2.8580E-01
         7.4048E-02  4.6407E-01  1.9958E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.6803E 01  7.7148E 00  1.2566E 01  4.0109E 00  9.2477E 00
         2.9344E 00  5.7935E 01


 IT =    5        WT =   3.1463E 07       DOS =   2.0003E 00

D(I)=    5.9147E-02  6.3109E-03  4.0056E-03  4.4961E-03  4.2181E-01  1.0526E-02  4.1955E-01  4.4555E-02  2.6939E-01
         6.8444E-02  4.8004E-01  2.1205E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.6870E 01  7.7951E 00  1.2329E 01  3.8219E 00  9.2176E 00
         3.0927E 00  5.7334E 01


 IT =    6        WT =   3.1379E 07       DOS =   2.0003E 00

D(I)=    6.3855E-02  6.2866E-03  3.9862E-03  4.4743E-03  4.1706E-01  1.0472E-02  4.1628E-01  4.5488E-02  2.6057E-01
         6.5357E-02  4.8658E-01  2.1951E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.6994E 01  7.8688E 00  1.2120E 01  3.6802E 00  9.2390E 00
         3.1880E 00  5.6704E 01


 IT =    7        WT =   3.1299E 07       DOS =   2.0003E 00

D(I)=    6.8410E-02  6.3186E-03  4.0029E-03  4.4931E-03  4.1657E-01  1.0510E-02  4.1273E-01  4.6603E-02  2.5675E-01
         6.3876E-02  4.8709E-01  2.2296E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7147E 01  7.9435E 00  1.1940E 01  3.5696E 00  9.2955E 00
         3.2372E 00  5.6063E 01


 IT =    8        WT =   3.1221E 07       DOS =   2.0003E 00

D(I)=    7.2956E-02  6.3764E-03  4.0363E-03  4.5306E-03  4.1809E-01  1.0591E-02  4.0941E-01  4.7846E-02  2.5581E-01
         6.3317E-02  4.8337E-01  2.2398E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7311E 01  8.0188E 00  1.1784E 01  3.4798E 00  9.3693E 00
         3.2583E 00  5.5416E 01


 IT =    9        WT =   3.1146E 07       DOS =   2.0003E 00

D(I)=    7.7624E-02  6.4461E-03  4.0773E-03  4.5765E-03  4.2054E-01  1.0692E-02  4.0627E-01  4.9147E-02  2.5631E-01
         6.3237E-02  4.7773E-01  2.2366E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7478E 01  8.0930E 00  1.1646E 01  3.4035E 00  9.4497E 00
         3.2641E 00  5.4764E 01
```

TABLE IV. - Continued. OPEX-II LISTING AND OUTPUT FOR SAMPLE PROBLEM

```
IT =  10      WT =  3.1073E 07      DOS =  2.0003E 00

D(I)=  8.2493E-02  6.5201E-03  4.1209E-03  4.6255E-03  4.2335E-01  1.0801E-02  4.0332E-01  5.0475E-02  2.5747E-01
       6.3402E-02  4.7113E-01  2.2261E-01

T(I)=  2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7641E 01  8.1653E 00  1.1525E 01  3.3365E 00  9.5312E 00
       3.2614E 00  5.4107E 01


IT =  11      WT =  3.1002E 07      DOS =  2.0003E 00

D(I)=  8.7612E-02  6.5948E-03  4.1650E-03  4.6750E-03  4.2625E-01  1.0911E-02  4.0058E-01  5.1818E-02  2.5889E-01
       6.3692E-02  4.6402E-01  2.2111E-01

T(I)=  2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7801E 01  8.2355E 00  1.1419E 01  3.2765E 00  9.6112E 00
       3.2538E 00  5.3444E 01


IT =  12      WT =  3.0933E 07      DOS =  2.0003E 00

D(I)=  9.3008E-02  6.6686E-03  4.2084E-03  4.7238E-03  4.2911E-01  1.1020E-02  3.9809E-01  5.3174E-02  2.6038E-01
       6.4050E-02  4.5660E-01  2.1929E-01

T(I)=  2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.7954E 01  8.3039E 00  1.1327E 01  3.2220E 00  9.6886E 00
       3.2427E 00  5.2778E 01


IT =  13      WT =  3.0867E 07      DOS =  2.0003E 00

D(I)=  9.8697E-02  6.7407E-03  4.2507E-03  4.7712E-03  4.3189E-01  1.1126E-02  3.9585E-01  5.4544E-02  2.6184E-01
       6.4451E-02  4.4855E-01  2.1721E-01

T(I)=  2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.8102E 01  8.3704E 00  1.1250E 01  3.1726E 00  9.7628E 00
       3.2288E 00  5.2107E 01


IT =  14      WT =  3.0802E 07      DOS =  2.0003E 00

D(I)=  1.0469E-01  6.8108E-03  4.2917E-03  4.8172E-03  4.3456E-01  1.1230E-02  3.9386E-01  5.5931E-02  2.6324E-01
       6.4884E-02  4.4111E-01  2.1489E-01

T(I)=  2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.8244E 01  8.4355E 00  1.1187E 01  3.1277E 00  9.8334E 00
       3.2124E 00  5.1433E 01


IT =  15      WT =  3.0740E 07      DOS =  2.0003E 00

D(I)=  1.1099E-01  6.8788E-03  4.3312E-03  4.8616E-03  4.3712E-01  1.1331E-02  3.9214E-01  5.7340E-02  2.6457E-01
       6.5341E-02  4.3308E-01  2.1234E-01

T(I)=  2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.8381E 01  8.4992E 00  1.1137E 01  3.0871E 00  9.9003E 00
       3.1935E 00  5.0756E 01


IT =  16      WT =  3.0680E 07      DOS =  2.0003E 00

D(I)=  1.1761E-01  6.9448E-03  4.3695E-03  4.9046E-03  4.3957E-01  1.1429E-02  3.9068E-01  5.8777E-02  2.6580E-01
       6.5823E-02  4.2487E-01  2.0954E-01

T(I)=  2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.8514E 01  8.5620E 00  1.1102E 01  3.0504E 00  9.9634E 00
       3.1719E 00  5.0077E 01


IT =  17      WT =  3.0622E 07      DOS =  2.0003E 00

D(I)=  1.2453E-01  7.0090E-03  4.4066E-03  4.9461E-03  4.4193E-01  1.1524E-02  3.8947E-01  6.0250E-02  2.6694E-01
       6.6325E-02  4.1647E-01  2.0652E-01

T(I)=  2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.8642E 01  8.6240E 00  1.1079E 01  3.0176E 00  1.0023E 01
       3.1477E 00  4.9396E 01
```

TABLE IV. - Continued. OPEX-II LISTING AND OUTPUT FOR SAMPLE PROBLEM

```
IT =  18        WT =    3.0566E 07        DOS =    2.0003E 00

C(I)=    1.3177E-01  7.0714E-03  4.4425E-03  4.9865E-03  4.4421E-01  1.1617E-02  3.8852E-01  6.1765E-02  2.6798E-01
         6.6849E-02  4.0787E-01  2.0324E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.8767E 01  8.6857E 00  1.1070E 01  2.9882E 00  1.0078E 01
         3.1205E 00  4.8713E 01


IT =  19        WT =    3.0512E 07        DOS =    2.0003E 00

C(I)=    1.3931E-01  7.1323E-03  4.4775E-03  5.0257E-03  4.4642E-01  1.1707E-02  3.8783E-01  6.3332E-02  2.6891E-01
         6.7392E-02  3.9906E-01  1.9972E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.8889E 01  8.7473E 00  1.1074E 01  2.9622E 00  1.0129E 01
         3.0904E 00  4.8030E 01


IT =  20        WT =    3.0460E 07        DOS =    2.0003E 00

C(I)=    1.4714E-01  7.1919E-03  4.5116E-03  5.0640E-03  4.4858E-01  1.1795E-02  3.8738E-01  6.4962E-02  2.6974E-01
         6.7954E-02  3.9004E-01  1.9596E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9008E 01  8.8092E 00  1.1092E 01  2.9391E 00  1.0175E 01
         3.0571E 00  4.7346E 01


IT =  21        WT =    3.0409E 07        DOS =    2.0003E 00

C(I)=    1.5526E-01  7.2505E-03  4.5451E-03  5.1016E-03  4.5071E-01  1.1882E-02  3.8718E-01  6.6664E-02  2.7045E-01
         6.8534E-02  3.8081E-01  1.9194E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9126E 01  8.8717E 00  1.1122E 01  2.9189E 00  1.0217E 01
         3.0205E 00  4.6662E 01


IT =  22        WT =    3.0361E 07        DOS =    2.0003E 00

C(I)=    1.6363E-01  7.3083E-03  4.5781E-03  5.1386E-03  4.5282E-01  1.1968E-02  3.8723E-01  6.8452E-02  2.7105E-01
         6.9131E-02  3.7136E-01  1.8766E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9243E 01  8.9352E 00  1.1165E 01  2.9012E 00  1.0255E 01
         2.9802E 00  4.5979E 01


IT =  23        WT =    3.0315E 07        DOS =    2.0003E 00

C(I)=    1.7223E-01  7.3656E-03  4.6108E-03  5.1753E-03  4.5493E-01  1.2052E-02  3.8752E-01  7.0340E-02  2.7152E-01
         6.9744E-02  3.6168E-01  1.8314E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9360E 01  9.0000E 00  1.1222E 01  2.8859E 00  1.0287E 01
         2.9362E 00  4.5296E 01


IT =  24        WT =    3.0271E 07        DOS =    2.0003E 00

C(I)=    1.8104E-01  7.4227E-03  4.6434E-03  5.2120E-03  4.5707E-01  1.2137E-02  3.8806E-01  7.2343E-02  2.7187E-01
         7.0373E-02  3.5179E-01  1.7836E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9477E 01  9.0666E 00  1.1291E 01  2.8726E 00  1.0314E 01
         2.8882E 00  4.4615E 01


IT =  25        WT =    3.0229E 07        DOS =    2.0003E 00

C(I)=    1.9002E-01  7.4800E-03  4.6762E-03  5.2488E-03  4.5926E-01  1.2221E-02  3.8883E-01  7.4478E-02  2.7209E-01
         7.1017E-02  3.4166E-01  1.7333E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9595E 01  9.1353E 00  1.1373E 01  2.8612E 00  1.0336E 01
         2.8360E 00  4.3936E 01
```

TABLE IV. - Concluded. OPEX-II LISTING AND OUTPUT FOR SAMPLE PROBLEM

```
 IT =  26        WT =   3.0189E 07        DOS =   2.0003E 00

D(I)=    1.9912E-01  7.5379E-03  4.7094E-03  5.2860E-03  4.6153E-01  1.2306E-02  3.8985E-01  7.6764E-02  2.7217E-01
         7.1675E-02  3.3131E-01  1.6806E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9715E 01  9.2064E 00  1.1469E 01  2.8515E 00  1.0353E 01
         2.7791E 00  4.3259E 01


 IT =  27        WT =   3.0150E 07        DOS =   2.0003E 00

D(I)=    2.0829E-01  7.5966E-03  4.7432E-03  5.3240E-03  4.6389E-01  1.2391E-02  3.9110E-01  7.9222E-02  2.7211E-01
         7.2347E-02  3.2075E-01  1.6255E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9838E 01  9.2805E 00  1.1577E 01  2.8433E 00  1.0363E 01
         2.7175E 00  4.2586E 01


 IT =  28        WT =   3.0114E 07        DOS =   2.0003E 00

D(I)=    2.1746E-01  7.6568E-03  4.7780E-03  5.3631E-03  4.6638E-01  1.2479E-02  3.9259E-01  8.1875E-02  2.7190E-01
         7.3031E-02  3.0998E-01  1.5682E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  1.9964E 01  9.3577E 00  1.1699E 01  2.8362E 00  1.0368E 01
         2.6508E 00  4.1916E 01


 IT =  29        WT =   3.0079E 07        DOS =   2.0003E 00

D(I)=    2.2657E-01  7.7186E-03  4.8139E-03  5.4034E-03  4.6902E-01  1.2568E-02  3.9430E-01  8.4747E-02  2.7156E-01
         7.3731E-02  2.9901E-01  1.5088E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  2.0095E 01  9.4386E 00  1.1835E 01  2.8303E 00  1.0366E 01
         2.5787E 00  4.1251E 01


 IT =  30        WT =   3.0046E 07        DOS =   2.0003E 00

D(I)=    2.3554E-01  7.7829E-03  4.8515E-03  5.4456E-03  4.7186E-01  1.2661E-02  3.9623E-01  8.7864E-02  2.7105E-01
         7.4440E-02  2.8784E-01  1.4475E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  2.0230E 01  9.5233E 00  1.1984E 01  2.8253E 00  1.0357E 01
         2.5011E 00  4.0590E 01


 IT =  31        WT =   3.0014E 07        DOS =   2.0003E 00

D(I)=    2.4428E-01  7.8489E-03  4.8903E-03  5.4891E-03  4.7486E-01  1.2756E-02  3.9839E-01  9.1257E-02  2.7042E-01
         7.5170E-02  2.7652E-01  1.3844E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  2.0371E 01  9.6125E 00  1.2146E 01  2.8212E 00  1.0342E 01
         2.4173E 00  3.9936E 01


 IT =  32        WT =   2.9985E 07        DOS =   2.0003E 00

D(I)=    2.5270E-01  7.9159E-03  4.9324E-03  5.5363E-03  4.7822E-01  1.2858E-02  4.0072E-01  9.4946E-02  2.6957E-01
         7.5857E-02  2.6503E-01  1.3159E-01

T(I)=    2.6000E 01  2.5000E 00  6.0000E-01  1.1000E 01  2.0519E 01  9.7056E 00  1.2323E 01  2.8175E 00  1.0321E 01
         2.3278E 00  3.9288E 01
401* UNIT05, EOF.                                                                                   REC=  00000 FIL=
```

CONCLUDING REMARKS

An optimization procedure for minimizing radiation shield weight has been described. The procedure, built around the steepest-descent code, OPEX-II, depends strongly on the validity of the dose-thickness relation. It has been observed that when care has been taken to accurately fit the dose-thickness relation, predictions of minimum weight configurations are quite good as verified by a necessary detailed proof calculation.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, January 8, 1969,
124-09-11-01-22.

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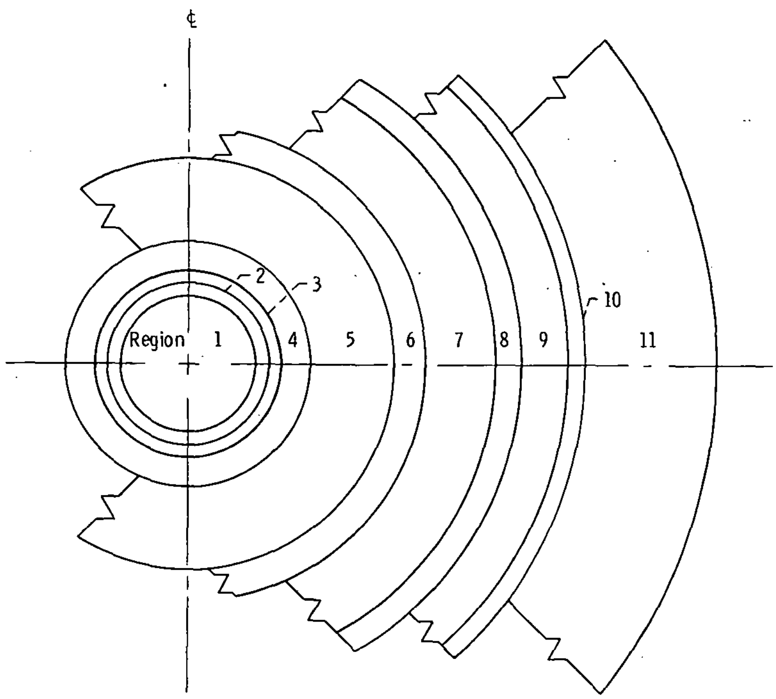

Figure 1. - Geometry for sample problem.

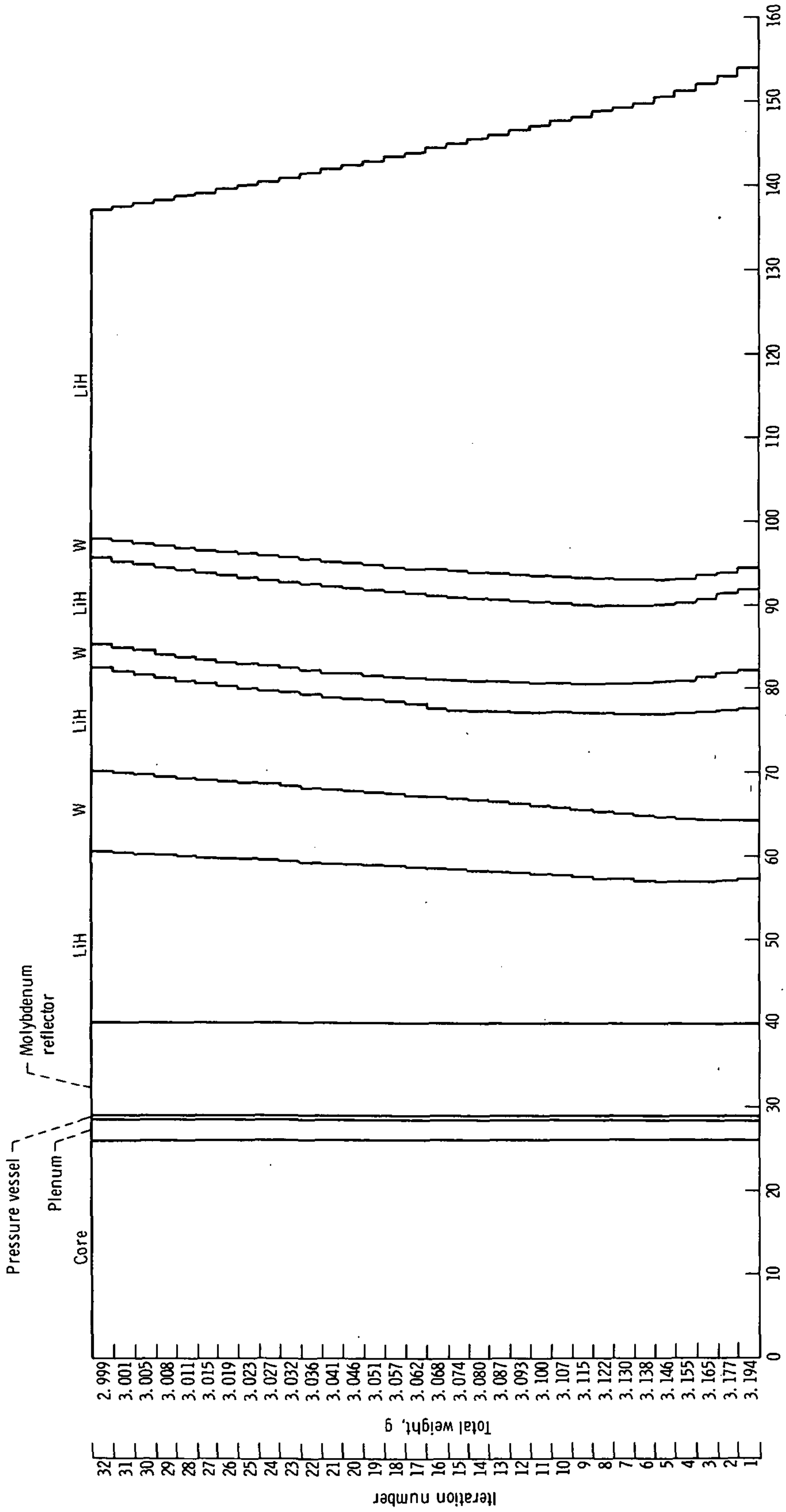

Figure 2. - Results of OPEX-II iterations.

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