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A FORTRAN CODE FOR COMPUTING THE PRINCIPLE MASS MOMENTS OF INERTIA OF COMPOSITE BODIES

by James E. Cake

Lewis Research Center

Cleveland, Ohio

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ABSTRACT

A mathematical model of the mass characteristics of composite bodies has been programmed for digital computer processing. The description and equation development of the mathematical model are presented. The technique is of particular interest for spacecraft whose size or mass prohibits the use of experimental measurement in an inertia laboratory. The computer program is written in the FORTRAN IV (version 13) language and is operational on the IBM 7094 Direct Couple System of the Lewis Research Center. A sample problem illustrates the input data requirements and the output format of the program. The use options of the program and program limitations are fully described.

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SUMMARY

A mathematical model of the mass characteristics of composite bodies has been programmed for digital computer processing. The description and equation development of the mathematical model are presented. The technique is of particular interest for spacecraft whose size or mass prohibits the use of experimental measurement in an inertia laboratory.

The computer program is written in the FORTRAN IV (version 13) language and is operational on the IBM 7094 II/7044 Direct Couple System of the Lewis Research Center. A sample problem illustrates the input data requirements and the output format of the program. The use options of the program and program limitations are fully described.

INTRODUCTION

Studies of the rotational dynamics of spacecraft require the knowledge of the mass characteristics of the spacecraft. These characteristics include spacecraft weight, location of the center of mass, principal mass moments of inertia, and the directions of the principal axes of inertia. Laboratories to experimentally measure these characteristics may not be readily available or cannot handle spacecraft of large size and mass. Therefore, a mathematical model which treats the spacecraft as a composite body is desirable.

This report describes the mathematical model which has been developed and programmed for digital computer processing. The computer program is written in the FORTRAN IV (version 13) language and is operational on the IBM 7094 II/7044 Direct Couple System of the Lewis Research Center.

The report places particular emphasis on data preparation for the computer program. A sample problem is included for both illustrative purposes and as a verification of the mathematical model. The program has been used to calculate the mass characteristics of the Space Electric Rocket Test vehicle (SERT II).

THE PROBLEM

The mathematical model of the mass characteristics of a composite body is based on the fact that the moments and products of inertia of composite bodies are found by summing the moments and products of inertia of the individual components or particles. This may be shown from a few fundamental principles of rigid body and particle dynamics.

From reference 1, the components of the angular momentum of a rigid body about a fixed point of rotation, 0, are:

$$\left.\begin{aligned} h_{x_0} &= I_{x_0}\omega_{x_0} - P_{xy_0}\omega_{y_0} - P_{xz_0}\omega_{z_0} \\ h_{y_0} &= -P_{yx_0}\omega_{x_0} + I_{y_0}\omega_{y_0} - P_{yz_0}\omega_{z_0} \\ h_{z_0} &= -P_{zx_0}\omega_{x_0} - P_{zy_0}\omega_{y_0} + I_{z_0}\omega_{z_0} \end{aligned}\right\} \tag{1}$$

where I_{x_0}, I_{y_0}, and I_{z_0} are the mass moments of inertia and P_{yz_0}, P_{zx_0}, etc., are the mass products of inertia of the body.

The total angular momentum of a system of particles is the sum of the angular momenta of the individual particles.

$$\left.\begin{aligned} H_{x_0} &= \sum_{i=1}^{n} h_{x_{0_i}} \\ H_{y_0} &= \sum_{i=1}^{n} h_{y_{0_i}} \\ H_{z_0} &= \sum_{i=1}^{n} h_{z_{0_i}} \end{aligned}\right\} \tag{2}$$

Substitute equation (1) into equation (2). By expanding the right side, the components of the angular momentum of the system can be expressed in a form similar to equation (1). The moments and products of inertia of the system of particles or composite are found to be the sum of the moments and products of inertia of the individual components.

Description of Mathematical Model

The spacecraft or other rigid body is composed of n discrete components of mass m_i, each with its center of mass at (x_i, y_i, z_i) in the reference axis system defined by figure 1.

Definition of the center of mass. - The center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the body with respect to the reference axis system is defined as

$$\left.\begin{aligned} \bar{x} &= \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \\ \bar{y} &= \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} \\ \bar{z} &= \frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i} \end{aligned}\right\} \tag{3}$$

Definition of the inertia tensor. - The mass moments of inertia of each component are $I_{x_i}, I_{y_i}, I_{z_i}$ which are defined about axes parallel to the reference axis system. The center of mass axis system as defined in figure 1 has its origin at the system center of mass and is parallel to the reference axis system. Then the system mass moments of inertia about the center of mass axis system are:

$$\left.\begin{aligned}
I_{\bar{x}} &= \sum_{i=1}^{n} \left\{ I_{x_i} + m_i \left[(\bar{z} - z_i)^2 + (\bar{y} - y_i)^2 \right] \right\} \\
I_{\bar{y}} &= \sum_{i=1}^{n} \left\{ I_{y_i} + m_i \left[(\bar{z} - z_i)^2 + (\bar{x} - x_i)^2 \right] \right\} \\
I_{\bar{z}} &= \sum_{i=1}^{n} \left\{ I_{z_i} + m_i \left[(\bar{x} - x_i)^2 + (\bar{y} - y_i)^2 \right] \right\}
\end{aligned}\right\} \tag{4}$$

The mass products of inertia defined about axes parallel to the reference axis system are P_{xy_i} P_{yx_i} , P_{yz_i} P_{zy_i} , and P_{xz_i} P_{zx_i} . The system mass products of inertia parallel to the reference system are

$$\left.\begin{aligned}
P_{\bar{x}\bar{y}} = P_{\bar{y}\bar{x}} &= \sum_{i=1}^{n} \left[m_i (\bar{x} - x_i)(\bar{y} - y_i) + P_{xy_i} \right] \\
P_{\bar{y}\bar{z}} = P_{\bar{z}\bar{y}} &= \sum_{i=1}^{n} \left[m_i (\bar{y} - y_i)(\bar{z} - z_i) + P_{yz_i} \right] \\
P_{\bar{x}\bar{z}} = P_{\bar{z}\bar{x}} &= \sum_{i=1}^{n} \left[m_i (\bar{x} - x_i)(\bar{z} - z_i) + P_{xz_i} \right]
\end{aligned}\right\} \tag{5}$$

The inertia tensor is defined as

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} = \begin{bmatrix} I_{\bar{x}} & -P_{\bar{x}\bar{y}} & -P_{\bar{x}\bar{z}} \\ -P_{\bar{y}\bar{x}} & I_{\bar{y}} & -P_{\bar{y}\bar{z}} \\ -P_{\bar{z}\bar{x}} & -P_{\bar{z}\bar{y}} & I_{\bar{z}} \end{bmatrix} \tag{6}$$

From reference 2, the principal mass moments of inertia are found from the determinant

$$\begin{vmatrix} (I_{11} - I) & I_{12} & I_{13} \\ I_{21} & (I_{22} - I) & I_{23} \\ I_{31} & I_{32} & (I_{33} - I) \end{vmatrix} = 0$$

Evaluating the determinant yields a cubic equation in I whose roots are I_1, I_2, I_3, the three principal mass moments of inertia.

Orientation of the principal axes. - If the products of inertia are nonzero in the inertia tensor, then as shown in reference 2, the following method can be used to solve for the directions of the principal axes.

$$(I_{22} - I)\frac{\omega_y}{\omega_x} + I_{23}\frac{\omega_z}{\omega_x} = -I_{21} \tag{7}$$

$$I_{32}\frac{\omega_y}{\omega_x} + (I_{33} - I)\frac{\omega_t}{\omega_x} = -I_{31} \tag{8}$$

Substitute each principal inertia I_j for I in equations (7) and (8). Solve for $\omega_{y_j}/\omega_{x_j}$ and $\omega_{z_j}/\omega_{x_j}$. By arbitrarily choosing $\omega_{x_j} = 1$, then $\omega_{y_j}/\omega_{x_j} = \omega_{y_j}$ and $\omega_{z_j}/\omega_{x_j} = \omega_{z_j}$. Then the magnitude of the angular velocity vector

$$r_j = \sqrt{\omega_{x_j}^2 + \omega_{y_j}^2 + \omega_{z_j}^2} = \sqrt{1 + \omega_{y_j}^2 + \omega_{z_j}^2}$$

The direction cosines are found to be

$$l_{A_1\bar{x}} = \frac{1}{r_1} \qquad l_{A_2\bar{x}} = \frac{1}{r_2} \qquad l_{A_3\bar{x}} = \frac{1}{r_3}$$

$$l_{A_1\bar{y}} = \frac{\omega_{y_1}}{r_1} \qquad l_{A_2\bar{y}} = \frac{\omega_{y_2}}{r_2} \qquad l_{A_3\bar{y}} = \frac{\omega_{y_3}}{r_3}$$

$$l_{A_1\bar{z}} = \frac{\omega_{z_1}}{r_1} \qquad l_{A_2\bar{z}} = \frac{\omega_{z_2}}{r_2} \qquad l_{A_3\bar{z}} = \frac{\omega_{z_3}}{r_3}$$

where A_1 is the axis about which I_1 is defined, etc.

The set of direction cosines represents the rotation matrix between the center of mass axis system and the principal axes of the body.

$$\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix} = \begin{bmatrix} l_{A_1\bar{x}} & l_{A_1\bar{y}} & l_{A_1\bar{z}} \\ l_{A_2\bar{x}} & l_{A_2\bar{y}} & l_{A_2\bar{z}} \\ l_{A_3\bar{x}} & l_{A_3\bar{y}} & l_{A_3\bar{z}} \end{bmatrix} \begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} \tag{9}$$

If four products of inertia in the inertia tensor are zero, an alternate method is required to solve for the direction cosines. Consider the case where $\bar{x}$ is a principal axis. Then the inertia tensor is

$$[I] = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & I_{23} \\ 0 & I_{32} & I_{33} \end{bmatrix} \tag{10}$$

The direction cosine matrix between the center of mass axis system and the principal axes represents a single rotation α in the $\bar{y}\bar{z}$ plane.

$$[l] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \tag{11}$$

From reference 2,

$$[I'] = [l]\,[I]\,[l]^T \tag{12}$$

where $[I']$ is the inertia tensor about the principal axes of inertia. Substituting (8) and (9) into (10) and solving for I'_{32},

$$I'_{32} = I_{32}\cos^2\alpha + \frac{1}{2}(I_{33} - I_{22})\sin 2\alpha - I_{23}\sin^2\alpha$$

and since $I_{32} = I_{23}$, then

$$I'_{32} = I_{32}\cos 2\alpha + \frac{1}{2}(I_{33} - I_{22})\sin 2\alpha$$

In the principal inertia tensor $I'_{32} = 0$. Therefore, reducing equation (11) yields

$$\alpha = \frac{1}{2}\tan^{-1}\frac{2I_{32}}{I_{22} - I_{33}}$$

A similar approach is used to find the direction cosines whenever $\bar{y}$ or $\bar{z}$ is a principal axis.

Accuracy of Mathematical Model

The accuracy of the mathematical model is dependent on the accuracy of the mass characteristics of the individual components. Ideally, accurate mass characteristics of each component would be found in a laboratory which measures the quantities experimentally. If this is not possible, the component may be assumed to be homogeneous in mass distribution and of a regular geometric shape. Standard formulae may be used to calculate the moments and products of inertia about axes parallel to the reference system. If the center of mass is not found experimentally, assuming a homogeneous mass distribution locates the center of mass at the geometric center of each component.

Less accurate results will be obtained if any of the above assumptions or simplifications are made in calculating the inertia tensor and center of mass rather than experimentally measuring these mass characteristics. For any method, the resulting component inertia tensor would be the input data to the computer program.

DESCRIPTION OF DIGITAL COMPUTER PROGRAM

The computer program is written in the FORTRAN IV (version 13) language and is operational on the IBM 7094 II/7044 Direct Couple System of the Lewis Research Center.

The main program, READ, controls the input and output functions desired by the user. Subroutine INRTIA calculates the principal mass moments of inertia and the directions of the principal axes. Subroutine OUT lists the input data for the individual components, the system inertia tensor, the principal inertias and directions of the principal axes, the system weight, and the system center of mass. Schematic flow diagrams of READ, INRTIA, and OUT are presented in figures 2 to 4, respectively. Subroutine EIGEN is called by INRTIA to calculate the eigenvalues (principal inertias) of the system inertia tensor, and subroutine CUBIC solves a cubic equation.

Common statement symbols used in the program are defined in appendix B, and a complete program listing is given in appendix C. An understanding of the input data preparation and output data interpretation is essential to using the program.

Program Use Options

The input and output portions of the program have been written to make its use simple and efficient. The initial input data to the program is placed on punch cards. This data may then be written on a magnetic tape. For future calculations, the data is input using the tape with any changes to the data array made by card input. The tape may then be changed to the new data array or retained as the original data set-up.

The input options, which are assigned the FORTRAN variable name NOPT, are:

(1) NOPT = 1; the data array is input from cards.

(2) NOPT = 2; the data array is input from magnetic tape. No changes or additions are made to the array.

(3) NOPT = 3; the data array is input from magnetic tape. Changes or additions are made to the array.

(4) NOPT = 4; the data array is not input through READ.

The output options, assigned the variable name NOUT, are:

(1) NOUT = 1; subroutine OUT is used for output data.

(2) NOUT = 2; subroutine OUT is not used.

The magnetic tape options, assigned the variable name NTAPE, are:

(1) NTAPE = 1; the new data array will be written on tape.

(2) NTAPE = 2; the tape is left unchanged from the previous data array.

Program Input Preparation

There are eight possible types of input cards to the program. The use of the card will vary with the program use option.

Card type	FORTRAN variable	Format	Card columns	Description	Units
1	NOPT	I5	1-5	Input option	-------
	NOUT	I5	6-10	Output option	-------
	NTAPE	I5	11-15	Tape option	-------
2	DESCRI	5A6	1-30	Component description	-------
	NCOMP	I5	31-35	Component number	-------
	IPART	I5	41-45	Part number	-------
	MODEL	I5	51-55	Model number	-------
	IDATE	I6	60-61	Month	-------
			62-63	Day	-------
			64-65	Year	-------
3	NCOMP	I5	1-5	Component number	-------
	MASS	E12.4	6-17	Component weight	lb
	IXC	E10.3	18-27	I_{x_i}	slug-ft^2
	IYC	E10.3	28-37	I_{y_i}	slug-ft^2
	IZC	E10.3	38-47	I_{z_i}	slug-ft^2
	XC	E10.3	48-57	x_i	inches
	YC	E10.3	58-67	y_i	inches
	ZC	E10.3	68-77	z_i	inches
4	PXYO	E10.3	1-10	P_{xy_i}	slug-ft^2
	PXZO	E10.3	11-20	P_{xz_i}	slug-ft^2
	PYZO	E10.3	21-30	P_{yz_i}	slug-ft^2

Card type	FORTRAN	Format	Card columns	Description	Units
5	IADD	I1	1	Change or addition indicator	-------
	N	I4	2-5	List number	-------
6	Same format as card 2, except all zeros punched in the entry fields				
7	Same format as card 3, except all zeros punched in the entry fields				
8	Same format as card 4, except all zeros punched in the entry fields				

Each component requires a set of three input cards (cards 2, 3, 4). For any component input by cards, a set of "zero field" data (cards 6, 7, 8) must be the last set read by the program.

Data Array Change Using Tape Input

The program assigns a list number to every component in the data array including the "zero field" data. When changing the data array after a tape input the following quantities must be specified on card 5:

(1) IADD = 0; the data of a component in the array is being changed.
= 1; a new component is being added and the length of the array is increased.

(2) N; the list number of the data to be changed or added.

When adding components, the list number of the first addition is assigned the list number of the "zero field" data which was on tape. For either component additions or changes, the new set of "zero field" data required to end the read phases is always considered an addition to the list.

The following examples illustrate the card set-up for the various input options:

Option 1: Cards 1, 2, 3, 4, 2, 3, 4, 2, 3, 4, 6, 7, 8

Option 2: Card 1

Option 3: Cards 1, 5, 2, 3, 4, 5, 6, 7, 8

Option 4: Card 1

Batches of inertia calculations may be run using the program.

Appendix D contains a sample problem illustrating the input data requirements and the output format of the program.

Program Limitations

Because of core limitations, the program has reserved storage locations for a composite body with a maximum of 850 components. For larger composites the inertia calculation should be made as separate jobs.

The magnetic tape is programmed to operate from logic unit number eight. Therefore, any control cards preceding the program must specify the tape to be "set-up" on that unit.

CONCLUDING REMARKS

The purpose of this report has been to describe a computer program capable of computing the mass characteristics of composite bodies. The input words and output data have been fully defined by the description of the mathematical model. A sample problem was used to illustrate the input options, input data preparation, and output data interpretation.

The accuracy of the mathematical model may be increased by experimental measurement of the mass characteristics of the component.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 6, 1968,
704-01-00-17-22.

APPENDIX A

MATHEMATICAL SYMBOLS

A_1, A_2, A_3	coordinate axes defining the principal mass moments of inertia
$H_{x_0}, H_{y_0}, H_{z_0}$	angular momentum of a system of components about x_0, y_0, and z_0, respectively, slug-ft^2/sec; kg-m^2/sec
$h_{x_0}, h_{y_0}, h_{z_0}$	angular momentum of a component about x_0, y_0, and z_0, respectively, slug-ft^2/sec; kg-m^2/sec
I	mass moment of inertia, slug-ft^2; kg-m^2
l	direction cosine
m	mass, slug; kg
P	mass product of inertia, slug-ft^2; kg-m^2
r	magnitude of the angular velocity vector, rad/sec
x_i, y_i, z_i	component center of mass in the reference axis system, ft; m
$\bar{x}, \bar{y}, \bar{z}$	coordinates of the system center of mass in the reference axis system, ft; m
α	angle of rotation, deg
ω	angular velocity, rad/sec

Subscripts:

x_0, y_0, z_0	reference axis coordinate system
$\bar{x}, \bar{y}, \bar{z}$	center of mass coordinate system
x', y', z'	principal axes coordinate system

Superscript:

T	matrix transpose

APPENDIX B

COMMON STATEMENT SYMBOLS

A	element of inertia tensor
DCSN(J, J)	direction cosines
DESCRI(J, J)	description of component
IDATE(J)	date of inclusion into inertia calculation
IPART(J)	part number of component
IXC(J)	moment of inertia about the x axis of the component
IYC(J)	moment of inertia about the y axis of the component
IZC(J)	moment of inertia about the z axis of the component
IXB(J)	component moment of inertia translated to the $\bar{x}$ axis
IYB(J)	component moment of inertia translated to the $\bar{y}$ axis
IZB(J)	component moment of inertia translated to the $\bar{z}$ axis
IXY(J)	component product of inertia translated to the $\bar{x}\bar{y}$ plane
IXZ(J)	component product of inertia translated to the $\bar{x}\bar{z}$ plane
IYZ(J)	component product of inertia translated to the $\bar{y}\bar{z}$ plane
MASS(J)	component weight
MODEL(J)	component model number
NCOMP(J)	component number
PI1, PI2, PI3	principal mass moments of inertia
PXYO(J)	product of inertia about the xy plane of the component
PXZO(J)	product of inertia about the xz plane of the component
PYZO(J)	product of inertia about the yz plane of the component
SWEIGH	total system weight
XBAR	system center of mass distance from x_0 axis
YBAR	system center of mass distance from y_0 axis
ZBAR	system center of mass distance from z_0 axis
XC(J)	component center of mass distance from x_0 axis

YC(J)	component center of mass distance from y_0 axis
ZC(J)	component center of mass distance from z_0 axis
XWRTCG(J)	distance from component to $\bar{x}$ axis
YWRTCG(J)	distance from component to $\bar{y}$ axis
ZWRTCG(J)	distance from component to $\bar{z}$ axis

APPENDIX C

FORTRAN LISTING

```
$IBFTC READ

C      ***************************************************************
C      USE OPTIONS
C          NOPT=1     READ INPUT FROM CARDS
C          NOPT=2     READ INPUT FROM TAPE, NO CHANGES
C          NOPT=3     READ INPUT FROM TAPE, MAKE CHANGES AND ADDITIONS
C          NOPT=4     INPUT THROUGH SUBROUTINE READ IS NOT USED
C
C          NOUT=1     USE OUTPUT SUBROUTINE OUT
C          NOUT=2     SUBROUTINE OUT IS NOT USED
C
C          NTAPE=1    NEW DATA ARRAY WRITTEN ON TAPE
C          NTAPE=2    TAPE LEFT UNCHANGED FROM PREVIOUS DATA ARRAY
C
C
C      REMARKS
C          IF THE INPUT AND OUTPUT ROUTINES ARE NOT USED, DATA TO
C          SUBROUTINE INRTIA IS TRANSMITTED THROUGH
C                COMMON/INPUT/----------
C                COMMON/OUTPT/-------------
C                COMMON/TENSOR/------------
C
C      DESCRIPTION OF AXES
C
C
C          1.   THE REFERENCE AXIS SYSTEM IS DEFINED BY THE USER
C          2.   THE CENTER OF MASS AXIS SYSTEM IS PARALLEL TO THE REFERENCE
C               SYSTEM WITH ORIGIN AT THE SYSTEM CM
C          3.   INPUT MOMENTS AND PRODUCTS OF INERTIA ARE ABOUT AXES
C               PARALLEL TO THE REFERENCE AXIS SYSTEM WITH ORIGIN AT THE
C               COMPONENT C.M.
C
C
C      DESCRIPTION OF PARAMETERS
C          MASS                    WEIGHT OF COMPONENT     (LBS)
C          XC,YC,ZC                LOCATION OF COMPONENT IN REFERENCE
C                                  AXIS SYSTEM (INCHES)
C          IXC,IYC,IZC             MOMENTS OF INERTIA OF COMPONENT IN
C                                  COMPONENT AXIS SYSTEM    (SLUG-FT2)
C          PXYO,PXZO,PYZO          PRODUCTS OF INERTIA OF COMPONENT IN
C                                  COMPONENT AXIS SYSTEM    (SLUG-FT2)
C          XBAR,YBAR,ZBAR          CENTER OF MASS       (INCHES)
C          XWRTCG,YWRTCG,ZWRTCG DISTANCE OF COMPONENT FROM C.M.  (INCHES)
C          IXB,IYB,IZB             MOMENTS OF INERTIA OF COMPONENT IN C.M.
C                                  AXIS SYSTEM  (SLUG-FT2)
C          IXY,IXZ,IYZ             PRODUCTS OF INERTIA OF COMPONENT IN C.M.
C                                  AXIS SYSTEM   (SLUG-FT2)
C          SWEIGH                  WEIGHT OF SYSTEM      (LBS)
C          PI1,PI2,PI3             PRINCIPAL MOMENTS OF INERTIA OF SYSTEM
C          COSD                    DIRECTION COSINE SET
C          A                       ELEMENT OF SYSTEM INERTIA TENSOR
```

```
C     ******************************************************************
      COMMON/INPUT/ DESCRI(850,5),NCOMP(850),IPART(850),MODEL(850),
     *IDATE(850),MASS(850),IXC(850),IYC(850),IZC(850),XC(850),YC(850),
     * ZC(850),PXYO(850),PXZO(850),PYZO(850)
      COMMON/OUTPT/XWRTCG(850),YWRTCG(850),ZWRTCG(850),IXB(850),IYB(850)
     *,IZB(850),IXY(850),IXZ(850),IYZ(850),COSD(3,3),PI1,PI2,PI3,SWEIGH,
     *XBAR,YBAR,ZBAR
      COMMON/TENSOR/A11,A12,A13,A22,A23,A33
      EQUIVALENCE(A12,A21),(A13,A31),(A23,A32)
      REAL MASS,IXC,IYC,IZC
    4 ICNT=0
      REWIND 8
C     SELECT INPUT AND OUTPUT OPTION
      READ(5,103) NOPT,NOUT,NTAPE
      GO TO(1,2,2,500),NOPT
C     CARD INPUT
    1 ICNT=ICNT+1
      N=ICNT
      READ(5,101)(DESCRI(N,J),J=1,5),NCOMP(N),IPART(N),MODEL(N),IDATE(N)
      READ(5,100) NCOMP(N),MASS(N),IXC(N),IYC(N),IZC(N),XC(N),YC(N),ZC(
     *N)
      READ(5,102) PXYO(N),PXZO(N),PYZO(N)
      IF(MASS(N).NE.0.0) GO TO 1
      GO TO 500
C     TAPE INPUT
    2 ICNT=ICNT+1
      N=ICNT
      READ(8)(DESCRI(N,J),J=1,5),NCOMP(N),IPART(N),MODEL(N),IDATE(N),
     *MASS(N),IXC(N),IYC(N),IZC(N),XC(N),YC(N),ZC(N),PXYC(N),PXZO(N),
     *PYZO(N)
      IF(MASS(N).NE.0.0) GO TO 2
      IF(NOPT.EQ.2) GO TO 500
      ICNT=ICNT-1
   10 READ(5,104) IADD,N
      READ(5,101)(DESCRI(N,J),J=1,5),NCOMP(N),IPART(N),MODEL(N),IDATE(N)
      READ(5,100) NCOMP(N),MASS(N),IXC(N),IYC(N),IZC(N),XC(N),YC(N),
     *ZC(N)
      READ(5,102) PXYO(N),PXZO(N),PYZO(N)
      IF(IADD.EQ.1) ICNT=ICNT+1
      IF(MASS(N).NE.0.0) GO TO 10
  500 CALL INRTIA(ICNT)
C     SELECT OUTPUT OPTIONS
      IF(NTAPE.EQ.0) GO TO 600
      REWIND 8
      DO 5 N=1,ICNT
    5 WRITE(8)(DESCRI(N,J),J=1,5),NCOMP(N),IPART(N),MODEL(N),IDATE(N),
     *MASS(N),IXC(N),IYC(N),IZC(N),XC(N),YC(N),ZC(N),PXYO(N),PXZO(N),
     *PYZO(N)
  600 IF(NOUT.EQ.1) CALL OUT(ICNT)
      GO TO 4
  100 FORMAT(I5,E12.4,6E10.3)
  101 FORMAT(5A6,I5,5X,I5,5X,I5,4X,I6)
  102 FORMAT(3E10.3)
  103 FORMAT(I5,I5,I5)
  104 FORMAT(I1,I4)
      END
```

```
$IBFTC INERTIA
      SUBROUTINE INRTIA(ICNT)
      COMMON/INPUT/ DESCRI(850,5),NCOMP(850),IPART(850),MODEL(850),
     *ICATE(850),MASS(850),IXC(850),IYC(850),IZC(850),XC(850),YC(850),
     * ZC(850),PXYO(850),PXZO(850),PYZO(850)
      COMMON/OUTPT/XWRTCG(850),YWRTCG(850),ZWRTCG(850),IXB(850),IYB(850)
     *,IZB(850),IXY(850),IXZ(850),IYZ(850),CCSD(3,3),PI1,PI2,PI3,SWEIGH,
     *XBAR,YBAR,ZBAR
      COMMON/TENSOR/A11,A12,A13,A22,A23,A33
      DIMENSION PI(3),Z(3),Y(3)
      EQUIVALENCE (PXY,PYX),(PYZ,PZY),(PXZ,PZX)
      EQUIVALENCE (A12,A21),(A23,A32),(A13,A31)
      EQUIVALENCE (A11,ROLL),(A22,PITCH),(A33,YAW)
      EQUIVALENCE(PI(1),PI1)
      REAL MASS,IXC,IYC,IZC,IXB,IYB,IZB,IXY,IXZ,IYZ
C
C     LOOP FOR CALCULATING SYSTEM CENTER OF MASS
C
      SXCM=0.0
      SYCM=0.0
      SZCM=0.0
      SMASS=0.0
      DO 5 N=1,ICNT
      MASS(N)=MASS(N)/32.174049
      XC(N)=XC(N)/12.0
      YC(N)=YC(N)/12.0
      ZC(N)=ZC(N)/12.0
      XCM=XC(N)*MASS(N)
      YCM=YC(N)*MASS(N)
      ZCM=ZC(N)*MASS(N)
      SMASS=SMASS+MASS(N)
      SXCM=SXCM+XCM
      SYCM=SYCM+YCM
    5 SZCM=SZCM+ZCM
      XBAR=SXCM/SMASS
      YBAR=SYCM/SMASS
      ZBAR=SZCM/SMASS
      N=ICNT
C
C     LOOP FOR CALCULATING SYSTEM MASS MOMENTS AND MASS PRODUCTS
C
      ROLL=0.0
      YAW=0.0
      PITCH=0.0
      PXY=0.0
      PXZ=0.0
      PYZ=0.0
      DO 3 K=1,N
      X=XBAR-XC(K)
      Y=YBAR-YC(K)
      Z=ZBAR-ZC(K)
```

```
      IXB(K)=IXC(K)+MASS(K)*(Z*Z+Y*Y)
      IYB(K)=IYC(K)+MASS(K)*(Z*Z+X*X)
      IZB(K)=IZC(K)+MASS(K)*(X*X+Y*Y)
      ROLL=ROLL+IXB(K)
      PITCH=PITCH+IYB(K)
      YAW=YAW+IZB(K)
      IXY(K)=MASS(K)*X*Y+PXYO(K)
      IXZ(K)=MASS(K)*X*Z+PXZO(K)
      IYZ(K)=MASS(K)*Y*Z+PYZO(K)
      PXY=PXY+IXY(K)
      PYZ=PYZ+IYZ(K)
      PXZ=PXZ+IXZ(K)
C
C     SET UP INERTIA TENSOR. OTHER ELEMENTS BY EQUIVALENCE
C
      A12=-PXY
      A23=-PYZ
      A13=-PXZ
    3 CONTINUE
      IF(A12.EQ.0.0.AND.A13.EQ.0.0.AND.A23.EQ.0.0) GO TO 24
      IF(A12.EQ.0.0.AND.A13.EQ.0.0) GO TO 21
      IF(A12.EQ.0.0.AND.A23.EQ.0.C) GO TO 22
      IF(A13.EQ.0.0.AND.A23.EQ.0.C) GO TO 23
C
C     SOLVE THE EIGENVALUE PROBLEM FOR THE PRINCIPAL INERTIAS
C
      CALL EIGEN(PI1,PI2,PI3)
      IF(PI1.EQ.0.0) GO TO 8
C
C     DERIVE DIRECTION COSINE MATRIX
C
      DO 9 I=1,3
      Z(I)=(-A21*A32+A31*(PITCH-PI(I)))/(A23*A32-(PITCH-PI(I))*
     1(YAW-PI(I)))
      Y(I)=(-A21-A23*Z(I))/(PITCH-PI(I))
      R=SQRT(1.0+Y(I)*Y(I)+Z(I)*Z(I))
      COSD(I,1)=1.0/R
      COSD(I,2)=Y(I)/R
      COSD(I,3)=Z(I)/R
    9 CONTINUE
      GO TO 91
C
C     DIRECTION COSINES AND PRINCIPAL MOMENTS WHEN XX IS A PRINCIPAL
C     AXIS.
C
   21 ALPHA=.5*  ATAN((2.0*A23)/(A22-A33))
      SA=SIN(ALPHA)
      CA=COS(ALPHA)
      COSD(1,1)=1.0
      COSD(2,1)=0.0
      COSD(3,1)=0.0
      COSD(1,2)=0.0
      COSD(2,2)=CA
      COSD(3,2)=-SA
      COSD(1,3)=0.0
      COSD(2,3)=SA
```

```
      COSD(3,3)=CA
      PI1=A11
      PI2=A22*CA*CA+A33*SA*SA+A23*SIN(ALPHA*2.0)
      PI3=A22*SA*SA+A33*CA*CA-A23*SIN(ALPHA*2.0)
      GO TO 91
C
C     DIRECTION COSINES AND PRINCIPAL MOMENTS WHEN YY IS A PRINCIPAL
C     AXIS
C
   22 ALPHA=.5*  ATAN((2.0*A13)/(A33-A11))
      CA=COS(ALPHA)
      SA=SIN(ALPHA)
      COSD(1,1)=CA
      COSD(2,1)=0.0
      COSD(3,1)=SA
      COSD(1,2)=0.0
      COSD(2,2)=1.0
      COSD(3,2)=0.0
      COSD(1,3)=-SA
      COSD(2,3)=0.0
      COSD(3,3)=CA
      PI1=A11*CA*CA+A33*SA*SA-A13*SIN(2.0*ALPHA)
      PI2=A22
      PI3=A11*SA*SA+A33*CA*CA+A13*SIN(2.0*ALPHA)
      GO TO 91
C
C     DIRECTION COSINES AND PRINCIPAL MOMENTS WHEN ZZ IS A PRINCIPAL
C     AXIS.
C
   23 ALPHA=.5*  ATAN((2.0*A12)/(A11-A22))
      CA=COS(ALPHA)
      SA=SIN(ALPHA)
      COSD(1,1)=CA
      COSD(2,1)=-SA
      COSD(3,1)=0.0
      COSD(1,2)=SA
      COSD(2,2)=CA
      COSD(3,2)=0.0
      COSD(1,3)=0.0
      COSD(2,3)=0.0
      COSD(3,3)=1.0
      PI1=A11*CA*CA+A22*SA*SA+A12*SIN(2.0*ALPHA)
      PI2=A11*SA*SA+A22*CA*CA-A12*SIN(2.0*ALPHA)
      PI3=A33
      GO TO 91
C
C     DIRECTION COSINES AND PRINCIPAL MOMENTS WHEN THE PRINCIPAL AXES
C     ARE ALIGNED WITH THE CENTER OF MASS SYSTEM
C
   24 COSD(1,1)=1.0
      COSD(2,1)=0.0
      COSD(3,1)=0.0
      COSD(1,2)=0.0
      COSD(2,2)=1.0
      COSD(3,2)=0.0
      COSD(1,3)=0.0
```

```
      COSD(2,3)=0.0
      COSD(3,3)=1.0
      PI1=A11
      PI2=A22
      PI3=A33
      GO TO 91
C
C     CONVERSIONS FOR OUTPUT BLOCK
C
   91 SWEIGH=SMASS*32.174049
      XBAR=XBAR*12.0
      YBAR=YBAR*12.0
      ZBAR=ZBAR*12.0
      DO 6 K=1,N
      MASS(K)=MASS(K)*32.174049
      XC(K)=XC(K)*12.0
      YC(K)=YC(K)*12.0
      ZC(K)=ZC(K)*12.0
      XWRTCG(K)=XBAR-XC(K)
      YWRTCG(K)=YBAR-YC(K)
    6 ZWRTCG(K)=ZBAR-ZC(K)
      GO TO 4
    8 WRITE(6,233)
  233 FORMAT(93H0EXECUTION HAS BEEN TERMINATED.  SOLUTION TO THE EIGENVA
     1LUE PROBLEM RESULTED IN COMPLEX ROOTS)
    4 CONTINUE
      RETURN
      END
```

```
$IBFTC OUT
      SUBROUTINE OUT(ICNT)
      COMMON/INPUT/ DESCRI(850,5),NCOMP(850),IPART(850),MODEL(850),
     *IDATE(850),MASS(850),IXC(850),IYC(850),IZC(850),XC(850),YC(850),
     * ZC(850),PXYO(850),PXZO(850),PYZO(850)
      COMMON/OUTPT/XWRTCG(850),YWRTCG(850),ZWRTCG(850),IXB(850),IYB(850)
     *,IZB(850),IXY(850),IXZ(850),IYZ(850),COSD(3,3),PI1,PI2,PI3,SWEIGH,
     *XBAR,YBAR,ZBAR
      COMMON/TENSOR/A11,A12,A13,A22,A23,A33
      EQUIVALENCE(A12,A21),(A13,A31),(A23,A32)
      REAL  IXY,IXZ,IYZ,MASS,IXC,IYC,IZC,IXB,IYB,IZB
      WRITE(6,213)
  213 FORMAT(1H1)
      WRITE(6,216)
  216 FORMAT(1HL)
      WRITE(6,215)
  215 FORMAT(30X,46H******  COMPONENT MASS CHARACTERISTICS   ******)
      WRITE(6,216)
C
C      INDIVIDUAL COMPONENT OUTPUT BLOCK
C
      N=ICNT
      JCNT=0
      DO 6 K=1,N
      JCNT=JCNT+1
      WRITE (6,200) JCNT,(DESCRI(K,J),J=1,5),NCOMP(K),IPART(K),MODEL(K),
     *IDATE(K)
  200 FORMAT(2X,9HLIST NO. ,I5,4X,5A6,9HCOMP. NO.,I6,10H PART NO. ,I6,
     111H MODEL NO. ,I6,19H    DATE OF ENTRY-   ,I6)
      WRITE(6,201) MASS(K),XC(K),YC(K),ZC(K)
 201  FORMAT(6X,8HWEIGHT= ,E12.4,18H    POSITION-    XC=,E11.4,
     *9H INCHES   ,6X,6H    YC=,E11.4,9H INCHES   ,6X,4H ZC=,E11.4,8H INCHE
     *S  )
      WRITE(6,251)  IXC(K),IYC(K),IZC(K)
  251 FORMAT(6X,38HINPUT MOMENT OF INERTIA-           IXX=,E11.4,
     *9H SLUG-FT2,8X,4HIYY=,E11.4,9H SLUG-FT2,6X,4HIZZ=,E11.4,9H SLUG-FT
     *2)
      WRITE(6,252) PXYO(K),PXZO(K),PYZO(K)
 252  FORMAT(6X,38HINPUT PRODUCT OF INERTIA-         PXYO=,E11.4,
     *9H SLUG-FT2,7X,5HPXZO=,E11.4,9H SLUG-FT2,5X,5HPYZO=,E11.4)
      WRITE(6,214) XWRTCG(K),YWRTCG(K),ZWRTCG(K)
  214 FORMAT(38H        COMPONENT POSITION WRT SYSTEM CM,7H CM-X= ,E10.3,
     19H INCHES   ,6X,7H CM-Y= ,E10.3,9H INCHES   ,6X,5HCMZ= ,E10.3,
     19H INCHES   )
      WRITE(6,212) IXB(K),IYB(K),IZB(K)
  212 FORMAT(38H        MOMENT OF INERTIA CONTRIBUTION -,7H  IXC= ,E10.3,
     19H SLUG-FT2,6X,7H  IYC= ,E10.3,9H SLUG-FT2,6X,5HIZC= ,E10.3,
     19H SLUG-FT2)
      WRITE(6,234) IXY(K),IXZ(K),IYZ(K)
  234 FORMAT(38H        PRODUCT OF INERTIA CONTRIBUTION-,7H  PXY= ,E10.3,
     19H SLUG-FT2,6X,7H  PXZ= ,E10.3,9H SLUG-FT2,6X,5HPYZ= ,E10.3,
```

```
     19H SLUG-FT2)
      WRITE(6,211)
  211 FORMAT(1HL)
    6 CONTINUE
C
C      INERTIA TENSOR OUTPUT BLOCK
C
      WRITE(6,213)
      WRITE(6,217)
  217 FORMAT(1H4)
      WRITE(6,218)
  218 FORMAT(40X,48HINERTIA  TENSOR  AT  SYSTEM  CENTER  OF  MASS
      WRITE(6,216)
      WRITE(6,219)
  219 FORMAT(15X,2H**,23X,2H**,11X,2H**,46X,2H**)
      WRITE(6,220)
  220 FORMAT(15X,2H* ,23X,2H *,11X,2H* ,46X,2H *)
      WRITE(6,221) A11,A12,A13
  221 FORMAT(15X,6H*  I11,6X,3HI12,6X,6HI13  *,11X,2H* ,E11.4,6X,
     1E11.4,6X,E11.4,3H  *)
      WRITE(6,220)
      WRITE(6,222) A21,A22,A23
  222 FORMAT(15X,6H*  I21,6X,3HI22,6X,6HI23  *,5X,1H=,5X,2H* ,E11.4,
     16X,E11.4,6X,E11.4,3H  *)
      WRITE(6,220)
      WRITE(6,223) A31,A32,A33
  223 FORMAT(15X,6H*  I31,6X,3HI32,6X,6HI33  *,11X,2H* ,E11.4,6X,
     1E11.4,6X,E11.4,3H  *)
      WRITE(6,220)
      WRITE(6,219)
      WRITE(6,217)
      WRITE(6,224)
  224 FORMAT(40X,37HPRINCIPAL  MASS  MOMENTS  OF  INERTIA)
      WRITE(6,211)
      WRITE(6,232)PI1,PI2,PI3
 232  FORMAT(8HOROOT1= ,E12.5,9H SLUG-FT2,6X,7HROOT2= ,E12.5,9H SLUG-FT2
     1,6X,7HROOT3= ,E12.5,9H SLUG-FT2)
C
C      DIRECTION COSINE OUTPUT BLOCK
C
      WRITE(6,225)
 225  FORMAT(1HL)
      WRITE(6,226)
  226 FORMAT(40X,32HDIRECTION COSINE ROTATION MATRIX)
      WRITE(6,216)
      WRITE(6,227)
      WRITE(6,228)
  227 FORMAT(25X,2H**,21X,2H**,13X,2H**,26X,2H**)
  228 FORMAT(25X,2H* ,21X,2H *,13X,2H* ,26X,2H *)
      WRITE(6,229) (COSD(1,J),J=1,3)
  229 FORMAT (25X,7H*  L1XB,4X,4HL1YB,4X,6HL1ZB *,13X,3H*  ,F7.4,1X,
     1F7.4,1X,F7.4,3X,1H*)
      WRITE(6,228)
      WRITE(6,230) (COSD(2,J),J=1,3)
  230 FORMAT(25X,7H*  L2XB,4X,4HL2YB,4X,6HL2ZB *,6X,1H=,6X,3H*
     1F7.4,1X,F7.4,1X,F7.4,3X,1H*)
```

```
      WRITE(6,228)
      WRITE(6,231) (COSD(3,J),J=1,3)
  231 FORMAT(25X,7H*  L3XB,4X,4HL3YB,4X,6HL3ZB *,13X,3H*  ,F7.4,1X,
     1F7.4,1X,F7.4,3X,1H*)
      WRITE(6,228)
      WRITE(6,227)
C
C     SYSTEM OUTPUT BLOCK
C
      WRITE(6,211)
      WRITE(6,209) SWEIGH
  209 FORMAT(16H SYSTEM WEIGHT= ,E12.5,7H POUNDS)
      WRITE(6,236)
      WRITE(6,205)
  205 FORMAT(30X,45HSYSTEM CENTER OF MASS WRT REFERENCE SYSTEM     )
      WRITE(6,206) XBAR,YBAR,ZBAR
  206 FORMAT(7H0XBAR= ,E12.5,7H INCHES,6X,7H YBAR= ,E12.5,7H INCHES,
     16X,7H ZBAR= ,E12.5,7H INCHES)
      WRITE(6,236)
      WRITE(6,208) N
  208 FORMAT(29H TOTAL NUMBER OF COMPONENTS= ,I6)
  236 FORMAT(1HJ)
      RETURN
      END
```

```
$IBFTC EIGEN
      SUBROUTINE EIGEN(EV1,EV2,EV3)
      EQUIVALENCE(A12,A21),(A31,A13),(A23,A32)
      COMMON/TENSOR/A11,A12,A13,A22,A23,A33
      COMMON/COEFF/P,Q,R
      P=-(A11+A22+A33)
      Q=-(-A11*A22-A11*A33-A22*A33+A12*A21+A13*A31+A32*A23)
      R=-(A11*A22*A33-A12*A21*A33+A12*A31*A23+A13*A21*A32-A13*A22*A31
     1-A32*A23*A11)
      CALL CUBIC(EV1,EV2,EV3)
      RETURN
      END
```

```
$IBFTC CUBIC
      SUBROUTINE CUBIC(R1,R2,R3)
      COMMON/COEFF/P,Q,R
      DATA CONRTD/57.2957795/
      A=(1.0/3.0)*(3.0*Q-P*P)
      B=(1.0/27.0)*(2.0*(P**3)-9.0*P*Q+27.0*R)
      IF((B*B/4.0+A*A*A/27.0).GT.0.0) GO TO 9
      IF((B*B/4.0+A*A*A/27.0).EQ.0.0) GO TO 7
C     THREE REAL AND UNEQUAL ROOTS
      PHI=ARCOS(-B/(2.0*SQRT(-(A*A*A)/27.0)))
      FACTOR=2.0*SQRT(-A/3.0)
      R1=FACTOR*COS(PHI/3.0)-P/3.0
      R2=FACTOR*COS((PHI/3.0)+(120.0/CONRTD))-P/3.0
      R3=FACTOR*COS((PHI/3.0)+(240.0/CONRTD))-P/3.0
      GO TO 5
C     THREE REAL ROOTS, AT LEAST TWO EQUAL
    7 AA=+(-B/2.0)**(1.0/3.0)
      R1=2.0*AA-P/3.0
      R2=-AA -P/3.0
      R3=-AA-P/3.0
      GO TO 5
C     ONE REAL ROOT, AND TWO CONJUGATE IMAGINARY
    9 R1=0.0
      R2=0.0
      R3=0.0
    5 RETURN
      END
```

APPENDIX D

SAMPLE CALCULATION

A sample problem and its solution are presented to demonstrate the preparation of input data and to illustrate the output format. Using the proper matrix relations, a check is made on the computed principal mass moments of inertia and the directions of the principal axes.

Problem

A composite body consists of six regular geometric shaped components located within a cylindrical shell. A top view of the body is presented in figure 5. Assume a reference system coincident with the geometric axes through the base of the shell. Component II has its principal axes rotated 45° from the reference axes. The principal moments of inertia of component II are

$$I_{x'} = 2.5 \text{ (slug-ft}^2\text{)}$$

$$I_{y'} = 2.5 \text{ (slug-ft}^2\text{)}$$

$$I_{z'} = 1.0 \text{ (slug-ft}^2\text{)}$$

The inertia tensor of component II must be found about axes parallel to the reference axis system. The relation between the principal moments of inertia and moments of inertia about axes parallel to the reference system is

$$[I] = [l]^T[I'][l] \tag{D1}$$

where $[l]$ is the direction cosine matrix between the principal axes and axes parallel to the reference system. For this case, $[l]$ represents a single rotation in the xy plane through α, or

$$[l] = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \tag{D2}$$

and

$$[I] = \begin{bmatrix} I_x & I_{xy} \\ I_{yx} & I_y \end{bmatrix} \tag{D3}$$

$$[I'] = \begin{bmatrix} I_{x'} & 0 \\ 0 & I_{y'} \end{bmatrix} \tag{D4}$$

Substituting equations (D2), to (D4) into equation (D1), then

$$\begin{bmatrix} I_x & I_{xy} \\ I_{yx} & I_y \end{bmatrix} = \begin{bmatrix} I_{x'} \cos^2\alpha + I_{y'} \sin^2\alpha & (I_{y'} - I_{x'})\sin \alpha \cos \alpha \\ (I_{y'} - I_{x'})\sin \alpha \cos \alpha & I_{x'} \sin^2\alpha + I_{y'} \cos^2\alpha \end{bmatrix} \tag{D5}$$

For component II, $\alpha = 45^\circ$, so equation (D5) becomes

$$\begin{bmatrix} I_x & I_{xy} \\ I_{yx} & I_y \end{bmatrix} = \begin{bmatrix} 1.3 & 0 \\ 0 & 1.3 \end{bmatrix}$$

The input data required for the problem is shown in table I. Table II shows the input card requirements.

The computer output for the problem is given in table III. The component mass characteristics are individually listed. The component moment of inertia contribution and product of inertia contribution represent the inertia of the component about the center of mass axis system. The inertia tensor is consistent with the definition of equation (6). The principal mass moments of inertia are numbered to correspond with the nomenclature of the direction cosine matrix defined by equation (13). In the output direction cosine matrix, L1XB represents the direction cosine between the axis about which ROOT1 is defined and the $\bar{x}$ axis of the center of mass coordinate system. The center of mass location is with respect to the reference axis system.

The matrix $[l]$ in equation (D1) is an orthogonal transformation and therefore premultiplying (D1) by $[l]$ and postmultiplying by $[l]^T$

$$[I'] = [l]\,[I]\,[l]^T \tag{D6}$$

From matrix theory, the trace of any square matrix is invariant under an orthogonal transformation. By inspection of the computer results, the trace of [I] equals the trace of [I']. As a further verification of the computer results, the matrices are substituted into equation (D6).

$$\begin{bmatrix} 0.2015 & 0.7473 & -0.6332 \\ .8440 & .1956 & .4994 \\ .4970 & -.6349 & -.5915 \end{bmatrix} \begin{bmatrix} 295.8 & -5.472 & -25.23 \\ -5.472 & 343.0 & -15.61 \\ -25.32 & -15.61 & 328.3 \end{bmatrix} \begin{bmatrix} 0.2015 & 0.8440 & 0.4970 \\ .7473 & .1956 & -.6394 \\ -.6332 & .4994 & -.5915 \end{bmatrix}$$

$$= \begin{bmatrix} 354.754 & 0.013 & -1.11 \\ .013 & 279.585 & -.252 \\ 1.083 & -.014 & 333.704 \end{bmatrix}$$

TABLE I. - SAMPLE PROBLEM INPUT DATA

Component	Description	Weight, lb	Center of mass, ft	Mass moment of inertia, slug-ft^2		
				I_{x_i}	I_{y_i}	I_{z_i}
I	Cylindrical shell: Length, 10 ft Inner radius, 24 in. Outer radius, 25 in.	640	(0, 0, 5)	208.4	208.4	83.4
II	Cube: 1 by 1 by 2 ft	160	(1, 1, 7)	1.3	1.3	1
III	Sphere: Radius, 1 ft	128	$\left(\frac{1}{2}, \frac{1}{2}, 3\right)$	1.6	1.6	1.6
IV	Thin plate: 1 by 1 ft	8	$\left(-\frac{1}{2}, -\frac{1}{2}, 5\frac{1}{2}\right)$	0.1	0.1	0.1
V	Cylinder: Length, 5 ft Radius, 1/2 ft	80	$\left(1, -1, 2\frac{1}{2}\right)$	5.4	5.4	0.3
VI	Cylinder: Length, 3 ft Radius, 1/2 ft	64	(-1, 1, 5)	1.6	1.6	0.3
VII	Cylinder: Length, 8 ft Radius, 1/2 ft	1024	$\left(-1\frac{1}{2}, 0, 4\right)$	4	4	174.6

TABLE II. - SAMPLE PROBLEM INPUT DATA

TITLE: SAMPLE PROBLEM INPUT DATA

PROJECT NUMBER | ANALYST | SHEET ____ OF ____

FORTRAN STATEMENT

```
00001000010000000
CYLINDRICAL SHELL             00001       00001      00001     090268
00001      0.0640+4    2.084+2 2.084+2     .834+2      .000+0     .000+0     .060+3
    .000+0     .000+0     .000+0
CUBE                          00002       00001      00001     090268
00002      0.0160+4     .013+2  .013+2     .001+3      .012+3     .012+3     .084+3
    .000+0     .000+0     .000+0
SPHERE                        00003       00001      00001     090268
00003      0.0128+4     .016+2  .016+2     .016+2      .006+3     .006+3     .036+3
    .000+0     .000+0     .000+0
THIN PLATE                    00004       00001      00001     090268
00004      0.0008+4     .100+0  .100+0     .100+0     -.006+3    -.006+3     .066+3
    .000+0     .000+0     .000+0
CYLINDER  L=5, R=1/2          00005       00001      00001     090268
00005      0.0008+5     .054+2  .054+2     .300+0      .012+3    -.012+3     .030+3
    .000+0     .000+0     .000+0
CYLINDER  L=3, R= 1/2         00006       00001      00001     090268
00006      0.0064+4     .016+2  .016+2     .300+0     -.012+3    +.012+3     .060+3
    .000+0     .000+0     .000+0
CYLINDER  L=8, R=1/2          00007       00001      00001     090268
00007      0.1024+4     .004+3  .004+3    1.746+2     -.018+3     .000+0     .048+3
    .000+0     .000+0     .000+0
END OF DATA                   00000       00000      00000     000000
00000      0.0000+0     .000+0  .000+0     .000+0      .000+0     .000+0     .000+0
    .000+0     .000+0     .000+0
```

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CD-6720

TABLE III. - SAMPLE PROBLEM OUTPUT LISTING

```
****** COMPONENT MASS CHARACTERISTICS ******

LIST NO.    1   CYLINDRICAL SHELL              COMP. NO.    1 PART NO.      1 MODEL NO.      1   DATE OF ENTRY-   90268
   WEIGHT=  0.6400E 03   POSITION-   XC= 0.           INCHES          YC= 0.           INCHES          ZC= 0.6000E 02 INCHES
   INPUT MOMENT OF INERTIA-          IXX= 0.2084E 03 SLUG-FT2        IYY= 0.2084E 03 SLUG-FT2        IZZ= 0.8340E 02 SLUG-FT2
   INPUT PRODUCT OF INERTIA-        PXYO= 0.          SLUG-FT2       PXZO= 0.          SLUG-FT2       PYZO= 0.
   COMPONENT POSITION WRT SYSTEM CM CM-X= -0.741E 01 INCHES          CM-Y=  0.116E 01 INCHES          CMZ= -0.659E 01 INCHES
   MOMENT OF INERTIA CONTRIBUTION -  IXC=  0.215E 03 SLUG-FT2         IYC=  0.222E 03 SLUG-FT2         IZC=  0.912E 02 SLUG-FT2
   PRODUCT OF INERTIA CONTRIBUTION-  PXY= -0.119E 01 SLUG-FT2         PXZ=  0.675E 01 SLUG-FT2         PYZ= -0.106E 01 SLUG-FT2

LIST NO.    2   CUBE                           COMP. NO.    2 PART NO.      1 MODEL NO.      1   DATE OF ENTRY-   90268
   WEIGHT=  0.1600E 03   POSITION-   XC= 0.1200E 02 INCHES          YC= 0.1200E 02 INCHES          ZC= 0.8400E 02 INCHES
   INPUT MOMENT OF INERTIA-          IXX= 0.1300E 01 SLUG-FT2        IYY= 0.1300E 01 SLUG-FT2        IZZ= 0.1000E 01 SLUG-FT2
   INPUT PRODUCT OF INERTIA-        PXYO= 0.          SLUG-FT2       PXZO= 0.          SLUG-FT2       PYZO= 0.
   COMPONENT POSITION WRT SYSTEM CM CM-X= -0.194E 02 INCHES          CM-Y= -0.108E 02 INCHES          CMZ= -0.306E 02 INCHES
   MOMENT OF INERTIA CONTRIBUTION -  IXC=  0.377E 02 SLUG-FT2         IYC=  0.466E 02 SLUG-FT2         IZC=  0.181E 02 SLUG-FT2
   PRODUCT OF INERTIA CONTRIBUTION-  PXY=  0.727E 01 SLUG-FT2         PXZ=  0.205E 02 SLUG-FT2         PYZ=  0.114E 02 SLUG-FT2

LIST NO.    3   SPHERE                         COMP. NO.    3 PART NO.      1 MODEL NO.      1   DATE OF ENTRY-   90268
   WEIGHT=  0.1280E 03   POSITION-   XC= 0.6000E 01 INCHES          YC= 0.6000E 01 INCHES          ZC= 0.3600E 02 INCHES
   INPUT MOMENT OF INERTIA-          IXX= 0.1600E 01 SLUG-FT2        IYY= 0.1600E 01 SLUG-FT2        IZZ= 0.1600E 01 SLUG-FT2
   INPUT PRODUCT OF INERTIA-        PXYO= 0.          SLUG-FT2       PXZO= 0.          SLUG-FT2       PYZO= 0.
   COMPONENT POSITION WRT SYSTEM CM CM-X= -0.134E 02 INCHES          CM-Y= -0.484E 01 INCHES          CMZ=  0.174E 02 INCHES
   MOMENT OF INERTIA CONTRIBUTION -  IXC=  0.106E 02 SLUG-FT2         IYC=  0.149E 02 SLUG-FT2         IZC=  0.722E 01 SLUG-FT2
   PRODUCT OF INERTIA CONTRIBUTION-  PXY=  0.179E 01 SLUG-FT2         PXZ= -0.645E 01 SLUG-FT2         PYZ= -0.233E 01 SLUG-FT2

LIST NO.    4   THIN PLATE                     COMP. NO.    4 PART NO.      1 MODEL NO.      1   DATE OF ENTRY-   90268
   WEIGHT=  0.8000E 01   POSITION-   XC=-0.6000E 01 INCHES          YC=-0.6000E 01 INCHES          ZC= 0.6600E 02 INCHES
   INPUT MOMENT OF INERTIA-          IXX= 0.1000E 00 SLUG-FT2        IYY= 0.1000E 00 SLUG-FT2        IZZ= 0.1000E 00 SLUG-FT2
   INPUT PRODUCT OF INERTIA-        PXYO= 0.          SLUG-FT2       PXZO= 0.          SLUG-FT2       PYZO= 0.
   COMPONENT POSITION WRT SYSTEM CM CM-X= -0.141E 01 INCHES          CM-Y=  0.716E 01 INCHES          CMZ= -0.126E 02 INCHES
   MOMENT OF INERTIA CONTRIBUTION -  IXC=  0.462E 00 SLUG-FT2         IYC=  0.377E 00 SLUG-FT2         IZC=  0.192E 00 SLUG-FT2
   PRODUCT OF INERTIA CONTRIBUTION-  PXY= -0.175E-01 SLUG-FT2         PXZ=  0.308E-01 SLUG-FT2         PYZ= -0.156E 00 SLUG-FT2

LIST NO.    5   CYLINDER L=5,R=1/2             COMP. NO.    5 PART NO.      1 MODEL NO.      1   DATE OF ENTRY-   90268
   WEIGHT=  C.8000E 02   POSITION-   XC= 0.1200E 02 INCHES          YC=-0.1200E 02 INCHES          ZC= 0.3000E 02 INCHES
   INPUT MOMENT OF INERTIA-          IXX= 0.5400E 01 SLUG-FT2        IYY= 0.5400E 01 SLUG-FT2        IZZ= 0.3000E 00 SLUG-FT2
   INPUT PRODUCT OF INERTIA-        PXYO= 0.          SLUG-FT2       PXZO= 0.          SLUG-FT2       PYZO= 0.
   COMPONENT POSITION WRT SYSTEM CM CM-X= -0.194E 02 INCHES          CM-Y=  0.132E 02 INCHES          CMZ=  0.234E 02 INCHES
   MOMENT OF INERTIA CONTRIBUTION -  IXC=  0.179E 02 SLUG-FT2         IYC=  0.214E 02 SLUG-FT2         IZC=  0.980E 01 SLUG-FT2
   PRODUCT OF INERTIA CONTRIBUTION-  PXY= -0.441E 01 SLUG-FT2         PXZ= -0.785E 01 SLUG-FT2         PYZ=  0.532E 01 SLUG-FT2

LIST NO.    6   CYLINDER    L=3,R=1/2          COMP. NO.    6 PART NO.      1 MODEL NO.      1   DATE OF ENTRY-   90268
   WEIGHT=  0.6400E 02   POSITION-   XC=-0.1200E 02 INCHES          YC= 0.1200E 02 INCHES          ZC= 0.6000E 02 INCHES
   INPUT MOMENT OF INERTIA-          IXX= 0.1600E 01 SLUG-FT2        IYY= 0.1600E 01 SLUG-FT2        IZZ= 0.3000E 00 SLUG-FT2
   INPUT PRODUCT OF INERTIA-        PXYO= 0.          SLUG-FT2       PXZO= 0.          SLUG-FT2       PYZO= 0.
   COMPONENT POSITION WRT SYSTEM CM CM-X=  0.459E 01 INCHES          CM-Y= -0.108E 02 INCHES          CMZ= -0.659E 01 INCHES
   MOMENT OF INERTIA CONTRIBUTION -  IXC=  0.382E 01 SLUG-FT2         IYC=  0.249E 01 SLUG-FT2         IZC=  0.221E 01 SLUG-FT2
   PRODUCT OF INERTIA CONTRIBUTION-  PXY= -0.686E 00 SLUG-FT2         PXZ= -0.418E 00 SLUG-FT2         PYZ=  0.987E 00 SLUG-FT2

LIST NO.    7   CYLINDER L=8,R=1/2             COMP. NO.    7 PART NO.      1 MODEL NO.      1   DATE OF ENTRY-   90268
   WEIGHT=  0.1024E 04   POSITION-   XC=-0.1800E 02 INCHES          YC= 0.           INCHES          ZC= 0.4800E 02 INCHES
   INPUT MOMENT OF INERTIA-          IXX= 0.4000E 01 SLUG-FT2        IYY= 0.4000E 01 SLUG-FT2        IZZ= 0.1746E 03 SLUG-FT2
   INPUT PRODUCT OF INERTIA-        PXYO= 0.          SLUG-FT2       PXZO= 0.          SLUG-FT2       PYZO= 0.
   COMPONENT POSITION WRT SYSTEM CM CM-X=  0.106E 02 INCHES          CM-Y=  0.116E 01 INCHES          CMZ=  0.541E 01 INCHES
   MOMENT OF INERTIA CONTRIBUTION -  IXC=  0.108E 02 SLUG-FT2         IYC=  0.352E 02 SLUG-FT2         IZC=  0.200E 03 SLUG-FT2
   PRODUCT OF INERTIA CONTRIBUTION-  PXY=  0.272E 01 SLUG-FT2         PXZ=  0.126E 02 SLUG-FT2         PYZ=  0.139E 01 SLUG-FT2

LIST NO.    8   END OF DATA                    COMP. NO.    0 PART NO.      0 MODEL NO.      0   DATE OF ENTRY-       0
   WEIGHT=  0.               POSITION-   XC= 0.           INCHES          YC= 0.           INCHES          ZC= 0.           INCHES
   INPUT MOMENT OF INERTIA-          IXX= 0.           SLUG-FT2        IYY= 0.           SLUG-FT2        IZZ= 0.           SLUG-FT2
   INPUT PRODUCT OF INERTIA-        PXYO= 0.           SLUG-FT2       PXZO= 0.           SLUG-FT2       PYZO= 0.
   COMPONENT POSITION WRT SYSTEM CM CM-X= -0.741E 01 INCHES          CM-Y=  0.116E 01 INCHES          CMZ=  0.534E 02 INCHES
   MOMENT OF INERTIA CONTRIBUTION -  IXC=  0.           SLUG-FT2         IYC=  0.           SLUG-FT2         IZC=  0.           SLUG-FT2
   PRODUCT OF INERTIA CONTRIBUTION-  PXY= -0.           SLUG-FT2         PXZ= -0.           SLUG-FT2         PYZ=  0.           SLUG-FT2
```

TABLE III. - Concluded. SAMPLE PROBLEM OUTPUT LISTING

```
                           INERTIA  TENSOR  AT  SYSTEM  CENTER  OF  MASS


   **                              **       **                                                       **
   *                                *       *                                                         *
   *  I11       I12       I13       *       *  0.2958E 03       -0.5472E 01       -0.2523E 02   *
   *                                *       *                                                         *
   *  I21       I22       I23       *   =   * -0.5472E 01        0.3430E 03       -0.1561E 02   *
   *                                *       *                                                         *
   *  I31       I32       I33       *       * -0.2523E 02       -0.1561E 02        0.3283E 03   *
   *                                *       *                                                         *
   **                              **       **                                                       **

                           PRINCIPAL  MASS  MOMENTS  OF  INERTIA


ROOT1=  0.35479E 03 SLUG-FT2       ROOT2=  0.27959E 03 SLUG-FT2       ROOT3=  0.33279E 03 SLUG-FT2


                           DIRECTION COSINE ROTATION MATRIX


              **                       **          **                              **
              *                         *          *                                *
              *  L1XB    L1YB    L1ZB   *          *  0.2015   0.7473  -0.6332   *
              *                         *          *                                *
              *  L2XB    L2YB    L2ZB   *    =     *  0.8440   0.1956   0.4994   *
              *                         *          *                                *
              *  L3XB    L3YB    L3ZB   *          *  0.4970  -0.6349  -0.5915   *
              *                         *          *                                *
              **                       **          **                              **


SYSTEM WEIGHT=  0.21040E 04 POUNDS
                       SYSTEM CENTER OF MASS WRT REFERENCE SYSTEM
XBAR= -0.74144E 01 INCHES       YBAR=  0.11635E 01 INCHES       ZBAR=  0.53407E 02 INCHES

TOTAL NUMBER OF COMPONENTS=        8
```

REFERENCES

1. Housner, George W.; and Hudson, Donald E.: Applied Mechanics: Dynamics. Second ed., D. Van Nostrand, Inc., 1959.
2. Greenwood, Donald T.: Principles of Dynamics. Prentice-Hall, Inc., 1965.
3. Beer, Ferdinand P.; and Johnston, E. Russel, Jr.: Vector Mechanics for Engineers. McGraw-Hill Book Co., Inc., 1962.

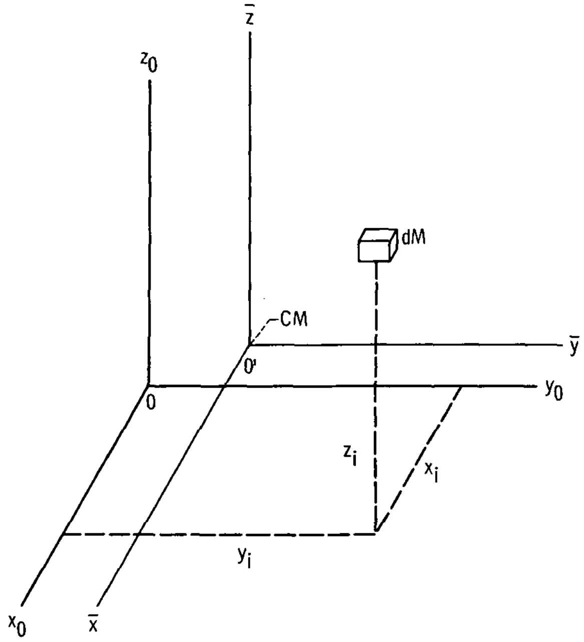

Figure 1. - Coordinate axes definition.

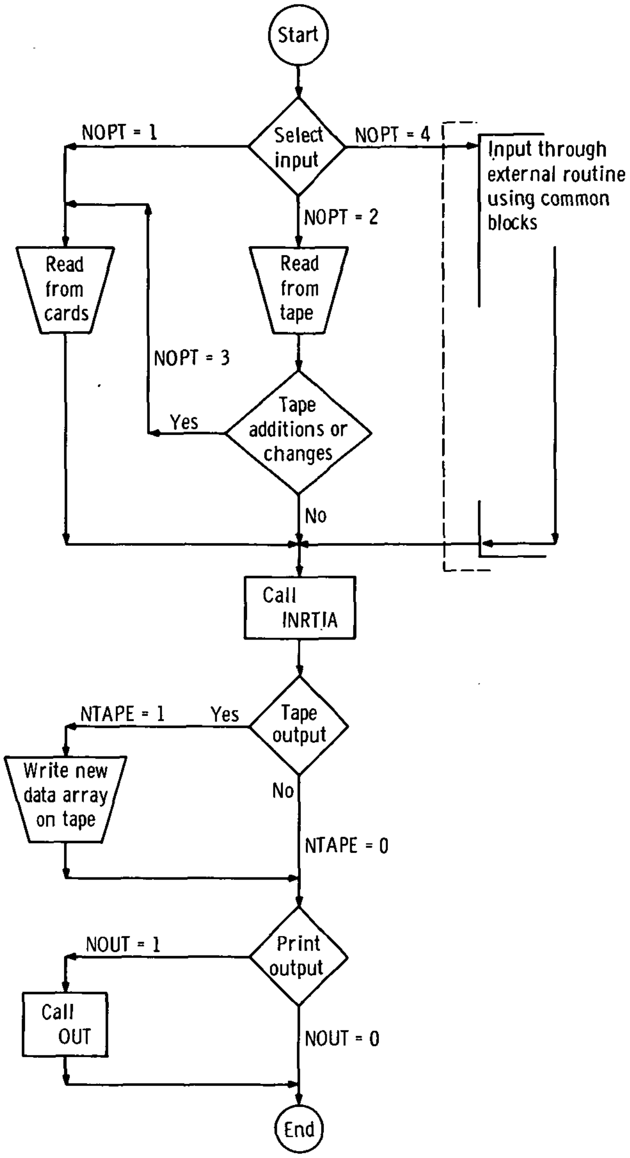

Figure 2. - READ is used as Main Program.

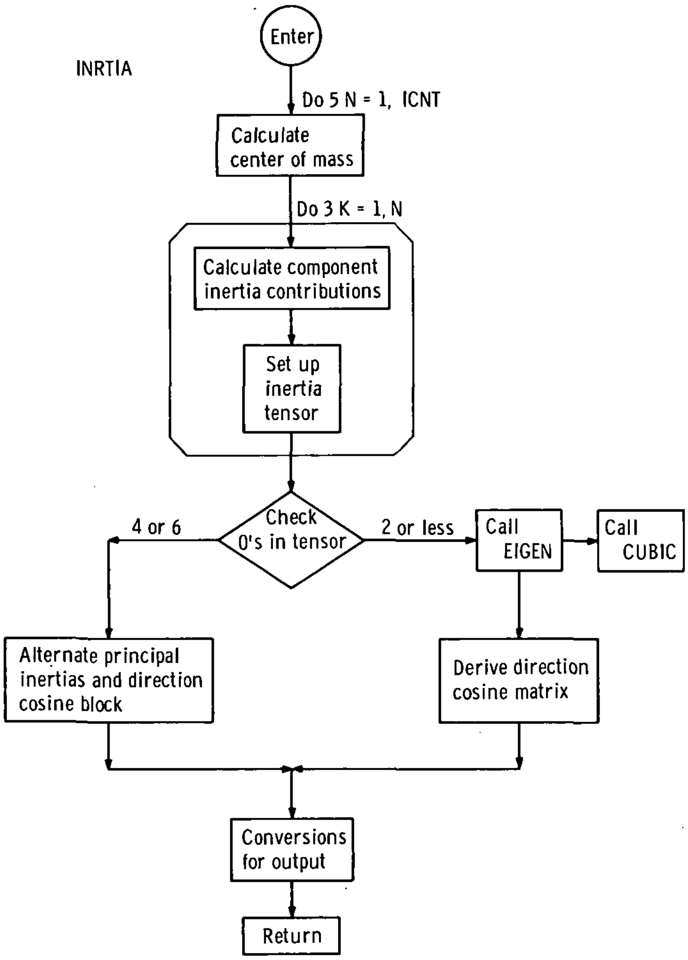

Figure 3. - INRTIA calculates principal moments of inertia and direction of principal axes.

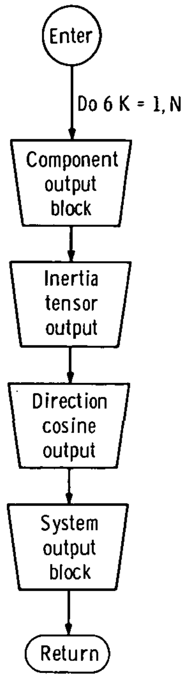

Figure 4. - OUT prints mass characteristics of each component and entire system.

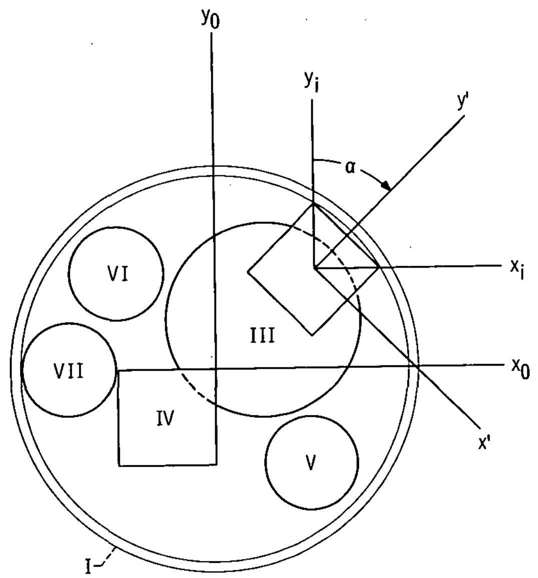

Figure 5. - Sample problem definition.

NASA-Langley, 1969 —— 23 E-4775

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