NASA TECHNICAL NOTE

NASA TN D-5105

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NASA TN D-5105

EFFECT OF GRAVITATIONAL-MODEL SELECTION ON ACCURACY OF LUNAR ORBIT DETERMINATION FROM SHORT DATA ARCS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1969

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SUMMARY

An analysis of orbit-determination accuracy has been made by using tracking data obtained during Lunar Orbiter Missions I and III. The analysis was performed mainly to study the effects of varying the lunar gravitational model and data-arc length on the accuracy of orbit determination. A brief study was also made of the effect of model selection on accuracy of predicting pericynthion altitude.

The results of the analysis are indicative of the accuracy of lunar orbit determination that may be expected with current gravitational models. Calculations are included which show that significant errors in determining spacecraft position and velocity are highly correlated with orbital-plane-orientation errors about the earth-moon vector; that is, the orientation of the lunar orbit cannot be precisely determined by earth-based Doppler measurements.

The analysis shows that both the choice of spherical harmonic coefficients used in the gravity-potential expansion and the choice of data-arc length can have significant effects on the accuracy of orbit determination. Simply solving for more coefficients can reduce the accuracy, as can indiscriminate use of longer data arcs. The best accuracy was obtained for a 10-hour data arc. Of the various sets of spherical harmonic coefficients solved for, there were several sets of 11 coefficients which appeared to produce equal accuracy.

The inability of a 21-coefficient model to improve the accuracy of orbit determination, together with the appearance of relatively large residuals at pericynthion in any solution, indicates that some radical change in the gravitational model may be required to represent the gravity field near the lunar surface accurately enough to take full advantage of the high inherent accuracy of the Doppler measurements. Another alternative for obtaining increased accuracy, as shown by the high correlation between uncertainty in state and uncertainty in orbit-plane orientation about the earth-moon vector, would be the use of measurements other than Doppler which would be sensitive to changes in orientation of the orbital plane about the earth-moon vector.

INTRODUCTION

After the five Lunar Orbiter missions in 1966 and 1967, a number of analyses to determine the characteristics of the lunar gravitational field were performed on the vast amounts of tracking data collected. (See, for example, refs. 1 to 5.) A knowledge of this field and its perturbative effects on a satellite is required to correctly define the orbit of the satellite. Although a rigorous description of the gravitational field may consist of an infinite number of coefficients, orbit-determination programs are limited as to the number of gravitational coefficients that can be determined in a solution. For example, the JPL (Jet Propulsion Laboratory) orbit-determination program (ref. 6) used to control the Lunar Orbiter spacecraft was limited to determining, in any one solution, up to only 11 coefficients. Error analyses of the Lunar Orbiter data (ref. 7) have shown up to 5-kilometer differences in locating a given lunar photographic site from different orbital passes and/or missions. This site-location error is directly related to the orbit-determination error.

The purpose of the present report is to give an insight into the accuracy of lunar orbit determination which may be expected with gravitational models having relatively few coefficients. The gravitational models investigated included the spherical harmonics through the fourth order. The results of the analysis were obtained by fitting tracking data from Lunar Orbiter Missions I and III. Relatively short data arcs of a few orbital periods were used in determining the orbit. For the most part, the JPL program was used for the data fits; additional results are presented in which up to 21 coefficients were determined from the Lungfish (lunar gravitational field in spherical harmonics) orbit-determination program.

The orbit-determination accuracy was analyzed by studying the effects of varying the gravitational model and the length of the data arc. In certain cases the gravitational coefficients were solved for and in others the coefficients were held fixed at predetermined values. Various criteria used to compare the cases included results from the fitted data arc as well as data predicted ahead of this arc.

SYMBOLS

$C_{n,m}, S_{n,m}$ coefficients of lunar gravitational potential harmonics (n is degree, m is order)

i inclination of orbital plane

$J_{n,m}$ zonal coefficient, $-C_{n,m}$

$P_{n,m}$	associated Legendre function
R	mean lunar radius (1738.09 kilometers)
r	distance from center of moon to satellite, $(x^2 + y^2 + z^2)^{1/2}$
$\overline{s}$	vector from moon to earth
U	lunar gravitational potential function
V	selenocentric velocity of satellite, $(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2}$
X,Y,Z	selenocentric Cartesian coordinate system in which X-axis is in the direction of Aries, XY-plane is parallel to earth equatorial plane, and Z-axis is in the direction of north celestial pole
x,y,z	satellite-position coordinates in X,Y,Z system
$\dot{x},\dot{y},\dot{z}$	satellite-velocity coordinates in X,Y,Z system
Δ	incremental value, for example, $\Delta x = x_i - x_r$
$\overline{\Delta r}$	position-deviation vector, $\Delta r = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}$
$\overline{\Delta V}$	velocity-deviation vector, $\Delta V = (\Delta \dot{x}^2 + \Delta \dot{y}^2 + \Delta \dot{z}^2)^{1/2}$
δ	angle between error vector and line perpendicular to angular-momentum vector
θ	rotation angle of orbital plane about earth-moon vector
λ	longitude of ascending node
μ	product of universal gravitational constant and mass of moon
σ	standard deviation in residuals of Doppler frequency, cps (1 cps = 1 hertz and 1 cps $\approx$ 0.07 meter/second)
ϕ	latitude, referenced from lunar equatorial plane

ψ	longitude, referenced counterclockwise from earth-moon vector

Subscripts:

a	actual
c	calculated
i	case number
r	selected as reference case
p	pericynthion

Notation:

$\| \|$	magnitude of vector or absolute value

A bar over a symbol indicates a vector.

METHOD OF ANALYSIS

The primary purpose of the analysis was to gain an indication of the accuracy of determining the lunar orbit. The state of the lunar satellite is defined by the position and velocity vectors at epoch, where epoch is defined as that point in time at the beginning of the data-fitting interval. The position and velocity vectors are given in a selenocentric (of date) Cartesian coordinate system. The satellite state is determined by a least-squares fit of the tracking measurements over a given interval of time (data arc). In this paper, short data arcs of about 1/2 day were chiefly used; longer data arcs up to several weeks were considered only for predicting pericynthion altitude. The determination of the orbital state and the solution of the gravitational field which consists of a number of coefficients constitute a single problem for which solutions may be obtained simultaneously. Alternatively, the gravitational coefficients can be held fixed in value and only the state solved for. Both methods are used in the present paper to show the corresponding effects on the orbit-determination results.

In either method the lunar gravitational field is approximated by a finite number of the coefficients in the infinite-series expansion of the lunar gravitational potential function in spherical harmonics, where m is the order and n is the degree of the harmonic

$$U = \frac{\mu}{r}\left[1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^n P_{n,m}(\sin \phi)\left(C_{n,m} \cos m\psi + S_{n,m} \sin m\psi\right)\right]$$

The ideal situation is such that enough of the $C_{n,m}$ and $S_{n,m}$ coefficients are known to give a sufficiently accurate representation of the gravitational field.

The JPL orbit-determination program (ref. 6) with some modifications was used for most of the present analysis. This program, along with the modifications, was used in real-time orbit determination for the Lunar Orbiter missions. Although it has the capacity of including 21 gravitational harmonic coefficients through the fourth order in the trajectory package, the program has the capability of solving for only 11 coefficients at a time. The so-called direct method of orbit determination is employed in which the actual observable (tracking measurement) is compared directly with the calculated observable. The calculated observable is determined by using a set of nominal (estimated) orbital and gravitational parameters. (When solving for gravitational parameters, their nominal values may be selected as zero.) The parameters are differentially corrected in the usual weighted least-squares manner to minimize the weighted sum of squares of the differences between the calculated and actual observable. Data for converged solutions only are presented in this paper, where convergence is defined as that point in the iterative process when the solution stabilizes.

Although range measurements were available, only Doppler measurements were used in the present analysis. The value of μ was held at a fixed value of 4902.58 km^3/sec^2.

Several cases which included differences in the gravitational model (number and values of coefficients considered) and in the length of data arc were investigated. (See table I.) In some cases the gravitational coefficients were solved for along with the state; in others their values were held fixed.

The different cases are analyzed by comparing the data within the fitted data arc as well as data predicted over periods of time which extend beyond the fitted arc. The data within the fitted arc include the converged initial-state values for the various cases, the associated values of the gravitational coefficients, the standard deviations of the fitted residuals (differences between predicted and observed Doppler data), some of the orbital parameters, and the time histories of the pericynthion residuals. The predicted data were obtained by using each solution to extend the orbit beyond the fitted data. These predictions include the standard deviations of the residuals for times up to about 2 days beyond epoch, values of predicted satellite longitude and latitude at a point in time near pericynthion about $1\frac{1}{2}$ days from epoch, and time histories of predicted pericynthion radius for periods up to several weeks. Also, calculations are presented to show how deviations in the state are related to the orbital-plane-orientation error about the earth-moon vector.

RESULTS AND DISCUSSION

Analysis of Orbit-Determination Accuracy

The orbit-determination-accuracy results were obtained primarily from Lunar Orbiter Mission III data and are shown in tables II to X and figures 1 to 6. Figure 1 shows the orientation of the lunar orbital plane with respect to the selenocentric coordinate system. Some pertinent angles are also shown in order to describe the orbit, which had a 209-minute period. At epoch the spacecraft was about 6.6° above the lunar equator and at pericynthion it was about 0.3° below the lunar equator. The earth-moon vector was 8.5° below the orbital plane and the angle between this line and $\bar{r}$ was 143.66°.

Figure 2 summarizes the tracking data used for the analysis. As previously noted, only the Doppler measurements were used and these were generally made at 1-minute intervals. Only the two-way Doppler (same station receiving as sending) data were used in analyzing the Mission III data, and no frequency bias existed in the data. For all cases epoch occurred at 16 hours and 15 minutes on February 17, 1967, and all the predict arcs were the same.

Lunar models.- The analysis basically consisted of the 23 cases in table I. For completeness several additional cases were investigated and their results are reported herein. In the first five cases, only the state $(x,y,z,\dot{x},\dot{y},\dot{z})$ was solved for with the gravitational coefficients held at fixed values as described in tables I and II. In case 5, about 25 percent of the tracking data were deleted; these data covered times of about 20 minutes before and after pericynthion passage. In cases 6 to 20, 11 coefficients were included in the solution and the remaining coefficients were fixed at Lungfish values or at zero. The coefficient $J_{2,0}$ was held fixed in all cases except case 20. Cases 17 and 18 differ only in that the unsolved coefficients were fixed at Lungfish values in case 17 and were fixed at zero in case 18.

The Lungfish orbit-determination program, which can be used to solve for a large number of gravitational coefficients, was specifically developed by NASA for analyzing the lunar gravitational field. (See refs. 1 to 3.) The set of coefficients shown in the second column of table II was determined from this Lungfish program and is based on 9 days of tracking data in the final orbit of Mission I. The coefficient $J_{2,0}$ was held fixed at a value of 2.07×10^{-4}. The other coefficients shown in table II are the converged solutions for the various cases. In most cases the coefficients which were not solved for were fixed at zero.

The results for case 23, in which more than 11 coefficients were solved for, were determined from the Lungfish orbit-determination program. This program and the JPL program, which was used for the other 22 cases, give comparable results, as illustrated in table III. Initial-position data determined by these two programs are included in figure 6 and show a similar comparison.

The TRW set of coefficients referred to in table I is a special set developed in reference 8 by TRW Systems. This set applies to low-inclination lunar orbits and contains, for the most part, the most sensitive coefficients; that is, those coefficients which are considered to have the greatest effect on the trajectory. The coefficients $J_{2,0}$, $J_{3,0}$, $J_{4,0}$, and $C_{2,2}$ are arbitrarily included for other reasons; for example, $J_{3,0}$ is known to have a relatively large effect on pericynthion-radius variation. (For example, see ref. 9.)

A converged solution could not be achieved when solving for the TRW set by using arcs greater than 600 minutes. A modified set was therefore chosen in case 8 and cases 12 to 15 wherein two different coefficients were used. (See table II.) This modified set produced lower correlation coefficients and convergence could be achieved with the longer arcs.

Effect of lunar model.- Table IV is presented to illustrate the large deviations in the state solution caused by the selection of lunar gravitational coefficients for the orbit determination. For all cases in this table, the data arc was 800 minutes and the coefficients used were held fixed at the Lungfish values (table II) during the iterative solution of state. The last five cases progress in order according to the sensitivity of the indicated coefficients (ref. 8); that is, the coefficient $S_{4,1}$ is the most sensitive, and so on. The selection of coefficients may cause differences up to 43 kilometers in radius and up to 14.5 m/sec in velocity.

Statistical results.- The statistics of the converged solutions are presented in tables V and VI. In table V the statistics of the fitted data are given, and in table VI the predicted statistics are given. Time histories of the fitted residuals for the 40-minute periods before and after pericynthion are shown for some of the cases in figure 3. These data depict the pericynthion fluctuations which were experienced in the orbit-determination procedures during the Lunar Orbiter missions. (For example, see ref. 2.) As indicated in table V and in figure 3, the lowest residuals, that is, the best fits, occur when solving for the TRW sets, the lowest order set, or the lowest degree set of coefficients with the shorter data arcs. (In general, see cases 9 to 14 and 17 to 20.) Case 23, in which all coefficients except $J_{2,0}$ were solved for, appears also to be a good fit; however, the data in table VI indicate that this case did not give as good a prediction for the longer arc.

Shown in table VI are the standard deviations in the Doppler residuals for tracking data beyond the fitted arc. (See fig. 2.) These results indicate how well the solution predicts the trajectory ahead in time and can be used as a measure of the authenticity of the orbit determined from the fitted data arc. Two prediction lengths were used and the beginning of each arc was taken 800 minutes after the epoch. The best long-term predictions occur for the cases in which both sets of TRW coefficients were solved for in the 500- and 600-minute data arcs (cases 10, 11, 13, and 14) and for the cases in which the 11 lowest order or 11 lowest degree coefficients were solved for in the 600-minute arc (cases 18 and 19). For cases 6, 7, and 8, in which coefficients in the longer data arc of 800 minutes were solved for, the prediction accuracy rapidly deteriorated as the predict arc was lengthened. For an indication of the effect of shortening the data arc, the 11 lowest order and 11 lowest degree coefficients were solved for in 300- and 400-minute data arcs. The prediction accuracies for these cases (not shown) were found to be poor. As shown in table VI, the prediction accuracy is comparatively poor and about the same, regardless of the fitted data-arc length, for cases 1 to 5, in which the coefficients were held fixed. When solving for some of the coefficients, the remaining coefficients should be fixed at zero rather than at some predetermined value, as shown by comparison of cases 17 and 18.

The prediction accuracy when solving for the four lowest degree coefficients (case 21) is as good as that for the better sets of 11 coefficients (cases 10, 11, 13, 14, 18, and 19), although the fit to the tracking data around the moon for case 21 is somewhat less accurate. (See fig. 3 and table V.) These results, however, imply that solving for fewer than 11 coefficients may yield an accuracy equivalent to the best cases shown in table VI. A case in which the four lowest order coefficients $\left(C_{2,1},\ C_{3,1},\ S_{2,1},\ S_{3,1}\right)$ were solved for was tried with a data arc of 600 minutes. Although the results of this case are not shown herein, they are poor compared with those of case 21, with the fitted and predicted standard deviations being about the same as those of case 1.

As shown in table VI, solving for all coefficients except $J_{2,0}$ in a data arc of 600 minutes (case 23) led to a larger standard deviation for the longer predict arc than the better 11-coefficient cases (cases 11, 14, 18, and 19). Also, the short-arc prediction accuracy was generally not improved. For this same case, but with a fitted data arc of 800 minutes (not shown in the table), the prediction standard deviation increased above that for the shorter fitted arc of 600 minutes. For a case which solved for all coefficients including $J_{2,0}$ in a fitted data arc of 600 minutes (also not shown in the table), the prediction standard deviation increased. Hence, there is no apparent advantage in solving for the extra coefficients.

Comparison of state.- The results on the initial-condition determination are shown in table VII, where the effects of varying the lunar gravitational model and length of data arc on the state solution are compared. In this table data are presented in terms of deviations from a reference case. The choice of the reference case was somewhat arbitrary. Case 14 was chosen because it contained low fitted and predicted residuals. (See tables V and VI.)

The importance of the values of the coefficients, at least in an incomplete gravitational model, is indicated by comparing data from cases 1 and 23 where the coefficients are treated differently. In case 1 the coefficients are fixed and in case 23 they are solved for. It is shown in table VII that values for state are similar for both cases; however, in table IV standard deviations are shown to be considerably different, especially for the shorter predict arc. The large effect of the coefficients on prediction accuracy is also shown in table VI by comparing the two sets of data for case 19. In making this comparison it is seen that the state for cases 2 and 19 (table VII) is about the same, but that when the Lungfish values used in case 2 are used in case 19, the prediction accuracy is decreased considerably.

In relation to the present investigation, several additional items of interest are noted concerning the length of the data arc:

1. Convergence was not achieved when solving for the TRW set of coefficients by using 700- and 800-minute data arcs but was achieved when the set was modified by changing two coefficients.

2. Convergence was not achieved when solving for the modified set of TRW coefficients in a 1200-minute data arc when data from three tracking stations were used. However, convergence was achieved for this case when all data from the tracking station at Goldstone were deleted. As shown in figure 2, the data deleted amounted to about 7 hours of tracking in the middle of the data arc.

3. Convergence was not achieved when solving for the modified set of TRW coefficients in a 1600-minute data arc when data from either two or three stations were used.

Based on these results and those presented in tables VI and VII, apparently the optimum data-arc length is about 600 minutes. Increases in the length beyond this point tend to be detrimental, as signified by the increase in magnitude of the prediction standard deviation. The analysis in reference 10 also indicates that three orbits of tracking (about 600 minutes) provide the maximum length practical for short-period fits.

Comparison of orbital parameters.- The orbital parameters which are shown for some of the cases in table VIII were derived for the time of pericynthion passage. The converged solution for the size (r_p and semimajor axis) and shape (eccentricity) has relatively small variation between the cases. Some of the deviations shown for the angles i and λ, however, are large in that they represent differences up to about 14 kilometers on the lunar surface. Variations in the predicted values of spacecraft latitude and longitude when the spacecraft is relatively near the moon (255 kilometers away) are given in table IX. The incremental-surface-distance data were determined from the resultant of $\Delta\phi$ and $\Delta\psi$ values and indicate the magnitude of the position error with respect to a point on the moon.

Orbital-plane rotation about earth-moon vector.- As shown in table VIII, converged solutions for various lunar models lead to noticeable differences in the angular orientations (i and λ) of the orbit. A comparison of calculated and actual magnitudes of state is presented in table X to show the relation between the orbital-plane-orientation error and the differences shown for state in table VII. Reference 11 shows that if the line from the tracking station to the center of the moon does not change its direction, that is, if the moon remains stationary, to completely determine the state of the satellite with range and/or range-rate measurements would be impossible. The orientation of the orbital plane of the satellite about the earth-moon vector could not be determined because the range or range-rate time histories would be the same for any other orientation. As the moon is allowed to rotate about the earth, this indeterminacy is alleviated by using long data arcs. The stationary-moon effect is also discussed in references 12 and 13. Data which show that the differences in cases 1 to 23 are generally correlated with this orientation error are presented in table X.

If the variations in $\bar{r}$ and $\bar{V}$ from the reference case correspond to an orbital-plane rotation about the earth-moon vector $\bar{s}$ (which has x-, y-, and z-components of -207296.04, -305389.63, and -144730.39 kilometers, respectively), then the cosine of the angle δ, which is illustrated in figure 4, will equal unity. This value implies δ equals zero.

The cosine of δ is given by

$$\cos\delta = \frac{\Delta(\bar{r} \times \bar{V}) \cdot \left[\bar{s} \times (\bar{r} \times \bar{V})\right]_r}{\left|\Delta(\bar{r} \times \bar{V})\right| \left|\left[\bar{s} \times (\bar{r} \times \bar{V})\right]_r\right|}$$

where the vector $\left[\bar{s} \times (\bar{r} \times \bar{V})\right]_r$ gives the direction of a change in the angular-momentum vector produced by a rotation about the earth-moon vector. The vector $\Delta(\bar{r} \times \bar{V})$ is the difference between the angular-momentum vector $(\bar{r} \times \bar{V})$ of the reference orbit and that of the case in question. As shown in table X, the angle δ is nearly equal to zero for most cases; hence, the errors (changes) in initial state are primarily due to a rotation of the orbital plane about the earth-moon vector.

The magnitude of the angular change between each case and the reference case is approximately $\frac{\left|\Delta(\bar{r} \times \bar{V})\right|}{\left|(\bar{r} \times \bar{V})_r\right|}$ and the angle of rotation of the orbital plane about the earth-moon vector is

$$\theta \approx \frac{\left|\Delta(\bar{r} \times \bar{V})\right|}{\left|(\bar{r} \times \bar{V})_r\right|} \cos \delta$$

$$\approx \frac{\Delta(\bar{r} \times \bar{V}) \cdot \left[\bar{s} \times (\bar{r} \times \bar{V})\right]_r}{\left|(\bar{r} \times \bar{V})_r\right| \left|\left[\bar{s} \times (\bar{r} \times \bar{V})\right]_r\right|}$$

The angle θ is shown for the different cases in table X; also shown are the calculated values of $\left|\overline{\Delta r}\right|$ and $\left|\overline{\Delta V}\right|$ which are the changes that this rotation would produce. These calculations are illustrated in figure 5. Comparison of the calculated values with the actual values indicates that most of the changes in $\bar{r}$ and $\bar{V}$ are correlated with this change in rotation. This correlation is further illustrated by figure 6, in which the Δy and Δz values of the position deviation (table VII) are plotted for each case. The position-deviation vectors can be shown in this two-dimensional manner inasmuch as the Δx component in each case is nearly zero. Also shown in figure 6 is a line representing the ratio of the Δy and Δz components ($\Delta x \approx 0$) of the cross product of $\bar{s}$ and $\bar{r}_r$, which is the direction of a change in $\bar{r}$ due to rotation about the earth-moon vector $\bar{s}$. Perfect agreement of the data points with the line would indicate errors solely due to rotation about $\bar{s}$. A comparison similar to that shown in figure 6 would be obtained for the velocity data of table VII; however, these data are not plotted because of the three dimensions involved.

It can be concluded that the differences in the state solution are directly attributed to the orbital-plane-orientation error about the earth-moon vector. For improved accuracy in orbit determination, even with a good knowledge of the gravitational model, other data-measuring methods (see ref. 14) may be required along with the Doppler measurements to increase the orientation accuracy of the orbit.

Pericynthion Variation

Some results from Mission I data are presented in figures 7 to 10 which show the long-time effect of lunar-model selection on pericynthion altitude. Although the analysis is brief, the results are of interest because of the direct relationship between the time history of predicted pericynthion altitude and satellite-lifetime prediction.

Lunar models.- The lunar gravitational models used for the analysis are given in table XI. The coefficients for the two models were derived from the initial high-orbit tracking data of Mission I (before the pericynthion was lowered from 189 to 56 kilometers). The coefficients for model 2 were derived from 4 days of tracking data by using the Lungfish program. The coefficients for model 1 are the TRW set of most sensitive coefficients and are the converged solutions from a 750-minute data arc. A summary of the tracking data fitted at 1-minute intervals for this model is shown in figure 7. The data available included 321 and 204 points of two-way Doppler measurements (same station receiving as sending) from tracking stations at Goldstone, California, and Woomera, Australia, respectively, and 42 points of three-way Doppler measurements (one station sending, another receiving) from the tracking station at Madrid, Spain. No frequency bias existed in the data.

Effect of lunar model.- Time histories of predicted pericynthion altitude based on the coefficients in table XI are shown in figures 8 to 10. The data in these figures are from the lower altitude orbit of Mission I and start at a time which is about 12 days beyond epoch of the fitting arcs used to derive the coefficients. In each figure the predicted results are compared with the measured time history of pericynthion altitude. The measured time history corresponds to real-time orbit determinations from short data arcs (about 1/2 day for each data point) and are considered very accurate for determining pericynthion radius. The pericynthion altitudes in the figures were obtained by subtracting the value of 1738.09 kilometers from the pericynthion-radius value. The initial values of state used for the prediction time histories in figures 8 to 10 were determined by fitting several weeks of tracking data obtained during the time span represented in the figures. The Lungfish orbit-determination program was used for the fit in which a 20-coefficient lunar model was solved for, and because of the long data arc, the accuracy of the state solution is considered adequate for predicting pericynthion radius.

Figure 8 is presented to show how adequate the shorter TRW set of coefficients (model 1) is for prediction as well as to show the large effect that one constant can have on orbit prediction. As shown in the figure, the predictions obtained by using the list of TRW coefficients given in table XI for model 1 compare favorably with the actual data up to about 10 days. By reducing to zero the value of $S_{4,1}$, considered to be the most sensitive coefficient in the TRW set, the pericynthion prediction is changed by 9 kilometers after 10 days. A similar change in $S_{2,1}$, the least sensitive coefficient, produced no effect on pericynthion altitude.

Figure 9 is presented to compare the predictions from the two models in table XI with the actual data. Model 1 was terminated when it deviated considerably from the real-time data. The data for model 2, which was the 20-coefficient lunar model, show good comparison with the actual data for the entire extent of the data presented. Hence, the gravitational model with more coefficients appears to be best for long-time prediction. This trend is in contrast to that shown in table VI for short-time prediction of 1 or 2 days.

Effect of spacecraft maneuvers.- During the Lunar Orbiter missions, some question arose concerning the effect on the lunar trajectory of spacecraft attitude maneuvers during the photographic orbits. The answer to this question determines whether or not fits can be made on data arcs containing some of these maneuvers without materially affecting the orbit-determination accuracy. In this connection, time histories of predicted pericynthion altitudes are shown in figure 10. One started on August 26, which was during the photographic orbits, and the other started August 29, which was after the photographic orbits. Comparison of the time histories predicted from these two dates shows that the effect of the attitude maneuvers is practically negligible.

CONCLUSIONS

An analysis of orbit-determination accuracy has been made with the use of tracking data obtained during Lunar Orbiter Missions I and III. The analysis was performed mainly to study the effect of varying the lunar gravitational model on the accuracy of determining the spacecraft orbit with relatively short data arcs (approximately 1/2 day). The lunar gravitational field included the first 21 spherical harmonic coefficients or less. The effect of the length of the data arc was also studied. The results are indicative of the accuracy of lunar orbit determination that may be expected with current gravitational models.

The following conclusions can be drawn from the analysis of statistical data on lunar orbit determination when low-altitude pericynthions occur:

1. Solving for a 21-coefficient representation for the lunar model did not yield better orbit-determination accuracy than solving for properly chosen 11-coefficient sets. The 11 coefficients of lowest degree or order were found to give good accuracy in the orbit-determination solution. Solving for a most sensitive set of 11 coefficients gave equally good results.

2. Regardless of the number of coefficients used in the lunar model, solving for them yielded better state determination than fixing the coefficients at predetermined values.

3. Results for a case in which only the four lowest degree coefficients were solved for showed fitted and predicted orbit-determination accuracies which were comparable with the 11-coefficient cases. This result indicates that considerably less than 11 coefficients in the lunar model may yield an accuracy equivalent to the best cases found in this report.

4. Data-arc length had a considerable effect on orbit-determination accuracy. An arc length of about 10 hours, which corresponds to about three orbital revolutions, was found best in the present analysis.

5. The variation of initial-condition solutions was shown to be highly correlated with orbital-plane-orientation errors about the earth-moon vector. Since Doppler measurements tend to be insensitive to this rotation, a considerable amount of orbit-determination error could be eliminated by including some other measurements more sensitive to this orbit-plane orientation angle.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 9, 1969,
814-11-00-03-23.

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TABLE I.- DESCRIPTION OF CASES FOR MISSION III

Case	Data arc, min[a]	Description
		State solved for with the following coefficients held fixed:
1	600	All 21 coefficients (Lungfish values)
2	800	All 21 coefficients (Lungfish values)
3	800	$J_{2,0}$ and 11 TRW coefficients (Lungfish values); all others zero
4	800	$J_{2,0}$ (Lungfish value); all others zero
5	800	All 21 coefficients (Lungfish values) with pericynthion data deleted
		State and the following coefficients solved for:
6	800	11 highest order coefficients
7	800	11 lowest order coefficients
8	800	Modified TRW set of 11 coefficients
9	400	TRW set of 11 most sensitive coefficients
10	500	TRW set of 11 most sensitive coefficients
11	600	TRW set of 11 most sensitive coefficients
12	400	Modified TRW set of 11 coefficients
13	500	Modified TRW set of 11 coefficients
14	600	Modified TRW set of 11 coefficients
15	700	Modified TRW set of 11 coefficients
16	600	11 highest order coefficients
17	600	11 lowest order coefficients
18	600	11 lowest order coefficients with unsolved coefficients zero
19	600	11 lowest degree coefficients
20	600	$J_{2,0}$ and 10 lowest degree coefficients
21	600	4 lowest degree coefficients
22	600	TRW set of 3 most sensitive coefficients
23	600	All 21 coefficients except $J_{2,0}$

[a]Interval of time.

TABLE II.- LUNAR GRAVITATIONAL COEFFICIENTS USED FOR PREDICTION

Coefficient	Fixed Lungfish values for cases –		Converged solutions for cases –							
	1,2,5	3	6	7	8	9	10	11	12	13
$J_{2,0}$	2.07×10^{-4}	2.07×10^{-4}	(a)	(a)	2.07×10^{-4}	2.07×10^{-4}	2.07×10^{-4}	2.07×10^{-4}	2.07×10^{-4}	2.07×10^{-4}
$J_{3,0}$	-.4461	-.4461	(a)	-10.6093×10^{-4}	2.6360	2.6270	2.5141	2.1427	-1.9243	1.2263
$J_{4,0}$	-.2089	-.2089	(a)	1.8365	2.4426	16.0658	4.9625	2.3876	1.1953	1.9637
$C_{2,1}$	.0881	0	(a)	-10.2260	-2.0511	0	0	0	-.5518	-1.1272
$C_{3,1}$	.4346	.4346	(a)	-.8790	-1.2549	-5.9238	-2.2076	-1.4180	-1.2275	-1.2427
$C_{4,1}$	-.0512	0	(a)	-5.3654	0	0	0	0	0	0
$C_{2,2}$	.2761	.2761	(a)	.1718	.2731	-.3080	.0701	.1440	.1120	.1693
$C_{3,2}$	-.0522	0	0.6261×10^{-4}	-.7335	.0337	0	0	0	-.0488	.0340
$C_{4,2}$	.0279	0	-.1014	(a)	0	0	0	0	0	0
$C_{3,3}$	.0091	0	.1095	(a)	0	0	0	0	0	0
$C_{4,3}$	-.0047	-.0047	-.0739	(a)	-.0400	-.0687	-.0483	-.0460	-.0126	-.0222
$C_{4,4}$	.0009	0	-.0173	(a)	0	0	0	0	0	0
$S_{2,1}$	-.4106	-.4106	(a)	$.5560 \times 10^{-4}$	0	5.2998	.8139	-.1590	0	0
$S_{3,1}$	.1701	.1701	(a)	1.3874	-3.1962	-4.8206	-3.6518	-3.1677	.1580	-1.8187
$S_{4,1}$	-.1018	-.1018	(a)	.2522	-.2161	.9082	-.2210	-.4955	-.4460	-.4503
$S_{2,2}$	-.0577	-.0577	$.5337 \times 10^{-4}$	-1.0005	0	.6924	.5016	.4732	0	0
$S_{3,2}$	.0187	0	-.4469	(a)	0	0	0	0	0	0
$S_{4,2}$	-.0834	-.0834	.8063	(a)	.5333	.8280	.6641	.6150	.2529	.3574
$S_{3,3}$	-.0335	0	.2082	(a)	0	0	0	0	0	0
$S_{4,3}$	-.0259	-.0259	.0621	(a)	-.0092	-.0112	-.0141	-.0165	-.0143	-.0114
$S_{4,4}$	.0017	0	-.0045	(a)	0	0	0	0	0	0

[a]Value same as that for cases 1, 2, 5.

TABLE II.- LUNAR GRAVITATIONAL COEFFICIENTS USED FOR PREDICTION – Concluded

Coefficient	Converged solutions for cases –									
	14	15	16	17	18	19	20	21	22	23
$J_{2,0}$	2.07×10^{-4}	2.07×10^{-4}	(a)	(a)	2.07×10^{-4}	2.07×10^{-4}	17.0466×10^{-4}	2.07×10^{-4}	2.07×10^{-4}	2.07×10^{-4}
$J_{3,0}$	.8072	2.3258	(a)	11.7606×10^{-4}	-3.1153	-1.4123	-1.0486	0	0	7.1515
$J_{4,0}$	1.7111	2.5052	(a)	5.3423	2.3405	-1.3298	18.6232	0	0	6.3838
$C_{2,1}$	-1.1253	-1.8846	(a)	-8.9408	-2.8270	-1.6631	-1.9654	-2.6680	0	2.1665
$C_{3,1}$	-1.2121	-1.3874	(a)	1.6490	-1.2616	.5070	-2.2312	0	-.3547	-2.6679
$C_{4,1}$	0	0	(a)	-5.5928	-.8098	0	0	0	0	1.2788
$C_{2,2}$	.1614	.2207	(a)	.4099	.1762	.2926	.1193	.4434	0	1.0411
$C_{3,2}$	.0294	.0336	0.6456×10^{-4}	-.8098	-.0821	-.0026	-.0625	0	0	.7465
$C_{4,2}$	0	0	.2768	(a)	0	0	0	0	0	.4004
$C_{3,3}$	0	0	.1555	(a)	0	.0460	0	0	0	.1598
$C_{4,3}$	-.0227	-.0367	-.0685	(a)	0	0	0	0	0	-.2134
$C_{4,4}$	0	0	.0040	(a)	0	0	0	0	0	-.0108
$S_{2,1}$	0	0	(a)	-2.0648	.9745	-.1246	1.6754	-.2747	0	.3530
$S_{3,1}$	-1.4952	-2.9574	(a)	2.1997	1.0825	.8476	.4986	0	0	-5.2392
$S_{4,1}$	-.4583	-.3392	(a)	-.2252	.0419	0	0	0	-.4850	.5615
$S_{2,2}$	0	0	$.6534 \times 10^{-4}$	-.6843	-.4504	-.3503	-.3776	-.5767	0	1.4524
$S_{3,2}$	0	0	-.7318	(a)	0	-.1225	.3043	0	0	-.4609
$S_{4,2}$	.3316	.5016	.6178	(a)	0	0	0	0	0	1.0662
$S_{3,3}$	0	0	.2115	(a)	0	0	0	0	0	.1988
$S_{4,3}$	-.0127	-.0112	-.0098	(a)	0	0	0	0	-.0208	.0972
$S_{4,4}$	0	0	-.0022	(a)	0	0	0	0	0	-.0256

[a]Value same as that for cases 1, 2, 5.

TABLE III.- COMPARISON OF RESULTS OF CASE 14 FROM TWO ORBIT-DETERMINATION PROGRAMS

Coefficient[a]	JPL operational program[b]		Lungfish program[c]	
	Value of coefficient	Standard deviation	Value of coefficient	Standard deviation
$J_{2,0}$	2.07×10^{-4}		2.07×10^{-4}	
$J_{3,0}$	.8072	0.232×10^{-4}	.4040	0.0736×10^{-4}
$J_{4,0}$	1.7111	.0977	1.5604	.0290
$C_{2,1}$	-1.1253	.0639	-1.0274	.0185
$C_{3,1}$	-1.2121	.0478	-1.1085	.0136
$C_{2,2}$	.1614	.0114	.1934	.00307
$C_{3,2}$	.0294	.00773	.0222	.00256
$C_{4,3}$	-.0227	.00128	-.0220	.000364
$S_{3,1}$	-1.4952	.166	-1.1562	.0515
$S_{4,1}$	-.4583	.0278	-.3874	.00798
$S_{4,2}$	.3316	.0139	.3021	.00403
$S_{4,3}$	-.0127	.000893	-.0087	.000282

[a]All other coefficients fixed at zero.
[b]372 points; σ of fit, 0.124.
[c]353 points; σ of fit, 0.139.

DIFFERENCE IN STATE[d]

x, km . . . . -0.867
y, km . . . . -0.938
z, km 1.291
$\dot{x}$, m/sec 0.197
$\dot{y}$, m/sec . . . . -0.640
$\dot{z}$, m/sec 0.136

[d]Lungfish value minus JPL operational value.

TABLE IV.- CONVERGED SOLUTIONS SHOWING INDIVIDUAL EFFECT OF GRAVITATIONAL COEFFICIENTS

[Lungfish values used for coefficients; data arc of 800 minutes]

Case description	Position component, km			Velocity component, km/sec		
	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
All coefficients zero	-284.2	3048.7	1709.8	-0.9008	-0.3578	0.1819
All coefficients zero except $J_{2,0}$	-286.1	3069.1	1671.7	-.9064	-.3490	.1719
All coefficients zero except $J_{2,0}$ and $S_{4,1}$	-286.5	3068.1	1673.7	-.9061	-.3495	.1723
All coefficients zero except $J_{2,0}$, $S_{4,1}$, and $C_{3,1}$	-286.7	3060.2	1688.9	-.9038	-.3531	.1762
All coefficients zero except $J_{2,0}$, $S_{4,1}$, $C_{3,1}$, and $S_{4,3}$	-286.3	3060.9	1687.3	-.9041	-.3527	.1758
$J_{2,0}$ and 11 TRW coefficients used; all others zero	-284.6	3061.0	1687.3	-.9042	-.3525	.1760
All 21 coefficients used	-285.7	3062.1	1685.1	-.9046	-.3520	.1751

TABLE V.- STATISTICS OF FITTED DATA ARCS

Case	Data arc, min	Number of points	σ, cps
1	600	372	0.876
2	800	502	1.73
3	800	502	1.76
4	800	502	1.32
5	800	370	1.09
6	800	502	.324
7	800	502	.414
8	800	502	.181
9	400	237	.099
10	500	316	.106
11	600	372	.126
12	400	237	.102
13	500	316	.102
14	600	372	.124
15	700	444	.171
16	600	372	.210
17	600	372	.141
18	600	372	.139
19	600	372	.140
20	600	372	.125
21	600	372	.278
22	600	372	.453
23	600	372	.117

TABLE VI.- STATISTICS OF PREDICT DATA ARCS

Case	Fitted data arc, min	σ, cps, for –	
		Predict arc of 1170 min with 706 points	Predict arc of 2340 min with 1305 points
1	600	40.2	77.8
2	800	32.8	66.7
3	800	32.7	66.8
4	800	31.7	60.7
5	800	32.1	66.3
6	800	16.2	61.7
7	800	17.3	66.5
8	800	15.7	60.6
9	400	10.1	36.5
10	500	8.0	18.3
11	600	8.1	17.7
12	400	10.0	23.4
13	500	6.6	16.9
14	600	6.3	18.2
15	700	12.2	51.7
16	600	8.9	26.4
17	600	7.0	28.5
18	600	6.6	19.0
19	600	8.3	15.5
[a]19	600	17.2	43.9
20	600	6.3	23.2
21	600	4.3	25.3
22	600	34.4	65.2
23	600	7.0	40.8

[a]Twenty-one Lungfish values used for coefficients in prediction.

TABLE VII.- VARIATION IN CONVERGED SOLUTIONS

[Increments are deviations in values of the initial state from those for case 14]

Case	Data arc, min	Δx, km	Δy, km	Δz, km	Δr, km	$\Delta\dot{x}$, m/sec	$\Delta\dot{y}$, m/sec	$\Delta\dot{z}$, m/sec	ΔV, m/sec
1	600	0.3	-2.8	5.7	6.4	0.9	-1.3	1.7	2.3
2	800	-.6	1.1	-1.2	1.7	-.3	.4	-.7	.9
3	800	.5	0	1.0	1.1	.1	-.1	.2	.2
4	800	-1.0	8.1	-14.6	16.7	-2.1	3.4	-3.9	5.6
5	800	.6	-4.8	9.6	10.7	1.5	-2.3	2.8	3.9
6	800	1.1	-6.4	11.9	13.6	1.7	-2.8	3.3	4.6
7	800	.1	-.8	1.6	1.8	.3	-.6	.8	1.0
8	800	.8	-3.0	5.6	6.4	.7	-1.1	1.3	1.8
9	400	1.7	-4.5	6.9	8.4	.5	-.7	.7	1.1
10	500	.5	-1.3	2.0	2.4	.1	-.3	.2	.4
11	600	.2	-.2	.3	.4	0	0	0	0
12	400	-.3	.5	-.8	1.0	-.1	0	-.1	.1
13	500	.2	-.5	1.0	1.1	.1	-.2	.2	.3
14	600	----	----	-----	----	----	----	----	---
15	700	.7	-2.4	4.5	5.1	.5	-.9	1.0	1.4
16	600	.5	-3.5	6.4	7.3	.9	-1.4	1.9	2.5
17	600	-1.1	3.9	-6.5	7.7	-.6	.8	-.8	1.3
18	600	-.3	.7	-1.4	1.6	-.1	.3	-.3	.4
19	600	-.4	1.0	-1.4	1.8	-.1	.2	-.2	.3
20	600	-.7	1.3	-1.6	2.2	-.1	.1	.2	.2
21	600	.8	-4.2	8.1	9.2	1.2	-1.8	2.2	3.1
22	600	-1.2	5.5	-10.2	11.7	-1.3	2.2	-2.4	3.5
23	600	1.4	-3.6	6.1	7.2	.4	-.5	.8	1.0

TABLE VIII.- ORBITAL PARAMETERS DERIVED FROM CERTAIN CONVERGED SOLUTIONS

[Increments are deviations from values for case 14]

Case	Pericynthion radius, r_p, km	Semimajor axis, km	Eccentricity	Argument of pericynthion, deg	Inclination, i, deg	Longitude of ascending node, λ, deg	Δi, deg	$\Delta\lambda$, deg
2	1782.98	2689.70	0.337110	180.776	20.868	36.738	-0.100	0.027
3	1782.91	2689.85	.337172	180.811	20.949	36.686	-.019	-.025
4	1783.01	2689.54	.337059	180.383	20.509	37.070	-.459	.359
5	1782.97	2689.71	.337114	181.043	21.223	36.501	.255	-.210
6	1783.00	2690.06	.337190	181.260	21.302	36.325	.334	-.386
7	1782.84	2689.20	.337038	181.100	21.102	36.480	.134	-.231
8	1783.07	2690.23	.337204	180.946	21.129	36.611	.161	-.100
9	1783.08	2690.15	.337180	181.032	21.086	36.524	.118	-.187
10	1783.12	2690.30	.337203	180.834	20.998	36.718	.030	.007
11	1783.14	2690.38	.337213	180.782	20.965	36.767	-.003	.056
12	1783.16	2690.01	.337134	180.863	20.960	36.659	-.008	-.052
13	1783.16	2690.19	.337161	180.837	20.992	36.699	.024	-.012
14	1783.17	2690.20	.337160	180.821	20.968	36.711	------	-----
15	1783.10	2690.24	.337195	180.919	21.096	36.633	.128	-.078

TABLE IX.- VARIATION IN PREDICTIONS APPROXIMATELY 33 HOURS FROM EPOCH

[Increments are deviations from values for case 14]

Case	$\Delta\phi$, deg	$\Delta\psi$, deg	Incremental surface distance, km
2	0.64	-0.12	19.7
3	.57	-.18	18.1
4	.90	-.10	27.4
5	.30	-.17	10.5
6	-.45	.06	13.8
7	-1.29	-.02	39.1
8	-.38	.14	12.3
9	.04	-.03	1.5
10	.19	0	5.8
11	.20	.01	6.1
12	-.04	-.06	2.2
13	.01	0	.3
14	----	-----	----
15	-.30	.11	9.7

TABLE X.- COMPARISON OF CALCULATED AND ACTUAL MAGNITUDES OF STATE

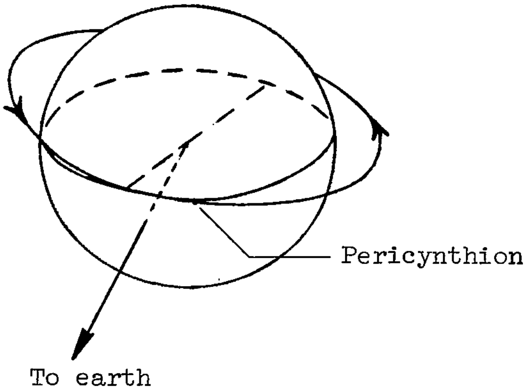
[Increments are deviations from values for case 14]

Case	cos δ	θ, deg	$\lvert\overline{\Delta r}\rvert_c$, km	$\lvert\overline{\Delta r}\rvert_a$, km	$\lvert\overline{\Delta V}\rvert_c$, m/sec	$\lvert\overline{\Delta V}\rvert_a$, m/sec
1	-0.99945	-0.180	6.5	6.4	2.2	2.3
2	.98815	.061	2.2	1.7	.8	.9
3	-.98834	-.023	.8	1.1	.3	.2
4	.99991	.445	16.2	16.7	5.6	5.6
5	-.99957	-.303	11.0	10.7	3.8	3.9
6	-.99999	-.368	13.4	13.6	4.6	4.6
7	-.95053	-.063	2.3	1.8	.8	1.0
8	-.99672	-.158	5.7	6.4	2.0	1.8
9	-.91989	-.146	5.3	8.4	1.8	1.1
10	-.92956	-.044	1.6	2.4	.5	.4
11	-.69736	-.005	.2	.4	.1	0
12	.91066	.017	.6	1.0	.2	.1
13	-.99157	-.027	1.0	1.1	.3	.3
14	--------	------	----	----	---	---
15	-.99524	-.125	4.5	5.1	1.6	1.4
16	-.99996	-.201	7.3	7.3	2.5	2.5
17	.95019	.145	5.3	7.7	1.8	1.3
18	.99296	.038	1.4	1.6	.5	.4
19	.95257	.034	1.2	1.8	.4	.3
20	.58047	.020	.7	2.2	.2	.2
21	-.99997	-.247	9.0	9.2	3.1	3.1
22	.99798	.293	10.6	11.7	3.7	3.5
23	-.93496	-.131	4.8	7.2	1.6	1.0

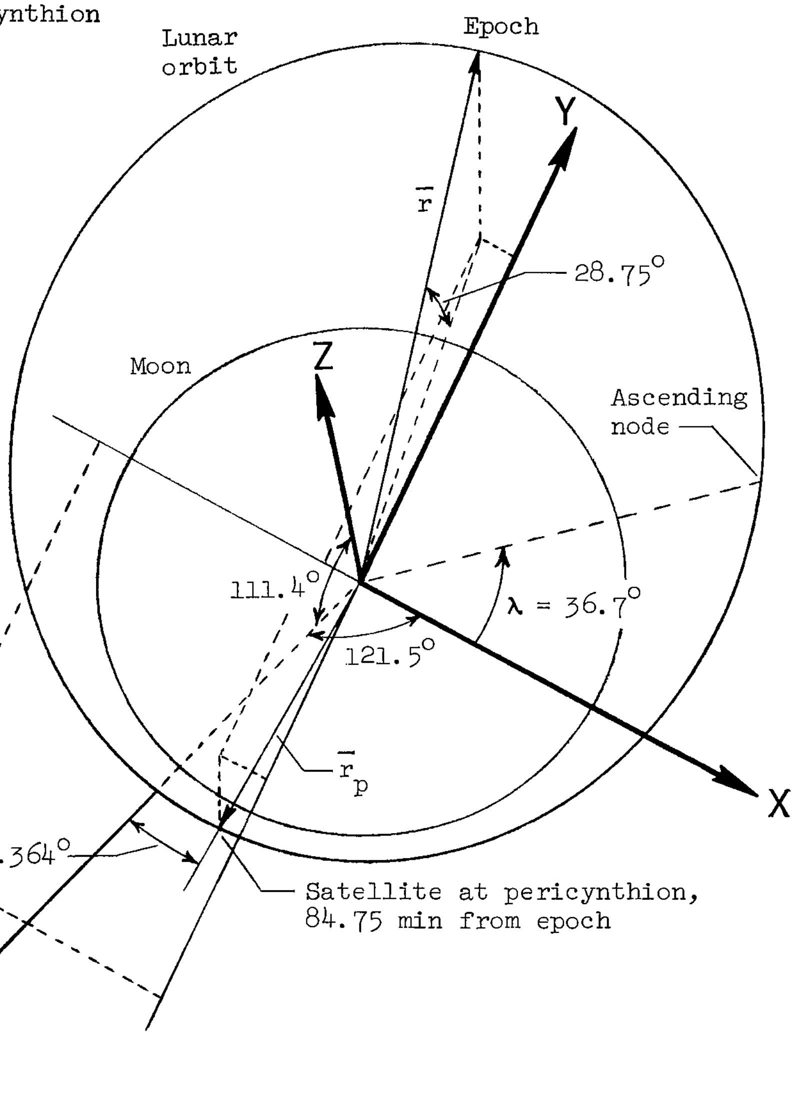
TABLE XI.- VALUES OF LUNAR HARMONIC COEFFICIENTS USED IN PERICYNTHION-ALTITUDE PREDICTION

Coefficients	Model 1[a]	Model 2[a]
$J_{2,0}$	2.048 $\times 10^{-4}$	2.048 $\times 10^{-4}$
$J_{3,0}$	-.19647839	-.15
$J_{4,0}$	-1.7533021	0
$C_{2,1}$	0	-.4138
$C_{3,1}$	.15838471	.3944
$C_{4,1}$	0	-.3695
$C_{2,2}$	.40878896	.2258
$C_{3,2}$	0	.2689
$C_{4,2}$	0	-.1384
$C_{3,3}$	0	.002056
$C_{4,3}$	-.0091994675	.01177
$C_{4,4}$	0	.000346
$S_{2,1}$	1.5652573	.2876
$S_{3,1}$	.091697821	.1862
$S_{4,1}$	1.2021058	.3364
$S_{2,2}$	-.044103304	.07012
$S_{3,2}$	0	.3611
$S_{4,2}$	-.042429673	.1769
$S_{3,3}$	0	-.06049
$S_{4,3}$	-.017627865	.004235
$S_{4,4}$	0	-.004147

[a]$J_{2,0}$ held fixed.



(a) View showing orientation of lunar orbit.



(b) View from above lunar orbital plane showing selenocentric coordinate system.

Figure 1.- Schematic views showing orbit of Lunar Orbiter III at time of data fits for present analysis.

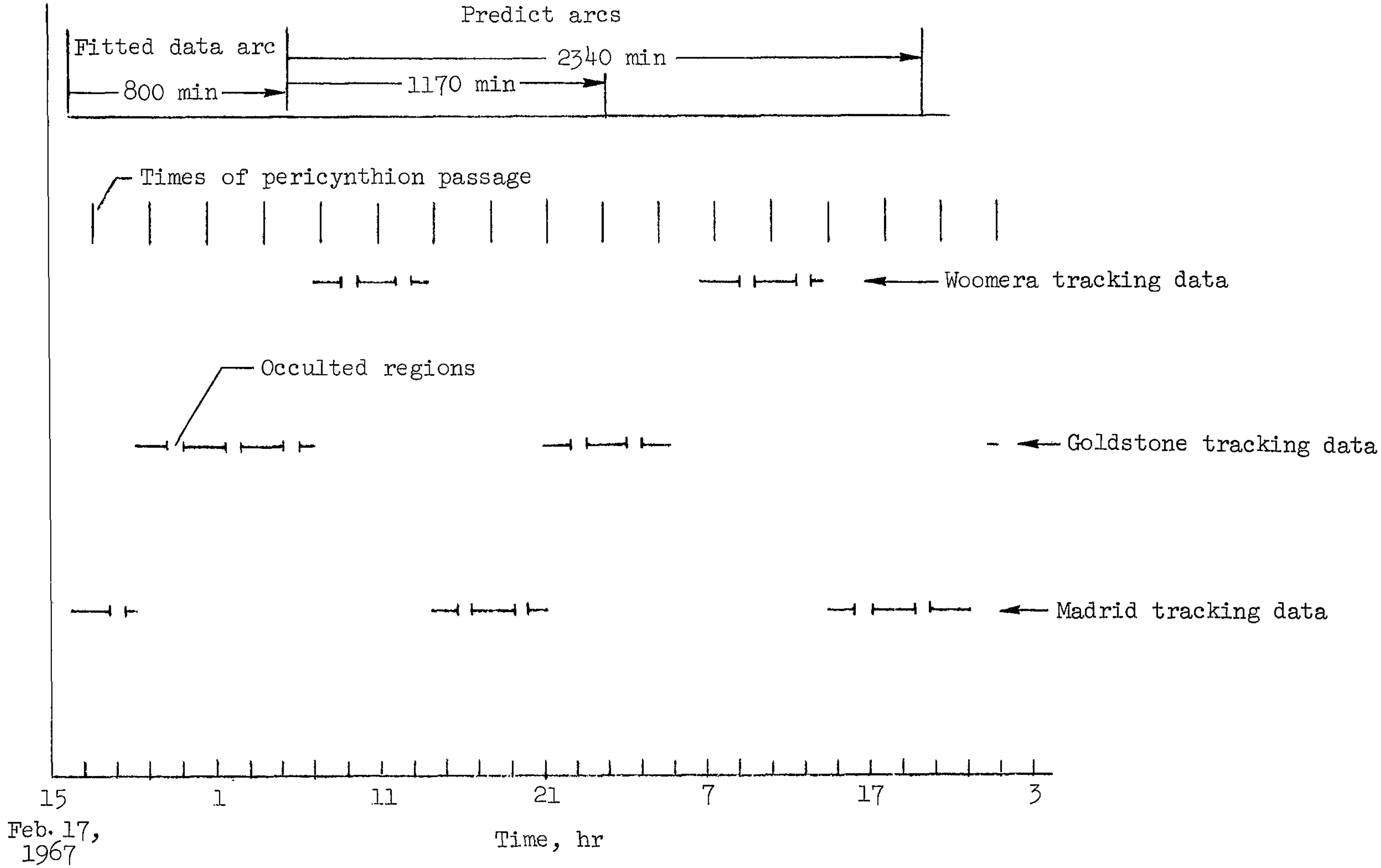


Figure 2.- Summary of tracking data used in the study of Mission III data.

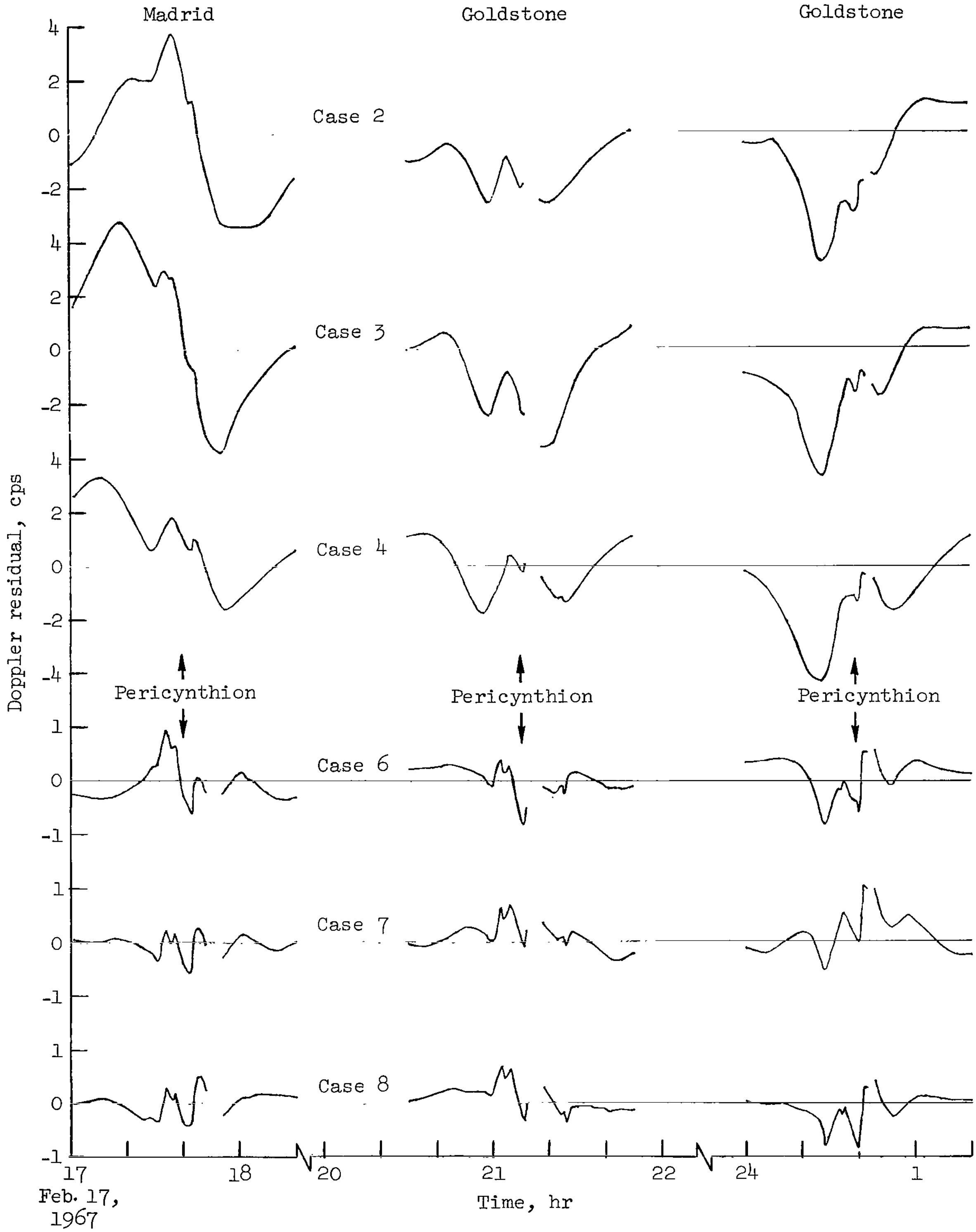


Figure 3.- Tracking data residuals for certain converged solutions.

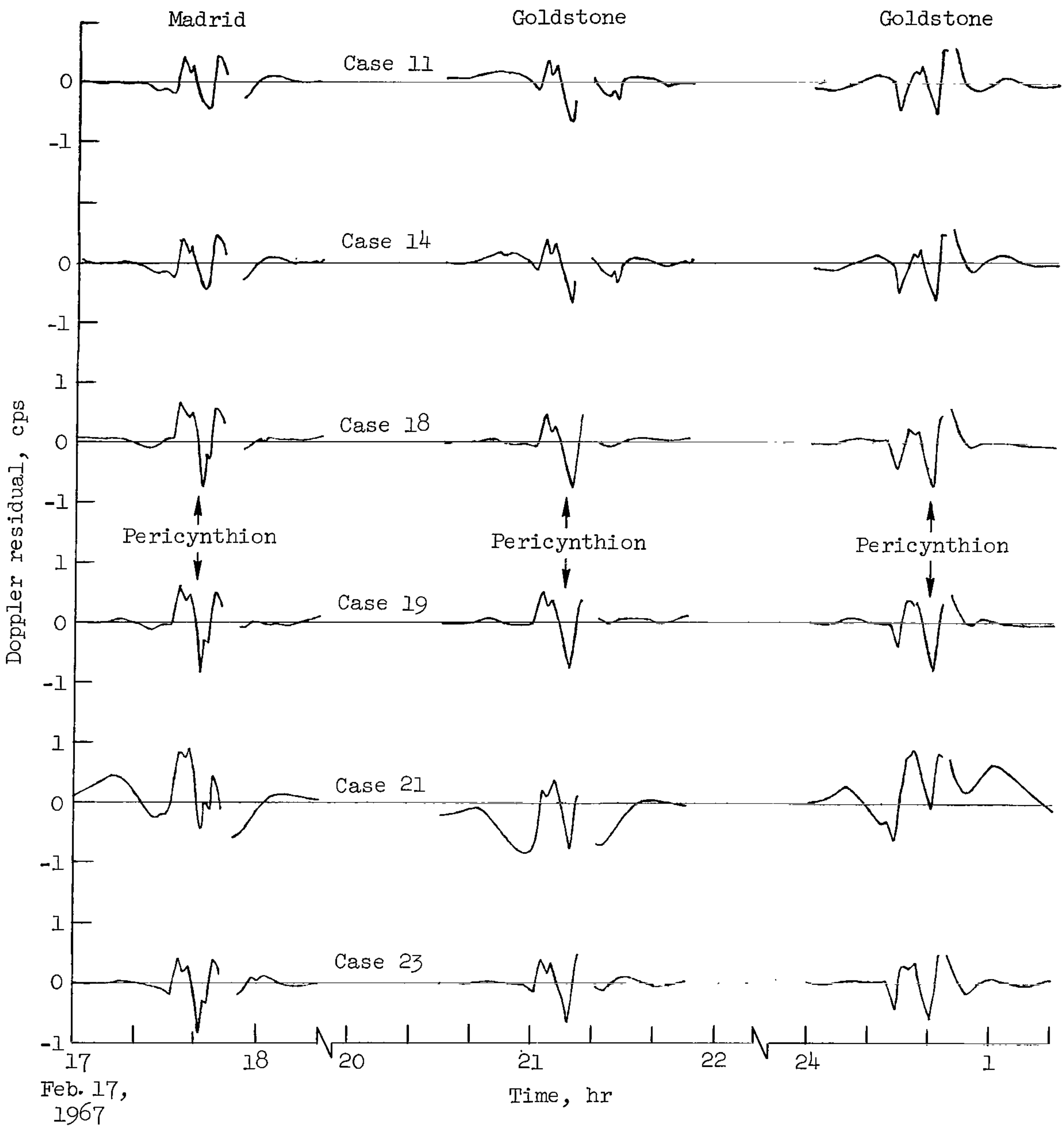


Figure 3.- Concluded.

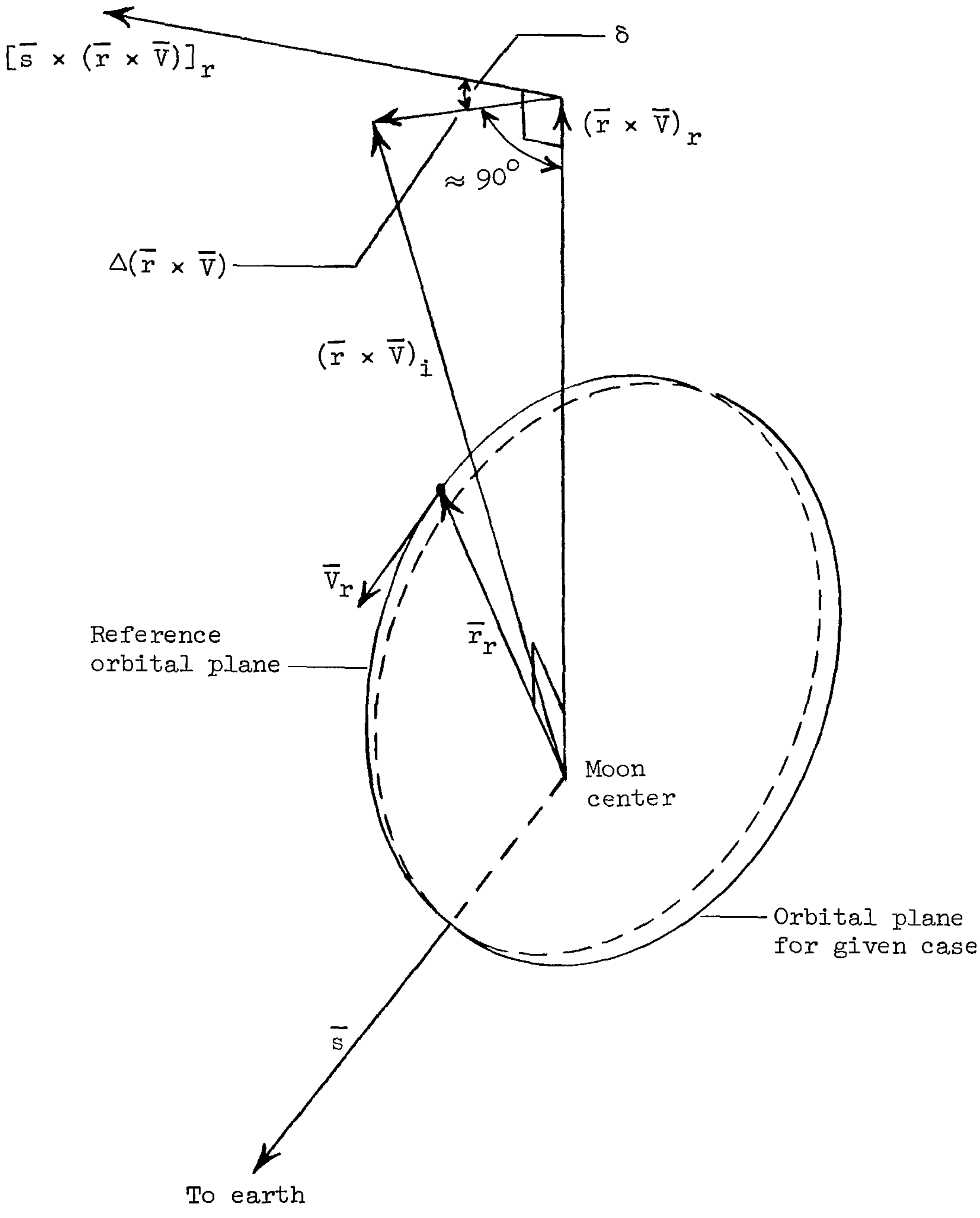


Figure 4.- Schematic drawing showing rotation of orbital plane about earth-moon vector.

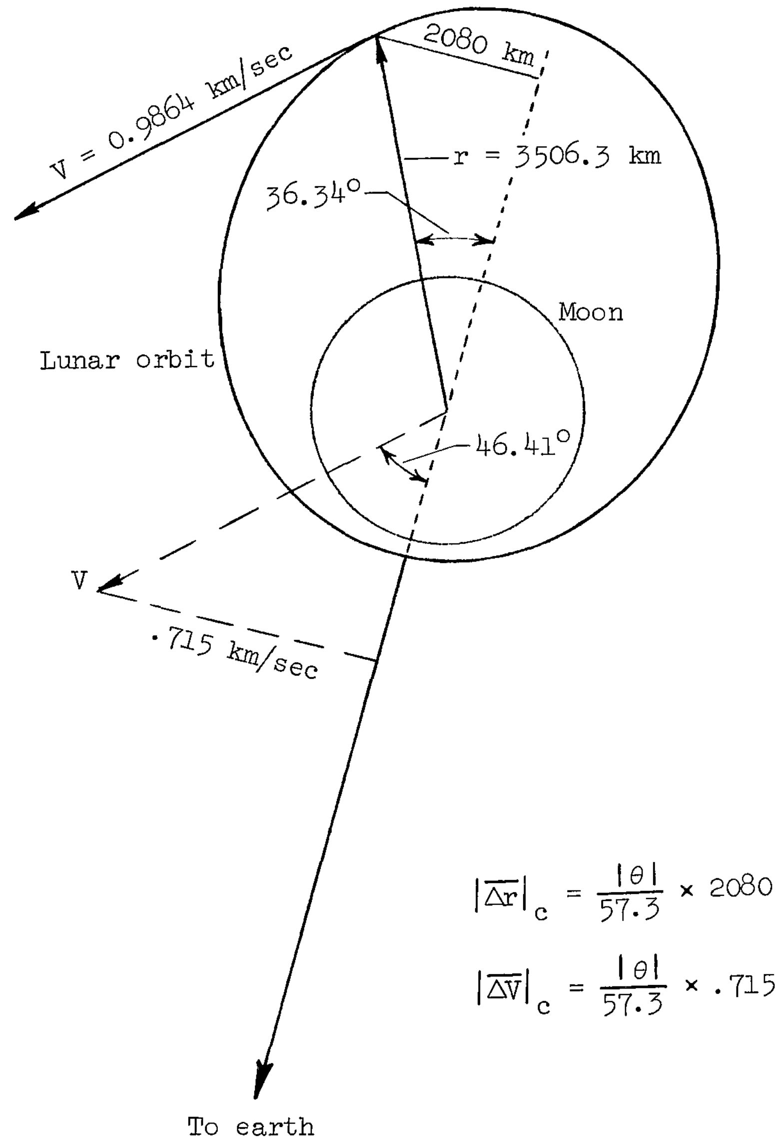


Figure 5.- Geometry showing calculation for position and velocity deviation caused by rotation about earth-moon vector.

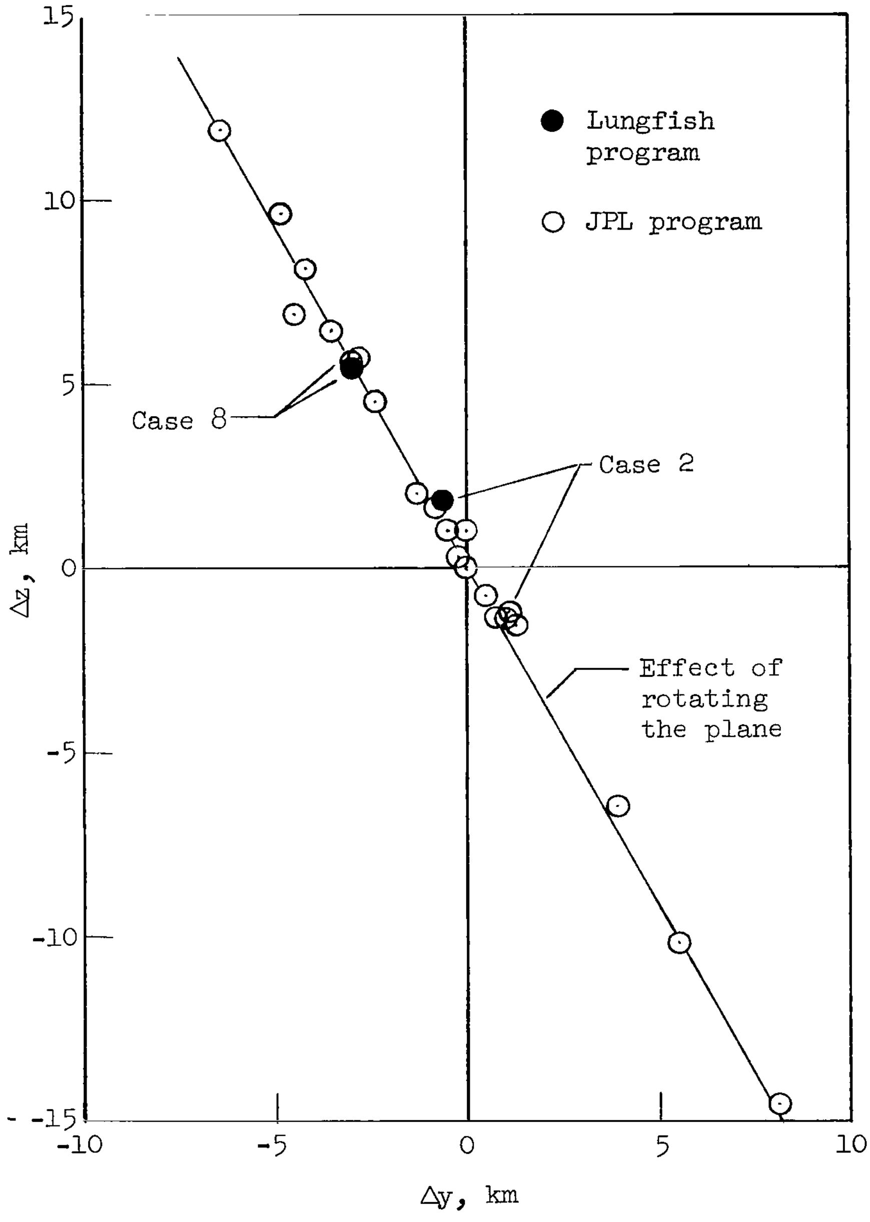


Figure 6.- Relation of deviation in initial conditions to rotation of orbital plane about earth-moon vector.

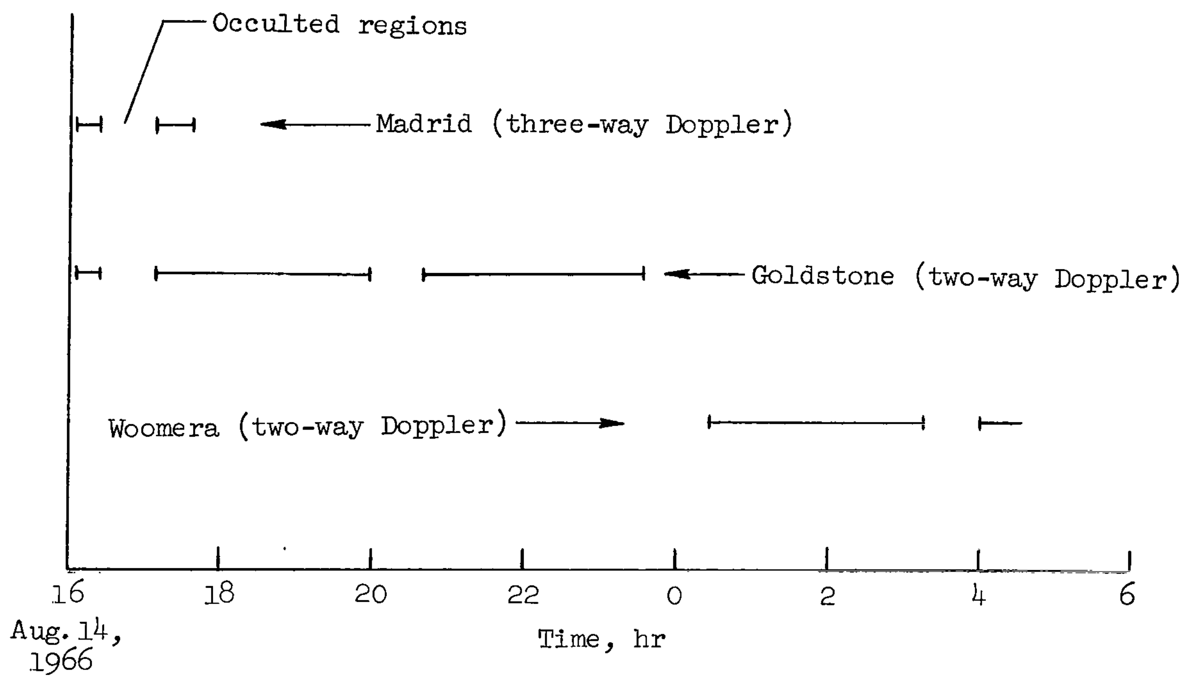


Figure 7.- Summary of Mission I tracking data fitted for model I.

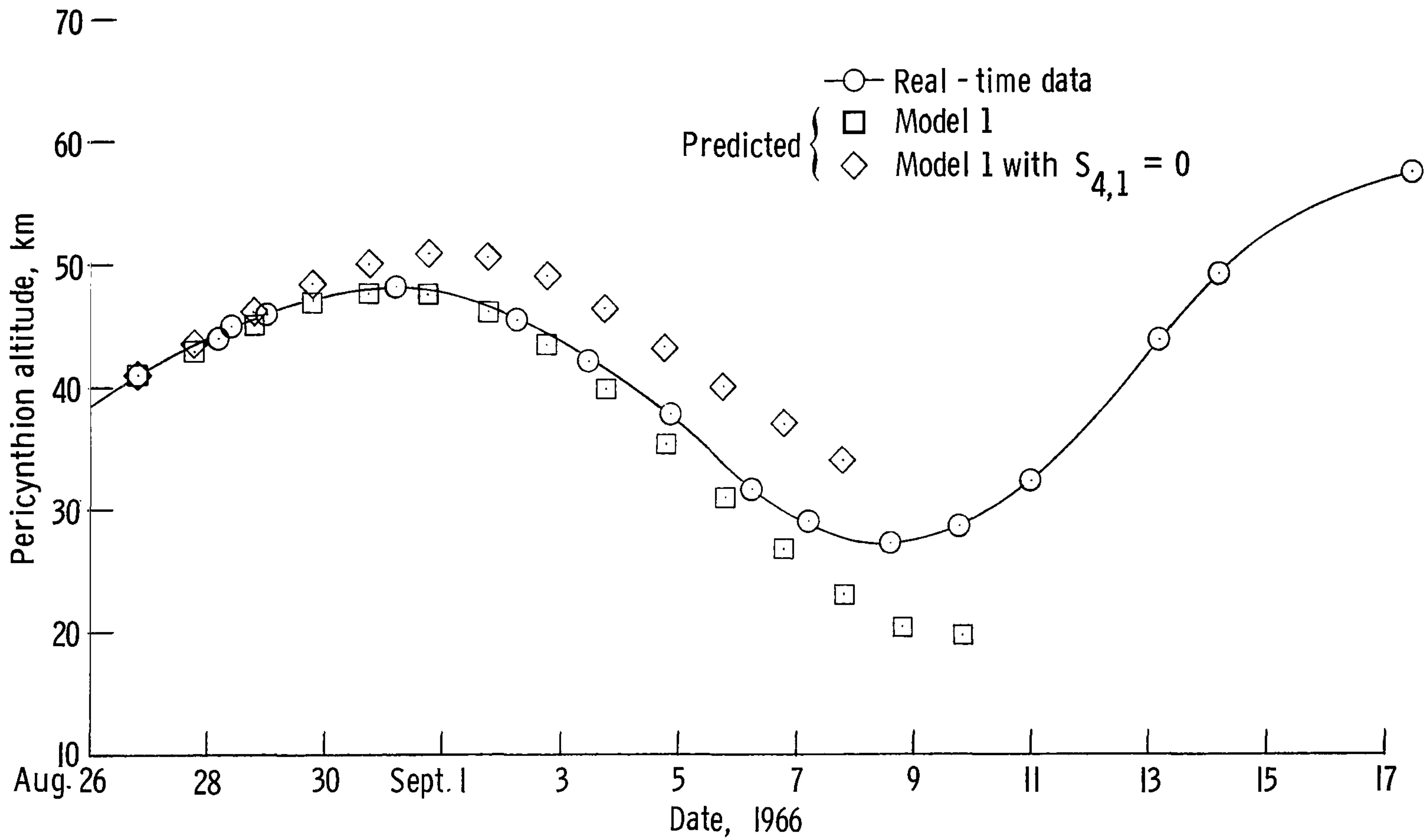


Figure 8.- Effect of change in value of one harmonic coefficient on pericynthion-altitude prediction.

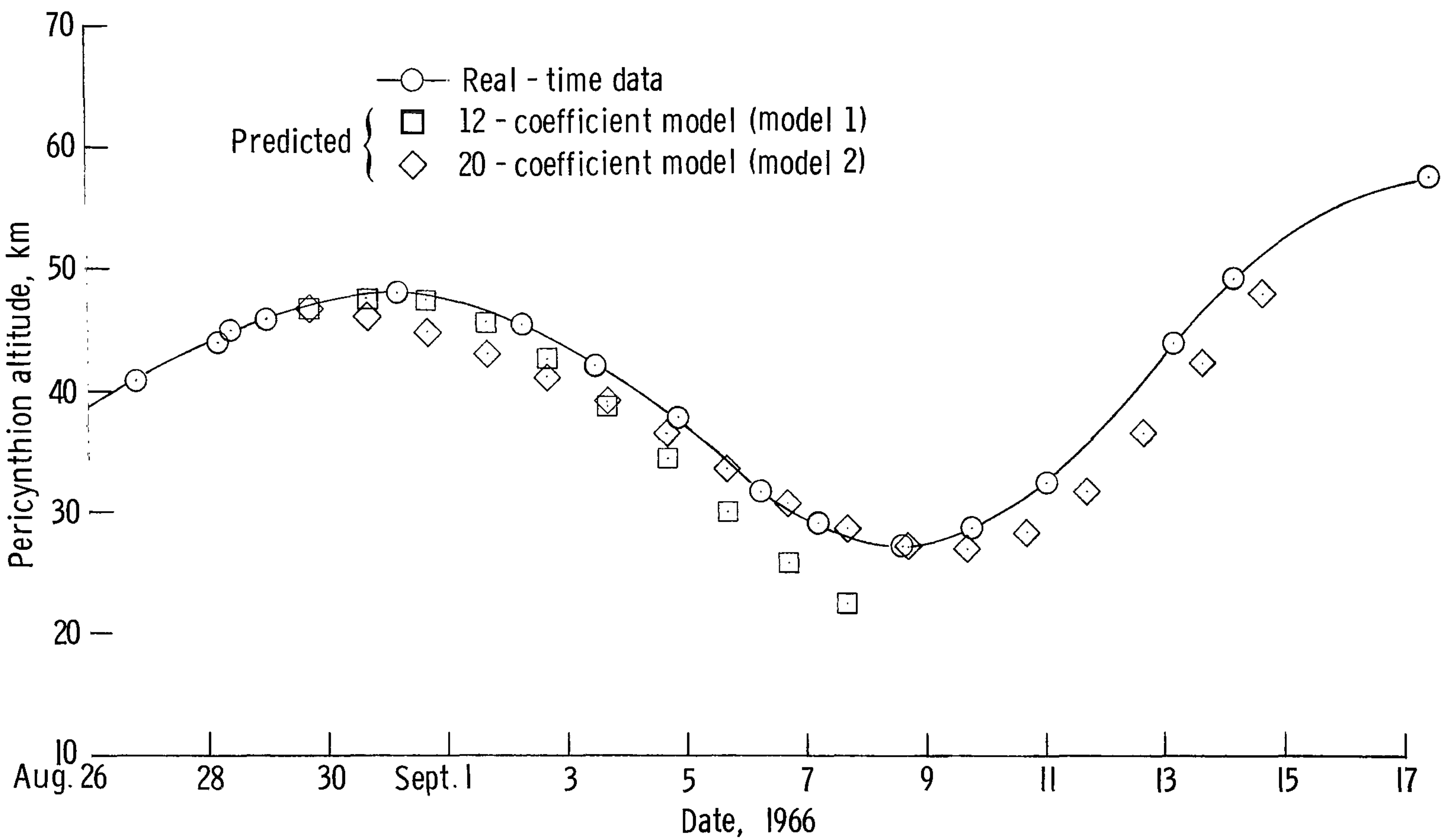


Figure 9.- Effect of lunar gravitational model on pericynthion-altitude prediction.

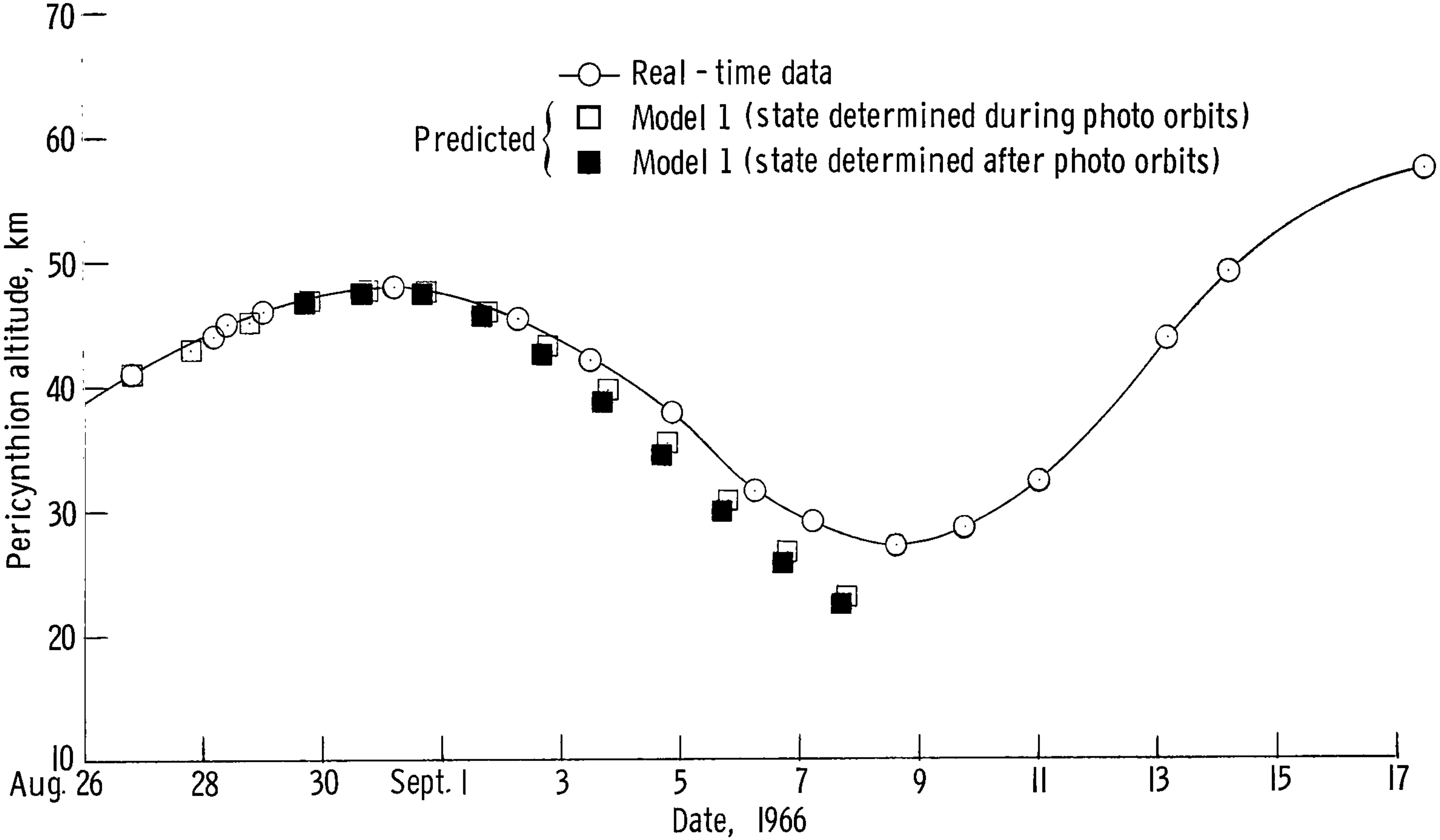


Figure 10.- Effect of spacecraft attitude maneuvers on pericynthion-altitude prediction.

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