

Department of Geodetic Science

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DATA ANALYSIS IN CONNECTION WITH THE NATIONAL
GEODETIC SATELLITE PROGRAM (II)

Third Semiannual Status Report

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Washington, D. C. 20546

The Ohio State University
Research Foundation
Columbus, Ohio 43212

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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and it is under the technical direction of Jerome D. Rosenberg, Project Manager, Geodetic Satellites Program, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C. 20546.

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1. STATEMENT OF WORK

The primary objective of the OSU investigation is the geometric analysis of geodetic satellite data. In order to fulfill this objective the following tasks are being attempted during the current grant period.

(1) Extension of the computer programs which can handle optical and/or range observations for range and range rate, Doppler, and laser systems.

(2) Development of data preprocessing systems to handle the data from the above observation methods.

(3) Analysis of observational data for the National Geodetic Satellite Program for the purpose of fulfilling the primary objective as observational data becomes available.

(4) The development of computer programs for the adjustment of optical, ranging and Doppler data in the short-arc mode.

(5) Theoretical investigations into sequential least squares adjustment of simultaneous satellite observations in combination with terrestrial data.

2. ACCOMPLISHMENTS DURING THE REPORT PERIOD

2.1 Extension of the Computer Programs for Range-Rate and Range-Difference Observations

Originally the problem was studied from two points of view: as adjustment where the observed quantities were to be range differences and adjustment with observed quantities being range rates. It was proposed that both cases would be treated as purely geometric and also using the short-arc method.

Further examinations revealed that the short-arc method for the range rate was too complex and also disadvantageous from an economical point of view. Besides, by numerical integration the observations of range rate may be directly converted into range differences

Consequently, it was decided to treat only range differences as observed quantities for both the purely geometric mode and the short-arc mode. In the latter, the orbit over a short arc is approximated by Keplerian orbit with six parameters to be determined. Optionally, ten parameters may also be used. The ten-parameter model is not exact either, but it is expected that the resulting discrepancies are not serious enough to introduce significant systematic errors. As for the observations, it was assumed that in order to compensate for eventual systematic frequency shifts, one parameter should be added for each satellite pass and each station. The number of unknown parameters for the geometric mode would thus be:

- (a) ground station coordinates
- (b) satellite coordinates
- (c) parameters due to frequency shifts.

For the short-arc mode:

- (a) ground station coordinates
- (b) orbital parameters (six or ten per pass)
- (c) parameters due to frequency shifts.

It is seen that for short-arc mode the satellite position parameters are replaced by orbital parameters, which reduces the number of unknowns and constraints that pass to an orbit (even though not exactly the actual orbit). Therefore, an improvement is expected using the short-arc mode.

The adjustment for both modes can be made using observation equations because each observation yields one equation. An additional verification of the theory and formulas is available using for the short-arc mode observation equations from the geometrical mode, supplemented by constraints between the satellite positions and orbital parameters (six or ten per orbit).

So far in this project, the theory and formulas were developed for both geometric and short-arc modes; the adjustment schemes were developed for both general and OSU's so-called "A" methods, including constraints, eventual weighting of parameters, adding of new observations (sequentially or

by normal equations), adding of constraints, and other features.

At present the programs are being written for these adjustment models, the purpose of which is to check the theory (including number of observations necessary) and the formulas.

The final report will be presented in the following parts:

- (1) theory and formulas pertaining to the geometric and short-arc modes,
- (2) adjustment schemes for the two modes, for their combination (i. e. , including the constraints), and other possibilities actually beyond the scope of the immediate need,
- (3) checking of the theory and formulas by several computer runs of generated data.

The main purpose of the program will be its eventual use in connection with Geociever observations.

2.2 Short-Arc Orbit Constraint Adjustment of Satellite Observations and Comparisons with Strictly Geometric Adjustment

The principle features and capabilities of the short-arc orbit constraint computer program were described in the First Semiannual Status Report (p. 4). During this reporting period, the experiments with BC-4 data that were described in the Second Semiannual Status Report (p. 25) were continued. These experiments concerned the feasibility of adjusting the BC-4 data in an orbital mode as well as various methods of treating the data in a geometric mode.

The data for these experiments was extracted from the tape deposited in the Data Center by ESSA in April, 1968. This tape contains 298 plates, 50 of which are erroneous (see section 2.32 of this report). A set of 12 plates, comprising five events from three stations, was extracted from this tape for the first series of experiments. The results of these experiments were reported in the Second Semiannual Status Report (p. 28). In order to confirm the results of the 12-plate series of experiments, the data was extended to include 45 plates, taken from four stations and comprising 21 events.

The data used is described below:

<u>Event</u>	<u>Date</u>	<u>Plates</u>	<u>Stations</u>
2492	Aug 18, 1966	B328, A404	2, 3
3173	Nov 18, 1966	B686, A478	1, 3
3185	Nov 19, 1966	B688, A479	1, 3
3539	Dec 17, 1966	E393, A491	2, 3
3538	Dec 17, 1966	B748, E392	1, 2
3560	Dec 19, 1966	B753, E394	1, 2
3561	Dec 19, 1966	B754, E395	1, 2
3795	Jan 14, 1967	B799, A513	1, 3
3837	Jan 20, 1967	B825, A521	1, 3
3935	Jan 31, 1967	B835, E416	1, 2
4061	Feb 14, 1967	B870, A544	1, 3
4182	Mar 1, 1967	E436, A557, U406	2, 3, 38
4196	Mar 3, 1967	A562, U408	3, 38
4212	Mar 5, 1967	A567, U411	3, 38
4236	Mar 8, 1967	E422, A571	2, 3
4244	Mar 9, 1967	E444, U417	2, 38
4251	Mar 10, 1967	B890, E446, U419	1, 2, 38
4259	Mar 11, 1967	E448, U421	2, 38
4267	Mar 12, 1967	E450, A574, U423	2, 3, 38
4276	Mar 13, 1967	A576, U425	3, 38
4292	Mar 15, 1967	B893, U427	1, 38

Stations:

- 1 Thule, Greenland
- 2 Beltsville, Maryland
- 3 Moses Lake, Washington
- 38 Revilla Gigedo Island, Mexico

The data for each plate consisted of the right ascension and declination of each image in the satellite trail, or about 200-500 right ascension-declination pairs per plate. All images on each plate were corrected for phase angle, parallax refraction, and parallax aberration (light-travel time). The data was processed as follows:

(a) A fifth-order polynomial in time was fit to the corrected right-ascensions and declinations for each satellite trail, and a single "fictitious image" was taken from the curve fit for each plate. These fictitious images were taken to be representative of those formed by the curve fitting technique

of ESSA. They were adjusted in a geometric mode only (experiment 3).

(b) Approximately 20 individual images, depending on the length of the trail, were selected from each plate. So that the same observations could be used in both the geometric and orbital modes, the individual images were reduced to simultaneity by second-order interpolation. These 20 images per plate were adjusted in both the geometric and short-arc modes (experiments 1 and 4). In addition, subsets of 10 individual images per plate were adjusted in both modes (experiments 2 and 5).

(c) 20 uncorrected images, taken directly from the tape, were adjusted in the orbital mode (experiment 6).

The results of these experiments are given in Table 2.1. In all experiments the coordinates of Beltsville on the SAO C-5 datum were held fixed and the distance from Beltsville to Moses Lake was constrained. Thus the solution coordinates refer to the origin defined by the SAO C-5 datum.

All of the experiments in the 45-plate series yielded reasonable estimates of the unit variance, so that the results are directly comparable. Many of the observations made about the 12-plate experiments are corroborated by the results of the larger set. The single fictitious image appears to contain about as much information as 10 individual images. The 20 individual image adjustments in both modes appear to be stronger than either the adjustments of 10 individual images or the one fictitious image, and the improvement is about a factor of $\sqrt{2}$. This indicates that information is indeed lost in the curve fitting procedure: more information is available on the plate than is carried by the single fictitious image. It also appears that if only ten images, rather than several hundred, were measured the same amount of information could be obtained from the plate. If 20 images are measured, even more information may be obtained.

The adjustments with the 20 uncorrected images, taken directly from the NSSDC tape, were performed for both the 12 plate and the 45 plate data sets in order to determine the significance of the corrections for parallactic

Table 2.1
Results of the 45 Plate Series of Experiments

Table 2.1 Results of the 45 Plate Series of Experiments														
Experiment	Moses Lake			Revilla Ggedo			Thule			Degrees of Freedom	A posteriori stan. dev. of a single obs.			
	X	Y		Z	X	Y		Z	X			Y		Z
		m	+			m	+					m	+	
	-2127800 m +	-3785800 m +	4656000 m +	-2160900 m +	-5642700 m +	2035300 m +	546500 m +	-1390000 m +	6180200 m +					
Geometric mode														
1. 20 individual images	-31.1 (2.9)	-105.8 (11.1)	32.2 (9.3)	-62.6 (5.4)	-21.1 (9.0)	60.4 (12.1)	87.9 (9.9)	-11.3 (11.5)	35.3 (15.6)	448	1.9			
2. 10 individual images	-30.7 (4.1)	-85.3 (16.0)	35.0 (13.7)	-63.0 (7.7)	-12.1 (12.6)	45.7 (17.6)	82.2 (14.4)	-26.3 (16.7)	29.1 (23.0)	223	2.0			
3. 1 fictitious image	-26.7 (4.6)	-88.2 (19.0)	26.1 (14.9)	-56.5 (9.7)	-57.6 (14.1)	78.7 (19.3)	67.1 (14.4)	-2.3 (16.6)	47.0 (23.2)	13	0.7			
Orbital Mode														
4. 20 individual images	-20.6 (3.6)	-85.3 (12.5)	51.7 (8.2)	-55.4 (5.8)	-16.3 (10.1)	77.3 (11.1)	88.9 (8.8)	-0.2 (10.8)	41.2 (13.6)	1568	1.7			
5. 10 individual images	-32.8 (5.2)	-129.2 (19.6)	61.7 (13.0)	-65.1 (8.6)	-43.1 (15.6)	91.9 (16.7)	105.3 (13.4)	-26.8 (16.8)	110.1 (21.6)	742	1.8			
6. 20 uncorrected images	-22.5 (4.1)	-79.8 (15.1)	31.5 (9.7)	-77.6 (6.4)	-32.4 (11.4)	79.8 (12.7)	91.8 (9.2)	-18.5 (14.8)	13.5 (14.6)	1192	1.7			

Beltsville is held fixed on the SAO C-5 datum. The first row of each entry gives the adjusted coordinate, and the second row gives the standard deviation.

refraction, phase angle, and parallax aberration. Surprisingly, smaller residuals were obtained than with most of the adjustments involving corrected images. In view of the spreads of the adjusted coordinates provided by the other adjustments, the solutions provided by the uncorrected observations appear to be quite reasonable in most coordinates. This is only an indication that the effect of the systematic error in the observations is less than the effect of sample variation and is certainly no justification for allowing known systematic errors to remain in the observations.

It appears clear from the a posteriori standard deviations of the adjustments that the quality of a single image from a BC-4 plate is about $1''.7$. A careful analysis of the residuals from the orbital mode adjustments indicated that two statistical populations were present: the standard deviation of a single image from a plate taken with a 300 mm lens is about $2''.0$, and that for a 450 mm lens is about $1''.4$. These results agree well with technological analyses of the error components. The standard deviation of a fictitious image taken from a fifth-degree curve fit to the entire satellite trail appears to be about $0''.7$, which agrees well with the results obtained by ESSA.

Although the 20 individual images appear to contain more information than the single fictitious image, there is a slight problem in that these 20 images must be correlated. The correlation arises because the same set of plate constants was used to transform the (x,y) plate coordinates of each image to right ascension and declination. This means that each right ascension is correlated with every other right ascension on the same plate since they all contain the same errors in the plate constants. However, this type of correlation cannot be handled by either the geometric or orbital mode adjustment programs used in these experiments (nor can it be handled by most programs described by other investigators in available reports). Since the same considerations apply to the MOTS and PC-1000 data deposited in the Data Center, the effect of neglecting these correlations was investigated in some depth. The worsening of the solution caused by neglecting these correlations was found to be a function only of the number of images per plate and the correlation

coefficient r between different coordinates on the same plate. This worsening effect is tabulated in Table 2.2 below. The interpretation of this table is that the uncertainty of the solution obtained by neglecting correlation between images is no worse than the tabulated value times the uncertainty of the minimum variance solution.

Table 2.2 Worsening Factors for the Effect of the Neglect of Correlations Between Observations		
Number of Images per Plate	Worsening Factor	
	$r = 0.06$	$r = 0.1$
1	1.0	1.0
2	1.0036	1.010
3	1.007	1.021
4	1.013	1.034
5	1.019	1.049
10	1.062	1.146
15	1.117	1.260
20	1.179	1.383
25	1.245	1.511

If the uncertainty of the direction of the optical axis for a BC-4 plate is taken to be $0''.5$, then the correlation between different images is about 0.06. Thus the worsening of the solution is 1.062 and 1.179 when the number of individual images taken from each plate is 10 and 20 respectively. A correlation coefficient of $r = 0.1$ is more representative of the situation from MOTS and PC-1000 photography.

It appears that the most information may be obtained from the BC-4 data deposited in the Data Center by processing 20 individual images per

plate in either the geometric or short-arc mode, although some allowance must be made for the worsening of the solution if the correlation between different images is neglected. The use of more than 20 individual images per plate is not recommended since then the images would have to be taken so close together in time that the effect of serial correlation would become appreciable.

For those groups that are measuring their own plates and for whom cost considerations are important, the recommended procedure is to perform separate astrometric plate reductions for several different areas of the plate and to form one fictitious image for each area. This method is investigated in the reports by Hornbarger and by Veach [Reports of the Department of Geodetic Science, Nos. 106 and 110]. The images formed by this method will be correlated only to the extent that the sets of stars used in performing the astrometric plate reductions overlap.

The results of the experiments in the short-arc mode indicate that the simple Keplerian orbit is a sufficient model for arcs lasting under five minutes, at least for optical data (for arcs over ten minutes, or for electronic data with its greater inherent precision, a more sophisticated orbit model may be necessary). Because the arcs recorded by the BC-4 plates were so short, some difficulty was experienced with ill-conditioned sub-matrices in the normal equations. When this difficulty was investigated, it was found that the condition of these matrices improved only moderately when longer arcs were used. The use of epoch position and velocity vectors X and $\dot{X}$ rather than Keplerian elements as unknowns did not seem to help the conditioning. However, being careful to select a reference epoch that lies near or in the data span does seem to alleviate the difficulty of ill-conditioning for very short arcs.

A report describing the short-arc orbit constraint program and the investigations into various methods of adjusting the BC-4 data is currently being prepared for printing. The results described in this section are also discussed in greater detail there [Report of the Department of Geodetic Science, No. 118].

2.3 Data Handling

2.31 Data Received and Being Analyzed

Essentially all of the simultaneous optical data (PC-1000, MOTS, and BC-4) that was in the Data Center as of July 1, 1968, has been received, the times converted to UT1, and the correction for parallactic refraction has been made. In addition, all of the SECOR data from the Pacific net, as of July 1, 1968, has been received. Altogether approximately 5000 MOTS, 2000 PC-1000, 55,000 SECOR observations are being investigated.

An attempt was made to tie the MOTS station network to the Coast and Geodetic Survey worldwide net, but it was impossible because of the small amount of data available. A letter was sent to Mr. Harris exploring the problem.

One of the main areas of concern at the present time is to perform a simultaneous adjustment using PC-1000, MOTS, and SECOR data. Approximately 2000 ranges that were observed from the SE United States quad have been received from the Army Map Service, and there are several ties between these stations and PC-1000 camera stations. A data screening program to eliminate the bad observations from the adjustment is in process. Both the PC-1000 and MOTS data have been screened to the extent that they are ready for the combined adjustment.

It seems that the PC-1000 data received from the Data Center has many very bad observations (complete blunders), much more so percentage-wise than the MOTS data (approximately 30% vs. 10%). In some cases the bad observations are on just one or two flashes of a sequence, but thirteen flash sequences have been located where all of the observations are bad from at least one station. This information is given in Table 2.3. On five of these passes the bad station has been determined.

Table 2.3 Flash Sequences Where Bad PC-1000 Observations Have Been Detected				
Date		Starting Time	Stations Involved	Comments
1	4 Jan 66	6 ^h 20 ^m	3401, 3405	
2	15 Jan 66	2 56	3401, 3657	3657 is bad
3	20 Jan 66	3 24	3407, 3861	
4	23 Jan 66	3 36	3405, 3406	
5	26 Jan 66	1 45	3401, 3404, 3405, 3406, 3407	3405 is bad
6	29 Jan 66	1 55	3649, 3657	3649 is bad
7	1 Feb 66	0 05	3406, 3407, 3657	3406 is bad
8	1 Mar 66	9 14	3402, 3657	
9	30 Mar 66	0 49	3106, 3406	
10	30 Mar 66	2 53	3401, 3406	
11	31 Mar 66	0 45	3106, 3407	
12	31 Mar 66	3 02	3106, 3402	
13	26 Apr 66	7 16	3401, 3402, 3861	3402 is bad

2.32 BC-4 Data Tape Processing

This work concerns the magnetic tape Astro #221 (Serial #3380) which was deposited with the NSSDC by ESSA on April 15, 1968. A copy of this tape was furnished to OSU by the Data Center. This tape contains 298 files, each file corresponding to a single BC-4 plate and containing the right ascensions and declinations of 200-600 satellite images.

Processing of this tape at OSU is as follows:

(1) Deletion of 50 files. Of these, 29 files are duplicates of others, 20 plates contain errors in the "Date of Observation" (or are the companion plates

in the event with such plates), and one plate has no companion on the tape (event 2460). These 50 files have already been identified by the NGSP Project Manager.

(2) All observations will be corrected for parallactic refraction, phase angle, and parallactic aberration (light-travel time). This will be done using existing programs at OSU. The orbits used by ESSA in their satellite predictions will be used to generate the range to the satellite.

(3) All files will be unblocked and the file marks will be removed. This will make the tape compatible with systems that have no provisions for handling file marks and short records (e.g., the Fortran IV input-output routines under the IBM 7090/94 IJOB monitor). The end of each plate will still be marked by a pseudo-end-of-file character inserted in column 80 of the last record of each plate when the tape was generated by the USCGS.

(4) The corrected and reformatted tape will be sent to the Data Center.

Work on this phase of the project began in the last week of the reporting period.

2.33 Conversion of Existing Computer Programs

The Ohio State University Computer Center has recently installed an IBM 360/75 computer. It is expected that OSU's 7094 will not be available after June, 1969. This means that all computer programs written under this contract must be converted to the 360 operating system and Fortran IV language. For programs that were originally written in Fortran IV for the 7094, the conversion is not difficult. However, the programs that were originally written in the SCATRAN language must be rewritten.

Specifically, it is necessary to rewrite the group of programs known as "Least Squares Adjustment of Satellite Observations for Simultaneous Directions and Ranges" and described in Reports of the Department of Geodetic Science Nos. 86, 87, and 88. The use of the Optical Adjustment program during the last year has uncovered a few areas in which this program is not sufficiently flexible. Therefore, several features will be either modified or added to this program when it is rewritten for the System 360. These features

include the following:

(1) The method of computing an approximate satellite position will be changed to use the observations from all stations participating in the event.

The equation by which this is done is

$$S^0 = [\sum_i (U_i U_i^T - I)]^{-1} [\sum_i (U_i U_i^T - I) X_i]$$

where S^0 is the approximate satellite position, U_i is the unit vector in the direction of the observation from the i th station, X_i is the position of the i th station, and the sums are taken over all stations participating in the event. The approximate satellite position formed in this way is not a rigorous least squares estimate; however, it will always be a sufficiently accurate position around which to linearize the observation equation. Moreover, it will serve well for the detection of blunders in the observations.

(2) The observation equation will be rewritten to be linear in the residuals rather than the unknowns, i. e., the observation equation is presently of the form

$$X + BV + W = 0.$$

However, the B matrix is nonsingular, so that this may be rewritten

$$B^{-1}X + V + B^{-1}W = 0.$$

This form will not complicate the formation of the normal equations and will facilitate obtaining the residuals of the adjustment.

(3) All observation equations will be saved on external storage for subsequent formation and analysis of residuals.

(4) The range, optical, and solution programs will be combined into one program, with the option to execute only one module or all modules.

(5) Constraint equations will be added to fix the azimuth between stations or the relative position between stations. The relative position constraint is especially necessary. When different networks are being combined, the equipment of the different systems are often collocated. However, this usually means that the equipment are located close together, not precisely at the same location.

3. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time

Edward J. Krakiwsky, Research Associate, part time, until December

Georges Blaha, Research Associate, part time

Charles R. Schwarz, Research Associate, part time

James P. Reilly, Research Assistant, part time

Jeanne C. Preston, Research Aide, part time

Irene B. Tesfai, Technical Assistant, full time

John R. Miller, Research Aide, part time, until August

Jane B. Sims, Technical Assistant, part time, until November

William H. Wright, Research Aide, part time

4. TRAVEL

Trips made by Project Supervisor Ivan I. Mueller during the report period are:

Washington, D. C., August 20, 1968

to attend discussions at NASA Headquarters concerning the project

San Antonio and Houston, September 29 - October 3, 1968

to attend Semi-Annual Meeting of the American Society of Photogrammetry and to visit Manned Spacecraft Center

Washington, D. C., October 25, 1968

to attend informal meeting to discuss the consequences of changes in the coordinates of the pole at the U. S. Naval Observatory

San Francisco, November 30 - December 4, 1968

to attend the Seventh Western National Meeting of the American Geophysical Union.

5. REPORTS PUBLISHED TO DATE

OSU Department of Geodetic Science Reports published under Grant

No. NSR 36-008-003:

- 70 The Determination and Distribution of Precise Time
by Hans D. Preuss
April, 1966
- 71 Proposed Optical Network for the National Geodetic Satellite Program
by Ivan I. Mueller
May, 1966
- 82 Preprocessing Optical Satellite Observations
by Frank D. Hotter
April, 1967
- 86 Least Squares Adjustment of Satellite Observations for Simultaneous
Directions or Ranges, Part 1 of 3: Formulation of Equations
by Edward J. Krakiwsky and Allen J. Pope
September, 1967
- 87 Least Squares Adjustment of Satellite Observations for Simultaneous
Directions or Ranges, Part 2 of 3: Computer Programs
by Edward J. Krakiwsky, George Blaha, Jack M. Ferrier
August, 1968
- 88 Least Squares Adjustment of Satellite Observations for Simultaneous
Directions or Ranges, Part 3 of 3: Subroutines
by Edward J. Krakiwsky, Jack Ferrier, James P. Reilly
December, 1967
- 93 Data Analysis in Connection with the National Geodetic Satellite Program
by Ivan I. Mueller
November, 1967

OSU Department of Geodetic Science Reports published under Grant

No. NGR 36-008-093:

- 100 Preprocessing Electronic Satellite Observations
by Joseph Gross
March, 1968
- 106 Comparison of Astrometric and Photogrammetric Plate Reduction Techniques
for a Wild BC-4 Camera
by Daniel H. Hornbarger
March, 1968

- 110 Investigations into the Utilization of Passive Satellite Observational Data
 by James P. Veach
 June, 1968
- 114 Sequential Least Squares Adjustment of Satellite Triangulation and
 Trilateration in Combination with Terrestrial Data
 by Edward J. Krakiwsky
 October, 1968
- 118 The Use of Short Arc Orbital Constraints in the Adjustment of Geodetic
 Satellite Data
 by Charles R. Schwarz
 December, 1968