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THE USE OF A GROUND-BASED MULTIPLE-BEAM DETECTOR IN CROSSED-BEAM ATMOSPHERIC EXPERIMENTATION

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ABSTRACT

Experimentation is planned at MSFC test sites to measure near-ground winds both by tower-mounted anemometers and by a crossed-beam system combining a single-beam with a multiple-beam detector. The pertaining beam geometry is studied in the present report, its range of physical applicability is examined, and a proposal is made of what is considered a best detector arrangement for observing horizontal winds at different heights simultaneously.

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RESEARCH AND DEVELOPMENT OPERATIONS

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THE USE OF A GROUND-BASED MULTIPLE-BEAM DETECTOR IN CROSSED-BEAM ATMOSPHERIC EXPERIMENTATION

SUMMARY

This report is concerned with establishing a suitable experimental layout for a system consisting of a single-beam and a multiple-beam detector receiving, say, 2n beam pairs. Both are assumed as sitting on level ground. Such a system is capable of measuring horizontal winds simultaneously near n test heights provided the beam couples are properly arranged relative to the single beam. In particular, they must not be contained in one plane, forming a fan there.

Since all the couples are subject to the same or quite analogous conditions, it suffices to deal with one pair only. (Two different heights are considered in a numerical example towards the end of the paper.) The most pressing of these conditions are nearness of the three beams in the region under observation and the limitation of the transmitted errors that spring from the expressions for the two velocity components. These depend on the inverse times needed by (point-like) eddies to move from one (lineal) beam to the next. The experimental error generally is assumed to lie within ±0.1 second of the true travel time. The parameters entering the velocity expressions have been determined so as to answer best the above conditions. A detector design criterion has also been taken into account.

The fact that two beams are received by the same detector impairs the adaptability of the system. It can hardly handle other than horizontal winds blowing into or out of an azimuthal angle range of about 45 degrees.

Detector arrangement guidelines are given in section VI.

I. INTRODUCTION

In atmospheric experimentation three single-beam detectors suffice to monitor winds blowing, near a given height, from a certain compass of directions. Fundamentals and details of the system's performance have been studied in reference 1.* The guiding ideas and some of the mathematical formulations apply with the present problem as well. The numbers identifying the latter will be given by adding the symbol A in front.

Multi-beam detectors primarily aim at observing winds at different heights simultaneously; they are made to receive light from several directions.** It can be shown that, for the purpose intended, these "beams" must not fan out in one plane, desirable as this may be for the hardware design. The mathematical proof, although not difficult, is somewhat long-winded and is not spelled out here. From geometric inspection, it is fairly clear that, if the one detector's single beam and a beam pair of the other approach each other at a preselected height, as they must do to insure trustworthy measurement, a second pair sideways in the same plane cannot achieve the same end, the single beam missing it by a wide margin. Rather, the pairs must be stacked one above the other for measuring at different heights, so that they can be kept close there to the first detector's line of sight.

How to adapt the pair inclinations to the various heights will be brought out by the investigation, which, however, can and will be concerned with the handling of one exemplary pair only; the height dependency of others is of course analogous.

In two ways the combination of a single-beam and a two-beam detector differs from the setup envisaged in reference 1. It violates the condition that the beams cannot be allowed to intersect. In addition, there is a loss of free parameters caused by having, so to speak, two detectors at the same location. Both these deviations tend to restrict the applicability of the arrangement, as indeed they will be shown to do.

II. VELOCITY COMPONENTS

When operating three separate detectors receiving lineal beams a, b, c from the directions $\underline{\alpha}$, $\underline{\beta}$, $\underline{\gamma}$, respectively, one can obtain the eddy transit times τ^*_{ab}, τ^*_{bc}, τ^*_{ca} in between beams and then compute the wind vector components, V_i, from the system

* W. Heybey, "Wind Vector Calculation Using Crossed-Beam Data and Detector Arrangement for Measuring Horizontal Winds," NASA TM X-53754, July 11, 1968.

** Chief designer is E. Klugman, IITRI.

$$\left.\begin{aligned}
\frac{\Delta_{ab}}{\tau^*_{ab}} &= \begin{vmatrix} \alpha_2 & \alpha_3 \\ \beta_2 & \beta_3 \end{vmatrix} V_1 + \begin{vmatrix} \alpha_3 & \alpha_1 \\ \beta_3 & \beta_1 \end{vmatrix} V_2 + \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} V_3 \\
\frac{\Delta_{bc}}{\tau^*_{bc}} &= \begin{vmatrix} \beta_2 & \beta_3 \\ \gamma_2 & \gamma_3 \end{vmatrix} V_1 + \begin{vmatrix} \beta_3 & \beta_1 \\ \gamma_3 & \gamma_1 \end{vmatrix} V_2 + \begin{vmatrix} \beta_1 & \beta_2 \\ \gamma_1 & \gamma_2 \end{vmatrix} V_3 \\
\frac{\Delta_{ca}}{\tau^*_{ca}} &= \begin{vmatrix} \gamma_2 & \gamma_3 \\ \alpha_2 & \alpha_3 \end{vmatrix} V_1 + \begin{vmatrix} \gamma_3 & \gamma_1 \\ \alpha_3 & \alpha_1 \end{vmatrix} V_2 + \begin{vmatrix} \gamma_1 & \gamma_2 \\ \alpha_1 & \alpha_2 \end{vmatrix} V_3
\end{aligned}\right\} \qquad \text{(A12)}$$

where the α_i, β_i, γ_i are the beam direction cosines and the Δ are given by the expressions (A11), which depend on the detector locations P_1, P_2, P_3.

Suppose now the beams b and c are received by the same (multiple) detector at $P_2 = P_3$. With such a configuration, eddy trains leaving the beam b, for instance, will not in general encounter the beam c on their courses, so that the transit time τ^*_{bc} cannot be measured. When in an exceptional case the wind direction is such that an eddy train can intersect with the beam c, many more neighboring parallel trains will also arrive at c, making τ^*_{bc} an indeterminate quantity.*

Another aspect of the same predicament appears in the solution for V_i of the system (A12):

$$\left.\begin{aligned}
(\alpha\beta\gamma)V_1 &= \gamma_1 L_1 + \alpha_1 L_2 + \beta_1 L_3 \\
(\alpha\beta\gamma)V_2 &= \gamma_2 L_1 + \alpha_2 L_2 + \beta_2 L_3 \\
(\alpha\beta\gamma)V_3 &= \gamma_3 L_1 + \alpha_3 L_2 + \beta_3 L_3
\end{aligned}\right\} \qquad \text{(A13)}$$

---

*Note that in deriving (A12) it had been assumed that the wind is constant near the observation height.

where

$$L_2 = \frac{\Delta_{bc}}{\tau^*_{bc}}$$

is zero now, as $x_2 = x_3$, $y_2 = y_3$, $z_2 = z_3$.* As a consequence, the solutions (A13) would give the wind vector as

$$\underline{i}V_1 + \underline{j}V_2 + \underline{k}V_3 = \frac{1}{(\alpha\beta\gamma)} (\underline{\gamma}L_1 + \underline{\beta}L_3); \tag{1}$$

that is, it would be parallel to the plane of the beams b and c. Such a wind, if present, can be measured, since whatever finite values τ^*_{bc} may have, L_2 is always zero.**

If an anemometer can be placed at the height of interest, its vane would suggest a suitable vertical plane for the two beams. The third, piercing through it, must be geared to the selected height. This configuration permits the determination of all three wind components, but it has the disadvantage (aside from its need for outside support) that during observation the eddy trains have to remain parallel to the established plane to satisfy the condition imposed on the flow by having $L_2 = 0$. In other words, the eddy lines must always intersect with both the b- and c-beams. Atmospheric motion can rarely be expected to stay sufficiently put to achieve this adequately.*** Furthermore, any additional beam pair would have to lie in the same plane, since for it, $L_2 = 0$ as before. This precludes simultaneous observation at different heights, unless vanes at those heights indicate the use of other vertical planes. Even then, the various observed space parts must have rather definite lateral positions, so that the single a-beam can meet requirements in every one of them.

---

* To avoid the above ambiguities reference 1 requires non-intersecting beams, $L_i \neq 0$.

** With the multiple detector at height zero, τ^*_{bc} can be zero only for a ground level wind.

*** Determination of the horizontal component alone can be attempted with the use of two single beams whose common normal is parallel to the (approximately) known wind direction. Deviation from the normal will not prevent interception. Indeed, measurements with this arrangement have been carried out and reportedly met with some success.

In view of these technical complications and physical uncertainties, it seems best to forgo the determination of the vertical wind component. Since it appears in the equations, we will have to assign it a definite value, however.

Over level ground outside storm clouds or other instability regions, the wind can be presumed to blow largely horizontally. In these circumstances, the assumption $V_3 = 0$ seems justified.*

The system (A13), though still formally correct with $V_3 \equiv 0$, cannot be maintained, for the last line would imply that

$$\frac{\tau^*_{ca}}{\tau^*_{ab}} = - \frac{\beta_3}{\gamma_3} \frac{\Delta_{ca}}{\Delta_{ab}} = \text{const.}$$

Relation (1), derived from it, is no longer valid; there is no connection between the wind vector and the (b,c)-plane, which thus can be freely chosen.

For a similar reason the middle line of the system (A12) is to be dropped; it would call for a fixed value of the ratio V_2/V_1, which is quite as inadmissible as the above fixed ratio of transit times. We are thus left with

$$\frac{1}{\alpha_3\beta_3} \frac{\Delta_{ab}}{\tau^*_{ab}} = (a_2 - b_2)\, V_1 + (b_1 - a_1)\, V_2$$

$$\frac{1}{\gamma_3\alpha_3} \frac{\Delta_{ca}}{\tau^*_{ca}} = (c_2 - a_2)\, V_1 + (a_1 - c_1)\, V_2$$

where the abbreviations

$$a_k = \frac{\alpha_k}{\alpha_3}, \quad b_k = \frac{\beta_k}{\beta_3}, \quad c_k = \frac{\gamma_k}{\gamma_3}; \qquad k = 1,\ 2 \tag{2}$$

---

*With three single-beam detectors, the validity of this assumption can be checked by observation (it cannot be here). It was considered as ascertained in the later parts of reference 1, although merely for reasons of convenience. Work is now in progress to study the use of single-beam detectors when V_3 is not negligibly small.

have been introduced. On solving for the wind components V_i, an equivalent of the system (A13) emerges as

$$\left.\begin{aligned}\begin{vmatrix} b_1 - a_1 & b_2 - a_2 \\ c_1 - a_1 & c_2 - a_2 \end{vmatrix} V_1 &= (c_1 - a_1)\frac{\Delta_1}{\tau^*_{ab}} + (b_1 - a_1)\frac{\Delta_3}{\tau^*_{ca}} \\ \begin{vmatrix} b_1 - a_1 & b_2 - a_2 \\ c_1 - a_1 & c_2 - a_2 \end{vmatrix} V_2 &= (c_2 - a_2)\frac{\Delta_1}{\tau^*_{ab}} + (b_2 - a_2)\frac{\Delta_3}{\tau^*_{ca}}\end{aligned}\right\} . \quad (3)$$

By the definitions (A18),

$$\left.\begin{aligned}\Delta_1 &= \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & 0 \\ a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \end{vmatrix} \\ \Delta_3 &= \begin{vmatrix} x_1 - x_2 & y_1 - y_2 & 0 \\ c_1 & c_2 & 1 \\ a_1 & a_2 & 1 \end{vmatrix}\end{aligned}\right\} . \quad (4)$$

Here, the detectors (at P_1 and at $P_2 = P_3$) are assumed at ground level ($z_1 = z_2 = z_3 = 0$). Let us further agree to place the a-detector at the origin of a (right-handed) (x,y,z)-system ($x_1 = y_1 = z_1 = 0$). Earlier results with three single beams suggest the second quadrant for the point P_2, provided that horizontal winds in the first and third quadrants are to be monitored with sufficient accuracy. However, when the lines b and c now cross each other at z = 0, the concomitant loss of two free position parameters,* and therefore of flexibility, was found to aggravate the transmitted errors, many of them in a quite intolerable degree. It was inferred that the azimuthal wind angle (counted from the positive x-axis in the positive sense) could no longer be permitted to move through

*The third coordinate is z = 0 as before.

an entire quadrant ($0 \leqq \varphi \leqq 90°$). Although, even with $0 \leqq \varphi \leqq 45°$, the worst possible analytical errors can still be appreciable in some cases, the situation was judged not serious enough to demand further curtailing of the angle interval. Best results were obtained when the line $\overline{P_1P_2}$ was taken as the normal to the bisector of the azimuthal range (φ = 22 1/2°). It is evident that an equivalent setup for the range $-22.5° \leqq \varphi \leqq 22.5°$ would work as well if the point P_2 was located on the y-axis. This version was finally adopted for the mathematical simplicity it offers. For instance, the determinants (4) become

$$\Delta_1 = y_2(b_1 - a_1)$$

$$\Delta_3 = y_2(c_1 - a_1),$$

and the solutions (3) can be put into the form

$$\left.\begin{aligned} FV_1 &= \frac{1}{\tau_1} + \frac{1}{\tau_3} \\ FV_2 &= \frac{c_2 - a_2}{c_1 - a_1}\frac{1}{\tau_1} + \frac{b_2 - a_2}{b_1 - a_1}\frac{1}{\tau_3} \end{aligned}\right\}, \tag{5}$$

where

$$F = \frac{1}{y_2}\begin{vmatrix} 1 & \dfrac{b_2 - a_2}{b_1 - a_1} \\ 1 & \dfrac{c_2 - a_2}{c_1 - a_1} \end{vmatrix} \tag{6}$$

and

$$\tau_1 \equiv \tau^*_{ab}, \qquad \tau_3 \equiv \tau^*_{ca}. \tag{7}$$

The bearing of the beam direction cosines on the velocity components (and their errors) is indicated here by the compact combinations

$$\frac{b_2 - a_2}{b_1 - a_1} \equiv \frac{\alpha_3\beta_2 - \alpha_2\beta_3}{\alpha_3\beta_1 - \alpha_1\beta_3}, \quad \frac{c_2 - a_2}{c_1 - a_1} = \frac{\alpha_3\gamma_2 - \alpha_2\gamma_3}{\alpha_3\gamma_1 - \alpha_1\gamma_3}. \tag{8}$$

The right sides follow from the definitions (2). These two ratios are the significant parameters in the set (5). There had been three (C, Q_∞, Q_0) in the equivalent equation (A33) derived on the basis of three single-beam detectors. Impaired flexibility is here directly seen.

III. TRANSIT HEIGHTS

For proper wind identification, only those eddy paths are permissible that, within a narrow space region, are capable of connecting beam a with beam b, beam c with beam a. Most important are the heights of these paths; in reference 1 they have been designated by z_1^* and z_5^*, respectively, since they are the z-components of the vectors $\underline{r}_1^*$ and $\underline{r}_5^*$ appearing in the set (A15). Evaluation with $x_1 = x_2 = x_3 = 0$; $y_2 = y_3$; $z_1 = z_2 = z_3 = 0$ gives

$$\left.\begin{aligned} z_1^*(q) &= \frac{y_2}{(a_2 - b_2) - q(a_1 - b_1)} \\ z_5^*(q) &= \frac{y_2}{(a_2 - c_2) - q(a_1 - c_1)} \end{aligned}\right\}, \tag{9}$$

where

$$q = \frac{V_2}{V_1} = \tan \varphi. \tag{10}$$

The curves $z_i^*(q)$ are equilateral hyperbolas with common horizontal asymptotes $z^* = 0$ and the vertical asymptotes

$$q = q_1 = \frac{a_2 - b_2}{a_1 - b_1} \quad \text{and} \quad q = q_5 = \frac{a_2 - c_2}{a_1 - c_1}. \tag{11}$$

Of physical interest are those segments only that stretch within the interval

$$-q_o \leqq q \leqq q_o = \tan 22.5°. \tag{12}$$

If the infinite discontinuities at $q = q_1$ and $q = q_5$ are far outside it, the hyperbolas will run more or less parallel to $z^* = 0$ within. A wind blowing at any azimuth in $-22.5° \leqq \varphi \leqq 22.5°$ can be detected, provided it is constant over the height interval $|z_5^*(q) - z_1^*(q)|$, which consequently must be sufficiently small. It will shrink to zero at one point, if the hyperbolas intersect within the relevant interval, and will presumably remain narrow in its neighborhood, as one wishes it to be.*

More specifically, the height interval is required to extend around a preselected reference height h, especially so at the terminals $q = -q_o$ and $q = q_o$. If we put

$$\left.\begin{aligned} z_1^*(-q_o) &= hF_1, \quad & z_1^*(q_o) &= hG_1 \\ z_5^*(-q_o) &= hF_5, \quad & z_5^*(q_o) &= hG_5 \end{aligned}\right\} \tag{13}$$

the four parameters must therefore be chosen close to unity. Intersection as desired will occur when taking

$$F_1 > 1, \quad G_1 < 1, \quad F_5 < 1, \quad G_5 > 1. \tag{14}$$

(The opposite inequalities evidently would do as well.) If one allows for the fact that the heights (9) have to be positive, the first two inequalities (14) require that

$$0 < (a_2 - b_2) + q_o(a_1 - b_1) < (a_2 - b_2) - q_o(a_1 - b_1),$$

---

*One can force non-intersecting hyperbolas to lie close beside each other in the interval; however, errors have been shown to grow too large for certain wind directions.

therefore, that

$$(a_1 - b_1) < 0, \quad (a_2 - b_2) > 0, \quad q_1 = \frac{a_2 - b_2}{a_1 - b_1} < 0. \tag{15a}$$

Similarly, the second pair of the inequalities (14) gives

$$(a_1 - c_1) > 0, \quad (a_2 - c_2) > 0, \quad q_5 = \frac{a_2 - c_2}{a_1 - c_1} > 0. \tag{15b}$$

Let us then put

$$q_1 = -p_1 q_o, \quad q_5 = p_5 q_o. \tag{16}$$

If the positive parameters p_1 and p_5 here introduced are sufficiently large, the discontinuity points (11) of the functions (9) will be far outside the meaningful interval (12), one on either side of it.

IV. ERROR TRANSMISSION FINDINGS

Indications of what values to assign to the parameters p_1 and p_5 will issue from the analytical errors incurred when using the set (5) which, with the aid of the expressions (11) and (16), assumes the form

$$\begin{aligned} \frac{q_o}{y_2}(p_1 + p_5)\, V_1 &= \frac{1}{\tau_1} + \frac{1}{\tau_3} \\ \frac{q_o}{y_2}(p_1 + p_5)\, V_2 &= q_o\left(\frac{p_5}{\tau_1} - \frac{p_1}{\tau_3}\right) \end{aligned} \quad . \tag{17}$$

Note that q_o(= tan 22.5°) is not a free parameter at right.

The maximum observational time error was fixed at ±0.1 second. Extended investigations on its bearing on the components V_1 and V_2 led to the following conclusions:

(1) The common factor at left plays no role in error transmission.

(2) Although, in principle, the three beams should be rather closely bundled up in the zone of measurement, the times needed for the eddy trains to connect a with b, c with a, cannot be allowed to drop, say, below 1 second,* since the errors increase in inverse proportion to τ_1 and τ_3. The "worst" errors tabulated below are therefore based on either $\tau_1 = 1$ second, or $\tau_3 = 1$ second.

(3) To keep both the worst strength errors and the worst angle errors at low values is harder to accomplish than when operating with three single-beam detectors. Bracketing difficulties were pointed out earlier and traced to the loss of flexibility.

(4) The errors are rather sensitive to variations of p_1 and p_5. The pairs 10, 12 and 12, 10, e.g., are markedly inferior to the pair $p_1 = p_5 = 11$, which was found as one of the two best combinations. It produces equal, tolerably low maximal strength errors at $q = -q_o$ and $q = q_o$, and was chosen for that reason. A second pair ($p_1 = 8$, $p_5 = 20$) evolved from the desire to curtail certain large angle errors that occurred with the first pair. However, it generates relatively high strength errors in some circumstances. Details are given in the table to follow (slide rule computation). If V' and φ' are the faulty strength and azimuth results emerging with the (largest) observational error of $\pm$ 0.1 second, the percent error in terms of the true value V will be

$$p = (\frac{V'}{V} - 1) \times 100\%,$$

while $\Delta\varphi = \varphi' - \varphi$ gives the angle aberration.

Of the "true" values, τ_1 and τ_3, one is generally taken as unity and is assumed to have been read off as 1.1 second. With the other one, two faulty readings have been considered, 0.1 second too small and too high. In one exceptional instance where the τ's must be equal, the true values have been taken as $\tau_1 = \tau_3 = 1.1$ second to escape a reading-off below 1 second.

*It will be seen later, how this stipulation also enters into the determination of the detector span y_2, which is irrelevant as far as error transmission is concerned.

TABLE OF WORST EXPECTED ERRORS

	A. $p_1 = p_5 = 11$		B. $p_1 = 8$, $p_5 = 20$	
$\varphi = -22.5°$	$\tau_1 = \frac{6}{5}$	$\tau_3 = 1$	$\tau_1 = 3$	$\tau_3 = 1$
	$\tau' = \frac{11}{10}$	$\tau'_3 = \frac{11}{10}$	$\tau'_1 = \frac{29}{10}$	$\tau'_3 = \frac{11}{10}$
	$p = -8.4\%$	$\Delta\varphi = 22.5°$	$p = -12.6\%$	$\Delta\varphi = 15.4°$
	$\tau'_1 = \frac{13}{10}$	$\tau'_3 = \frac{11}{10}$	$\tau'_1 = \frac{31}{10}$	$\tau'_3 = \frac{11}{10}$
	$p = -9.5\%$	$\Delta\varphi = 1.8°$	$p = -11.5\%$	$\Delta\varphi = 7°$
$\varphi = 0°$	$\tau_1 = \frac{11}{10}$	$\tau_3 = \frac{11}{10}$	$\tau_1 = \frac{5}{2}$	$\tau_3 = 1$
	$\tau'_1 = 1$	$\tau'_3 = \frac{12}{10}$	$\tau'_1 = \frac{12}{5}$	$\tau'_3 = \frac{11}{10}$
	$p = 9.1\%$	$\Delta\varphi = 22.5°$	$p = -0.6\%$	$\Delta\varphi = 18.3°$
	$\tau'_1 = \frac{12}{10}$	$\tau'_3 = \frac{12}{10}$	$\tau'_1 = \frac{13}{5}$	$\tau'_3 = \frac{11}{10}$
	$p = -8.5\%$	$\Delta\varphi = 0°$	$p = -6.8\%$	$\Delta\varphi = 7.7°$
$\varphi = 22.5°$	$\tau_1 = 1$	$\tau_3 = \frac{6}{5}$	$\tau_1 = \frac{19}{9}$	$\tau_3 = 1$
	$\tau'_1 = \frac{11}{10}$	$\tau'_3 = \frac{11}{10}$	$\tau'_1 = \frac{181}{90}$	$\tau'_3 = \frac{11}{10}$
	$p = -8.4\%$	$\Delta\varphi = -22.5°$	$p = 12.2\%$	$\Delta\varphi = 15.7°$
	$\tau'_1 = \frac{11}{10}$	$\tau'_3 = \frac{13}{10}$	$\tau'_1 = \frac{199}{90}$	$\tau'_3 = \frac{11}{10}$
	$p = -9.5\%$	$\Delta\varphi = -1.8°$	$p = 0.4\%$	$\Delta\varphi = 5.8°$

V. HEIGHT INTERVALS AND TRAVEL PATH LENGTHS; DETECTOR REQUIREMENTS

Somewhat at a loss to decide which of the parameter combinations (A and B, see table above) offers the more attractive error complex, we set out to consult criteria not examined so far. These include:

(1) Limiting the height difference between any two parallel, beam-connecting eddy courses so that they can be taken as in fact belonging to the same wind.

(2) Providing reasonable transit path lengths. They must neither be too short (travel times below 1 second make for errors too large), nor too great lest eddies lose identity or decay when journeying from one beam to the next. This would destroy the correlation that lies at the root of the crossed-beam method.

(3) Practical considerations, as (a) the wish to have the detectors not too far apart and (b) ease in adjusting them.

Access to the first item is gained by combining expressions (9), (13), (15), and (16):

$$\left.\begin{aligned} z_1^*(-q_o) &= hF_1 = -\frac{1}{a_1 - b_1}\,\frac{y_2}{q_o(p_1 - 1)} \\ z_1^*(q_o) &= hG_1 = -\frac{1}{a_1 - b_1}\,\frac{y_2}{q_o(p_1 + 1)} \\ z_5^*(-q_o) &= hF_5 = \frac{1}{a_1 - c_1}\,\frac{y_2}{q_o(p_5 + 1)} \\ z_5^*(q_o) &= hG_5 = \frac{1}{a_2 - c_2}\,\frac{y_2}{q_o(p_5 - 1)} \end{aligned}\right\} . \tag{18}$$

This set of two pairs of relations yields the ratios

$$\frac{G_1}{F_1} = \frac{p_1 - 1}{p_1 + 1}\,, \qquad \frac{G_5}{F_5} = \frac{p_5 + 1}{p_5 - 1}\,, \tag{19}$$

which can be used to judge the terminal height intervals

$$\left.\begin{aligned}\Delta h(-q_o) &= (F_1 - F_5)h \\ \Delta h(q_o) &= (G_5 - G_1)h\end{aligned}\right\} \tag{20}$$

in terms of p_1 and p_5. These are the largest encountered, for the z^*-hyperbolas were made to intersect in between stations $q = -q_o$ and $q = q_o$. From their general course, it can be inferred that if, at one terminal, the height interval is cut down, it will grow larger at the other. Let us agree therefore to postulate the terminal height spans as equal. Relations (19) and (20) then give

$$F_1 = \frac{p_5}{p_1}\frac{p_1 + 1}{p_5 - 1} F_5 \tag{21}$$

or

$$\Delta h = (F_1 - F_5)h = \frac{p_1 + p_5}{p_1(p_5 - 1)} F_5 h.$$

If we adopt 0.9 as the smallest value admissible for F_5, we see that

$$\Delta h = 0.18h \quad \text{with} \quad p_1 = p_5 = 11$$

$$\Delta h = 0.166h \quad \text{with} \quad p_1 = 8,\ p_5 = 20.$$

At a great measuring height, these figures perhaps overtax the capability of the wind to stay constant in the vertical. Regrettably, nothing much can be done to improve on this situation. Lowering F_5 means to lower the three other height parameters as well, so that, in effect, one may be measuring beneath the reference height. With $F_5 = 0.9$ it already follows that, in the case (B), $G_5 = 0.995$, contradicting conditions (14), which intend to keep the hyperbolas near h. With $F_5 = 0.95$ the quantity G_5 rises to the acceptable value 1.05, but Δh, now = 0.175h, does not differ significantly from $\Delta h = 0.18h$ as found in the case (A). Still, one may be inclined to judge the latter slightly inferior to (B) on account of somewhat greater height spans.

For examining, secondly, the path lengths to be expected, we use the formulas (A16), which simplify into

$$R^*_{ab} = \frac{y_2 \sqrt{1 + q^2}}{|p_1 q_o + q|} , \qquad R^*_{ca} = \frac{y_2 \sqrt{1 + q^2}}{|p_5 q_o - q|} .$$

These functions attain the minima

$$R^*_{ab}\Big|_{min} = \frac{y_2}{\sqrt{1 + (p\ q_o)^2}} , \qquad R^*_{ca}\Big|_{min} = \frac{y_2}{\sqrt{1 + (p_5 q_o)^2}}$$

at

$$q_{min} = \frac{1}{p_1 q_o} \quad \text{and} \quad q_{min} = - \frac{1}{p_5 q_o} ,$$

respectively. Both of them, with the value combinations adopted, are well within the interval $< -q_o, q_o >$.

In case (A) ($p_1 = p_5 = 11$), the minimum values are equal:

$$R^*\Big|_{min} = \frac{y_2}{\sqrt{21.7636}} .$$

This result suggests a figure for the separation distance, y_2, which will grow larger if stronger winds are to be measured. Suppose one wishes to monitor winds up to 40 knots $\approx$ 20 m/sec. $R^*|_{min}$ then should be at least equal to 20 m, so that the eddy transit time reaches at least 1 second (to keep errors low). Hence,

$$y_2 = 93.3 \text{ m} \approx 94 \text{ m}.$$

The longest travel path is

$$R^*_{ca}\Big|_{q=q_o} = y_2 \frac{\sqrt{1 + q_o^2}}{q_o (p_5 - 1)} = 24.6 \text{ m}.$$

There is not much difference in path lengths here.

In the case (B), the shortest travel is connected with

$$R^*_{ca}\Big|_{min} = \frac{y_2}{\sqrt{69.64}} ,$$

leading to

$$y_2 = 167 \text{ m}$$

for 40-knots winds. The longest path becomes

$$R^*_{ab}\Big|_{q=-q_o} = y_2 \frac{\sqrt{1 + q_o^2}}{q_o(p_1 - 1)} = 62.4 \text{ m.}$$

This is not a forbiddingly large figure, although case (A) should provide for better correlations on the whole. The shorter base line, too, may speak in favor of it.

For ease of handling the multiple detector, the beams b and c should be symmetric to the vertical plane through the y-axis on which the detector is sitting. Such a requirement can be allowed for, since the conditions obtained up to now for the six independent beam direction parameters are only two in number:

$$\frac{a_2 - b_2}{a_1 - b_1} = -p_1 q_o, \quad \frac{a_2 - c_2}{a_1 - c_1} = p_5 q_o. \tag{22}$$

They follow from the expressions (11) and (16). We add to them the conditions

$$\gamma_1 = -\beta_1, \quad \gamma_2 = \beta_2, \; \gamma_3 = \beta_3, \tag{23}$$

necessary and sufficient for the symmetry desired. Inserting them into equations (22) (recalling the definitions (2)), one finds that the relation

$$\frac{a_1 - b_1}{a_1 + b_1} = - \frac{p_5}{p_1} \tag{24}$$

must be satisfied. Since now $-c_1 = b_1$, the first and third of the height expressions (18) yield

$$\frac{a_1 - b_1}{a_1 + b_1} = - \frac{F_5}{F_1} \frac{p_5 + 1}{p_1 - 1} .$$

On eliminating the fraction at left,

$$\frac{F_1}{F_5} = \frac{p_5 + 1}{p_5} \frac{p_1}{p_1 - 1} .$$

A different expression for this ratio appears through condition (21). Comparison leads to the demand

$$p_1 = p_5.$$

It is seen that if one wishes to provide for (1) the above beam symmetry and (2) equal terminal height intervals, one must adopt the case (A) where both p's have the same value 11. The decision, long in the balance, is finally made.

As a consequence, relation (24) entails that a_1 is to be taken as zero. The a-beam then runs in the symmetry plane of the beam couple b,c. The simplicity of this scheme throws additional support to (A); it should alleviate adjustment labors and minimize the peril of misaligning.

The technically more complicated case (B) might be preferred when wind direction is more important than wind strength.

VI. BEAM DIRECTIONS AND OPTICAL AXIS; OBSERVED VOLUME

In case (A) equations (23) reduce to the single relationship

$$b_2 + 11q_o b_1 = a_2, \tag{25}$$

while any of the four expressions (18) gives

$$b_1 = \frac{1}{10F_1}\frac{1}{q_o}\frac{y_2}{h}, \tag{26}$$

where y_2 = 94m, $q_o = \sqrt{2} - 1$, $F_1 = 1.08$ (as follows, with $F_5 = 0.9$, from the relation (21)). The direction parameters b_1 and $c_1 = -b_1$ by now depend on the test height h alone* and can be considered known. Viewed from the positive x-axis, beam c runs behind beam b (as $b_1 > 0$, $c_1 < 0$); in between them, the a-beam pursues a middle course. Eddy trains in the general direction of the negative x-axis will move from b to a ($\tau^*_{ab} \equiv \tau_1 < 0$), from a to c ($\tau^*_{ca} \equiv \tau_3 < 0$), so that the first line of the set (17) yields $V_1 < 0$ as it should do.

Of the two direction parameters a_2 and b_2, one remains undetermined. If we select

$$a_2 = \pm\sqrt{\frac{1}{\alpha_3{}^2} - 1},$$

we see that the elevation angle

$$\chi_a = 90° - \text{arc cos } \alpha_3$$

can be considered a free parameter. However, its value, as will be shown immediately, is not wholly discretionary, because the present design of the multiple detector does not permit spreads over ω_{bc} = 50° between the b- and c-beams. Thus,

$$1 > \cos\,\omega_{bc} = -\beta_1{}^2 + \beta_2{}^2 + \beta_3{}^2 = 1 - 2\beta_1{}^2 \geqq \cos 50° \approx 0.766.$$

Beam definition is provided by the elevation angle alone (a-beam) or in conjunction with the angle ψ made by both the b- and c-beams with their bisecting line ("optical axis"). For this angle, one has

*Note that a_1 is zero, i.e., h-independent.

$$\left.\begin{aligned} \cos\psi &= \cos\frac{1}{2}\,\omega_{bc} = \sqrt{1-\beta_1^2} \\ 1 > \cos\psi &\geqq \cos 25° = 0.90631 \end{aligned}\right\}. \tag{27}$$

Rearranging to write

$$\cos^2\psi = 1 - b_1^2\,\beta_3^2 = 1 - \frac{b_1^2}{1 + b_1^2 + b_2^2}\,,$$

one arrives at

$$b_2^2 = (b_1 \operatorname{cotg}\psi)^2 - 1. \tag{28}$$

There is a condition on b_1:

$$b_1 > \tan\psi \;(> 0),$$

which tells us that, ψ given, we cannot measure at any height we may wish to; for expression (26) requires that

$$h < \frac{y_2}{10.8q_o}\operatorname{cotg}\psi \approx 21 \operatorname{cotg}\psi.$$

Conversely, large test heights in general call for small angles ψ. This is understandable. Even at such heights the b- and c-beams must move not far apart in order to ensure proper measurement.

By relations (25) and (28)

$$\cot^2\psi = \frac{1}{b_1^2} + \left(11q_o - \frac{a_2}{b_1}\right)^2 \tag{29}$$

The value chosen for a_2 must comply with the requirement that ψ should remain below 25°.

Planned exploratory experimentation will monitor winds near the test heights

$$h_1 = 11.43\text{m}, \quad h_2 = 30.48\text{m}$$

(and therefore needs two b,c-couples). By expression (26), corresponding figures for b_1 follow with $q_o = 0.41421$, $y_2 = 94$ m, $F_1 = 1.08$. The elevation angle $\chi_a = 45°$ gives $a_2 = \pm 1$. With the positive sign, it proves to be too small for technically useable values of ψ. The larger angle $\chi_a = 60°$ yields results well below the upper bound: $\psi_1 = 13.5°$, $\psi_2 = 14.5°$, even if again the positive choice,

$$a_2 = + \frac{1}{\sqrt{3}} ,$$

is made. Since α_3 is always positive, $a_2 > 0$ means that $\alpha_2 > 0$. The a-beam then makes an acute angle with the positive y-axis. Regarding the two other beams, we may consult relations (25) and (26) to find

$$b_2 = a_2 - \frac{11}{10.8} \frac{y_2}{h} .$$

With both values of h, the subtrahend is larger than unity, rendering $b_2 < 0$ when

$$a_2 = \frac{1}{\sqrt{3}} .$$

Hence, $\beta_2 < 0$ and, by conditions (23), $\gamma_2 < 0$. The b- and c-beams thus make equal obtuse angles with the positive y-axis.

By definitions (2), the orthogonality condition

$$\beta_1^2 + \beta_2^2 + \beta_3^2 = 1$$

may be written as

$$\beta_3^2 = \frac{1}{1 + b_1^2 + b_2^2} .$$

Since b_1 and b_2 are fixed values by now, the equal elevation angles of the b- and c-beams can be computed. More useful, however, for practical application is the elevation angle, $\chi_{b,c}$, of the optical axis which has the direction of the vector sum

$$\underline{\beta} + \underline{\gamma} = \underline{i}(\beta_1 + \gamma_1) + \underline{j}(\beta_2 + \gamma_2) + \underline{k}(\beta_3 + \gamma_3).$$

Again applying conditions (23), we arrive at

$$\underline{\beta} + \underline{\gamma} = 2\beta_2\underline{j} + 2\beta_3\underline{k}.$$

This result shows that the optical axis is in the symmetry plane, and yields its direction cosines as

$$0, \quad \frac{\beta_2}{\sqrt{\beta_2^2 + \beta_3^2}} = \frac{b_2}{\sqrt{1 + b_2^2}}, \quad \frac{\beta_3}{\sqrt{\beta_2^2 + \beta_3^2}} = \frac{1}{\sqrt{1 + b_2^2}} = \frac{1}{b_1 \operatorname{cotg} \psi}.$$

They show that its angle with the positive y-axis is obtuse (like those of the beams whose angle it bisects). From

$$\chi_{b,c} = 90° - \text{arc cos}\,\frac{1}{b_1 \operatorname{cotg} \psi},$$

one finds that, with the numerical values adopted above,

$$\chi_{b,c} = \begin{cases} 7.5° & \text{if } h = h_1 \\ 22° & \text{if } h = h_2. \end{cases}$$

Although $h_2 > h_1$, the segment, h cosec $\chi_{b,c}$, of the optical axis between the detector and the two height levels is shorter (81.4m) for $h = h_2$ than for $h = h_1$ (87.6m). This may explain the fact that ψ_2 turned out somewhat larger than ψ_1.

With the aid of the angles χ_a, ψ, $\chi_{b,c}$, the detectors can be mounted for observation. A general idea of the prevailing wind direction is required so as to fix the line P_1P_2 approximately normal to it. On it, the detector seats are separated by y_2 m. The a-beam and the two "optical axes" are contained in the vertical plane through the base line, facing each other at specified elevation angles.* In two planes defined by those axes and the normal direction of the vertical (symmetry) plane, the beam couples are set by the deviation angles ψ_1 and ψ_2, respectively.

The beam direction cosines are not needed in practice, yet are given here for the sake of theoretical completeness ($F_1 = 1.08$):

$$\alpha_1 = 0, \qquad \alpha_2 = a_2\alpha_3, \qquad \alpha_3 = \frac{1}{\sqrt{1 + a_2^2}}$$

$$\beta_1 = \sin\psi, \qquad \beta_2 = \left(a_2 - \frac{11}{10F_1}\frac{y_2}{h}\right)\beta_3, \qquad \beta_3 = 10F_1q_o\frac{h}{y_2}\sin\psi$$

$$\gamma_1 = -\beta_1, \qquad \gamma_2 = \beta_2, \qquad \gamma_3 = \beta_3.$$

Those of the bisector (optical axis) are

$$\vartheta_1 = 0, \qquad \vartheta_2 = \frac{\beta_2}{\cos\psi}, \qquad \vartheta_3 = \frac{\beta_3}{\cos\psi}.$$

Excepting h and a_2, the parameters here have definite values settled upon by the preceding argumentation, which is also responsible in part for the form of the expressions. For a given observation height, the choice of a_2 determines the angle ψ as well as the direction cosines and therefore the elevation angles. It should be emphasized, however, that it does not reflect upon error transmission, nor on the travel path lengths (22), nor on the general shape of the space volume investigated (especially not on the F's and G's). The latter merely shifts parallel to the y-axis when a_2 is varied. This can be seen by the coordinates of the path terminals, which are the end points of the pertinent position vectors in the set (A15). Those of the vectors $\underline{r}_1^*$ and $\underline{r}_2^*$ are on a and b, respectively, those of the vectors $\underline{r}_5^*$ and $\underline{r}_6^*$ lie on c and a. Note that $p_1 = p_5 = 11$, also, that $c_1 = -b_1$, $c_2 = b_2$. The final results are:

*They intersect at the (here obtuse) angles $180° - (\chi_a + \chi_{b,c})$.

$$x_1^* = 0 \qquad y_1^* = a_2 z_1^* \qquad z_1^* = \frac{1}{b_1} \frac{y_2}{11q_o + q}$$

$$x_2^* = \frac{y_2}{11q_o + q} \qquad y_2^* = y_2 + b_2 z_1^* \qquad z_2^* = z_1^*$$

$$x_5^* = - \frac{y_2}{11q_o - q} \qquad y_5^* = y_2 + b_2 z_5^* \qquad z_5^* = \frac{1}{b_1} \frac{y_2}{11q_o - q}$$

$$x_6^* = 0 \qquad y_6^* = a_2 z_5^* \qquad z_6^* = z_5^* .$$

These coordinates vary with the direction, q, of the wind.* The two zero values of x^* were to be expected, both the points P_1^* and P_6^* lying on the a-beam; x_2^* and x_5^* confirm that c is "behind" b. The quantity b_1 is given by formula (26) and is a constant after settling for a test height h. Evidently, the x- and z-coordinates of all the path terminals do not depend on the value chosen for a_2. While, on the contrary, all y^*'s do vary with it (making for the shift indicated), the transit path lengths again do not. Indeed, by using relation (25), one derives

$$y_2^* - y_1^* = \frac{y_2 q}{11q_o + q} , \qquad y_6^* - y_5^* = \frac{y_2 q}{11q_o - q},$$

which differences are not affected by the value one may select for a_2; nor as a consequence, are the lengths $\overline{P_5^* P_6^*} \equiv R_{ab}^*$, $\overline{P_5^* P_6^*} \equiv R_{ca}^*$. This was already apparent from their expressions (22) (which can easily be rederived with the terminal coordinates now at hand).

VII. TRAVEL TIME RESTRICTIONS AND CONCLUSION

Atmospheric experimentation is threatened by the perplexing possibility that a few eddy trains may connect beams outside the space part under observation. The corresponding transit times elicited from the intensity records must be discarded; they would lead to spurious winds. In two dimensions ($V_3 = 0$) this can be done after establishing meaningful travel time ratios that do occur with transits in the volume singled out for measurement. It is highly improbable (although not outright impossible) that a pair of "wrong" connections should give rise to a

---

*One recalls that q is restricted to the interval $< -q_o, q_o >$. Other winds cannot be measured without violation of (at least one of the) basic requirements.

meaningful ratio. In the present instance, the range of the latter is rather limited.

With $p_1 = p_5 = 11$, the solutions (17) give

$$\frac{V_2}{V_1} \equiv q = 11q_o \frac{\frac{\tau_3}{\tau_1} - 1}{\frac{\tau_3}{\tau_1} + 1} , \qquad (30)$$

so that

$$\frac{\tau_3}{\tau_1} = \frac{11q_o + q}{11q_o - q} .$$

The fraction at right increases with q. From

$$-q_o \leqq q \leqq q_o,$$

$$\frac{5}{6} \leqq \frac{\tau_3}{\tau_1} \leqq \frac{6}{5} . \qquad (31)$$

Observed ratios outside this narrow interval should be dismissed on suspicion they might be owing to unrelated winds. If they are near the boundaries, one may be inclined to be lenient; however, one should inquire into error transmission and the height interval in which wind constancy would have to have prevailed.

Suppose, e.g., that record evaluation has given

$$\tau_1 = 4 \text{ sec.}, \qquad \tau_3 = 5 \text{ sec.},$$

so that the ratio, 1.25, is somewhat too large. By expression (30), the affiliated value of q becomes

$$q = \frac{11}{9} q_o > q_o.$$

The height difference (taking $a_1 = 0$, $c_1 = -b_1$, $p_1 = p_5 = 11$) is found as

$$\Delta h(q) \equiv |z_5^*(q) - z_1^*(q)| = 20F_1 q_o h \left|\frac{q}{(11q_o)^2 - q^2}\right|.$$

With $q_o = \sqrt{q} - 1$, $F_1 = 1.08$, and the above value of q, this gives $\Delta h \approx 0.221h$, a result considerably larger than 0.18h (the constancy interval at $q = q_o$). If it is thought acceptable with a presumably well-behaved wind, the observation may be considered valid on account of the rather large values of τ_1 and τ_3. Indeed, even if the observational error limit is doubled to ±0.2 second, the worst analytical errors can be shown to remain within reasonable bounds.

The narrow margins (31) indicate that, after correlating the experimental records, one has to look for (a,b) and (c,a) correlation maxima occurring at approximately equal delay times (both positive or both negative). Moreover, one pair of such maxima ought to be detectable in any event. If it is not, a careful check of all underpinnings of the experimentation would seem to be in order. These include basic concepts of the method, multitudinous aspects of delay time acquisition, detector layout, design, and adjustment, required properties of the atmospheric state and motion. Into the more fundamental ones, one would of course go only when repeated failure cannot be explained otherwise.

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THE USE OF A GROUND-BASED MULTIPLE-BEAM DETECTOR IN CROSSED-BEAM ATMOSPHERIC EXPERIMENTATION

by W. H. Heybey

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