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THEORETICAL STUDIES OF COMPRESSOR NOISE

by M. V. Lowson

Prepared by
WYLE LABORATORIES
Huntsville, Ala.
for Langley Research Center

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A	Compressor area $= \pi R_T^2$
A	Arbitrary constant
A_B	Area of blades
A_e	Effective area of compressor $= \pi R^2$
A_λ	Complex Fourier coefficient of wake velocity (see Equation 18)
B	Number of rotor blades
C_D	Drag coefficient
D	Compressor diameter in inches
D	Drag (circumferential force component)
D_λ	$= D_{\lambda R} + i\, D_{\lambda I}$ complex drag harmonic (see Equations 50 and 51)
F_i	Force (three component vector)
F_r	Component of force in the direction of the observer
J_n	Bessel function of first kind and nth order
K_n	Modified Bessel function of second kind and nth order
L	Lift per unit span
M	Mach number and rotational Mach number in compressor
M_r	Component of convection Mach number in direction of observer
Q	Mass source intensity ($q = \partial Q/\partial t$)
R	Effective radius of action of point source
R_T	Compressor tip radius
S	Lift response function (see Equations 77 and 79)

LIST OF SYMBOLS (Continued)

T	Thrust
T_λ	$= T_{\lambda R} + i\ T_{\lambda I}$ complex thrust harmonic (see Equations 50 and 51)
U_1	Relative velocity at blades
V	Number of stator vanes
V_R	Rotational velocity at effective radius R
V_T	Mechanical tip velocity of rotor blades (feet per second)
W	Power level
$W_{n\lambda}$	Power in nth sound harmonic due to λ harmonic source input
a_0	Speed of sound in free air
$a_{\lambda T}, b_{\lambda T}, a_{\lambda D}, b_{\lambda D}$	Harmonic components of thrust and drag (see Equation 51)
b	Blade spacing = $2\pi R/V$
c	Blade chord
f	Arbitrary function
h	Typical dimension for velocity defect (wake displacement thickness)
i	$\sqrt{-1}$ and suffix for tensor notation
j	Suffix for tensor notation
k	Order of compressor source input harmonic ($\lambda = kV$)
m	Order of compressor sound harmonic ($n = mB$)
n	Sound harmonic number ($n = mB$)
p	Fluctuating pressure ($a_0^2\ \rho$)
q	Mass source intensity (second rate of introduction of mass)

r	Distance from observer to source
r	Power summation parameter
r_1	Distance from observer to compressor hub
s	Blade span
t	Time
v	Upwash
w	Velocity of wake (in direction of mean flow)
x,y,z	Cartesian coordinates, x along compressor axis out of compressor
x_i, y_i	Cartesian coordinates of observer and source locations, respectively

Greek Symbols

α	My/r
α	Angle of blade to x-axis (see Figure 5)
β	Angle of blade to wake flow (see Figure 5)
γ_λ	$= \alpha_\lambda + i\,\beta_\lambda$ complex Fourier coefficient of wake velocity (see Eq. 18)
η	Coordinates following source
θ	Angle around rotor ($\theta = 0$ on y axis)
λ	Source input harmonic ($\lambda = kV$)
μ	Modal order $= \lvert n - \lambda \rvert$, and phase factor
ν	Summation parameter over stator vanes
ξ	Circumferential coordinate
π	3.141 ...

LIST OF SYMBOLS (Continued)

ρ	Density perturbation
ρ_0	Density of free air
σ	Non-dimensional frequency over blade = $\omega c/U$
τ	Retarded time $\tau = t - r/a_0$
ϕ	Reference angle between observer and zeroth blade
Ψ	Angle around source ($\Psi = 0$ along x-axis)
ω	Circular frequency
Ω	Angular velocity of compressor rotor

Suffices

$(\underset{\sim}{\ })$	Indicates vector quantity
$\lvert\ \rvert$	Modulus of
$(\ )_n$	Refers to result for nth harmonic sound only
$(\ )_\lambda$	Refers to result for λth source harmonic only
$(\ )_\mu$	Refers to result for μth mode input only

THEORETICAL STUDIES OF COMPRESSOR NOISE

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ABSTRACT

A comprehensive theoretical study of the discrete frequency noise generated by compressors and fans is presented. Analytical results for both sound pressure and sound power are given for four separate source mechanisms in the compressor, including rotating and stationary force and mass fluctuations. Each mechanism is modeled by a cascade of point sources, one on each blade, with appropriate phase relations between them. Duct effects are ignored. It is shown that many phenomena generally associated with the duct, such as cutoff frequency effects, are in fact due to source effects, so that the present results are of general applicability to all hard wall duct cases. The results demonstrate considerable similarity between the rotor and stator noise radiation characteristics, so that the rotor and stator are equally significant in the noise generation process.

Extensive numerical results for both sound power and directivity based on the analysis are presented and discussed. The leading features are the demonstration of equivalent cutoff effects without a duct, the particularly unfavorable acoustic power and directivity characteristics near to cutoff, and the favorable effects of low order modes on the sound. The theoretical prediction of reduced noise for a compressor with equal numbers of rotors and stators is verified by existing data. The principal problem in the prediction of noise is in predicting the aerodynamic characteristics of the wake flows in the compressor, and this is discussed. A crude model for the wake flow is used to derive results which give acceptable comparison with experiment. The model gives a diameter squared times relative velocity to the fifth power law for the sound radiation due to the fluctuating force terms, which are found to be significantly larger than the siren (mass source) terms in the compressor. The power law derived is in agreement with experimental data. The effects of increasing rotor-stator and stator-rotor separation are discussed. In general it appears that the theory is in fairly good agreement with experiment, and can therefore be used for design studies. The principal errors arise in predicting the aerodynamic characteristics of the wakes in the compressor.

1.0 INTRODUCTION

Noise radiation from jet-engined aircraft has two principal components; the typically broad band roar due to the jet exhaust and the typically discrete frequency whine due to the compressor and turbine. Jet exhaust noise, which used to dominate the observed sound field has been successfully attacked, partly as a result of theoretical suggestions, but perhaps more as the consequence of enlightened engineering design. Unfortunately the community noise problems caused by noise radiation from jet engines have still grown in magnitude in recent years. The cacophonous roar of the high speed exhaust flows of early jet aircraft is now being replaced by the irritating whine of enlarged compressor and fan stages. This latter is of increased significance for at least three reasons; firstly, the discrete frequencies typically occur in the range of maximum auditory response; secondly, the subjective effects of a discrete frequency sound are found to be greater than a broad band noise of the same intensity; and thirdly, the sound is generally most offensive on approach when the aircraft is close to the ground, and there is little possibility of controlling the noise by changing flight profile. With the imminent introduction of very high bypass ratio fan-jet engines, the problems caused by compressor and fan noise must be expected to become more severe.

There is a clear need for further experimental and theoretical work on the problem, both to suggest methods for controlling the noise, and to give reliable prediction techniques, so that compressor noise can be included as a parameter in the initial design stages of the aircraft. Some work has already been accomplished and has given several important results for compressor noise control (for example, Refs. 1-12). Unfortunately much of the experimental data is unpublished, so that only limited results are available in the open literature. Several theoretical studies have been published (Refs. 1, 4, 6, 9, 10 and 12). These have, in general, concentrated on the effects of the inlet duct on the radiated noise and on broad studies of trends. It appears that only Hetherington (Ref. 6) and Slutsky (Ref. 12) have attempted to predict actual sound radiation levels directly from theory.

The problems associated with either experimental or theoretical studies of compressor noise are certainly large. This is because the observed noise is the end product of a long chain of cause and effect, to be discussed in more detail in the next section. Each link in this chain is under the influence of several parameters, and the large number of links leads to a dependence on an excessively large number of parameters. Thus it is extremely difficult to achieve complete coverage of the parameters involved. As will be seen in Section 8 of this report, simple empirical predictions are frequently in error by ± 10 dB, and even this margin is small when it is realized that effects such as rotor-stator spacing and duct cutoff can cause differences in noise radiation of up to 20 dB and 40 dB, respectively.

Nevertheless, it is clearly desirable to obtain a more complete theoretical understanding of compressor noise radiation. In this report a simplified case is studied which ignores the effect of a hard wall duct. This approach would be expected to be valid for duct lengths of less than a half wavelength. However, it will be shown in the report that the analysis is, in fact, more generally valid and appears to apply to all practical hard wall duct cases irrespective of length. Naturally, ducts with acoustically treated walls are excluded from a direct application of the theory.

It may be noted that the calculation of acoustic radiation from a compressor is basically a simpler problem than calculation for the jet exhaust. The fluctuating force and mass sources present within the compressor are definable in terms of the compressor geometry and aerodynamics without too much complication although comparatively little source data is presently available. In contrast, it is extremely difficult to define with any degree of precision the acoustic stress tensor T_{ij} which is the source of jet noise. It is therefore realistic to hope for good accuracy from a basic theory in the present case, providing information becomes available to define the source terms well.

In the report, first of all the basic characteristics of noise generation are discussed, and it is shown how four separate source mechanisms can be identified. Succeeding sections give theoretical descriptions of each of these mechanisms from first principles and methods for predicting the magnitude of the source terms are discussed. Fairly extensive numerical results are then presented, including directionality and overall power levels as a function of basic acoustic parameters. In a final section an attempt is made to condense all the detailed results for a broad comparison with experiment. The effect of axial motion on the sound output is discussed in an Appendix.

2.0 MECHANISMS OF NOISE GENERATION

2.1 Physical Basis

It is worthwhile, before starting the mathematical developments, to review the basic physical mechanisms by which noise is generated. It is important to realize that sound is a wave phenomenon. Sound travels through the air at a well defined wave speed, and because of this, sound heard at any one instant must have been generated at some earlier instant. As is illustrated on Figure 1 the exact earlier time at which the sound was generated is dependent on the distance from the observer to the sound source. Thus, for an extended source, sound heard at the same time t will, in general, have been generated at different "retarded times" τ, where $\tau = t - r/a_0$ represents the distance from the observer to the point under consideration, and a_0 is the speed of sound in the undisturbed atmosphere. It is therefore vitally important to include the retarded time effects over the source if any accurate solution for the acoustic characteristics is to be obtained.

In the present case the compressor disc is the extended source, and over the compressor disc exist fluctuating mass sources and forces which can give rise to acoustic radiation. Because of compressor geometry and aerodynamics, all the sources (source here includes both mass source and force terms) exist with definite phase relations around the disc. Phase is an extremely important parameter in acoustics. Referring back to Figure 1, it can be seen that if the signals from the two sources arrive in phase they will add, while if they are out of phase, they will subtract. The phase difference is dependent on the phasing at the source, but this phasing must be evaluated at the proper retarded times appropriate to the point of observation. Alternatively, the process can be visualized as an incoming spherical wave, centered on the observation point, sweeping through the source region gathering up the individual contributions from each part. Again, if the contributions are out of phase, cancellation will occur, while if they are in phase each will add into the total.

Now, in general, the lines of equal phase in the source region are in motion. Thus the phase velocity in the source region becomes of considerable importance. As an example consider the situation shown in Figure 2a, with sound being radiated away from a line source with a phase velocity V. It can be seen that $V = a_0/\cos\theta$. Thus the phase velocity which couples into the wave is greater than the speed of sound. As is shown in Figures 2b and 2c, increases in phase velocity can be accommodated by increases in the angle of radiation θ. But when the phase velocity is reduced below the speed of sound, no coupled acoustic wave is possible. For the infinitely long system illustrated here, the sound radiation from a source with a subsonic phase velocity would be identically zero, all contributions cancelling identically. However, a compressor has finite dimensions. For this reason cancellation is incomplete and subsonic phase velocities in a compressor can still give rise to some radiated sound. But it is found that the amount of sound radiated is extremely small in such cases, as will be shown later in the report. Conversely, when the phase velocity is supersonic, the acoustic radiation can be large. The same effect has been studied extensively for radiation within a compressor duct (Refs. 1 and 4), where the effects are referred to as "cutoff". However, it is important to note that the basic mechanism is not dependent on the duct boundary conditions, but only on the source phase characteristics.

It should also be noted that the phase velocity of the source is only rarely equal to its physical velocity. Figure 3 gives an example. Suppose that the two rows represent the rotor and stator of a compressor. Suppose also that a pressure pulse is generated when any two members of the rows are in line as shown by the arrows - this is closely analogous to the real situation. If the rotor row moves a pulse will occur, as shown, when the second pair comes in line. However, it can be seen that the effective phase velocity corresponding to these successive pulses is much higher (triple in this case) than the actual velocity of the rotor. Both the rotor and stator act as "phased arrays" with a high effective velocity. Therefore even though the rotor is moving at subsonic speed, the phase velocities of the pressure fluctuations can be supersonic, and this causes efficient and undesirable radiation from the compressor. It might also be noted that, when the rotor itself is moving at supersonic speeds, it can couple directly into the sound field without any requirement for these interaction effects, so that supersonically moving rotors are particularly efficient generators of sound. Subsonic rotors on the other hand generate sound only by the interaction process described above, and are therefore more amenable to treatment at early stages of design.

All these phasing effects will become evident in the analytical results of this report. Each of the sources of sound present in the compressor has similar phasing effects, with the exact phase velocity being related to the number of rotor and stator blades and circumferential speed. In the next section a more detailed description of each of these sources is given.

2.2 Noise Generation by the Compressor

The mechanisms underlying compressor noise radiation were described by the author in an earlier paper (Ref. 11). Figure 4 shows the chain of cause and effect which leads to sound radiation by compressors. The aerodynamics of the compressor necessarily involve unsteady flow components. These unsteady flows cause unsteady forces to act upon the compressor blades, so that dipole type sound is radiated from each blade. The radiation from each blade combines to give the sound output of the rotor or stator disc. The efficiency of propagation of this sound down the inlet or outlet duct is governed by the acoustic modes of the duct, and the final sound which reaches the observer is the result of radiation from the end of this duct.

In the present report a simpler problem is studied, radiation of sound from a compressor with negligible duct effects. Many current jet engine compressors have duct lengths of less than a diameter, and it seems unnecessary to consider the detailed duct mode effects in the calculation. Some duct phenomena could be important for noise control, as discussed in Reference 11, but even for long (hard wall) ducts it is found that the zero duct approximation will give a reasonable idea of the overall acoustic results. Indeed the present approximations are probably a better description than the baffled right angle inlet model used in many studies.

A study of the effects at work within the compressor suggests that there are at least four separate mechanisms which contribute towards the acoustic radiation. These may be understood by reference to Figure 5, which gives a diagrammatic representation of the inside of the compressor annulus. A series of wakes stream from the initial stator cascade and are intersected by the rotor, which in turn generates a series of wakes, traveling with the rotor, which are intersected by the second stator row. These fluctuating wakes have two effects; they are the cause of both fluctuating forces and of mass fluctuations at the blade rows.

Consider first the rotor row shown in Figure 5. The stator wakes contain slower moving fluid. Thus the interception of the stator wakes by the rotor will result in a fluctuating mass source at the rotor row. The stator-rotor combination thus acts like an inefficient siren. The velocity defect in the stator wakes has another effect, which may be understood by reference to the velocity diagram shown in Figure 5. The net velocity vector through the rotor row is the result of a vector addition of the rotational (ΩR) and wake (w) velocity components. The effect of a reduction in wake velocity is a small change in relative velocity magnitude but a large change in angle of attack, as shown by the dashed vector in Figure 5. This change in angle of attack can produce substantial fluctuating forces on the rotor blades. Alternatively, this effect may be considered as a time varying downwash at the blade. In both the fluctuating mass and fluctuating force cases, each blade does not radiate at the same time. Thus important phase velocity effects occur, dependent on the relative number of rotor and stator blades, for the same reasons as were discussed in the previous section.

At the second stator row, the same two phenomena also exist. The interception of the rotor wakes by the stator again gives rise to fluctuating mass sources, and consequent siren type radiation. In the same way the rotor wake velocity defect causes fluctuating angles of attack or downwash and therefore fluctuating forces on these stator blades.

Thus, within the compressor, there are at least four separate acoustic source mechanisms at work, two monopole type arrays, due to the mass fluctuations, one located at the stator, and one moving with the rotor, and two dipole type source arrays due to the force fluctuations, again with one located on the stator and one moving with the rotor. Of these four source mechanisms, only one has been the subject of detailed study. This is the case of the stationary monopole source array. Embleton and Thiessen (Ref. 13) have published a thorough study of the characteristics of such an array, and they included the important effects of phase velocity around the array. Several of their results may be expected to be of more general application to the other forms of source considered here, and are discussed below.

Embleton and Thiessen found that the number of individual sources (i.e., stator blades) present was only important when these were less than about eight. For higher numbers the results corresponded closely to an equivalent ring source with the same circumferential phase variation. Thus only the phase velocity effects were found to be important. This conclusion may be expected to be of general application, and, indeed, it will be shown later to be true for all four types of sources discussed above. This result is of particular significance because it suggests that detailed description of the acoustic source characteristics is probably unnecessary providing the gross characteristics such as phase velocity are properly described. For instance, it will be seen that all the analysis in the present paper reduces the force and mass inputs to point sources (delta functions) of appropriate phasing and direction. This approximation affords several simplifications to the analysis. Even in the absence of the results discussed above, the approximation would be acceptable if the physical dimension of the source in question were less than half the wavelength of the sound generated. For instance, the chordwise lift distribution will rarely require consideration because it acts essentially over only a small part of the blade chord. However, the results above suggest that the delta function approximation even for the spanwise loading will usually be adequate. This was certainly found in a recent study of helicopter rotor noise (Ref. 14). The most probable source of errors in the present analysis lie in the uncertain estimates of wake geometry and aerodynamics, as will be discussed in more detail in Section 6.3.

2.3 Frequency Effects in the Compressor

Several important effects related to the frequency characteristics of the compressor radiation may also be deduced from a study of Figure 5. Consider first of all the rearmost stator row. Each blade is stationary, and undergoes a force fluctuation due to the velocity profile in the rotor wakes. The time variation of these fluctuations is governed by the rotor speed. Thus the frequency of the noise radiated by the stator is directly related to the rotor frequency. If the rotor has B blades and rotates at angular velocity Ω then the fundamental frequency of the stator radiation will be $B\Omega$.

The wake velocity field behind the rotor can be analyzed into spatial Fourier components. Because of the rotation, each spatial harmonic of the rotor wake is transformed into a single temporal harmonic of the stator radiation. Now immediately behind the rotor the wake velocity profile is very sharp, and thus contains all spatial frequencies, but as the wake expands downstream it becomes smoother and less intense, so that at large rotor-stator separations only the first spatial harmonic of the wake may be significant. Thus it may be predicted that the stator radiation for small rotor-stator separations will contain substantial noise at all multiples of the blade passage frequency $B\Omega$, and that the effect of increasing rotor-stator separation will be to preferentially reduce the higher harmonics of the sound radiated by the stator.

The effects at the rotor, however, are rather different. The rotor field passes through what is essentially a stationary velocity field due to the wakes from the first stator (see Figure 5). The frequency of the fluctuating forces on, and thus the acoustic radiation by, the rotor is governed by the rotor speed. Thus the angular velocity of the rotor governs temporal frequencies at both the rotor and the stator. Clearly, the stator wake can still be analyzed into spatial Fourier components, and their relative magnitude will depend on stator-rotor separation in the same way as discussed above; i.e., at large separations only the first spatial harmonic of the stator wake will be significant. Furthermore successive spatial harmonics of the stator wake will give rise to successive loading harmonics on the rotor. However, in contrast to the stator case, each loading harmonic on the rotor gives rise to more than one sound harmonic in the radiation field.

This is due to the motion of the rotor blades. As is well known, relative motion between source and observer gives rise to Doppler frequency shifts, with the observed frequency rising as the source approaches the observer and reducing as the source recedes. Thus the rotation of a particular frequency source moving with the rotor causes a periodic variation of the frequency observed at a fixed point. The effects are very similar to those of frequency modulated radio signals. It is found that frequency modulation causes any single frequency input to be observed as a series of frequencies each displaced from the input frequency by some multiple of the modulation frequency. Thus fluctuating forces at one harmonic will cause radiation in all harmonics. For this reason increase of stator-rotor separation may be expected to reduce the noise basically in all the sound harmonics radiated by the rotor. This may be contrasted with the predominant reduction in the higher harmonics predicted above for the equivalent stator radiation effect.

It will be shown later on in the report that each loading harmonic (λ) gives rise to a different mode of radiation from the rotor, in a very similar manner to that described in the work of Tyler and Sofrin (Ref. 1). Each mode will have different acoustic radiation characteristics, so that the observed radiation levels from the rotor is dependent on both the acoustic radiation efficiency of each mode, and the relative magnitude of its contribution from the various loading harmonics λ. The overall radiation pattern is thus given by the sum of all the modes.

3.0 GENERAL EQUATIONS FOR SOUND GENERATION

3.1 The Basic Equation

In this section the general equations for sound generation will be derived from first principles. Throughout this section Tensor notation with the summation convention will be used. This notation is particularly convenient for reducing the volume of mathematics. For example, the continuity equation may be written

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_i)}{\partial x_i} = Q \tag{1}$$

where ρ = the density

t = time

Q = the rate of introduction of mass per unit volume, which may vary in any desired way over space

x_i = a three-dimensional cartesian coordinate representing x_1, x_2, x_3 as i takes on the values 1,2,3.

v_i = a velocity component representing v_1, v_2, v_3 as i takes on the values 1,2,3.

The Einstein summation convention used here requires that when the same index is repeated in any term, then the summation of that term over all values of the index is necessary. Thus the second term in the above equation can be expanded as

$$\frac{\partial (\rho v_i)}{\partial x_i} = \frac{\partial (\rho v_1)}{\partial x_1} + \frac{\partial (\rho v_2)}{\partial x_2} + \frac{\partial (\rho v_3)}{\partial x_3}$$

The simplicity achieved by using this notation is apparent. Similarly the equation for the conservation of momentum can be written

$$\frac{\partial (\rho v_i)}{\partial t} + \frac{\partial (\rho v_i v_j)}{\partial x_j} + \frac{\partial p_{ij}}{\partial x_j} = F_i \tag{2}$$

where F_i = (i = 1,2,3) the components of external force per unit volume acting over the fluid.

p_{ij} = the nine component stress tensor which includes the viscous and internal pressure forces acting on the fluid.

In Equation 2 the suffix i can again take on the values 1,2,3, but here each different value implies a separate equation. Equation 2 is Reynolds' form of the compressible Navier-Stokes equations. Both the second and the third terms on the left hand side of Equation 2 have a repeated index j, and therefore requires summing. Writing out Equation 2 in the long conventional notation would require a total of three equations, each with 8 terms. Again the advantages of the present Tensor notation are clear.

Differentiating Equation 1 with respect to t and Equation 2 with respect to x_i and subtracting gives

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial Q}{\partial t} - \frac{\partial F_i}{\partial x_i} + \frac{\partial^2 (\rho v_i v_j + p_{ij})}{\partial x_i \partial x_j}$$

Again, differentiation of Equation 2 with respect to x_i introduced a double suffix i, which requires summing. Doing the same operation without Tensor notation would have required differentiation of three original equations by three different variables followed by summing, but in the present notation the result is obvious. In order to derive the equation for sound generation, the term

$$a_0^2 \, \partial^2 \rho / \partial x_i^2$$

is subtracted from each side, first done by Lighthill (Ref. 15), giving finally the Basic Equation for Sound Generation as

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial Q}{\partial t} - \frac{\partial F_i}{\partial x_i} + \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{3}$$

where $T_{ij} = \rho v_i v_j + p_{ij} - a_0^2 \rho \delta_{ij}$

$\delta_{ij} = 1, \ i = j; = 0, \ i \neq j$ (The Kronecker δ)

a_0 = the speed of sound in the undisturbed fluid.

The left hand side of Equation 3 is the wave equation ($\partial^2/\partial x_i^2 \equiv \nabla^2$), and the right hand side can be regarded as a collection of acoustic source terms. The wave equation is given in terms of a (fluctuating) density ρ but, if desired, can be easily converted to pressure p by putting $p = a_0^2 \rho$, which will apply in most practical problems. It may be observed that ρ (or equivalently p) occurs on both sides of Equation 3 so that in principle Equation 3 cannot be solved directly. In practice the terms on the right hand side may be regarded as known and expressions for the sound field obtained using the well known solutions to the Wave Equation.

Each term on the right hand side of Equation 3 represents a different possible acoustic source mechanism. The first, $\partial Q/\partial t$, gives the effect of mass introduction. In the compressor it was shown above that such sources are present at both the rotor and the stator blade rows. Fluctuating mass sources are the most efficient radiators of sound at low speeds. The second term $\partial F_i/\partial x_i$ (note the necessary summation) gives the effect of fluctuating forces acting on the air. Again it was shown above that such forces may be expected to act on both the rotor and stator blade rows. The third term $\partial^2 T_{ij}/\partial x_i \partial x_j$ incorporates rather a large number of effects, the most important of which, in general, is the direct sound radiation by turbulence. T_{ij} may be regarded as an acoustic stress tensor. Note that since i and j may independently take on 3 values T_{ij} actually has nine components. It is possible that this acoustic stress term could have a more direct effect on compressor noise. Ffowcs Williams and Hawkings (Ref. 16) have pointed out that the steady and fluctuating stress fields around a rotating blade will produce discrete frequency sound radiation. The overall features of such noise, such as phase velocity effects, dependence on compressor design features, and so on, may be expected to be very similar to the force terms studied here. Unfortunately calculation of the actual magnitude of the source term due to the acoustic stresses is not straightforward. The approach adopted in the present report is to consider only the force and mass source terms in the radiation field. The fact that preliminary agreement with experiment is obtained tends to validate this simpler approach.

It should be pointed out that the basic equation (3) is for sound radiation in a uniform medium. Although the sources on the right hand side are allowed to move through the acoustic medium, direct use of the equation does not include any acoustic effects of velocity of the medium itself. Thus effects such as refraction and diffraction of the sound by the velocity or temperature fields existing within the compressor are not included in the present analysis. This can lead to errors at high axial Mach number. For instance, no effects due to phenomena such as choking would be predicted. Fortunately, these deficiencies in the analysis are not in general of great significance for the prediction of basic source characteristics. A further discussion is given in the Appendix.

3.2 Solution of the Basic Equation

The solution to the wave equation is well known. If the right hand side of Equation 3 is written as $g(\underset{\sim}{y})$, the solution to (3) is

$$\rho = \frac{1}{4\pi a_0^2} \int \left[\frac{g}{r}\right] d\underset{\sim}{y} \tag{4}$$

where ρ = a fluctuating density

r = the distance from source to observer

$\underset{\sim}{y}$ = the coordinate of the source position.

The symbol ~ under y denotes y as a vector quantity. This symbol is used because it requires a printer to use heavy (Clarendon) type, as is usual for vectors. The square brackets around the g/r term are of extreme importance, since they imply evaluation of their contents at "retarded" time $\tau = t - r/a_0$. As was discussed in Section 2, proper account of the retarded time effects is essential if accurate results are to be obtained.

Equation 4 gives the basic result, but requires manipulation to appear in a form suitable for calculations. Using the results in Equation 3, the expression for radiation by a fluctuating mass source is obtained directly by substitution, as

$$p = \frac{1}{4\pi a_0^2} \int \left[\frac{1}{r}\frac{\partial Q}{\partial t}\right] d\underset{\sim}{y} \tag{5}$$

and will be left in this form for the time being. In the same way the expression for the sound generation by the fluctuating forces is given directly by

$$p = \frac{-1}{4\pi a_0^2} \int \left[\frac{1}{r}\frac{\partial F_i}{\partial y_i}\right] d\underset{\sim}{y} \tag{6}$$

It is convenient to change the differentiations from source ($\underset{\sim}{y}$) to observer ($\underset{\sim}{x}$) variables. In order to achieve this, consider any function f. Then

$$[f(t)] = f(t - r/a_0)$$

So, by the chain rule

$$\frac{\partial}{\partial y_i}[f] = \left[\frac{\partial f}{\partial y_i} + \frac{x_i - y_i}{a_0 r}\frac{\partial f}{\partial t}\right]$$

also

$$\frac{\partial}{\partial x_i}[f] = \left[\frac{\partial f}{\partial x_i} - \frac{x_i - y_i}{a_0 r}\frac{\partial f}{\partial t}\right]$$

So that addition gives

$$\frac{\partial}{\partial x_i}[f] = \left[\frac{\partial f}{\partial x_i}\right] + \left[\frac{\partial f}{\partial y_i}\right] - \frac{\partial}{\partial y_i}[f]$$

Putting $f = F_i/r$ in this equation, where F_i is a function of $\underset{\sim}{y}$ only, gives

$$\frac{\partial}{\partial x_i}\left[\frac{F_i}{r}\right] = \left[\frac{1}{r}\frac{\partial F_i}{\partial y_i}\right] - \frac{\partial}{\partial y_i}\left[\frac{F_i}{r}\right]$$

On integration over all $\underset{\sim}{y}$ space, the last term vanishes by the divergence theorem, giving

$$\int \frac{\partial}{\partial x_i}\left[\frac{F_i}{r}\right] d\underset{\sim}{y} = \int \left[\frac{1}{r}\frac{\partial F_i}{\partial y_i}\right] d\underset{\sim}{y} \tag{7}$$

Thus, using the result of Equation 7, Equation 1 can be rewritten as

$$\rho = \frac{-1}{4\pi a_0^2}\frac{\partial}{\partial x_i}\int \left[\frac{F_i}{r}\right] d\underset{\sim}{y} \tag{8}$$

which is thus an alternative expression for the sound radiation by a distribution of fluctuating forces. This equation could also be derived directly from consideration of solutions of the wave equation, as was given in Ref. 14, see also Lighthill's original paper (Ref. 15).

3.3 The Effects of Motion

If we now assume that the sources are in motion, it is convenient to specify the forces in a moving frame of reference, for example, on the rotor. Suppose coordinates measured in this frame are defined by $\underset{\sim}{\eta}$, and the origin of these coordinates is moving with velocity $a_0 \underset{\sim}{M}$. Then, at any instant the $\underset{\sim}{\eta}$ and $\underset{\sim}{y}$ coordinate systems are connected via

$$\underset{\sim}{\eta} = \underset{\sim}{y} - \underset{\sim}{M} a_0 t$$

However, in Equation 5 or 8 we do not require to evaluate the integral at any instant t. Instead the integral must be evaluated at the appropriate retarded time $\tau = t - r/a_0$, i.e., over

$$\underset{\sim}{\eta} = \underset{\sim}{y} + \underset{\sim}{M} r - \underset{\sim}{M} a_0 t$$

Thus, in the coordinate transformation from fixed to moving axes it is appropriate to use, as first suggested by Lighthill (Ref. 15)

$$\underset{\sim}{\eta} = \underset{\sim}{y} + \underset{\sim}{M} r \tag{9}$$

This axis transformation also affects the volume element of the integration, and the integral must be divided by the Jacobian of the transformation

$$J = \begin{vmatrix} \partial\eta_1/\partial y_1 & \partial\eta_2/\partial y_1 & \partial\eta_3/\partial y_1 \\ \partial\eta_1/\partial y_2 & \partial\eta_2/\partial y_2 & \partial\eta_3/\partial y_2 \\ \partial\eta_1/\partial y_3 & \partial\eta_2/\partial y_3 & \partial\eta_3/\partial y_3 \end{vmatrix} = 1 - M_r$$

where $M_r = M_i(x_i - y_i)/r = \left\{ M_1(x_1 - y_1) + M_2(x_2 - y_2) + M_3(x_3 - y_3) \right\} /r$
is the component of the convection Mach number in the direction of the observer.

Thus, for mass sources in motion, Equation 5 may be rewritten to give

$$\rho = \frac{1}{4\pi a_0^2} \int \left[\frac{\partial Q/\partial t}{r(1 - M_r)} \right] d\underset{\sim}{\eta} \tag{10}$$

and for the fluctuating forces in motion we obtain, from Equation 8,

$$\rho = \frac{-1}{4\pi a_0^2} \frac{\partial}{\partial x_i} \int \left[\frac{F_i}{r(1 - M_r)} \right] d\underset{\sim}{\eta} \tag{11}$$

The derivative $\partial/\partial x_i$ is operating on both an integral over $\underset{\sim}{\eta}$ and a retarded time operator, both of which are functions of x. For any function f(t)

$$[f(t)] = [f(t - r/a_0)]$$

Thus the partial derivative with respect to x_i, keeping $\underset{\sim}{\eta}$ constant of f is given by the chain rule as

$$\frac{\partial}{\partial x_{i(\eta)}}\left[f\right] = \left[\frac{\partial f}{\partial x_{i(\eta)}} - \frac{\partial r}{\partial x_{i(\eta)}} \frac{1}{a_0} \frac{\partial f}{\partial t}\right] \tag{12}$$

Now $$\frac{\partial r}{\partial x_{i(\eta)}} = \frac{\partial}{\partial x_{i(\eta)}}\left\{x_j - y_j\right\}^2 = \frac{x_j - y_j}{r} \frac{\partial}{\partial x_{i(\eta)}}(x_j - y_j)$$

From Equation 10 $$\frac{\partial}{\partial x_{i(\eta)}}(x_j - y_j) = \delta_{ij} + M_j \frac{\partial r}{\partial x_{i(\eta)}}$$

Hence $$\frac{\partial r}{\partial x_{i(\eta)}} = \frac{x_i - y_i}{r(1 - M_r)}$$

and using this in Equation 12

$$\frac{\partial}{\partial x_{i(\eta)}}\left[f\right] = \left[\frac{\partial f}{\partial x_{i(\eta)}} - \frac{x_i - y_i}{r(1 - M_r)} \frac{1}{a_0} \frac{\partial f}{\partial t}\right] \tag{13}$$

The two terms in Equation 13 have different dimensional dependence. The first term is essentially proportional to $1/r$ where r is the distance of the observer from the source, while the second is proportional to $1/\lambda$ where λ is the typical wavelength of the acoustic radiation emitted. Thus, many wavelengths away from the source region, the first term will become negligibly small. The region over which this holds will be termed the acoustic far field. Thus using Equation 13, the acoustic far field approximation for Equation 11 is

$$\rho = \frac{1}{4\pi a_o^2} \int \left[\frac{x_i - y_i}{a_o r(1 - M_r)} \frac{\partial}{\partial t}\left(\frac{F_i}{r(1 - M_r)}\right)\right] d\underset{\sim}{\eta} \tag{14}$$

where $\partial/\partial t$ is now a differential following the source. This result was previously obtained by the author for the special case of a point force, following a longer analysis (Ref. 17).

This more natural form of the expression for the sound field of a force in arbitrary motion is found to be very convenient for the development of the general results for a harmonically varying system. This will be done in Sections 4.2 and 5.2. The equations given above are general and will apply to any source in motion. However, in the applications made in the body of this report, it is always assumed that the compressor hub is stationary with respect to the free air. Modifications which allow the effects of gross compressor motion to be accounted for in a simple way are discussed in the Appendix.

4.0 SOUND RADIATION BY MASS FLUCTUATIONS

It was shown in Section 2.0 that mass flow fluctuations occur in the compressor at both the stator and rotor blade rows. In Section 3.0, a theoretical basis was given for calculating the noise radiation, for mass fluctuations in both fixed and moving systems. The application of this theoretical analysis to the compressor case is made in this section.

4.1 Mass Fluctuations at the Stator

At the stator row, there exists a rotating velocity profile due to the wakes from the rotor. Since the velocity profile is steady in a frame of reference moving with the rotor, it does not, by itself, produce any sound. But after the profile is intercepted by the stator, local fluctuating velocities exist in any frame of reference, so that sound can be radiated. The effect is shown, diagrammatically, in Figure 6. Suppose that the stator causes a velocity defect immediately behind itself. Thus the incoming rotor wake field will be periodically intercepted as it moves past the stator vanes. In order to estimate the magnitude of the fluctuating mass sources occurring, we subtract the rotor wake component (which does not, by itself, produce any noise) from the combined flow field. This will result in a negative velocity source, attached to the stator blade, whose amplitude is equal to the local rotor wake velocity in the absence of the stator. This source can be considered to extend over a distance equal to the wake displacement thickness. This fluctuating source can be further specialized to an equivelent point mass source located at the stator blade. This model is essentially a high frequency approximation to the sound radiated. When the sound wavelength is long compared to the blade spacing the effects of the small velocity increment between the wakes will cancel the sound due to the defect in the wakes. At high frequencies the velocity increment is extended over several wavelengths and does not radiate efficiently, while the more compact velocity defect will radiate as discussed here. Thus, under the present model, the stator row can be considered, for acoustic purposes, as an array of point mass sources.

An analysis of this case has been presented by Embleton and Thiessen (Ref. 13), and the theory presented below may be regarded as basically an extension of their work. The source system may be described mathematically with the aid of Figure 7a. Suppose there are V simple sources corresponding to the V stator vanes, equally spaced around a circle of radius R. Using Equation 5, the total sound pressure is given by the sum of all the sound outputs.

$$p = \sum_{\nu=0}^{V-1} p_\nu = \sum_{\nu=0}^{V-1} \left[\frac{\partial Q_\nu / \partial t}{4\pi \, r_\nu} \right] \tag{15}$$

where the suffix ν has been used to denote the variables which are a function of the source location. The rotor wake velocity field may be analyzed into a Fourier series, in complex form, defined by

$$w = \sum_{\lambda=-\infty}^{+\infty} A_\lambda \exp(-i\lambda\theta_R) \tag{16}$$

θ_R is the angle around the rotor, measured with respect to rotating coordinates and λ is the wake spatial harmonic. Clearly λ must be a multiple of B as there are B wakes streaming from the B rotor blades. The fundamental wake period is given when $\lambda = B$. A_λ is a complex Fourier coefficient. In terms of conventional Fourier components the wake velocity can be defined as

$$w = \alpha_0 + \sum_{\lambda=1}^{\infty} \alpha_\lambda \cos\lambda\theta_R + \beta_\lambda \sin\lambda\theta_R \tag{17}$$

So that the relation between the real and complex coefficients is given by

$$A_\lambda = (\alpha_\lambda + i\beta_\lambda)/2 \qquad A_{-\lambda} = A_\lambda^* = (\alpha_\lambda - i\beta_\lambda)/2 \qquad |\gamma_\lambda|^2 = \alpha_\lambda^2 + \beta_\lambda^2 \tag{18}$$

Since the rotor wake is rotating at angular velocity Ω, the relation between θ_S fixed in the stator and θ_R fixed in the rotor is $\theta_S = \theta_R + \Omega t$. Thus, fixing the phase origin in the zeroth blade, put $\theta_S = 2\pi\nu/V$, giving $\theta_R = 2\pi\nu/V - \Omega t$. Note how a spatial variation in the rotor wake is transformed into time variation at the stator row through the effects of rotor wake rotation. It may also be noted, that the model used for the mass source definition, as discussed above, requires that $Q = w h s \rho_0$ where h is the wake displacement thickness and s is blade span. Note that h is measured normal to the wake. If velocity is measured axially, h is measured normal to the axis with the same overall mass defect result. Using these results together with Equations 15 and 16 gives the sound pressure as

$$p = \sum_{\nu=0}^{V-1} \sum_{\lambda=-\infty}^{+\infty} \frac{-i\lambda\Omega h s \rho_0 A_\lambda}{4\pi r_\nu} \exp\left\{-i\lambda\left[\frac{2\pi\nu}{V} - \Omega\left(t - \frac{r}{a_0}\right)\right]\right\} \tag{19}$$

where the retarded time effects have now been included explicitly in the equation. Referring back to Figure 7a, it can be seen that r_ν, the distance to the νth source, is

$$r_\nu^2 = x^2 + \left\{y - R\cos\left(\phi + \frac{2\pi\nu}{V}\right)\right\}^2 \tag{20}$$

where x,y,z are Cartesian coordinates with their origin at the compressor hub, x, along the axis and y in the plane of the observer. R is the source radius, and ϕ is the reference angle between the zeroth source and the y-axis. Equation 20 can be rewritten as

$$r_\nu = \left\{(x^2 + y^2) - 2\,y\,R\cos\left(\phi + \frac{2\pi\nu}{V}\right) + R^2\cos^2\left(\phi + \frac{2\pi\nu}{V}\right)\right\}^{0.5} \tag{21}$$

The first term is equal to r_1, the distance of the observer from the hub (note that r_1 here is not the distance to the first source), and the last term may be neglected when $r_1 >> R$. Neglect of the last term is a purely geometrical result so that the region over which the approximation is valid can be referred to as the geometric far field. This should be contrasted with the acoustic far field discussed in Section 3.3. Note that both approximations will always apply sufficiently far from the source region. Using this geometric far field approximation, Equation 21 can be simplified using the binominal theorem to give the approximation

$$r_\nu \sim r_1 - \frac{yR}{r_1}\cos\left(\phi + \frac{2\pi\nu}{V}\right) \tag{22}$$

Equations 19 and 22 now give

$$p = \sum_{\nu=0}^{V-1}\sum_{\lambda=-\infty}^{+\infty} \frac{-i\lambda\Omega h s \rho_0 A_\lambda}{4\pi r_1} \exp\left\{-i\lambda\left[\frac{2\pi\nu}{V} - \Omega t + \frac{\Omega r_1}{a_0} - \frac{My}{r_1}\cos\left(\phi + \frac{2\pi\nu}{V}\right)\right]\right\} \tag{23}$$

where $M = \Omega R/a_0$ is the rotational Mach number of the rotor Since the phase effects of r_ν are of first order significance in the result these must be retained. However, the amplitude effect of the r_ν in the denominator of Equation 19 was of second order (essentially it is a geometric near field term) so that it has now been replaced by r_1.

In order to further evaluate Equation 23 it is useful to expand part of the exponential as a series of Bessel functions using Formula 44 of Maclachlan's book (Ref. 18)

$$\exp i z \cos\theta = \sum_{n=-\infty}^{+\infty} i^n J_n(z)\cos n\theta \tag{24}$$

With this result, Equation 23 gives

$$p = \sum_{\lambda=-\infty}^{+\infty} \frac{-i\lambda\,\Omega h s \rho_0 A_\lambda}{\Delta\pi r_1} \exp\left\{ i\lambda \left(\Omega t - \frac{\Omega r_1}{a_0}\right)\right\}$$

$$\times \sum_{\nu=0}^{V-1} \exp\left(\frac{-2\pi i \lambda \nu}{V}\right) \sum_{\mu=-\infty}^{+\infty} i^\mu J_\mu\left(\frac{\lambda M y}{r_1}\right) \cos\mu\left(\phi + \frac{2\pi\nu}{V}\right) \tag{25}$$

The second line of Equation (25) is of key interest. Summations of geometric functions obey "orthogonality" relations, (see for example Ref. 19) so that most of the terms vanish identically. We can write

$$\sum_{\nu=0}^{V-1} \exp\left\{\frac{2\pi i \nu}{V} m\right\} = V \sum_{k=-\infty}^{+\infty} \delta\,(m + k V) \tag{26}$$

δ here is the discrete analog of the Dirac delta function, equal to unity when its argument is zero, and equal to zero otherwise. (It is of interest to note the relation between the Kronecker function, and the modified Dirac: $\delta_{ij} = \delta(i - j)$). Thus the summation in Equation (26) is identically zero unless m is an integral multiple of V. From Equation (26) may also be deduced

$$\sum_{\nu=0}^{V-1} \exp\,(-2\pi i\, \lambda\nu/V) \cos\,\mu\,(\phi + 2\pi\nu/V)$$

$$= \frac{1}{2} V \sum_{k=-\infty}^{+\infty} \exp i\,(\lambda - kV)\,\phi \left\{\delta(\mu - \lambda + kV) + \delta(-\mu - \lambda + kV)\right\} \tag{27}$$

As mentioned above, if all the wakes from the rotor are uniform only values of $\lambda = mB$ occur, where B is the number of blades and m is a harmonic order. In the present stator problem the parameter λ also operates on time in the imaginary exponential so that mB is the temporal frequency parameter. This corresponds to the discussion of Section 2.3. The summation over λ in Equation 25 therefore becomes a summation over time here. Putting the complex magnitude of the mth sound harmonic as $c_m = a_m + i\,b_m$, we can use analogs of the results in Equations 18 to express the sound pressure directly as a harmonic. Therefore, Equations 25 and 27 give the mth harmonic of the sound radiation in the rotor-stator mass source interaction as

$$c_m = \sum_{k=-\infty}^{+\infty} \frac{-i\,mBV\,\Omega\, h s \rho_0 A_n}{2\pi r_1}\, i^{mB-kV}\, J_{mB-kV}\left(\frac{mBMy}{r_1}\right) \tag{28}$$

In Equation 28 the phase factor $\exp i\left((mB - kV)\phi - mB\,\Omega r_1/a_0\right)$ has been dropped and the result $J_{-n}(z) = (-1)^n J_n(z)$ has also been used (Ref. 18).

4.2 Rotor Mass Sources

The same basic mechanisms as discussed in the previous section will also apply to the rotor mass sources. Using the same arguments as before, the velocity field of the stator wake may be subtracted from the combined field to leave a negative velocity source at the rotor. Once again this may be approximated to a point mass source, here attached to the rotor blade. The key difference now is that the rotor blade is moving. Thus, using the equations for moving sources developed in Section 3.3, the acoustic field radiated can be determined. First of all a basic expression for radiation from a fluctuating mass source moving in an arbitrary harmonic manner will be calculated. The result would apply to a mass source repetitively over any arbitrary path. Recalling $p = a_0^2\,\rho$, the sound pressure radiated by a point mass source moving with the rotor can be written down from Equation 10 as

$$p = \left[\frac{\partial Q/\partial t}{4\pi r\,(1 - M_r)}\right] \tag{29}$$

Note that, in this formulation of the result, although the point source is in motion the value of $\partial Q/\partial t$ is measured with respect to the stationary atmosphere. If the differential with respect to time were taken following the source, then it would be necessary to introduce additional terms to account for the effective momentum output of the moving mass source. This point is discussed in more detail in Refs. 17 and 20. Here the simpler approach of evaluating the source magnitude with respect to the fixed axes will be used. If the source is assumed to be harmonic, then the sound output will be harmonic also, and may be evaluated as a Fourier series. Using the complex notation for the magnitude of the Fourier coefficients, the value of the nth harmonic of the sound radiation can be written down from Equation 29 as

$$c_n = a_n + i\,b_n = \frac{\omega}{\pi}\int \left[\frac{\partial Q/\partial t}{4\pi r(1 - M_r)}\right] \exp i\,n\,\omega\,t\; dt \tag{30}$$

where the integral is over any period $\omega/2\pi$. It will be found to be convenient to evaluate the integral in retarded source time τ, where $\tau = t - r/a_0$, $dt = (1 - M_r)\,d\tau$. Using this transformation gives

$$c_n = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \left[\frac{\partial Q/\partial t}{4\pi r}\right] \exp\left[i\,n\omega(\tau + r/a_0)\right]\,d\tau \tag{31}$$

which is a general result for an arbitrary harmonic variation in the mass source. As discussed above, the model used in the present analysis reduces to a negative mass source attached to the rotor. To evaluate this it is necessary to define the velocity field arriving from the stator. This can be done in much the same way as for the rotor wake velocity field in Section 4.1.

$$w = \sum_{\lambda=-\infty}^{+\infty} A_\lambda \exp(-i\lambda\theta_S) \tag{32}$$

This is an expression for the wake field in terms of complex Fourier coefficients A_λ. The same relations as given in Equation 18 apply for the connection between the complex and real Fourier coefficients.

Then, following the arguments above, the magnitude of the fluctuating point mass source is

$$Q = -\rho h s \sum_{\lambda=-\infty}^{+\infty} A_\lambda \exp(-i\lambda\theta_S) \tag{33}$$

This mass source is measured relative to the stator coordinates, as required by Equation 29. However we require to evaluate the relevant integrals in the moving rotor frame, so that, as before, we put $\theta_S = \theta_R + \Omega t$. We further assume that $\theta_R = 0$ for the particular blade under consideration and particularize the mass source to a delta function of equal total strength, as in the previous section. Equation 30 can now be used directly, giving

$$c_n = \sum_{\lambda=-\infty}^{+\infty} -\frac{\Omega}{\pi} \int_0^{2\pi/\Omega} \frac{i\rho_0 h s \lambda \Omega A_\lambda}{4\pi r} \exp(-i\lambda\Omega\tau) \exp\left\{in\Omega(\tau + r/a_0)\right\} d\tau \tag{34}$$

As before we use geometrical far field approximations for a rotating source. These follow essentially the same arguments as in the derivation of Equation 22. We find

$$r \simeq r_1 - \frac{yR}{r_1}\cos\theta \tag{35}$$

Use of this result gives the expression for the Fourier coefficients as

$$c_n = \sum_{\lambda=-\infty}^{+\infty} \int_0^{2\pi} - \frac{\rho_o hs\lambda\Omega A_\lambda}{4\pi^2 r_1} \exp i \left\{ (n-\lambda)\theta - \frac{nMy}{r_1} \cos\theta \right\} d\theta \tag{36}$$

where unnecessary phase factors have been ignored. The integral is one of the standard forms for Bessel functions. Maclachlan (Ref. 18) gives

$$\int_0^{2\pi} \exp \left\{ i\,(n\theta - z\cos\theta) \right\} d\theta = 2\pi \bar{i}^n J_n(z) \tag{37}$$

Using this result Equation 36 gives

$$c_n = \sum_{\lambda=-\infty}^{+\infty} - \frac{i\rho_o hs\lambda\Omega A_\lambda}{2\pi r_1} \; i^{-(n-\lambda)} \; J_{n-\lambda}\left(\frac{nMy}{r_1}\right) \tag{38}$$

This is the desired result (in complex form) for the sound output due to mass fluctuations at the rotor. Equation 38 only applied to a single rotating blade. To obtain the result for B blades we must sum. After summation it is found that all harmonics which are not exact multiples of the number of rotor blades B will cancel, as given in Equation 26. Furthermore, for the model in this section we can identify $\lambda = kV$, since only stator wake harmonics which are integral multiples of the number of stator vanes (V) will be present if the stator flows are uniform. Thus for the full stator-rotor mass source interaction we find

$$c_n = \sum_{k=-\infty}^{+\infty} \frac{i\rho_0 hskVB\Omega A_k}{2\pi r_1} \; i^{-(nB-kV)} \; J_{nB-kV}\left(\frac{mBMy}{r_1}\right) \tag{39}$$

4.3 Comments on the Results

Equations 28 and 39 will be observed to be remarkably similar. The Bessel function term occurs to the same order and argument, and the multiplication terms outside the Bessel function are quite closely the same. In other words, the principal features of the sound radiation from the mass sources in a compressor are essentially the same for both stator-rotor and rotor-stator interactions. This result is somewhat unexpected and of quite clear significance.

In fact there are some detailed variations between the two results. Equation 28 for the rotor-stator interaction contains $m A_m$ as a multiplier while Equation 39 for the stator-rotor case shows $k A_k$. In the stator case the radiated frequencies are determined by the spatial harmonic content of the rotor wake as was discussed in Section 2.3. Equation 28 shows that each sound harmonic from the stator is composed of the total of all k values in the summation with the same coefficient A_m. The rotor radiation case is rather different. Here, increasing stator-rotor separation affects the coefficients, A_k, and in the case of extreme separation only one value of A_k may be significant. However this will radiate in all harmonics with an efficiency depending on the value of the Bessel function. Thus, as was discussed in Section 2.3, it appears that increasing separation for the rotor-stator interaction will tend primarily to reduce the higher harmonics of the noise, while increasing separation for the stator-rotor case will reduce all harmonics, although not necessarily equally because of the effects of the Bessel function. Each value of k may be considered to give rise to a single "mode" of radiation in the same way as discussed by Tyler and Sofrin (Ref. 1).

Both equations are given in terms of complex Fourier coefficients A. They can be expressed in terms of ordinary Fourier coefficients by means of Equations 18. Thus, for example, Equation 39 gives

$$c_n = a_n + i b_n = \sum_{\lambda=1}^{\infty} - i^{-(n-\lambda-1)} \frac{\rho_0 h s \lambda \Omega}{4\pi r_1} \left\{ \alpha_\lambda \left(J_{n-\lambda}\left(\frac{nMy}{r_1}\right) + (-1)^\lambda J_{n+\lambda}\left(\frac{nMy}{r_1}\right) \right) \right.$$

$$\left. + i\beta_\lambda \left(J_{n-\lambda}\left(\frac{nMy}{r_1}\right) - (-1)^\lambda J_{n+\lambda}\left(\frac{nMy}{r_1}\right) \right) \right\} \qquad (40)$$

For the great majority of practical cases Equation 40 can be simplified, since the $J_{n-\lambda}$ terms are very much greater than the $J_{n+\lambda}$ terms providing neither n or λ is small. This may be attributed to the fact that the $n-\lambda$ terms correspond to a rapidly rotating source pattern while the $n+\lambda$ terms correspond to a slowly moving. As suggested by the discussion in Section 2.1, the more slowly moving source pattern is usually an extremely inefficient noise radiator. This point is covered in more detail on page 746 of Morse and Ingard's book (Ref. 21). It also corresponds to the finding of Embleton and Thiessen (Ref. 13) that the number of sources present was only significant if it were less than about eight. For numbers less than this value the effects of the $n+\lambda$ terms could be important. But, by ignoring these terms, Equation 40 can be rewritten as

$$c_n = \sum_{\lambda=1}^{\infty} (-i)^{-(n-\lambda-1)} \frac{\rho_0 \, h s \lambda \Omega}{4\pi \, r_1} (\alpha_\lambda + i \, \beta_\lambda) \, J_{n-\lambda} \left(\frac{n M y}{r_1}\right) \tag{41}$$

This equation for the stator-rotor interaction and its equivalent for the rotor-stator case can be used with very good accuracy in virtually all practical compressor problems.

:.4 Power Radiated

One of the single most important parameters in the description of an acoustic source is the overall acoustic power radiated. Knowledge of overall power enables pressure at any point to be estimated by the use of the spherical spreading law. It is thus of considerable interest to be able to predict the acoustic power radiation in the present case. If the sound pressure at any field point in the nth harmonic is given by p_n, then the mean power W_n in that harmonic is given by a space-time integral around the source,

$$W_n = \frac{1}{T} \iint \frac{p_n^2}{\rho_o a_o} \, dA \; dt = \frac{1}{2} \int_0^{\pi} \int_0^{2\pi} \frac{|c_n|^2}{\rho_o a_o} \; r_1^2 \sin \Psi \; d\Psi \; d\phi \tag{42}$$

The factor of 1/2 is the result of the time integration and corresponds to the rms value of pressure being $1/\sqrt{2}$ times the amplitude for sinusoidal fluctuations.

In Equation 42, ϕ is the angle around the source, ($\phi = 0$ on the y-axis) and Ψ is an angle over the top of the source, with $\Psi = 0$ along the compressor (x) axis. The ϕ integration may be performed directly. It will be observed that in both Equations 28 and 39 there is no effect of ϕ on the amplitude of the sound radiation, although it does enter the phase terms. Since ϕ is only a phase variable the mean square value of the fluctuating pressure field will be the same at any point, and the ϕ integration is trivial. Thus

$$W_n = \frac{\pi \, r_1^2}{\rho_0 \, a_0} \int_0^{\pi} |c_n|^2 \sin \psi \; d\psi \tag{43}$$

Direct analytical evaluation of the overall power using Equations 28 and 39 does not give results in a convenient form, although the necessary integration can be performed on a computer. However, a fairly simple analytic result can be obtained after some approximation.

First of all we use the result discussed in Section 4.3 that the n+λ terms can be neglected compared with the n−λ terms. We then calculate the overall power radiated by a single term in the summations of Equations 28 and 39, that is, for a single mode. For the rotor radiation case this corresponds to the power radiated by a single spatial harmonic k, and will correspond closely to the actual power radiated at large stator-rotor separations. For the stator radiation case the calculation is simply an analytic convenience. The analysis below is given in terms of the rotor parameters. Exactly equivalent results also apply to the stator. The power radiated by a single mode is therefore

$$W_{n\lambda} = \frac{\pi r_1^2}{\rho_0 a_0} \int_0^{\pi} \left| c_{n\lambda} \right|^2 \sin\Psi \, d\Psi \tag{44}$$

which gives, using Equation 39

$$W_{n\lambda} = \frac{\rho_0 (h s \lambda \Omega)^2}{16\pi a_0} \gamma_\lambda^2 \int_0^{\pi} J_{n-\lambda}^2 (\sin\phi) \sin\Psi \, d\Psi \tag{45}$$

γ_λ is the absolute magnitude of the wake harmonic and was defined in Equations 18. Embleton and Thiessen (Ref. 13) give the result

$$\int_0^{\pi/2} J_n^2 (z \sin\theta) \sin\theta \, d\theta = \frac{1}{z} \sum_{\nu=0}^{\infty} J_{2n+2\nu+1} (2z) \tag{46}$$

which can be used if desired. However, for the present purposes, a slightly different form of the result is more convenient. This is

$$\int_0^{\pi/2} J_n^2 (z \sin\theta) \sin\theta \, d\theta = \sum_{r=0}^{\infty} \frac{(-1)^r z^{2n+2r}}{r! \, (2n+r)! \, (2n+2r+1)} \tag{47}$$

where r is simply a summation parameter.

Equation 45 is symmetric about $\Psi = \pi/2$, so that writing $|n - \lambda| = \mu$, Equation 45 may be evaluated to give

$$W_{n\lambda} \approx \frac{\lambda^2 h^2 s^2 \Omega^2 \rho_0 \left|\gamma_\lambda\right|^2}{8\pi a_0} \sum_{r=0}^{\infty} \frac{(-r)^r (n M)^{2\mu + 2r}}{r!\,(2\mu + r)!\,(2\mu + 2r + 1)} \qquad (48)$$

This equation for the power output applies directly to the rotor radiation case. For the stator radiation the same equation applies.

The series in Equation 48 is found to converge in about 3 n M terms for $|n - \lambda| = 0$, so that power results may be computed rapidly in comparison to the direct integral approach using Equation 45. Note however that the series in Equation 48 contains alternating terms of large magnitude, so that computational precautions are necessary to ensure accuracy.

Calculations of power from a real compressor requires the integral to be taken over the sum of several loading harmonics for any given sound harmonic. There is therefore the possibility of contributions to the power from the cross terms of the loading harmonics. Series expansions of these cross terms can be made in a similar manner to that above, but this would result in much additional computational complexity. Since, on the average, the contributions of the cross terms may be expected to cancel out, it is suggested that direct summation of the individual contributions of each loading input given by Equation 48 be used to determine overall power levels. Note also that sound power results for a B bladed rotor require the multiplication of Equation 48 by B^2 and the replacement of n by m B throughout. The thrust and drag terms are then the thrust and drag per blade.

5.0 SOUND RADIATION BY FLUCTUATING FORCES

5.1 Sound Radiated by Fluctuating Forces on the Stator

Calculation of the sound radiated by the fluctuating forces on the stator cascade is a straightforward variation on the results presented in Section 4.1 for the fluctuating mass sources at the stator. Since the stator row is stationary, Equation 14 may be specialized to give the noise from a point force as

$$p = \left[\frac{(x_i - y_i)}{4 \pi a_0 r^2} \; \frac{\partial F_i}{\partial t} \right] \tag{49}$$

This result for a stationary dipole is well known. In order to apply Equation 49 to the case of compressor noise radiation it is necessary first to specify the forces F_i existing at the rotor face. This is done as for the mass sources in Section 4.1. Define the complex cyclic (point) thrust and drag forces on the blades by the Fourier series

$$\left.\begin{aligned} T &= \sum_{\lambda=-\infty}^{+\infty} T_\lambda \exp(-i \lambda \Omega t) \\ D &= \sum_{\lambda=-\infty}^{+\infty} D_\lambda \exp(-i \lambda \Omega t) \end{aligned}\right\} \tag{50}$$

where Ω is the angular velocity of the rotor blades and λ gives the order of loading harmonic. T_λ and D_λ are complex quantities and must be specially defined for a consistent analysis. If the ordinary Fourier expression for thrust is

$$T = a_{oT} + \sum_{\lambda=1}^{\infty} a_{\lambda T} \cos \lambda \Omega t + b_{\lambda T} \sin \lambda \Omega t$$

then

$$T_\lambda = T_{\lambda R} + i T_{\lambda I} = (a_{\lambda T} + i b_{\lambda T})/2 \tag{51}$$

and

$$T_{-\lambda} = T_\lambda^* = T_{\lambda R} - i T_{\lambda I} = (a_{\lambda T} - i b_{\lambda T})/2$$

so that the two Fourier series expressions are equivalent. Note that $T_0 = a_{0T}$ = mean thrust. Similar expressions can be written down for the drag terms.

Figure 7b shows that

$$\left.\begin{aligned} F_i &= -\ T,\ -\ D\sin\theta,\ D\cos\theta \\ x_i - y_i &= x,\ y - R\cos\theta,\ -\ R\sin\theta \end{aligned}\right\} \qquad (52)$$

Note that the force on the air acts in the opposite direction to that on the blade. Thus Equations 49, 50 and 52 combine to give the sound radiation from any one blade due to a fluctuating force as

$$p = \sum_{\lambda} \left[\frac{-i\lambda\Omega(xT_\lambda + yD_\lambda \sin\theta)\ \exp(-i\lambda\Omega t)}{4\pi a_0 r_1^2} \right] \qquad (53)$$

θ is the angle around the stator.

The key requirement now is to define the phase variation of the fluctuating forces on the blades in the stator cascade. To do this, results to be presented in more detail in Section 6.2 of this report will be utilized. There it is shown that the phase of the fluctuating forces is dependent on the phase of the fluctuating velocity at the blade plus a phase angle dependent on blade chord and frequency. However, for any given sound harmonic, this additive phase angle will be the same for all blades in the stator cascade. Hence it will be ignored, and only the variations in phase from blade to blade due to the effects of the rotor wake need be considered. Thus, the same arguments as used for the stator mass sources (Section 4.1) can be used again, so that Equation 53 generalizes for the multiple blade case to give

$$p = \sum_{\nu=0}^{V-1} \sum_{\lambda=-\infty}^{+\infty} \frac{-i\lambda\Omega}{4\pi a_0 r_1^2} \left\{ xT_\lambda + yD_\lambda \sin(\phi + 2\pi\nu/V) \right\} \exp\left\{ -i\lambda\left(\frac{2\pi\nu}{V} - \Omega\left(\frac{t-r}{a_0}\right)\right)\right\}$$

(54)

As before the geometric far field approximation for r (Equation 22) can be used in the expression, and this can in turn be rewritten into a Bessel function form using Equation 24. This gives

$$p = \sum_{\nu=0}^{V-1} \sum_{\lambda=-\infty}^{+\infty} \frac{-i\lambda\Omega}{4\pi a_0 r_1^2} \left\{x T_\lambda + y D_\lambda \sin(\phi + 2\pi\nu/V)\right\} \exp\left\{i\lambda(\Omega t - \Omega r_1/a_0)\right\}$$

$$\times \exp\left(\frac{-2\pi i\lambda\nu}{V}\right) \sum_{\mu=-\infty}^{+\infty} i^\mu J_\mu\left(\frac{\lambda M y}{r_1}\right) \cos\mu\left(\phi + \frac{2\pi\nu}{V}\right) \tag{55}$$

Again, as before, the orthogonality relations in the ν summation may be utilized to eliminate many of the terms. The T_λ term above may be observed to have an identical form to that of Equation 25, and the reduction there can be used directly. However the D_λ term differs from that occurring before through the sine factor. Equation 26 can be used to derive an appropriate relation, which is

$$\sum_{\nu=0}^{V-1} \exp\frac{-2\pi i\lambda\nu}{V} \sum_{\mu=-\infty}^{+\infty} \cos\mu\left(\phi + \frac{2\pi\nu}{V}\right) \sin\left(\phi + \frac{2\pi\nu}{V}\right)$$

$$= \frac{V}{4i} \sum_{k=-\infty}^{+\infty} \exp -i(kV-\lambda)\phi \left\{\delta(kV-\lambda+1+\mu) - \delta(kV-\lambda-1-\mu)\right.$$

$$\left. + \delta(kV-\lambda+1-\mu) - \delta(kV-\lambda-1+\mu)\right\} \tag{56}$$

From this result it will be found that

$$\sum_{\nu=0}^{V-1} y D_\lambda \sin(\phi + 2\pi\nu/V) \exp(-2\pi i\lambda\nu/V) \sum_{\mu=-\infty}^{+\infty} i^\mu J_\mu\left(\frac{\lambda M y}{r_1}\right) \cos\mu\left(\theta + \frac{2\pi\nu}{V}\right)$$

$$= \sum_{k=-\infty}^{+\infty} y D_\lambda V \frac{i^{kV-\lambda}}{2} \left\{J_{kV-\lambda-1}\left(\frac{\lambda M y}{r_1}\right) + J_{kV-\lambda+1}\left(\frac{\lambda M y}{r_1}\right)\right\} \exp i(\lambda - kV)\phi \tag{57}$$

Now a well known result for Bessel functions (Formula 28 of Maclachan (Ref. 18)) is

$$J_{n-1}(z) + J_{n+1}(z) = \frac{2n}{z} J_n(z) \tag{58}$$

Equation 55 may now be written down in a more concise form, by using the relations previously derived in Equation 27, and the new results of Equations 57 and 58. Also note, as before, that $\lambda = mB$. This gives the complex magnitude of the mth sound harmonic radiated by the stator forces due to rotor-stator interaction as

$$c_m = \sum_{k=-\infty}^{+\infty} \frac{-imBV\Omega}{2\pi a_0 r_1} \left\{ \frac{x}{r_1} T_m - \frac{(mB-kV)}{mBM} D_m \right\} i^{mB-kV} J_{mB-kV}\left(\frac{mBMy}{r_1}\right) \tag{59}$$

where the phase factor has been deleted as in Section 4.1.

5.2 Sound Generated by Fluctuating Forces at the Rotor

As in Section 4.2, we will first find the result for the sound generated by an arbitrary harmonic force. Here it is assumed that the force and motion periodically repeat some otherwise arbitrary variation. The observed sound field thus contains a series of harmonics. Writing $p = a_0^2 \rho$, and specializing to a point force, Equation 14 gives

$$p = \left[\frac{(x_i - y_i)}{(1 - M_r)\, a_o r} \quad \frac{\partial}{\partial t} \left\{ \frac{F_i}{4\pi r (1 - M_r)} \right\} \right] \tag{60}$$

Note that Equation 60 must be evaluated at retarded time τ but it is desired to calculate sound harmonics in the observers time t, where $\tau = t - r/a_0$.

Defining the complex magnitude of the nth sound harmonic in the usual way gives

$$c_n = a_n + i\, b_n = \frac{\omega}{\pi} \int \left[\frac{x_i - y_i}{(1 - M_r)\, a_o r} \quad \frac{\partial}{\partial t} \left(\frac{F_i}{4\pi r (1 - M_r)} \right) \right] \exp in\omega t \, dt. \tag{61}$$

The integral is over any period $2\pi/\omega$.

Changing variables back to retarded time τ gives, using $dt = (1 - M_r)\, d\tau$,

$$c_n = \frac{\omega}{\pi} \int_0^{2\pi/\omega} \left[\frac{(x_i - y_i)}{a_o r} \quad \frac{\partial}{\partial t} \left(\frac{F_i}{4\pi r (1 - M_r)} \right) \right] \exp\left\{ in\omega\, (\tau + r/a_o) \right\} d\tau$$

and integrating by parts

$$c_n = -\frac{\omega}{4\pi^2 r}\int_0^{2\pi/\omega}\left[\frac{i n \omega F_r}{a_o} + \frac{F_i}{1 - M_r}\left(\frac{-M_i}{r} + \frac{(x_i - y_i)}{r^2} M_r\right)\right]$$

$$\times \exp\left\{ i n \omega (\tau + r/a_o) \right\} d\tau$$

where $F_r = F_i (x_i - y_i)/r$, the component of force in the direction of the observer. The second term is important only in the acoustic near field, because of the additional factors r in the denominator, so that the result for the far field harmonics becomes simply

$$c_n = \frac{-\omega}{4\pi^2 r}\int_0^{2\pi/\omega}\left[\frac{i n \omega F_r}{a_o}\right] \exp\left\{ i n \omega (\tau + r/a_o)\right\} d\tau \tag{62}$$

Again, in order to use this result for calculations of the noise, it is necessary to define the fluctuating force field. As in Equation 60 put

$$T = \sum_\lambda T_\lambda \exp(-i\lambda\Omega t), \quad D = \sum_\lambda D_\lambda \exp(-i\lambda\Omega t) \tag{63}$$

Furthermore, Equation 52 can be used to give

$$\left.\begin{aligned} F_r &= -\frac{x\,T}{r} - \frac{y\,D}{r} \sin\theta \\ r &\approx r_1 - \frac{y\,R}{r_1} \cos\theta \end{aligned}\right\} \tag{64}$$

Using all these results, Equation 62 gives the harmonics of the far field sound radiation from the rotor as

$$c_n = \frac{i n \Omega}{4\pi^2 a_o r}\int_0^{2\pi} \sum_{\lambda=-\infty}^{+\infty} \left\{ \frac{x T_\lambda}{r_1} + \frac{y D_\lambda}{r_1} \sin\theta \right\}$$

$$\times \exp\left\{ i (n - \lambda)\theta - i n \alpha \cos\theta \right\} d\theta \tag{65}$$

where $\alpha = \Omega\, Ry/a_0\, r_1 = My/r_1$ where $M = \Omega\, R/a_0$ is the rotational Mach number of the compressor.

Using the Bessel function formula given in Equation 37 together with the following result which can be readily derived from it,

$$\int_0^{2\pi} \exp\left\{ i\,(n\,\theta - z\cos\theta) \right\} \sin\theta\, d\theta = -2\pi\, i^{-n}\, \frac{n}{z}\, J_n(z) \tag{66}$$

Equation 65 may be evaluated directly to give the result for the radiation from a single rotor blade as

$$c_n = \frac{i\,n\,\Omega}{2\pi\, a_0\, r_1} \sum_{\lambda=-\infty}^{+\infty} (-i)^{n-\lambda} \left\{ \frac{x\,T_\lambda}{r_1} - \frac{n-\lambda}{n}\, \frac{D_\lambda}{M} \right\} J_{n-\lambda}\left(\frac{n\,My}{r_1} \right) \tag{67}$$

As before, we may sum over all the B rotor blades present, and obtain substantial cancellation by use of Equation 26. In this case we may also identify $\lambda = kV$. Thus the final result for the complex magnitude of the mth sound harmonic of the noise radiated by the fluctuating forces on the rotor is

$$c_m = \frac{i\,m\,B^2\,\Omega}{2\pi\, a_0\, r_1} \sum_{k=-\infty}^{+\infty} (-i)^{mB-kV} \left\{ \frac{x\,T_k}{r_1} - \frac{mB-kV}{mBM}\, D_k \right\} J_{mB-kV}\left(\frac{m\,B\,My}{r_1} \right) \tag{68}$$

5.3 Comments on the Results

It will be observed that the general form of Equations 28, 39, 59 and 68 are remarkably similar. All contain the same Bessel function term to the same argument and order. Since the effects due to the Bessel function dominate the results, it may be concluded that the gross features of all four types of sound radiation will be very similar. The similarity between Equations 28 and 39 for the mass source cases has already been commented upon in Section 4.3. Comparison of Equations 59 and 68 shows again that the force terms are nearly identical, so that little difference can be expected between the stator and rotor radiation fields. Again the thurst and drag terms are essentially the same except that the harmonic orders of the force terms are dependent on m and k in the two cases. A slight additional difference is that the stator equation (59) is multiplied by the term BV while the rotor Equation 68 is multiplied by B^2. This corresponds to the fact that the force terms in each equation are the forces per blade. Thus a V in Equation 59 and B in Equation 68 can be taken inside the force terms so that they then represent the total fluctuating force over the stator or rotor disc.

All the discussion of the effects of blade row separation in Section 4.3 applies again in the present fluctuating force case. Increase of rotor-stator spacing will tend principally to reduce the higher harmonics of the noise while increase of stator-rotor spacing will reduce all the noise harmonics. Again each kth term may be considered as a single "mode".

Equations 59 and 68 give the results in terms of complex Force coefficients T_m and T_k. The results can be rewritten in terms of ordinary Fourier coefficients. Equation 68 gives the result

$$c_n = \frac{\Omega}{4\pi a_o r_1} \sum_{\lambda=0}^{\infty} (-i)^{n-\lambda-1} \left\{ \frac{n x a_{\lambda T}}{r_1} \left(J_{n-\lambda} + (-1)^{\lambda} J_{n+\lambda}\right) + \right.$$

$$+ \frac{i n x\, b_{\lambda T}}{r_1} \left(J_{n-\lambda} - (-1)^{\lambda} J_{n+\lambda}\right) \quad - \; .....$$

$$- \frac{a_{\lambda D}}{M} \left\{ (n-\lambda) J_{n-\lambda} + (n+\lambda) (-1)^{\lambda} J_{n+\lambda} \right\}$$

$$\left. - \frac{i b_{\lambda D}}{M} \left\{ (n-\lambda) J_{n-\lambda} - (n+\lambda) (-1)^{\lambda} J_{n+\lambda} \right\} \right\} \tag{69}$$

where the argument of all the Bessel functions is nMy/r_1. $a_{\lambda T}$, $b_{\lambda T}$, $a_{\lambda D}$, $b_{\lambda D}$, are ordinary Fourier coefficients of the fluctuating thrust and drag related to the T_λ and D_λ as in Equations 51.

As discussed for Equation 40, in virtually all practical compressor noise cases, the $J_{n+\lambda}$ terms will be negligible compared to the $J_{n-\lambda}$ terms, so that a simplified version of Equation 69 which should be accurate for nearly all calculations, is

$$c_m = \frac{mB^2\Omega}{4\pi a_0 r_1} \sum_{k=0}^{-\infty} (-i)^{mB-kV-1} \left\{ \frac{x}{r_1} (a_{\lambda T} + i b_{\lambda T}) - \frac{mB-kV}{mBM} (a_{\lambda D} + i b_{\lambda D}) \right\} J_{mB-kV}\left(\frac{mBMy}{r_1}\right) \tag{70}$$

It is also of interest to consider the special case when only steady forces exist on the rotor. This corresponds to putting $\lambda = 0$ in Equation 67 with the result

$$c_n = \frac{(-i)^{n-1} n\Omega}{2\pi a_o r} \left\{ \frac{x\, T_o}{r} - \frac{D_o}{M} \right\} J_n \left(\frac{n\, M y}{r} \right) \tag{71}$$

This is the result obtained by Gutin (Ref. 22) for the case of propeller noise radiation (see also Ref. 17). Reduction to this classical solution provides a useful test of the result. The same result for the rotor noise radiation was also obtained by a more direct method which was an extension of that presented in Ref. 10, and agrees with that obtained by another alternative method in Ref. 14.

Furthermore after the preliminary draft of this report had been prepared Morse and Ingard's new book (Ref. 21) was published. Pages 738-747 of this book develop equations for noise radiation from a propeller in unsteady flow which is, of course, one of the cases treated here. Equation 11.3.20 of their book agrees with Equation 68 of this report. Their results were an extension of earlier unpublished consulting work by Ingard. A report by Arnold et al. (Ref. 23) also gives an analysis of a similar case, and here frequencies other than harmonics of the rotational speed were allowed to occur. As expected from the discussion of Section 2.3, they found that single frequency input could give multiple frequency output. This is the result of the modulated Doppler frequency shift due to rotor source motion.

5.4 Power Calculations

Calculation of the acoustic power radiated by the fluctuating force distributions again requires the integration of the pressure over the field, and thus follows the same lines as in Section 4.4. The single mode acoustic output of the rotor radiation is, in this case, via Equations 43 and 70,

$$W_{n\lambda} \approx \frac{\Omega^2}{16\pi\, a_o^3\, \rho_o} \int_0^{\pi} \left\{ n^2\, c_{\lambda T}^2 \cos^2 \Psi + \frac{(n-\lambda)^2\, c_{\lambda D}^2}{M^2} \right\} J_{n-\lambda}^2 \,(n\, M \sin \Psi) \sin \Psi \, d\Psi \tag{72}$$

where $c_{\lambda T}^2 = a_{\lambda T}^2 + b_{\lambda T}^2$, $c_{\lambda D}^2 = a_{\lambda D}^2 + b_{\lambda D}^2$. The cross terms between the thrust and drag vanish identically on integration. To evaluate the integral we require a further result, established by Embleton (Ref. 24).

$$\int_0^{\pi/2} J_n^2 (z \sin \theta) \cos^2\theta \sin \theta \, d\theta = \sum_{r=0}^{\infty} \frac{(-1)^r \; z^{2n+2r}}{r! \, (2n+r)! \, (2n+2r+1) \, (2n+2r+3)}$$

So that, together with Equation 47 we find that

$$W_{n\lambda} \approx \frac{1}{8\pi a_0 \rho_0 R^2} \sum_r \frac{(-1)^r \, (nM)^{2\mu + 2r}}{r! \, (2\mu + r)! \, (2\mu + 2r + 1)}$$

$$\times \left\{ \frac{(Mn)^2 \, c_{\lambda T}^2}{2\mu + 2r + 3} + \mu^2 \, c_{\lambda D}^2 \right\} \tag{73}$$

gives an approximation to the power radiated by a specific rotor loading harmonic. As before this approximation will be most accurate when $|n - \lambda| << |n + \lambda|$ i.e., when $\lambda >> 0$, $n >> 0$, which will usually be the case for a compressor.

6.0 DEFINITION OF SOURCE TERMS

6.1 Potential and Wake Interactions

Before the sound radiation can be calculated it is necessary to define the forces and mass fluctuation at the rotor and stator blade rows. The mass source fluctuations can be derived directly from a knowledge of the wake profiles, but definition of the more significant fluctuating force terms is more difficult. Fortunately, the prediction of the fluctuating force levels on the blades has been the subject of studies by Kemp and Sears (Refs. 25 and 26). They showed that two effects should be considered; the potential flow interactions between the blades, and the effect of the viscous wakes from an upstream row impinging on a downstream row. Solution of the potential interaction problem is not straightforward. Account must be taken of the interference of the fluctuating trailing vortex sheet from the first blade with the downstream row, and also of the mutual effects of the bound vorticity on each blade. The viscous wake problem, illustrated in Figure 5, is more straightforward as there is only a single effect to consider; the passage of the fluctuating velocity field of the wake through the downstream row.

Now the velocity field of a vortex is proportional to the inverse square of distance while the velocity decrement in a wake is approximately proportional to the inverse of the distance. Thus, at large blade separations only the viscous interference effects may be expected to be of significance, and conversely at small separations only the potential interactions are probably significant. Kemp and Sears (Ref. 26) found that, in a typical case, the viscous interference effects were of the same order as the potential interactions for a rotor-stator separation equal to 0.1 of the rotor chord.

In order to reduce noise and vibration most modern compressors operate at a rotor-stator separation of more than 1.0 chord lengths. Thus it seems unlikely that the potential flow interactions will play any significant role in determining the fluctuating forces on the rotor and stator. Therefore, attention may be confined solely to the viscous interactions. An added advantage of this is that this interaction is far easier to analyze. Some additional comments on this point are given in the published discussion to Reference 9. In the next section some of the basic features of the wakes in a compressor and the forces acting on an airfoil in an oscillatory slipstream will be reviewed, following the methods of Kemp and Sears (Ref. 26).

6.2 Definition of the Fluctuating Forces in the Blades

In their analysis, Kemp and Sears (Ref. 26) assumed that each blade acts independently, so that mutual interference effects in the cascade are eliminated. They also assumed that the wake velocity decrement is small, so that second order perturbation terms can be removed. Both these assumptions are probably reasonable for the relatively large axial and circumferential blade spacings typical of current compressors.

Suppose now that the velocity profiles in the trailing wakes can be defined. Then the coefficients α and β in Equation 17 for the wake spatial harmonics will also be determined. The effect of these varying wake velocities convected relative to the blades will be to cause a time varying upwash at the blades, as shown in Figure 5 and discussed in Section 2.0

Consider first the stator-rotor interaction. Suppose that the wake velocity profiles vary around the circumference in a way defined by

$$w = \alpha_0 + \sum_{\lambda=1}^{\infty} \alpha_\lambda \cos \lambda \theta \tag{74}$$

The sine terms in Equation 74 have been neglected since the wake may be assumed to be symmetrical. This approximation was discussed by Kemp and Sears (Ref. 26). Also since the rotor is moving at angular velocity Ω we can put $\theta = \Omega t$.

From Figure 5, if the velocity at a point in the stator wake is w, then the upwash at the rotor is $v = w \sin \beta$. Sears (Ref. 27) has analyzed the case of a wing entering a sinusoidal gust. He showed that if a thin airfoil experiences a non-steady upwash of the form

$$v = \mathrm{Re} \left\{ v' \exp i \omega (t - x/U_1) \right\} \tag{75}$$

then the lift per unit span acting at the quarter chord point, is given by

$$L = \mathrm{Re} \left\{ \pi c \rho U_1 v' S(\sigma) \exp i \omega t \right\} \tag{76}$$

where the lift response function S is given by

$$S(\sigma) = \left\{ i \sigma \left(K_0 (i \sigma) + K_1 (i \sigma) \right) \right\}^{-1} , \quad \sigma = \omega c / U_1 \tag{77}$$

where K is a modified Bessel function of the second kind (Ref. 18). This particular form for S was given by Kemp (Ref. 28), and gives the phase relative to the half chord point. Thus, combining Equations 74 to 77, the fluctuating lift per unit span at the rotor is given by

$$L = \mathrm{Re} \left\{ \pi c \rho_0 U_1 \sum_{\lambda=1}^{\infty} - \alpha_\lambda \sin \beta \, S(\sigma) \exp i \lambda \Omega t \right\} \tag{78}$$

As discussed by Kemp and Sears (Refs. 25 and 26) this equation can be applied to the compressor blading provided there is reasonable circumferential separation. In fact, for moderate non-dimensional frequencies ($\sigma > \pi$) a simplified expression for S can be found. From the standard forms for Bessel functions (Ref. 18)

$$K_0(i\sigma) = -\pi/2\, i \left\{J_0(\sigma) - i\, Y_0(\sigma)\right\}$$

$$K_1(i\sigma) = -\pi/2 \left\{J_1(\sigma) - i\, Y_1(\sigma)\right\}$$

So that

$$S(\sigma) = \left[(\pi\sigma/2)\left\{J_0 - Y_1 - i\,(Y_0 + J_1)\right\}\right]^{-1}$$

Again, from the standard asymptotic forms for Bessel functions (Ref. 18)

$$J_o(\sigma) \approx \sqrt{2/\pi\sigma}\, \cos(\sigma - \pi/4), \quad J_1(\sigma) \approx \sqrt{2/\pi\sigma}\, \cos(\sigma - 3\pi/4)$$

$$Y_o(\sigma) \approx \sqrt{2/\pi\sigma}\, \sin(\sigma - \pi/4), \quad Y_1(\sigma) \approx \sqrt{2/\pi\sigma}\, \sin(\sigma - 3\pi/4)$$

Using these expressions, expanding, and rearranging therefore gives

$$S = \frac{\cos(\sigma - \pi/4) + i\,\sin(\sigma - \pi/4)}{(2\pi\,\sigma)^{\frac{1}{2}}} \tag{79}$$

A comparison of the exact and approximate expressions is given in Figure 8. It can be observed that the approximate form is probably sufficiently accurate for nearly all practical applications. Equation 79 may be used in Equation 78 to give an expression for the fluctuating lift on the blade as

$$L = \mathrm{Re}\left\{-\pi\, c\, \rho_0\, s\, U_1 \sum_{\lambda=1}^{\infty} \frac{\alpha_\lambda \sin\beta}{(2\pi\sigma)^{\frac{1}{2}}} \exp i\,(\lambda\,\Omega\, t + \sigma - \pi/4)\right\} \tag{80}$$

Thus if the complex spatial Fourier coefficients α_λ of the wake velocity profile can be established, the fluctuating lift on the blade can be calculated. Note that the lift is proportional to the inverse square root of σ, so that the blade operates to some extent as a damper of the higher frequency input. Equation 80 also shows how the phase of L is basically dependent on the phase of the fluctuating velocity on the blade. The additional phase effects due to the imaginary exponential are dependent on blade chord and frequency but will not vary from blade to blade. This feature has already been assumed in the application of the fluctuating force phasing relationships in Section 5. To complete the specification of the noise radiation we need only *to define the coefficients* α_λ. *This is the most difficult task of the study.*

Kemp and Sears (Ref. 26) gave numerical estimates for the wake parameters in their paper, and these can be used if desired. However, review of the characteristics of wakes in practical compressors suggests that these estimates may easily be grossly misleading.

6.3 Characteristics of Wakes in a Real Compressor

Kemp and Sears (Ref. 26) applied the results of Silverstein et al. (Ref. 29) on the wakes behind an isolated airfoil. Indeed this appears to be the only comprehensive airfoil wake data available. However, it seems unlikely that the isolated airfoil, particularly that investigated by Silverstein et al., is representative of the conditions inside a compressor, which is complicated by at least four factors

- axial pressure gradient
- radial pressure gradient
- varying swirl angle
- Mach number effects.

A substantial axial pressure rise occurs across each compressor stage. Pressure ratios above 1.5 are typical of modern bypass fan stages. This pressure gradient must be expected to cause a substantial thickening of the blade wake compared to the isolated airfoil case, because the low momentum fluid in the wake will lose comparatively more velocity in overcoming the same pressure rise. An associated effect is the high camber typical of compressor blading which will also tend to generate wakes of increased thickness, and possibly even local separation regions.

The radial pressure gradient has additional important effects. The basic design characteristics of the compressor blades lead to a substantial radial outflow of the low momentum fluid in the boundary layer, and the same characteristics will lead to a radial outflow of the fluid in the wake. Total head surveys clearly show the collection of the low momentum fluid near the outer casing behind the rotor. Behind the stator the reverse effect occurs and stagnant fluid collection on the inboard side is observed. This is shown in Figure 9, taken from Reference 30.

Compressor blading is virtually always designed to perform equal work on the fluid over the whole annulus. Because of the reduction in rotor velocity near the hub this requirement is met by applying a greater turning angle to the flow at this point.

The wake therefore has a differential swirl as it leaves the rotor and will have a higher circumferential velocity near to the hub than at the tip. Again this effect can be clearly seen in total head surveys such as shown here in Figure 9. It is interesting to note that this leads to an effective skewing of the wake relative to the next row, and this has been suggested as a possible method for reducing compressor noise output (Ref. 11). It is clear that this effect will lead to phase differences of the fluctuating force over the blade span, which could have significant effects on the results.

Most modern compressors run at relative Mach numbers greater than 0.8. This is well above the drag rise Mach number of the blades, and significant increases in wake thickness may be expected over the low speed results of Silverstein et al. (Ref. 29). Results shown in Figure 153 of Reference 30 suggest the momentum thickness may be as much as a factor of 4 larger, although this is probably unusual.

Many additional features which offer additional complications can occur. Part span reinforcements of the blades are common. Blade chord is rarely constant over the annulus. Tip and root effects in addition to those discussed above may be important, and in some cases additional asymmetries due to inlet design or upstream struts can be significant. Furthermore the off-design operation of the compressor introduces a host of possible new features - rotating stall being the most dramatic. Unfortunately there appears to be little comprehensive data on wakes in compressors, basically because they are only of secondary significance in performance. The review of Compressor Design Methods by Lewis Research Staff (Ref. 30) gives many examples of the problems which may occur. Certainly, in view of the discussion above, it seems unlikely that Silverstein et al's data (Ref. 29) for an isolated airfoil will be applicable. However, one possible crude model is put forward in Section 8 as a basis for some calculations.

7.0 RESULTS AND DISCUSSION

7.1 General

The present theory gives results for each of four possible sources of noise, rotor and stator forces, and rotor and stator fluctuating mass output. Furthermore, the acoustic output from any one source will generally contain contributions from several "modes". Clearly, for a real compressor, the observed noise field from each source and from each mode will not be distinguishable in any simple way. However, in order to understand the underlying mechanisms, it is helpful to study each possible mode of each source separately. This will also suggest the most promising avenues of attack for reducing the noise.

There are two features of the sound output which are important; the directionality and the acoustic power output, both of which are predicted by the present theories. Acoustic power is the principal gross feature of the sound, and is affected by the gross features of engine design, such as mass flow and revolution rate. However directionality can also play a significant role, especially since it is more readily modified by detail design and because it is generally the sound radiated radially which has the most significant effect on community noise. This fact is often overlooked in discussions of compressor noise. These two features will be discussed separately below.

Before beginning the discussion it is helpful to summarize the principal parameters involved. Firstly n is the sound harmonic number and is equal to mB where m is the harmonic order and B is the number of rotor blades. Thus first harmonic sound radiation, at the rotor blade passage frequency corresponds to $m = 1$, or $n = B$. The parameter λ is equal to kV where k is an integer and V is the number of stator vanes. For the rotor λ corresponds to the loading harmonic order. For instance, $\lambda = 0$ gives the steady load condition, but on the stator λ is simply a summation parameter. The modal order μ is defined by $\mu = |n - \lambda| = |mB - kV|$ and is one of the key parameters governing the characteristics of the sound radiation. The modulus sign in the above definition has been introduced for convenience in writing some of the equations. The rotational speed (Ω) of the compressor is introduced through the rotational Mach number M ($= \Omega R/a_0$) of the point source at radius R which represents the blade. The basic frequency parameter for the radiation is nM ($= mBM$), so that much of the discussion will be given in terms of the effects of frequency parameter nM and modal order μ. It may be noted that $nM = kR$ where k is the wave number of the radiation and R is the typical radius, and in this alternate form may be more familiar to acousticians. nM has been used here because of its more direct interpretation in terms of compressor parameters.

7.2 Acoustic Power Levels

First of all the acoustic power levels for various forms of compressor noise radiation will be investigated. Equations 48 and 73 gave the sound power in the nth harmonic due to a single mode of the fluctuating mass and force sources, respectively. We may rewrite Equation 73 as

$$W_{n\mu} \approx \frac{1}{8\pi a_0 \rho_0 R^2} \left\{ \mu^2 c_{\lambda D}^2 \hat{D} + (Mn)^2 c_{\lambda T}^2 \hat{T} \right\} \tag{81}$$

where

$$\hat{D} = \sum_{r=0}^{\infty} \frac{(-1)^r (nM)^{2\mu+2r}}{r!\,(2\mu+r)!\,(2\mu+2r+1)} \tag{82}$$

and

$$\hat{T} = \sum_{r=0}^{\infty} \frac{(-1)^r (nM)^{2\mu+2r}}{r!\,(2\mu+r)!\,(2\mu+2r+1)\,(2\mu+2r+3)} \tag{83}$$

Equation 81 as written gives the result for the rotor radiation case, but the functional form of the result for the stator radiation is identical, so that all conclusions drawn from the present study of Equation 81 will apply equally to the rotor or stator radiation. The differences come essentially in the way the various modes add together for each case.

Comparison of Equation 82 with Equation 48 will show that $\hat{D}$ also gives the basic form of the mass source radiation, requiring only a multiplying factor to give the final result. Figure 10 gives computations of this summation (Equation 82) for various values of μ and nM and therefore gives the effects on both the drag and mass source terms. The calculations agree with those presented by Embleton and Thiessen (Ref. 13) for the same problem using a different series (Equation 46) but cover a slightly wider range of parameters. For clarity only curves for selected modes have been shown.

In Figure 10 it can be seen that, for any given modal order μ, sound radiation is very inefficient for low frequencies, but becomes significant for values of the frequency parameter nM roughly greater than the modal order μ. The efficient and inefficient radiation conditions in Figure 10 correspond respectively to supersonic and subsonic phase velocities for the fluctuating forces as was suggested in Section 2.1. It can also be seen that the radiation efficiencies of all modes are equivalent, to within

2 dB, if the frequency is sufficiently high. The asymptotic value of the series in Equation 82 is given quite accurately by 1/2 nM, and this curve is also shown in Figure 10.

The series in Equation 82 are badly conditioned, and meaningful results were unobtainable in the present computations for nM > 20 in spite of the use of double precision arithmetic. Thus the use of Equation 82 for sound power predictions at the high frequencies typical of a compressor (nM ~ 50) may only be practicable via the asymptotic solution mentioned above. In using this solution it is also of interest to show where the results for any given mode first approach the asymptotic line. It is reasonable to suppose that the first peak observed for each mode in Figure 10 corresponds roughly to the passing of the first peak of the Bessel functions which generated it (Equation 47). The values of nM at which $J_\mu(nM)$ reaches its first peak for various μ are shown on the asymptotic curve in Figure 8 and can be seen to correspond closely to the points at which the various modal orders first approach the asymptotic line. A reasonably accurate approximation to these first peak values is given by (Ref. 31)

$$nM = \mu + 0.809\ \mu^{1/3} \qquad (84)$$

and the use of this expression together with the asymptotic curve given above appears to offer a fairly accurate prediction method for overall power levels for any given mode above the first peak. Below the value of nM given by Equation 84, the sound drops off at roughly 6 μ dB per halving of frequency. This last approximation does somewhat underestimate the sound power, particularly at high values of μ.

Figure 11 gives equivalent results for the Thrust summation $\hat{T}$ in Equation 83. All the remarks applied to Figure 10 apply again. It appears that 1/4 nM gives the asymptotic curve for this case. However it can be seen that the accuracy of both the asymptotic solution and the approximation of Equation 84 are reduced in this case.

Figure 12 gives results for combined thrust and drag power levels, from the expression given in brackets in Equation 81 with $c_{\lambda D} = c_{\lambda T} = 1$, which would apply if the rotor blades were lying at about 45° to the x axis. Figure 12 therefore corresponds to μ^2 times Figure 10 plus $(nM)^2$ times Figure 11, and is representative of the radiation by the force terms in a real compressor. Figure 12 can be used directly for the calculation of acoustic power. Figure 12 plots the summation in chain brackets in Equation 81 divided by the square of the fluctuating force harmonic. If the magnitude of this harmonic can be estimated, either from experiment or theory, then the acoustic power radiated can be calculated directly by using Equation 81 and Figure 12 in combination. The equation applies to either rotor or stator radiation as previously discussed. If several modes are present then the acoustic power may be assumed to be given by the sum of the contribution of each. This effectively ignores power radiation by cross-terms.

Further study of the results which are plotted in Figure 12 allows several important conclusions to be drawn about parameter trends in compressor noise. Note firstly that the radiation efficiency increases with frequency. At sufficiently high frequencies the thrust terms dominate and give the asymptotic form as $nM/4$. This result will be used in a comparison with experiment in Section 8. It may be observed that $nM > \mu$ gives a good approximation to the efficient radiation region for the overall compressor case. Another point of interest is the lower efficiency of the lower order modes at high frequencies. This arises from the lessened efficiency of the drag terms for the low modes (see Equation 81). Lower order modes have generally been assumed to be undesirable acoustically, and Figure 12 shows how, at low frequencies ($nM < 1$) where most previous acoustic problems have been encountered, the lower modes are indeed far more efficient radiators of sound. But at the high frequencies typical of a compressor the lower modes are desirable for minimizing acoustic power output. For $nM = 16$ the peak effect is over 3 dB. Furthermore sideline radiation patterns for the lower order modes are lower, as will be shown in Section 7.3. Practical results from an experimental compressor supporting a reduction of power output at $\mu = 0$ have been reported by Crigler and Copeland (Ref. 7) and Figure 13 is reproduced from their report. Acoustic power levels have been calculated and are also shown in the figure. It will be observed that the acoustic power radiated for the 53 guide vanes $\mu = 0$ case is about 8 dB less than for the 31 guide vane and 7 dB less than for the 62 guide vane case. Extrapolation of Figure 12 suggests a maximum effect of only 4 dB so that there are probably other effects present in Crigler and Copeland's results. Nevertheless the major reduction in sound radiation occurring for the $\mu = 0$ case here is obviously of practical significance. Note on Figure 13 how the peak levels for the $\mu = 0$ case are higher, but carry very little acoustic power, so that power levels are low.

It is also of interest to study the effects of choosing various stator vane numbers on the noise output. For the purposes of argument suppose a 16-blade rotor ($B = 16$) operating at $M = 0.5$ is chosen. Figure 14 shows the result for the effect on the first and second harmonic noise ($m = 1,2$) of the first and second mode radiation ($k = 1,2$) due to the rotor force terms. As has been pointed out previously (Section 2.3) the effect of large stator-rotor separation will be to emphasize the contribution of the $k = 1$ term, so that the case $k = 1$, $m = 1$ is of principal interest. Recalling that $\mu = |mB-kV|$, Figure 14 can be seen to be consistent with Figure 12, but the effect of variation of vane number V is now made explicit. For the first mode first harmonic case ($k = m = 1$) the curves are symmetrical about $V = 16$, the $\mu = 0$ case. For $k = m = 1$, Figure 14 also shows how for small numbers of stator blades efficiency is very low. At $V = 8$ we start to move into an efficient radiation region, reaching a maximum at $V = 10$, then there is a slight lowering of efficiency as stator blade numbers are increased up to 16. Past this point efficiency again increases up to a maximum of about 2.6 dB additional at $V = 22$. It may also be noted that $V = 10, 22$ corresponds to the most unfavorable sideline directivity pattern as well as the highest power, as will be shown in the next section. For $V = 10$ the peak would be in front of the compressor and for $V = 22$ behind. For further increases in blade number efficiency then drops off rapidly.

The effect of second harmonic loading on first harmonic noise ($k = 2$, $m = 1$) may also be seen in Figure 14. Substantially the same effects occur, except the range of efficient radiation is for $4 < V < 12$. Likewise, the effects of first harmonic loading on second harmonic noise ($k = 1$, $m = 2$) is seen to be greatest for $16 < V < 48$, and for second harmonic loading on second harmonic noise ($k = 2$, $m = 2$) $8 < V < 24$. Now the actual fluctuating loadings occurring for $k = 2$ and higher can be reduced to a minimum by choice of large rotor-stator separations. Thus it can be concluded that increase in rotor stator separation will be particularly effective at reducing first harmonic noise when there are less stators than rotors. A second interesting conclusion is that choice of many more stators than rotors will tend to increase second harmonic noise. These general conclusions should apply to any compressor.

Clearly the optimum way to utilize the results shown in Figure 14 is to choose the stator vane number such that radiation is in the inefficient region of the curves. Unfortunately this is difficult on practical compressors. Apart from the effects of multi-mode input and higher harmonics the high idling Mach number of modern compressors is very restrictive. Figure 15 shows the effect of rotational Mach number. It can be seen that the region of efficient radiation spreads out considerably, and reaches down even to zero vane numbers for the $M = 1$ case. (Note that the $V = 0$ cases are underestimated by 6 dB in Figure 15 because of the neglect of the $n + \lambda$ terms, as discussed in Section 4.3.) Thus excessive numbers of stator vanes are necessary to achieve low radiation efficiency unless the engine has been specifically designed to run at a low rotational Mach number. On the other hand, the potential of utilizing low values of μ for minimizing noise would apply at any subsonic Mach number.

A further important conclusion may be drawn from Figure 14. The $V = 0$ case corresponds to the steady loading, which will be an order of magnitude greater than any fluctuating harmonic loading level. Thus at supersonic rotational speeds the primary source of the noise radiation at the blade passage frequency is due to the steady loadings, and design of a viable quiet engine operating at these speeds becomes extremely difficult. In contrast, at subsonic speeds, noise is governed by the fluctuating loadings and the noise is therefore considerably more amenable to reduction at source.

Another way of looking at the blade number problem is to consider a fixed number of stator vanes, and a variable number of rotor blades. The effects are shown in Figure 16. Increase in number of rotor blades (B) increases radiation efficiency, and vice versa, so that the curves are not symmetrical about the $B = 16$ point. It can be seen that an increase in number of rotor blades pays far less dividends than a decrease. On the other hand having a larger number of rotor than stator blades enables advantage to be taken of other acoustic effects associated with the duct, as discussed in Reference 11, so that the most desirable choice for the designer is not entirely clear.

The basic similarities between the present results and those of Tyler and Sofrin (Ref. 1) for acoustic propagation in an open circular duct are apparent. Tyler and Sofrin showed how the radiation of any given mode down a circular duct was dependent on its frequency. Above a critical "cutoff" frequency the modes propagated directly with unit efficiency, but below their cutoff frequency, the modes decayed exponentially within the duct. In fact the theoretical cutoff frequency is given by the approximation of Equation 84, which has already been shown to correspond closely to the changeover between efficient and inefficient radiation for the present case. Furthermore, in the next section, it will be shown how the broad features of the directionality of the source radiation terms were similar to those of the duct radiation given by Tyler and Sofrin (Ref. 1).

It is concluded that many of the features of compressor noise sometimes thought to be associated with the duct are in reality reflections of the source input characteristics. Furthermore it can be seen that a source radiating at conditions well below the duct cutoff frequency would project so little sound to the far field that the effects of decay within the duct will usually be of small practical significance. Clearly, if the duct is treated with acoustical attenuating treatments, it will have to be considered in detail, but for the hard wall duct case it is thought that the present analysis will be of general application to both long and short duct radiation problems.

7.3 Directionality Effects

The directivity pattern of compressor noise radiation is often important, since it is generally the sound radiated sideways from the compressor that causes the principal community noise problems. Equations have been given which enable the sound pressure at any field point to be calculated. In order to discuss directionality effects it is worthwhile to repeat the relevant parts of these equations.

Stator and Rotor Fluctuating Force Terms (Equations 59 and 68)

$$\left|c_{n\lambda}\right| = A \left\{ \frac{x\, c_{\lambda T}}{r_1} - \frac{n-\lambda}{nM}\, c_{\lambda D} \right\} J_{n-\lambda}\left(\frac{nMy}{r_1}\right) \tag{85}$$

Stator and Rotor Fluctuating Mass Terms (Equations 28 and 39)

$$\left|c_{n\lambda}\right| = A\, J_{n-\lambda}\left(\frac{nMy}{r_1}\right) \tag{86}$$

Both equations give the sound radiation in a single sound harmonic n due to the action of a single value of λ (see discussion in Section 7.1 for a definition of the parameters involved). Both the expressions above are approximations which apply for large blade numbers, so that $J_{n+\lambda}$ terms have been ignored compared to the $J_{n-\lambda}$ terms, as previously suggested. In each case, the constant A represents terms which do not contribute to the directionality pattern, but which require detailed consideration in calculations of overall power, as discussed in the previous section.

As has already been pointed out in Section 5.3 the basic forms of Equations 85 and 86 are very similar. The Bessel function term occurs to the same order and argument in each. Indeed the directivity effects of the drag terms in Equation 85 are identical to those of the mass sources in Equation 86, paralleling the similarity noted for the power radiation in the last section. In fact Equation 86 also applies to the radiation from an annular duct. Thus it can be anticipated that the broad features of the mass and force directivity patterns will be the same.

In order to evaluate the directionality effects, a matrix of directivity patterns has been prepared for both the force and mass term cases (Equations 85 and 86) and these are shown in Figures 17 and 18. The two basic variables are again the frequency parameter nM and the modal order μ. In each of these figures the compressor axis is pointing upwards (but see discussion below), and the directivity factor is given in dB as a function of direction from the compressor hub. The directivity factor is the ratio of the observed sound pressure level at any point to the mean sound pressure level at the same distance from the source. This latter can be calculated from the acoustic power output of the source assuming that it radiates uniformly in all directions. The dotted lines on the figures give lines of equal ground noise level. Because the sound radiated at small angles to the axis travels further before reaching the ground than that travelling downwards it undergoes additional inverse square loss due to spherical spreading. The dotted lines are equal ground noise contours calculated allowing for this effect and assuming the compressor axis is parallel to the ground. The sound level relative to these contours will generally be the most significant parameter from the community noise point of view.

The curves in Figure 17 for the sound radiation due to point forces are not symmetrical about the compressor disc. This is because for small values of λ the thrust and drag terms cancel in the forward quadrants but are additive in the rear quadrant, so that more sound is heard behind the compressor disc than in front. This effect is well known in propeller noise theory (Refs. 22 and 32). However, as can be seen from Equation 85 the situation is reversed when $\lambda > n$; i.e., $kV > mB$. For this case more sound is radiated forward out of the compressor inlet than rearwards. The effect of the $\lambda > n$ case is given quite accurately by simply inverting the directionality patterns shown in Figure 17. The effect could have practical application, since it gives a method by which the designer can choose the more intense sound to radiate forwards or rearwards into any available sound absorbing device in the duct.

Since the mass source directionality pattern is entirely symmetric only the forward radiation lobe is shown in Figure 18. It can be seen that in both Figures 17 and 18 the sound radiation is predominantly sideways for small values of the frequency parameter nM and large values of the modal order μ while for large nM and small μ the radiation is greater along the axis. The figures may be interpreted in terms of several parameters. Increase in tip Mach number corresponds to a left to right movement in the directivity matrices. Also $n = mB$ where B is the number of rotor blades and m is the harmonic number. Successive harmonics therefore occupy successive positions from left to right in the matrix. Assuming that there are more rotor blades than stator vanes, increase in stator vane number corresponds to movement upwards in the matrix. On the other hand rotor blade number affects both axes. Increase in the rotor blade number implies a movement downward and to the right in the matrices.

The various directivity patterns may also be related to the overall power radiation for the same case. It will be recalled that efficient radiation occurred roughly for $nM > \mu$. Thus the lobed patterns towards the top right hand corners of the matrices in Figures 17 and 18 correspond to an efficient radiation, while the sideways radiation patterns towards the bottom left hand corner correspond to inefficient radiation conditions. Thus the least efficient radiation cases from the acoustic power point of view have the worst directivity pattern from the point of view of community noise. Fortunately the effects of the decreased efficiency for $nM < \mu$ will generally overcome any directivity effects. However there are other significant features. As can be seen in Figure 12 or 14, there is a maximum in the radiation efficiency close to the condition $\mu = nM$. Figures 17 and 18 show how this case corresponds to strong sideline radiation. Thus it is desirable from the point of both directivity and power to design compressors away from the $\mu = nM$ point. Conversely, Figures 12 and 14 also showed how it was desirable to go to low order modes ($\mu \rightarrow 0$) to reduce sound power. Figures 17 and 18 show how this also has favorable directivity effects with minimal sideline radiation. Figure 13 gave experimental results due to Crigler and Copeland (Ref. 7) which support both the reduced power and the improved directivity pattern of the $\mu = 0$ case.

It is also of interest to compare these ring source directivity approximations to the radiation from a complete duct. Figure 19 shows a matrix of directivity patterns, calculated from the formulae given by Tyler and Sofrin (Ref. 1), and assuming radiation is in the first radial mode. It may be observed that the same broad effects occur for the open duct case as in the ring source. The principal difference is at high frequency parameter (nM) where the radiation patterns for the complete duct are more substantially forward. This effect gives a general idea of what may be expected if results were found by integration over a complete compressor annulus, assuming the sources were in phase, rather than in the present effective ring approximation.

The similarity between the radiation patterns for the duct and the present calculations where no duct is present is significant. It shows again how many of the phenomena which are sometimes considered to be due to the duct are simply the result of the source parameters. The similarity of the duct "cutoff" effect to the present case with no duct was discussed in the previous section. The results above re-emphasize how detailed geometrical features of the compressor are unlikely to be important providing the key effects of source magnitude and (particularly) phase are included.

Before concluding this discussion it is useful to review the rather large number of approximations made in the study. Expressions for four separate source terms have been given and evaluated, and most of the conclusions apply essentially to radiation from a single source. Interactions between the four sources have not been considered. All cases have been reduced to the problem of point sources circling the origin. A real compressor consists of a collection of three-dimensionally extended sources which can have phase as well as amplitude variation. Nevertheless it is felt that most of the general conclusions drawn here should be equally valid for the practical compressor with a finite annulus width. This was found for the helicopter rotor case (Ref. 14). The effects of compressor motion relative to the free air have been ignored in the present analysis. However, the Appendix shows how a simple modification can be used to analyze the case of compressor motion, as required. This analysis does not include possible acoustic refraction effects due to velocity gradients in the compressor. This would be an important source of error at high flow Mach numbers (> 0.8).

The results for the power output are a further approximation. The neglect of the $J_{n+\lambda}$ terms compared to the $J_{n-\lambda}$ terms is only valid for n and λ large. For the propeller case when $\lambda = 0$, that approximation will be 6 dB low. However, in general, the approximation is expected to be valid for compressor noise. The accuracy of the asymptotic values for the power can be clearly defined from the curves. The most important approximation in compressor noise has not been explicitly used here. That is, the assumption that cross-terms in the power integral for multimode input will vanish. This may be expected to yield some errors in any practical calculation, but this is expected to be of the order of a dB or so unless an exceptional case is studied. In general, it is thought that the most significant errors in practical application will arise in the specification of the aerodynamic source terms, particularly the wake characteristics.

Note also that many of the general conclusions drawn are valid only for subsonic rotors. As discussed in Section 7.2 the direct radiation from the steady loads will dominate the noise from supersonic rotors, at least at the blade passage frequency and its harmonics. Thus effects of variations in spacing and blade/vane numbers may be comparatively insignificant for the supersonic case.

8.0 COMPARISON WITH EXPERIMENT

It should be clear from the discussion to this point that there are an extremely large number of parameters which can have significant effects on the noise radiated by a compressor. Nevertheless it is of considerable interest to attempt to apply the present theory to make predictions of the noise levels that might be expected. Unfortunately, any detailed prediction of the noise requires a detailed knowledge of compressor configuration and such information is not generally available. In the present section an attempt will be made to provide a prediction method for subsonic rotors based on the theory. Many of the details discussed in previous sections are not considered here, so that all the conclusions suggested there must be carefully reviewed before they are applied to any particular case. Nevertheless it is thought that the broad features of the results are of general revelance and should be of assistance to the designer.

8.1 Sound Due to Fluctuating Force Terms

It was shown in previous sections how many interaction modes could contribute to the noise radiation. Here we specialize to the case of single mode stator-rotor interaction. This enables the results of Equation 81 to be used. The discussion in Section 7.2 showed how the asymptotic value of the summation at high frequencies was $(mBM/4)\, c_{\lambda T}^2$, where $c_{\lambda T}$ is the amplitude of a single loading harmonic. Equation 80 enables the asymptotic value of $c_{\lambda T}$ to be written down as

$$\left|c_{\lambda T}\right| = \pi c \rho_0 s U_1 \left|\alpha_\lambda\right| \sin\beta \left\{\frac{U_1}{2\pi V \Omega c}\right\}^{\frac{1}{2}}$$

Here U is the relative velocity at the rotor blade, and s the blade span. The nondimensional frequency σ used in Equation 80 is a result of passage through the stator wakes, thus giving $\sigma = V\Omega c/U_1$. Note that the elimination of the phase effects in the above result applies to all blades equally. The relative phasing of the blades has already been taken into account by Equation 81. Thus using Equation 81 gives the power in the first harmonic due to unit mode input at the rotor as

$$W_{1\lambda} \approx \frac{B^2}{8\pi a_0 \rho_0 R^2} \left(\frac{BM}{4}\right) \left(\pi c s \rho_0 U_1 \left|\alpha_\lambda\right| \sin\beta\right)^2 \left\{\frac{U_1}{2\pi V\Omega c}\right\}$$

$$\approx \frac{\rho_0 c s^2 B^2 \sin\beta\, U_1^3}{64 a_0^2 R V} \alpha_\lambda^2 \qquad (87)$$

Thus the requirement now is for an estimate of α_λ; the amplitude of the wake spatial harmonic. In view of the discussion of Section 6.2, the difficulties of specifying α_λ are apparent. In order to obtain some estimate it will be assumed that only a fundamental mode sinusoidal variation occurs in the wake, and that this variation has a momentum deficit equal to that generated at the blade. This broadly corresponds to a wide separation of stator and rotor, but in a real compressor under these conditions much of the blade momentum loss could be in the form of the mean pressure loss rather than wake effects. Thus this model should give a comparatively high numerical value for the harmonic, which probably more than compensates for the single mode assumed.

Thus suppose

$$w = U_1 \left(1 + \frac{\alpha_1}{U_1} \cos \frac{2\pi \xi}{b}\right)$$

when ξ is a circumferential coordinate and b is the blade spacing. The drag of the blade may be estimated by a wake integration, using

$$C_D = \frac{2}{c} \int_0^b \left(1 - \frac{w}{U_1}\right) \frac{w}{U_1} \, d\xi$$

which gives

$$\left(\frac{|\alpha_1|}{U_1}\right)^2 = \frac{C_D c}{b}$$

thus relating the wake amplitude to the drag coefficient of the blade and a solidity term. Using this result in Equation 87 gives

$$W_{11} \approx \frac{C_D \rho_0 c^2 s^2 B^3 U_1^5}{128 a_0^2 \pi R^2} \tag{88}$$

where $\sin \beta$ has been put equal to unity.

Equation 88 can be rewritten as

$$W_{11} \approx \frac{\rho_0 C_D B A_B^2 U_1^5}{128 A_e a_0^2} \tag{89}$$

where A_B is the total area of the blades and A_e is the effective area based on the effective radius R ($A_e = \pi R^2$).

It is of interest to attempt to relate this expression to data which is available. Figure 20 gives a recently proposed method (Ref. 32) for calculating the maximum sound pressure level in the fundamental frequency at a 200 ft sideline. Several sets of data are also shown. The graph uses compressor overall diameter D (in inches) and mechanical tip speed (V_T) as non-dimensionalizing parameters. Equation 89 may be put in this form. U_1 is the relative (effective) velocity at some effective radius. Thus, if R_T is the tip radius, $U_1 = R\, V_T/R_T \sqrt{2}$ assuming a 45° angle between U_1 and V_T as suggested by Figure 5. This angle is probably of the right order.

Thus

$$W_{11} \approx \frac{\rho_0 C_D B}{128 a_0^2} \left(\frac{A_B}{A}\right)^2 \left(\frac{R}{R_T}\right)^3 \frac{\pi D^2}{4} \left(\frac{V_T}{\sqrt{2}}\right)^5 \tag{90}$$

where A is the total compressor area. Typical values of the parameters in the equation are

ρ = 0.00238 slugs/ft^3

a_0 = 1117 ft/sec

B = 30

A_B/A = 0.5

R/R_T = 0.8

C_D = 0.02

Using these gives

$$W_{11} \approx 1.08 \times 10^{-15} \ D^2 \ V_T^5 \quad \text{lb ft/sec}$$

where D is measured in inches. To obtain the values at a 200 ft sideline, spherical radiation will be assumed, so that multiplying W_n by $\rho \, a_0 \, A_s^{-1}$, where A_s is the area of the sphere, gives the pressure squared as

$$p^2 = 0.57 \times 10^{-20} \ D^2 \ V_T^5 \quad \text{lb}^2/\text{ft}^4$$

Referring to 0 dB (4.177×10^{-7} lb/ft^2 = 0.0002 dynes/cm^2) and taking logs gives the predicted sound pressure level at a 200 ft sideline as

$$p = 50 \log_{10} V_T + 20 \log_{10} D - 75 \ \text{dB} \tag{91}$$

This curve is also plotted in Figure 20.

In view of the very large number of assumptions made it is very encouraging that the line even falls on the data points, and perhaps the agreement between theory and experiment occurring in Figure 20 should be regarded as somewhat fortuitous. Nevertheless several features of this analysis do seem to be of general validity.

The variation of sound power with compressor dimension squared certainly seems correct. The variation of power with the fifth power of the relative velocity seems to be an important result, and the theoretical assumptions underlying this seem well founded since they are based on the asymptotic values of the various expressions. The result differs from the usual sixth power law for point dipoles, basically because the asymptotic value of the frequency term is proportional to V^1 in the extended source case as opposed to V^2 for the point dipole. Several investigators have found this fifth power variation in experiments on compressor noise. However, it is of particular interest that the basic theory suggests the relative velocity rather than the tip velocity as the dominant parameter. Since the tip velocity controls frequency, it is somewhat surprising that its direct effects disappear in the final result. However, several investigators have used relative velocity with some success as an empirical parameter, and the theory would certainly seem to support this.

Beranek (Ref. 33) gives the following formula for the overall sound power of small ventilating fans (re: 10^{-13} watt)

$$\text{PWL} = 135 + 20 \log_{10} \text{HP} - 10 \log_{10} q$$

where HP is the rated motor Horse Power and q the fan discharge in ft^3/min. Since $HP \sim D^2 V^3$ and $q \sim D^2 V$ this formula also implies a $D^2 V^5$ law. In fact, making equivalent assumptions to those used above gives the following formula for the sound pressure level at a 200 ft sideline.

$$SPL = 50 \log_{10} V_T + 20 \log_{10} D - 78 \tag{92}$$

This is in remarkable agreement with the experimental results, as shown in Figure 20. Note, however, that the Beranek formula applies to overall power while previous results refer to first harmonic levels. This apparent agreement between an empirical formula for low speed ventilating fans and measurements for compressors also suggests that the effect of any siren type noise terms is low. In spite of the obvious deficiencies in the models used, the agreement within 3 dB of the rather crude theoretical model, Equation 91, and an extension of a well established empirical formula is also very satisfying.

Returning to the basic equation, (90), it can be seen that overall power depends on a solidity factor (A_B/A) squared. This suggests a possible method for reducing noise by reducing solidity, particularly since the compressor mechanical power is only dependent on the first power of A_B. However, caution should be exercised in any application of these solidity results, since they are dependent on the exact form assumed for the wake. For instance, direct application of the results of Silverstein, et al. (29) leads to the introduction of what is effectively a solidity cubed term.

8.2 Mass Source Terms

The second feature of interest in the compressor is the mass source terms. Following the same model as in the force case, but substituting in Equation 48, and using the asymptotic value of the summation (1/2 nM) gives

$$W_{11} \approx \frac{B^2 V^2 \Omega^2 h^2 s^2 \rho_0 \alpha_\lambda^2}{8\pi a_0} \left(\frac{1}{2nM}\right)$$

$$\approx \frac{B V^2 h^2 s^2 \rho_0 U_1^2 V_R}{16\pi R^2} \left(\frac{\alpha_\lambda}{U_1}\right)^2 \tag{93}$$

where V_R is the rotational velocity of the rotor at station R. Division by Equation 87 gives the ratio of mass to force sound as

$$\frac{W_M}{W_F} = 8\, a_0^2 \left(\frac{V}{B}\right)^2 \left(\frac{V_R}{U_1^3}\right) \frac{h^2}{bc} \tag{94}$$

The same assumptions as in the previous section can be used for the relation of U to V_T. In addition, assume $V = B$, $b = c$, $h/c = 0.01$. (The assumption for displacement thickness $h/c = 0.01$ is consistent with $C_D = 0.02$, see Ref. 30, pp. 201-204.) These assumptions give

$$10 \log_{10} \frac{W_M}{W_F} = 36.5 - 20 \log_{10} V_T$$

so that the equation for the acoustic power output due to the mass sources at the 200 ft sideline point becomes

$$p = 30 \log_{10} V_T + 20 \log_{10} D - 38.5 \tag{95}$$

This curve is also shown in Figure 20. It can be seen that these "siren" sources are of minor significance at the tip speeds typical of practical compressors, but could possibly become important at low tip speeds. It should also be noted that off design conditions - low tip speed for example - can lead to significant increases in wake displacement thickness which would immediately be reflected in increased siren noise.

However, as in the case of the fluctuating forces, care must be exercised in drawing conclusions from these results. As was pointed out earlier the model used is conservative, and will overestimate the noise when the wavelength is large compared to the blade spacing. It will be observed that the exact value of the wake harmonic cancelled out of the power ratio expression so the comparative magnitudes may be of approximately the correct order, providing the value chosen for h/c is realistic. The siren sources obey a velocity cubed law and, again, this appears to be based on reasonable assumptions. But in the present case the overall velocity dependence is on relative velocity (U_1) squared times rotational velocity, so that the result of dependence only on relative velocity in the force case is not repeated here.

Again the difficulties of adequate specification of the source terms prohibit any definitive conclusions about the effects of solidity, or the accuracy of the absolute values of source level predicted by Equation 95. It does appear that increase in solidity will have a pronounced effect on the noise, but that even this increase would be unlikely to be of practical significance compared to the force terms at typical compressor conditions.

8.3 Discussion

Before concluding this section on the preliminary comparison of theory and experiment it is as well to emphasize again the detail effects not accounted for in the estimates of Sections 8.1 and 8.2. Figure 20 shows how any simple collapse of the data can give errors of ± 10 dB. The reason for this, as discussed in the Introduction, is the very large number of detail parameters that can affect the sound output. Many of these effects can be included in the theory, and some have already been discussed in Section 7.

Parameters which can be included in the theory include:

- Rotational speed
- Blade/Vane numbers
- Separation between rotor and stator
- Broad features of blade and vane geometry
- Stage aerodynamic and performance parameters
- Multistage radiation
- Compressor hub velocity (see Appendix)

Effects which are not included in the theory include:

- Duct treatment
- Passage of sound through blade rows
- Effects of flow velocity on sound propagation
- Noise radiation due to thickness or acoustic stress

The theory also suggests that the following features will not have substantial effects on the sound.

- Hard wall duct phenomena
- Detailed compressor geometry.

The key problem is in predicting the aerodynamic characteristics of the shed wakes. Once these are established the theory enables, at least in principle, very comprehensive predictions of the acoustic radiation to be made. Even off-design conditions including such effects as rotating stall will be described by the general theory, although in such cases the summations of Equation 26 will no longer apply and sound can be radiated at any harmonic of the rotational speed, rather than only at harmonics of the blade passage frequency. Thus at the present time the key requirement is for comprehensive information on the unsteady aerodynamics of the compressor.

The primary effect on the noise radiation not included in the theory is the acoustic effect of finite flow velocity. Clearly choked flows would not be predicted correctly using the present results. However, lesser, but still significant effects, are possible. For instance the theory shows how rotor-stator or stator-rotor interactions are equivalent. On a real engine the rotor radiation field will propagate through the comparatively slowly moving velocity field of the stator. But the stator radiation field propagates out of the inlet against the high velocity field due to the rotor. Thus, in practical engines, where high subsonic Mach numbers behind the rotor are possible, the stator will radiate less efficiently than the rotor out of the inlet. Such effects are not included in the present theory.

In view of the detail effects listed above, and of the current aerodynamic uncertainties, Figure 3 must be regarded as giving encouraging agreement between theory and experiment. The $D^2\ U^5$ law for the fluctuating forces and $D^2\ U^3$ law for the mass sources are thought to be well founded. The equations presented in Sections 8.1 and 8.2 were not basically intended for prediction purposes, but their agreement with data does allow some confidence to be placed in the analysis. Figure 13 also supported some of the analytical predictions. Thus trends established by the theory are thought to be significant, and of practical value to improving compressor noise predictions and control techniques.

9.0 CONCLUSIONS

The sound radiated by an axial flow compressor has been analyzed theoretically. Four possible mechanisms have been considered, all of which are associated with rotor-stator type interactions in the compressor. Each has been reduced to the effect of a cascade of point sources, one attached to each blade, with appropriate phase factors between them. Acoustic effects of the compressor duct are ignored. Explicit analytic expressions have been obtained both for sound pressure as a function of position and for overall sound power, and these have been used with a crude source model to give general predictions for compressor noise radiation which agrees surprisingly well with experiment. The principal practical conclusions of the study are:

(1) The stator and rotor are basically of equal significance in noise radiation.

(2) Increasing rotor-stator separation will preferentially reduce the higher harmonic radiation by the stator, while increase of stator-rotor separation will reduce all harmonics of noise radiation by the rotor.

(3) The predominant radiation mechanism is via the fluctuating forces on the blades. Mass source (siren) terms are insignificant in practical compressors.

(4) The gross features of the noise radiation characteristics (overall power and directionality) are basically governed by the source parameters, rotational speed and blade/vane numbers, and are not substantially dependent on hard wall duct effects.

(5) In particular, compressor noise at frequencies below "cutoff" for a duct, would radiate very inefficiently even in the absence of a duct.

(6) To achieve inefficient acoustic radiation, it is desirable to reduce rotational speed below a critical value and generally to maximize the difference between the number of rotor and stator blades. Several further detail effects are discussed in the text.

(7) If rotational speed cannot be reduced below the critical value, then it is desirable to minimize the difference between the number of rotor and stator blades.

(8) In particular, equal numbers of rotor and stator blades give a minimum acoustic power radiation coupled with favorable directivity patterns.

(9) Sound radiation at conditions close to the critical (cutoff) case is particularly unfavorable having increased acoustic power and undesirable directivity characteristics.

(10) Use of the theory with a crude model for the fluctuating force levels gives a $D^2 V^5$ law for the noise radiation, in good agreement with both the trends and absolute levels from experiment.

(11) Above the cutoff condition the relative (effective) velocity at the blade appears to be more significant in the noise radiation than the mechanical tip speed and should therefore be minimized in a quiet engine. The results also show that increase in solidity increases noise.

(12) On compressors operating at supersonic tip speeds the steady loads radiate efficiently, and dominate the blade passage frequency sound. Noise control measures suggested for subsonic rotors are unlikely to be of particular value at supersonic speeds.

The results given enable acoustic power and directionality patterns to be calculated from a knowledge of compressor geometry and aerodynamics. None of the approximations used in the theory are thought to be unreasonable, and it seems that the most likely source of error in the application of the theory is in the specification of the aerodynamic source terms, particularly the wake characteristics.

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APPENDIX A

EFFECTS OF AXIAL MOTION

Exact account of all possible effects of the axial motion of the compressor is complex. Finite flow velocities across the compressor disc will result in increased sound radiation to the rear, and beyond the limiting case of sonic velocities across the disc no sound is radiated forwards out of the inlet duct. However, once the sound emerges from either the inlet, or exit, plane of the compressor it will undergo refraction by the local velocity gradients, which will tend to cancel the effects of the flow at the disc. In the case where the compressor disc operates in a short duct, below sonic velocities, these local velocity variations may be expected to have a comparatively small effect, and it seems probable that the only significant acoustic effect is due to relative motion between the compressor hub and the free air. For this case, a simple modification to the equations seems adequate.

Equation 8 can be written as

$$p = \left[\frac{(x_i - y_i)}{a_o \left\{ r - (x_i - y_i) M_i \right\}} \frac{\partial}{\partial t} \left\{ \frac{F_i}{4\pi \left\{ r - (x_i - y_i) M_i \right\}} \right\} \right] \tag{A1}$$

If the compressor hub moves at velocity M_0, then the equation can be rewritten as

$$p = \left[\frac{(x_i - y_i)}{a_o \left\{ r (1 - M_{or}) - (x_i - y_i) M_i \right\}} \frac{\partial}{\partial t} \left\{ \frac{F_i}{4\pi \left\{ r(1 - M_{or}) - (x_i - y_i) M_i \right\}} \right\} \right] \tag{A2}$$

where the velocities M_i are still measured relative to the hub, and M_{0r} is the component of the hub convection velocity in the direction of the observer. Comparison of the two equations above suggests that the effect of hub velocity can be taken into account by simply replacing r in any results for a static case by $r(1 - M_{0r})$ to give the moving case.

As an example, consider the results obtained by Garrick and Watkins (Ref. 32) for the case of a propeller moving at Axial Mach Number M_1. They found

$$\left| c_n \right| = \frac{n\,\Omega}{2\pi a_o S} \left\{ T_o \left\{ M_1 + \frac{x'}{S} \right\} \frac{1}{\beta^2} - \frac{D_o}{M} \right\} J_n \left(\frac{nMy}{S} \right) \tag{A3}$$

where the expression is given in the present notation, and

$$\left.\begin{aligned} S &= r - M_1 x \\ x' &= x - M_1 r \\ \beta^2 &= 1 - M_1^2 \end{aligned}\right\} \qquad \text{(A4)}$$

The above expressions relating the Garrick and Watkins parameters to those used here are derived below. Substituting these expressions, the Garrick and Watkins result may be written as

$$\left| c_n \right| = \frac{m B \Omega}{2 \pi a_o (r - M_1 x)} \left\{ \frac{x T_o}{r - M_1 x} - \frac{D_o}{M} \right\} J_n \left(\frac{n M y}{r - M_1 x} \right) \qquad \text{(A5)}$$

This is exactly the result which would be derived from the Gutin expression (Equation 71) for a stationary propeller, on replacing r in (71) by $r\,(1 - M_1\, x/r)$ throughout, as suggested above.

Thus it appears that all the results obtained in the present report for a static compressor can be generalized to the moving case by application of the rule stated above. Note however that this is only an approximate result for gross compressor motion and changes due to local velocity variations in and around the compressor are not accounted for. In addition, Doppler frequency shift changes by the same factor will, of course, occur at the stationary observer.

The results used above (Equations A4) may be derived straightforwardly. Garrick (Ref. 34) and others quoted by him, derived an expression for the source term of a linearly moving dipole as

$$p = \frac{-1}{4\pi} \frac{\partial}{\partial x_i} \left[\frac{F_i}{S} \right] \qquad \text{(A6)}$$

where

$$S^2 = x'^2 + \beta^2 (y'^2 + z'^2) \qquad \text{(A7)}$$

$$\beta^2 = 1 - M^2 \qquad \text{(A8)}$$

The Lighthill (Ref. 15) result for the linearly moving dipole, as used in the present report, is

$$p = \frac{1}{4\pi} \frac{\partial}{\partial x_i} \left[\frac{F_i}{r(1 - M_r)} \right] \qquad \text{(A9)}$$

The difference between the two expressions arises from the differing coordinate systems used in each. In both expressions the differentiation is performed following the source, but in Garrick's result the coordinate system (x', y', z') is fixed in the moving source, whereas in Lighthill's result coordinates are fixed in space. Although it is clear that any two correct derivations must be equivalent, it is of interest to make the comparison in detail. This also enables the results of Garrick and Watkins (Ref. 32), for the moving propeller, to be interpreted in terms of the present coordinate system as above.

The basic geometry is shown in Figure 21. Sound is emitted at point A and heard by the observer at point B, the distance from A to B being r. When the sound is heard the source has moved a distance M_1 r along the x-axis to point C. Thus the fixed and moving coordinates at B are related by

$$x', y', z' = x - M_1 r, y, z \tag{A10}$$

If the source position C is at (x_1, y_1, z_1) in the space coordinates then Equation A7 may be rewritten as

$$S^2 = (x - x_1)^2 + \beta^2 \left\{(y - y_1)^2 + (z - z_1)^2\right\} \tag{A11}$$

Consider now the triangle EBC in Figure 12.

$$EB^2 = BC^2 - EC^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 - (M r \sin \theta)^2$$

Triangle ABD shows $(r \sin \theta)^2 = (y - y_1)^2 + (z - z_1)^2$, so that

$$EB^2 = (x - x_1)^2 + \beta^2 \left\{(y - y_1)^2 + (z - z_1)^2\right\}$$

$$= S^2$$

from Equation A11, thus identifying EB with S, as shown in Figure 21. Thus, also

$$r = S + M_1 r \cos \theta$$

that is,

$$S = r - M_1 x = r (1 - M_r) \tag{A12}$$

thus proving the equivalence of Equations A6 and A9 in the present case, and supplying the results quoted in Equations A4.

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I. Experimental and Empirical Studies

II. Theoretical Analysis

III. Compressor Aerodynamics

IV. Miscellaneous

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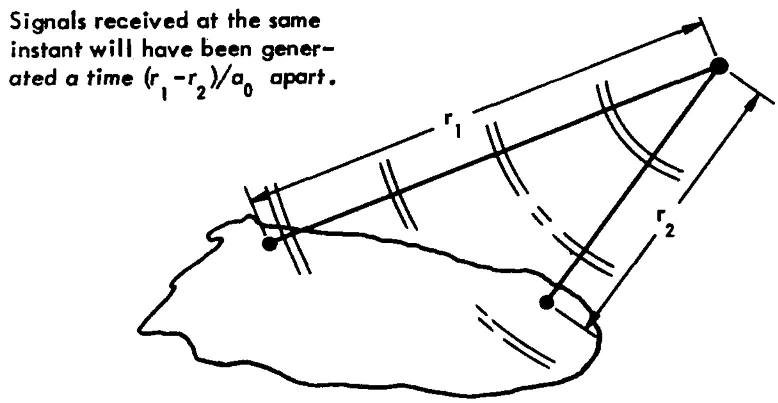


Figure 1. Effects of Retarded Time

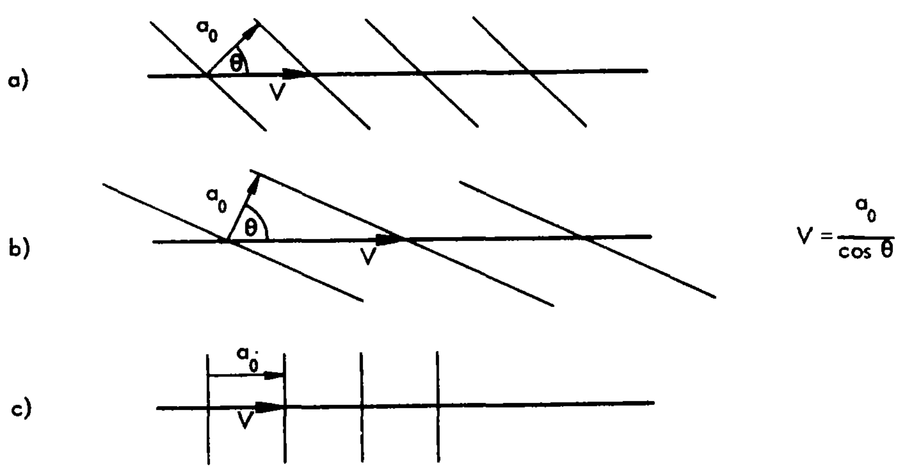


Figure 2. Effects of Source Velocity

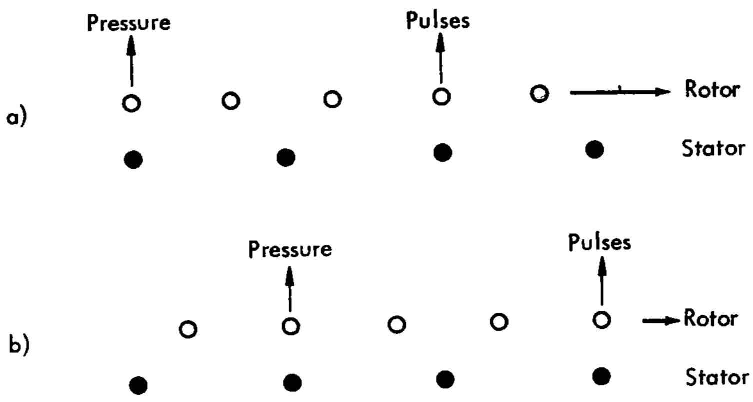


Figure 3. Rotor-Stator Phase Effects

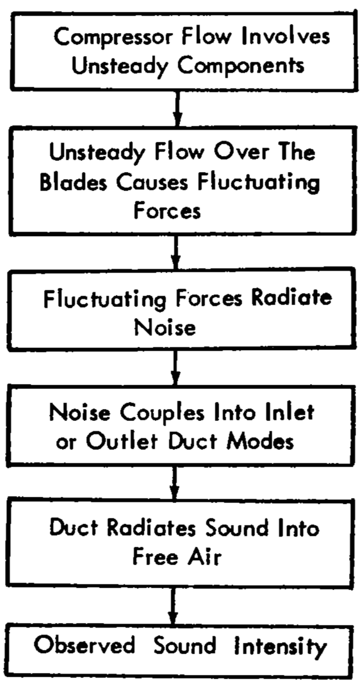


Figure 4. The Cause and Effect Chain of Compressor Noise Radiation

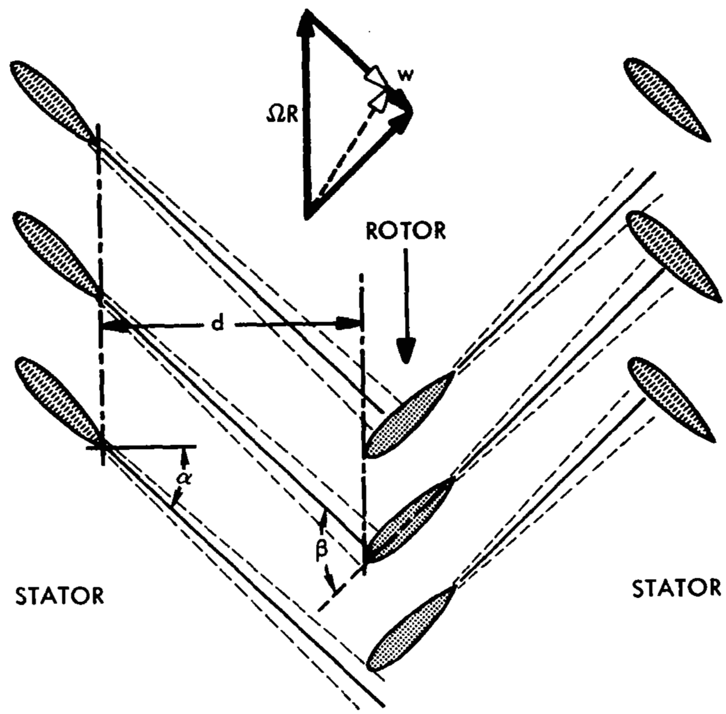


Figure 5. Basic Wake Geometry

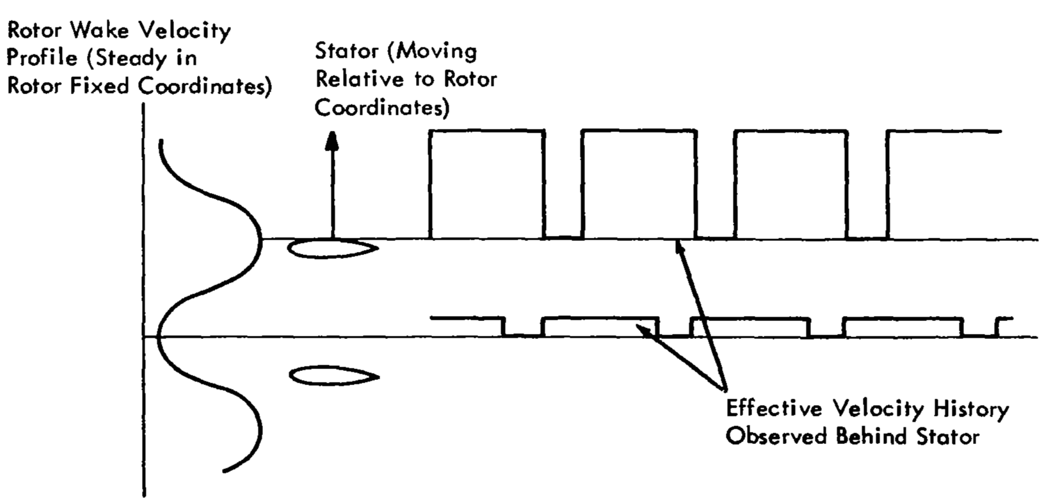
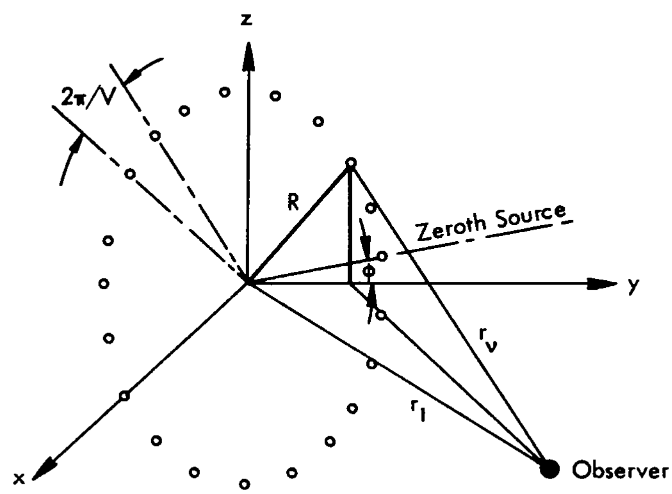
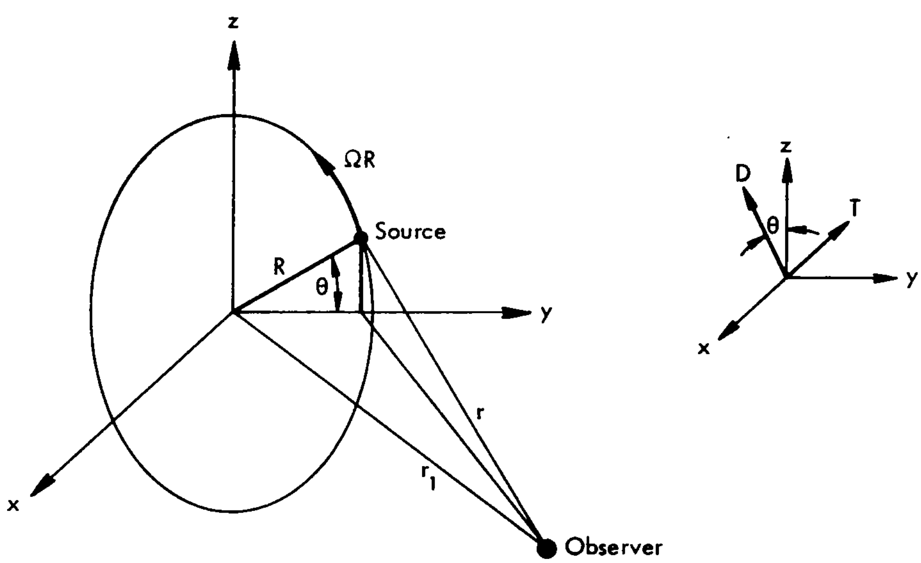


Figure 6. Model Used for Wake Mass Fluctuations



a) Coordinates for Stator Radiation



b) Coordinates for Rotor Radiation

Figure 7. Coordinate Systems for Compressor Noise Analysis

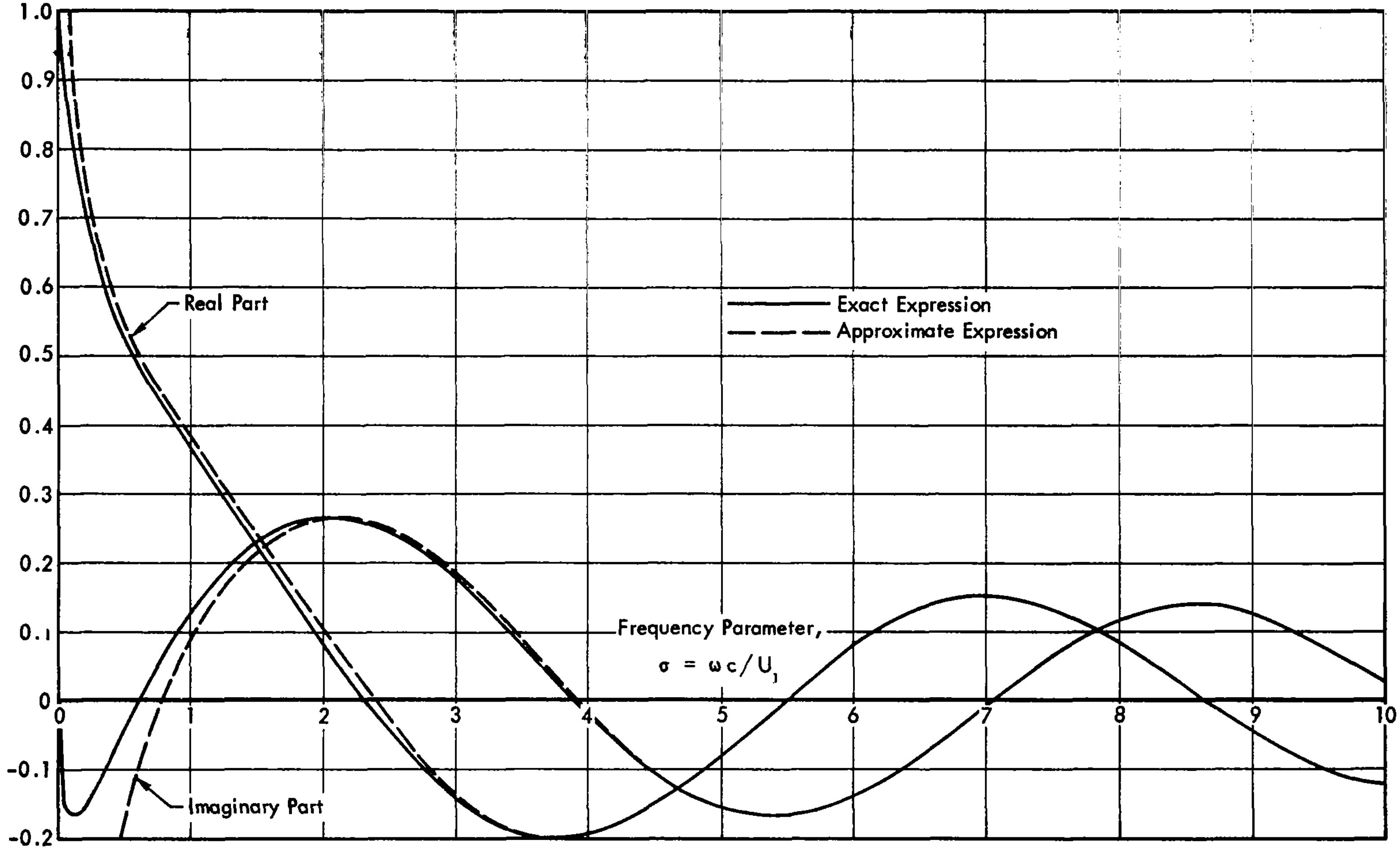


Figure 8. Exact and Approximate Lift Function for a Sinusoidal Gust

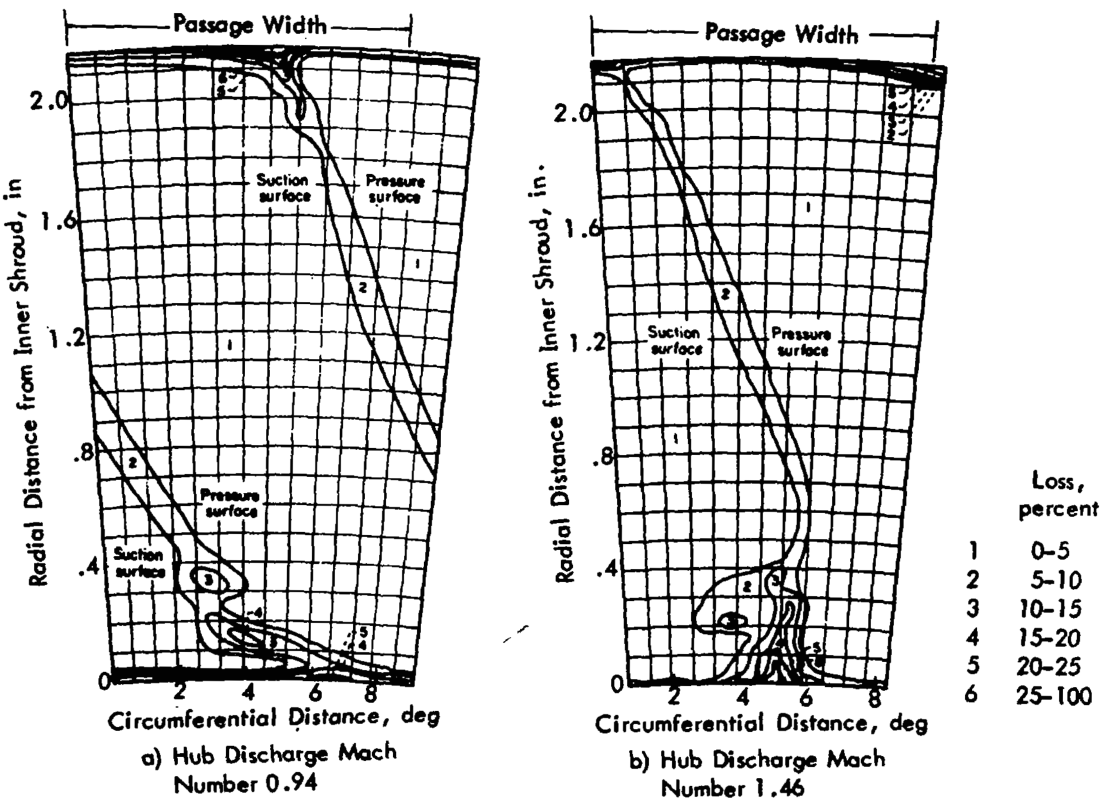


Figure 9. Contours of Kinetic Energy Loss at Exit of Stator Row (from Ref. 30)

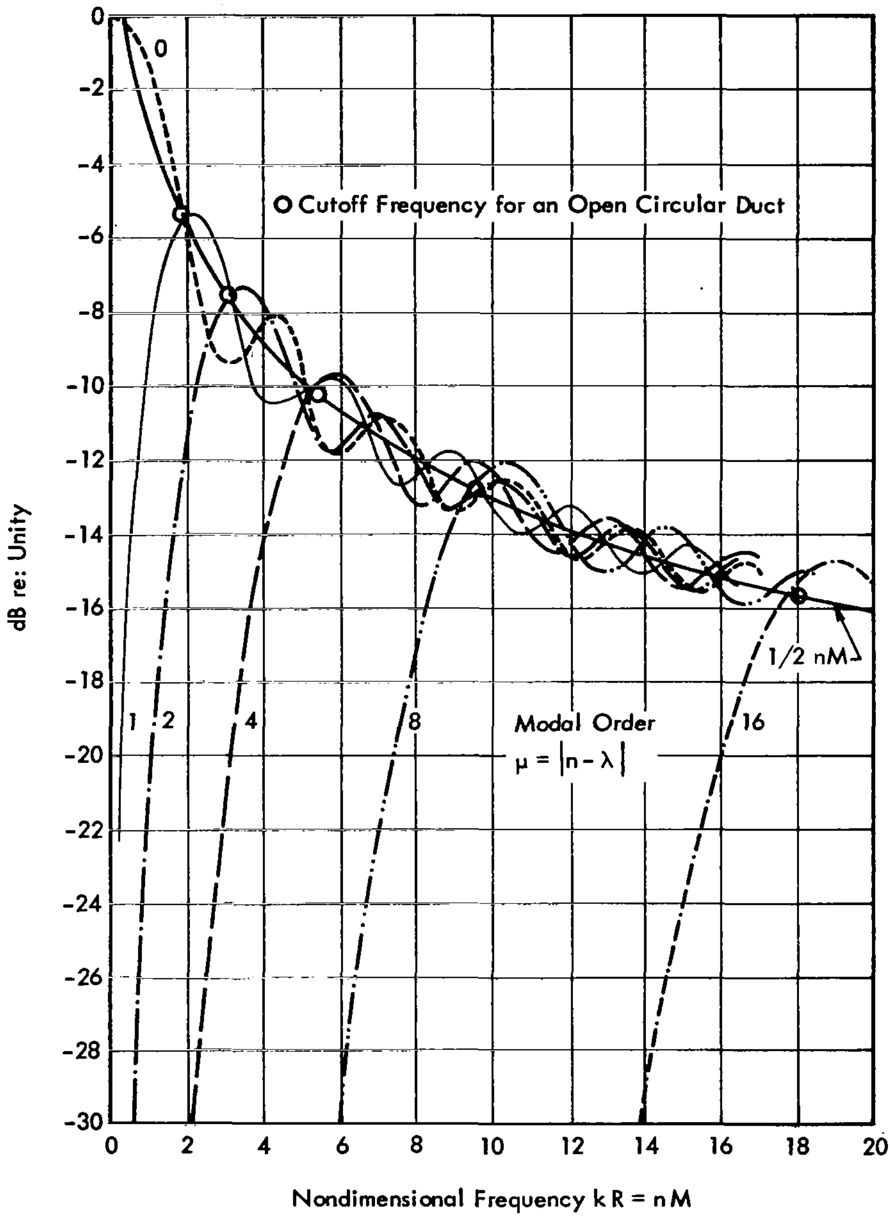


Figure 10. Contribution to Power of Mass and Drag Terms;

$$\text{Value of } \hat{D} = \sum_{r=0}^{\infty} \frac{(-1)^r \, nM^{2\mu+2r}}{r!\,(2\mu+r)!\,(2\mu+2r+1)}$$

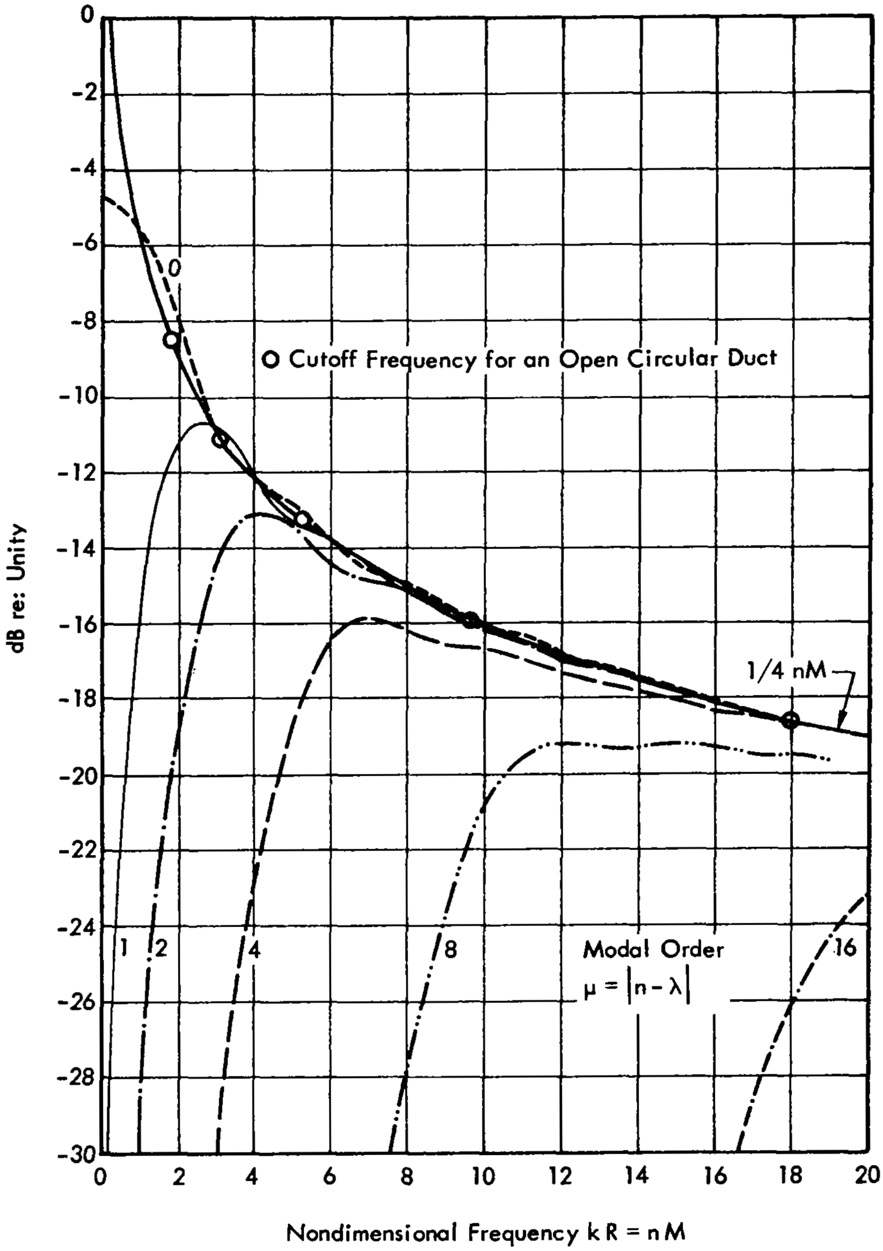


Figure 11. Contribution to Power of Thrust Terms;

$$\text{Value of } \hat{T} = \sum_{r=0}^{\infty} \frac{(-1)^r \, nM^{2\mu+2r}}{r!\,(2\mu+r)!\,(2\mu+2r+1)(2\mu+2r+3)}$$

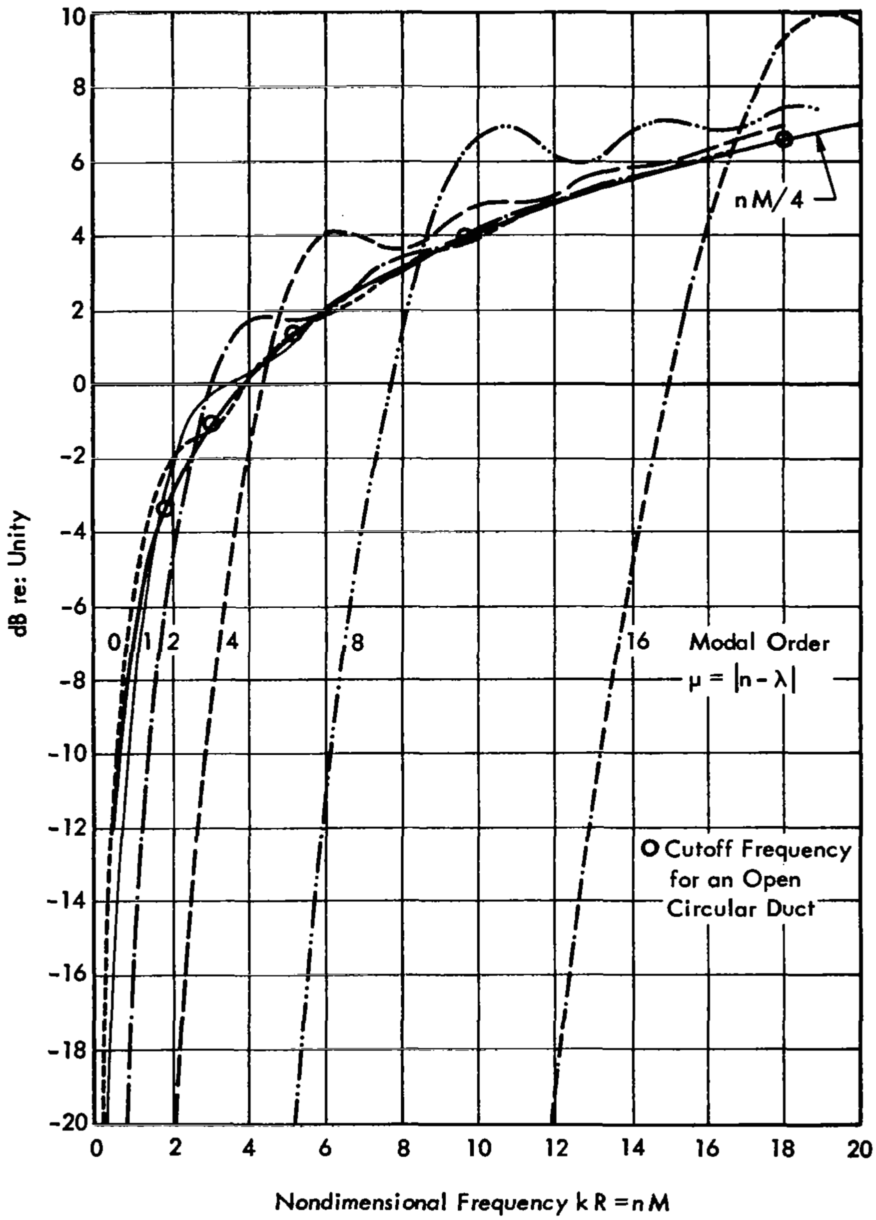


Figure 12. Contribution of Typical Compressor Force Term to Power; Value of $\mu^2\,\hat{D} + (nM)^2\,\hat{T}$

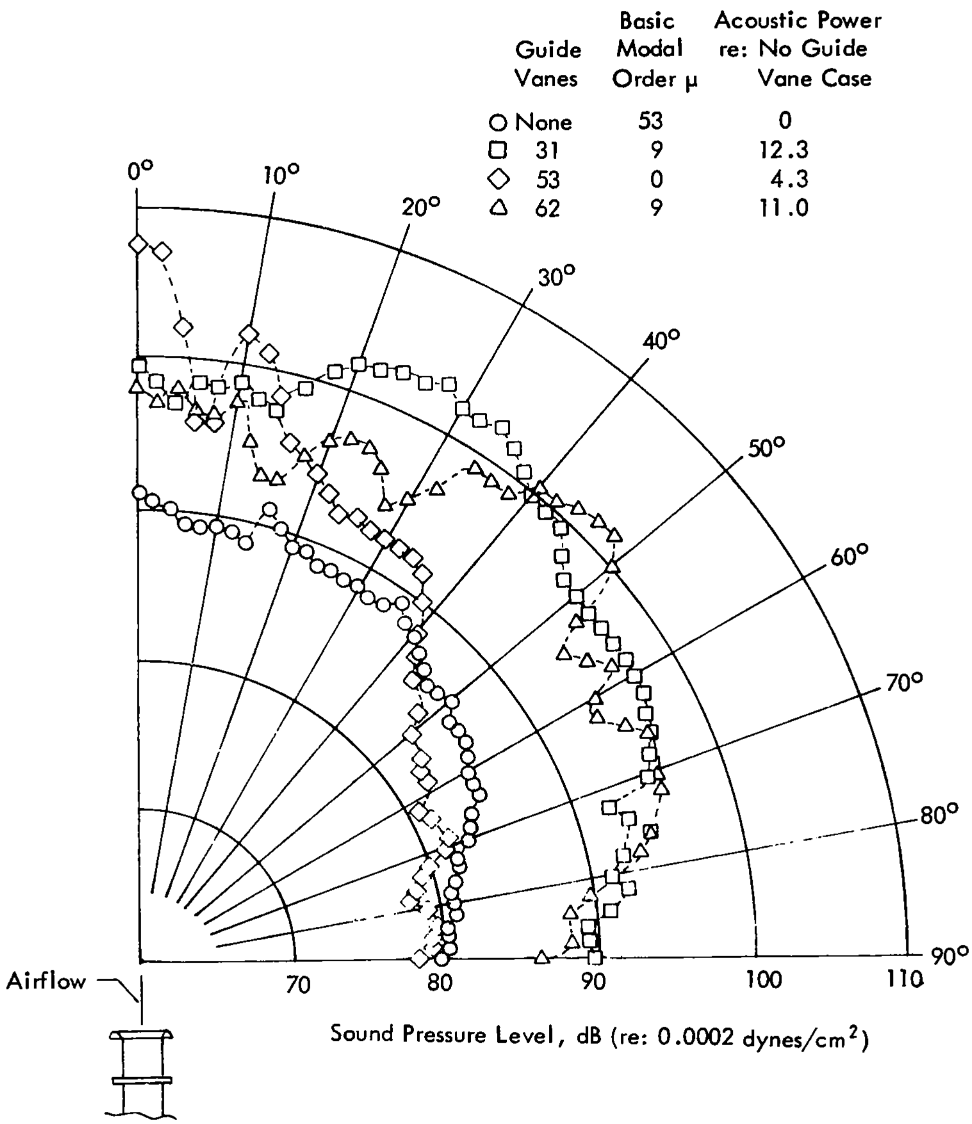


Figure 13. Measured One-Third Octave (at Blade Passage Frequency) Radiation Patterns for Various Rotor - Guide-Vane Configurations. $M_t = 0.346$. From Crigler and Copeland (Ref. 7).

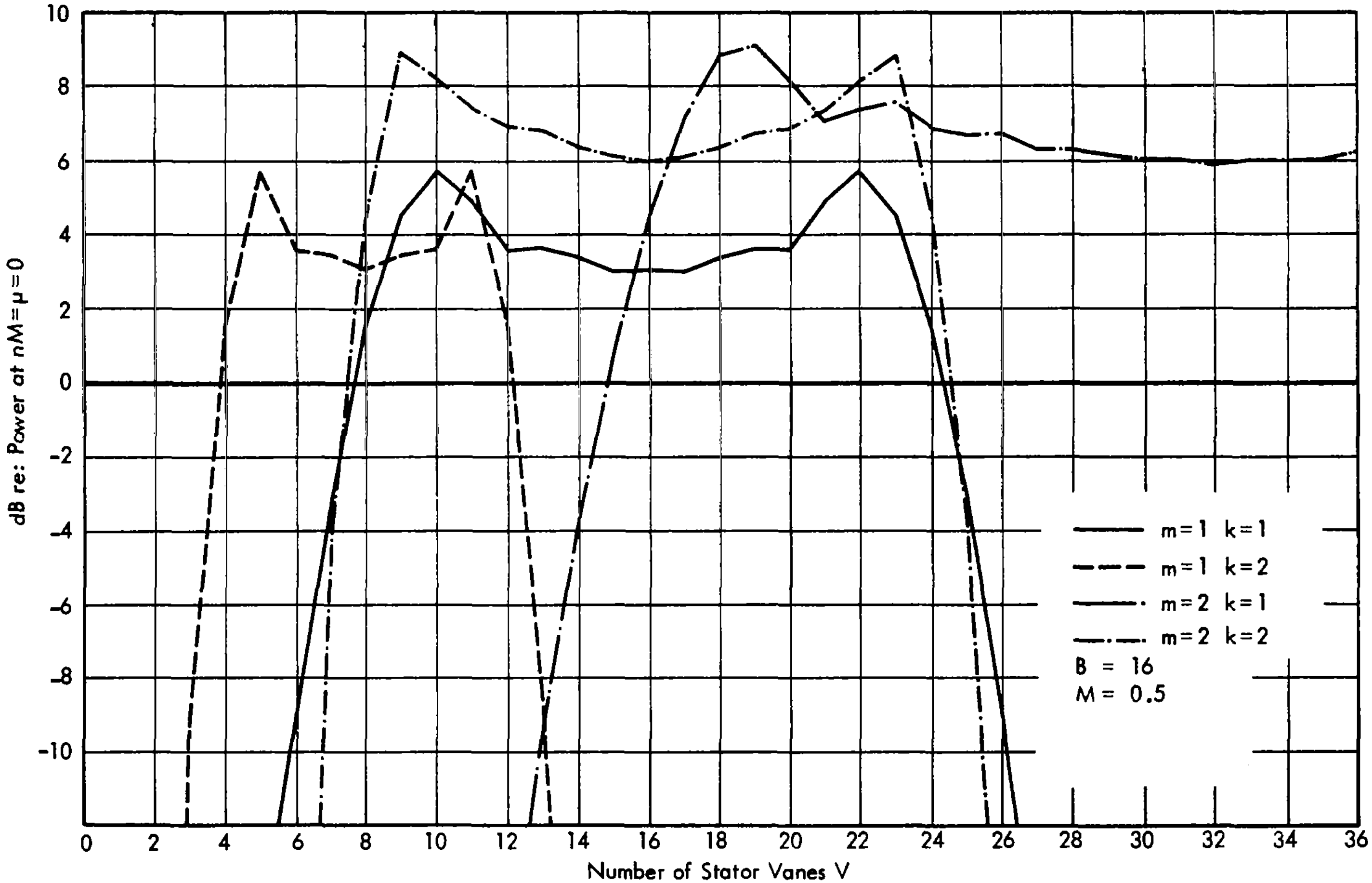


Figure 14. Acoustic Power Radiation by a Compressor
Variation with Stator Vane Number

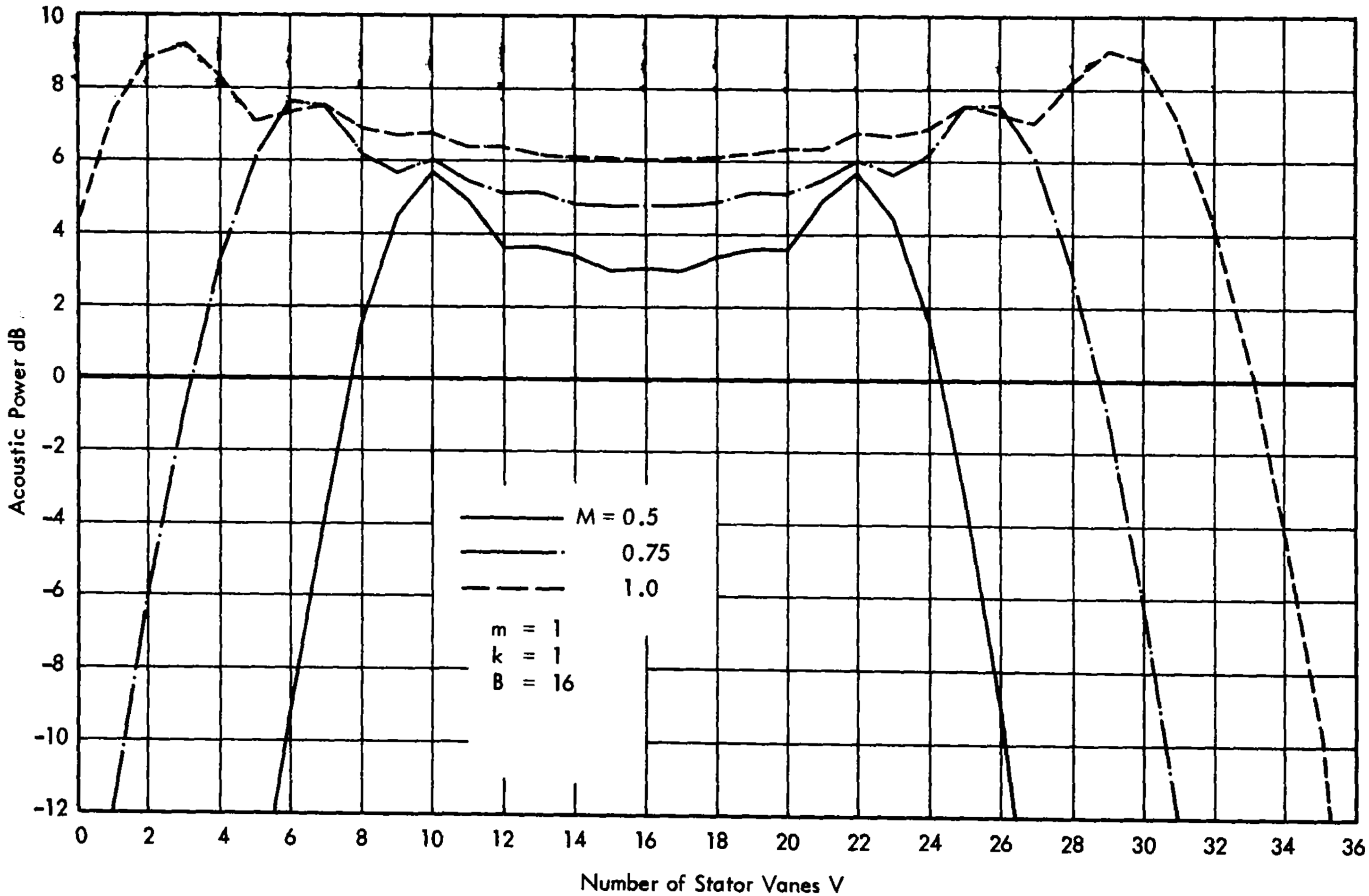


Figure 15. Effect of Rotational Mach Number on Acoustic Power

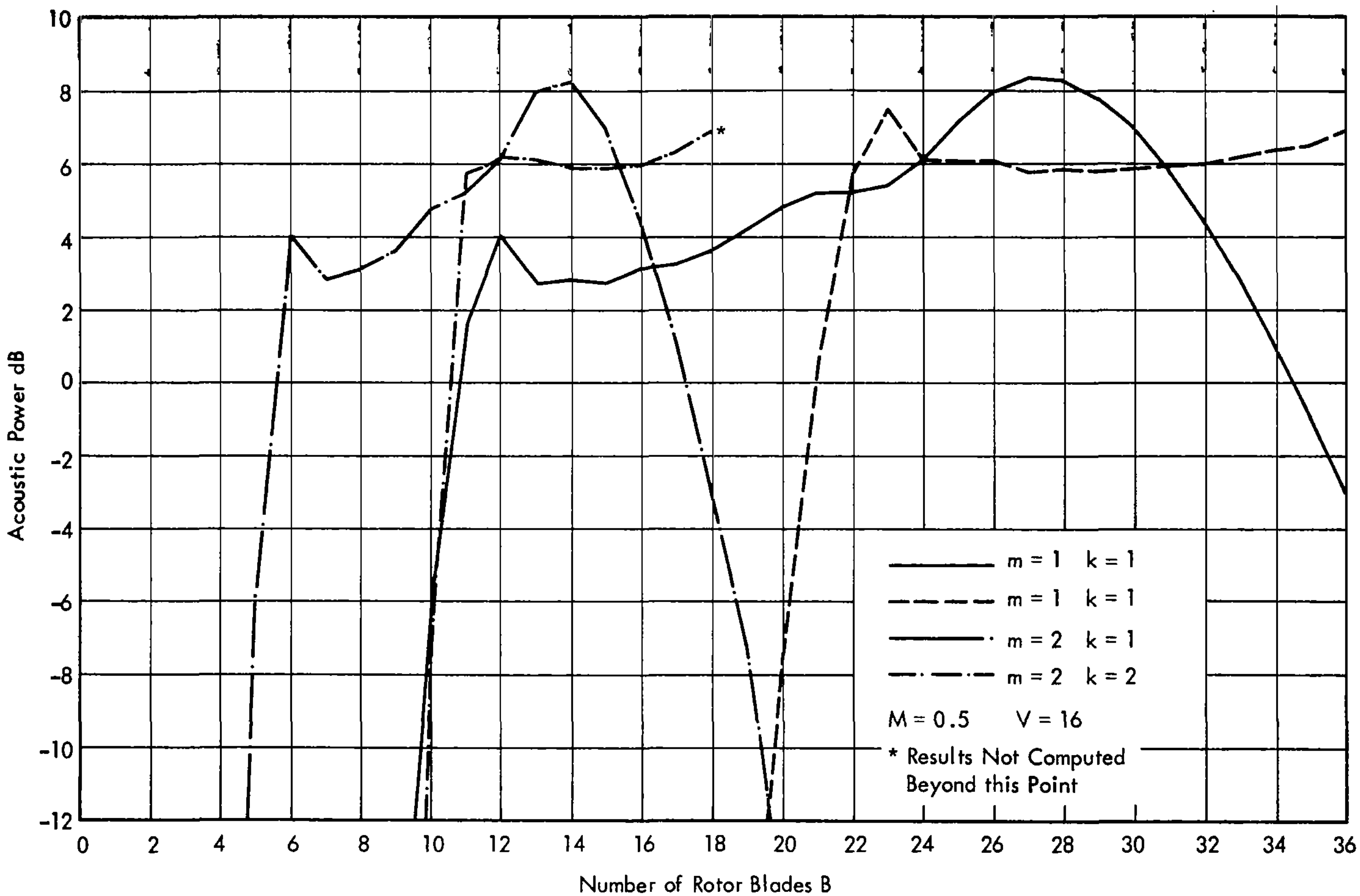


Figure 16. Effect of Number of Rotor Blades on Acoustic Power

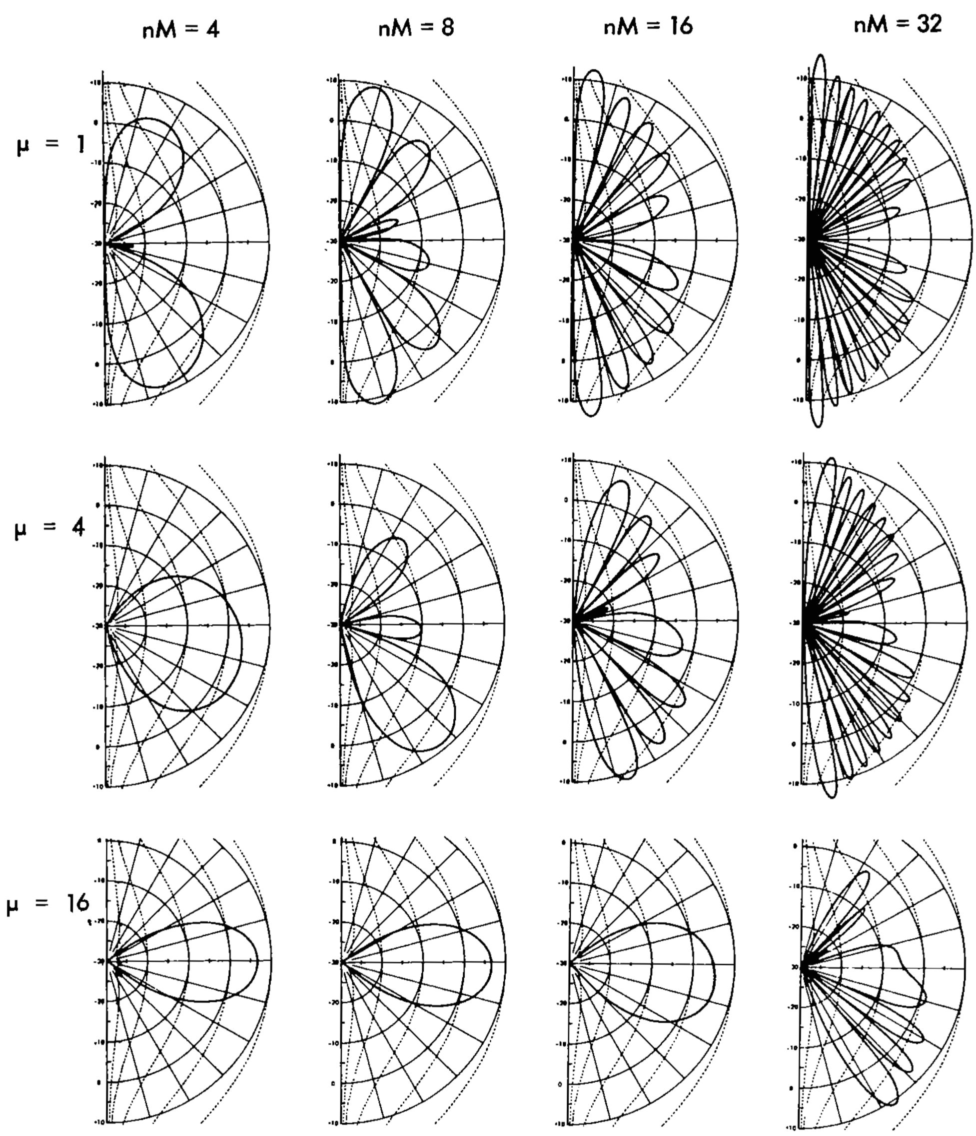


Figure 17. Directivity Patterns for Fluctuating Force Terms in Stator-Rotor Interactions. Up is forward for $\lambda < n$

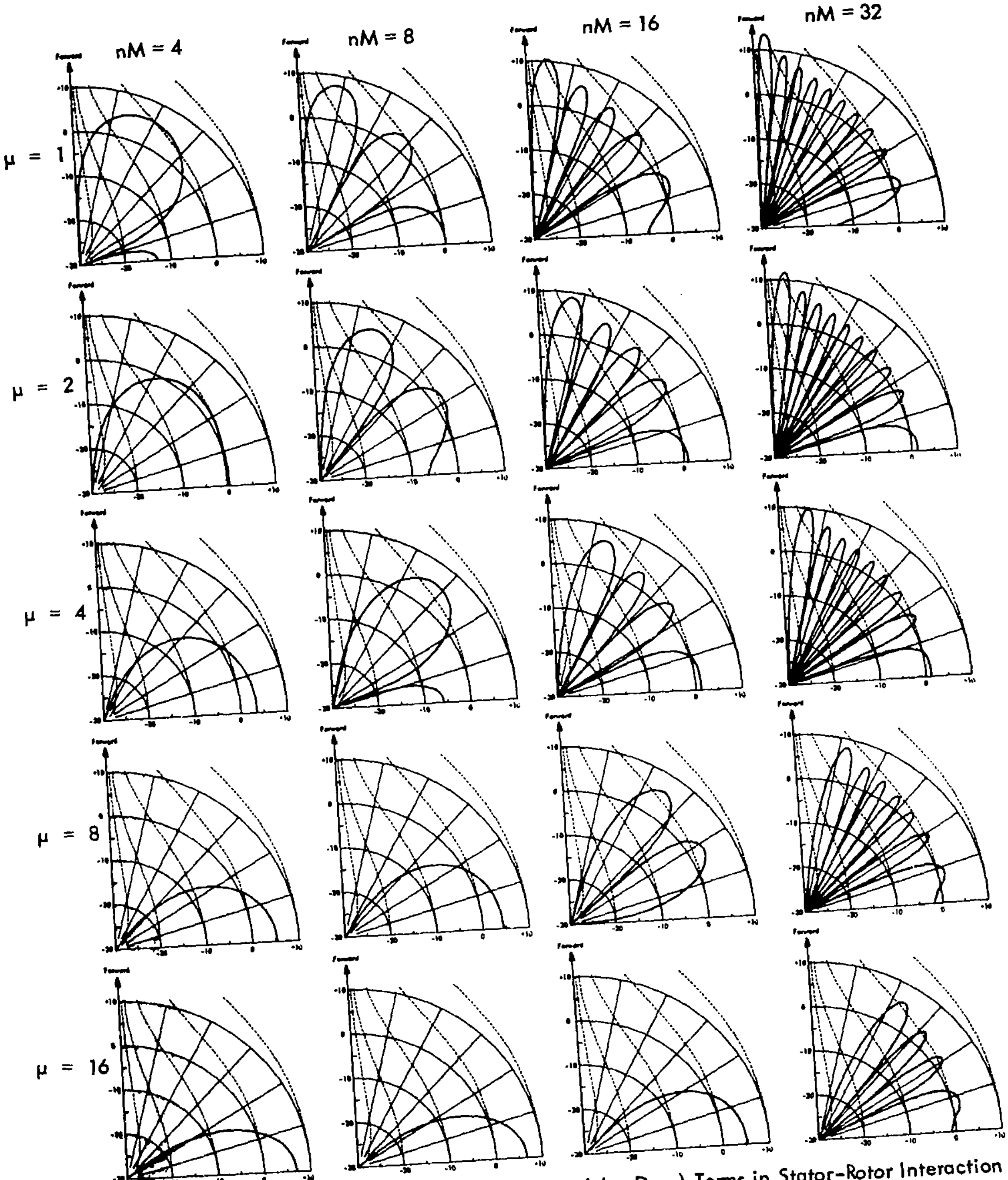


Figure 18. Directivity Patterns for Mass Source (also Drag) Terms in Stator-Rotor Interaction

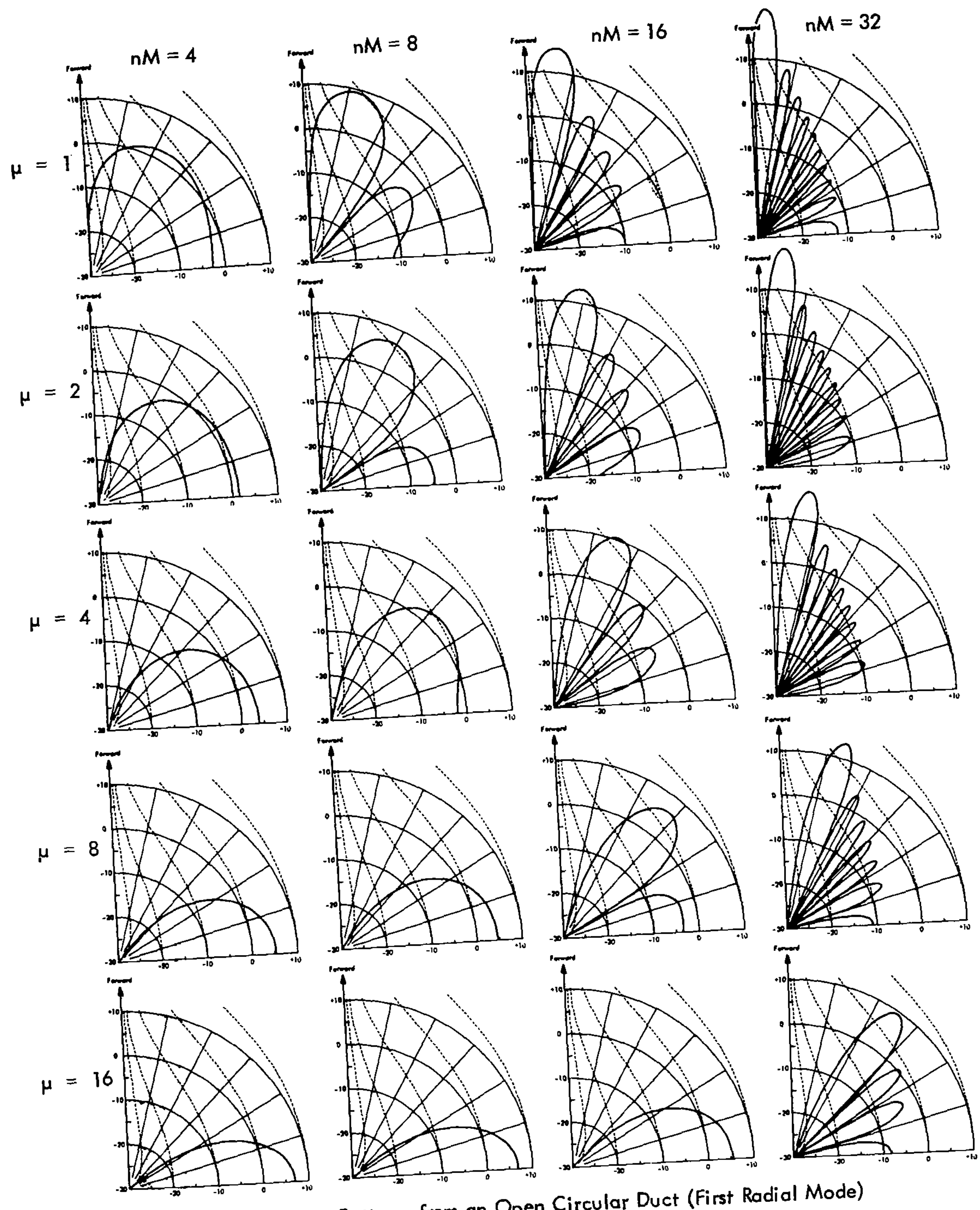


Figure 19. Directivity Patterns from an Open Circular Duct (First Radial Mode)

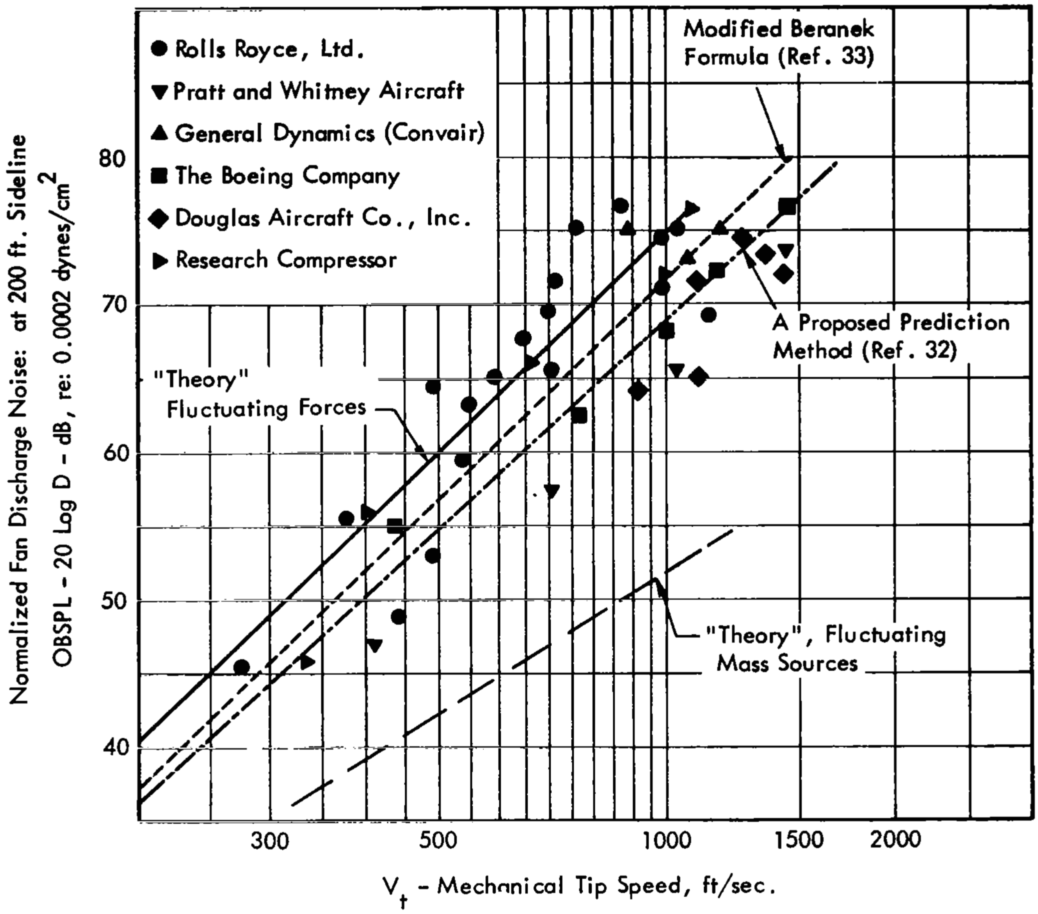


Figure 20. Comparison of "Theory" and Experiment. Maximum Fan Discharge Noise in Octave Band Containing Fundamental Blade Passage Frequency (After Wintermeyer and McKaig, Ref. 32)

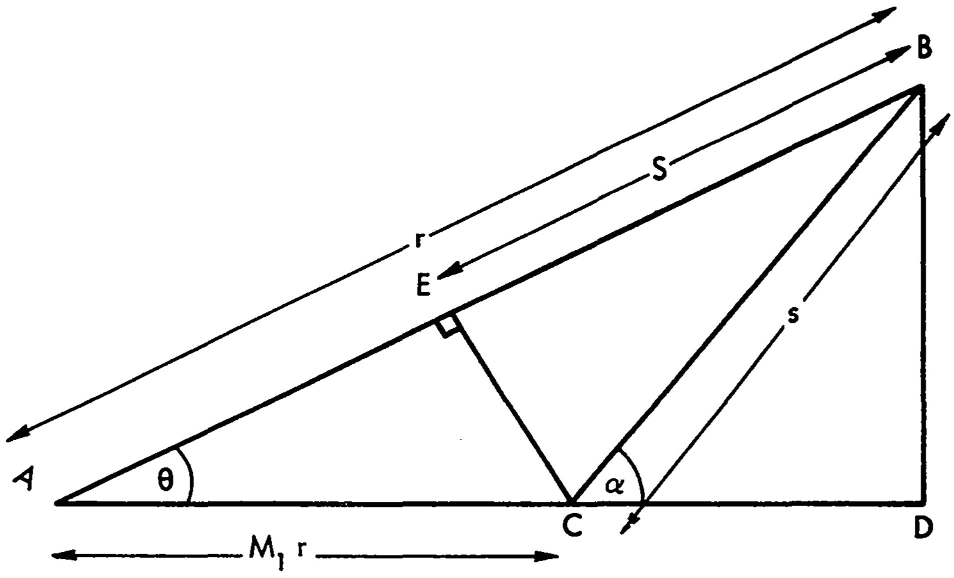


Figure 21. Basic Geometry for Moving Source Transformation

NASA-Langley, 1969 —— 28 CR-1287