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SUMMARY

This note considers more particularly the scattering of electromagnetic waves on small-scale wave turbulences due to the presence of electromagnetic fields in the plasma and characterized by a wide assortment of wave numbers.

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A series of works were devoted to the propagation of electromagnetic waves in a turbulent medium (see, for example, [1, 2]. Scattering on large-scale turbulences, subject to the Kolmogorov law, were mainly considered in these works. In the present work we are investigating the scattering of high-frequency electromagnetic waves on small-scale wave turbulences caused by the presence in the plasma of electromagnetic fields and characterized by a wide assortment of various wave numbers. Considered in particular is the interaction of high-frequency electromagnetic waves with specific oscillations existing in the region of frequent collisions, which was not taken into account earlier. Note that the scattering of electromagnetic waves on small-scale turbulences in the chromosphere without accounting for the collisions was considered in [3].

1. The effects of nonlinear interactions of longitudinal waves, taking into account the pair collisions were discussed in [4], and general expressions were obtained for nonlinear polarizability of second ($S_1(k_-, k_1, -k_2)$; $S_2(k_1, k_2, k_-)$)

and third orders [$\Sigma(k_1, k_2, k_1, -k_2)$ and $\Sigma(k_1, k_2, -k_2, k_1)$] at the condition(*)

$$| \omega_- - k_- v_{T_\alpha} | \ll \nu_\alpha \ll | \omega_{1,2} - k_{1,2} v_{T_\alpha} | \qquad (\alpha = e, i) \qquad (1)$$

The nonlinear currents obtained in [4] allow us to compute and analyze the nonlinear interactions of high-frequency oscillations and low-frequency hydrodynamic waves ($\omega_- \ll \nu_e, \nu_i$). Such interactions offer interest for numerous astrophysical problems, for they permit us to judge on the mutual relationship of plasma and hydrodynamic turbulences.

The aim of the present work is the generalization of the results obtained for the interaction of transverse waves with longitudinal ones and also of transverse with transverse waves, and to examine, in particular, the problem of the scattering of transverse waves on low-frequency hydrodynamic turbulence.

We shall limit ourselves to the case when the virtual wave is longitudinal. On the one hand, such a process predominates over the interaction through a transverse virtual wave [5]. On the other hand, the very same nonlinear currents describe the effects of disintegrating interactions of transverse and acoustic hydrodynamic waves. It is easy to see that in this case there occurs convolution of S_1 and S_2 with longitudinal orts $k_{-i}/|\vec{k}_-|$. This indicates that one may utilize the solutions for $V_k^{(1)}$ found in [4], and that only the expression for currents by $V_k^{(1)}$ ought to be refined. As a result, we shall obtain the following expressions for the polarizbility of second order:

$$\frac{k_{-l}}{|k_-|} S_{ijl}(k_-, k_1, -k_2) = -i \frac{1{,}71 |k_-| n_0 e^3 \nu_e}{m_e^2 \omega_1 \omega_2 \Omega \Omega_e} \delta_{jl},$$

$$S_{ijl}(k_1, k_2, k_-) \frac{k_{-l}}{|k_-|} = \frac{i e^3 n_0 |k_-|}{m_e^2 \omega_1 \varkappa \omega_e \omega_-} \delta_{ij}. \qquad (2)$$

When computing Σ_{ijls} there arises $n_{k_-}^{(2)} = (k_- V_k^{(2)})/\omega_-$, i. e. again only the londitudinal component $V_{k_-}^{(2)}$. This allows us to utilize the expression for $V_{k_-}^{(2)}$ obtained in [4], refining only the relationship between $j_k^{(2)}$ AND $V_k^{(2)}$.

As a result, we find

$$\frac{1}{2} [\Sigma(k_1, k_2, k_1, -k_2) + \Sigma(k_1, k_2, -k_2, k_1)] = \frac{1{,}71 n_0 e^4 k_-^2 \nu_e}{\omega_- m_e^3 \omega_1^2 \omega_2 \Omega \Omega_e} \delta_{ij} \delta_{ls}$$

(*) Here and further we used the denotations of [1].

Contracting with one longitudinal and one transverse ort $e^t_{k_2}$, we obtain that the interaction formulas of longitudinal and transverse waves differ from interaction formulas of longitudinal waves among themselves by the substitution of $(k_1 k_2)^2/k_1^2 k_2^2$ by $(k_1 e^t_{k_2}/k_1)^2$. Upon averaging with respect to polarizations the last coefficient has the form $[k_1 k_2]^2/k_1^2 k_2^2$. Analogously, for the interaction of transverse waves with transverse ones, instead of $(k_1 k_2)^2/k_1^2 k_2^2$ we obtain upon averaging by polarizations

$$\frac{1}{2}\left(1+\frac{(k_1 k_2)^2}{k_1^2 k_2^2}\right).$$

2. As an example we shall consider the scattering of transverse waves on acoustic oscillations (*). As is not too difficult to show, the interaction of acoustic and transverse waves has the character of scattering, provided the following inequality is satisfied:

$$k_s v_{\text{rp}} \gg \nu_r \left(\frac{\omega_{0e}}{\omega}\right)^2, \tag{3}$$

where k_s is the wave number of acoustic oscillations, v_{rp} is the group velocity of transverse waves $\left(v_{\text{rp}} = \frac{\partial \omega}{\partial k}\right.$, where ω is the frequency of transverse waves), ω_{0e} is the electron Langmuir frequency.

The condition (3) stems from the possibility of writing denominators of the type $[\omega_k - \omega_{k-k_1} + \omega_s]^{-1}$ by means of $\delta(\omega_k - \omega_{k-k_1} + \omega_s)$, which expresses the laws of conservation at scattering.

At the same time, one may take advantage of the general expressions for the probabilities of scattering w_{k', k, k_s} [7]:

$$w_{k', k, k_s} = 4\pi (2\pi)^6 \left| e^t_{ik'} S_{ijl}(k', k, k_s) \frac{k_{sl}}{|k_s|} e^t_{jk} \right| \times \frac{\omega^2}{\frac{\partial}{\partial \omega'}[\omega'^2 \varepsilon(k', \omega')] \frac{\partial}{\partial \omega}[\omega^2 \varepsilon(k, \omega)] \frac{\partial}{\partial \omega_s}[\varepsilon^s(k_s, \omega_s)]}, \tag{4}$$

where $e^t_{k'}$, e^t_k are respectively the orts of the scattered and scattering transverse waves. Utilizing formulas (2), (4) and the expressions for the diffusion coefficient (concerning the general formulas for diffusive and elastic scatterings see [8])

$$D_{ij} = \int \frac{dk_s}{(2\pi)^3} k_{si} k_{sj} w_{k', k, k_s} N^s_{k_s} \delta(\omega_s - k_s v_{rp})$$

(where $N^s_{k_s}$ is the density of the number of quanta $\underline{s}$), we shall obtain the estimate

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(*) We call "acoustic oscillations" the sound oscillations in the region $\omega_s \ll \nu_e, \nu_i$ (ω_s is the frequency of acoustic oscillations), which may exist in an isothermic plasma (for details see [6]).

of the characteristic inverse time of diffusive scattering on an angle of the order of the unity:

$$\gamma \sim \frac{1}{\tau} \sim \frac{D}{\omega^2} \sim \frac{W^s}{n_0 m_e v_{T_e}^2} \frac{T_e}{T_i} \left(\frac{\omega_{0e}}{\omega}\right)^4 \frac{k_s}{k} \frac{\omega_{0e}}{1 + T_e/T_i} \tag{5}$$

at $\omega_s \gg k_s^2 v_{T_e}^2 / \nu_e$ ("low-frequency" acoustic sound [3]) and

$$\gamma \sim \frac{1}{\tau} \sim \frac{D}{\omega^2} \sim \frac{W^s}{n_0 m_e v_{T_e}^2} \frac{T_e}{T_i} \left(\frac{\omega_{0e}}{\omega}\right)^4 \frac{k_s}{k} \frac{\omega_{0e}}{5/3 + T_e/T_i} \tag{6}$$

at $\omega_s \ll k_s^2 v_{T_e}^2 / \nu_e$ ("high-frequency" acoustic sound [6]). Here W^s is the energy density of acoustic quanta.

If the scattering is diffusive, the inequality (3) in the "low-frequency" region assumes the form $\omega_{0e} \gg ck \gg \omega_{0e} (v_{T_e}/c) \sqrt{m_i/m_e}$ or $v_{T_e}/c \ll \sqrt{m_e/m_i}$, if $\omega \simeq \omega_{0e}$, and $v_{T_e}/c \ll \sqrt{m_e/m_i}\,(\omega/\omega_{0e})^2$, if $\omega \simeq ck$. Analogously, in the "high-frequency" region $\gg \omega_{0e} (v_{T_e}/c)$ for $\omega \simeq \omega_{0e}$ and $(v_{T_e}/c) \ll (\omega/\omega_{0e})^2$ for $\omega \simeq ck$, which is always fulfilled.

Let us now estimate the inverse time at single scattering by an angle of the order of the unity (k_s k):

$$\gamma \sim \int \gamma(n, n')\, dn' \sim \frac{W^s}{n_0 m_e v_{T_e}^2} \left(\frac{\omega_{0e}}{\omega}\right)^3 \omega_{0e} \frac{T_e}{T_i} \frac{1}{1 + (T_e/T_i)} \tag{7}$$

– in the low-frequency region for $\omega \simeq ck$,

$$\gamma \sim \int \gamma(n, n')\, dn' \sim \frac{W^s}{n_0 m_e v_{T_e}^2} \left(\frac{\omega_{0e}}{\omega}\right)^3 \omega_{0e} \frac{T_e}{T_i} \frac{1}{5/3 + T_e/T_i} \tag{8}$$

– in the "high-frequency" region for $\omega \simeq ck$.

For a single scattering in the "low-frequency"region condition (3) assumes the form $N_{D_e} \ll c^2 m_e / v_{T_e}^2 m_i$ for $\omega \simeq \omega_{0e}$ and $\nu_e / v_{T_e} > k_s > \omega_{0e}/c$ or $N_{D_e} \ll c/v_{T_e}$, for $\omega \sim ck$. In the "high-frequency" region $N_{D_e} \ll c^2/v_{T_e}^2$, provided $\omega \simeq \omega_{0e}$, and $N_{D_e} \ll c/v_{T_e}$ if $\omega \sim ck$. Here N_{De} is the number of electrons in the Debye sphere.

The obtained formulas offer interest for astrophysical applications. Scattering on ion-acoustic oscillations may take place on wave fronts, where the nonisothermicity condition $T_e \gg T_i$ is fulfilled. The scattering effect considered is also possible in an isothermic plasma.

Moreover, the intensity of hydrodynamic turbulence may be determined in a series of cases by the results of observations. Such, for example, is the situation for the hydrodynamic turbulence of the Sun. This is why the results obtained here offer interest for the interpretation of experimental data on radar location of th Sun.

*** T H E E N D ***

REFERENCES

1. V. I. TATARSKIY. Rasprostraneniye voln v turbulentnoy atmosfere. (Wave Propagation in a Turbulent Atmosphere). Iz."NAUKA", 1967.
2. V. V. ZHELEZNYAKOV, Radioizlucheniye Solntsa i planet (Sun's and planets' Radioemission). Izd-vo "NAUKA", 1964.
3. S. A. KAPLAN, V.N. TSYTOVICH. Attronom. Zh., 44, 1036, 1967.
4. V. G. MAKHAN'KOV, V. N. TSYTOVICH. ZhETF, 53, 1789, 1967.
5. V. N. TSYTOVICH. UFN, 90, 435, 1966.
6. V. G. MAKHAN'KOV, V. N. TSYTOVICH. ZhETF (in press).
7. V. N. TSYTOVICH. Nelineynyye effekty v plazme (Nonlinear Effects in Plasma). Izd-vo "NAUKA", 1967.
8. D. M. SAKHOKIYA, V. N. TSYTOVICH. Yadernyy sintez (Nuclear Synthesis), 8, 241, 1968

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