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CURRENTS OF QUASI-TRAPPED PARTICLES HAVING DISCONTINUITIES ON THE DAY AND NIGHT SIDES OF THE EARTH

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SUMMARY

The effect is investigated of the presence in the drift current of a collisionless plasma of a source or of a discontinuity. Applied to the geomagnetic field, the estimate is conducted of the possible effects linked with discontinuities in the currents of quasi-trapped particles on the day and nighttime sides of the Earth.

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The motion of a low-density ionized gas in a nonuniform magnetic field was investigated by Parker [1]. In this work Parker considered a stationary system of particles situated in a magnetic field without taking into account the possible presence of a source region, or particle vanishing in the current. Therefore, the solution found by Parker corresponds to a drift of particles along a closed magnetic shell at invariable distribution of particles by velocities and pitch-angles on the shell. As a consequence of that, the distribution of particle density along the line of force will not depend on longitude (i.e. the longitudinal pressure gradient $\partial p/\partial \Phi = 0$), and the flux of particles directed downward (toward the specular points), will be equal to the flux of particles proceeding upward (in the equatorial plane) at any point of the field line, i.e. current will be absent along the field lines.

In its application to the geomagnetic field, such a statement of the problem is valid only at computation of the effects of the closed ring current (see, for example, [2]). In the case of drift current of quasi-trapped particles, having which drift of particles takes place (point A) ; the second limit azimuth of the source will be called the inner boundary. As previously only particles of minimum pitch-angle $\theta_0 \sim 0°$ will be present on the line AB.

However, on the line BC particles of all pitch-angles from 0 to $\pi/2$ arriving here either from various parts of the source, or the mirror-point reflected ones, will fix themselves (Fig.2).

In the sector ABD the reflected particles are totally absent, all particles moving here in a single direction, from the source toward mirror points and their density increases in the direction of the drift (as a result of the dependence $v_{dr} = f(\theta_0)$. The reflected particles fix themselves only at transition through the line BD and, as they approach the line BC, their density increases. At the outset there will be appearing particles with great values of initial pitch-angles θ_0, then to them will add up particles with smaller and smaller θ_0 and, finally, reflected particles will appear on the line BC, with minimum possible pitch-angles $\theta = 0 + \xi$. Therefore, it is possible to fix on any element S of the line BC particles with pitch-angles $0 < \theta < \pi/2$, proceeding from the source toward the side of mirror points with $\theta = \pi/2$ (their initial pitch-angle $\theta_0{}^*$ may be found from the condition $\sin\theta_0{}^* = q^{1/2}$) and particles with $0 < \theta < \pi/2$, moving toward the inverse side (to the initial plane) after reflection in mirror points.

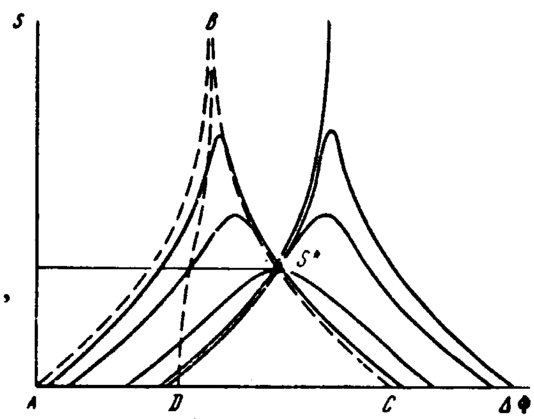

Fig.2

It is easy to find the density of these particles and the pressure induced by them if we know the distribution function by pitch-angles. Assume that

$$f_0(\theta) = k_{\alpha\beta} \times \sin^{\alpha+1}\theta_0 \cos^\beta \theta_0 \ [3, 5],$$

where α and β are the parameters characterizing the anisotropy of articles in the source; $k_{\alpha\beta}$ is a constant. Then the density of particles on an element S of the field line will be

$$n(S,\theta^*) = n_e q^{\alpha/2} \frac{\int_{\theta^*}^{\theta^*+d\theta} \sin^{\alpha+1}\theta (1 - q\sin^2\theta)^{\beta/2} d\theta}{\int_0^{\pi/2} \sin^{\alpha+1}\theta \cos^\beta \theta d\theta} \qquad (1)$$

The minimum transverse pressure on the line AB may be taken for zero, as $\theta_{min} \sim 0$. The cross pressure on the line BC will be determined by the expression

$$p_\perp(S) = \frac{mv^2 n_e}{2} q^{\alpha/2} \frac{2\int\limits_0^{\pi/2} \sin^{\alpha+3}\theta (1 - q\sin^2\theta)^{\beta/2} d\theta}{\int\limits_0^{\pi/2} \sin^{\alpha+1}\theta \cos^\beta \theta d\theta}. \tag{2}$$

Consequently on the outer boundary the transverse pressure between the lines AB and BC will vary from 0 to $p_\perp$ (S). For the determination of the longitudinal interval over which this variation is observed we have $\Delta p_\perp(S) = p_\perp(S)$ and it is sufficient to find $\Delta\Phi = AC$, that is, the magnitude of azimuthal displacement of particles with pitch angle $\theta_{min} = 0 + \xi$ for a half-period of oscillation between the mirror points. For simplicity we may assume that the pressure on the outer boundary is linearly dependent on longitude; then the azimuthal gradient will be

$$\nabla p_\perp(S) = K_3 \frac{\partial p_\perp(S)}{\partial\Phi} = K_3 \frac{p_\perp(S)}{\Delta\Phi(S)}. \tag{3}$$

Here and further K_1, K_2, K_3 are metric coefficients.

Behind the line BC multiply reflected particles will add up to the above considered particles; the transverse pressure induced by them will gradually grow until attaining the maximum near the inner boundary of the source. Subsequently the transverse pressure remains invariable along the drift trajectory up to the region of particle destruction, where the pressure gradient will have the opposite sign. In the case when the extension of the source Δl = AC, the pressure remains invariable from the outer boundary (line BC) to the region of destruction.

The appearance in the magnetic field of longitudinal pressure gradients perpendicular to the field results in emergence of currents

$$i = \frac{c}{B^2}[\mathbf{B} \times \nabla p_\perp], \tag{4}$$

flowing in the direction of the normal to the magnetic field and to pressure gradient.

The total intensity of such currents may be found by integrating over the area $d\sigma = dS \cdot d\Phi/K_1K_3$ (dS being an element of the field line)

$$I = \int\limits_0^S \frac{c}{B^2} \frac{[\mathbf{B} \times \nabla p_\perp(S)]}{K_1} dS. \tag{5}$$

In application to the geomagnetic field let us compute the strength of currents induced by longitudinal pressure gradients, limiting ourselves to the outer boundary of the source. Starting from the hypothesis on particle penetration into the magnetosphere through the neutral points and their subsequent escape into the Earth's magnetic tail [6], we must place the source of particles near the noon meridian, and the region of destruction in the night sector of the magnetosphere.

For particles arriving from the tail the inverse pattern will be valid: the source on the night side and the region of destruction on the daytime side.

In the first approximation we shall consider the geomagnetic field as a dipole, and we shall denote:

$r_e = f_e a$ as the distance from the center of the Earth to the field line (in the equatorial plane), $\underline{a}$ as the radius of the Earth, ϕ as the geomagnetic latitude of the point on the field line, B_0 as the field on the ground at the equator. Then,

$$K_1 = \frac{1}{r_e(1+3\sin^2\varphi)^{1/2}\cos\varphi},$$

$$K_2 = \frac{(1+3\sin^2\varphi)^{1/2}}{\cos^3\varphi},$$

$$K_3 = \frac{1}{r_e\cos^3\varphi},$$

$$q = \frac{\cos^6\varphi}{\sqrt{1+3\sin^2\varphi}}.$$

The density of currents conditioned by longitudinal gradients, in the dipole field will be

$$i = \frac{c}{B(\varphi)r_e\cos^3\varphi}\frac{\partial p_n}{\partial\Phi} = \frac{cf_e^2\cos^3\varphi}{B_0 a(1+3\sin^2\varphi)^{1/2}}\frac{\partial p_n}{\partial\Phi}. \tag{6}$$

The relation between the longitudinal displacement of the particle and the shift along the field line was obtained for the dipole field by Alfvén [7]:

$$\Delta\Phi(\varphi) = \left(\frac{r_e}{c_{at}}\right)^2\Delta I_1(\varphi),$$

$$\Delta I_1(\varphi) = \int_0^\varphi \frac{3\cos^2\varphi(1+\sin^2\varphi)\left[1-\dfrac{\sin^2\theta_0}{2q}\right]}{(1+3\sin^2\varphi)^{3/2}\left[1-\dfrac{\sin^2\theta_0}{q}\right]}d\varphi. \tag{7}$$

Taking into account (6) and (7), the total intensity of currents (5) may be represented in the form

$$I = i_0 a^2 f_e^4 \int_0^{\varphi} D(\varphi)\, d\varphi ,$$

where $i_0 = cmv^2 n_e / aB_0 = 1.57 \cdot 10^{-16} n_0 E$ a/cm^2, n_0E is the energy density expressed in kev/cm^3, $D(\phi)$ is a coefficient characterizing the variation of current's intensity with latitude.

The dependence $D(\phi)$, computed for certain cases of angular anisotropy, is plotted in Fig.3. Calculations show that the total force of currents conditioned by pressure gradients may reach at the outer boundary of the source the magnitude $I_0 = 1.25\ 10^7\ \underline{a}$ (at the condition $n_0E = 100$ kev/ /cm^3, $f_e = 10$). As the distance from the center of the Earth decreases, the current strength decreases by $\sim f_e^4$. The direction of currents depends on the sign of particles. In the region of the source the current will be directed inside the magnetosphere for negative particles, and outside for the posive ones.

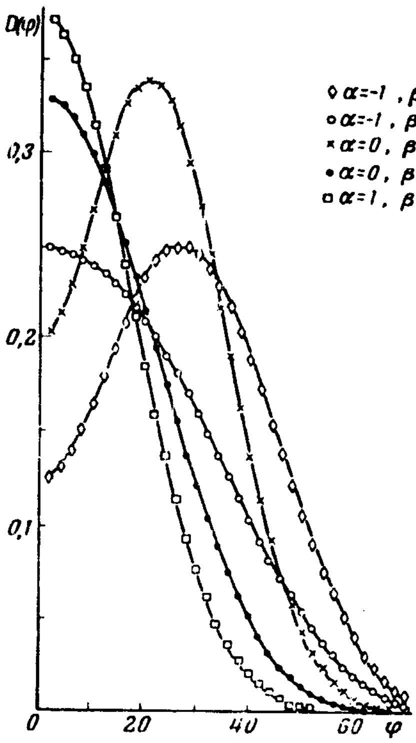

Fig.3

If the source is not stationary, the magnitude of the transverse pressure will be a function of longitude (and time) beyond the limits of the source also. The consideration of temporal effects is contemplated for a later date. Here we shall note only that with the increase of source's activity a rise in transverse pressure will be observed from the outer boundary of the source to the inner one, with subsequent drop in the direction of the region of destruction, which will apparently result in the settling of a system of currents of the form shown in Fig.4.

Besides the above considered currents, other ones, directed along the field lines will also be observed at the boundary of the source. This effect is the result of the fact that the direct (from source to reflection points and the reflected particle fluxes are displaced relative to one another in longitude. Because of that there takes place near the source's boundary a noncompensated charge transfer directed toward the reflection points. As the range from the boundary line AB in the direction of the drift increases (see Fig.1), the direct and reflected fluxes are equalizing and the charge transfer ceases. The density of the longitudinal currents on an element S of the line of force are easy to

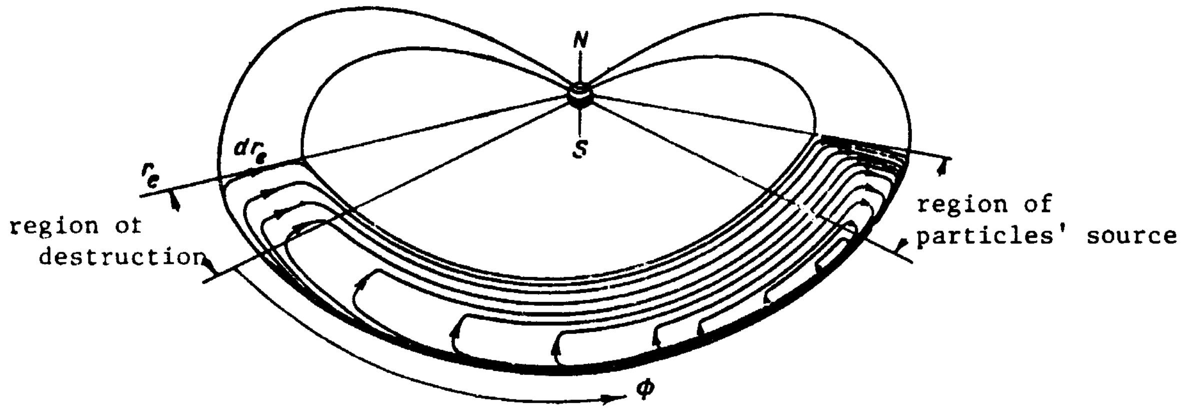

Fig.2

estimate if the initial density n_e of particles and the character of angular anisotropy are known

$$j = en(S)\,v_{\parallel}(S) = en_e v_e q^{\alpha/2}\,\frac{\int_0^{\frac{\pi}{2}} \sin^{\alpha+1}\theta\cos\theta\,(1-q\sin^2\theta)^{\beta/2}\,d\theta}{\int_0^{\frac{\pi}{2}} \sin^{\alpha+1}\theta\cos^{\beta}\theta\,d\theta}.$$

However, the total intensity of longitudinal currents $I = \int jdS$ of such an origin is found to be negligibly small because of the fact that the currents flow only in a narrow boundary layer.

The effect induced by particles with pitch-angles lying in the cone of losses may result to be more interesting in the application to the geomagnetic field. These particles will be fully absorbed in the Earth's atmosphere and, consequently, the direct flux of such particles will not be compensated by the inverse flux. (It is assumed here that the condition of particle flux's quasi-neutrality is not fulfilled).

Let now the cone of losses be determined by the critical angle $\varepsilon_{0c} = 2°$, and the intensity of the source be $n_0 v \sim 10^8$ cm^{-2}·sec^{-1}; then for an angular anisotropy of the form $(\alpha = -1, \beta = 0; \alpha = -1, \beta = 2)$ the density of currents along the line of force will have a magnitude

$$j \sim e n_0 v_0 q^{-1/2} \sin 2° \approx q^{-1/2} \cdot 0{,}2 \cdot 10^{-2} \text{ CGSE}.$$

Multiplying by the area of the source

$$dS = q \cdot a^2 f_e df_e d\Phi$$

and assuming $d\Phi = 2°$, $f_{e1} = 6$, $f_{e2} = 10$, we shall obtain that the intensity of currents flowing along the field lines will vary from $I \sim 2 \cdot 10^5$ a at the equator to $I \sim 10^4$ a at ionosphere heights.

**** T H E E N D ****

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