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COMPARISON OF THE THEORY OF MOTION OF REMOTE ARTIFICIAL EARTH'S SATELLITES WITH OBSERVATIONS

by

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(USSR)

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COMPARISON OF THE THEORY OF MOTION OF REMOTE ARTIFICIAL EARTH'S SATELLITES WITH OBSERVATIONS*

Vestnik Moskovskogo Universiteta
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by V. P. Dolgachev

SUMMARY

The analytical expressions for secular and long-period perturbations of orbits of remote AES, earlier obtained by the present author**, are utilized for the computation of perturbations of certain real AES, whose results of observations are processed and published. The results of calculations by analytical dependences are further compared with the results of numerical integration of satellite's equations of motion in the gravitational field of the Moon.

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1. STATEMENT OF THE PROBLEM

During the motion of remote artificial Earth's satellites (AES), the main perturbations are induced by Earth's nonsphericity and by attraction of the Moon and the Sun. Since the influence of Earth's nonsphericity is studied in detail, only the perturbing action of the Moon and of the Sun are examined in the present work, these heavenly bodies being considered as material points moving along elliptical orbits. In view of the fact that we shall subsequently conduct comparison with the results of numerical integration of differential equations of motion, while the obtained expressions for perturbations are for those of first order, it will be sufficient to consider the perturbing influence of any one body.

To be specific we shall assume the Moon as being the perturbing body.

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* SRAVNENIYE TEORII DVIZHENIYA DALEKIKH ISZ S NABLYUDENIYAMI
** see [1] and our own translation ST–CM–10687 of March 1968

Let Oxyz be a rectangular equatorial fixed system of coordinates with origin at the center of Earth's inertia, whose axis Ox is directed at the point of spring equinox, the axis Oz – along the Earth's axis of rotation, and axis Oy completes the system to the right-handed one. Then, in the field of forces under consideration, the satellite's differential equations of motion will have the form

$$\begin{aligned} \frac{d^2x}{dt^2} &= -\frac{\mu x}{r^3} - \mu_L\left(\frac{x_L - x}{\Delta^3} - \frac{x_L}{r_L^3}\right), \\ \frac{d^2y}{dt^2} &= -\frac{\mu y}{r^3} - \mu_L\left(\frac{y_L - y}{\Delta^3} - \frac{y_L}{r_L^3}\right), \\ \frac{d^2z}{dt^2} &= -\frac{\mu z}{r^3} - \mu_L\left(\frac{z_L - z}{\Delta^3} - \frac{z_L}{r_L^3}\right), \end{aligned} \tag{1}$$

where $\mu = fm_E$, $\mu_L = fm_L$, f is the gravitational constant, m_E is the mass of the Earth, m_L is that of the Moon, x, y, z, x_L, y_L, z_L are respectively the coordinates of the satellite and of the Moon, $r^2 = x^2 + y^2 + z^2$ and $\Delta^2 = (x - x_L)^2 + (y - y_L)^2 + (z - z_L)^2$.

System (1) with the assigned initial conditions was integrated by the Runge-Kutta method with a precision to the fourth power of the step on a computer. The integration step was chosen automatically by the method proposed by Yegorov in 1953. The integration was conducted over a time interval corresponding to revolutions of the Moon, which constitutes some 64 AES orbits. The values of constants entering in (1) are as follows (km is the unit of length, the unit of time is the sec): $\mu = 398620$ and $\mu_L = 4889$.

The initial values of the coordinates and of Moon's velocity components were taken for 1 March 1960:

$$x_L^{(0)} = -341358{,}58; \quad y_L^{(0)} = 199\,024{,}27;$$
$$z_L^{(0)} = 80\,363{,}89; \quad \dot{x}_L^{(0)} = -0{,}50\,141\,736;$$
$$\dot{y}_L^{(0)} = 0{,}80\,299\,933; \quad \dot{z}_L^{(0)} = -0{,}25\,048\,302.$$

The calculations were conducted for a satellite with the following equatorial elements: $a = 38\,142$ км, $e = 2/3$, $i = 34°36'$, $\omega = 71°46'28''$, $\Omega = 119°51'25''$, $u = 26°46'40''$.

According to these initial values of elements the equatorial coordinates and the satellite's velocity components were found, being taken alongside with $x_L^{(0)}$, $y_L^{(0)}$, $z_L^{(0)}$, $\dot{x}_L^{(0)}$, $\dot{y}_L^{(0)}$, $\dot{z}_L^{(0)}$ for the initial conditions at integration of system (1).

The values of Kepler osculating equatorial elements of the satellite were computed by the well known formulas of the problem of two bodies for time intervals $\Delta t = P / 4$, where P is the initial value of satellite's rotation period.

At the same time, the perturbed values of elements were computed for the same moments of time by the earlier obtained formulas [1] (ST-CM-10687).

It should be noted that in formulas for perturbations, the elements are related to Moon's orbit plane, while equatorial elements are obtained by way of numerical integration. The transition from equatorial elements i, Ω, ω to selenoorbital ones i_1, Ω_1, ω_1, the equatorial elements of the Moon, i_L, Ω_L, ω_L being known, is realized by formulas:

$$
\begin{aligned}
\cos i_1 &= \cos i \cos i_L + \sin i \sin i_L \cos(\Omega - \Omega_L),\\
\sin \Omega_1^* &= \frac{\sin i \sin(\Omega - \Omega_L)}{\sin i_1},\\
\cos \Omega_1^* &= \frac{\cos i_L \cos i_1 - \cos i}{\sin i_L \sin i_1},\\
\Omega_1 &= \Omega_1^* - \omega_L,\\
\sin(\omega - \omega_1) &= \frac{\sin i_L - \cos i_1 \cos i}{\sin i},\\
\cos(\omega - \omega_1) &= \frac{\cos i_L - \cos i_1 \cos i}{\sin i_1 \sin i}.
\end{aligned}
\qquad (2)
$$

For known selenorobital elements i_1, Ω_1, ω_1 the equatorial elements i, Ω, ω are computed by formulas

$$
\begin{aligned}
&\Omega_1^* = \Omega_1 + \omega_L,\\
&\cos i = \cos i_L \cos i_1 - \sin i_L \sin i_1 \cos \Omega_1^*,\\
&\sin \Omega = \frac{\sin i_1 \sin \Omega_1^*}{\sin i}, \quad \cos \Omega = \frac{\cos i_1 - \cos i \cos i_L}{\sin i \sin i_L},\\
&\sin(\omega - \omega_1) = \frac{\sin i_L \sin \Omega_1^*}{\sin i}, \quad \cos(\omega - \omega_1) = \frac{\cos i_L - \cos i_1 \cos i}{\sin i_1 \sin i}.
\end{aligned}
\qquad (3)
$$

The equatorial elements of the Moon's orbit are computed either by the given coordinates and velocity components of the Moon, or (in the absence of such) by the mean elements of the lunar orbit related to the ecliptic, which are compiled in the "USSR Astronomical Yearbook".

2. CHOICE OF ARBITRARY CONSTANTS

The analytical expressions for the secular and long-period satellite orbit perturbations from an external body, obtained in [1], have the form

$$\begin{aligned} e &= e_0 + f_1(v, a_0, e_0, i_0, \omega_0, \Omega_0, M_{00}), \\ i &= i_0 + f_2(v, a_0, e_0, i_0, \omega_0, \Omega_0, M_{00}), \\ \Omega &= \Omega_0 + f_3(v, a_0, e_0, i_0, \omega_0, \Omega_0, M_{00}), \\ \omega &= \omega_0 + f_4(v, a_0, e_0, i_0, \omega_0, \Omega_0, M_{00}), \\ M_0 &= M_{00} + f_5(v, a_0, e_0, i_0, \omega_0, \Omega_0, M_{00}), \end{aligned} \tag{4}$$

where v is the unperturbed true anomaly of the satellite, $a_0, l_0, i_0, \Omega_0, \omega_0, M_{00}$ are the integration constants, f_i $(i = 1, 2, \ldots 5)$ are certain quite specific function of v. For brevity we shall denote the systen (4) in the form

$$\ni_i = \ni_{i0} + f_i(v, a_0, \ni_{0j}) \qquad (i, j = 1, 2, \ldots, 5).$$

The numerical integration of system (1) was conducted at following initial conditions: $v = v_0$ & $\ni_{0j} = \ni_j^{(0)}$.

Then

$$\ni_i^{(0)} = \ni_{i0} + f_i(v_0, a_0, \ni_{0j}) \tag{5}$$

and the $\ni_{i0}$ sought for must be found by resolving the nonlinear system (5). However, when considering the perturbations of first order one may postulate the value $\ni_i = \ni_i^{(0)}$ in $f_i(v_0, a_0, \ni_{0j})$ and this is why

$$\ni_{i0} = \ni_i^{(0)} - f_i(v_0, a_0, \ni_j^{(0)}). \tag{6}$$

In such a case

$$\begin{aligned} \ni_{i\Phi} &= \ni_i^{(0)} - f_i(v_0, a_0, \ni_j^{(0)}) + \delta_1 \ni_i, \\ \ni_{ir} &= \ni_i^{(0)} + \delta e. \end{aligned}$$

where $\ni_{i\Phi}$ denotes the value of any element obtained by calculation using the formula, and $\ni_{ir}$ is the value obtained by numerical integration.

3. COMPARISON OF THE RESULTS OF COMPUTATION BY FORMULAS WITH THE RESULTS OF NUMERICAL INTEGRATION

The results of computations with the aid of formulas and those of numerical integration of Eqs (1) are plotted in graphs of Figs 1 - 3.

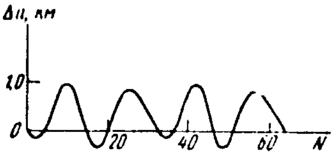

Fig.1. Perturbation of the major semiaxis a in perigee after the results of numerical integration

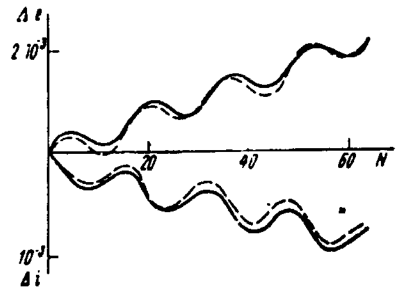

Fig.2. Perturbation of eccentricity e and inclination i in perigee. Solid line shows the calculations by formulas, and the dashed line – the numerical integration

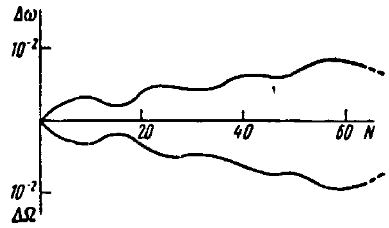

Fig.3. Perturbation of perigee ω's and ascending node Ω's longitude in perigee. The denotations are the same as in Fig.2

The value of perturbations of equatorial elements at time of passage by the satellite of the perigee is shown in all the graphs. As is well known, the satellite orbit's major semiaxis has no secular or long-period perturbations from an external body, which is illustrated in Fig.1. It may be seen from this figure that the major semiaxis has only periodical perturbations with oscillation amplitude of about 1 km. The perturbation in perigee of eccentricity e and inclination i is represented in Fig.2, where one may see that e and i have a secular drift, respectively equal to $1.1 \cdot 10^{-3}$ and $-3 \cdot 10^{-4}$ for a revolution period of the Moon.

Shown in Fig.3 is the longitude perturbation of perigee ω and longitude of the ascending node Ω. The secular variation of these elements is so great that in the graph's scale, the long-period perturbations are hardly perceived.

For Ω the secular drift for the period of Moon's revolution constitutes: $-4.7 \cdot 10^{-3}$ and for ω: $4.1 \cdot 10^{-3}$. The period of long-period element perturbations equals one half of the revolution period of the Moon.

The difference in the values of elements obtained by numerical integration and computation by formulas nowhere exceeds 15", which assures the precision of coordinate determination of the considered satellite to 3 km at most.

If one utilizes for the computations of perturbations the formulas obtained by the author, which take into account the eccentricities of satellite's orbit and of the lunar orbit and also the parallactic terms, a higher precision may be obtained.

4. COMPARISON WITH THE OBSERVATIONS

Comparison of numerical results obtained by formulas with the results of observation of concrete AES is beset with specific difficulties. These difficulties are the result of the fact that there are only very few works, in which the values of perturbations from the Moon and the Sun, obtained from observations, are separate from other perturbing factors. Presented here are the results of observations and the magnitudes of lunar-solar perturbations for two satellites: VANGUARD-1 (1958β_2) and the carrier-rocket of AES ECHO-1 (1960 j_2). The elements of these satellites are published in [2]. For the satellite 1958β_2 the perturbation of perigee height from the Moon and the Sun (dashed line) and also of perigee height from the Moon, the Sun and solar pressure (dash-dotted line) are brought out in the work [3] (see Fig.4). These curves are plotted on the basis of calculations with the aid of formulas obtained by the authors. The values of prrigee height, obtained from observations, are marked by circles. The values of perigee height, computed by analytical dependences derived by us are plotted in Fig.4 for a time interval of 150 days by solid line.

In his review paper [4], Kozai brings forth the results of processing of satellite 1960 j_2 observations obtained by Izhak.

The dashed line in Fig.5 shows the perturbation of orbit eccentricity of this satellite from the Moon and the Sun in accord with the Izhak calculations.

If we add to these perturbations those due to light pressure, we shall have, according to [4], a good coincidence with the observations.

[See Figures 4 and 5 on the next page].

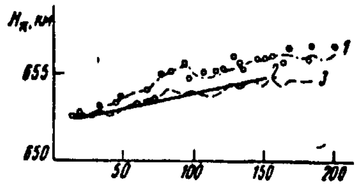

Fig.4. Perturbation of the perigee height H_π of AES "VANGUARD-1" :

1) perturbation due to the Moon, the Sun and the solar pressure according to work [3]; 2) values of perturbations from the Moon and the Sun, obtained by the author; 3) perturbation from the Moon and the Sun according to the work [3]. The circles indicate the values of H_π obtained from the observations

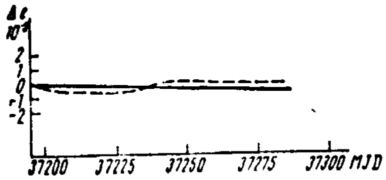

Fig.5. Perturbation of the eccentricity of the 1960 j_2 satellite. The straight line indicates the values of perturbations, obtained by Izhak, e_0 = 0.011414

The results of calculations by our own analytical dependences are plotted in Fig.5 by a solid line. The values of perturbations from the Moon and the Sun of the remaining elements, obtained from observations, the authors, referred to, have unfortunately failed to bring forth.

*** T H E E N D ***

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