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National Aeronautics and Space Administration  
Goddard Space Flight Center  
Contract No. NAS-5-12487

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ST -CR -IP -10814

**N 69-22887**  
(ACCESSION NUMBER)  
7  
(PAGES)  
CR 100 530  
(NASA CR OR TMX OR AD NUMBER)

(THRU)  
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(CODE)  
29  
(CATEGORY)

FACILITY FORM 006

INTERACTION OF COSMIC PROTONS WITH INTERPLANETARY PLASMA

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4 MARCH 1969

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Vestnik Moskovskogo Universiteta  
Fizika, Astronomiya  
No.1, 113- 116,  
Izd-vo Moskovskogo Universiteta, 1969

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SUMMARY

This note considers the energy losses of a nonrelativistic proton beam to the excitation of longitudinal electrostatic oscillations, omitting the possible case of strongly low-frequency magnetohydrodynamic oscillations; accounting of spectrum's low-frequency branch likely to alter the basic results.

\* \* \*

The interaction of cosmic (galactic and solar) protons with the non-isothermic plasma of the solar wind determines a series of complex physical effects, to which the following could be referred: beam instabilities, plasma agitation, energy losses of protons to excitation of plasma wave spectrum etc.

We shall consider the energy losses of a nonrelativistic proton beam to the excitation of longitudinal electrostatic oscillations of the interplanetary plasma [1].

There exists a uniform, nonisothermic, boundless and fully ionized plasma. A cylindrical proton beam of radius  $R$ , length  $L$  with charge density  $\rho(\vec{x} - \vec{v}t)$  moves along the axis  $z$  with constant velocity  $\vec{v}$ . The velocity of protons is high, by comparison with the motion velocity of quiet solar wind; this is why we neglect the motion of the plasma. The motion in the entire space is described in the kinetic approximation by a system of equations of collisionless plasma and a system of Maxwellian equations, taking into account the fulfillment of quasi-neutrality condition ( $N_e = N_i = N$  - the concentrations of electrons and protons are identical). Let us superimpose small perturbations

to that system. The applicability of the linear approximation may be justified by the fact that the energy density of galactic cosmic rays for particles with densities  $\leq 10^9$  ev constitutes  $\leq 10^{-11}$  erg/cm<sup>3</sup> and the energy density of solar cosmic rays is  $\sim 10^{-9}$  erg/cm<sup>3</sup> even at time of flare maximum, which is substantially below the energy density of the quiet solar wind, which is  $10^{-8}$  erg/cm<sup>3</sup>.

It is well known [3] that charges, moving in a plasma with a velocity greater than the wave's phase velocity ( $v > \omega/k$ ), may be decelerated by the electric field of the wave and modify their direction of motion relative to the wave. In electrodynamics, the condition of linear approximation requires that the energy variation  $\Delta W$  of the charge at wavelength distance be  $\Delta W \ll W$  ( $W$  being the initial energy of the charge) and  $\vec{k}$  is the wave vector.

In the linear approximation the perturbed equations have the form

$$\frac{\partial f_e}{\partial t} + \vec{u} \frac{\partial f_e}{\partial x} - \frac{e}{m} \left( \vec{E} \frac{\partial f_{e0}}{\partial u} \right) = 0, \quad \frac{\partial f_i}{\partial t} + \vec{u} \frac{\partial f_i}{\partial x} + \frac{e}{M} \left( \vec{E} \frac{\partial f_{i0}}{\partial u} \right) = 0, \quad (1)$$

$$\Delta \varphi = -4\pi e \left( \int_{-\infty}^{+\infty} f_e \vec{u} du - \int_{-\infty}^{+\infty} f_i \vec{u} du \right) - 4\pi q \vec{x} \cdot \vec{\alpha},$$

here  $f_e$ ,  $f_{ei}$  is the perturbation of the distribution function,  $f_{e0}$ ,  $f_{i0}$  are the distribution functions of the ground, unperturbed state,  $m$ ,  $M$  are respectively the masses of electrons and protons;  $\varphi$  is the electrostatic potential,  $\vec{E}$  is the electric field,  $e$  is the charge of the electron and  $q$  is the total electric charge of the beam.

We shall seek the solution of system (1) in the form of  $f_e$ ,  $f_i$  expansion into Fourier integral. For Fourier-amplitudes we have

$$\varphi_{\vec{k}} = \frac{\frac{4\pi q}{(2\pi)^3} \rho(\vec{k})}{k^2 - \frac{4\pi e^2}{m} \int \frac{\left( \vec{k} \frac{\partial f_{e0}}{\partial u} du \right)}{(k u - k v)} - \frac{4\pi e^2}{M} \int \frac{\left( \vec{k} \frac{\partial f_{i0}}{\partial u} du \right)}{(k u - k v)}}. \quad (2)$$

In formula (2) the denominator determines the so-called dispersion equation of excitation of plasma oscillations responsible for energy losses of the charges. Note that in the absence of perturbations the solar wind plasma may

be in a state of thermodynamic equilibrium; then we shall take  $f_{0e}$ ,  $f_{0i}$  in the form

$$f_{0e} = N \left( \frac{m}{2\pi\theta_e} \right)^{3/2} \exp \left( -\frac{mu^2}{2\theta_e} \right), \quad f_{0i} = N \left( \frac{M}{2\pi\theta_i} \right)^{3/2} \exp \left( -\frac{Mu^2}{2\theta_i} \right).$$

where  $\theta_e$ ,  $\theta_i$  are the temperature of electrons and ions (in ev units).

Let us integrate the dispersion equation over all transverse components ( $\vec{u} = (\vec{u}_\perp, u_\parallel)$ ,  $\vec{k}u_\perp = ku$ ,  $\vec{k}u_\parallel = 0$ )

$$k^2 \left[ 1 - \frac{\omega_{pe}^2}{k^2} \cdot \frac{m}{\theta_e} \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{u \exp \left( -\frac{u^2}{2} \right)}{\beta_e - u} du - \frac{\omega_{pi}^2}{k^2} \cdot \frac{M}{\theta_i} \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{u \exp \left( -\frac{u^2}{2} \right)}{\beta_i - u} du \right] = 0.$$

Following were the denotations introduced:

$$\omega = \vec{k}\vec{v}, \quad \xi = ku, \quad \beta_e = \sqrt{\frac{m}{\theta_e}} \cdot \frac{\omega}{k}, \quad \beta_i = \sqrt{\frac{M}{\theta_i}} \cdot \frac{\omega}{k}, \quad v_e^2 = \frac{3\theta_e}{m}, \quad v_i^2 = \frac{3\theta_i}{M}.$$

$\omega_{e0}$ ,  $\omega_{i0}$  being respectively the Langmuir frequencies of electrons and protons. The integrals are analytically computed for the case, when the wave's phase velocities are great by comparison with thermal velocities:  $\beta_e \gg 1$ ,  $\beta_i \gg 1$ . Expanding the expression  $1/(\beta - u)$  in series, we shall take the integrals by parts. With a precision to terms of second order of smallness, the dispersion equation takes the form

$$-\frac{k^2}{\omega^2} (\omega_0^2 + k^2 A^2 - \omega^2) = 0, \\ \omega_0^2 = \omega_{e0}^2 + \omega_{i0}^2, \quad A^2 = \left( \frac{\omega_{e0}}{\omega_0} \right)^2 v_e^2 + \left( \frac{\omega_{i0}}{\omega_0} \right)^2 v_i^2.$$

In the cylindrical system of coordinates  $(r, z)$ ,  $\vec{k} = (\kappa, \omega)$  we have for the beam

$$\rho(\vec{x} - \vec{v}t) = \begin{cases} \frac{q}{4\pi R^2} & |z - vt| < \frac{R}{2}, \quad r < R, \\ 0 & , \quad r > R, \end{cases}$$

and we integrate over all wave numbers, starting from values  $kh_e < 1$  ( $h_e$  being the Debye radius of electrons).

.../...

$$\varphi(\vec{r}, t) = \frac{q}{\pi v} \int_{-\infty}^{+\infty} \frac{\left(\frac{2v}{\omega l} \sin \frac{\omega l}{2v}\right) e^{i \frac{\omega}{v}(z-vt)} \omega^2}{R(\omega^2 - \omega_0^2)} d\omega \left\{ \int_0^{\infty} \frac{I_0(xr) I_1(xR)}{x^2 + \frac{\omega^2}{v^2}} dx - \right. \\ \left. - \int_0^{\infty} \frac{I_0(xr) I_1(xR)}{x^2 + b^2} dx \right\}, \quad (3)$$

$$E_z = -\frac{q}{\pi v^2} \int_{-\infty}^{+\infty} \frac{\left(\frac{2v}{\omega l} \sin \frac{\omega l}{2v}\right) e^{i \frac{\omega}{v}(z-vt)} i\omega^3}{R(\omega^2 - \omega_0^2)} d\omega \left\{ \int_0^{\infty} \frac{I_0(xr) I_1(xR)}{x^2 + \frac{\omega^2}{v^2}} dx - \right. \\ \left. - \int_0^{\infty} \frac{I_0(xr) I_1(xR)}{x^2 + b^2} dx \right\}, \quad (4)$$

where  $I_0, I_1$  are Bessel functions of zero and first orders

$$b^2 = d^2(\Omega^2 - \omega^2), \quad d^2 = \left(\frac{1}{A^2} - \frac{1}{v^2}\right), \quad \Omega^2 = \frac{\omega_0^2}{1 - \frac{A^2}{v^2}}.$$

The expression for energy loss of proton beam per unit of path and in the unit of time may be written in the form [4]

$$-\frac{dW}{dz} = \frac{q}{\pi R^2 l} \int_0^R 2\pi r dr \int_{-l/2}^{+l/2} E_z(r, z-vt) d(z-vt). \quad (5)$$

We are interest only in the real part of this expression. After appropriate transformations, which we omit, we shall have

$$-\frac{dW}{dz} = \frac{q^2}{\pi R^2} \left\{ Re \int_{-\infty}^{+\infty} \frac{\left(\frac{2v}{\omega l} \sin \frac{\omega l}{2v}\right)^2 i\omega}{\omega^2 - \omega_0^2} \left[ 1 - 2I_1\left(\frac{\omega}{v} R\right) K_1\left(\frac{\omega}{v} R\right) \right] d\omega - \right. \\ \left. - \frac{1}{v} Re \int_{-\infty}^{+\infty} \frac{\left(\frac{2v}{\omega l} \sin \frac{\omega l}{2v}\right)^2 i\omega^3}{\omega^2 - \omega_0^2} \times \right. \\ \left. \times [1 - 2I_1(d\sqrt{\Omega^2 - \omega^2} R) K_1(d\sqrt{\Omega^2 - \omega^2} R)] \frac{d\omega}{d^2(\Omega^2 - \omega^2)} \right\}.$$

Here  $I_1, K_1$  are Bessel functions from a purely imaginary argument.

When investigating and computing improper integrals, we apply the properties of  $\delta$ -functions [5]

$$\frac{1}{\omega^2 - \omega_0^2} \rightarrow \frac{p.v.}{\omega^2 - \omega_0^2} \pm i\pi \frac{\omega}{|\omega|} \delta(\omega^2 - \omega_0^2).$$

Omitting the intermediate operations, we shall obtain

$$\begin{aligned} -\frac{dW}{dz} = & \frac{2q^2}{R^2} \left( \frac{2v}{\omega_0 l} \sin \frac{\omega_0 l}{2v} \right)^2 \left[ 1 - 2I_1 \left( \frac{\omega_0}{v} R \right) K_1 \left( \frac{\omega_0}{v} R \right) \right] + \\ & + \frac{q^2}{\pi v^2} R^2 \int_{\omega > \Omega}^{\infty} \left[ \frac{2I_1(d\sqrt{\omega^2 - \Omega^2} R)}{d\sqrt{\omega^2 - \Omega^2} R} \right]^2 \left( \frac{2v}{\omega l} \sin \frac{\omega l}{2v} \right)^2 \frac{\omega^2 d\omega}{\omega^2 - \omega_0^2}. \end{aligned} \quad (6)$$

It may be concluded from expression (6) that energy losses are intense in frequencies near  $\omega_0$  [6], where  $(\omega_0^2 = \omega_{0e}^2 + \omega_{0u}^2)$ . The integral near  $\omega_0$  is finite; it is possible to discern that its value is smaller in magnitude than the first term of expression (6). When  $\omega$  strongly differs from  $\omega_0$ , the product of the interference factors decreases rapidly and the entire integral is small. A nonlinear consideration of similar questions may be found in the appropriate literature [7, 8].

In the present work we considered the questions of excitation of longitudinal electrostatic oscillations; however, in interplanetary plasma the excitation of strongly low-frequency magnetohydrodynamic oscillations is possible [9]. The accounting of the low-frequency branch of the spectrum may significantly alter the basic results (6); however, the discussion of this question is no longer within the framework of the present paper.

In conclusion I extend my thanks to B. A. Tverskoy and V. P. Yakimenko for their valuable discussions.

\*\*\*\* T H E E N D \*\*\*\*

CONTRACT No. NAS-5-12487  
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Translated by ANDRE L. BRICHANT  
on 3 March 1969

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