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A COMPUTER CODE FOR ONE-DIMENSIONAL ELASTIC WAVE PROBLEMS

by Richard W. Mortimer and James F. Hoburg

Prepared by
DREXEL INSTITUTE OF TECHNOLOGY
Philadelphia, Pa.
for Langley Research Center

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ABSTRACT

A general purpose computer code for solving one-dimensional elastic wave problems is presented. The code involves the application of the method of characteristics to the displacement governing differential equations of motion of the structures. Some of the structures which can be analyzed by this program include shells, Mindlin plates, bars, and Timoshenko beams; subroutines for these structures have been prewritten for the user. The code is capable of handling boundary conditions which may be specified as step, ramp, exponential, and sinusoidal time functions; these have also been prewritten for the user. Detailed instructions for the use of the program, its computational procedure, and its limitations are given. An example involving the impacting of a conical shell is included.

SYMBOLS

x - Spacial Coordinate

x_o - x coordinate of the boundary

Δx - Mesh size

t - Time

u_1, u_2, u_3 - Displacement Variables

$f_1 \ldots f_6$, $g_1 \ldots g_6$, $h_1 \ldots h_6$ - Coefficients in governing differential equations (correspond to α_{ij}'s and β_{ij}'s in Refs. 1 and 2.)

c_1 - Leading wave speed

c_2 - Second wave speed

$A_1 \ldots A_7$, $B_1 \ldots B_7$, $C_1 \ldots C_7$ - Coefficients of variables in boundary condition equations

b_1, b_2, b_3 - Time functions in boundary condition equations

M_o - Number of lines to be evaluated

[ ] - Bracket represents jump in the enclosed variable

Remaining symbols defined in appropriate appendices

I. INTRODUCTION

In Ref. [1], Chou and Mortimer showed that a large number of elastic wave problems involving one space variable may be treated, in a unified manner, by a system of second-order hyperbolic partial differential equations. The dependent variables of this system were seen to be the generalized displacements; the coefficients appearing in the equations were functions of the spatial variables. This system of n equations was analyzed by the method of characteristics, yielding closed form equations for the physical characteristics, the characteristic equations, and the propagation of discontinuities. Among the elastic wave problems that may be represented by this unified approach are structures such as shells, Mindlin plates, bars, and Timoshenko beams.

The purpose of this report is to describe MCDIT 21, a general purpose computer code (or program), designed to solve elastic wave problems which may be represented by this unified approach. In general, this program is capable of solving wave problems governed by one, two, or three coupled, second-order hyperbolic differential equations involving one or two distinct wave speeds. A familiarity with Refs. [1] and [2] will aid the reader in understanding the theoretical aspects of the method of characteristics and the numerical procedure utilized in this program.

This program offers the user two alternatives for solving problems. First, subroutines for some common structures, such as conical and cylindrical shells, bars, Timoshenko beams, etc., (see Fig. 1) have been prewritten, so the user need specify only the structure, dimensions, elastic constants, and type of boundary loadings (e.g. step, ramp, sinusoidal, etc.). Second, for other elastic wave problems the user must write the subroutines which specify the coefficients of the governing equations and/or the boundary loading functions.

This report begins with a description of the general capabilities of MCDIT 21. The details of the calculation procedures used by the program are then given. Instructions for the use of MCDIT 21 for solving specific elastic wave problems then follow. Appendix A is a listing of the MCDIT 21 Main Program Deck. Appendices B and C include the listings of the prewritten subroutines to be used for the common structures, and the boundary loading functions (step, ramp, etc.). Instructions for writing subroutines for those structures or boundary loadings which have not been prewritten are included in Appendix D. Appendix E includes the characteristic and continuity equations used in the method of characteristics solution. The application of MCDIT 21 to an impacted conical shell and the detailed input required for this example are presented in Appendix F.

All runs of MCDIT 21 have been tested on an IBM 360, Model 65, computer with running times of approximately 2.5 minutes for $M_o = 150$.

II GENERAL CAPABILITIES OF MCDIT 21

The MCDIT 21 program is capable of solving elastic wave problems whose governing equations are of the following forms:

$$\frac{\partial^2 u_1}{\partial x^2} - \frac{1}{c_1^2}\frac{\partial^2 u_1}{\partial t^2} = f_1 \frac{\partial u_1}{\partial x} + f_2 u_1 \quad , \tag{II-1}$$

or

$$\begin{aligned}
\frac{\partial^2 u_1}{\partial x^2} - \frac{1}{c_1^2}\frac{\partial^2 u_1}{\partial t^2} &= f_1 \frac{\partial u_1}{\partial x} + f_2 u_1 + f_5 \frac{\partial u_3}{\partial x} + f_6 u_3 \\
\frac{\partial^2 u_3}{\partial x^2} - \frac{1}{c_2^2}\frac{\partial^2 u_3}{\partial t^2} &= h_1 \frac{\partial u_1}{\partial x} + h_2 u_1 + h_5 \frac{\partial u_3}{\partial x} + h_6 u_3 \ ,
\end{aligned} \tag{II-2}$$

or

$$\begin{aligned}
\frac{\partial^2 u_1}{\partial x^2} - \frac{1}{c_1^2}\frac{\partial^2 u_1}{\partial t^2} &= f_1 \frac{\partial u_1}{\partial x} + f_2 u_1 + f_3 \frac{\partial u_2}{\partial x} + f_4 u_2 + f_5 \frac{\partial u_3}{\partial x} + f_6 u_3 \\
\frac{\partial^2 u_2}{\partial x^2} - \frac{1}{c_1^2}\frac{\partial^2 u_2}{\partial t^2} &= g_1 \frac{\partial u_1}{\partial x} + g_2 u_1 + g_3 \frac{\partial u_2}{\partial x} + g_4 u_2 + g_5 \frac{\partial u_3}{\partial x} + g_6 u_3 \\
\frac{\partial^2 u_3}{\partial x^2} - \frac{1}{c_2^2}\frac{\partial^2 u_3}{\partial t^2} &= h_1 \frac{\partial u_1}{\partial x} + h_2 u_1 + h_3 \frac{\partial u_2}{\partial x} + h_4 u_2 + h_5 \frac{\partial u_3}{\partial x} + h_6 u_3
\end{aligned} \tag{II-3}$$

where u_1, u_2, and u_3 are generalized displacements. The wave speeds c_1 and c_2 must be distinct and are considered as known (and constant) with $c_1 > c_2$. The coefficients $f_1 \ldots f_6$, $g_1 \ldots\ldots g_6$, and $h_1 \ldots\ldots h_6$ are functions of x which the user may specify.

Common Structures whose governing equations are in the form of eqs. (II-1), (II-2), and (II-3) are listed in Fig. 1 under Structure choice. The governing equations of the cylindrical and spherical dilatation, and the rotary and longitudinal shears are in the form of eq. (II-1). The beam, plate, bar, and sheet structures have governing equations of the form of eqs. (II-2): the shell structures are in the form of eqs. (II-3). Each of the subroutines specifying the governing equations of these thirteen structures have been prewritten for the MCDIT 21 program.

In this section we will discuss the most general case, equations (II-3), since problems governed by either of equations (II-1) or (II-2) can be considered as special cases of (II-3).

Referring to equations (II-3), we see that six initial conditions and six boundary conditions are required in order to obtain a solution. The MCDIT 21 program treats problems involving zero initial conditions and semi-infinite mediums, therefore, only three boundary conditions need be specified (regularity will be required at infinity). These conditions are specified by the user along the pertinent boundary line, $x = x_o$, in the following form:

$$\begin{aligned}
A_1 \frac{\partial u_1}{\partial x} + A_2 u_1 + A_3 \frac{\partial u_2}{\partial x} + A_4 u_2 + A_5 \frac{\partial u_3}{\partial x} + A_6 u_3 + A_7 \frac{\partial u_1}{\partial t} &= b_1(t) \\
B_1 \frac{\partial u_1}{\partial x} + B_2 u_1 + B_3 \frac{\partial u_2}{\partial x} + B_4 u_2 + B_5 \frac{\partial u_3}{\partial x} + B_6 u_3 + B_7 \frac{\partial u_2}{\partial t} &= b_2(t) \qquad \text{(II-4)} \\
C_1 \frac{\partial u_1}{\partial x} + C_2 u_1 + C_3 \frac{\partial u_2}{\partial x} + C_4 u_2 + C_5 \frac{\partial u_3}{\partial x} + C_6 u_3 + C_7 \frac{\partial u_3}{\partial t} &= b_3(t)
\end{aligned}$$

where $A_1 \ldots A_7$, $B_1 \ldots B_7$, $C_1 \ldots C_7$ are constants and $b_1(t)$, $b_2(t)$, and $b_3(t)$ are functions of time at $x = x_o$. Five boundary condition time functions have been prewritten for MCDIT 21 and are listed in Fig. 1. With the governing equations (in the form of II-3) stipulated, and the boundary conditions, (II-4), together with the zero initial conditions prescribed, the mathematical system may be solved. This solution is obtained by utilizing the method of characteristics to determine the values of the quantities u_1, $\frac{\partial u_1}{\partial x}$, $\frac{\partial u_1}{\partial t}$, u_2, $\frac{\partial u_2}{\partial x}$, $\frac{\partial u_2}{\partial t}$, u_3, $\frac{\partial u_3}{\partial x}$, and $\frac{\partial u_3}{\partial t}$ at the mesh points (dots) of the network in the physical $(x, c_1 t)$ plane depicted in Fig. 2. Detailed construction of this network is discussed in Refs. [1] and [2].

The method of characteristics can solve problems involving discontinuities of quantities which are functions of the displacement derivatives (e.g. stresses and velocities). A detailed and mathematical discussion of discontinuities, the equations governing the magnitude of discontinuities as they propagate, and the speed with which discontinuities propagate is contained in Ref. [1] and the details will not be repeated here. If discontinuities in $\frac{\partial u_1}{\partial x}$ and $\frac{\partial u_1}{\partial t}$ or $\frac{\partial u_2}{\partial x}$ and $\frac{\partial u_2}{\partial t}$ occur, respectively, at point A (Fig. 2) they will propagate along line AB, hereafter known as the first discontinuity line. Similarly, if discontinuities in $\frac{\partial u_3}{\partial t}$ and $\frac{\partial u_3}{\partial x}$ occur at point A, they will propagate along line AC, hereafter known as the second discontinuity line. Any of these discontinuities (or combination) are readily handled by this program.

III METHODS OF CALCULATION IN MCDIT 21

The method of calculating the variables at ordinary mesh points consists essentially of solving a system of simultaneous equations for a corresponding number of variables. The computational procedure can best be described with the aid of Figure 2; the details of equations and auxiliary conditions to be used will appear later in this section. The essence of the procedure is as follows:

1. From the initial conditions and boundary conditions the values of the nine variables u_1, $\partial u_1/\partial x$, $\partial u_1/\partial t$ .... $\partial u_3/\partial t$ are known at points 2 and 1. The values of the variables at point 3 are then computed through a simultaneous solution of the governing characteristic equations and boundary conditions.
2. The computation then proceeds to the $\frac{dx}{c_1 dt} = -1$ characteristic line through point 4. The values of the nine variables are known now at points 3, 2, and 4. The values of the nine variables are then computed at point 5.
3. With the values of the nine variables known at points 3 and 5, the variables are computed at point 6 through a simultaneous solution of the governing characteristic equations and boundary conditions.
4. The computation then proceeds to the $\frac{dx}{c_1 dt} = -1$ characteristic line through point 7. Knowing the values of the variables at points 5, 4, and 7, the values are then computed at point 8.
5. This process continues by solving for the values of the variables at points 8, 9, and 10, respectively. Again, the computation shifts to the next $\frac{dx}{c_1 dt} = -1$ characteristic line and solves for the values of the variables at the mesh points along this line (e.g., 12, 13, 14 and 15). This procedure continues until the values along the $\frac{dx}{c_1 dt} = -1$ characteristic through the M_o^{th} point on AB are obtained.

The actual program consists of a main program and several subroutines (see figure 3). These subroutines may be divided into two separate levels; the first level and the second level subroutines. Each of the first level subroutines is used to evaluate one of the different types of points in the physical plane. The second level subroutines are general in nature. Their purpose is to define quantities or perform tasks which are needed for more than one type of point in the physical plane. Some second level subroutines remain the same regardless of the type of problem or boundary conditions being specified. For example, the simultaneous solution subroutine is used to solve a system of simultaneous equations for each new point at which unknowns must be determined. Its form remains unchanged for all problems. This type of subroutine is termed invariant. Other second level subroutines are used to define the problem and boundary conditions to be run. Thus, these subroutines are completely dependent upon the nature of the particular run desired and are termed user-specified.

For each new point the main program decides the point type and calls the corresponding first level subroutine. Each first level subroutine, in turn, calls those second level subroutines necessary to evaluate quantities at the new point (see figure 3). A description of each of the second level subroutines, followed by a description of the main program and first level subroutines follows.

Second Level Subroutines

Boundary Condition Time Functions Subroutine (user specified)

This subroutine, which actually consists of three Fortran "SUBROUTINE"s, is used to specify the 3 functions b_1, b_2, and b_3 which form the right-hand sides of the 3 boundary condition equations. Any function of t may be specified for each of the three.

Discontinuity Values Subroutine (user specified)

This subroutine, which actually consists of two Fortran "SUBROUTINE"s, is used to specify the values of the discontinuities in $\frac{\partial u_1}{\partial x}$, $\frac{\partial u_1}{\partial t}$, $\frac{\partial u_2}{\partial x}$, and $\frac{\partial u_2}{\partial t}$ which occur along the first discontinuity line and the values of the discontinuities in $\frac{\partial u_3}{\partial x}$ and $\frac{\partial u_3}{\partial t}$ which occur along the second discontinuity line. Any function of x may be specified for each of the six discontinuities.

Governing Equation Coefficient Definitions Subroutine (user specified)

This subroutine, which actually consists of three Fortran "SUBROUTINE"s, is used to specify the coefficients $f_1 \ldots f_6$, $g_1 \ldots g_6$, and $h_1 \ldots h_6$ in the governing differential equations. Any function of x may be specified for each of the 18 coefficients.

Printout Quantities Subroutine (user specified)

This subroutine, which consists of one Fortran "SUBROUTINE", is used to specify the output desired. Any of the quantities calculated at the mesh points and/or any functions of those quantities at any of the mesh points may be printed out, as specified by this subroutine.

Solution Matrix Subroutine (invariant)

This subroutine sets up the coefficients of the solution matrix for the quantities at certain types of points to be evaluated in terms of known quantities at points previously evaluated.

Simultaneous Solution Subroutine (invariant)

This subroutine uses the solution matrix to solve the simultaneous equations at the points to be evaluated.

Main Program and First Level Subroutines:

The main program begins by reading in the values of M_o, x_o, Δx, c_1, c_2, $A_1 \ldots A_7$, $B_1 \ldots B_7$, and $C_1 \ldots C_7$ from the data cards. A preliminary printout then lists, at the beginning of the output, all of the quantities read in. The main program then begins calling first level subroutines depending upon the type of point to be evaluated next. All first level subroutines are invariant.

First Point Subroutine

The first point subroutine is used to calculate the quantities at point 1 in Figure 4 (or point 3 in Fig. 2). The subroutine is called only once, at the beginning of evaluation of quantities in the physical plane. This type of point can occur only at the beginning of evaluation, since it involves both the crossing of the second discontinuity line (2-5) and the satisfaction of boundary conditions (at point 1) as shown in Fig. 4. For such a point to occur more than once, the slope of the second discontinuity line in the physical (x, c_1t) plane must exceed the value of 3. This program is not equipped to handle such a case.

To calculate the quantities at point 1 of Fig. 4, one needs the information at point 2. It is, however, not possible to solve for the quantities at point 2 independently of those at point 1, since quantities at point 6 must be used in the calculation of those at point 2. Quantities at point 6 must be expressed, using a linear interpolation, in terms of those at point 5 and the unknowns at point 1. Thus, the quantities at points 1 and 2 are evaluated simultaneously. A total of 18 unknowns exist: 9 at point 1 and 9 at point 2. The 18 needed equations are obtained as follows:

2 characteristic equations* along 1-2.
1 characteristic equation along 1-7.
2 characteristic equations along 2-3.
1 characteristic equation along 2-4.
2 characteristic equations along 2-6.
1 characteristic equation along 2-5.
3 continuity equations* along 1-5.
2 continuity equations along 2-3.
1 continuity equation along 2-4.
3 boundary conditions at point 1.

All 6 continuity equations are used to eliminate the 3 displacement variables u_1, u_2, and u_3 at point 1 and the same 3 at point 2, leaving a system of 12 equations in 12 unknowns. The solution matrix is set up within the first point subroutine and is then solved, using the second level subroutine for solution of n equations in n unknowns. After a solution for the 12 derivatives is obtained, the 6 continuity equations are used to calculate the displacement variables at points 1 and 2. The first point subroutine calls the following second level subroutines during its execution (see figure 3).

A) The Boundary Condition Time Function subroutine to specify the three boundary conditions to be satisfied at point 1.

B) The Discontinuity Values subroutine to specify the values of $\frac{\partial u_1}{\partial x}$, $\frac{\partial u_1}{\partial t}$, $\frac{\partial u_2}{\partial x}$, $\frac{\partial u_2}{\partial t}$, $\frac{\partial u_3}{\partial x}$, and $\frac{\partial u_3}{\partial t}$ at point 5, the values of $\frac{\partial u_1}{\partial x}$, $\frac{\partial u_1}{\partial t}$, $\frac{\partial u_2}{\partial x}$, and $\frac{\partial u_2}{\partial t}$ at point 3, and the jumps in $\frac{\partial u_3}{\partial x}$ and $\frac{\partial u_3}{\partial t}$ at point 2.

F) The Governing Equation Coefficient Definitions subroutine to be used in specifying the characteristic equations.

C) The Simultaneous Solution subroutine to solve the 12 equations in 12 unknowns.

D) The Printout Quantities subroutine to print out desired quantities at points 3, 5, and 1.

*See Appendix E for these equations.

Input Point Subroutine

The input point subroutine is called at the beginning of each new $\frac{dx}{c_1 dt} = -1$ characteristic line. It is used to define and print out the quantities specified at a point on the first discontinuity line. (The points 1, 2, 4, 7, 11 and 16 in Fig. 2 are all input points.) Only 2 second level subroutines need be called during execution: (see Fig. 3).

B) The Discontinuity Values subroutine to specify the values of $\frac{\partial u_1}{\partial x}$, $\frac{\partial u_1}{\partial t}$, $\frac{\partial u_2}{\partial x}$, and $\frac{\partial u_2}{\partial t}$ at the input point.

D) The Printout Quantities subroutine to print out desired quantities at the input point.

Boundary Point Subroutine

The boundary point subroutine is called at the end of each line. It is used to calculate quantities at the points on the boundary which satisfy the equations along left running characteristic directions and which satisfy the boundary condition equations. (The boundary points are referred to as points 6, 10, 15, and 21 in Fig. 2. One example is also shown in Fig. 5). The 9 equations used to calculate the 9 unknowns at point 1 of Fig. 5 are obtained as follows:

2 characteristic equations along 1-3.
1 characteristic equation along 1-4.
2 continuity equations along 1-3.
1 continuity equation along 1-4.
3 boundary conditions at point 1.

All 3 continuity equations are used to eliminate the 3 displacement variables at point 1, leaving a system of 6 equations in 6 unknowns. The solution matrix is set up within the boundary point subroutine. The resulting system is solved using the simultaneous solution subroutine. After a solution for the 6 derivatives is obtained, the 3 continuity equations are used to calculate the displacement variables at point 1. The boundary point subroutine calls the following second level subroutines during execution: (see Figure 3).

A.) The Boundary Condition Time Function subroutine to specify the three boundary conditions to be satisfied at point 1.

F.) The Governing Equation Coefficient Definitions subroutine to be used in specifying the characteristic equations.

C.) The Simultaneous Solution subroutine to solve the 6 equations in 6 unknowns.

D.) The Printout Quantities subroutine to print out desired quantities at point 1.

Ordinary Point Subroutine

The ordinary point subroutine is used for each point after an input point and before a boundary point, except for points complicated by the crossing of the second discontinuity line. Thus, the ordinary points are referred to as the points, 9, 14, 17, 19 and 20 in Fig. 2. The 9 equations used to calculate the 9 unknowns at point 1 of Fig. 6 are obtained as follows:

2 characteristic equations along 1-3.
1 characteristic equation along 1-4.
2 characteristic equations along 1-9.
1 characteristic equation along 1-6.
2 continuity equations along 1-3.
1 continuity equation along 1-4.

As in the case of the Boundary Point Subroutine, the 3 continuity equations are used to eliminate the 3 displacement variables at point 1, leaving a system of 6 equations in 6 unknowns. This system is solved and the continuity equations are used to calculate the displacement variables at point 1. The ordinary point subroutine calls the same second level subroutines as does the boundary point subroutine except that the boundary condition time function subroutine, is not used in this case. Also, the Solution Matrix subroutine is used to set-up the system of simultaneous equations (see Figure 3).

Case I Subroutine

The Case I subroutine is used for points complicated by the crossing of the second discontinuity line in the manner shown in Figure 7. The points include those marked 5, 8, and 18 in Fig. 2. Quantities are first calculated at point 1 (Fig. 7b). The discontinuities in $\frac{\partial u_3}{\partial x}$ and $\frac{\partial u_3}{\partial t}$ at point 1 are then added to the calculated values of $\frac{\partial u_3}{\partial x}$ and $\frac{\partial u_3}{\partial t}$ respectively. Finally, quantities are calculated at point 1' (Fig. 7c), a mesh point. For each of the two points, 1 and 1', a system of 9 equations in 9 unknowns is solved, just as for an ordinary point. Exactly the same second level subroutines are called during execution as are called from the ordinary point subroutine, with one addition, i.e., the Discontinuity Value Subroutine is called to specify the jumps in $\frac{\partial u_3}{\partial x}$ and $\frac{\partial u_3}{\partial t}$ at point 1. Of course, each of the subroutines, except the specifications of printout quantities, which is used only at point 1', must, in this case be called twice; once for evaluation of quantities at point 1 and once for point 1' of Fig. 7.

Case II Subroutine

The Case II subroutine is used for a set of points complicated by the crossing of the second discontinuity line in the manner shown in Figure 8a. The points include those marked 12 and 13 in Fig. 2. Quantities are first calculated at point 1 of the first block (Fig. 8b). The discontinuities in $\frac{\partial u_3}{\partial x}$ and $\frac{\partial u_3}{\partial t}$ at point 1 are then added to the calculated values of $\frac{\partial u_3}{\partial x}$ and $\frac{\partial u_3}{\partial t}$ respectively. Quantities are then calculated at point 1' (Fig. 8c) of the first block, a mesh point. Then the quantities at point 1 (Fig. 8d) of the second block, are calculated. Discontinuities in $\frac{\partial u_3}{\partial x}$ and $\frac{\partial u_3}{\partial t}$ are added as before, and finally, quantities are evaluated at point 1' (Fig. 8e) of the second block, another mesh point. Exactly the same second level subroutines are called during execution as are called from the Case I subroutine, with each being called twice as many times, since two blocks must be evaluated.

IV INSTRUCTIONS FOR USE OF MCDIT 21

The MCDIT 21 program offers the user two choices for solving problems. First, the problem to be solved by the user consists of a structure which is identical to one of the thirteen prewritten common structure packages listed in Fig. 1 and the user's boundary conditions are identical to any of the five prewritten boundary condition packages listed in Fig. 1. Second, the user's structure and/or the user's boundary conditions have not been included as prewritten packages, but, the user's governing equations are of the form of equations (II-1), (II-2), or (II-3) and the boundary conditions of the form of (II-4). Each of these choices for utilization of MCDIT 21 will now be discussed.

A. Prewritten Common Structure and Boundary Condition Packages

1. Input to Machine

a. Main Program Deck - invariant

b. Common Structure Package - user chooses prewritten structure package pertinent to his problem and key punches this package with necessary physical and geometrical constants as described in Appendix B.

c. Boundary Condition Packages - user chooses the three boundary condition packages pertinent to his problem and key punches these packages with necessary magnitude constants as described in Appendix C.

d. Printout Quantities Subroutine Specification - user writes and key punches a subroutine to specify the points at which he desires data printed out and the quantities to be printed out at those points, as described in Appendix D.

e. Seven (7) Main Program Input Data Cards - user specifies following quantities per format listed.

M_o, x_o, Δx, c_1, c_2	(I4,4E15.8)
A_1, A_2, A_3, A_4, A_5	(5E15.8)
A_6, A_7	(2E15.8)
B_1, B_2, B_3, B_4, B_5	(5E15.8)
B_6, B_7	(2E15.8)
C_1, C_2, C_3, C_4, C_5	(5E15.8)
C_6, C_7	(2E15.8)

2. <u>Output from Machine</u>

The output of the program will include a preliminary printout listing all input data which was read into the main program. The printout of the quantities at mesh points will then begin as specified by the user.

B. Structure and/or Boundary Conditions Not Prewritten

1. <u>Input to Machine</u>

a. Main Program Deck - invariant

b. Structure Package - user chooses one of the prewritten common structure packages, if applicable, and follows details of Appendix B. If not applicable, user writes his own structure package as detailed in Appendix D.

c. Boundary Condition Packages - user chooses or writes the three boundary condition packages pertinent to his problem. Instructions for use of prewritten packages are in Appendix C. Instructions for writing a new package are in Appendix D.

d. Printout Quantities Subroutine Specification - user writes and key punches a subroutine to specify the points at which he desires data printed out and the quantities to be printed out at those points, as described in Appendix D.

e. Seven (7) Main Program Input Data Cards - user specifies following quantities per format listed.

M_o, x_o, Δx, c_1, c_2	(I4,4E15.8)
A_1, A_2, A_3, A_4, A_5	(5E15.8)
A_6, A_7	(2E15.8)
B_1, B_2, B_3, B_4, B_5	(5E15.8)
B_6, B_7	(2E15.8)
C_1, C_2, C_3, C_4, C_5	(5E15.8)
C_6, C_7	(2E15.8)

2. Output from Machine

The output of the program will include a preliminary printout listing all input data which was read into the main program. The printout of the quantities at mesh points will then begin as specified by the user.

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CR-1306

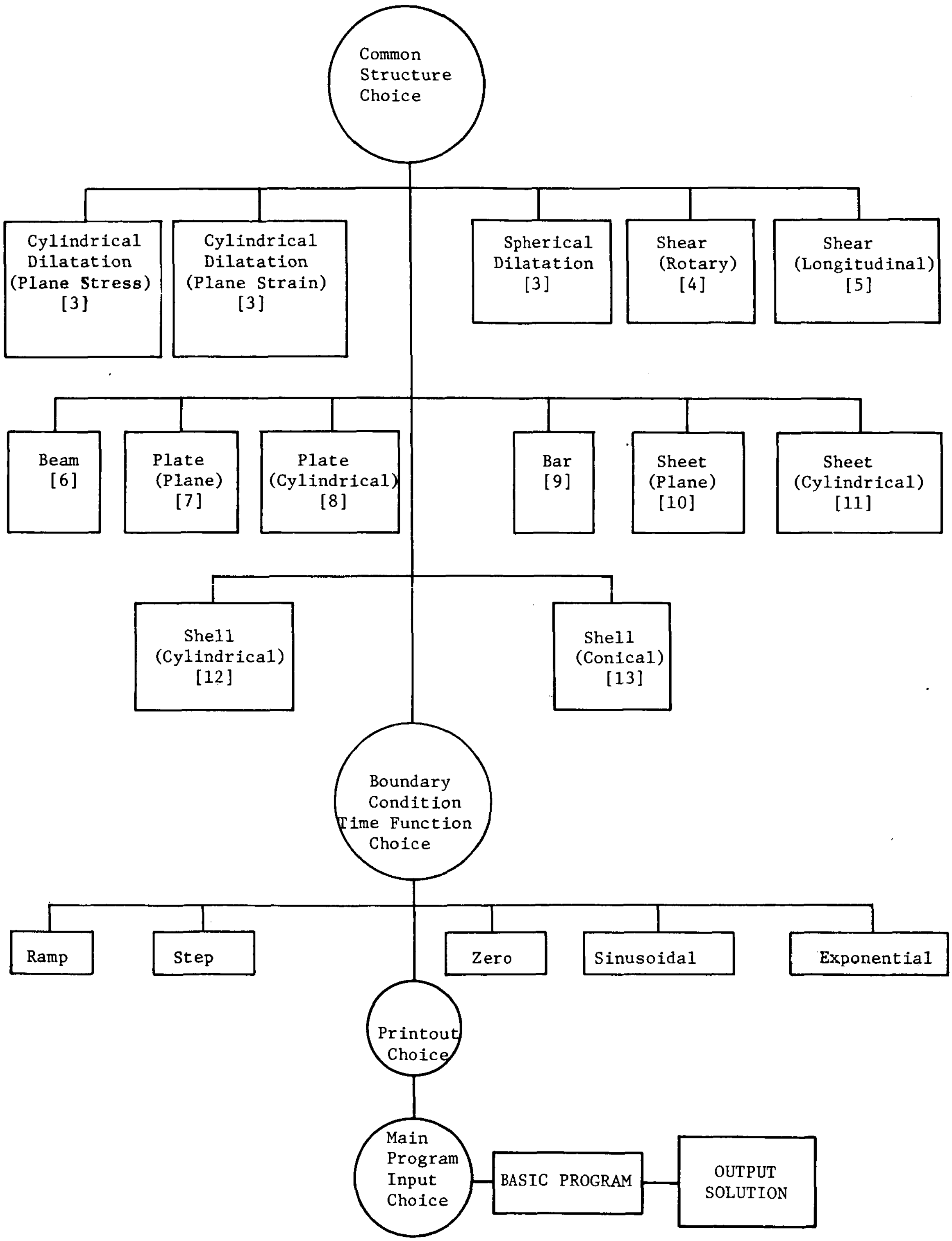


Figure 1. Prewritten Packages for User Specified Subroutines

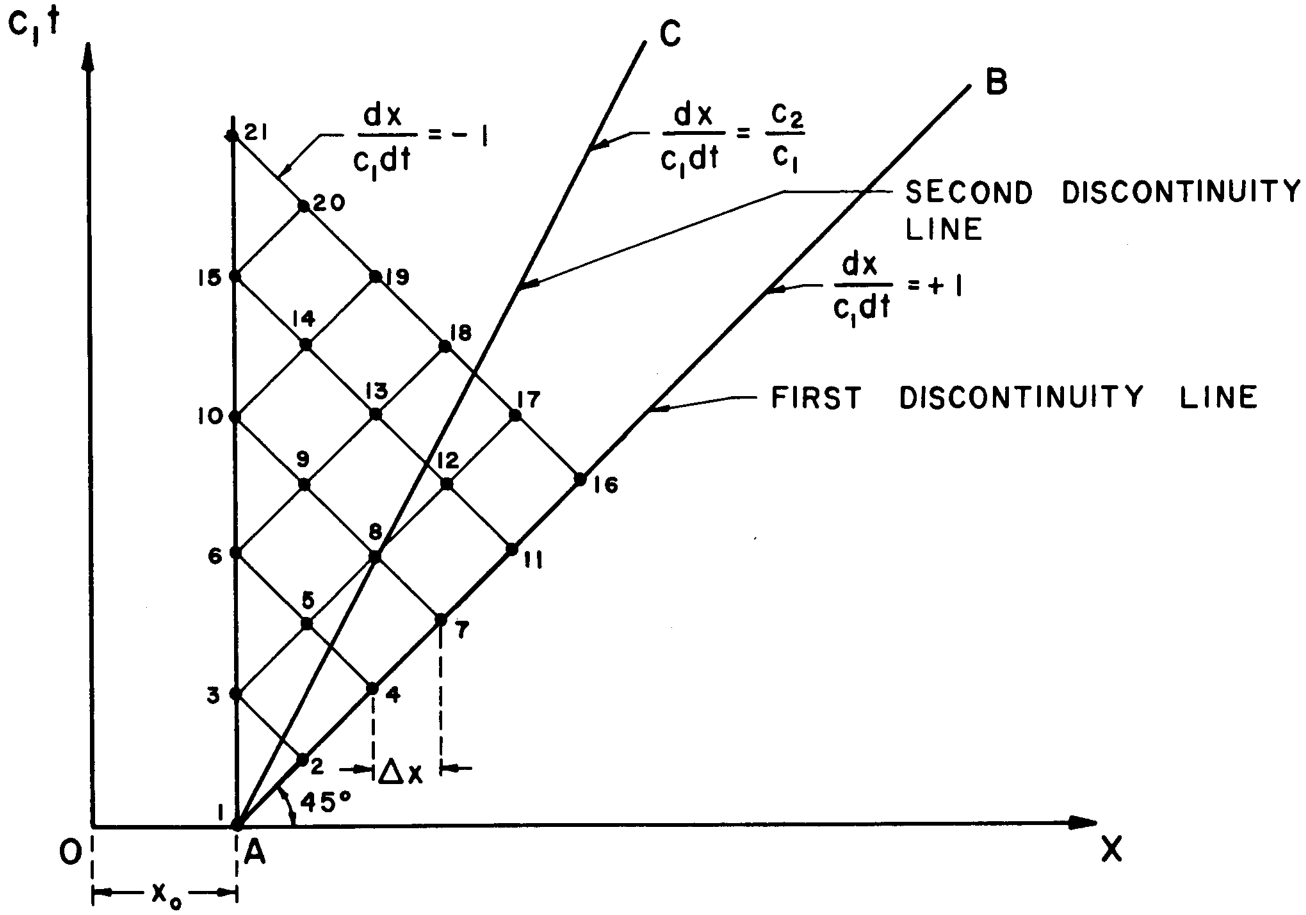


Figure 2. The Physical Plane

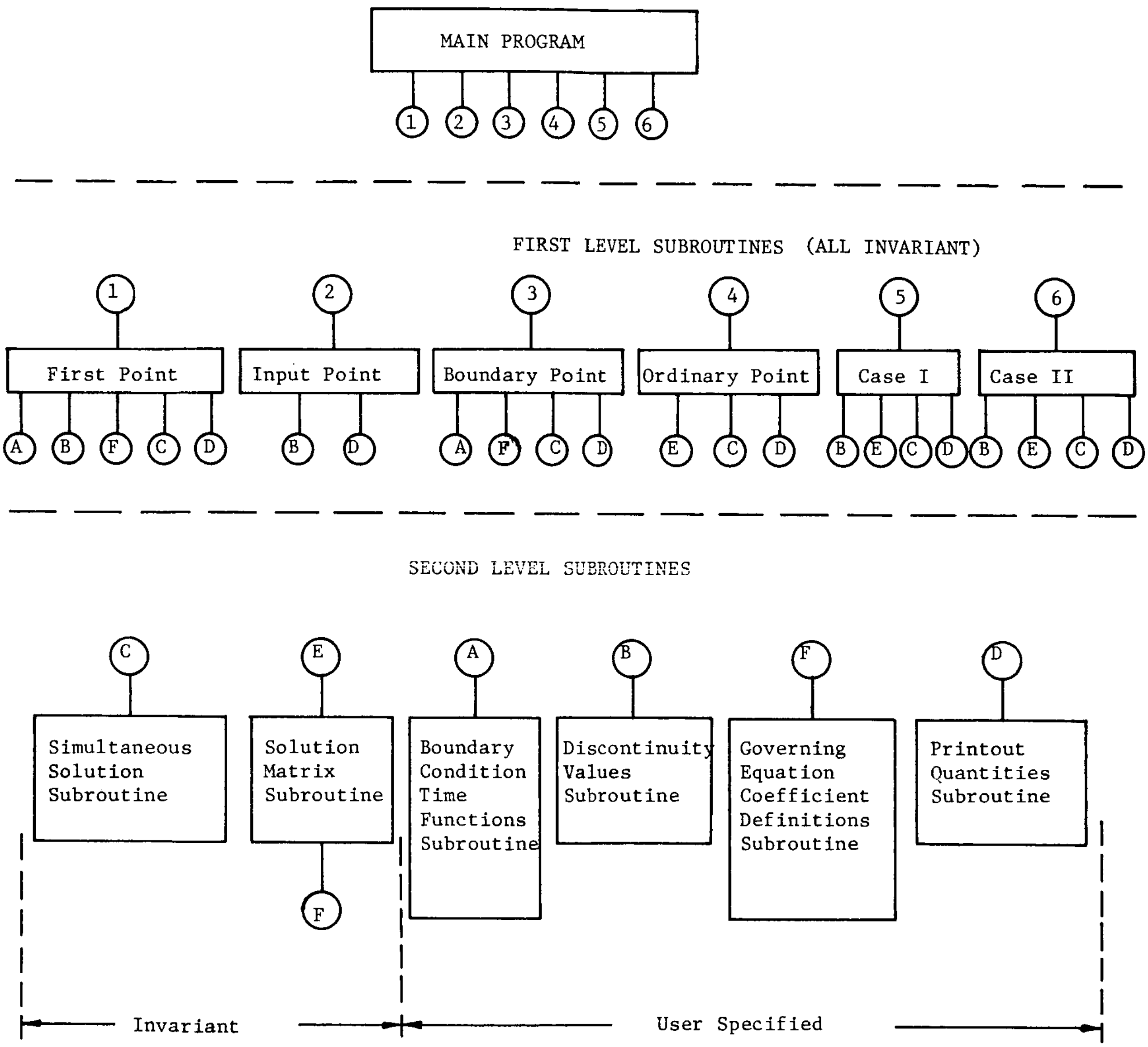


Figure 3. The Program

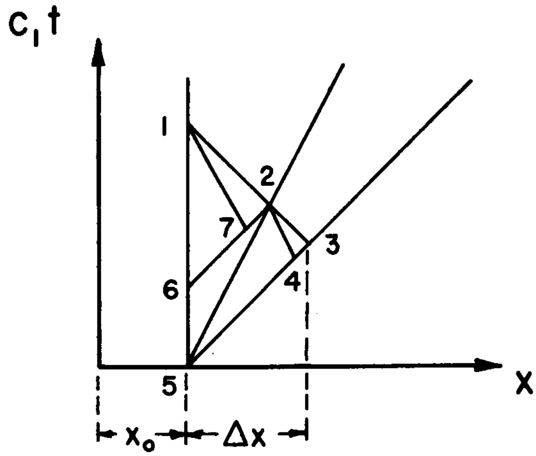


Figure 4. The First Point

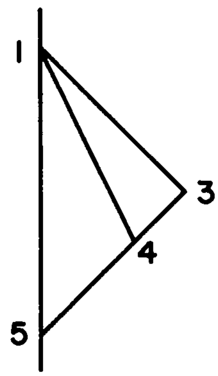


Figure 5. A Boundary Point

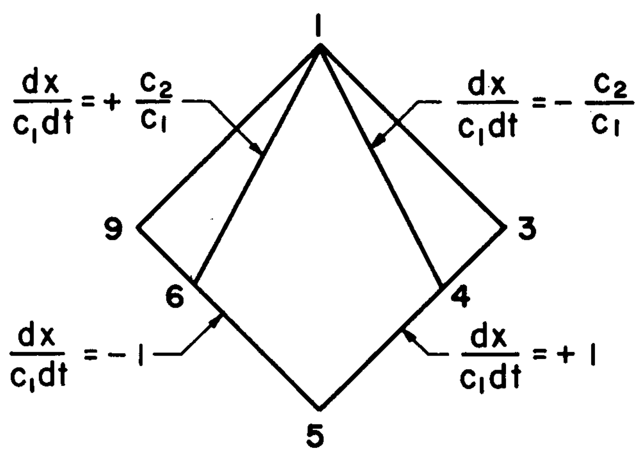
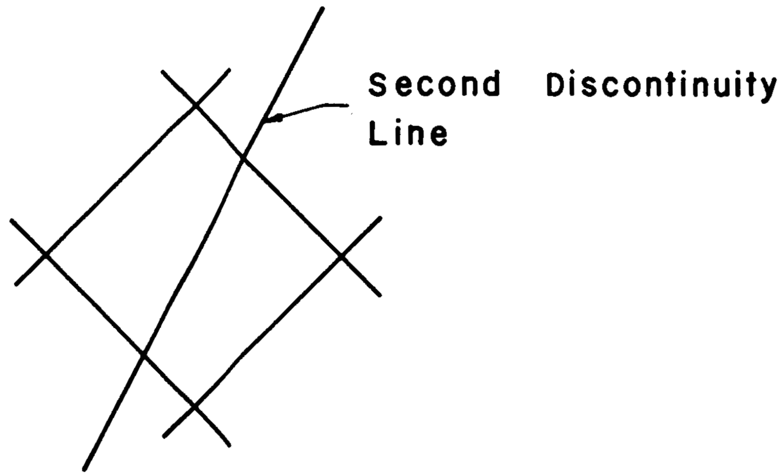
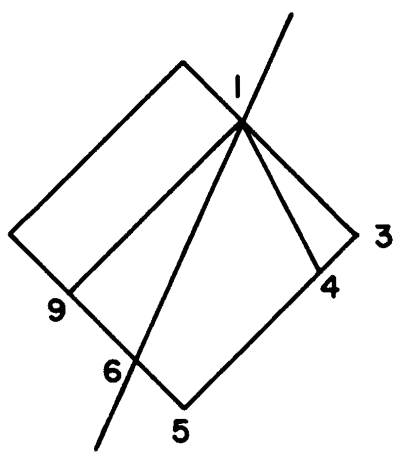


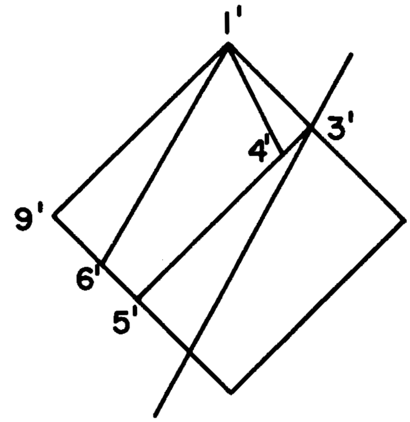
Figure 6. An Ordinary Point



a. Occurence of a Case I Point

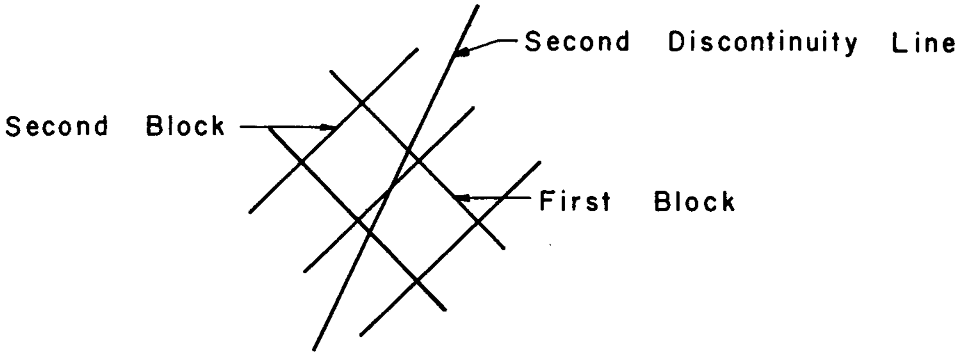


b. Evaluation at Point 1

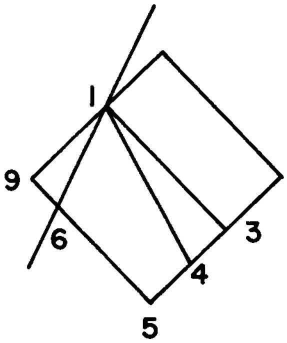


c. Evaluation at Point 1'

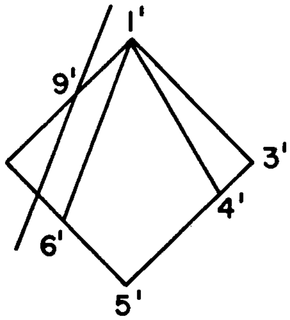
Figure 7. A Case I Point



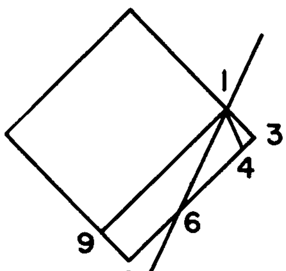
a. Occurence of a Case II Set of Points



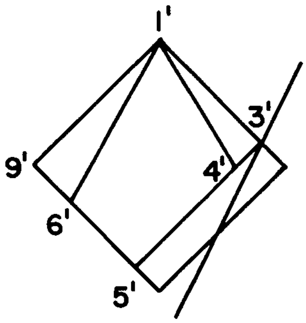
b. Evaluation at Point 1, First Block



c. Evaluation at Point 1', First Block



d. Evaluation at Point 1, Second Block



e. Evaluation at Point 1', Second Block

Figure 8. A Case II Point

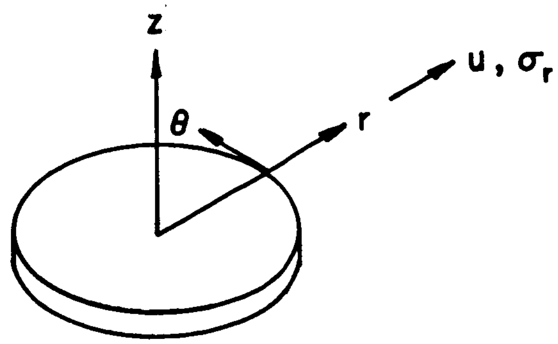


Figure 9. Cylindrical Dilatation (Plane Stress)

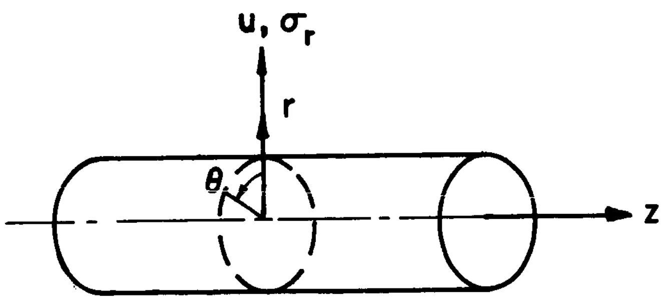


Figure 10. Cylindrical Dilatation (Plane Strain)

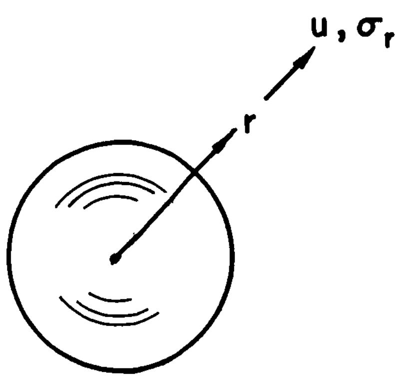


Figure 11. Spherical Dilatation

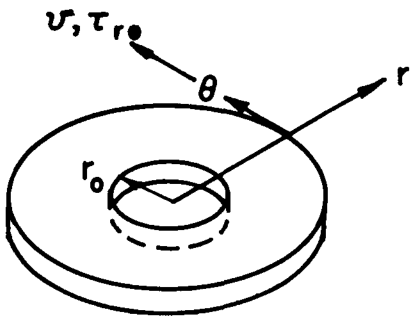


Figure 12. Shear (Rotary)

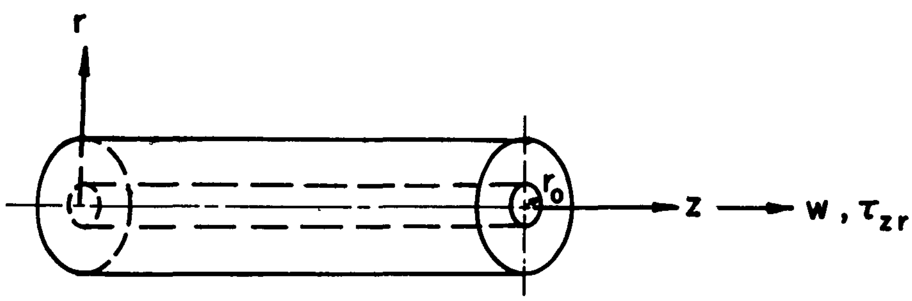


Figure 13. Shear (Longitudinal)

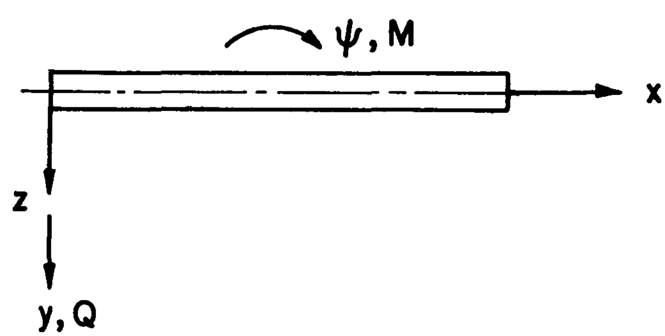


Figure 14. Timoshenko Beam

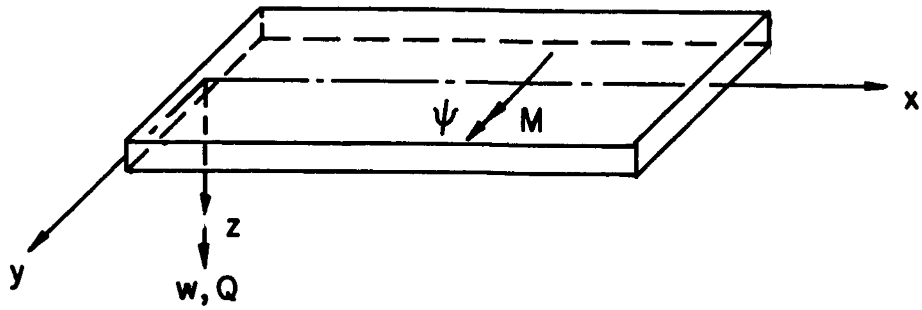


Figure 15. Plate (Plane)

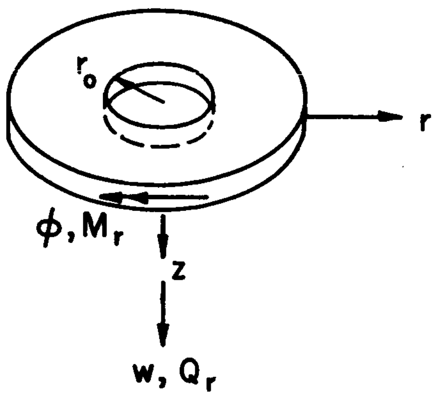


Figure 16. Plate (Cylindrical)

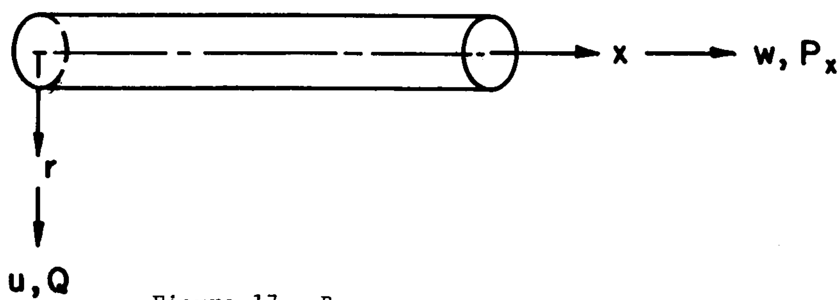


Figure 17. Bar

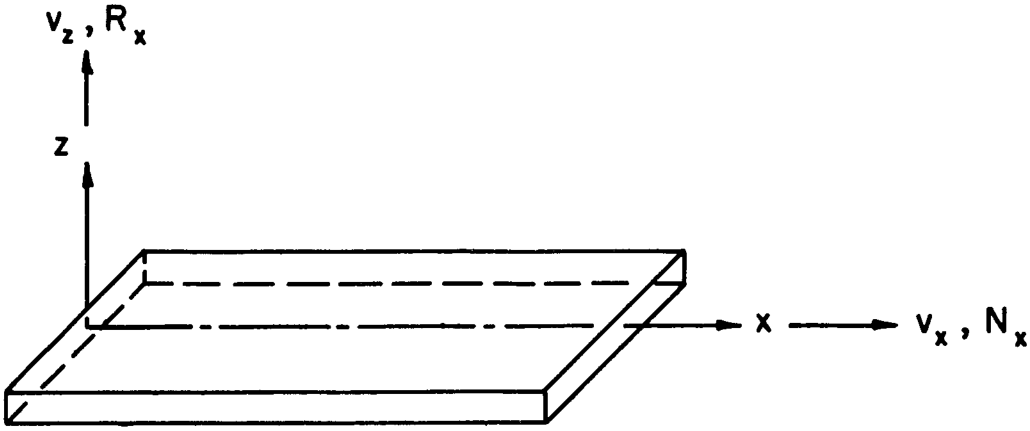


Figure 18. Sheet (Plane)

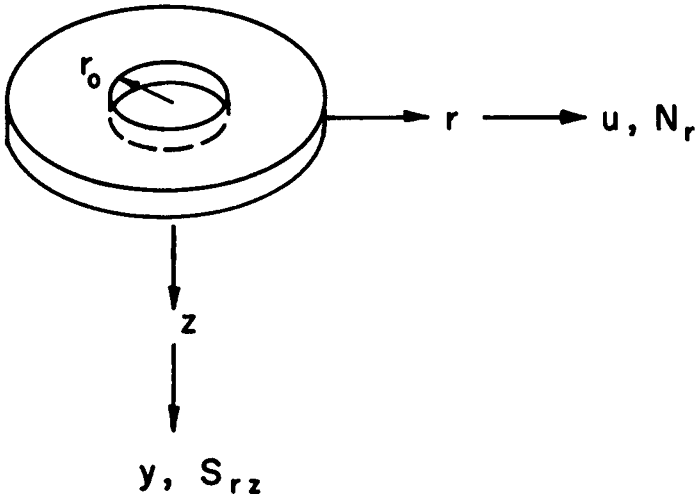


Figure 19. Sheet (Cylindrical)

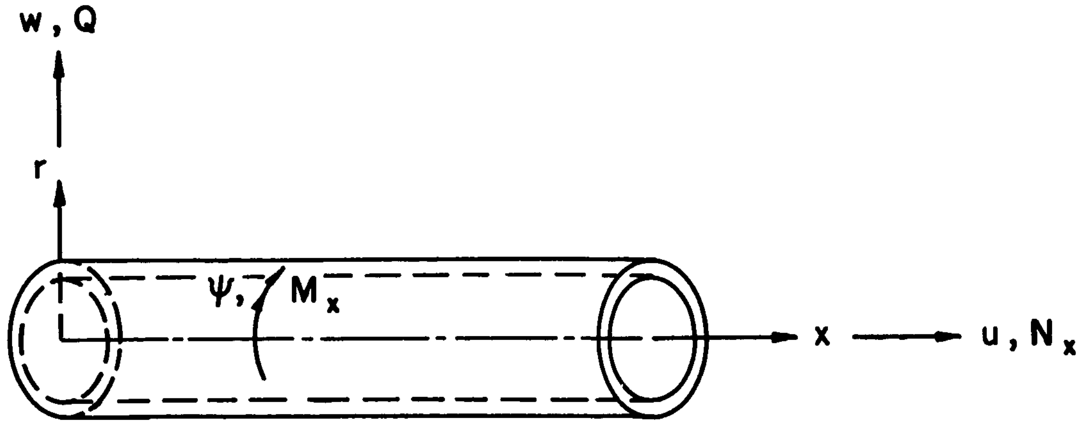


Figure 20. Cylindrical Shell

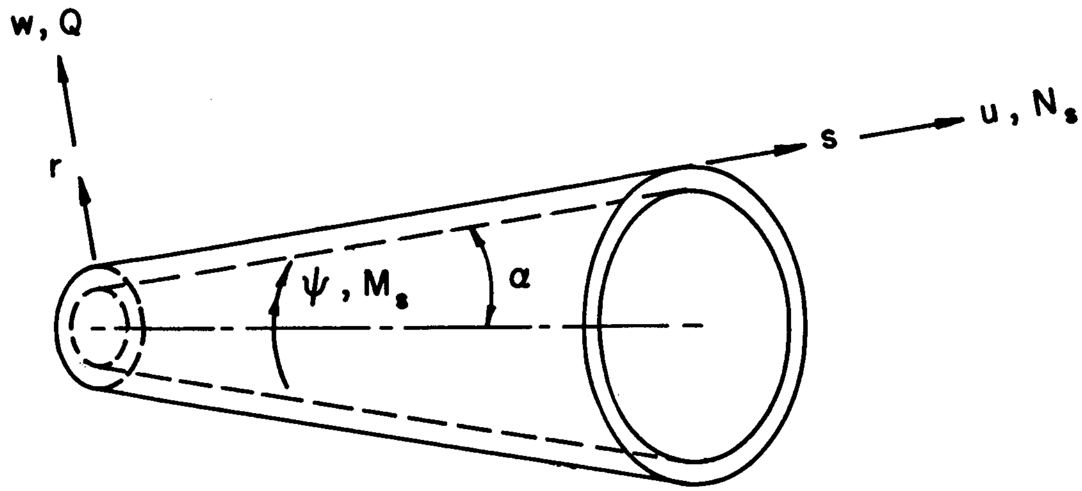


Figure 21. Conical Shell

APPENDIX A: LISTING OF MCDIT 21 MAIN PROGRAM AND INVARIANT SUBROUTINES

MAIN PROGRAM

```
      COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
     1U(9),V(5),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
    1 FORMAT(I4,4E15.8)
    2 FORMAT(2E15.8)
    3 FORMAT(5E15.8)
    4 FORMAT(1H ,38HNUMBER OF POINTS ALONG LEADING WAVE = ,I4)
    5 FORMAT(1H -8HXZERO = ,E15.8,5X,9HDELTAX = ,E15.8)
    6 FORMAT(1H ,5HC1 = ,E15.8,8X,5HC2 = ,E15.8)
    7 FORMAT(1H ,1H(,E15.8,7H)*U1X+(,E15.8,6H)*U1+(,E15.8,7H)*U2X+(,E15.
     18,6H)*U2+(,E15.8,7H)*U3X+(,E15.8,4H)*U3)
    8 FORMAT(1H ,4H  +(,E15.8,37H)*U1T = BOUNDARY CONDITION FUNCTION 1)
    9 FORMAT(1H ,4H  +(,E15.8,37H)*U2T = BOUNDARY CONDITION FUNCTION 2)
   10 FORMAT(1H ,4H  +(,E15.8,37H)*U3T = BOUNDARY CONDITION FUNCTION 3)
   24 FORMAT(1H ,43HSLOPE OF II+ LINE EXCEEDS OR EQUALS MAXIMUM)
   25 FORMAT(1H ,41HVALUE OF 3.0 COMPATIBLE WITH THIS PROGRAM)
   37 FORMAT(1H ,14HERROR IN LOGIC)
   69 FORMAT(1H ,////)
      READ1,MZERO,XZERO,PINC,C1,C2
      READ3,A(1),A(2),A(3),A(4),A(5)
      READ2,A(6),A(7)
      READ3,B(1),B(2),B(3),B(4),B(5)
      READ2,B(6),B(7)
      READ3,C(1),C(2),C(3),C(4),C(5)
      READ2,C(6),C(7)
      PRINT4,MZERO
      PRINT5,XZERO,PINC
      PRINT6,C1,C2
      PRINT7,A(1),A(2),A(3),A(4),A(5),A(6)
      PRINT8,A(7)
      PRINT7,B(1),B(2),B(3),B(4),B(5),B(6)
      PRINT9,B(7)
      PRINT7,C(1),C(2),C(3),C(4),C(5),C(6)
      PRINT10,C(7)
      EM=C1/C2
      IF(EM-3.)22,23,23
   23 PRINT24
      PRINT25
      GOTO9999
   22 PRINT69
      XLI=1.
      CALLFIRSTP
   91 LI=2
      IYZ=1.-2./(EM+1.)
      GOTO26
   27 XLI=LI
      I=1
      IYZZ=IYZ
      IYZ=XLI-(2.*XLI)/(EM+1.)
      CALLINPUTP
   35 IF(I-LI)28,29,29
   28 IF(I-1-IYZZ)30,31,32
   31 IF(IYZZ-IYZ)33,34,34
   32 IF(I-1-IYZ)36,36,30
```

```
   36 PRINT37
      GOTO9999
   29 CALLBOUNDP
   92 LI=LI+1
   26 IF(LI-MZERO)27,9999,9999
   30 CALLORDINP
      GOTO35
   33 CALLCASE32
      GOTO35
   34 CALLCASE1P
      GOTO35
 9999 CALLEXIT
      END
```

SIMULTANEOUS SOLUTION SUBROUTINE

```
      SUBROUTINEMASUB
      COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
     1U(9),V(9),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
      N=M-1
      DO 5200 NN=1,N,1
      NNN=NN+1
      DO 5100 JJ=NNN,M,1
      FRAC=-Y(JJ,NN)/Y(NN,NN)
      DO 5050 KK=NN,M,1
 5050 Y(JJ,KK)=FRAC*Y(NN,KK)+Y(JJ,KK)
 5100 Z(JJ)=FRAC*Z(NN)+Z(JJ)
 5200 CONTINUE
      DO 5500 NN=1,N,1
      NNN=M-NN
      JJ=NNN+1
      DO 5400 KK=1,NNN,1
 5400 Z(KK)=-Z(JJ)*(Y(KK,JJ)/Y(JJ,JJ))+Z(KK)
 5500 CONTINUE
      DO 5600 KKK=1,M,1
 5600 UU(KKK)=Z(KKK)/Y(KKK,KKK)
 9999 RETURN
      END
```

FIRST POINT SUBROUTINE

```
 SUBROUTINEFIRSTP
 COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
1U(9),V(9),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
 DIMENSIONHOLD(12)
 ZERO=0.
 PHI=(EM-1.)/(EM+1.)
 ALPH=1.-((4.*EM)/((1.+EM)**2))
 SLOP=PHI
 X5=XZERO
 T5=0.
 X1=XZERO
 T1=2.*PINC/C1
 X3=XZERO+PINC
 T3=T1/2.
 X2=XZERO+(2.*PINC)/(1.+EM)
 X4=ALPH*(X5-X3)+X3
 X6=XZERO
 X7=SLOP*(X6-X2)+X2
 CALLJUMPI(X5,UX5,UT5,VX5,VT5)
 CALLJUMPII(XZERO,WX5,WT5)
 CALLJUMPI(X3,UX3,UT3,VX3,VT3)
 CALLJUMPI(X4,UX4,UT4,VX4,VT4)
 CALLGECOFF(1,X2,X3)
 CALLGECOFF(2,X2,X6)
 CALLGECOFF(3,X1,X2)
 CALLGECOFG(1,X2,X3)
 CALLGECOFG(2,X2,X6)
 CALLGECOFG(3,X1,X2)
 CALLGECOFH(1,X2,X4)
 CALLGECOFH(2,X2,X5)
 CALLGECOFH(3,X1,X7)
 CALLBCTF1(T1,R1)
 CALLBCTF2(T1,R2)
 CALLBCTF3(T1,R3)
 CALLJUMPII(X2,DU(8),DU(9))
 DX23=X2-X3
 DX24=X2-X4
 DX26=X2-X6
 DT15=T1-T5
 DX25=X2-X5
 DX12=X1-X2
 DX17=X1-X7
 Y(1,1)=A(1)
 Y(1,2)=A(2)*DT15/2.+A(7)
 Y(1,3)=A(3)
 Y(1,4)=A(4)*DT15/2.
 Y(1,5)=A(5)
 Y(1,6)=A(6)*DT15/2.
 Y(1,7)=0.
 Y(1,8)=0.
 Y(1.9)=0.
 Y(1,10)=0.
 Y(1,11)=0.
```

```
Y(1,12)=0.
Z(1)=R1-A(2)*DT15*UT5/2.-A(4)*DT15*VT5/2.-A(6)*DT15*WT5/2.
Y(2,1)=C1*(1.-F(1,3)*DX12/2.)
Y(2,2)=1.-C1*DX12*DT15*F(2,3)/4.
Y(2,3)=-C1*DX12*F(3,3)/2.
Y(2,4)=-C1*DX12*DT15*F(4,3)/4.
Y(2,5)=-C1*DX12*F(5,3)/2.
Y(2,6)=-C1*DX12*DT15*F(6,3)/4.
Y(2,7)=C1*(-1.-F(1,3)*DX12/2.-F(2,3)*DX12*DX23/4.)
Y(2,8)=-1.+F(2,3)*DX12*DX23/4.
Y(2,9)=(-C1*DX12/2.)*(F(3,3)+F(4,3)*DX23/2.)
Y(2,10)=F(4,3)*DX12*DX23/4.
Y(2,11)=(-C1*DX12/2.)*(F(5,3)+F(6,3)*DX24/2.)
Y(2,12)=C1*F(6,3)*DX12*DX24/(4.*C2)
Z(2)=(C1*DX12/2.)*(F(2,3)*DT15*UT5/2.+F(2,3)*DX23*(UX3-UT3/C1)/2.+
1F(4,3)*DT15*VT5/2.+F(4,3)*DX23*(VX3-VT3/C1)/2.+F(6,3)*DT15*WT5/2.+
2F(6,3)*DX24*(-DU(8)+DU(9)/C2)/2.)
Y(3,1)=B(1)
Y(3,2)=B(2)*DT15/2.
Y(3,3)=B(3)
Y(3,4)=B(4)*DT15/2.+B(7)
Y(3,5)=B(5)
Y(3,6)=B(6)*DT15/2.
Y(3,7)=0.
Y(3,8)=0.
Y(3,9)=0.
Y(3,10)=0.
Y(3,11)=0.
Y(3,12)=0.
Z(3)=R2-B(2)*DT15*UT5/2.-B(4)*DT15*VT5/2.-B(6)*DT15*WT5/2.
Y(4,1)=-C1*DX12*G(1,3)/2.
Y(4,2)=-C1*DX12*DT15*G(2,3)/4.
Y(4,3)=C1*(1.-G(3,3)*DX12/2.)
Y(4,4)=1.-C1*DX12*DT15*G(4,3)/4.
Y(4,5)=-C1*DX12*G(5,3)/2.
Y(4,6)=-C1*DX12*DT15*G(6,3)/4.
Y(4,7)=(-C1*DX12/2.)*(G(1,3)+G(2,3)*DX23/2.)
Y(4,8)=G(2,3)*DX12*DX23/4.
Y(4,9)=C1*(-1.-G(3,3)*DX12/2.-G(4,3)*DX12*DX23/4.)
Y(4,10)=-1.+G(4,3)*DX12*DX23/4.
Y(4,11)=(-C1*DX12/2.)*(G(5,3)+G(6,3)*DX24/2.)
Y(4,12)=C1*DX12*DX24*G(6,3)/(4.*C2)
Z(4)=(C1*DX12/2.)*(G(2,3)*DT15*UT5/2.+G(2,3)*DX23*(UX3-UT3/C1)/2.+
1G(4,3)*DT15*VT5/2.+G(4,3)*DX23*(VX3-VT3/C1)/2.+G(6,3)*DT15*WT5/2.+
2G(6,3)*DX24*(-DU(8)+DU(9)/C2)/2.)
Y(5,1)=(-C2*DX17/2.)*H(1,3)*(1.+SLOP*PHI)
Y(5,2)=(-C2*DX17*DT15/4.)*H(2,3)*(1.+SLOP*PHI)
Y(5,3)=(-C2*DX17/2.)*H(3,3)*(1.+SLOP*PHI)
Y(5,4)=(-C2*DX17*DT15/4.)*H(4,3)*(1.+SLOP*PHI)
Y(5,5)=C2*(1.-SLOP*PHI-(H(5,3)*DX17/2.)*(1.+SLOP*PHI))
Y(5,6)=1.-SLOP*PHI-(C2*DX17*DT15*H(6,3)/4.)*(1.+SLOP*PHI)
Y(5,7)=(-C2*DX17/2.)*(H(1,3)*(1.-SLOP)+(H(2,3)*DX23/2.)*(1.-SLOP))
Y(5,8)=(C2*DX17*DX23*H(2,3)/(4.*C1))*(1.-SLOP)
```

```
 Y(5,9)=(-C2*DX17/2.)*(H(3,3)*(1.-SLOP)+(H(4,3)*DX23/2.)*(1.-SLOP))
 Y(5,10)=(C2*DX17*DX23*H(4,3)/(4.*C1))*(1.-SLOP)
 Y(5,11)=C2*(SLOP-1.-(H(5,3)*DX17/2.)*(1.-SLOP)-(H(6,3)*DX17*DX24/4
1.)*(1.-SLOP))
 Y(5,12)=SLOP-1.+(H(6,3)*DX17*DX24/4.)*(1.-SLOP)
 Z(5)=SLOP*(WT5*(1.-PHI))+C2*SLOP*WX5*(1.-PHI)+(C2*DX17/2.)*(H(1,3)
1*SLOP*UX5*(1.-PHI)+(H(2,3)*DT15*UT5/2.)*(1.+SLOP*PHI)+(H(2,3)*DX23
2*(UX3-UT3/C1)/2.)*(1.-SLOP)+H(3,3)*SLOP*VX5*(1.-PHI)+(H(4,3)*DT15*
3VT5/2.)*(1.+SLOP*PHI)+(H(4,3)*DX23*(VX3-VT3/C1)/2.)*(1.-SLOP)+H(5,
43)*SLOP*WX5*(1.-PHI)+(H(6,3)*DT15*WT5/2.)*(1.+SLOP*PHI)+(H(6,3)*DX
524*(-DU(8)+DU(9)/C2)/2.)*(1.-SLOP))
 Y(6,1)=C(1)
 Y(6,2)=C(2)*DT15/2.
 Y(6,3)=C(3)
 Y(6,4)=C(4)*DT15/2.
 Y(6,5)=C(5)
 Y(6,6)=C(6)*DT15/2.+C(7)
 Y(6,7)=0.
 Y(6,8)=0.
 Y(6,9)=0.
 Y(6,10)=0.
 Y(6,11)=0.
 Y(6,12)=0.
 Z(6)=R3-C(2)*DT15*UT5/2.-C(4)*DT15*VT5/2.-C(6)*DT15*WT5/2.
 Y(7,1)=0.
 Y(7,2)=0.
 Y(7,3)=0.
 Y(7,4)=0.
 Y(7,5)=0.
 Y(7,6)=0.
 Y(7,7)=C1*(1.-F(1,1)*DX23/2.-F(2,1)*DX23**2/4.)
 Y(7,8)=1.+F(2,1)*DX23**2/4.
 Y(7,9)=(-C1*DX23/2.)*(F(3,1)+F(4,1)*DX23/2.)
 Y(7,10)=F(4,1)*DX23**2/4.
 Y(7,11)=(-C1*DX23/2.)*(F(5,1)+F(6,1)*DX24/2.)
 Y(7,12)=C1*DX23*DX24*F(6,1)/(4.*C2)
 Z(7)=UT3+C1*UX3+C1*DX23*(F(1,1)*UX3/2.+F(2,1)*DX23*(UX3-UT3/C1)/4.
1+F(3,1)*VX3/2.+F(4,1)*DX23*(VX3-VT3/C1)/4.+F(5,1)*(-DU(8))/2.+F(6,
21)*DX24*(-DU(8)+DU(9)/C2)/4.)
 Y(8,1)=C1*PHI*(1.+F(1,2)*DX26/2.)
 Y(8,2)=PHI*(-1.+C1*DX26*DT15*F(2,2)/4.)
 Y(8,3)=C1*DX26*F(3,2)*PHI/2.
 Y(8,4)=C1*DX26*DT15*F(4,2)*PHI/4.
 Y(8,5)=C1*DX26*F(5,2)*PHI/2.
 Y(8,6)=C1*DX26*DT15*F(6,2)*PHI/4.
 Y(8,7)=C1*(-1.+F(1,2)*DX26/2.+F(2,2)*DX26*DX23/4.)
 Y(8,8)=1.-F(2,2)*DX26*DX23/4.
 Y(8,9)=(C1*DX26/2.)*(F(3,2)+F(4,2)*DX23/2.)
 Y(8,10)=-F(4,2)*DX26*DX23/4.
 Y(8,11)=(C1*DX26/2.)*(F(5,2)+F(6,2)*DX24/2.)
 Y(8,12)=-C1*DX26*DX24*F(6,2)/(4.*C2)
 Z(8)=UT5*(1.-PHI)+C1*UX5*(PHI-1.)-(C1*DX26/2.)*(F(1,2)*UX5*(1.-PHI
1)+F(2,2)*DX23*(UX3-UT3/C1)/2.+F(2,2)*DT15*PHI*UT5/2.+F(3,2)*VX5*(1
```

```
2.-PHI)+F(4,2)*DX23*(VX3-VT3/C1)/2.+F(4,2)*DT15*PHI*VT5/2.+F(5,2)*W
3X5*(1.-PHI)+F(6,2)*DX24*(-DU(8)+DU(9)/C2)/2.+F(6,2)*DT15*PHI*WT5/2
4.)
 Y(9,1)=C1*PHI*G(1,2)*DX26/2.
 Y(9,2)=C1*DX26*DT15*G(2,2)*PHI/4.
 Y(9,3)=C1*PHI*(1.+G(3,2)*DX26/2.)
 Y(9,4)=PHI*(-1.+C1*DX26*DT15*G(4,2)/4.)
 Y(9,5)=C1*G(5,2)*DX26*PHI/2.
 Y(9,6)=C1*DX26*DT15*G(6,2)*PHI/4.
 Y(9,7)=(C1*DX26/2.)*(G(1,2)+G(2,2)*DX23/2.)
 Y(9,8)=-G(2,2)*DX26*DX23/4.
 Y(9,9)=C1*(-1.+G(3,2)*DX26/2.+G(4,2)*DX26*DX23/4.)
 Y(9,10)=1.-G(4,2)*DX26*DX23/4.
 Y(9,11)=(C1*DX26/2.)*(G(5,2)+G(6,2)*DX24/2.)
 Y(9,12)=-C1*DX26*DX24*G(6,2)/(4.*C2)
 Z(9)=VT5*(1.-PHI)+C1*VX5*(PHI-1.)-(C1*DX26/2.)*(G(1,2)*UX5*(1.-PHI
1)+G(2,2)*DX23*(UX3-UT3/C1)/2.+G(2,2)*DT15*PHI*UT5/2.+G(3,2)*VX5*(1
2.-PHI)+G(4,2)*DX23*(VX3-VT3/C1)/2.+G(4,2)*DT15*PHI*VT5/2.+G(5,2)*W
3X5*(1.-PHI)+G(6,2)*DX24*(-DU(8)+DU(9)/C2)/2.+G(6,2)*DT15*PHI*WT5/2
4.)
 Y(10,1)=0.
 Y(10,2)=0.
 Y(10,3)=0.
 Y(10,4)=0.
 Y(10,5)=0.
 Y(10,6)=0.
 Y(10,7)=(-C1*DX23/2.)*(G(1,1)+G(2,1)*DX23/2.)
 Y(10,8)=G(2,1)*DX23**2/4.
 Y(10,9)=C1*(1.-G(3,1)*DX23/2.-G(4,1)*DX23**2/4.)
 Y(10,10)=1.+G(4,1)*DX23**2/4.
 Y(10,11)=(-C1*DX23/2.)*(G(5,1)+G(6,1)*DX24/2.)
 Y(10,12)=C1*DX23*DX24*G(6,1)/(4.*C2)
 Z(10)=VT3+C1*VX3+C1*DX23*(G(1,1)*UX3/2.+G(2,1)*DX23*(UX3-UT3/C1)/4
1.+G(3,1)*VX3/2.+G(4,1)*DX23*(VX3-VT3/C1)/4.+G(5,1)*(-DU(8))/2.+G(6
2,1)*DX24*(-DU(8)+DU(9)/C2)/4.)
 Y(11,1)=0.
 Y(11,2)=0.
 Y(11,3)=0.
 Y(11,4)=0.
 Y(11,5)=0.
 Y(11,6)=0.
 Y(11,7)=(-C2*DX24/2.)*(H(1,1)+H(2,1)*DX23/2.)
 Y(11,8)=C2*DX24*DX23*H(2,1)/(4.*C1)
 Y(11,9)=(-C2*DX24/2.)*(H(3,1)+H(4,1)*DX23/2.)
 Y(11,10)=C2*DX24*DX23*H(4,1)/(4.*C1)
 Y(11,11)=C2*(1.-H(5,1)*DX24/2.-H(6,1)*DX24**2/4.)
 Y(11,12)=1.+H(6,1)*DX24**2/4.
 Z(11)=DU(9)+C2*DU(8)+C2*DX24*(H(1,1)*UX4/2.+H(2,1)*DX23*(UX3-UT3/C
11)/4.+H(3,1)*VX4/2.+H(4,1)*DX23*(VX3-VT3/C1)/4.+H(5,1)*(-DU(8))/2.
2+H(6,1)*DX24*(-DU(8)+DU(9)/C2)/4.)
 Y(12,1)=0.
 Y(12,2)=0.
 Y(12,3)=0.
```

```
      Y(12,4)=0.
      Y(12,5)=0.
      Y(12,6)=0.
      Y(12,7)=(C2*DX25/2.)*(H(1,2)+H(2,2)*DX23/2.)
      Y(12,8)=-C2*DX25*DX23*H(2,2)/(4.*C1)
      Y(12,9)=(C2*DX25/2.)*(H(3,2)+H(4,2)*DX23/2.)
      Y(12,10)=-C2*DX25*DX23*H(4,2)/(4.*C1)
      Y(12,11)=C2*(-1.+H(5,2)*DX25/2.+H(6,2)*DX25*DX24/4.)
      Y(12,12)=1.-H(6,2)*DX25*DX24/4.
      Z(12)=WT5-C2*WX5-(C2*DX25/2.)*(H(1,2)*UX5+H(2,2)*DX23*(UX3-UT3/C1)
     1/2.+H(3,2)*VX5+H(4,2)*DX23*(VX3-VT3/C1)/2.+H(5,2)*WX5+H(6,2)*DX24*
     2(-DU(8)+DU(9)/C2)/2.)
      M=12
      IF(Y(1,1))1,2,1
    2 DO3J=1,12
      HOLD(J)=Y(1,J)
      Y(1,J)=Y(2,J)
    3 Y(2,J)=HOLD(J)
      CEEP=Z(1)
      Z(1)=Z(2)
      Z(2)=CEEP
    1 IF(Y(3,3))4,5,4
    5 DO6J=1,12
      HOLD(J)=Y(3,J)
      Y(3,J)=Y(4,J)
    6 Y(4,J)=HOLD(J)
      CEEP=Z(3)
      Z(3)=Z(4)
      Z(4)=CEEP
    4 IF(Y(6,6))98,8,98
    8 DO9J=1,12
      HOLD(J)=Y(6,J)
      Y(6,J)=Y(5,J)
    9 Y(5,J)=HOLD(J)
      CEEP=Z(6)
      Z(6)=Z(5)
      Z(5)=CEEP
   98 CALLMASUB
   99 UP(2)=UU(7)
      UP(3)=UU(8)
      UP(5)=UU(9)
      UP(6)=UU(10)
      UP(8)=UU(11)
      UP(9)=UU(12)
      U(2,2)=UU(1)
      U(3,2)=UU(2)
      U(5,2)=UU(3)
      U(6,2)=UU(4)
      U(8,2)=UU(5)
      U(9,2)=UU(6)
      U(1,2)=(U(3,2)+UT5)*DT15/2.
      U(4,2)=(U(6,2)+VT5)*DT15/2.
      U(7,2)=(U(9,2)+WT5)*DT15/2.
```

```
      UP(1)=((UP(2)+UX3)/2.-(UP(3)+UT3)/(2.*C1))*DX23
      UP(4)=((UP(5)+VX3)/2.-(UP(6)+VT3)/(2.*C1))*DX23
      UP(7)=((UP(8)-DU(8))/2.-(UP(9)-DU(9))/(2.*C2))*DX24
      U(1,1)=0.
      U(2,1)=UX3
      U(3,1)=UT3
      U(4,1)=0.
      U(5,1)=VX3
      U(6,1)=VT3
      U(7,1)=0.
      U(8,1)=0.
      U(9,1)=0.
      CALLPRINTO(X5,T5,ZERO,UX5,UT5,ZERO,VX5,VT5,ZERO,WX5,WT5,XLI)
      CALLPRINTO(X3,T3,U(1,1),U(2,1),U(3,1),U(4,1),U(5,1),U(6,1),U(7,1),
     1U(8,1),U(9,1),XLI)
      CALLPRINTO(X1,T1,U(1,2),U(2,2),U(3,2),U(4,2),U(5,2),U(6,2),U(7,2),
     1U(8,2),U(9,2),XLI)
 9999 RETURN
      END
```

INPUT POINT SUBROUTINE

```
      SUBROUTINEINPUTP
      COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
     1U(9),V(9),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
      X=XZERO+XLI*PINC
      T=XLI*PINC/C1
      V(1)=0.
      V(4)=0.
      V(7)=0.
      V(8)=0.
      V(9)=0.
      CALLJUMPI(X,V(2),V(3),V(5),V(6))
      CALLPRINTO(X,T,V(1),V(2),V(3),V(4),V(5),V(6),V(7),V(8),V(9),XLI)
  290 RETURN
      END
```

BOUNDARY POINT SUBROUTINE

```
      SUBROUTINEBOUNCP
      COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
     1U(9),V(9),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
      DIMENSIONHCLD(12)
      XI=I
      X1=XZERO
      T=(XLI+XI)*PINC/C1
      SMUK=2./(EM+1.)
      X3=X1+PINC
      X4=X1+SMUK*PINC
      DO10J=1,9
      W(J,3)=V(J)
   10 W(J,4)=U(J,I)+SMUK*(V(J)-U(J,I))
      WX4A=W(8,4)
      WT4A=W(9,4)
      CALLGECOFF(1,X1,X3)
      CALLGECOFG(1,X1,X3)
      CALLGECOFH(1,X1,X4)
      CALLBCTF1(T,R1)
      CALLBCTF2(T,R2)
      CALLBCTF3(T,R3)
      DX13=X1-X3
      DX14=X1-X4
      Y(2,1)=C1*(1.-F(1,1)*DX13/2.)
      Y(2,2)=1.+F(2,1)*DX13**2/2.
      Y(2,3)=-C1*F(3,1)*DX13/2.
      Y(2,4)=F(4,1)*DX13**2/2.
      Y(2,5)=-C1*F(5,1)*DX13/2.
      Y(2,6)=F(6,1)*DX13**2/2.
      Z(2)=W(3,3)+C1*W(2,3)+C1*DX13*(F(1,1)*W(2,3)+F(2,1)*(W(1,3)+U(1,I)
     1-U(3,I)*DX13/C1)+F(3,1)*W(5,3)+F(4,1)*(W(4,3)+U(4,I)-U(6,I)*DX13/C
     21)+F(5,1)*W(8,3)+F(6,1)*(W(7,3)+U(7,I)-U(9,I)*DX13/C1))/2.
      Y(4,1)=-C1*G(1,1)*DX13/2.
      Y(4,2)=G(2,1)*DX13**2/2.
      Y(4,3)=C1*(1.-G(3,1)*DX13/2.)
      Y(4,4)=1.+G(4,1)*DX13**2/2.
      Y(4,5)=-C1*G(5,1)*DX13/2.
      Y(4,6)=G(6,1)*DX13**2/2.
      Z(4)=W(6,3)+C1*W(5,3)+C1*DX13*(G(1,1)*W(2,3)+G(2,1)*(W(1,3)+U(1,I)
     1-U(3,2)*DX13/C1)+G(3,1)*W(5,3)+G(4,1)*(W(4,3)+U(4,I)-U(6,I)*DX13/C
     21)+G(5,1)*W(8,3)+G(6,1)*(W(7,3)+U(7,I)-U(9,I)*DX13/C1))/2.
      DT=-2.*DX13/C1
      Y(5,1)=-C2*H(1,1)*DX14/2.
      Y(5,2)=-C2*H(2,1)*DT*DX14/4.
      Y(5,3)=-C2*H(3,1)*DX14/2.
      Y(5,4)=-C2*H(4,1)*DT*DX14/4.
      Y(5,5)=C2*(1.-H(5,1)*DX14/2.)
      Y(5,6)=1.-C2*H(6,1)*DT*DX14/4.
      Z(5)=W(9,4)+C2*W(8,4)+C2*DX14*(H(1,1)*W(2,4)+H(2,1)*(W(1,4)+U(1,I)
     1+U(3,I)*DT/2.)+H(3,1)*W(5,4)+H(4,1)*(W(4,4)+U(4,I)+U(6,I)*DT/2.)+H
     2(5,1)*W(8,4)+H(6,1)*(W(7,4)+U(7,I)+U(9,I)*DT/2.))/2.
      Y(1,1)=A(1)
      Y(1,2)=A(7)+A(2)*DT/2.
```

```
      Y(1,3)=A(3)
      Y(1,4)=A(4)*DT/2.
      Y(1,5)=A(5)
      Y(1,6)=A(6)*DT/2.
      Z(1)=R1-A(2)*(U(1,I)+U(3,I)*DT/2.)-A(4)*(U(4,I)+U(6,I)*DT/2.)-A(6)
     1*(U(7,I)+U(9,I)*DT/2.)
      Y(3,1)=B(1)
      Y(3,2)=B(2)*DT/2.
      Y(3,3)=B(3)
      Y(3,4)=B(7)+B(4)*DT/2.
      Y(3,5)=B(5)
      Y(3,6)=B(6)*DT/2.
      Z(3)=R2-B(2)*(U(1,I)+U(3,I)*DT/2.)-B(4)*(U(4,I)+U(6,I)*DT/2.)-B(6)
     1*(U(7,I)+U(9,I)*DT/2.)
      Y(6,1)=C(1)
      Y(6,2)=C(2)*DT/2.
      Y(6,3)=C(3)
      Y(6,4)=C(4)*DT/2.
      Y(6,5)=C(5)
      Y(6,6)=C(7)+C(6)*DT/2.
      Z(6)=R3-C(2)*(U(1,I)+U(3,I)*DT/2.)-C(4)*(U(4,I)+U(6,I)*DT/2.)-C(6)
     1*(U(7,I)+U(9,I)*DT/2.)
      IF(Y(1,1))1,2,1
    2 DO3J=1,6
      HOLD(J)=Y(1,J)
      Y(1,J)=Y(2,J)
    3 Y(2,J)=HOLD(J)
      CEEP=Z(1)
      Z(1)=Z(2)
      Z(2)=CEEP
    1 IF(Y(3,3))4,5,4
    5 DO6J=1,6
      HOLD(J)=Y(3,J)
      Y(3,J)=Y(4,J)
    6 Y(4,J)=HOLD(J)
      CEEP=Z(3)
      Z(3)=Z(4)
      Z(4)=CEEP
    4 IF(Y(6,6))99,8,99
    8 DO9J=1,6
      HOLD(J)=Y(6,J)
      Y(6,J)=Y(5,J)
    9 Y(5,J)=HOLD(J)
      CEEP=Z(6)
      Z(6)=Z(5)
      Z(5)=CEEP
   99 CALLMASUB
      DO11J=1,3
      V(3*J-1)=UU(2*J-1)
   11 V(3*J)=UU(2*J)
      V(1)=U(1,I)+(U(3,I)+V(3))*DT/2.
      V(4)=U(4,I)+(U(6,I)+V(6))*DT/2.
      V(7)=U(7,I)+(U(9,I)+V(9))*DT/2.
```

```
      DO12J=1,9
      U(J,I)=W(J,3)
   12 U(J,I+1)=V(J)
      CALLPRINTO(X1,T,V(1),V(2),V(3),V(4),V(5),V(6),V(7),V(8),V(9),XLI)
 9999 RETURN
      END
```

ORDINARY POINT SUBROUTINE

```
      SUBROUTINEORDINP
      COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
     1U(9),V(9),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
      XI=I
      X1=XZERO+(XLI-XI)*PINC
      T=(XLI+XI)*PINC/C1
      SMUK=2./(EM+1.)
      X3=X1+PINC
      X9=X1-PINC
      X4=X1+SMUK*PINC
      X6=X1-SMUK*PINC
      DO1J=1,9
      W(J,3)=V(J)
      W(J,9)=U(J,I+1)
      W(J,4)=U(J,I)+SMUK*(V(J)-U(J,I))
      W(J,6)=U(J,I)+SMUK*(U(J,I+1)-U(J,I))
    1 U(J,I)=V(J)
      WX4A=W(8,4)
      WT4A=W(9,4)
      CALLGECOFF(1,X1,X3)
      CALLGECOFF(2,X1,X9)
      CALLGECOFG(1,X1,X3)
      CALLGECOFG(2,X1,X9)
      CALLGECOFH(1,X1,X4)
      CALLGECOFH(2,X1,X6)
      DX13=X1-X3
      DX14=X1-X4
      DX19=X1-X9
      DX16=X1-X6
      CALLSOLMAT(WX4A,WT4A,DX13,DX14,DX19,DX16)
      CALLMASUB
   99 DO2J=1,3
      V(3*J-1)=UU(2*J-1)
    2 V(3*J)=UU(2*J)
      V(1)=W(1,3)+(W(2,3)+V(2)-(W(3,3)+V(3))/C1)*DX13/2.
      V(4)=W(4,3)+(W(5,3)+V(5)-(W(6,3)+V(6))/C1)*DX13/2.
      V(7)=W(7,4)+(W(8,4)+V(8)-(W(9,4)+V(9))/C2)*DX14/2.
      CALLPRINTO(X1,T,V(1),V(2),V(3),V(4),V(5),V(6),V(7),V(8),V(9),XLI)
  290 I=I+1
 9999 RETURN
      END
```

CASE I POINT SUBROUTINE

```
      SUBROUTINECASE1P
      COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
     1U(9),V(9),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
      XI=I
      T=(XLI+XI)*PINC/C1
      X1=XZERO+(2.*PINC*XLI)/(EM+1.)
      X9=X1-PINC
      X3=XZERO+(XLI-XI+1.)*PINC
      X4=XZERO+(4.*PINC*EM*XLI)/(EM+1.)**2-(2.*PINC*(XI-1.))/(EM+1.)
      X6=X1-2.*PINC/(EM+1.)
      SMUK9=(1.-2./(EM+1.))/(2.*(XLI-1.)/(EM+1.)-(XLI-XI-1.))
      SMUK4=(XLI-XI+1.)-(4.*EM*XLI/(EM+1.)**2)+(2.*(XI-1.)/(EM+1.))
      DO1J=1,9
      W(J,3)=V(J)
      W(J,9)=UP(J)+SMUK9*(U(J,I+1)-UP(J))
      W(J,4)=V(J)+SMUK4*(U(J,I)-V(J))
      W(J,6)=UP(J)
    1 U(J,I)=V(J)
      CALLJUMPII(X1,DU(8),DU(9))
      W(8,3)=W(8,3)-DU(8)
      WX4A=W(8,4)+DU(8)
      WT4A=W(9,4)+DU(9)
      W(8,4)=W(8,4)-DU(8)
      W(9,4)=W(9,4)-DU(9)
      CALLGECOFF(1,X1,X3)
      CALLGECOFF(2,X1,X9)
      CALLGECOFG(1,X1,X3)
      CALLGECOFG(2,X1,X9)
      CALLGECOFH(1,X1,X4)
      CALLGECOFH(1,X1,X6)
      DX13=X1-X3
      DX14=X1-X4
      DX19=X1-X9
      DX16=X1-X6
      CALLSOLMAT(WX4A,WT4A,DX13,DX14,DX19,DX16)
      CALLMASUB
   99 DO2J=1,3
      W(3*J-1,3)=UU(2*J-1)
    2 W(3*J,3)=UU(2*J)
      W(1,3)=V(1)+(V(2)+W(2,3)-(V(3)+W(3,3))/C1)*DX13/2.
      W(4,3)=V(4)+(V(5)+W(5,3)-(V(6)+W(6,3))/C1)*DX13/2.
      W(7,3)=W(7,4)+(W(8,4)+W(8,3)-(W(9,4)+W(9,3))/C2)*DX14/2.
      X3=X1
      X1=XZERO+(XLI-XI)*PINC
      X9=X1-PINC
      X4=XZERO+(PINC/(EM+1.))*(XLI+XI+EM*(XLI-XI)-2.*EM*XLI/(EM+1.)+2.*X
     1LI/(EM+1.))
      X6=XZERO+(PINC*(XLI-XI-2.)+EM*PINC*(XLI-XI))/(EM+1.)
      SMUK4=(2.*XLI*(EM-1.)/(EM+1.)**2)-(XLI-XI)*(EM-1.)/(EM+1.)
      SMUK6=((XLI-XI-2.+EM*(XLI-XI))/(EM+1.)-(XLI-XI-1.))/(2.*(XLI-1.)/(
     1EM+1.)-(XLI-XI-1.))
      DO3J=1,9
      W(J,4)=W(J,3)+SMUK4*(W(J,9)-W(J,3))
```

```
      UP(J)=W(J,3)
      W(J,9)=U(J,I+1)
    3 W(J,6)=W(J,9)+SMUK6*(W(J,6)-W(J,9))
      WX4A=W(8,4)
      WT4A=W(9,4)
      CALLGECOFF(1,X1,X3)
      CALLGECOFF(2,X1,X9)
      CALLGECOFG(1,X1,X3)
      CALLGECOFG(2,X1,X9)
      CALLGECOFH(1,X1,X4)
      CALLGECOFH(2,X1,X6)
      DX13=X1-X3
      DX14=X1-X4
      DX19=X1-X9
      DX16=X1-X6
      CALLSOLMAT(WX4A,WT4A,DX13,DX14,DX19,DX16)
      CALLMASUB
   98 DO4J=1,3
      V(3*J-1)=UU(2*J-1)
    4 V(3*J)=UU(2*J)
      V(1)=W(1,3)+(W(2,3)+V(2)-(W(3,3)+V(3))/C1)*DX13/2.
      V(4)=W(4,3)+(W(5,3)+V(5)-(W(6,3)+V(6))/C1)*DX13/2.
      V(7)=W(7,4)+(W(8,4)+V(8)-(W(9,4)+V(9))/C2)*DX14/2.
      CALLPRINTO(X1,T,V(1),V(2),V(3),V(4),V(5),V(6),V(7),V(8),V(9),XLI)
  290 I=I+1
 9999 RETURN
      END
```

CASE II POINT SUBROUTINE

```
      SUBROUTINECASE32
      COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
     1U(9),V(9),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
      XI=I
      T=(XLI+XI)*PINC/C1
      X1=XZERO+(2.*PINC*XI)/(EM-1.)
      X3=X1+PINC
      X9=XZERO+(XLI-XI-1.)*PINC
      X6=XZERO+2.*PINC*(XLI-1.)/(EM+1.)
      DO1J=1,7
    1 W(J,6)=UP(J)
      W(8,6)=UP(8)-DU(8)
      W(9,6)=UP(9)-DU(9)
      SMUK4=XLI-XI+1.-(4.*EM*XI)/((EM+1.)*(EM-1.))+(2.*(XI-1.)/(EM+1.))
      IF(SMUK4-1.)302,302,306
  302 X4=XZERO+(4.*EM*PINC*XI)/((EM+1.)*(EM-1.))-2.*PINC*(XI-1.)/(EM+1.)
      DO2J=1,9
    2 W(J,4)=V(J)+SMUK4*(U(J,I)-V(J))
      GOTO308
  306 X4=XZERO+(4.*EM*PINC*XI/(EM-1.)**2)-(2.*PINC*(XLI-1.)/(EM-1.))
      SMUK4=(XLI-XI-4.*EM*XI/(EM-1.)**2+2.*(XLI-1.)/(EM-1.))/(XLI-XI-2.*
     1(XLI-1.)/(EM+1.))
      DO3J=1,9
    3 W(J,4)=U(J,I)+SMUK4*(W(J,6)-U(J,I))
  308 SMUK3=XLI-(XI*(EM+1.)/(EM-1.))
      DO4J=1,9
      W(J,3)=V(J)+SMUK3*(U(J,I)-V(J))
    4 W(J,9)=U(J,I+1)
      CALLJUMPII(X1,DU(8),DU(9))
      W(8,9)=W(8,9)+DU(8)
      WX4A=W(8,4)
      WT4A=W(9,4)
      CALLGECOFF(1,X1,X3)
      CALLGECOFF(2,X1,X9)
      CALLGECOFG(1,X1,X3)
      CALLGECOFG(2,X1,X9)
      CALLGECOFH(1,X1,X4)
      CALLGECOFH(2,X1,X6)
      DX13=X1-X3
      DX14=X1-X4
      DX19=X1-X9
      DX16=X1-X6
      CALLSOLMAT(WX4A,WT4A,DX13,DX14,DX19,DX16)
      CALLMASUB
   99 DO5J=1,3
      W(3*J-1,9)=UU(2*J-1)
    5 W(3*J,9)=UU(2*J)
      W(1,9)=W(1,3)+(W(2,3)+W(2,9)-(W(3,3)+W(3,9))/C1)*DX13/2.
      W(4,9)=W(4,3)+(W(5,3)+W(5,9)-(W(6,3)+W(6,9))/C1)*DX13/2.
      W(7,9)=W(7,4)+(W(8,4)+W(8,9)-(W(9,4)+W(9,9))/C2)*DX14/2.
      X9=X1
      X1=XZERO+(XLI-XI)*PINC
      X3=X1+PINC
```

```
      X4=X1+2.*PINC/(EM+1.)
      X6=X1-2.*PINC/(EM+1.)
      SMUK6=(XLI-XI-(XLI-XI-2.+EM*(XLI-XI))/(EM+1.))/(XLI-XI-2.*(XLI-1.)
     1/(EM+1.))
      SMUK4=(EM-1.)/(EM+1.)
      DO7J=1,9
      W(J,3)=V(J)
      W(J,6)=U(J,I)+SMUK6*(W(J,6)-U(J,I))
      W(J,4)=V(J)+SMUK4*(U(J,I)-V(J))
    7 U(J,I)=V(J)
      WX4A=W(8,4)
      WT4A=W(9,4)
      CALLGECOFF(1,X1,X3)
      CALLGECOFF(2,X1,X9)
      CALLGECOFG(1,X1,X3)
      CALLGECOFG(2,X1,X9)
      CALLGECOFH(1,X1,X4)
      CALLGECOFH(2,X1,X6)
      DX13=X1-X3
      DX14=X1-X4
      DX19=X1-X9
      DX16=X1-X6
      CALLSOLMAT(WX4A,WT4A,DX13,DX14,DX19,DX16)
      CALLMASUB
   98 DO8J=1,3
      V(3*J-1)=UU(2*J-1)
    8 V(3*J)=UU(2*J)
      V(1)=W(1,3)+(W(2,3)+V(2)-(W(3,3)+V(3))/C1)*DX13/2.
      V(4)=W(4,3)+(W(5,3)+V(5)-(W(6,3)+V(6))/C1)*DX13/2.
      V(7)=W(7,4)+(W(8,4)+V(8)-(W(9,4)+V(9))/C2)*DX14/2.
      CALLPRINTO(X1,T,V(1),V(2),V(3),V(4),V(5),V(6),V(7),V(8),V(9),XLI)
  290 I=I+1
      XI=I
      T=(XLI+XI)*PINC/C1
      X1=XZERO+(2.*PINC*XLI)/(EM+1.)
      X9=X1-PINC
      X3=XZERO+(XLI-XI+1.)*PINC
      X4=XZERO+(4.*PINC*EM*XLI)/(EM+1.)**2-(2.*PINC*(XI-1.))/(EM+1.)
      X6=XZERO+(2.*PINC*(XI-1.))/(EM-1.)
      DO9J=1,7
    9 W(J,6)=W(J,9)
      W(8,6)=W(8,9)+DU(8)
      W(9,6)=W(9,9)+DU(9)
      SMUK9=XLI-XI+1.-2.*XLI/(EM+1.)
      SMUK4=((XLI-XI+1.)-(4.*EM*XLI/(EM+1.)**2)+(2.*(XI-1.)/(EM+1.)))/(X
     1LI-XI+1.-(2.*(XI-1.)/(EM-1.)))
      DO10J=1,9
      W(J,3)=V(J)
      W(J,4)=V(J)+SMUK4*(W(J,9)-V(J))
   10 W(J,9)=U(J,I)+SMUK9*(U(J,I+1)-U(J,I))
      CALLJUMPII(X1,DU(8),DU(9))
      W(8,3)=W(8,3)-DU(8)
      WX4A=W(8,4)+DU(8)
```

```
      WT4A=W(9,4)+DU(9)
      W(8,4)=W(8,4)-DU(8)
      W(9,4)=W(9,4)-DU(9)
      CALLGECOFF(1,X1,X3)
      CALLGECOFF(2,X1,X9)
      CALLGECOFG(1,X1,X3)
      CALLGECOFG(2,X1,X9)
      CALLGECOFH(1,X1,X4)
      CALLGECOFH(2,X1,X6)
      DX13=X1-X3
      DX14=X1-X4
      DX19=X1-X9
      DX16=X1-X6
      CALLSOLMAT(WX4A,WT4A,DX13,DX14,DX19,DX16)
      CALLMASUB
   97 DO11J=1,3
      W(3*J-1,3)=UU(2*J-1)
   11 W(3*J,3)=UU(2*J)
      W(1,3)=V(1)+(V(2)+W(2,3)-(V(3)+W(3,3))/C1)*DX13/2.
      W(4,3)=V(4)+(V(5)+W(5,3)-(V(6)+W(6,3))/C1)*DX13/2.
      W(7,3)=W(7,4)+(W(8,4)+W(8,3)-(W(9,4)+W(9,3))/C2)*DX14/2.
      X3=X1
      X1=XZERO+(XLI-XI)*PINC
      X9=X1-PINC
      X6=X1-2.*PINC/(EM+1.)
      X4=XZERO+(PINC/(EM+1.))*(XLI+XI+EM*(XLI-XI)-2.*EM*XLI/(EM+1.)+2.*X
     1LI/(EM+1.))
      SMUK4=(2.*XLI*(EM-1.)/(EM+1.)**2)-(XLI-XI)*(EM-1.)/(EM+1.)
      SMUK6=(EM-1.)/(EM+1.)
      DO12J=1,9
      UP(J)=W(J,3)
      W(J,4)=W(J,3)+SMUK4*(W(J,9)-W(J,3))
      W(J,9)=U(J,I+1)
      W(J,6)=W(J,9)+SMUK6*(U(J,I)-W(J,9))
   12 U(J,I)=V(J)
      WX4A=W(8,4)
      WT4A=W(9,4)
      CALLGECOFF(1,X1,X3)
      CALLGECOFF(2,X1,X9)
      CALLGECOFG(1,X1,X3)
      CALLGECOFG(2,X1,X9)
      CALLGECOFH(1,X1,X4)
      CALLGECOFH(2,X1,X6)
      DX13=X1-X3
      DX14=X1-X4
      DX19=X1-X9
      DX16=X1-X6
      CALLSOLMAT(WX4A,WT4A,DX13,DX14,DX19,DX16)
      CALLMASUB
   96 DO13J=1,3
      V(3*J-1)=UU(2*J-1)
   13 V(3*J)=UU(2*J)
      V(1)=W(1,3)+(W(2,3)+V(2)-(W(3,3)+V(3))/C1)*DX13/2.
```

```
      V(4)=W(4,3)+(W(5,3)+V(5)-(W(6,3)+V(6))/C1)*DX13/2.
      V(7)=W(7,4)+(W(8,4)+V(8)-(W(9,4)+V(9))/C2)*DX14/2.
      CALLPRINTO(X1,T,V(1),V(2),V(3),V(4),V(5),V(6),V(7),V(8),V(9),XLI)
  293 I=I+1
 9999 RETURN
      END
```

SOLUTION MATRIX SUBROUTINE

```
      SUBROUTINESOLMAT(WX4A,WT4A,DX13,DX14,DX19,DX16)
      COMMONU(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3),Z(12),UU(12),D
     1U(9),V(9),UP(9),A(7),B(7),C(7),PINC,XLI,EM,C1,C2,XZERO,I,M
      Y(1,1)=C1*(-1.+F(1,2)*DX19/2.+F(2,2)*DX19*DX13/4.)
      Y(1,2)=1.-F(2,2)*DX19*DX13/4.
      Y(1,3)=C1*(F(3,2)*DX19/2.+F(4,2)*DX19*DX13/4.)
      Y(1,4)=-F(4,2)*DX19*DX13/4.
      Y(1,5)=C1*(F(5,2)*DX19/2.+F(6,2)*DX19*DX14/4.)
      Y(1,6)=-C1*F(6,2)*DX19*DX14/(4.*C2)
      Z(1)=W(3,9)-C1*W(2,9)-(C1*DX19/2.)*(F(1,2)*W(2,9)+F(2,2)*DX13*(W(2
     1,3)-W(3,3)/C1)/2.+F(2,2)*(W(1,3)+W(1,9))+F(3,2)*W(5,9)+F(4,2)*DX13
     2*(W(5,3)-W(6,3)/C1)/2.+F(4,2)*(W(4,3)+W(4,9))+F(5,2)*W(8,9)+F(6,2)
     3*DX14*(W(8,4)-W(9,4)/C2)/2.+F(6,2)*(W(7,4)+W(7,9)))
      Y(3,1)=C1*(G(1,2)*DX19/2.+G(2,2)*DX19*DX13/4.)
      Y(3,2)=-G(2,2)*DX19*DX13/4.
      Y(3,3)=C1*(-1.+G(3,2)*DX19/2.+G(4,2)*DX19*DX13/4.)
      Y(3,4)=1.-G(4,2)*DX19*DX13/4.
      Y(3,5)=C1*(G(5,2)*DX19/2.+G(6,2)*DX19*DX14/4.)
      Y(3,6)=-C1*DX19*DX14*G(6,2)/(4.*C2)
      Z(3)=W(6,9)-C1*W(5,9)-(C1*DX19/2.)*(G(1,2)*W(2,9)+G(2,2)*DX13*(W(2
     1,3)-W(3,3)/C1)/2.+G(2,2)*(W(1,3)+W(1,9))+G(3,2)*W(5,9)+G(4,2)*DX13
     2*(W(5,3)-W(6,3)/C1)/2.+G(4,2)*(W(4,3)+W(4,9))+G(5,2)*W(8,9)+G(6,2)
     3*DX14*(W(8,4)-W(9,4)/C2)/2.+G(6,2)*(W(7,4)+W(7,9)))
      Y(6,1)=C2*(H(1,2)*DX16/2.+H(2,2)*DX16*DX13/4.)
      Y(6,2)=-C2*DX16*DX13*H(2,2)/(4.*C1)
      Y(6,3)=C2*(H(3,2)*DX16/2.+H(4,2)*DX16*DX13/4.)
      Y(6,4)=-C2*DX16*DX13*H(4,2)/(4.*C1)
      Y(6,5)=C2*(-1.+H(5,2)*DX16/2.+H(6,2)*DX16*DX14/4.)
      Y(6,6)=1.-H(6,2)*DX16*DX14/4.
      Z(6)=W(9,6)-C2*W(8,6)-(C2*DX16/2.)*(H(1,2)*W(2,6)+H(2,2)*DX13*(W(2
     1,3)-W(3,3)/C1)/2.+H(2,2)*(W(1,3)+W(1,6))+H(3,2)*W(5,6)+H(4,2)*DX13
     2*(W(5,3)-W(6,3)/C1)/2.+H(4,2)*(W(4,3)+W(4,6))+H(5,2)*W(8,6)+H(6,2)
     3*DX14*(W(8,4)-W(9,4)/C2)/2.+H(6,2)*(W(7,4)+W(7,6)))
```

```
      Y(2,1)=C1*(1.-F(1,1)*DX13/2.-F(2,1)*DX13**2/4.)
      Y(2,2)=1.+F(2,1)*DX13**2/4.
      Y(2,3)=C1*(-F(3,1)*DX13/2.-F(4,1)*DX13**2/4.)
      Y(2,4)=F(4,1)*DX13**2/4.
      Y(2,5)=C1*(-F(5,1)*DX13/2.-F(6,1)*DX13*DX14/4.)
      Y(2,6)=C1*DX13*DX14*F(6,1)/(4.*C2)
      Z(2)=W(3,3)+C1*W(2,3)+C1*DX13*(F(1,1)*W(2,3)/2.+F(2,1)*DX13*(W(2,3
     1)-W(3,3)/C1)/4.+F(2,1)*W(1,3)+F(3,1)*W(5,3)/2.+F(4,1)*DX13*(W(5,3)
     2-W(6,3)/C1)/4.+F(4,1)*W(4,3)+F(5,1)*W(8,3)/2.+F(6,1)*DX14*(W(8,4)-
     3W(9,4)/C2)/4.+F(6,1)*(W(7,4)+W(7,3))/2.)
      Y(4,1)=C1*(-G(1,1)*DX13/2.-G(2,1)*DX13**2/4.)
      Y(4,2)=G(2,1)*DX13**2/4.
      Y(4,3)=C1*(1.-G(3,1)*DX13/2.-G(4,1)*DX13**2/4.)
      Y(4,4)=1.+G(4,1)*DX13**2/4.
      Y(4,5)=C1*(-G(5,1)*DX13/2.-G(6,1)*DX13*DX14/4.)
      Y(4,6)=C1*DX13*DX14*G(6,1)/(4.*C2)
      Z(4)=W(6,3)+C1*W(5,3)+C1*DX13*(G(1,1)*W(2,3)/2.+G(2,1)*DX13*(W(2,3
     1)-W(3,3)/C1)/4.+G(2,1)*W(1,3)+G(3,1)*W(5,3)/2.+G(4,1)*DX13*(W(5,3)
     2-W(6,3)/C1)/4.+G(4,1)*W(4,3)+G(5,1)*W(8,3)/2.+G(6,1)*DX14*(W(8,4)-
     3W(9,4)/C2)/4.+G(6,1)*(W(7,4)+W(7,3))/2.)
      Y(5,1)=C2*(-H(1,1)*DX14/2.-H(2,1)*DX14*DX13/4.)
      Y(5,2)=C2*DX14*DX13*H(2,1)/(4.*C1)
      Y(5,3)=C2*(-H(3,1)*DX14/2.-H(4,1)*DX14*DX13/4.)
      Y(5,4)=C2*DX14*DX13*H(4,1)/(4.*C1)
      Y(5,5)=C2*(1.-H(5,1)*DX14/2.-H(6,1)*DX14**2/4.)
      Y(5,6)=1.+H(6,1)*DX14**2/4.
      Z(5)=WT4A+C2*WX4A+C2*DX14*(H(1,1)*W(2,4)/2.+H(2,1)*DX13*(W(2,3)-W(
     13,3)/C1)/4.+H(2,1)*(W(1,3)+W(1,4))/2.+H(3,1)*W(5,4)/2.+H(4,1)*DX13
     2*(W(5,3)-W(6,3)/C1)/4.+H(4,1)*(W(4,3)+W(4,4))/2.+H(5,1)*W(8,4)/2.+
     3H(6,1)*DX14*(W(8,4)-W(9,4)/C2)/4.+H(6,1)*W(7,4))
      M=6
      RETURN
      END
```

APPENDIX B: PREWRITTEN COMMON STRUCTURE PACKAGES

This appendix details the prewritten packages for the common structures shown in Figure 1. Each of these packages includes the coefficient specification, discontinuity magnitude specification, and output specification subroutines. In each of these packages the user need only fill in the constants which are dependent upon the material, dimensions, and discontinuity magnitude. The symbols representing these constants are underlined within each package; the user substitutes a floating point number for each. Constants depending upon the discontinuity magnitude are defined in each problem section; others are defined in the following list of symbols.

a,R	-	bar radius, cylindrical shell radius
A	-	Area
c_b	-	bar velocity = $(E/\rho)^{1/2}$
c_d	-	dilatational (or irrotational) velocity = $\{(\lambda+2G)/\rho\}^{1/2}$
c_e	-	equivoluminal (or distortional) velocity = $(G/\rho)^{1/2}$
c_p	-	plate velocity = $\{E/\rho(1-\nu^2)\}^{1/2}$
c_s	-	shear velocity = $k\ c_e$
D	-	flexural rigidity = $Eh^3/12(1-\nu^2)$
E	-	modulus of elasticity
G	-	shear modulus = $E/2(1+\nu)$
h	-	thickness
I	-	moment of inertia
k^2	-	shear correction factor
K, K_1	-	correction factors
E_p	-	$E/(1-\nu^2)$
λ	-	Lame's constant of elasticity = $\nu E/(1+\nu)\ (1-2\nu)$
ν	-	Poisson's ratio
r,s	-	radial distance, meridional distance
r_o,s_o	-	r at boundary, s at boundary
ρ	-	density
M	-	bending moment
N	-	normal stress resultant averaged across sheet
P	-	bar stresses
Q	-	shear stress resultant

η, F_2, g $-h^2/12R$, $1-\eta/R$, $k^2(1-\nu)/2$, respectively

Problem 1 - Cylindrical Dilatation (Plane Stress) [3]: See Figure 9.

Governing Equation: $$\frac{\partial^2 u}{\partial r^2} - \frac{1}{c_p^2}\frac{\partial^2 u}{\partial t^2} = \frac{1}{r^2}u - \frac{1}{r}\frac{\partial u}{\partial r} \; ; \; u = u_1$$

Generalized Stress: $$\sigma_r = \frac{E\nu}{r(1-\nu^2)}u + \frac{E}{1-\nu^2}\frac{\partial u}{\partial r}$$

Discontinuities $$[\frac{\partial u}{\partial r}] = k' \, r^{-1/2} \; ; \; [\frac{\partial u}{\partial t}] = -k' \, c_p \, r^{-1/2}$$

where $k' = \sqrt{r_o}\,\frac{1-\nu^2}{E}\left[\sigma_r\right]_{r = r_o}$ if σ_r boundary condition

$k' = -\sqrt{r_o}\,(\frac{1}{c_p})\left[\frac{\partial u}{\partial t}\right]_{r = r_o}$ if $\frac{\partial u}{\partial t}$ boundary condition

Problem Package:

```
      SUBROUTINE JUMP I(X,DU1X, DU1T, DU2X, DU2T)
      DU1X = k'/X**(.5)
      DU1T = -DU1X * cp
      DU2X = 0.
      DU2T = 0.
      RETURN
      END
      SUBROUTINE JUMP II(X, DU3X, DU3T)
      DU3X = 0.
      DU3T = 0.
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1,ID) = -1./X
      F(2,ID) = 1./X**2
      DO1J = 3,6
    1 F(J,ID) = 0.
      RETURN
      END
      SUBROUTINE GECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1J = 1,6
    1 G(J,ID) = 0.
      RETURN
      END
```

```
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.
      DO 1J = 1,6
    1 H(J,ID) = 0.
      RETURN
      END
```

Problem 2 - Cylindrical Dilatation (Plane Strain) [3]: See Figure 10.

Governing Equation: $\frac{\partial^2 u}{\partial r^2} - \frac{1}{c_d^2}\frac{\partial^2 u}{\partial t^2} = \frac{1}{r^2} u - \frac{1}{r}\frac{\partial u}{\partial r}$; $u = u_1$

Generalized Stress: $\sigma_r = \frac{\lambda}{r} u + (\lambda+2G)\frac{\partial u}{\partial r}$

Discontinuities: $[\frac{\partial u}{\partial r}] = k' \, r^{-1/2}$; $[\frac{\partial u}{\partial t}] = - k' \, c_d \, r^{-1/2}$

where $k' = \sqrt{r_o} \, \frac{1}{\lambda+2G}\left[\sigma_r\right]_{r = r_o}$ if σ_r boundary condition

$k' = - \sqrt{r_o} \, (\frac{1}{c_d}) \left[\frac{\partial u}{\partial t}\right]_{r = r_o}$ if $\frac{\partial u}{\partial t}$ boundary condition

Problem Package:

```
      SUBROUTINEJUMP I(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k' /X** (.5)
      DU1T = -DU1X * cd
      DU2X = 0.
      DU2T = 0.
      RETURN
      END

      SUBROUTINEJUMPII(X, DU3X, DU3T)
      DU3X = 0.
      DU3T = 0.
      RETURN
      END
```

```
      SUBROUTINEGECOFF (ID, XA, XB)
      COMMON U(9,300), Y(12, 12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1,ID) - 1./X
      F(2,ID) = 1./X**2
      DO 1 J=3,6
    1 F(J,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1 J=1,6
    1 G(J,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.
      DO 1 J=1,6
    1 H(J,ID) = 0.
      RETURN
      END
```

Problem 3 - Spherical Dilatation [3]: See Figure 11

Governing Equation: $\frac{\partial^2 u}{\partial r^2} - \frac{1}{c_d{}^2}\frac{\partial^2 u}{\partial t^2} = \frac{2}{r^2} u - \frac{2}{r}\frac{\partial u}{\partial r}$; $u = u_1$

Generalized Stress: $\sigma_r = \frac{2\lambda}{r} u + (\lambda+2G)\frac{\partial u}{\partial r}$

Discontinuities: $[\frac{\partial u}{\partial r}] = k' \, r^{-1}$; $[\frac{\partial u}{\partial t}] = -k' \, c_d \, r^{-1}$

where $k' = \frac{r_o}{\lambda+2G}\left[\sigma_r\right] r = r_o$ if σ_r boundary condition

$k' = -\frac{r_o}{c_d}\left[\frac{\partial u}{\partial t}\right] r = r_o$ if $\frac{\partial u}{\partial t}$ boundary condition

Problem Package:

```
      SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k'/X
      DU1T = -DU1X *Cd
      DU2X = 0.
      DU2T = 0.
      RETURN
      END
      SUBROUTINEJUMPII(X, DU3X, DU3T)
      DU3X = 0.
      DU3T = 0.
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1,ID) = -2./X
      F(2,ID) = 2./X**2
      DO 1 J = 3,6
    1 F(J,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 G(J,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6.3), H(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 H(J, ID) = 0.
      RETURN
      END
```

Problem 4 - Shear (Rotary) [4]: See Figure 12.

Governing Equation: $\frac{\partial^2 v}{\partial r^2} - \frac{1}{c_e^2} \frac{\partial^2 v}{\partial t^2} = \frac{1}{r^2} v - \frac{1}{r} \frac{\partial v}{\partial r}$; $v = u_1$

Generalized Stress: $\tau_{r\theta} = -\frac{G}{r} v + G \frac{\partial v}{\partial r}$

Discontinuities: $[\frac{\partial v}{\partial r}] = k' \, r^{-1/2}$; $[\frac{\partial v}{\partial t}] = -k' \, c_e \, r^{-1/2}$

where $k' = \sqrt{r_o} \, \frac{1}{G} \left[\tau_{r\theta}\right]_{r = r_o}$ if $\tau_{r\theta}$ boundary condition

$k' = -\frac{\sqrt{r_o}}{c_e} \left[\frac{\partial v}{\partial t}\right]_{r = r_o}$ if $\frac{\partial v}{\partial t}$ boundary condition

Problem Package:

```
      SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k'/X**(.5)
      DU1T = - DU1X * ce
      DU2X = 0.
      DU2T = 0.
      RETURN
      END
      SUBROUTINEJUMPII(X,DU3X, DU3T)
      DU3X = 0.
      DU3T = 0.
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1,ID) = -1./X
      F(2,ID) = 1./X**2
      DO 1 J = 3,6
    1 F(J,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 G(J,ID) = 0 .
      RETURN
      END
```

```
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12, 12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 H(J, ID) = 0.
      RETURN
      END
```

Problem 5 - Shear (Longitudinal) [5]: See Figure 13.

Governing Equation: $\frac{\partial^2 w}{\partial r^2} - \frac{1}{c_e^2}\frac{\partial^2 w}{\partial t^2} = -\frac{1}{r}\frac{\partial w}{\partial r}$; $w = u_1$

Generalized Stress: $\tau_{zr} = G\frac{\partial w}{\partial r}$

Discontinuities: $[\frac{\partial w}{\partial r}] = k' \; r^{-1/2}$; $[\frac{\partial w}{\partial t}] = -\,k' \; c_e \; r^{-1/2}$

where $k' = \sqrt{r_o}\,\frac{1}{G}\left[\tau_{zr}\right]_{r = r_o}$ if τ_{zr} boundary condition

$k' = -\sqrt{r_o}\,(\frac{1}{c_e})\left[\frac{\partial w}{\partial t}\right]_{r = r_o}$ if $\frac{\partial w}{\partial t}$ boundary condition

Problem Package:

```
      SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k /X** (.5)
      DU1T = -DU1X * ce
      DU2X = 0.
      DU2T = 0.
      RETURN
      END
      SUBROUTINEJUMPII(X, DU3X, DU3T)
      DU3X = 0.
      DU3T = 0.
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1,ID) = -1./X
      DO 1 J = 2,6
    1 F(J,ID) = 0.
      RETURN
      END
```

```
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 G(J,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 H(J,ID) = 0.
      RETURN
      END
```

Problem 6 - Beam (Timoshenko) [6]: See Figure 14.

Governing Equations: $\frac{\partial^2\psi}{\partial x^2} - \frac{1}{c_b^2}\frac{\partial^2\psi}{\partial t^2} = \frac{AGk^2}{EI}\psi - \frac{AGk^2}{EI}\frac{\partial y}{\partial x}$; $\psi = u_1$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c_s^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial\psi}{\partial x}$$

Generalized Stresses: $M = -EI\frac{\partial\psi}{\partial x}$; $y = u_3$

$$Q = k^2AG\left(\frac{\partial y}{\partial x} - \psi\right)$$

Discontinuities: $\left[\frac{\partial\psi}{\partial x}\right] = k'$; $\left[\frac{\partial\psi}{\partial t}\right] = -k'\,c_b$

$$\left[\frac{\partial y}{\partial x}\right] = k'' \quad ; \quad \left[\frac{\partial y}{\partial t}\right] = -k''\,c_s$$

where $k' = -\frac{[M]}{EI}\ x = x_o$ if M boundary condition

$k' = -\frac{1}{c_b}\left[\frac{\partial\psi}{\partial t}\right]\ x = x_o$ if $\frac{\partial\psi}{\partial t}$ boundary condition

$k'' = \frac{[Q]\ x = x_o}{k^2AG}$ if Q boundary condition

$k'' = -\frac{1}{c_s}\left[\frac{\partial y}{\partial t}\right]\ x = x_o$ if $\frac{\partial y}{\partial t}$ boundary condition

Problem Package:

```
      SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k'
      DU1T = -DU1X * c_b
      DU2X = 0.
      DU2T = 0.
      RETURN
      END

      SUBROUTINEJUMPII(X, DU3X, DU3T)
      DU3X = k"
      DU3T = -DU3X * c_s
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1,ID) = 0.

      F(2,ID) = AGk^2/EI
      F(3,ID) = 0.
      F(4,ID) = 0.

      F(5,ID) = - AGk^2/EI
      F(6,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      DO 1 J = 1,6
    1 G(J,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.
      H(1, ID) = 1.
      DO 1 J = 2,6
    1 H(J, ID) = 0.
      RETURN
      END
```

Problem 7 - Plate (Plane) (Mindlin) [7]: See Figure 15.

Governing Equations:
$$\frac{\partial^2\psi_x}{\partial x^2} - \frac{1}{c_p^{\,2}}\frac{\partial^2\psi_x}{\partial t^2} = \frac{hGk^2}{D}\psi_x + \frac{hGK^2}{D}\frac{\partial\bar{w}}{\partial x} \quad ; \psi_x = u_1$$

$$\frac{\partial^2\bar{w}}{\partial x^2} - \frac{1}{c_s^{\,2}}\frac{\partial^2\bar{w}}{\partial t^2} = -\frac{\partial\psi_x}{\partial x} \quad ; \bar{w} = u_3$$

Generalized Stresses: $M_x = D\dfrac{\partial\psi_x}{\partial x}$

$$Q_x = k^2hG(\psi_x + \frac{\partial\bar{w}}{\partial x})$$

Discontinuities:
$$[\frac{\partial\psi_x}{\partial x}] = k' \quad ; \quad [\frac{\partial\psi_x}{\partial t}] = -k'c_p$$

$$[\frac{\partial w}{\partial x}] = k'' \quad ; \quad [\frac{\partial\bar{w}}{\partial t}] = -k''c_s$$

where
$$k' = \frac{1}{D}\left[M_x\right]_{x=x_o} \quad \text{if } M_x \text{ boundary condition}$$

$$k' = -\frac{1}{c_p}\left[\frac{\partial\psi_x}{\partial t}\right]_{x=x_o} \quad \text{if } \frac{\partial\psi_x}{\partial t} \text{ boundary condition}$$

$$k'' = \frac{1}{k^2hG}\left[Q_x\right]_{x=x_o} \quad \text{if } Q_x \text{ boundary condition}$$

$$k'' = -\frac{1}{c_s}\left[\frac{\partial\bar{w}}{\partial t}\right]_{x=x_o} \quad \text{if } \frac{\partial\bar{w}}{\partial t} \text{ boundary condition}$$

Problem Package:

```
SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
DU1X = k'
DU1T = -DU1X * c_p
DU2X = 0.
DU2T = 0.
RETURN
END
SUBROUTINEJUMPII(X, DU3X, DU3T)
DU3X = k"
DU3T = -DU3X * c_s
RETURN
END
SUBROUTINEGECOFF(ID, XA, XB)
COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
X = (XA + XB)/2.
F(1,ID)= 0.

F(2,ID) = hGk^2/D
F(3,ID) = 0.
F(4,ID) = 0.

F(5,ID) = hGk^2/D
F(6,ID) = 0.
RETURN
END
```

```
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      DO 1 J = 1,6
    1 G(J, ID) = 0.
      RETURN
      END

      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.
      H(1, ID) = -1.
      DO 1 J = 2,6
    1 H(J, ID) = 0.
      RETURN
      END
```

<u>Problem 8</u> - Plate (Cylindrical) (Chou and Koenig) [8]: See Figure 16.

Governing Equations:

$$\frac{\partial^2\phi}{\partial r^2} - \frac{1}{c_p^{\,2}}\frac{\partial^2\phi}{\partial t^2} = -\frac{1}{r}\frac{\partial\phi}{\partial r} + \left[\frac{1}{r^2} + \frac{k^2Gh}{D}\right]\phi + \frac{k^2Gh}{D}\frac{\partial w}{\partial r} \quad ; \phi = u_1$$

$$\frac{\partial^2 w}{\partial r^2} - \frac{1}{c_s^{\,2}}\frac{\partial^2 w}{\partial t^2} = -\frac{\partial\phi}{\partial r} - \frac{1}{r}\phi - \frac{1}{r}\frac{\partial w}{\partial r} \quad ; w = u_2$$

Generalized Stresses:

$$M_r = \frac{D\nu}{r}\phi + D\frac{\partial\phi}{\partial r}$$

$$Q_r = k^2Gh\left(\phi + \frac{\partial w}{\partial r}\right)$$

Discontinuities:

$$\left[\frac{\partial\phi}{\partial r}\right] = k' \, r^{-1/2} \quad ; \quad \left[\frac{\partial\phi}{\partial t}\right] = -k' \, c_p \, r^{-1/2}$$

$$\left[\frac{\partial w}{\partial r}\right] = k'' \, r^{-1/2} \quad ; \quad \left[\frac{\partial w}{\partial t}\right] = -k'' \, c_s \, r^{-1/2}$$

where

$$k' = \frac{\sqrt{r_o}}{D}\left[M_r\right]_{r = r_o} \quad \text{if } M_r \text{ boundary condition}$$

$$k' = \frac{\sqrt{r_o}}{c_p}\left[\frac{\partial\phi}{\partial t}\right]_{r = r_o} \quad \text{if } \frac{\partial\phi}{\partial t} \text{ boundary condition}$$

$$k'' = \sqrt{r_o}\,\frac{[Q]_{r = r_o}}{k^2Gh} \quad \text{if } Q \text{ boundary condition}$$

$$k'' = \sqrt{r_o}/c_s\left[\frac{\partial w}{\partial t}\right]_{r = r_o} \quad \text{if } \frac{\partial w}{\partial t} \text{ boundary condition}$$

Problem Package:

```
      SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k'  /X ** (.5)
      DU1T = -DU1X * c_p
      DU2X = 0.
      DU2T = 0.
      RETURN
      END
      SUBROUTINEJUMPII(X, DU3X, DU3T)
      DU3X = k" /X ** (.5)
      DU3T = -DU3X * c_s
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1,ID) = -1./X
      F(2, ID) = 1./X ** 2 + k_2^2Gh/D
      F(3, ID) = 0.
      F(4, ID) = 0.
      F(5, ID) = k_2^2Gh/D
      F(6, ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 G(J, ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3),  H(6,3)
      X = (XA + XB) /2.
      H(1, ID) = -1.
      H(2, ID) = -1./X
      H(3, ID) = 0.
      H(4, ID) = 0.
      H(5, ID) = -1./X
      H(6, ID) = 0.
      RETURN
      END
```

Problem 9 - Bar (Mindlin and Herrmann) [9]: See Figure 17

Governing Equations: $\frac{\partial^2 w}{\partial x^2} - \frac{1}{c_d^2}\frac{\partial^2 w}{\partial t^2} = -\frac{2\lambda}{a\rho\ c_d^2}\frac{\partial u}{\partial x}$; $w = u_1$

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c_s^2}\frac{\partial^2 u}{\partial t^2} = \frac{4\lambda K_1^2}{aG\ k^2}\frac{\partial w}{\partial x} + \frac{8\ K_1^2(\lambda+G)}{a^2 G\ k^2} u \ ; \ u = u_3$$

Generalized Stresses: $P_x = a\lambda u + \frac{a^2(\lambda+2G)}{2}\frac{\partial w}{\partial x}$

$$Q = \frac{k^2 a^2 G}{4}\frac{\partial u}{\partial x}$$

Discontinuities: $[\frac{\partial w}{\partial x}] = k'$; $[\frac{\partial w}{\partial t}] = -k'\ c_d$

$$[\frac{\partial u}{\partial x}] = k'' \ ; \ [\frac{\partial u}{\partial t}] = -k''\ c_s$$

where

$k' = \frac{2\left[P_x\right]}{a^2(\lambda+2G)}\ x = x_o$ if P_x boundary condition

$k' = -\frac{1}{c_d}\left[\frac{\partial w}{\partial t}\right]\ x = x_o$ if $\frac{\partial w}{\partial t}$ boundary condition

$k'' = \frac{4\left[Q\right]}{k^2 a^2 G}\ x = x_o$ if Q boundary condition

$k'' = -\frac{1}{c_s}\left[\frac{\partial u}{\partial t}\right]\ x = x_o$ if $\frac{\partial u}{\partial t}$ boundary condition

Problem Package:

```
      SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k'
      DU1T = -DU1X * cd
      DU2X = 0.
      DU2T = 0.
      RETURN
      END
      SUBROUTINEJUMPII(X, DU3X, DU3T)
      DU3X = k"
      DU3T = -DU3X * cs
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,4
    1 F(J, ID) = 0.
      F(5,ID)= - 2λ/aρcd²
      F(6,ID) = 0.
      RETURN
      END
```

```
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 G(J, ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.

      H(1, ID) = 4λK1^2/aGk^2
      DO 1 J = 2,5
    1 H(J, ID) = 0.

      H(6, ID) = 8K1^2(λ+G)/a^2Gk^2
      RETURN
      END
```

Problem 10 - Sheet (Plane) (Kane and Mindlin) [10]: See Figure 18.

Governing Equations:

$$\frac{\partial^2 v_x}{\partial x^2} - \frac{1}{c_d^2}\frac{\partial^2 v_x}{\partial t^2} = -\frac{K\lambda}{h\rho\, c_d^2}\frac{\partial v_z}{\partial x} \qquad ; \; v_x = u_1$$

$$\frac{\partial^2 v_z}{\partial x^2} - \frac{1}{c_e^2}\frac{\partial^2 v_z}{\partial t^2} = \frac{3\lambda K}{hG}\frac{\partial v_x}{\partial x} + \frac{3K^2}{h^2}\left(\frac{c_d}{c_e}\right)^2 v_z \; ; \; v_z = u_3$$

Generalized Stresses:

$$N_x = 2K\lambda\, v_z + 2h(\lambda+2G)\frac{\partial v_x}{\partial x}$$

$$R_x = \frac{2}{3}h^2 G\frac{\partial v_z}{\partial x}$$

Discontinuities: $\left[\frac{\partial v_x}{\partial x}\right] = k'$; $\left[\frac{\partial v_x}{\partial t}\right] = -k'c_d$

$\left[\frac{\partial v_z}{\partial x}\right] = k''$; $\left[\frac{\partial v_z}{\partial t}\right] = -k''c_e$

where

$$k' = \frac{\left[N_x\right]}{2h(\lambda+2G)}\quad x = x_o \qquad \text{if } N_x \text{ boundary condition}$$

$$k' = -\frac{1}{c_e}\left[\frac{\partial v_x}{\partial t}\right]\quad x = x_o \qquad \text{if } \frac{\partial v_x}{\partial t} \text{ boundary condition}$$

$$k'' = \frac{\left[R_x\right]}{2h^2G}\quad x = x_o \qquad \text{if } R_x \text{ boundary condition}$$

$$k'' = -\frac{1}{c_e}\left[\frac{\partial v_z}{\partial t}\right]\quad x = x_o \qquad \text{if } \frac{\partial v_z}{\partial t} \text{ boundary condition}$$

Problem Package:

```
      SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k'
      DU1T = -DU1X * c_d
      DU2X = 0.
      DU2T = 0.
      RETURN
      END
      SUBROUTINEJUMPII(X,DU3X, DU3T)
      DU3X = k"
      DU3T - - DU3X * c_e
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,4
    1 F(J,ID) = 0.
      F(5,ID) = - Kλ/hρc_d^2
      F(6,ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 G(J,ID) = 0.
      RETURN
      END
```

```
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.

      H(1, ID) =  3λK/hG

      DO 1 J = 2,5
    1 H(J, ID) = 0.
      H(6, ID) = 3K²(cd/ce)²/h²

      RETURN
      END
```

Problem 11 - Sheet (Cylindrical) (Jahsman) [11]: See Figure 19.

Governing Equations:
$$\frac{\partial^2 u}{\partial r^2} - \frac{1}{c_d^2}\frac{\partial^2 u}{\partial t^2} = \frac{u}{r^2} - \frac{1}{r}\frac{\partial u}{\partial r} - \frac{\nu K_1}{h(1-\nu)}\frac{\partial y}{\partial r} \quad ; \; u = u_1$$

$$\frac{\partial^2 y}{\partial r^2} - \frac{1}{c_e^2}\frac{\partial^2 y}{\partial t^2} = \frac{24K_1\nu}{h(1-2\nu)r}\,u + \frac{24K_1\nu}{h(1-2\nu)}\frac{\partial u}{\partial r} + \frac{24K_1^2(1-\nu)}{h^2(1-2\nu)}\,y - \frac{1}{r}\frac{\partial y}{\partial r} \quad y = u_3$$

Generalized Stresses:
$$N_r = \frac{h\lambda}{r}u + K_1\lambda\,y + h(\lambda+2G)\frac{\partial u}{\partial r}$$

$$S_{rz} = \frac{Gh^2}{12}\frac{\partial y}{\partial r}$$

Discontinuities:
$$[\frac{\partial u}{\partial r}] = k'\,r^{-1/2} \quad ; \quad [\frac{\partial u}{\partial t}] = -k'\,c_d\,r^{-1/2}$$

$$[\frac{\partial y}{\partial r}] = k''\,r^{-1/2} \quad ; \quad [\frac{\partial y}{\partial t}] = -k''c_e\,r^{-1/2}$$

where

$$k' = \sqrt{r_o}\,\frac{1}{h(\lambda+2G)}\left[N_r\right] \quad r = r_o \quad \text{if } N_r \text{ boundary condition}$$

$$k' = -\sqrt{r_o}\,(\frac{1}{c_d})\left[\frac{\partial u}{\partial t}\right] \quad r = r_o. \quad \text{if } \frac{\partial u}{\partial t} \text{ boundary condition}$$

$$k'' = \sqrt{r_o}\,\frac{12}{Gh^2}\left[S_{rz}\right] \quad r = r_o \quad \text{if } S_{rz} \text{ boundary condition}$$

$$k'' = -\sqrt{r_o}\,(\frac{1}{c_e})\left[\frac{\partial y}{\partial t}\right] \quad r = r_o \quad \text{if } \frac{\partial y}{\partial t} \text{ boundary condition}$$

Problem Package:

```
      SUBROUTINEJUMPI(X, DU1X, DU1T, DU2X, DU2T)
      DU1X = k' /X **(.5)
      DU1T = -DU1X * cd
      DU2X = 0.
      DU2T = 0.
      RETURN
      END
      SUBROUTINEJUMPII(X, DU3X, DU3T)
      DU3X = k" /X** (.5)
      DU3T = - DU3X * ce
      RETURN
      END
      SUBROUTINEGECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12, 12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1, ID) = -1./X
      F(2, ID) = 1./X **2
      F(3, ID) = 0.
      F(4, ID) = 0.
      F(5, ID) = νK1/h(1-ν)
      F(6, ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFG(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      DO 1 J = 1,6
    1 G(J, ID) = 0.
      RETURN
      END
      SUBROUTINEGECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.
      H(1, ID) = 24K1ν/h(1-2ν)
      H(2, ID) = 24K1ν/h(1-2ν) /X
      H(3, ID) = 0.
      H(4, ID) = 0.
      H(5, ID) = -1./X
      H(6, ID) = 24K1^2(1-ν)/h^2(1-2ν)
      RETURN
      END
```

Problem 12 - Cylindrical shell (axially symmetric): See Figure 20

$$u_1 = u\ , \quad u_2 = \psi, \quad u_3 = w\ , \; c_1 = c_p\ , \; c_2 = c_s$$

Governing Equations:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c_p^{\,2}} \frac{\partial^2 u}{\partial t^2} = -\frac{\nu}{R} \frac{\partial w}{\partial x}$$

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{c_p^{\,2}} \frac{\partial^2 \psi}{\partial t^2} = \frac{g}{R\eta F_2} \psi + \frac{g+\eta\nu/R}{R\eta\ F_2} \frac{\partial w}{\partial x}$$

$$\frac{\partial^2 w}{\partial x^2} - \frac{1}{c_s^{\,2}} \frac{\partial^2 w}{\partial t^2} = \frac{\nu}{Rg} \frac{\partial u}{\partial x} - \left(1 + \frac{\eta\nu}{gR}\right) \frac{\partial \psi}{\partial x} + \frac{(1 + \eta/R)}{R^2 g}\ w$$

Generalized Stresses:

$$N_x = hE_p \frac{\partial u}{\partial x} + \frac{hE_p\ \nu}{R} w$$

$$M_x = D(1-\eta) \frac{\partial \psi}{\partial x} - \frac{D\ \nu}{R^2} w$$

$$Q = k^2 Gh\ \psi + k^2 Gh \frac{\partial w}{\partial x}$$

Discontinuities:

$$\left[\frac{\partial u}{\partial x}\right] = K' c_p^{\,-1/2} \qquad \left[\frac{\partial u}{\partial t}\right] = -c_p \left[\frac{\partial u}{\partial x}\right]$$

$$\left[\frac{\partial \psi}{\partial x}\right] = \left[\frac{\partial \psi}{\partial t}\right] = 0$$

$$\left[\frac{\partial w}{\partial x}\right] = K''' c_s^{\,-1/2} \qquad \left[\frac{\partial w}{\partial t}\right] = -c_s \left[\frac{\partial w}{\partial x}\right]$$

where:

$$K' = \frac{c_p^{\,1/2}}{hE_p} [N_x]_{x = x_o} \quad \text{if } N_x \text{ boundary condition}$$

$$K' = -\frac{1}{c_p^{\,1/2}} [\dot{u}]_{x = x_o} \quad \text{if } \frac{\partial u}{\partial t} \text{ boundary condition}$$

$$K''' = \frac{c_s^{\,1/2}}{K^2 Gh} [Q]_{x = x_o} \quad \text{if } Q \text{ boundary condition}$$

$$K''' = -\frac{1}{c_s^{\,1/2}} [\dot{w}]_{x = x_o} \quad \text{if } \frac{\partial w}{\partial t} \text{ boundary condition}$$

Problem Package:

```
      SUBROUTINE JUMP I (X,DU1X,DU1T,DU2X,DU2T)
      DU1X=K'/c_p^(1/2)
      DU1T=-DU1X* c_p
      DU2X=0.
      DU2T=0.
      RETURN
      END
      SUBROUTINE JUMP II (X,DU3X,DU3T)
      DU3X=K'''/c_s^(1/2)
      DU3T=-DU3X* c_s
      RETURN
      END
      SUBROUTINE GECOFF (ID,XA,XB)
      COMMON U(9,300),Y(12,12),W(9,9),F(6,3)
      X=(XA+XB)/2.
      DO 1 J=1,4
    1 F(J,ID)=0.
      F(5,ID)= - ν/R
      F(6,ID)=0.
      RETURN
      END
      SUBROUTINE GECOFG (ID,XA,XB)
      COMMON U(9,300),Y(12,12),W(9,9),F(6,3),G(6,3)
      X=(XA+XB)/2.
      G(1,ID)=0.
      G(2,ID)=0.
      G(3,ID)=0.
      G(4,ID)= g/R η F2
      G(5,ID)= (g + η ν/R)/R η F2
      G(6,ID)=0.
      RETURN
      END
      SUBROUTINE GECOFH (ID,XA,XB)
      COMMON U(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3)
      X=(XA+XB)/2.
      H(1,ID)= ν/R g
      H(2,ID)=0.
      H(3,ID)= - (1 + ην/gR)
      H(4,ID)=0.
      H(5,ID)=0.
      H(6,ID)= (1 + η/R)/R^2 g
      RETURN
      END
```

Problem 13 Conical Shell: See Figure 21

$u_1 = u,\ u_2 = \psi,\ u_3 = w,\quad c_1 = c_p,\ c_2 = c_s$

Governing Equations:

$$\frac{\partial^2 u}{\partial s^2} - \frac{1}{c_p^{\,2}}\frac{\partial^2 u}{\partial t^2} = -\frac{1}{s}\frac{\partial u}{\partial s} - \frac{h^2\cot^2\alpha}{12\ s}\frac{\partial \psi}{\partial s} - \frac{\nu\cot\alpha}{s}\frac{\partial w}{\partial s}$$

$$\frac{\partial^2 \psi}{\partial s^2} - \frac{1}{c_p^{\,2}}\frac{\partial^2 \psi}{\partial t^2} = \left\{\frac{1}{1 - \frac{h^2\cot^2\alpha}{12\ s^2}}\left[\frac{1}{s^2}\left(1 + \frac{h^2\cot^2\alpha}{3\ s^2}\right) + \frac{12g}{h^2}\right]\right\}\psi$$

$$+ \frac{1}{\frac{h^2}{12}\left(1 - \frac{h^2\cot^2\alpha}{12\ s^2}\right)}\left(g + \frac{\nu h^2\cot^2\alpha}{12\ s^2}\right)\frac{\partial w}{\partial s}$$

$$\frac{\partial^2 w}{\partial s^2} - \frac{1}{c_s^{\,2}}\frac{\partial^2 w}{\partial t^2} = \frac{1}{s}\ \frac{\nu\cot\alpha}{g}\frac{\partial u}{\partial s} - \left(1 + \frac{h^2\nu\cot^2\alpha}{12\ gs^2}\right)\frac{\partial \psi}{\partial s}$$

$$+ \frac{1}{s^2}\ \frac{\cot^2\alpha}{g}\left(1 + \frac{h^2\cot^2\alpha}{12\ s^2}\right) w$$

Generalized Stresses:

$$N_s = E_p\ \nu h\left(\frac{1}{s}\right)u + \frac{E_p h^3\cot\alpha}{12}\left(\frac{1}{s^2}\right)(1-\nu)\ \psi + E_p\ \nu h\ \cot\alpha\ \left(\frac{1}{s}\right)\ w$$

$$+ E_p h\ \frac{\partial u}{\partial s}$$

$$M_s = -\frac{E_p h^3\nu\cot\alpha}{12}\left(\frac{1}{s^2}\right)u + \frac{E_p h^3\nu\cot\alpha}{12}\left(\frac{1}{s^2}\right)\left(\frac{h^2\cot\alpha}{12s} + s\ \tan\alpha\right)\psi$$

$$- \frac{E_p h^3\nu\cot^2\alpha}{12}\left(\frac{1}{s^2}\right)w + \frac{E_p h^3\cot\alpha}{12}\left(\frac{1}{s}\right)\left(s\ \tan\alpha - \frac{h^2\cot\alpha}{12s}\right)\frac{\partial \psi}{\partial s}$$

$$Q = k^2 Gh\ \psi + k^2 Gh\ \frac{\partial w}{\partial s}$$

Discontinuities:

$$\left[\frac{\partial u}{\partial s}\right] = K' \, s^{-1/2} + K'' \, s^{-3/2}$$

$$\left[\frac{\partial u}{\partial t}\right] = -c_p \left[\frac{\partial u}{\partial s}\right]$$

$$\left[\frac{\partial \psi}{\partial s}\right] = \frac{24 \, K'' \tan\alpha}{h^2} \, s^{-1/2}$$

$$\left[\frac{\partial \psi}{\partial t}\right] = -c_p \left[\frac{\partial \psi}{\partial s}\right]$$

$$\left[\frac{\partial w}{\partial s}\right] = K''' \, c_s^{-1/2} s^{-1/2}$$

$$\left[\frac{\partial w}{\partial t}\right] = -c_s \left[\frac{\partial w}{\partial s}\right]$$

where

$$K' = \frac{s_o^{1/2}}{E_p h} \left\{[N_s]_{s = s_o} - \frac{[M_s]_{s = s_o}}{2\left(s_o \tan\alpha - \frac{h^2 \cot\alpha}{12 s_o}\right)}\right\}, \text{ if } N_s \text{ and } M_s \text{ are boundary conditions}$$

$$K' = \frac{s_o^{1/2}}{c_p} \left\{\frac{h^2}{s_o \, 24 \tan\alpha} [\dot{\psi}]_{s = s_o} - [\dot{u}]_{s = s_o}\right\}, \text{ if } \frac{\partial u}{\partial t} \text{ and } \frac{\partial \psi}{\partial t} \text{ are boundary conditions}$$

$$K'' = \frac{s_o^{3/2} [M_s]_{s = s_o}}{2 \, E_p h \left(s_o \tan\alpha - \frac{h^2 \cot\alpha}{12 s_o}\right)}, \text{ if } M_s \text{ is boundary condition}$$

$$K'' = -\frac{h \, s_o^{1/2}}{24 \, c_p \tan\alpha} [\dot{\psi}]_{s = s_o}, \text{ if } \frac{\partial \psi}{\partial t} \text{ is boundary condition}$$

$$K''' = \frac{c_s^{1/2} s_o^{1/2}}{K^2 G h} [Q]_{s = s_o}, \text{ if } Q \text{ is boundary condition}$$

$$K''' = -\frac{s_o^{1/2}}{c_s^{1/2}} [\dot{w}]_{s = s_o}, \text{ if } \frac{\partial w}{\partial t} \text{ is boundary condition}$$

PROBLEM PACKAGE:

```
      SUBROUTINE JUMPI (X, DU1X, DU1T, DU2X, DU2T)
      DU1X = K'/X**.5 + K''/X**1.5
      DU1T = -DU1X* c_p
      DU2X = 24K'' tan α/h^2 /X** .5
      DU2T = -DU2X* c_p
      RETURN
      END
      SUBROUTINE JUMPII (X, DU3X, DU3T)
      DU3X = K''' c_s^(-1/2) /X**.5
      DU3T = -DU3X* c_s
      RETURN
      END
      SUBROUTINE GECOFF(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3)
      X = (XA + XB)/2.
      F(1,ID) = -1./X
      F(3,ID) = -h^2cot^2α/12 /X
      F(5,ID) = -νcotα /X
      DO1I = 2,6,2
    1 F(I,ID) = 0.
      RETURN
      END
      SUBROUTINE GECOFG(ID,XA,XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
      X = (XA + XB)/2.
      Q = 1./(1. - h^2cot^2α/12 /X**2)
      Z = h^2cot^2α/3 /X**2
      DO1I = 1,3
    1 G(I,ID) = 0.
      G(4,ID) = Q *((1.+Z)/X**2 + 12g/h^2)

      G(5,ID) = 12/h^2 *  Q* (g + ν/4 * Z)
      G(6,ID) = 0.
      RETURN
      END
      SUBROUTINE GECOFH(ID, XA, XB)
      COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3), H(6,3)
      X = (XA + XB)/2.
      H(1,ID) = νcot α/g /X
      H(2,ID) = 0.
      H(3,ID) = -1. - h^2 νcot^2 α/12g /X**2
      H(4,ID) = 0.
      H(5,ID) = 0.
      H(6,ID) = cot^2 α/g * (1. + h^2cot^2α/12 /X**2) /X**2
      RETURN
      END
```

APPENDIX C: PREWRITTEN BOUNDARY CONDITION PACKAGES

This appendix consists of several packages, each of which includes a subroutine for the specification of a particular function of time along the boundary. Notice that the "Boundary Conditions Time Functions Subroutine" actually consists of three "SUBROUTINES" in the programming sense. For each of these, the user should substitute a boundary condition package; the first to specify b_1, the second to specify b_2, and the third to specify b_3. Thus, a complete specification of the Boundary Conditions Time Functions Subroutine consists of three boundary condition packages. Each of the following packages defines a variable F<u>i</u>. The user should substitute either 1, 2, or 3 for <u>i</u>, depending upon whether the package is being used to specify b_1, b_2, or b_3 respectively. In some of the packages, it is necessary for the user to fill in constants to exactly specify the time function. Such constants are defined prior to each package, and as in the problem packages, are underlined within the packages themselves.

In these packages, the magnitude of the function is assumed to be unity. If a magnitude other than unity is desired, merely multiply the righthand side of the F<u>i</u> statement by the desired value. For example, if the user has a boundary condition involving the sine with amplitude 10.0, then his F<u>i</u> statement in the Sinusoidal package should read

F<u>i</u> = 10. * SIN(ANGLE)

Boundary Condition 1: Step:

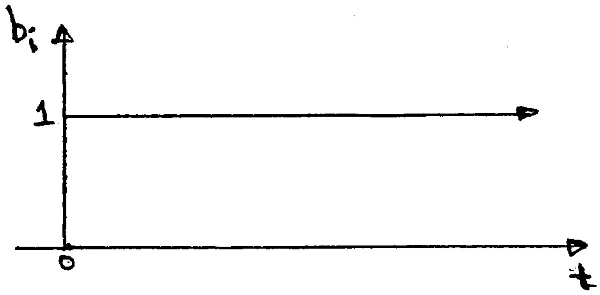


Figure C1: Step boundary condition

```
SUBROUTINEBCTFi (T, Fi)
Fi = 1.
RETURN
END
```

Boundary Condition 2: Ramp:

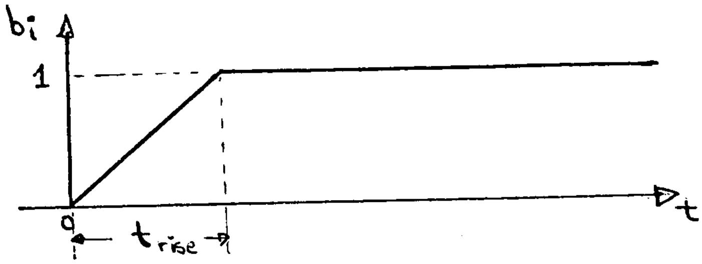


Figure C2: Ramp boundary condition

```
   SUBROUTINEBCTFi(T, F i)
   IF (T - trise) 1, 1,2
1  Fi = T/trise
   GOTO3
2  Fi = 1.
3  RETURN
   END
```

Boundary Condition 3: Sinusoidal:

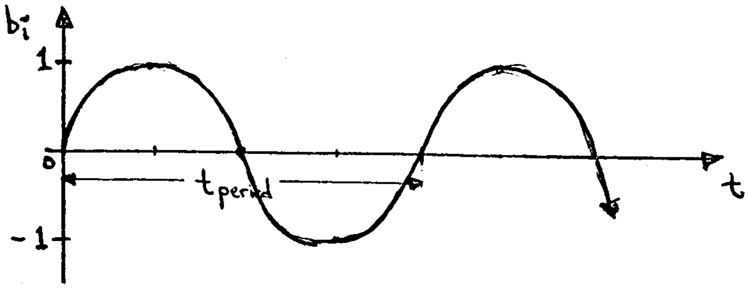


Figure C3: Sinusoidal boundary condition

```
SUBROUTINEBCTFi (T, F i)
BOP = T/tperiod
N = BOP
ZN = N
ANGLE = (BOP - ZN) * 6.2831853
Fi = SIN(ANGLE)
RETURN END
```

Boundary Condition 4: Exponential:

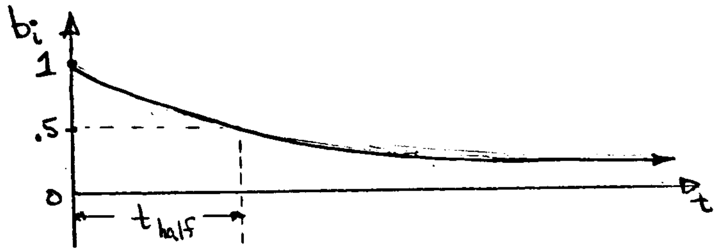


Figure C4: Exponential boundary condition

```
SUBROUTINEBCTFi (T, F i)
RAISE = +.69315/thalf
Fi = EXP(RAISE)
RETURN
END
```

Boundary Condition 5: Zero:

```
SUBROUTINEBCTFi (T, Fi)
Fi = 0.
RETURN
END
```

APPENDIX D: INSTRUCTIONS FOR WRITING PRINTOUT SUBROUTINE AND USER SPECIFIED STRUCTURE AND BOUNDARY CONDITION PACKAGES

This appendix includes the instructions for writing each of the necessary subroutines for a structure or boundary condition not included in the prewritten packages. To use MCDIT 21, for problems not prewritten, the following conditions on the user's structure must be remembered.

1. The governing differential equations of the structure must be in the form of equations (II-1), (II-2), or (II-3).
2. The program treats semi-infinite regions only.
3. The initial conditions utilized by the program are zero.

In the discussion of Sections II, III, and IV, a system of three equations ($n = 3$) corresponding to equations (II-3) was considered. The utilization of MCDIT 21 for problems governed by equations of the form of equations (II-1) or (II-2) is straightforward, as is now discussed.

<u>$n = 1$ structure-equation (II-1)</u>

1. User sets $f_3 \ldots f_6$, $g_1 \ldots g_6$, and $h_1 \ldots h_6$ all equal to zero.
2. User specifies $u_2 = u_3 = 0$ along the boundary (for all t) as two of the three boundary conditions.
3. User specifies $c_1 = c_2$ = wave speed in $n = 1$ problem.
4. Solution will include the desired solution for u_1 and its derivatives, as well as the trivial solutions for u_2, u_3 and their derivatives.

<u>$n = 2$ structure-equations (II-2)</u>

1. User sets f_3, f_4, h_3, h_4, and $g_1 \ldots g_6$, all equal to zero.
2. User specifies $u_2 = 0$ along the boundary (for all t) as one of the three boundary conditions.

3. User specifies c_1 = leading wave speed and c_2 = second wave speed in n = 2 problem.

4. Solution will include the desired solution for u_1, u_2, and their derivatives, as well as the trivial solutions for u_2 and its derivatives.

In the subroutine description which follows, notice that most of the subroutines actually specify more than one "SUBROUTINE" in the programming sense. However, this is of no concern in the conceptual view of the program. The quantities shown below on the left are represented by the corresponding FORTRAN variables on the right.

number of $\frac{dx}{c_1 dt} = -1$ lines already evaluated - XLI

x at the point being evaluated				-	X
t "	"	"	"	-	T
u_1 "	"	"	"	-	U1
u_2 "	"	"	"	-	U2
u_3 "	"	"	"	-	U3
$\frac{\partial u_1}{\partial x}$	"	"	"	-	U1X
$\frac{\partial u_1}{\partial t}$	"	"	"	-	U1T
$\frac{\partial u_2}{\partial x}$	"	"	"	-	U2X
$\frac{\partial u_2}{\partial t}$	"	"	"	-	U2T
$\frac{\partial u_3}{\partial x}$	"	"	"	-	U3X
$\frac{\partial u_3}{\partial t}$	"	"	"	-	U3T

jump in	$\frac{\partial u_1}{\partial x}$	along	first	discontinuity	line	-	DU1X
" "	$\frac{\partial u_1}{\partial t}$	"	"	"	"	-	DU1T
" "	$\frac{\partial u_2}{\partial x}$	"	"	"	"	-	DU2X

jump in $\frac{\partial u_2}{\partial t}$ along first discontinuity line - DU2T

" " $\frac{\partial u_3}{\partial x}$ along second " " - DU3X

" " $\frac{\partial u_3}{\partial t}$ " " " " - DU3T

x_a and x_b for averaging governing equation coefficients between two points - XA and XB

$f_1 \ldots f_6$ - F(1,ID)...F(6,ID)

$g_1 \ldots g_6$ - G(1,ID)...G(6,ID)

$h_1 \ldots h_6$ - H(1,ID)...H(6,ID)

b_1, b_2, and b_3 - F1, F2, and F3

Following are descriptions of the content of each of the user specified subroutines (Use Appendices B and C as examples):

A. Subroutines replacing Common Structure Package

1. Discontinuity Value Specification Subroutine

   a. a fortran statement: SUBROUTINE JUMP I (X, DU1X, DU1T, DU2X, DU2T)

   b. a series of fortran statements which, at its conclusion has defined DU1X, DU1T, DU2X, and DU2T in terms of X.

   c. the fortran statements:
```
RETURN
END
SUBROUTINE JUMP II (X, DU3X, DU3T)
```

   d. a series of fortran statements which, at its conclusion, has defined DU3X and DU3T in terms of X.

   e. the fortran statements:
```
RETURN
END
```

2. Governing Equation Coefficient Values Subroutine

   a. the fortran statements:
```
SUBROUTINE GECOFF (ID, XA, XB)

COMMON U(9,300), Y(12,12), W(9,9), F(6,3)

X = (XA + XB)/2.
```

b. a series of fortran statements which, at its conclusion, has defined F(1,ID), F(2,ID), F(3,ID), F(4,ID), F(5,ID), and F(6,ID) in terms of X.

c. the fortran statements:
```
RETURN
END
SUBROUTINEGECOFG(ID, XA, XB)
COMMON U(9,300), Y(12,12), W(9,9), F(6,3), G(6,3)
X = (XA + XB)/2.
```

d. a series of fortran statements which, at its conclusion, has defined G(1,ID), G(2,ID), G(3,ID), G(4,ID), G(5,ID), and G(6,ID) in terms of X.

e. the fortran statements:
```
RETURN
END
SUBROUTINE GECOFH(ID, XA, XB)
COMMON U(9,300), Y(12,12), W(9,9), F(6,3),
  G(6,3), H(6,3)
X = (XA + XB)/2.
```

f. a series of fortran statements which, at its conclusion, has defined H(1,ID), H(2,ID), H(3,ID), H(4,ID), H(5,ID), and H(6,ID) in terms of X.

g. the fortran statements:
```
RETURN
END
```

B. Subroutine Replacing Boundary Condition Packages

a. a fortran statement: `SUBROUTINE BCTF1(T,F1)`

b. a series of fortran statements, which, at its conclusion, has defined F1 in terms of T.

c. the fortran statements:
```
RETURN
END
SUBROUTINE BCTF2(T,F2)
```

d. a series of fortran statements, which, at its conclusion, has defined F2 in terms of T.

e. the fortran statements:
```
RETURN
END
SUBROTUINE BCTF3(T,F3)
```

f. a series of fortran statements, which, at its conclusion, has defined F3 in terms of T.

g. the fortran statements:
```
RETURN
END
```

C. Printout Quantities Specification Subroutine

a. the fortran statement: SUBROUTINEPRINTO(X, T, U1, U1X, U1T, U2, U2X, U2T, U3, U3X, U3T, XLI)

b. a series of fortran statements defining quantities (involving or functions of the 12 variables enclosed in parenthesis of part a.) to be printed out at a particular point.

For example, if the user desires to print out the values of x, t, and $\partial u_3/\partial x$ for all points on the first 20 $\frac{dx}{c_1 dt} = -1$ lines, the subroutine should include the following statements:

```
3 FORMAT(1H, 4HX = ,  E15.8, 2X, 4HT = ,  E15.8, 2X, 6HU3X = , E15.8)
  IF(XLI - 20.) 1, 2, 2
1 PRINT 3, X, T, U3X
2 CONTINUE
```

If the user desires to print out quantities only at specific predetermined points an appropriate IF statement must be used as illustrated below. In this case, care must be taken since the computer may only carry 4 or 5 significant figures in a floating point representation of X or T. For example, if printout is desired at all points along the line X = .5, and the computed values of X are carried as X = .4999, an ordinary IF statement will not cause printout. In order to avoid this situation the user should use the following technique:

Define in the subroutine a number TOL = $\frac{\Delta x}{10}$ and have the computer test to determine if the absolute value of the difference between the computer value and the stipulated printout value of X or T is less than TOL.

The following example illustrates this printout technique. The quantity

$$S = (.2)\ u_1 + (.3)\ \partial u_2/\partial t$$

is to be printed out, together with t, at all points along X (spatial coordinate) = .5 and 1.0. Assume the mesh size to be $\Delta x = .01$. The printout quantities specification subroutine is as follows:

```
  SUBROUTINEPRINTO(X,T,U1,U1X,U1T,U2,U2X,U2T,U3,U3X,U3T,XLI)
1 FORMAT(1H, 4HX = ,E15.8,2X,4HT = ,E15.8,2X,4HS = ,E15.8)
  TOL=+0.1E-02
  IF (ABS(X-.5)-TOL)2,2,3
3 IF (ABS(X-1.)-TOL)2,2,4
2 S=(.2)*U1+(.3)*U2T
  PRINT 1,X,T,S
4 RETURN
  END
```

Another example of this subroutine is included in Appendix F.

APPENDIX E: CHARACTERISTIC AND CONTINUITY EQUATIONS

The characteristic equations used for calculating the variables $u_1, \frac{\partial u_1}{\partial x}, \frac{\partial u_1}{\partial t}, u_2, \frac{\partial u_2}{\partial x}, \frac{\partial u_2}{\partial t}, u_3, \frac{\partial u_3}{\partial x}$, and $\frac{\partial u_3}{\partial t}$ by the method of characteristics are as follows (where $\frac{\partial u_1}{\partial x} = u_{1,x}$, $\frac{\partial u_1}{\partial t} = u_{1,t}$, etc.):

$$d(u_{1,t}) - c_1 d(u_{1,x}) + c_1[f_1\, u_{1,x} + f_2\, u_1 + f_3\, u_{2,x} + f_4 u_2 + f_5\, u_{3,x} + f_6\, u_3]dx = 0 \qquad \text{(E-1)}$$

$$\text{Along } \frac{dx}{c_1 dt} = +1$$

$$d(u_{1,t}) + c_1 d(u_{1,x}) - c_1[f_1\, u_{1,x} + f_2\, u_1 + f_3\, u_{2,x} + f_4\, u_2 + f_5\, u_{3,x} + f_6\, u_3]dx = 0 \qquad \text{(E-2)}$$

$$\text{Along } \frac{dx}{c_1 dt} = -1$$

$$d(u_{2,t}) \mp c_1 d(u_{2,x}) \pm c_1[g_1\, u_{1,x} + g_2\, u_1 + g_3\, u_{2,x} + g_4\, u_2 + g_5\, u_{3,x} + g_6\, u_3]dx = 0 \qquad \text{(E-3)}$$

$$\text{Along } \frac{dx}{c_1 dt} = \pm 1, \text{ respectively}$$

$$d(u_{3,t}) \mp c_2 d(u_{3,x}) \pm c_2[h_1\, u_{1,x} + h_2\, u_1 + h_3\, u_{2,x} + h_4\, u_2 + h_5\, u_{3,x} + h_6\, u_3]dx = 0 \qquad \text{(E-4)}$$

$$\text{Along } \frac{dx}{c_1 dt} = \pm \frac{c_2}{c_1}, \text{ respectively}$$

The continuity equations used for the calculations are

$$du_i = u_{i,x}\, dx + u_{i,t}\, dt \qquad i = 1, 2, 3$$

along any direction. (E-5)

The derivation of these equations along with the finite difference form of these equations used in the numerical computations may be found in Refs. 1 or 2.

APPENDIX F: CONICAL SHELL EXAMPLE

Consider a conical shell problem with the governing equations given under Problem Package 13 in Appendix B, and the following numerical values of the constants used in that problem.

$\alpha = 45°$; $h = 0.1$; $\nu = 1/3$; $k^2 = .87$; $c_p = 1$; $c_s = .53851646$; $s_o = 1.4142351$

The boundary conditions to be specified are:

$$\left.\begin{array}{ll} \frac{\partial u}{\partial t} = 0,\ t < 0 \quad ; & \frac{\partial u}{\partial t} = \cos\alpha\ ,\ t > 0 \\ \frac{\partial \psi}{\partial t} = 0,\ \text{all } t & \\ \frac{\partial w}{\partial t} = 0,\ t < 0 \quad ; & \frac{\partial w}{\partial t} = -\sin\alpha,\ t > 0 \end{array}\right\} s = s_o$$

which is equivalent to a conical shell impacted at one end by a flat plate with a constant axial velocity of one.

It is desired to run this problem at a mesh size of $\Delta x = .01$ for $M_o = 200$ points and to print out the values of $s, t, u, \frac{\partial u}{\partial s}, \frac{\partial u}{\partial t}, \psi, \frac{\partial \psi}{\partial s}, \frac{\partial \psi}{\partial t}, w, \frac{\partial w}{\partial s}$, and $\frac{\partial w}{\partial t}$ for all points along the first 6 lines. Also, we want to print out the values of $s, t, \frac{\partial u}{\partial t}, \frac{\partial w}{\partial t}, \frac{Ns}{hE_p}, \frac{Ms}{hE_p}$ and $\frac{Q}{hE_p}$ only at points along the lines $t = 1$ and $t = 2$.

First, we see that for a conical shell we must use package 13 in Appendix B. All that need be done by the user is to calculate the values of the underlined coefficients in this package and then punch the entire package where the numerical values just calculated are utilized for the respective underlined coefficients.

We then note that the first and third boundary conditions are simply step functions of time, and we can make use of the prewritten boundary condition package 1 in Appendix C. The second boundary condition is a zero function of time, prewritten as boundary condition 5 in Appendix C. We write the printout subroutine, following the instructions given in Appendix D. Thus, the user specified subroutines, as they are read into the computer, should appear as follows:

```
      SUBROUTINEJUMPI(X,DU1X,DU1T,DU2X,DU2T)
      DU1X=-0.84090296E+00/X**.5
      DU1T=-DU1X
      DU2X=0.
      DU2T=0.
      RETURN
      END
```

```
      SUBROUTINEJUMPII(X,DU3X,DU3T)
      DU3X=+0.15615176E+01/X**.5
      DU3T=-DU3X*(+0.53851646E+00)
      RETURN
      END
```

```
      SUBROUTINEGECOFF(ID,XA,XB)
      COMMON U(9,300),Y(12,12),W(9,9),F(6,3)
      X=(XA+XB)/2.
      F(1,ID)=-1./X
      F(2,ID)=0.
      F(3,ID)=-0.83333333E-03/X
      F(4,ID)=0.
      F(5,ID)=-0.33333333E+00/X
      F(6,ID)=0.
      RETURN
      END
```

```
      SUBROUTINEGECOFG(ID,XA,XB)
      COMMON U(9,300),Y(12,12),W(9,9),F(6,3),G(6,3)
      X=(XA+XB)/2.
      DO1J=1,3
    1 G(J,ID)=0.
      A=1.-(+0.83333333E-03)/X**2
      G(4,ID)=(1./A)*(1.+0.33333333E-02/X**2)/(X**2)+0.34800000E+03
      G(5,ID)=(+0.29+0.27777778E-03/X**2)/(+0.83333333E-03*A)
      G(6,ID)=0.
      RETURN
      END
```

```
      SUBROUTINEGECOFH(ID,XA,XB)
      COMMON U(9,300),Y(12,12),W(9,9),F(6,3),G(6,3),H(6,3)
      X=(XA+XB)/2.
      H(1,ID)=+1.14942529E+00/X
      H(2,ID)=0.
      H(3,ID)=-(1.+0.95785000E-03/X**2)
      H(4,ID)=0.
      H(5,ID)=0.
      H(6,ID)=(+3.44827586)*(1.+0.83333333E-03/X**2)/X**2
      RETURN
      END
```

BOUNDARY CONDITION PACKAGES

```
      SUBROUTINEBCTF1(T,F1)
      F1=+0.70710678E+00
      RETURN
      END
```

```
      SUBROUTINEBCTF2(T,F2)
      F2=0.
      RETURN
      END
```

```
      SUBROUTINEBCTF3(T,F3)
      F3=-0.70710678E+00
      RETURN
      END
```

PRINTOUT QUANTITIES SUBROUTINE SPECIFICATIONS

```
      SUBROUTINEPRINTO(X,T,U1,U1X,U1T,U2,U2X,U2T,U3,U3X,U3T,XLI)
    1 FORMAT(1H ,4HS = ,E15.8,2X,4HT = E15.8)
   10 FORMAT(1H ,6HU1T = ,E15.8,2X,6HU3T = ,E15.8)
    2 FORMAT(1H ,5HNS = ,E15.8,2X,5HMS = ,E15.8,2X,4HQ = ,E15.8,/)
    3 FORMAT(1H ,3(E15.8,2X),/)
    4 FORMAT(1H ,4(E15.8,2X))
      TOL=+0.1E-02
      IF(XLI-5.)5,5,6
    5 PRINT 4,X,T,U1,U1X
      PRINT 4,U1T,U2,U2X,U2T
      PRINT 3,U3,U3X,U3T
    6 IF(ABS(T-1.)-TOL)7,7,8
    8 IF(ABS(T-2.)-TOL)7,7,9
    7 PRINT 1,X,T
      PRINT 10,U1T,U3T
      A=+0.33333333E+00/X
      B=+0.55555555E-03/X**2
      SNS=A*U1+B*U2+A*U3+U1X
      C=+0.27777778E-03/X**2
      D=+0.83333333E-03/X
      SMS=-C*U1+C*(D+X)*U2-C*U3+D*(X-D)*U2X
      Q=(+0.29)*(U2+U3X)
      PRINT 2,SNS,SMS,Q
    9 RETURN
      END
```

The input data cards, as defined on page 17, should appear as follows for this particular problem:

```
+200+0.14142351E+01+0.10000000E-01+0.10000000E+01+0.53851646E+00
+0.00000000E+00+0.00000000E+00+0.00000000E+00+0.00000000E+00+0.00000000E+00
+0.00000000E+00+0.10000000E+01
+0.00000000E+00+0.00000000E+00+0.00000000E+00+0.00000000E+00+0.00000000E+00
+0.00000000E+00+0.10000000E+01
+0.00000000E+00+0.00000000E+00+0.00000000E+00+0.00000000E+00+0.00000000E+00
+0.00000000E+00+0.10000000E+01
```

The output data obtained will then appear as follows: (The first two pages include the preliminary printout and the values of quantities at points along the first six lines. The third page is a sampling of the output of the first fourteen points at t = 1.0. The fourth page is a sampling of the output of the first fourteen points at t = 2.0.

```
NUMBER OF POINTS ALONG LEADING WAVE =  200
XZERO =  0.14142351E 01      DELTAX =  0.99999979E-02
C1 =  0.10000000E 01         C2 =  0.53851646E 00
( 0.0            )*U1X+( 0.0          )*U1+( 0.0          )*U2X+
( 0.0            )*U2+( 0.0           )*U3X+( 0.0         )*U3
     +( 0.10000000E 01)*U1T = BOUNDARY CONDITION FUNCTION 1
( 0.0            )*U1X+( 0.0          )*U1+( 0.0          )*U2X+
( 0.0            )*U2+( 0.0           )*U3X+( 0.0         )*U3
     +( 0.10000000E 01)*U2T = BOUNDARY CONDITION FUNCTION 2
( 0.0            )*U1X+( 0.0          )*U1+( 0.0          )*U2X+
( 0.0            )*U2+( 0.0           )*U3X+( 0.0         )*U3
     +( 0.10000000E 01)*U3T = BOUNDARY CONDITION FUNCTION 3
```

0.14142351E 01	0.0	0.0	-0.70710719E 00
0.70710719E 00	0.0	0.0	0.0
0.0	0.13130646E 01	-0.70710719E 00	
0.14242344E 01	0.99999979E-02	0.0	-0.70462036E 00
0.70462036E 00	0.0	0.0	0.0
0.0	0.0	0.0	
0.14142351E 01	0.19999996E-01	0.14142126E-01	-0.70994049E 00
0.70710677E 00	0.0	-0.31897001E 01	0.0
-0.14142126E-01	0.13108006E 01	-0.70710677E 00	
0.14342346E 01	0.19999996E-01	0.0	-0.70215935E 00
0.70215935E 00	0.0	0.0	0.0
0.0	0.0	0.0	
0.14242344E 01	0.29999994E-01	0.14109708E-01	-0.70941812E 00
0.70570242E 00	-0.24884671E-01	-0.22082222E 00	-0.15634260E 01
-0.80280453E-02	0.13164978E 01	-0.70346302E 00	
0.14142351E 01	0.39999992E-01	0.28283231E-01	-0.71278167E 00
0.70710677E 00	0.0	-0.62935820E 01	0.0
-0.28283231E-01	0.12978601E 01	-0.70710677E 00	
0.14442348E 01	0.29999994E-01	0.0	-0.69972432E 00
0.69972432E 00	0.0	0.0	0.0
0.0	0.0	0.0	

0.14342346E 01	0.39999992E-01	0.14068179E-01	-0.70890439E 00
0.70431387E 00	-0.35771128E-01	0.27066164E 01	-0.30912304E 01
-0.19070844E-02	0.13023710E 01	-0.69254458E 00	
0.14242344E 01	0.49999990E-01	0.28222829E-01	-0.71220940E 00
0.70570886E 00	-0.55649795E-01	-0.33116255E 01	-0.14898005E 01
-0.22069555E-01	0.13254557E 01	-0.70652765E 00	
0.14142351E 01	0.59999987E-01	0.42424336E-01	-0.71561998E 00
0.70710677E 00	0.0	-0.92335043E 01	0.0
-0.42424336E-01	0.12753191E 01	-0.70710677E 00	
0.14542351E 01	0.39999992E-01	0.0	-0.69731444E 00
0.69731444E 00	0.0	0.0	0.0
0.0	0.0	0.0	
0.14442348E 01	0.49999990E-01	0.14018625E-01	-0.70707673E 00
0.70222390E 00	-0.35752952E-01	0.36200743E 01	-0.35310392E 01
0.16554911E-03	-0.15360195E-01	0.17200094E-01	
0.14342346E 01	0.59999987E-01	0.28153546E-01	-0.71168834E 00
0.70433259E 00	-0.96197188E-01	-0.38464469E 00	-0.29219503E 01
-0.15766829E-01	0.13294678E 01	-0.69775224E 00	
0.14242344E 01	0.69999933E-01	0.42336091E-01	-0.71498245E 00
0.70571268E 00	-0.84633172E-01	-0.62331610E 01	-0.13828888E 01
-0.36121193E-01	0.13261671E 01	-0.71105361E 00	
0.14142351E 01	0.79999983E-01	0.56565441E-01	-0.71846193E 00
0.70710677E 00	0.0	-0.11943664E 02	0.0
-0.56565441E-01	0.12436190E 01	-0.70710677E 00	
0.14642344E 01	0.49999990E-01	0.0	-0.69492930E 00
0.69492930E 00	0.0	0.0	0.0
0.0	0.0	0.0	
0.14542351E 01	0.59999987E-01	0.13970308E-01	-0.70460838E 00
0.69979858E 00	-0.34821451E-01	0.35310278E 01	-0.34337692E 01
0.20390295E-03	-0.21015473E-01	0.20055845E-01	
0.14442348E 01	0.69999933E-01	0.28075680E-01	-0.71117908E 00
0.70296347E 00	-0.12197936E 00	0.24601555E 01	-0.42987700E 01
-0.94064288E-02	0.13128910E 01	-0.68121260E 00	
0.14342346E 01	0.79999983E-01	0.42239293E-01	-0.71443516E 00
0.70435232E 00	-0.15277719E 00	-0.33001232E 01	-0.27012224E 01
-0.29706903E-01	0.13524904E 01	-0.70573759E 00	
0.14242344E 01	0.89999974E-01	0.56449462E-01	-0.71773463E 00
0.70571935E 00	-0.11140227E 00	-0.89339781E 01	-0.12572975E 01
-0.50195388E-01	0.13179007E 01	-0.71525198E 00	
0.14142351E 01	0.99999964E-01	0.70706546E-01	-0.72129732E 00
0.70710677E 00	0.0	-0.14390627E 02	0.0
-0.70706546E-01	0.12050152E 01	-0.70710677E 00	

```
S =   0.14142351E 01   T =   0.99999976E 00
U1T =   0.70710677E 00   U3T = -0.70710677E 00
NS = -0.81937504E 00   MS = -0.24636276E-01   Q =   0.19964623E 00

S =   0.14342346E 01   T =   0.99999976E 00
U1T =   0.70545286E 00   U3T = -0.69820625E 00
NS = -0.80139863E 00   MS = -0.21387510E-01   Q =   0.17010182E 00

S =   0.14542351E 01   T =   0.99999976E 00
U1T =   0.70376515E 00   U3T = -0.68758911E 00
NS = -0.79352427E 00   MS = -0.18218216E-01   Q =   0.15986311E 00

S =   0.14742346E 01   T =   0.99999976E 00
U1T =   0.70207989E 00   U3T = -0.67376709E 00
NS = -0.78582472E 00   MS = -0.15041091E-01   Q =   0.15015870E 00

S =   0.14942350E 01   T =   0.99999976E 00
U1T =   0.70038003E 00   U3T = -0.65786612E 00
NS = -0.77834988E 00   MS = -0.12248114E-01   Q =   0.14216459E 00

S =   0.15142345E 01   T =   0.99999976E 00
U1T =   0.69862705E 00   U3T = -0.63896435E 00
NS = -0.77107632E 00   MS = -0.96380524E-02   Q =   0.13424098E 00

S =   0.15342350E 01   T =   0.99999976E 00
U1T =   0.69681150E 00   U3T = -0.61646265E 00
NS = -0.76395786E 00   MS = -0.70048347E-02   Q =   0.12605911E 00

S =   0.15542345E 01   T =   0.99999976E 00
U1T =   0.69493121E 00   U3T = -0.59082639E 00
NS = -0.75703961E 00   MS = -0.45181289E-02   Q =   0.11880171E 00

S =   0.15742350E 01   T =   0.99999976E 00
U1T =   0.69297278E 00   U3T = -0.56267697E 00
NS = -0.75031990E 00   MS = -0.23600888E-02   Q =   0.11264151E 00

S =   0.15942345E 01   T =   0.99999976E 00
U1T =   0.69090521E 00   U3T = -0.53091586E 00
NS = -0.74373716E 00   MS = -0.19003637E-03   Q =   0.10576689E 00

S =   0.16142349E 01   T =   0.99999976E 00
U1T =   0.68874323E 00   U3T = -0.49491727E 00
NS = -0.73731774E 00   MS =   0.20337568E-02   Q =   0.98879337E-01

S =   0.16342344E 01   T =   0.99999976E 00
U1T =   0.68647879E 00   U3T = -0.45534104E 00
NS = -0.73110181E 00   MS =   0.38552617E-02   Q =   0.93317330E-01

S =   0.16542349E 01   T =   0.99999976E 00
U1T =   0.68405157E 00   U3T = -0.41021740E 00
NS = -0.72500265E 00   MS =   0.56532435E-02   Q =   0.85968673E-01

S =   0.16742344E 01   T =   0.99999976E 00
U1T =   0.68148381E 00   U3T = -0.35855532E 00
NS = -0.71902144E 00   MS =   0.74557923E-02   Q =   0.77051699E-01
```

```
S =   0.14142351E 01   T =   0.19999990E 01
U1T =   0.70710677E 00   U3T = -0.70710677E 00
NS = -0.88346136E 00   MS = -0.33302285E-01   Q =   0.28184670E 00

S =   0.14342346E 01   T =   0.19999990E 01
U1T =   0.70615530E 00   U3T = -0.69461149E 00
NS = -0.84635288E 00   MS = -0.29161643E-01   Q =   0.21308839E 00

S =   0.14542351E 01   T =   0.19999990E 01
U1T =   0.70511925E 00   U3T = -0.67991853E 00
NS = -0.83804649E 00   MS = -0.25237471E-01   Q =   0.19597703E 00

S =   0.14742346E 01   T =   0.19999990E 01
U1T =   0.70408654E 00   U3T = -0.66087252E 00
NS = -0.82986009E 00   MS = -0.21312010E-01   Q =   0.17975426E 00

S =   0.14942350E 01   T =   0.19999990E 01
U1T =   0.70301956E 00   U3T = -0.64085084E 00
NS = -0.82205021E 00   MS = -0.18047541E-01   Q =   0.16578442E 00

S =   0.15142345E 01   T =   0.19999990E 01
U1T =   0.70183623E 00   U3T = -0.61899656E 00
NS = -0.81448162E 00   MS = -0.15114337E-01   Q =   0.15191048E 00

S =   0.15342350E 01   T =   0.19999990E 01
U1T =   0.70057595E 00   U3T = -0.59433961E 00
NS = -0.80704314E 00   MS = -0.12141392E-01   Q =   0.13826114E 00

S =   0.15542345E 01   T =   0.19999990E 01
U1T =   0.69927913E 00   U3T = -0.56834865E 00
NS = -0.79983395E 00   MS = -0.94185024E-02   Q =   0.12648612E 00

S =   0.15742350E 01   T =   0.19999990E 01
U1T =   0.69788831E 00   U3T = -0.54253936E 00
NS = -0.79295641E 00   MS = -0.72838031E-02   Q =   0.11602587E 00

S =   0.15942345E 01   T =   0.19999990E 01
U1T =   0.69637346E 00   U3T = -0.51485652E 00
NS = -0.78619367E 00   MS = -0.51350370E-02   Q =   0.10502267E 00

S =   0.16142349E 01   T =   0.19999990E 01
U1T =   0.69477379E 00   U3T = -0.48530406E 00
NS = -0.77954364E 00   MS = -0.28425895E-02   Q =   0.95289230E-01

S =   0.16342344E 01   T =   0.19999990E 01
U1T =   0.69314718E 00   U3T = -0.45709813E 00
NS = -0.77329689E 00   MS = -0.12729492E-02   Q =   0.87857902E-01

S =   0.16542349E 01   T =   0.19999990E 01
U1T =   0.69134390E 00   U3T = -0.42747766E 00
NS = -0.76713330E 00   MS =   0.31113881E-03   Q =   0.79517007E-01

S =   0.16742344E 01   T =   0.19999990E 01
U1T =   0.68946409E 00   U3T = -0.39585626E 00
NS = -0.76107323E 00   MS =   0.20439313E-02   Q =   0.71668863E-01
```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