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SUMMARY

An analytical and experimental investigation was conducted to determine the heave (vertical) response characteristics of an open-plenum air-cushion suspension to simulated guideway inputs. In addition, passive and active control concepts for attenuating undesirable heave responses were considered. Analog-computer solutions of the linear and nonlinear equations of motion were obtained and compared with experimental data. Results of analysis and experiment defined cushion resonant response conditions at which guideway inputs may be greatly amplified in the absence of cushion control. With control, however, the analysis suggests that heave responses may be significantly attenuated for a relatively wide range of guideway inputs.

INTRODUCTION

The tracked air-cushion vehicle (TACV) appears to be a promising mode of ground transportation where speeds up to 300 miles per hour (482.7 km/hr) or more are desired. During operation, such a vehicle is supported vertically and guided laterally along a suitable guideway (track) by means of an air-cushion suspension system. The performance, operating economy, and ride quality of the TACV is highly dependent upon interactions between the vehicle, the air cushions, and the guideway which is a primary source of harmonic or random dynamic disturbances. Since an air cushion is essentially a mass supported by an air spring (generally nonlinear), the TACV has characteristic resonant frequencies which, depending upon operating conditions, can be well within the range of expected guideway disturbance frequencies and human comfort sensitivities. A detailed understanding of the nonlinear air-cushion dynamics is essential for the design and/or control of the cushions to achieve acceptable levels of vehicle response.

The French aerotrain system (ref. 1) has demonstrated the operational feasibility of using air cushions to support a high-speed vehicle moving along a relatively rigid track. Both theoretical and experimental research (refs. 2 to 6, for example) have been performed to study the heave (vertical) motion and/or stability of various air-cushion configurations. The theoretical studies (refs. 2, 3, 4, and 6) have been based primarily upon approaches which employ linear analyses to predict air-cushion heave responses. Since air cushions are inherently nonlinear, the use of linear analyses may be insufficient to predict fully the air-cushion dynamic-response behavior. Furthermore, any nonlinearities in air-cushion response will have a direct bearing upon the design of active or passive control systems to minimize the cushion response.

This paper presents the results of an analytical and experimental investigation to determine the dynamic behavior (in heave) of a simple plenum air-cushion suspension system in response to steady-state and transient-input disturbances. The linear and nonlinear behavior of the system is examined and the feasibility of employing active and passive control techniques to reduce undesirable responses is explored.

SYMBOLS

The units used for the physical quantities in this report are given both in the U.S. Customary Units and in the International System of Units (SI). Any consistent system of units may be used in the analysis.

A effective cushion support area

A_1 plate area of passive control device

a_1 magnitude of overshoot to applied step

C_d plenum discharge coefficient

C_1 damping coefficient of passive control device

d plenum base diameter

G_c total feedback gain of active control system

G_o valve gain of passive control device

g acceleration due to gravity

h distance between air-cushion base and guideway

$K(\omega)$	linearized air-cushion spring stiffness
K_h	linearized mass flow coefficient, $\left.\frac{\partial w_o}{\partial h}\right]_{\substack{p=\bar{p} \\ h=\bar{h}}}$
K_p	linearized mass flow coefficient, $\left.\frac{\partial w_o}{\partial p}\right]_{\substack{p=\bar{p} \\ h=\bar{h}}}$
K_1	spring stiffness of passive control device
m	plate mass of passive control device
m_p	plenum mass
p	absolute cushion pressure
p_a	atmospheric pressure
R	universal gas constant
s	Laplace transform variable
T	absolute temperature
t	time
T_a	acceleration response ratio
T_x	displacement response ratio
u	plate displacement of passive control device
V	total cushion volume
V_o	dead volume of air cushion
W	weight of air in air cushion
w_c	active control weight flow

w_{cp}	passive control weight flow
w_i	supply weight flow rate to cushion
w_o	weight flow rate out of air cushion
x	absolute displacement of plenum mass
z	amplitude of input disturbance
z_o	maximum input disturbance (single amplitude)
γ	polytropic gas constant
δ	log decrement
ζ	equivalent viscous damping ratio
ϵ	linear viscous damping ratio
ρ	mass density of air
$\lambda(\omega)$	air-cushion damping factor
τ_N	spring time delay
τ_n	parameter defined by equation (A11)
τ_1	active control time constant
ω	excitation frequency
ω_n	undamped natural frequency

A Δ before a symbol denotes a perturbation about equilibrium, a bar over a symbol denotes an average (equilibrium) value, and a dot denotes differentiation with respect to time.

EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus is shown schematically in figure 1 and photographically in figure 2. This apparatus consists of a 12.25 pound (54.5 newtons) simple plenum-type air cushion supplied by a constant-pressure air source through a number of flexible hoses. The air cushion is restrained to move only vertically as it rides over a flat ground plate which serves as a guideway. Guideway irregularities are reproduced by means of a hydraulic vibration exciter attached to the underside of the ground plate as shown in figure 1. A linear-displacement transducer mounted to a rigid framework was used to monitor continuously the absolute cushion displacement. Pertinent dimensions of the plenum are given in figure 3.

During operation, air is delivered by the flexible hoses to the upper chamber of the plenum and passes to the lower chamber through a perforated circular plate and a fine mesh wire screen which smooths the flow delivered to the lower chamber. Air-cushion operating (or hover) height is set by opening the input flow valve and monitoring the output of the position transducer. Once the desired hover height is reached, the hydraulic vibration exciter is actuated to oscillate the ground plate; these oscillations correspond to simulated periodic displacements of the guideway. By maintaining a constant-amplitude input displacement, varying the input frequency, and monitoring plenum displacement response, the ratio of output to input displacement of the system was determined for the particular hover height under consideration. Displacement responses for other operating heights were obtained in a similar manner.

MATHEMATICAL ANALYSIS

This section develops the equations governing the heave (vertical) response of a simple plenum-type air cushion to simulated guideway irregularities and describes the method of solution. In addition, the concept of using active and passive control techniques to minimize undesirable plenum response is introduced and discussed.

Development of System Equations

The coordinate system and nomenclature used in the development of the nonlinear equation of motion is shown in figure 4. Summing the forces acting on the plenum mass gives

$$m_p\ddot{x} = A(p - p_a) - m_p g \tag{1}$$

where m_p is the mass of the plenum device, A is the plenum support area, x is the absolute displacement of the plenum mass, p_a is ambient pressure, p is the absolute cushion support pressure, and g is the acceleration due to gravity. An equation for cushion pressure p can be developed by applying the law of conservation of mass flow rate to the air-cushion volume; that is,

$$\frac{d}{dt}(\rho V) = (w_i - w_o)\frac{1}{g} \tag{2}$$

where w_i is the supply weight flow rate and w_o is the weight flow rate out of the cushion volume. Performing the differentiation indicated in equation (2) gives

$$\dot{\rho} V + \rho \dot{V} = (w_i - w_o)\frac{1}{g}$$

or equivalently

$$\frac{\partial \rho}{\partial p}\dot{p} V + \rho \dot{V} = (w_i - w_o)\frac{1}{g} \tag{3}$$

It is assumed that the thermodynamic processes occur sufficiently rapid to obviate any significant dissipation of heat. Thus, the isentropic gas law is used to calculate the partial derivative in equation (3). The isentropic gas law states that

$$\frac{p}{\rho^{\gamma}} = \text{Constant} \tag{4}$$

where γ is the polytropic exponent ($\gamma = 1.4$ for isentropic process). By utilizing equation (4), the following partial derivative is obtained:

$$\frac{\partial \rho}{\partial p} = \frac{1}{\gamma}\frac{\rho}{p} \tag{5}$$

Substituting equation (5) into equation (3) yields the following pressure equation:

$$\dot{p} = \gamma(w_i - w_o)\frac{p}{\rho V g} - \frac{\gamma p \dot{V}}{V} \tag{6}$$

By applying the perfect gas law ($p/\rho g = RT$), equation (6) becomes

$$\dot{p} = \frac{\gamma(w_i - w_o)RT}{V} - \frac{\gamma p \dot{V}}{V} \tag{7}$$

For this analysis the supply flow rate is assumed to be constant or

$$w_i = \text{Constant} \tag{8}$$

The equation for weight flow rate out of the periphery of the air cushion is

$$w_o = C_d \pi d h g \sqrt{2\rho(p - p_a)} \tag{9}$$

where C_d is the discharge coefficient, d is the outside diameter of the plenum base, and ρ is the mass density of the air contained in the cushion volume.

The cushion volume V can be divided into two parts: a "dead" volume V_o which is independent of hover height h and a "live" volume which is a function of hover height; that is,

$$V = V_o + Ah \tag{10}$$

The final equation necessary to describe completely the motion of the system is the constraint relationship between coordinates. By referring to figure 4, this relationship can be written as

$$h = x - z \tag{11}$$

where $z = z_o \sin \omega t$ for harmonic disturbances and $z = z_o \delta(t - t_o)$ for step disturbances.

Solution of System Equations

Because of the nonlinear nature of the system equations, analog computer techniques were utilized to obtain solutions. The analog computer block diagram representing equations (1), (7), (8), (9), (10), and (11) is presented in figure 5. By using this analog diagram, properly scaled, the heave motions of the fluid suspension system can be determined for a broad range of input and air-cushion operating parameters. The numerical data used in this analysis are based upon the physical properties of the experimental model and are listed in table I.

Very good estimates of air-cushion response for small input disturbances can be obtained by linearizing the system equations and then deriving the linearized transfer functions describing the system behavior. The actual linearizing process and resulting equations are discussed in detail in the appendix.

Passive Control of Plenum Response

In general, the fluid-suspension heave responses are characterized by large response amplitudes occurring at the resonant frequency. The accelerations of the plenum mass associated with these responses will usually have an adverse effect on overall system ride quality and economy. Two promising approaches to the problem of alleviating the resonant response condition are considered analytically in this paper. The first approach involves the use of a passive control device shown schematically in figure 6. The passive device is basically a spring-loaded plate (with damper) whose motion in response to positive increases in cushion pressure will open an orifice and allow additional air to escape from the cushion volume. In reality, this device is equivalent to a spring-loaded relief valve. For the air cushion, the dynamic characteristics of the escape valve (both natural frequency and damping) are very important. To behave as a damper in the region of cushion resonance, the spring rate and damping coefficient must be tuned so that the phase shift between plate displacement and cushion pressure is 90° at cushion resonance. The equation describing plate motion is

$$m\ddot{u} + C_1\dot{u} + K_1 u = A_1 p(t) \tag{12}$$

or in transfer function form

$$\frac{u(s)}{p(s)} = \frac{A_1/K_1}{\frac{m}{K_1}s^2 + \frac{C_1}{K_1}s + 1} \tag{13}$$

The flow out of the air cushion (plenum) through the orifice area is assumed to be proportional to positive displacements of the plate; that is,

$$\left.\begin{aligned} w_{cp} &= G_o u \qquad && (u > 0) \\ w_{cp} &= 0 \qquad && (u \leqq 0) \end{aligned}\right\} \tag{14}$$

where G_o is the constant relating flow rate to plate displacement. The operational block diagram representing equations (13) and (14) is shown in figure 7(a) and the corresponding analog computer circuit diagram is presented in figure 7(b).

Active Control of Plenum Response

The active cushion control concept is illustrated schematically in figure 8. The plenum acceleration response is monitored by a suitable accelerometer, the output of which is applied to an acceleration controller for electronic compensation. The controller output in turn activates a control valve that modulates the input flow in such a manner as to oppose, or null, the plenum acceleration response. The block diagram of the active control system is shown in figure 9(a) and the corresponding analog computer diagram in figure 9(b). The accelerometer and control-valve transfer functions are assumed to be unity for the frequency range of interest. The transfer function relating control flow to plenum acceleration is

$$\frac{w_c(s)}{\ddot{x}(s)} = \frac{G_c}{\left(1 + \tau_1 s\right)^2} \tag{15}$$

where G_c is the total feedback gain (accelerometer, controller, and control valve) and $\tau_1 = 1/\omega_n$. The denominator of equation (15) is the transfer function representing the electronic compensation applied to the accelerometer signal. The denominator may be rewritten as

$$D(s) = \tau_1^2\left(\frac{1}{\tau_1^2} + \frac{2}{\tau_1} s + s^2\right) \tag{16}$$

By choosing $\tau_1 = 1/\omega_n$, the compensation network represents a critically damped second-order system tuned to have an undamped natural frequency of ω_n. Thus, at suspension system resonance ($\omega = \omega_n$), the controller shifts the phase of the acceleration signal by 90^o, and thereby applies a signal to the control valve that is proportional to the velocity of the plenum mass. The resultant modulation of supply flow results in a force applied to the mass that is proportional to the velocity of the mass and is, therefore, a damping force. Above resonance, the controller output will be greatly attenuated and will be 180^o out of phase with the acceleration signal. In this region, the active control system is relatively ineffective.

RESULTS AND DISCUSSION

Analytical and experimental studies were conducted to determine the steady-state and transient response behavior of the plenum air cushion model over a broad range of operating parameters and disturbance inputs. The disturbance inputs for both studies were restricted to sinusoidal and step displacements of the guideway. The results are presented and discussed in four sections. The first section discusses the static characteristics of the air-cushion system and the second section describes the dynamic-response behavior of the air-cushion suspension system to small disturbances with emphasis on establishing the general nature of the response. The constraint of small disturbance amplitudes is next relaxed and the resulting nonlinear dynamic behavior of the air-cushion suspension system is qualitatively described. Finally, the results of applying the passive and active control concepts to minimize or eliminate suspension-system resonances are discussed.

The forced response of the fluid suspension system is defined in terms of two non-dimensional parameters; one is a measure of the displacement response and the other, a measure of the acceleration response of the plenum mass. The first parameter is called the dynamic displacement response ratio and is defined as the ratio of maximum upward displacement about the undisturbed hover height to the maximum input displacement (see fig. 10); that is,

$$T_x = \left|\frac{x - \bar{h}}{z}\right|_+ \tag{17}$$

where $\left| \; \right|_+$ denotes the positive peak values only. The second parameter is the absolute acceleration response ratio of the system and is defined as the ratio of the peak absolute acceleration of the plenum mass to the peak input acceleration; that is,

$$T_a = \left| \frac{\ddot{x}}{\ddot{z}} \right|_+ \tag{18}$$

Figure 10(a) shows a typical time history of the cushion absolute displacement as predicted by the linearized analysis given in the appendix. The absolute displacement response is symmetrical about the undisturbed equilibrium height and maintains its sinusoidal wave shape. However, for large input disturbances, the cushion displacements are highly unsymmetrical about the undisturbed equilibrium position as illustrated in figure 10(b). The acceleration peaks corresponding to the displacement time history of figure 10(b) will obviously be much larger than the acceleration peaks corresponding to the displacement time history of figure 10(a). For the linear analysis, the displacement and acceleration transmissibilities are identical.

Static Characteristics

The important static properties of the air-cushion system were obtained from the analog computer simulation and are presented in figures 11 and 12. The relationship between input (or supply) flow rate and hover height as predicted by equation (9) is shown in figure 11. The force-deflection characteristics of the fluid suspension system for various values of supply flow rate are illustrated in figure 12. For an initial plenum weight of 12.25 pounds (54.5 newtons), note the nonlinear nature of the force-deflection curves and their dependence upon the value of initial supply flow rate. The slope of these curves at any point corresponds to the spring stiffness of the system at that particular point. Thus, the spring stiffness is nonlinear – hardening considerably for downward deflections about equilibrium and softening for upward displacements about equilibrium.

Air-Cushion Response to Small Disturbances

The analytical and experimental steady-state displacement response characteristics of the air cushion to small disturbances are presented in figure 13. The displacement response ratio is shown as a function of input frequency for several values of undisturbed hover height. The air-cushion suspension response is seen to be very similar to that exhibited by a conventional, single-degree-of-freedom mechanical spring-mass-damper system. The figure also shows that as the hover height is increased, there is a marked decrease in air-cushion-suspension resonant frequency and a corresponding increase in the dynamic amplification of the input at resonance. These data are summarized in figure 14 which presents the variation of suspension frequency and response amplitude with hover height.

Cushion Response to Large Disturbances

The analog computer and experimental data indicate that, in general, the fluid-suspension-system dynamic behavior (that is, resonant frequency, displacement transmissibility, acceleration transmissibility, and damping) depends significantly upon the amplitude of the steady-state and transient input. Furthermore, it is very difficult to determine specific sets of operating or input parameters that define boundaries at which air-cushion response can be described by a linear analysis. Therefore, the results introduced in this section are presented with the primary intent of indicating, in a qualitative manner only, the basic nonlinear response characteristics of the air cushion and the relative importance of these responses.

Steady-state inputs.- The analog computer and experimental data indicate that the fluid-suspension-system resonant frequency tends to decrease with increasing steady-state input amplitude. A typical trend is illustrated in figure 15(a) where the suspension-system resonant frequency is presented as a function of the ratio of input amplitude to undisturbed hover height for an undisturbed hover height of 0.10 inch (0.254 cm). The resonant frequency is defined as the frequency at which maximum response occurs and, as indicated in this figure, is dependent upon the direction of frequency sweep as well as input amplitude. The decrease in suspension-system frequency is attributed to an increase in the effective hover height resulting from the larger input amplitudes.

From the standpoint of economy and ride quality, the important dynamic parameters are the displacement and acceleration response ratios occurring at suspension-system resonance. The variation of these parameters with input amplitude (in percent of undisturbed hover height) is presented in figures 15(b) and 15(c). The displacement, as shown in figure 15(b), tends to increase slightly with input amplitude and then level off whereas the acceleration response (fig. 15(c)) continues to increase rapidly with input amplitude. The linear analysis would not predict either of these effects. Larger inputs would result in even larger disparities between displacement and acceleration response.

An example of perhaps the most interesting nonlinear phenomenon to occur is illustrated by the fluid-suspension-system displacement response characteristics presented in figure 16. These results were obtained for hover heights of 0.10 inch (0.254 cm) and 0.50 inch (1.27 cm). The input amplitude for each case was equal to approximately 20 percent of the corresponding undisturbed hover height and was held constant with respect to input frequency. As indicated in figure 16, the displacement response of the system is characterized by two large peaks instead of the one peak that would result for a linear system. The low frequency peak represents the response of the fluid suspension system at its fundamental resonant frequency and the high frequency resonance corresponds to subharmonic oscillations of the fluid suspension system, the actual frequency of oscillation being the first even subharmonic of the input frequency (that is, one-half of the excitation frequency). Similar subharmonic responses were generated for other ranges of input and operating parameters and, although no attempt was made to define subharmonic response boundaries for those parameters, several general comments regarding this phenomena are of interest. First, subharmonic oscillations did not occur for input amplitudes less than approximately 5 percent of the equilibrium hover height. The range of input frequencies (input frequency bandwidth) at which subharmonic resonances occurred generally increased with increasing input amplitude. However, there are upper and lower bounds to the subharmonic frequency bandwidth beyond which subharmonic resonances do not occur regardless of the magnitude of input amplitude. The lowest subharmonic frequency of oscillation is generally lower than the linear natural frequency. The subharmonic frequency bandwidth is dependent to a great extent upon the dwell time at each discrete input frequency. A rapid input frequency sweep will result in a much narrower subharmonic frequency bandwidth than that produced by a slow frequency sweep. Finally, the peak magnitude of the subharmonic oscillations usually occurs at an input frequency equal to twice the natural frequency of the fluid suspension system.

Transient inputs.- The transient response behavior of the fluid suspension was investigated analytically by applying step disturbances to the system and recording the resultant response time histories. The step disturbances consisted of both upward and downward guideway displacements and it was assumed that the total cushion support area experienced the disturbance instantaneously. Thus the effect of a finite penetration time of the step disturbance has been neglected.

The nomenclature used in describing the transient response behavior is illustrated by the typical response time history of figure 17. The parameters of primary interest include the percentage overshoot of the plenum mass and the equivalent viscous damping ratio. The percentage overshoot is defined as follows:

$$\text{Percent overshoot} = \frac{a_1}{z_o}(100) \tag{19}$$

where z_o is the input step amplitude.

The percent overshoot is presented in figure 18 as a function of the applied step amplitude for several equilibrium hover heights and for both upward and downward steps. This figure indicates that the overshoot response depends significantly upon the direction of the applied step. This behavior is a result of the nonlinear characteristics of the fluid-suspension spring rate; that is, the spring rate stiffens considerably for downward displacements about equilibrium and softens for upward displacements.

The equivalent viscous damping ratio is given by (see ref. 7)

$$\zeta = \frac{\delta}{2\pi} \tag{20}$$

where δ is the log decrement of the response. The equivalent viscous damping ratio was calculated on the basis of the first cycle of the transient-response time history resulting from the application of an upward step displacement. Damping calculations were limited to the first cycle of response because the air-cushion system is highly damped and the transient oscillations decayed very rapidly (3 to 5 cycle maximum duration). The variation of the equivalent viscous damping ratio with the input step amplitude is presented in figure 19 together with the damping ratio as predicted by linear theory (eq. (A12)). A comparison of linear and nonlinear theory indicates that the linear theory tends to overestimate the damping for the smaller input ratios.

Passive and Active Control of Heave Response

The large-amplitude resonant responses and the possible occurrence of large subharmonic oscillations are certainly detrimental to overall ride quality, operational economy, and vehicle stability. This section discusses the results of the application to the air-cushion system of the active and passive control concepts developed in the analysis section. The study of these concepts was conducted with the analog computer in an effort to demonstrate their feasibility. Experimental verification of these concepts using actual hardware is necessary in order to prove their feasibility fully.

Figure 20 presents typical analog results illustrating the reduction in suspension-system displacement response attainable with the use of the active and passive control systems. The uncontrolled response curve corresponds to a hover height of 0.50 inch (1.27 cm) and an input amplitude displacement of 0.10 inch (0.254 cm). With the use of the passive control system, the primary resonant peak was shifted to a somewhat lower frequency value and the primary response amplitude was reduced by a factor greater than five. The subharmonic oscillation was eliminated. Both the primary and secondary resonant peaks were essentially eliminated with the use of the active control system. The active system provided additional isolation capability in the region near and below suspension system resonance. It was observed that the control flow requirement necessary to achieve this particular level of attenuation was approximately 17 percent of the equilibrium cushion flow.

The acceleration response for the same operating conditions is shown in figure 21 together with the reduction in response resulting from application of the active and passive control systems. The accelerations corresponding to the uncontrolled condition are very high, especially during the subharmonic resonance condition. The active and passive control systems proved very effective in attenuating both the fundamental and subharmonic acceleration response of the system. The active control system performed somewhat better since it provided isolation over the total frequency range whereas the passive control system tended to amplify slightly the input acceleration at the lower frequencies. Regardless of the control mechanism, however, a level of acceleration equal to approximately 0.5g was still experienced by the plenum mass for frequencies above approximately 10 Hz.

CONCLUDING REMARKS

An investigation was conducted to study analytically and experimentally the steady-state and transient dynamic behavior of a simple-plenum air cushion in the heave mode and to examine analytically the effectiveness of active and passive control techniques to reduce cushion resonant response.

For the range of variables considered in this study, the analytical description of the uncontrolled air-cushion fluid suspension was verified by the experimental findings. The air-cushion dynamic behavior was characterized by a fundamental resonant response at which guideway inputs were significantly amplified. For relatively low level inputs, the linearized theory and experimental data indicate that the fundamental resonant frequency varies inversely with static hover height whereas the system transmissibility increases with increasing static hover height. When the input disturbance levels were increased, a significant departure from the linear behavior was observed, particularly in the air-cushion accelerations which the linear theory may grossly underestimate. Probably the most important nonlinear phenomenon observed was the large-amplitude subharmonic oscillations that occurred for certain combinations of input frequency and displacement.

Analytical studies of cushion control techniques suggest that substantial reductions in cushion response may be achieved with active and passive control devices. Passive control of the cushion response by means of a tuned passive relief valve appears to be very effective in reducing resonant response and even further reduction in response appears to be possible with the use of an active control system to modulate cushion supply flow.

Langley Research Center,
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APPENDIX

LINEARIZED SYSTEM EQUATIONS

The equations describing the heave motion of the plenum fluid suspension are linearized by assuming that the system variables undergo small excursions about equilibrium and also that such parameters as cushion volume, temperature, and density can be replaced by their average values. These linearized equations are then utilized to develop the transfer functions representing the system spring rate, damping, and transmissibility characteristics. The equations for the system variables are

$$\left.\begin{aligned} x &= \bar{x} + \Delta x \\ h &= \bar{h} + \Delta h \\ w_o &= \bar{w}_o + \Delta w_o \\ V &= \bar{V} + \Delta V \\ p &= \bar{p} + \Delta p \end{aligned}\right\} \tag{A1}$$

where $\Delta(\ )$ denotes a perturbation about equilibrium and $(\bar{\ })$ denotes an equilibrium (constant) value. By using equations (A1), equation (1) becomes

$$m_p \, \Delta\ddot{x} = A \, \Delta p = \Delta F \tag{A2}$$

where ΔF is the force acting on the plenum mass.

Equation (6) becomes (second-order terms being neglected)

$$\Delta\dot{p} = -\frac{\gamma RT \, \Delta w_o}{\bar{V}} - \frac{\gamma A\bar{p} \, \Delta\dot{h}}{\bar{V}} \tag{A3}$$

Assuming small variations in cushion outflow w_o gives

$$\Delta w_o = K_h \, \Delta h + K_p \, \Delta p \tag{A4}$$

where K_h and K_p are the linearized weight flow coefficients defined as

$$K_h = \left.\frac{\partial w_o}{\partial h}\right]_{\substack{p=\bar{p} \\ h=\bar{h}}} = \frac{w_o}{\bar{h}} \tag{A5}$$

and

$$K_p = \left.\frac{\partial w_o}{\partial p}\right]_{\substack{p=\bar{p} \\ h=\bar{h}}} = \frac{w_o}{2(p - p_a)} \tag{A6}$$

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The coordinate constraint equation is

$$\Delta x = \Delta h + z \tag{A7}$$

where z is now assumed to be small. Figure 22(a) is a representation in block diagram form of equations (A1) to (A7). Figure 22(b) is a rearrangement and simplification of figure 22(a) and relates the forces acting on the plenum mass to plenum displacement and velocity. Figure 22(a) is used to develop the transfer functions corresponding to the absolute and relative displacement response ratios (transmissibilities). These ratios are for the absolute displacement response ratio:

$$\frac{\Delta x}{z} = \frac{2\epsilon \tau_n s + 1}{\tau_n{}^2 \tau_N s^3 + \tau_n{}^2 s^2 + 2\epsilon \tau_n s + 1} \tag{A8}$$

and for the relative displacement response ratio:

$$\frac{\Delta h}{z} = \frac{-s^2 \tau_n{}^2 (\tau_N s + 1)}{\tau_n{}^2 \tau_N s^3 + \tau_n{}^2 s^2 + 2\epsilon \tau_n s + 1} \tag{A9}$$

where

$$\tau_N = \frac{\bar{V}}{\gamma R T K_p} \tag{A10}$$

$$\tau_n{}^2 = \frac{m_p K_p}{A K_h} = \frac{\bar{h}}{2g} \tag{A11}$$

$$\epsilon(\text{damping ratio}) = \frac{A\bar{p}\sqrt{g}}{R\bar{T}K_h\sqrt{2\bar{h}}} \tag{A12}$$

The transfer function $\Delta F/\Delta h$ as obtained from figure 22(b) contains in-phase (real) components and out-of-phase (imaginary) components. The in-phase component is defined as an equivalent spring rate $K(\omega)$ and the out-of-phase component is defined as an equivalent damping coefficient $\lambda(\omega)$. These components are

$$K(\omega) = \frac{\gamma A^2 \bar{p} (\tau_N \omega)^2}{\bar{V}\left[(\tau_N \omega)^2 + 1\right]} + \frac{2}{\bar{h}} \frac{(\bar{p} - p_a)A}{\left[(\tau_N \omega)^2 + 1\right]} \tag{A13}$$

and

$$\lambda(\omega) = \left\{\frac{\gamma A^2 \bar{p}}{\bar{V}} + \frac{2(p - p_a)A}{\bar{h}} \frac{\tau_N}{\left[(\tau_N \omega)^2 + 1\right]}\right\} \tag{A14}$$

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For the frequency range of interest in this analysis, the term $\tau_N \omega$ in these equations is small and can be neglected. Thus, the undamped natural frequency of the suspension system as calculated from equation (A13) is

$$\omega_n = \sqrt{\frac{2g}{\bar{h}}} \tag{A15}$$

Comparison of equations (A11) and (A15) yields the additional relationship

$$\omega_n = \frac{1}{\tau_n} \tag{A16}$$

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TABLE I.- NUMERICAL VALUES USED IN ANALYSIS

Parameter	Value
Isentropic exponent, γ	1.4
Universal gas constant, R	$640 \dfrac{\text{inches-pounds force}}{\text{pounds mass-}^{\circ}\text{R}}$ $(8.3143\ \text{J}^{\circ}\text{K}^{-1}\text{mol}^{-1})$
Cushion base diameter, d	14.4 inches (36.59 cm)
Cushion base area, A	162.7 inch^2 (0.105 m^2)
Plenum weight	12.25 pounds (54.5 newtons)
Equilibrium air density, $\bar{\rho}$	$11.5 \times 10^{-8} \dfrac{\text{lb-sec}^2}{\text{ft}^4}$ $\left(59.25 \times 10^{-6} \dfrac{\text{kilogram}}{\text{meter}^3}\right)$
Atmospheric pressure, p_a	$14.7 \dfrac{\text{pounds}}{\text{inch}^2}$ $\left(101.4 \times 10^3 \dfrac{\text{newtons}}{\text{meter}^2}\right)$
Plenum discharge coefficient, C_d . .	0.61
Air-cushion dead volume, V_o	244 inch^3 (4×10^{-3} meter^3)
Average cushion temperature, $\overline{T}$ . . .	520° R (289° K)

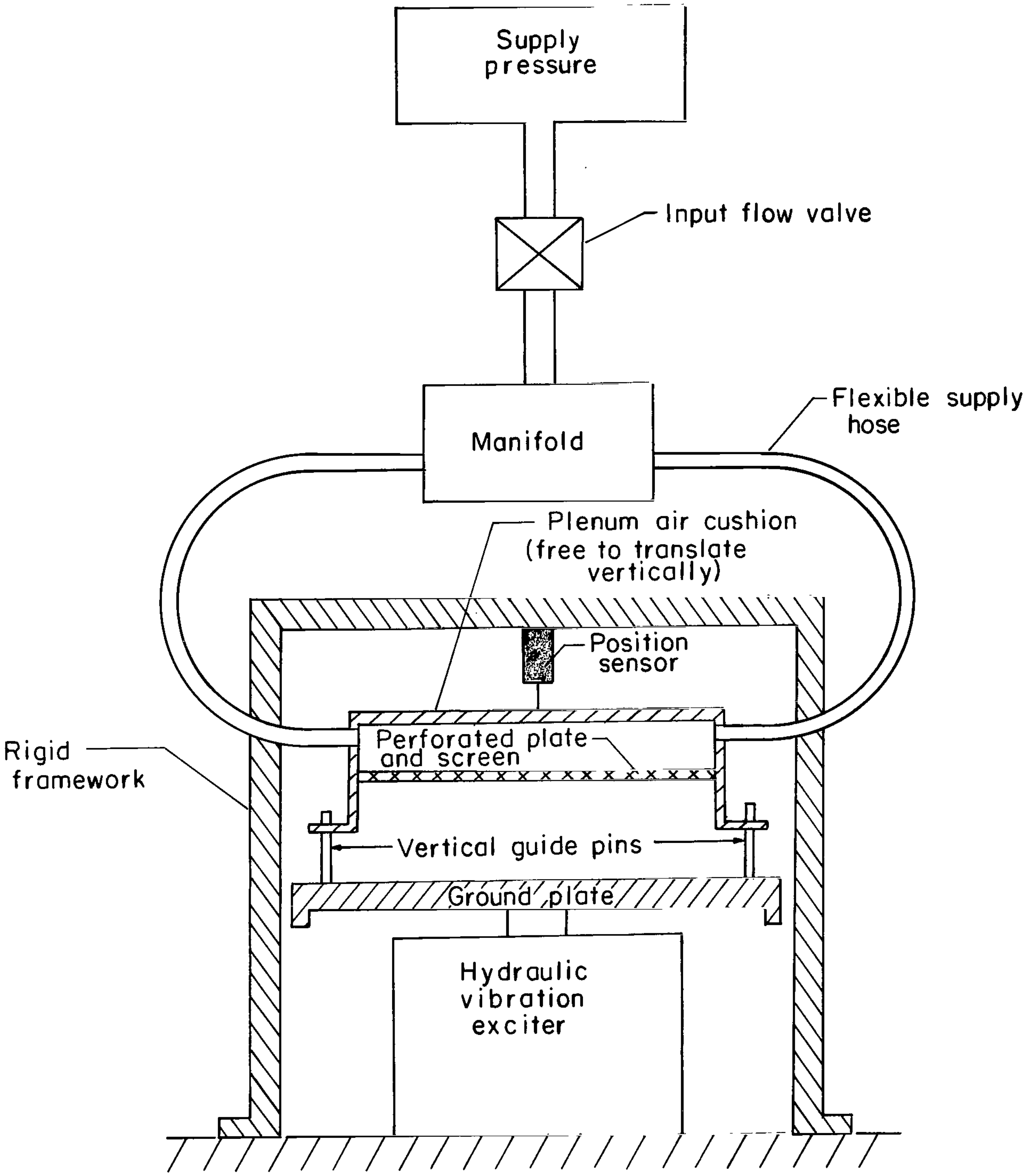


Figure 1.- Schematic of experimental apparatus.

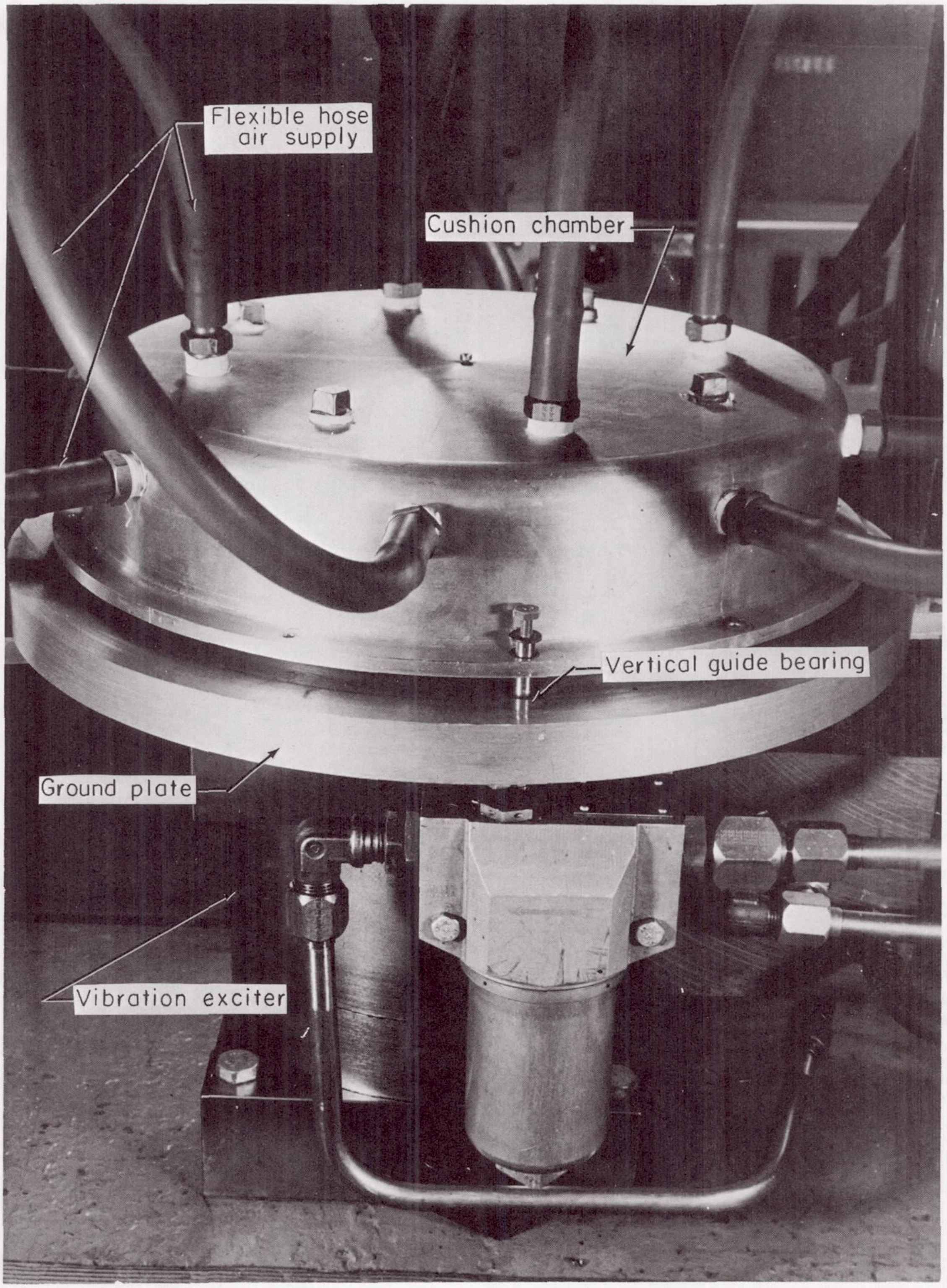


Figure 2.- Photograph of experimental apparatus. L-67-9965

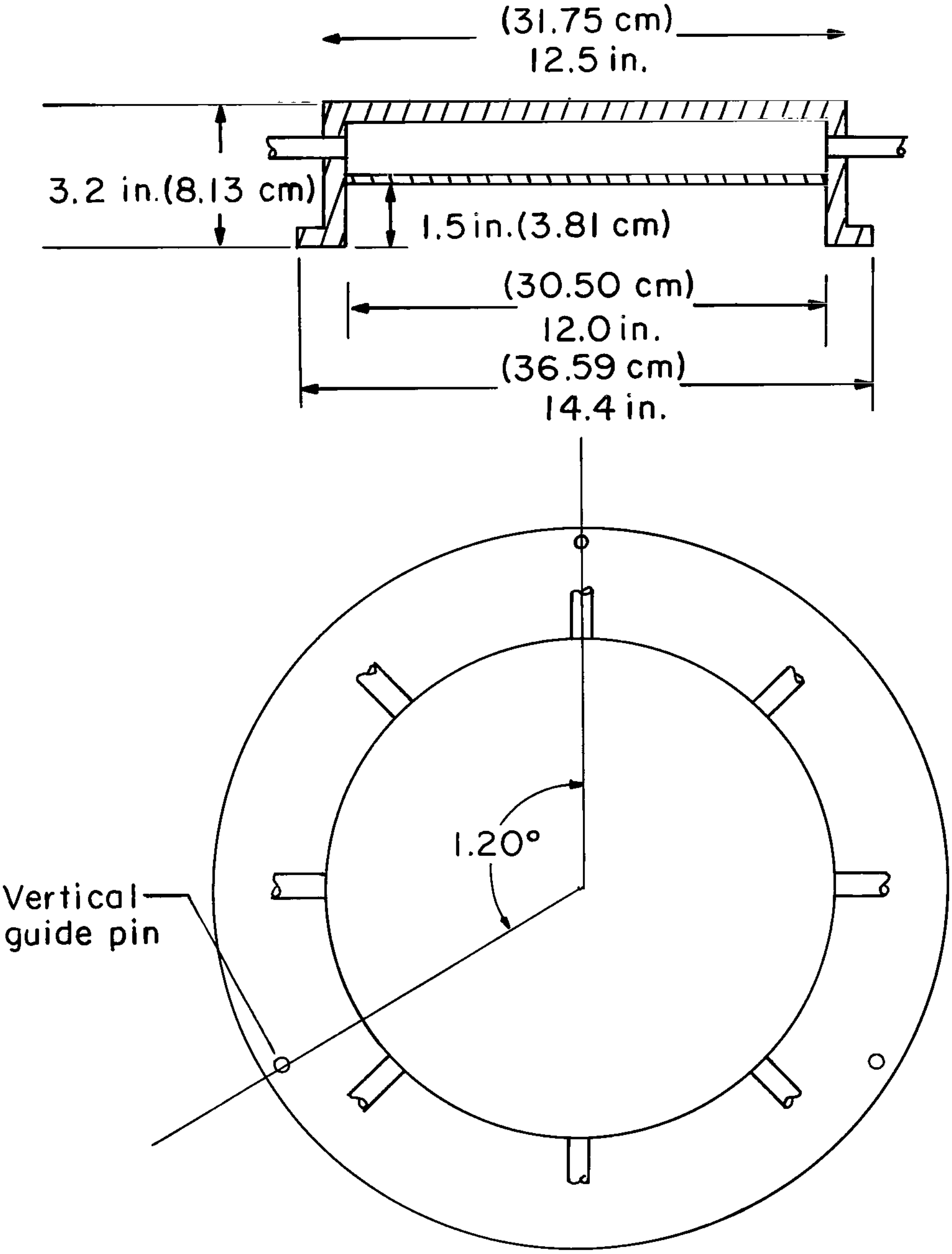


Figure 3.- Dimensions of experimental plenum-type air cushion.

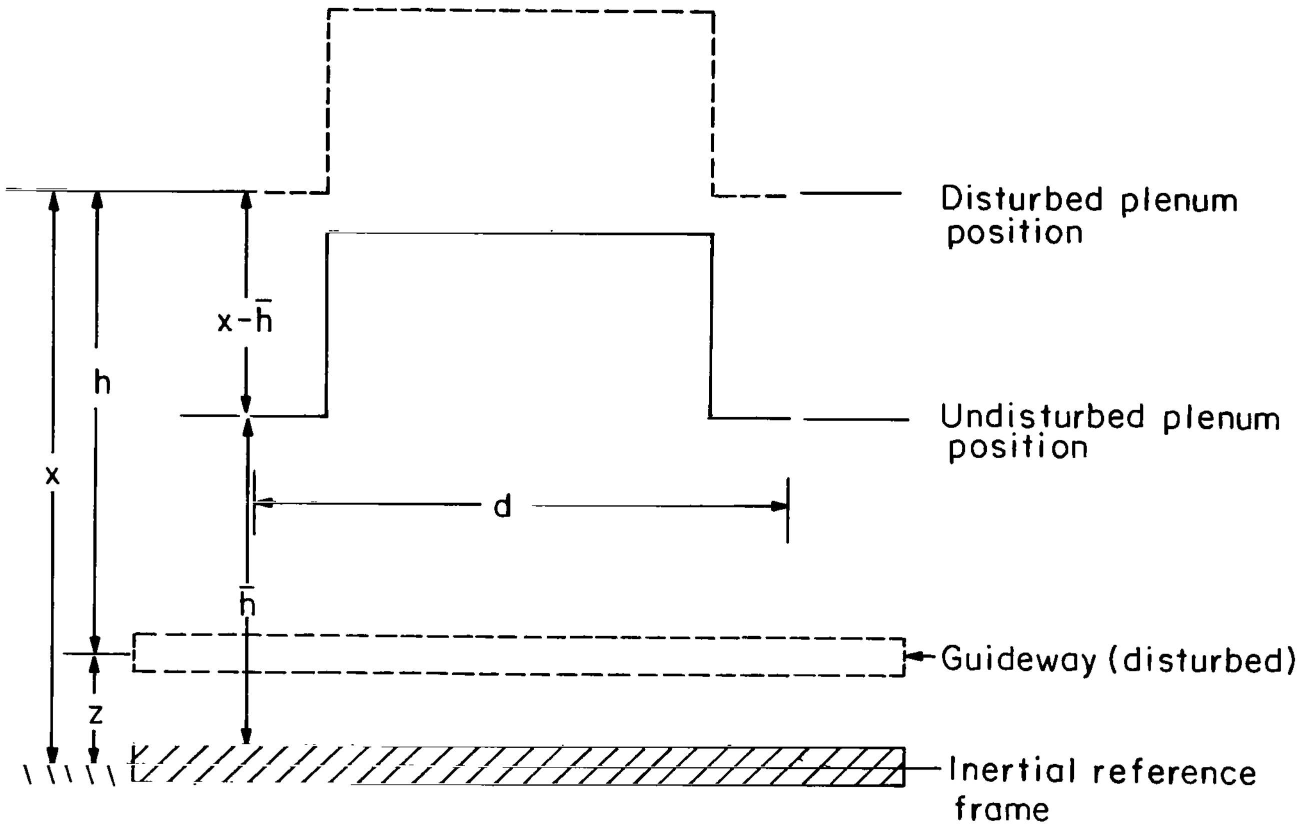


Figure 4.- Plenum-air-cushion suspension coordinates and nomenclature.

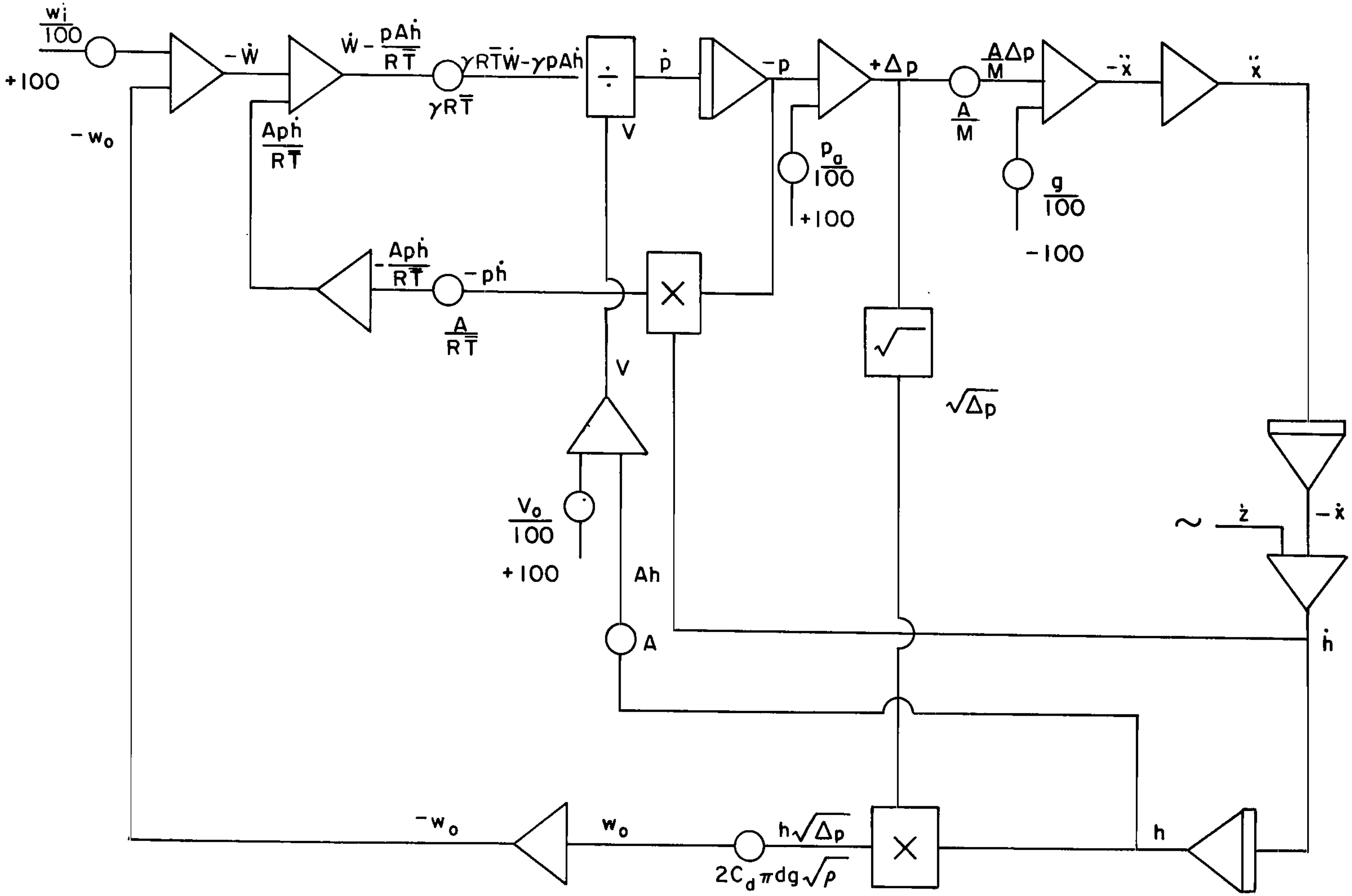


Figure 5.- Analog computer circuit diagram representation of the nonlinear equations.

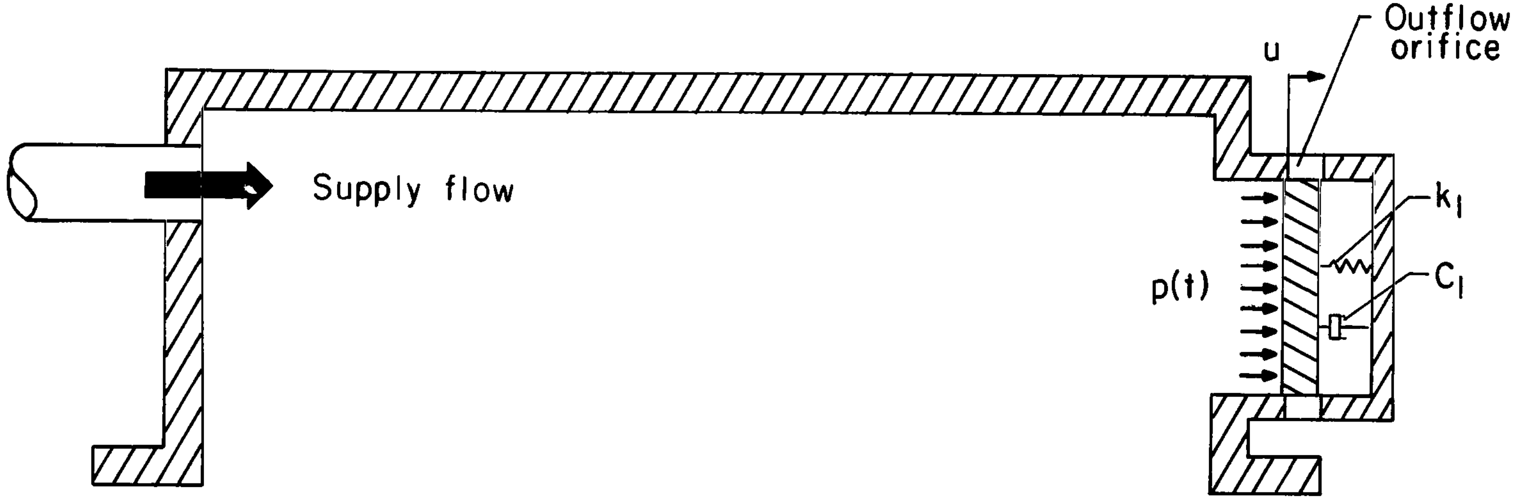
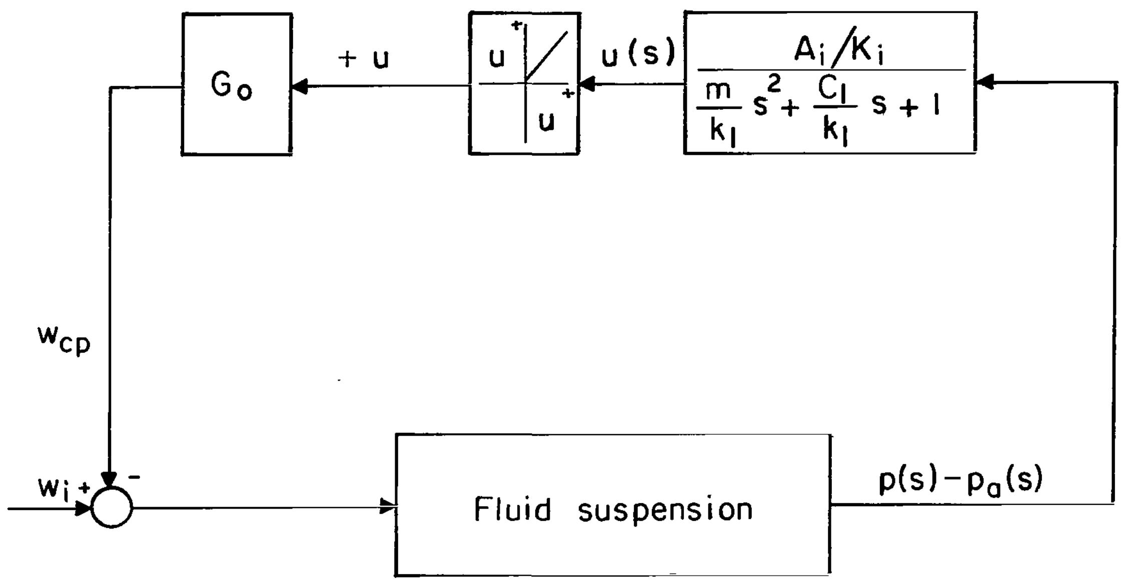
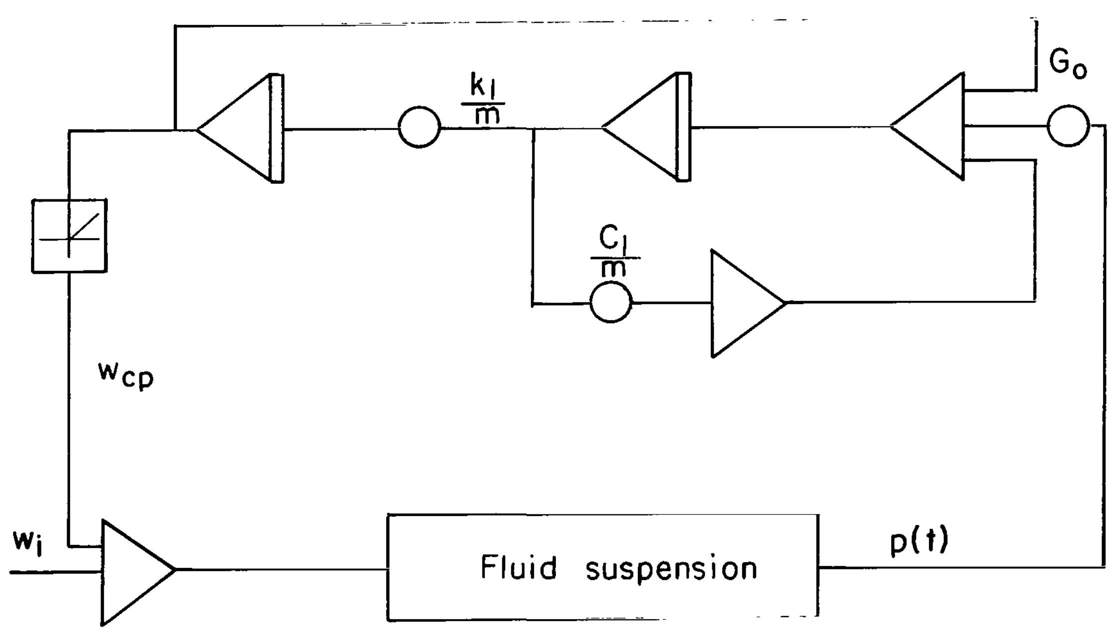


Figure 6.- Schematic of open plenum with passive control device.



(a) Operational block diagram.



(b) Analog computer diagram.

Figure 7.- Analog computer and operational block diagrams for passive suspension control.

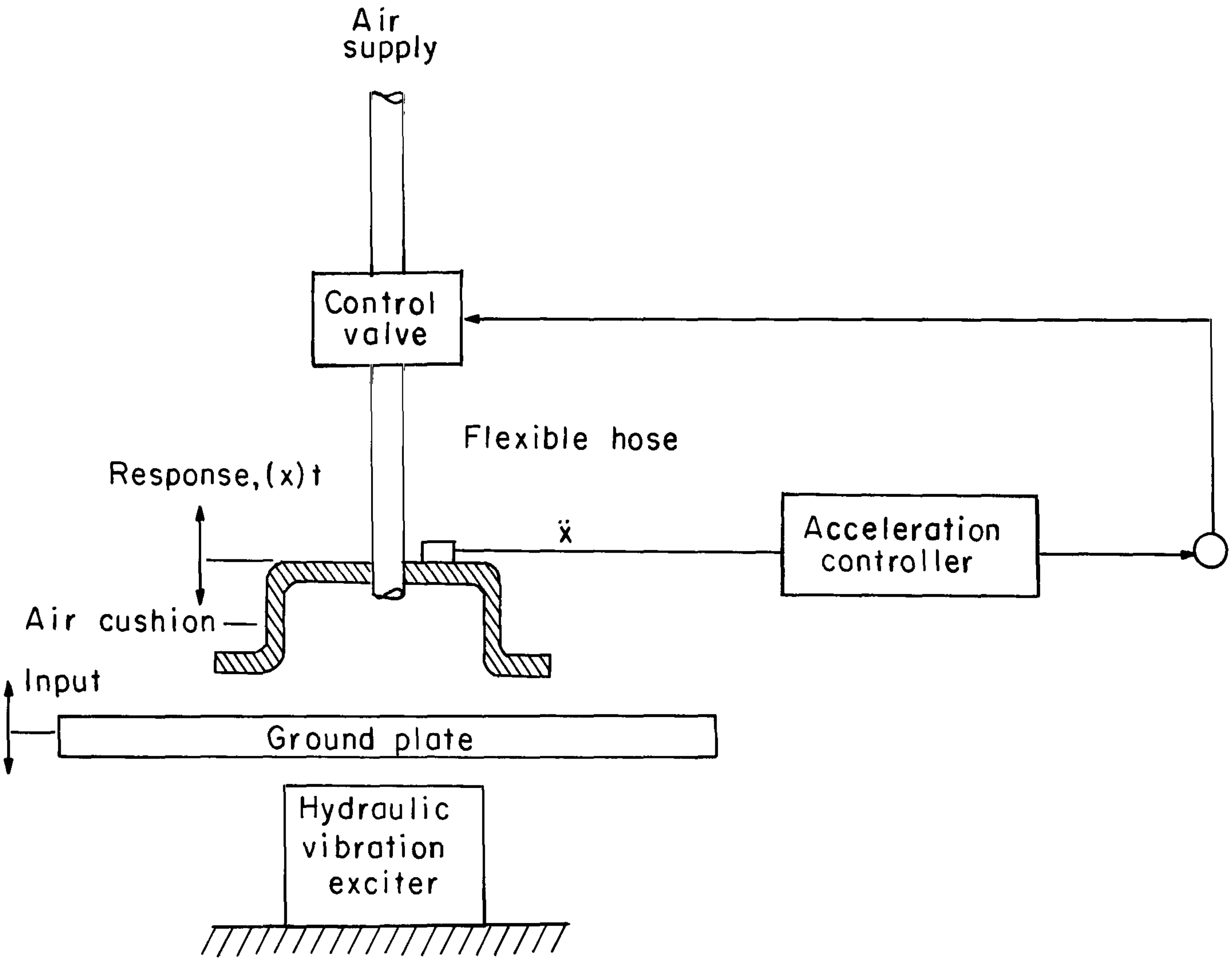
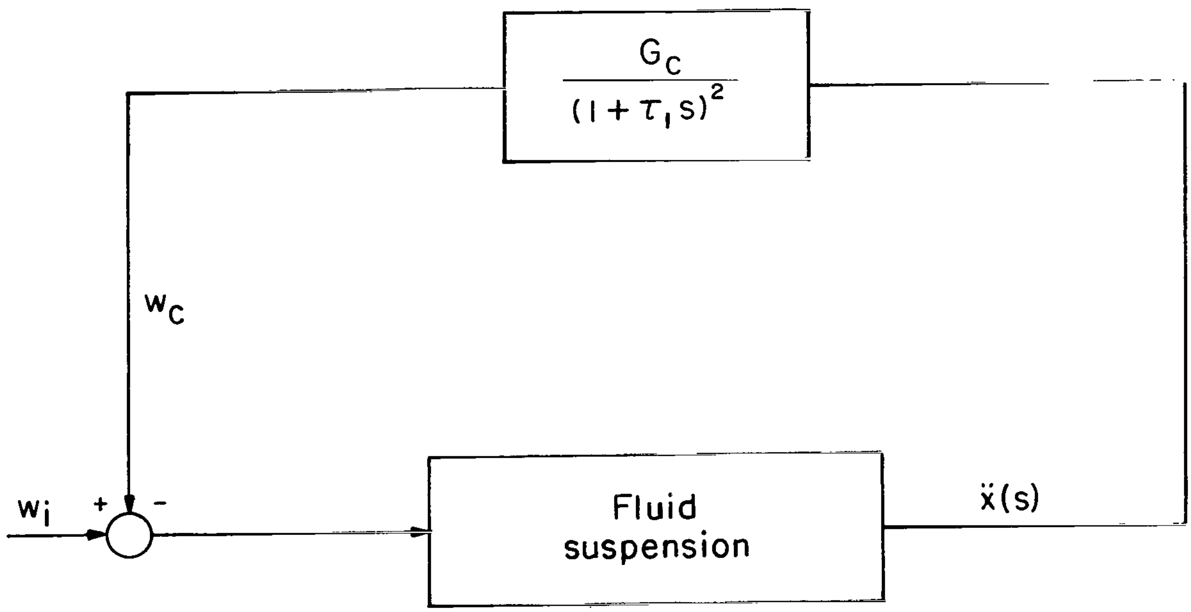
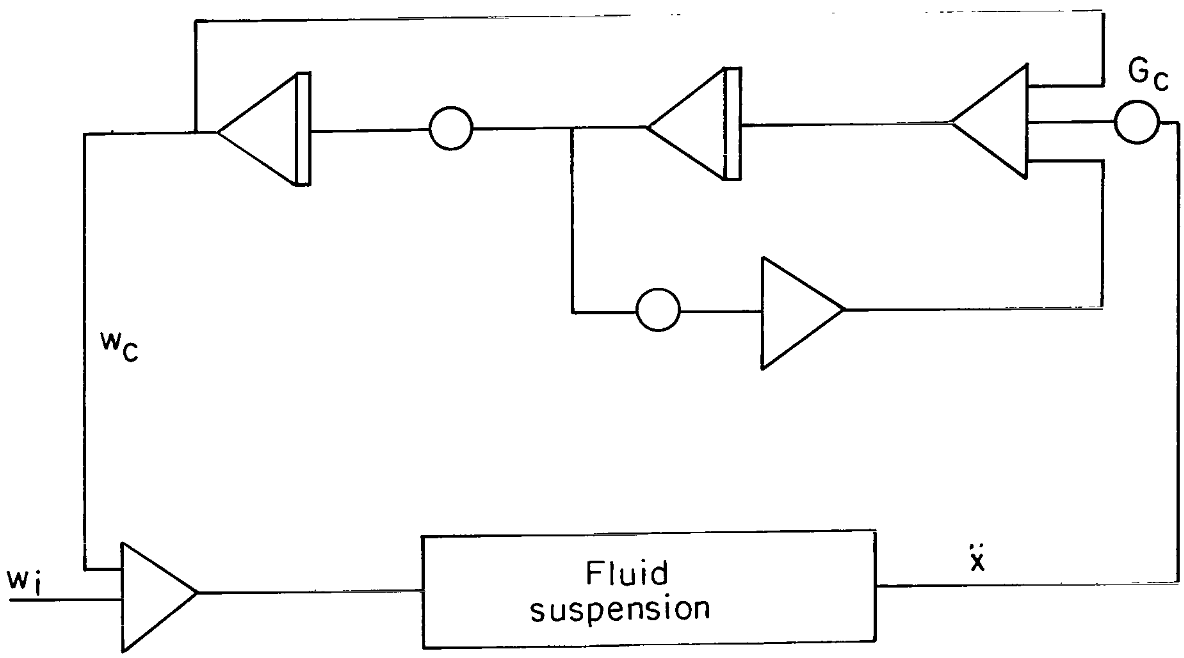


Figure 8.- Schematic of air-cushion active control device.

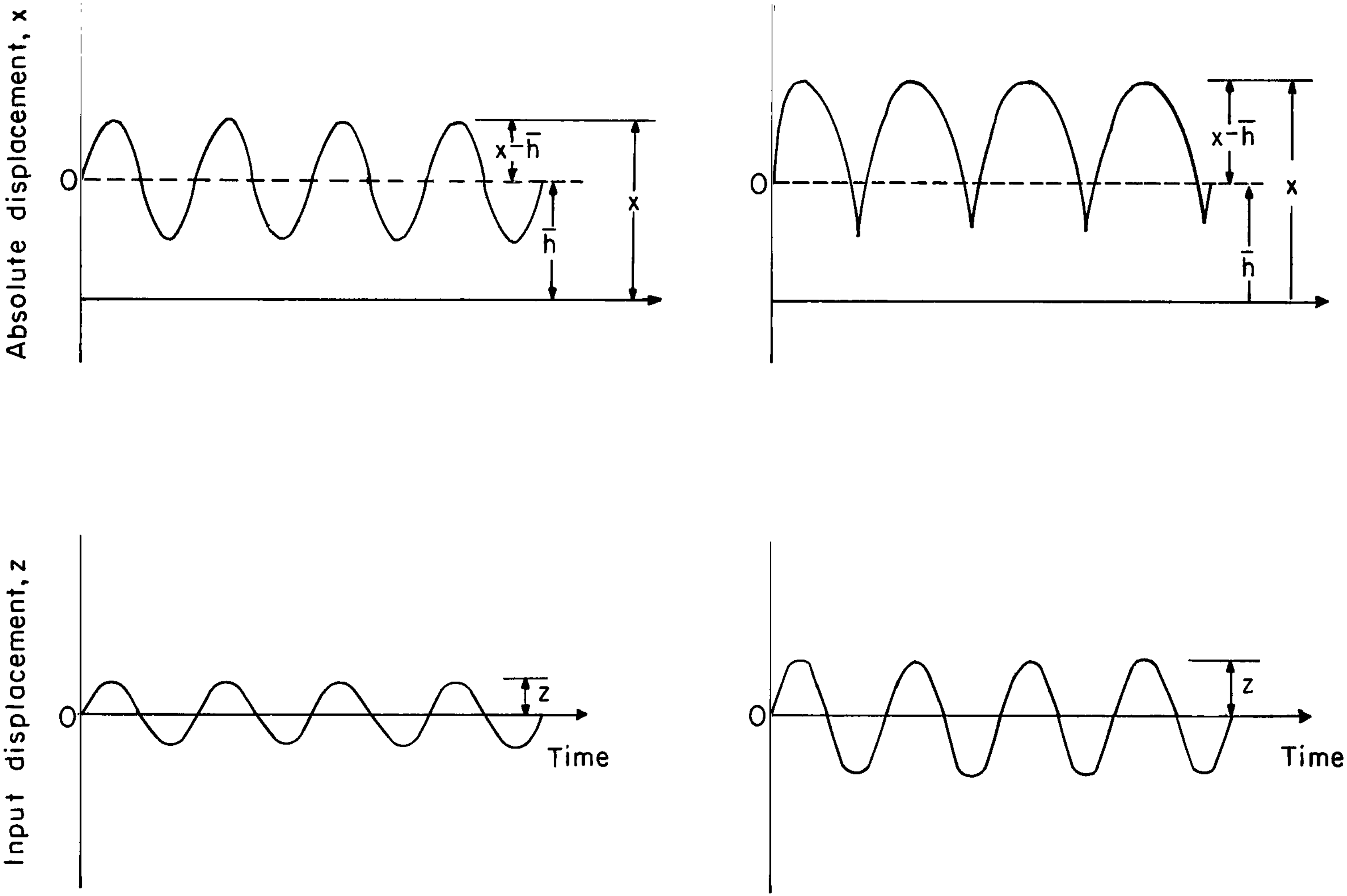


(a) Block diagram.



(b) Analog computer diagram.

Figure 9.- Analog computer and operational block diagrams for active suspension control.



(a) Linearized analysis.

(b) Nonlinear analysis.

Figure 10.- Typical resonant response time histories based upon linear and nonlinear analyses.

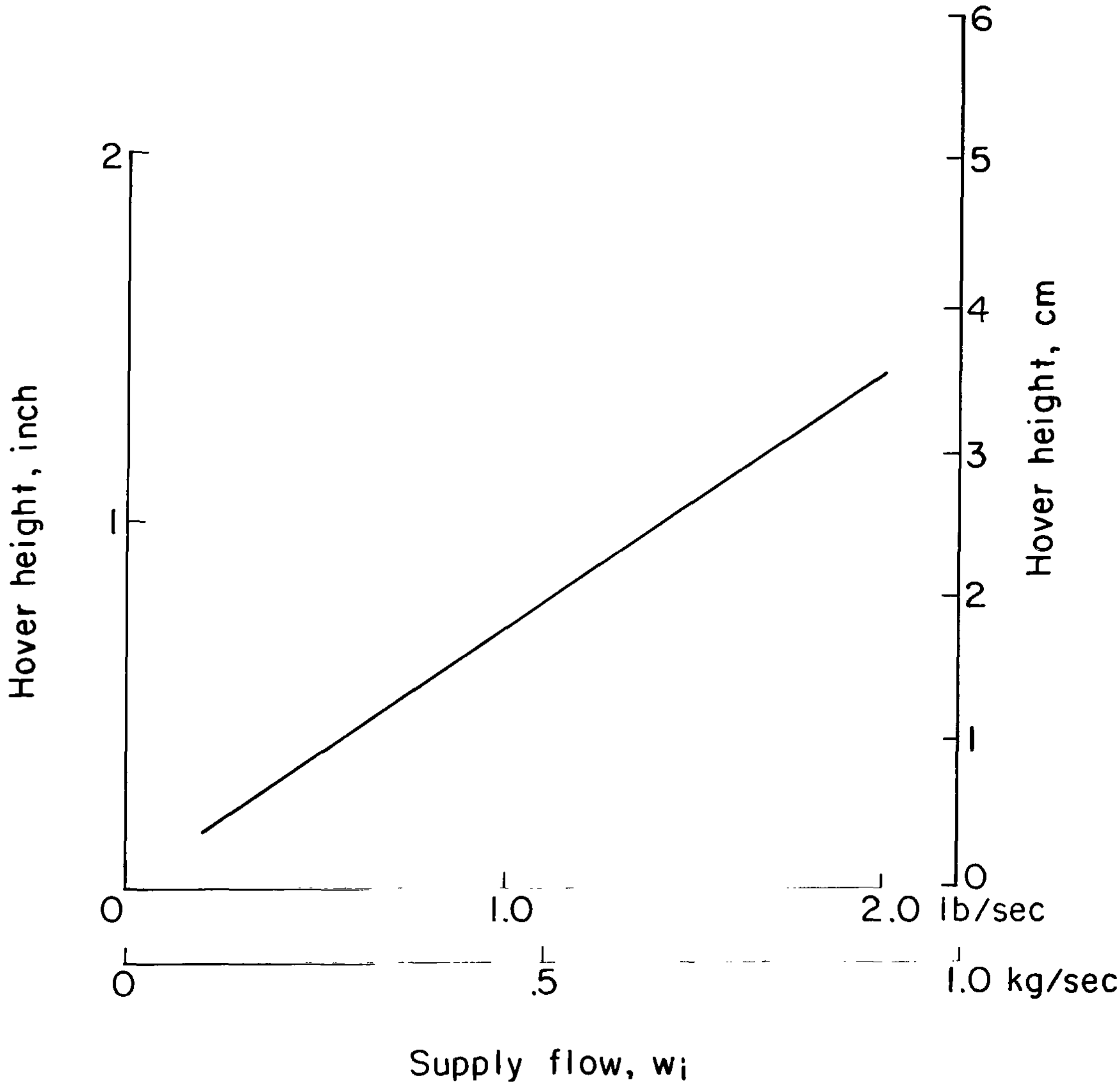


Figure 11.- Variation of hover height with supply flow rate for simple plenum analytical model. Plenum weight, 12.25 lb (54.5 newtons); cushion support area, 162.7 in^2 (0.105 meter^2).

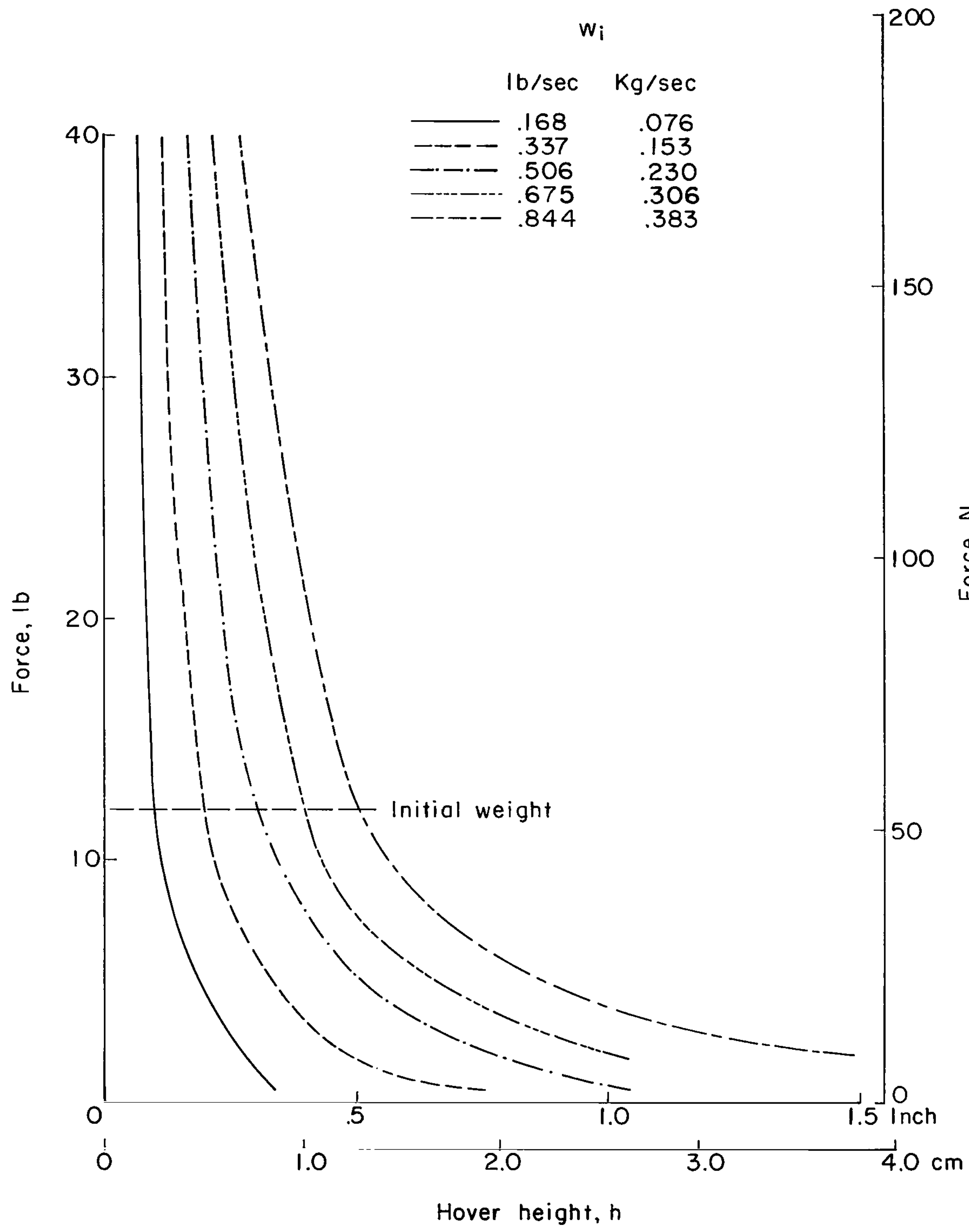


Figure 12.- Static force-deflection characteristics of simple plenum analytical model for several values of initial supply flow rate. Plenum weight, 12.25 lb (54.5 newtons); cushion support area, 162.7 in^2 (0.105 meter^2).

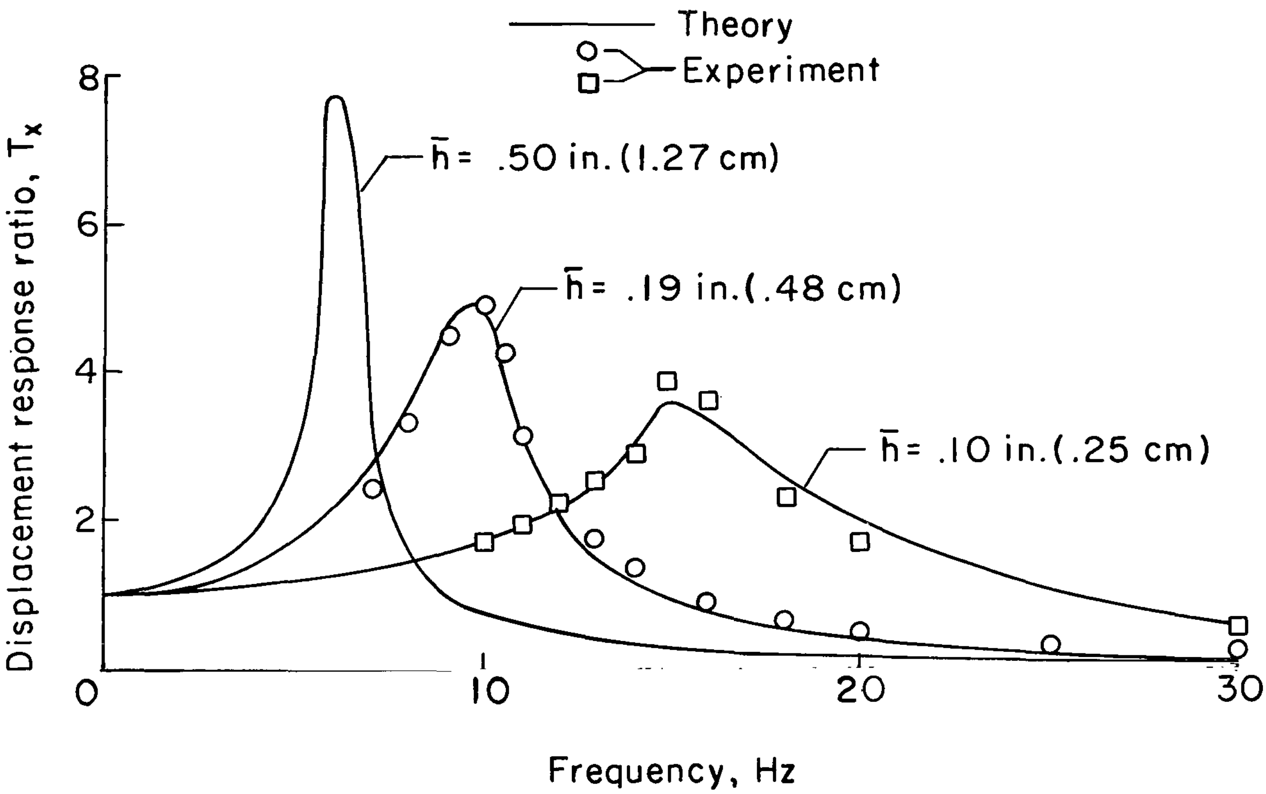


Figure 13.- Typical plenum response characteristics for input single amplitudes less than 5 percent of the equilibrium hover height. Plenum weight, 12.25 lb (54.5 newtons); cushion support area, 162.7 in^2 (0.105 meter^2).

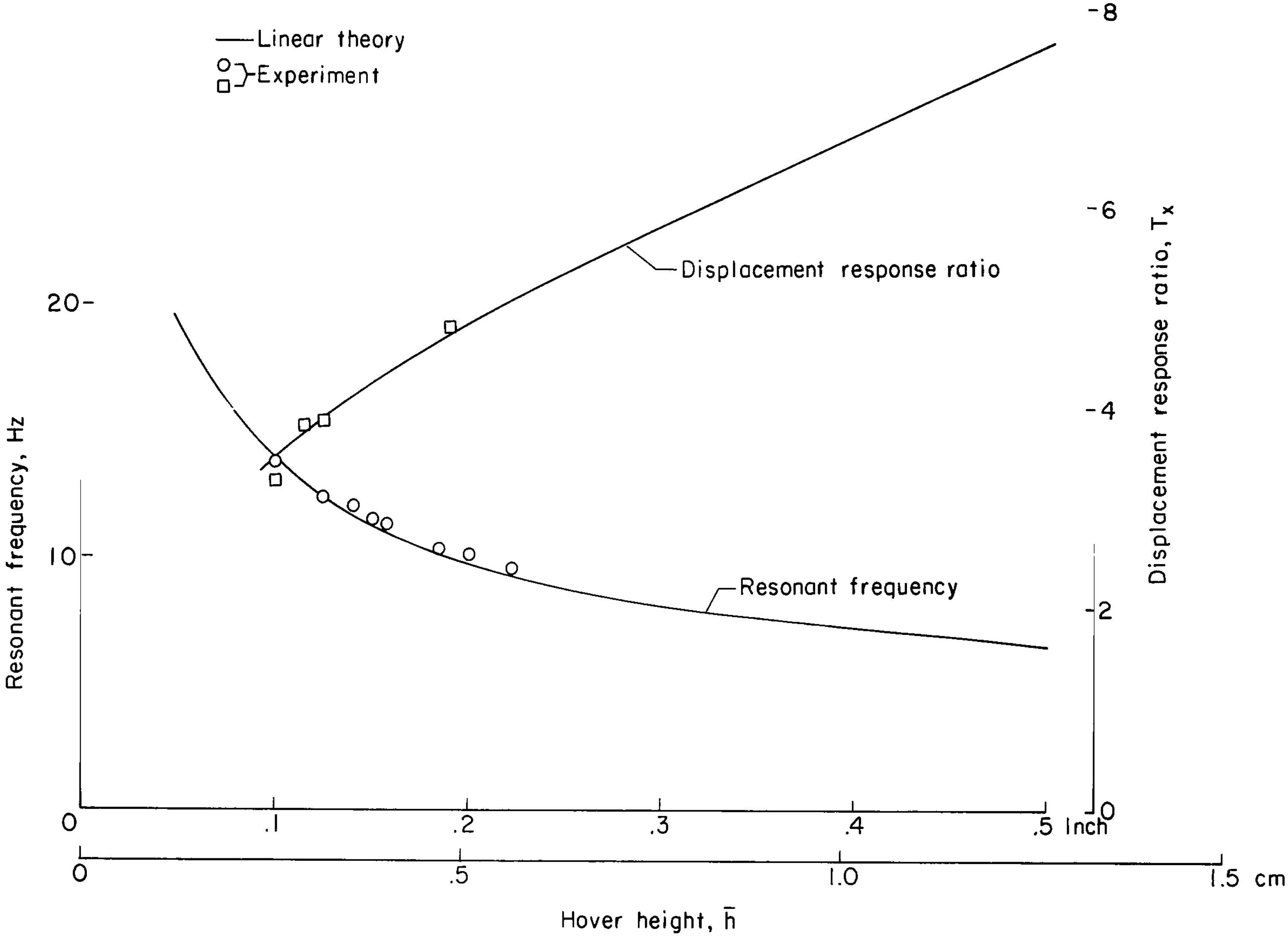


Figure 14.- Variation of resonant frequency and displacement response ratio with equilibrium hover height for input amplitudes equal to 10 percent of the equilibrium hover height.

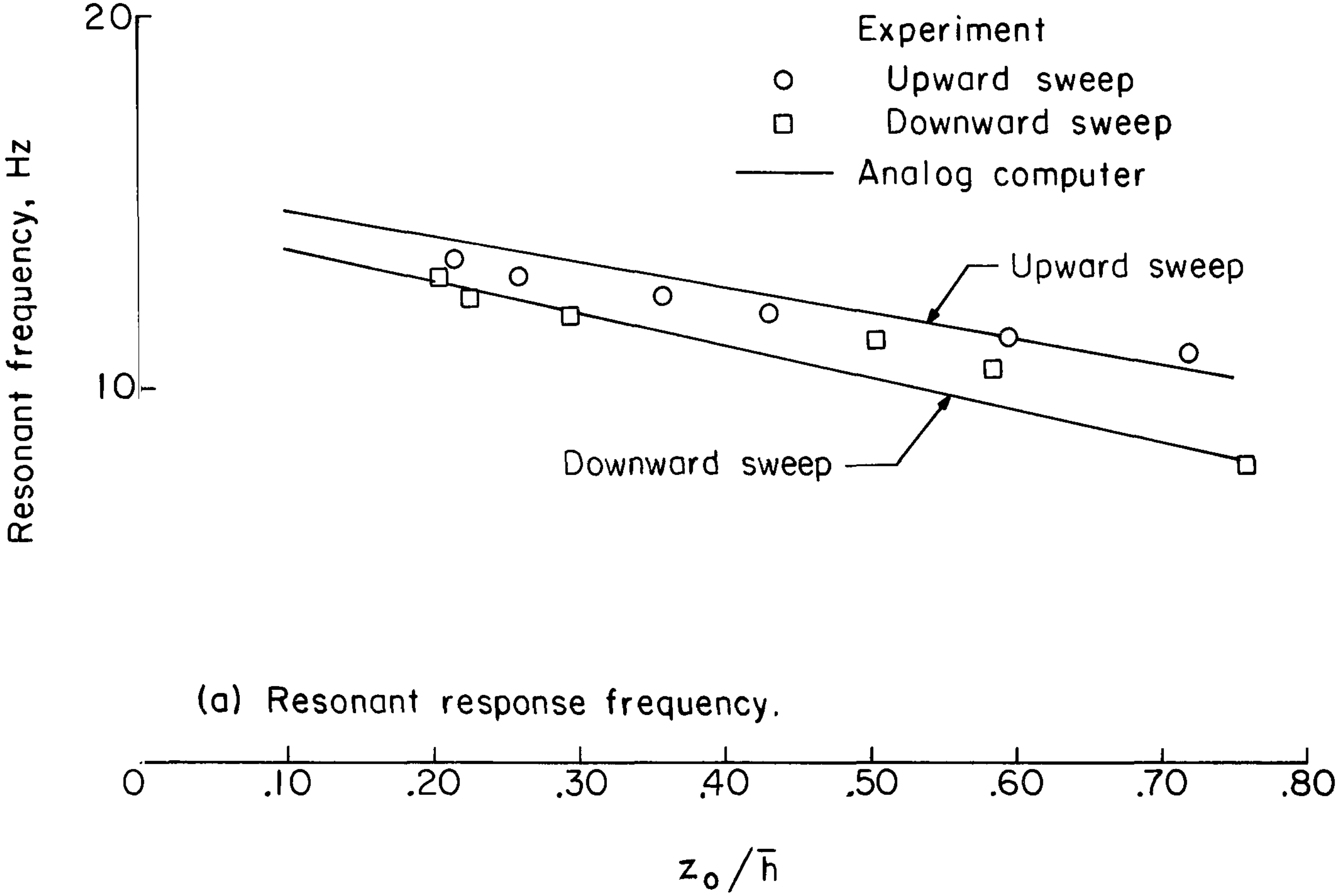


Figure 15.- Effect of input amplitude on air-cushion resonant response characteristics for a hover height of 0.10 inch (0.254 cm).

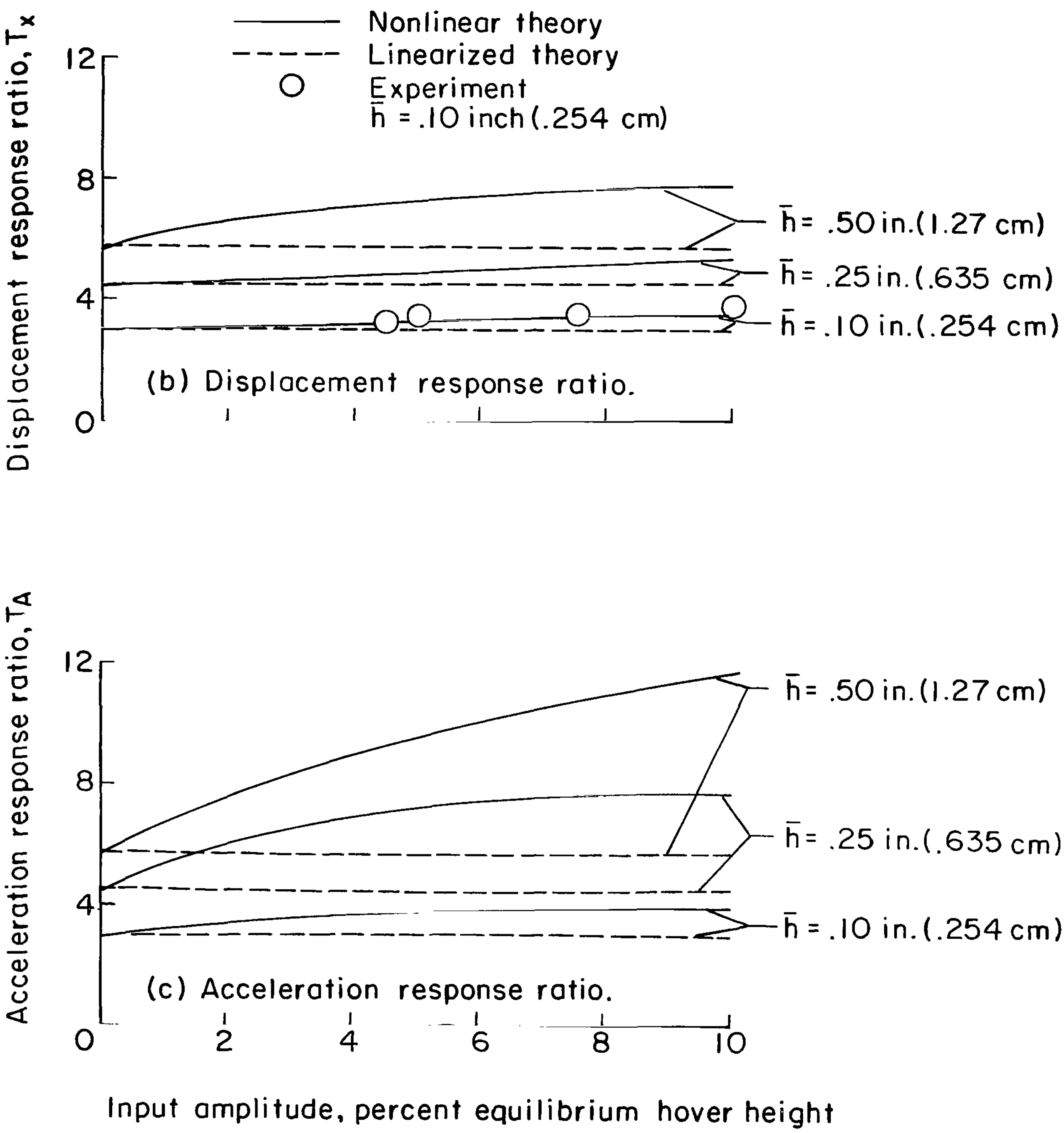


Figure 15.- Concluded.

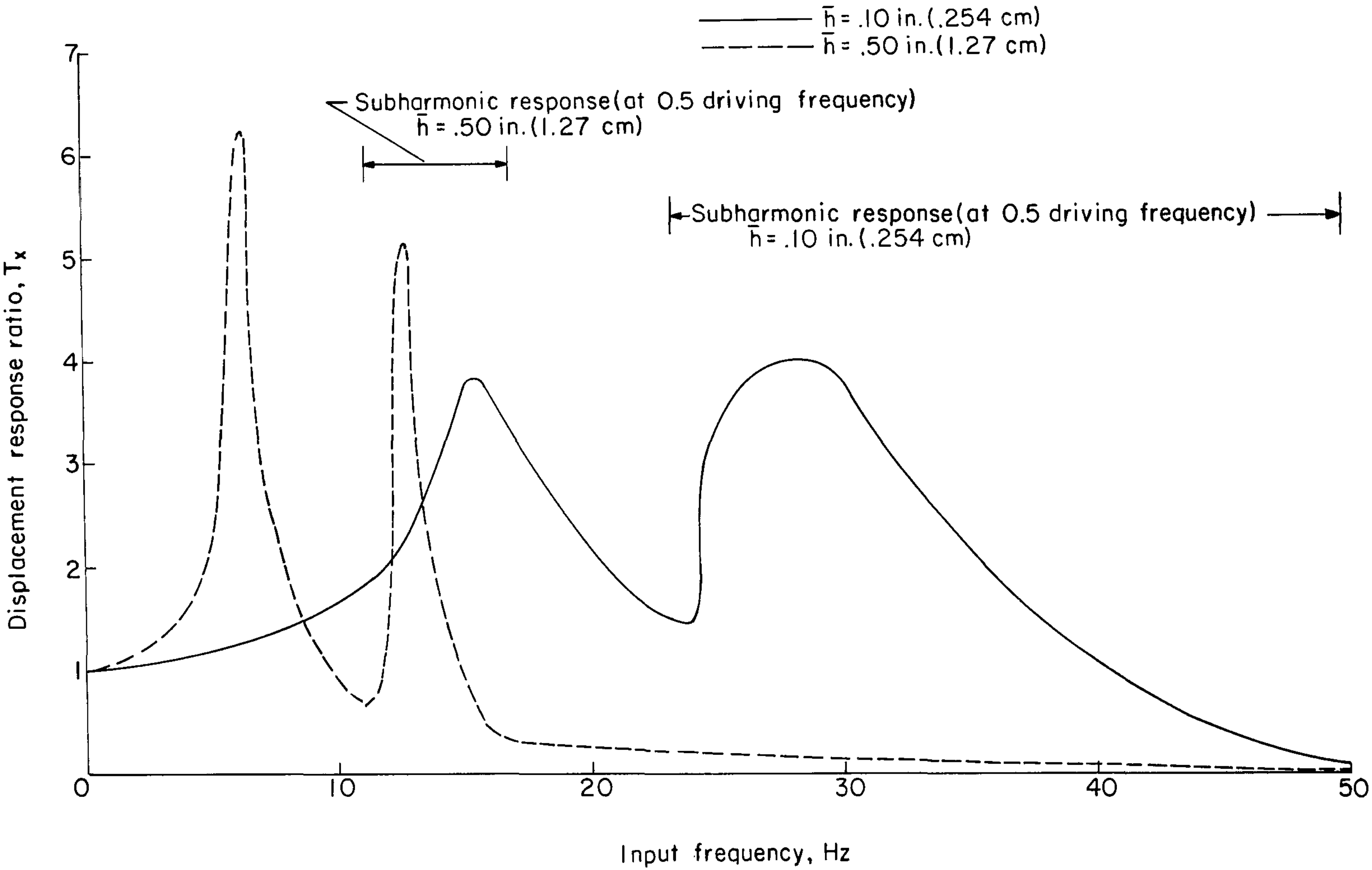


Figure 16.- Nonlinear fluid-suspension-system displacement response for $z_0/\bar{h}$ = 0.20.

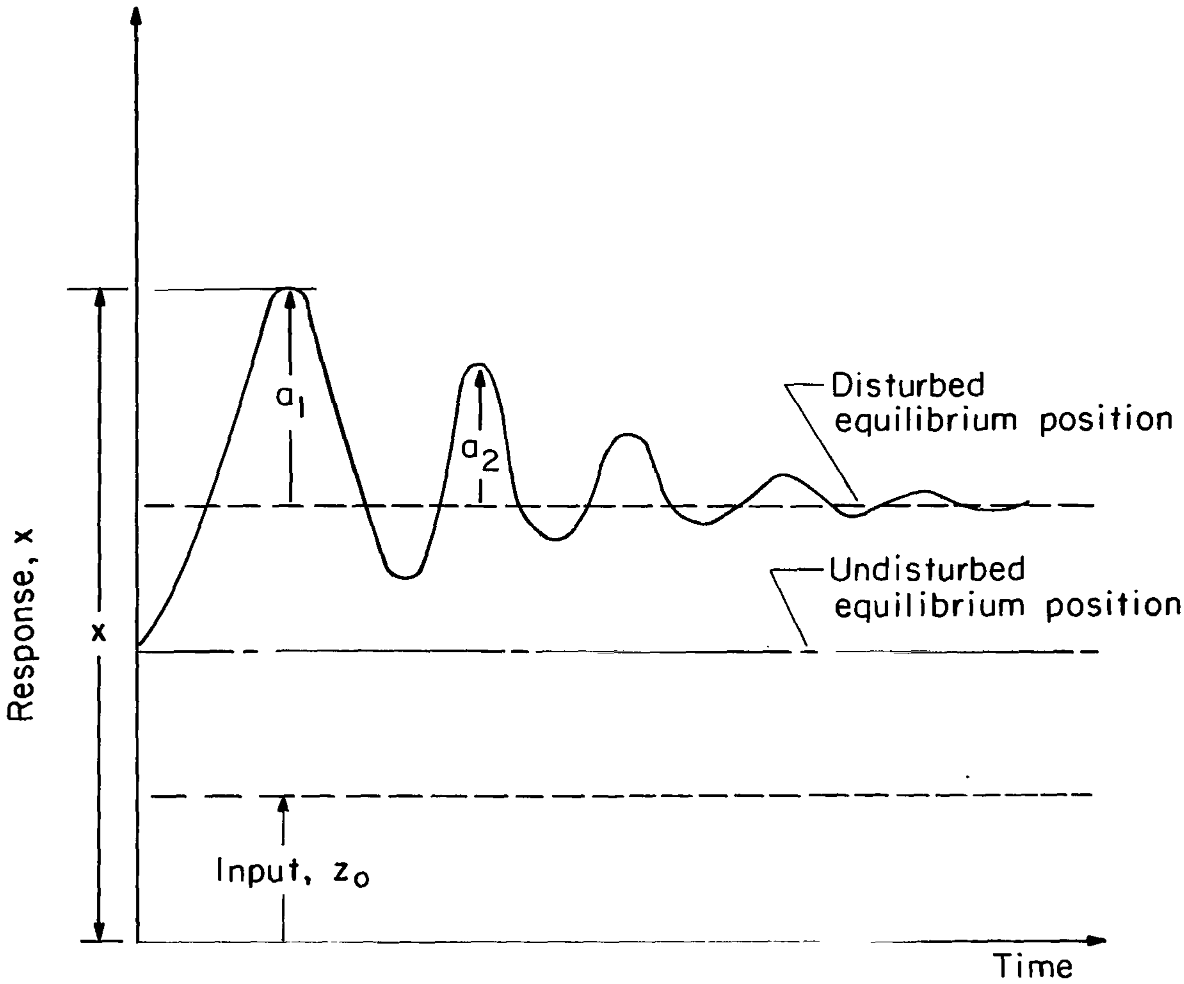


Figure 17.- Typical transient response time history.

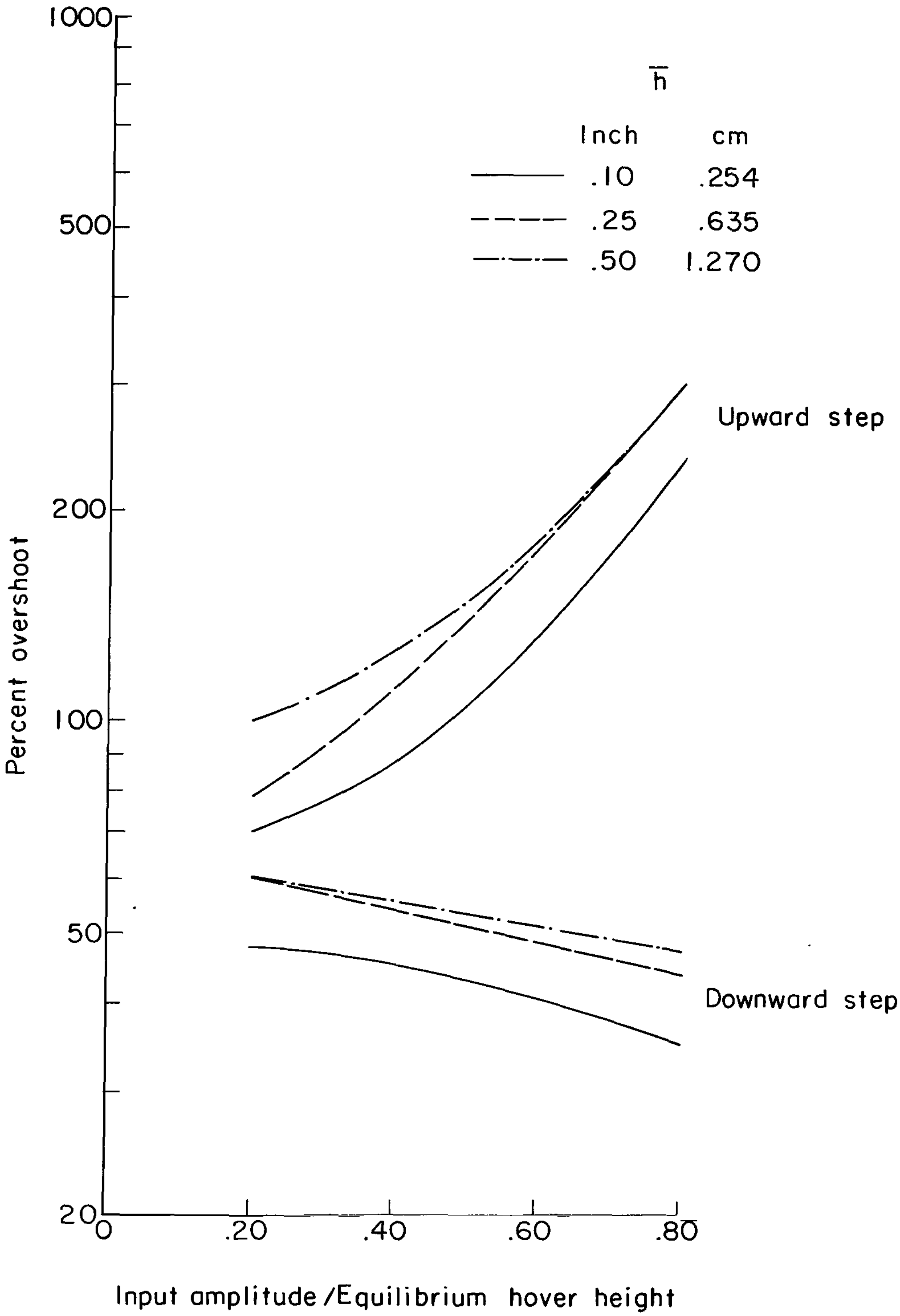


Figure 18.- Plenum fluid-suspension overshoot characteristics. Plenum weight, 12.25 lb (54.5 newtons); cushion support area, 162.7 in^2 (0.105 meter^2).

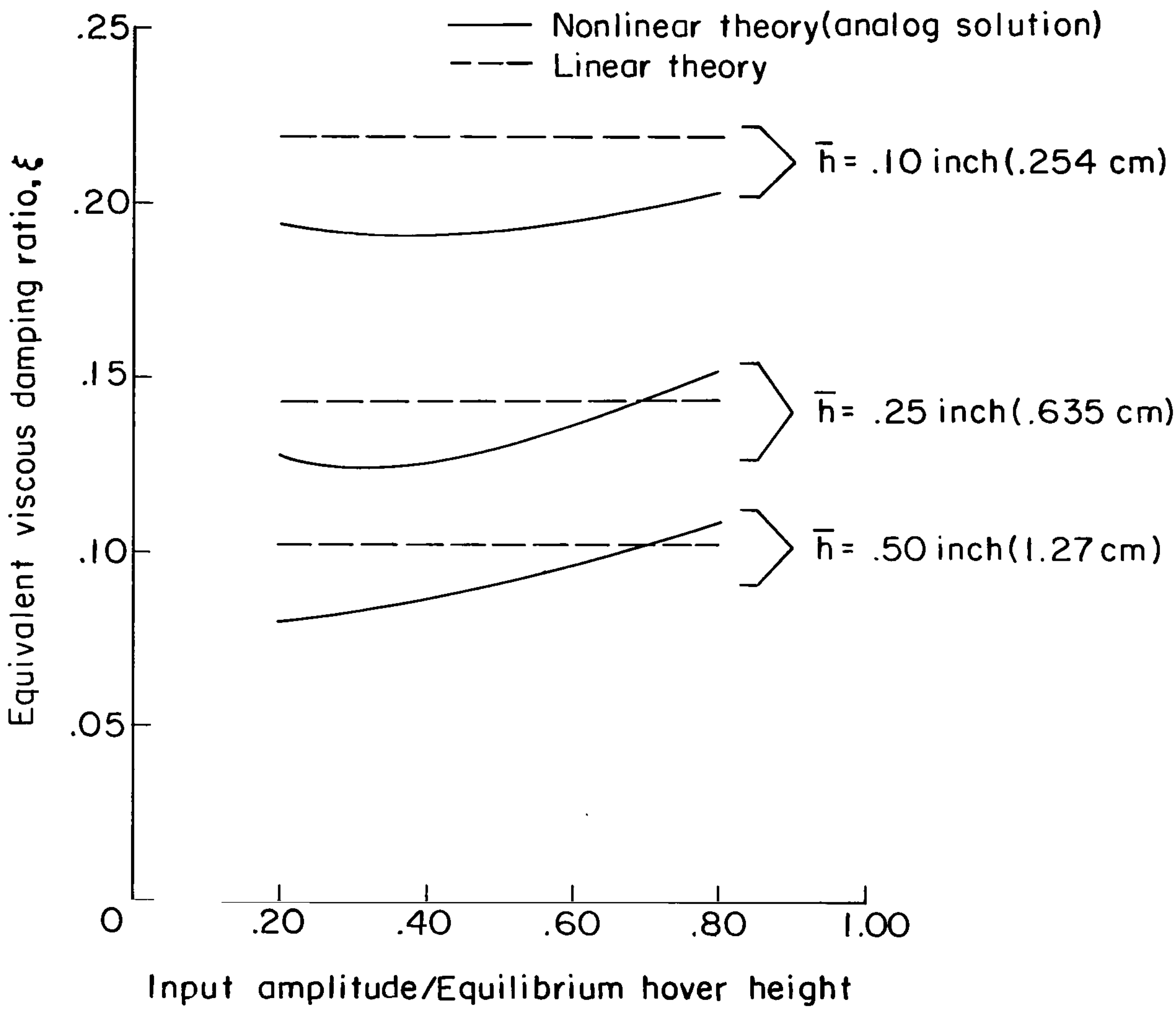


Figure 19.- Variation of fluid suspension-system damping with input amplitude for first response cycle. Plenum weight, 12.25 lb (54.5 newtons); cushion support area, 162.7 in^2 (0.105 meter^2).

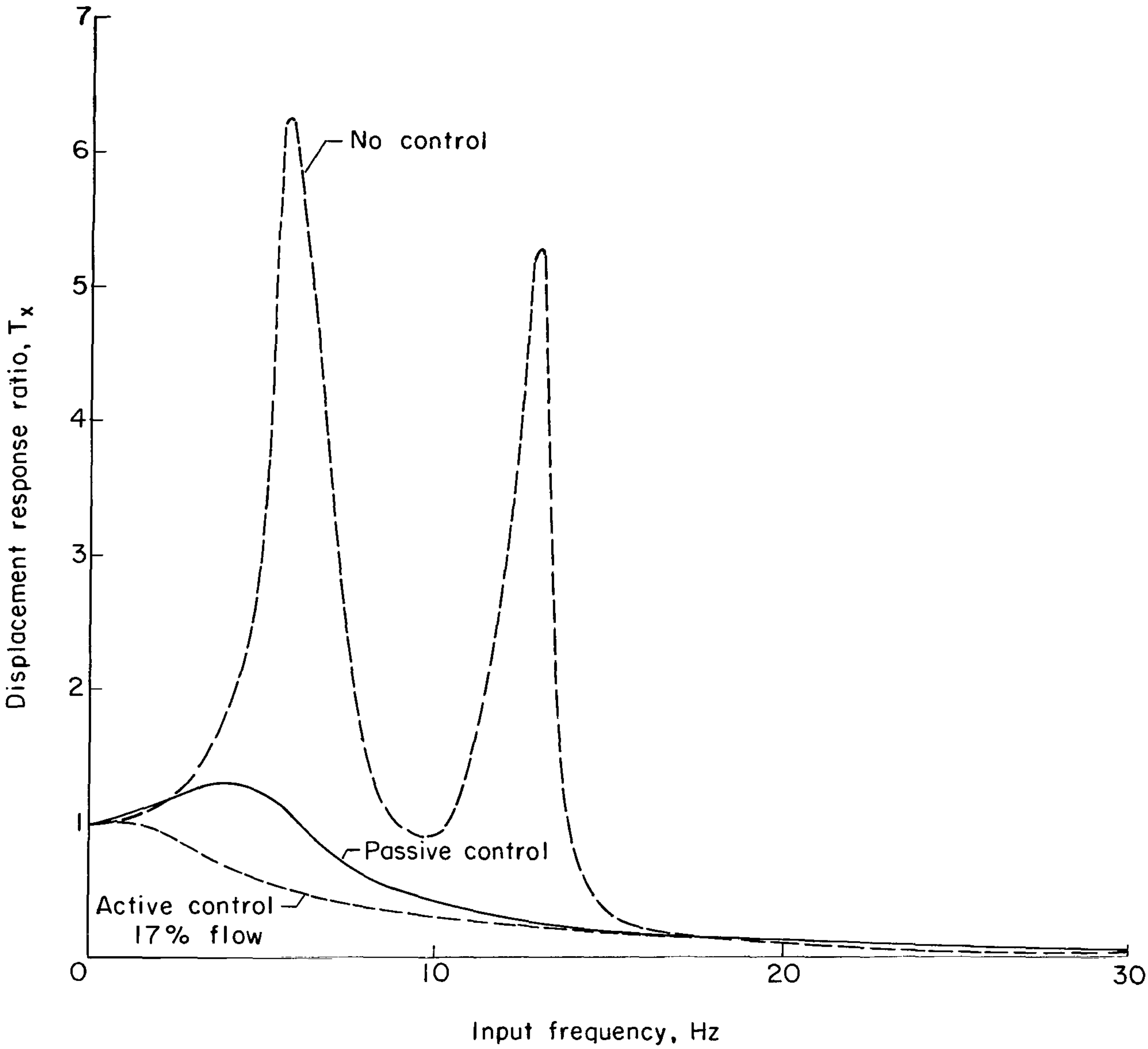


Figure 20.- Typical reduction in plenum response with active and passive control. $\bar{h}$ = 0.50 inch (1.27 cm), $\frac{z_0}{\bar{h}}$ = 0.20 .

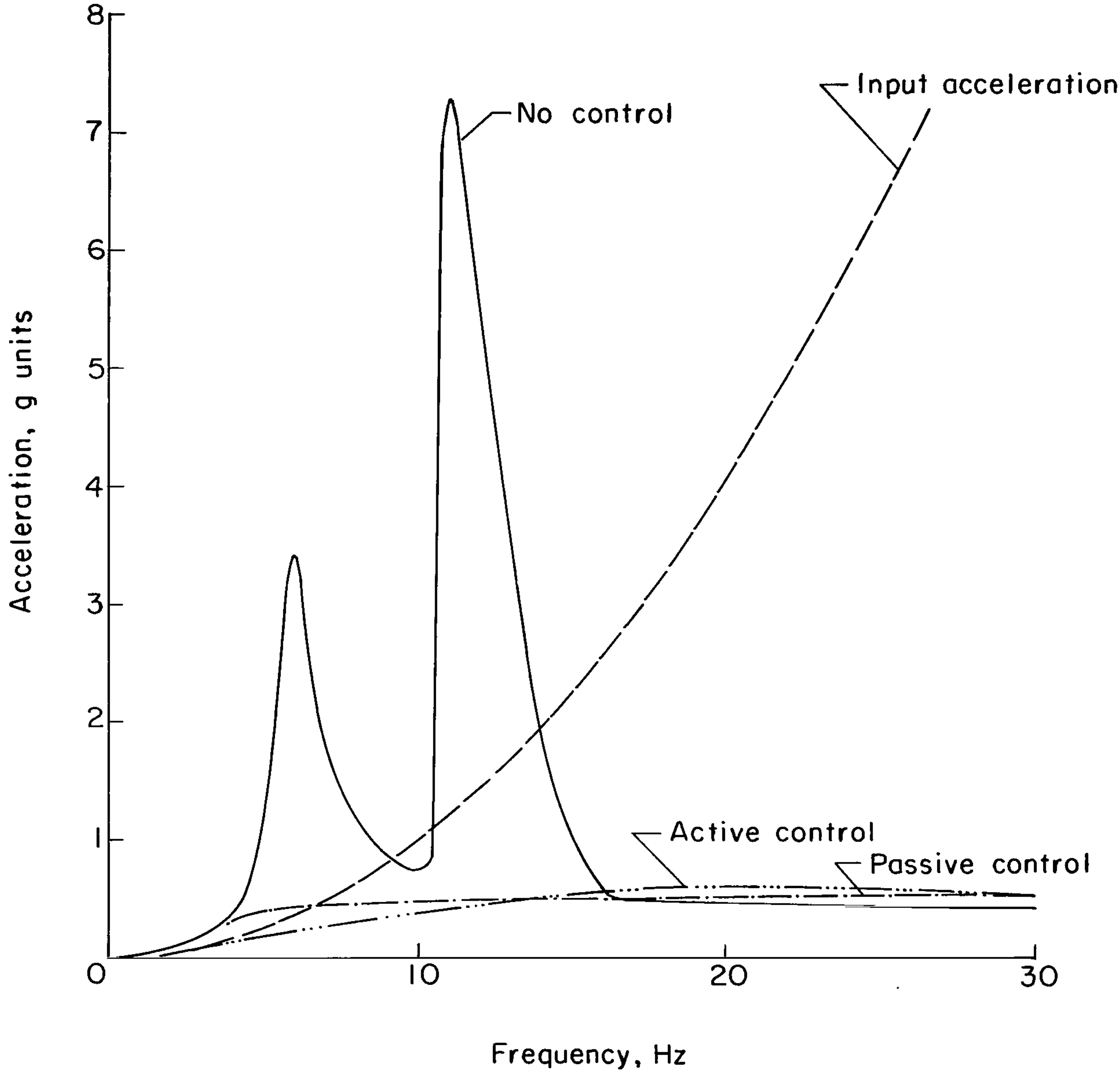
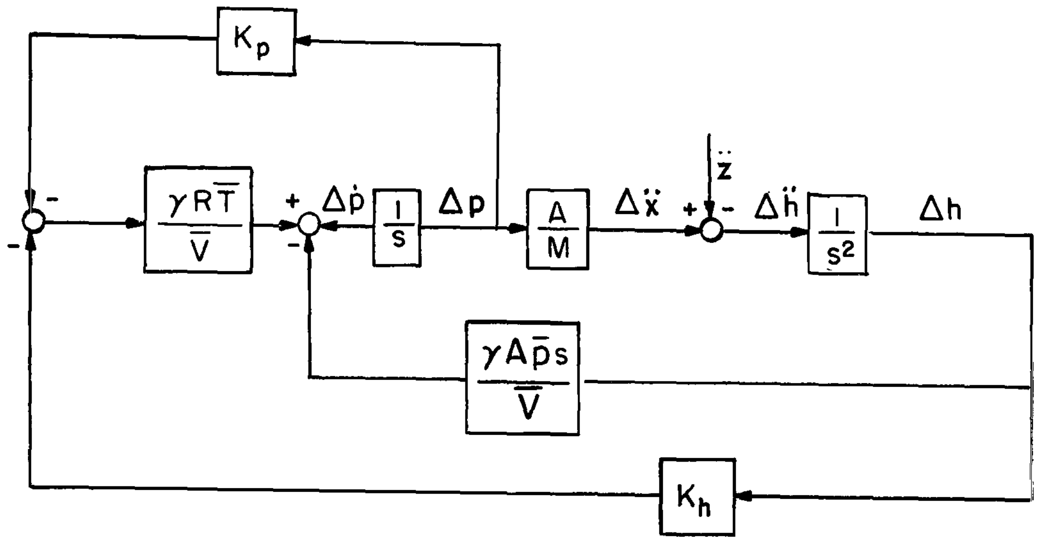
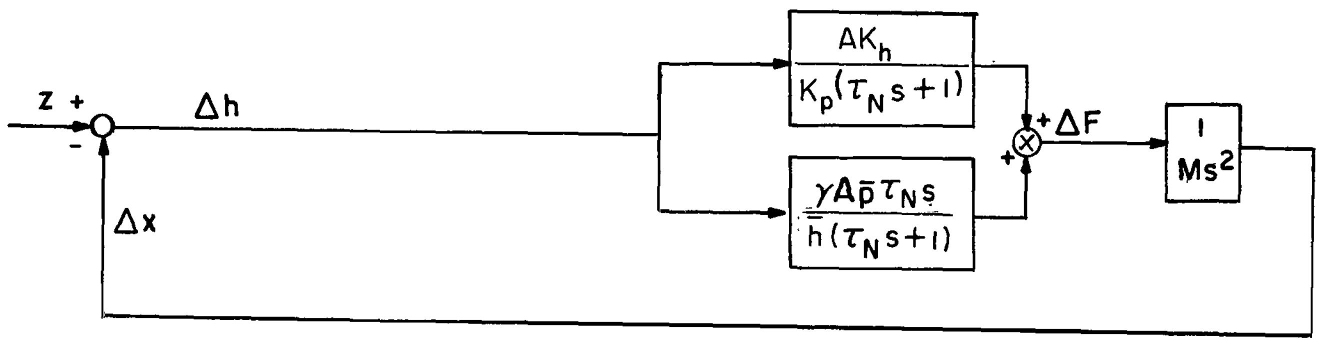


Figure 21.- Fluid-suspension-system acceleration response with and without active and passive control for constant-amplitude input. $\bar{h}$ = 0.50 inch (1.27 cm); $\frac{z_0}{\bar{h}} = 0.20$.



(a) Fluid-suspension block diagram.



(b) Force block diagram.

Figure 22.- Linearized block diagrams.

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