

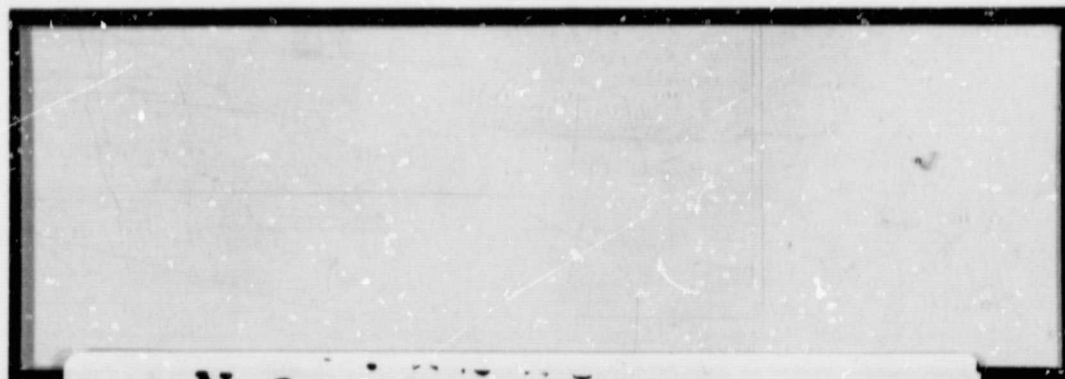
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AUBURN, ALABAMA



DIGITAL COMPENSATION OF THE THRUST
VECTOR CONTROL SYSTEM

PREPARED BY

SAMPLED-DATA CONTROL SYSTEMS GROUP

AUBURN UNIVERSITY

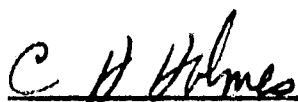
C. L. Phillips, Project Leader

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FOREWORD

This document is a technical summary of the progress made since October 1, 1967, by the Auburn University Electrical Engineering Department toward fulfillment of Phase IV of contract No. NAS8-11274. This contract was awarded to Auburn Research Foundation, Auburn, Alabama, May 28, 1964, and was extended September 28, 1966 by the George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Huntsville, Alabama.

SUMMARY

Analysis techniques are developed for a sampled-data control system using a multirate controller.

The simulation of a continuous time system containing a digital controller is accomplished through the use of discrete-time techniques.

LIST OF PERSONNEL

The following named staff members of Auburn University have actively participated on this project:

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I. INTRODUCTION

The problem of digitally controlling a process in which the outputs are sampled at a fixed rate with an independent, special purpose, digital device is considered in Chapter II. In particular, the problem of obtaining an open-loop transfer function for the case where the digital controller is operating at a sampling frequency which is an integer multiple of the basic system sampling frequency is studied.

In Chapter III, a modified version of the third-order digital compensator given in [5] is presented and its suitability for the stabilization of the Saturn S1-C attitude control system is discussed. In order to study the time-domain behavior of the vehicle with this and other compensators in the attitude control loop, it is necessary to simulate the resulting hybrid system. Since a general purpose digital element is not presently available for interfacing with the Auburn analog computer, it is necessary that the simulation be performed on a digital computer. In order to accomplish this objective, the vehicle differential equations must be discretized with respect to time. A method for performing this discretization is given in Chapter III along with an example of the response which results when an initial condition is applied to the closed loop system.

II. ANALYSIS OF SAMPLED-DATA CONTROL SYSTEMS USING MULTIRATE CONTROLLERS

Introduction

In sampled-data control systems for which the information sampling rate is required to be within a certain range, improved performance can often be obtained by operating the digital compensator at a higher sampling rate. In this chapter analysis techniques are developed for a sampled-data control system using a multirate controller. The techniques are directly applicable to the attitude control system of the Saturn V S1-C.

Analysis of the Multirate System[1],[2],[3]

The multirate system to be analyzed is shown in Figure II-1. The error

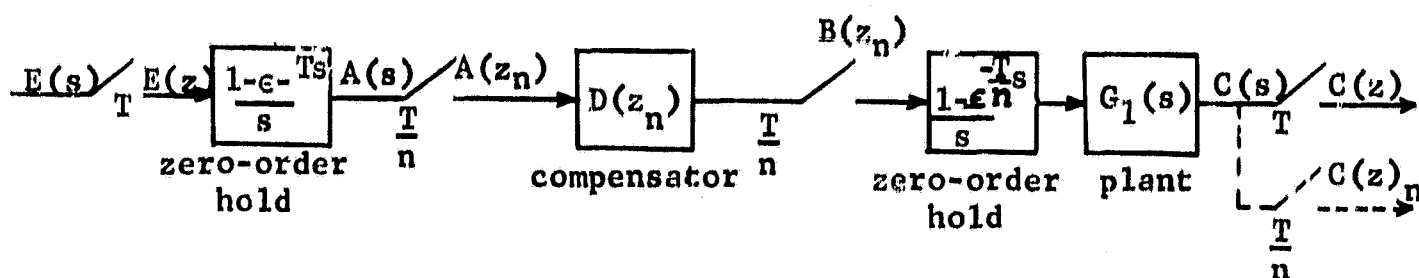


Figure II-1. Multirate sampled-data system.

signal is sampled every T seconds. The digital compensator, whose transfer function is $D(z_n)$, operates every T/n seconds. Thus

$$z_n = e^{\frac{T}{n}s} = (e^{Ts})^{\frac{1}{n}} = z^{\frac{1}{n}}, \quad (\text{II-1})$$

since $z = e^{sT}$. The input to the digital compensator is held at the sampled value for T seconds. This operation is shown in Figure II-1 by the first

zero-order hold. Thus, in Figure II-1,

$$E(z) = e(0) + e(T)z^{-1} + e(2T)z^{-2} + \dots, \quad (\text{II-2})$$

and

$$\begin{aligned} A(z)_n &= e(0) + e(0)z^{-\frac{1}{n}} + e(0)z^{-\frac{2}{n}} + \dots + e(0)z^{-\frac{n-1}{n}} \\ &\quad + e(T)z^{-1} + e(T)z^{-\frac{n+1}{n}} + \dots + e(T)z^{-\frac{2n-1}{n}} + e(2T)z^{-2} + \dots \end{aligned} \quad (\text{II-3})$$

The notation $E(z)$ indicates that the signal is sampled every T seconds, while the notation $A(z)_n$ indicates that the signal is sampled every T/n seconds. Thus $A(z)_n$ contains fractional powers of z , i.e., $z^{\frac{1}{n}}$. Note that $A(z)_n$ is obtained from $A(z)$ by the substitution given in (II-1); or,

$$A(z)_n = A(z) \left|_{z^{\frac{1}{n}} = z_n} \right. \quad (\text{II-4})$$

Then (II-3) can also be written as

$$\begin{aligned} A(z)_n &= e(0) + e(0)z_n^{-1} + e(0)z_n^{-2} + \dots + e(0)z_n^{-(n-1)} \\ &\quad + e(T)z_n^{-n} + \dots + e(T)z_n^{-(2n-1)} + e(2T)z_n^{-2n} + \dots \end{aligned}$$

To continue the analysis of the system of Figure II-1, we can express $B(z)_n$ as

$$\begin{aligned} B(z)_n &= A(z)_n D(z)_n \\ &= D(z)_n [e(0) + e(T)z^{-1} + \dots] \\ &\quad + D(z)_n z^{-\frac{1}{n}} [e(0) + e(T)z^{-1} + \dots] + \dots \end{aligned}$$

$$+ D(z)_n z^{-\frac{n-1}{n}} [e(0) + e(T)z^{-1} + \dots] , \quad (II-5)$$

or

$$B(z)_n = E(z)D(z)_n [1 + z^{-\frac{1}{n}} + \dots + z^{-\frac{n-1}{n}}] \quad (II-6)$$

Let

$$D_1(z)_n = D(z)_n [1 + z^{-\frac{1}{n}} + \dots + z^{-\frac{n-1}{n}}] \quad (II-7)$$

Then

$$B(z)_n = E(z)D_1(z)_n \quad (II-8)$$

$B(z)_n$ can be expressed as

$$B(z)_n = b(0) + b\left(\frac{T}{n}\right)z^{-\frac{1}{n}} + b\left(\frac{2T}{n}\right)z^{-\frac{2}{n}} + \dots \quad (II-9)$$

The output $c(t)$ is

$$c(t) = b(0)g(t) + b\left(\frac{T}{n}\right)g\left(t - \frac{T}{n}\right) + b\left(\frac{2T}{n}\right)g\left(t - \frac{2T}{n}\right) + \dots , \quad (II-10)$$

where

$$g(t) = \mathcal{L}^{-1} \left[\frac{1-e^{-\frac{Ts}{n}}}{s} G_1(s) \right] \quad (II-11)$$

The z -transform of the output, $C(z)$, will be found by first deriving the submultiple z -transform of the output, $C(z)_n$. The output $c(t)$ at any submultiple sampling instant $t = \frac{k}{n} T$, k an integer, is, from (II-10),

$$c\left(\frac{kT}{n}\right) = b(0)g\left(\frac{kT}{n}\right) + b\left(\frac{T}{n}\right)g\left(\frac{kT}{n} - \frac{T}{n}\right) + \dots + b\left(\frac{kT}{n}\right)g(0) . \quad (II-12)$$

From Figure II-1 and equations (II-9) and (II-12),

$$C(z)_n = B(z)_n G(z)_n = c(0) + c\left(\frac{T}{n}\right)z^{-\frac{1}{n}} + c\left(\frac{2T}{n}\right)z^{-\frac{2}{n}} + \dots \quad (II-13)$$

where, [3]

$$G(z)_n = G(z) \left| \begin{array}{l} T = T/n \\ z = z^{\frac{1}{n}} \end{array} \right.$$

Or, from (II-12) and (II-13),

$$\begin{aligned} C(z)_n &= b(0)g(0) + [b(0)g\left(\frac{T}{n}\right) + b\left(\frac{T}{n}\right)g(0)]z^{-\frac{1}{n}} + \dots \\ &+ [b(0)g(T) + b\left(\frac{T}{n}\right)g\left(T-\frac{T}{n}\right) + \dots + b(T)g(0)]z^{-1} + \dots \end{aligned} \quad (\text{II-14})$$

In order to obtain $C(z)$ from $C(z)_n$, the following identity is useful: [2]

$$\begin{aligned} &\frac{1}{n} [C(z)_n + C(z\epsilon^{j2\pi})_n + C(z\epsilon^{j4\pi})_n + \dots + C(z\epsilon^{j2\pi(n-1)})_n] \\ &= \frac{1}{n} \left[nc(0) + c\left(\frac{T}{n}\right)z^{-\frac{1}{n}} \left(1 + \epsilon^{-j\frac{2\pi}{n}} + \epsilon^{-j\frac{4\pi}{n}} + \dots + \epsilon^{-j\frac{2\pi(n-1)}{n}}\right) \right. \\ &+ c\left(\frac{2T}{n}\right)z^{-\frac{2}{n}} \left(1 + \epsilon^{-j\frac{4\pi}{n}} + \epsilon^{-j\frac{8\pi}{n}} + \dots + \epsilon^{-j\frac{4\pi(n-1)}{n}}\right) + \dots \\ &\left. + c(T)z^{-1} (1 + \epsilon^{-j2\pi} + \epsilon^{-j4\pi} + \dots + \epsilon^{-j4n\pi}) \dots \right], \end{aligned} \quad (\text{II-15})$$

where

$$C(z\epsilon^{j2\pi})_n = c(0) + c\left(\frac{T}{n}\right) \left[\epsilon^{j2\pi} z \right]^{-\frac{1}{n}} + c\left(\frac{2T}{n}\right) \left[\epsilon^{j2\pi} z \right]^{-\frac{2}{n}} + \dots$$

By use of the relationship

$$1 + \epsilon^{-j\frac{k2\pi}{n}} + \epsilon^{-j\frac{k4\pi}{n}} + \dots + \epsilon^{-j\frac{k2\pi(n-1)}{n}} = \begin{cases} 0 & k \neq mn \\ n & k = mn \end{cases}, \quad m=0,1,2,\dots \quad (\text{II-16})$$

equation (II-15) can be reduced to

$$\frac{1}{n} [C(z)_n + C(z\epsilon^{j2\pi})_n + \dots + C(z\epsilon^{j2\pi(n-1)})_n]$$

$$= c(0) + c(T)z^{-1} + c(2T)z^{-2} + \dots = C(z) \quad (\text{II-17})$$

It is seen then, from (II-8), (II-13), and (II-17) that

$$C(z) = E(z) \left\{ \frac{1}{n} [D_1(z)_n G(z)_n + D_1(z\epsilon^{j2\pi})_n G(z\epsilon^{j2\pi})_n + \dots + D_1(z\epsilon^{j2\pi(n-1)})_n G(z\epsilon^{j2\pi(n-1)})_n] \right\} \quad (\text{II-18})$$

The open-loop transfer function for the system of Figure II-1 is then seen to be

$$\frac{C(z)}{E(z)} = \frac{1}{n} [D_1(z)_n G(z)_n + D_1(z\epsilon^{j2\pi})_n G(z\epsilon^{j2\pi})_n + \dots + D_1(z\epsilon^{j2\pi(n-1)})_n G(z\epsilon^{j2\pi(n-1)})_n] \quad (\text{II-19})$$

Analysis is more easily performed if (II-19) is expressed as a function of z_n , obtained using the substitution $z^{1/n} = z_n$.

$$\frac{C(z_n)}{E(z_n)} = \frac{1}{n} [D_1(z_n)G(z_n) + D_1(z_n \epsilon^{j\frac{2\pi}{n}}) G(z_n \epsilon^{j\frac{2\pi}{n}}) + \dots + D_1(z_n \epsilon^{j\frac{2\pi(n-1)}{n}}) G(z_n \epsilon^{j\frac{2\pi(n-1)}{n}})] \quad (\text{II-20})$$

Note that when (II-20) is reduced to its simplest form for a given system, z_n will always appear as $(z_n)^{kn}$; $k=0,1,2,\dots$.

Nyquist Diagram

The Nyquist diagram for the system of Figure II-1 is obtained from (II-20) by allowing z_n to vary along the unit circle in the z_n -plane. As an example, consider the case of (II-20) for $n = 3$.

$$\frac{C(z_3)}{E(z_3)} = \frac{1}{3} [D_1(z_3)G(z_3) + D_1(z_3 e^{j\frac{2\pi}{3}})G(z_3 e^{j\frac{2\pi}{3}}) + D_1(z_3 e^{j\frac{4\pi}{3}})G(z_3 e^{j\frac{4\pi}{3}})] \quad (\text{II-21})$$

As z_3 varies from $1 \angle 0^\circ$ to $1 \angle 120^\circ$, $z_3 e^{j\frac{2\pi}{3}}$ will vary from $1 \angle 120^\circ$ to $1 \angle 240^\circ$, and $z_3 e^{j\frac{4\pi}{3}}$ will vary from $1 \angle 240^\circ$ to $1 \angle 360^\circ$. Further, as z_3 varies from $1 \angle 120^\circ$ to $1 \angle 240^\circ$, $z_3 e^{j\frac{2\pi}{3}}$ will vary from $1 \angle 240^\circ$ to $1 \angle 360^\circ$, and $z_3 e^{j\frac{4\pi}{3}}$ will vary from $1 \angle 0^\circ$ to $1 \angle 120^\circ$. Thus, by the symmetry of (II-21), the Nyquist diagram obtained by varying z_3 from $1 \angle 0^\circ$ to $1 \angle 120^\circ$ is repeated as z_3 varies from $1 \angle 120^\circ$ to $1 \angle 240^\circ$, and is repeated again as z_3 varies from $1 \angle 240^\circ$ to $1 \angle 360^\circ$. Thus the entire Nyquist diagram can be obtained from (II-21) by varying z_3 from $1 \angle 0^\circ$ to $1 \angle 120^\circ$.

In using the Nyquist diagram to determine stability, it is seen that every pole of $D_1(z_3)G(z_3)$ will appear as a pole of $D_1(z_3 e^{j\frac{2\pi}{3}})G(z_3 e^{j\frac{2\pi}{3}})$, but rotated through an angle of $-\frac{2\pi}{3}$ radians. Also, every pole of $D_1(z_3)G(z_3)$ will appear as a pole of $D_1(z_3 e^{j\frac{4\pi}{3}})G(z_3 e^{j\frac{4\pi}{3}})$, but rotated through an angle of $-\frac{4\pi}{3}$ radians. Then the number of poles of the open-loop transfer function (II-21) outside the unit circle in the z_3 plane is three times the number of poles of $D_1(z_3)G(z_3)$ that occur outside the unit circle. If N is the number of encirclements of the -1 point as seen from a plot of the Nyquist diagram obtained by varying z_3 from $1 \angle 0^\circ$ to $1 \angle 120^\circ$ along the unit circle, then

$$3N = Z - 3P \quad (\text{II-22})$$

where Z is the number of zeros of the system characteristic equation that lie outside the unit circle in the z_3 -plane, and P is the number of poles

of $D_1(z_3)G(z_3)$ that lie outside the unit circle. For the general case that the fast-rate sampling occurs every T/n seconds, (II-22) becomes

$$nN = Z^{-n}P \quad (\text{II-23})$$

The above techniques will be applied to the analysis of the attitude control system of the Saturn V S1-C. Problems may develop in attempting to use (II-20) in design, since the compensation function cannot be factored from the open-loop transfer function. These problems will be studied during the next few months.

III. THE DIGITALLY COMPENSATED SYSTEM

A. Introduction

In chapter II of [5], a third order compensation function, $D(z)$, was developed which stabilizes the "rate-derived" thrust vector control system. A refinement of this function is discussed in this chapter.

B. The Development of the Function

One of the undesirable characteristics of the compensation function presented on page 5 of [5] is that the third bending mode is phase stabilized. It was decided to modify the original compensation function by using a second-order numerator term to create a notch effect between the second and third bending modes. This accomplished gain stabilization of the second and third bending modes. The w -plane representation of the modified function is :

$$D(w) = \frac{a_0 \left[w/.0101 + 1 \right] \left[w^2/ (.3)^2 + 2(.1) w/.3 + 1 \right]}{\left[w/.0989 + 1 \right]^2 \left[w/.1494 + 1 \right]}, \quad (\text{III-1})$$

where

$$\begin{aligned} a_0 &= .2969 && \text{for } 0 \leq T < 140 \text{ seconds,} \\ a_0 &= .178 && \text{for } 140 \leq T \leq 158.6 \text{ seconds.} \end{aligned}$$

An equivalent z-plane representation of this function is :

$$D(z) = \frac{1.3472 a_0 (z - .98)(z^2 - 1.583z + .8957)}{(z - .82)^2 (z - .74)} \quad (\text{III-2})$$

An s-plane frequency response with $a_0 = .2969$ is given by Figure III-1. The Nyquist plots for the T. V. C. system which result when the $D(z)$ given by (III-2) is used are given for selected flight times in Appendix A. Based on these plots, it appears that satisfactory stability margins can be obtained by implementation of the function given by (III-2). The major drawback associated with this function is that a relatively low value of a_0 is used. As effort is underway to obtain satisfactory stability margins with an a_0 near unity.

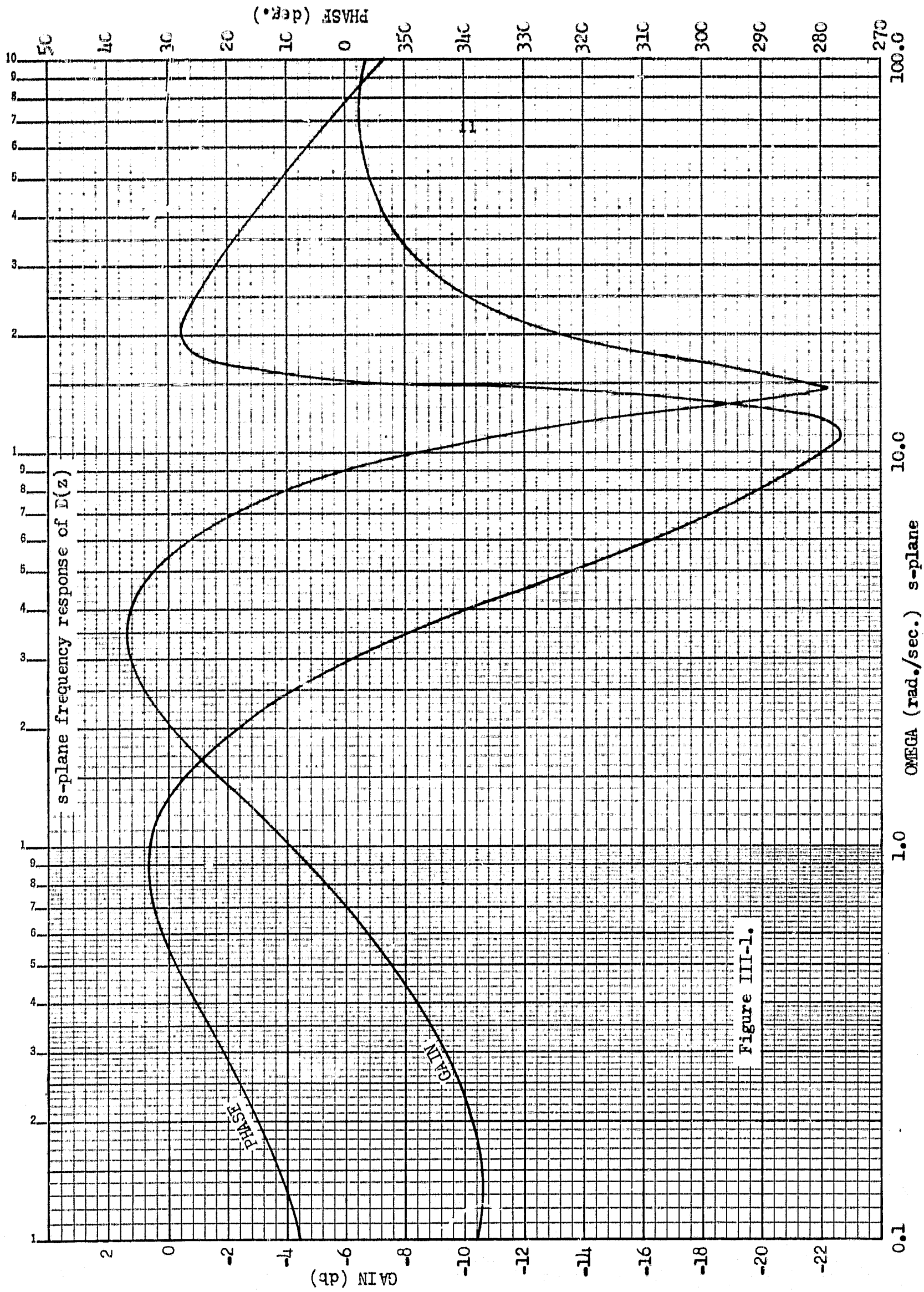
C. Discrete-Time Simulation

In order to evaluate the design of a digital compensation function for a continuous time system, it is necessary to simulate the complete system in some manner. One method of obtaining a model for a hybrid system is to use discrete-time techniques.

The linear differential equations which describe that Saturn V S1-C at a particular time of flight may always be written in the form of first-order differential equations as shown below:

$$\dot{\underline{x}}(t) = A\underline{x}(t) + b\underline{\beta}_c(t), \quad (\text{III-3})$$

$$\phi_D(t) = \underline{c}^T \underline{x}(t), \quad (\text{III-4})$$



where $\beta_c(t)$ represents commanded engine angle and $\phi_D(t)$ represents the vehicle attitude error as measured at the instrument unit.

The simplified equations of motion are:

Controller dynamics:

$$\frac{\beta_E}{\beta_c} = \frac{(34.48)^2}{s^2 + 2(.434)(34.48)s + 34.48^2} \quad (\text{III-5})$$

Moment equation:

$$\ddot{\phi} = -c_1 \dot{\phi} - c_2 \beta_E \quad (\text{III-6})$$

Bending mode equations:

$$\ddot{\eta}_i + 2\xi_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i = (RP \cdot Y\beta_i / GM_i) \beta_E \quad (\text{III-7})$$

Attitude error equation:

$$\phi_D = \phi + \sum_{i=1}^4 YPD_i \eta_i \quad (\text{III-8})$$

A signal flow graph for the system represented by the above equations is shown by Figure III-2. From the signal flow graph of Figure III-2, the A , b and c matrices of (III-3) and (III-4) may be developed. The elements of these matrices are given in Appendix B.

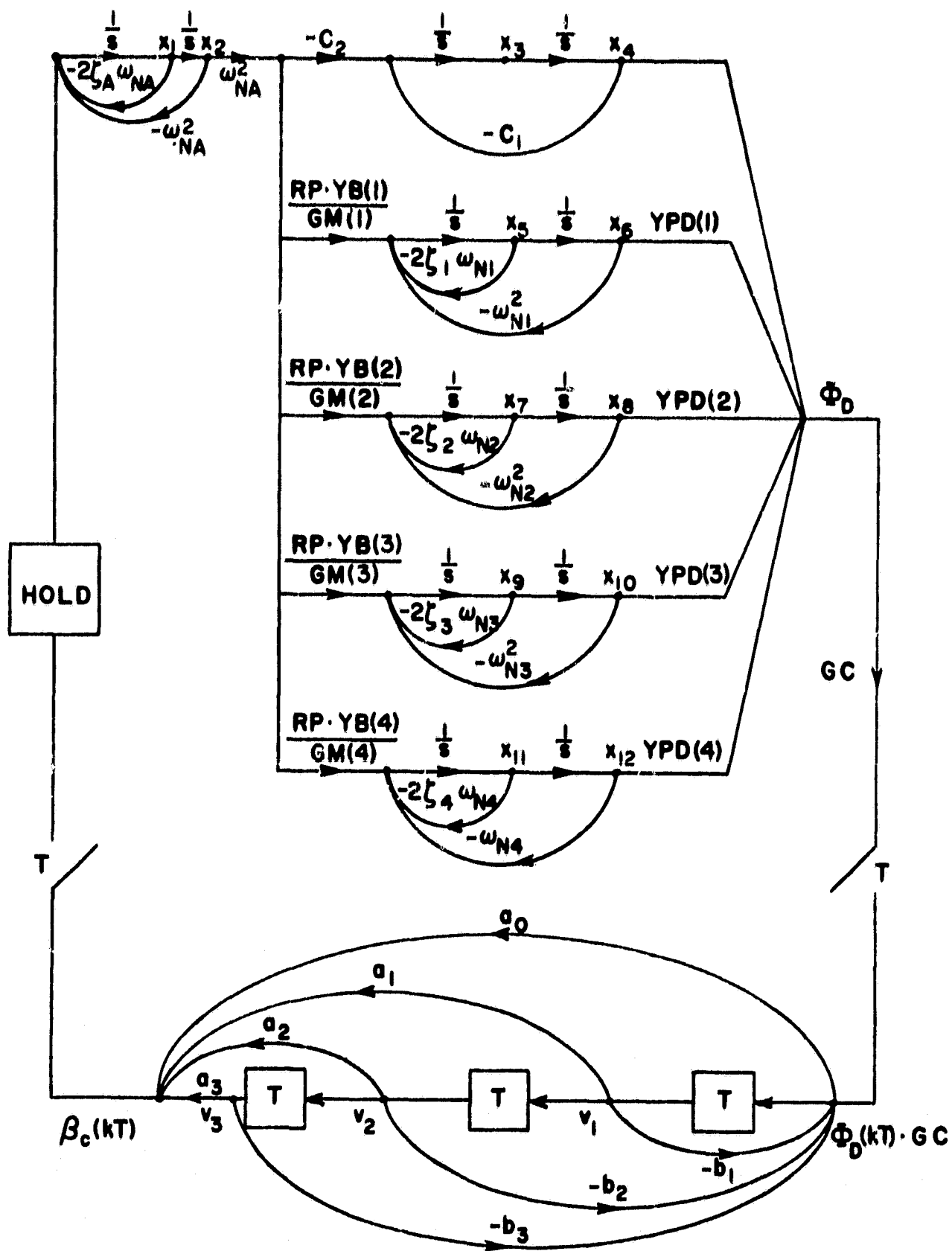


Figure III-2.--Signal flow diagram for the hybrid T.V.C. System.

The solution of equation (III-3) is: [4]

$$\underline{x}(t) = \phi(t - t_0) \underline{x}(t_0) + \int_{t_0}^t \phi(t-\tau) \underline{b} \beta_c(\tau) d\tau \quad (\text{III-9})$$

where $\phi(t-t_0)$ is a square matrix given by:

$$\phi(t-t_0) = \exp[\Lambda (t-t_0)] = \sum_{k=0}^{\infty} \frac{\Lambda^k (t-t_0)^k}{k!} \quad (\text{III-10})$$

The matrix $\phi(t-t_0)$ is called the state transition matrix [1] since it maps an initial state $\underline{x}(t_0)$ into the state $\underline{x}(t)$ at any later time $t \geq t_0$.

Now, since a digital controller is used, the output of the system, $\phi_D(t)$, is measured only every T seconds, where T is the system sampling rate. Further, the engine command signal, $\beta_c(t)$, can change only every T seconds and must remain constant between sampling instants. Using this information, the solution of the state equations can be discretized in the following manner. A property of the state transition matrix is:

$$\int_{\tau=0}^T \phi(T-\tau) d\tau = \int_{\tau=0}^T \phi(\tau) d\tau \quad (\text{III-11})$$

Proof: Let $\sigma = T-\tau$, then $d\sigma = -d\tau$ and

$$\int_{\tau=0}^T \phi(T-\tau) d\tau = \int_{\sigma=T}^0 \phi(\sigma) (-d\sigma) = \int_{\sigma=0}^T \phi(\sigma) d\sigma.$$

Now, suppose that the system state $\underline{x}(kT)$ and $\beta_c(kT)$ are known and it is desired to determine the system state, $\underline{x}[(k+1)T]$. From (III-9) it

is evident that

$$\underline{x}[(k+1)T] = \phi[(k+1)T - kT] \underline{x}(kT) + \int_{\tau=kT}^{(k+1)T} \phi[(k+1)T - \tau] \underline{b}\beta_c(\tau) d\tau. \quad (\text{III-12})$$

Note that $\beta_c(\tau) = \beta_c(kT)$ for $kT \leq \tau < (k+1)T$.

Thus (III-12) becomes

$$\underline{x}[(k+1)T] = \phi(T) \underline{x}(kT) + \left[\int_{\tau=kT}^{(k+1)T} \phi[(k+1)T - \tau] \right] \underline{b}\beta_c(kT). \quad (\text{III-13})$$

Using the property that

$$\int_{\tau=0}^t f(\tau) d\tau = \int_{\tau=\Delta}^{t+\Delta} f(\tau - \Delta) d\tau, \quad (\text{III-14})$$

and that

$$\phi(t_1 + t_2) = \phi(t_1) \phi(t_2)$$

we can write

$$\phi(T) \int_{\tau=kT}^{(k+1)T} \phi[-(\tau - kT)] d\tau = \int_{\tau=0}^T \phi[-(\tau - T)] d\tau = \int_{\tau=0}^T \phi(T - \tau) d\tau. \quad (\text{III-15})$$

Then from (III-11) and (III-15), (III-13) can be expressed as

$$\underline{x}[(k+1)T] = \phi(T) \underline{x}(kT) + \left[\int_{\tau=0}^T \phi(\tau) d\tau \right] \underline{b}\beta_c(kT). \quad (\text{III-16})$$

the matrix $\phi(T)$ and the matrix $\left[\int_{\tau=0}^T \phi(\tau) d\tau \right]$ are constant coefficient matrices which need to be evaluated only once. The Taylor expansion of

these matrix functions can be used to obtain numerical results for the matrix coefficients. The expansions are:

$$\phi(T) = I + AT + \frac{A^2 T^2}{2!} + \frac{A^3 T^3}{3!} + \dots \quad (\text{III-17})$$

and

$$\int_0^T \phi(\tau) d\tau = IT + \frac{AT^2}{2!} + \frac{A^2 T^3}{3!} + \frac{A^3 T^4}{4!} + \dots \quad (\text{III-18})$$

It was found for the system under consideration that $\phi(T)$ and $\int_0^T \phi(\tau) d\tau$ can be accurately computed by using 25 terms of (III-17) and (III-18) respectively. Let $A_1 = \phi(T)$, and $\underline{b}_1 = \left[\int_0^T \phi(\tau) d\tau \right] \underline{b}$. Therefore from (III-16)

$$\underline{x}[(k+1)T] = A_1 \underline{x}(kT) + b_1 \beta_c(kT) \quad (\text{III-19})$$

$$\phi_D(kT) = \underline{c}^T \underline{x}(kT) \quad (\text{III-20})$$

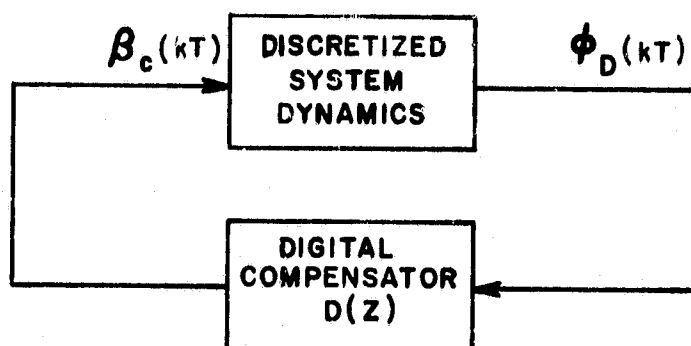


Figure III-3.--Block diagram of discretized system.

The digital compensation function of (III-2) may be written as

$$D(z) = \frac{\beta_c(z)}{\phi_D(z)} = GC \left[\frac{z^3 - 2.563z^2 + 2.44704z - .877786}{z^3 - 2.38z + 1.886z - .497576} \right] \quad (\text{III-21})$$

This transfer function represents a system which is characterized by difference equations. One set of first-order difference equations whose transfer function is (III-21) is:

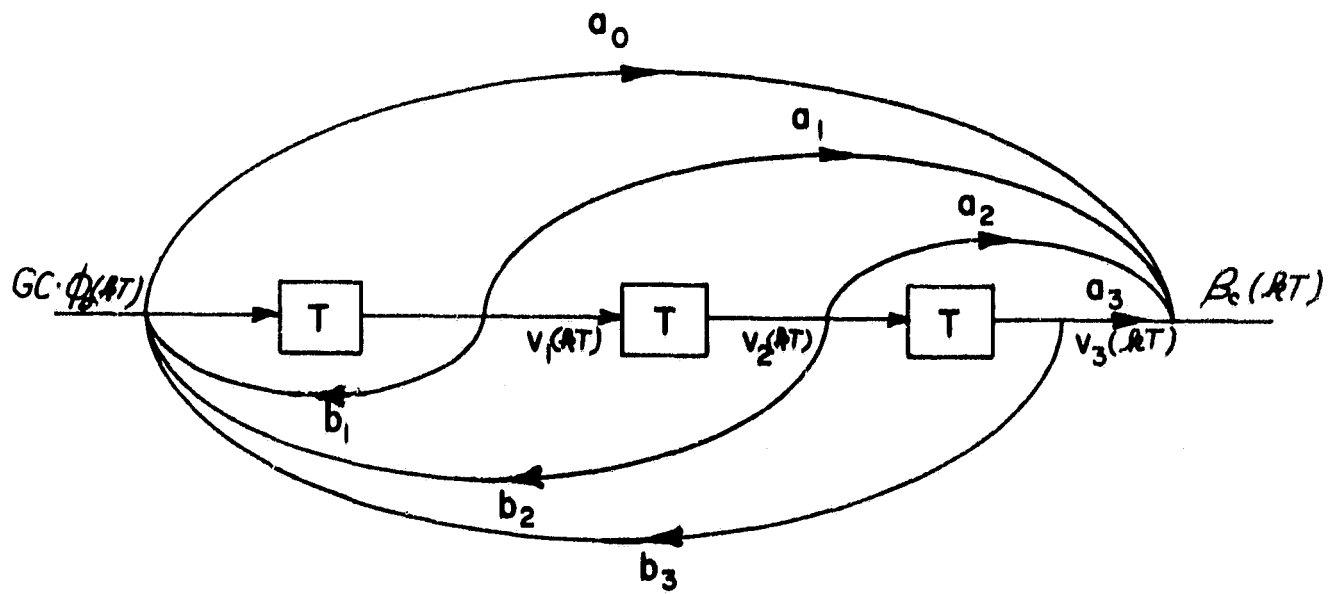
$$\begin{aligned} v_1[(k+1)T] &= -b_1 v_1(kT) - b_2 v_2(kT) - b_3 v_3(kT) + \phi_D(kT) \cdot GC \\ v_2[(k+1)T] &= v_1(kT) \\ v_3[(k+1)T] &= v_2(kT) \\ \beta_c(kT) &= (a_1 - b_1 \cdot a_0) v_1(kT) + (a_2 - b_2 \cdot a_0) v_2(kT) + (a_3 - b_3 \cdot a_0) \\ &\quad v_3(kT) + \phi_D(kT) \cdot GC \cdot a_0 \end{aligned} \quad (\text{III-22})$$

where $a_0 = 1.0$, $a_1 = -2.563$, $a_2 = 2.44704$, $a_3 = -0.877786$, $b_1 = -2.38$, $b_2 = 1.886$, and $b_3 = -0.497576$.

Equations (III-22) can be mechanized as shown by Figure III-4.

The calculation of the response of the system is accomplished as follows: Initial states are chosen for the vehicle, $\underline{x}(0)$ and the compensation, $\underline{y}(0)$. Then $\phi_D(0)$ and $\beta_c(0)$ may be calculated by (III-20) and (III-23) respectively. Next, (III-19) and (III-22) are used to calculate $\underline{x}(T)$ and $\underline{y}(T)$. Then $\phi_D(t)$ and $\beta_c(T)$ are computed. The process is repeated for all kT of interest.

The computer program given in Appendix C was used to implement the



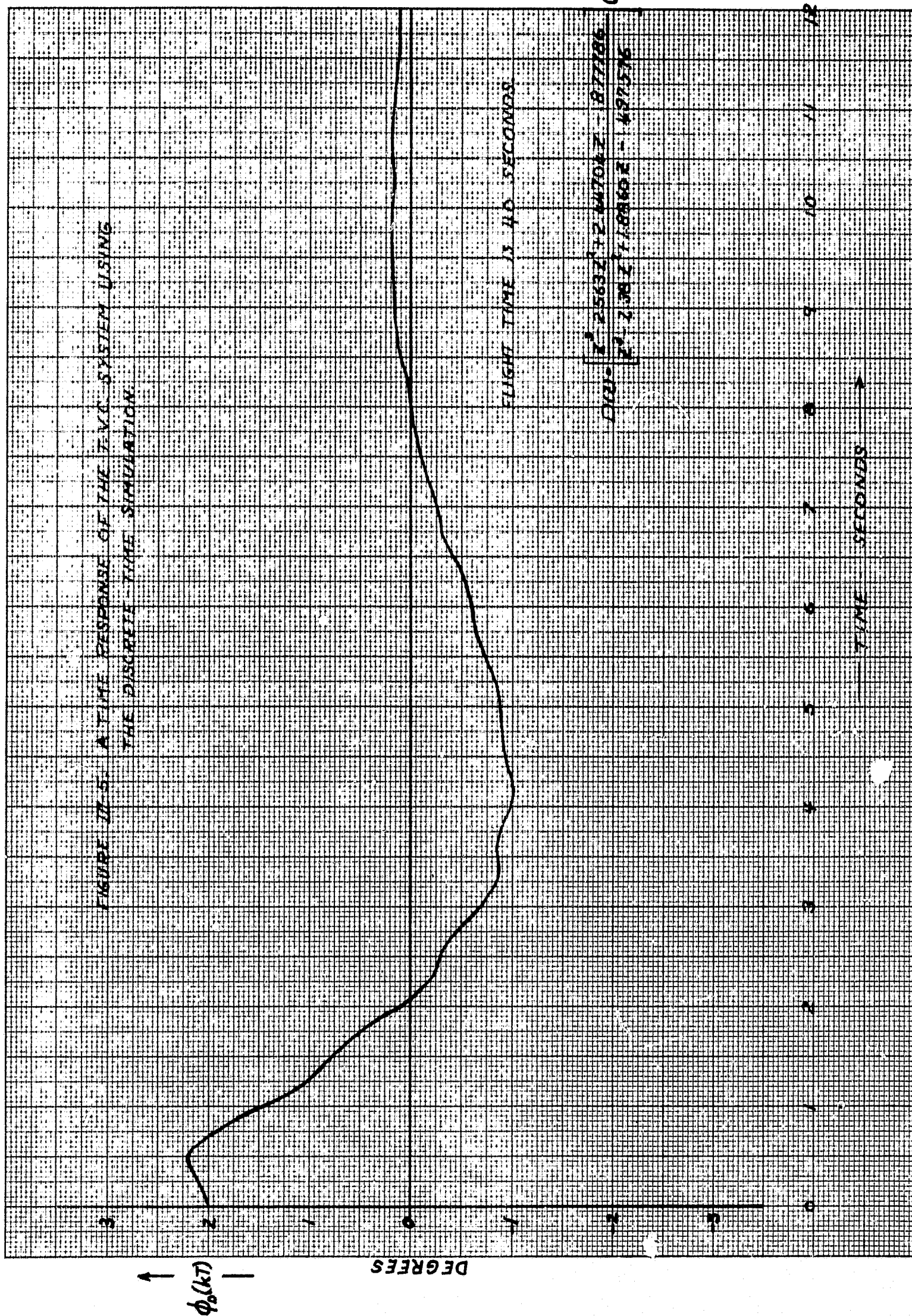
$$\begin{aligned} a_0 &= 1.0 \\ a_1 &= -2.563 \\ a_2 &= 2.44704 \\ a_3 &= -0.877786 \\ b_1 &= -2.38 \\ b_2 &= 1.886 \\ b_3 &= -0.497576 \end{aligned}$$

Figure III-4.--A Realization of $D(z)$

above steps. A time response for $\phi_D(kT)$ using the $D(z)$ of (III-21) with a control gain, GC, of 0.4 and the vehicle data at 40 seconds of flight is given in Figure (III-5).

The approach discussed above is particularly useful in studying the time domain behavior of the attitude control system when considering the effects of quantization in the implementation of $D(z)$. Studies of quantization effects using this approach will be included in future reports.

FIGURE 17-5. A TIME RESPONSE OF THE T-M-C SYSTEM USING
THE DISCRETE TIME SIMULATION.



IV. CONCLUSIONS

An open loop transfer function was derived in Chapter II for a hybrid control system in which the digital controller is operating at a sampling frequency that is an integer multiple of the basic system sampling rate. However, from a design standpoint, it is unfortunate that the open loop transfer function cannot be written as the product of the digital controller transfer function and the plant transfer function.

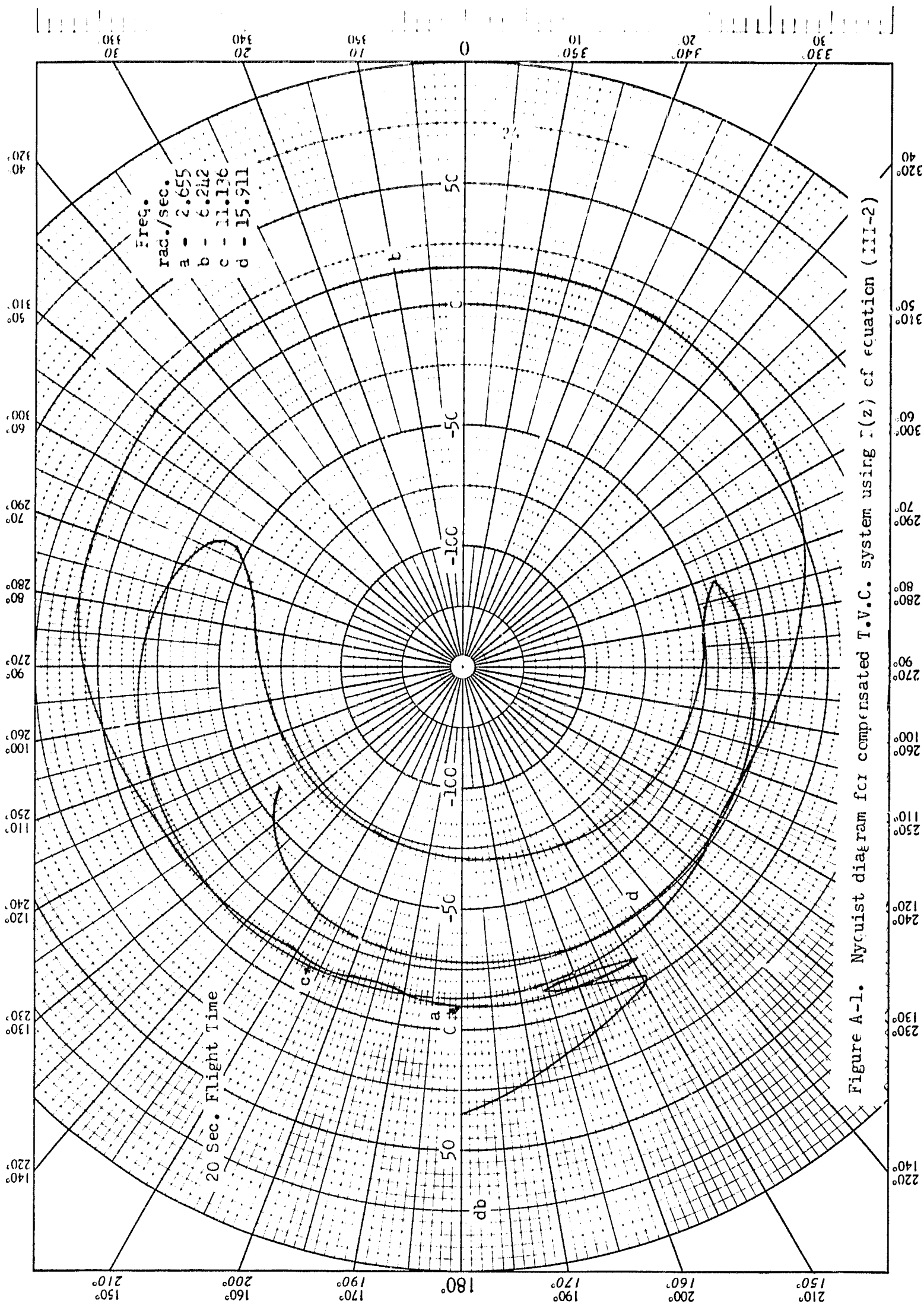
A modified third-order compensation function is discussed in Chapter III and Nyquist diagrams for the compensated system are given in Appendix A. It appears that this function provides acceptable stability margins for the T.V.C. system. (The data used in generating these diagrams is for vehicle AS-504 and was provided to Auburn during the fall of 1966.) The major disadvantage of the modified compensation function is that a small value of a_0 (.296) is used. The problem of stabilizing the system with an a_0 near unity is under study.

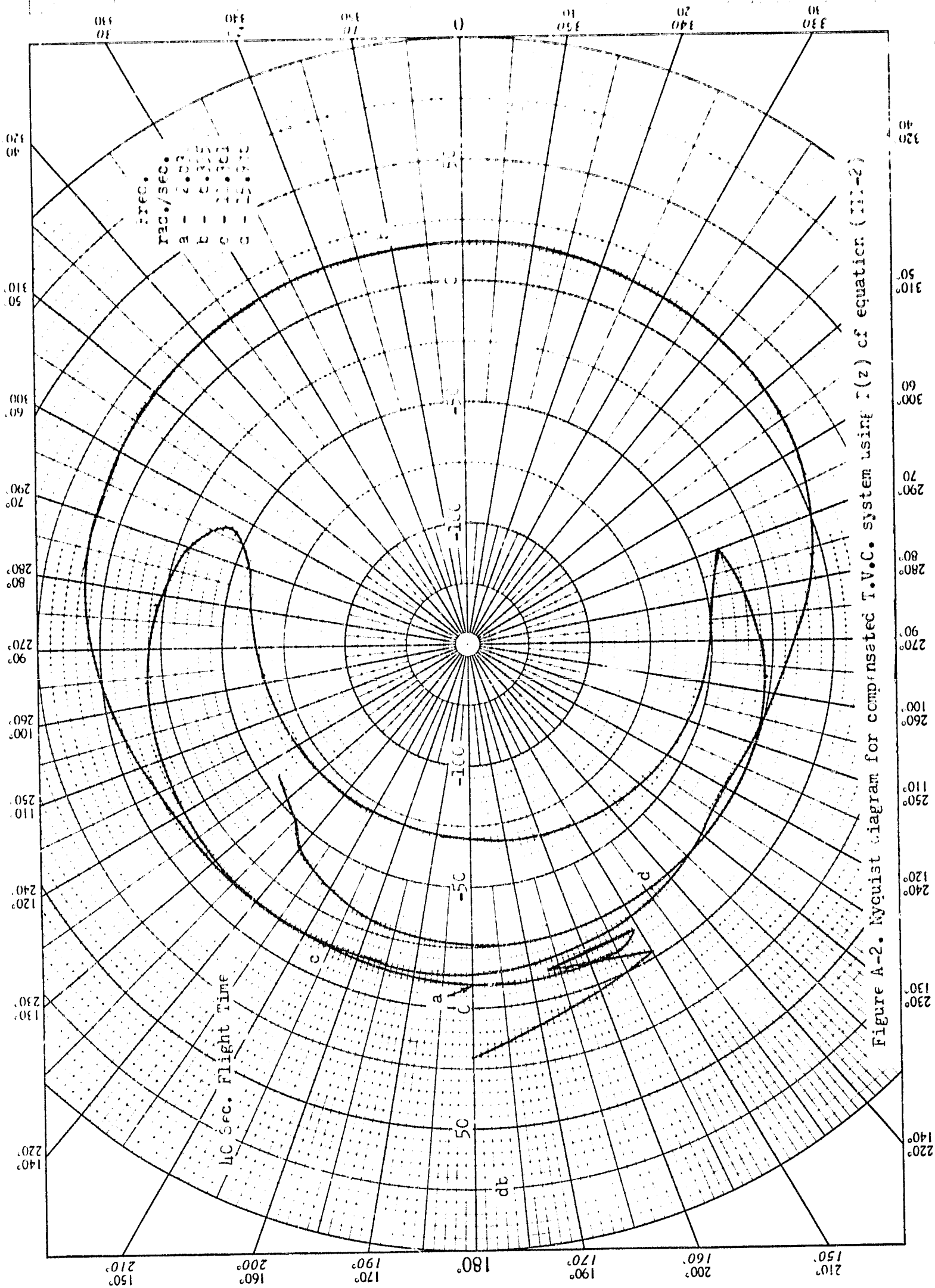
A method for developing a discrete-time model for the vehicle dynamics is also given in Chapter III. This was accomplished by the utilization of a Taylor series expansion for the state transition matrix of a set of first order differential equation which characterize the motions of the vehicle. The discretized model of the system dynamics was used in conjunction with the difference equations which represent the digital controller in order to study the time domain behavior of the closed loop system. The digital computer program which was used to perform this study is given in Appendix C.

REFERENCES

1. Kranc, G. M., "Compensation of an Error-Sampled System by a Multirate Controller," Trans. A.I.E.E., Part 2, July, 1957, pp. 149-159.
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3. Kuo, B. C., Analysis and Synthesis of Sampled-Data Control Systems, Englewood Cliffs, N. J., Prentice-Hall, Inc., 1963; Chapter 10.
4. Derusso, P. M., Roy, R. J., and Close, C. M., "State Variables for Engineers," John Wiley & Sons, Inc., New York, London, and Sydney 1965, pp. 360-362.
5. Phillips, C. L., et.al, Digital Compensation of the Thrust Vector Control System, Eighth Technical Report, Auburn Research Foundation, Auburn, Alabama, May 1967.

APPENDIX A





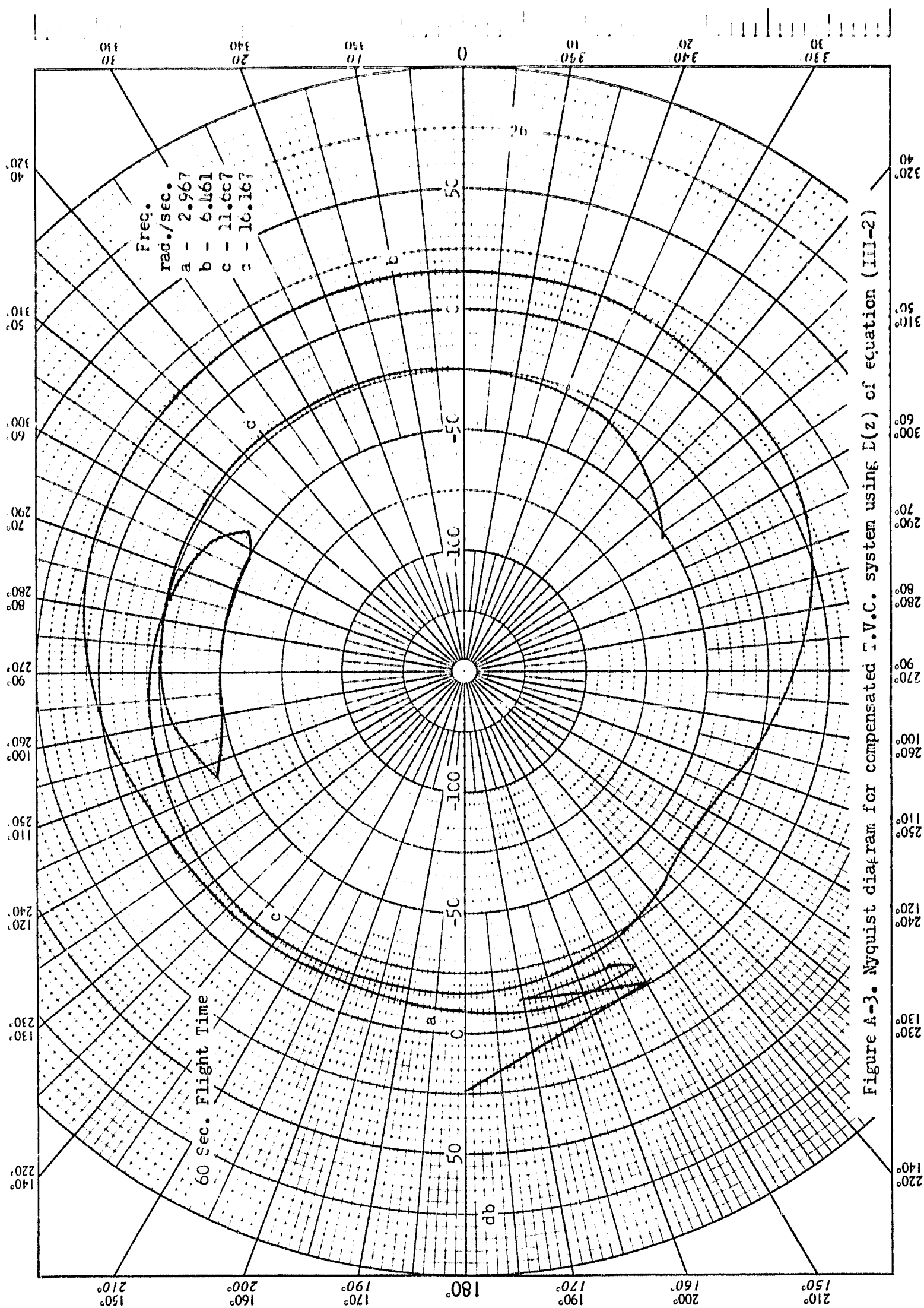
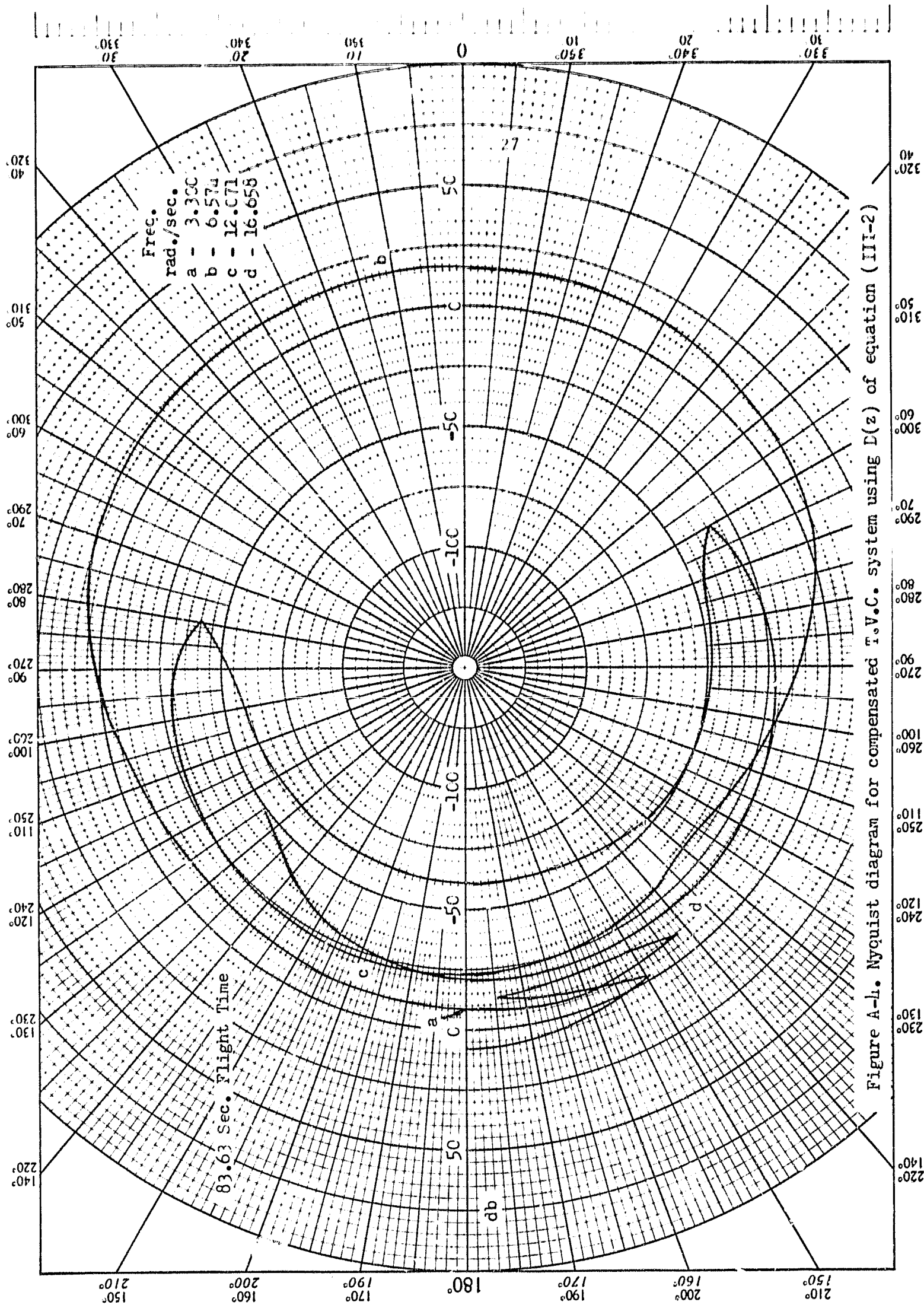


Figure A-3. Nyquist diagram for compensated T.V.C. system using $D(z)$ of equation (III-2)



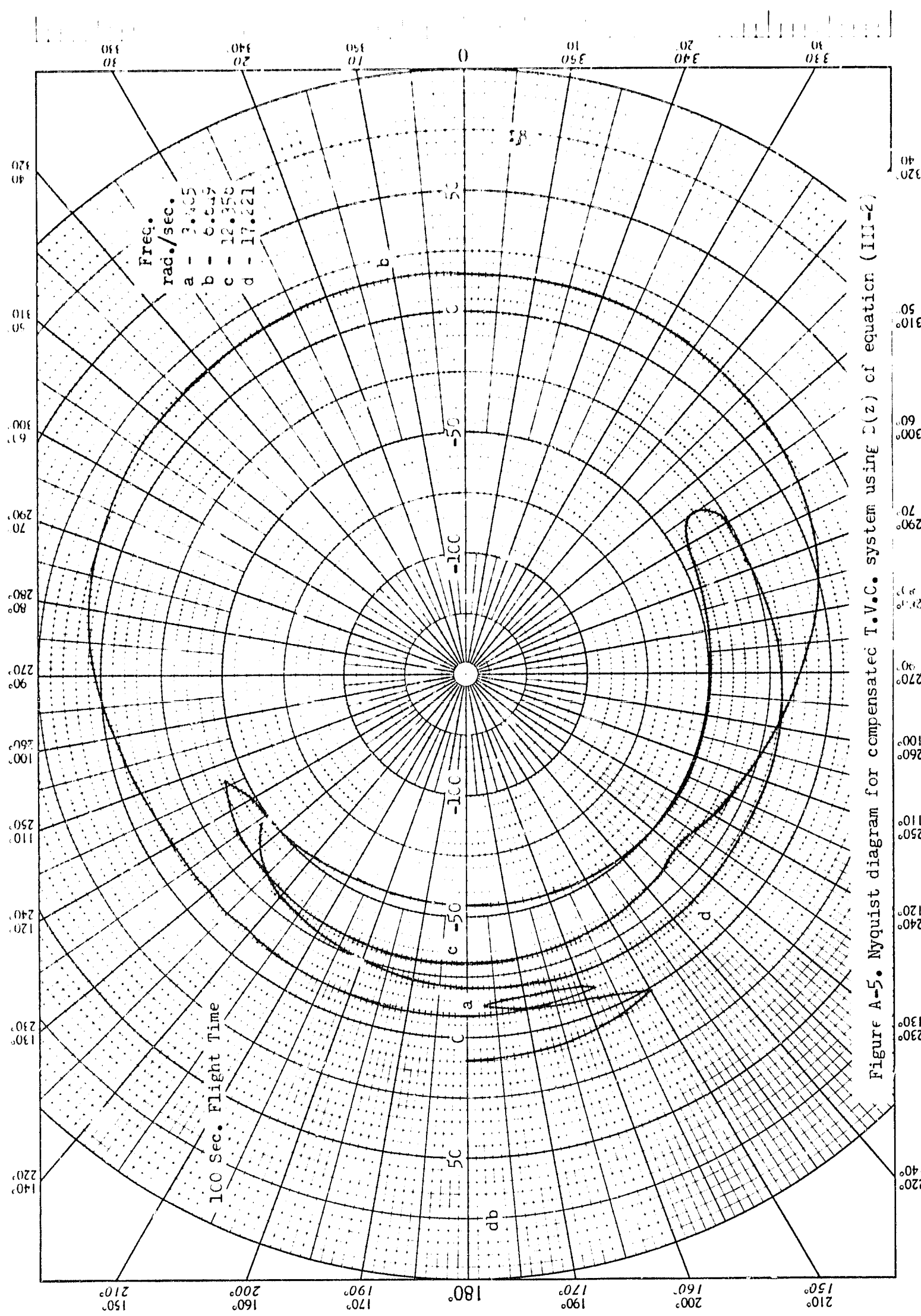
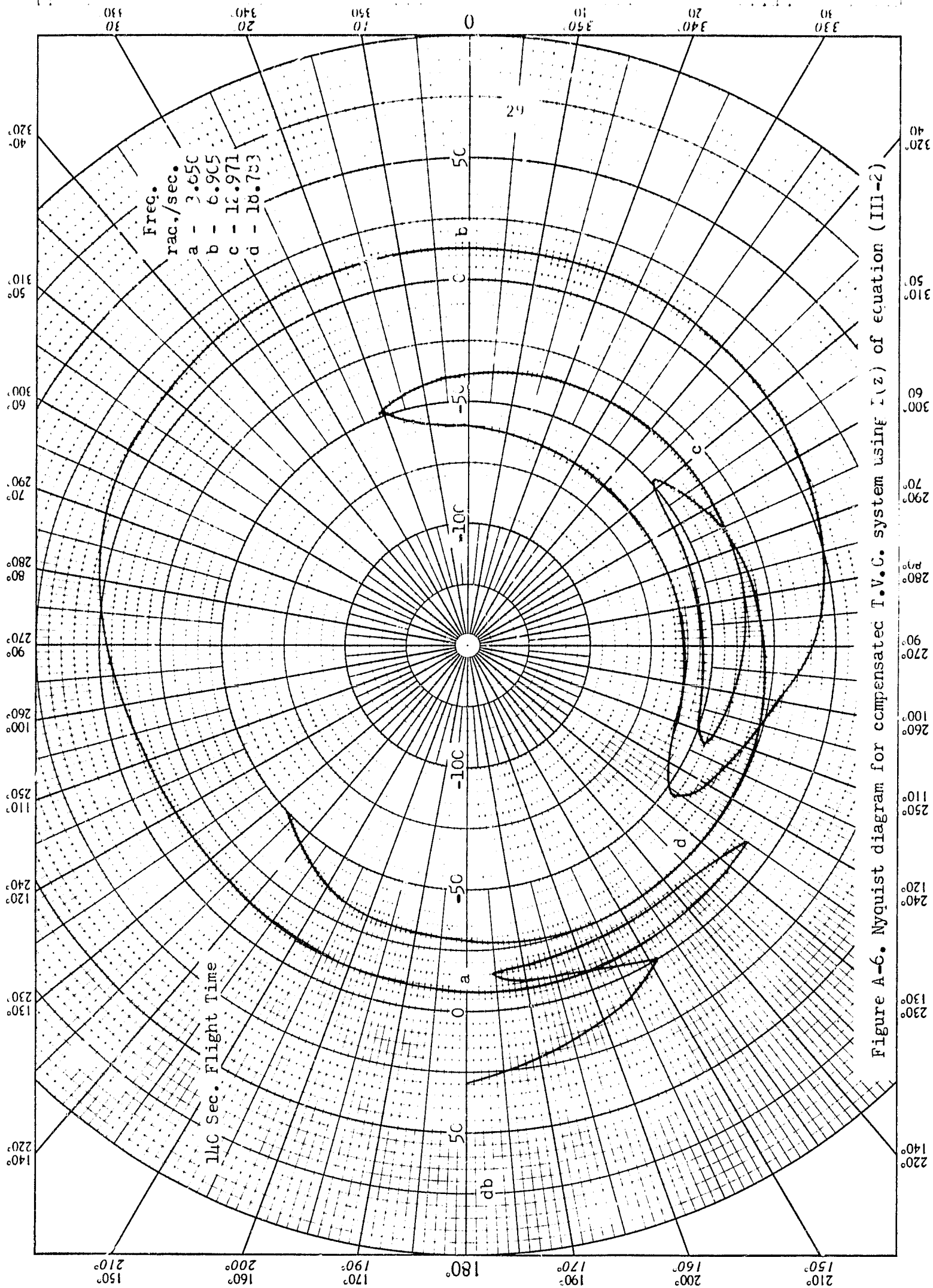
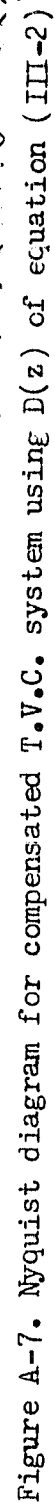


Figure A-5. Nyquist diagram for compensated T.V.C. system using $D(z)$ of equation (III-2)





APPENDIX B

A is a 12 x 12 matrix

b is a 12 x 1 matrix

c is a 12 x 1 matrix

$A(1,1) = -2(.434)(34.48)$
 $A(1,2) = -(34.48)^2$
 $A(2,1) = 1$
 $A(4,3) = 1$
 $A(6,5) = 1$
 $A(8,7) = 1$
 $A(10,9) = 1$
 $A(12,11) = 1$
 $A(3,2) = -C2(34.48)^2$
 $A(3,4) = -C1$
 $A(5,5) = -2ZETA(1)WB(1)$
 $A(5,6) = -WB(1)^2$
 $A(7,7) = -2ZETA(2)WB(2)$
 $A(7,8) = -WB(2)^2$
 $A(9,9) = -2ZETA(3)WB(3)$
 $A(9,10) = -WB(3)^2$
 $A(11,11) = 2ZETA(4)WB(4)$
 $A(11,12) = -WB(4)^2$
 $A(5,2) = RP\ YB(1)\ (34.48)^2/GM(1)$
 $A(7,2) = RP\ YB(2)\ (34.48)^2/GM(2)$
 $A(9,2) = RP\ YB(3)\ (34.48)^2/GM(3)$
 $A(11,2) = RP\ YB(4)\ (34.48)^2/GM(4)$

All the other elements are 0.

$b(1) = 1$

All the other elements are 0.

$C(4) = 1$
 $C(6) = YPD(1)$
 $C(8) = YPD(2)$
 $C(10) = YPD(3)$
 $C(12) = YPD(4)$

All the other elements are 0.

\$IBJOB
\$IBFTC

APPENDIX C

DIMENSION WB(4),ZETA(4),GM(4),YB(4),YPB(4),YPD(4),A(12,12),AN(12,
112,25),B(12,12),C(12,12),AI(12,12),AT(12,12),AIN(12,12,25),AANT(
2 12,12),PD(12),PHI(15,15),X(15),X1(15),CONTRL(2001)

D(Z) UNCOUPLED FROM DISCRETIZED SYSTEM TRANSITION MATRIX

QUANTIZATION EXCLUDED

COMMON/COM1/XP(3,2)
CALL TRAP

STORE INITIAL CONDITIONS ON STATES OF D(Z)

DO 666 I=1,3
666 XP(I,1)=0.0
GC=0.24
10 FORMAT(3(5X,E15.8))
11 FORMAT(4(5X,E15.8))
50 CONTINUE
READ(5,10)AL0,AL1,B0
READ(5,10)B1,B2,B3
READ(5,10)B4,B5,B6
READ(5,11)B7,C1,C2,FC
READ(5,11)ALCG, AIXX, AIE, T
READ(5,11)ASE, AK3, AK4, P
READ(5,11)AK7, AM, RP, V
READ(5,11)WB(1), WB(2), WB(3), WB(4)
READ(5,11)ZETA(1), ZETA(2), ZETA(3), ZETA(4)
READ(5,11)GM(1), GM(2), GM(3), GM(4)
READ(5,11)YB(1), YB(2), YB(3), YB(4)
READ(5,11)YPB(1), YPB(2), YPB(3), YPB(4)
READ(5,11)YPD(1), YPD(2), YPD(3), YPD(4)
DO 9 I=1,4
9 WB(I)=WB(I)*2.0*3.1415927
33 FORMAT(1H1,4X,10HINPUT DATA,/))
PRINT 33
PRINT 10, AL0, AL1, B0
PRINT 10, B1, B2, B3
PRINT 10, B4, B5, B6
PRINT 11, B7, C1, C2, FC
PRINT 11, ALCG, AIXX, AIE, T
PRINT 11, ASE, AK3, AK4, P
PRINT 11, AK7, AM, RP, V
PRINT 11, WB(1), WB(2), WB(3), WB(4)
PRINT 11, ZETA(1), ZETA(2), ZETA(3), ZETA(4)
PRINT 11, GM(1), GM(2), GM(3), GM(4)
PRINT 11, YB(1), YB(2), YB(3), YB(4)
PRINT 11, YPB(1), YPB(2), YPB(3), YPB(4)
PRINT 11, YPD(1), YPD(2), YPD(3), YPD(4)
ITEP=25
PRINT 17,ITER
17 FORMAT(1H1,2HN=I3,/))
PRINT 26,T

26 FORMAT(15X,12HFLIGHT TIME=F5.1,1X,7HSECONDS,/))

N IS THE ORDER OF THE SYSTEM A MATRIX

N=12

T IS THE SAMPLING PERIOD

T=.04

DEFINE ELEMENTS OF A MATRIX

```

DO 1 I=1,N
DO 1 J=1,N
1 A(I,J)=0.0
WA=34.48
ZETAA=.434
A(1,1)=-2.*ZETAA*WA
A(1,2)=-WA**2
DO 2 I=2,12,2
I1=I-1
2 A(I,I1)=1.0
A(3,2)=-C2*WA**2
A(3,4)=-C1
A(5,5)=-2.*ZETA(1)*WB(1)
A(5,6)=-WB(1)**2
A(7,7)=-2.*ZETA(2)*WB(2)
A(7,8)=-WB(2)**2
A(9,9)=-2.*ZETA(3)*WB(3)
A(9,10)=-WB(3)**2
A(11,11)=-2.*ZETA(4)*WB(4)
A(11,12)=-WB(4)**2
A(5,2)=RP*YB(1)*WA**2/GM(1)
A(7,2)=RP*YB(2)*WA**2/GM(2)
A(9,2)=RP*YB(3)*WA**2/GM(3)
A(11,2)=RP*YB(4)*WA**2/GM(4)

```

COMPUTE PHI(T), THE STATE TRANSITION MATRIX

ITER=NUMBER OF TERMS USED IN TAYLOR SERIES EXPANSION TO CALCULATE PHI(T)

```

DO 3 I=1,N
DO 3 J=1,N
AN(I,J,1)=A(I,J)*T
B(I,J)=A(I,J)*T
3 C(I,J)=A(I,J)*T
DO 4 LL=2,ITER
DO 5 I=1,N
DO 5 J=1,N
5 A(I,J)=C(I,J)/FLOAT(LL)
CALL MATMUL(A,B,N,C)
DO 6 I=1,N
DO 6 J=1,N
6 AN(I,J,LL)=C(I,J)

```

```

4  CONTINUE
   DO 7 I=1,N
   DO 7 J=1,N
7  AI(I,J)=0.0
   DO 8 I=1,N
8  AI(I,I)=1.0
   DO 12 I=1,N
   DO 12 J=1,N
12 AT(I,J)=AI(I,J)
   DO 15 I=1,N
   DO 15 J=1,N
   DO 15 LL=1,ITER
15 AT(I,J)=AT(I,J)+AN(I,J,LL)
   DO 13 I=1,N
   DO 13 J=1,N
13 PRINT 14,I,J,AT(I,J)
14 FORMAT(5X,3HAT(,I2,1H,,I2,2H)=E20.8)

```

C
C
C
COMPUTE THE INTEGRAL OF THE STATE TRANSITION MATRIX

```

   DO 18 I=1,N
   DO 18 J=1,N
   AIN(I,J,1)=AN(I,J,1)*T/2.
   AI(I,J)=AI(I,J)*T
   E(I,J)=AN(I,J,1)
13 C(I,J)=AIN(I,J,1)
   DO 20 LI=2,ITER
   LIL=LI+1
   DO 19 I=1,N
   DO 19 J=1,N
19 A(I,J)=C(I,J)/FLOAT(LIL)
   CALL MATMUL(A,B,N,C)
   DO 21 I=1,N
   DO 21 J=1,N
21 AIN(I,J,LI)=C(I,J)
20 CONTINUE
   DO 22 I=1,N
   DO 22 J=1,N
22 AANT(I,J)=AI(I,J)
   DO 23 I=1,N
   DO 23 J=1,N
   DO 23 LI=1,ITER
23 AANT(I,J)=AANT(I,J)+AIN(I,J,LI)
   DO 24 I=1,N
   DO 24 J=1,N
24 PRINT 25,I,J,AANT(I,J)
25 FORMAT(10X,5HAANT(,I2,1H,,I2,2H)=E20.8)

```

C
C
C
BD(I) IS THE PRODUCT OF THE INTEGRATED PHI(T) MATRIX AND THE B MATRIX

```

   DO 27 I=1,N
27 BD(I)=0.0
   DO 28 I=1,N
28 BD(I)=AANT(I,1)

```

PRINT 49

49 FORMAT(1H1)

35

DEFINE INITIAL CONDITIONS FOR THE SYSTEM STATES

DO 29 I=1,N

29 X(I)=0.0

X(4)=2.0

DEFINE A DO-LOOP TO UPDATE THE SYSTEM STATES AND PRINT THE OUTPUT STATES
DESIRED AT EACH SAMPLING INSTANT

DO 999 M=1,2001

KK=M-1

TIME=T*FLOAT(KK)

DO 31 I=1,N

31 X1(I)=0.0

DO 32 I=1,N

DO 32 K=1,N

32 X1(I)=X1(I)+AT(I,K)*X(K)

PHID=X(4)+YPD(1)*X(6)+YPD(2)*X(8)+YPD(3)*X(10)+YPD(4)*X(12)

CSRM = YPD(2)*X(8)

CONTRL(M) = PHID*GC

CALL DIGCOM(CONTRL(M),BETAC)

PRINT 35,PHID,CONTRL(M),BETAC,TIME,CSRM

DO 34 I=1,N

34 X1(I)=X1(I)+PD(I)*BETAC

DO 36 I=1,N

36 X(I)=X1(I)

999 CONTINUE

35 FORMAT(1 X,5HPHID=E16.8,3X,7HBETAIN=E16.8,3X,6HBETAC=E16.8,3X,

17HDEGREES,2X,5HTIME=F6.2,3X,4HWD2=E13.8)

GO TO 50

16 STOP

END

\$IBFTC DIGCOM

SUBROUTINE DIGCOM(UI,YP)

COMMON/COM1/XP(3,2)

A0=1.0

A1 = -2.563

A2 = 2.44704

A3 = -.877786

B1 = -2.38

B2 = 1.8860

B3 = -.497576

XP(1,2)=-B1*XP(1,1)-B2*XP(2,1)-B3*XP(3,1)+UI

XP(2,2)=XP(1,1)

XP(3,2)=XP(2,1)

YP=(A1-A0*B1)*XP(1,1)+(A2-A0*B2)*XP(2,1)

1 +(A3-A0*B3)*XP(3,1)+UI*A0

DO 1 I=1,3

1 XP(I,1)=XP(I,2)

RETURN

END

\$IBFTC MATMUL

```

SUBROUTINE MATMUL(A,B,N,C)
DIMENSION A(N,N),B(N,N),C(N,N)
CALCULATE C(I,J) COEFFICIENTS
DO 3 I=1,N
DO 3 J=1,N
C(I,J)=0.0
DO 4 K=1,N
4 C(I,J)=C(I,J)+A(I,K)*B(K,J)
3 CONTINUE
RETURN
END

```

\$IDMAP TRAP

```

ENTRY TRAP
AXT **,4
TRA **
SXA TRAP-1,4
CLA 8
STA RESET+1
CLA FIX
TSX S.SCCR,4
STO 8
TRA TRAP-1

```

RESET

```

PLT 0
TRA **
CLA 0
ARS 20
LPT
TRA *+2
TRA* RESET+1
SXA OUT,4
TSX S.WRIT,4
PZE 3,MES

```

OUT

```

AXT **,4
ZAC
LES 35
TRA* 0

```

FIX

```

TRA RESET

```

MES

```

PCI 3, **** UNDERFLOW
END

```

\$ENTRY

0.10000000E 01	0.25000000E 00	0.10000000E 01	
0.29200000E 00	0.12400000E-01	0.40200000E-03	
0.56400000E-05	0.49800000E-07	0.16000000E-09	
0.17600000E-12	-0.24435000E-01	0.91660000E 00	0.34970928E 08
0.27851710E 02	0.85009766E 09	0.18259200E 06	0.40000000E 02
0.51590400E 05	0.25475000E 00	0.20545000E 00	0.10000000E 01
0.20889000E 01	0.23766675E 07	0.27976742E 08	0.12699737E 03
0.10112000E 01	0.18092000E 01	0.25440000E 01	0.33347000E 01
0.50000000E-02	0.50000000E-02	0.50000000E-02	0.50000000E-02
0.12852000E 05	0.50590000E 04	0.32280000E 04	0.81260000E 04
0.98960000E-01	-0.68610000E-01	0.42170000E-01	-0.45720000E-01
-0.49000000E-02	0.33000000E-02	-0.30000000E-02	0.36500000E-02
0.15160000E-01	0.36000000E-02	-0.88300000E-02	-0.14460000E-0

\$IDSYS