General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

May, 1968

Final Report

Contract NAS8-20172

Modification Number 4

BUREAU OF ENGINEERING RESEARCH

N69-24380
(ACCESSION NUMBER) (PAGES) (THRU) 1 (CODE) 07 (CATEGORY)
CR-98468 (NASA CR OR TMX OR AD NUMBER)
FACILITY FORM 602

FEASIBILITY STUDY OF A POLARIZATION-CONTROLLABLE ANTENNA

by

Dr. Harold Mott
Professor of Electrical Engineering
Project Director

and

David N. McQuiddy, Jr.
Research Associate

Prepared for

George C. Marshall Space Flight Center
Huntsville, Alabama

RECEIVED FEB 1969 NASA STI FACILITY INPUT BRANCH

BER

COLLEGE OF ENGINEERING

UNIVERSITY OF ALABAMA

UNIVERSITY ALABAMA

SQT-55541

University of Alabama
Bureau of Engineering Research
University, Alabama

FEASIBILITY STUDY OF A POLARIZATION-CONTROLLABLE ANTENNA

Final Report
Contract NAS8-20172 Modification Number 4

by

Dr. Harold Mott
Professor of Electrical Engineering
Project Director

and

David N. McQuiddy, Jr.
Research Associate

Prepared for

George C. Marshall Space Flight Center
Huntsville, Alabama

TABLE OF CONTENTS

TABLES AND ILLUSTRATIONS .....

ACKNOWLEDGEMENT .....

ABSTRACT .....

Chapter I INTRODUCTION 1

Polarization Characteristics 1
Polarization Matching 2
Possible Applications 3
Essential Components for Polorization Control .. 4

Chapter II THEORETICAL DEVELOPMENT OF TECHNIQUE FOR POLARIZATION ANALYSIS 7

Mathematical Representation 7
Relation Between Polarization Parameters 10
Contours of Constant Amplitude and Phase 13
Antenna Complex Vector Length 18
Polarization Mismatch Loss 19
Contours of Constant Loss 23

Chapter III AN ANALYSIS OF AN ANTENNA SYSTEM 31

Basic System Elements 31
A Mathematical Model 33
Equations Governing System Operation 36
Effect of Phase Shifter Loss 43
Experimental Study of System Operation 48

Chapter IV THE CONTROL PROCESS 58

An Examination of the System Equations 58
System Parameters for Commonly Used Antenna Polarization 60
The Search Stage of Controller Operation 65
Lock-On and Track 67
Summary 70

Chapter V	A DESCRIPTION OF THE SYSTEM COMPONENTS	73
	The Radiating Elements	73
	The Digital Phase Shifters	75
	The Digital Controller	77
	Phase Measurement Equipment	79
Chapter VI	CONCLUSIONS AND RECOMMENDATIONS	84
	REFERENCES	87
Appendix A	A SIMPLE WAVEGUIDE SYSTEM FOR RADIATING ELLIPTICALLY-POLARIZED WAVES	89
	REFERENCES	100
Appendix B	FLOW CHART FOR DIGITAL CONTROLLER LOGIC	101
Appendix C	CROSS-POLARIZED RECEPTION	112
	BIBLIOGRAPHY	114

TABLES AND ILLUSTRATIONS

Table		Page
2-1	Polarization Characteristics in Terms of q	11
4-1	Parameters for Common Polarization States	60
4-2	Normalized System Output	61
4-3	Range of Values for System Parameters	62
4-4	Output Phase Behavior	69

Figure		
1	Smith Chart with various polarization states shown	12
2	Contours of constant linear polarization ratio and constant phase	17
3	Matched and crossed polarization states for a transmit-receive antenna configuration	22
4	Loss contours for circular polarization	27
5	Loss contours for linear horizontal polarization	28
6	Loss contours for an antenna with an axial ratio of 3 and a tilt angle of 0 degrees	29
7	System configuration in block diagram form	32
8	Signal flow graph of circuit shown in Fig. 7	34
9	Polyrod antenna dimensions	49
10	Photograph of the experimental antenna system	51
11	Photograph of polarization measuring equipment	53
12	Smith Chart representation of polarization states when system phase shifters are of the 5 bit digital design	59

13	Loss contours for linear horizontal, linear at 45 degrees tilt angle, and circular polarization.....	64
14	Crosscorrelation Phase measuring device.....	81
A-1	Waveguide system component arrangement.....	90
A-2	Coordinate representation of waveguide system.....	92
A-3	Polarization contour for waveguide system.....	96
A-4	Polarization contour for "variable-wave" plate system.....	98

ACKNOWLEDGEMENT

This report has been prepared for the Instrumentation Branch, Astrionics Division, George C. Marshall Space Flight Center, Huntsville, Alabama, under Contract NAS8-20172, Modification Number 4.

The authors wish to express their appreciation to Mr. Donald Stone and Mr. Paul Swindall of the Astrionics Division for their helpful advice and discussions. Appreciation is also expressed to Mr. Jack J. Flood of the University of Alabama for his competent assistance in the fabrication of the waveguide devices.

ABSTRACT

This study describes the characteristics of a radiating structure and feed system. The polarization properties of the field radiated by this structure can be controlled by electronic means. An analysis is presented that develops the polarization properties in terms of the controlled parameters. Requirements on the operating specifications of the system components are derived. A control process required to allow adaptive mode operation is described.

A second system is described where two angles of rotation or one rotation angle and a variable wave plate in a waveguide establish any desired polarization on the axis of a circularly symmetric radiator.

CHAPTER I

INTRODUCTION

This report describes an analytical study, the objectives of which are to evaluate the major problems associated with developing a polarization-controllable antenna. Polarization-controllable implies a radiating structure consisting of components that allow electronic control of the polarization characteristics of its radiated field. The study is conducted both from an analytical and an experimental standpoint. The analytical study defines the required system components and operating parameters while the experimental work verifies certain aspects of the analytical study. The study is concerned, primarily, with defining the system configuration required to achieve polarization control and not with determining possible applications for the system. The feasibility of achieving polarization control is found to be based on sound theoretical engineering principles, but unequivocal proof of feasibility must also be based on the intended system application.

Polarization Characteristics

The trace of the electric field vector of an electromagnetic wave defines its polarization. Polarization is linear, circular, or elliptic, where the first two are special cases of the third. If a wave is elliptically polarized, the electric field vector, which lies in a plane

perpendicular to the direction of propagation, traces out an elliptical helix in space. If the "helix" progresses in the direction of a right-hand screw thread, the sense of polarization is right-elliptical. The opposite direction of rotation characterizes left-elliptical polarization. The ratio of the major to the minor axis of the elliptical cross section of the "helix" defines the axial ratio, and the orientation of the major axis with respect to horizonal defines the polarization orientation angle (tilt angle τ). These three, sense of rotation, axial ratio, and polarization orientation angle, are the polarization characteristics of an electromagnetic wave.

Polarization Matching

In order for an antenna to receive maximum possible power from an incident electromagnetic wave, its polarization characteristics in the direction of incidence must be identical to those of the incident wave. Obviously, it is not possible to perform this polarization matching for all incident waves if the receiving antenna has fixed polarization characteristics. The polarization of electromagnetic waves received by an antenna may be fixed, variable, or random. There are several reasons for this. First, the incident wave may be produced by several different transmitting antennas having different polarization characteristics. Second, a transmitting antenna designed for a specified polarization generally will maintain that polarization only over a relatively narrow beamwidth, even for low gain antennas. If the transmitting antenna changes its aspect angle during transmission, such as would occur on a missile in flight, there will result an incident wave

of variable polarization. Third, it is well known that transmission through the ionsphere or reflections from it causes polarization changes to the transmitted wave which cannot, in general, be calculated in advance. If the transmitting or receiving antenna moves with respect to the ionsphere, there would result an incident wave with changing polarization.

The effect of polarization mismatch can be described as a decrease in the received signal-to-noise ratio equal to the mismatch loss. Typical examples of mismatch loss follow: (1) The mismatch loss, defined as the ratio of received power to available power expressed in db, is -3 db if the receiving antenna is linearly polarized and the incident wave is circularly polarized, or vice versa. (2) If the receiving antenna has a linear polarization characteristic with any tilt angle and the incident wave linear polarization with a perpendicular tilt angle, or if one is left-circularly polarized and the other is right-circularly polarized the loss is infinite and no signal will be received.

Possible Applications for a Polarization-Controllable Antenna

The applications for polarization-controllable antennas can be divided into two major areas. The first area includes those applications concerned with extracting a maximum possible power from an incident wave. The requirement for a two-way communications circuit to be polarization matched is met, provided one of the two terminals consists of a polarization-controllable antenna with the control information derived from the polarization characteristics of the incident wave. In this application the polarization characteristics are matched

at both terminals resulting in a best system signal-to-noise ratio (in the polarization sense). An antenna system meeting these needs is termed a polarization adaptive antenna. The second area of application consists of those systems deriving information from the polarization characteristics of the incident wave where the wave can be produced by a scattering action or an independent source. Grouped in this area would be polarization agility for countermeasure systems, a test set for automatic recording of the polarization characteristics of antennas, and radar system applications when additional target information can be obtained from the polarization of the scattered field.

Essential Components for Polarization Control

The foregoing discussion has been based on the assumption that the antenna polarization characteristics can be changed at will. The radiating system required to provide this capability can be determined by recalling that an elliptically polarized wave can be expressed as a sum of two orthogonal linearly polarized waves with a certain amplitude and phase relation [1]. This fact suggests a structure consisting of two radiating elements orthogonal in space each having a linear polarization characteristic. The energy collected by these two elements must be brought into time phase and then combined in an impedance matched combiner (four port variable coupler). Since an elliptically polarized wave can also be expressed as the sum of two circularly polarized waves of opposite sense with a certain amplitude and phase relationship [2] it is apparent that the two radiating elements can have a circular polarization characteristic with opposite rotation sense.

The polarization-controllable antenna system requires a phase shifter, a variable power combiner, and a pair of cross polarized (linear, circular) radiating elements. It is not necessary to use two physically different radiating elements. One may use a single square horn fed by a square waveguide. The waveguide is fed by two sources so that it propagates simultaneously two orthogonal modes such as the TE_{01} and TE_{10} modes. The same can be said for a circular guide supporting two orthogonal TE_{11} modes feeding a circular horn or polyrod radiating element. Other types of cross polarized radiating elements may be used, with or without reflectors and with phase centers coincident or separated. Arrays of elements may be used. The requirement is that one-half of the elements be capable of radiating a field cross polarized with respect to the polarization of the other half. In this case each array is treated as one of the two radiating elements.

For most applications of the polarization-controllable antenna it is necessary that the directional gain of the two radiating elements be identical at all expected angles of incidence. This is normally true for identical elements at boresight; however, at an angle off boresight this is not true unless the elements are identical and have a conical radiation pattern. Since most practical applications require operation of an antenna at an incident angle near boresight in order to utilize the directional gain properties of the antenna, no additional requirement in this respect is imposed on the system when implementing the polarization-controllable feature. Naturally small differences in the element gains will have little effect on the system operation. The effect of unequal element gain is to introduce a change in the polarization characteristics of the incident wave as detected by the

polarization controllable antenna system. This is more detrimental to polarization-adaptive antenna applications than to the other system applications.

A brief description of the properties of a polarization-controllable antenna has been considered in introductory form. The following chapters are concerned with a comprehensive analysis of the system components required to provide the polarization-controllable capability. Chapter II develops the fundamental concepts required for a mathematical description of polarization. The development is extended to include a graphical procedure for evaluating the polarization mismatch loss between elliptically polarized antennas. Chapter III establishes the necessary component configuration and develops a mathematical model from which are determined the equations governing system operation. Chapter IV defines the control process required to enable adaptive mode operation. A description of the hardware required for system implementation is contained in Chapter V. The final chapter discusses system applications and the conclusions and recommendations resulting from this study.

CHAPTER II

THEORETICAL DEVELOPMENT OF TECHNIQUES FOR POLARIZATION ANALYSIS

The ability to handle problems concerned with the polarization aspects of electromagnetic fields is limited only by the mathematical model chosen to represent its polarization characteristics. A description similar to that considered by Rumsey [3] is developed in this chapter. The procedure for graphically displaying the information contained in the mathematical expressions is included. The advantages of this method for describing polarization characteristics will become evident in this and following chapters.

Mathematical Representation of Plane Wave Polarization

Any elliptically polarized plane wave traveling in the positive z direction can be expressed mathematically in terms of two orthogonal linear polarized components as

$$\underline{E}^i = E_o(a\,\underline{u}_x + b\,e^{j\phi_1}\,\underline{u}_y) \qquad (2\text{-}1)$$

where a and b are positive real quantities and

$$a^2 + b^2 = 1 \ .$$

The instantaneous field is found from (2-1) by

$$\underline{\mathcal{E}}^{i} = \sqrt{2}\ \mathrm{Re}\,(\underline{E}^{i}\, e^{j(\omega t - kz)})\,. \tag{2-2}$$

It is advantageous to define left and right rotating vectors of constant length as

$$\underline{\omega}_{\ell} = \underline{u}_x + j\,\underline{u}_y \tag{2-3a}$$

and

$$\underline{\omega}_{r} = \underline{u}_x - j\,\underline{u}_y \tag{2-3b}$$

where $\underline{\omega}_{\ell}(\underline{\omega}_{r})$, if multiplied by $e^{j\omega t}$ and the real part taken, represents physically a left (right) hand time rotating vector of unit amplitude. It is possible to express an elliptically polarized plane wave traveling in the z direction in terms of these rotating components. This expression is

$$\underline{E}^{i} = E_o(L\,\underline{\omega}_{\ell} + R e^{j\theta}\,\underline{\omega}_{r})\,, \tag{2-4}$$

where L and R are positive real and

$$L^2 + R^2 = 1/2\,.$$

The quantity $E_o(x,y,z)$ of equation (2-1) and (2-4) is determined by the power density of the plane wave. Since equation (2-1) and (2-4) can represent the same plane wave, they can be equated to yield the relation between the linear polarization parameters a, b, and ϕ_1 and the circular polarization parameters L, R, and θ. Performing this operation yields

$$a\,\underline{u}_x + b e^{j\phi_1}\,\underline{u}_y = L\,\underline{\omega}_{\ell} + R e^{j\theta}\,\underline{\omega}_{r}\,. \tag{2-5}$$

Using Eqn. (2-3) and equating coefficients of the x-directed and y-directed unit vectors results in the following equations

$$a = L + Re^{j\theta}$$

and

$$be^{j\phi_1} = j(L - Re^{j\theta}) \quad .$$

These expressions can be manipulated into the following form,

$$2Re^{j\theta} = a + jbe^{j\phi_1}$$

and

$$2L = a - jbe^{j\phi_1} \quad .$$

Dividing the second expression by the first produces the desired relation between the linear and circular polarization parameters. That is

$$\frac{Le^{-j\theta}}{R} = \frac{a - jbe^{j\phi_1}}{a + jbe^{j\phi_1}} \quad . \qquad (2\text{-}6)$$

The x-z coordinate plane will be arbitrarily chosen to lie in the horizontal plane throughout this study. Thus, it is seen from Eqns. (2-1) and (2-4), that the ratio of the magnitude of the vertical linear polarization component to the horizontal linear component is given by b/a, and that ϕ_1 is the phase angle by which the vertical component leads the horizontal component. Likewise, the ratio of the magnitude of the left circular polarization component to the right circular component is L/R, and θ denotes the phase angle by which the right circular component leads the left circular component. Time reference has been

chosen such that the horizontal (x-directed) linear component is a maximum at t = o and z = o, and such that the left circular component is x-directed at t = o.

Relation Between Polarization Parameters

The definitions presented below for p and q follow the notation used by Rumsey [3] in his paper on polarization. They are

$$q = \frac{L\, e^{-j\theta}}{R} \tag{2-7a}$$

and

$$p = j\,\frac{b}{a}\, e^{j\phi_1} \quad . \tag{2-7b}$$

Equation (2-6) can be simplified by substituting Eqn. (2-7) for the appropriate terms. The result is

$$q = \frac{1 - p}{1 + p} \quad . \tag{2-8}$$

Table 2-1 contains the relation between the polarization parameter q and the polarization characteristics. This relation results from the definition Eqn. (2-7a).

The transformation from p to q, as given by Eqn. (2-8), is identical to the transformation from current reflection coefficient to normalized impedance that occurs in transmission line theory. The simplest type of impedance transformation is that due to a uniform length of transmission line, which causes a phase change at constant amplitude of the reflection coefficient. The simplest type of polarization transformation, as described by linear and circular components, is that due to

rotation of the antenna producing the elliptically polarized wave, producing a phase change ($\Delta\theta$) at constant amplitude (L/R) of q. It, therefore, follows that q should be considered the analogue of current reflection coefficient. This leads naturally to the use of the Smith Chart as a means of graphical analysis for polarization problems [3].

Table 2-1

Polarization Characteristics in Terms of q

Polarization Characteristics		Equivalent	Parameters	
Item	Range		Item	Range
Tilt angle τ	$-\pi/2 \leq \tau \leq \pi/2$	$\longrightarrow$	$\theta/2$	$-\pi < \theta < \pi$
Axial ratio	$1 \leq AR \leq \infty$	$\longrightarrow$	$\left\|\frac{\|q\| + 1}{\|q\| - 1}\right\|$	$0 \leq \|q\| \leq \infty$
Rotation Sense	Left	$\longrightarrow$	$\|q\| > 1$	—
	Right	$\longrightarrow$	$\|q\| < 1$	—

Figure 1 shows the orientation and shape of the polarization ellipse represented by its individual q point (center point of the ellipse) on the Smith Chart. Each point on the chart represents two polarization states with the same axial ratio and tilt angle but opposite rotation sense. The same representation is required in transmission line theory when negative resistance devices are present [4]. For polarization representation, this ambiguity is resolved by recognizing that right elliptic rotation (RER) requires the real part of p to be positive, and left elliptic rotation (LER) results when the real part of p is negative (cf. 2-8). The polarization ellipse shown in Fig. 1 is for a wave traveling in the +z direction and is thus receding

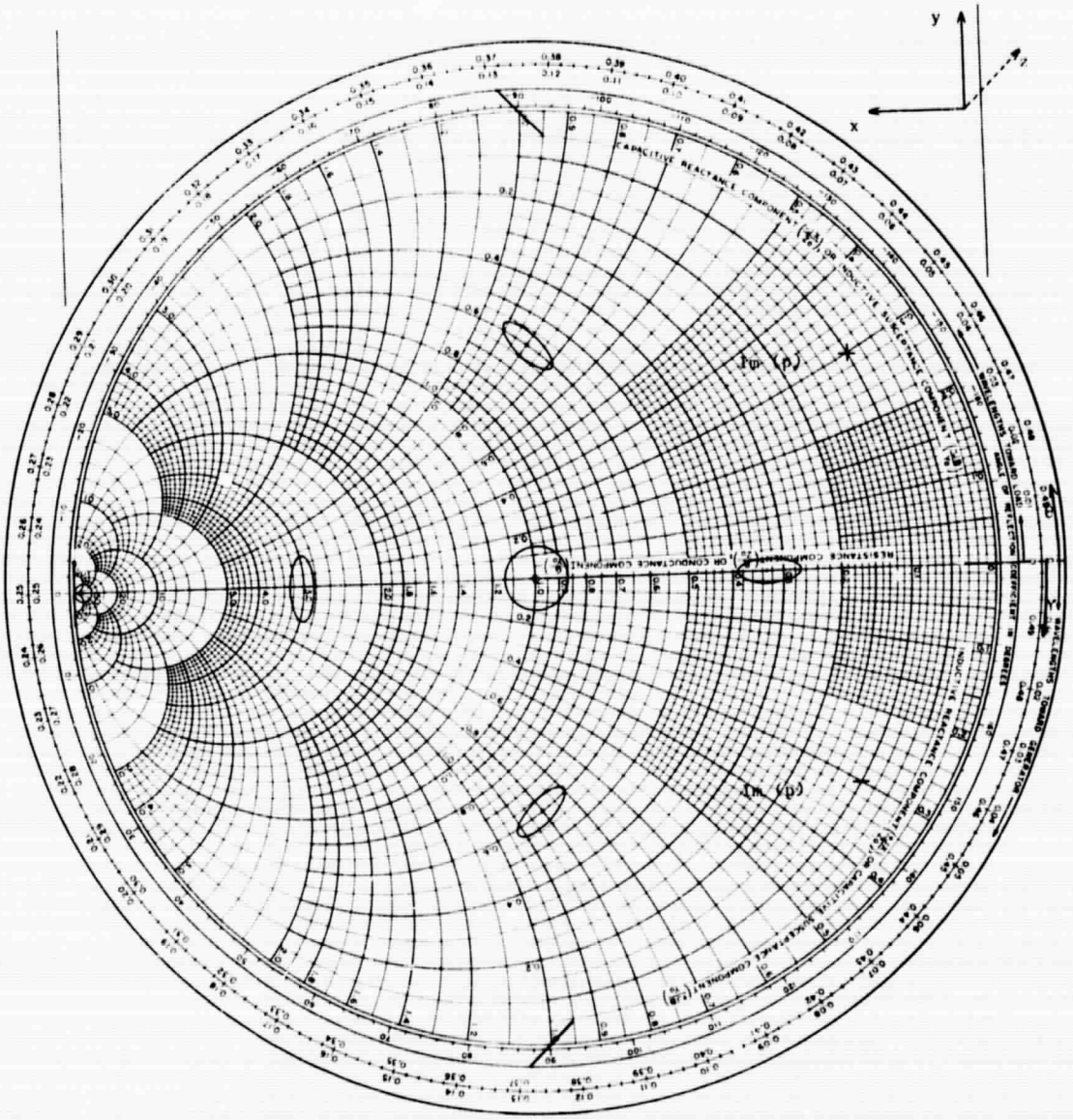


Figure 1. Smith Chart with various polarization states shown.

from the observer. It follows from the definition of q (reflection coefficient) that radial lines (θ = const.) on the Smith Chart are the loci of polarization states with the same tilt angle, and in fact the tilt angle is equal to $\theta/2$. Constant axial ratio contours are identical to the circles of constant SWR on a Smith Chart impedance plot.

Contours of Constant Amplitude and Phase

Certain operating characteristics of the polarization-controllable antenna system can be readily determined provided a plot of contours of constant linear polarization ratio, $|p|$, and phase angle, ϕ_1, is available. These contours are derived by solving Eqn. (2-8) for p (merely interchange p and q) and substituting Eqn. (2-7) for p and q. The result is

$$\frac{b}{a} e^{j(\phi_1 + \pi/2)} = \frac{1 - \frac{L}{R} e^{-j\theta}}{1 + \frac{L}{R} e^{-j\theta}} .$$

In order to simplify the notation the following changes of variable are performed:

$$K = b/a$$

$$Q = L/R$$

$$\gamma = -\theta$$

so that

$$K e^{j(\pi/2 + \phi_1)} = \frac{1 - Q e^{j\gamma}}{1 + Q e^{j\gamma}} . \qquad (2\text{-}9)$$

Multiplying Eqn. (2-9) by its complex conjugate yields

$$K^2 = \frac{1 - 2Q\cos\Upsilon + Q^2}{1 + 2Q\cos\Upsilon + Q^2}$$

or

$$(1 - K^2)Q^2 - 2(1 + K^2)Q\cos\Upsilon + (1 - K^2) = 0$$

resulting in

$$Q^2 - 2(\cos\Upsilon)\left(\frac{1 + K^2}{1 - K^2}\right)Q + \left(\frac{1 + K^2}{1 - K^2}\right)^2 = \left(\frac{1 + K^2}{1 - K^2}\right)^2 - 1 \ . \tag{2-10}$$

Recall that the equation in ρlane polar coordinates (Q,Υ) of a circle of radius r with the center at (ℓ, α) is given by

$$Q^2 - 2[\cos(\Upsilon - \alpha)]\ell Q + \ell^2 = r^2 \ .$$

Hence, it is seen that Eqn. (2-10) represents a circle of radius

$$r = \left[\left(\frac{1 + K^2}{1 - K^2}\right)^2 - 1\right]^{1/2} \tag{2-11}$$

with a center at

$$Q = \frac{1 + K^2}{1 - K^2}$$

$$\Upsilon = 0 \ . \tag{2-12}$$

Contours of constant linear polarization ratio (K) can be plotted on the Smith Chart with the aid of Eqns. (2-11) and (2-12).

To determine the contours of constant phase add to Eqn. (2-9) the conjugate of Eqn. (2-9). This yields

$$2K \cos(\pi/2 + \phi_1) = \frac{2(1 - Q^2)}{1 + 2Q \cos \Upsilon + Q^2} \quad . \tag{2-13}$$

The next step requires that the conjugate of Eqn. (2-9) be substracted from Eqn. (2-9) to obtain

$$2K \sin(\pi/2 + \phi_1) = \frac{-4Q \sin \Upsilon}{1 + 2Q \cos \Upsilon + Q^2} \quad . \tag{2-14}$$

If Eqn. (2-14) is divided by Eqn. (2-13), the result will be

$$\tan(\pi/2 + \phi_1) = \frac{-2Q \sin \Upsilon}{1 - Q^2} \quad .$$

Let

$$C = \cot \phi_1$$

then

$$C = \frac{2Q \sin \Upsilon}{1 - Q^2} \quad . \tag{2-15}$$

Equation (2-15) can be manipulated into the following form

$$Q^2 - \frac{2Q}{C} \cos(\Upsilon + \pi/2) + \frac{1}{C^2} = 1 + \frac{1}{C^2} \quad . \tag{2-16}$$

Equation (2-16) represents a circle of radius

$$r = \left| \sec \phi_1 \right|$$

with a center at

$$Q = \tan \phi_1$$
$$\gamma = -\pi/2 \qquad (2\text{-}17)$$

where ϕ_1 must be restricted to the range

$$\pi < \phi_1 < 2\pi \ .$$

Equation (2-17) is valid for right elliptical rotation, i.e., for $Q < 1$. For left elliptical rotation ($Q > 1$), it is necessary to replace Q by

$$1/Q$$

in Eqn. (2-15). The same manipulations used above for the case $Q < 1$, can be used to obtain the following contour parameters,

$$r = \left|\sec \phi_1\right|$$
$$Q = \tan \phi_1 \qquad (2\text{-}18)$$
$$\gamma = \pi/2$$

where ϕ_1 is restricted to the range

$$0 < \phi_1 < \pi \ .$$

It should be noted that a negative Q, as given by Eqns. (2-17) and (2-18), increases γ by π radians. Figure 2 shows a plot of selected values of constant polarization ratio and phase superimposed on a Smith Chart. The contours representing polarization states with constant phase are the set of circles passing through the points $q = \pm 1$ (the short and open-circuit points); the contours for constant polarization ratio are orthogonal to this set. The values for the constant phase contours are displayed along the vertical axis, with those values on the left

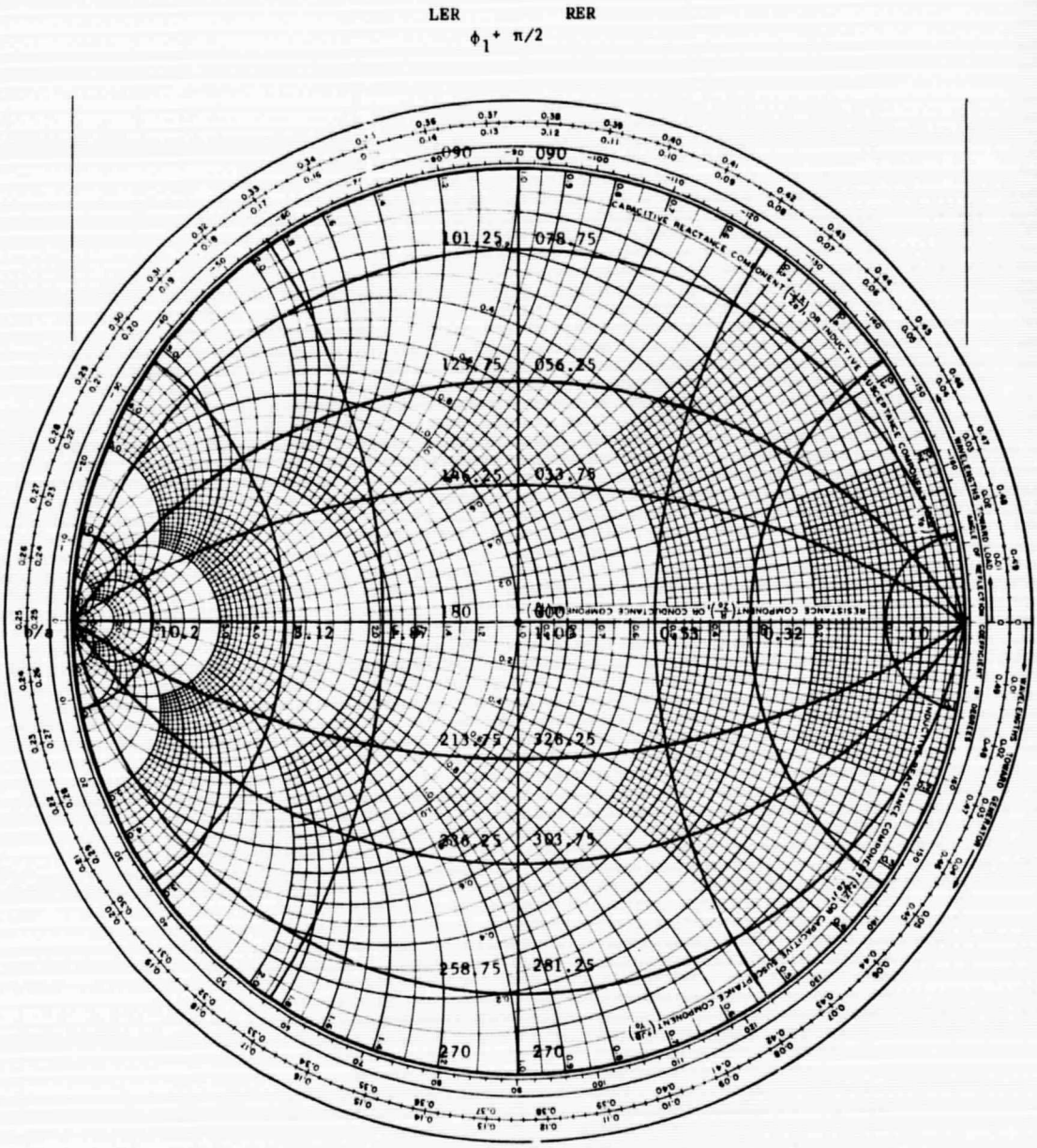


Figure 2. Contours of constant linear polarization ratio and constant phase.

representing LER states and values on the right representing RER states. The values for the constant amplitude contours are displayed along the horizontal axis. Note that the phase values are plotted for constant $\phi_1 + \pi/2$ rather than ϕ_1.

Antenna Complex Vector Length

A generalized vector effective length [5] may be defined for any antenna by the equation,

$$\underline{E}^t = \frac{j\eta I\, e^{-jkr}}{2\lambda r}\ \underline{h}\ (\theta, \phi) \qquad (2\text{-}19)$$

where $\underline{E}^t$ is the radiation field when the antenna transmits,

$\underline{h}(\theta,\phi)$ is the vector effective length,

η is the intrinsic impedance of free space,

I is the antenna terminal current,

k is the free space propagation constant, and

λ is the wavelength.

Since Eqn. (2-19) is applicable to any antenna having an elliptical polarization characteristic, it is necessary that $\underline{h}(\theta,\phi)$ be a complex vector. It may be shown by reciprocity [6] that the open-circuit terminal voltage of the same antenna when used for reception is

$$V = \underline{h} \cdot \underline{E}^i \qquad (2\text{-}20)$$

where $\underline{E}^i$ is the electric field incident on the antenna. In Eqn. (2-20) it is necessary to work only with two components of $\underline{h}$ and $\underline{E}^i$ if the coordinate system is set up properly, since the radiation field has

no radial component. Then

$$V = h_x E_x^i + h_y E_y^i \tag{2-21}$$

where the subscripts refer to the x and y components.

Polarization Mismatch Loss

It is possible to define a polarization mismatch loss [7] between any two antennas operating in a transmit-receive configuration (two-way circuit). This loss is expressed mathematically as

$$\rho = \frac{\left| \underline{E}^i \cdot \underline{h} \right|^2}{\left| \underline{E}^i \right|^2 \left| \underline{h} \right|^2} \tag{2-22}$$

where ρ ranges from zero (infinite loss) to unity (matched). Equation (2-1) can be written, using Eqn. (2-7b), as

$$\underline{E}^i = E_o^1 \left(\underline{u}_x + e^{-j\,\pi/2}\, p_1\, \underline{u}_y \right) . \tag{2-23}$$

The subscript 1 designates antenna 1 with polarization parameter p_1 as the source of the incident field. In a similar manner the vector length of the receiving antenna, antenna 2, can be expressed in terms of p_2 by expanding Eqn. (2-19) into the form of Eqn. (2-1). This results in

$$\underline{h} = h_o \left(\underline{u}_x + e^{-j\,\pi/2}\, p_2\, \underline{u}_{y-} \right) \tag{2-24}$$

where $\underline{u}_{y-}$ is a unit vector in the minus y direction and h_o is some constant determined by the characteristics of antenna 2. The minus y-directed unit vector is required since the z axes of the transmitting and receiving antennas point toward each other along the path of

propagation. For the sake of simplicity, the x axes of the two antennas have been chosen parallel, pointing in the same direction; therefore, the y axes are parallel, pointing in the opposite directions. This requirement on the coordinate axes is consistent with the description of the polarization characteristics of an electromagnetic field as viewed from the source (wave receding). It should be emphasized that both p_1 and p_2 are functions of the polar angles measured between the respective antenna's boresight and the path of propagation (z-axis) or any other convenient reference.

Using Eqns. (2-23) and (2-24) to evaluate the terms in Eqn. (2-22) yields,

$$\underline{E}^i \cdot \underline{h} = E_o^1 \, h_o \, (1 + p_1 p_2)$$

and

$$\left|\underline{E}^i \cdot \underline{h}\right|^2 = \left|E_o^1 \, h_o\right|^2 (1 + p_1 p_2)(1 + p_1^* p_2^*) ,$$

also

$$\left|\underline{E}^i\right|^2 = \left|E_o^1\right|^2 (1 + p_1 p_1^*)$$

and

$$\left|\underline{h}\right|^2 = \left|h_o\right|^2 (1 + p_2 p_2^*)$$

so that,

$$\rho = \frac{(1 + p_1 p_2)(1 + p_1^* p_2^*)}{(1 + p_1 p_1^*)(1 + p_2 p_2^*)} \quad . \tag{2-25}$$

It is of interest to examine the significance of Eqn. (2-25) for the two extreme values of ρ before proceeding to the general case. Two antennas in a transmit-receive configuration are said to be cross-polarized if electromagnetic energy launched by the transmitting antenna is present at the receiving antenna location, but none of the available

energy is collected by the receiving antenna. Antennas 1 and 2, as described above by their p's, are cross-polarized when $\rho = o$. Let $\rho = o$ in Eqn. (2-25) to obtain

$$p_1 = -\frac{1}{p_{2C}} \quad . \tag{2-26}$$

This is the relation between the p's of two cross-polarized antennas.

The two antennas are polarization matched when their p's are such that $\rho = 1$. This requires that

$$(1 + p_1 p_1^*)(1 + p_2 p_2^*) = (1 + p_1 p_2)(1 + p_1^* p_2^*)$$

which simplifies to

$$p_1 p_1^* + p_2 p_2^* = p_1 p_2 + p_1^* p_2^*$$

or

$$(p_2^* - p_1)(p_1^* - p_2) = 0 \quad . \tag{2-27}$$

Equation (2-27) is satisfied when

$$p_{2m}^* = p_1 \quad . \tag{2-28}$$

Equation (2-28) is the relation between the p's of two polarization-matched antennas in a transmit-receive configuration.

The information contained in Eqns. (2-26) and (2-28) is used to graphically display in Fig. 3 the cross-polarized state p_{2C} and the polarization-matched state p_{2m} of polarization state p_1. Note that these states are easily located by imaging in the $\gamma = o, \pi$ line for the matched state and in the $\gamma = \pm \pi/2$ line for the crossed state.

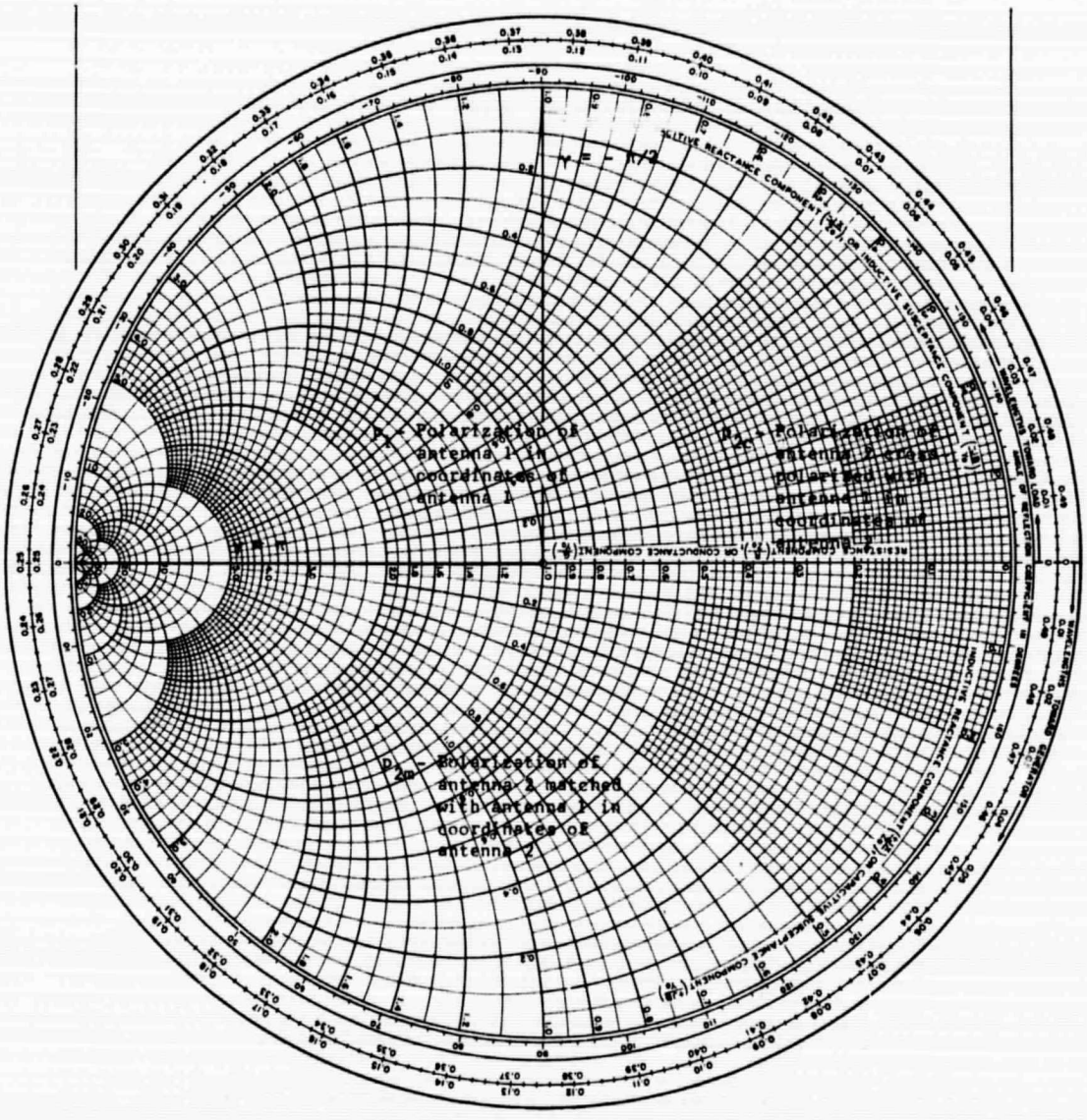


Figure 3. Matched and crossed polarization states for a transmit-receive antenna configuration.

The crossed state (p_{2C}) has a real part whose sign is the negative of the sign of the real part of p_1, i.e., the sense of rotation is opposite that of p_1.

Contours of Constant Loss

The information contained in Eqn. (2-25) can be best utilized when displayed as contours of constant polarization mismatch loss on the Smith Chart. The equations for these contours are readily determined by substituting Eqn. (2-8) into Eqn. (2-25) for p and performing the mathematical operations shown below. Equation (2-25) becomes

$$\rho = \frac{(1 + q_1 q_2)(1 + q_1^* q_2^*)}{(1 + q_1 q_1^*)(1 + q_2 q_2^*)} \tag{2-29}$$

so that substituting

$$q = Q\, e^{j\gamma}$$

results in

$$(1 + Q_1^2 + Q_2^2 + Q_1^2 Q_2^2)\rho = 1 + 2Q_1 Q_2 \cos(\gamma_1 + \gamma_2) + Q_1^2 Q_2^2 \tag{2-30}$$

or

$$Q_2^2 - \frac{2Q_1 Q_2}{(Q_1^2 + 1)\rho - Q_1^2} \cos(\gamma_1 + \gamma_2) + \frac{(Q_1^2 + 1)\rho - 1}{(Q_1^2 + 1)\rho - Q_1^2} = 0$$

which is the same as

$$Q_2^2 - \frac{2Q_1 \cos(\gamma_1 + \gamma_2)}{(Q_1^2 + 1)\rho - Q_1^2} Q_2 + \frac{Q_1^2}{[(Q_1^2 + 1)\rho - Q_1^2]^2} = \frac{(1 - \rho)\rho}{\left[\rho - \frac{Q_1^2}{1 + Q_1^2}\right]^2} . \tag{2-31}$$

Equation (2-31) is recognized as the equation of a circle with its center at

$$Q_2 = \frac{Q_1}{(Q_1^2 + 1)\rho - Q_1^2} \tag{2-32a}$$

$$\Upsilon_2 = - \Upsilon_1 , \tag{2-32b}$$

and having a radius of

$$r = \left| \frac{[(1 - \rho)\rho]^{1/2}}{\rho - \frac{Q_1^2}{1 + Q_1^2}} \right| . \tag{2-32c}$$

It must be remembered that it was necessary to map both the right half and left half complex p planes into the unit circle in order to graphically display all polarization states. This operation results in all points within the unit circle representing two different polarization states. For this reason it is necessary to apply certain restrictions when interpreting the results of Eqns. (2-32). The operation required to map polarization states having left elliptical rotation into the unit circle simply consists of inverting the magnitude of q while maintaining the same angle. Thus, that part of the circle described by Eqns. (2-32), that falls outside the unit circle must be mapped back into the unit circle as a contour representing the same polarization loss but for polarizations states with opposite rotation sense. The equation for this contour can be obtained by replacing Q_2 with $1/Q_2$ in Eqn. (2-30). Thus

$$(1 + Q_1^2 + \frac{1}{Q_2^2} + \frac{Q_1^2}{Q_2^2})\rho = 1 + 2\frac{Q_1}{Q_2}\cos(\gamma_1 + \gamma_2) + \frac{Q_1^2}{Q_2^2}$$

which, after algebraic manipulation becomes

$$Q_2^2 - 2Q_2\cos(\gamma_1+\gamma_2)\frac{Q_1}{\rho(1+Q_1^2)-1} + \frac{Q_1^2}{[\rho(1+Q_1^2)-1]^2} = \frac{(1-\rho)\rho}{\left[\rho - \frac{1}{Q_1^2+1}\right]^2}$$

Again, this is recognized as the equation of a circle with its center at

$$Q_2 = \frac{Q_1}{(Q_1^2+1)\rho - 1} \qquad (2\text{-}33a)$$

$$\gamma_2 = -\gamma_1 \ , \qquad (2\text{-}33b)$$

and having a radius of

$$r = \left| \frac{[(1-\rho)\rho]^{1/2}}{\rho - \frac{1}{Q_1^2+1}} \right| \ . \qquad (2\text{-}33c)$$

The procedure to follow in plotting the constant loss contours, as described by Eqns. (2-32) and (2-33), is summarized below: The q representing the polarization characteristics of the known antenna yields Q_1 and γ_1. If Q_1 is less than unity (RER), substitute Q_1 and γ_1 into Eqns. (2-32) to find the center and radius for the desired loss ρ, if the contour (circle) is not completely contained within the unit circle then that part outside represents polarization states with LER and with the same loss factor ρ. This contour is obtained from

Eqns. (2-33) by using the same Q_1, γ_1 and ρ as that used above. Only that part of the contour falling within the unit circle is of interest. If Q_1 is greater than unity (LER) enter equation (2-32) with $1/Q_1$, γ_1, and ρ. If the contour is not completely contained within the unit circle then that part outside represents polarization states of right elliptical rotation. This contour is obtained from Eqns. (2-33) using $1/Q_1$, γ_1, and ρ. Each contour, consisting of a closed curve, represents all polarization states having the same polarization mismatch loss when used in a transmit-receive configuration with the antenna whose polarization parameter is q_1.

The loss contours for three important polarization states have been plotted in Figs. 4, 5 and 6. Figure 4 shows the loss contours for circular polarization. If $q_1 = 0$ (right circular) then all contours representing less than 3 db loss have RER and greater than 3 db ($\rho < .5$) have LER. The converse is true for $q_1 = \infty$ (left circular). Figure 5 displays the contours for linear horizontal polarization ($q_1 = 1$). The contours for any other value of linear polarization ($q_1 = e^{j\gamma_1}$) can be obtained by rotating the contours of Fig. 5 about the $q = 0$ point through an angle equal to $-\gamma_1$. The loss contours for an antenna with an axial ratio of 3 and a tilt angle of zero degrees are shown in Fig. 6. The contours are labeled for $q_1 = \frac{1}{2} e^{j\sigma}$ (cf. Table 2-1); however, the same contours are valid for $q_1 = 2e^{j\sigma}$(LER) if the labels RER and LER are interchanged. The contours for polarization states with the same axial ratio but different tilt angles can be obtained by rotating the contours of Fig. 6 about the $q = 0$ point through an angle of $\gamma_1 = 2\tau$ degrees.

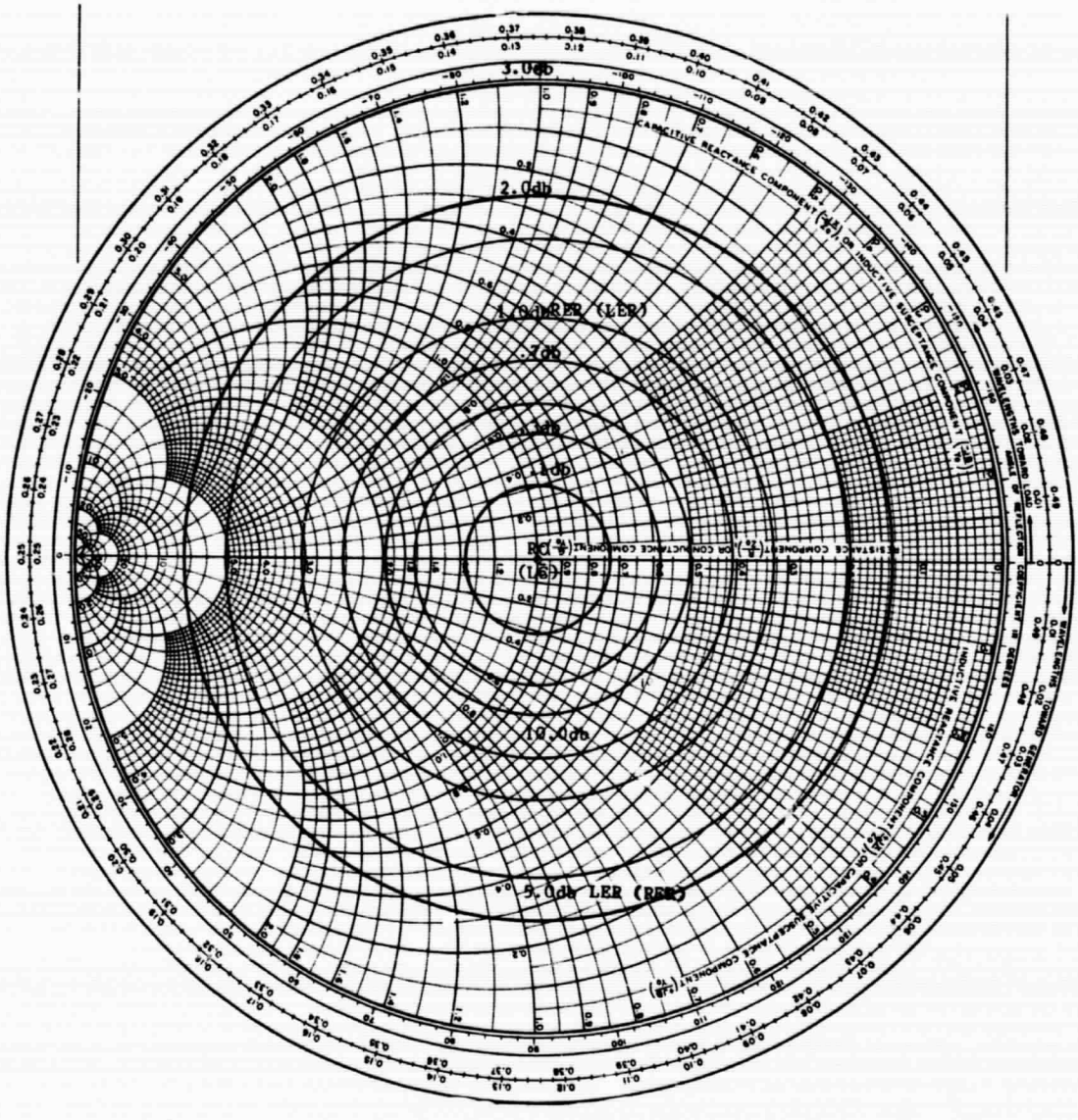


Figure 4. Loss contours for circular polarization.

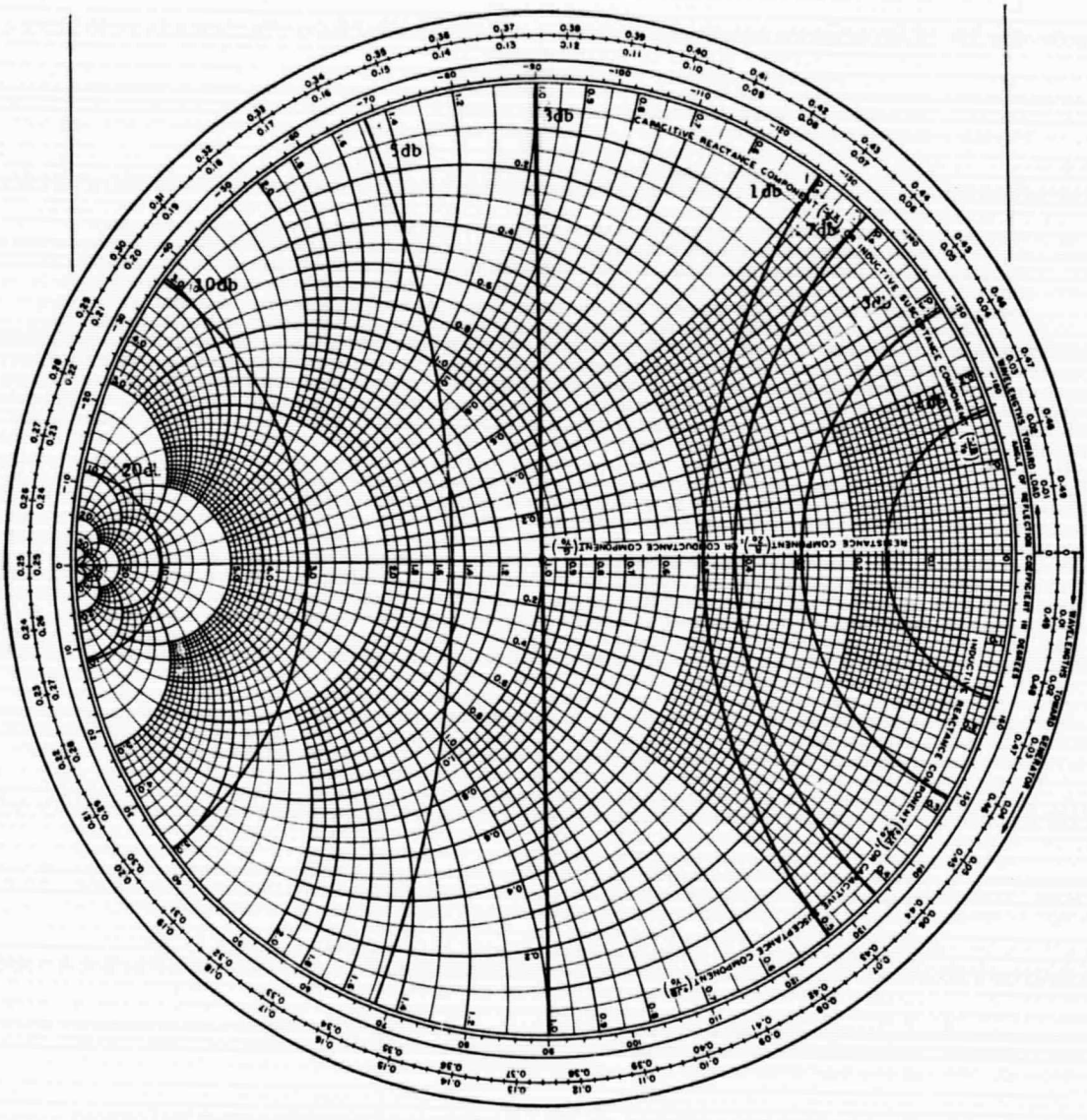


Figure 5. Loss contours for linear horizontal polarization.

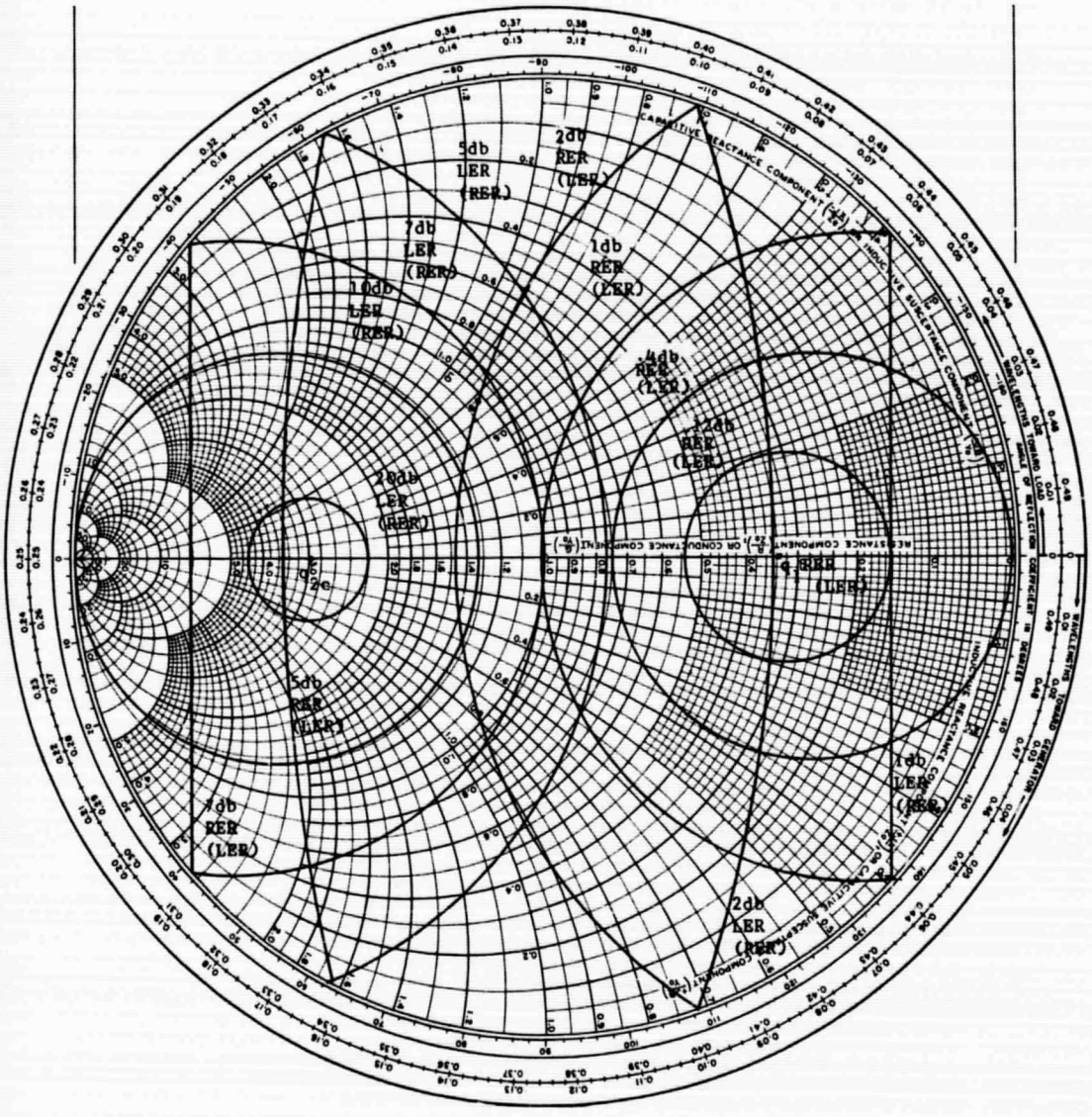


Figure 6. Loss contours for an antenna with an axial ratio of 3 and a tilt angle of 0 degrees.

The value of the techniques developed in this chapter will become apparent through their use in the remaining chapters of this study. One application of this method for describing the polarization properties of a radiating structure is shown in Appendix A, where two angles of rotation or one rotation angle and a variable phase shift in a waveguide system establish any desired polarization on the axis of a circularly symmetric radiator.

CHAPTER III

AN ANALYSIS OF AN ANTENNA SYSTEM

Basic System Elements

In Chapter I an intuitive approach was used to define the basic components required to provide a radiating system with polarization control. The basic components were; a matched variable power combiner, a phase shifter, and two radiating elements. The purpose of this chapter is to present a mathematical analysis of a polarization controllable antenna system in order that the operating characteristics of the system be determined.

Figure 7 displays in block diagram form the circuit components and configuration of the purposed system. The components shown are of waveguide construction; however, the analysis is equally valid for their low frequency counterpart. The matched power combiner consists of two 3 db hybrids (magic tees) and the phase shifter ψ, and is similar to one described by Teeter and Bushore [8]. The second phase shifter, μ, is required to establish the correct electrical phase between the two received signals. The two radiating elements are linearly polarized, establishing orthogonal field vectors. The system will also operate with 90 degree hybrids and with circularly polarized radiating elements; however, this requires a minor modification in the circuit analysis. This modification is due to the difference

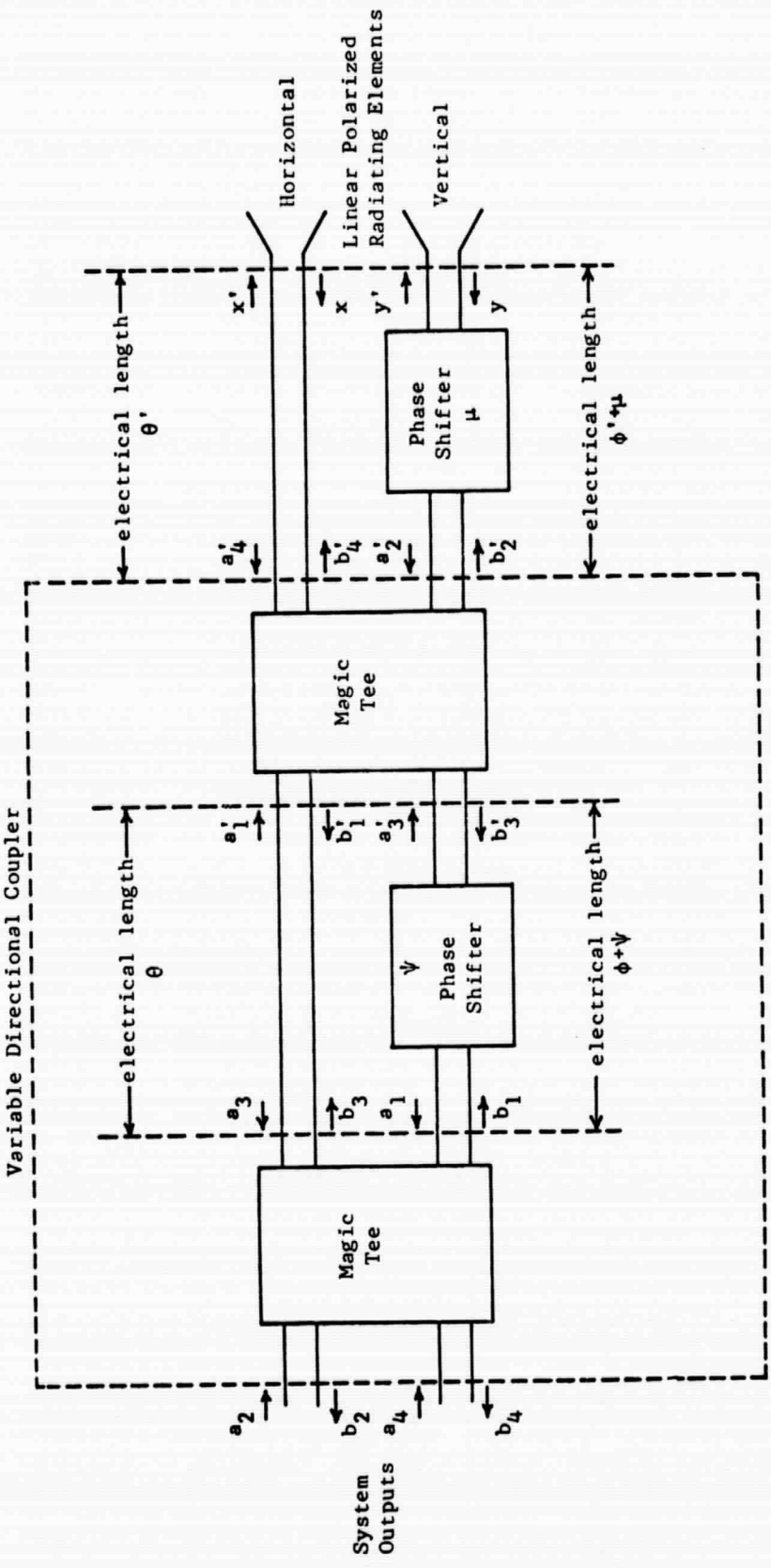


Figure 7. System configuration in block diagram form.

in the phase relation between arms for the two types of hybrids.

A Mathematical Model

A signal flow graph of the circuit of Fig. 7 is shown in Fig. 8. The components are considered ideal, i.e., they are matched and lossless. Each phase shifter is capable of reciprocal operation. The circuit has 4 external ports and as such can be described by a 4 by 4 scattering matrix. It can be shown that any completely matched junction of four transmission lines is a directional coupler [9]. The scattering matrix of a directional coupler is of the form

$$S = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix} . \qquad (3\text{-}1)$$

The zero elements on the principal diagonal indicate that the junction is completely matched. Since the circuit elements as reciprocal it is necessary that the matrix be symmetrical, i.e.,

$$S_{31} = S_{13}$$
$$S_{32} = S_{23}$$
$$S_{41} = S_{14}$$
$$S_{42} = S_{24} \quad .$$

The matrix is unitary since it represents a lossless junction [10].

The value of the four elements of the scattering matrix can be determined from the signal flow graph and the matrix equation,

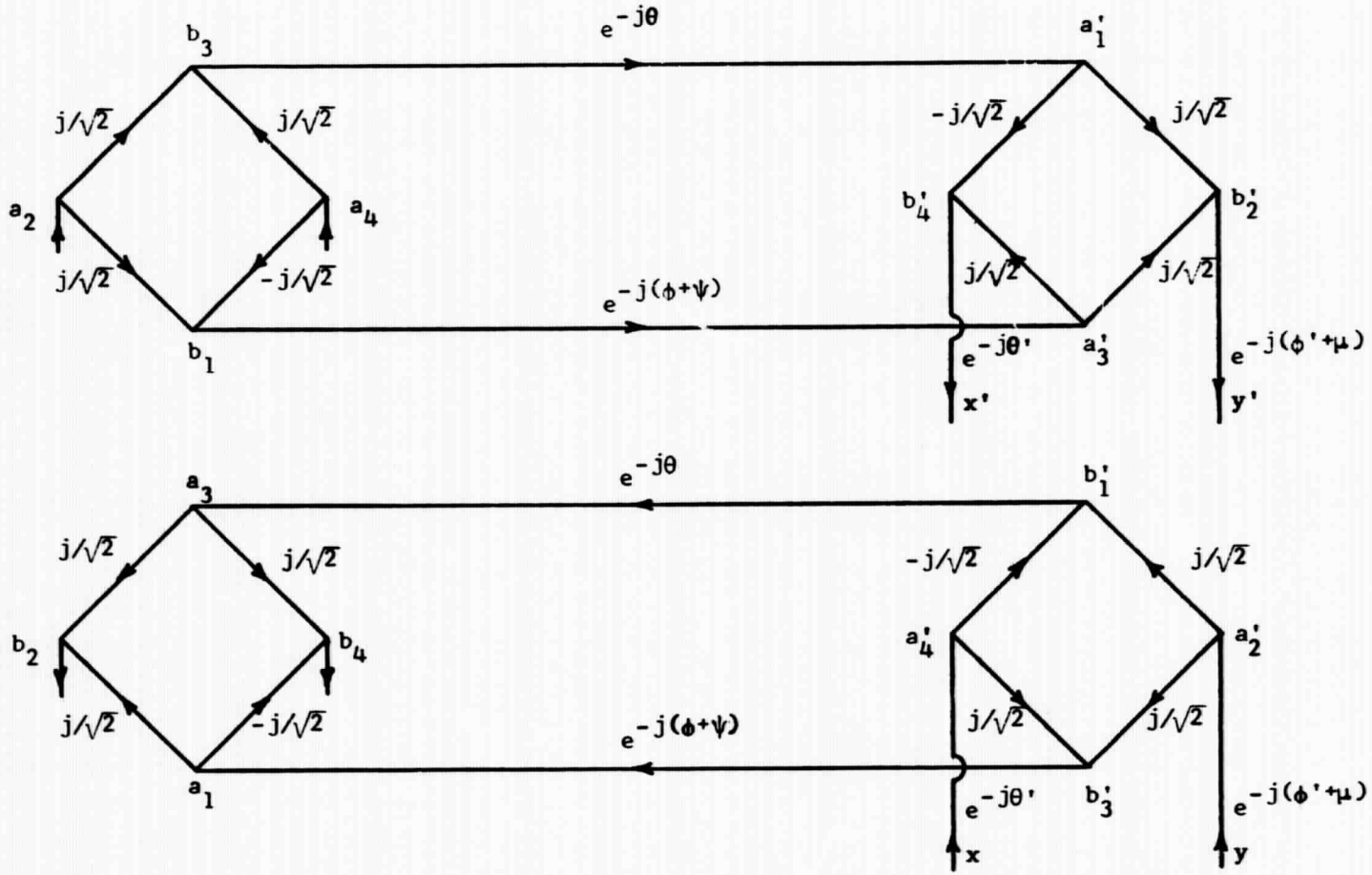


Figure 8. Signal flow graph of circuit shown in Fig. 7.

$$\begin{bmatrix} b_2 \\ b_4 \\ x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & 0 \\ S_{14} & S_{24} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_2 \\ a_4 \\ x \\ y \end{bmatrix} . \quad (3\text{-}2)$$

The element S_{13} is the value of b_2 due to an incident wave of value unity at x, with all other arms terminated in matched loads. Thus, from Fig. 8,

$$S_{13} = e^{-j\theta'} \left[\frac{e^{-j\,\pi/2}}{\sqrt{2}} \cdot e^{-j\theta} \cdot \frac{e^{j\,\pi/2}}{\sqrt{2}} + \frac{e^{j\,\pi/2}}{\sqrt{2}} e^{-j(\phi + \psi)} \cdot \frac{e^{j\,\pi/2}}{\sqrt{2}} \right]$$

$$= \frac{1}{2} e^{-j\theta'} \left[e^{-j\theta} - e^{-j(\phi + \psi)} \right] . \quad (3\text{-}3)$$

Likewise,

$$S_{14} = -\frac{1}{2} e^{-j(\phi' + \mu)} \left[e^{-j\theta} + e^{-j(\phi + \psi)} \right] , \quad (3\text{-}4)$$

$$S_{23} = \frac{1}{2} e^{-j\theta'} \left[e^{-j\theta} + e^{-j(\phi + \psi)} \right] , \quad (3\text{-}5)$$

$$S_{24} = \frac{1}{2} e^{-j(\phi' + \mu)} \left[-e^{-j\theta} + e^{-j(\phi + \psi)} \right] . \quad (3\text{-}6)$$

It will be of value to examine some properties of the matrix

$$R = \begin{bmatrix} S_{13} & S_{14} \\ S_{23} & S_{24} \end{bmatrix} .$$

It is obvious from an investigation of Eqns. (3-4) and (3-5) that R is

not symmetrical. Equation (3-1) can be written in the form

$$S = \begin{bmatrix} 0 & R \\ R^T & 0 \end{bmatrix}$$

where R^T is the transpose of R. Since S is symmetrical and unitary,

$$SS^{*T} = SS^* = I$$

where I is the unit matrix. Therefore

$$\begin{bmatrix} 0 & R \\ R^T & 0 \end{bmatrix} \begin{bmatrix} 0 & R^* \\ R^{*T} & 0 \end{bmatrix} = \begin{bmatrix} RR^{*T} & 0 \\ 0 & R^T R^* \end{bmatrix} = I$$

which leads to the result

$$RR^{*T} = I$$

and R is unitary.

Equations Governing System Operation

Correct system operation in the receive mode is achieved when the phase values μ and ψ are such that all the energy collected by the two radiators is available at arm 2. This requires that no energy be available at arm 4 ($b_4 = 0$) regardless of the quantities, x and y. It is seen from Eqn. (3-2) that $b_4 = 0$ when

$$S_{23}x = - S_{24}y$$

or

$$e^{-j\theta'}\left[e^{-j\theta} + e^{-j(\phi+\psi)}\right]x = -e^{-j(\phi'+\mu)}\left[-e^{-j\theta} + e^{-j(\phi+\psi)}\right]y$$

so that

$$1 = j\frac{y}{x}e^{-j(\phi'+\mu-\theta')}\tan(\psi/2+\phi/2-\theta/2)\ . \qquad (3\text{-}7)$$

Equation (3-7) is satisfied when

$$|x/y| = \tan(\psi/2+\phi/2-\theta/2) \qquad (3\text{-}8a)$$

and

$$\angle y/x + \pi/2 = \mu + \phi' - \theta' \ . \qquad (3\text{-}8b)$$

The quantities x and y are determined by the polarization parameter p of the wave incident on the system and the vector lengths of the two linearly polarized radiators. To show this, recall from Chapter II that

$$V_x = h_x E_x^i = a\, h_x$$

and

$$V_y = h_y E_y^i = b\, e^{j\phi_1} h_y$$

where h_x and h_y are the vector lengths of the two system linear radiators. Let

$$x = g\, V_x$$

$$y = g\, V_y$$

where g relates the open circuit terminal voltage to the waveguide transverse electric field representation x, y. Therefore

$$\left| x/y \right| = a/b \left| h_x/h_y \right| \tag{3-9a}$$

and

$$\angle y/x = \phi_1 + \angle h_y - \angle h_x \ . \tag{3-9b}$$

The system is considered to be adapted to receive the incident plane wave with polarization, $p = j\, b/a\, e^{j\phi_1}$, when phase shifters μ and ψ are adjusted to satisfy Eqn. (3-8). Note that phase shifter ψ must be capable of 180 degrees differential phase shift while μ requires 360 degrees of differential phase shift.

The system operates in the transmit mode when energy is applied at arm 2. The polarization parameter of the transmitted wave can be determined from Eqn. (3-2) with $a_4 = 0$. This leads to the result

$$x' = S_{13}\, a_2$$

$$y' = S_{14}\, a_2$$

so that

$$E_{x'} = g'\, x'\, h_x = g'\, S_{13}\, h_x\, a_2$$

and

$$E_{y'} = -g'\, y'\, h_y = -g'\, S_{14}\, h_y\, a_2$$

where g' relates the waveguide transverse electric field to the antenna terminal current (cf. 2-19). The minus sign is required with the $E_{y'}$ term since the direction of propagation has been reversed (cf. 2-24). Hence

$$P_t = j \frac{E_{y'}}{E_{x'}} = -j \frac{S_{14}}{S_{13}} \cdot \frac{h_y}{h_x}$$

or

$$P_t = \frac{h_y}{h_x} e^{-j(\phi' + \mu - \theta')} \cot (\psi/2 + \phi/2 - \theta/2) \quad . \qquad (3\text{-}10)$$

It is evident from Eqn. (3-10) that any desired polarization can be generated by selecting the proper values for μ and ψ, and that the range of values for μ and ψ are 360 and 180 degrees, respectively.

The preceding paragraphs have considered system operation in the receive mode and in the transmit mode. A principal application for the system consists of adapting (in the polarization sense) to an incident wave and then transmitting a wave having the correct polarization characteristics for maximum reception at the source of the incident wave (polarization-adaptive operation). The following mathematical analysis determines the necessary system requirements for this mode of operation. The system is adapted for reception when $b_4 = 0$. For transmission, the energy is inserted at arm 2, i.e., $a_4 = 0$. Let

$$c = \begin{bmatrix} b_2 \\ 0 \end{bmatrix}$$

$$d = \begin{bmatrix} a_2 \\ 0 \end{bmatrix}$$

$$e = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f = \begin{bmatrix} x' \\ y' \end{bmatrix},$$

then from Eqn. (3-2)

$$c = R\,e \tag{3-11}$$

$$f = R^T d,$$

or

$$f^* = R^{*T} d^*$$

so that

$$Rf^* = d^*, \tag{3-12}$$

since R is unitary. Assume some relation between the received signal b_2 and the transmitted signal a_2 such as

$$b_2 = z\,a_2^* \tag{3-13}$$

where z is any complex number. Then

$$c = z\,d^*,$$

or upon substituting Eqn. (3-12)

$$c = z\,Rf^*$$

which yields after using Eqn. (3-11)

$$e = z\,f^*$$

so that

$$x = z\,x'^*$$

and

$$y = z\,y'^* \quad .$$

Dividing yields

$$y/x = (y'/x')^* \tag{3-14}$$

so that z is arbitrary from the polarization standpoint. It can be determined from Eqn. (3-9) that

$$y/x = -j\, h_y/h_x\, p_r \tag{3-15}$$

where p_r is the polarization parameter of the received wave. The polarization parameter of the transmitted wave is

$$p_t = j \frac{E_{y'}}{E_{x'}} = -j \frac{y'}{x'} \frac{h_y}{h_x} \quad . \tag{3-16}$$

Substituting Eqns. (3-15) and (3-16) into Eqn. (3-14) yields

$$p_r = \frac{|h_x|^2}{|h_y|^2} p_t^* \quad . \tag{3-17}$$

Equation (2-28) gives the polarization matched requirement between the p's of two antennas in a two-way receive-transmit circuit. Thus if,

$$|h_x| = |h_y| \tag{3-18}$$

it is seen from Eqn. (3-17) that the system meets this requirement. The vector lengths of the two radiators are functions of the direction of incidence, thus the magnitude of the vector lengths must be identical at all expected angles of incidence. Note that there is no requirement imposed on the phase of the radiators and hence their phase centers can be physically different.

The phase terms θ, ϕ, θ' and ϕ' are functions of frequency since they represent the time delay (in electrical degrees) associated with a physical length of transmission line. Obviously the system bandwidth will be a maximum when $\theta = \phi$ and $\theta' = \phi'$. Since this is not an imposing restriction and should actually be a design criterion it will be assumed that $\theta = \phi$ and $\theta' = \phi'$. This will require that any phase difference due to unequal propagation paths when the radiators are not phase coincident be attributed to the phase of h_x or h_y. These terms enter the receive and transmit modes of operation as shown in Eqns. (3-8), (3-9), and (3-10). Based on the above discussion the equations governing system operation in the receive mode reduce to

$$\tan \Psi/2 = a/b \frac{|h_x|}{|h_y|} \tag{3-19a}$$

and

$$\mu = \phi_1 + \pi/2 + \angle h_y - \angle h_x \tag{3-19b}$$

where the polarization parameter of the received wave is

$$p_r = j\, b/a\, e^{j\phi_1} .$$

For the transmit mode

$$p_t = \frac{|h_y|}{|h_x|} \cot \Psi/2\, e^{-j(\mu + \angle h_y - \angle h_x)} \tag{3-20}$$

defines the polarization parameter of the transmitted wave.

Effect of Phase Shifter Loss

The system operation was derived in terms of ideal components. Obviously some consideration for the effects of nonideal components is in order. The realization of matched tee's and phase shifters is possible at a single frequency and in fact over a limited range of frequencies, thus the assumption of matched components will be considered valid over a limited bandwidth. It is not possible to build phase shifters without loss, however, and this restriction requires additional consideration. It should be evident that a loss associated with phase shifter μ produces the same effect as a decrease in the magnitude of h_y relative to the magnitude of h_x. Thus it will be sufficient to examine the system behavior when losses accompany phase shifter ψ.

The signal flow graph of Fig. 8 can be easily modified to include a loss introduced with phase shifter ψ. This modification consists of replacing the terms $e^{-j(\phi + \psi)}$ with $re^{-j(\phi + \psi)}$ where r is a positive real number such that

$$0 \leq r \leq 1 \quad .$$

The matrix elements (Eqns. (3-3) thru (3-6)) are modified in the same manner. For the analysis that follows it will be assumed that the system has been adapted, i.e., Eqn. (3-19) is satisfied and furthermore that $h_x = h_y$. These assumptions will simplify the mathematics without restricting the interpretation of the results. The procedure for determining the modified receiving mode operation will be to compute the quantities b_2 and b_4 under the above assumptions. Now

$$b_4 = S_{23}\, x + S_{24}\, y$$

or

$$b_4 = \frac{1}{2}\, e^{-j(\theta' + \theta)}\, [(1 + re^{-j\psi})x + e^{-j\mu}\, (-1 + re^{-j\psi})y] \ ,$$

but from Eqn. (3-19) and the assumption that $h_x = h_y$

$$\mu = \phi_1 + \pi/2$$

$$x = a = \sin \psi/2$$

$$y = be^{j\phi_1} = \cos \psi/2 \cdot e^{j\phi_1}$$

(note that the condition $|x| = a$, $|y| = b$ effectively normalizes the incident power to unity) hence

$$b_4 = \frac{1}{2}\, e^{-j(\theta' + \theta)}\, [\sin \psi/2 + j \cos \psi/2 + r e^{-j\psi}\, (\sin \psi/2 - j \cos \psi/2)]$$

which is

$$b_4 = e^{-j(\theta' + \theta + \psi/2 - \pi/2)}\ \frac{(1 - r)}{2} \ . \qquad (3\text{-}21)$$

It is seen that $b_4 = 0$ for the lossless case ($r = 1$), and this essentially confirms the derivation of Eqn. (3-19). Next,

$$b_2 = S_{13}\, x + S_{14}\, y$$

so that using the modified matrix elements

$$b_2 = \frac{1}{2}\, e^{-j(\theta' + \theta)}\, [(1 - re^{-j\psi})x - e^{-j\mu}\, (1 + re^{-j\psi})y\,]$$

or as above

$$b_2 = \frac{1}{2} e^{-j(\theta' + \theta)} \left[\sin \psi/2 + j \cos \psi/2 - r e^{-j\psi} (\sin \psi/2 - j \cos \psi/2) \right]$$

which reduces to

$$b_2 = e^{-j(\theta' + \theta + \psi/2 - \pi/2)} \frac{(1 + r)}{2} \quad . \tag{3-22}$$

Note that the collected power $|b_2|^2$ is unity for the lossless case. A typical loss for an electronically controlled 180 degree phase shifter is .5 db. This corresponds to a value for r of .943. Evaluating Eqn. (3-22) for this value of loss yields an overall system loss (phase shifter plus arm 4) of .26 db. The power at arm 4 as shown by Eqn. (3-21) is 31 db below that available in the incident wave.

The effect of phase shifter loss on the transmit mode of operation is best shown by considering the polarization parameter of the transmitted wave. From previous considerations it is known that

$$p_t = -j \frac{S_{14}}{S_{13}} \frac{h_y}{h_x}$$

or

$$p_t = j \frac{h_y}{h_x} e^{-j(\mu + \phi' - \theta')} \cdot \frac{e^{-j\theta} + r e^{-j(\phi + \psi)}}{e^{-j\theta} - r e^{-j(\phi + \psi)}} \quad .$$

If

$$\theta = \phi$$

and

$$\theta' = \phi' \ ,$$

then

$$p_t = j \frac{h_y}{h_x} e^{-j\mu} \frac{1 - r^2 - 2jr \sin \psi}{1 + r^2 - 2r \cos \psi}$$

or

$$p_t = j \frac{h_y}{h_x} e^{-j(\mu + \tan^{-1} \frac{2r \sin \psi}{1 - r^2})} \frac{(1 - 2r^2 + r^4 + 4r^2 \sin^2 \psi)^{1/2}}{1 + r^2 - 2r \cos \psi} . \quad (3\text{-}23)$$

It is easily shown that Eqn. (3-23) reduces to Eqn. (3-20) when r = 1.

In order to summarize the results obtained for the case when loss is associated with the ψ phase shifter it is necessary to recognize certain features of the lossless case. The equation to be satisfied for the receiving mode (Eqn. 3-8) requires independent control of the two system variables (μ, ψ) with regard to the two independent variables of the polarization parameter p, i.e., the value ψ is completely determined by the magnitude of p, and the value of μ is a function of the phase of p alone. The system operation is altered when this loss is present with the primary change being one of finite power existing at arm 4. However, for low loss ($\leq$ .5 db), the power at arm 4 ($|b_4|^2$) is well below the output power ($|b_2|^2$) and the system loss is approximately one half the phase shifter loss. The process of adapting the system, i.e., satisfying Eqn. (3-8) must utilize the null behavior at b_4 as one of the two required sources of control information. The process of system control is the topic of Chapter IV and will not be discussed further here.

The polarization parameter of a transmitted wave is given by Eqn. (3-10) for the lossless case. It is seen that the magnitude of the polarization parameter is a function of ψ, while the phase is determined by the value of μ. Equation (3-23) determines the polarization parameter that results from a loss associated with the ψ phase

shifter. The magnitude of the polarization parameter is a function of ψ, however, the phase now depends on the value of both μ and ψ. For the case of low loss ($r \approx 1$) deviation from the ideal case is slight. The major effect is on the phase of the polarization parameter. This effect is maximum for ψ near 0 and 180 degrees. These values of ψ correspond to "almost" vertical linear and horizontal linear polarization respectively. Very little change in the polarization characteristics (axial ratio and tilt angle) accompanies large changes in the phase of the polarization parameter for polarization states near the vertical and horizontal linear states (this is easily seen by referring to Fig. 2).

In some applications it might be necessary to determine accurately in a simple form the polarization properties of the system. The simplicity of the equations for the lossless case make them desirable for this application. To achieve this condition when phase shifter loss is present consider the effect of introducing an equal loss in the other transmission line connecting the two magic tees (Fig. 7). The matrix elements (Eqns. (3-3) thru (3-6)) are then modified by replacing the terms,

$$e^{-j\theta} \longrightarrow r e^{-j\theta}$$

and

$$e^{-j(\phi + \psi)} \longrightarrow r e^{-j(\phi + \psi)} .$$

The same result obtains if the matrix S is multiplied by r. Since the operating parameters of the system are determined in terms of ratios of the matrix elements the dependence on r vanishes and system operation

is described by the equations derived for the lossless case; however, the system operates with a fixed loss equal to r^2 rather than no loss.

Experimental Study of Systems Operation

The final verdict regarding the feasibility of any system rests with successful operation of a prototype model. For this reason it was considered necessary to construct an experimental model that would enable evaluation of the basic features of system operation. The circuit configuration shown in Fig. 7 was assembled using the x band waveguide items given below.

Item	Manufacturer	Model
Magic Tee	Waveline Inc.	658
ψ Phase Shifter	Waveline Inc.	608-C
μ Phase Shifter	Hewlett Packard	X 885 A
H Plane Elbows	Waveline Inc.	633
E Plane Elbows	Waveline Inc.	632

The linear radiating elements were developed as a single structure. It consists of a rectangular-to-circular waveguide transducer with orthogonal rectangular feeds. The design of the transducer is similar to the design described by Lompkins [11]. The radiator, supporting two orthogonal TE_{11} circular waveguide modes, is a 6.5 wavelength cylindrical polyrod antenna operating at 9450 MH_z. The experimental results obtained by Mueller and Tyrrell [12] were used in the polyrod design. The polyrod was machined from Rexolite 1422 dielectric rod stock to the dimensions shown in Fig. 9. The electrical characteristics

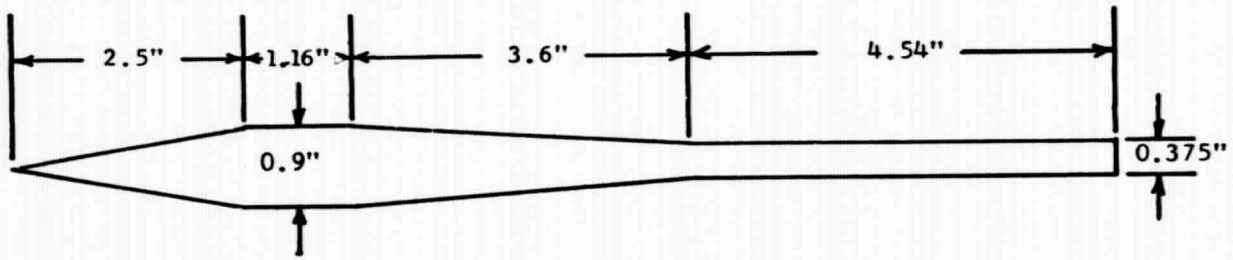


Figure 9. Polyrod antenna dimensions (6.5λ at 9450 MHz).

of the components were measured at 9450 MH_z. The results are shown below.

Item	VSWR	Attenuation	Gain	Isolation
Magic Tee	< 1.1	——	——	50 db
ψ Phase Shifter	1.04	.3 db	——	——
μ Phase Shifter	1.03	.3 db	——	——
Transducer:				
Colinear Arm	1.05	——	——	48 db
Orthogonal Arm	1.06	——	——	48 db
Polyrod	——	——	16.9 db	——

Figure 10 is a photograph of the completely assembled experimental model. Note that two 608-C phase shifters are present, one was used to correct the electrical line length such that $\theta - \phi = 0$, the other was the variable phase shifter ψ.

A polarization measuring test unit was fabricated in order that the polarization properties of the experimental system be readily determined. The polarization measuring equipment (PME) consists of a rectangular standard gain horn connected to a rotary joint and geared to a sine-cosine potentiometer. The assembly is driven by a variable speed motor. The rectangular horn is electrically connected to a receiver which provides a d.c. voltage proportional to the field incident on the horn. This d.c. voltage is used to excite the sine-cosine potentiometer so that an X-Y recorder can be used to plot field strength vs. angular position of the rectangular horn, i.e., the polarization pattern of the incident field. Mounted directly above

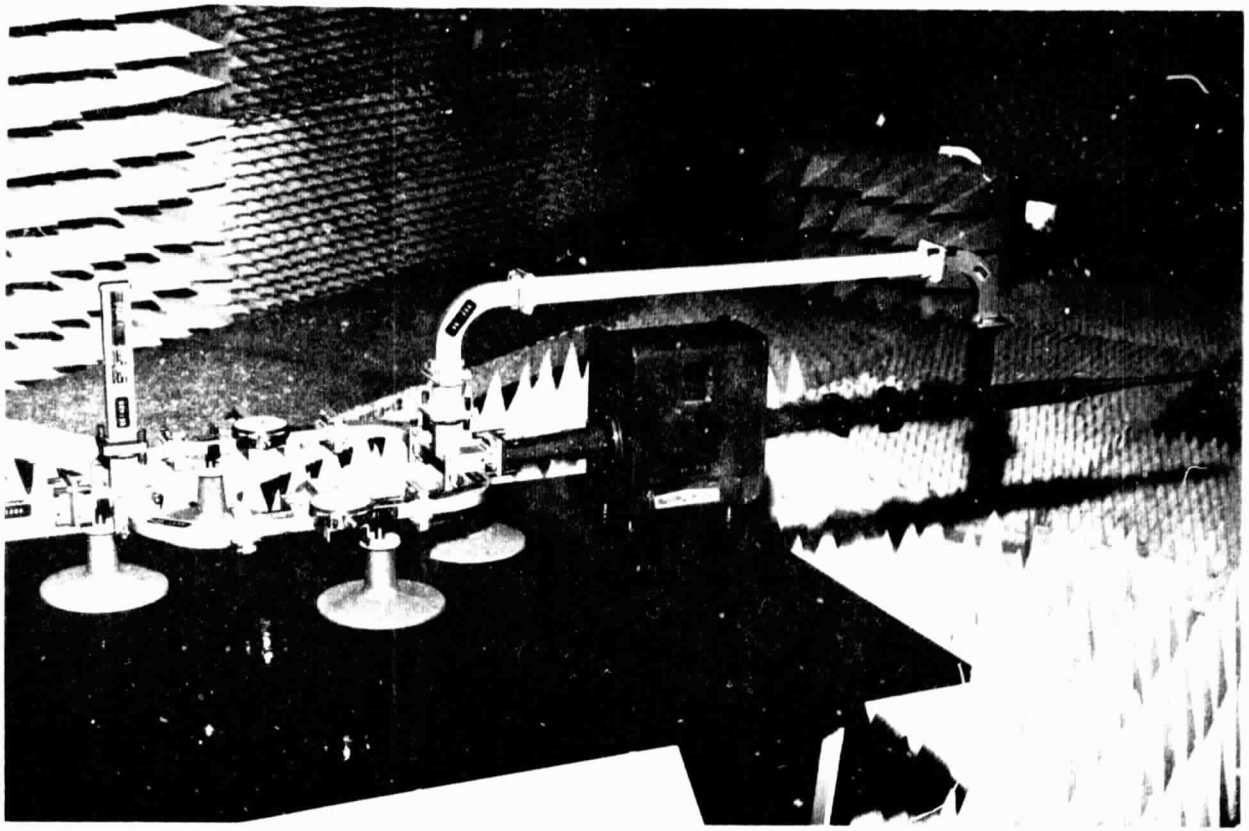


Figure 10. Photograph of the experimental antenna system.

the rotating rectangular horn is a 15 db circular horn with a connecting circular waveguide (.9 inch ID) containing a quarter-wave dielectric plate. The circular horn is manually rotated to receive either right circular or left circular polarization. Thus the polarization rotation sense of the incident field is determined by the larger of the right circular or left circular outputs of the circular horn. Thus it is seen that the PME provides the polarization ellipse and sense of rotation of an incident field. Figure 11 is a photograph of the PME described above.

The PME and the experimental system were collimated and separated by 3 meters. This was necessary since it was desirable to have the system radiator vector lengths equal, i.e., $h_x(0,0) = h_y(0,0)$ as is obviously the case for the polyrod. The system was calibrated by adjusting the PME rectangular horn to receive a linear polarized field with a tilt angle of -45 degrees. The system phase shifters μ and ψ were adjusted with the system transmitting until the rectangular horn output reached a null. Thus the system was radiating a field cross-polarized with the polarization of the horn. It is seen from Eqn. (3-20) that the value of the system phase shifters must be

$$\psi = \pi/2$$

$$\mu = 3\ \pi/2$$

in order to transmit a field of this polarization. The index on the phase shifters was adjusted to read these values, and the system was considered calibrated. The validity of Eqn. (3-20) was tested by changing μ and ψ to produce eight different values of linear

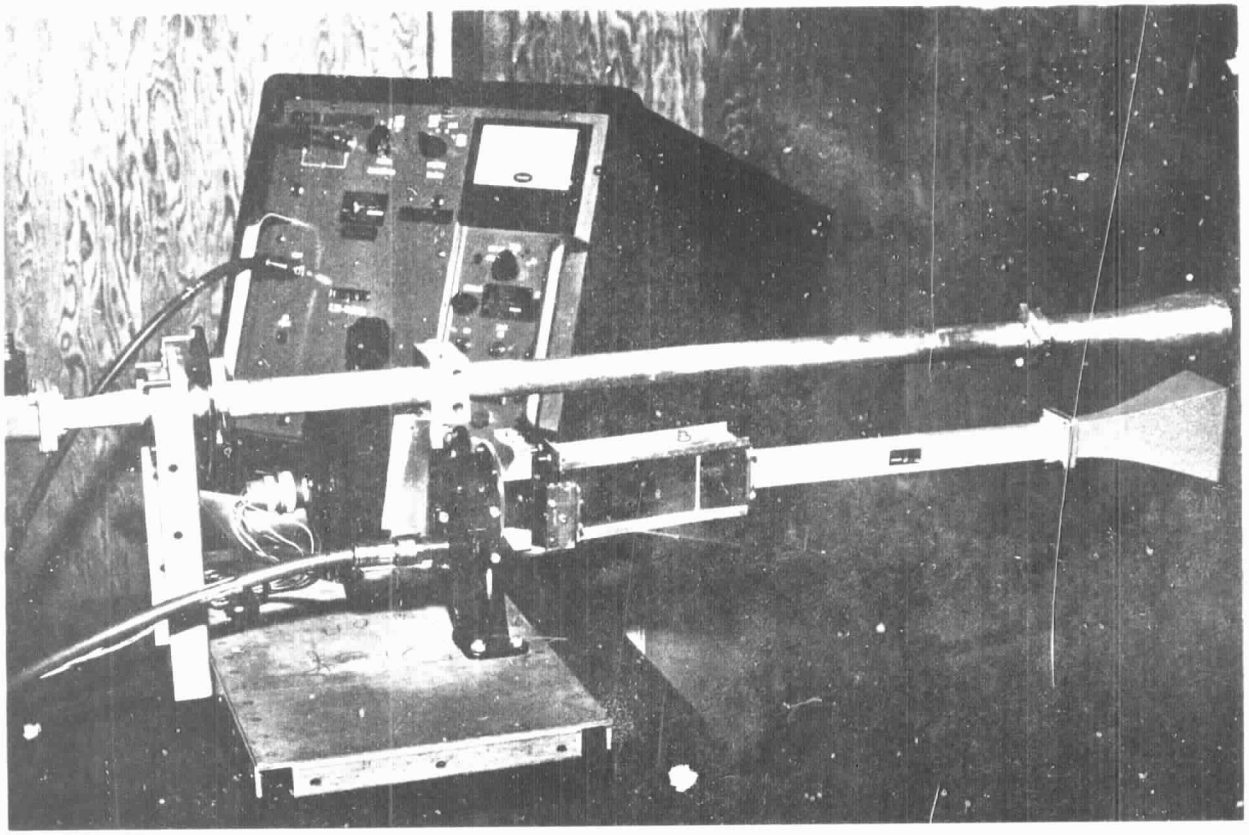


Figure 11. Photograph of the polarization measuring equipment.

polarization. The rectangular horn was adjusted to receive the cross-polarized field. This field strength was at least 40 db below that field strength received at the polarization matched positions for all eight values. The phase shifters were next set to the values required to produce right circular and left circular polarization. The axial ratio was less than 1.06 in both cases. This is within the accuracy of the PME. It is concluded from these results that system operation is accurately described in the transmit mode by Eqn. (3-20).

To check the validity of the equations describing system operation in the receive mode (3-19) the PME was used to transmit a wave of known polarization and the system phase shifters were adjusted to obtain a null at arm 4 (b_4). The phase values for the null setting were within ± 2 degrees of the theoretical values as shown by Eqn. (3-19). This procedure was repeated for a number of different polarizations. Again the results are within the performance specifications for the phase shifters.

In summary, this chapter has presented an analysis of a polarization controllable-antenna system, first by deriving the equations describing the operating characteristics of the system, and then by presenting experimental verification of these equations. It should be stated that the experimental system consisted of non-ideal components, yet the results corresponded closely to the theoretical results obtained by considering all components ideal.

CHAPTER IV

THE CONTROL PROCESS

A study of the system behavior from the standpoint of adaptive operation was performed in Chapter III. It was shown that the system was capable of this mode of operation provided Eqn. (3-18) is satisfied. In order to achieve adaptive mode operation it is necessary to introduce a control unit, capable of sampling the dynamic state of the system, and capable of using this information to adjust the controllable system elements, phase shifters μ and ψ, in such a manner that Eqn. (3-19) is satisfied. Hence, the purpose of this chapter is to investigate the operating characteristics of this control unit.

An Examination of the System Equations

It should be obvious that information required for system control must be obtained from the amplitude and phase behavior of the signals at arm 2 and arm 4 since these are the only externally available ports of the system. For this reason it is desirable to determine the expressions for these two signals in terms of the incident wave polarization parameter p. In Chapter III it was found that

$$b_2 = \frac{1}{2} e^{-j(\theta' + \theta)}[(1 - re^{-j\psi})x - e^{-j\mu}(1 + re^{-j\psi})y] \qquad (4\text{-}1)$$

and

$$b_4 = \frac{1}{2} e^{-j(\theta' + \theta)}[(1 + re^{-j\psi})x + e^{-j\mu}(-1 + re^{-j\psi})y] \qquad (4\text{-}2)$$

where it was assumed that the system electrical lengths had been adjusted such that

$$\theta' = \phi'$$

$$\theta = \phi \ .$$

Furthermore, it will be assumed that Eqn. (3-18) is satisfied and also that

$$\angle h_y = \angle h_x \ . \qquad (4\text{-}3)$$

If Eqn. (4-3) is not satisfied for a particular system design, the effect will be a shift in the phase of the polarization parameter of the incident wave as viewed by the system. Mathematically, this can be resolved by replacing $\phi_1 + \angle h_y - \angle h_x$ with ϕ_1'. In any case the effect can be measured and compensated for when necessary. It is mathematically expedient to require arbitrarily that

$$|g\, h_x| = 1 \qquad (4\text{-}4a)$$

$$|g\, h_y| = 1 \ . \qquad (4\text{-}4b)$$

This normalizes the power collected by the system to unity and in no way imposes limitations on the development that follows. Finally, it is necessary to avoid undue complications by restricting consideration to the lossless case (r = 1). Under these conditions Eqns. (4-1) and

(4-2) become

$$b_2 = \frac{1}{2} e^{-j(\theta' + \theta)}[(1 - e^{-j\psi})a - e^{-j(\mu - \phi_1)}(1 + e^{-j\psi})b]$$

and

$$b_4 = \frac{1}{2} e^{-j(\theta' + \theta)}[(1 + e^{-j\psi})a + e^{-j(\mu - \phi_1)}(-1 + e^{-j\psi})b]$$

or, simplifying

$$b_2 = e^{-j(\theta' + \theta + \psi/2 - \pi/2)}[a \sin \psi/2 - b e^{-j(\mu + \pi/2 - \phi_1)} \cos \psi/2] \quad (4\text{-}5)$$

and

$$b_4 = e^{-j(\theta' + \theta + \psi/2)}[a \cos \psi/2 + b e^{-j(\mu + \pi/2 - \phi_1)} \sin \psi/2] \; . \quad (4\text{-}6)$$

Equations (4-5) and (4-6) yield the amplitude and phase of the system outputs due to an incident wave with polarization parameter

$$p = j \frac{b}{a} e^{j\phi_1} \; .$$

A cursory examination of Eqn. (4-5) reveals that the amplitude of b_2 is a function of both μ and ψ, and also that the phase of b_2 depends on these quantities. The same is true of b_4. This behavior seems to preclude the possibility that a simple control process exists, i.e., the ability to control a system variable, say ψ, from amplitude measurements and the other (μ) from some phase relation existing between outputs b_2 and b_4. The complexity of the control problem seems to dictate the use of digital logic in the controller. The advantage of digital logic is enhanced by the availability of high speed microwave (and lower frequency) digital phase shifters. Hence, the remainder of this chapter will be primarily concerned with the task of extracting control information from Eqns. (4-5) and (4-6)

favorable to digital processing.

If digital phase shifters are used the system is no longer capable of an infinity of polarization states; rather, a fixed number of discrete states exist. These states can be displayed as the points of intersection of the contours of constant amplitude and phase of the polarization parameter. It is common to find 5 bit phase shifters, capable of 360 degree differential phase shift, available as "off-the-shelf" items from several manufacturers. A 5 bit phase shifter is capable of a 11.25 degree phase increment in a 360 degree range. It is only required that the ψ shifter have a range of 180 degrees; however, it is necessary to have the full 180 degree capability (not available with a 4 bit 11.25 degree shifter). The 4 bit configuration would be desirable from a loss standpoint since loss is directly related to the number of bits. It would seem that a good system design requires 5 bit 360 degree phase shifters. Figure 12 is a graphical display of the polarization states of the system assuming 5 bit phase shifters. The contours are labeled with the appropriate values of ψ and μ. These values are found by recalling that (from Eqn. 3-19)

$$b/a = \cot \psi/2 \tag{4-7a}$$

and

$$\phi_1 + \pi/2 = \mu \quad . \tag{4-7b}$$

The contours are determined by Eqns. (2-11), (2-12), (2-17) and (2-18). It should be pointed out that the values of μ to the left of the vertical axis are for LER and those to the right for RER and that all intersections represent two polarization states with the exception

μ
LER RER

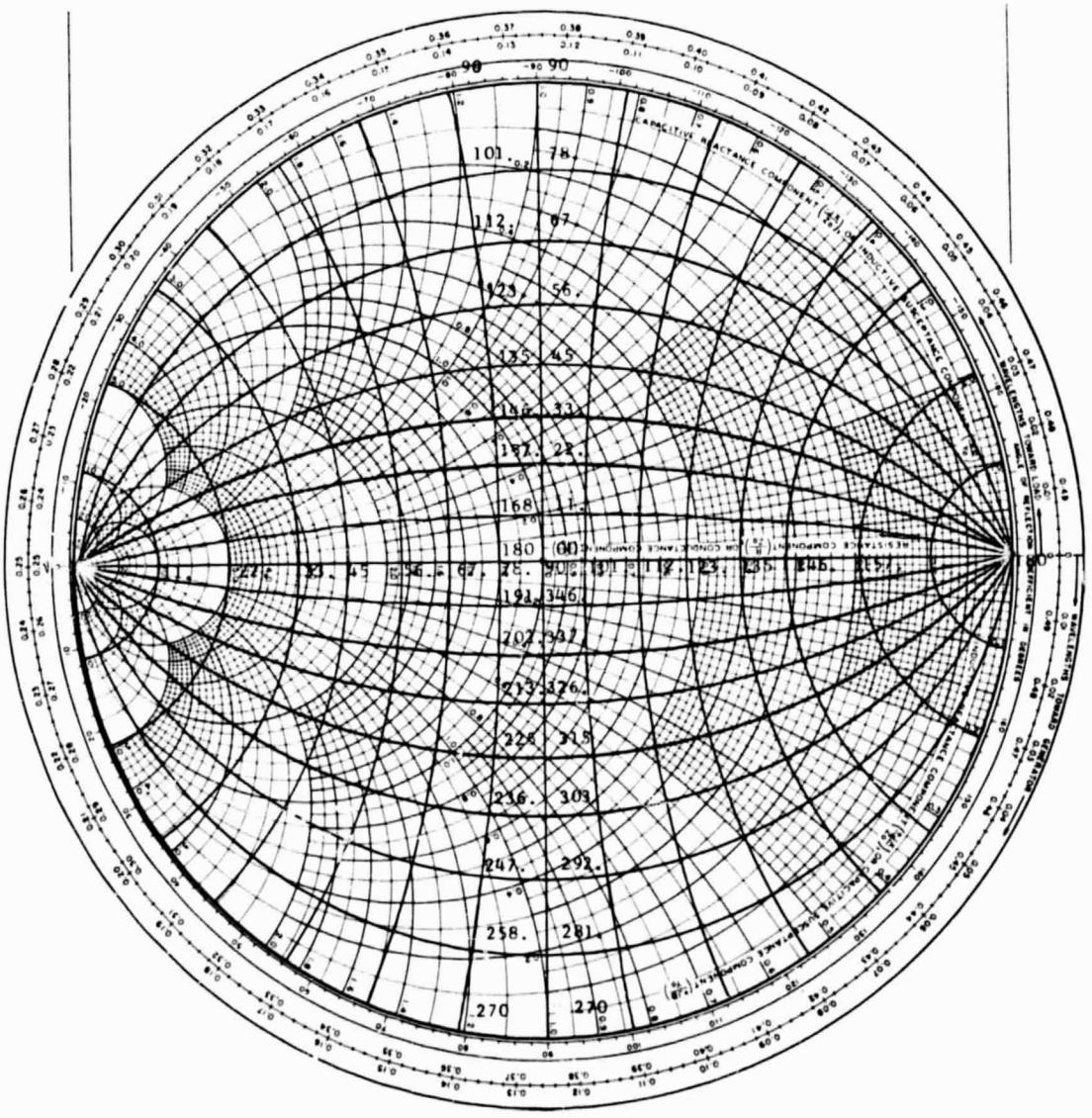


Figure 12. Smith Chart representation of polarization states when system phase shifters are of the 5 bit digital design.

of the points on the circumference. To reiterate, the discrete values of μ and ψ establish polarization states, shown by points of intersection, that the system is capable of receiving with zero polarization mismatch loss. The points determine the polarization parameter of the incident wave, which is in the coordinate system of the source.

System Parameters for Commonly Used Antenna Polarization

It should be evident that a certain amount of information, concerning the polarization of an incident wave, can be obtained by comparing the system responses for selected polarization states. Six common polarization states and the antenna system parameters required to receive the six states are shown in Table 4-1.

Table 4-1

Parameters for Common Polarization States

State	Polarization	Incident Wave Parameter		Antenna System Parameter	
		b/a	ϕ_1	ψ	μ
1	Linear Vertical	∞	——	0	——
2	Linear Horizontal	0	——	π	——
3	Linear 45°	1	0	$\pi/2$	$\pi/2$
4	Circular Left	1	$\pi/2$	$\pi/2$	π
5	Linear -45°	1	π	$\pi/2$	$3\pi/2$
6	Circular Right	1	$3\pi/2$	$\pi/2$	0

The system parameters are given by Eqn. (4-7).

The magnitude squared value of the antenna system output (normalized output power for the system receiving) is obtained from Eqn. (4-5) as,

$$|b_2|^2 = a^2 \sin^2 \psi/2 + b^2 \cos^2 \psi/2 + 2ab \sin \psi/2 \cos \psi/2 \sin(\mu - \phi_1) \ . \tag{4-8}$$

Equation (4-8) is used to obtain the output for each of the six polarization states as a function of the incident wave parameters. The results are shown in Table 4-2.

Table 4-2

Normalized System Output

State	Polarization	$\|b_2\|^2$
1	Linear Vertical	b^2
2	Linear Horizontal	a^2
3	Linear 45°	$1/2 + ab \cos \phi_1$
4	Circular Left	$1/2 + ab \sin \phi_1$
5	Linear -45°	$1/2 - ab \cos \phi_1$
6	Circular Right	$1/2 - ab \sin \phi_1$

The information contained in Table 2 can be used to determine limits on the incident wave parameter when it is given that the system response (b_2) is greater for one state than any of the others. This information is displayed in Table 4-3.

Table 4-3

Range of Values for System Parameters

State	If	Then	System Parameters Should Be
1	Linear Vertical > Others	$b > .924$	$0 < \psi < 45°$
2	Linear Horizontal > Others	$a > .924$	$135° < \psi < 180°$
3	Linear 45° > Others	$.383 < a < .924$ $.383 < b < .924$ $0 \leq \phi_1 < \pi/4$ $7\pi/4 < \phi_1 \leq 2\pi$	$45° < \psi < 135°$ $\pi/4 < \mu < 3\pi/4$
4	Circular Left > Others	$.383 < a < .924$ $.383 < b < .924$ $\pi/4 < \phi_1 < 3\pi/4$	$45° < \psi < 135°$ $3\pi/4 < \mu < 5\pi/4$
5	Linear -45° > Others	$.383 < a < .924$ $.383 < b < .924$ $3\pi/4 < \phi_1 < 5\pi/4$	$45° < \psi < 135°$ $5\pi/4 < \mu < 7\pi/4$
6	Circular Right > Others	$.383 < a < .924$ $.383 < b < .924$ $5\pi/4 < \phi_1 < 7\pi/4$	$45° < \psi < 135°$ $7\pi/4 < \mu \leq 2\pi$ $0 \leq \mu \leq \pi/4$

An examination of Table 4-3 reveals that if, e.g., state 3 yields an output greater than the output for any other state, then the optimum value of μ and ψ is within the limits shown in the table for state 3. Note that states 1 and 2 yield information only on the value of ψ.

The data displayed in Table 4-3 are available in a more general form in Chapter 2, i.e., through the use of the equations for the loss contours for circular and linear polarization. The equations can be

used to draw the loss contours for linear horizontal, linear at 45 degrees, and circular polarizations. These contours are shown in Fig. 13. If a transparency of Fig. 13 is made and superimposed on Fig. 12, it will be seen that the fourth column of Table 4-3 can be obtained. It is necessary to rotate the transparency through an angle (Υ) of 90, 180, and 270 degrees in order to obtain all the data. It should be observed from this procedure that when one polarization state produces a greater output, this output is, at least, within 1 db of the maximum possible output. Note that it is possible that two states, .7 to 1 db below the maximum, and even three states, 1 db below the maximum, have equal outputs.

The phase relation between the signals at arm 2 and arm 4 can be determined by examining Eqns. (4-5) and (4-6). The phase factor common to both equations is

$$\theta + \theta' + \psi/2 - \pi/2 \quad .$$

If this factor is removed from both equations, the equations that are obtained,

$$b_2' = a \sin \psi/2 - b \cos \psi/2 \; e^{-j(\mu - \phi_1 + \pi/2)} \tag{4-9}$$

and

$$b_4' = e^{-j\,\pi/2}[a \cos \psi/2 + b \sin \psi/2 \; e^{-j(\mu - \phi_1 + \pi/2)}] \quad , \tag{4-10}$$

contain all the information necessary to specify the phase difference. The following definition is made

$$\Phi + \pi = \mu - \phi_1 + \pi/2 \quad , \tag{4-11}$$

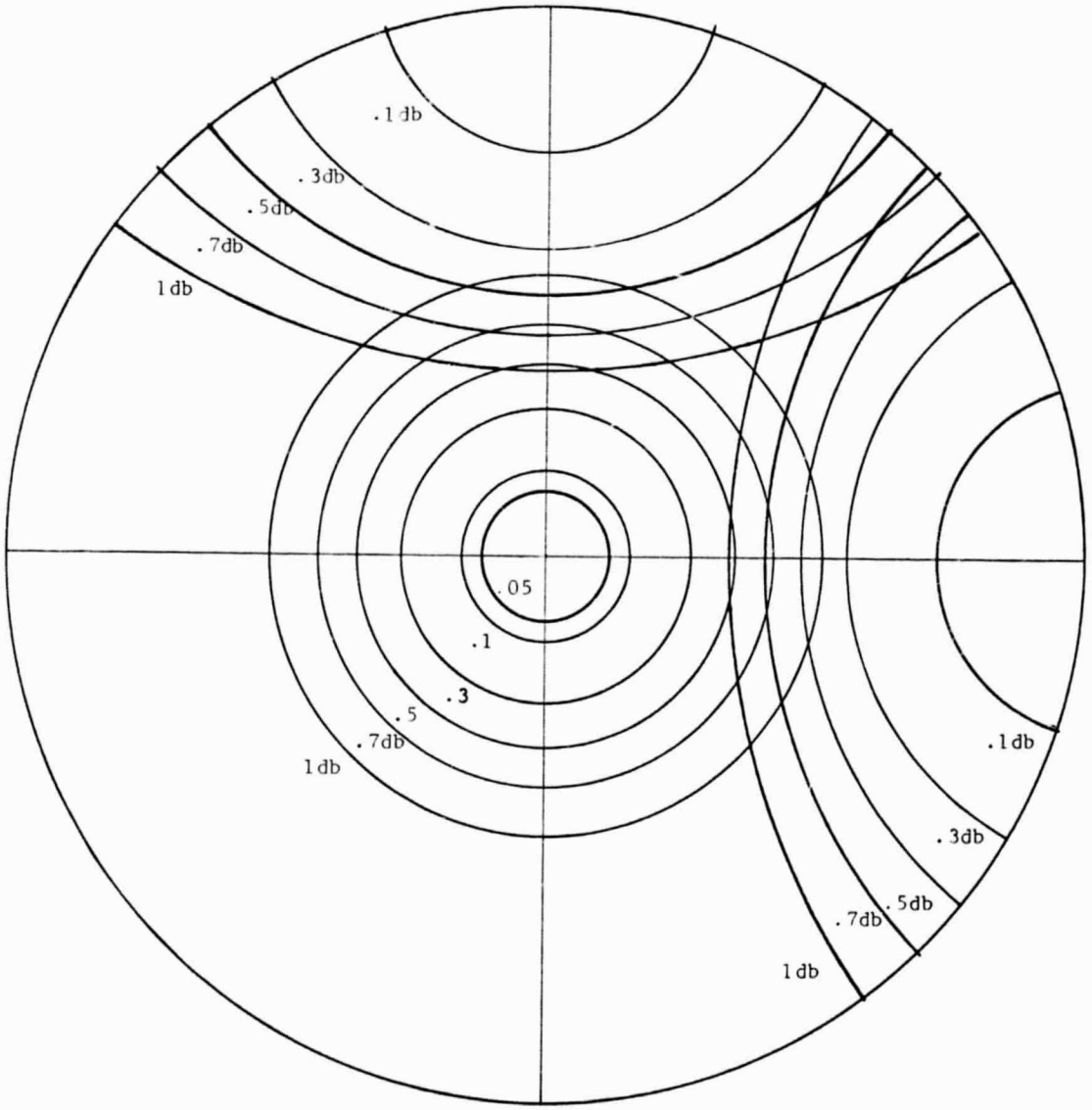


Figure 13. Loss contours for linear horizontal, linear at 45 degrees tilt angle, and circular polarization.

with the observation that when $\Phi = 0$ the equation becomes Eqn. (4-7b), i.e., $\Phi = 0$ is one of the requirements for adaptive operation. Using Eqn. (4-11), it is seen that Eqns. (4-9) and (4-10) become

$$b_2' = a \sin \psi/2 + b \cos \psi/2 \cos \Phi - jb \cos \psi/2 \sin \Phi \qquad (4\text{-}12)$$

and

$$b_4' = e^{-j\,\pi/2}[a \cos \psi/2 - b \sin \psi/2 \cos \Phi + jb \sin \psi/2 \sin \Phi] \; . \qquad (4\text{-}13)$$

The Search Stage of Controller Operation

The information available from a comparison of the amplitude of the output signal for the six polarization states, forms the basis for the control unit operation in the search stage of controller operation. The search stage is defined as the initial stage of the adaptive process. The objective of the search stage is to obtain sufficient information, concerning the polarization of the incident wave, in order that the system variables, μ and ψ, can be adjusted near their correct operating values (Eqn. 3-19). An examination of Table 4-3 in conjunction with Eqn. (4-11) shows that if the output for any one of the last four states is greater than the output for the other states, then the value of μ for that state (Table 4-1) defines Φ to within the range

$$-\pi/4 < \Phi < \pi/4 \; .$$

The range of ψ is

$$\pi/4 < \psi < 3\pi/4 \; .$$

When the output for state 1 or 2 is greater than all others, the correct value for μ can be found by observing the phase difference with ψ set for the greater output (Table 4-1). For state 1 greater ψ is equal to zero so that Eqns. (4-12) and (4-13) become,

$$b_2' = b \cos \Phi - jb \sin \Phi$$

or

$$b_2' = b\, e^{-j\Phi}$$

and

$$b_4' = a\, e^{-j\,\pi/2} \quad .$$

The phase difference is

$$\Phi_d = \Phi - \pi/2$$

or

$$\Phi_d = \mu - \phi_1 - \pi \quad .$$

Hence the correct value for μ is determined by changing μ until the difference in phase between arm 4 and arm 2, referenced to arm 4, is $-\pi/2$ radians. For state 2 the greater, the correct value for μ is obtained when

$$\Phi_d = \pi/2 \quad .$$

If the controller operation is based on the discussion given above, it is seen that the search stage obtains sufficient data to allow the adjusting of phase shifters μ and ψ to within $\pm\ \pi/4$ radians of their correct values.

Lock-On and Track

The lock-on stage is initiated at the termination of the search stage. The purpose of the lock-on stage is to examine the data obtained during the search stage and based on this examination adjust the phase shifters as close as possible to their correct values. The track stage of controller operation relies, primarily, on the information contained in the phase relation that exists between the signals b_2 and b_4. The track stage must adjust the phase shifters as close as possible to the polarization matched condition, and must maintain this condition in the presence of an incident wave with varying polarizations characteristics.

The phase difference for the general case is most easily determined from a study of Eqns. (4-12) and (4-13). Recall that the matched condition is obtained when

$$\Phi = 0$$

and

$$\cot \psi/2 = b/a$$

or

$$a \cos \psi/2 - b \sin \psi/2 = 0 \ .$$

Also,

$$a \sin \psi/2 + b \cos \psi/2 \cos \Phi > 0$$

for

$$0 \leq \psi \leq \pi$$

and

$$-\pi/4 < \Phi < \pi/4 \ .$$

If the matched value for ψ is labeled ψ_o, then

$$b = \cos \psi_o/2$$

and

$$a = \sin \psi_o/2 \quad .$$

The quantity

$$a \cos \psi/2 - b \sin \psi/2 \cos \Phi$$

becomes

$$\frac{1}{2} \left[\sin \frac{\psi_o + \psi}{2} (1 - \cos \Phi) + \sin \frac{\psi_o - \psi}{2} (1 + \cos \Phi)\right]$$

which is

$$\frac{1 + \cos \Phi}{2} \left[\sin \frac{\psi_d + 2\psi}{2} \tan^2 \Phi + \sin \frac{\psi_d}{2}\right] \qquad (4\text{-}14)$$

where

$$\psi_d = \psi_o - \psi \quad . \qquad (4\text{-}15)$$

It is seen that (4-14) is positive for

$$\psi_d \text{ approx. } > \; - \frac{\Phi^2}{115} \text{ degrees} \qquad (4\text{-}16)$$

when ψ_d and Φ are small.

The behavior of the phase of b_2' ($\angle b_2'$) as Φ approaches zero from positive values and from negative values is listed below:

$\angle b_2' < \pi/4$ for $\Phi = -\pi/4$

and approaches 0 as $\Phi \longrightarrow 0$,

$\angle b_2' > -\pi/4$ for $\Phi = \pi/4$

and approaches 0 as $\Phi \longrightarrow 0$.

The value of the phase of b_4' depends on the magnitude and sign of expression (4-14) and also the magnitude and sign of Φ. This

dependence is shown below:

$$\angle b_4' > -\pi \quad \text{for} \quad \Phi = -\pi/4 \quad \text{if} \quad (4\text{-}14) \quad \text{is} +$$
$$\text{and} \longrightarrow -\pi/2 \quad \text{as} \quad \Phi \longrightarrow 0,$$
$$\angle b_4' < 0 \quad \text{for} \quad \Phi = +\pi/4 \quad \text{if} \quad (4\text{-}14) \quad \text{is} +$$
$$\text{and} \longrightarrow -\pi/2 \quad \text{as} \quad \Phi \longrightarrow 0,$$
$$\angle b_4' < \pi \quad \text{for} \quad \Phi = -\pi/4 \quad \text{if} \quad (4\text{-}14) \quad \text{is} -$$
$$\text{and} \longrightarrow +\pi/2 \quad \text{as} \quad \Phi \longrightarrow 0,$$
$$\angle b_4' > 0 \quad \text{for} \quad \Phi = +\pi/4 \quad \text{if} \quad (4\text{-}14) \quad \text{is} -$$
$$\text{and} \longrightarrow +\pi/2 \quad \text{as} \quad \Phi \longrightarrow 0.$$

The information presented above is sufficient to develop the phase difference

$$\Phi_d = \angle b_4' - \angle b_2',$$

and its relation to ψ_d and Φ. This is presented in Table 4-4.

Table 4-4

Output Phase Behavior

Φ Approaching Zero From $\pm$ Values	Value of $a \cos \psi/2 - b \sin \psi/2 \cos \Phi$	Φ_d Approaches as $\Phi \longrightarrow 0$
Φ^-	< 0	$\pi \longrightarrow \pi/2$
Φ^+	< 0	$0 \longrightarrow \pi/2$
Φ^-	> 0	$-\pi \longrightarrow -\pi/2$
Φ^+	> 0	$0 \longrightarrow -\pi/2$
Φ^-	$= 0$	π
Φ^+	$= 0$	0

It should be noted that when $\psi_d > 0$ the phase difference approaches $-\pi/2$ radians as Φ approaches zero. Thus the phase shifter ψ must be decreased to reach ψ_o, if the phase difference is negative. Also, the value of μ must be increased if the phase difference is less than $-\pi/2$ and decreased if Φ_d is greater than $-\pi/2$. If $\psi_d < 0$ the phase difference approaches $\pi/2$ radians as Φ approaches zero, and the phase shifter must be increased to reach ψ_o. Actually, ψ_d should be greater than and less than approximately $-\Phi^2/2$ radians rather than zero; however, for the values of Φ for which this behavior is important, Φ is small. The small error that results can be compensated for in the controller logic. The relations displayed in Table (4-4) and discussed above form the logic for the track stage of the digital controller operation. A flow chart of the digital controller logic is presented in Appendix B.

Summary

The results obtained in this chapter were based on the restriction that phase shifter ψ is lossless. The same results are obtained when loss is present, provided an equal loss is inserted in the second transmission line connecting the magic tees of Fig. 7. Even if this is not done, it is not expected that the results will be significantly altered provided the loss is small (< .5 db).

The requirement to measure the amplitude of the output for the six polarization states and determine which state has the greater output is fundamental to the controller operation. The measuring device will have some increment within which all amplitudes will appear equal.

Hence, there will be areas on the polarization chart, rather than points, that represent polarization states of an incident wave, producing "equal" amplitude response. These areas can be determined from Fig. 13 for a given increment and transposed to Fig. 12 to determine the polarization states involved. The digital controller must be programmed to include this effect.

The six polarization states considered (Table 4-3) form three cross-polarized pairs. Any one state is 3 db below all other states except its cross-polarized state. It was seen from Table 4-2 and also from Fig. 13 that an incident wave of any polarization would produce an output no more than 1 db below maximum for at least one of the six states. Hence, the threshold for acquisition is never more than 1 db above the system threshold.

This chapter has considered in detail the phase difference (Φ_d) existing between the two system output ports. The behavior of Φ_d, as the system parameters μ and ψ approach their correct values, provides data required for system control. In Chapter III it was required that the signal at b_4 vanish when the system parameters reached their correct value. It was also seen in Chapter III that a loss accompanying phase shifter ψ prevented the signal at b_4 from reaching zero. In any respect, it is to be expected that non-ideal system components will prevent the signal at b_4 from ever reaching more than 50 db below the signal at b_2. This will occur only when the phase shifters are within a couple of degrees of their correct value. Thus it is evident that a phase measurement is required on signals separated in amplitude by as much as 50 db. This dynamic range requirement on the phase

measurement can be somewhat reduced by programming the digital controller to expect a loss of phase information near the correct operating point. A method for obtaining the phase measurement will be considered in the next chapter.

CHAPTER V

A DESCRIPTION OF THE SYSTEM COMPONENTS

The result of the analysis presented in the previous chapters has served to define necessary conditions on the system components. It is the purpose of this chapter to consider in more detail the individual system components.

The Radiating Elements

The linear polarized radiating elements were described mathematically by their complex vector lengths

$$h_x(\theta,\phi)\ \underline{u}_x$$

and

$$h_y(\theta,\phi)\ \underline{u}_y\ ,$$

where the angles θ and ϕ are the polar coordinate angles referred to the z and x axes respectively. The radiating elements can be any structures that will radiate a linear polarized field. An array of dipoles or slots is one such structure. If the phase centers of the two elements are not coincident, it is necessary to assign the phase, resulting from the difference in electrical length between phase centers, to the phase of one of the vector lengths referred to the other.

The polarization-adaptive mode of operation requires that the vector lengths have equal magnitude (Eqn. 3-18). This is a rather stringent requirement when it is considered that the magnitudes are functions of the angle of arrival of the incident wave. Crossed dipoles have vector lengths with equal magnitude provided the dipoles are identical and the direction of incidence is perpendicular to the plane of the dipoles. At any other angle of incidence the magnitudes are unequal. It is well known that the conical horn antenna, operating in the dominant TE_{11} mode, has effectively a tapered aperture distribution in the electric plane. For this reason its beamwidths in the electric and magnetic planes are more nearly equal than those with a square pyramidal horn [13]. Hence a conical horn excited with orthogonal TE_{11} modes will more nearly satisfy the equal magnitude requirement, for angles off boresight, than will a square horn excited with the TE_{01} and TE_{10} modes.

Potter [14] has presented a technique which simultaneously results in complete beamwidth equalization in all planes, complete phase center coincidence, and at least 30 db sidelobe suppression. The technique, referred to as the "dual-mode conical horn", utilizes a conical horn excited at the throat region in both the dominant TE_{11} mode and the higher-order TM_{11} mode. This structure has the appropriate symmetry to support two orthogonal TE_{11} modes and as such is suitable as the radiating elements (each mode considered as one element) for the polarization-adaptive application. It satisfies the following relation;

$$h_x(\theta,\phi) = h_y(\theta,\phi)$$

for all θ and ϕ within a 15 db beamwidth.

A "finloaded horn antenna" [15] has the characteristics necessary to satisfy the equal magnitude requirement. The finloaded horn differs from the conventional pyramidal horn in that it has thin metal fins perpendicular to all four sides. The fins modify the field in the horn, and, therefore, this horn has several advantages over the simple pyramidal horn, such as:

a) By proper selection of the fin size the electric and magnetic plane patterns can be made equal.

b) The square shaped horn retains its two symmetry planes, and therefore behaves identically for the two orthogonal modes.

The system will operate with two circularly polarized elements (one right circular and the other left circular); however, the analysis presented in Chapter III must be modified. Most circularly polarized antennas, with the exception of the helical class of antennas, obtain their polarization characteristics from the simultaneous illumination of orthogonal linear modes and are, therefore, more complex than linear polarized antennas. It was for this reason that the analysis was performed on a system utilizing linear radiating elements.

The Digital Phase Shifters

The polarization-controllable antenna utilizes two phase shifters. One phase shifter requires a 180 degree differential phase shift range while the second requires a 360 degree range. The phase shifters must be capable of reciprocal operation, i.e., the same phase must be introduced regardless of the direction of propagation.

The advantage gained by using digital phase shifters was considered in Chapter IV. These phase shifters normally use PIN diodes or ferrite toroids to produce the necessary phase steps. They are capable of switching speeds near a microsecond.

System operation in the adaptive mode requires the polarization tracking of an incident wave with varying polarization characteristics. This tracking should be accomplished with a minimum reaction on the transmission properties of the system (since tracking is performed concurrent with the recovery of information from the modulation content of the incident wave). An examination of Fig. 12 shows that a continuous change in the incident wave polarization introduces the possibility that the μ phase shifter will be required to change 348.75 degrees in one step. This occurs when the incident polarization track (as displayed on the Smith Chart) crosses the positive real p axis (cf. Fig. 12). The effect of switching through a phase step of this magnitude is one of introducing a loss (amplitude and phase modulation by the system), during the switching, in the transmission properties of the system. The same problem is inherent in the operation of the digital phase shifters. Consider the operation of a digital phase shifter when changing from the 168.75 degree value to the 180 degree value. The 11.25, 22.5, 45, and 90 degree phase increments must be simultaneously reduced to zero while the 180 degree phase increment is increased from its zero state to the 180 degree state. It can be expected that the behavior will differ considerably from the ideal, which is a smooth transition from the 168.75 degree state to the 180 degree state. Thus to reduce the system interaction with the signal, it is necessary to use high

switching speeds. It can be concluded that it is impossible to perform the tracking operation, using digital phase shifters, without introducing system generated noise into the received signal. It is necessary to take into consideration this source of interference when investigating applications for adaptive operation.

The Digital Controller

The digital controller is a special purpose digital computer designed to follow a sequence of logical instructions based on the amplitude and phase (Φ_d) relations discussed in Chapter IV. The sequence of instructions evolve by introducing a change in the polarization state of the system and then measuring the effect this change has on the output amplitude and phase.

The operation of the controller progresses through three distinct stages. The first stage, the search stage, determines if a signal is incident on the antenna system with an intensity exceeding a predetermined threshold level. The polarization state or states which produce an output that exceeds the threshold level are stored along with the magnitude of the output referenced to the threshold level. The second stage, the lock-on stage, examines the data stored by the search stage and determines which of the six states is the greater. As previously noted, it is possible for two and even three polarization states to produce the same output magnitude. The lock-on stage examines each of these possibilities to determine which applies. The results of this examination provides the values for phase shifters μ and ψ within the limits shown in Table 4-3. The operation of the

third stage, the tracking stage, is based on the data presented in Table 4-4. The tracking stage continually samples the value of Φ_d. Phase shifter μ is changed until Φ_d is near $\pm \pi/2$, and then ψ is adjusted so that Φ_d switches between $+ \pi/2$ and $- \pi/2$.

A flow chart representing the logical operation of the digital controller is shown in Appendix B. The operation of the search and lock-on stages follow step by step the procedure outlined above. In an earlier chapter it was pointed out that near the correct operating values for the phase shifters the signal at b_4 is small. For this reason it is possible that a phase difference measurement cannot be performed. Thus those logical operations calling for a phase difference measurement could receive a reply indicating that a phase measurement is impossible. The next logical step should be a check to insure that the output is above threshold. If this is the case the phase measurement is again attempted. An output below threshold returns operational control to the control console. The control console can reinstate the search stage, set the system to some perferred polarization state, or perform any other desired operation.

The track stage logic must be modified to include the possibility that phase shifter μ could cause the phase differenc (Φ_d) to change sign (in the ideal situation Φ_d changes sign only when Ψ passes Ψ_d). This effect was discussed in the paragraph following Table 4-4. The modification consists of following each operation that changes the state of the μ phase shifter with an operation that determines if Φ_d has changed sign. This condition is possible only when the phase shifters are within one bit of their correct

value.

It should be obvious from Fig. 12 that two polarization states, linear vertical, and linear horizontal are associated with an arbitrary value of phase (ϕ_1). If the system is tracking a changing polarization and the track passes through either of these two states, it is extremely likely that Φ will exceed the limits $\pm \pi/4$. Since the tracking logic requires that Φ be within these limits for proper operation the system will break-lock. To prevent this from occurring it is necessary to examine the state of Ψ each time it is changed. If it has been changed to the 0 degree state or the 180 degree state the control logic switches back to the lock-on stage (location numbers 3 and 4).

There is a rate of change of the incident polarization that will exceed the tracking response time of the system. A cursory view of the controller logic leaves the impression that a time consuming procedure is involved. This is not true, e.g., only one of eighteen possible parallel paths is traversed in the lock-on stage of operation. Considering the fact that high speed digital logic is currently available, it is concluded that the system response will be limited by the time required to perform the phase difference measurement (Φ_d).

Phase Measurement Equipment

A wide variety of techniques for obtaining microwave phase measurements have been considered by Dyson [16]. The techniques that apply to the problem at hand hetrodyne the microwave signal

to a low frequency (less than 1MHz) where the phase measurement can be obtained from a time interval measurement. The bandwidth of the low frequency circuit is determined by the minimum signal level (threshold level) from which phase information is desired. The time required to perform the phase measurement is inversely proportional to the bandwidth. The measuring scheme that provides the maximum sensitivity utilizes a minimum bandwidth which in turn requires a very stable local oscillator and signal source. Thus it can be concluded that the time required to perform a phase measurement is dependent on the required system threshold level. It is also apparent that applications for the system which require a maximum sensitivity will require a more complex phase measuring technique. Under this condition the system response time will be a maximum, i.e., it will track an incident wave polarization only if the rate of change of polarization is slow. It can be stated that a trade-off between system sensitivity and response time is necessary when considering applications involving the adaptive mode of system operation.

It is of interest to consider a measuring scheme that will allow a maximum sensitivity. It is necessary to measure the phase difference between two signals separated in magnitude by as much as 30 db. Since the system threshold is determined by the minimum acceptable signal at b_2 it is necessary to obtain reliable phase information from a signal (b_4) 30 db below the threshold. The phase measuring device is shown in Fig. 14 in block diagram form. It is a crosscorrelation phase measurement

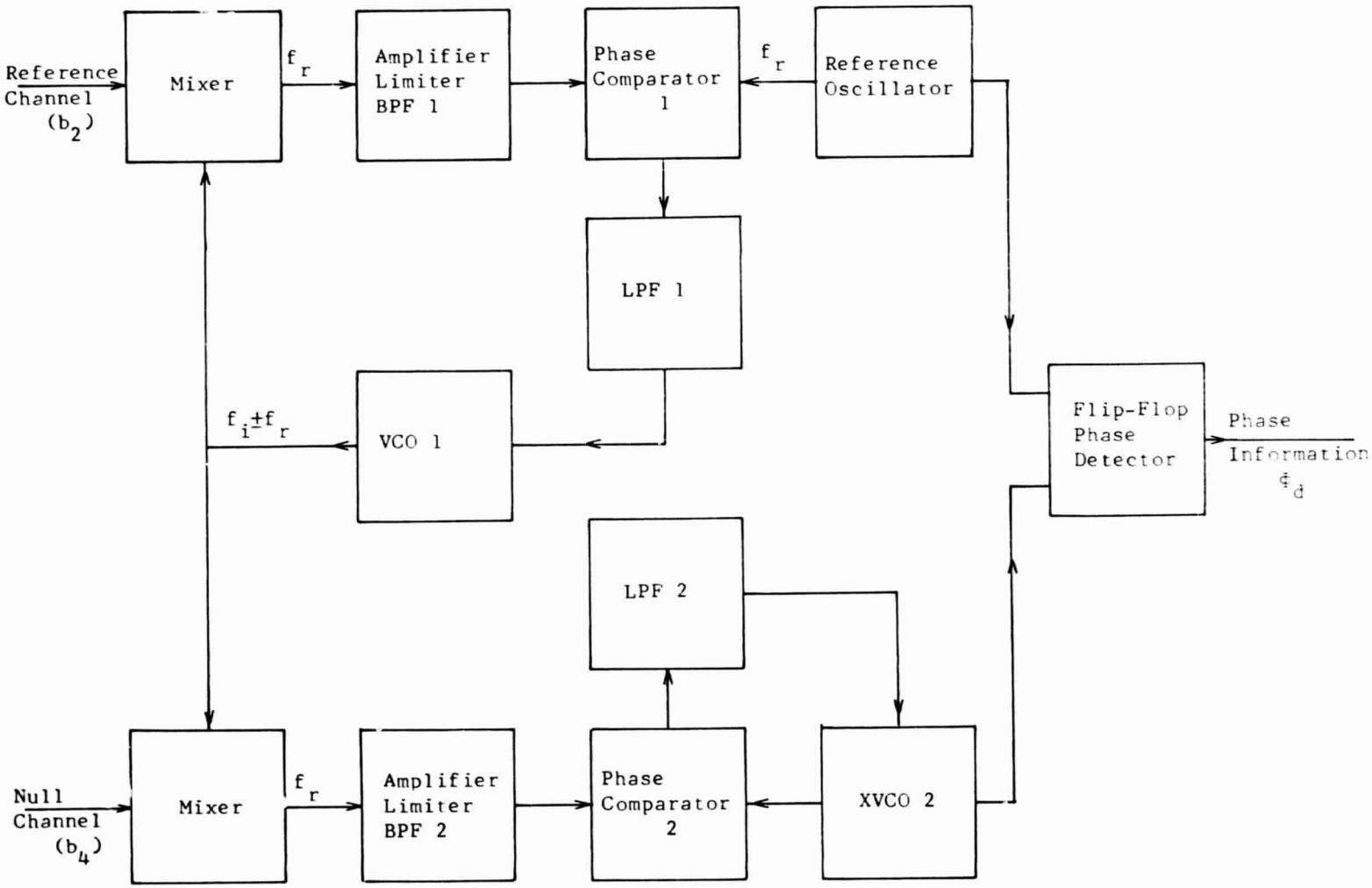


Figure 14. Crosscorrelation phase measuring device.

scheme that requires two phaselock loops, one in the reference channel (b_2), and one in the null channel (b_4). The phase measurement is only possible when both loops are phaselocked. The reference channel loop, loop 1, is phasedlocked to a crystal controlled oscillator which also supplies the reference signal to the phase detector. The loop 1 bandwidth should be as wide as possible to permit rapid attainment of the phaselocked condition. This bandwidth will be determined by the reference channel threshold and the type of modulation on the signal. The null channel loop (loop 2) is required to track the phase difference between the two channels. This loop consists of a crystal voltage controlled oscillator with the same center frequency as the reference oscillator. In fact the two crystals can be enclosed in the same temperature controlled oven to minimize relative temperature drift. It is, however, necessary to insure a maximum of isolation between the two oscillators in order to prevent injection locking. For proper operation the loop 2 bandwidth should be one thousandth of the loop 1 bandwidth. This means a narrow loop 2 bandwidth and a long phaselock response time. Limiters are shown with both loops to provide an effective increase in loop bandwidth for signals above the threshold level. These amplifier limiter combinations must possess identical phase shift characteristics, independent of the signal level. This measuring scheme provides the stable reference signals required for minimum bandwidth applications.

The phase measuring device must be capable of deciding if a phase measurement is possible. This can be accomplished in the

relatively wide bandwidth devices by establishing a threshold level in the null channel. When the signal in the null channel drops below this level the phase measuring device will indicate that a phase measurement is not possible. The crosscorrelation scheme will supply phase information when the loops are locked. The output of the loop filters can be examined for the locked condition and the results used to indicate the availability of phase information.

The purpose of this section has been to describe the factors that should influence the choice of a device capable of automatic phase measurements. It was not intended to describe a particular device satisfactory for all applications, but rather to demonstrate that at least on paper the phase measuring requirements can be satisfied.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The applications for polarization-controllable antennas have been arbitrarily divided into two classes. The first class consists of those applications that would benefit from the adaptive feature of the polarization-controllable antenna. The second class concerns applications that derive information from the polarization characteristics of the incident wave.

The polarization-controllable antenna system has been analyzed to determine system requirements for the adaptive mode of operation. The results of this analysis are summarized below:

(1) the magnitude of the vector effective lengths of the two linear radiators must be equal at the angle of incidence;

(2) the complex form of the signals at the system output arms require a complicated control process;

(3) the control process must introduce a random phase modulation on the incident signals in order to obtain the control information and;

(4) an automatic phase measurement is required between two signals that will vary over a wide dynamic range.

Two antennas of the horn type construction satisfy the "equal magnitude" restriction. These antennas are broad beamwidth low gain structures. They are, however, suitable as the feed structures for the high gain narrow beamwidth reflector class of antennas. A special purpose digital computer can perform the control process required for adaptive operation. The requirement for an automatic phase measurement presents the major problem that will be encountered in the implementation of an adaptive system.

It can be concluded that the polarization-controllable antenna is capable of adaptive operation. The system is feasible for those applications that could encounter severe polarization mismatch loss; however, the complexity of the system, its "slow" response to a changing polarization, and the interference introduced by the random phase modulation must be considered before any conclusion can be reached regarding a particular application. For certain applications it is conceivable that polarization tracking would not be required, but that it would be necessary to search and obtain a good (within .7db) polarization match. This operation could be performed without phase measurements and thus could be performed in a short period of time by the digital controller.

The equations that govern system operation in the transmit and receive mode of operation have been derived. The system parameters required to transmit or receive a wave of any given polarization characteristics can be determined from these equations. It is shown in Appendix C that the polarization-controllable antenna allows simultaneous reception of the matched and cross-polarized components of an incident

wave. The cross-polarized component is available as a system output at arm 4 (b_4).

The polarization-controllable antenna can be used as a laboratory instrument for antenna pattern measurements for arbitrary polarization states. For radar applications the system is capable of transmitting a wave of any polarization characteristics and then receiving simultaneously the matched and cross-polarized components of the reflected wave. This provides additional information concerning the nature of the reflecting object. In fact, a polarization signature could be obtained by changing the system polarization state (in a few microseconds) between successive illuminations of the object. The rotation rate (provided it is not about an axis of revolution of the object) of a reflecting object could be determined by transmitting a linear polarized wave and then switching the system state immediately to receive a circular polarized wave. The received signals at b_2 and b_4 will differ in frequency by an amount equal to twice the rotation frequency.

In general it can be concluded that the polarization-controllable antenna is feasible. It is recommended that the problems associated with automatic phase measurements receive careful evaluation. A simple solution to these problems would eliminate the undesirable aspects of system operation in the adaptive mode.

REFERENCES

1. Kraus, J. D., Antennas, McGraw-Hill Book Company, New York, p. 466; 1950.

2. ibid, p. 478.

3. Rumsey, V. H., "Transmission between elliptically polarized antennas," Proc. IRE, vol. 39, no. 5, pp. 535-540; May, 1951.

4. Smith, P. H., "A new negative Smith Chart," Microwave J., vol. 8, no. 6, pp. 83-92; June, 1965.

5. Sinclair, G., "The transmission and reception of elliptically polarized waves," Proc. IRE, vol. 38, no. 2, pp. 148-151; 1950

6. Schelkunoff, S. A., Electromagnetic Waves, D. Van Nostrand Co., New York, p. 479; 1943.

7. Tai, C. T., "On the definition of the effective apertures of antennas," IRE Trans. on Antennas and Propagation, vol. AP-9, no. 2, pp. 224-225; March, 1961.

8. Teeter, W. L. and Bushore, K. R., "A variable-ratio microwave power divider and multiplexer," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, no. 4, pp. 277-229; October, 1957.

9. Montgomery, C. G., et.al., Principles of Microwave Circuits, McGraw-Hill Book Company, New York, pp. 302-303; 1948.

10. ibid, p. 149

11. Tompkins, R. D., "A broad-band dual-made circular waveguide transducer," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, no. 3, pp. 181-183; July, 1956.

12. Mueller, G. E. and Tyrrell, W. A., "Polyrod Antennas," Bell System Tech. J., vol. 26, pp. 837-851; October, 1947.

13. Potter, P. D., "A new horn antenna with suppressed sidelobes and equal beamwidths," Microwave J., vol. 6, no. 6, pp. 71-78.

14. ibid.

15. Komlos, S. G., et.al., "Feed System for clockwise and counter-clockwise circular polarizations," IRE Trans. on Antennas and Propagation, vol. AP-9, no. 6, pp. 577-578; November, 1961.

16. Dyson, J. D., "The measurement of phase at UHF and microwave frequencies," IEEE Trans. an Microwave Theory and Techniques, vol. MTT-14, no. 9, pp. 410-423; September 1966.

APPENDIX A

A SIMPLE WAVEGUIDE SYSTEM FOR RADIATING ELLIPTICALLY-POLARIZED WAVES

In a study of the polarization properties of a microwave antenna system it is often beneficial to have available an antenna capable of transmitting a field with any desired polarization. An antenna system which can provide this capability consists of a circular waveguide horn fed by a circular guide loaded with a quarter-wave plate, a circular waveguide-to-circular waveguide collinear rotary joint, a rectangular-to-circular waveguide (TE_{11} mode) transducer, and a rectangular-to-rectangular waveguide collinear rotary joint. This component arrangement is shown in Fig. A-1.

In operation the TE_{11} mode is established in the first circular guide section with a plane of symmetry dependent on the orientation of the input rectangular guide. The quarter-wave plate, whose orientation is independent of the symmetry plane of the mode, then establishes a phase shift for one component of this mode with respect to the orthogonal component.

For reference purposes the broadwall of the input waveguide is taken parallel to the horizontal plane and considered the x axis of the fixed coordinate system. The broadwall of the rotatable rectangular waveguide serves as a reference for the angular displaced axis x' with angle of displacement Υ. The angle between the y' axis and the plane

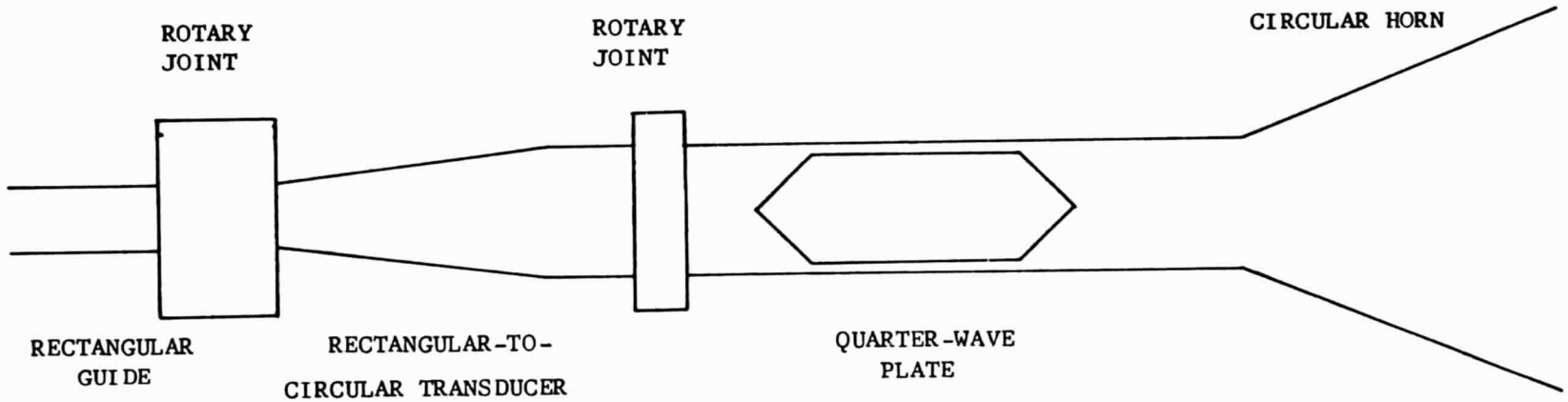


Figure A-1. Waveguide system component arrangement.

of the quarter-wave plate (plane in which linear component is delayed in phase by 90°) is denoted by δ. The unit vectors u_{11} and $u_{\perp}$ are in the plane and perpendicular to the plane of the quarter-wave plate respectively and both are transverse to the axis of revolution of the circular horn. The coordinate representation is shown in Fig. A-2.

It is obvious that the first rectangular guide section, apart from serving a transmission function, merely establishes a reference frame. It may be omitted and the rectangular-to-rectangular rotary joint replaced by, e.g., a moveable coaxial-to-rectangular transducer. Also, any antenna with circular symmetry may be used in place of the horn, e.g., a cylindrical polyrod.

In terms of this representation the far field transmitted by the antenna is expressible as

$$\underline{E}(\gamma, \delta) = E_{11}\, \underline{u}_{11} + e^{j\pi/2}\, E_{\perp}\, \underline{u}_{\perp} \qquad \text{(A-1)}$$

The quantities E_{11} and $E_{\perp}$ are the relative-field strengths, in the plane and perpendicular to the plane of the quarter-wave plate respectively, on the axis of the horn at the far field point. From Fig. A-2 it is seen that

$$\underline{u}_{11} = \cos(\gamma + \delta)\, \underline{u}_{y} - \sin(\gamma + \delta)\, \underline{u}_{x} \qquad \text{(A-2)}$$

$$\underline{u}_{\perp} = \sin(\gamma + \delta)\, \underline{u}_{y} + \cos(\gamma + \delta)\, \underline{u}_{x} \qquad \text{(A-3)}$$

also that

$$E_{11} = E_{y'} \cos \delta \qquad \text{(A-4)}$$

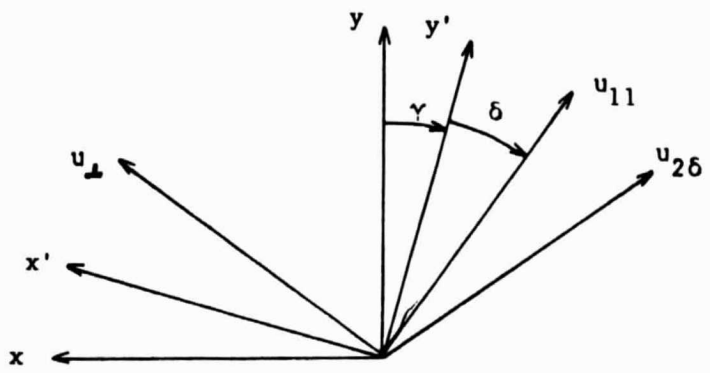


Figure A-2. Coordinate representation of waveguide system.

and

$$E_{\perp} = E_{y'} \sin \delta \ . \qquad \text{(A-5)}$$

For convenience Eqn. (A-1) can be normalized to unity by requiring $\left|E_{y'}\right| = 1$. Using Eqn. (A-2) thru (A-5), Eqn. (A-1) becomes

$$\underline{E}(\gamma, \delta) = \cos \delta \, [\cos(\gamma + \delta)\underline{u}_y - \sin(\gamma + \delta)\underline{u}_x]$$

$$+ e^{j\pi/2} \sin \delta \, [\sin(\gamma + \delta)\underline{u}_y + \cos(\gamma + \delta)\underline{u}_x] \ . \qquad \text{(A-6)}$$

Simple trigonometric manipulations of Eqn. (A-6) yield

$$\underline{E}(\gamma,\delta) = 1/2 \left\{ [\cos \gamma \underline{u}_y - \sin \gamma \underline{u}_x] + [\cos(\gamma + 2\delta)\underline{u}_y - \sin(\gamma + 2\delta)\underline{u}_x] \right.$$

$$\left. + e^{j\pi/2} \, [\cos \gamma \underline{u}_y - \sin \gamma \underline{u}_x] + e^{-j\pi/2} \, [\cos(\gamma + 2\delta)\underline{u}_y - \sin(\gamma + 2\delta)\underline{u}_x] \right\} \ . \qquad \text{(A-7)}$$

The first and third bracketed terms in Eqn. (A-7) are identified as unit vectors in the y' direction ($\underline{u}_{y'}$). The second and fourth terms describe a unit vector leading $\underline{u}_{y'}$ by angle 2δ ($\underline{u}_{2\delta}$), as shown in Fig. A-2.

The linearly polarized fields of Eqn. (A-7) may be expressed in terms of right circular and left circular rotating components.

$$\underline{u}_{\ell} = \varepsilon^{-j(\frac{\pi}{2} + \gamma)} \, \underline{\omega}_{\ell}$$

$$\underline{u}_{r} = \varepsilon^{+j(\frac{\pi}{2} + \gamma)} \, \underline{\omega}_{r}$$

where

$$\underline{\omega}_{\ell} = (\underline{u}_x + \varepsilon^{j\pi/2}\ \underline{u}_y)$$

$$\underline{\omega}_{r} = (\underline{u}_x + \varepsilon^{-j\pi/2}\ \underline{u}_y)$$

Physically, $\underline{u}_{\ell}$ and $\underline{u}_{r}$, if multiplies by $\varepsilon^{j\omega t}$ and the real part taken, represent left and right rotating vectors of constant length. Their phase angles are chosen so that at the time origin they coincide with $\underline{u}_{y'}$. Then

$$\underline{u}_{y'} = 1/2\ (\underline{u}_{\ell} + \underline{u}_{r})\ .$$

Making similar substitutions for $\underline{u}_{2\delta}$ causes Eqn. (A-7) to become

$$\underline{E}(\Upsilon,\delta) = \frac{\sqrt{2}}{4}\, e^{j\pi/4}\, [(e^{-j(\pi/2 + \Upsilon)} + e^{-j(\pi + \Upsilon + 2\delta)})\, \underline{\omega}_{\ell}$$

$$+ (e^{j(\pi/2 + \Upsilon)} + e^{j(\Upsilon + 2\delta)})\, \underline{\omega}_{r}\,] \qquad \text{(A-8)}$$

Additional simplification yields

$$\underline{E}(\Upsilon,\delta) = \frac{\sqrt{2}}{2}\, e^{j\pi/4}\, [e^{-j(\Upsilon + \delta + \frac{3\pi}{4})} \cos(\pi/4 + \delta)\, \underline{\omega}_{\ell}$$

$$+ e^{j(\Upsilon + \delta + \pi/4)} \cos(\delta - \pi/4)\, \underline{\omega}_{r}]\ . \qquad \text{(A-9)}$$

Equation (A-9) is in a form suitable for plotting a polarization contour on Rumsey's [1] q plane. Thus, it is seen from Eqn. (A-9) that

$$q = e^{-j(2\Upsilon + 2\delta + \pi)} \cot(\delta + \pi/4)\ . \qquad \text{(A-10)}$$

A plot of Eqn. (A-10) is shown in Fig. A-3 for $\Upsilon = 0$. It is evident that all possible axial ratios are included as δ increases from $-\pi/4$ to 0 (left circular to linear vertical polarization), with the field vector rotating in the left hand sense. The polarization changes from linear vertical to right circular as δ increases from 0 to $+\pi/4$ with all possible axial ratios included. The real part of the complex p plane is taken as negative for left-elliptical representation ($-\pi/4 \leq \delta < 0$). In terms of the more familiar quantities, axial ratio, tilt angle, and circulation sense, Eqn. (A-10) yields

$$AR = \cot \delta \qquad -\pi/4 \leq \delta \leq \pi/4$$

$$\tau = \pi/2 + \Upsilon + \delta$$

where the tilt angle (τ) is with respect to the horizontal (x axis) and the axial ratio is negative for left-elliptical polarization.

The procedure for generating any desired polarization consists of determining the magnitude of δ from the desired axial ratio and the sign of δ from the circulation sense; then the desired tilt angle determines Υ.

A logical change in this system is the replacement of the quarter-wave plate with a "variable-wave" plate, such as a ferrite slab biased transverse to the direction of propagation and parallel to the slab, which introduces a phase delay θ into the field component parallel to the plate. Physical consideration fixes the angle of polarization inclination δ at $\pi/4$ radians. This configuration is amenable to the same analysis applied to the quarter-wave plate, with the result

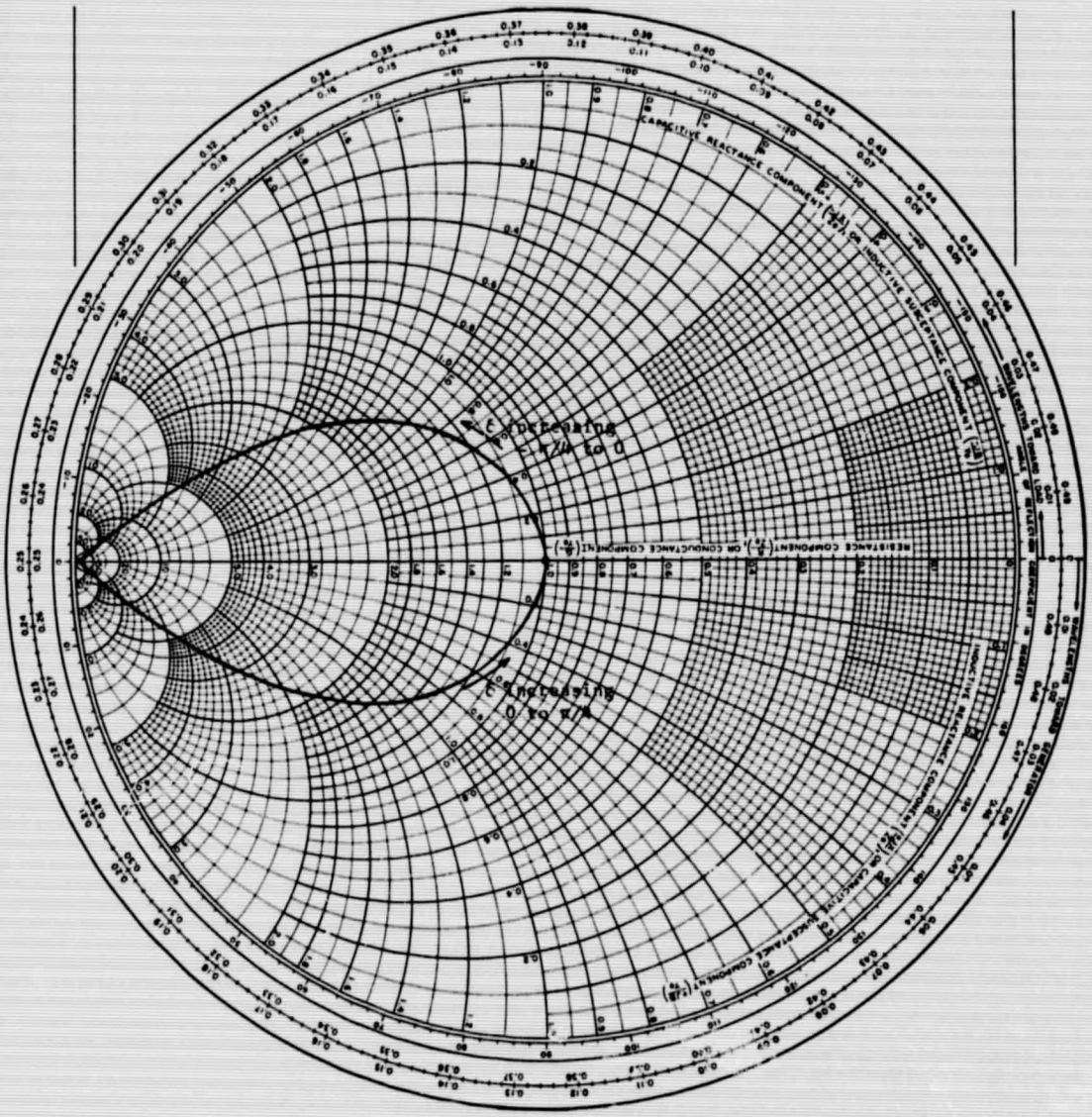


Figure A-3. Polarization contour for waveguide system.

$$q = e^{-j(\pi + 2\gamma)} \cot\left(\frac{\theta}{2} + \frac{\pi}{2}\right) \qquad \text{(A-11)}$$

leading to

$$\begin{aligned} AR &= \cot\frac{\theta}{2} \\ & \qquad -\pi/2 \leq \theta \leq \pi/2 \qquad \text{(A-12)} \\ \tau &= \frac{\pi}{2} + \gamma \end{aligned}$$

$$\begin{aligned} AR &= \tan\frac{\theta}{2} \\ & \qquad \pi/2 \leq \theta \leq 3\pi/2 \quad . \qquad \text{(A-13)} \\ \tau &= \gamma \end{aligned}$$

A plot of the polarization contour described by Eqn. (A-11) is shown in Fig. A-4 for $\gamma = 0$. The values of θ above the contour represent values for left elliptical rotation and the values below the contour are right elliptical rotation, i.e., for $0 < \theta < \pi$ the polarization generated is of RER. It should be evident that the range of θ given in Eqn. (A-12) or (A-13) is sufficient to generate any desired polarization.

The operation of the two systems described above requires the rotation of the impenging electric field vector with respect to the horizontal reference plane. The quarter-wave plate of the first system must also undergo an additional rotation with respect to the electric field vector. It is not necessary that this rotation be of a mechanical nature. Brooks [2] has described a technique to obtain an electrical rotation of the plane of a quarter-wave plate. It is also possible to perform, electrically, the rotation of the electric field vector by utilizing the Faraday rotation effect produced by a ferrite rod concentric with the circular waveguide, and biased with an axial magnetic field. However, the system will not be reciprocal.

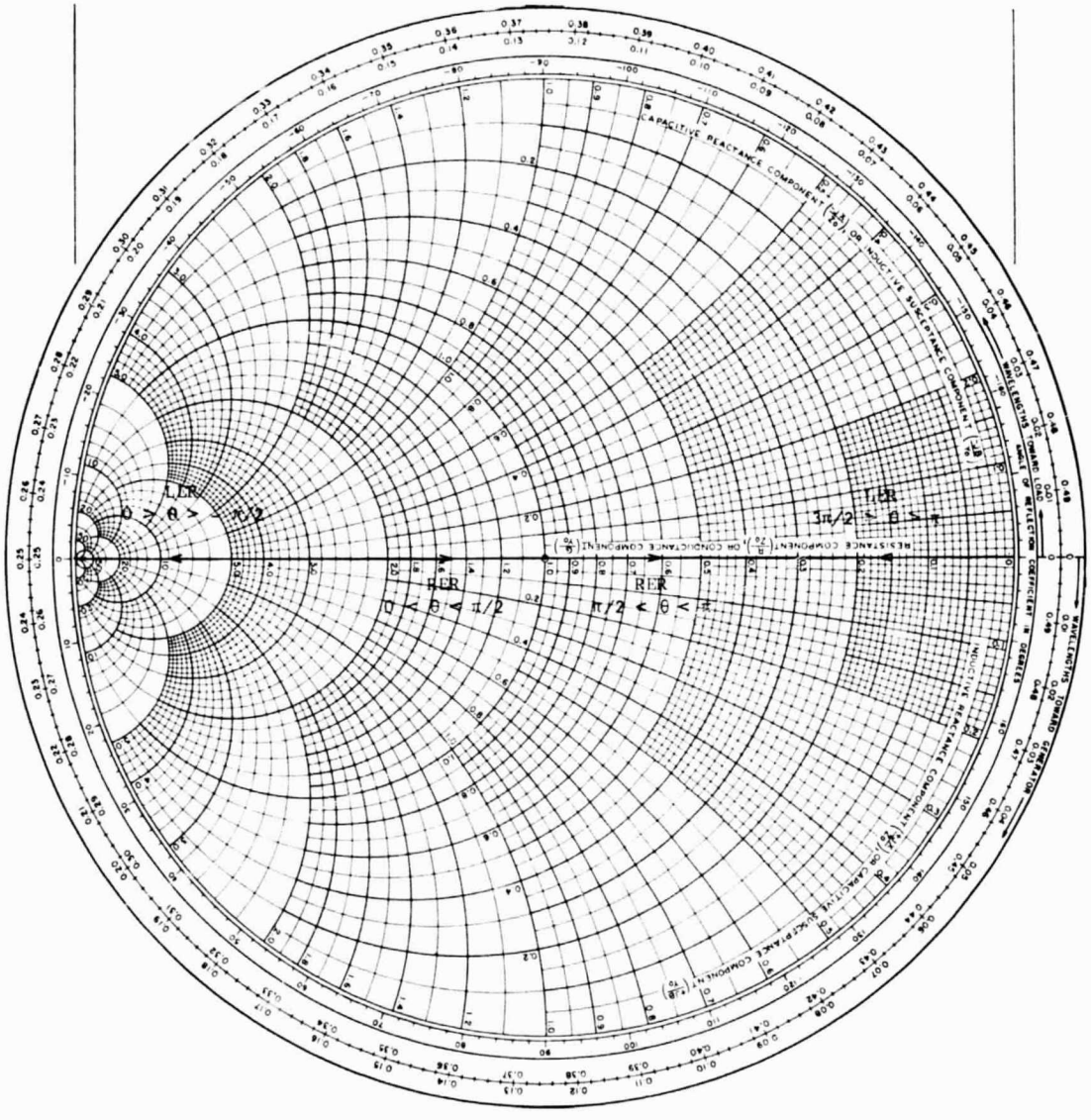


Figure A-4. Polarization contour for "variable-wave" plate waveguide system. $\gamma = 0$

Thus, it is possible to control the polarization of the transmitted field with an electrical system that involves no mechanical components. A study of the control parameters for this system could be accomplished following the techniques developed in Chapter III.

REFERENCES

1. Rumsey, V. H., "Transmission between elliptically polarized antennas," Proc. IRE, Vol. 39, pp. 535-540, May, 1951.

2. Brooks, R. O., Unpublished work, Raytheon Co., Bedford, Mass., August, 1963.
Referenced by:
Kummer, W. H., Microwave Scanning Antennas, Academic Press, New York, Vol. III, pp. 98-101, 1966.

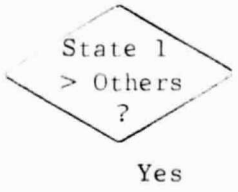
APPENDIX B

FLOW CHART FOR DIGITAL CONTROLLER LOGIC

Flow Chart Symbols

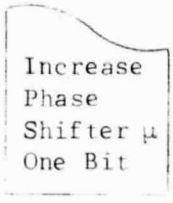
1. Compare

   Make the comparison shown in the symbol.



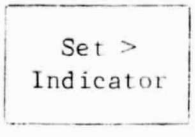
2. Output Command

   Execute the command stated in the symbol.



3. Operation Statement

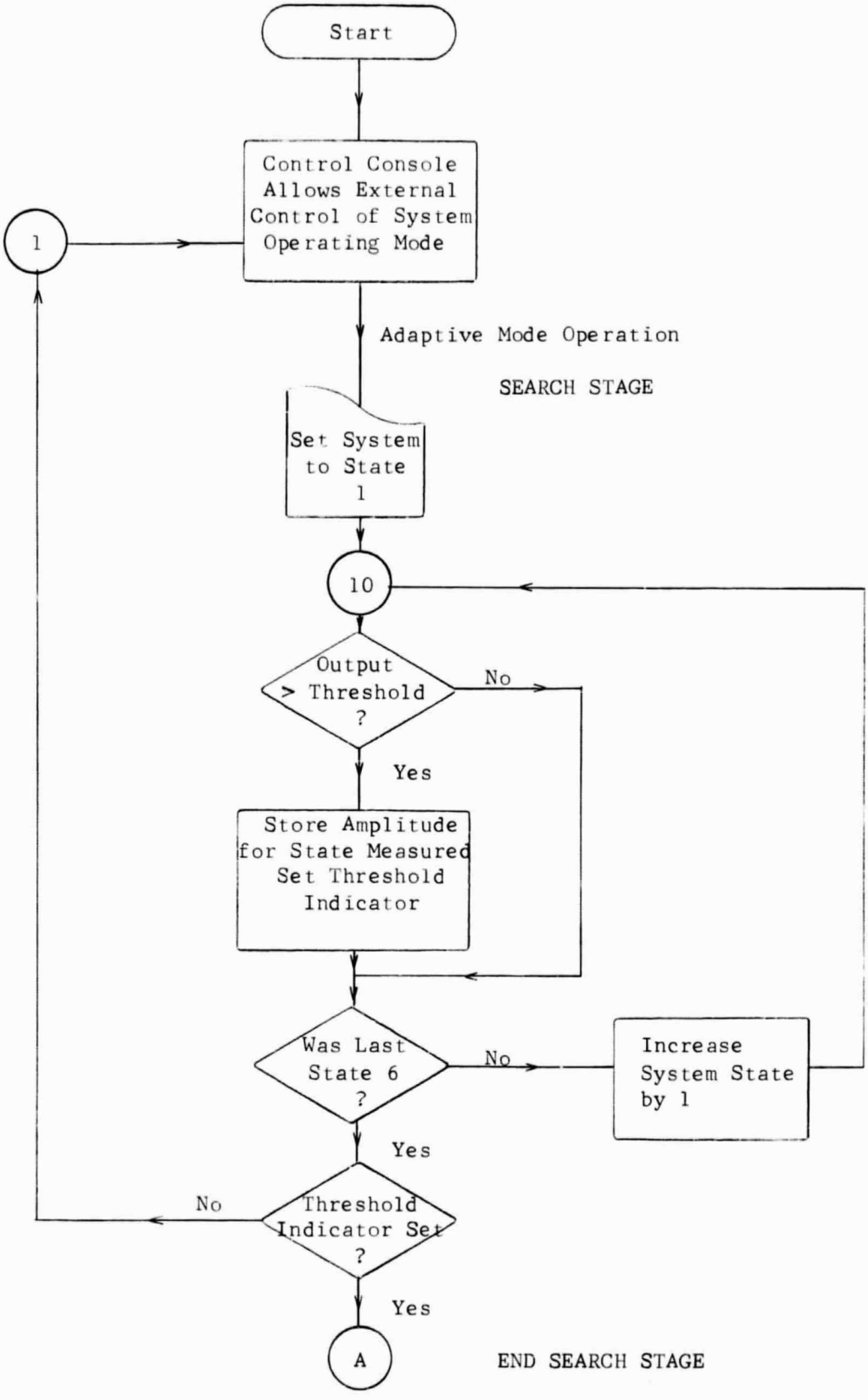
   Perform the indicated operations.

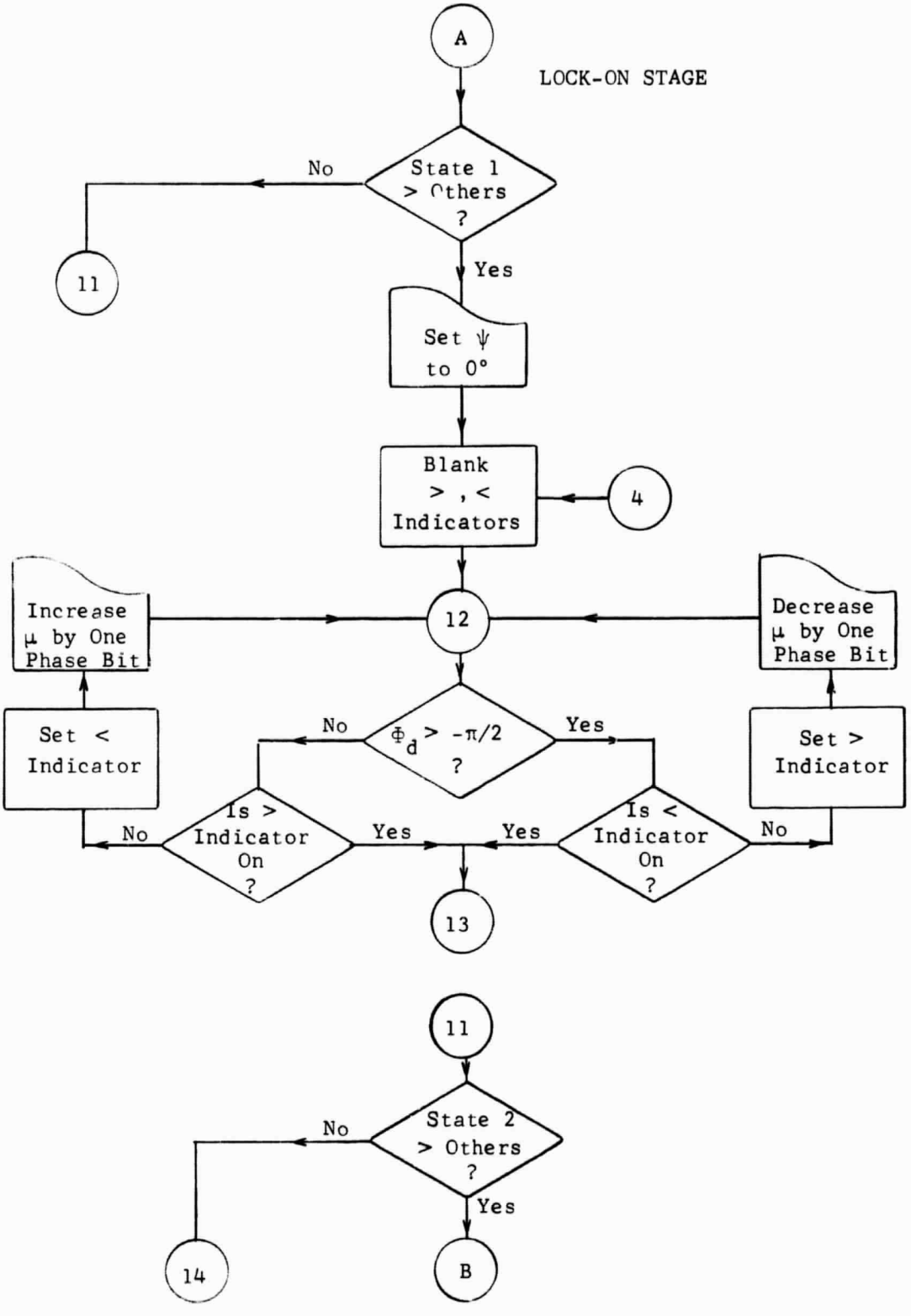


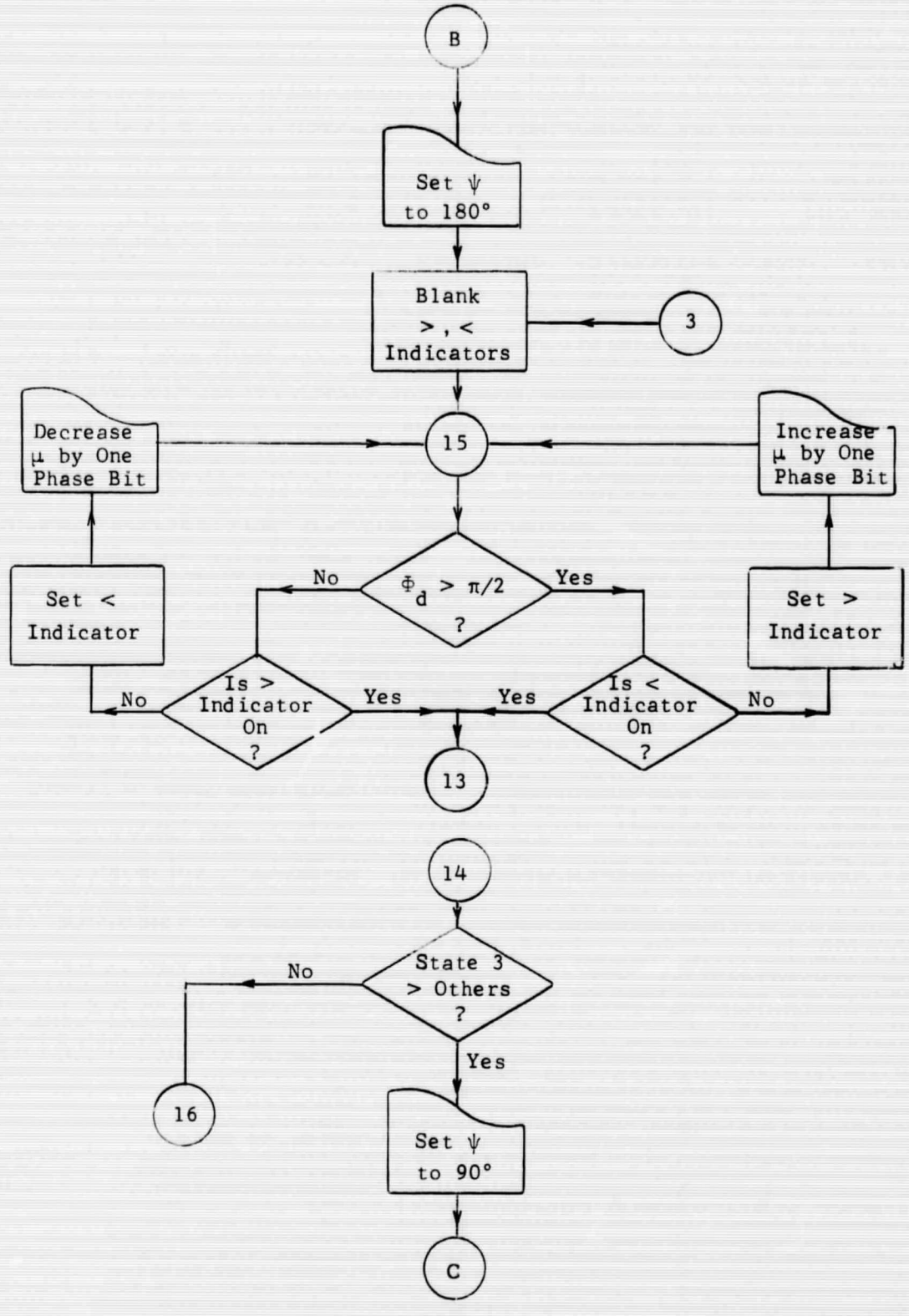
4. Location Number

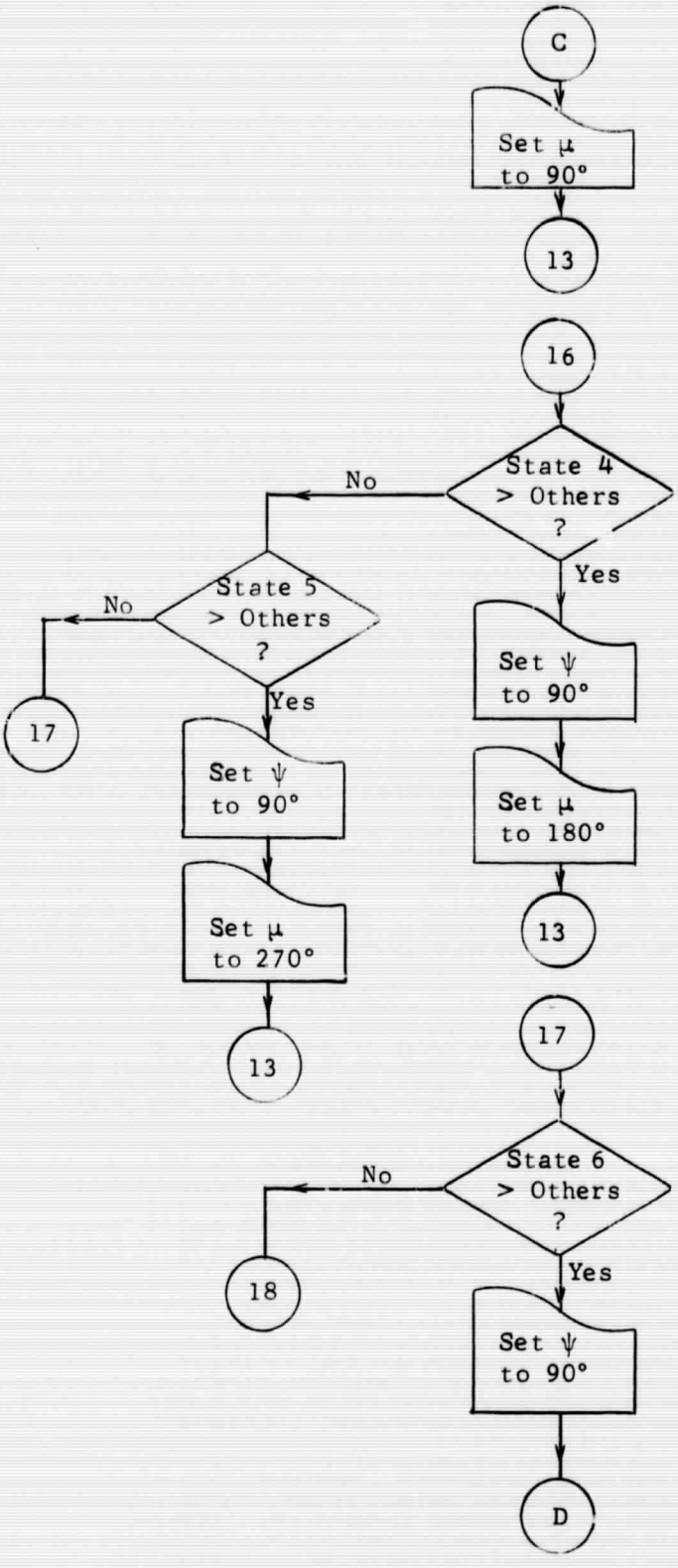
   1 This identifies the location referred as number 1.

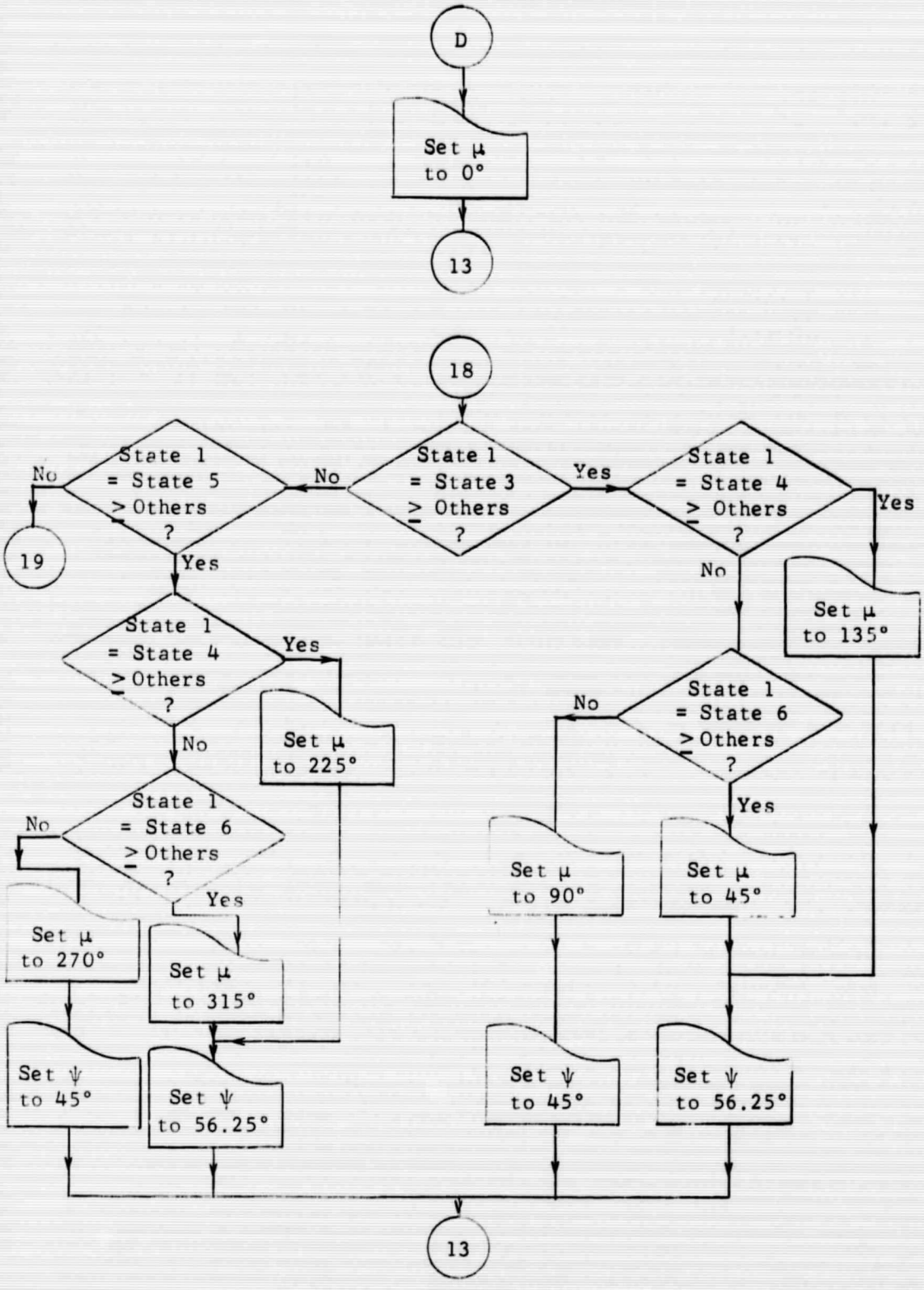


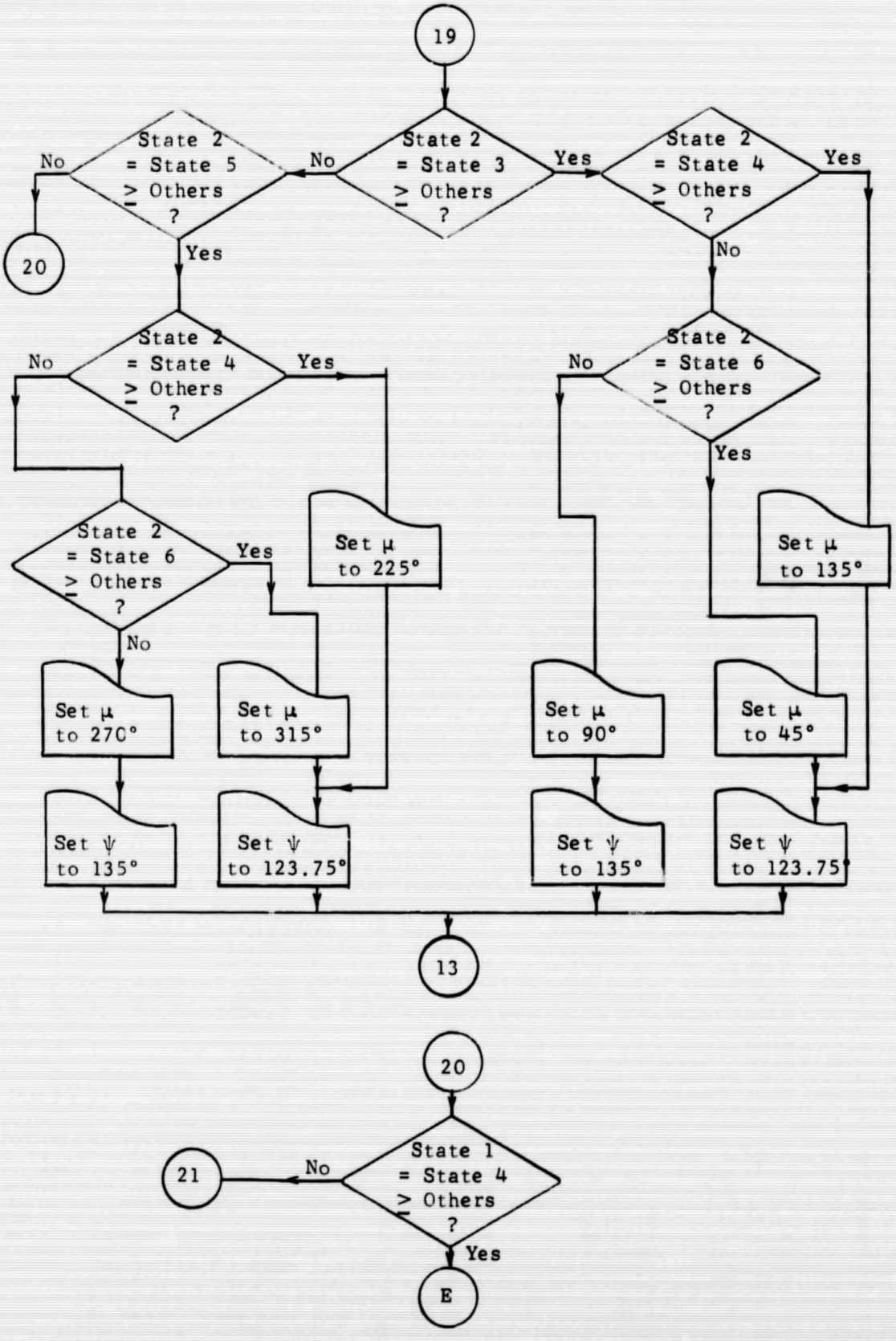

Start
Control Console Allows External Control of System Operating Mode
1
Adaptive Mode Operation
SEARCH STAGE
Set System to State 1
10
Output > Threshold ?
No
Yes
Store Amplitude for State Measured Set Threshold Indicator
Was Last State 6 ?
No
Increase System State by 1
Yes
Threshold Indicator Set ?
No
Yes
A
END SEARCH STAGE

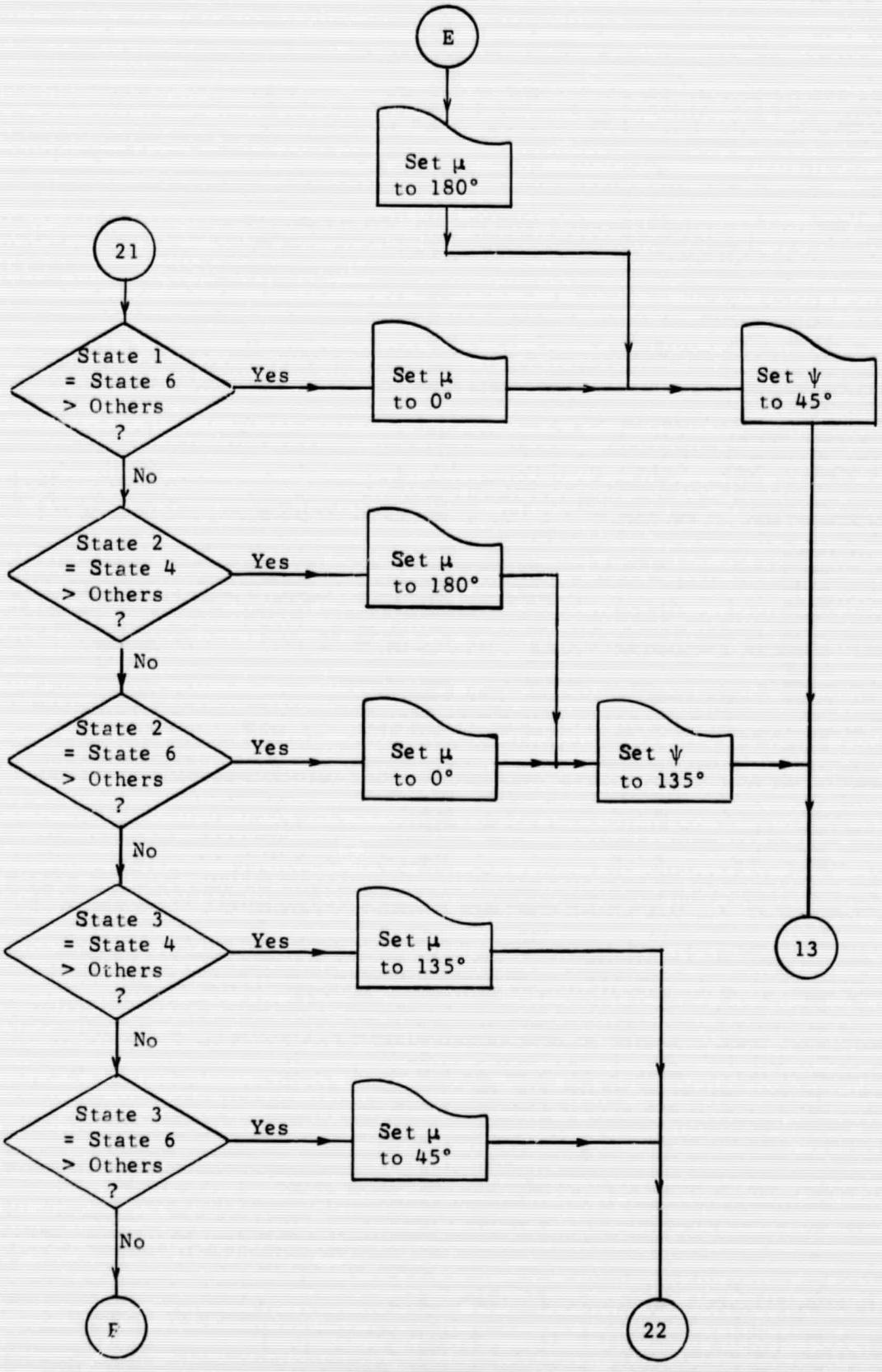

A
LOCK-ON STAGE
State 1 > Others ?
No
Yes
11
Set ψ to 0°
Blank > , < Indicators
4
Increase μ by One Phase Bit
12
Decrease μ by One Phase Bit
Set < Indicator
No
Φd > -π/2 ?
Yes
Set > Indicator
Is > Indicator On ?
No
Yes
Yes
Is < Indicator On ?
No
13
11
State 2 > Others ?
No
Yes
14
B

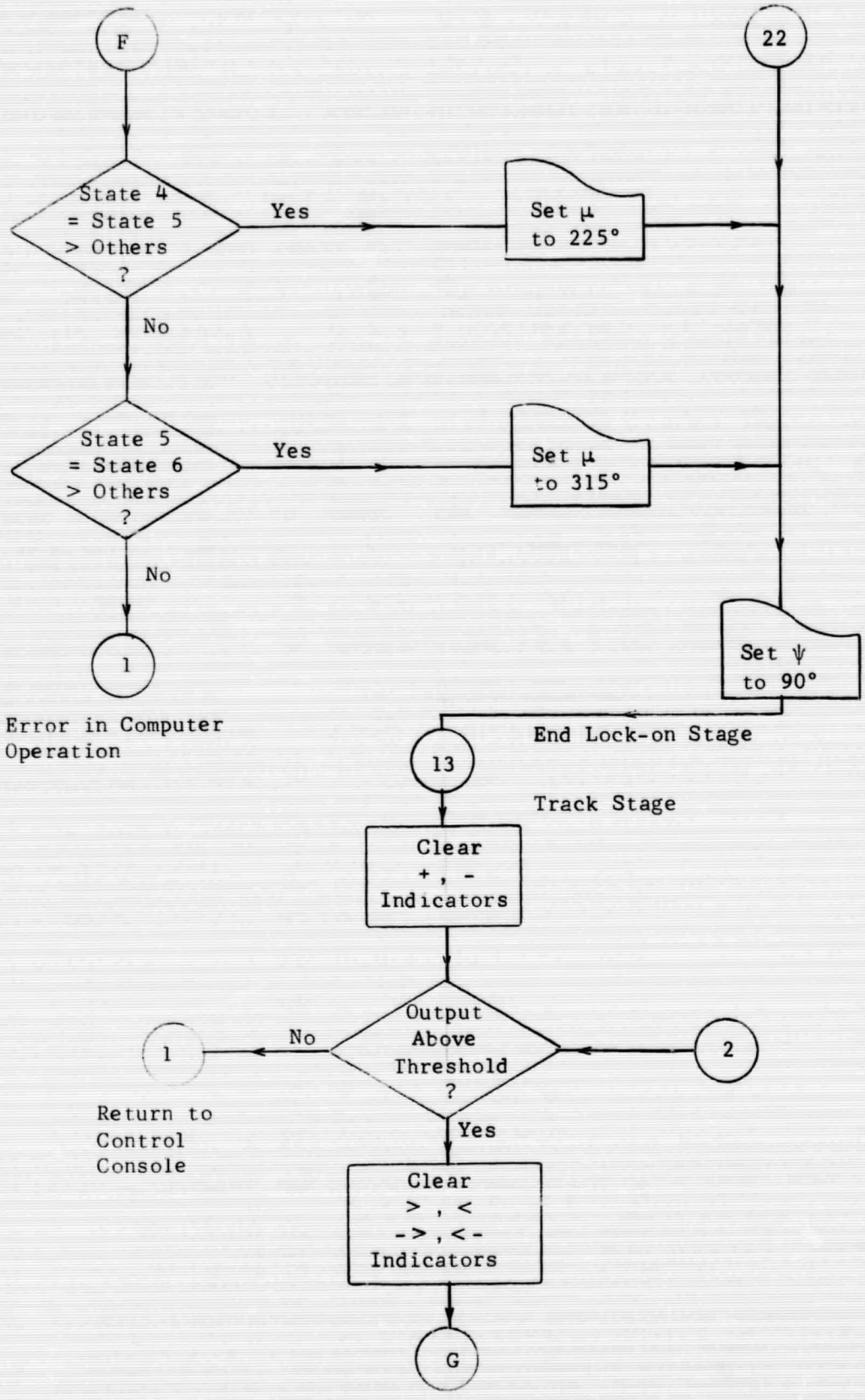

B
Set ψ to 180°
Blank >, < Indicators
3
Decrease μ by One Phase Bit
15
Increase μ by One Phase Bit
No
Φd > π/2 ?
Yes
Set < Indicator
Set > Indicator
No
Is > Indicator On ?
Yes
Yes
Is < Indicator On ?
No
13
14
No
State 3 > Others ?
Yes
16
Set ψ to 90°
C

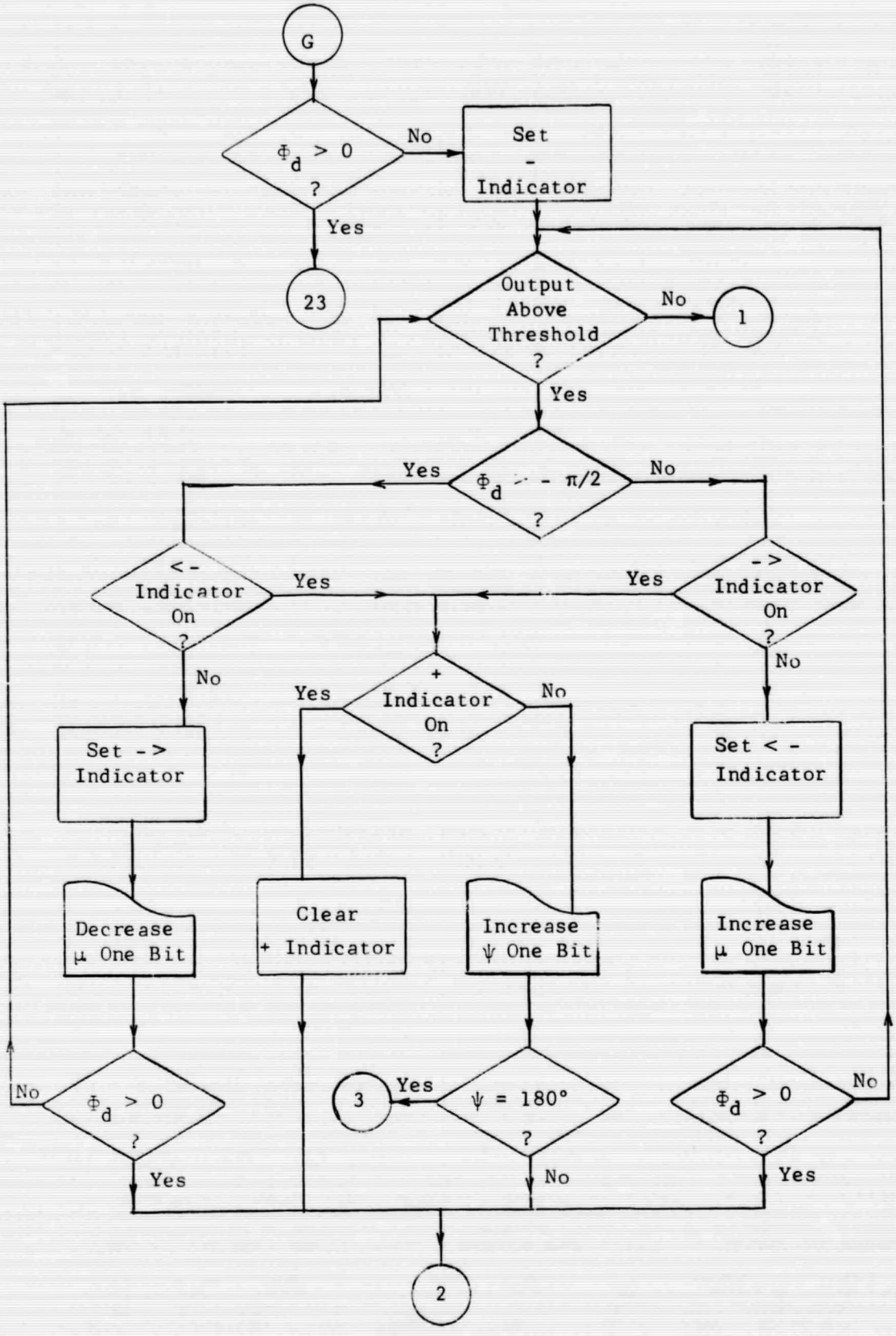

C
Set μ to 90°
13
16
State 4 > Others ?
No
Yes
State 5 > Others ?
No
Yes
17
Set ψ to 90°
Set μ to 270°
13
Set ψ to 90°
Set μ to 180°
13
17
State 6 > Others ?
No
Yes
18
Set ψ to 90°
D

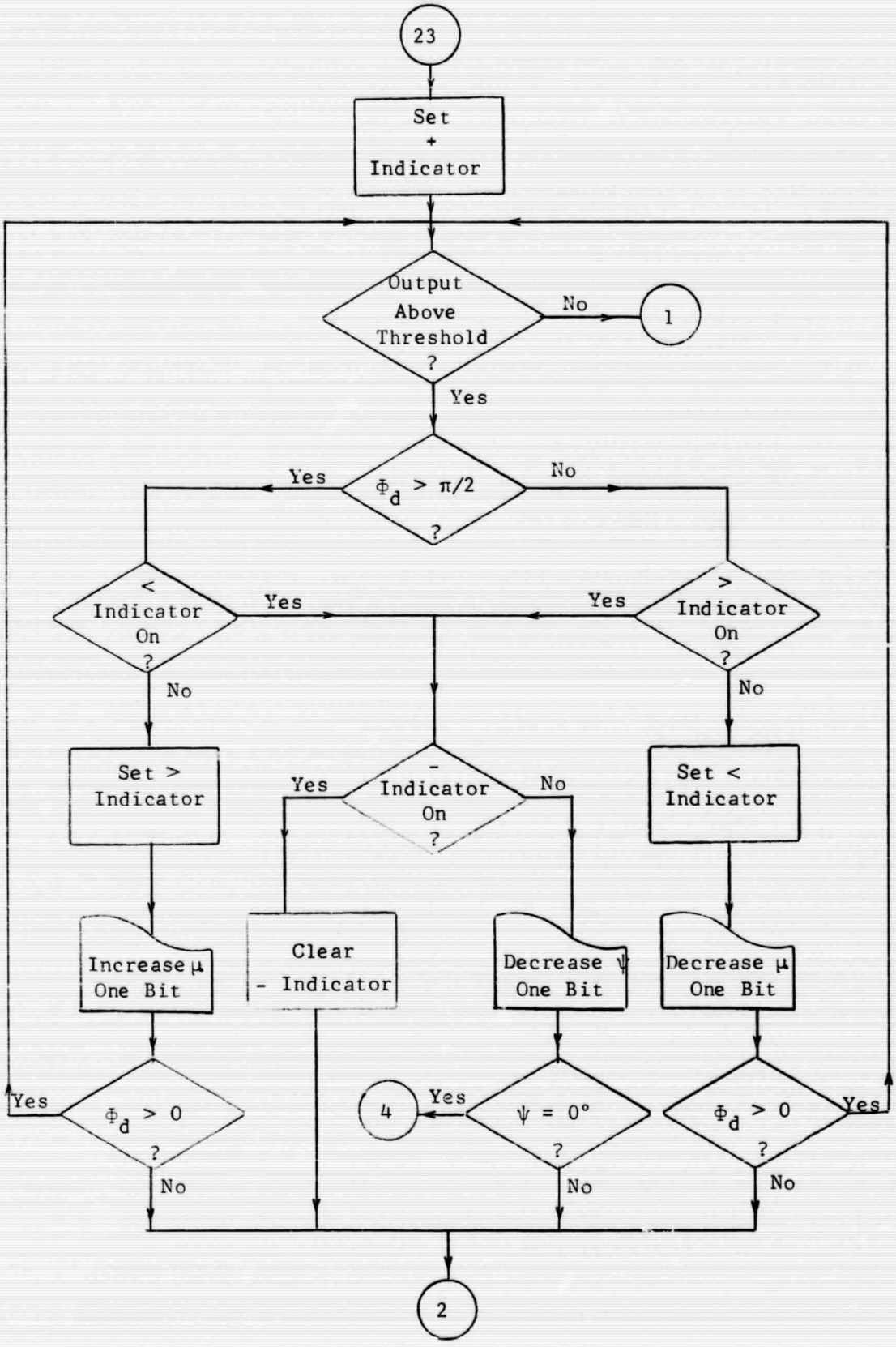

D
Set μ to 0°
13
18
State 1 = State 3 ≥ Others ?
No
Yes
State 1 = State 5 ≥ Others ?
No
19
Yes
State 1 = State 4 ≥ Others ?
Yes
No
Set μ to 135°
State 1 = State 4 ≥ Others ?
Yes
No
Set μ to 225°
State 1 = State 6 ≥ Others ?
No
Yes
State 1 = State 6 ≥ Others ?
No
Yes
Set μ to 90°
Set μ to 45°
Set μ to 270°
Set μ to 315°
Set ψ to 45°
Set ψ to 56.25°
Set ψ to 45°
Set ψ to 56.25°
13


19
State 2 = State 3 ≥ Others ?
No
Yes
State 2 = State 5 ≥ Others ?
No
Yes
20
State 2 = State 4 ≥ Others ?
Yes
No
State 2 = State 4 ≥ Others ?
No
Yes
State 2 = State 6 ≥ Others ?
No
Yes
State 2 = State 6 ≥ Others ?
Yes
No
Set μ to 225°
Set μ to 135°
Set μ to 270°
Set μ to 315°
Set μ to 90°
Set μ to 45°
Set ψ to 135°
Set ψ to 123.75°
Set ψ to 135°
Set ψ to 123.75°
13
20
State 1 = State 4 ≥ Others ?
No
Yes
21
E


E
Set μ to 180°
21
State 1 = State 6 > Others ?
Yes
Set μ to 0°
Set ψ to 45°
No
State 2 = State 4 > Others ?
Yes
Set μ to 180°
No
State 2 = State 6 > Others ?
Yes
Set μ to 0°
Set ψ to 135°
No
State 3 = State 4 > Others ?
Yes
Set μ to 135°
13
No
State 3 = State 6 > Others ?
Yes
Set μ to 45°
No
F
22


F
State 4 = State 5 > Others ?
Yes
Set μ to 225°
No
State 5 = State 6 > Others ?
Yes
Set μ to 315°
No
1
Error in Computer Operation
22
Set ψ to 90°
End Lock-on Stage
13
Track Stage
Clear +, - Indicators
Output Above Threshold ?
No
1
Return to Control Console
2
Yes
Clear >, < ->, <- Indicators
G


G
$\Phi_d > 0$?
No
Set - Indicator
Yes
23
Output Above Threshold ?
No
1
Yes
Φ_d - $\pi/2$?
Yes
No
<- Indicator On ?
Yes
Yes
-> Indicator On ?
No
No
+ Indicator On ?
Yes
No
Set -> Indicator
Set < - Indicator
Decrease μ One Bit
Clear + Indicator
Increase ψ One Bit
Increase μ One Bit
No
$\Phi_d > 0$?
3
Yes
$\psi = 180°$?
No
$\Phi_d > 0$?
No
Yes
Yes
2


23
Set + Indicator
Output Above Threshold ?
No
1
Yes
$\Phi_d > \pi/2$?
Yes
No
< Indicator On ?
Yes
Yes
> Indicator On ?
No
No
Set > Indicator
Yes
- Indicator On ?
No
Set < Indicator
Increase μ One Bit
Clear - Indicator
Decrease ψ One Bit
Decrease μ One Bit
Yes
$\Phi_d > 0$?
4
Yes
$\psi = 0°$?
$\Phi_d > 0$?
Yes
No
No
No
2

APPENDIX C

CROSS-POLARIZED RECEPTION

Consider the system transmitting a field with a polarization parameter of

$$p_t = -j \frac{S_{14}}{S_{13}} \frac{h_y}{h_x} . \qquad \text{(C-1)}$$

Observe that if this field is reflected from some far field object and returns to the antenna system, the energy recovered at arm 2 will result from the reflected wave with a polarization component

$$p_r = \frac{|h_x|^2}{|h_y|^2} p_t^* . \qquad \text{(C-2)}$$

The energy available at arm 4 is due to the polarization component of the reflected wave that is cross-polarized to that of the transmitted wave. To show this recall that the R matrix is unitary thus,

$$S_{13}S_{23}^* + S_{14}S_{24}^* = 0. \qquad \text{(C-3)}$$

Since

$$b_2 = S_{13}x_c + S_{14}y_c$$

it is seen that b_2 is zero when x_c and y_c are such that

$$x_c = S_{23}^*$$

and

$$y_c = S_{24}^* .$$

For this condition b_4 is a maximum. The polarization parameter of an incident wave with these characteristics is given by Eqn. (3-15). It is

$$\frac{y_c}{x_c} = -j \frac{h_y}{h_x} p_c$$

or

$$\frac{S^*_{24}}{S^*_{23}} = -j \frac{h_y}{h_x} p_c . \qquad (C-4)$$

Substitute Eqn. (C-3) into Eqn. (C-4) to obtain

$$p_c = -j \frac{S_{13}}{S_{14}} \frac{h_x}{h_y} . \qquad (C-5)$$

The desired result is obtained by substituting Eqn. (C-1) into Eqn. (C-5). That is

$$p_c = -\frac{1}{p_t} . \qquad (C-6)$$

It is seen that Eqn. (C-6) is identical to Eqn. (2-26).

BIBLIOGRAPHY

BOOKS

Collin, R.E., Field Theory of Guided Waves, McGraw-Hill Book Co., New York, 1962.

Gardner, F.M., Phaselock Techniques, John Wiley and Sons, Inc., New York, 1966.

Harrington, R.F., Time-Harmonic Electromagnetic Fields, McGraw-Hill Book Co., New York, 1961.

Jackson, J.D., Classical Electrodynamics, John Wiley and Sons, Inc., New York, 1962.

Jenkins, F.A. and H.E. White, Fundamentals of Optics, McGraw-Hill Book Co., New York, 1957.

Kraus, J.D., Antennas, McGraw-Hill Book Co., New York, 1950.

Kummer, W.H., Microwave Scanning Antennas, vol. III, R.C. Hansen, Ed., McGraw-Hill Book Co., New York, 1966.

Levy, R., Advances in Microwave, L. Young, Ed., Academic Press, New York, 1966.

Matthaei, G.L. and E.G. Cristal, Advances in Microwaves, L. Young, Ed., Academic Press, New York, 1967.

Montgomery, C.G., Technique of Microwave Measurements, McGraw-Hill Book Co., New York, 1947.

Montgomery, C.G., et.al., Principles of Microwave Circuits, McGraw-Hill Book Co., New York, 1948.

Oliner, A.A. and R.G. Malech, Microwave Scanning Antennas, vol. II, R.C. Hansen, Ed., McGraw-Hill Book Co., New York, 1966.

Schelkunoff, S.A., Electromagnetic Waves, D. VanNostrand Co., New York, 1943.

Silver, S., Microwave Antenna Theory and Design, McGraw-Hill Book Co., New York, 1949.

Skolnik, M.I., Introduction to Radar Systems, McGraw-Hill Book Co., New York, 1962.

Stratton, J.A., Electromagnetic Theory, McGraw-Hill Book Co., New York, 1941.

Walter, C.H., Traveling Wave Antennas, McGraw-Hill Book Co., New York, 1965.

ARTICLES

Allen, P.J. and R.D. Tompkins, "An instantaneous microwave polarimeter," Proc. IRE, vol. 47, pp. 1231-1237; July 1959.

Allen, P.J., "Generating a rotating polarization," Proc. IRE, vol. 48, pp. 941; May 1960.

Breithauph, R.W., "Cross polarization in a general two plate polarizer," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-15, pp. 55-57; Jan. 1967.

Chu, T.S. and R.G. Kouyoumjian, "An analysis of polarization variation and its application to circularly-polarized radiators," IRE Trans. on Antennas and Propagation, vol. AP-10, pp. 188-192; March 1962.

Cohen, M.H., "Radio astronomy polarization measurements," Proc. IRE, vol. 46, pp. 172-183; Jan. 1958.

Copeland, J.R., "Radar target classification by polarization properties," Proc. IRE, vol. 48, pp. 1290-1296; July 1960.

Culter, C.C., et.al., "Microwave antenna measurements," Proc. IRE, vol. 35, pp. 1462-1471; Dec. 1947.

Desoer, C.A., "The optimum formula for the gain of a flow graph or a simple derivation of Coates' Formula," Proc. IRE, vol. 48, pp. 883-889; May 1960.

Elfving, C., et.al., "Methods of automatic analysis and synthesis of polarization of electromagnetic waves," Microwave J., vol. 3, pp. 57-63; Nov. 1960.

Ernst, W.P., "A technique for measuring phase modulation or rapid phase changes of a microwave signal," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-13, pp. 70-76; Jan. 1965.

Friis, H.T., "A note on a simple transmission formula," Proc. IRE, vol. 34, pp. 254-256; May 1946.

Goebels, F.J. and K.C. Kelly, "Arbitrary polarization from annular slot planar antennas," IRE Trans. on Antennas and Propagation, vol. AP-9, pp. 342-349; July 1961.

Graves, C.D., "Radar polarization power scattering matrix," Proc. IRE, vol. 44, pp. 248-252; Feb. 1956.

Hunton, J.K., "Analysis of microwave measurement techniques by means of signal flow graphs," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 206-212; March 1960.

Mason, S.J., "Feedback theory - further properties of signal flow graphs," Proc. IRE, vol. 44, pp. 920-926; July 1956.

Meyer, M.A., "Applications of the turnstile junction," IRE Trans. on Microwave Theory and Techniques, vol. Mtt-3, pp. 40-45; Dec. 1955.

Mittra, R., "An automatic phase measuring circuit at microwaves," IRE Trans. on Instrumentation, vol. I-6, pp. 238-240; Dec. 1957.

Morgan, M.G. and W.R. Evans, "Synthesis and analysis of elliptic polarization loci in terms of space-quadrature sinusoidal components," Proc. IRE, vol. 39, pp. 552-556; May 1951.

Nolen, J.C., "Phased array polarization agility," IEEE Trans. on Antennas and Propagation, vol. AP-13, pp. 820-821; Sept. 1965.

Park, D.J.M., "Scattering diagrams and some new nonreciprocal waveguide circuits," Microwave J., vol. 6, pp. 65-70; June 1963.

Pedersen, B.O., "Phase-sensitive detection with multiple frequencies," IRE Trans. on Instrumentation, vol. I-9, pp. 349-355; Dec. 1960.

Reggia, F. and T. Mak, "Reciprocal latching phase modulation for microwave frequencies," IEEE Trans. on Magnetics, vol. MAG-2, pp-269-273; Sept. 1966.

Reuss, M.L., "A nonreciprocal circular polarizer," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-15, pp. 37-41; Jan. 1967.

Riblet, H.J., "The short-slot hybrid junction," Proc. IRE, vol. 40, pp. 180-184; Feb. 1952.

Rumsey, V.H., "The reaction concept in electromagnetic theory," Phys. Rev., vol. 94, pp. 1483-1491; June 1954.

Rumsey, V.H., G.A. Deschamps, M.L. Kales, and J.L. Bohnert, "Techniques for handling elliptically polarized waves with special reference to antennas," Proc. IRE, vol. 39, pp. 533-552; May 1951.

Schafer, G.E., "Mismatch errors in microwave phase shift measurements," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 617-622; Nov. 1960.

Scott, W.G. and K.M. Soo Hoo, "A theorem on the polarization of null-free antennas," IEEE Trans. on Antennas and Propagation, vol. AP-14, pp. 587-590; Sept. 1966.

Vaillancourt, R.M., "Analysis of the variable-ratio microwave power divider," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 238-239; April 1958.

Vogt, G., "An electronic method for steering the beam and polarization of HF antennas," IRE Trans. on Antennas and Propagation, vol. AP-10, pp. 193-200; March 1962.