SIMULATION OF SERVICE LOAD DISTRIBUTIONS UTILIZING STATIONARY, NON-GAUSSIAN RANDOM TIME HISTORIES

by

HERBERT ARTHUR LEYBOLD

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APPROVED BY:

L. W. Royster M. H. Clayton

J. K. Whitfield F. D. Hart

Chairman of Advisory Committee

ABSTRACT

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The objective of the study is to determine the feasibility of generating a non-Gaussian, stationary, random load time history which can be used to simulate service load histories during fatigue testing of structural parts.

To achieve this objective, five random load time histories of arbitrary amplitude and each having a different, mathematically describable, non-Gaussian, amplitude probability density function, were generated with the aid of a CDC 6000 series digital computer and FORTRAN IV programming. Cumulative peak distributions were obtained from each of the resulting random load time histories and compared with cumulative peak service load distributions. Five random load time histories are investigated in order to assure that the best possible duplication of the cumulative peak service load distribution is obtained.

Of primary importance in the investigation was the matching of predicted, that is, digitally generated and service load cumulative peak distributions. Of secondary importance was the technique used for digitally generating the five different, non-Gaussian, random load time histories.

Cumulative peak distributions of two of the generated random time histories approximated the cumulative peak

service distributions for aircraft. The remaining three cumulative peak distributions approximated the cumulative peak service distributions of ground transportation, that is, car, truck, rail, overhead travelling crane, etc.

To my wife, Pat, and children, Richard, Laura, Alan, Karen, John, and Mary for their help, encouragement, patience and understanding during the past two years.

BIOGRAPHY

The author was born on April 15, 1934, in New York City. He received his elementary and high school education in the public schools of that city, graduating in January of 1952. In June of 1955 he received the Bachelor of Science degree in Civil Engineering from the Polytechnic Institute of Brooklyn, N.Y.

From June 1955 to the present time he has been employed as an Aerospace Technologist by the Langley Research Center of the National Aeronautics and Space Administration (NASA), formerly the National Advisory Committee for Aeronautics (NACA), Langley Station, Hampton, Virginia 23365. At NASA, he has conducted research in the field of fatigue of metals and has acted as a consultant on writing test specifications and testing of spacecraft in the shock and vibration environments. During the course of his work, he has authored 10 publications and coauthored 2 publications. In addition, he enrolled in the NASA Langley Research Center graduate study program and received a Master of Science degree in Engineering Mechanics from the Virginia Polytechnic Institute in June 1963.

The author married Miss Patricia Ann Peterson in 1955, and they have six children, Richard Herbert, twelve years old; Laura Ann, ten years old; Alan William, nine years old; Karen Ann, eight years old; John Edward, six years old; and Mary Ann, three years old.

ACKNOWLEDGMENTS

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1. INTRODUCTION

Over the past several years the trend has been towards random load fatigue testing of aircraft structural parts rather than the constant amplitude and programmed constant amplitude fatigue testing which has been going on for years. The advent of more sophisticated fatigue testing machines capable of applying complex load-time histories which more nearly simulate actual service experience has been the major factor towards this trend. Within this framework of random load fatigue testing there has evolved two schools of thought on simulating aircraft service experience. The first utilizes a quasi-stationary Gaussian random process to simulate service experience. This is accomplished by programming, in a random fashion, the root-mean-square level of a Gaussian random process. The second utilizes a stationary non-Gaussian random process to simulate service experience. It is the purpose of this thesis to show that this second approach is feasible.

Toward this end a random number generator was used to generate random numbers having a uniform or constant amplitude probability density function between the limits of zero and one. These numbers were then shaped such that the amplitude probability density of a discrete random time history composed of these random numbers had one of the following density functions: Poisson, Binomial, Log-Normal, Weibull, and Exponential. These density functions were arbitrarily selected because they are widely discussed in the literature and are mathematically describable. The resulting random time histories are analyzed to determine their peak statistics. It is generally believed that the peak statistic is the significant fatigue inducing parameter of a random time history. It is also the statistic most frequently obtained during service experience. Cumulative peak distributions are computed and compared with cumulative peak service distributions. For aircraft the distribution is a straight line on semi-log paper.

It is shown that the random time histories having Exponential and Log-Normal amplitude probability density functions can be used to simulate the service load experience of aircraft. It is also shown that the other three random time histories having Binomial, Poisson and Weibull amplitude probability density functions can be used to simulate service load experience of ground transportation such as cars, trucks, railroads, and overhead travelling cranes.

2. REVIEW OF LITERATURE

It has become increasingly more apparent in recent years that more and more fatigue tests are being conducted using random loads. There appears to be several reasons for this trend. First, considerably more research has been done in recent years in this area as evidenced by the number of articles being published. These articles have undoubtedly influenced the fatigue engineers philosophy or outlook on fatigue testing. Secondly, the fatigue engineer has come to realize that service loadings are complex loadings and that he is incapable of predicting fatigue life under their influence. Hence he must resort to a much more accurate simulation of the service loadings in order to determine fatigue life. Finally, he now has at his disposal testing equipment which will permit him to apply any complex load history he desires to test specimens and structures. The above mentioned items have brought us to the current state of the art in random load fatigue testing. Random process theory upon which random load fatigue testing is based cam be found in such texts as Bendat and Piersol (1966), Crandall and Mark (1963), Davenport and Root (1958) and Robson (1964).

Swanson (1968) presents an excellent review of the literature on random load fatigue testing. In this reference he starts with the testing to develop the well known S-N curve and goes on to the block tests wherein constant amplitude cycles are stratified at various stress levels in an arbitrary sequence so as to approximate the relative frequency of service loads from the service load spectrum. In addition he discusses the actual random process and/or an analogous random process to the one encountered in service which may be either stationary, quasi-stationary or non-stationary and result in the same load frequency as given by the service load spectrum. Swanson also indicates that there are basically two approaches to fatigue testing under random loading: the "digital" approach and the analogue approach. The digital approach will necessarily result in a random process that is stationary in nature while the analogue approach may result in either a stationary or quasi-stationary process.

The author (1963) was involved in obtaining statistics for the conduct of both block and random type tests. Based on these statistics, Leybold and Naumann (1963) conducted tests to evaluate the effects of both block and random type tests on the fatigue life of aluminum alloys. Leybold (1963) noted that the primary statistic used to conduct fatigue tests was the peak statistic. A mathematical method for predicting the peak distribution or peak statistic has been developed by Rice (1954) for a Gaussian, stationary random process. Broch (1963 & 1968) discussed the work developed by Rice and applies it to the case of fatigue under random loading. Schijve (1963) tried to determine which statistic of a random process was most significant with regard to fatigue. In most cases the random process investigated and used for testing was Gaussian and stationary in nature. Two primary reasons for using this type of a random process were: (1) a Gaussian stationary random process is easily generated with a commercially available random noise generator and (2) the peak distribution of a Gaussian, stationary random process was easily computed based on Rice's work. However, the drawback to using this type of a random process is that service load data show that they are rarely stationary in nature and may or may not be Gaussian in nature. Based on this fact, Swanson (1963 & 1965) has proposed the Gaussian non-stationary random process for fatigue testing. Using this method of testing the rms of a stationary random process is programmed to vary in a random fashion such that the resulting peak distribution will match that of service load data. Service load data are usually presented in the form of cumulative peak distributions, that is, cumulative number of peaks exceeding a given level. Service load cumulative peak distributions for aircraft usually plot as straight lines on semi-log paper. To cite just two of the publications available with aircraft service load data we have Coleman (1968) who presents his data in terms of g's (acceleration due to gravity) and Bullen (1967) who presents his data in terms of gust velocities.

The quasi-stationary random fatigue test proposed by Swanson is one way of simulating the service load peak distribution. Another method also proposed by Swanson (1968) was to find a stationary non-Gaussian, random process, which exhibits a cumulative peak distribution the same as an aircraft service load cumulative peak distribution, that is, a straight line on semi-log paper. Thus two different ways have been proposed for simulating service load time histories: (1) the quasi-stationary method where the random process is Gaussian in nature and the root-mean-square levels are programmed in a random fashion and (2) the stationary method, that is, constant root-mean-square level, where the random process is non-Gaussian in nature. This thesis deals with the latter method.

3. GENERATION AND ANALYSIS OF RANDOM TIME HISTORIES

3.1 General

In this chapter random numbers are generated and shaped so that the resulting discrete random time history has a non-Gaussian amplitude probability density function. Actual and theoretical amplitude probability density functions are compared. In addition peak and cumulative peak distributions are calculated. The computations are broken down into three parts as shown in the next three sections. The computer programs for carrying out these computations can be found in the Appendices, sections 8.1 and 8.2. More will be said about these programs in Chapter 4 on computer programs.

Five different amplitude probability density functions were used in order to insure that the best possible agreement between the digitally generated data and the service load data would be attained.

It should be noted that the data presented are only a small portion of the data generated with the aid of the computer. Parametric studies were made using all five amplitude probability density functions. The resulting data were just too voluminous to include in this thesis. Only those data which show trends or agreement with service load data have been included.

3.2 Generation of Random Numbers

Random numbers bounded by the limits zero and one were generated with a standard random number generator having a uniform or constant amplitude probability density function. The numbers were then shaped such that the resulting discrete random time history would have some arbitrarily selected amplitude probability density function. The following amplitude probability density functions were chosen because they are mathematically describable and discussed thoroughly in the literature: Poisson, Binomial, Log-Normal, Weibull and Exponential. The procedure for digitally generating the shaped random numbers was as follows: Let $F(x)$ be the amplitude probability density function and $CF(x)$ the cumulative amplitude probability density function. $CF(x)$ is related to the amplitude probability density function by the relation

$$dCF(x) = F(x)dx \tag{3.1}$$

or by integrating

$$CF(x) = \int_{-\infty}^{x} F(\alpha)d\alpha \tag{3.2}$$

which is bounded by the limits zero and one when x takes on the values of $-\infty$ and $+\infty$, respectively. Thus, random numbers having a uniform probability density distribution with limits between zero and one can be generated with a random number generator and substituted for $CF(x)$ in equation (3.2). This equation can then be evaluated for x. As many random numbers can be generated in this manner as are needed and they will necessarily have the desired amplitude probability density function. For this work each random time history consisted of 10000 random numbers.

The five random time histories generated in this manner had the following amplitude probability density functions: Poisson, Binomial, Log-Normal, Weibull and Exponential. In the first of these time histories, whose amplitude probability density function is discrete, the following recurrence formula was used to compute the cumulative probability density:

$$F(x + 1) = \frac{\mu}{x + 1} F(x) \tag{3.3}$$

The cumulative frequency, CF(x), was compared with a digitally generated uniformly distributed random number Y having a value between zero and one.

The random variable x was found by determining when Y fell between the limits CF(x) and CF(x + 1), that is,

$$Y = \sum_{0}^{x} F(x) \tag{3.4}$$

The second time history was generated in a similar manner using the binomial recurrence formula:

$$F(x + 1) = \frac{(n - x)}{(x + 1)} \frac{p}{q} F(x) \tag{3.5}$$

The third time history or log-normal one was generated by solving equation (3.2) with a numerical approximation of the integral (see section 8.2.2.3).

In the case of the last two time histories equation (3.2) could be integrated directly and resulted in the following random variables x: For the Weibull probability density function:

$$F(x) = \alpha\beta x^{\alpha-1} \exp(-\beta x^{\alpha}) \tag{3.6}$$

and

$$CF(x) = 1 - \exp(-\beta x^{\alpha}) \tag{3.7}$$

solving equation (3.7) for x we get

$$x = \left\{-\frac{1}{\beta} \log_e\left[1 - CF(x)\right]\right\}^{\frac{1}{\alpha}}$$

For the Exponential probability density function:

$$F(x) = \theta \exp(-\theta x) \tag{3.8}$$

and

$$CF(x) = 1 - \exp(-\theta x) \tag{3.9}$$

solving equation (3.9) for x we get

$$x = -\frac{1}{\theta} \log_e\left[1 - CF(x)\right]$$

It should be noted that the Exponential distribution is a special case of the Weibull distribution when $\alpha = 1$.

The first 100 random numbers generated for each of the five different amplitude probability density functions used are shown in Figures 3.1-3.10. The mean, root-mean-square and standard deviation are shown for each time history. The mean indicates the average or expected value of the random variable, say x. The mean is mathematically expressed as

$$E(x) = \int_{-\infty}^{\infty} xF(x)dx \tag{3.10}$$

The root-mean-square is a measure of the amplitude of a time history and is the square root of the mean of the squares of a random variable x. The mean square is mathematically expressed as

$$E(x^2) = \int_{-\infty}^{\infty} x^2F(x)dx \tag{3.11}$$

The standard deviation is a measure of the dispersion from the mean and is the square root of the variance of a random variable x. The variance is defined mathematically as

$$\sigma^2 = E\left\{\left[x - E(x)\right]^2\right\} \tag{3.12}$$

Crandall and Mark (1963) show that this can be reduced to

$$\sigma^2 = E(x^2) - \left[E(x)\right]^2 \tag{3.13}$$

All of the above mentioned properties were calculated for each of the five random time histories generated. The

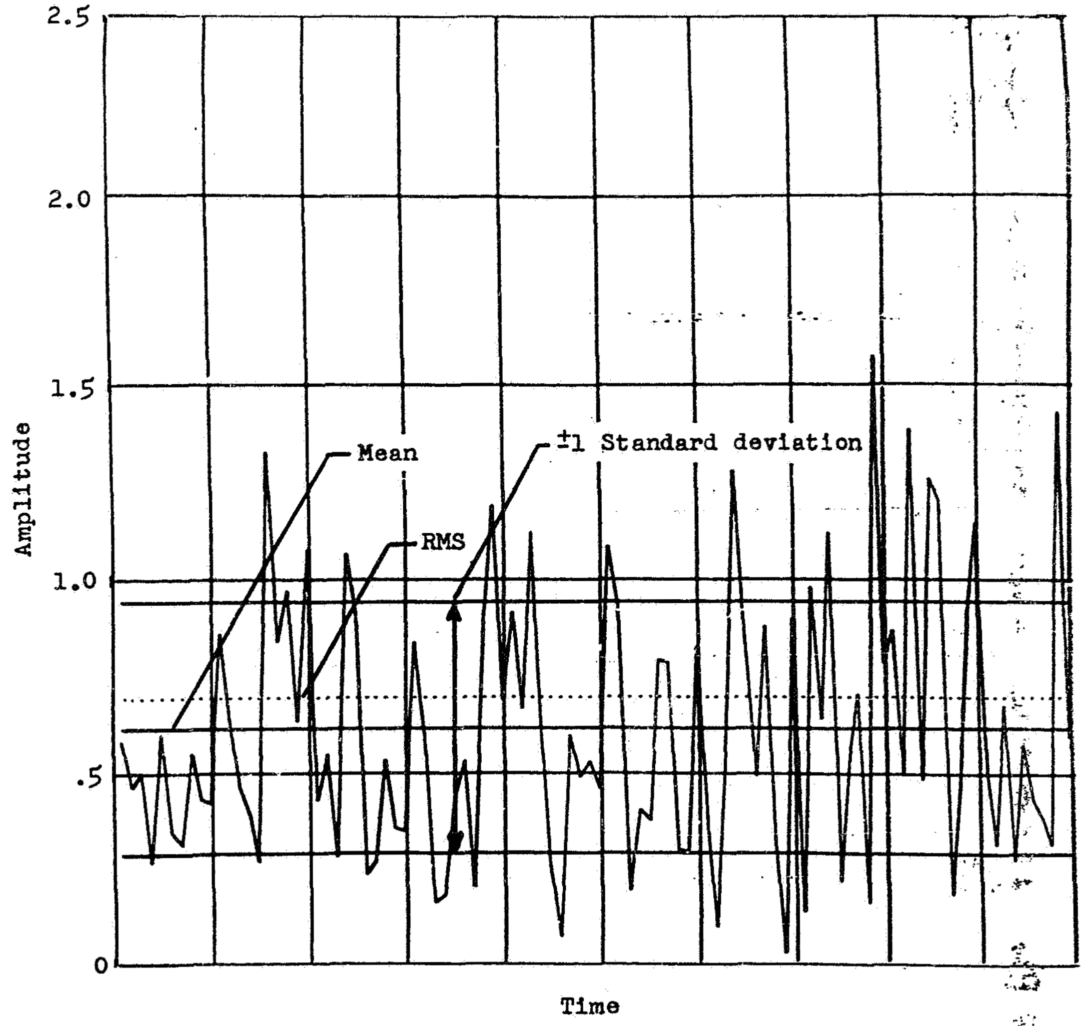

Figure 3.1. Time history of random numbers having a Weibull amplitude probability density: $\alpha = 2$; $\beta = 2$

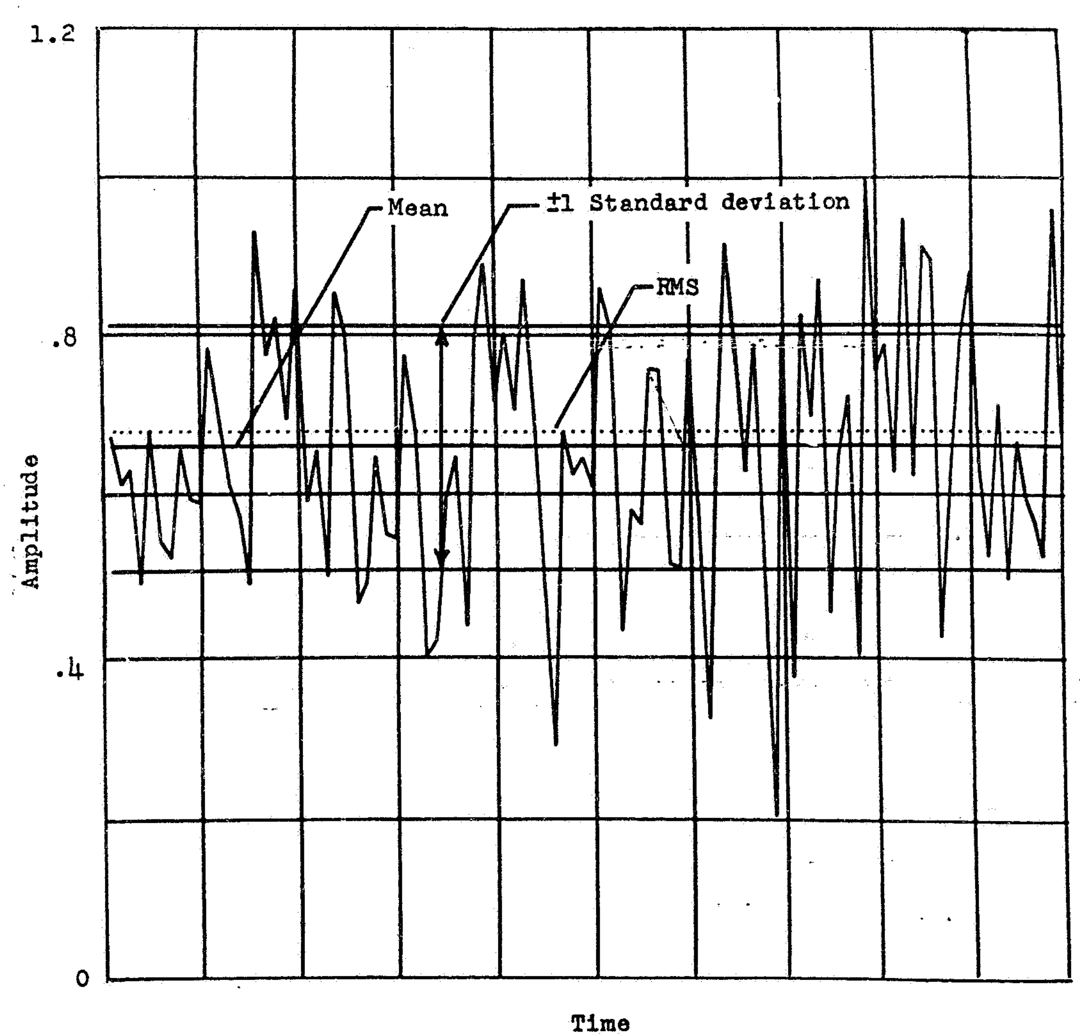

Figure 3.2. Time history of random numbers having a Weibull amplitude probability density: $\alpha = 5$; $\beta = 5$

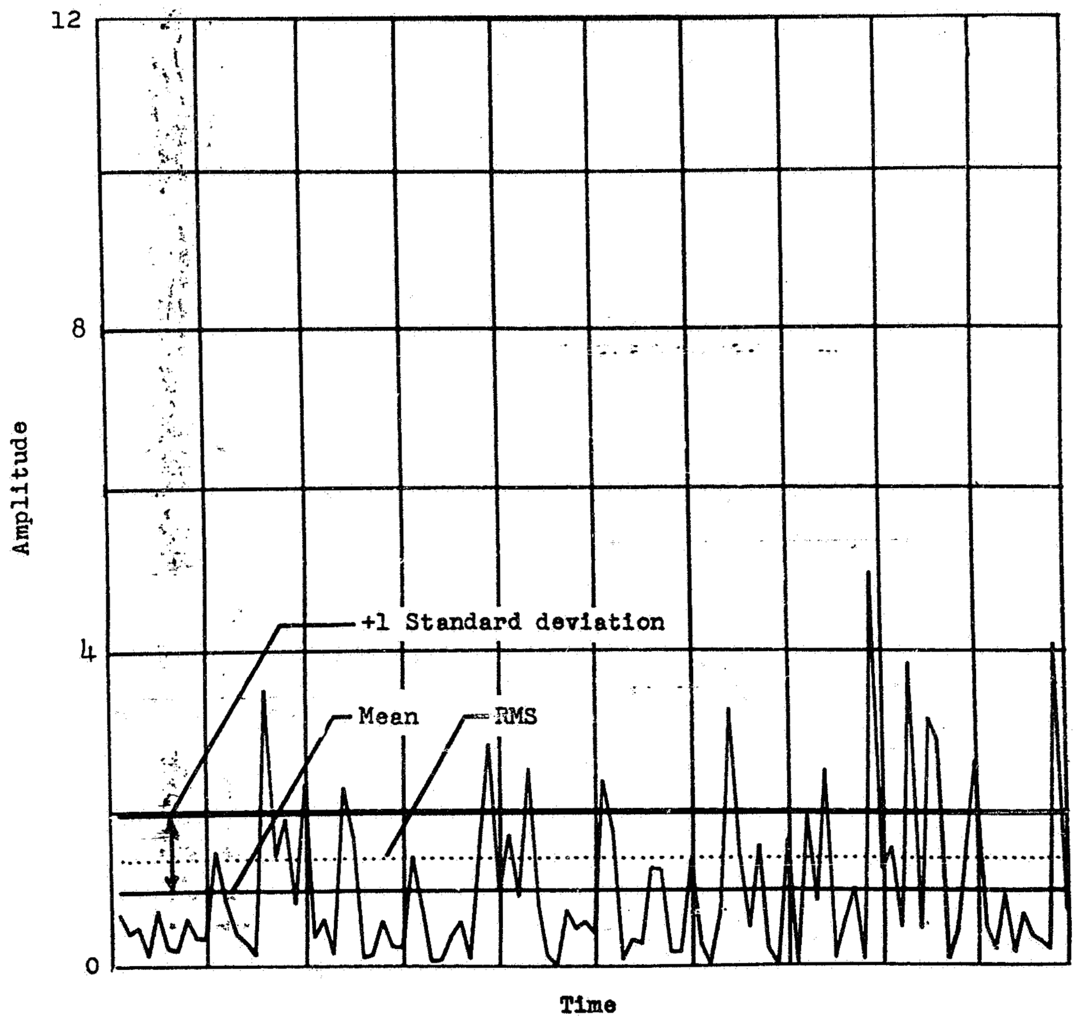

Figure 3.3. Time history of random numbers having an Exponential amplitude probability density: $\theta = 1$

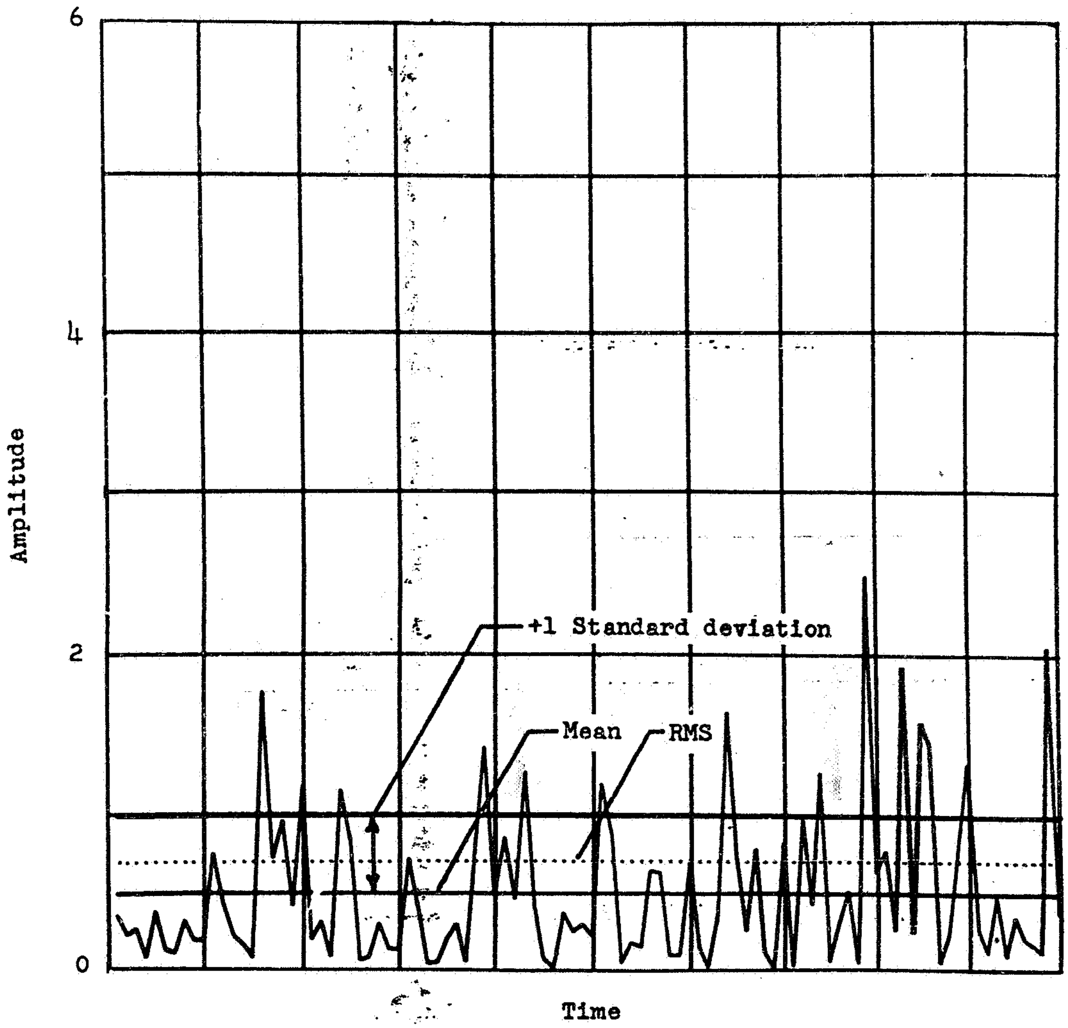

Figure 3.4. Time history of random numbers having an Exponential amplitude probability density: $\theta = 2$

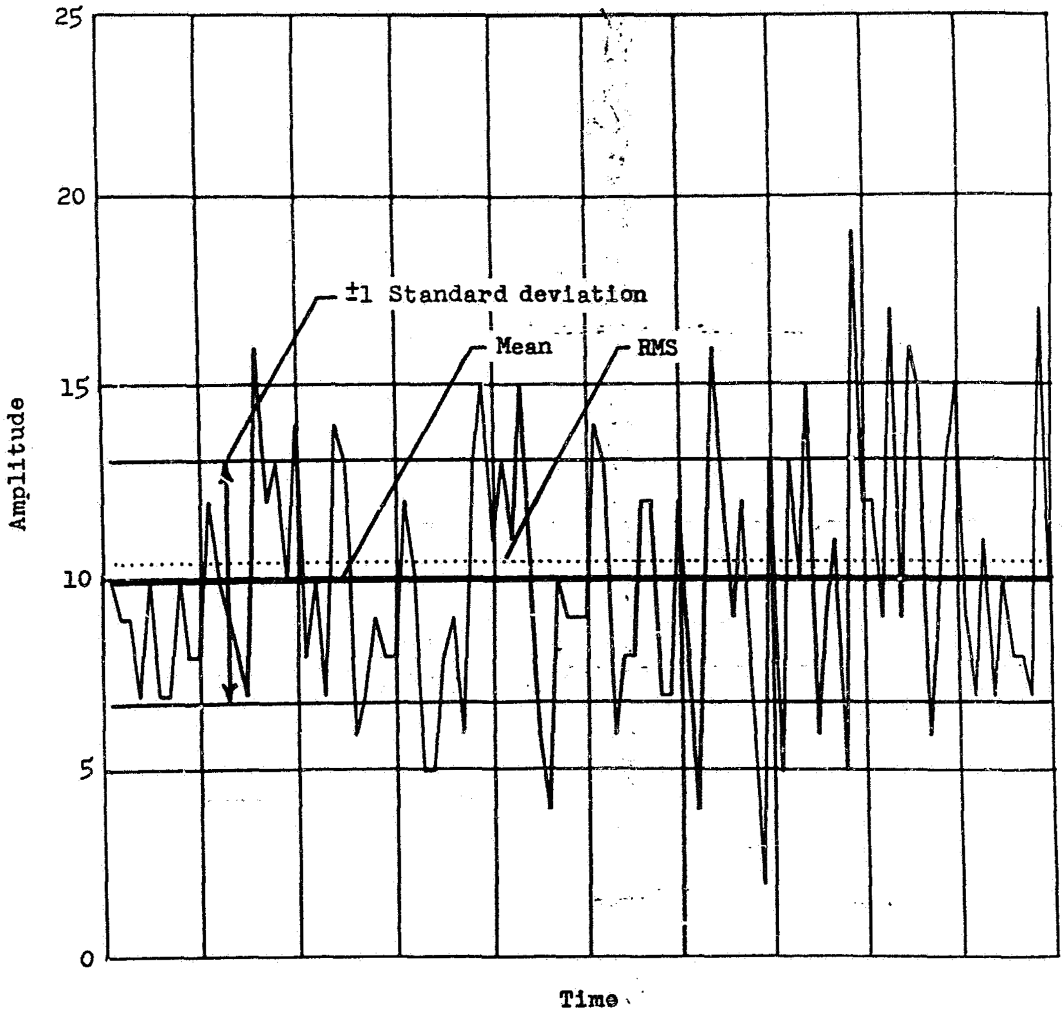

Figure 3.5. Time history of random numbers having a Poisson amplitude probability density: $\mu = 10$

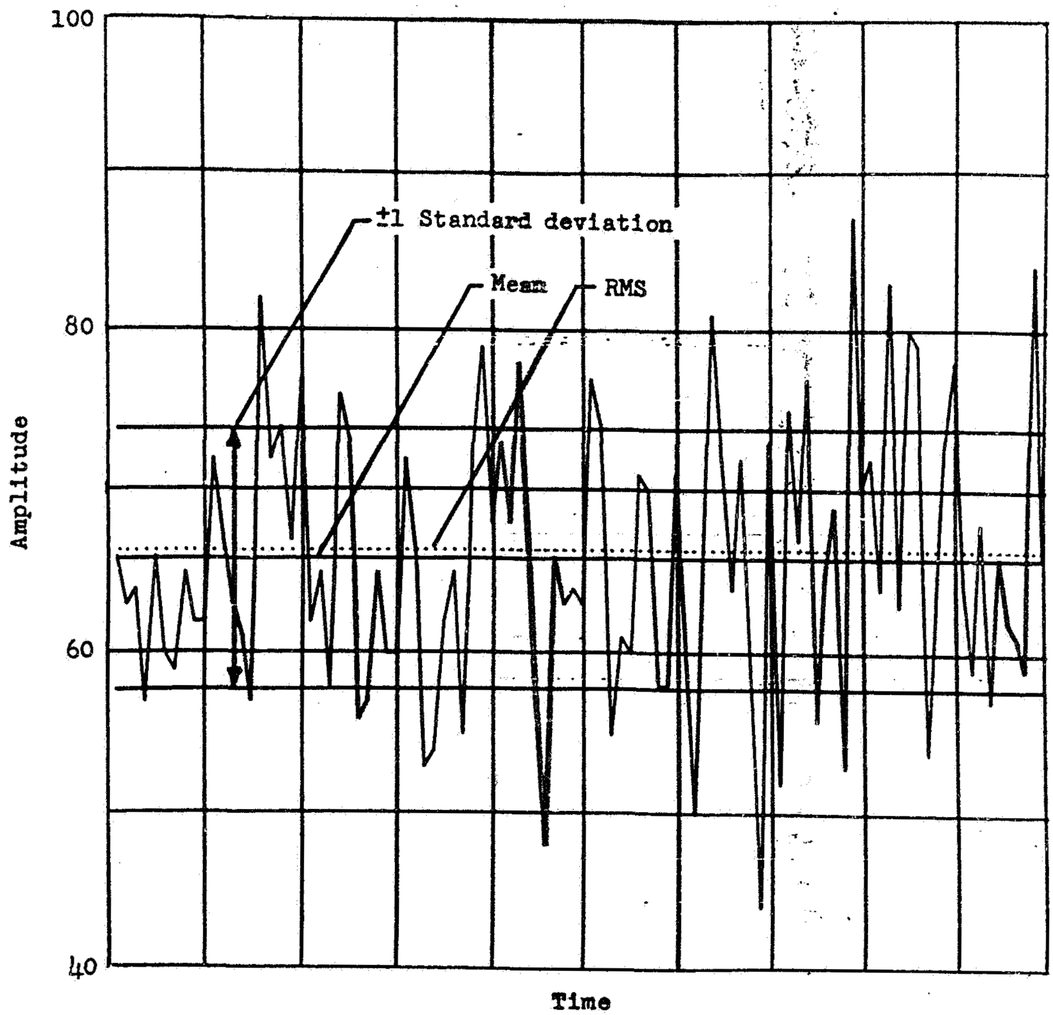

Figure 3.6. Time history of random numbers having a Poisson amplitude probability density: $\mu = 66$

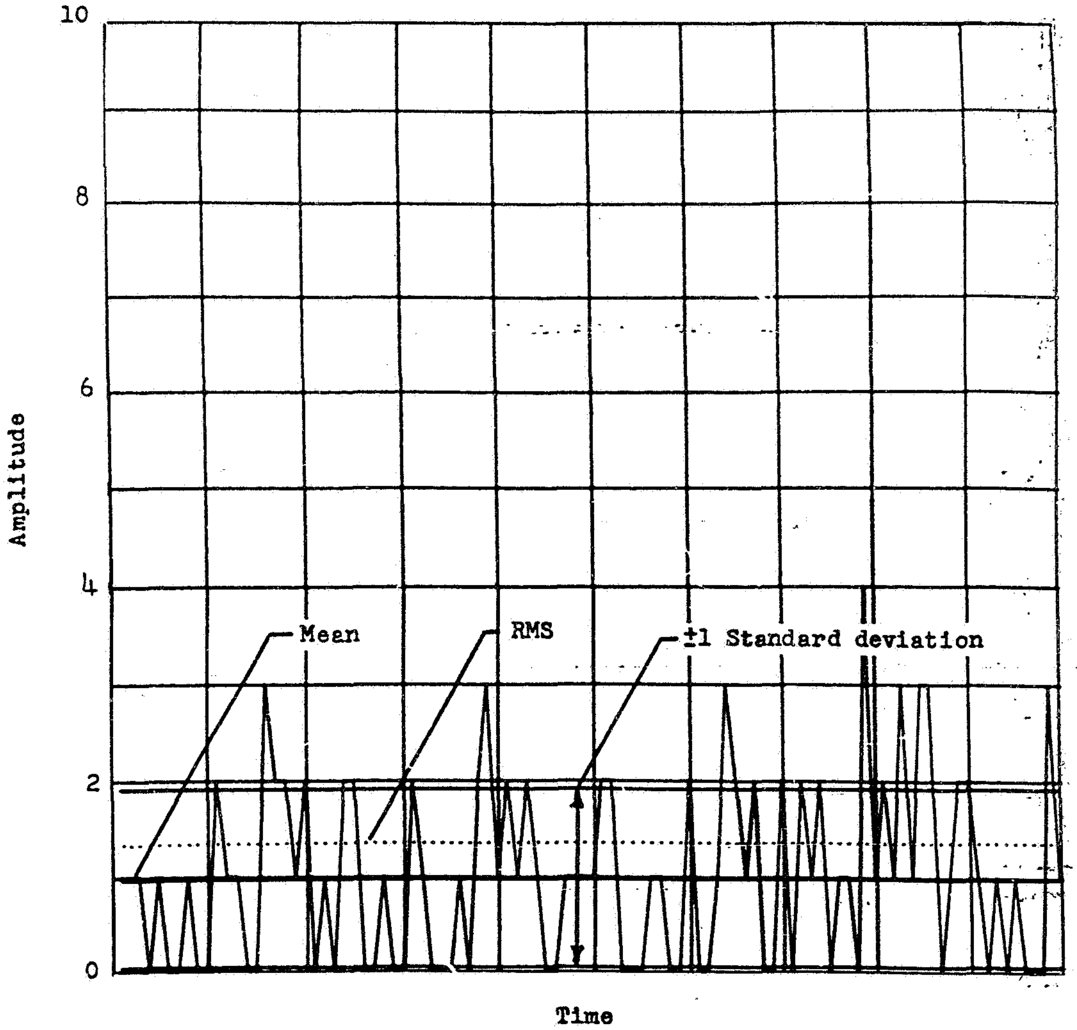

Figure 3.7. Time history of random numbers having a Binomial amplitude probability density: n = 10; p = 0.1

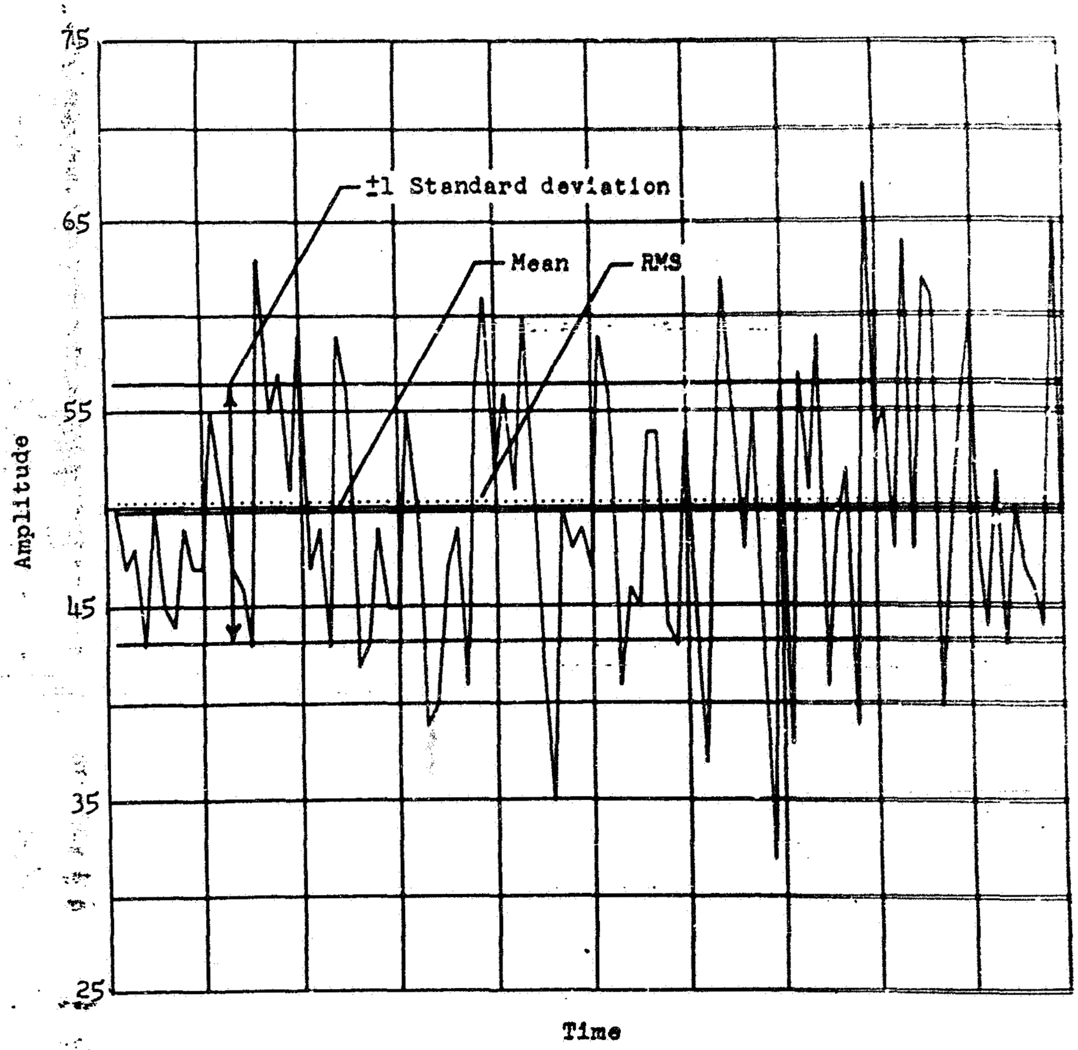

Figure 3.8. Time history of random numbers having a Binomial amplitude probability density: n = 500; p = 0.1

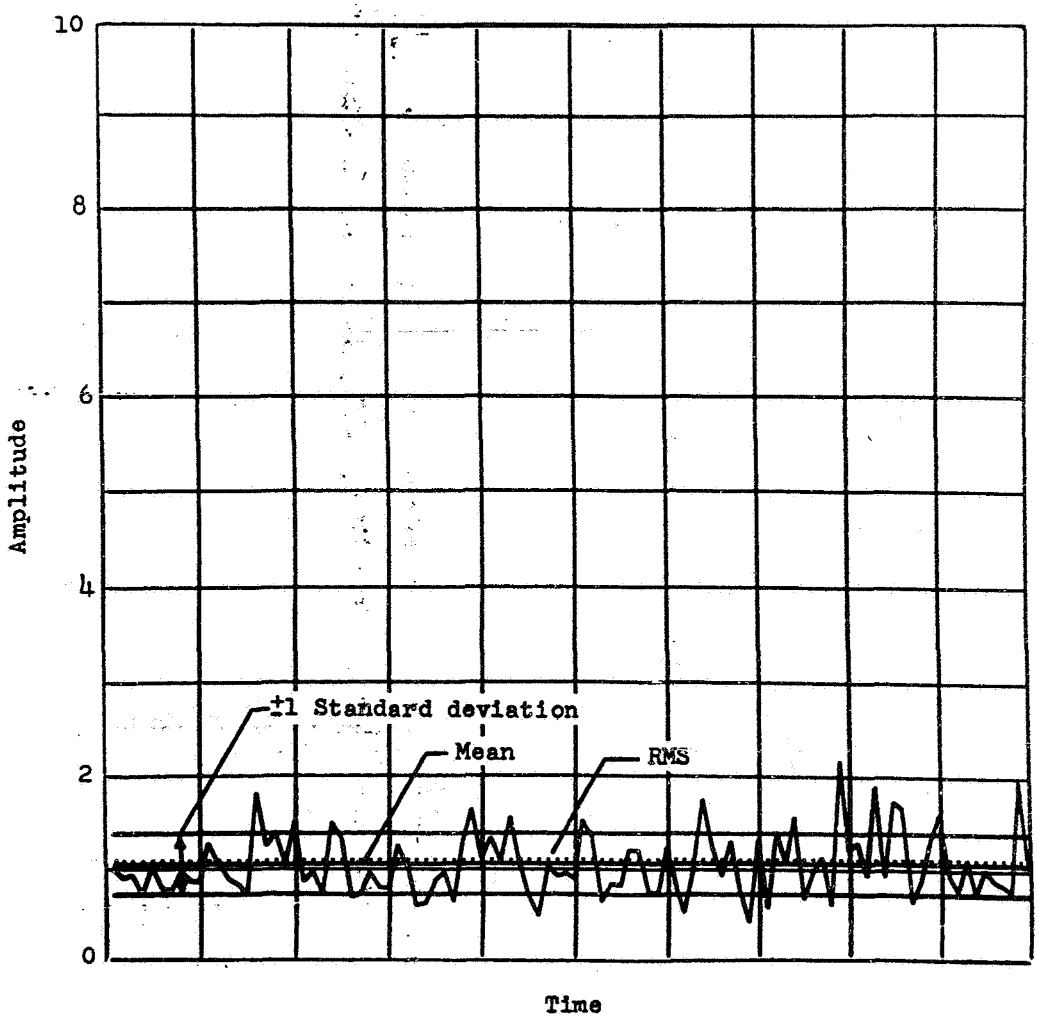

Figure 3.9. **Time history of random numbers having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.1$**

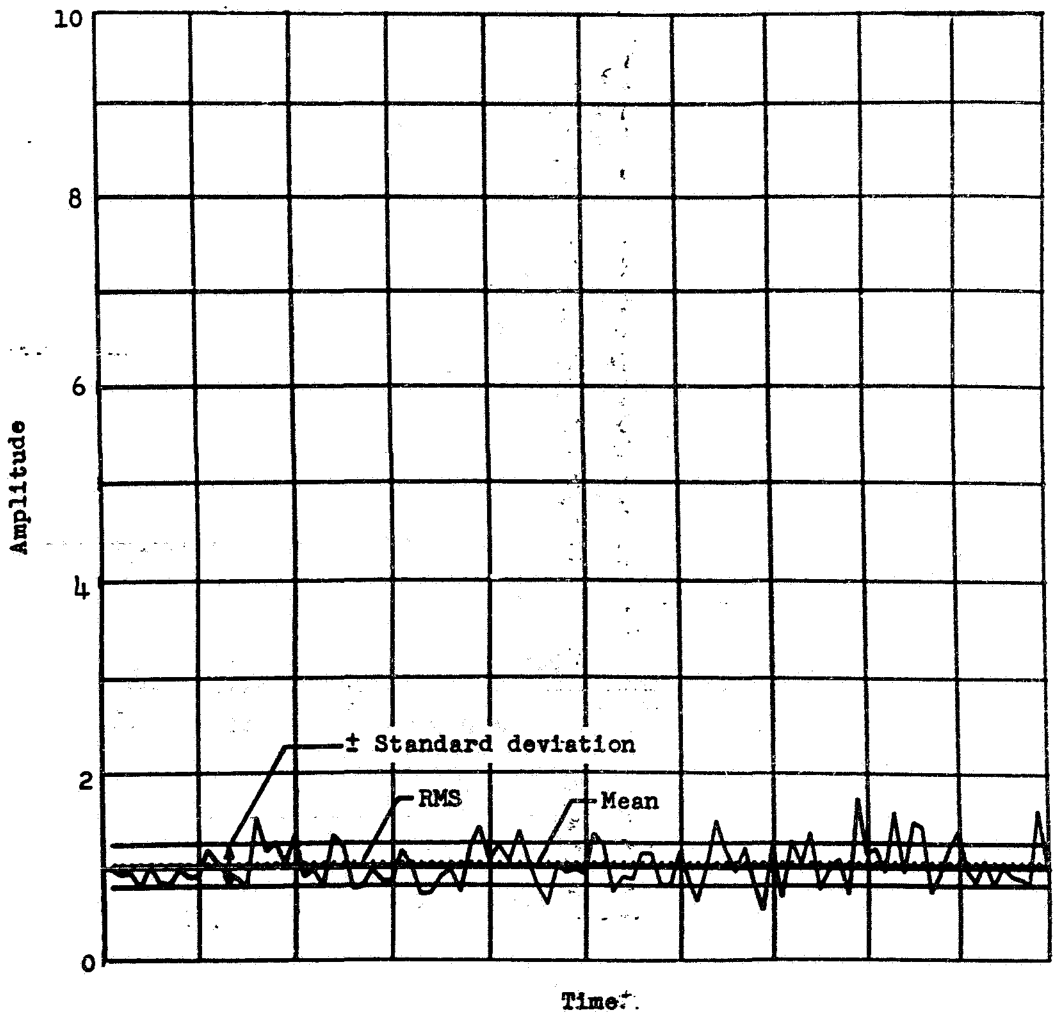

Figure 3.10. Time history of random numbers having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.05$

results of these calculations are shown in Table 3.1. In order to check these values with the theoretical ones, the author scanned the literature to obtain the theoretical equations for the mean, mean-square and variance of the five density functions used. To his dismay all were not readily available. Thus, the basic statistical equations and density functions were used to derive the desired properties. These derivations are shown in Appendix 8.5 and the results are summarized in Table 3.2. The theoretical properties were calculated and showed good agreement with the properties tabulated in Table 3.1.

3.3 Calculation and Comparison of Computed and Theoretical Histograms

After each of the five random time histories were digitally generated, the amplitude scale of each was divided up into equal intervals and the random numbers falling in each interval were counted for all 10,000 numbers of each time history. The resulting frequency of occurrence of the random numbers were plotted as histograms in Figures 3.11-3.22. In addition the theoretical histograms were also plotted and compared with the actual histograms in these figures. It can be seen that good agreement was obtained. A chi-squared test was performed to compare the actual histogram with the theoretical one. In all cases the calculated chi-square value was well below the theoretical chi-squared value at the 95 percent

Table 3.1. Comparison of computed and theoretical properties of density functions

Density function	Parameters	Mean, E(x)		Root-mean-square, RMS		Standard deviation, σ	
		Computed	Theoretical	Computed	Theoretical	Computed	Theoretical
Weibull	$\alpha = 2, \beta = 2$	0.6207	0.6267	0.7004	0.7071	0.3245	0.3276
	$\alpha = 5, \beta = 5$	0.6631	0.6655	0.6801	0.6827	0.1511	0.1524
Exponential	$\theta = 1$	0.9811	1.0	1.3917	1.4142	0.9870	1.0
	$\theta = 2$	0.4906	0.5	0.6959	0.7071	0.4935	0.5
Poisson	$\mu = 10$	9.948	10.0	10.436	10.488	3.130	3.162
	$\mu = 66$	65.867	66.0	66.365	66.498	8.032	8.124
Binomial	$n = 10, p = 0.1$	0.979	1.0	1.357	1.378	0.941	0.949
	$n = 500, p = 0.1$	49.892	50.0	50.340	50.448	6.626	6.708
Log-Normal	$\left.\begin{array}{l}\mu_{log} = 0 \\ \sigma^2_{log} = 0.1\end{array}\right\}$	1.0453	1.0513	1.0983	1.1052	0.3372	0.3409
	$\left.\begin{array}{l}\mu_{log} = 0 \\ \sigma^2_{log} = 0.05\end{array}\right\}$	1.0214	1.0253	1.0468	1.0513	0.2296	0.2322

Table 3.2. Summary of properties of density functions

Density function	Mean, $E(x)$	Mean-square, $E(x^2)$	Variance, σ^2
Poisson	μ	$\mu(\mu + 1)$	μ
Binomial	np	$n(n - 1)p^2 + np$	$np(1 - p)$
Exponential	$\frac{1}{\theta}$	$\frac{2}{\theta^2}$	$\frac{1}{\theta^2}$
Weibull	$\left(\frac{1}{\beta}\right)^{1/\alpha}\Gamma\left(\frac{\alpha + 1}{\alpha}\right)$	$\left(\frac{1}{\beta}\right)^{2/\alpha}\Gamma\left(\frac{\alpha + 2}{\alpha}\right)$	$\left(\frac{1}{\beta}\right)^{2/\alpha}\left\{\Gamma\left(\frac{\alpha + 2}{\alpha}\right) - \left[\Gamma\left(\frac{\alpha + 1}{\alpha}\right)\right]^2\right\}$
Log-Normal	$\exp\left(\mu_{\log} + \frac{1}{2}\sigma^2_{\log}\right)$	$\exp\left(2\mu_{\log} + 2\sigma^2_{\log}\right)$	$\exp\left(2\mu_{\log} + \sigma^2_{\log}\right)\left[\exp\left(\sigma^2_{\log}\right) - 1\right]$

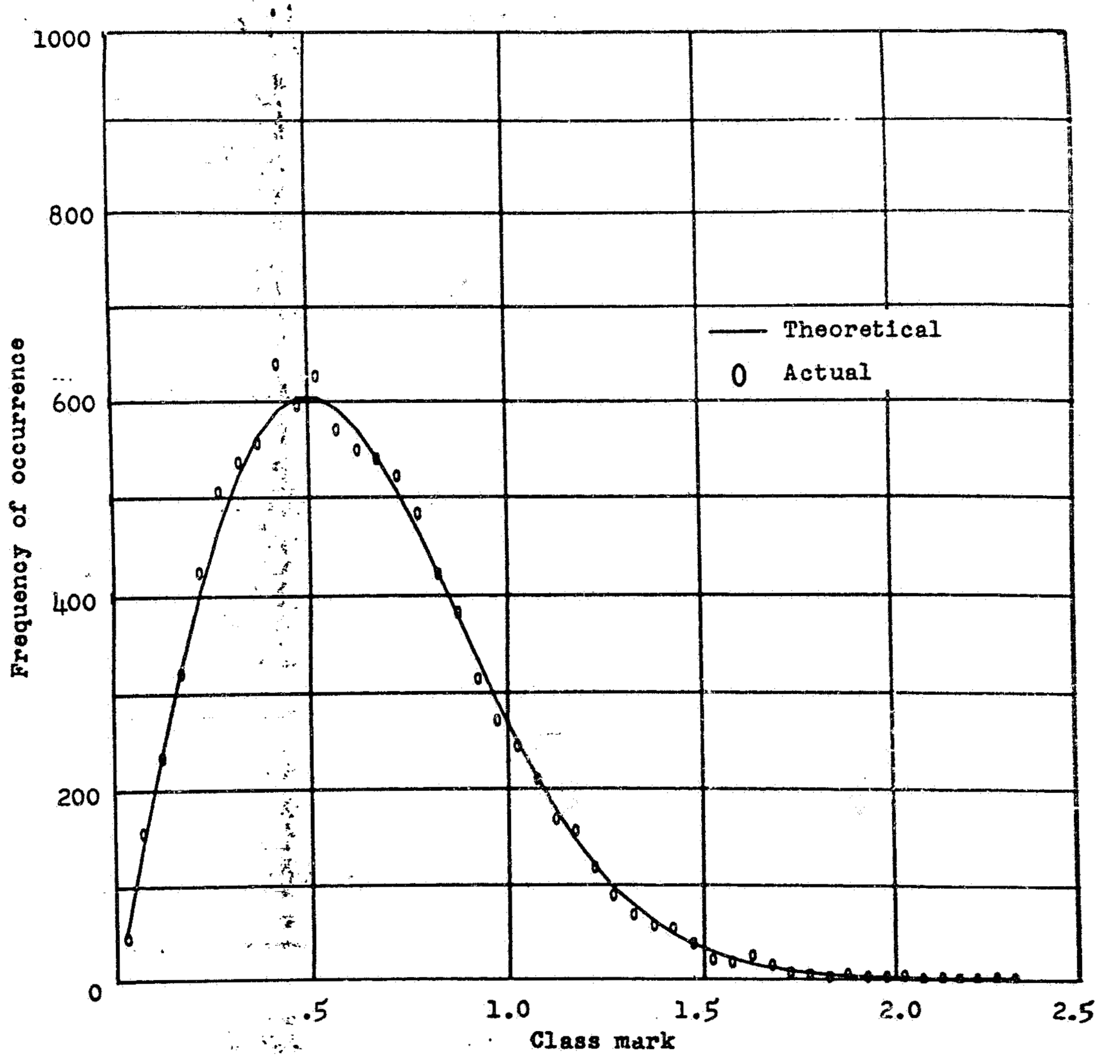

Figure 3.11. Actual and theoretical histograms for a Weibull amplitude probability density: $\alpha = 2$; $\beta = 2$

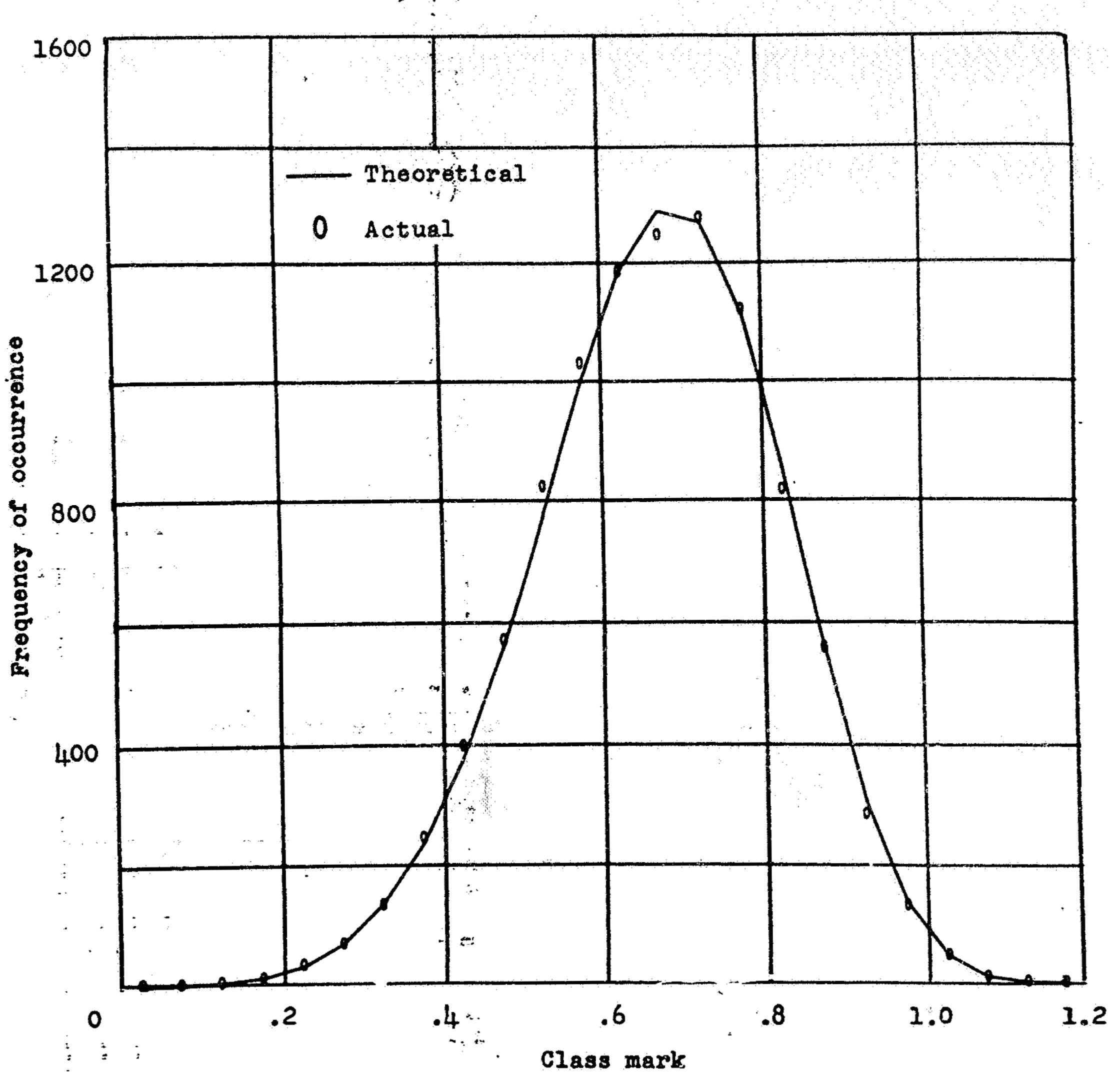

Figure 3.12. Actual and theoretical histograms for a Weibull amplitude probability density: $\alpha = 5$; $\beta = 5$

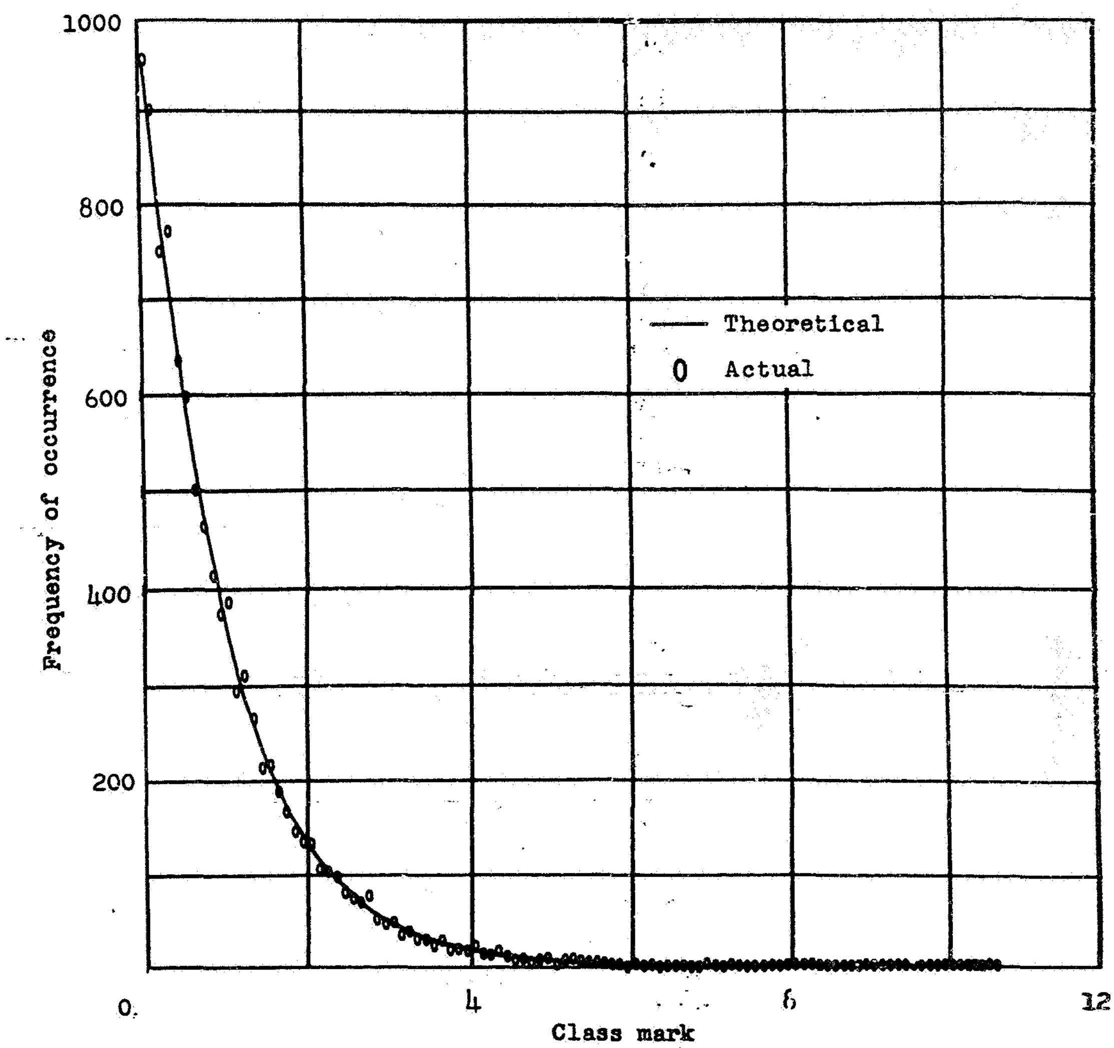

Figure 3.13. Actual and theoretical histograms for an Exponential amplitude probability density: $\theta = 1$

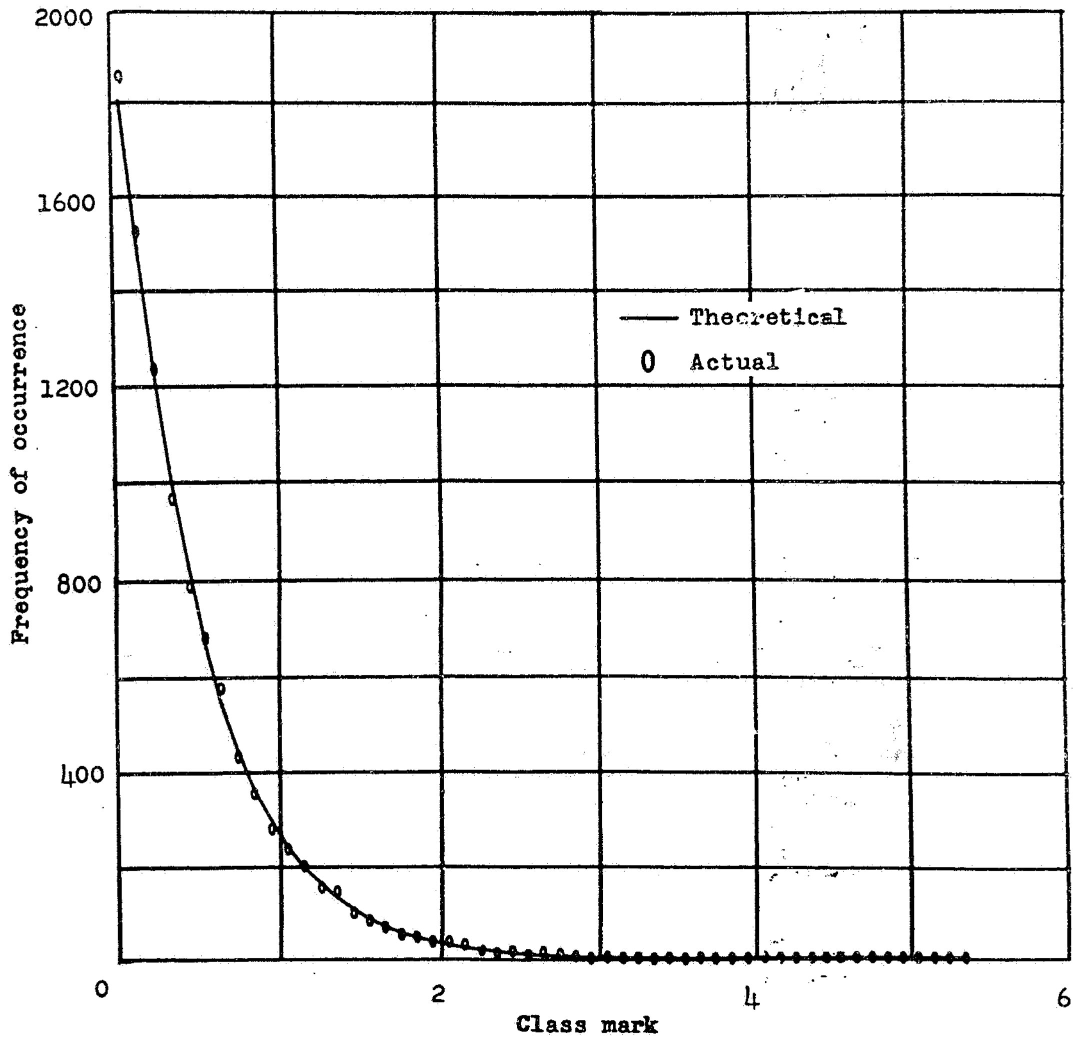

Figure 3.14. Actual and theoretical histograms for an Exponential amplitude probability density: $\theta = 2$

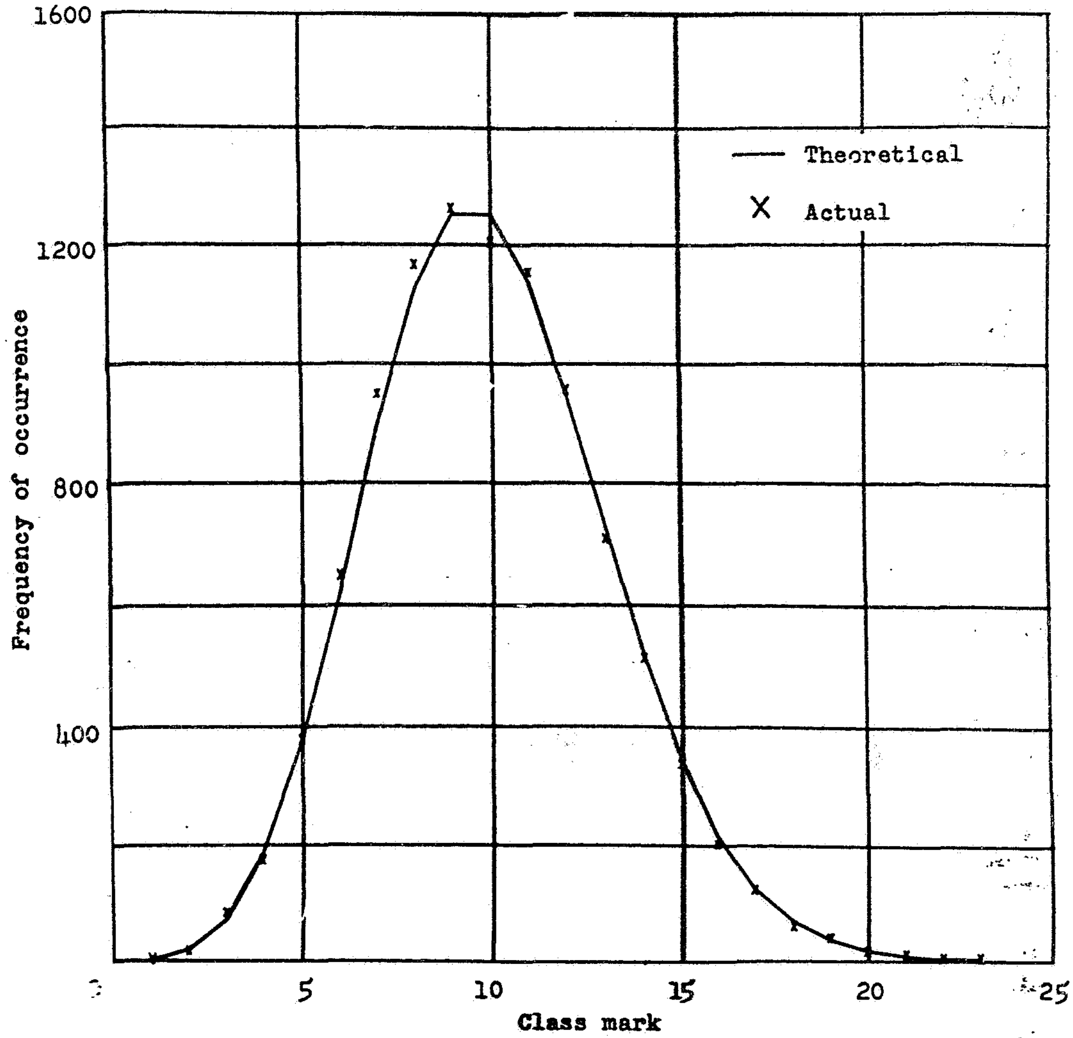

Figure 3.15. Actual and theoretical histograms for a Poisson amplitude probability density: $\mu = 10$

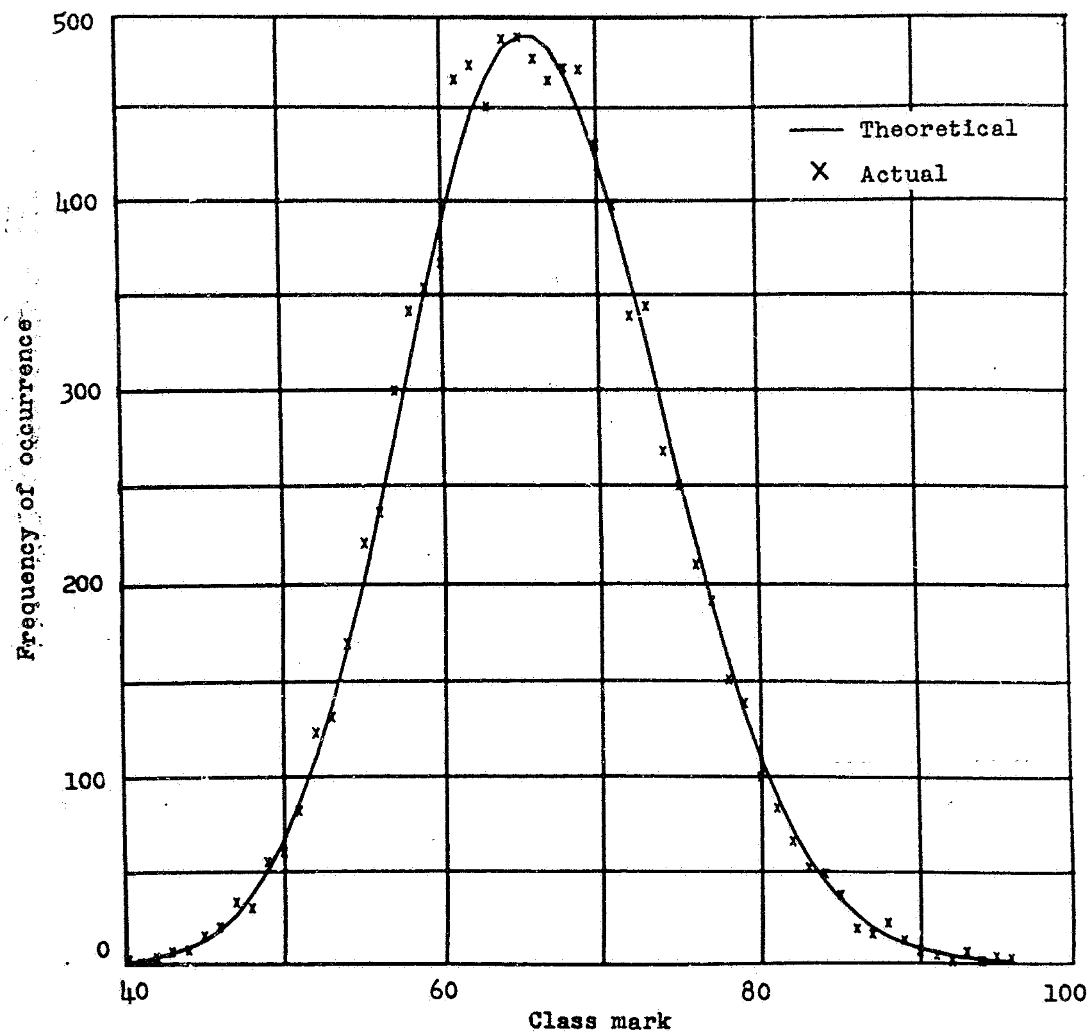

Figure 3.16. Actual and theoretical histograms for a Poisson amplitude probability density: $\mu = 66$

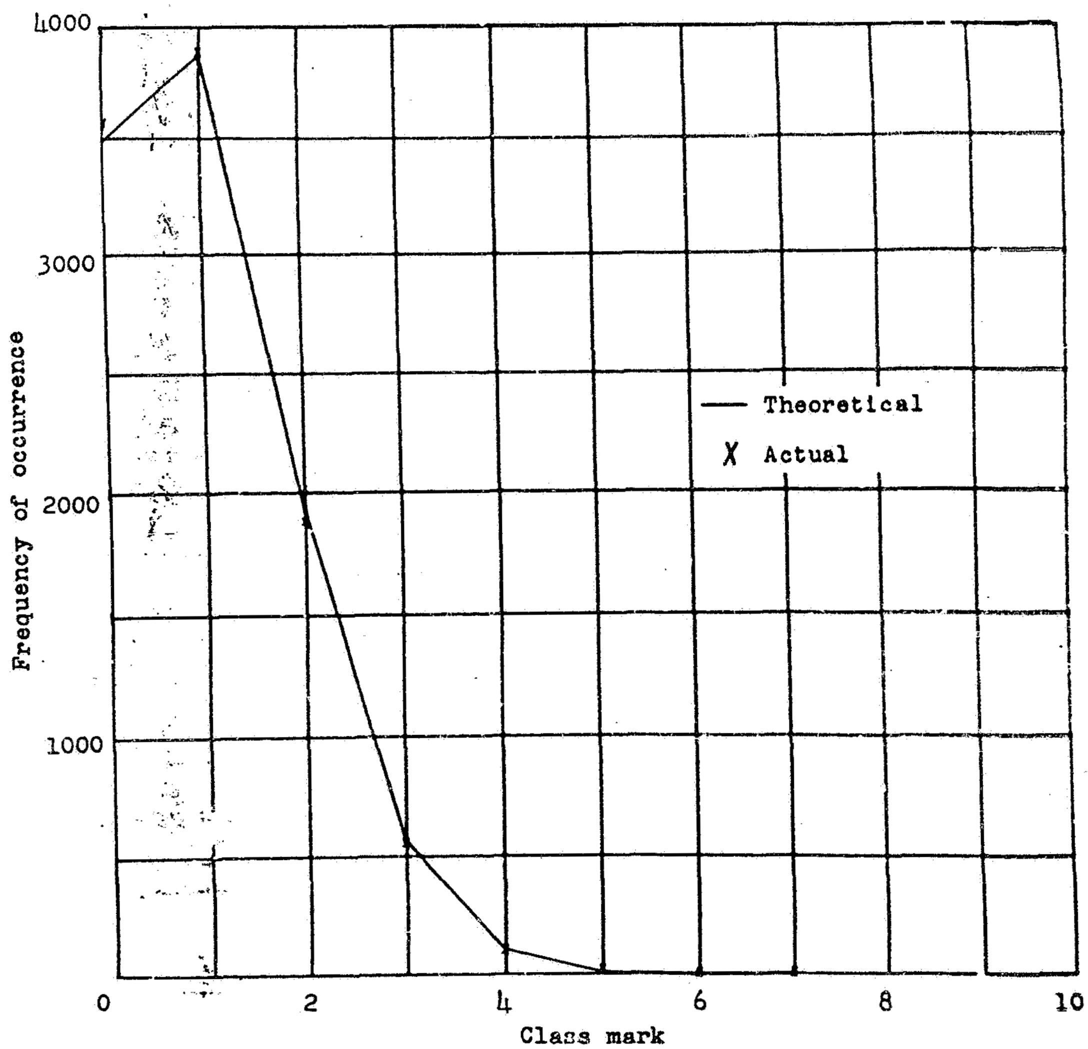

Figure 3.17. Actual and theoretical histograms for a Binomial amplitude probability density: $n = 10$; $p = 0.1$

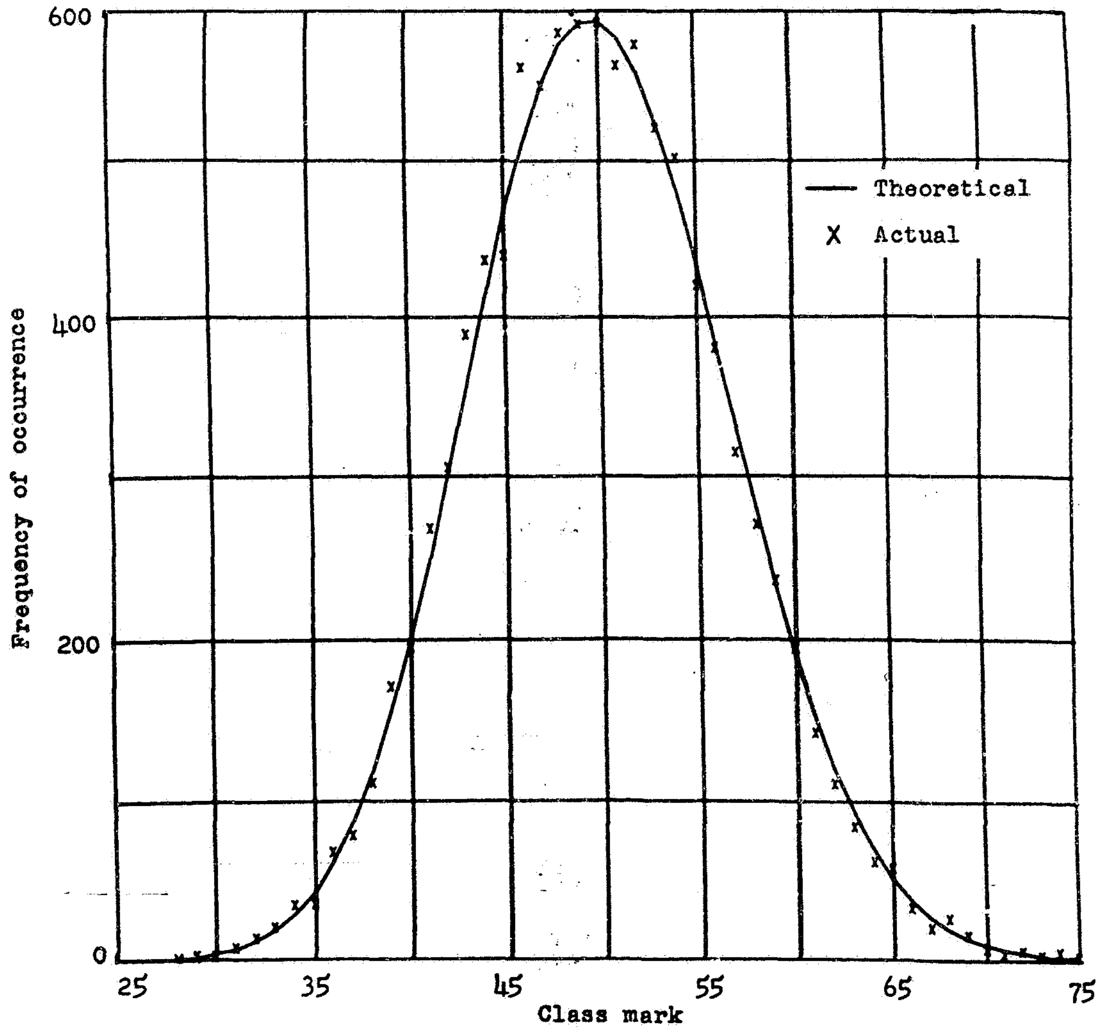

Figure 3.18. Actual and theoretical histograms for a Binomial amplitude probability density: n = 500; p = 0.1

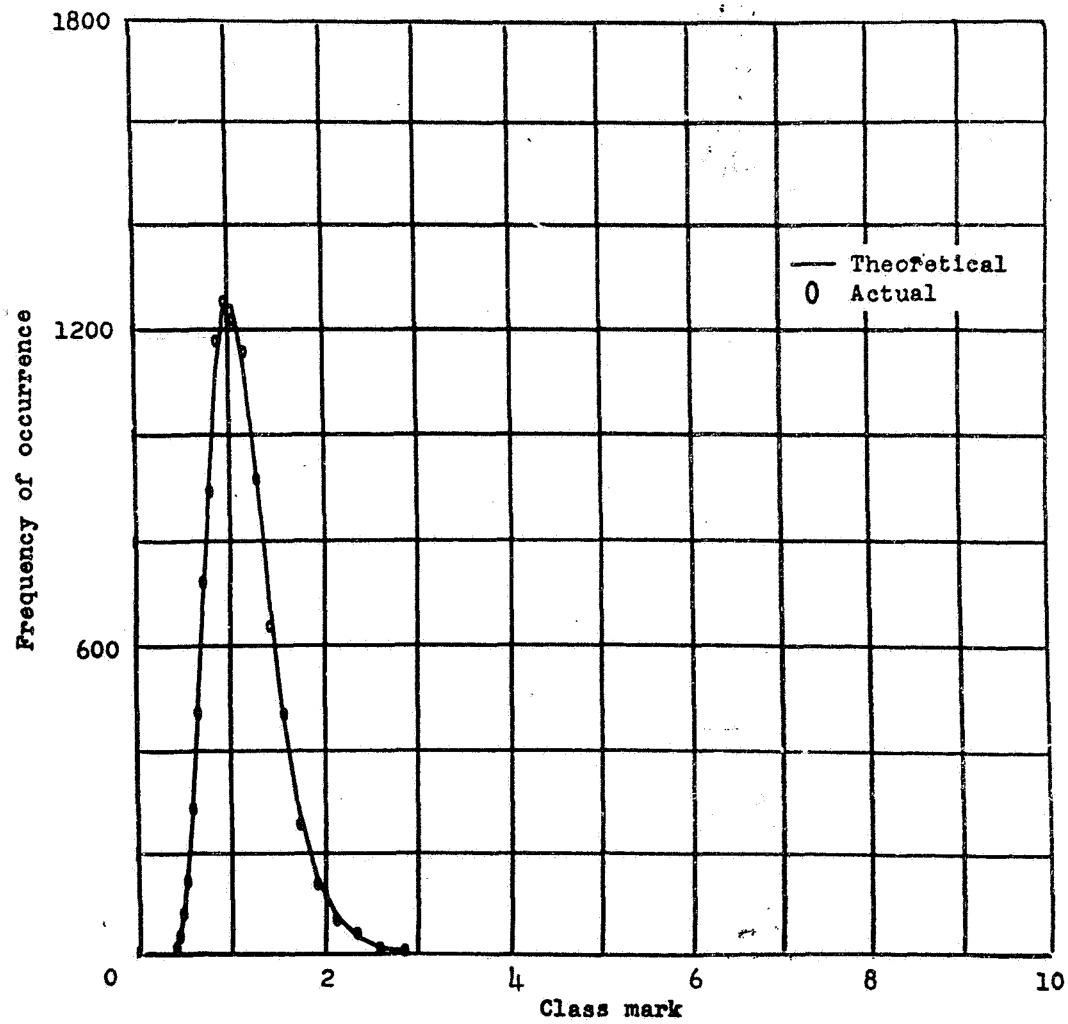

Figure 3.19. Actual and theoretical histograms for a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.1$

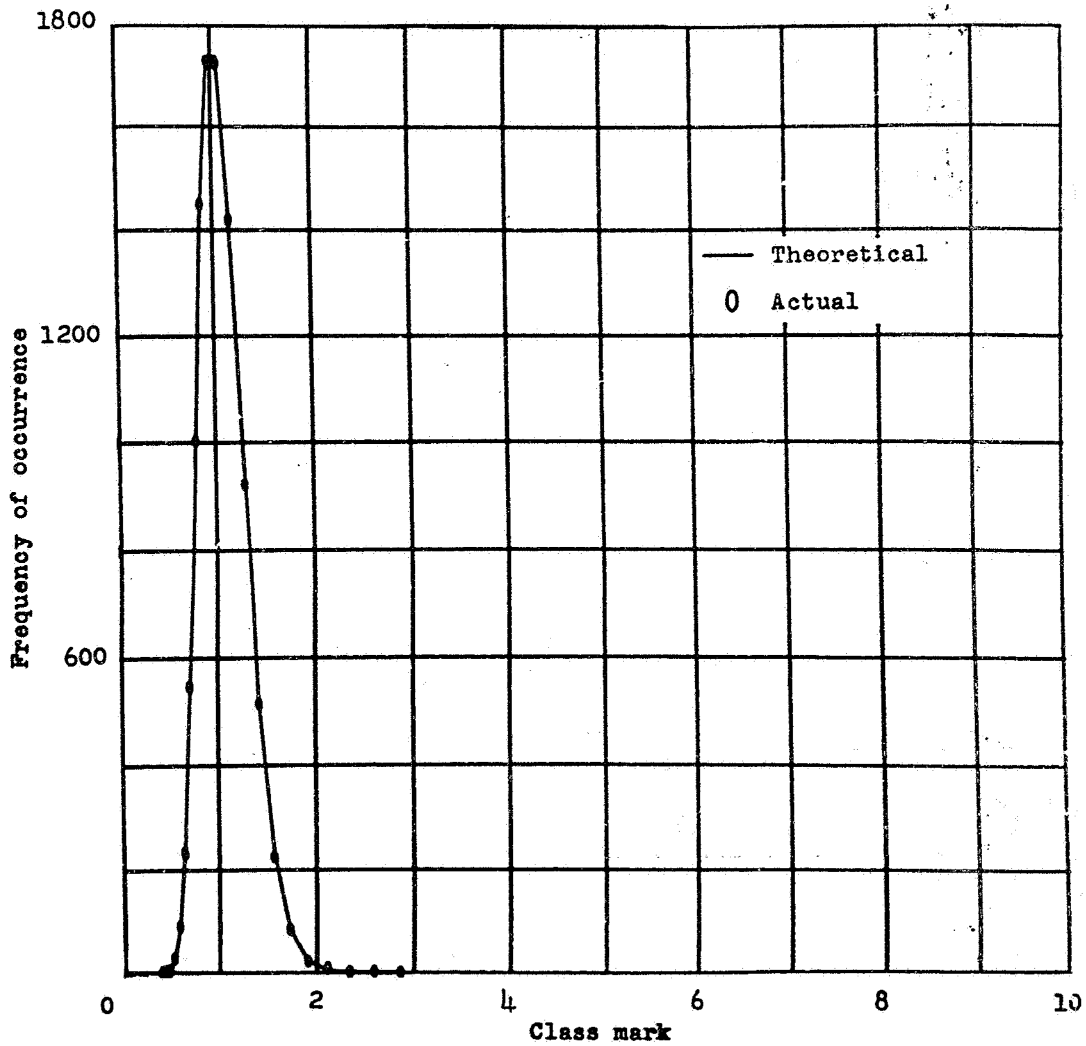

Figure 3.20. Actual and theoretical histograms for a Log-Normal amplitude probability density: $\sigma^2_{log} = 0.05$

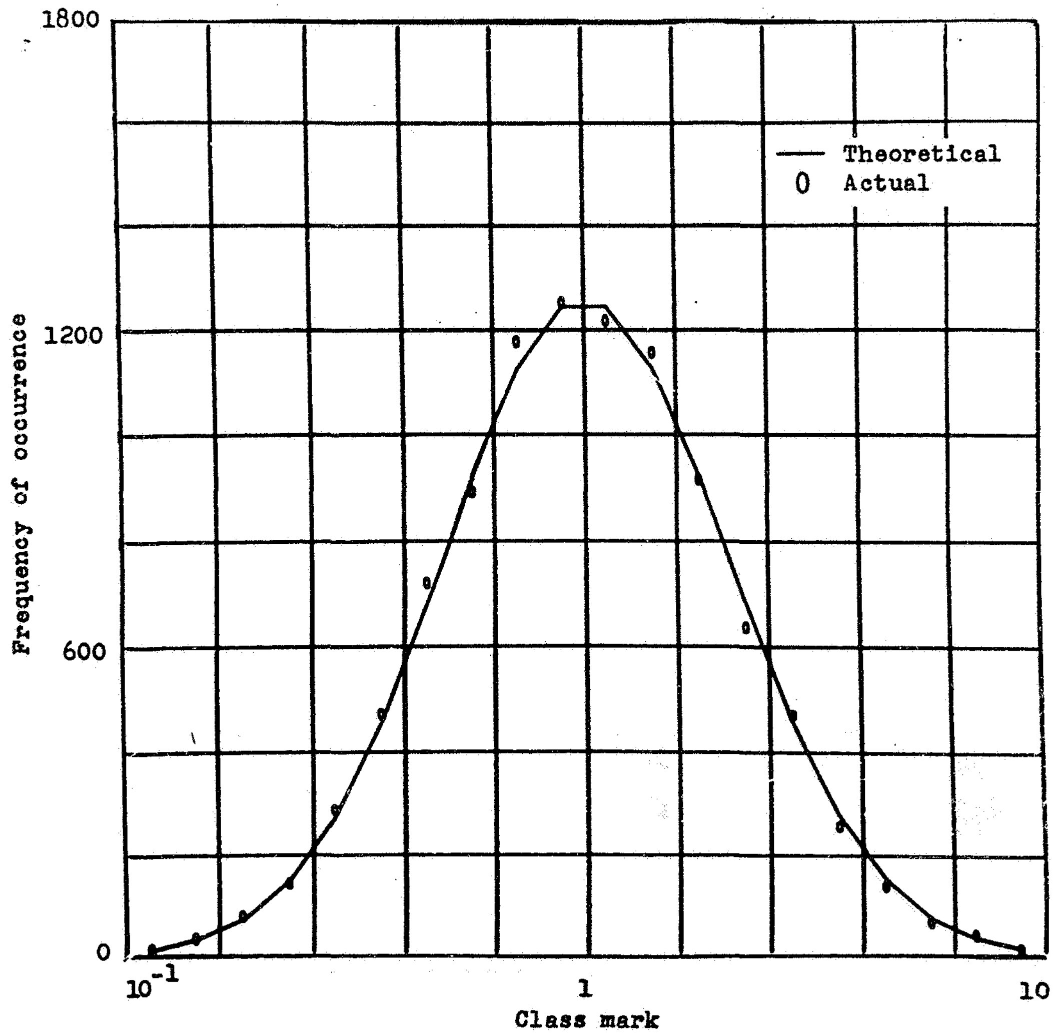

Figure 3.21. Actual and theoretical logarithmic histograms for a Log-Normal amplitude probability density: $\sigma_{\log}^2 = 0.1$

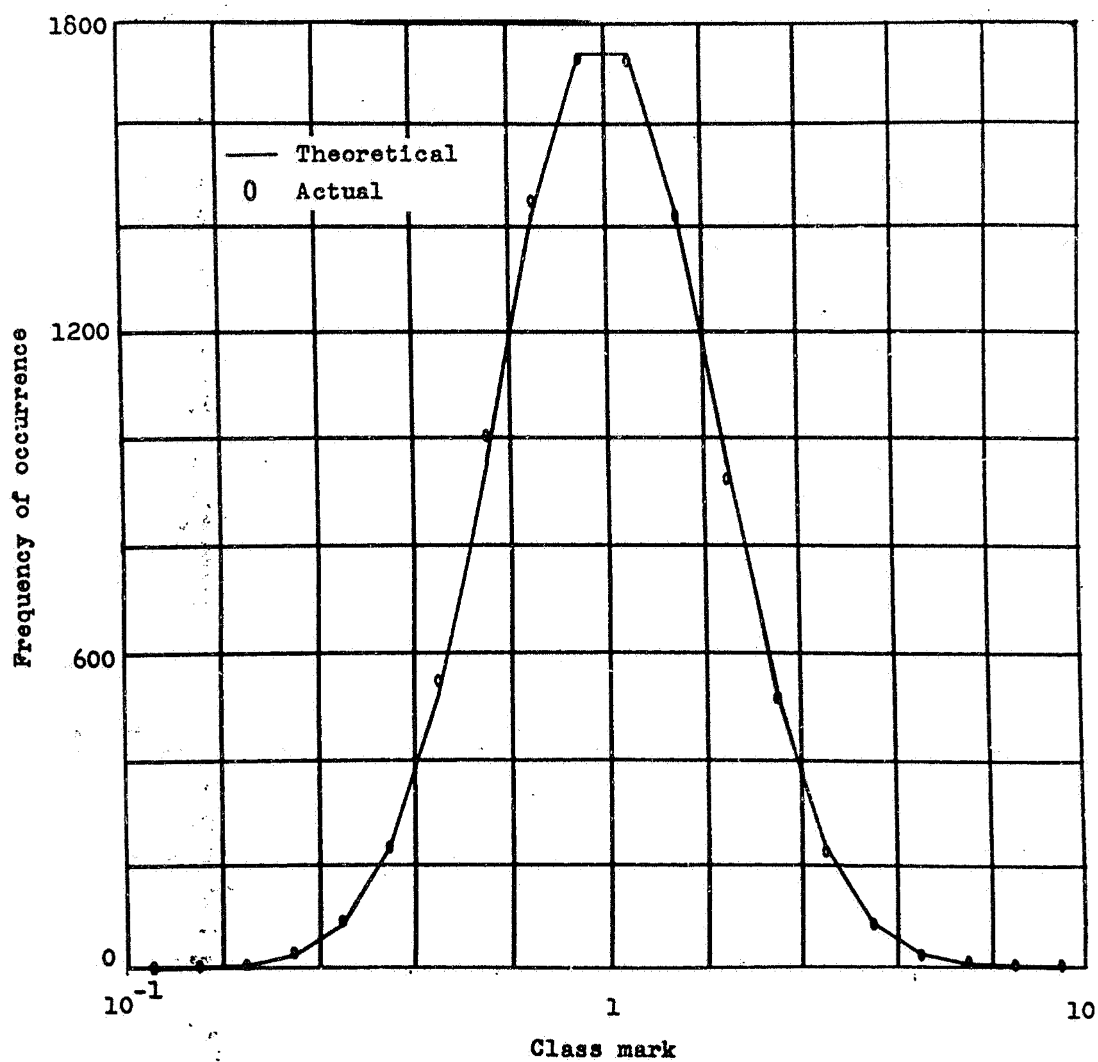

Figure 3.22. Actual and theoretical logarithmic histograms for a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.05$

confidence level. This exercise was done in order to show that the digitally generated random time histories do indeed have the desired amplitude probability density functions.

As indicated in the title of this thesis all time histories generated were to be non-Gaussian in nature. However, certain values for the various parameters in the density functions used tend to make the distributions approach a Gaussian distribution. It is well known that some of these distributions can be approximated with a normal or Gaussian distribution. This fact becomes obvious when looking at Figures 3.11-3.22. The α and β parameters of the Weibull distribution were each varied in unit increments from 2 to 5. Figure 3.11 where $\alpha = 2$, $\beta = 2$ shows a highly non-Gaussian shape while Figure 3.12 with $\alpha = 5$, $\beta = 5$ shows a shape approaching Gaussian. The θ parameter of the Exponential distribution was varied from 1/4 through 2 by doubling each θ in succession. This distribution will always be highly non-Gaussian as shown in Figures 3.13 and 3.14. The Poisson distribution tends toward the Gaussian distribution even for small mean values μ as illustrated in Figures 3.15 and 3.16. The Binomial distribution tends to be highly non-Gaussian for small n (see Figure 3.17) and Gaussian for large n (see Figure 3.18). In the Log-Normal distribution, the parameters μ_{log} and σ^2_{log} were varied as follows:

$\mu_{\log} = 0, 1$; $\sigma^2_{\log} = 2, 1, 0.5, 0.1, 0.05$. A horizontal translation of the distribution occurred when $\mu_{\log}$ was increased from zero to one. Figures 3.19 and 3.20 show the highly non-Gaussian shape of the distribution while Figures 3.21 and 3.22 show the Gaussian shape when plotted on semi-log coordinates.

3.4 Determination of Peak and Cumulative Peak Distributions for Both Maximum and Minimum Peak Values

As with the histograms, the amplitude scale of the time histories were divided up into equal intervals and the peaks, both maximum and minimum, were counted in each interval. Maximum peaks and minimum peaks are defined as shown in Figure 3.23. The maximum peak distributions are plotted in Figures 3.24-3.33 while the minimum peak distributions are shown in Figures 3.34-3.43. Since service load data are based on maximum peaks, we are primarily interested in the maximum peak distributions of the generated time histories. However, it is also of interest to see whether or not the peaks, both maximum and minimum, are distributed symmetrically about the mean of a random time history. Thus the minimum peak distributions are also plotted. One can check for symmetry by visualizing a maximum peak distribution overlaid on a minimum peak distribution. The two distributions will intersect at the

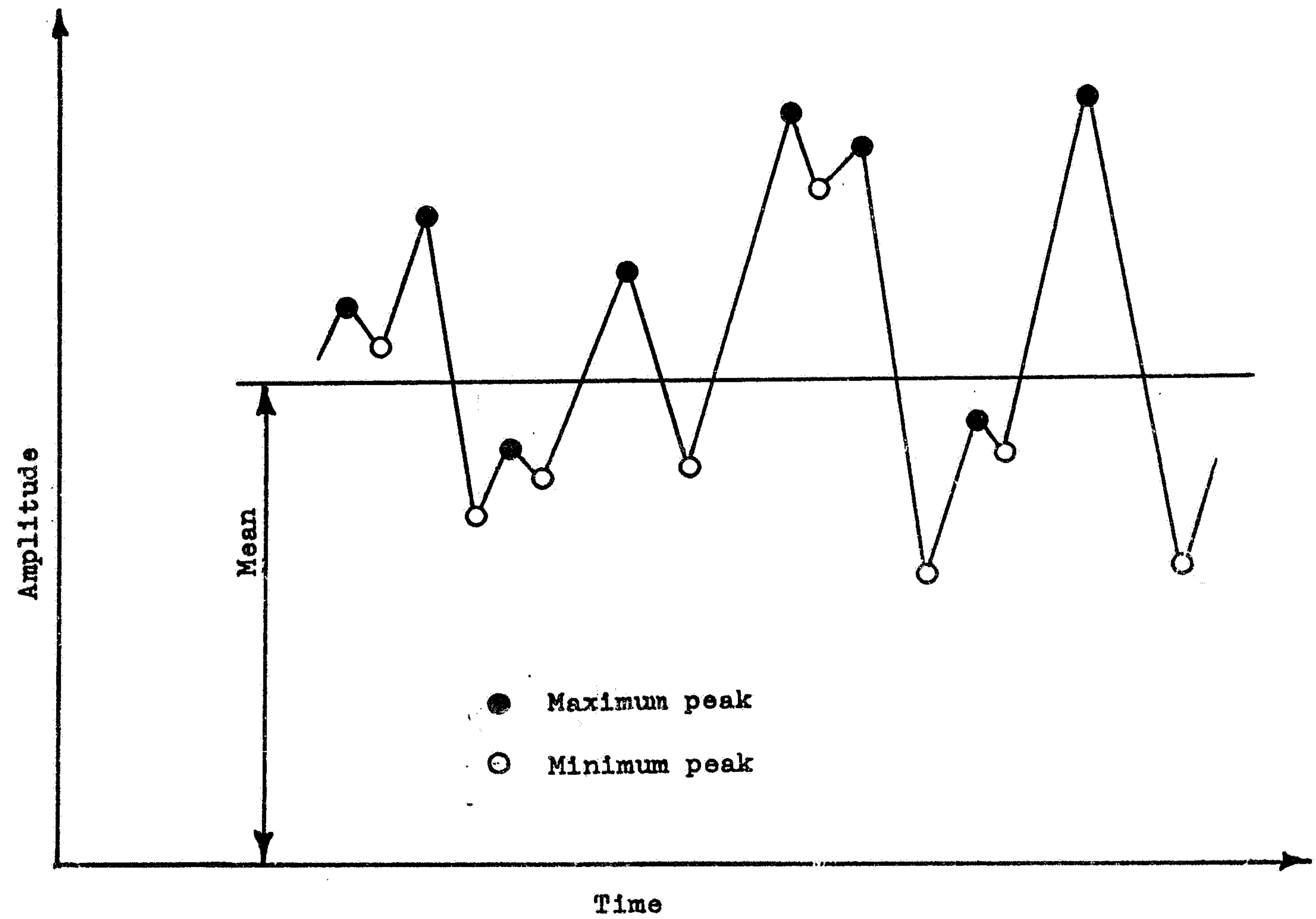

Figure 3.23. Description of maximum and minimum peaks

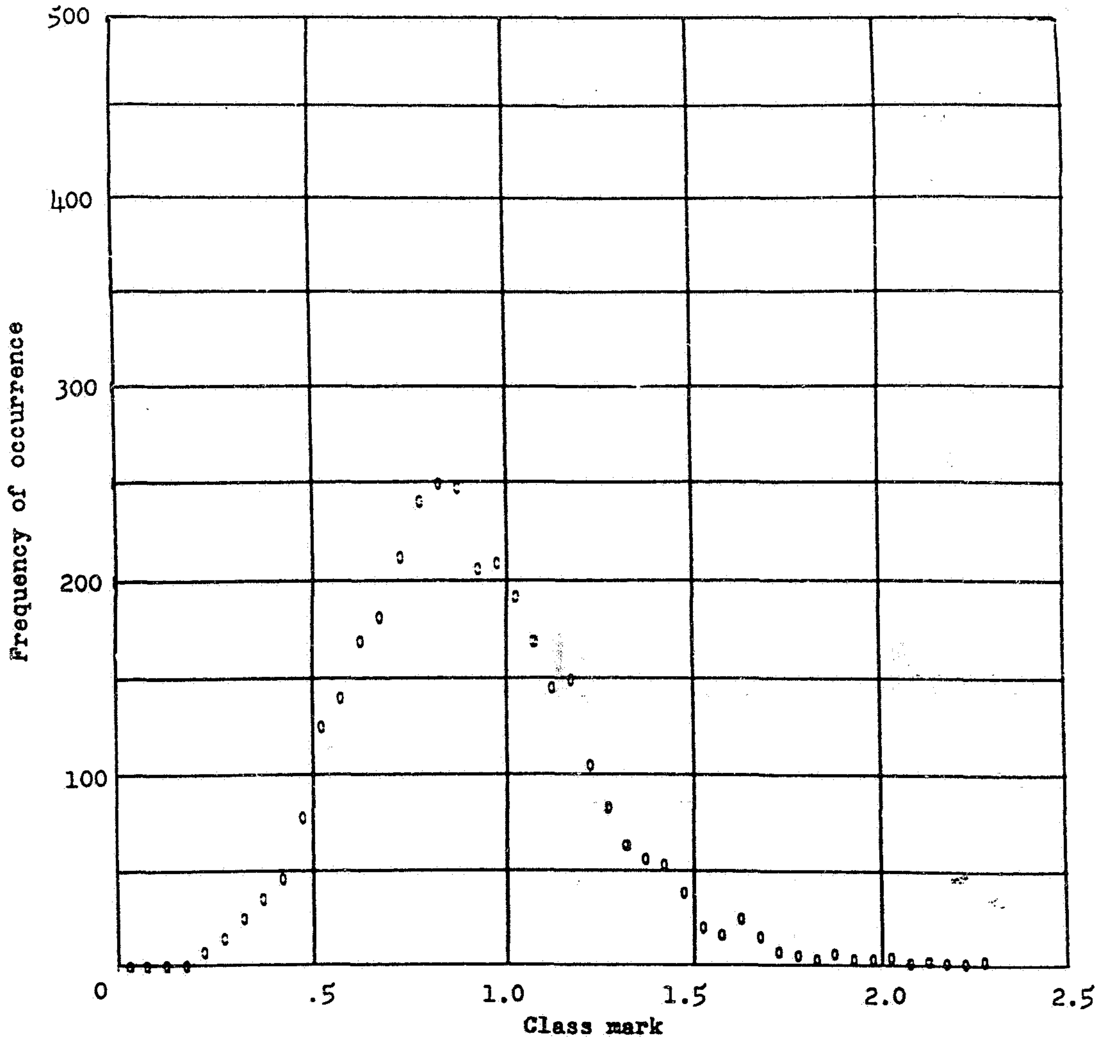

Figure 3.24. Maximum peak distribution of a time history having a Weibull amplitude probability density: $\alpha = 2$; $\beta = 2$

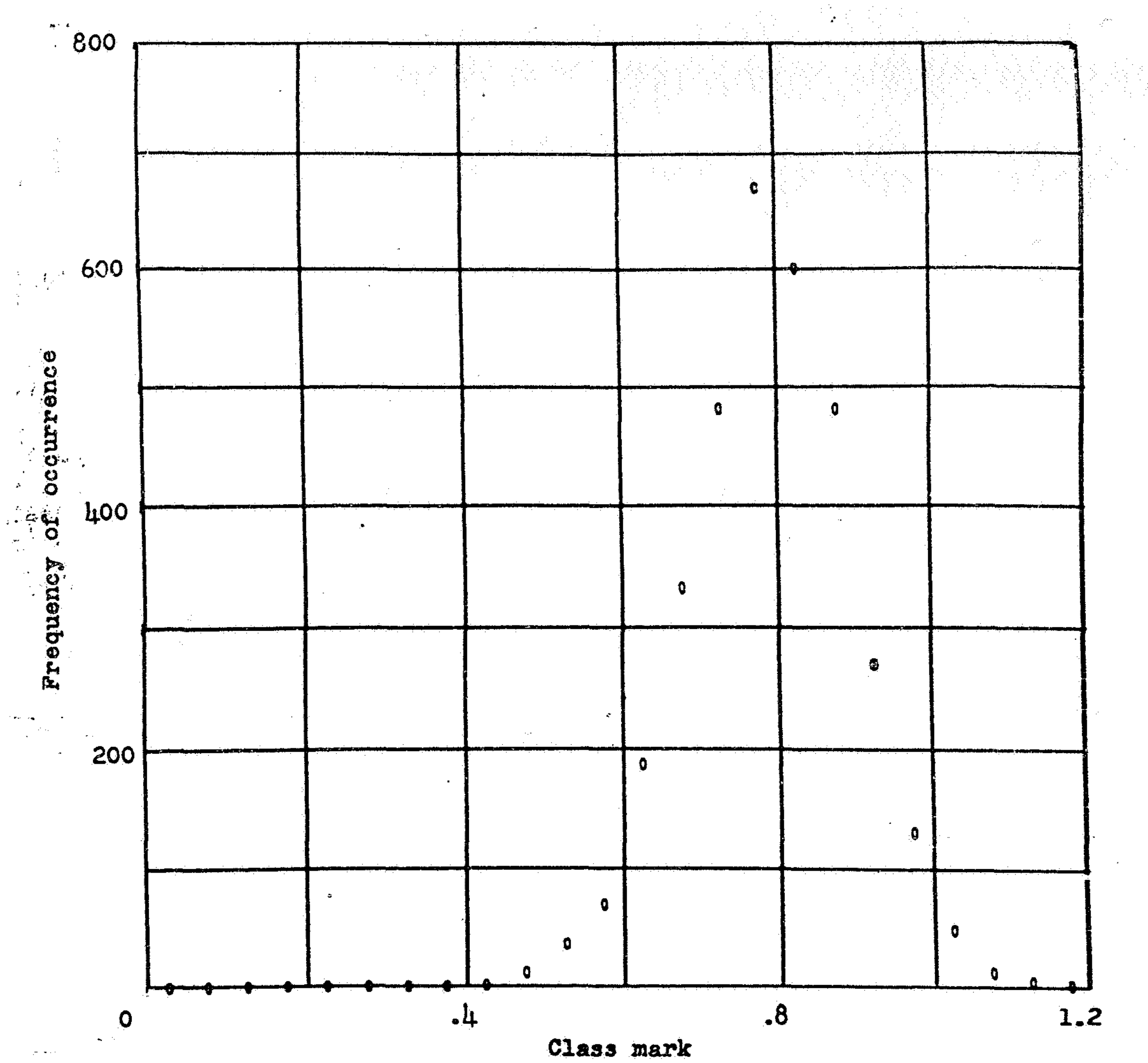

Figure 3.25. Maximum peak distribution of a time history having a Weibull amplitude probability density: $\alpha = 5$; $\beta = 5$

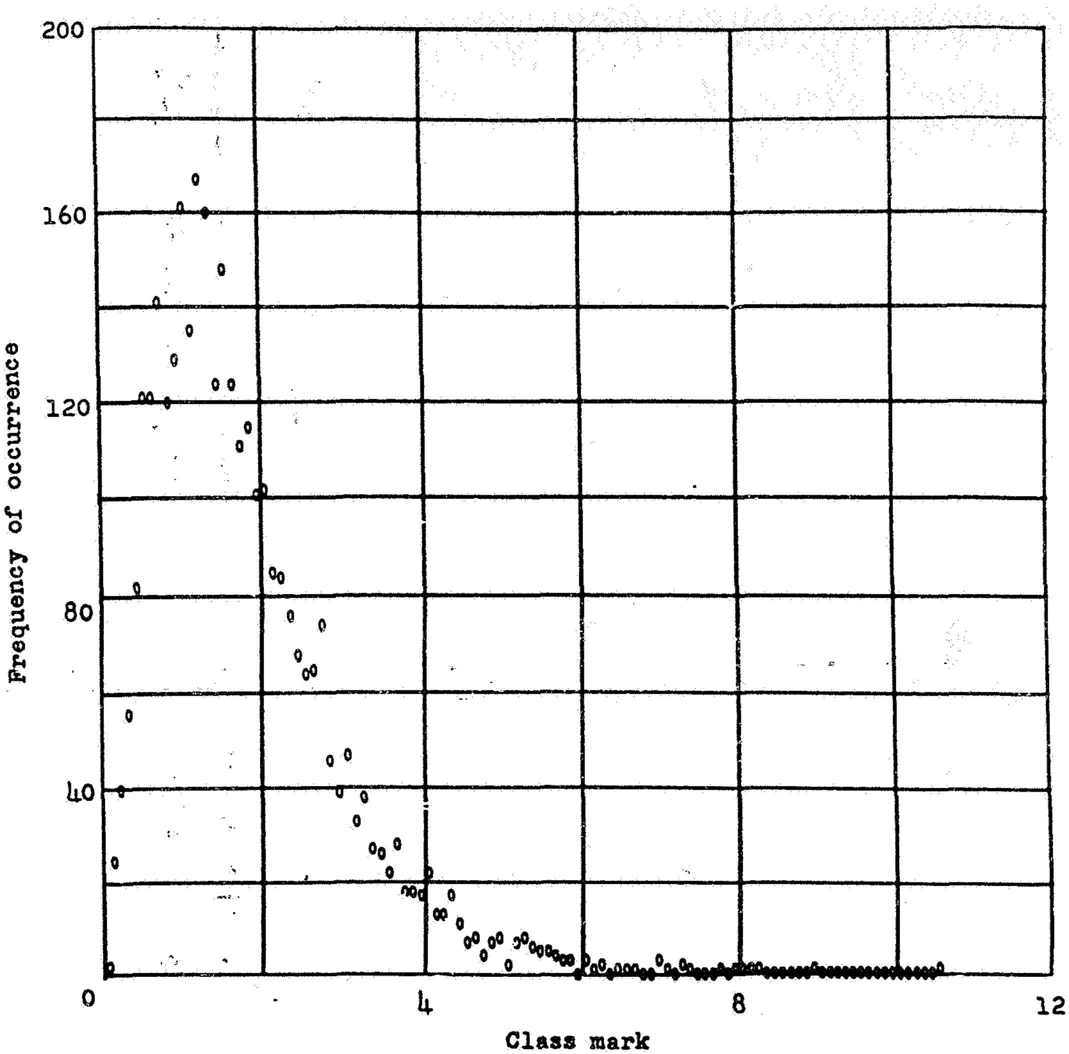

Figure 3.26. Maximum peak distribution of a time history having an Exponential amplitude probability density: $\theta = 1$

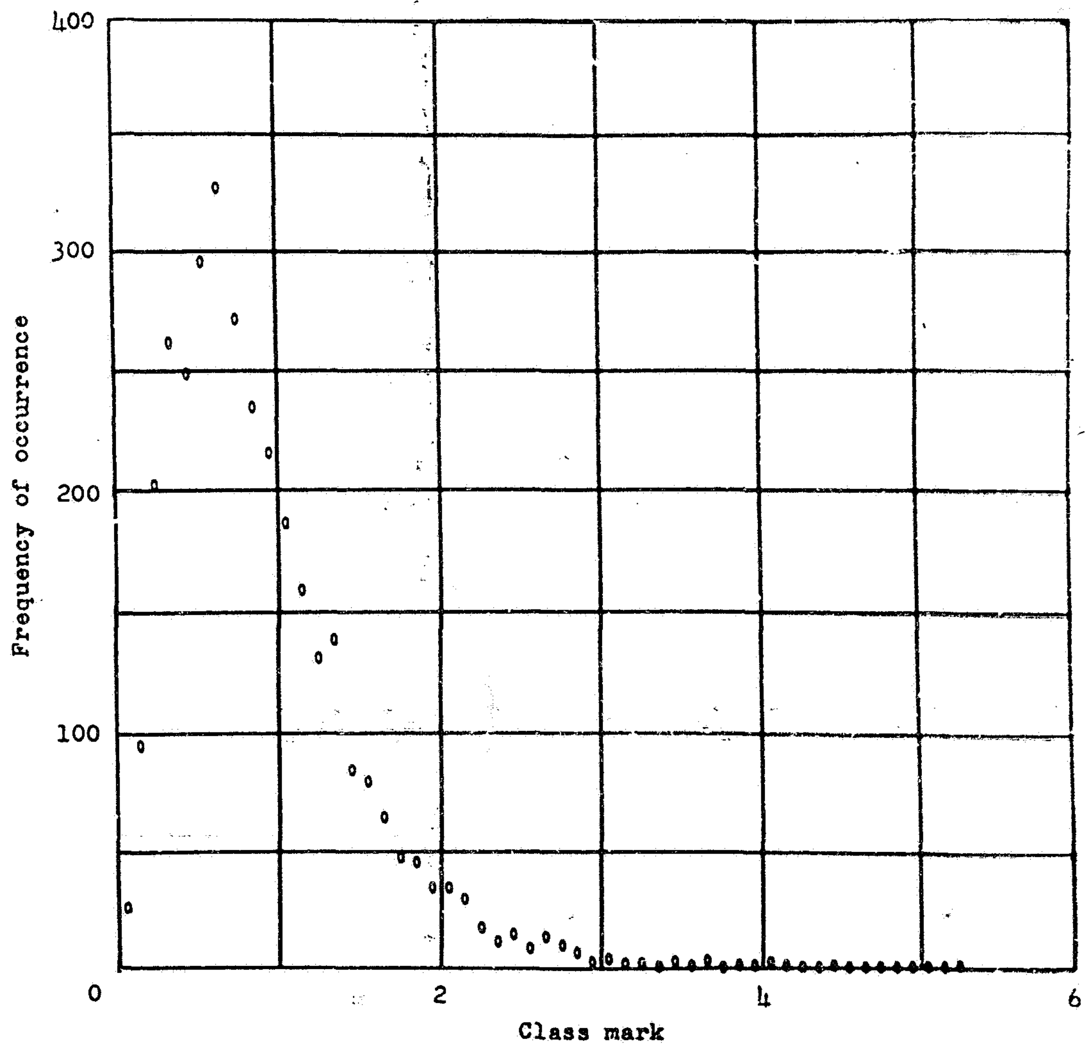

Figure 3.27. Maximum peak distribution of a time history having an Exponential amplitude probability density: $\theta = 2$

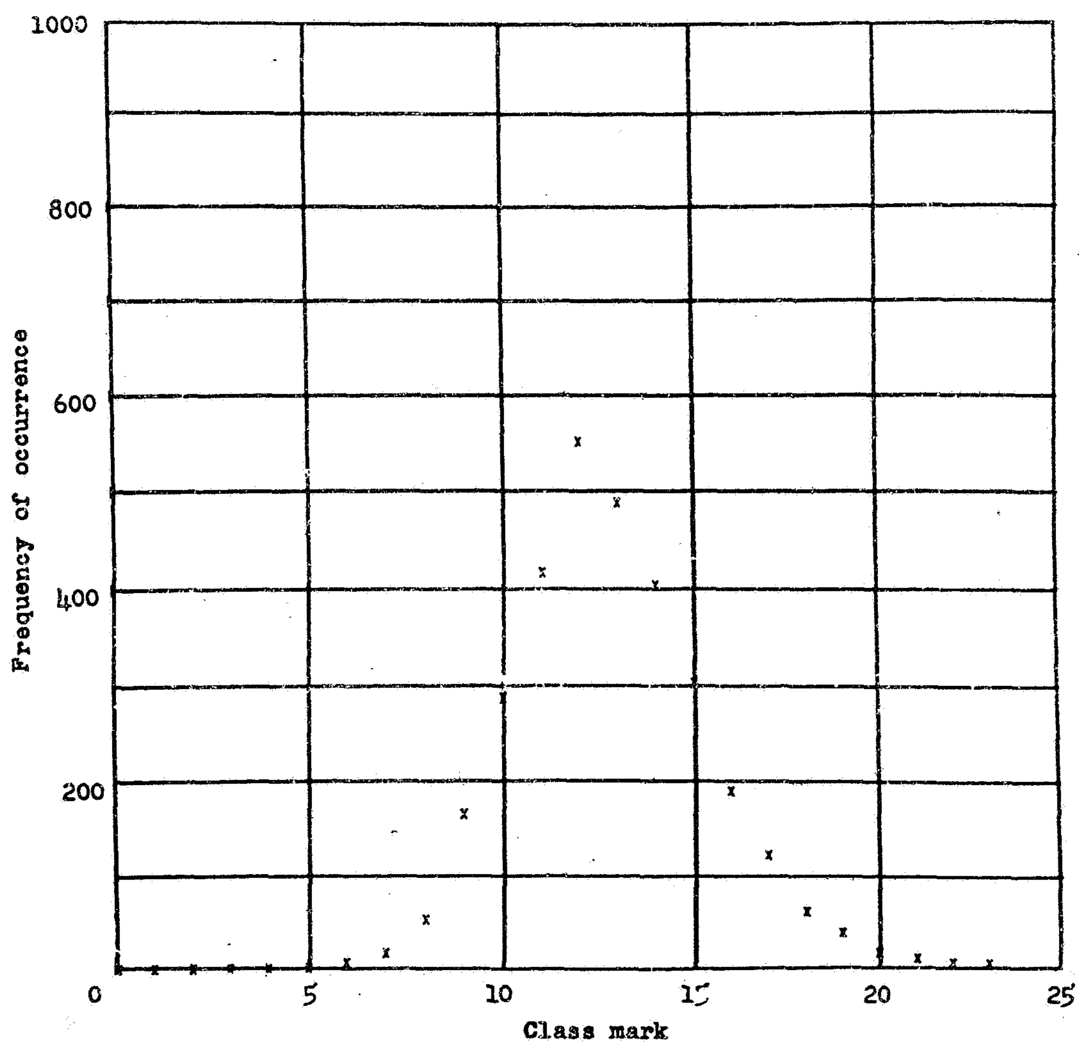

Figure 3.28. Maximum peak distribution of a time history having a Poisson amplitude probability density: $\mu = 10$

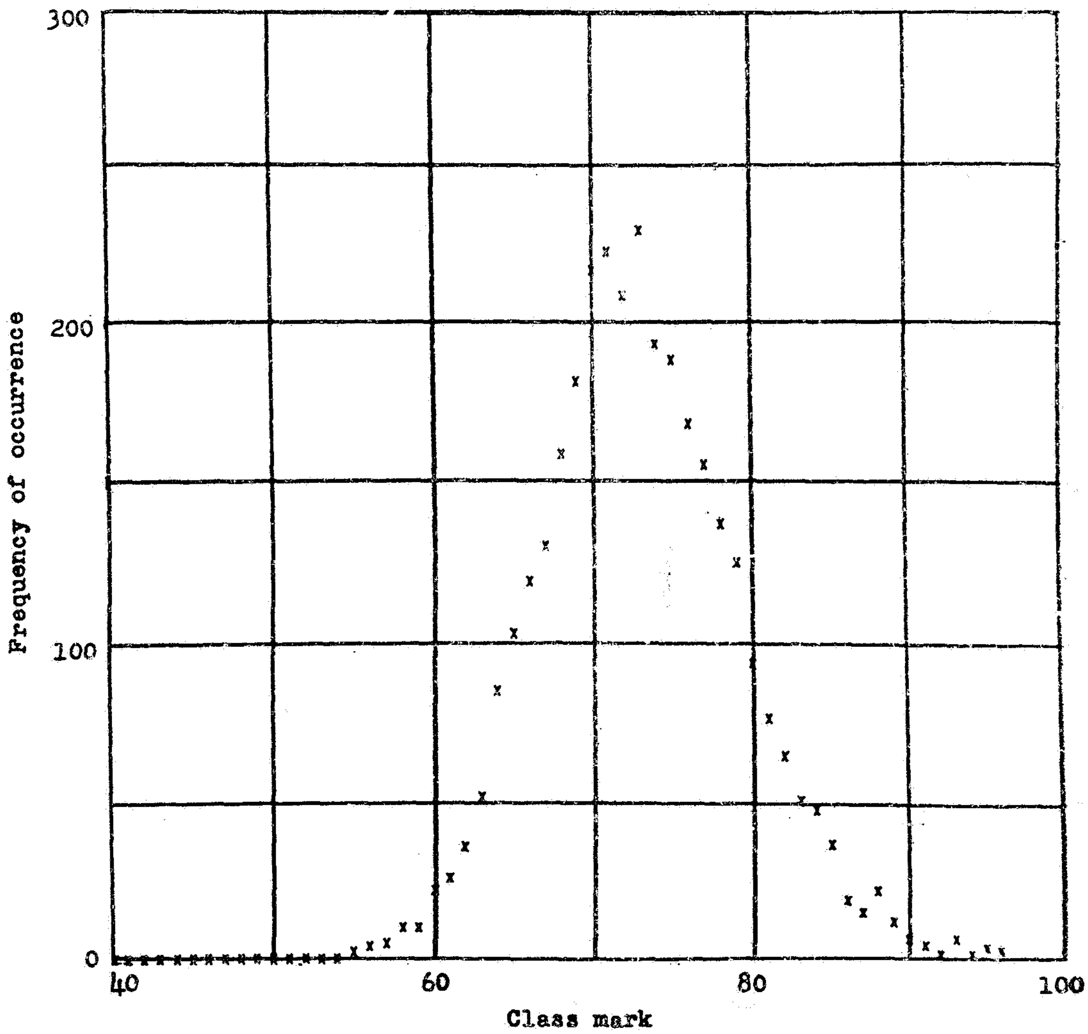

Figure 3.29. Maximum peak distribution of a time history having a Poisson amplitude probability density: $\mu = 66$

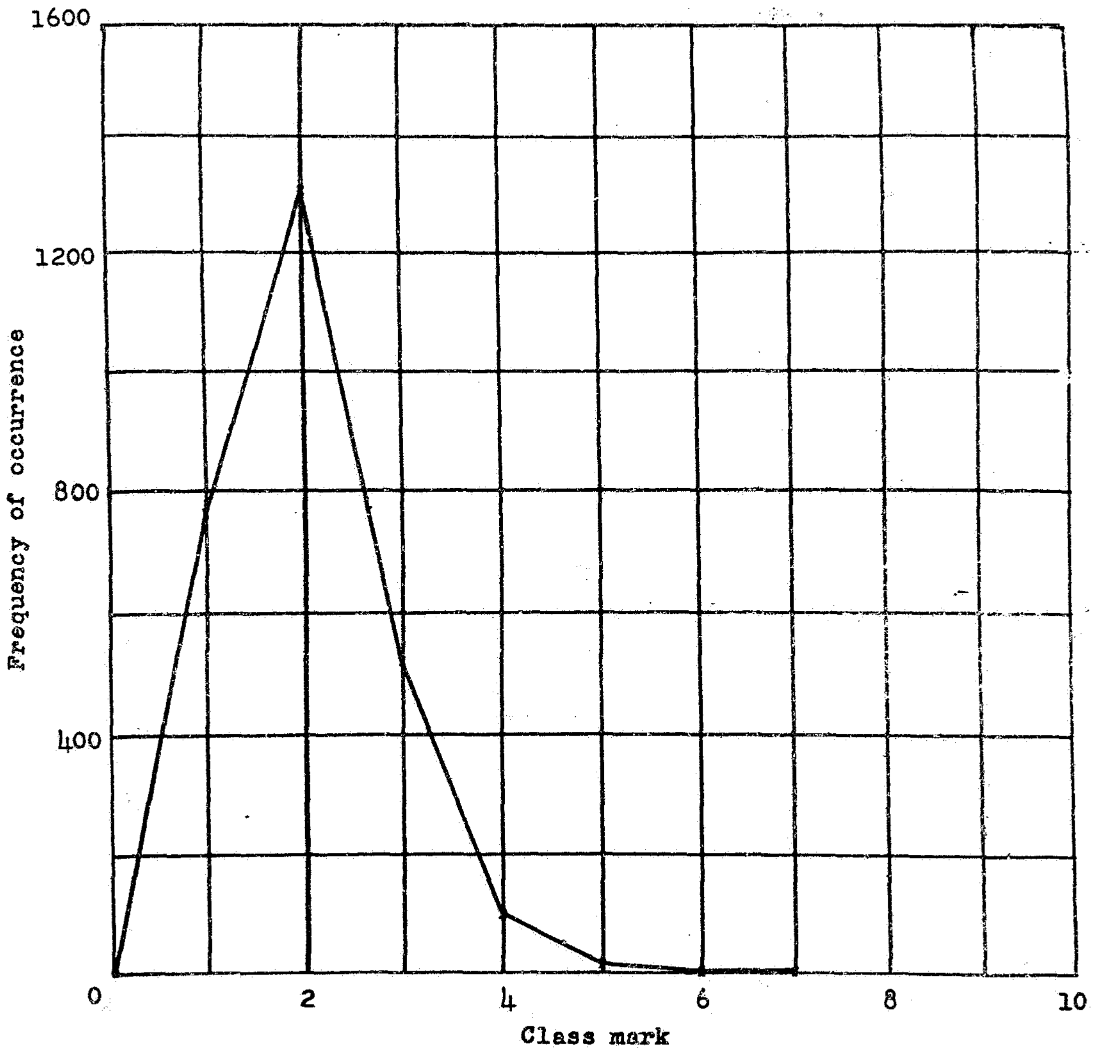

Figure 3.30. Maximum peak distribution of a time history having a Binomial amplitude probability density: $n = 10$; $p = 0.1$

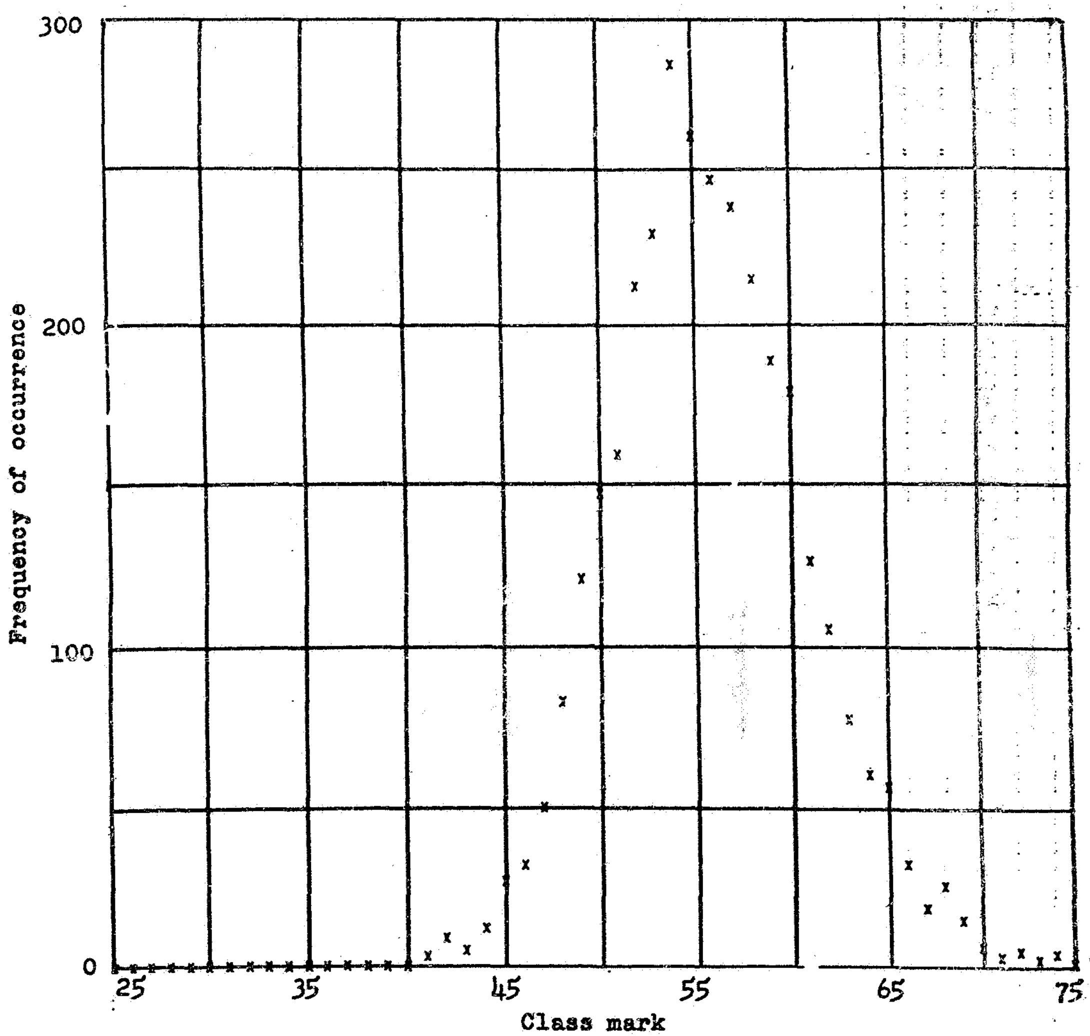

Figure 3.31. Maximum peak distribution of a time history having a Binomial amplitude probability density: n = 500; p = 0.1

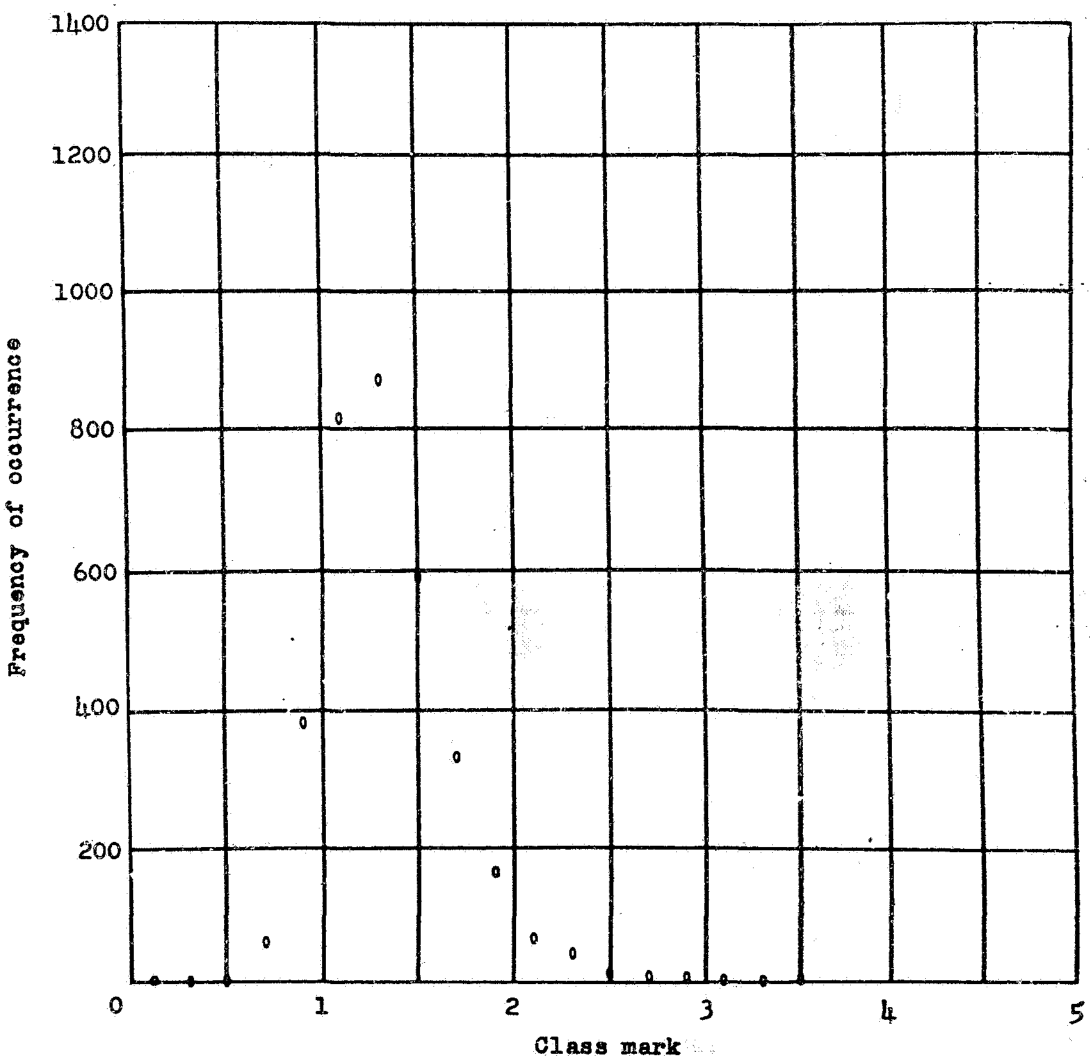

Figure 3.32. Maximum peak distribution of a time history having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.1$

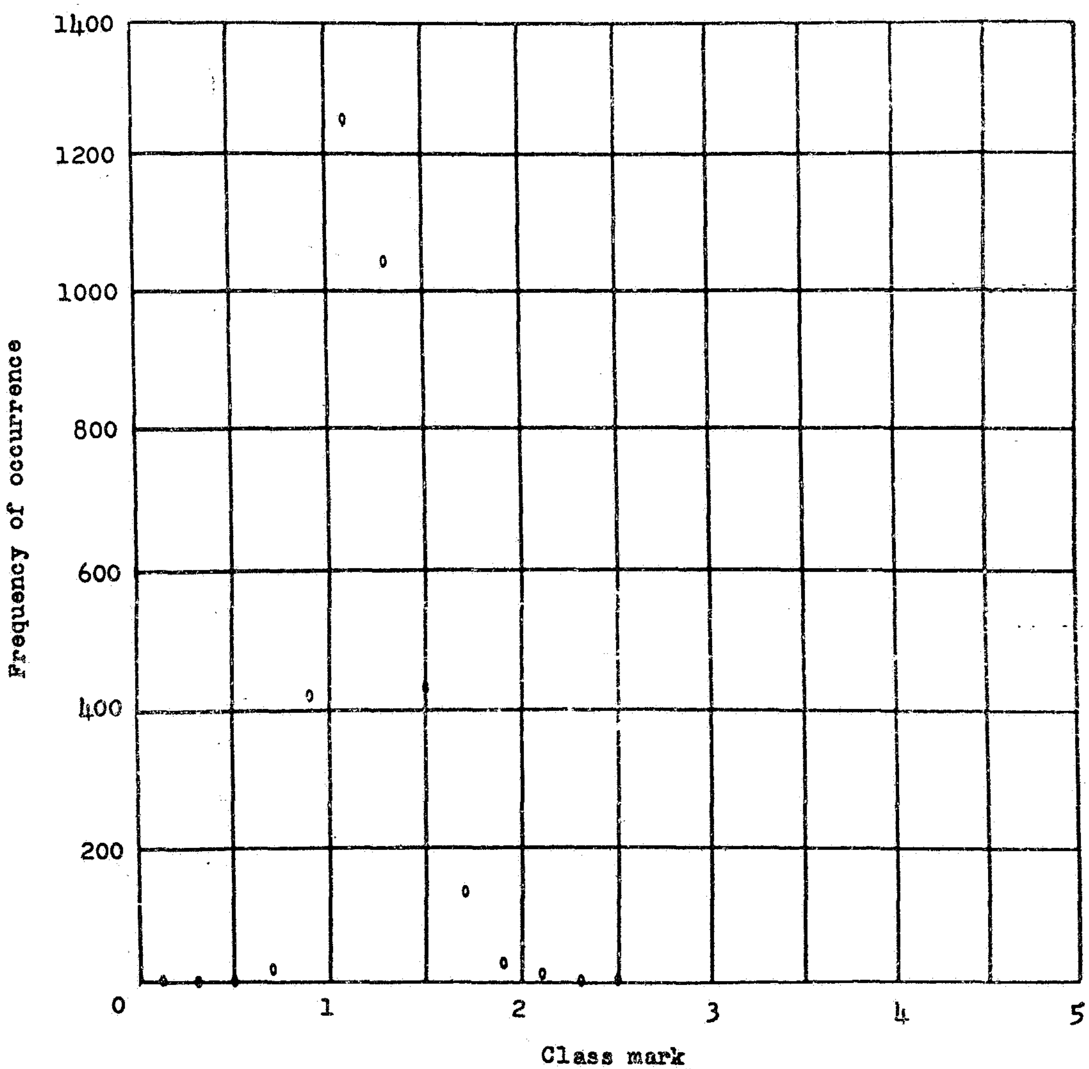

Figure 3.33. Maximum peak distribution of a time history having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.05$

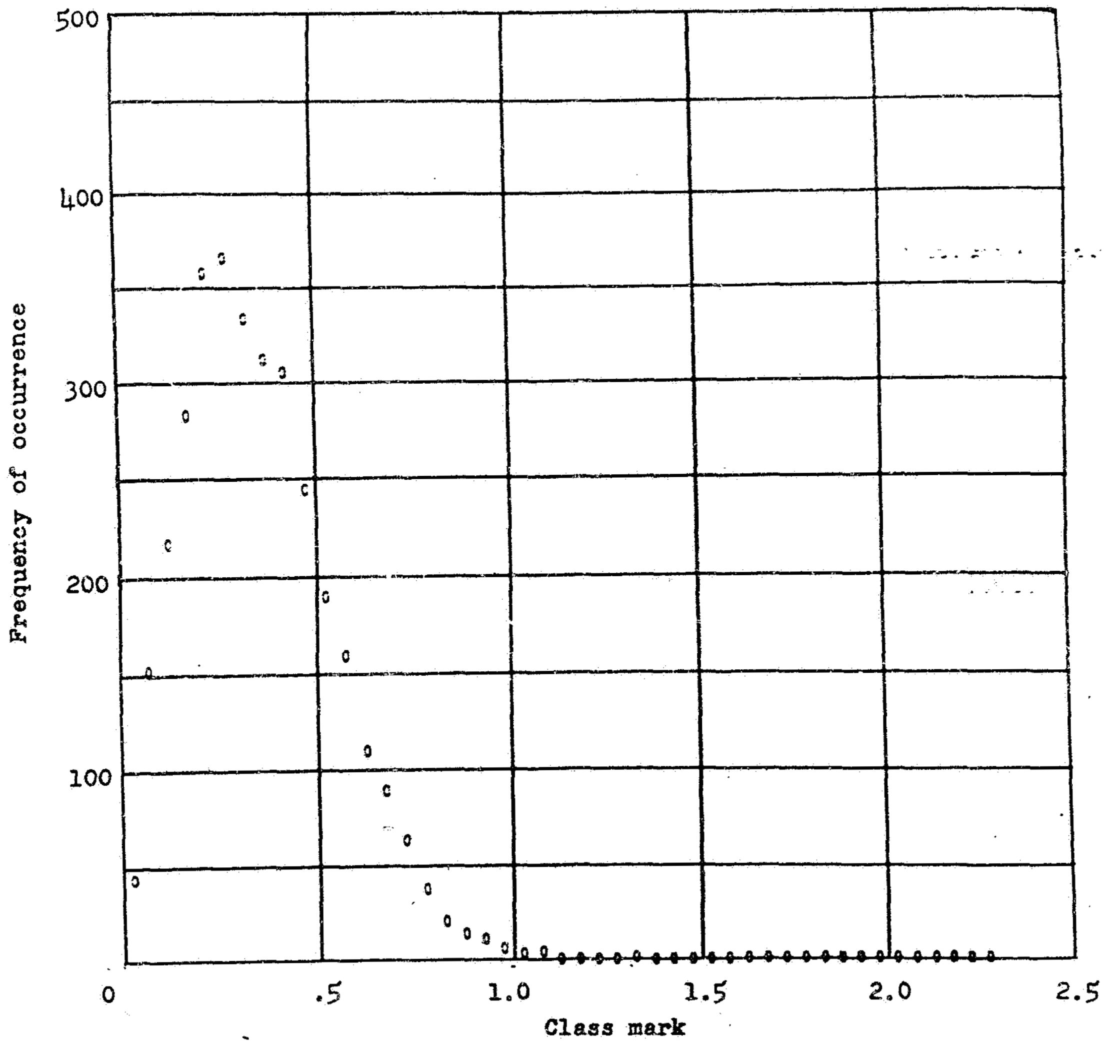

Figure 3.34. Minimum peak distribution of a time history having a Weibull amplitude probability density: $\alpha = 2$; $\beta = 2$

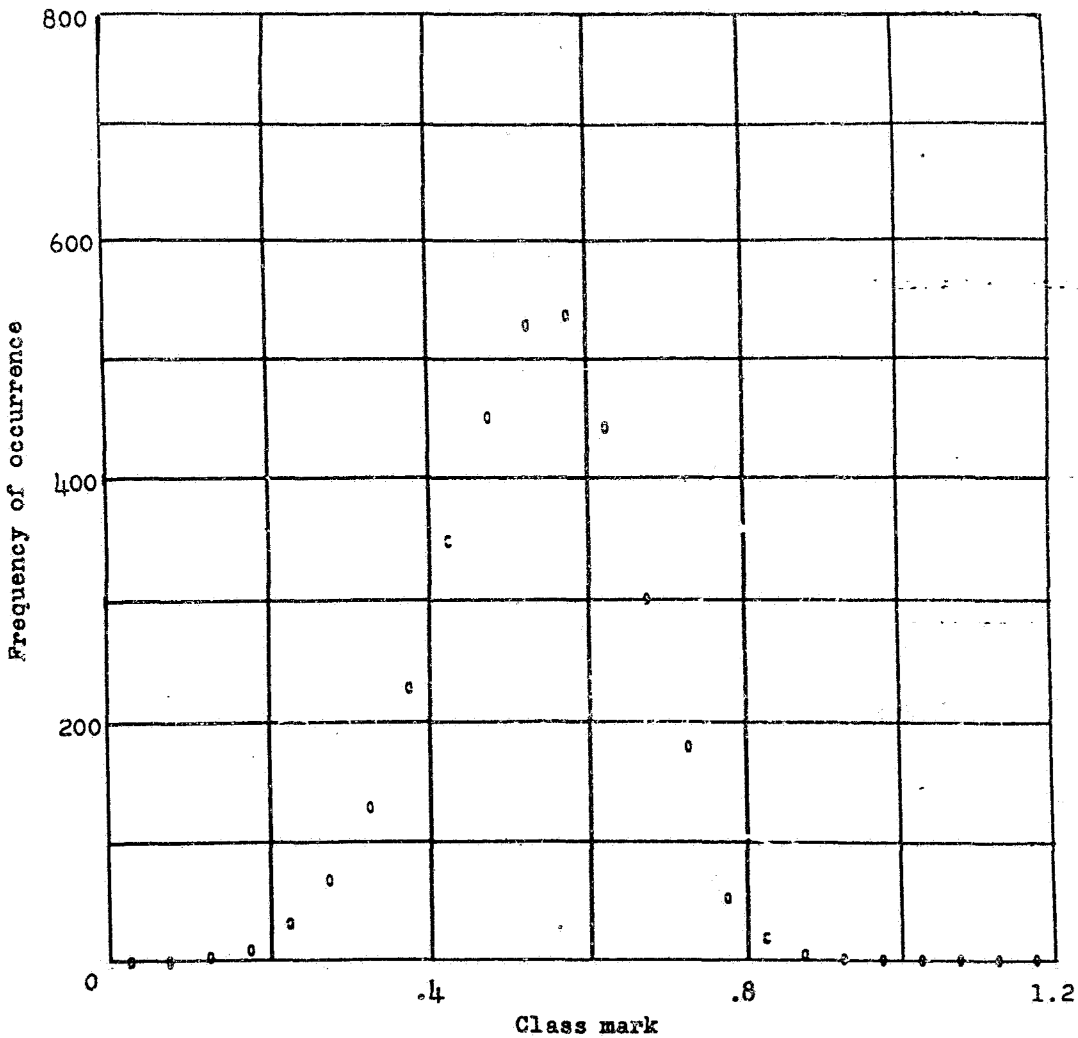

Figure 3.35. Minimum peak distribution of a time history having a Weibull amplitude probability density: $\alpha = 5$; $\beta = 5$

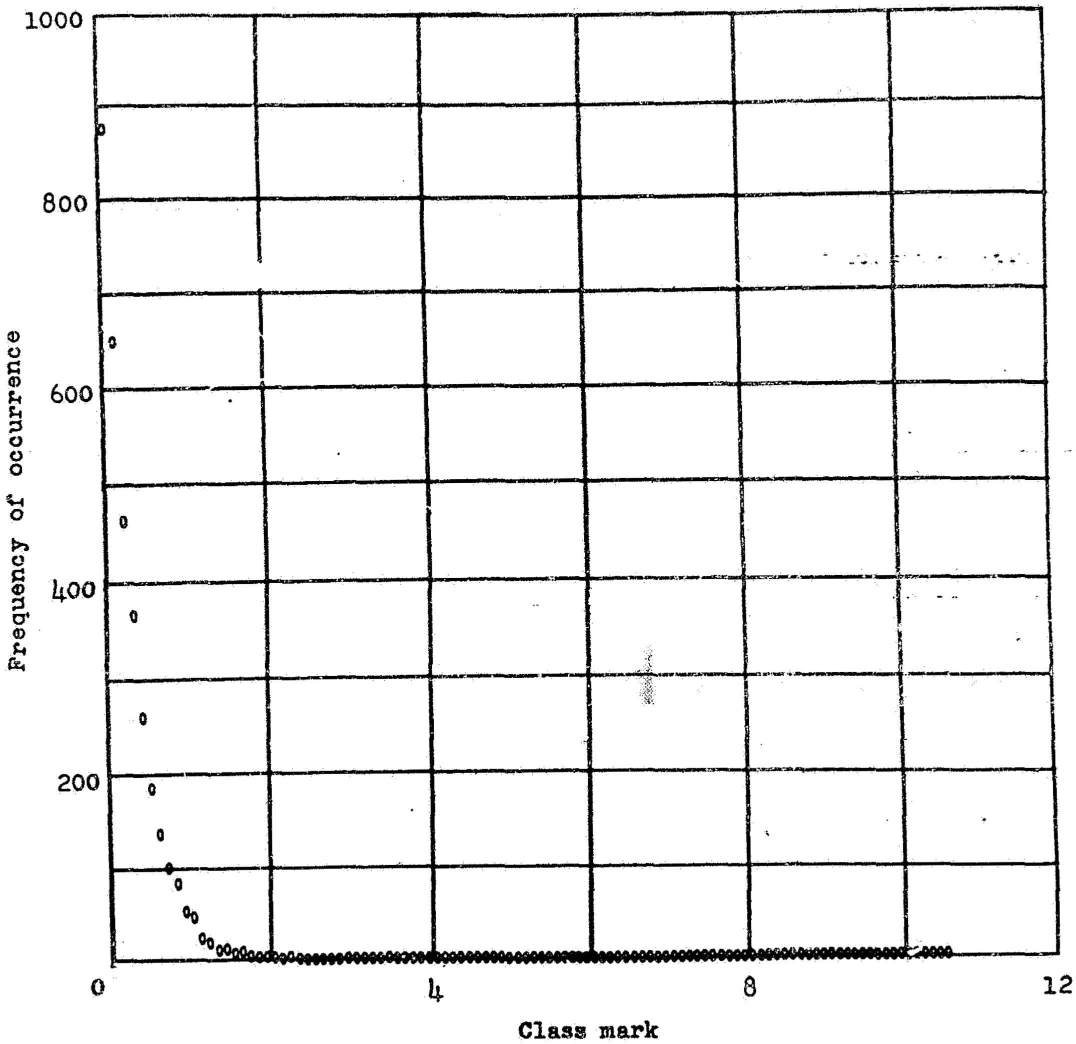

Figure 3.36. Minimum peak distribution of a time history having an Exponential amplitude probability density: $\theta = 1$

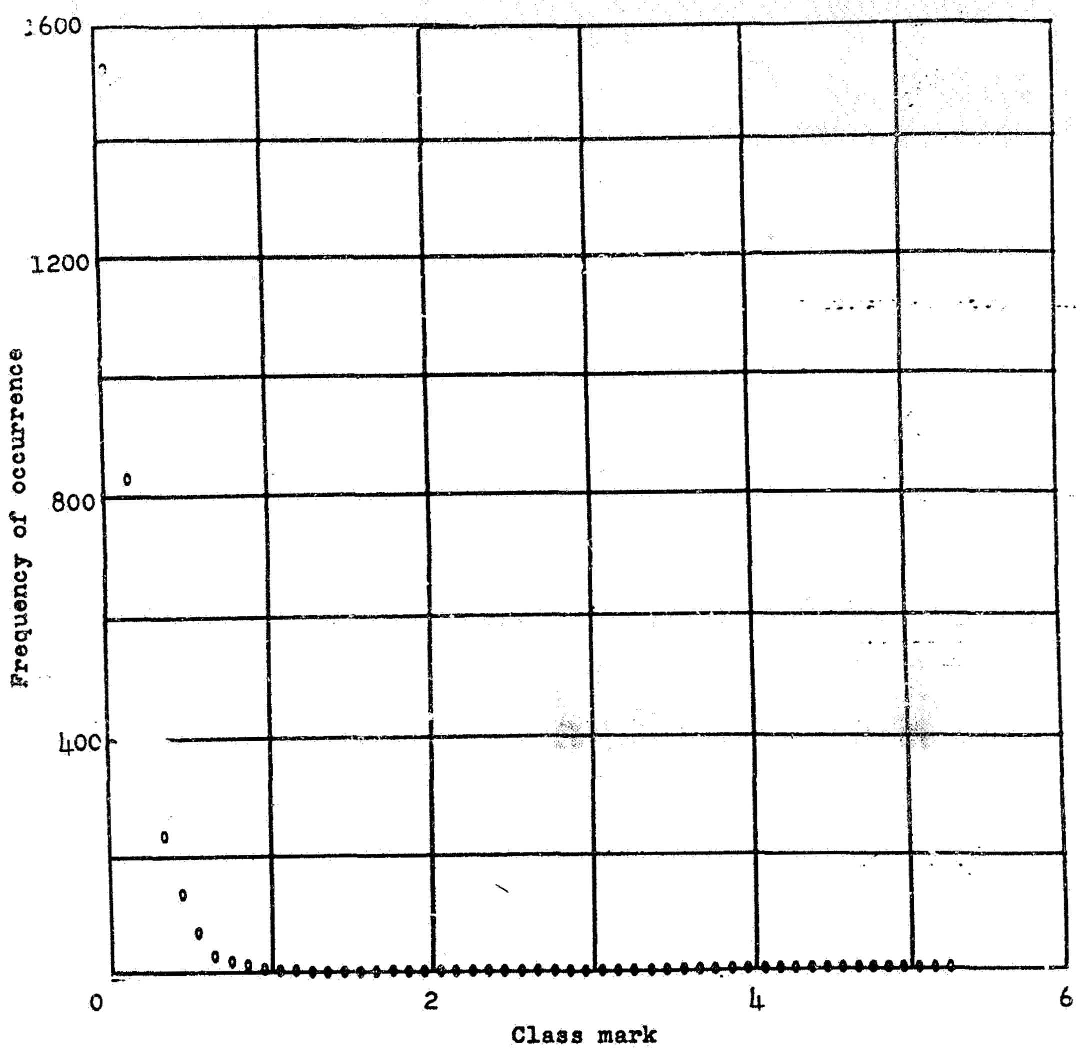

Figure 3.37. Minimum peak distribution of a time history having an Exponential amplitude probability density: $\theta = 2$

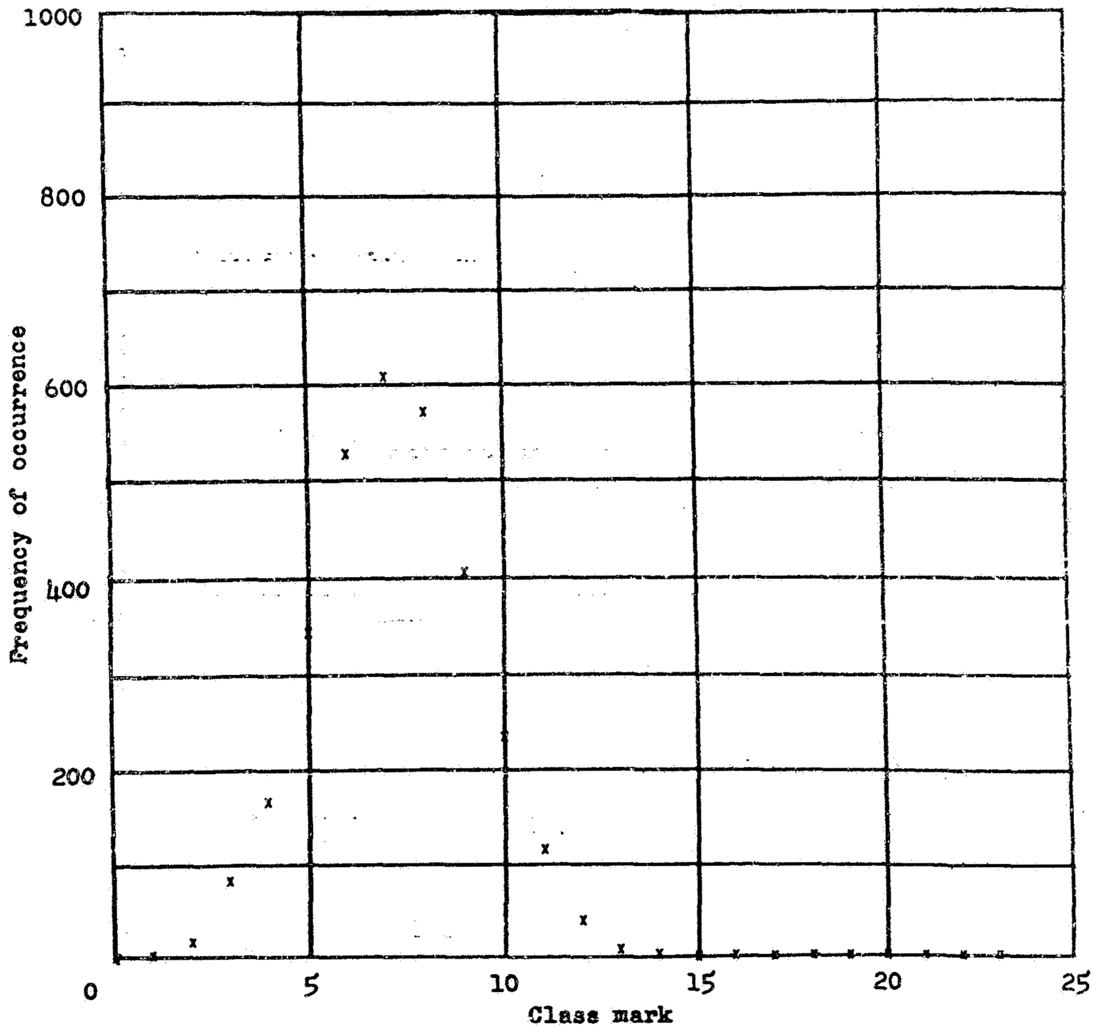

Figure 3.38. Minimum peak distribution of a time history having a Poisson amplitude probability density: $\mu = 10$

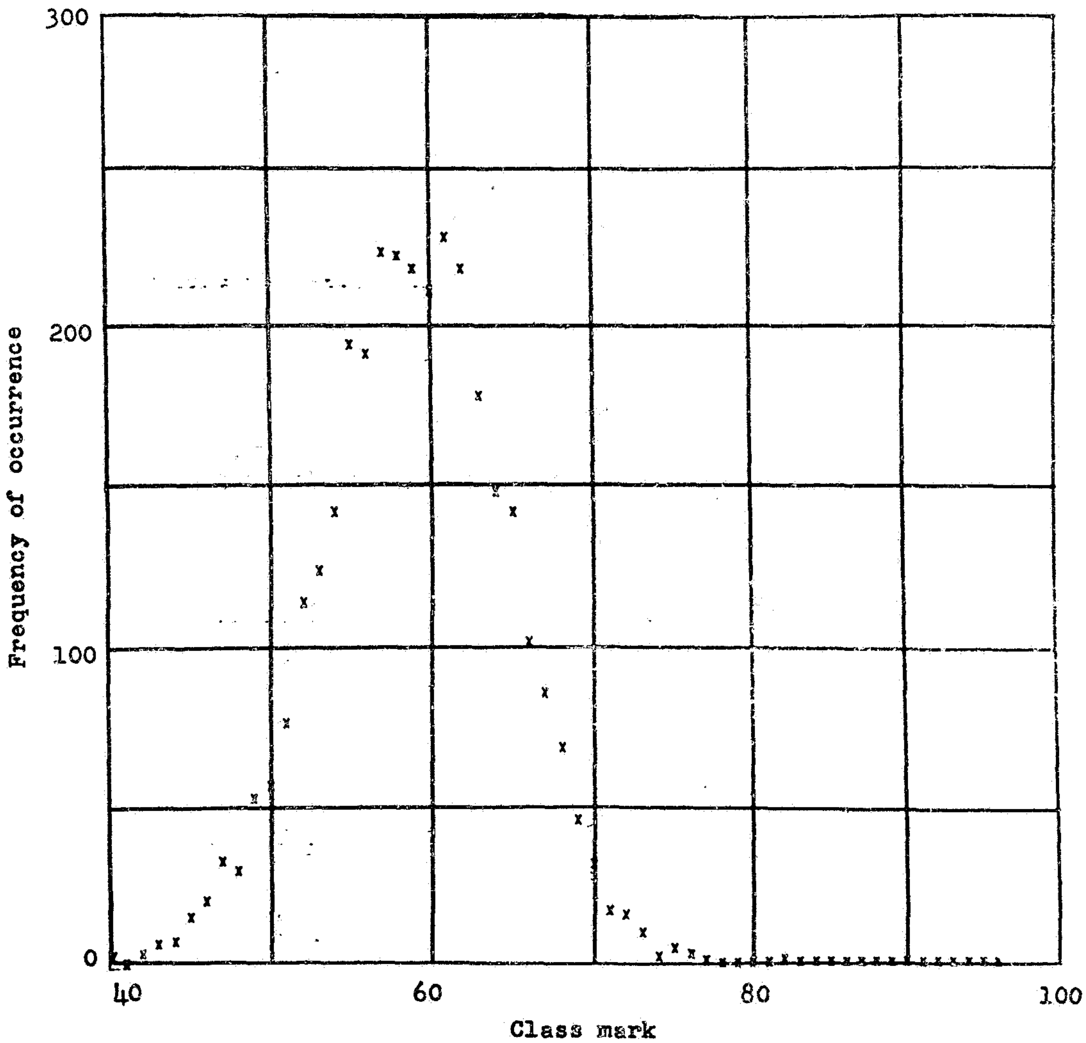

Figure 3.39. Minimum peak distribution of a time history having a Poisson amplitude probability density: $\mu = 66$

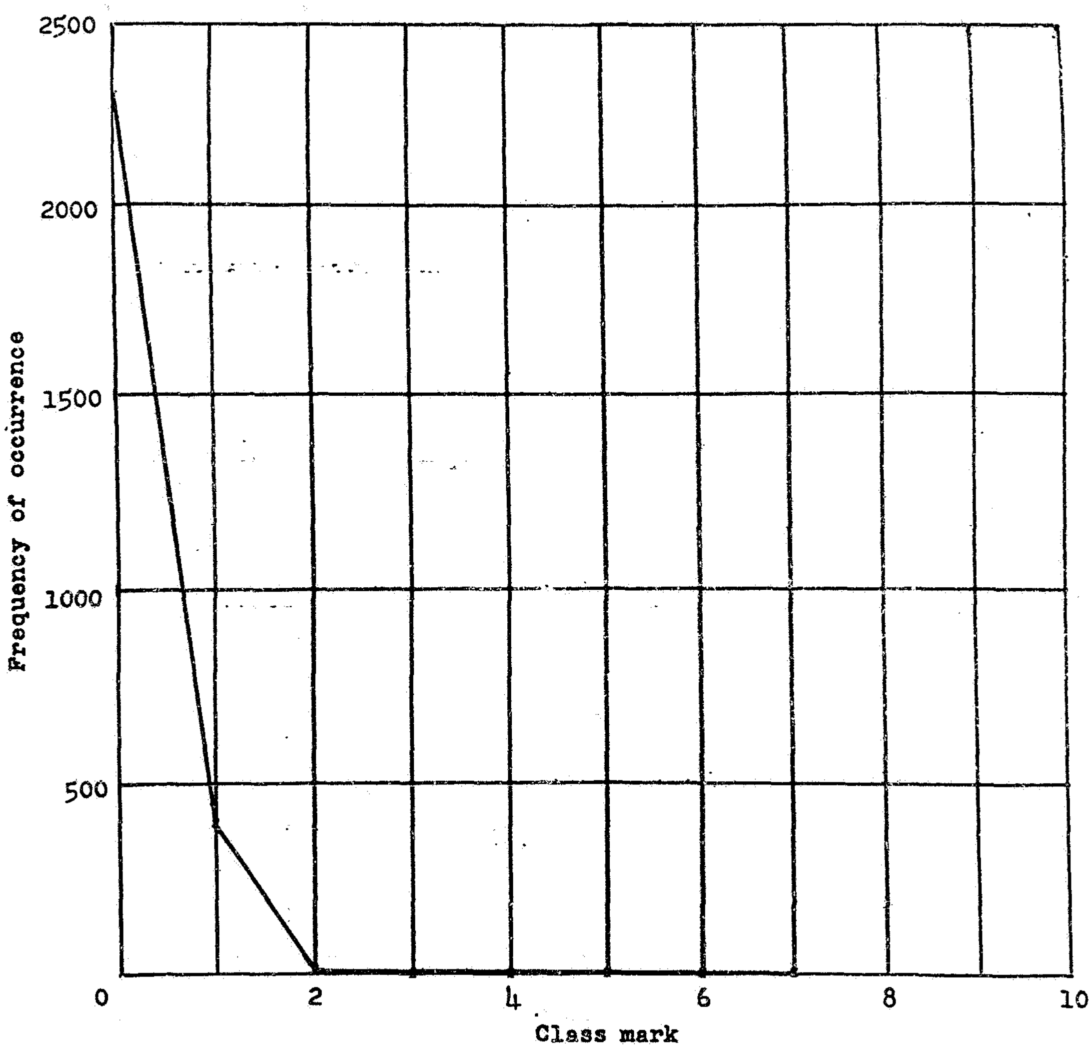

Figure 3.40. Minimum peak distribution of a time history having a Binomial amplitude probability density: $n = 10$; $p = 0.1$

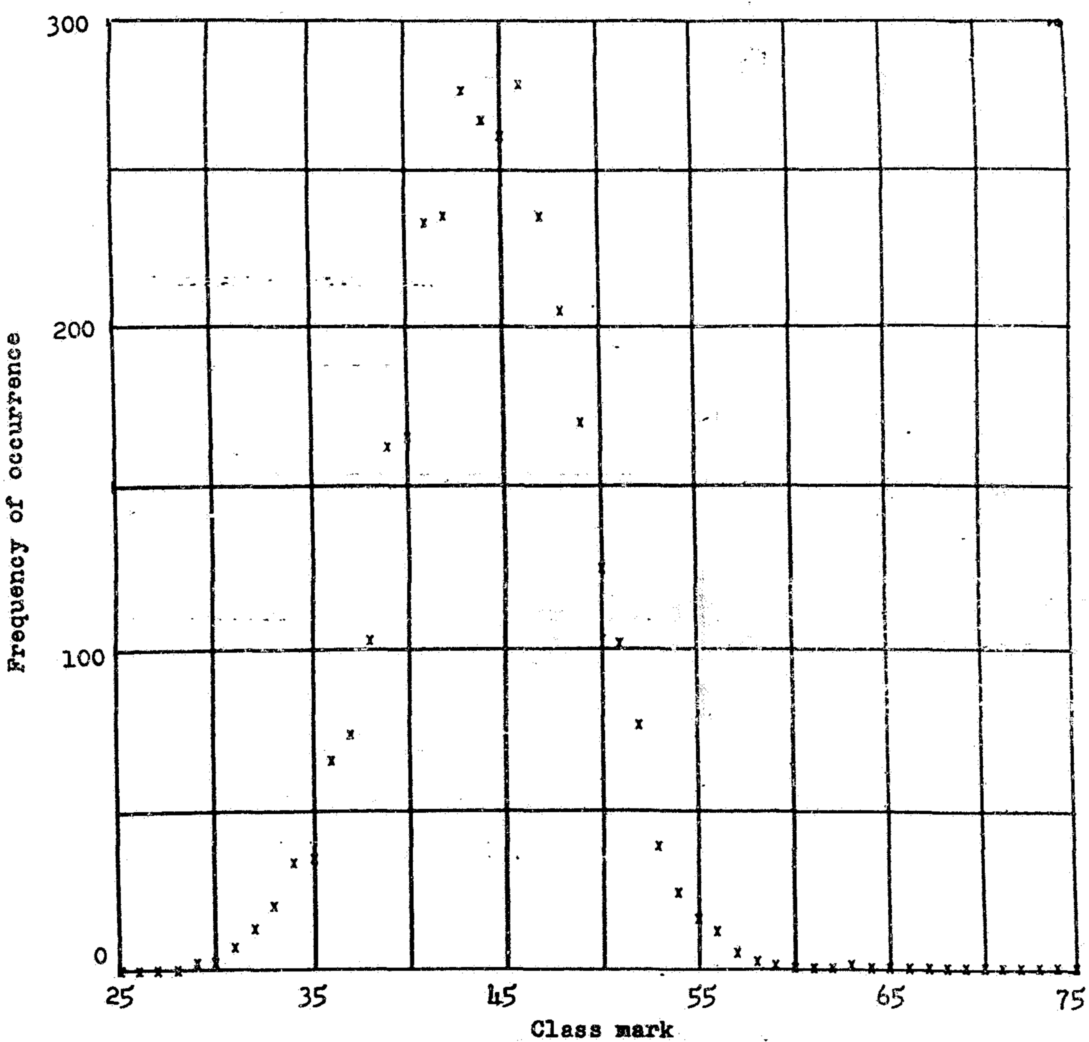

Figure 3.41. Minimum peak distribution of a time history having a Binomial amplitude probability density: n = 500; p = 0.1

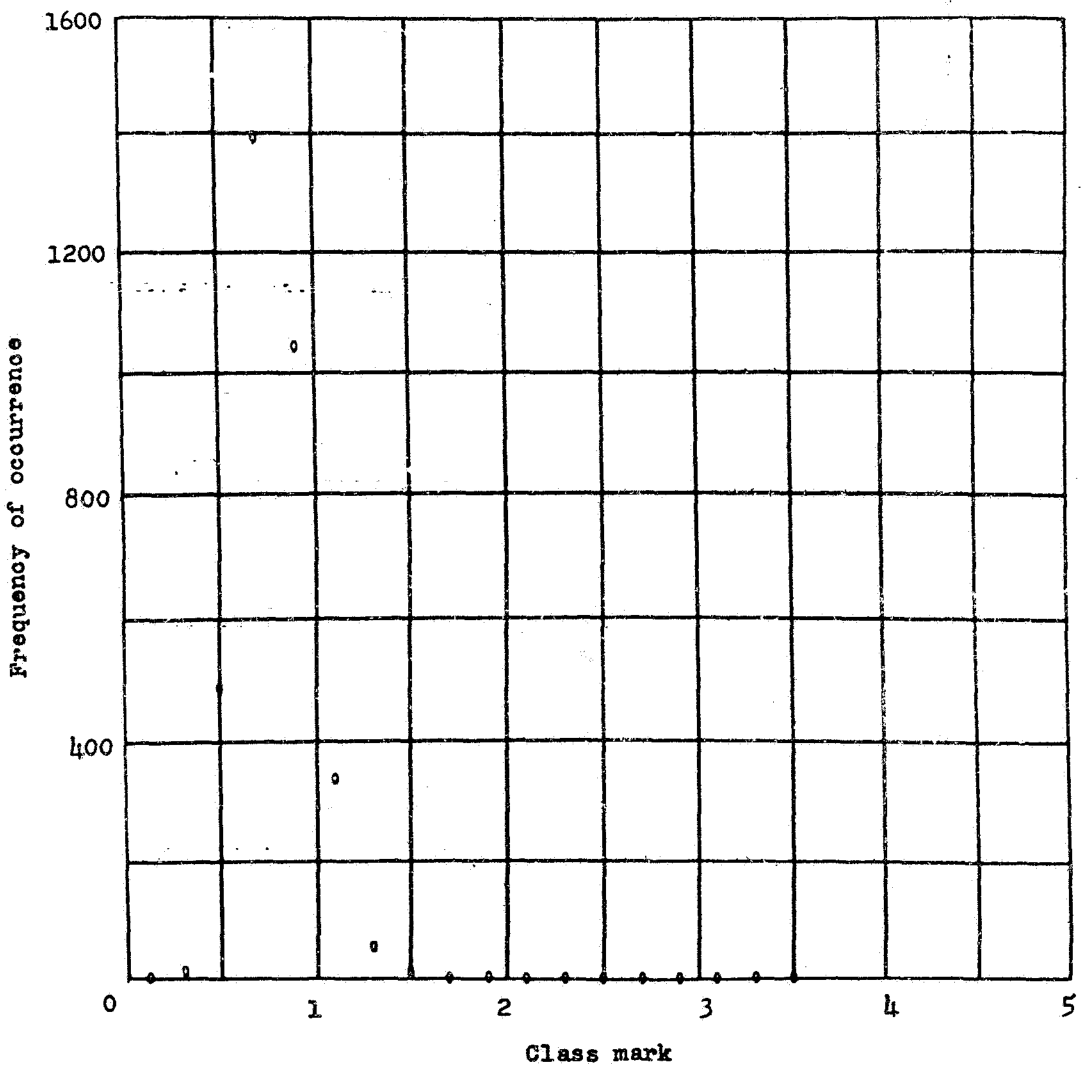

Figure 3.42. Minimum peak distribution of a time history having a Log-Normal amplitude probability density: $\sigma^2_{log} = 0.1$

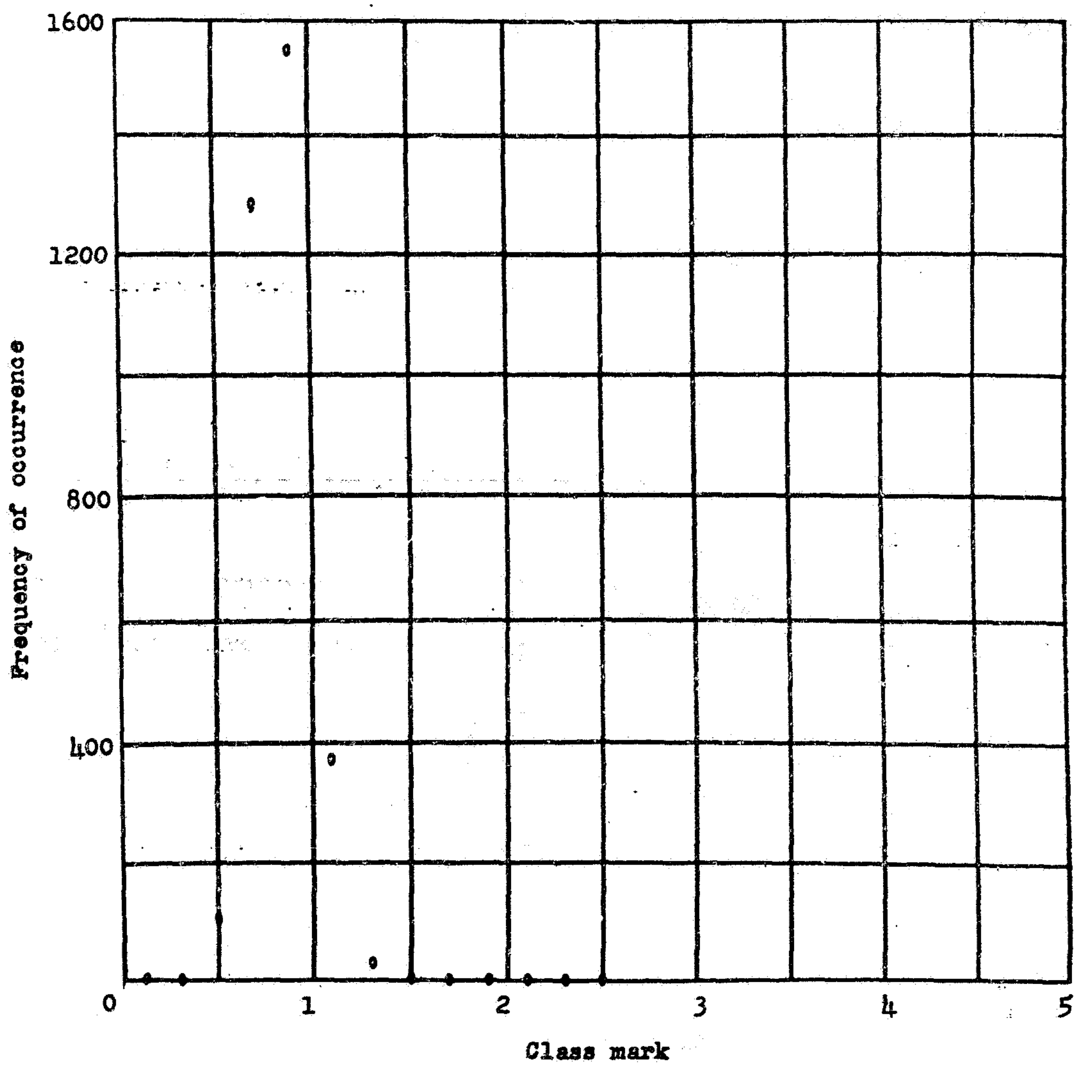

Figure 3.43. Minimum peak distribution of a time history having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.05$

histories presented in Figures 3.1-3.10. It can be seen from the peak distributions which are not symmetrical about the mean that there are a greater number of maximum peaks at the lower amplitudes and a correspondingly smaller number of peaks at the higher amplitudes. This is exactly the same condition that is encountered in aircraft. There are a relatively high number of gusts of low velocity with a corresponding decrease in the number of gusts at high velocity. Maximum peaks are measured as the number of exceedances of a given amplitude level above the mean or 1g stress of an aircraft. Thus the peaks measured in service are presented as a cumulative distribution. Thus in this investigation cumulative peak distributions were computed for both maximum and minimum peaks and are presented in Figures 3.44-3.53 and Figures 3.54-3.63, respectively. The maximum cumulative peak distributions are seen to start rolling off at the mean value of the time history. Only in the case of the Exponential and Log-Normal density function do these distributions fall off from the mean on a straight line. This fact is most significant in that this is the manner in which service data behaves. More will be said about this point in chapter 5.

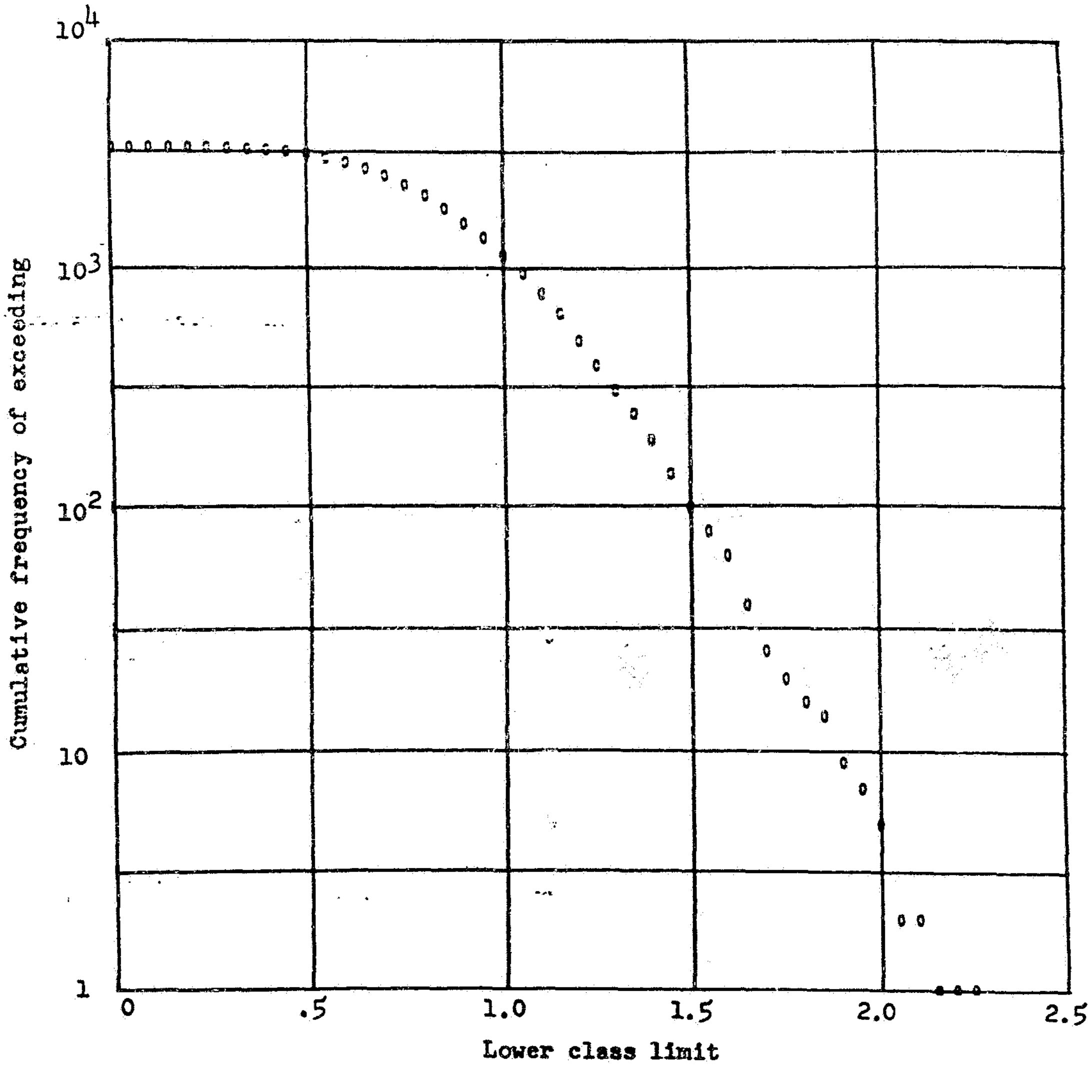

Figure 3.44. Log of cumulative maximum peaks of a time history having a Weibull amplitude probability density: $\alpha = 2$; $\beta = 2$

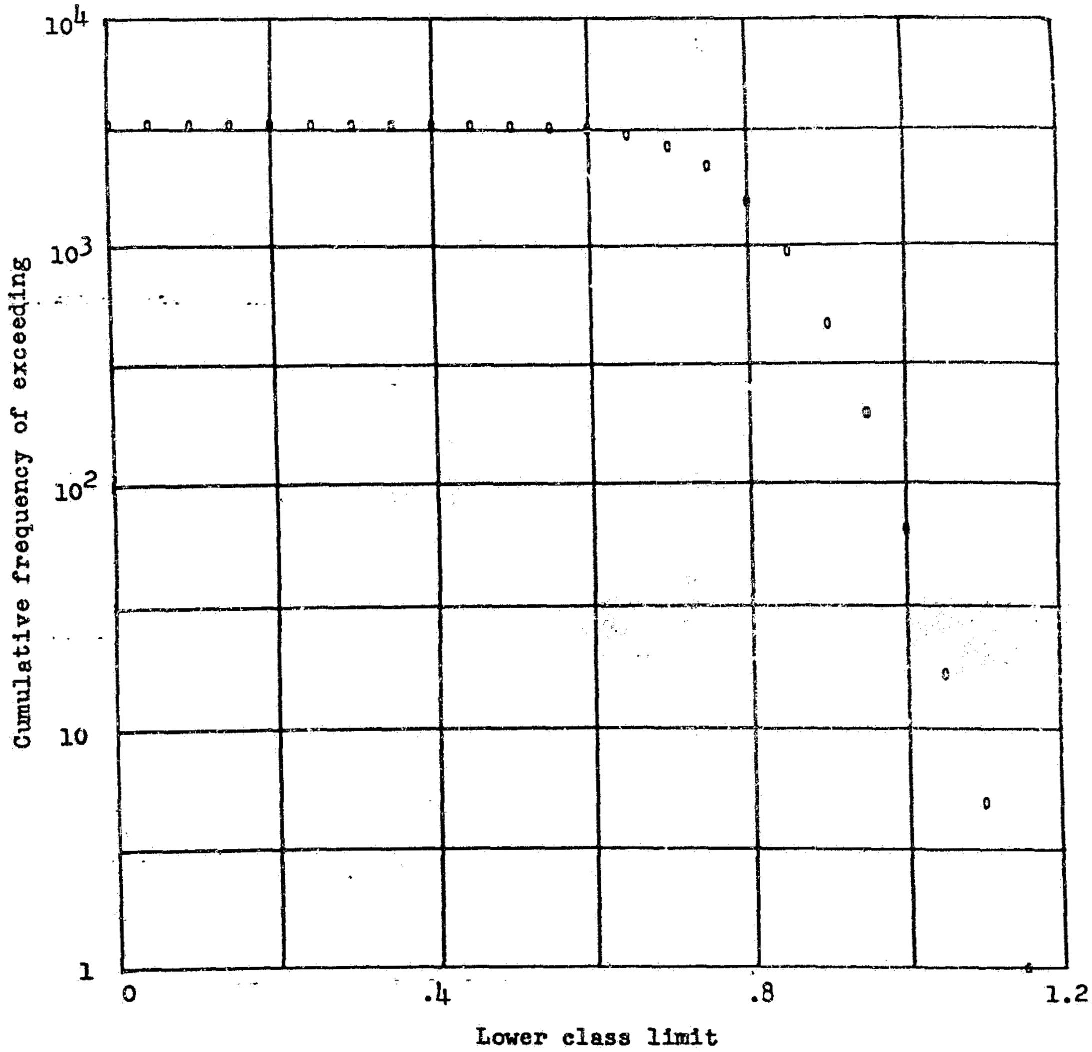

Figure 3.45. Log of cumulative maximum peaks of a time history having a Weibull amplitude probability density: $\alpha = 5$; $\beta = 5$

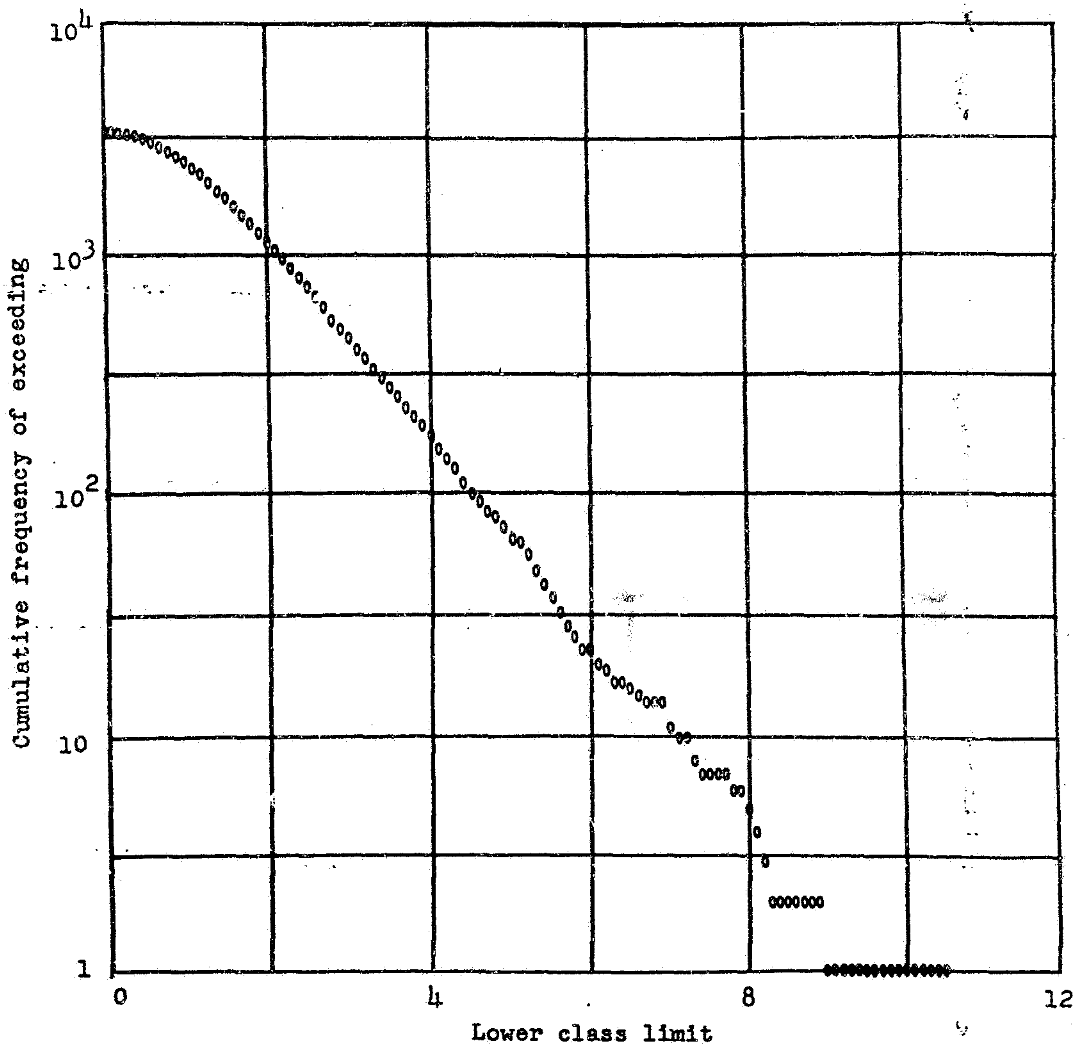

Figure 3.46. Log of cumulative maximum peaks of a time history having an Exponential amplitude probability density: $\theta = 1$

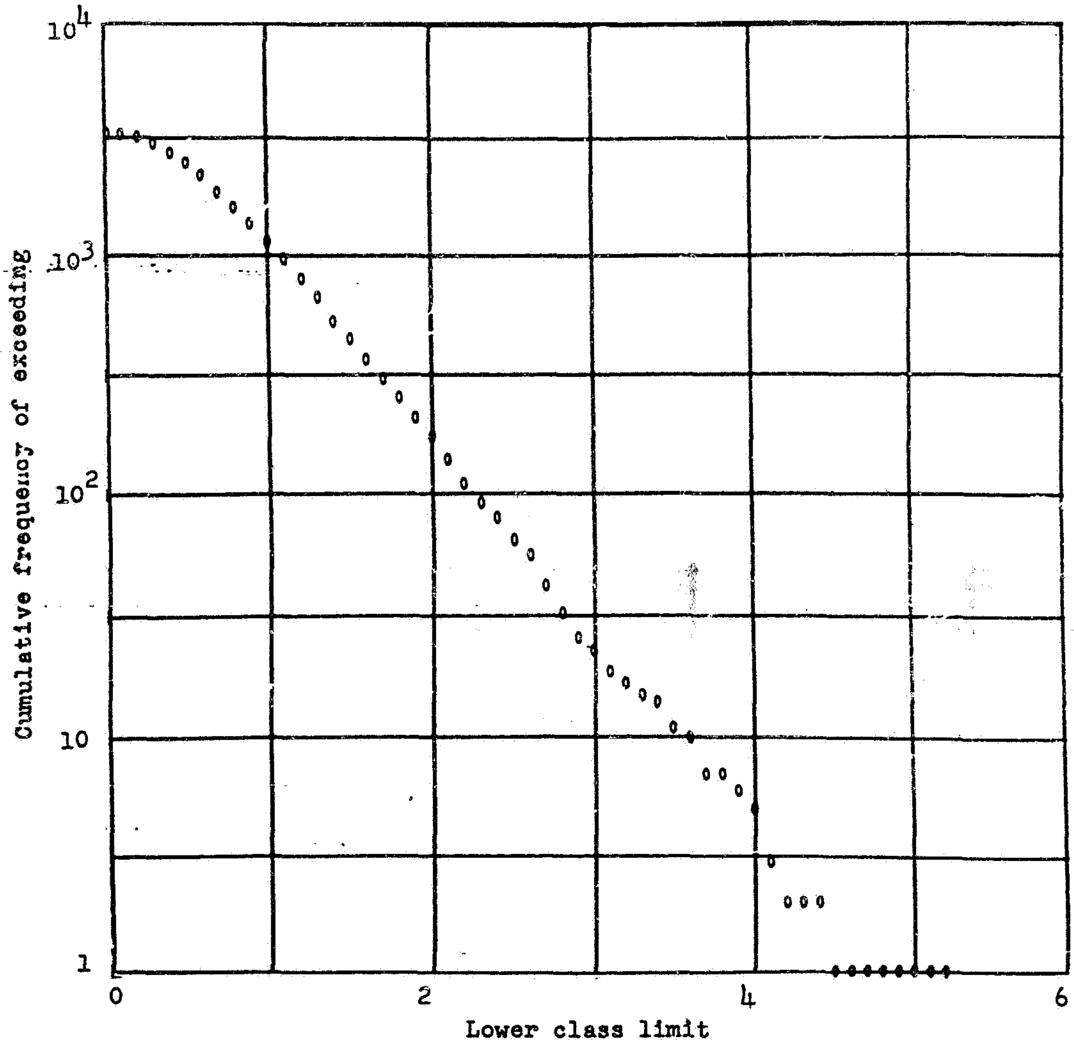

Figure 3.47. Log of cumulative maximum peaks of a time history having an Exponential amplitude probability density: $\theta = 2$

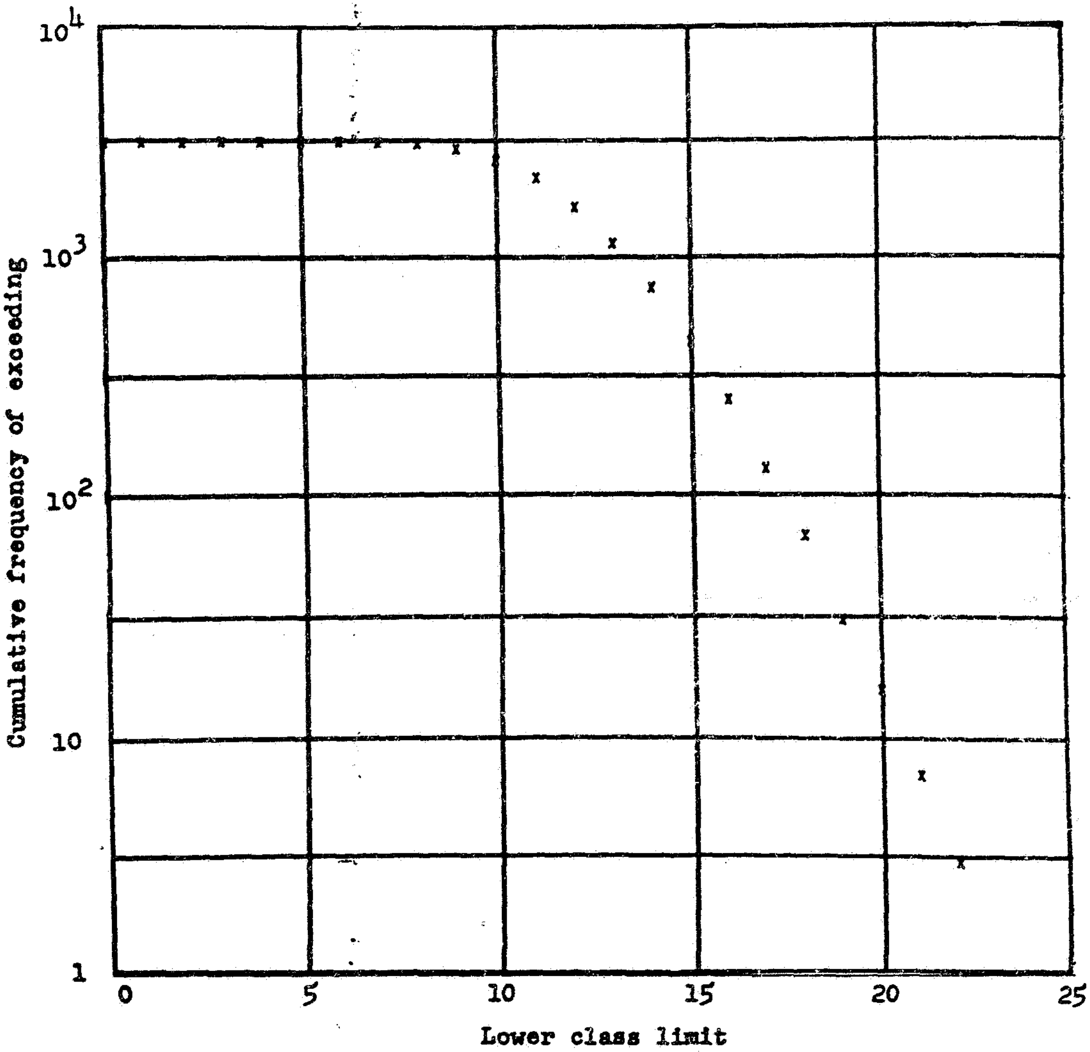

Figure 3.48. Log of cumulative maximum peaks of a time history having a Poisson amplitude probability density: $\mu = 10$

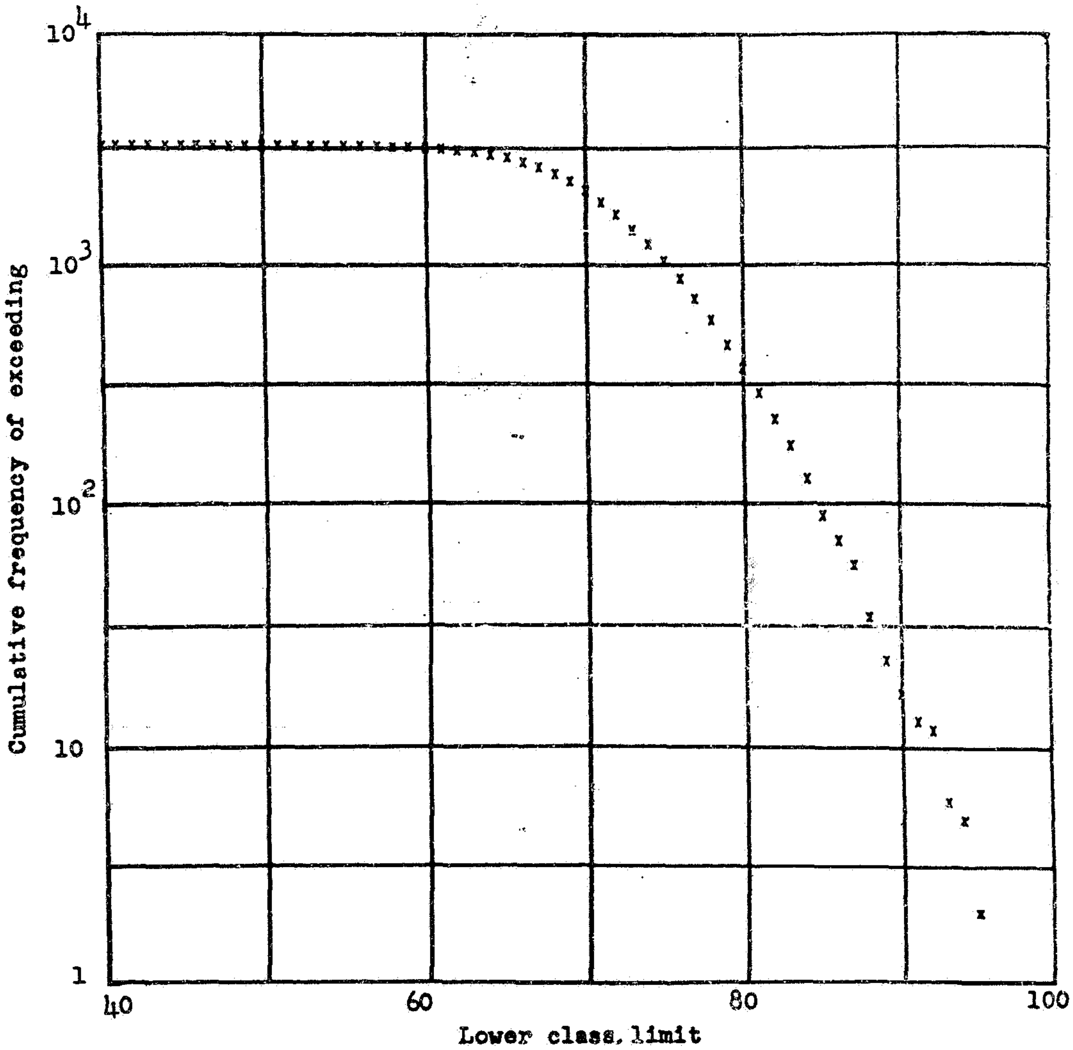

Figure 3.49. Log of cumulative maximum peaks of a time history having a Poisson amplitude probability density: $\mu = 66$

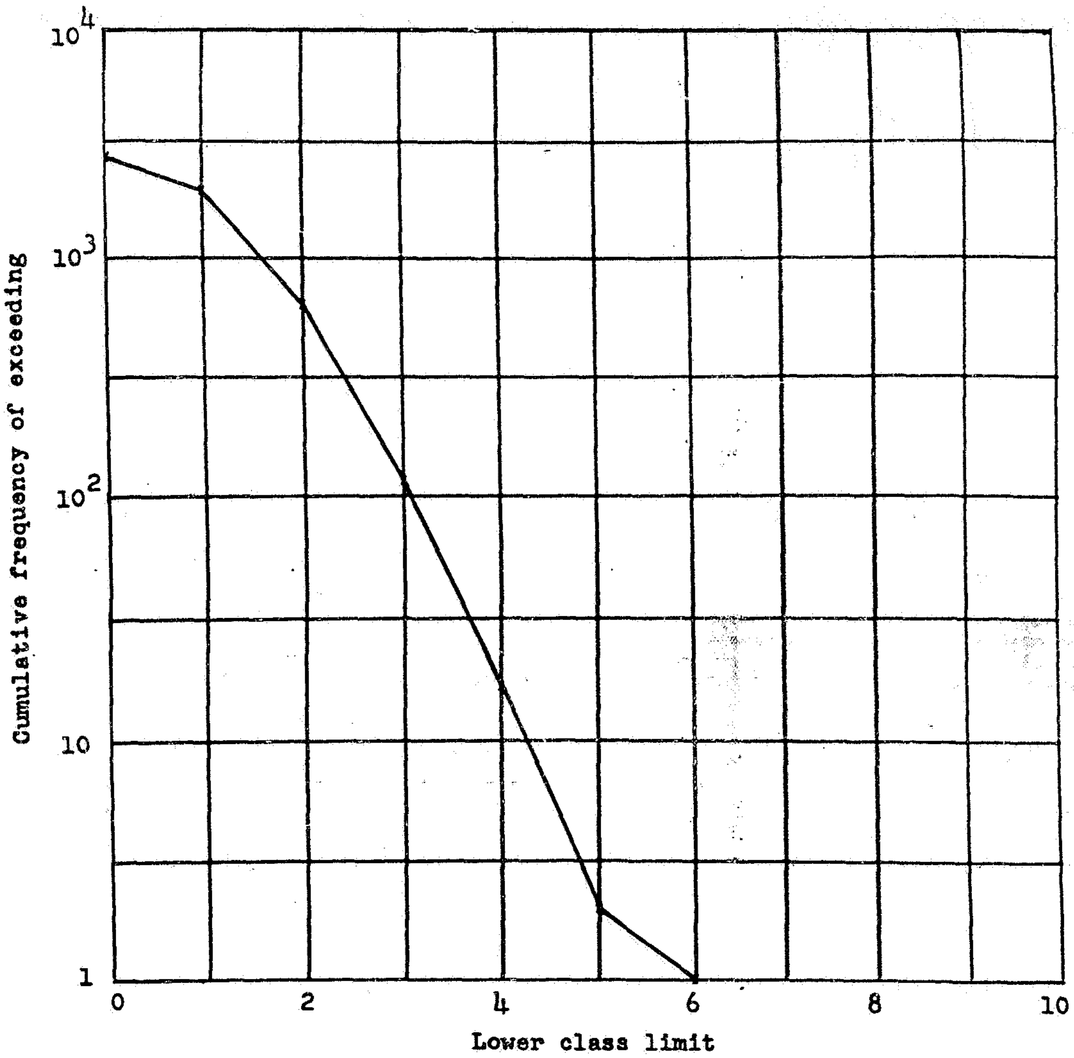

Figure 3.50. Log of cumulative maximum peaks of a time history having a Binomial amplitude probability density: $n = 10$; $p = 0.1$

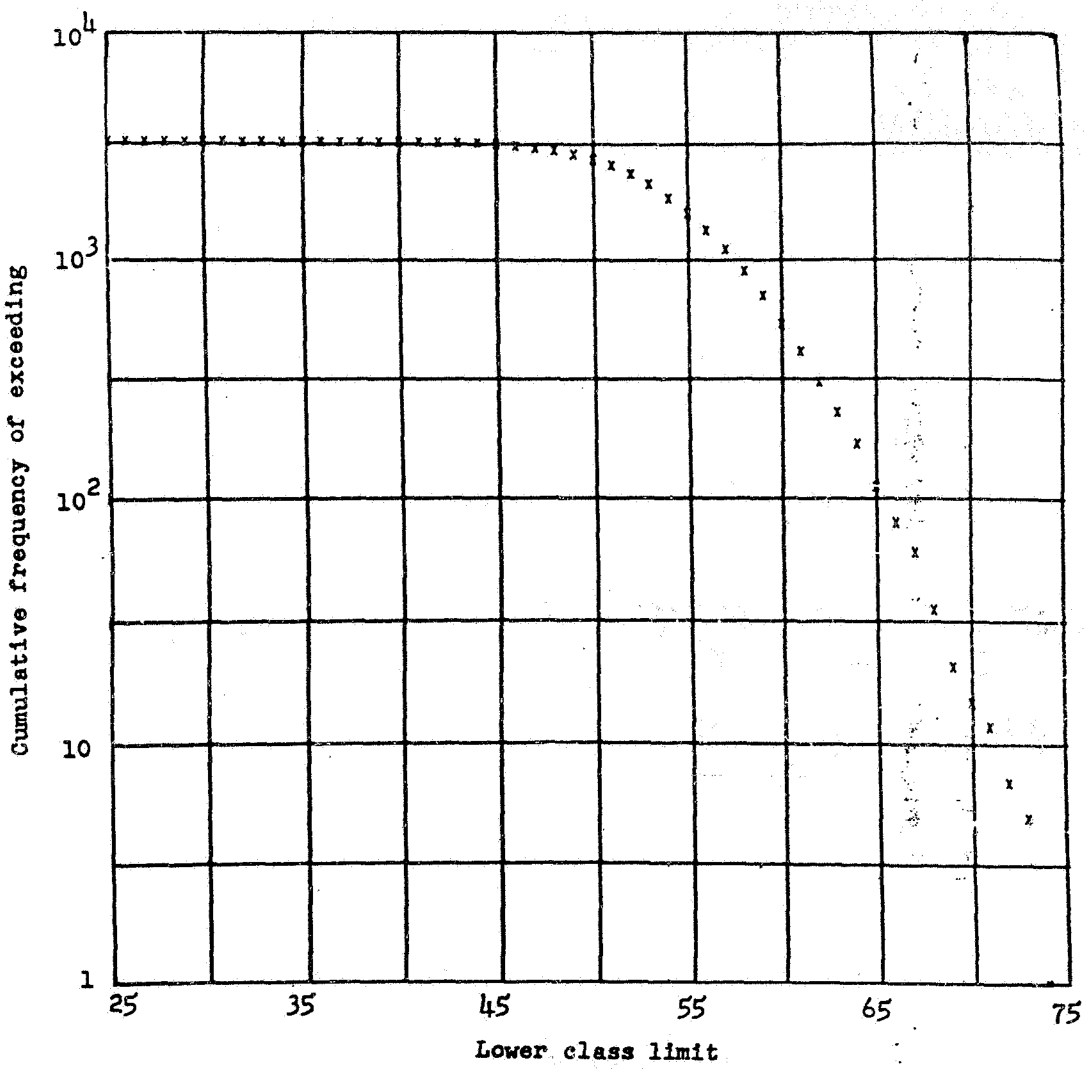

Figure 3.51. Log of cumulative maximum peaks of a time history having a Binomial amplitude probability density: n = 500; p = 0.1

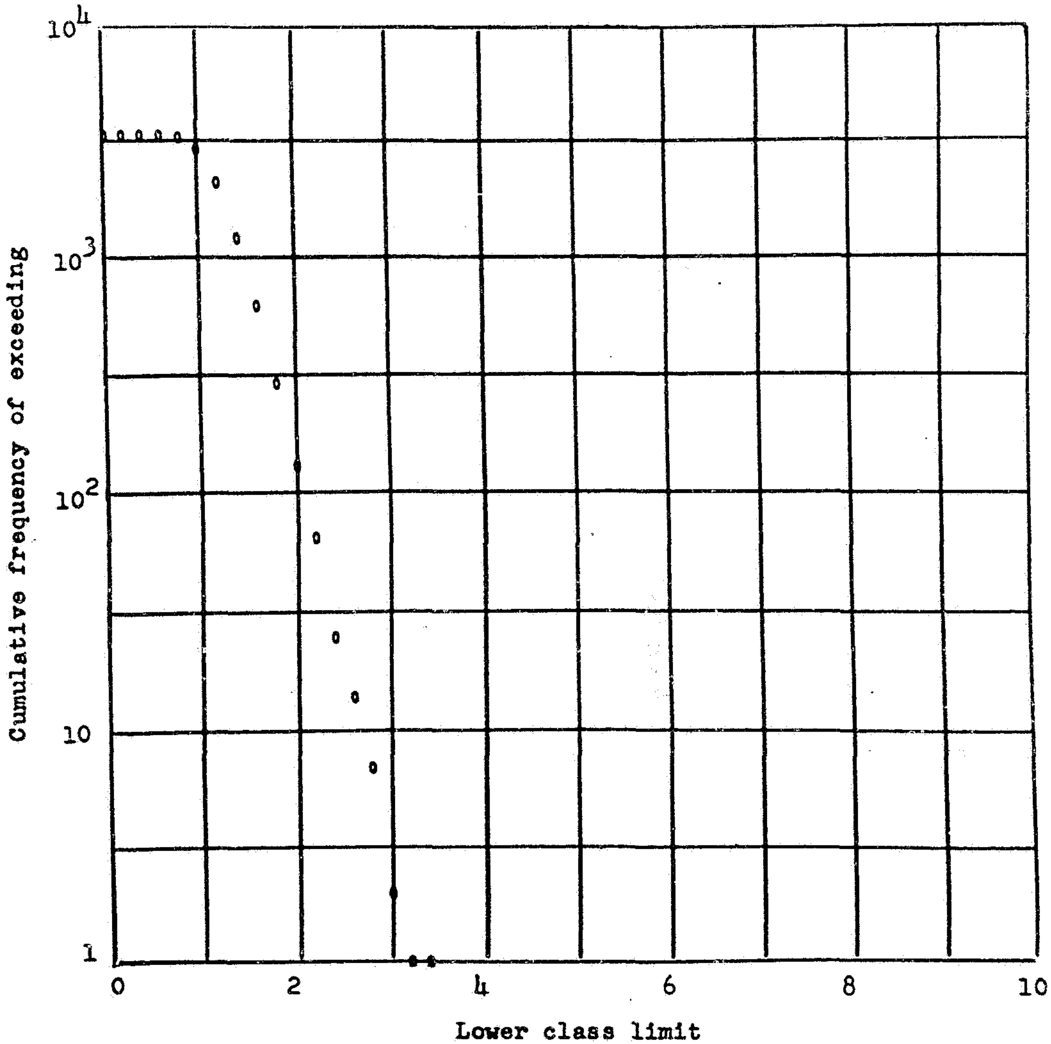

Figure 3.52. Log of cumulative maximum peaks of a time history having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.1$

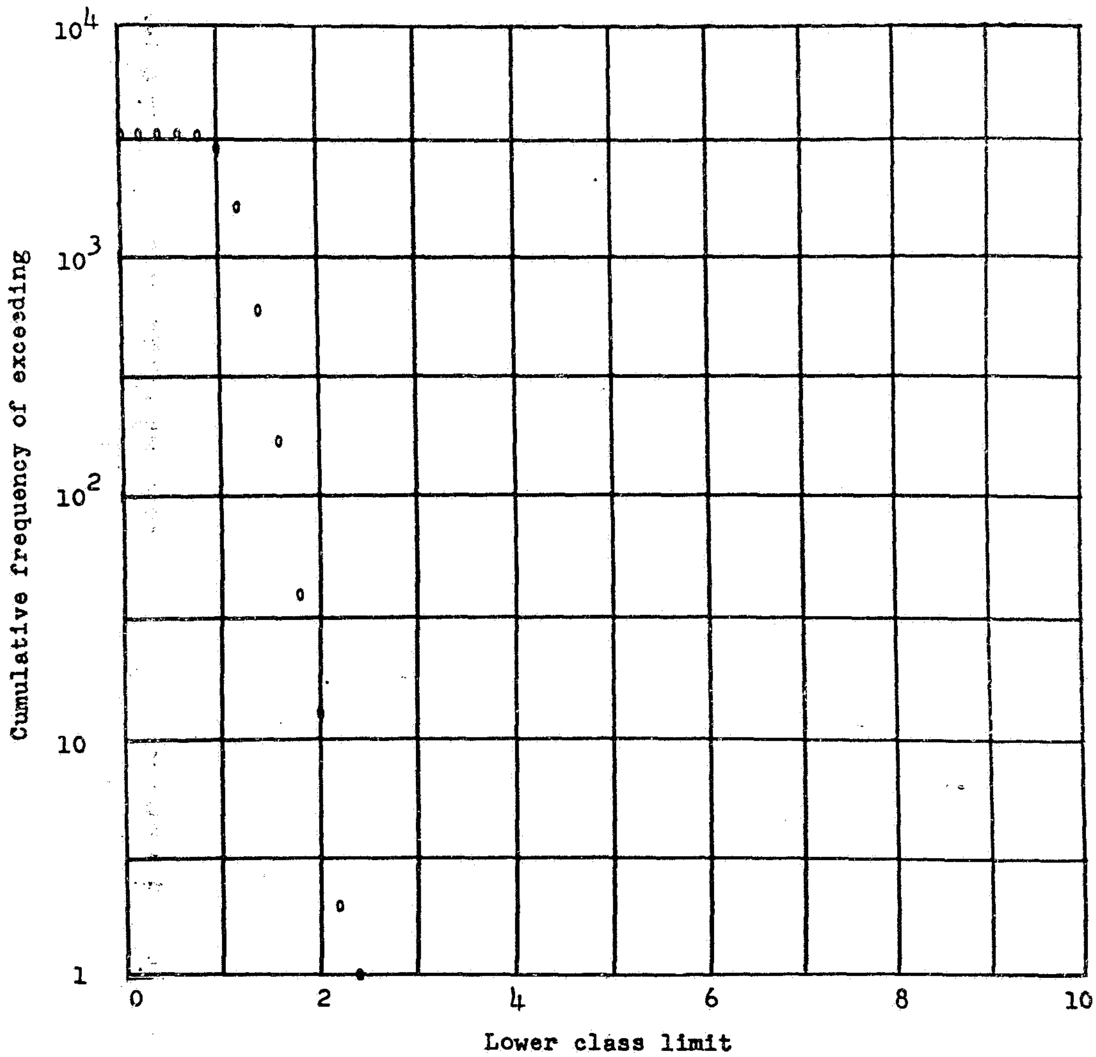

Figure 3.53. Log of cumulative maximum peaks of a time history having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.05$

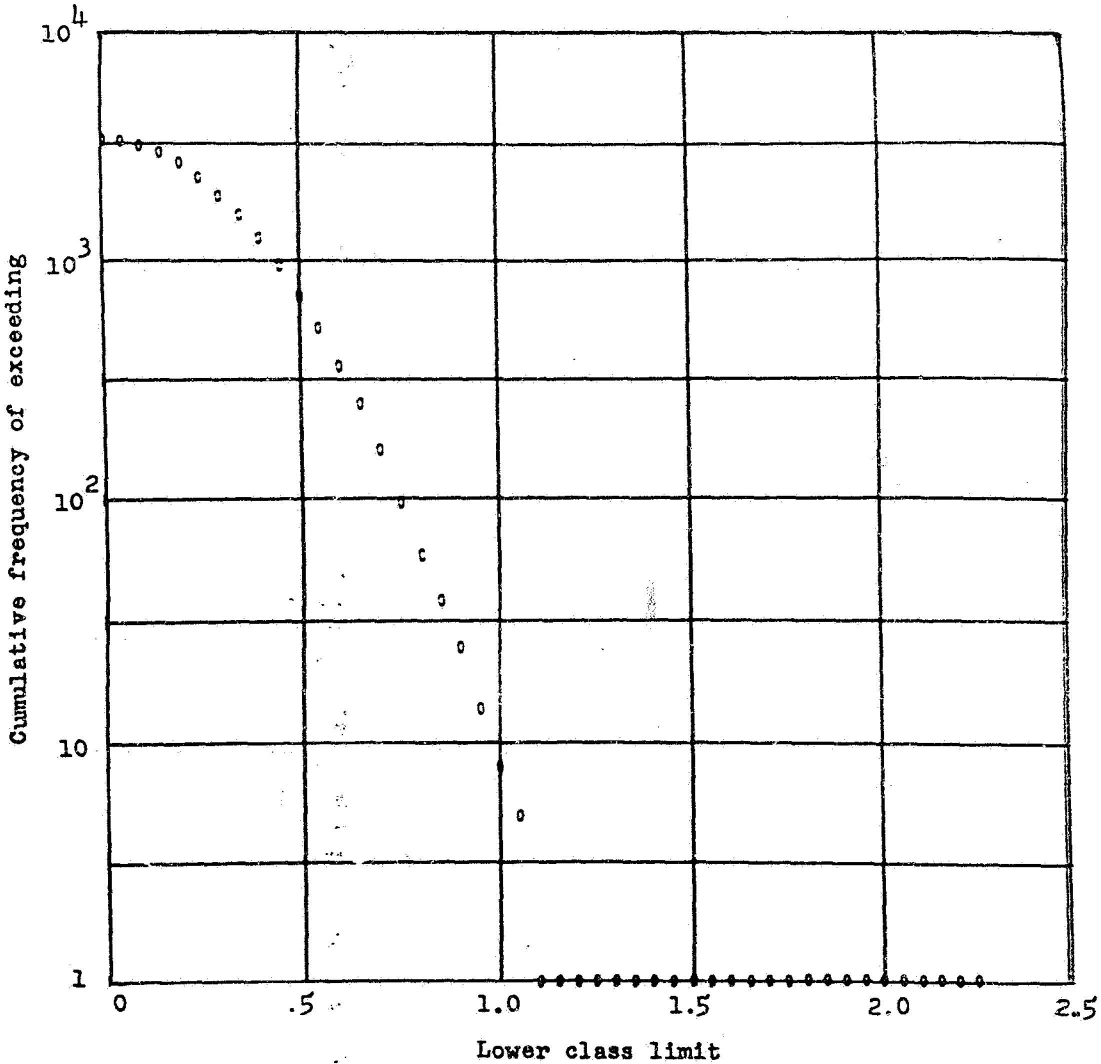

Figure 3.54. Log of cumulative minimum peaks of a time history having a Weibull amplitude probability density: $\alpha = 2$; $\beta = 2$

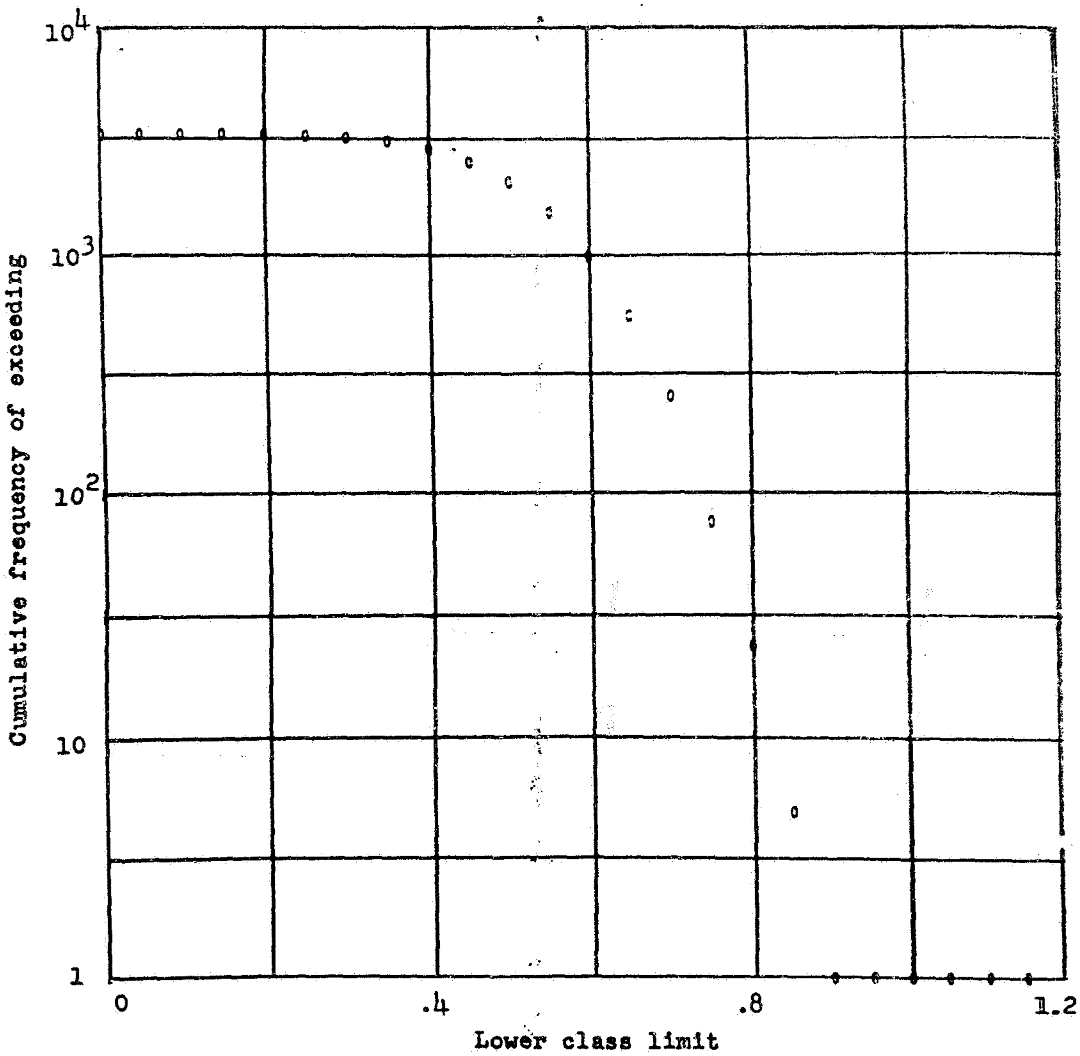

Figure 3.55. Log of cumulative minimum peaks of a time history having a Weibull amplitude probability density: $\alpha = 5$; $\beta = 5$

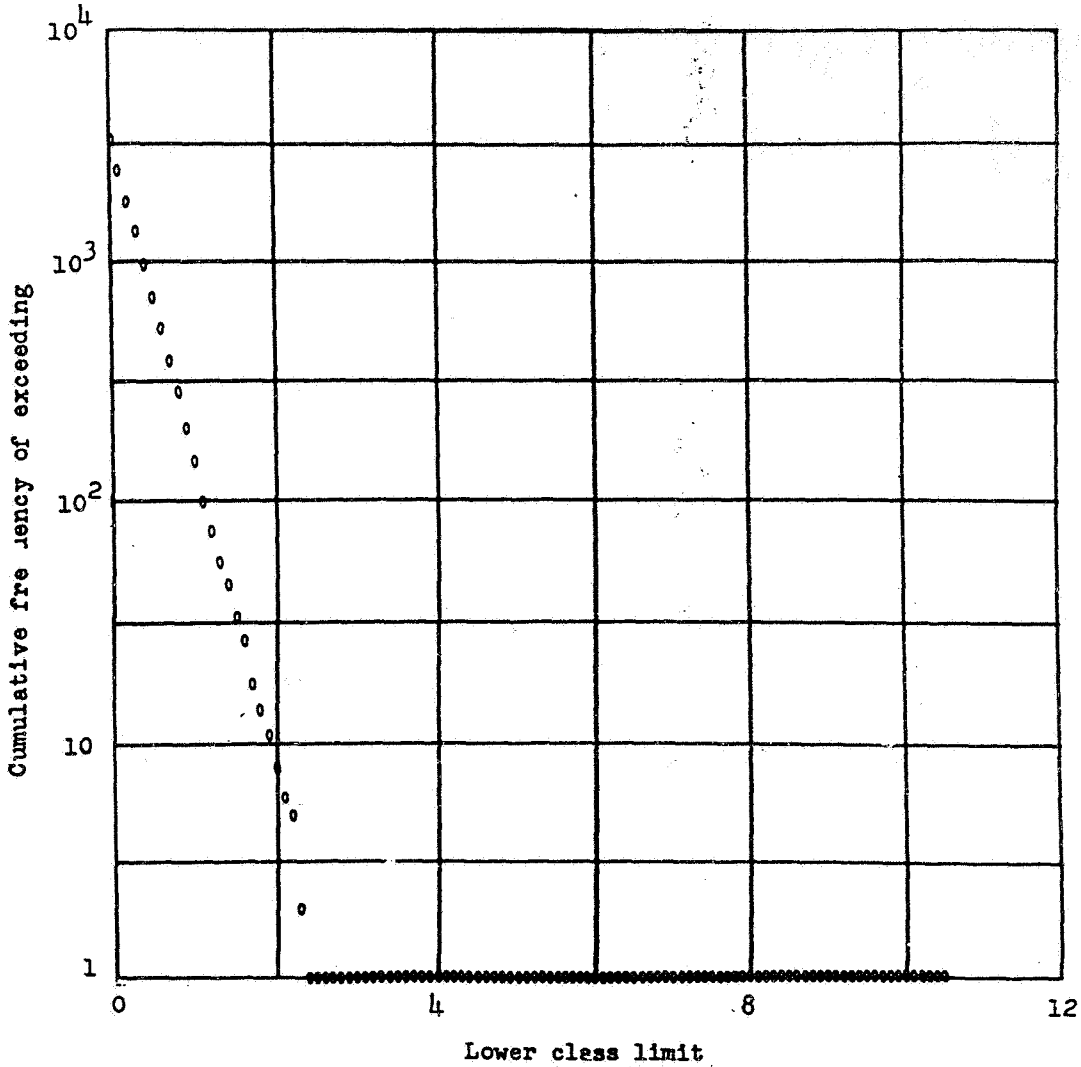

Figure 3.56. Log of cumulative minimum peaks of a time history having an Exponential amplitude probability density: $\theta = 1$

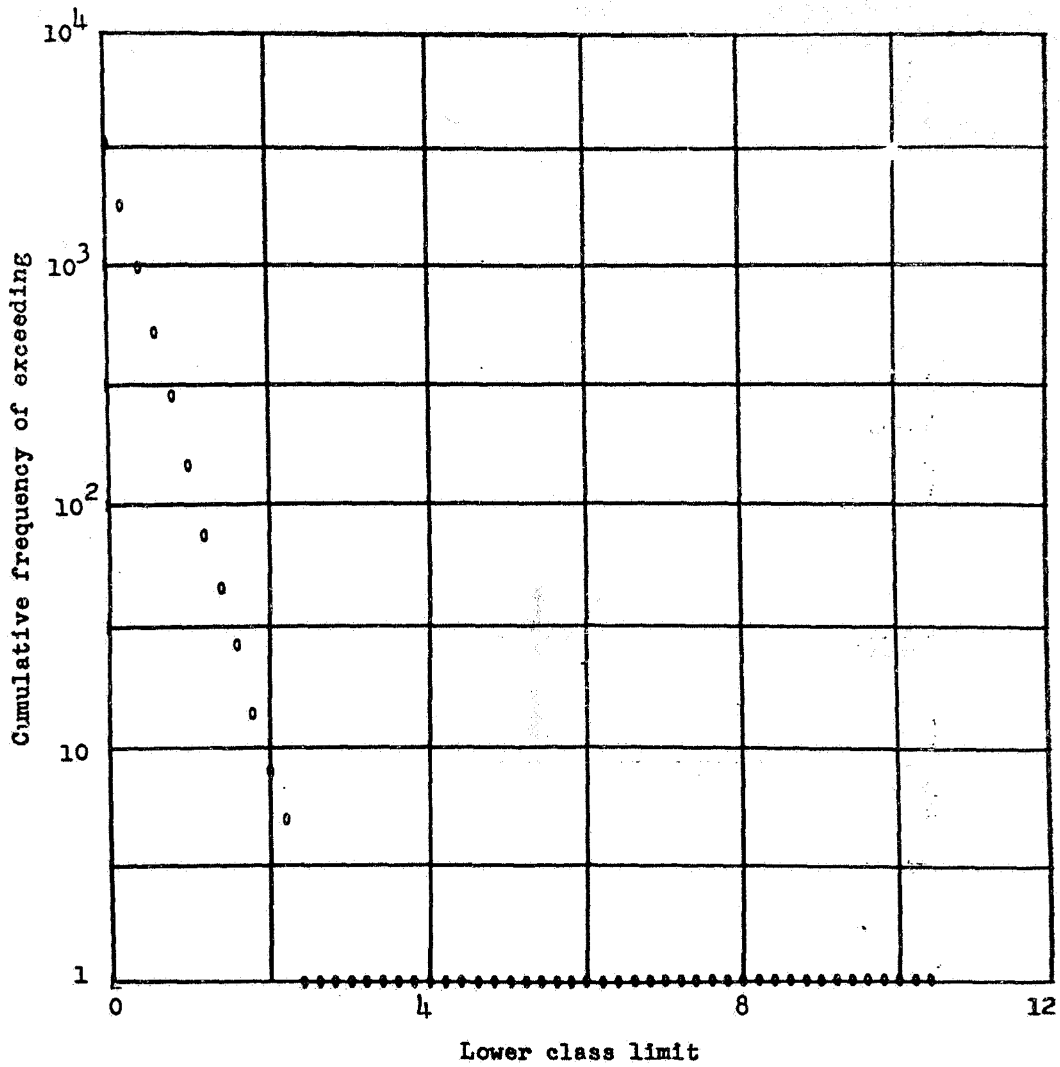

Figure 3.57. Log of cumulative minimum peaks of a time history having an Exponential amplitude probability density: $\theta = 2$

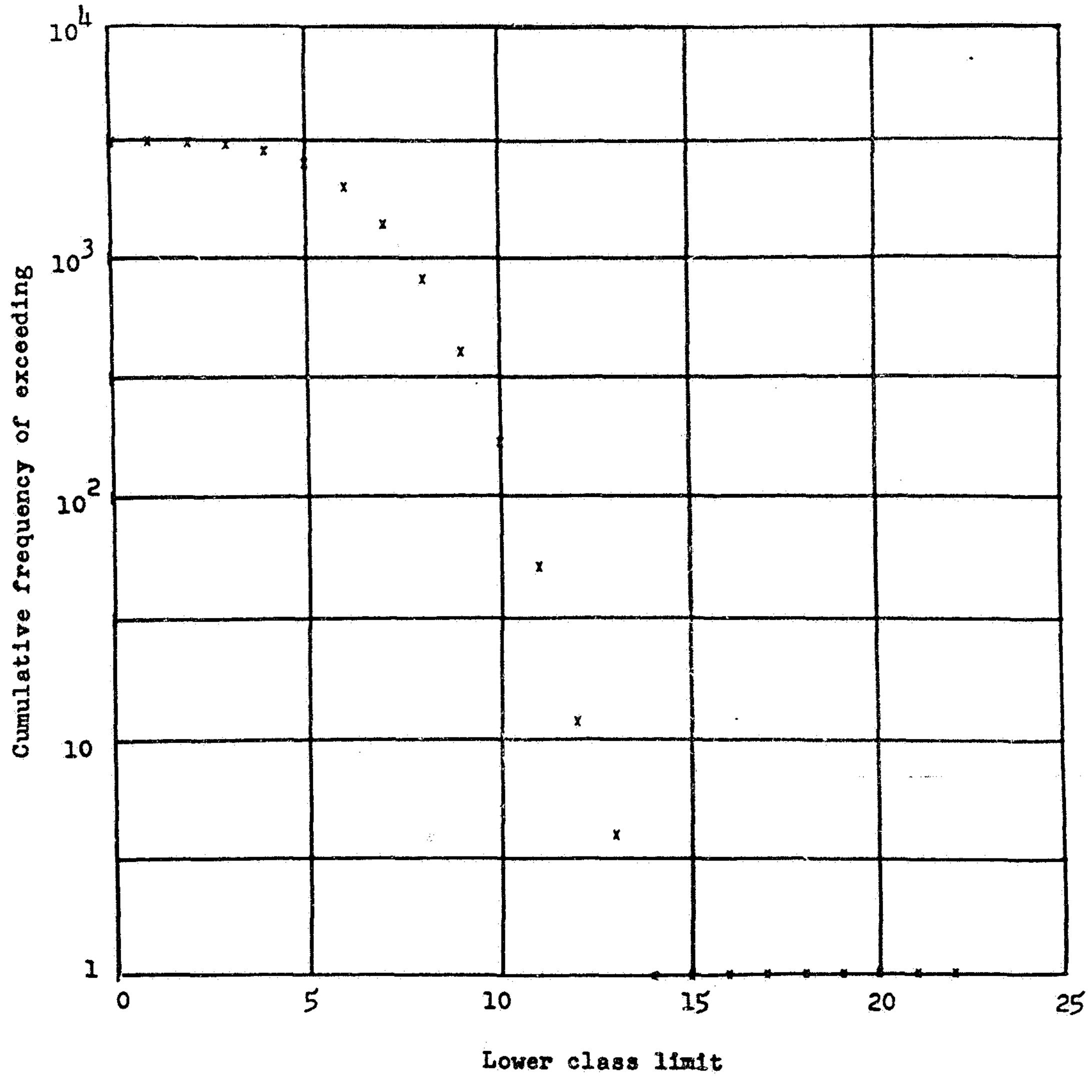

Figure 3.58. Log of cumulative minimum peaks of a time history having a Poisson amplitude probability density: $\mu = 10$

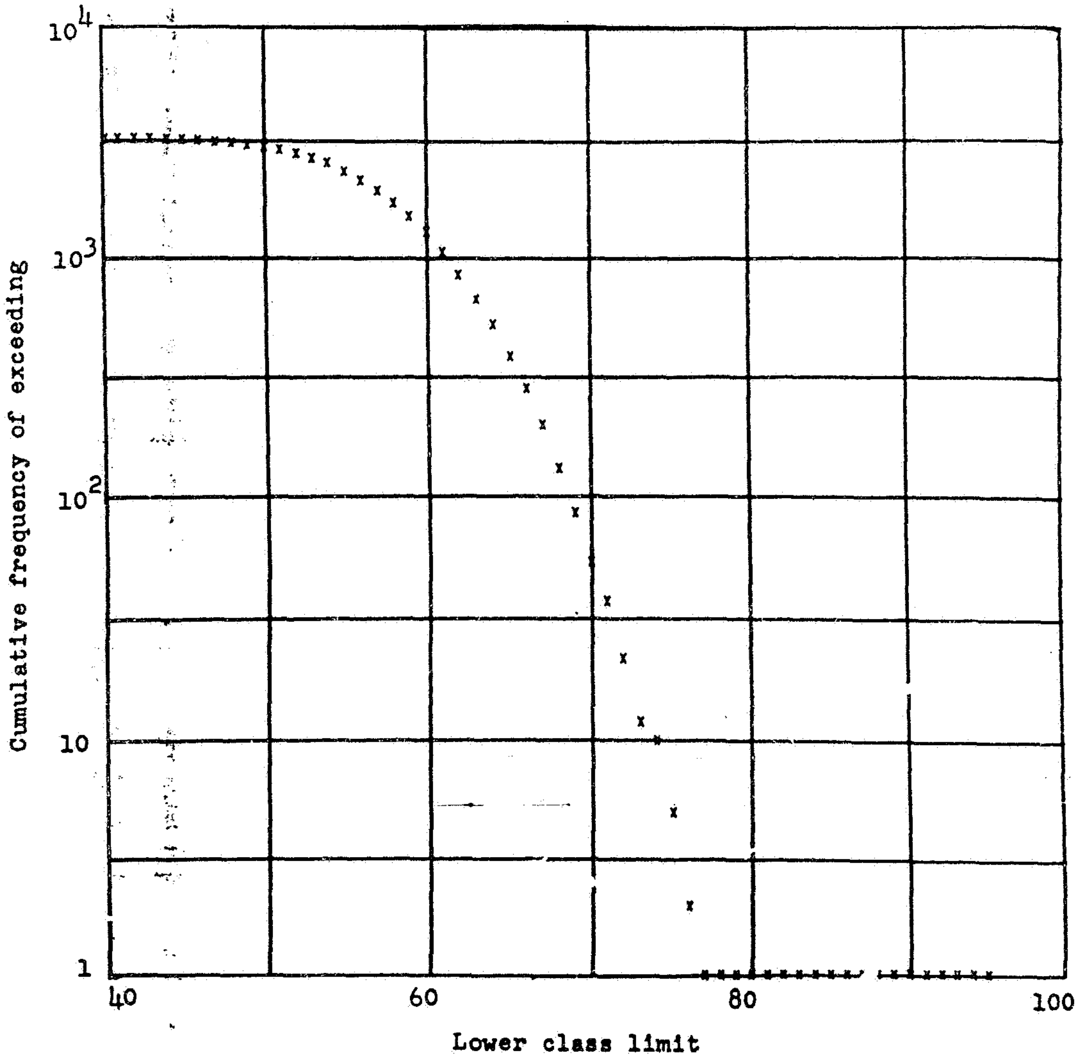

Figure 3.59. Log of cumulative minimum peaks of a time history having a Poisson amplitude probability density: $\mu = 66$

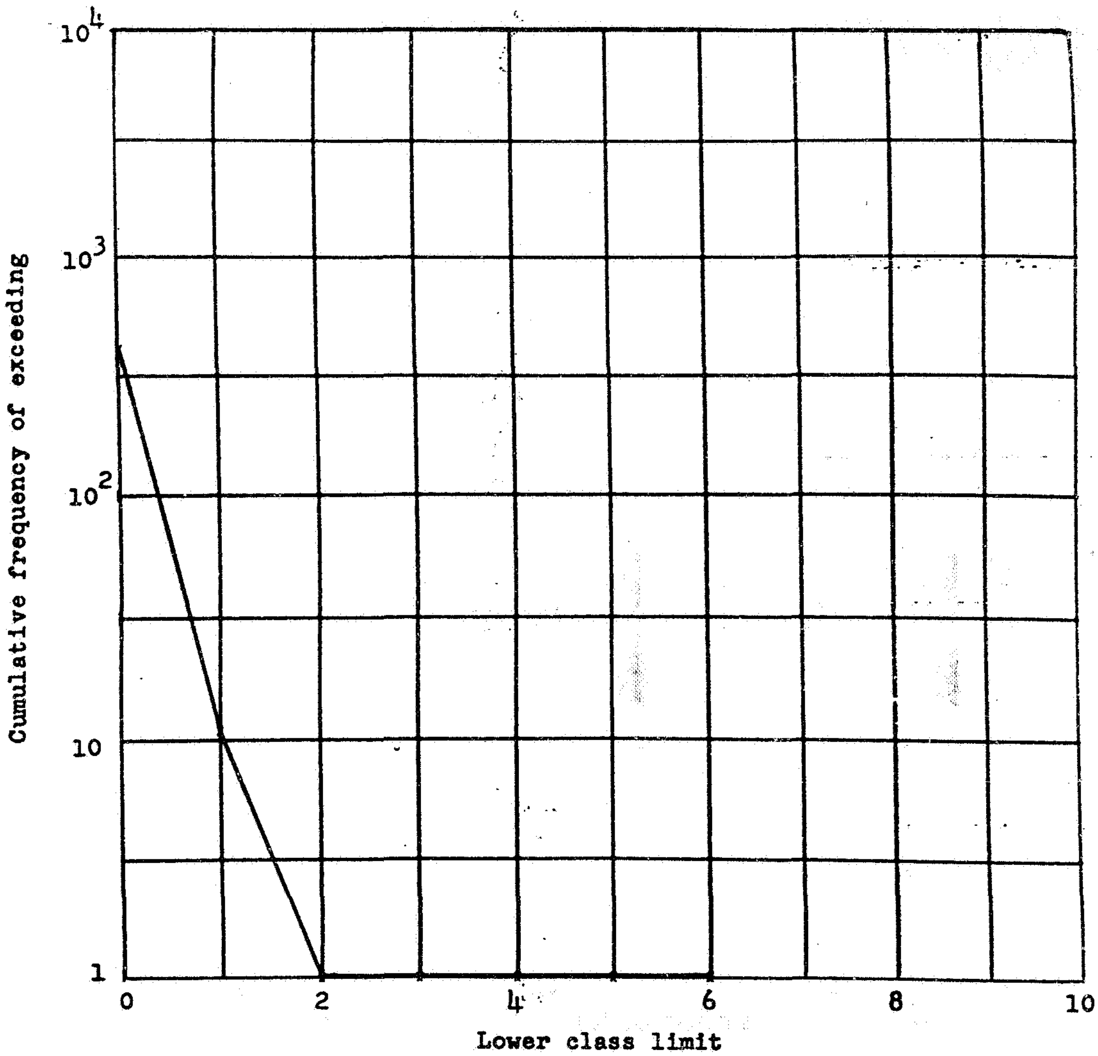

Figure 3.60. Log of cumulative minimum peaks of a time history having a Binomial amplitude probability density: $n = 10$; $p = 0.1$

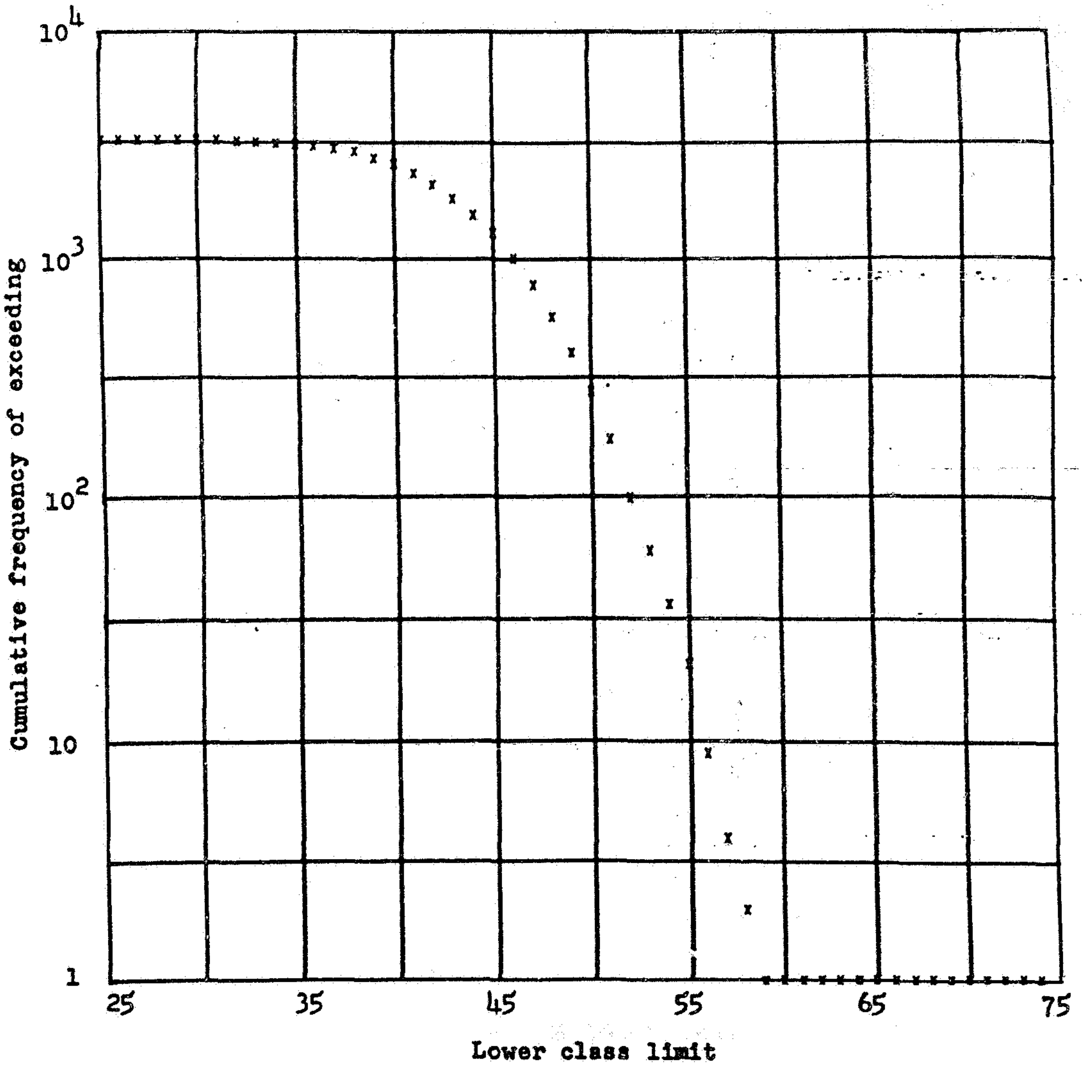

Figure 3.61. Log of cumulative minimum peaks of a time history having a Binomial amplitude probability density: n = 500; p = 0.1

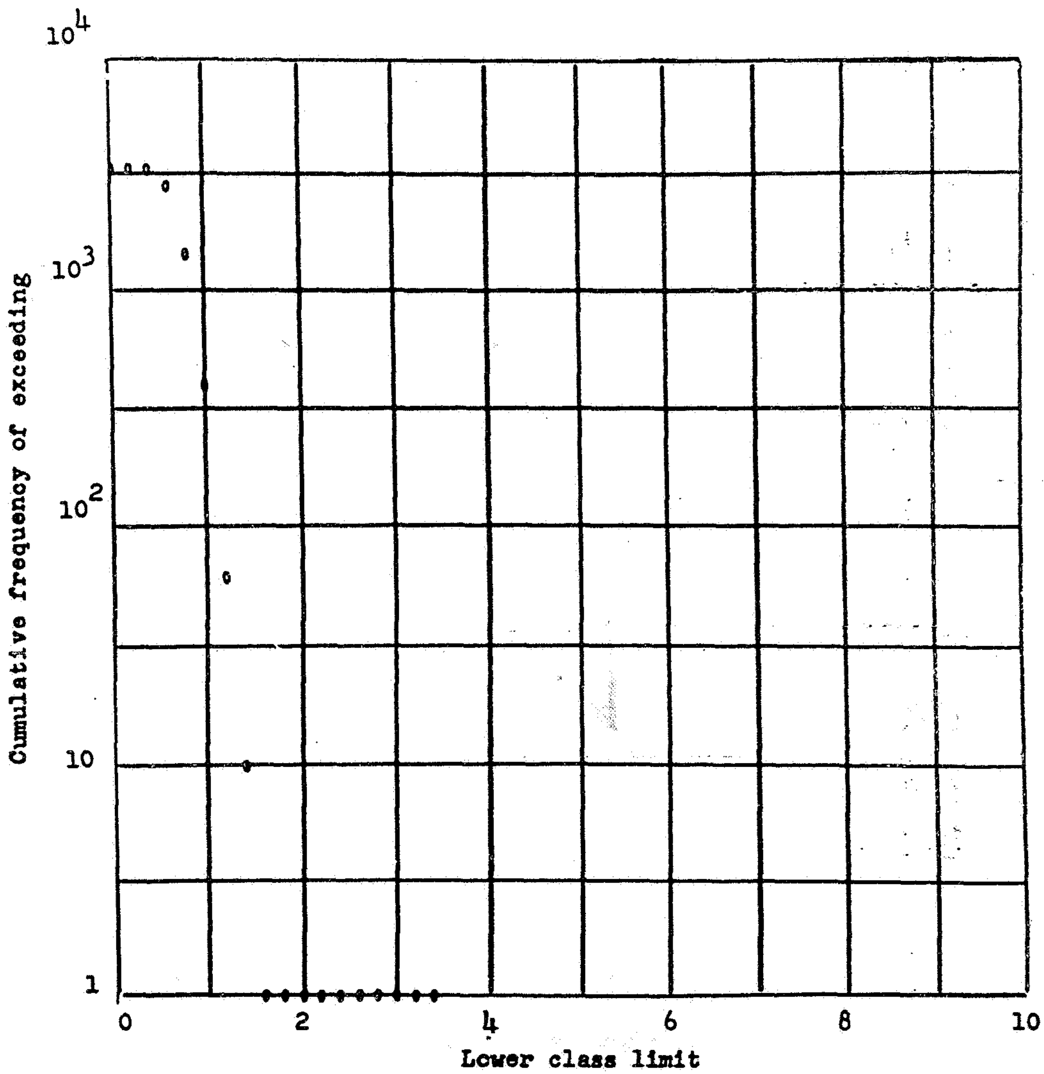

Figure 3.62. Log of cumulative minimum peaks of a time history having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.1$

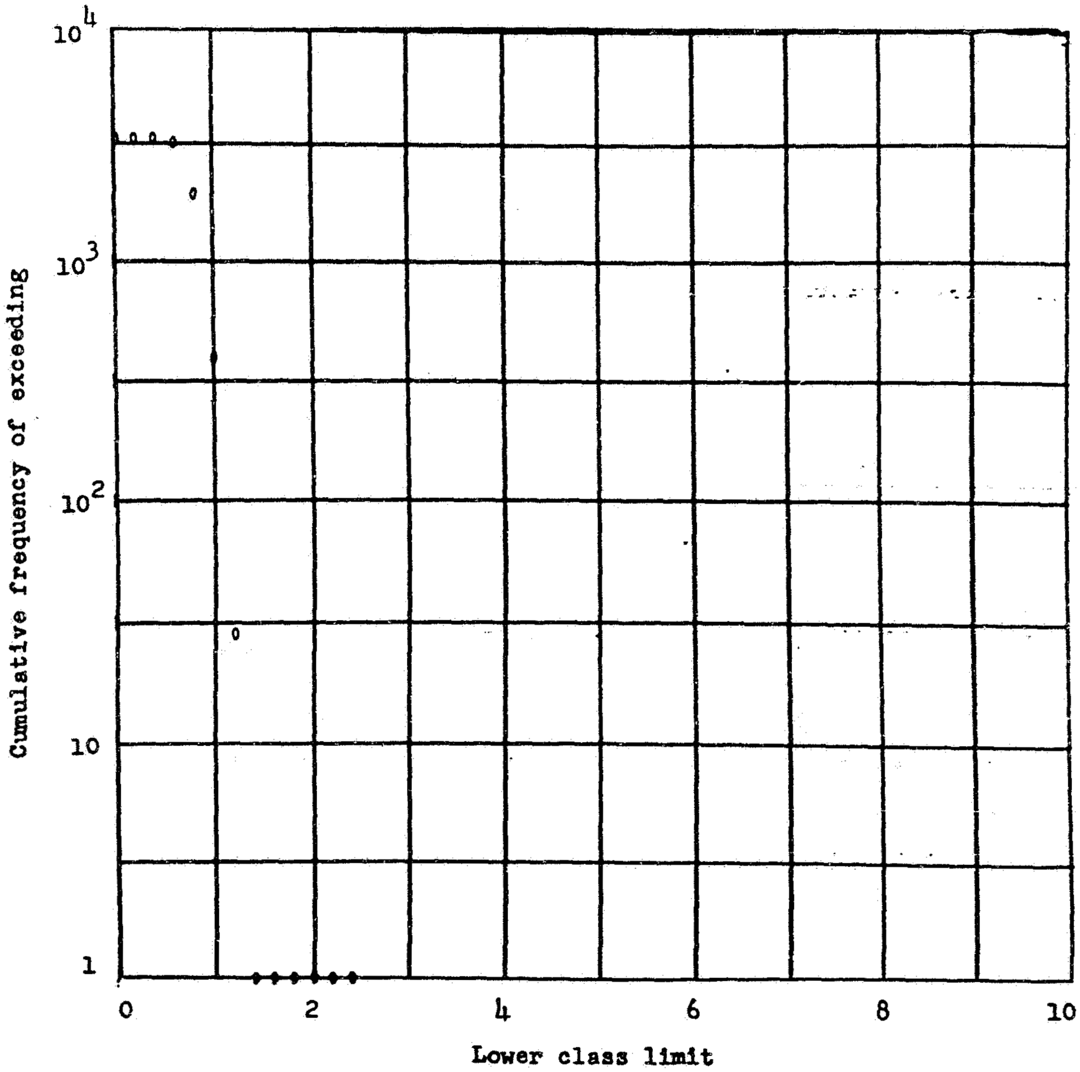

Figure 3.63. Log of cumulative minimum peaks of a time history having a Log-Normal amplitude probability density: $\sigma^2_{\log} = 0.05$

4. COMPUTER PROGRAMS

4.1 General

The programs presented here were developed in order to digitally generate stationary, non-Gaussian random time histories which have mathematically describable amplitude probability density functions. The programs were written in FORTRAN IV language and ran on a CDC 6000 series digital computer. Compile time for the programs was something less than 10 seconds with an execution or run time of approximately 30 seconds per each parameter variation.

4.2 Program Listings and Typical Block Diagram

The main programs are listed in Appendix 8.1 as follows:

8.1.1 Binomial Distribution

8.1.2 Poisson Distribution

8.1.3 Weibull Distribution

8.1.4 Exponential Distribution

8.1.5 Log-Normal Distribution

Subroutines are listed in Appendix 8.2 as follows:

8.2.1.1 Random Number Generator RANF

8.2.1.2 Plotting Subroutines DDIPLT

8.2.2.1 Function XWEIB

8.2.2.2 Function XEXP

8.2.2.3 Function XLOGNL

A typical block diagram for these computer programs is shown in Appendix 8.3.

4.3 Input Data

Required input data are defined in comment cards near the beginning of each program listing (see section 8.1).

4.4 Output Data

The output of the programs which have been discussed in section 3 are summarized in plot form. Specifically, the outputs are time histories, histograms, peak distributions and cumulative peak distributions.

5. COMPARISON OF CUMULATIVE PEAK DISTRIBUTIONS WITH SERVICE LOAD DATA

5.1 Applicability

Thus far the discussion has been centered around the simulation of service load data for aircraft. In most cases this type of data is presented for gust loads on either a "g" (acceleration due to gravity) or velocity basis. The data are usually plotted on semi-log paper with the cumulative number of exceedances being plotted on the log scale and the "g" or velocity level on the linear scale. In first approximation this curve plots as a straight line. This is illustrated in Figure 5.1 which has been taken from Bullen (1967). It should be pointed out that service load data are based on maximum peak values. If this typical gust load spectrum is compared with the digitally generated cumulative peak distributions, it can be seen that the data presented in Figures 3.46, 3.47, 3.52 and 3.53 could very easily be used to approximate the service load data by appropriate scaling of the abscissa. As an example, the digitally generated data of Figure 3.52 has been replotted in Figure 5.2 along with the gust load data of Figure 5.1. The abscissa of Figure 3.52 was scaled to agree with the abscissa of Figure 5.1. It can be seen that the digitally generated data agrees very well with the service load data. In the case of the gust load data presented by Coleman (1968), where the cumulative frequency of

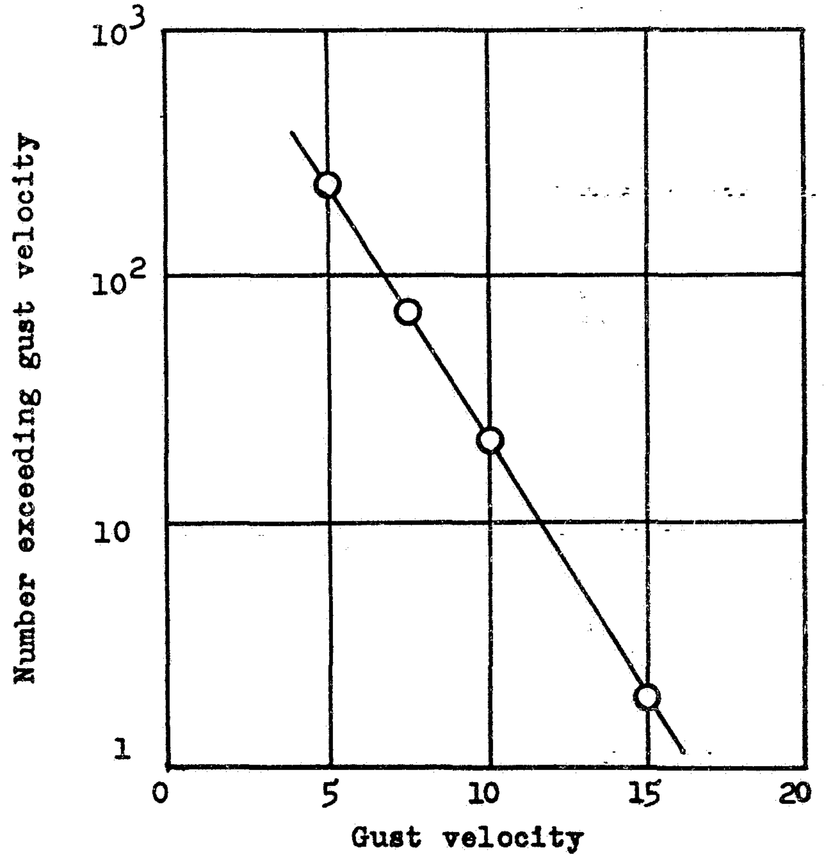

Figure 5.1. Typical aircraft service loading spectrum (Bullen 1967)

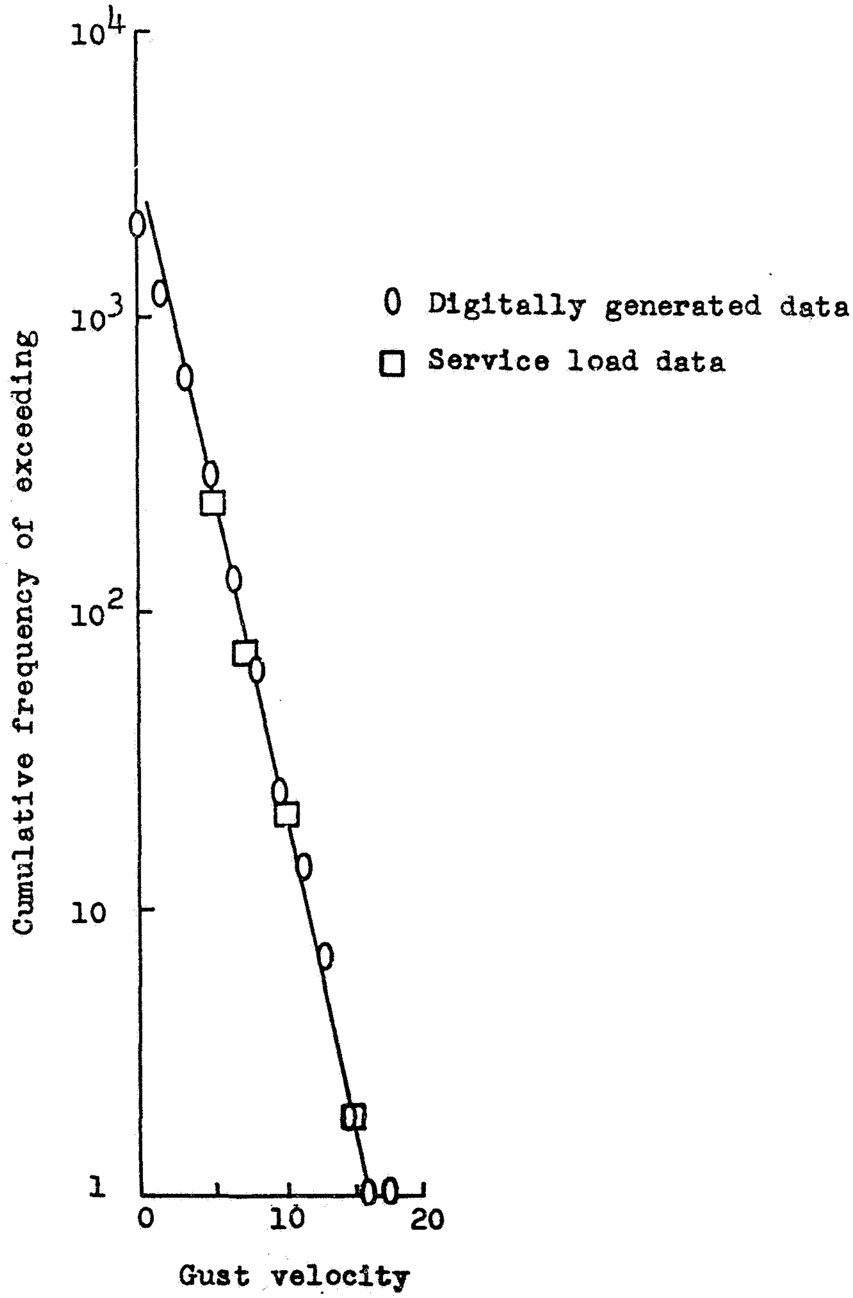

Figure 5.2. Comparison of digitally generated data and service load data

exceeding an acceleration of a given level is given on a per mile or per flight basis, all that is needed is an estimate of the total number of miles or flights to be flown in the aircraft lifetime. With this information these curves become comparable with other cumulative peak distributions found in the literature.

With regard to the other cumulative peak distributions which were digitally generated, they may be compared favorably with service load data for other than aircraft. Prothro (1968) and Jacoby (1967) present service load data for ground transportation items such as cars, trucks, and railroads and also for traveling overhead cranes.

Figure 5.3 shows typical loading spectra for motor cars where the load time history was measured over varying distances from 1 km to 1250 km. The shorter the measuring distance the more apt one is to encounter only one specific road condition, that is, a concrete highway, a tar surface, a country road, etc. One surface condition usually means the load is distributed normally and hence the cumulative distribution is rounded as shown by the two lower curves in Figure 5.3. The longer the measuring distance is the more one is apt to get a mixture of surface conditions in the measured interval and hence the distribution changes from a normal one to a straight line distribution as indicated by the upper two curves in Figure 5.3. It should be noted that the spring displacement at the intersection with the ordinate is the mean spring displacement, that is, the

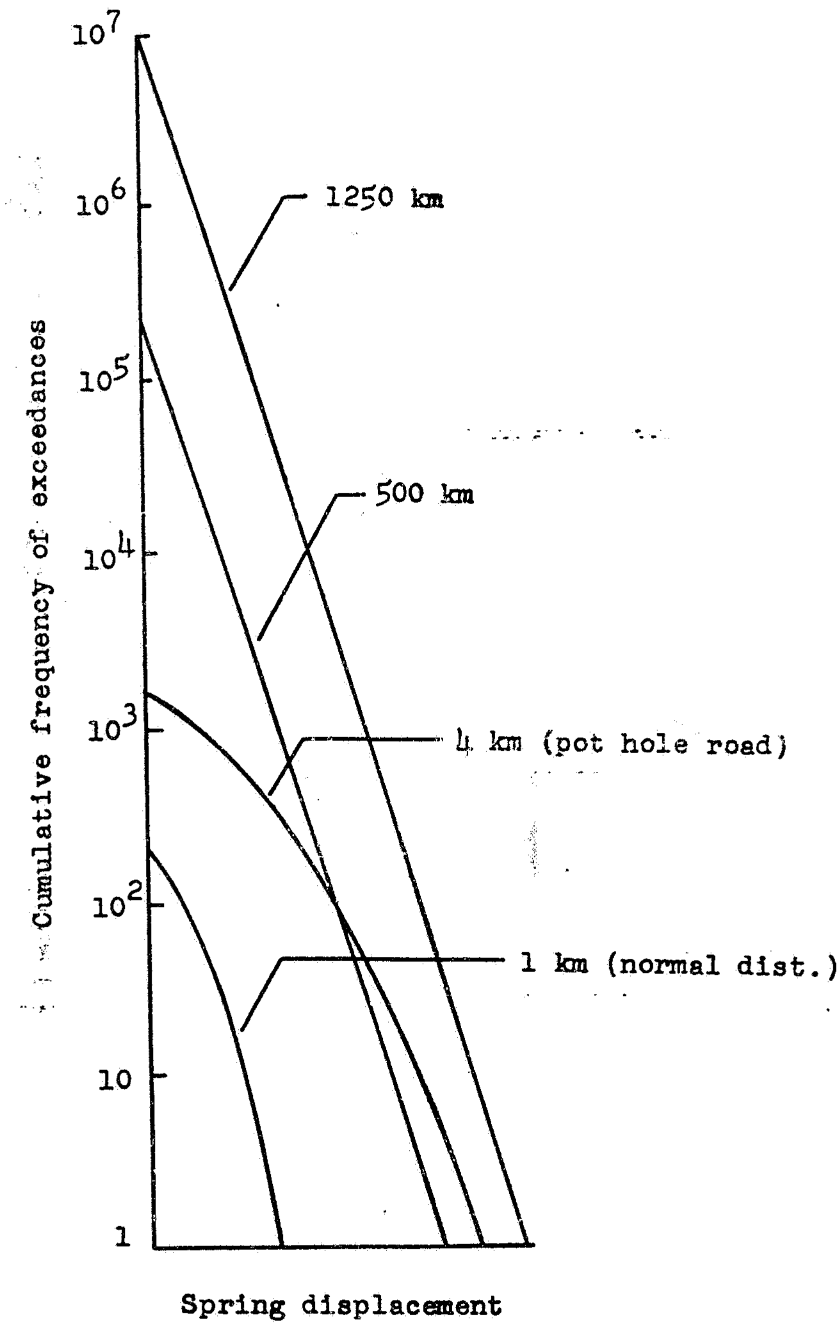

Figure 5.3. Typical motor car service loading spectrum (Jacoby 1967)

displacement due to the weight of the car on the springs. It can be seen that these curves compare favorably with the cumulative maximum peak distributions shown in Figures 3.44-3.53.

Figure 5.4 is a typical service load spectrum for the front axle of a truck, this time in terms of bending moment. Again, as was discussed in Figure 5.3, the distribution starts at the mean or static load position. The dashed curve represents what happens if the loads on the front axle of the truck are such that the springs "bottom out." This phenomenon is not observed with the digitally generated distributions but the basic cumulative frequency curve is similar to some of those shown in Figures 3.44-3.53.

Figure 5.5 shows a typical load spectrum for a railroad car travelling at 65 mph. The measured distance was 173 miles. Although one might assume that the surface condition of a track would be constant, Prothro (1968) points out that track is classified as smooth and rough. In fact 74 miles of the 173 measured miles was classified as smooth. Since we can classify track as to rough, smooth or somewhere in between it is logical to assume that in the 173 measured miles there was a mixing of the various track conditions. Using the same reasoning as for motor cars we would expect a straight line distribution. We do indeed get a straight line distribution above the mean

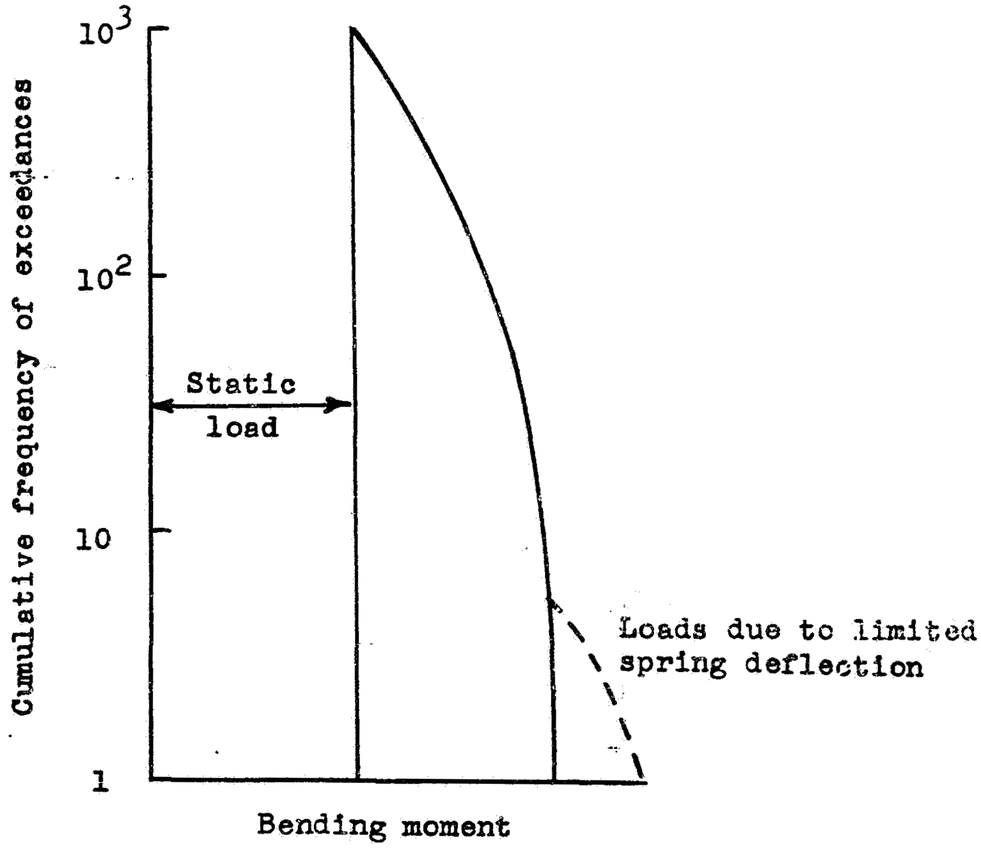

Figure 5.4. Typical truck front axle service loading spectrum (Jacoby 1967)

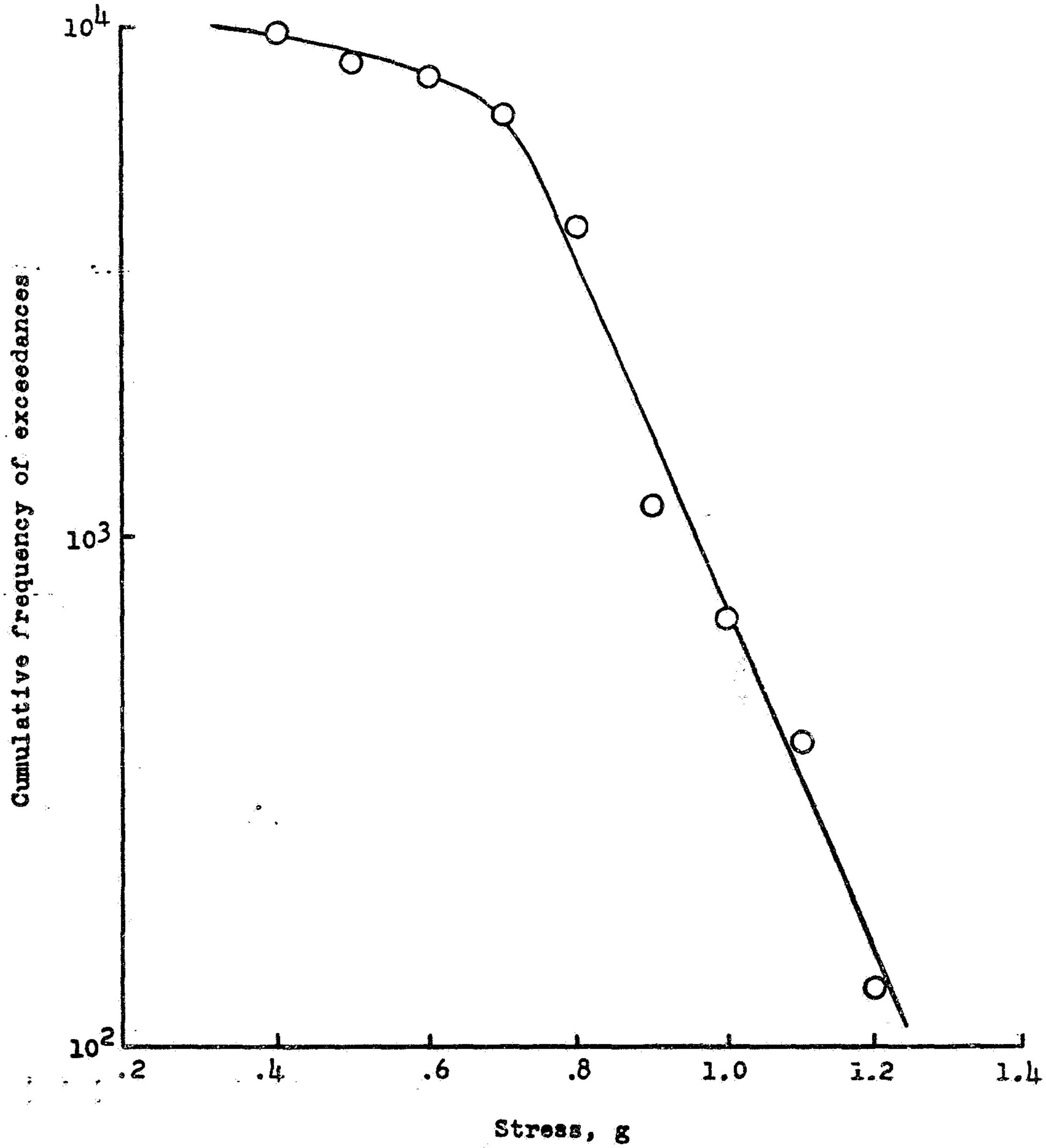

Figure 5.5. Typical railroad car service loading spectrum (Prothro 1968)

stress which is approximately 0.78g. Again this curve compares favorably with some of the digitally generated cumulative distributions shown in Figures 3.44-3.53.

Finally, Figure 5.6 shows a typical service load spectrum for a travelling overhead crane girder. Those different loadings are superimposed on this curve, all starting out from the mean stress. The basic loading forms the familiar curved distribution indicating a somewhat Gaussian distribution of loads. This should have been anticipated since the travelling surface of the crane would not be expected to vary much in roughness and the measured distance is small. The vibrations indicated are of small magnitude and relatively high frequency while the overload stresses are of larger magnitude and account for relatively few loadings. As with the other specimens discussed the shape of the basic loading curve is similar to some of these shown in Figures 3.44-3.53.

At this point it is clearly obvious that service load data from many sources, aircraft, car, truck, railroad and crane can be simulated using digitally generated non-Gaussian stationary random time histories. Once the basic shape of the cumulative distribution is obtained the ordinates and abscissas of these distributions can be adjusted to coincide with the appropriate service load data.

The comparison of the cumulative peak distributions of time histories having Weibull, Poisson and Binomial

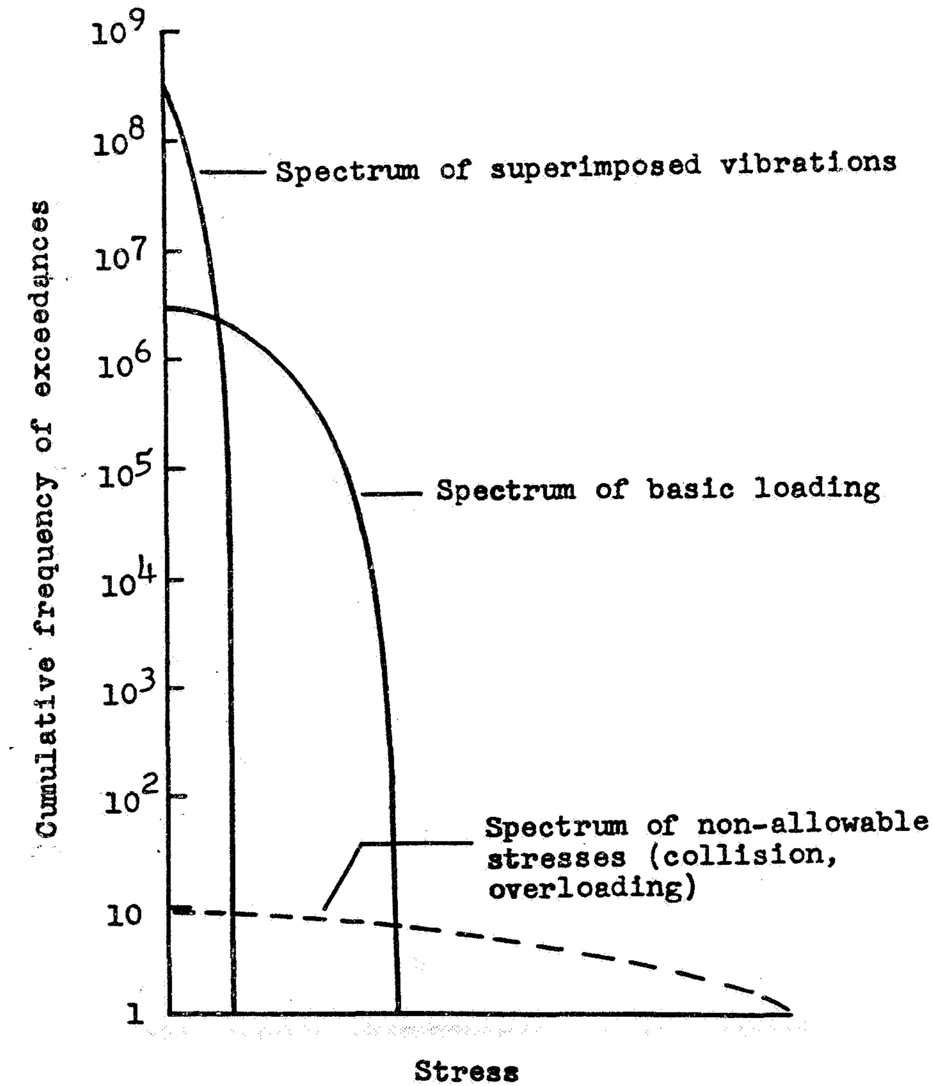

Figure 5.6. Typical overhead travelling crane girder service loading spectrum (Jacoby 1967)

probability density functions was not made with the service load data from ground transportation because of the varied number of cumulative peaks exhibited (see Figures 5.3-5.6). The number of peaks is a direct function of the sample length of the time history. In the present investigation the sample length was fixed arbitrarily at 10,000 random numbers which resulted in 3,336 maximum peaks. The sample length could just as easily be adjusted to result in 5,000 or 10,000 peaks or any other number of peaks so as to match the service load data. The primary purpose here was to show that the shape of the cumulative peak distribution matched the service load data.

5.2 Utilization

It has been pointed out previously that two methods have been proposed for simulating, in a random fashion, the service load histories of aircraft. The first method proposes the generation of a "quasi-stationary" random time history whose cumulative peak distribution simulates that found in service. The second method, described herein, proposes a stationary, non-Gaussian random time history whose cumulative peak distribution also simulates that found in service. Either or both methods can be used to conduct random load fatigue tests under simulated service conditions.

Proponents of the quasi-stationary method of testing argue that the root-mean-square stress or load encountered in service is varying continuously or is at least only constant for short periods of time. The use of a stationary Gaussian time history from a random noise generator whose root-mean-square level is varied in a random fashion is employed to simulate the service conditions. The number of root-mean-square levels chosen to represent the conditions encountered in service is of primary importance. Too few levels will not yield the required service distribution. Too many will complicate the test setup. Of course, this method of testing necessarily uses an analogue signal as its input either to an electrodynamic or an electrohydraulic shaker. It is also said that this method enables the wave form of the analogue signal to be preserved, that is, the frequency characteristics of the signal will be applied to the specimen as generated. However, Clevenson and Steiner (1967) have shown that the shape of the power spectral during curve of a random time history, that is, the frequency content of the random signal, has little or no effect on the fatigue life.

The stationary approach described herein, which is necessarily digital, can be cited for its ease of testing. A punched tape or punched card can be used to actuate a hydraulic servo valve to apply the peak loads in their proper sequence. The root-mean-square level is held constant so that no adjustment is required throughout a test. Wave form is lost with this method but as mentioned previously is considered of no consequence.

Analytically both methods result in the same cumulative peak distribution. In addition, both methods apply the loads in a random failure. Based on a linear accumulation of damage both methods would necessarily result in the same fatigue life.

It is the author's opinion that the digital or stationary approach to testing is the more practical of the two proposed methods for the following reasons: (1) all the peak statistics are determined analytically prior to testing, (2) input to testing machine is in the form of punched or magnetic tape, and (3) a constant root-mean-square level is maintained throughout the test. However, it would be beneficial to conduct both stationary and quasi-stationary tests in order to provide additional incite into the important fatigue inducing parameters involved in fatigue testing under random loading.

6. CONCLUSIONS AND RECOMMENDATIONS

Five random time histories having mathematically describable, non-Gaussian, amplitude probability density functions have been generated with the aid of a digital computer and their peak statistics determined. The following conclusions have been drawn from an analysis of the data obtained: (1) Stationary random time histories having Exponential and/or Log-Normal amplitude probability density functions can be used to simulate the peak load distributions encountered by aircraft in service. (2) Peak load distributions encountered in service by ground transportation and handling equipment, that is, car, truck, rail and overhead crane, can be approximated using the remaining three time histories investigated, namely those having Weibull, Poisson, and Binomial amplitude probability density functions. (3) The resulting time histories can be used to conduct stationary random and/or programmed block fatigue tests in which they simulate service load conditions. (4) Maximum and minimum peak distributions are unsymmetrical about the mean for non-Gaussian probability density functions but approach a symmetrical condition as the probability density function approaches the normal or Gaussian distribution.

It is recommended that a comparison based on fatigue testing be made between the proposed "stationary" method of simulating service conditions and the proposed "quasi-stationary" method of simulating the same conditions. These tests could provide additional incite into the important fatigue inducing parameters involved in fatigue testing under random loading.

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8. APPENDICES

8.1 FORTRAN Programs for Generating Non-Gaussian Random Time Histories and Determining Certain Peak Statistics

8.1.1 Binomial Distribution

```
C
C     RANDOM TIME HISTORY HAVING A BINOMIAL PROBABILITY
C     DENSITY FUNCTION
C
      PROGRAM PLOT      (INPUT,OUTPUT,TAPE21,TAPE5=INPUT,
     1TAPE6=OUTPUT)
      DIMENSION P(1000),LZ(10000),JCOUNT(1000),F(1000)
      DIMENSION LL(1000),NF(1000),P1(1000)
      DIMENSION LZMAX(5000),LZMIN(5000),ZYM(3)
      DIMENSION CZMAX(1000),CZMIN(1000),XSDBOT(100)
      DIMENSION AMID(500),YJ1(1000),YJ(1000),YLOGJSM(500)
      DIMENSION PCOUNT(100),RANDOM(100),YBINTV(500)
      DIMENSION XMY(1),XXM(1),IN(2),XM(2),YM(2),YYM(3)
      DIMENSION XRMS(100),XXMEAN(100),XSDTOP(100)
      DIMENSION XAM(4),YAM(2),ZAM(4),YLOGISM(500)
C
C     INPUT DATA AND CONSTANTS
C
      DATA XM/11HPOINT COUNT/
      DATA YM/13HRANDOM NUMBER/
      DATA IN/7HHAL SRD,8HBINOMIAL/
      DATA XMY/9HFREQUENCY/
      DATA XXM/10HCLASS MARK/
      DATA YYM/24HMAXIMUM PEAK VALUE COUNT/
      DATA ZYM/24HMINIMUM PEAK VALUE COUNT/
      DATA XAM/40HLOG OF CUMULATIVE FREQUENCY FOR MAXIMUMS/
      DATA YAM/17HLOWER CLASS LIMIT/
      DATA ZAM/40HLOG OF CUMULATIVE FREQUENCY FOR MINIMUMS/
      ITAPE=6LTAPE21
C
C     INPUT VARIABLES DEFINED
C
C     N=NUMBER OF EVENTS
C     M=NUMBER OF RANDOM NUMBERS TO BE GENERATED
C     PEE=PROBABILITY OF SUCCESS IN A GIVEN EVENT
C     BOTTOM=LOWER LIMIT OF RANDOM VARIABLE,LZ
C     TOP=UPPER LIMIT OF RANDOM VARIABLE,LZ
C     FREQM1=MAXIMUM VALUE OF THEORETICAL AND ACTUAL
C     FREQUENCY OF OCCURRENCE
C     FREQM2=LARGEST PEAK VALUE (MINIMUM)
C     FREQM3=LARGEST PEAK VALUE (MAXIMUM)
C
    2 READ(5,5) N,M,PEE,BOTTOM,TOP,FREQM1,FREQM2,FREQM3
```

```
    5 FORMAT(2I10,6F10.6)
      IF(EOF,5) 991,990
  990 CONTINUE
      AN=N
      P(1)=0.0
      Q=1.-PEE
      F(1)=Q**N
      KIT=N+1
      WRITE(6,8) N,PEE
    8 FORMAT(1H1,3X,26HBINOMIAL DISTRIBUTION    N=I5,5X,2HP=
     1,F10.6//)
      WRITE(6,19)
   19 FORMAT(9X,1HX,10X,1HF,14X,1HP,12X,3H1-P/)
      LABCDE=0
      M=M+1
      J1=2
      J=0
      AM=M
      KOUNT=0
C
C     BINOMIAL PROBABILITY AND CUMULATIVE PROBABILITY
C
      DO 110 I=1,KIT
      AI=I
      AI1=I-1
      F(I+1)=((AN-AI1)/AI)*(PEE/Q)*F(I)
      P(I+1)=P(I)+F(I)
      P1(I)=1.-P(I)
      IF(P(I+1).LT.0.00000001) GO TO 120
      GO TO 30
  120 J1=I+2
      GO TO 110
   30 CONTINUE
      IF(P(I+1).GT.0.99999999) GO TO 40
      GO TO 110
   40 K=I
      GO TO 111
  110 CONTINUE
  111 J1=J1-2
      K=K-2
      IF(J1.EQ.0) GO TO 999
      WRITE(6,1)(I,F(I+1),P(I+2),P1(I+1),I=J1,K)
    1 FORMAT(I10,3F15.8)
      GO TO 998
  999 CONTINUE
      KK=K+1
      DO 996 I=1,KK
      II=I-1
  996 WRITE(6,997) II,F(I),P(I+1),P1(I)
  997 FORMAT(I10,3F15.8)
  998 CONTINUE
      J1=J1+1
      K=K+1
```

```
C
C     CALCULATION OF THEORETICAL FREQUENCY OF OCCURRENCE
C
      DO 112 I=J1,K
      NF(I)=F(I)*AM
      IF(NF(I).EQ.0)  GO TO 112
      KOUNT=KOUNT+1
      LL(KOUNT)=I
  112 CONTINUE
      MIN=LL(1)
      MAX=LL(KOUNT)
      WRITE(6,63) MIN,MAX
   63 FORMAT(5HOMIN=I6,6H  MAX=I6)
C
C     GENERATION OF RANDOM NUMBERS
C
      X=34521637721.
      LSQ=0
      LSUM=0
   53 Y=RANF(X)
      X=0.0
      IF(LABCDE.EQ.M) GO TO 50
      DO 51 I=MIN,MAX
      IF(Y.GE.P(I).AND.Y.LT.P(I+1)) GO TO 52
      GO TO 51
   52 LABCDE=LABCDE+1
      LZ(LABCDE)=I-1
      LSUM=LSUM+LZ(LABCDE)
      LSQ=LSQ+LZ(LABCDE)**2
      GO TO 53
   51 CONTINUE
      GO TO 53
   50 CONTINUE
      XMEAN=FLOAT(LSUM)/FLOAT(M)
      RMS=SQRT(XMEAN*(1.-PEE)+XMEAN**2)
      XMEANSQ=FLOAT(LSQ)/FLOAT(M)
      STDDEV=SQRT(XMEANSQ-XMEAN**2)
C
C     TIME HISTORY PLOT OF FIRST 100 RANDOM NUMBERS
C
      MIN=MIN-1
      MAX=MAX-1
      DO 700 I=1,100
      XSDTOP(I)=XMEAN+STDDEV
      XSDBOT(I)=XMEAN-STDDEV
      XRMS(I)=RMS
      XXMEAN(I)=XMEAN
      PCOUNT(I)=I
  700 RANDOM(I)=LZ(I)
      CALL DDIPLT(0,IN,100,PCOUNT,XRMS,1.0,100.,BOTTOM,TOP,
     12,XM,2,YM,13,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XXMEAN,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
```

```
      CALL DDIPLT(0,IN,100,PCOUNT,XSDTOP,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XSDBOT,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(1,IN,100,PCOUNT,RANDOM,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      KCOUNT=0
C
C     THEORETICAL HISTOGRAM
C
      WRITE(6,54)
   54 FORMAT(22HOTHEORETICAL HISTOGRAM//)
      WRITE(6,55)
   55 FORMAT(1HO,8X,1HX,7X,11HOCCURRENCES/)
      IF(MIN.EQ.0) GO TO 899
      WRITE(6,9)(I,NF(I+1),I=MIN,MAX)
    9 FORMAT(I10,I15)
      GO TO 898
  899 CONTINUE
      MAX=MAX+1
      DO 896 I=1,MAX
      II=I-1
  896 WRITE(6,897) II,NF(I)
  897 FORMAT(I10,I15)
  898 CONTINUE
C
C     DETERMINATION OF ACTUAL HISTOGRAM
C
      WRITE(6,13)
   13 FORMAT(1HO//,3X,16HACTUAL HISTOGRAM//)
      MIN=MIN+1
      MAX=MAX+1
      DO 11 I=MIN,MAX
      ICOUNT=0
      DO 10 J=1,M
      IF(LZ(J).EQ.I-1) ICOUNT=ICOUNT+1
   10 CONTINUE
   11 JCOUNT(I)=ICOUNT
      WRITE(6,55)
      DO 15 I=MIN,MAX
      L=I-1
   15 WRITE(6,12)L,JCOUNT(I)
   12 FORMAT(I10,I15)
      WRITE(6,59) RMS,XMEAN,STDDEV
   59 FORMAT(1H1,3X,4HRMS=F10.3,3X,5HMEAN=F10.3,3X,15HSTD.
     1DEVIATION=F10.3)
      WRITE(6,56)
   56 FORMAT(1HO//,8X,1HX,7X,11HTHEO. FREQ.,4X,12HACTUAL
     1FREQ.)
      CHI=0
      LA=MAX
      DO 16 I=MIN,MAX
      L=I-1
```

```
      CHI=(((F(I)*AM)-FLOAT(JCOUNT(I)))**2/(F(I)*AM))+CHI
      KCOUNT=KCOUNT+1
      AMID(KCOUNT)=L
      YJ1(KCOUNT)=NF(I)
      YJ(KCOUNT)=JCOUNT(I)
   16 WRITE(6,17) L,NF(I),JCOUNT(I)
   17 FORMAT(I10,2I15)
C
C     PLOTS OF COMPUTED AND THEORETICAL HISTOGRAMS
C
      CALL DDIPLT(0,IN,KCOUNT,AMID,YJ1,BOTTOM,TOP,0,FREQM1,
     11,XXM,1,XMY,14,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,YJ,BOTTOM,TOP,0,FREQM1,1
     1,XXM,1,XMY,12,ITAPE)
C
C     DETERMINATION OF PEAK AND CUMULATIVE PEAK DISTRIBU-
C     TIONS FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      MAX=0
      MIN=0
      J=1
      I=0
  200 CONTINUE
      I=I+1
      IF(I.GT.M-1) GO TO 205
      IF(I.NE.1) GO TO 1036
  201 J=J+1
      IF(LZ(I).NE.LZ(J)) GO TO 202
      GO TO 201
  202 CONTINUE
      IF(LZ(I).GT.LZ(J)) GO TO 1037
      GO TO 1038
 1037 MAX=MAX+1
      LZMAX(MAX)=LZ(I)
      GO TO 1036
 1038 MIN=MIN+1
      LZMIN(MIN)=LZ(I)
 1036 CONTINUE
  204 J=J+1
      IF(LZ(I+1).NE.LZ(J)) GO TO 203
      GO TO 204
  203 IF(LZ(I+1).GT.LZ(J).AND.LZ(I+1).GT.LZ(I)) GO TO 1021
      IF(LZ(I+1).LT.LZ(J).AND.LZ(I+1).LT.LZ(I)) GO TO 1022
      GO TO 206
 1021 MAX=MAX+1
      LZMAX(MAX)=LZ(I+1)
      GO TO 206
 1022 MIN=MIN+1
      LZMIN(MIN)=LZ(I+1)
  206 I=J-2
      GO TO 200
  205 CONTINUE
      WRITE(6,1034)
```

```
1034 FORMAT(1H1,22X,16HPEAK VALUE COUNT)
     WRITE(6,1039) MAX,MIN
1039 FORMAT(1H0,10X,14HMAXIMUM PEAKS=I5,2X,14HMINIMUM PEAK
    1S=I5///4X,8HINTERVAL,3X,3HMAX,3X,3HMIN,4X,4HCMAX,4X,4
    2HCMIN,2X,8HLOG CMAX,3X,8HLOG CMIN/)
     LUINTV=0
     ISUM=0
     JSUM=0
     KCOUNT=0
1030 LUINTV=LUINTV+1
     IF(LUINTV.GT.LA) GO TO 1035
     ICOUNT=0
     JKOUNT=0
     LUINTV=LUINTV-1
     LBINTV=LUINTV-1
     DO 1031 I=1,MAX
     IF(LZMAX(I).LE.LUINTV.AND.LZMAX(I).GT.LBINTV) ICOUNT=
    1ICOUNT+1
1031 CONTINUE
     IF(ISUM.EQ.0) GO TO 1050
     ISOM=MAX-ISUM
     GO TO 1051
1050 ISOM=MAX
1051 ISUM=ISUM+ICOUNT
     IF(ISOM.EQ.0) GO TO 500
     XLOGISM=ALOG10(FLOAT(ISOM))
     GO TO 510
 500 XLOGISM=0
 510 DO 1032 J=1,MIN
     IF(LZMIN(J).LE.LUINTV.AND.LZMIN(J).GT.LBINTV) JKOUNT=
    1JKOUNT+1
1032 CONTINUE
     IF(JSUM.EQ.0) GO TO 1052
     JSOM=MIN-JSUM
     GO TO 1053
1052 JSOM=MIN
1053 JSUM=JSUM+JKOUNT
     IF(JSOM.EQ.0) GO TO 501
     XLOGJSM=ALOG10(FLOAT(JSOM))
     GO TO 511
 501 XLOGJSM=0
 511 WRITE(6,1033) LBINTV,LUINTV,ICOUNT,JKOUNT,ISOM,JSOM,X
    1LOGISM,XLOGJSM
1033 FORMAT(4I6,2I8,F9.4,F11.4)
C
C     PLOTS OF PEAK AND LOG CUMULATIVE PEAK DISTRIBUTIONS
C     FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
     KCOUNT=KCOUNT+1
     YLOGISM(KCOUNT)=XLOGISM
     YLOGJSM(KCOUNT)=XLOGJSM
     YBINTV(KCOUNT)=LBINTV
     CZMAX(KCOUNT)=ICOUNT
```

```
      CZMIN(KCOUNT)=JKOUNT
      AMID(KCOUNT)=LUINTV
      LUINTV=LUINTV+1
      GO TO 1030
 1035 CONTINUE
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGISM,BOTTOM,TOP,0,4
     1.,2,YAM,4,XAM,12,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGJSM,BOTTOM,TOP,0,4
     1.,2,YAM,4,ZAM,12,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,CZMAX,BOTTOM,TOP,0,FREQM
     13,1,XXM,3,YYM,12,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,CZMIN,BOTTOM,TOP,0,FREQM
     12,1,XXM,3,ZYM,12,ITAPE)
      GO TO 2
  991 STOP
      END
```

8.1.2 Poisson Distribution

```
C
C     RANDOM TIME HISTORY FOR A POISSON PROBABILITY DENSITY
C     FUNCTION
C
      PROGRAM PLOT     (INPUT,OUTPUT,TAPE21,TAPE5=INPUT,
     1TAPE6=OUTPUT)
      DIMENSION F(300),P(300),JCOUNT(300),LZ(10000),LL(300)
      DIMENSION LZMAX(5000),LZMIN(5000),NF(300)
      DIMENSION CZMAX(1000),CZMIN(1000),ZYM(3),XSDBOT(100)
      DIMENSION AMID(500),YJ1(1000),YJ(1000),YLOGJSM(500)
      DIMENSION PCOUNT(100),RANDOM(100),YBINTV(500)
      DIMENSION XMY(1),XXM(1),IN(2),XM(2),YM(2),YYM(3)
      DIMENSION XRMS(100),XXMEAN(100),XSDTOP(100)
      DIMENSION XAM(4),YAM(2),ZAM(4),YLOGISM(500)
C
C     INPUT DATA AND CONSTANTS
C
      DATA XM/11HPOINT COUNT/
      DATA YM/13HRANDOM NUMBER/
      DATA IN/7HHAL SRD,7HPOISSON/
      DATA XMY/9HFREQUENCY/
      DATA XXM/10HCLASS MARK/
      DATA YYM/24HMAXIMUM PEAK VALUE COUNT/
      DATA ZYM/24HMINIMUM PEAK VALUE COUNT/
      DATA XAM/40HLOG OF CUMULATIVE FREQUENCY FOR MAXIMUMS/
      DATA YAM/17HLOWER CLASS LIMIT/
      DATA ZAM/40HLOG OF CUMULATIVE FREQUENCY FOR MINIMUMS/
      ITAPE=6LTAPE21
C
C     INPUT VARIABLES DEFINED
C
C     M=NUMBER OF RANDOM NUMBERS TO BE GENERATED
```

```
C       U=MEAN OF RANDOM VARIABLE
C       BOTTOM=LOWER LIMIT OF RANDOM VARIABLE,LZ
C       TOP=UPPER LIMIT OF RANDOM VARIABLE,LZ
C       FREQM1=MAXIMUM VALUE OF THEORETICAL AND ACTUAL
C       FREQUENCY OF OCCURRENCE
C       FREQM2=LARGEST PEAK VALUE (MINIMUM)
C       FREQM3=LARGEST PEAK VALUE (MAXIMUM)
C
      2 READ(5,3) M,U,BOTTOM,TOP,FREQM1,FREQM2,FREQM3
      3 FORMAT(I10,6F10.6)
        IF(EOF,5) 991,990
    990 CONTINUE
        LABCDE=0
        M=M+1
        N=499
        J1=2
        J=0
        AM=M
        KOUNT=0
        IF(U.LE.21.) XLA=50.
        IF(U.GT.21..AND.U.LE.54.) XLA=100.
        IF(U.GT.54..AND.U.LE.92.) XLA=150.
        IF(U.GT.92..AND.U.LE.130.) XLA=200.
        IF(U.GT.130..AND.U.LE.170.) XLA=250.
        IF(U.GT.170..AND.U.LE.205.) XLA=300.
        WRITE(6,102) U,XLA
    102 FORMAT(1H1,3X,29HPOISSON DISTRIBUTION,    MEAN=F12.7,
       110H    XUPPER=F6.0//)
        F(1)=EXP(-U)
        P(1)=0.0
        WRITE(6,19)
     19 FORMAT(9X,1HX,9X,4HPROB,9X,8HCUM PROB/)
C
C       POISSON PROBABILITY AND CUMULATIVE PROBABILITY
C
        DO 110 I=1,N
        F(I+1)=(U/(I))*F(I)
        P(I+1)=P(I)+F(I)
        IF(P(I+1).LT.0.00000001) GO TO 120
        GO TO 30
    120 J1=I+2
        GO TO 110
     30 CONTINUE
        IF(P(I+1).GT.0.99999999) GO TO 40
        GO TO 110
     40 K=I
        GO TO 111
    110 CONTINUE
    111 J1=J1-2
        K=K-2
        IF(J1.EQ.0) GO TO 999
        WRITE(6,1)(I,F(I+1),P(I+2),I=J1,K)
```

```
    1 FORMAT(I10,2F15.8)
      GO TO 998
  999 CONTINUE
      KK=K+1
      DO 996 I=1,KK
      II=I-1
  996 WRITE(6,997) II,F(I),P(I+1)
  997 FORMAT(I10,2F15.8)
  998 CONTINUE
      J1=J1+1
      K=K+1
C
C     CALCULATION OF THEORETICAL FREQUENCY OF OCCURRENCE
C
      DO 112 I=J1,K
      NF(I)=F(I)*AM
      IF(NF(I).EQ.0) GO TO 112
      KOUNT=KOUNT+1
      LL(KOUNT)=I
  112 CONTINUE
      MIN=LL(1)
      MAX=LL(KOUNT)
      WRITE(6,63) MIN,MAX
   63 FORMAT(5HOMIN=I6,6H   MAX=I6)
C
C     GENERATION OF RANDOM NUMBERS
C
      X=34521637721.
      LSQ=0
      LSUM=0
   53 Y=RANF(X)
      X=0.0
      IF(LABCDE.EQ.M) GO TO 50
      DO 51 I=MIN,MAX
      IF(Y.GE.P(I).AND.Y.LT.P(I+1)) GO TO 52
      GO TO 51
   52 LABCDE=LABCDE+1
      LZ(LABCDE)=I-1
      LSUM=LSUM+LZ(LABCDE)
      LSQ=LSQ+LZ(LABCDE)**2
      GO TO 53
   51 CONTINUE
      GO TO 53
   50 CONTINUE
      XMEAN=FLOAT(LSUM)/FLOAT(M)
      RMS=SQRT(XMEAN*(XMEAN+1))
      XMEANSQ=FLOAT(LSQ)/FLOAT(M)
      STDDEV=SQRT(XMEANSQ-XMEAN**2)
C
C     TIME HISTORY PLOT OF FIRST 100 RANDOM NUMBERS
C
      MIN=MIN-1
```

```
      MAX=MAX-1
      DO 700 I=1,100
      XSDTOP(I)=XMEAN+STDDEV
      XSDBOT(I)=XMEAN-STDDEV
      XRMS(I)=RMS
      XXMEAN(I)=XMEAN
      PCOUNT(I)=I
  700 RANDOM(I)=LZ(I)
      CALL DDIPLT(0,IN,100,PCOUNT,XRMS,1.0,100.,BOTTOM,TOP,
     12,XM,2,YM,13,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XXMEAN,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XSDTOP,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XSDBOT,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(1,IN,100,PCOUNT,RANDOM,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
C
C     THEORETICAL HISTOGRAM
C
      KCOUNT=0
      WRITE(6,54)
   54 FORMAT(22HOTHEORETICAL HISTOGRAM//)
      WRITE(6,55)
   55 FORMAT(1HO,8X,1HX,7X,11HOCCURRENCES/)
      IF(MIN.EQ.0) GO TO 899
      WRITE(6,9)(I,NF(I+1),I=MIN,MAX)
    9 FORMAT(I10,I15)
      GO TO 898
  899 CONTINUE
      MAX=MAX+1
      DO 896 I=1,MAX
      II=I-1
  896 WRITE(6,897) II,NF(I)
  897 FORMAT(I10,I15)
  898 CONTINUE
C
C     DETERMINATION OF ACTUAL HISTOGRAM
C
      WRITE(6,13)
   13 FORMAT(1HO//,3X,16HACTUAL HISTOGRAM//)
      MIN=MIN+1
      MAX=MAX+1
      DO 11 I=MIN,MAX
      ICOUNT=0
      DO 10 J=1,M
      IF(LZ(J).EQ.I-1) ICOUNT=ICOUNT+1
   10 CONTINUE
   11 JCOUNT(I)=ICOUNT
      WRITE(6,55)
      DO 15 I=MIN,MAX
```

```
      L=I-1
   15 WRITE(6,12)L,JCOUNT(I)
   12 FORMAT(I10,I15)
      WRITE(6,59) RMS,XMEAN,STDDEV
   59 FORMAT(1H1,3X,4HRMS=F10.3,3X,5HMEAN=F10.3,3X,15HSTD.
     1DEVIATION=F10.3)
      WRITE(6,56)
   56 FORMAT(1H0//,8X,1HX,7X,11HTHEO. FREQ.,4X,12HACTUAL
     1FREQ.)
      CHI=0
      DO 16 I=MIN,MAX
      L=I-1
      CHI=(((F(I)*AM)-FLOAT(JCOUNT(I)))**2/(F(I)*AM))+CHI
      KCOUNT=KCOUNT+1
      AMID(KCOUNT)=L
      YJ1(KCOUNT)=NF(I)
      YJ(KCOUNT)=JCOUNT(I)
   16 WRITE(6,17) L,NF(I),JCOUNT(I)
   17 FORMAT(I10,2I15)
C
C     PLOTS OF COMPUTED AND THEORETICAL HISTOGRAMS
C
      CALL DDIPLT(0,IN,KCOUNT,AMID,YJ1,BOTTOM,TOP,0,FREQM1,
     11,XXM,1,XMY,14,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,YJ,BOTTOM,TOP,0,FREQM1,1
     1,XXM,1,XMY,12,ITAPE)
C
C     DETERMINATION OF PEAK AND CUMULATIVE PEAK DISTRIBU-
C     TIONS FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      MAX=0
      MIN=0
      J=1
      I=0
  200 CONTINUE
      I=I+1
      IF(I.GT.M-1) GO TO 205
      IF(I.NE.1) GO TO 1036
  201 J=J+1
      IF(LZ(I).NE.LZ(J)) GO TO 202
      GO TO 201
  202 CONTINUE
      IF(LZ(I).GT.LZ(J)) GO TO 1037
      GO TO 1038
 1037 MAX=MAX+1
      LZMAX(MAX)=LZ(I)
      GO TO 1036
 1038 MIN=MIN+1
      LZMIN(MIN)=LZ(I)
 1036 CONTINUE
  204 J=J+1
      IF(LZ(I+1).NE.LZ(J)) GO TO 203
```

```
      GO TO 204
 203  IF(LZ(I+1).GT.LZ(J).AND.LZ(I+1).GT.LZ(I)) GO TO 1021
      IF(LZ(I+1).LT.LZ(J).AND.LZ(I+1).LT.LZ(I)) GO TO 1022
      GO TO 206
1021  MAX=MAX+1
      LZMAX(MAX)=LZ(I+1)
      GO TO 206
1022  MIN=MIN+1
      LZMIN(MIN)=LZ(I+1)
 206  I=J-2
      GO TO 200
 205  CONTINUE
      WRITE(6,1034)
1034  FORMAT(1H1,22X,16HPEAK VALUE COUNT)
      WRITE(6,1039) MAX,MIN
1039  FORMAT(1H0,10X,14HMAXIMUM PEAKS=I5,2X,14HMINIMUM
     1PEAKS=I5///4X,8HINTERVAL,3X,3HMAX,3X,3HMIN,4X,4HCMAX,
     24X,4HCMIN,2X,8HLOG CMAX,3X,8HLOG CMIN/)
      LUINTV=0
      ISUM=0
      JSUM=0
      KCOUNT=0
      LA=XLA
1030  LUINTV=LUINTV+1
      ICOUNT=0
      JKOUNT=0
      LUINTV=LUINTV-1
      LBINTV=LUINTV-1
      DO 1031 I=1,MAX
      IF(LZMAX(I).LE.LUINTV.AND.LZMAX(I).GT.LBINTV) ICOUNT=
     1ICOUNT+1
1031  CONTINUE
      IF(ISUM.EQ.0) GO TO 1050
      ISOM=MAX-ISUM
      GO TO 1051
1050  ISOM=MAX
1051  ISUM=ISUM+ICOUNT
      IF(ISOM.EQ.0) GO TO 500
      XLOGISM=ALOG10(FLOAT(ISOM))
      GO TO 510
 500  XLOGISM=0
 510  DO 1032 J=1,MIN
      IF(LZMIN(J).LE.LUINTV.AND.LZMIN(J).GT.LBINTV) JKOUNT=
     1JKOUNT+1
1032  CONTINUE
      IF(JSUM.EQ.0) GO TO 1052
      JSOM=MIN-JSUM
      GO TO 1053
1052  JSOM=MIN
1053  JSUM=JSUM+JKOUNT
      IF(JSOM.EQ.0) GO TO 501
      XLOGJSM=ALOG10(FLOAT(JSOM))
```

```
      GO TO 511
  501 XLOGJSM=0
      IF(ISOM.EQ.0.AND.JSOM.EQ.0) GO TO 1035
  511 WRITE(6,1033) LBINTV,LUINTV,ICOUNT,JKOUNT,ISOM,JSOM,X
     1LOGISM,XLOGJSM
 1033 FORMAT(4I6,2I8,F9.4,F11.4)
C
C     PLOTS OF PEAK AND LOG CUMULATIVE PEAK DISTRIBUTIONS
C     FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      KCOUNT=KCOUNT+1
      YLOGISM(KCOUNT)=XLOGISM
      YLOGJSM(KCOUNT)=XLOGJSM
      YBINTV(KCOUNT)=LBINTV
      CZMAX(KCOUNT)=ICOUNT
      CZMIN(KCOUNT)=JKOUNT
      AMID(KCOUNT)=LUINTV
      LUINTV=LUINTV+1
      GO TO 1030
 1035 CONTINUE
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGISM,BOTTOM,TOP,0,4
     1.,2,YAM,4,XAM,12,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGJSM,BOTTOM,TOP,0,4
     1.,2,YAM,4,ZAM,12,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,CZMAX,BOTTOM,TOP,0,FREQM
     13,1,XXM,3,YYM,12,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,CZMIN,BOTTOM,TOP,0,FREQM
     12,1,XXM,3,ZYM,12,ITAPE)
      GO TO 2
  991 STOP
      END
```

8.1.3 Weibull Distribution

```
C
C     RANDOM TIME HISTORY HAVING A WEIBULL PROBABILITY
C     DENSITY FUNCTION
C
      PROGRAM PLOT     (INPUT,OUTPUT,TAPE21,TAPE5=INPUT,
     1TAPE6=OUTPUT)
      DIMENSION Z(10000),ZMIN(5000),ZMAX(5000),XSDBOT(100)
      DIMENSION XMID(100),CZMAX(1000),CZMIN(1000),ZYM(3)
      DIMENSION AMID(100),YJ1(1000),YJ(1000),YLOGJSM(500)
      DIMENSION PCOUNT(100),RANDOM(100),YBINTV(500)
      DIMENSION XMY(1),XXM(1),IN(2),XM(2),YM(2),YYM(3)
      DIMENSION XRMS(100),XXMEAN(100),XSDTOP(100)
      DIMENSION XAM(4),YAM(2),ZAM(4),YLOGISM(500)
C
C     INPUT DATA AND CONSTANTS
C
      DATA XM/11HPOINT COUNT/
```

```
      DATA YM/13HRANDOM NUMBER/
      DATA IN/7HHAL SRD,10HWEIB.DIST./
      DATA XMY/9HFREQUENCY/
      DATA XXM/10HCLASS MARK/
      DATA YYM/24HMAXIMUM PEAK VALUE COUNT/
      DATA ZYM/24HMINIMUM PEAK VALUE COUNT/
      DATA XAM/40HLOG OF CUMULATIVE FREQUENCY FOR MAXIMUMS/
      DATA YAM/17HLOWER CLASS LIMIT/
      DATA ZAM/40HLOG OF CUMULATIVE FREQUENCY FOR MINIMUMS/
      ITAPE=6LTAPE21
C
C     INPUT VARIABLES DEFINED
C
C     XINT=CLASS INTERVAL
C     ALPHA=PARAMETER OF WEIBULL DISTRIBUTION
C     BETA=PARAMETER OF WEIBULL DISTRIBUTION
C     ULIMIT=MAXIMUM VALUE OF RANDOM VARIABLE,Z
C     M=NUMBER OF RANDOM NUMBERS TO BE GENERATED
C     FREQM1=LARGEST PEAK VALUE (MINIMUM)
C     FREQM2=LARGEST PEAK VALUE (MAXIMUM)
C
  600 READ(5,601) XINT,ALPHA,BETA,ULIMIT,M,FREQM1,FREQM2
  601 FORMAT(4F10.3,I10,2F10.3)
      IF(EOF,5) 602,603
  603 CONTINUE
      X2=0
      SUMN=0
      M=M+1
      X=34521637721.
      CHI=0
      AM=M
      WRITE(6,1) ALPHA,BETA
    1 FORMAT(31H1WEIBULL DISTRIBUTION,    ALPHA=F10.6,8H
     1BETA=F10.6///)
C
C     GENERATION OF RANDOM NUMBERS
C
      DO 2 I=1,M
      Y=RANF(X)
      X=0.0
      RWEIB=XWEIB(Y,ALPHA,BETA)
      Z(I)=RWEIB
      SUMN=SUMN+Z(I)
    2 X2=X2+Z(I)**2
      XMEAN=SUMN/AM
      RMS=SQRT(X2/AM)
      XMEANSQ=X2/AM
      STDDEV=SQRT(XMEANSQ-XMEAN**2)
      WRITE(6,400) RMS,XMEAN,STDDEV
  400 FORMAT(1H1,3X,4HRMS=F8.4,4X,5HMEAN=F8.4,3X,15HSTD. DE
     1VIATION=F8.4)
```

```
C
C     TIME HISTORY PLOT OF FIRST 100 RANDOM NUMBERS
C
      BLIMIT=0
      TOP=ULIMIT
      BOTTOM=BLIMIT
      DO 700 I=1,100
      XSDTOP(I)=XMEAN+STDDEV
      XSDBOT(I)=XMEAN-STDDEV
      XRMS(I)=RMS
      XXMEAN(I)=XMEAN
      PCOUNT(I)=I
  700 RANDOM(I)=Z(I)
      CALL DDIPLT(0,IN,100,PCOUNT,XRMS,1.0,100.,BOTTOM,TOP,
     12,XM,2,YM,13,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XXMEAN,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XSDTOP,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XSDBOT,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(1,IN,100,PCOUNT,RANDOM,1.0,100.,BOTTOM,
     1TOP,2,XM,2,YM,14,ITAPE)
C
C     CALCULATION AND COMPARISON OF COMPUTED AND
C     THEORETICAL HISTOGRAMS
C
      KCOUNT=0
      J=0
      K=0
      AMDPT=XINT/2.
      WRITE(6,210)
  210 FORMAT(1H0//,4X,10HCLASS MARK,3X,11HTHEO. FREQ.,2X,12
     1HACTUAL FREQ.,7X,4HF(X)/)
      GO TO 7
    6 BLIMIT=BLIMIT+XINT
      IF(BLIMIT.GT.ULIMIT) GO TO 8
      AMDPT=AMDPT+XINT
      J=0
    7 CONTINUE
      DO 4 I=1,M
      IF(Z(I).GT.BLIMIT.AND.Z(I).LT.(BLIMIT+XINT)) GO TO 5
      GO TO 4
    5 J=J+1
    4 CONTINUE
      ALPHA1=ALPHA-1.
      FX=ALPHA*BETA*(AMDPT**ALPHA1)*EXP(-BETA*(AMDPT**ALPHA
     1))
      AJ1=FX*XINT*AM
      IF(AJ1.EQ.0) GO TO 8
      AJ=J
      J1=AJ1
```

```
      CHI=((AJ-AJ1)**2)/AJ1+CHI
      WRITE(6,11) AMDPT,J1,J,FX
   11 FORMAT(F12.4,4X,I10,I15,F16.6)
C
C     PLOTS OF COMPUTED AND THEORETICAL HISTOGRAMS
C
      K=K+J
      KCOUNT=KCOUNT+1
      AMID(KCOUNT)=AMDPT
      YJ1(KCOUNT)=J1
      YJ(KCOUNT)=J
      GO TO 6
    8 CONTINUE
      CALL DDIPLT(0,IN,KCOUNT,AMID,YJ1,BOTTOM,TOP,0,FREQM1,
     11,XXM,1,XMY,14,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,YJ,BOTTOM,TOP,0,FREQM1,1
     1,XXM,1,XMY,1,ITAPE)
      WRITE(6,10)
   10 FORMAT(1H ///)
      N=M-K
      WRITE(6,10)
C
C     DETERMINATION OF PEAK AND CUMULATIVE PEAK DISTRIBU-
C     TIONS FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      MAX=0
      MIN=0
      M=M-2
      DO 20 I=1,M
      IF(I.NE.1) GO TO 36
      IF(Z(I).GT.Z(I+1)) GO TO 37
      GO TO 38
   37 MAX=MAX+1
      ZMAX(MAX)=Z(I)
      GO TO 36
   38 MIN=MIN+1
      ZMIN(MIN)=Z(I)
   36 CONTINUE
      IF(Z(I+1).GT.Z(I).AND.Z(I+1).GT.Z(I+2)) GO TO 21
      IF(Z(I+1).LT.Z(I).AND.Z(I+1).LT.Z(I+2)) GO TO 22
      GO TO 20
   21 MAX=MAX+1
      ZMAX(MAX)=Z(I+1)
      GO TO 20
   22 MIN=MIN+1
      ZMIN(MIN)=Z(I+1)
   20 CONTINUE
      WRITE(6,34)
   34 FORMAT(1H1,22X,16HPEAK VALUE COUNT)
      WRITE(6,39) MAX,MIN
   39 FORMAT(1H0,10X,14HMAXIMUM PEAKS=I5,2X,14HMINIMUM PEAK
     1S=I5///4X,8HINTERVAL,3X,3HMAX,3X,3HMIN,4X,4HCMAX,4X,4
     2HCMIN,2X,8HLOG CMAX,3X,8HLOG CMIN/)
```

```
      ISUM=0
      JSUM=0
      UINTV=0
      KCOUNT=0
   30 UINTV=UINTV+XINT
      ICOUNT=0
      JCOUNT=0
      BINTV=UINTV-XINT
      DO 31 I=1,MAX
      IF(ZMAX(I).LE.UINTV.AND.ZMAX(I).GT.BINTV) ICOUNT=ICOU
     1NT+1
   31 CONTINUE
      IF(ISUM.EQ.0) GO TO 50
      ISOM=MAX-ISUM
      GO TO 51
   50 ISOM=MAX
   51 ISUM=ISUM+ICOUNT
      IF(ISOM.EQ.0) GO TO 500
      XLOGISM=ALOG10(FLOAT(ISOM))
      GO TO 510
  500 XLOGISM=0
  510 DO 32 J=1,MIN
      IF(ZMIN(J).LE.UINTV.AND.ZMIN(J).GT.BINTV) JCOUNT=JCOU
     1NT+1
   32 CONTINUE
      IF(JSUM.EQ.0) GO TO 52
      JSOM=MIN-JSUM
      GO TO 53
   52 JSOM=MIN
   53 JSUM=JSUM+JCOUNT
      IF(JSOM.EQ.0) GO TO 501
      XLOGJSM=ALOG10(FLOAT(JSOM))
      GO TO 511
  501 XLOGJSM=0
      IF(ISOM.EQ.0.AND.JSOM.EQ.0) GO TO 35
  511 WRITE(6,33) BINTV,UINTV,ICOUNT,JCOUNT,ISOM,JSOM,XLOGI
     1SM,XLOGJSM
   33 FORMAT(F6.2,2H -,F4.2,2I6,2I8,2F11.4)
C
C     PLOTS OF PEAK AND LOG CUMULATIVE PEAK DISTRIBUTIONS
C     FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      KCOUNT=KCOUNT+1
      YLOGISM(KCOUNT)=XLOGISM
      YLOGJSM(KCOUNT)=XLOGJSM
      YBINTV(KCOUNT)=BINTV
      IF(KCOUNT.EQ.1) XMID(KCOUNT)=XINT/2.
      IF(KCOUNT.GT.1) XMID(KCOUNT)=XMID(KCOUNT-1)+XINT
      CZMAX(KCOUNT)=ICOUNT
      CZMIN(KCOUNT)=JCOUNT
      GO TO 30
   35 CONTINUE
```

```
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGISM,BOTTOM,TOP,0,4
     1.,2,YAM,4,XAM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGJSM,BOTTOM,TOP,0,4
     1.,2,YAM,4,ZAM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,XMID,CZMAX,BOTTOM,TOP,0,FREQM
     12,1,XXM,3,YYM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,XMID,CZMIN,BOTTOM,TOP,0,FREQM
     12,1,XXM,3,ZYM,1,ITAPE)
      GO TO 600
  602 STOP
      END
```

8.1.4 Exponential Distribution

```
C
C     RANDOM TIME HISTORY HAVING AN EXPONENTIAL
C     PROBABILITY DENSITY DISTRIBUTION
C
      PROGRAM PLOT     (INPUT,OUTPUT,TAPE21,TAPE5=INPUT,
     1TAPE6=OUTPUT)
      DIMENSION Z(10000),ZMIN(5000),ZMAX(5000),XSDBOT(100)
      DIMENSION XMID(500),CZMAX(1000),CZMIN(1000),ZYM(3)
      DIMENSION AMID(500),YJ1(1000),YJ(1000),YLOGJSM(500)
      DIMENSION PCOUNT(100),RANDOM(100),YBINTV(500)
      DIMENSION XMY(1),XXM(1),IN(2),XM(2),YM(2),YYM(3)
      DIMENSION XRMS(100),XXMEAN(100),XSDTOP(100)
      DIMENSION XAM(4),YAM(2),ZAM(4),YLOGISM(500)
C
C     INPUT DATA AND CONSTANTS
C
      DATA XM/11HPOINT COUNT/
      DATA YM/13HRANDOM NUMBER/
      DATA IN/7HHAL SRD,10HEXP. DIST./
      DATA XMY/9HFREQUENCY/
      DATA XXM/10HCLASS MARK/
      DATA YYM/24HMAXIMUM PEAK VALUE COUNT/
      DATA ZYM/24HMINIMUM PEAK VALUE COUNT/
      DATA XAM/40HLOG OF CUMULATIVE FREQUENCY FOR MAXIMUMS/
      DATA YAM/17HLOWER CLASS LIMIT/
      DATA ZAM/40HLOG OF CUMULATIVE FREQUENCY FOR MINIMUMS/
      ITAPE=6LTAPE21
C
C     INPUT VARIABLES DEFINED
C
C     M=NUMBER OF  [illegible]DOM N[illegible]MBERS TO BE GENERATED
C     THETA=MEAN   [illegible]ANDOM [illegible]ARIABLE
C     FREQM1=MAX[illegible]  [illegible]ALUE OF THEORETICAL AND ACTUAL
C    1FREQUENCY    [illegible]CCURRENCE
C     FREQM2=LA[illegible]  [illegible]ST PEAK VALUE (MINIMUM)
C     XINT=CLAS[illegible] INTERVAL
C     ULIMIT=UPPER LIMIT OF RANDOM VARIABLE,Z
```

```
C     FREQM3=LARGEST PEAK VALUE (MAXIMUM)
C
  600 READ(5,601) M,THETA,FREQM1,FREQM2,XINT,ULIMIT,FREQM3
  601 FORMAT(I10,F10.3,2F10.0,3F10.3)
      IF(EOF,5) 602,603
  603 CONTINUE
      X2=0
      SUMN=0
      M=M+1
      X=34521637721.
      CHI=0
      AM=M
      WRITE(6,1) THETA
    1 FORMAT(61H1EXPONENTIAL DISTRIBUTION,  F(X)=THETA*EXP(
     1-THETA*X)    THETA=F10.6///)
C
C     GENERATION OF RANDOM NUMBERS
C
      DO 2 I=1,M
      Y=RANF(X)
      X=0.0
      REXP=XEXP(Y,THETA)
      Z(I)=REXP
      SUMN=SUMN+Z(I)
    2 X2=X2+Z(I)**2
      XMEAN=SUMN/AM
      RMS=SQRT(X2/AM)
      XMEANSQ=X2/AM
      STDDEV=SQRT(XMEANSQ-XMEAN**2)
      WRITE(6,400) RMS,XMEAN,STDDEV
  400 FORMAT(1H1,3X,4HRMS=F8.4,4X,5HMEAN=F8.4,3X,15HSTD. DE
     1VIATION=F8.4)
C
C     TIME HISTORY PLOT OF FIRST 100 RANDOM NUMBERS
C
      J=0
      K=0
      AMDPT=XINT/2.
      BLIMIT=0
      TOP=ULIMIT
      BOTTOM=BLIMIT
      DO 500 I=1,100
      XSDTOP(I)=XMEAN+STDDEV
      XSDBOT(I)=XMEAN-STDDEV
      XRMS(I)=RMS
      XXMEAN(I)=XMEAN
      PCOUNT(I)=I
  500 RANDOM(I)=Z(I)
      CALL DDIPLT(0,IN,100,PCOUNT,XRMS,1.0,100.,BOTTOM,TOP,
     12,XM,2,YM,13,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XXMEAN,1.0,100.,BOTTOM,TO
     1P,2,XM,2,YM,14,ITAPE)
```

```
      CALL DDIPLT(0,IN,100,PCOUNT,XSDTOP,1.0,100.,BOTTOM,TO
     1P,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XSDBOT,1.0,100.,BOTTOM,TO
     1P,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(1,IN,100,PCOUNT,RANDOM,1.0,100.,BOTTOM,TO
     1P,2,XM,2,YM,14,ITAPE)
C
C     CALCULATION AND COMPARISON OF COMPUTED AND
C     THEORETICAL HISTOGRAMS
C
      WRITE(6,210)
  210 FORMAT(1HO//,4X,10HCLASS MARK,3X,11HTHEO. FREQ.,2X,12
     1HACTUAL FREQ.,7X,4HF(X)/)
      KCOUNT=0
      GO TO 7
    6 BLIMIT=BLIMIT+XINT
      IF(BLIMIT.GT.ULIMIT) GO TO 8
      AMDPT=AMDPT+XINT
      J=0
    7 CONTINUE
      DO 4 I=1,M
      IF(Z(I).GT.BLIMIT.AND.Z(I).LT.(BLIMIT+XINT)) GO TO 5
      GO TO 4
    5 J=J+1
    4 CONTINUE
      FX=THETA*EXP(-AMDPT*THETA)
      AJ1=FX*XINT*AM
      IF(AJ1.EQ.0) GO TO 8
      AJ=J
      J1=AJ1
      CHI=((AJ-AJ1)**2)/AJ1+CHI
      WRITE(6,11) AMDPT,J1,J,FX
   11 FORMAT(F12.4,4X,I10,I15,F16.6)
C
C     PLOTS OF COMPUTED AND THEORETICAL HISTOGRAMS
C
      K=K+J
      KCOUNT=KCOUNT+1
      AMID(KCOUNT)=AMDPT
      YJ1(KCOUNT)=J1
      YJ(KCOUNT)=J
      GO TO 6
    8 CONTINUE
      CALL DDIPLT(0,IN,KCOUNT,AMID,YJ1,BOTTOM,TOP,0,FREQM1,
     11,XXM,1,XMY,14,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,YJ,BOTTOM,TOP,0,FREQM1,1
     1,XXM,1,XMY,1,ITAPE)
      WRITE(6,10)
   10 FORMAT(1H ///)
      N=M-K
      WRITE(6,10)
C
```

```
C     DETERMINATION OF PEAK AND CUMULATIVE PEAK DISTRI-
C     BUTIONS FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      MAX=0
      MIN=0
      M=M-2
      DO 20 I=1,M
      IF(I.NE.1) GO TO 36
      IF(Z(I).GT.Z(I+1)) GO TO 37
      GO TO 38
   37 MAX=MAX+1
      ZMAX(MAX)=Z(I)
      GO TO 36
   38 MIN=MIN+1
      ZMIN(MIN)=Z(I)
   36 CONTINUE
      IF(Z(I+1).GT.Z(I).AND.Z(I+1).GT.Z(I+2)) GO TO 21
      IF(Z(I+1).LT.Z(I).AND.Z(I+1).LT.Z(I+2)) GO TO 22
      GO TO 20
   21 MAX=MAX+1
      ZMAX(MAX)=Z(I+1)
      GO TO 20
   22 MIN=MIN+1
      ZMIN(MIN)=Z(I+1)
   20 CONTINUE
      WRITE(6,34)
   34 FORMAT(1H1,22X,16HPEAK VALUE COUNT)
      WRITE(6,39) MAX,MIN
   39 FORMAT(1H0,10X,14HMAXIMUM PEAKS=I5,2X,14HMINIMUM PEAK
     1S=I5///4X,8HINTERVAL,3X,3HMAX,3X,3HMIN,4X,4HCMAX,4X,4
     2HCMIN,2X,8HLOG CMAX,3X,8HLOG CMIN/)
      ISUM=0
      JSUM=0
      KCOUNT=0
      UINTV=0
   30 UINTV=UINTV+XINT
      ICOUNT=0
      JCOUNT=0
      BINTV=UINTV-XINT
      DO 31 I=1,MAX
      IF(ZMAX(I).LE.UINTV.AND.ZMAX(I).GT.BINTV) ICOUNT=ICOU
     1NT+1
   31 CONTINUE
      IF(ISUM.EQ.0) GO TO 50
      ISOM=MAX-ISUM
      GO TO 51
   50 ISOM=MAX
   51 ISUM=ISUM+ICOUNT
      IF(ISOM.EQ.0) GO TO 503
      XLOGISM=ALOG10(FLOAT(ISOM))
      GO TO 510
  503 XLOGISM=0
```

```
  510 DO 32 J=1,MIN
      IF(ZMIN(J).LE.UINTV.AND.ZMIN(J).GT.BINTV) JCOUNT=JCOU
     1NT+1
   32 CONTINUE
      IF(JSUM.EQ.0) GO TO 52
      JSOM=MIN-JSUM
      GO TO 53
   52 JSOM=MIN
   53 JSUM=JSUM+JCOUNT
      IF(JSOM.EQ.0) GO TO 501
      XLOGJSM=ALOG10(FLOAT(JSOM))
      GO TO 511
  501 XLOGJSM=0.
      IF(ISOM.EQ.0.AND.JSOM.EQ.0) GO TO 35
  511 WRITE(6,33) BINTV,UINTV,ICOUNT,JCOUNT,ISOM,JSOM,XLOGI
     1SM,XLOGJSM
   33 FORMAT(2F6.1,2I6,2I8,F9.4,F11.4)
C
C     PLOTS OF PEAK AND LOG CUMULATIVE PEAK DISTRIBUTIONS
C     FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      KCOUNT=KCOUNT+1
      YLOGISM(KCOUNT)=XLOGISM
      YLOGJSM(KCOUNT)=XLOGJSM
      YBINTV(KCOUNT)=BINTV
      IF(KCOUNT.EQ.1) XMID(KCOUNT)=XINT/2.
      IF(KCOUNT.GT.1) XMID(KCOUNT)=XMID(KCOUNT-1)+XINT
      CZMAX(KCOUNT)=ICOUNT
      CZMIN(KCOUNT)=JCOUNT
      GO TO 30
   35 CONTINUE
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGISM,BOTTOM,TOP,0,4
     11.,2,YAM,4,XAM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGJSM,BOTTOM,TOP,0,4
     11.,2,YAM,4,ZAM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,XMID,CZMAX,BOTTOM,TOP,0,FREQM
     13,1,XXM,3,YYM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,XMID,CZMIN,BOTTOM,TOP,0,FREQM
     12,1,XXM,3,ZYM,1,ITAPE)
      GO TO 600
  602 STOP
      END
```

8.1.5 Log-Normal Distribution

```
C
C     RANDOM TIME HISTORY HAVING A LOG NORMAL PROBABILITY
C     DENSITY DISTRIBUTION
C
      PROGRAM PLOT     (INPUT,OUTPUT,TAPE21,TAPE5=INPUT,
     1TAPE6=OUTPUT)
```

```
      DIMENSION Z(10000),W(10000),ZMAX(5000),ZMIN(5000)
      DIMENSION CZMAX(1000),CZMIN(1000),ZYM(3),XXX(2)
      DIMENSION AMID(500),YJ1(1000),YJ(1000),BMID(500)
      DIMENSION PCOUNT(100),RANDOM(100),YBINTV(500)
      DIMENSION XMY(1),XXM(1),IN(2),XM(2),YM(2),YYM(3),
      DIMENSION XRMS(100),XXMEAN(100),XSDTOP(100),
     1XSDBOT(100)
      DIMENSION XAM(4),YAM(2),ZAM(4),YLOGISM(500),
     1YLOGJSM(500)
C
C     INPUT DATA AND CONSTANTS
C
      DATA XM/11HPOINT COUNT/
      DATA YM/13HRANDOM NUMBER/
      DATA IN/7HHAL SRD,10HLOG NORMAL/
      DATA XMY/9HFREQUENCY/
      DATA XXX/17HLOG OF CLASS MARK/
      DATA XXM/10HCLASS MARK/
      DATA YYM/24HMAXIMUM PEAK VALUE COUNT/
      DATA ZYM/24HMINIMUM PEAK VALUE COUNT/
      DATA XAM/40HLOG OF NUMBER OF EXCEEDANCES....MAXIMUMS/
      DATA YAM/17HLOWER CLASS LIMIT/
      DATA ZAM/40HLOG OF NUMBER OF EXCEEDANCES....MINIMUMS/
      ITAPE=6LTAPE21
C
C     INPUT VARIABLES DEFINED
C
C     M=   BER OF RANDOM NUMBERS TO BE GENERATED
C     XMU  EAN OF LOGARITHMS OF RANDOM VARIABLE,Z
C     SIGMAS=VARIANCE OF LOGARITHMS OF RANDOM VARIABLE,Z
C     XINT=CLASS INTERVAL OF LOGARITHMS OF RANDOM VARIABLE,
C    1Z
C     YINT=CLASS INTERVAL OF PEAKS
C     TOP=UPPER LIMIT OF RANDOM VARIABLE,Z
C     BOTTOM=LOWER LIMIT OF RANDOM VARIABLE,Z
C     BLIMIT=LOGARITHM OF BOTTOM
C     ULIMIT=LOGARITHM OF TOP
C     FREQM1=MAXIMUM VALUE OF THEORETICAL AND ACTUAL FREQUE
C    1NCY OF OCCURRENCE
C     FREQM2=LARGEST PEAK VALUE (MINIMUM)
C     FREQM3=LARGEST PEAK VALUE (MAXIMUM)
C
   41 READ(5,40) M,XMU,SIGMAS,XINT,YINT,BLIMIT,ULIMIT,BOTTO
     1M,TOP,FREQM1,FREQM2,FREQM3
   40 FORMAT(I10,2F10.6,2F5.2,4F10.6/3F10.6)
      IF(EOF,5) 991,990
  990 CONTINUE
      PI=3.14159265
      AM=M
      M=M+1
      X=34521637721.
      BOTTUM=BLIMIT
```

```
      TIP=ULIMIT
      SIGMA=SQRT(SIGMAS)
      WRITE(6,2) XMU,SIGMAS,SIGMA
    2 FORMAT(89H1LOG NORMAL DISTRIBUTION,  F(X)=(1/(SIGMA*S
     1QRT(2*PI)))*EXP(-(LN(X)-MEAN)**2/(2*SIGMA**2))//6H ME
     2AN=F5.2,16H    SIGMA SQUARE=F5.2,9H    SIGMA=F5.2///)
C
C     GENERATION OF RANDOM NUMBERS
C
      SUMN=0
      X2=0
      DO 1 I=1,M
      Y=RANF(X)
      X=0.0
      RLOGNL=XLOGNL(Y,XMU,SIGMAS)
      Z(I)=RLOGNL
      W(I)=ALOG(Z(I))
      SUMN=SUMN+Z(I)
      X2=X2+Z(I)**2
    1 CONTINUE
      XMEAN=SUMN/AM
      RMS=SQRT(X2/AM)
      XMEANSQ=X2/AM
      STDDEV=SQRT(XMEANSQ-XMEAN**2)
      WRITE(6,400) RMS,XMEAN,STDDEV
  400 FORMAT(1H1,3X,4HRMS=F8.4,4X,5HMEAN=F8.4,3X,15HSTD. DE
     1VIATION=F8.4)
C
C     TIME HISTORY PLOT OF FIRST 100 RANDOM NUMBERS
C
      J=0
      K=0
      DO 700 I=1,100
      XSDTOP(I)=XMEAN+STDDEV
      XSDBOT(I)=XMEAN-STDDEV
      XRMS(I)=RMS
      XXMEAN(I)=XMEAN
      PCOUNT(I)=I
  700 RANDOM(I)=Z(I)
      CALL DDIPLT(0,IN,100,PCOUNT,XRMS,1.0,100.,BOTTOM,TOP,
     12,XM,2,YM,13,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XXMEAN,1.0,100.,BOTTOM,TO
     1P,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XSDTOP,1.0,100.,BOTTOM,TO
     1P,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(0,IN,100,PCOUNT,XSDBOT,1.0,100.,BOTTOM,TO
     1P,2,XM,2,YM,14,ITAPE)
      CALL DDIPLT(1,IN,100,PCOUNT,RANDOM,1.0,100.,BOTTOM,TO
     1P,2,XM,2,YM,14,ITAPE)
C
C     CALCULATION AND COMPARISON OF COMPUTED AND
C     THEORETICAL HISTOGRAMS
```

```
C
      WRITE(6,210)
  210 FORMAT(1H0//,4X,7HMID-PT.,2X,11HACTUAL FREQ,3X,24HLOG
     1MID-PT.   THEO. FREQ/)
      KCOUNT=0
      GO TO 7
    6 BLIMIT=BLIMIT+XINT
      IF(BLIMIT.GT.ULIMIT) GO TO 8
      J=0
    7 CONTINUE
      DO 4 I=1,M
      IF(W(I).GT.BLIMIT.AND.W(I).LT.(BLIMIT+.1)) GO TO 5
      GO TO 4
    5 J=J+1
    4 CONTINUE
      PTLOG=BLIMIT+XINT/2.
      PT=EXP(PTLOG)
      FX=(EXP(-(PTLOG-XMU)**2/(2.*SIGMAS)))/SQRT(2.*PI*SIGM
     1AS)
      J1=FX*XINT*AM
      WRITE(6,11) PT,J,PTLOG,J1
   11 FORMAT(F10.4,I11,F14.2,I13)
C
C     PLOTS OF COMPUTED AND THEORETICAL HISTOGRAMS
C
      K=K+J
      KCOUNT=KCOUNT+1
      AMID(KCOUNT)=PT
      BMID(KCOUNT)=PTLOG
      YJ1(KCOUNT)=J1
      YJ(KCOUNT)=J
      GO TO 6
    8 CONTINUE
      CALL DDIPLT(0,IN,KCOUNT,AMID,YJ1,BOTTOM,TOP,0,FREQM1,
     11,XXM,1,XMY,14,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,YJ,BOTTOM,TOP,0,FREQM1,1
     1,XXM,1,XMY,1,ITAPE)
      CALL DDIPLT(0,IN,KCOUNT,BMID,YJ1,BOTTUM,TIP,0,FREQM1,
     12,XXX,1,XMY,14,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,BMID,YJ,BOTTUM,TIP,0,FREQM1,2
     1,XXX,1,XMY,1,ITAPE)
      N=M-K
      WRITE(6,9) N
    9 FORMAT(1H1,2X,2HN=I5)
C
C     DETERMINATION OF PEAK AND CUMULATIVE PEAK DISTRIBU-
C     TIONS FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      MAX=0
      MIN=0
      M=M-2
      DO 20 I=1,M
```

```
      IF(I.NE.1) GO TO 36
      IF(Z(I).GT.Z(I+1)) GO TO 37
      GO TO 38
   37 MAX=MAX+1
      ZMAX(MAX)=Z(I)
      GO TO 36
   38 MIN=MIN+1
      ZMIN(MIN)=Z(I)
   36 CONTINUE
      IF(Z(I+1).GT.Z(I).AND.Z(I+1).GT.Z(I+2)) GO TO 21
      IF(Z(I+1).LT.Z(I).AND.Z(I+1).LT.Z(I+2)) GO TO 22
      GO TO 20
   21 MAX=MAX+1
      ZMAX(MAX)=Z(I+1)
      GO TO 20
   22 MIN=MIN+1
      ZMIN(MIN)=Z(I+1)
   20 CONTINUE
      WRITE(6,34)
   34 FORMAT(1H1,22X,16HPEAK VALUE COUNT)
      WRITE(6,39) MAX,MIN
   39 FORMAT(1H0,10X,14HMAXIMUM PEAKS=I5,2X,14HMINIMUM PEAK
     1S=I5///4X,8HINTERVAL,3X,3HMAX,3X,3HMIN,4X,4HCMAX,4X,4
     2HCMIN,2X,8HLOG CMAX,3X,8HLOG CMIN/)
C **  COUNTS PEAK VALUES **
      UINTV=0
      ISUM=0
      JSUM=0
      KCOUNT=0
   30 UINTV=UINTV+YINT
      ICOUNT=0
      JCOUNT=0
      BINTV=UINTV-YINT
      PTLOG=BINTV+YINT/2.
      DO 31 I=1,MAX
      IF(ZMAX(I).LE.UINTV.AND.ZMAX(I).GT.BINTV) ICOUNT=ICOU
     1NT+1
   31 CONTINUE
      IF(ISUM.EQ.0) GO TO 50
      ISOM=MAX-ISUM
      GO TO 51
   50 ISOM=MAX
   51 ISUM=ISUM+ICOUNT
      IF(ISOM.EQ.0) GO TO 500
      XLOGISM=ALOG10(FLOAT(ISOM))
      GO TO 510
  500 XLOGISM=0
  510 DO 32 J=1,MIN
      IF(ZMIN(J).LE.UINTV.AND.ZMIN(J).GT.BINTV) JCOUNT=JCOU
     1NT+1
   32 CONTINUE
      IF(JSUM.EQ.0) GO TO 52
```

```
      JSOM=MIN-JSUM
      GO TO 53
   52 JSOM=MIN
   53 JSUM=JSUM+JCOUNT
      IF(JSOM.EQ.0) GO TO 501
      XLOGJSM=ALOG10(FLOAT(JSOM))
      GO TO 511
  501 XLOGJSM=0
      IF(ISOM.EQ.0.AND.JSOM.EQ.0) GO TO 35
  511 WRITE(6,33) BINTV,UINTV,ICOUNT,JCOUNT,ISOM,JSOM,XLOGI
     1SM,XLOGJSM
   33 FORMAT(2F6.1,2I6,2I8,F9.4,F11.4)
C
C     PLOTS OF PEAK AND LOG CUMULATIVE PEAK DISTRIBUTIONS
C     FOR BOTH MAXIMUM AND MINIMUM PEAK VALUES
C
      KCOUNT=KCOUNT+1
      YLOGISM(KCOUNT)=XLOGISM
      YLOGJSM(KCOUNT)=XLOGJSM
      YBINTV(KCOUNT)=BINTV
      CZMAX(KCOUNT)=ICOUNT
      CZMIN(KCOUNT)=JCOUNT
      AMID(KCOUNT)=PTLOG
      GO TO 30
   35 CONTINUE
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGISM,BOTTOM,TOP,0,4
     1.,2,YAM,4,XAM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,YBINTV,YLOGJSM,BOTTOM,TOP,0,4
     1.,2,YAM,4,ZAM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,CZMAX,BOTTOM,TOP,0,FREQM
     13,1,XXM,3,YYM,1,ITAPE)
      CALL DDIPLT(1,IN,KCOUNT,AMID,CZMIN,BOTTOM,TOP,0,FREQM
     12,1,XXM,3,ZYM,1,ITAPE)
      GO TO 41
  991 STOP
      END
```

8.2 FORTRAN Program Subroutines

8.2.1 Library Subroutines

8.2.1.1 Random Number Generator RANF.

```
          IDENT   RANF
PROGRAM LENGTH
BLOCKS
PROGRAM*    LOCAL
ENTRY POINTS
   000001 RANF
          ENTRY   RANF
+         VFD 42/CHRANF,18/1
RANF      DATA    0
          SA2     RANNO                 .GET LAST RANDOM NUMBER.
          SA1     B1                    .GET ARG.
          BX6     X2
          ZR      X1,RANDOM             .ARG=0, MAKE NEW RANDOM NUMBER.
          NG      X1,RANF               .ARG.LT.0, REPORT LAST RANNO.
          SB2     -60B                  .ARG.GT.0, START NEW SEQUENCE.
          PX6     B2,X1                 .ENSURE GIVEN NUMBER HAS EXP
          MX2     59                      .OF 1717
          BX6     -X2+X6                  .AND IS ODD.
          SA6     A2
          ZR      B0,RANF               .RETURN.
RANDOM    SA1     RANMLT                .FORM NEW RANDOM NUMBER
          DX6     X1*X2                   .BY FORMING LOW-ORDER
          SA6     A2                      .PRODUCT BITS.
          NX6     B0,X6
          ZR      B0,RANF               .RETURN.
REL       EQU     **1+1
RANNO     DATA    17171274321477413155B
RANMLT    DATA    20000000000005536454B
          END
UNUSED STORAGE          24 STATEMENTS          5 SYMBOLS
```

8.2.1.2 Plotting Subroutine DDIPLT

```
      SUBROUTINE DDIPLT (IEC,IN,N,X,Y,XMIN,XMAX,YMIN,YMAX,
     1                  NXM,XM,NYM,YM,ISYM,DDITPE)
C       **********
C         IEC  DISPLAY END CODE
C              0     DATA INCOMPLETE
C              1     DISPLAY COMPLETE
C         IN   PROGRAMMER INITIALS AND JOB IDENTIFICATION (
C    1         2 WORD ARRAY)
C         N    NUMBER OF DATA POINTS TO BE PLOTTED
C         X    ARRAY CONTAINING X COORDINATES
C         Y    ARRAY CONTAINING Y COORDINATES
C         XMIN ****
C         XMAX    *** DATA RANGE TO BE PLOTTED. IF VALUES
C         YMIN    *** ARE ZERO THE RANGE WILL BE DETERMINED
C                 *** ON FIRST CURVE OF DISPLAY
C         YMAX ****
C         NXM  NUMBER OF WORDS IN HORIZONTAL MESSAGE ARRAY
C         XM   HORIZONTAL MESSAGE
C         NYM  NUMBER OF WORDS IN VEFTICAL MESSAGE ARRAY
C         YM   VERTICAL MESSAGE
C         ISYM SYMBOL CODE
C         DDITPE TAPE NAME IN PROGRAM CARD
C
      DIMENSION IN(2),ID(3),INIT(9),PLOT(100),IPLOT(100)
      DIMENSION X(N), Y(N), ISTAB(14), SM(3)
      DATA (ISTAB(I),I=1,14)/07620365533757575757575,
     1                       07620365534757575757575,
     2                       07620365520757575757575,
     3                       07620365553757575757575,
     4                       07620365554757575757575,
     5                       07620365540757575757575,
     6                       07620365561757575757575,
     7                       07620365515757575757575,
     8                       07620365516757575757575,
     9                       07620365513757575757575,
     1                       07620365514757575757575,
     2                       07620365532757575757575,
     3                       07620365552757575757575,
     4                       07660557575757575757575/
      DATA (SM(I),I=1,3)/07775757575757575757575,075757575757
     15757575757,07575757575757575757575/
      DIMENSION XGRID(14), YGRID(14),IXGRID(14),IYGRID(14)
      DIMENSION XSC(14),IXPOS(14),YSC(14),IXSC(14),IYSC(14)
      EQUIVALENCE (PLOT,IPLOT)
      EQUIVALENCE (XGRID,IXGRID),(YGRID,IYGRID)
      EQUIVALENCE  (XSC,IXSC),(YSC,IYSC)
      INTEGER DDITPE
      DATA NPAGE/0/,IEND/1/,NCAM/07600227575757575757575/
      DATA (ID(I),I=1,3)/07600227654123717377 5,00,075757575
     17575757600 44/
```

```
      DATA (INIT(I),I=1,9)/07640341337377575757575,00,0764034
     1133717757575757 5,00,07575757575757575757540,00,0764034133
     2617757575757 5,00,0757575757575757575760044/
C
C     TRANSFER ON FIRST ENTRY
C
      IF (INIT(2).EQ.0) GO TO 100
C
C     TRANSFER FOR START OF NEW DISPLAY (IEND=1) OR SET
C    1ICODE = 1
C
      IF (IEND.EQ.1) GO TO 110
      K=1
      GO TO 140
C
C     SEND INITIAL IDENTIFICATION FRAME
C
  100 CALL DISBCD (IN(1),INIT(2),1)
      ID(2)=INIT(2)
      WRITE (DDITPE) ID
C
C     INITIALIZE FOR A DISPLAY
C
  110 NPAGE=NPAGE+1
      LPCT=0
      IPLOT(1) = NCAM
      K=2
      CALL DISBCD (IN(2),INIT(4),1)
C
C     CHECK FOR X MAX-MIN VALUES
C
      IF (XMIN.EQ.0.0.AND.XMAX.EQ.0.0) GO TO 120
      XMN=XMIN
      XMX=XMAX
      GO TO 125
  120 XMN=X(1)
      XMX=X(1)
      DO 122 I=2,N
      IF (X(I).LT.XMN) XMN=X(I)
      IF (X(I).GT.XMX) XMX=X(I)
  122 CONTINUE
C
C     CHECK FOR Y MAX-MIN VALUES
C
  125 IF (YMIN.EQ.0.0.AND.YMAX.EQ.0.0) GO TO 127
      YMN=YMIN
      YMX=YMAX
      GO TO 130
  127 YMN=Y(1)
      YMX=Y(1)
      DO 129 I=2,N
      IF (Y(I).LT.YMN) YMN=Y(I)
```

```
      IF (Y(I).GT.YMX) YMX=Y(I)
  129 CONTINUE
C
C     SETUP HORIZONTAL(X) MESSAGE
C
  130 IPOS=520-NXM*40
      IHIGH=IPOS/40B
      ILOW=IPOS-IHIGH*40B
      IPLOT(K)=7575757576400000000B+(IHIGH*100B+ILOW)*1000
     1OB
      K=K+1
      CALL DISBCD (XM,PLOT(K),NXM)
      K=K+NXM
C
C     SETUP VERTICAL(Y) MESSAGE
C
      IPOS=520-NYM*40
      IHIGH=IPOS/40B
      ILOW=IPOS-IHIGH*40B
      IPLOT(K)=7575757576420000000B+IHIGH*100B+ILOW
      K=K+1
      CALL DISBCD (YM,PLOT(K),NYM)
      K=K+NYM
C
C     STORE MINIMUM VALUES FOR DISPLAY
C
      ENCODE (10,1,XMNT) XMN
    1 FORMAT (E10.3)
      ENCODE (10,1,YMNT) YMN
      GO TO 150
*
*     ADDITIONAL DATA FOR DISPLAY
*     CHECK X MAX-MIN VALUES
*
  140 IF (XMIN.NE.0.0.OR. XMAX.NE.0.0) GO TO 142
      ICODE2=0
      GO TO 145
  142 XMN=XMIN
      XMX=XMAX
      ICODE2=1
C
C     CHECK Y MAX-MIN VALUES
C
  145 IF (YMIN.NE.0.0.OR. YMAX.NE.0.0) GO TO 147
      IF (ICODE2) 150,1247,150
  147 YMN=YMIN
      YMX=YMAX
C
C     CHECK DATA RANGE, ADJUST IF ZERO
C
  150 XRANGE=XMX-XMN
      YRANGE=YMX-YMN
```

```
      IF (XRANGE.NE.0.0) GO TO 154
      IF (XMN.EQ.0.0) GO TO 152
      XMN=XMN-.1*ABS(XMN)
      XMX=XMX+.1*ABS(XMX)
      GO TO 154
  152 XMN=-1.0
      XMX=1.0
  154 IF (YRANGE.NE.0.0) GO TO 160
      IF (YMN.EQ.0.0) GO TO 156
      YMN=YMN-.1*ABS(YMN)
      YMX=YMX+.1*ABS(YMX)
      GO TO 160
  156 YMN=-1.0
      YMX=1.0
C
C     FIND INCREMENT AND ADJUST MAX-MIN VALUES
C
  160 XINC=(XMX-XMN)/10.0
      I=0
  162 IF (XINC.LT.10.0) GO TO 164
      I=I+1
      XINC=XINC/10.0
      GO TO 162
  164 IF (XINC.GE.1.0) GO TO 166
      I=I-1
      XINC=XINC*10.0
      GO TO 164
  166 IF (XINC.EQ.1.0) GO TO 1167
      IF (XINC.GT.2.0) GO TO 167
      XINC=2.0
      GO TO 169
 1167 XINC=1.0
      GO TO 169
  167 IF (XINC.GT.5.0) GO TO 168
      XINC=5.0
      GO TO 169
  168 XINC=10.0
  169 IIX=I
  170 IF (I) 171,173,172
  171 I=I+1
      XMN=XMN*10.0
      XMX=XMX*10.0
      GO TO 170
  172 I=I-1
      XINC=XINC*10.0
      GO TO 170
  173 XR=AINT(XMN/XINC)*XINC
      IF (XR.GT.XMN) XR=XR-XINC
      XMN=XR
      XR=AINT(XMX/XINC)*XINC
      IF (XMX.GT.XR) XR=XR+XINC
      XMX=XR
```

```
C
      YINC=(YMX-YMN)/10.0
      I=0
  174 IF (YINC.LT.10.0) GO TO 175
      I=I+1
      YINC=YINC/10.0
      GO TO 174
  175 IF (YINC.GE.1.0) GO TO 176
      I=I-1
      YINC=YINC*10.0
      GO TO 175
  176 IF (YINC.EQ.1.0) GO TO 1177
      IF (YINC.GT.2.0) GO TO 177
      YINC=2.0
      GO TO 179
 1177 YINC=1.0
      GO TO 179
  177 IF (YINC.GT.5.0) GO TO 178
      YINC=5.0
      GO TO 179
  178 YINC=10.0
  179 IIY=I
  180 IF (I) 181,183,182
  181 I=I+1
      YMN=YMN*10.0
      YMX=YMX*10.0
      GO TO 180
  182 I=I-1
      YINC=YINC*10.0
      GO TO 180
  183 YR=AINT(YMN/YINC)*YINC
      IF (YR.GT.YMN) YR=YR-YINC
      YMN=YR
      YR=AINT(YMX/YINC)*YINC
      IF (YMX.GT.YR) YR=YR+YINC
      YMX=YR
C
C     TABLE FOR X AND Y GRID
C
      XSCALE=(XMX-XMN)/983.0
      YSCALE=(YMX-YMN)/983.0
      IF (IEND.EQ.0) GO TO 246
      IX=1
      XGRID(1)=XMN
  184 IX=IX+1
      XGRID(IX)=XGRID(IX-1)+XINC
      IF (XGRID(IX).LT.XMX) GO TO 184
C
      IY=1
      YGRID(1)=YMN
  186 IY=IY+1
      YGRID(IY)=YGRID(IY-1)+YINC
```

```
      IF (YGRID(IY).LT.YMX) GO TO 186
C
C     TABLE FOR X AND Y LABELS
C
      DO 190 I=1,IX
  190 IXSC(I)=XGRID(I)
  192 IF (IABS(IXSC(1)).LT.1000.AND.IABS(IXSC(IX)).LT.1000)
     1GO TO 196
      DO 193 I=1,IX
  193 IXSC(I)=IXSC(I)/10
      GO TO 192
  196 DO 205 I=1,IX
      IF (IABS(IXSC(I)).GT.9) GO TO 197
      IXPOS(I)=-24
      GO TO 200
  197 IF (IABS(IXSC (I)).GT. 99) GO TO 198
      IXPOS(I)= -20
      GO TO 200
  198 IXPOS(I)=-16
  200 ENCODE (10,2,IXSC(I))IXSC(I)
    2 FORMAT (I4)
  205 CONTINUE
      IXPOS(IX)=-24
      CALL DISBCD (IXSC(1),IXSC(1),IX)
      DO 210 I=1,IY
  210 IYSC(I)=YGRID(I)
  212 IF (IABS(IYSC(1)).LT.1000.AND.IABS(IYSC(IY)).LT.1000)
     1GO TO 216
      DO 213 I=1,IY
  213 IYSC(I)=IYSC(I)/10
      GO TO 212
  216 DO 220 I=1,IY
      ENCODE (10,2,IYSC(I))IYSC(I)
  220 CONTINUE
      CALL DISBCD (IYSC(1),IYSC(1),IY)
      DO 230 I=1,IX
  230 IXGRID(I)=(XGRID(I)-XMN)/XSCALE+40.5
      DO 235 I=1,IY
  235 IYGRID(I)=(YGRID(I)-YMN)/YSCALE+32.5
C
C     DRAW GRID LINES
C
  238 IA1=IXGRID(1)/40B
      IA2=IXGRID(1)-IA1*40B
      IA3=IXGRID(IX)/40B
      IA4=IXGRID(IX)-IA3*40B
      IA5=766000000000000000000B+(IA1*100B+IA2)*100000000000
     10B+(IA3*100B+IA4)*10000B
      IA6=7640001400007575757575B
      DO 240 I=1,IY
      IA1=IYGRID(I)/40B
      IA2=IYGRID(I)-IA1*40B
```

```
      IA3=IA1*100B+IA2
      IPLOT(K)=IA5+IA3*100000000B+IA3
      IPLOT(K+1)=IA6+IA3*100000000B
      IPLOT(K+2)=IYSC(I)
  240 K=K+3
      IA1=IYGRID(1)/40B
      IA2=IYGRID(1)-IA1*40B
      IA3=IYGRID(IY)/40B
      IA4=IYGRID(IY)-IA3*40B
      IA5=7660000000000000000B+(IA1*100B+IA2)*100000000B+(
     1IA3*100B+IA4)
      IA6=7640000001775757575B
      DO 245 I=1,IX
      IA1=IXGRID(I)/40B
      IA2=IXGRID(I)-IA1*40B
      IA3=IA1*100B+IA2
      IA7=IXGRID(I)+IXPOS(I)
      IA8=IA7/40B
      IA9=IA7-IA8*40B
      IA10=IA8*100B+IA9
      IPLOT(K)=IA5+IA3*1000000000000B+IA3*10000B
      IPLOT(K+1)=IA6+IA10*1000000000000B
      IPLOT(K+2)=IXSC(I)
  245 K=K+3
  246 IF (IIX.GE.0) GO TO 247
      IIX=IIX+1
      XMN=XMN/10.0
      XMX=XMX/10.0
      XSCALE=XSCALE/10.0
      XINC=XINC/10.0
      GO TO 246
  247 IF (IIY.GE.0) GO TO 1180
      IIY=IIY+1
      YMN=YMN/10.0
      YMX=YMX/10.0
      YSCALE=YSCALE/10.0
      YINC=YINC/10.0
      GO TO 247
C
C     DISPLAY MINIMUM VALUES AND INCREMENT
C
 1180 IF (IEND.NE.1) GO TO 1247
      IPLOT(K)=7640002437377575606OB
      IPLOT(K+1)=6060676044314560136OB
      CALL DISBCD (XMNT,IPLOT(K+2),1)
      IPLOT(K+3)=6031452351254425456 3B
 1185 ENCODE (10,1,IPLOT(K+4)) XINC
      CALL DISBCD (IPLOT(K+4),IPLOT(K+4),1)
      IPLOT(K+5)=6060706044314560136OB
      CALL DISBCD (YMNT,IPLOT(K+6),1)
      IPLOT(K+7)=603145235125442545 63B
 1195 ENCODE (10,1,IPLOT(K+8)) YINC
```

```
      CALL DISBCD (IPLOT(K+8),IPLOT(K+8),1)
      K=K+9
C
C     SET SYMBOL AND SCALE DATA TO PLOT
C
 1247 IPLOT(K)=ISTAB(ISYM)
      K=K+1
      DO 250 I=1,N
      IF(X(I).GT.XMX) GO TO 301
      IF(X(I).LT.XMN) GO TO 302
      IF(Y(I).GT.YMX) GO TO 303
      IF(Y(I).LT.YMN) GO TO 304
  305 IA1=(X(I)-XMN)/XSCALE+40.5
      IA2=IA1/40B
      IA3=IA1-IA2*40B
      IA4=(Y(I)-YMN)/YSCALE+32.5
      IA5=IA4/40B
      IA6=IA4-IA5*40B
      IPLOT(K)=75757575757500000000B+(IA2*100B+IA3)*10000B+
     1IA5*100B+IA6
      K=K+1
      GO TO 249
  301 IF(X(I).EQ.0.) GO TO 248
      IF(ABS((X(I)-XMX)/X(I)).GT.5.E-14) GO TO 248
      GO TO 305
  302 IF(X(I).EQ.0.) GO TO 248
      IF(ABS((XMN-X(I))/X(I)).GT.5.E-14) GO TO 248
      GO TO 305
  303 IF(Y(I).EQ.0.) GO TO 248
      IF(ABS((Y(I)-YMX)/Y(I)).GT.5.E-14) GO TO 248
      GO TO 305
  304 IF(Y(I).EQ.0.) GO TO 248
      IF(ABS((YMN-Y(I))/Y(I)).GT.5.E-14) GO TO 248
      GO TO 305
  248 LPCT=LPCT+1
  249 IF (K.LT.101) GO TO 250
      WRITE (DDITPE)(IPLOT(J),J=1,100)
      K=1
  250 CONTINUE
      IF (K.EQ.1) GO TO 252
      K=K-1
      WRITE (DDITPE) (IPLOT(I),I=1,K)
  252 IF (IEC.EQ.1) GO TO 255
      IEND=0
      GO TO 1000
  255 ENCODE (10,2,INIT(6))NPAGE
      CALL DISBCD (INIT(6),INIT(6),1)
      IEND=1
      IF (LPCT.EQ.0) GO TO 256
      ENCODE (10,2,INIT(8))LPCT
      CALL DISBCD (INIT(8),INIT(8),1)
      GO TO 260
```

```
 256 INIT(8)=6060606060606060606060B
 260 WRITE (DDITPE)(INIT(I),I=1,9)
     WRITE (DDITPE)(SM(I),I=1,3)
     IEND=1
1000 RETURN
     END
```

8.2.2 Function Subprograms

8.2.2.1 Function XWEIB

```
      FUNCTION XWEIB(FX,ALPHA,BETA)
      ALPHA1=1./ALPHA
      XWEIB=((-1./BETA)*ALOG(1.-FX))**(ALPHA1)
      RETURN
      END
```

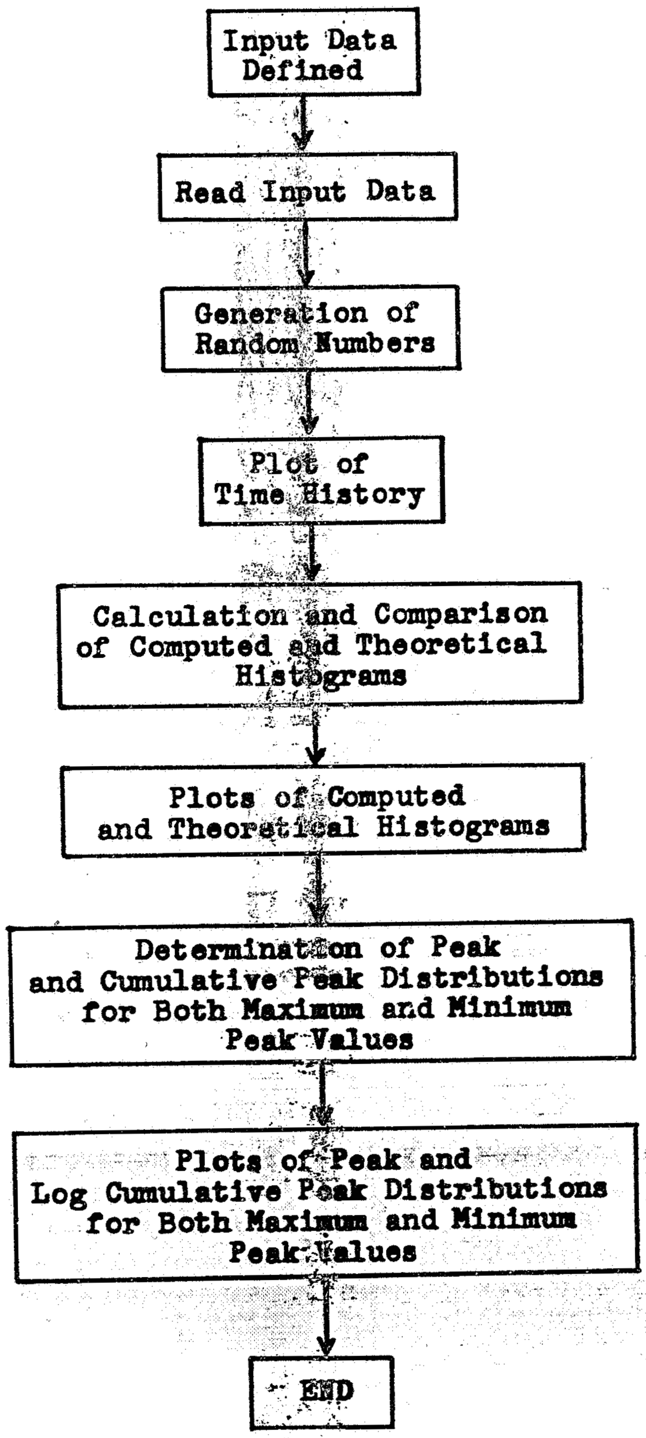
8.2.2.2 Function XEXP

```
      FUNCTION XEXP(FX,THETA)
      XEXP=XWEIB(FX,1.,THETA)
      RETURN
      END
```

8.2.2.3 Function XLOGNL

```
      FUNCTION XLOGNL(FX,XMU,SIGMA)
      SUM1=FX
      IF(SUM1.GT..5) SUM1=1.0-SUM1
      XNU=SQRT(ALOG(1.0/(SUM1*SUM1)))
      Z=XNU-((2.515517+.802853*XNU+.010328*XNU*XNU)/(1.0+1.
     1432788*XNU+.189269*XNU*XNU+.001308*XNU**3.0))
      IF(FX.LT..5) Z=-Z
      XLOGNL=EXP(SQRT(SIGMA)*Z+XMU)
      RETURN
      END
```

8.3 Typical FORTRAN Program Block Diagram



8.4 List of Symbols

$CF(x)$	- Cumulative probability density function
$F(x)$	- Probability density function
MEAN	- Arithmetic mean of random variable
m	- Integer
n	- Number of events (positive integer)
p	- Probability of success
q	- Probability of failure
RMS	- Root-mean-square
X, Z, LZ	- Random variables
Y	- Uniformly distributed random number
α	- Parameter of Weibull probability density function
β	- Parameter of Weibull probability density function
$\Gamma(\ )$	- Gamma function
θ	- Parameter of Exponential probability density function
μ	- Parameter of Poisson probability density function
μ_{log}	- Mean of logarithms
σ_{log}	- Standard deviation of logarithms

8.5 Derivation of Theoretical Equations Describing Properties of Density Functions

8.5.1 General

Mean: $E(x) = \int_{-\infty}^{\infty} xF(x)dx$

Mean square: $E(x^2) = \int_{-\infty}^{\infty} x^2F(x)dx$

Variance: $\sigma^2 = E(x^2) - \left[E(x)\right]^2$

8.5.2 Exponential Amplitude Probability Density Function

Probability density function: $F(x) = \theta \exp(-\theta x)$

Mean: $E(x) = \int_{0}^{\infty} x\theta \exp(-\theta x)dx = \frac{1}{\theta}$

Mean square: $E(x^2) = \int_{0}^{\infty} x^2\theta \exp(-\theta x)dx = \frac{2}{\theta^2}$

Variance: $\sigma^2 = \frac{1}{\theta^2}$

8.5.3 Weibull Amplitude Probability Density Function

Probability density function: $F(x) = \alpha\beta x^{\alpha-1}\exp(-\beta x^{\alpha})$

Mean: $E(x) = \int_{0}^{\infty} \alpha\beta x^{\alpha}\exp(-\beta x^{\alpha})dx$

Let

$$\beta x^{\alpha} = z$$

then

$$\left(\frac{z}{\beta}\right)^{1/\alpha} = x$$

and

$$dx = \frac{dz}{\alpha\beta x^{\alpha-1}}$$

Substituting we get

$$E(x) = \int_0^{\infty} \alpha z \exp(-z) \frac{dz}{\alpha\beta x^{\alpha-1}}$$

Simplifying we get

$$E(x) = \left(\frac{1}{\beta}\right)^{1/\alpha} \int_0^{\infty} (z)^{1/\alpha}\exp(-z)dz$$

From Ryshik and Gradstein (1963) equation (3.225) we get

$$E(x) = \left(\frac{1}{\beta}\right)^{1/\alpha} \Gamma\left(\frac{\alpha + 1}{\alpha}\right)$$

Mean square: $$E(x^2) = \int_0^{\infty} \alpha\beta x^{\alpha+1}\exp(-\beta x^{\alpha})dx$$

Let

$$z = \beta x^{\alpha}$$

then

$$\left(\frac{z}{\beta}\right)^{1/\alpha} = x$$

and

$$dx = \frac{dz}{\alpha\beta x^{\alpha-1}}$$

Substituting and simplifying we get

$$E(x^2) = \left(\frac{1}{\beta}\right)^{2/\alpha} \int_0^{\infty} z^{2/\alpha}\exp(-z)dz$$

From Ryshik and Gradstein (1963) equation (3.225) we get

$$E(x^2) = \left(\frac{1}{\beta}\right)^{2/\alpha} \Gamma\left(\frac{\alpha + 2}{\alpha}\right)$$

Variance: $\sigma^2 = \left(\frac{1}{\beta}\right)^{2/\alpha}\left\{\Gamma\left(\frac{\alpha+2}{\alpha}\right) - \left[\Gamma\left(\frac{\alpha+1}{\alpha}\right)\right]^2\right\}$

8.5.4 Log-Normal Amplitude Probability Density Function

Probability density function:

$$F(x) = \frac{1}{x\sigma_{log}\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_{log}^2}(\log_e x - \mu_{log})^2\right]$$

Let

$$z = \log_e x$$

then

$$F(x) = G(\log_e x) \equiv G(z)$$

$$E(x^n) = \int_0^\infty x^n dF(x)$$

Substituting we get

$$E(x^n) = \int_{-\infty}^\infty \exp(nz)dG(z)$$

From Aitchison and Brown (1957) we get

$$E(x^n) = \exp\left(n\mu_{log} + \frac{1}{2} n^2\sigma_{log}^2\right)$$

Therefore

Mean: $E(x) = \exp\left(\mu_{log} + \frac{1}{2}\sigma_{log}^2\right)$

Mean square: $E(x^2) = \exp\left(2\mu_{log} + 2\sigma_{log}^2\right)$

Variance: $\sigma^2 = \exp\left(2\mu_{log} + 2\sigma^2_{log}\right) - \exp\left(2\mu_{log} + \sigma^2_{log}\right)$

Simplifying we get

$$\sigma^2 = \exp\left(2\mu_{log} + \sigma^2_{log}\right)\left[\exp\left(\sigma^2_{log}\right) - 1\right]$$

8.5.5 Poisson Amplitude Probability Density Function

Probability density function: $F(x) = \dfrac{e^{-\mu}\mu^x}{x!}$

Mean: $E(x) = \displaystyle\sum_{x=0}^{\infty} \frac{xe^{-\mu}\mu^x}{x!}$

But at $x = 0$ the first term of the series is zero, therefore

$$E(x) = \sum_{x=1}^{\infty} \frac{e^{-\mu}\mu^x}{(x - 1)!} = \mu \sum_{x=1}^{\infty} \frac{e^{-\mu}\mu^{x-1}}{(x - 1)!}$$

Let $z = x - 1$, then

$$E(x) = \mu \sum_{z=0}^{\infty} \frac{e^{-\mu}\mu^z}{z!}$$

but

$$\sum_{z=0}^{\infty} \frac{e^{-\mu}\mu^z}{z!} = 1$$

therefore

$$E(x) = \mu$$

Mean square: $E(x^2) = \sum_{x=0}^{\infty} x^2 \frac{e^{-\mu}\mu^x}{x!}$

But at $x = 0$ the first term of the series is zero, therefore

$$E(x^2) = \sum_{x=1}^{\infty} \frac{xe^{-\mu}\mu^x}{(x-1)!} = \sum_{x=1}^{\infty} \frac{(x-1)e^{-\mu}\mu^x}{(x-1)!} + \sum_{x=1}^{\infty} \frac{xe^{-\mu}\mu^x}{x!}$$

Substituting $E(x)$ for the last summation we get

$$E(x^2) = \sum_{x=1}^{\infty} \frac{(x-1)e^{-\mu}\mu^x}{(x-1)!} + E(x)$$

But at $x = 1$ the first term of the summation is zero, therefore

$$E(x^2) = \sum_{x=2}^{\infty} \frac{e^{-\mu}\mu^x}{(x-2)!} + \mu = \mu^2 \sum_{x=2}^{\infty} \frac{e^{-\mu}\mu^{x-2}}{(x-2)!} + \mu$$

Let $z = x - 2$, then

$$E(x^2) = \mu^2 \sum_{z=0}^{\infty} \frac{e^{-\mu}\mu^z}{z!} + \mu$$

but

$$\sum_{z=0}^{\infty} \frac{e^{-\mu}\mu^z}{z!} = 1$$

therefore

$$E(x^2) = \mu^2 + \mu = \mu(\mu + 1)$$

Variance: $\sigma^2 = (\mu^2 + \mu) - \mu^2 = \mu$

8.5.6 Binomial Amplitude Probability Density Function

Probability density function:

$$F(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

Mean: $$E(x) = \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

But at $x = 0$ the first term of the series is zero, therefore

$$E(x) = np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$$

Let $z = x - 1$ and $m = n - 1$, then

$$E(x) = np \left[\sum_{z=0}^{m} \frac{m!}{z!(m-z)!} p^z (1-p)^{m-z} \right]$$

but

$$\sum_{z=0}^{m} \frac{m!}{z!(m-z)!} p^z (1-p)^{m-z} = 1$$

therefore

$$E(x) = np$$

Mean square: $$E(x^2) = \sum_{x=0}^{n} \frac{x^2 n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

But at $x = 0$ the first term of the series is zero, therefore

$$E(x^2) = \sum_{x=1}^{n} \frac{xn!}{(x-1)!(n-x)!} p^x(1-p)^{n-x}$$

or

$$E(x^2) = \sum_{x=1}^{n} \frac{(x-1)n!}{(x-1)!(n-x)!} p^x(1-p)^{n-x}$$

$$+ \sum_{x=1}^{n} \frac{xn!}{x!(n-x)!} p^x(1-p)^{n-x}$$

The second summation is equal to $E(x) = np$, therefore

$$E(x^2) = \sum_{x=1}^{n} \frac{(x-1)n!}{(x-1)!(n-x)!} p^x(1-p)^{n-x} + np$$

but at $x = 1$ the first term of the summation is zero, therefore

$$E(x^2) = n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2}(1-p)^{n-x} + np$$

Let $z = x - 2$ and $m = n - 2$, then

$$E(x^2) = n(n-1)p^2 \sum_{z=0}^{m} \frac{m!}{z!(m-z)!} p^z(1-p)^{m-z} + np$$

but

$$\sum_{z=0}^{m} \frac{m!}{z!(m-z)!} p^z(1-p)^{m-z} = 1$$

Therefore

$$E(x^2) = n(n-1)p^2 + np$$

or

$$E(x^2) = np\left[(n - 1)p + 1\right]$$

Variance: $\sigma^2 = np\left[(n - p)p + 1\right] - (np)^2$

or

$$\sigma^2 = np(1 - p)$$