SD 68-309

TWO-IMPULSE TRANSFER IN THE THREE-BODY PROBLEM

April 30, 1968

Prepared by

D. F. Bender

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G. A. Etemad / for
C. R. Faulders, Manager
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FOREWORD

The computer program described and the preliminary results presented herein were developed under NASA contracts NAS8-20238 and NAS8-21077 and under the company-funded Research and Development Authorization TPA-1000 for 1967-1968.

ABSTRACT

A computer program for finding two-impulse transfers between given terminal points in a fixed time in the reduced three-body problem has been developed. The only restriction is that the third body, which is imagined to be a space ship exerts a negligible attraction on the two large centers. Single pass trajectories between any two points are presumed. The program generates a patched conic of the proper general shape to use as an initial trajectory and uses the quasilinearization process to correct the trajectory so that the two-point boundary value problem is solved. The program and its use are described in detail and some preliminary results showing excellent convergence properties and illustrating its application to an impulsive transfer problem are presented.

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NOMENCLATURE

A_1, A_2	Angles of Perigee and Perilune (Figure 4)
c_1, c_2, c_3	Combination Coefficients of Homogeneous Solution Vectors
D	Earth-Moon Distance (Semi-Major Axis)
g_i, g	General Symbol for Expression for Time Derivative of X_i, Column Vector
G	Universal Constant of Gravitation Times Mass of Earth plus Moon
H	Homogeneous Solution Vector (of Equation 19)
i, j, k	Unit Vectors Along x, y, z
I	Impulse
J	Jacobian Matrix $\frac{\partial g_i}{\partial X_j}$
K	Jacobi Integral
P	Particular Solution Vector (of Equation 13)
r	Distance to Earth
s	Distance to Moon
t	Time (Independent Variable)
T	Time of Travel on Transfer Trajectory
u, v, w, V	Velocity Vector Components and Magnitude
x, y, z	Position Vector Components
X_j, X	General Symbol for Dependent Variable, Column Vector of Dependent Variables

ρ	Convergence Values for X, a Column Vector
ξ	$\underset{T}{\text{MAX}} \left\| X^{(n)}(t) - X^{(n+1)}(t) \right\|$, a Column Vector
μ	The Ratio Mass Moon to Mass Earth plus Moon
ω	Angular Velocity of the Moon in its Orbit

SUBSCRIPTS

o	For Departure Point at t = 0
T	For Arrival Point at t = T
t	For Transfer Trajectory

Superscript represents the iteration number, e. g. $X^{(n)}$

Underline signifies a vector in three dimensions

I. INTRODUCTION

Suppose it is desired that a spaceship on some orbit in earth-moon space transfer to a new orbit by means of a two-impulse maneuver. While this problem is topologically similar to two-impulse transfer in the two-body problem there are two significant differences from the computational and analytic points of view. In the first place the terminal orbits are not generally cyclic and points along them cannot be represented by five orbital elements and an angle. The six components of position and velocity are used instead. The second major difference is that one cannot describe the orbits between two fixed points as a known function of any one parameter. In general there is a single infinity of orbits through two given points in a gravitational field and in the two-body case one can choose, for example, the semi-latus rectum of the transfer orbit as the parameter to generate the orbits. For this case it is possible to compute the orbit, the velocities at both ends, and the impulses. Note that multiple complete orbits are generally not considered. In the three-body case it was decided to span the infinity of orbits by using time to transfer as the parameter. One must specify also the general plan of the path; that is, for example, counterclockwise around the earth to clockwise around the moon as in Figure 1. Just as in the two-body case multiple orbits in the system are not considered. It is a major problem to obtain the orbit, the velocities at both ends, and the impulses and in fact it was the aim of this portion of the work to provide a computer program to obtain these quantities. That is, from the given departure point (B_O in Figure 1), the given arrival point (B_T in Figure 1), the given orbit plan, and the given time interval (T), the program is to determine the transfer orbit trajectory, the velocities at both ends, and the impulses. The type of motion to be described is known as that of the reduced three-body problem, that is the third body whose motion is being investigated does not affect the motion of the two primary bodies, which may move in either circular or elliptical orbits about their center of mass.

This computer program represents the first necessary step in a longer range problem which is the numberical analysis of two-impulse transfers in earth-moon space. Now that it has been developed systematic studies of two-impulse transfers can be undertaken.

II. PROBLEM FORMULATION

The computational problem involved is a highly non-linear two-point boundary value problem and the method of solution utilizes a generalized Newton-Raphson technique which is called quasilinearization.[1]

In the complete program as it is described below the input coordinate systems may be centered at the earth, the moon, or their barycenter and the systems may be rotating or inertial. However, the computations are all managed in the system centered at the barycenter and rotating with the moon and the problem will be described in this system (see Figure 1). The equations of motion are easily derived using the Lagrangian procedure. Note that terms involving $\dot{\omega}$ occur since the rotating system is assumed to follow the moon and thus does not rotate uniformly. We find that the equations of motion may be expressed as six first-order differential equations:

$$\dot{u} = 2\omega v + \omega^2 x - \frac{\mu G(x-(1-\mu)D)}{s^3} - \frac{(1-\mu)G(x+\mu D)}{r^3} + \dot{\omega} y \tag{1}$$

$$\dot{v} = -2\omega u + \omega^2 y - \frac{\mu G y}{s^3} - \frac{(1-\mu)Gy}{r^3} - \dot{\omega} x \tag{2}$$

$$\dot{w} = \frac{-\mu G z}{s^3} - \frac{(1-\mu)Gz}{r^3} \tag{3}$$

$$\dot{x} = u \tag{4}$$

$$\dot{y} = v \tag{5}$$

$$\dot{z} = w \tag{6}$$

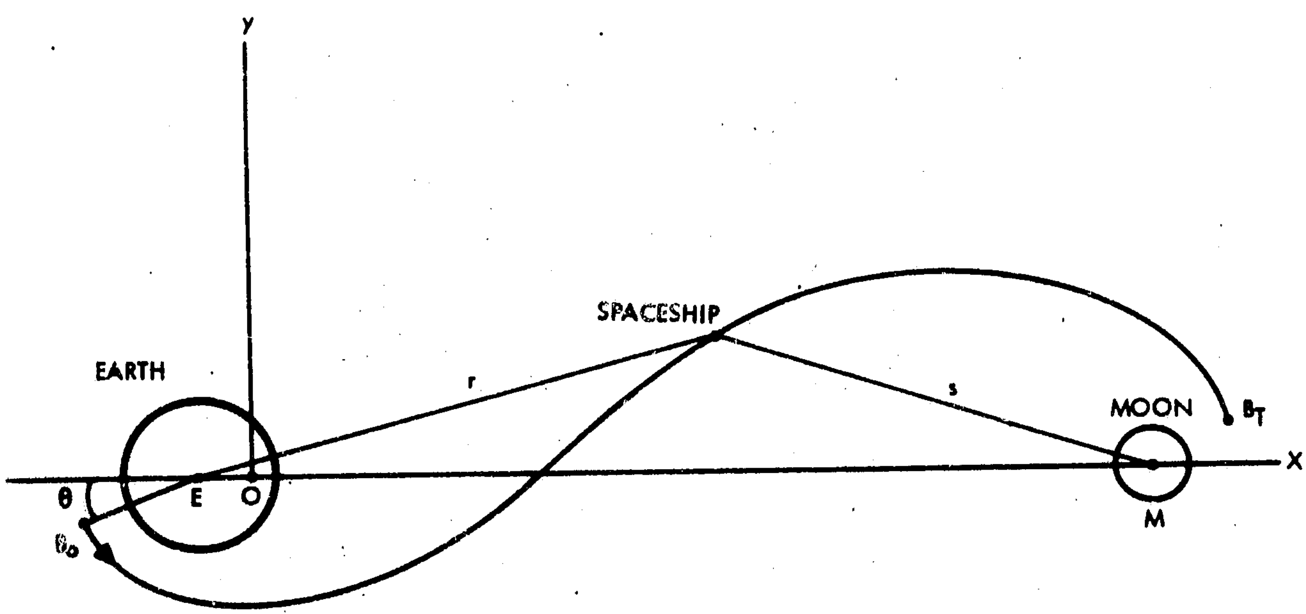
We represent the full set by the vector equation

$$\dot{X} = g(X) \tag{7}$$

As shown in Figure 1 the xy plane is the plane of the earth-moon motion and the x axis is directed toward the instantaneous position of the moon. G is the universal gravitational constant times the total mass of the earth-moon system, μ equals the mass of the moon divided by the total mass of the system, D is the earth-moon distance (or the semi-major axis of the orbit if it is elliptical), and ω is the angular velocity of the system. For the restricted problem in which the two primaries are assumed to be in circular orbits the equations simplify somewhat since $\dot{\omega} = 0$ and they possess a well known integral, the Jacobi integral, which is

$$K = -V^2 + \frac{2(1-\mu)G}{r} + \frac{2\mu G}{3} + \omega^2 \left(x^2 + y^2\right) \tag{8}$$

Two different unit systems are provided for in the program. One is based on the metric system using kilometers for distance and seconds for time. The second system uses lunar units for which $\omega = 1$ rad per unit time, D = 1 lunar distance, and $G = 1\ (\text{l.d})^3/(\text{u.t.})^2$



B_o = DEPARTURE $(u_o, v_o, w_o, x_o, y_o, z_o)$ EM = D

B_T = ARRIVAL $(u_T, v_T, w_T, x_T, y_T, z_T,)$ EO = μD

Figure 1. The Rotating Coordinate System

The computational problem can be described as follows: given the terminal positions (x_o, y_o, z_o and x_T, y_T, z_T), the transfer time (T) and the general shape of the trajectory, we wish to find the initial velocity (u_o, v_o, w_o) which will cause the vehicle to move from the initial point to the final point in the given time. The solution is to be accomplished using the quasilinearization procedure which is summarized below and in References 1 and 2. The set of equations (Equations 1 to 6) is of the proper form and to complete the analytical portion of the analysis it remains only to find expressions for the derivatives of the right hand sides (g_i) with respect to the six variables (X_j); i.e., to obtain the Jacobian matrix, J. This is easily accomplis d and the matrix for the general reduced three-body trajectory is

$$J = (J_{ij}) = \left(\frac{\partial g_i}{\partial X_j}\right) = \begin{bmatrix} 0 & 2\omega & 0 & \omega^2-A+E & \dot{\omega}+By & Bz \\ -2\omega & 0 & 0 & -\dot{\omega}+By & \omega^2-A+Cy^2 & Cyz \\ 0 & 0 & 0 & Bz & Cyz & -A+Cz^2 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

where

$$A = \left[\frac{1-\mu}{r^3} + \frac{\mu}{s^3}\right] G \tag{10}$$

$$B = \frac{3(1-\mu)G}{r^5}(x+\mu D) + \frac{3\mu G(x-(1-\mu)D)}{s^5} \tag{11}$$

$$C = \frac{3(1-\mu)G}{r^5} + \frac{3\mu G}{s^5} \tag{12}$$

$$E = +\frac{3(1-\mu)G(x+\mu D)^2}{r^5} + \frac{3\mu}{s^5} G\,(x-(1-\mu)D)^2 \tag{13}$$

The quasilinearization process may be summarized as follows: a crude approximation to the desired trajectory which satisfies the boundary conditions is assumed, and improved by successive corrections until it also satisfies the differential equations. The (n+1)'th iteration, $X^{(n+1)}$, is obtained from the n'th iteration, $X^{(n)}$, by integrating the approximate matrix differential equation:

$$\dot{X}^{(n+1)} = g(X^{(n)}) + J(X^{(n)}) \cdot (X^{(n+1)} - X^{(n)}) \tag{14}$$

It is obvious that when the process has converged, $X^{(N+1)} = X^{(n)}$ is a solution to Equation (7)

A particular integral of Equation 13, P, is obtained from the previous time history, $X^{(n)}$, by integrating Equation 13 starting with the initial values from the last iteration $(u_o^{(n)}, v_o^{(n)}, w_o^{(n)}, x_o, y_o, z_o)^*$. At the same time a set of three homogeneous solutions to the homogeneous part of Equation 13, that is to

$$\dot{H} = J(X^{(n)})\, H \tag{15}$$

are generated with initial conditions $(V_1, 0, 0, 0, 0, 0)^*$, $(0, V_2, 0, 0, 0, 0)^*$, and $(0, 0, V_3, 0, 0, 0)^*$. The linearity of Equation 7 in $X^{(n+1)}$ allows the new iteration to be expressed as P plus a linear combination of the three solutions, that is

$$X^{(n+1)} = P + c_1 H_1 + c_2 H_2 + c_3 H_3 \tag{16}$$

The coefficients (c_1, c_2, c_3) are determined by requiring this solution to satisfy the boundary condition on the final position (x_T, y_T, z_T). Thus it is seen that every iteration, $X^{(n+1)}$, satisfies the boundary conditions.

Convergence of the process is examined by evaluating the maximum change (at any point) between successive iterations in each coordinate.

$$\xi = \underset{T}{\mathrm{MAX}} \left| X^{(n)}(t) - X^{(n+1)}(t) \right| \tag{17}$$

Only when every component of ξ has satisfied given conditions is the procedure declared to have converged.

At the end of each iteration the impulses are computed at the beginning and at the end of the trajectory. We use

$$\underline{I}_1 = (u_o^{(n)} - u_o)\underline{i} + (v_o^{(n)} - v_o)\underline{j} + (w_o^{(n)} - w_o)\,\underline{k} \tag{18}$$

*The vectors X, g, H and ξ are really column vectors but the initial values are printed here as rows for convenience in typing.

$$\underline{I}_T = (u_T^{(n)} - u_T)\underline{i} + (v_T^{(n)} - v)\underline{j} + (w_T^{(n)} - w_o)\,\underline{k} \tag{19}$$

$$I = I_o + I_T = \sqrt{I_o \cdot I_o} + \sqrt{I_T \cdot I_T} \tag{20}$$

If the process converges within the assigned limit of iterations the program transfers to a mode in which the actual equations are integrated with the complete set of initial conditions as determined by QASLIN. The vector, ξ , now gives the differences between the final iteration and this integral. It may not measure how well this final integral represents the solution to the desired problem since the differences between the coordinates of the final point reached and those of the given end point are included in the comparison. Thus, if the integrated trajectory gets to the target point at a different time (or misses it a little ways) the vector, ξ , may not measure the closeness to which the orbit may approach the final point. The impulses computed on the basis of the integrated trajectory will be in error, while the impulses from the quasilinearization section will be very nearly correct. The discrepancy in the final point can be reduced by requiring tighter convergence. Thus, it occurs that the impulse to transfer from the QASLN portion converges on a value that is sufficiently accurate for most purposes before the position of the final point from the integrated portion will converge to the desired value. The example given in Section III, 7 below illustrates this phenomenon.

III. COMPUTER PROGRAM DESCRIPTION

A computer program written in double precision FORTRAN H language for the IBM 360 system has been developed for solving the problem of finding two-impulse transfers between given points in earth-moon space. It was adopted from an earlier FORTRAN IV version in order to make use of the more rapid double precision arithmetic of the IBM 360 system and its larger memory storage. It was developed to provide wide capability and has the following set of general properties:

a. The initial and final conditions can be given in any of a series of coordinate systems centered at the earth or the moon or the barycenter. Orbital elements for close earth orbits and/or close moon orbits are permissible.

b. Two unit systems are presently available: metric with kilometers and seconds or lunar units with $\omega = D = G = 1$ (see Equations 1 to 6 above). The total trip time, however, must be given in hours. Provision for a third system exists in the program. The actual constants used are provided in a single short subroutine and can easily be changed.

c. Real time with the initial epoch in modified Julian Days may be used and the moon's position will be determined by the program. The secular motion of the lunar orbit node and perigee during the trip time are neglected. Or, an artificial reference time may be used with the moon's initial position anywhere in its orbit.

d. The orbits of the earth and moon may be assumed to be circular or elliptical.

A block diagram of the overall computer program is shown in Figure 2. The general operation of the program is clear from this diagram and the details of using the program are described below under the following headings:

1. Input Data Coordinate Systems

2. Constants

3. Data Entry

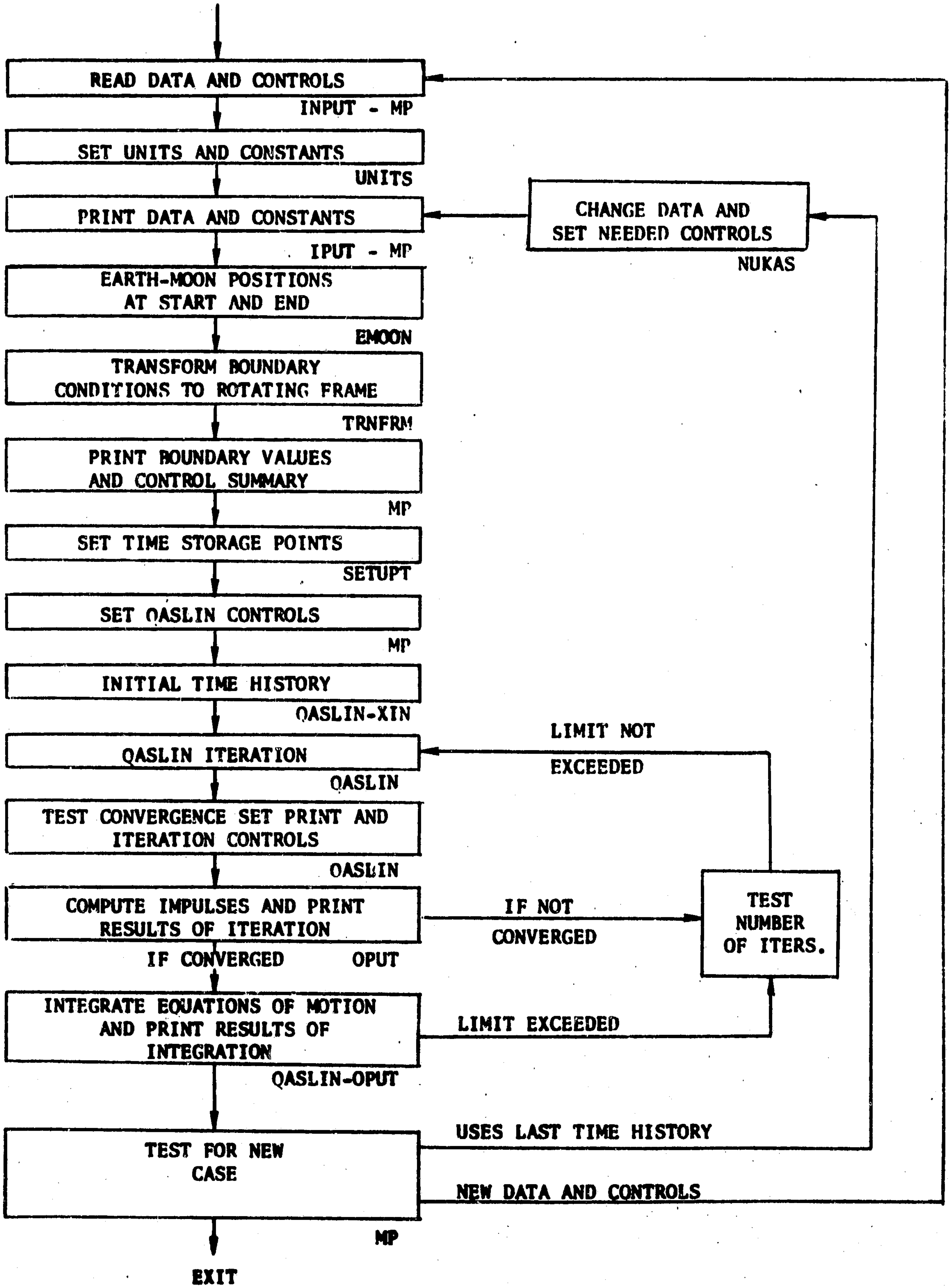


Figure 2. Flow Diagram for Two-Impulse Computation

4. Time History Storage

5. Initial Time History

6. Subroutine QASLN

7. Program Output

1. Input Data Coordinate Systems

As already mentioned, the initial and final boundary conditions may be given in earth centered, moon centered, or barycentric systems. These are specified by the values ICØRC(I) = J where: I = 1, 2 for initial, final conditions; and J = ±1, ±2, 3 for earth, moon, barycenter. A second integer IRØTS(I) further identifies the systems according to the scheme shown in Table 1. For cases in which the trajectory is to go from one center to the other, the sign on the value of J is used to indicate the direction of the trajectory around the center. The sign is that of the component of the angular momentum around the body in the Z direction. Thus for the trajectory of Figure 1 or Figure 4 ICØRC(1) = 1, ICØRC(2) = -2 if traversed from earth to moon and ICØRC(1) = 2, ICØRC(2) = -1 if transversed from moon to earth. In case the coordinate center for the given initial conditions is the barycenter ICØRC(1) = ICØRC(2) = 3 is required. SPARE(1) and SPARE(2) are then used to specify the trajectory directions.

2. Constants

The constants used in the program are listed in Table 2. Most of them were obtained from the American Ephemeris and Nautical Almanac. Mean rate (ω), semi-major axis (D), and G are adjusted to satisfy $\omega^2 = G/D^3$.

3. Data Entry

All data and controls are read and supplied to the program by means of the subroutine INPUT. This program reads up to 25 numbered cards each of which contains six data elements. The format of data on each of the twenty-five cards is determined by a set of cards read in at the very first. The particular arrangement required by the program is specified by the use of the data in the main program and other subroutines but not by the subroutine INPUT. Initially the entire data array is set to zero. If a data element is left blank on a card or if a card is omitted, the program will use the data of the previous case. The twenty-five cards, or whatever number is needed, can be in any order. The last one for a particular case must however carry a flag, a 1 in column 78 instead of a 0, to indicate the end of the file for the case.

Table 1. Coordinate System for Input Boundary Conditions

	IROTS(I) \ ICORC(I)		J=1* Earth Centered	J=2* Moon Centered	J=3 Barycenter Centered
Cartesian boundary conditions	Rotating System	1	x axis toward moon - xy plane is LOP	x axis opposite earth xy plane is LOP	x axis toward moon xy plane is LOP
Cartesian boundary conditions	non-rotating system	2	x axis toward moon's orbit node on ecliptic xy plane is LOP	x axis toward moon's orbit node on lunar equator xy plane is LOP	x axis toward moon's orbit node on ecliptic
Cartesian boundary conditions	non-rotating system	3	x axis toward vernal equinox xy plane is ecliptic	x axis toward node xy plane is lunar equator	not used
Cartesian boundary conditions	non-rotating system	4	x axis toward vernal equinox xy plane is equator	not used	not used
orbital elements for boundary conditions	non-rotating system	5	same as 2	same as 2	not used
orbital elements for boundary conditions	non-rotating system	6	same as 3	same as 3	not used
orbital elements for boundary conditions	non-rotating system	7	same as 4	same as 4	not used

*May be + or -

DATA ARRAY IMAGE

CARD NUMBER						
1	TEST CASE	D TO B, MOON	TO NEAR EAR	TH (3 DIMEN	.--XIEM, CON	IC)
2	2	1	-2	1	1	1
3	1	1	12	0	0	3
4	3.000000000D 03	-4.000000000D 03	1.000000000D 01	-1.732000000D 04	1.000000000D 04	3.000000000D 01
5	-7.900000000D-01	-5.920000000D-01	0.0	-2.230000000D 00	-3.960000000D 00	0.0
6	0.0	7.200000000D 01	4.000912500D 04	0.0	0.0	0.0
7	2	6	3	2	1	3
8	0.0	1.000000E 00	5.000000E 00	6.800000E 01	7.100000E 01	7.200000E 01
9	0.0	0.0	0.0	0.0	0.0	0.0
10	1	16	31	71	86	101
11	0	0	0	0	1	0
12	0.0	0.0	-1.500000E 01	0.0	9.000000E-01	0.0
13	0.0	2.061999E 00	-1.000000E 00	0.0	0.0	0.0
14	1.000000E-05	1.000000E-05	1.000000E-05	9.999999E-04	9.999999E-04	9.999999E-04
15	2.000000000D-02	2.000000000D-02	2.000000000D-02	2.000000000D-03	2.000000000D-03	2.000000000D-03
16	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0
20	0	0	0	0.0	0.0	0.0
21	19	0	0	6	0	0
22	0	0	0	0	0	0
23	1	2	0	0	2	0
24	0.0	0.0	0.0	9.700000000D 02	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0

Figure 3. Example of Data Card Summary

-C47

Table 2. Lunar Orbit and Astronomical Constants

Eccentricity	= .054900489
Mean Orbital Rate	= 13.17639 65268 °/day = ω
Incl. LOP to Ecliptic	= 5.1453964°
Incl. LOP to Lunar Equator	= 6.6804°
Incl. Ecliptic to Earth's Equator	= 23.44436° (Epoch of 1950)
Semi-major Axis	= 384747.87 km = D
Gravitational Constant Times Total Mass	= 403505.3 km^3/sec^2 = G
Ratio Mass Moon/Total Mass	= 1/82.30 = μ
Ref. Date JD 2,439,000.5	= Aug. 28, 1965 0 hr. U.T.
Asc. Node at Ref. Date	= 69.3226°
Asc. Nodal Rate	= -.0529539222°/day
Perigee at Ref. Date	= 56.5280°
Perigee Rate	= .16435 80025 °/day
Mean Anomaly at Ref. Date	= 41.1600°
Mean Anomaly Rate	= 13.06499 24465°/day

A summary of the card input format and the variable names is given in Table 3. Before returning the control to the main program the subroutine INPUT provides a complete list of the data (for all twenty-five cards). An example of this is given in Figure 3.

A brief description of each variable and how it is interpreted by the program is given in Table 4 where the variables are listed by card. It is possible that in the future a modification of an existing mechanism may be desired for some reason. Frequently this involves putting in a new bit of information, or transferring some data from one subroutine to another. In Table 5 the variables names that have been included but not used and hence that are available for such use are listed.

Table 3. Input Data and Control Card Format Summary

FØRMAT	FØRTRAN Symbol(s)	Card Number (Col. 79, 80)
18A4, 5X, I1, I2*	TITL(.18.)**	01
6I12	LØRM, IRAT, ICØRC(.2.), IRØTS(.2.)	02
6I12	ICØEL, IMØRTA, N(3), N(4), NCM, ND	03
6D12	PI(.3.), PF(.3.)	04
6D12	VI(.3.), VF(.3.)	05
6D12	TI, TF, EPØCH, ATMA(.3.)	06
6I12	NØSTEP, NINTP1, NDIF, JPRI, KFLA6, NTM	07
6E12	TSUBT(.6.)	08
6E12	TSUBT(7-10), XERR, EPSP	09
6I12	KSUB(.6.)	10
6I12	KSUB(7-10), ISET, NLS	11
6E12	SPARE(.6.)	12
6E12	SPARE(7-12)	13
6E12	RHØS(.6.)	14
6D12	XI(.6.)	15
6D12	TLS(.6.)	16
6D12	XLS(1, .6.)	17
6D12	XLS(2, .6.)	18
6D12	XLS(3, .6.)	19
3I12, 3D12	IXTRA(.3.), EXTRA(.3.)	20
6I12	NVX(.3., .2.)	21
6I12	NVX(.3., 3-4)	22
6I12	NIM(.4.), NKS, JSET	23
6D12	DVAR(.3., .2.)	24
6D12	DVAR(.3., 3-4)	25

* All cards have 5X, I1, I2 for columns 73-80

**(.N.) signifies that N data elements are included 1, 2, ... N

Table 4. Input Data and Controls Description

Card Number (Format)	FØRTRAN Symbol(s)	Description
1 (18A4)	TITL(. 18.)*	= Title, printed on first page of data along with the data
2 (6112)	LØRM	= 1 lunar unit, = 2 metric units, = 3 not defined
	IRAT	= 1 real time (EPØCH in modified Julian days is required), = 2 artificial time (moon's initial position is set arbitrarily by ATMA(3).
	ICØRC(. 2.)	Coordinate center for initial, final point data = ±1 earth, = ±2 moon, = 3 barycenter + means angular momentum vector to the North of LOP - means angular momentum vector to the South of LOP
	IRØTS(. 2.)	Coordinate system for input boundary conditions = 1 rotating system, = 2, 3, or 4 non-rotating systems, = 5, 6, or 7 orbital elements given in systems 2, 3, or 4. See Table 1 for complete description.
3 (6112)	ICØEL	= 1 moon's orbit circular, = 2 moon's orbit elliptical
	IMØRTA	Angle for moon's initial position given as: = 1 mean anomaly, = 2 true anomaly (artificial time only)

*(. N.) signifies that N data elements 1. 2. . . , N are being described

Table 4. Input Data and Controls Description (Cont)

Card Number (Format)	FØRTRAN Symbol(s)	Description
	N(3)	Maximum number of iterations allowed in QASLN
	N(4)	Print control in QASLN = 0, final solution is only printed = 1, time history at end of each iteration is added = 2, partial integral (P) and H's during integration are added Also for = -3 or less QASLN is by-passed after initial time history is obtained and printed.
	NCM	Not used
	ND	(= M2(5)) number of dimensions, = 2 planar case, = 3 three dimensional case
4 (6D12)	PI(.3.), PF(.3.)	X, Y, Z coordinates of initial, final point except if orbital elements are given these are p, e, i of initial, final orbit
5 (6D12)	VI(.3.), VF(.3.)	X, Y, Z velocities at initial, final point, except if orbital elements are given these are ω, Ω, f of initial, final orbit
6 (6D12)	TI, TF	Initial, final time <u>in hours</u> (from EPØCH in case of real time)
	EPØCH	Reference time in Julian Days - 2,400,000.0 (needed only for real time case)
	ATMA(.3.)	Moon's orbit angles and initial position: node, perigee and anomaly (see IMØRTA) in degrees

Table 4. Input Data and Controls Description (Cont)

Card Number (Format)	FØRTRAN Symbol(s)	Description
7 (6I12)	NØSTEP	The number of integration steps per storage internal. This can be as large as desired but a maximum of 3 is suggested since the second order interpolation on the previous time history uses values at the storage points. If not given XERR used, see XERR.
	NINTPI	The total time of the trajectory can be divided into as many as 9 sections in each of which the step-size interval is constant. NINTPI = number of such sections plus 1.
	NDIF	Number of differences used in the Adams-Moulton integration (3 maximum allowed). If larger than 3, 3 is assumed and 4th order Runge-Kutta-Gill integration will be used for NDIF steps before the program switches to the Adams-Moulton integration for the remainder of the time.
	JPRI	= 2 for printing time history when integrating after convergence or after completing N(3) iterations (otherwise JPRI can have any value) (= M2(7))
	KFLAG	KFLAG (for special problems) = 1 normal two impulse problem, = 2 special problem (dummy at present). KFLAG must = 1
	NTM	Maximum number of orthogonalizations allowed in the subroutine QASLN (maximum number permissible is 3)

Table 4. Input Data and Controls Description (Cont)

Card Number (Format)	FØRTRAN Symbol(s)	Description
8 (6E12)	TSUBT(. 6.)	The boundaries of the time sections in hours. TSUBT(1) does not have to be zero.
9 (6E12)	TSUBT(7-10)	TSUBT(NINTP1) is not used, rather the final time is obtained from (TF-TI)
	XERR	The step-size, equivalent to NØSTEP. If NØSTEP is not given NØSTEP will be determined from fifth root.
	EPSP	Volume limit of correction parallel-epiped (normalized). When volume is less than EPSP the orthogonalization procedure in QASLN is used.
10 (6I12)	KSUB(. 6.)	Step numbers corresponding to TSUBT. KSUB(1) must = 1 KSUB(NINTP1) must not exceed 200.
11 (6I12)	KSUB(7-10)	(KSUB(NINTP1 + 1) is set to 0 in program)
	ISET	Not used but is transcribed in main program.
	NLS	Number of linear segments used for initial time history. NLS must = 0 when linear segment method is not used. The maximum number of segments is 7, but only six data points may be entered. A scheme using PI and PF as end points is programmed with the given six as intermediate points for NLS = 6 or 7.

Table 4. Input Data and Controls Description (Cont)

Card Number (Format)	FØRTRAN Symbol(s)	Description
12 (6E12)	SPARE(.6.)	Single precision data entry used in various ways for first time history routines.
13 (6E12)	SPARE(7-12)	(a) for linear segments procedure SPARE(1-6) are X-velocity points SPARE(7-12) are Y-velocity points (b) for earth to moon or moon to earth patched conic procedure (1) (2) are used as ICØRC(1) (2) to give center and angular momentum direction when boundary data are given in barycenter system. (3) = AA1 angle of perigee of earth section ellipse (in degrees from opposite the moon at arrival) (4) = AA2 angle of perilune of moon centered conic (in degrees from opposite the earth at arrival) (5) = Maximum distance from earth for earth conic in lunar distance units (7) = 0, earth conic semi-major axis estimated from time of flight, = earth conic semi-major axis (8) Jacobi constant, used to correct magnitude of velocities on first time history if (9) is negative (c) for the single center case (both points near either earth or moon) (1) is used as ICØRC(1) to obtain the center and angular momentum vector direction when boundary point is in barycenter system.

Table 4. Input Data and Controls Description (Cont)

Card Number (Format)	FØRTRAN Symbol(s)	Description
14 (6E12)	RHØS(. 6.)	The values of the differences of the variables to be compared with the vector ϵ. Convergence is said to have occurred when for every component $\epsilon \leq \rho$. The units must correspond to those computed by the program as no conversions are used.
15 (6D12)	XI(. 6.)	The non-zero values used as initial values for the homogeneous solutions (only the first three are ever used. XI is dimensioned 10 in the program and thus free for transfer of data as desired).
16 (6D12)	TLS(. 6.)	Times of ends of linear segments for initial time history. These must be in the units of the problem (seconds or lunar time units) and must correspond to the independent variable scale. (See NLS on card 11)
17 (6D12)	XLS(1, . 6.)	X values at times of TLS (in rotating barycenter system only)
18 (6D12)	XLS(2, . 6.)	Y values at times of TLS (in rotating barycenter system only)
19 (6D12)	XLS (3, . 6.)	Z values at times of TLS (in rotating barycenter system only)
20	IXTRA(. 3.), EXTRA(. 3.) 3I12, 3D12	Integers entered into main program EXTRA(. 3.) entered into common PARAM none of these is used at present (note EXTRA (4) has special use, EXTRA(5-20) are not used)

Table 4. Input Data and Controls Description (Cont)

Card Number (Format)	FØRTRAN Symbol(s)	Description
21	NVX(.3.,.2.) 6I12	Number of the variable to be altered for two sets according to scheme (must be set to zero if not used) 1, 2, 3 for PI 4, 5, 6 for PF 7, 8, 9 for VI (would only be used 10, 11, 12 for VF when orbit ele- 13 for TI ments are given) 14 for TF 15, 16, 17 for ATMA 18 for EPØCH 19 moon's orbit circular to elliptical (Note that only three variables may be altered in one set)
22	NVX(.3., 3-4) 6I12	Same as above for 2 more sets
23	NIM(.4.), NKS, JSET 6I12	NIM = number of times to increment the variable for each of the four sets NKS = number of sets, JSET not used. Must be zero if none.
24	DVAR(.3.,.2.) 6D12	The increments in the variables called for by NVX. 2 sets
25	DVAR(.3., 3-4) 6D12	The same as above for 2 more sets

Table 5. Available Input and Transfer Variable Names for Future Modifications of Program

Variable	Input Position	Data Card Number	Where Available
IXTRA (.3.)	1, 2, 3	20	Main Program
EXTRA(.3.)	4, 5, 6	20	CØMMØN PARAM
EXTRA(5-20)	Not entered as data		CØMMØN PARAM
XI(4-6)	4, 5, 6	15	CØMMØN ZX
XI(7-10)	Not entered as data, set to zero in main at start		CØMMØN ZX
NCM	5	3	Main Program
ISET	5	11	Main Program
JSET	6	23	Main Program
RHØS(7-12)	Not entered as data, set to zero in main at start		CØMMØN ZØØ

4. Time History Storage

At the moment the maximum number of storage points is set at 200. Since the QASLN integration procedure is capable of handling a change in step-size it is permissible to divide the total time and the 200 points into as many as nine sections of constant step-size. This is desirable so that when near the earth or moon a small step-size can be used for accuracy since the forces of attraction are large and vary greatly over short distances. Over the intermediate space not near either center relatively longer step-sizes will give equivalent accuracy. The optimal use of the available steps remains a subject for further study. The total number of steps can be increased but this would increase the running time and it is not considered desirable at the present stage. Very large increases in step-size between sections has been found unsatisfactory in that the extrapolation for new large intervals is very much less accurate than the computation for the previous steps. This is indicated by a shift in the value of the Jacobi constant. Hence, it is best to keep the step-size change to a factor of two to four.

5. Initial Time History

In order to begin the quasilinearization process it is necessary to generate an initial guess for the time history of all the variables (X). In many cases this can be very crude but for highly sensitive and extremely non-linear problems the first guess may have to be carefully made in order to obtain convergence on the desired solution. Two-impulse transfer in the three-body problem presents such a situation if the trajectory sought is to transfer from the space near one body to that near the other.

The motions envisaged for study so far have been those fairly close to the lunar orbit plane (LOP). The motion out of the plane has been assumed to be linear and decoupled from the motion in the LOP, but a sinusoidal form could be utilized with very little additional programming if it is needed.

Three approaches for obtaining a first time history in the LOP have been utilized in this study. The first was an attempt to find a simple two-for-one Lissajous figure with variable amplitude to obtain an approximate trajectory. This form did succeed in the case tested but required several iterations before the trajectory was really close to convergence. In addition, it was felt that because one has to put in arbitrary variations in the velocity formulae to approximate the lunar trajectory velocities more realistically, the technique may not be very satisfactory.

With the hope of reducing the number of iterations required for convergence, patched conic procedures were then setup for the first time history along with controls so that the various types of earth to moon, earth-earth, moon-moon and moon to earth transfers could be studied. In addition a linear segmented time history program allowing up to 8 pairs of points to produce linear segments for all variables was adopted for use in this program. All three schemes have been made to yield satisfactory first time histories.

The patched conic scheme may have a sharp bend and velocity shift at the point where they are joined (L in Figure 4). In spite of this it has been found that the conic procedure generally converges with fewer iterations than the Lissajous figure scheme. Hence the latter is not recommended and is no longer available in the program.

As determined previously the linear segment procedure, even if started with about six points from the actual trajectory does not yield convergence in fewer iterations than does the patched conic method for near earth to near moon cases. Consequently, the use of linear segments is recommended only under special circumstances such as for trajectories to a Lagrange point.

The orbit plan or angular momentum direction information is entered by means of the signs of ICØRC(1) and (2) as described in Section III.I above. Other details to be supplied to the patched conic routine are the pericenter angles A_1, A2, and RLIM as shown in Figure 4.

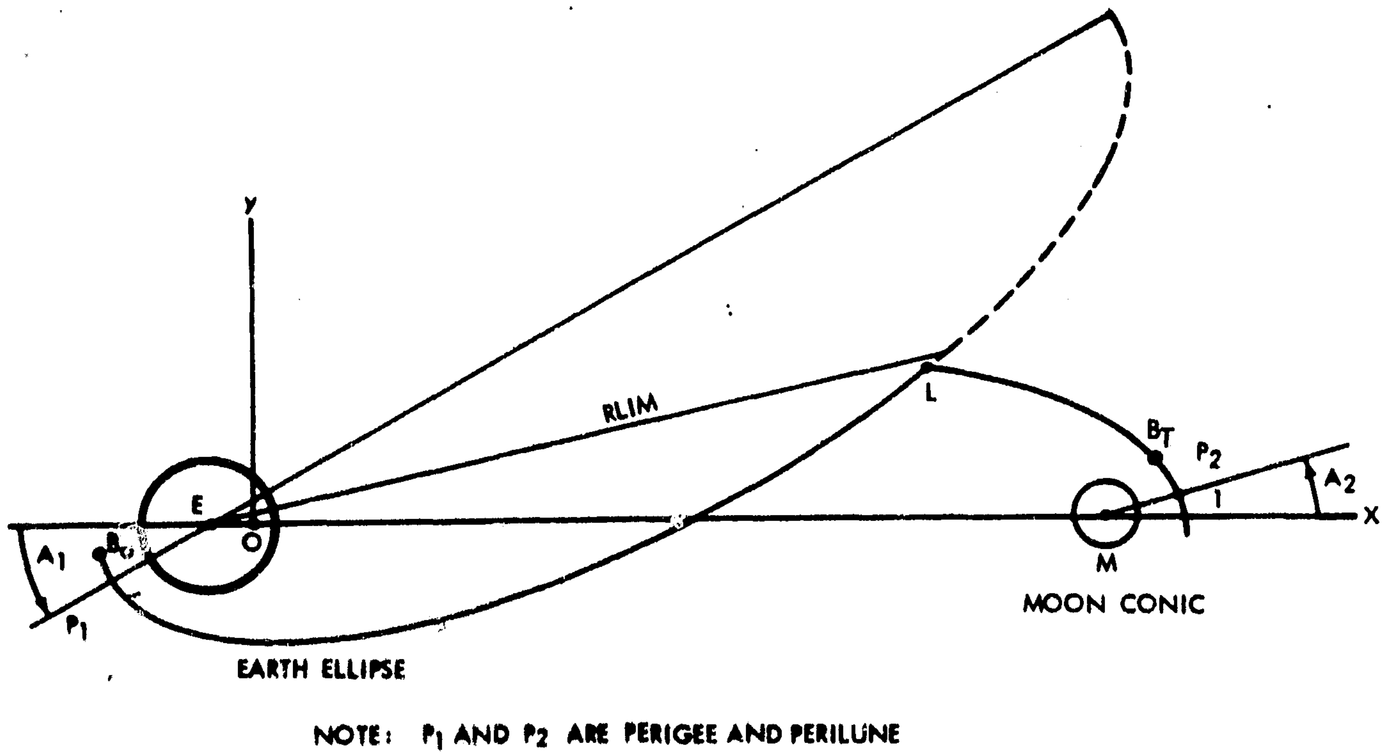


Figure 4. Patched Conic Geometry for Earth to Moon Transfers

These quantities are supplied through the use of SPARE as follows:

A_1 = SPARE(3) = Angle of Perigee (P_1) in Deg.

A_2 = SPARE(4) = Angle of Perilune (P_2) in Deg.

RLIM = SPARE(5)*D

The semi-major axis of the earth section conic can be inserted (SPARE(7)) or it can be chosen (SPARE(7) = 0) by means of the formula

a = (.712 - (TF - TI - 80)* .001.)*D l.u.

This was developed to give a reasonable value for an eighty hour trip to the moon and slightly higher semi-major axes for shorter trips. It was chosen rather than one assuming that the period should be proportional to the trip time because it calls for a variation in "a" of the proper direction as far as energy is concerned. Finally the eccentricity is determined from the initial point since the true anomaly is known.

The earth section conic is followed until the distance from the earth exceeds the value RLIM. Then the previous point (L) and the lunar boundary point B_T are used to determine the lunar conic. Since perilune has been chosen the lunar conic is now fixed and its elements are determined. The program can manage either an ellipse, or a hyperbola for this conic. The mean anomaly rate is taken to be the mean anomaly difference between L and A divided by the remaining time to go, thus assuring arrival at A at the proper time. Although this orbit may be traversed at an improper rate, the velocity time history contains the velocity components corresponding to the proper rate.

The sign convention and reference directions for specifying A_1 and A_2 are the same for all cases. In Figure 4 both are positive small angles as shown. It is, of course, essential that L and P_2 may be joined by a continuous conic and this will be affected by the choice of A_1 and A_2. Thus to go around the moon as shown $A_1 = 15°$ is a reasonable choice, while to go clockwise (ICØR(2) = 2) A_1 should be zero or negative so as to put L below the E. M. time. If this initial time history program fails the case is terminated with the printout: "XIN WAS UNABLE TO COMPUTE A FIRST TIME HISTORY,..."

A final input mechanism that has been introduced is the possibility of setting the value of the Jacobi constant for the trajectory of the first time history. This is entered as SPARE(8) and it is used if SPARE(9) is negative. The constant is used to adjust the value of the magnitude of the velocity at every point without changing the direction. The efficiency of this technique in speeding convergence has not been tested.

6. Subroutine QASLN

A double precision subroutine for managing a quasilinearization operation as outlined in Section II above has been developed by McCue and Radbill (Reference 2). The details of and controls for its use in the present program are included in Section 3(Table 4).

7. Program Output

An example of the output from the program is given in two series of consecutive pages. The first eight pages of the output are included in Figure 5 and the last eight pages are in Figure 6. Figure 5 contains the input data controls, various outputs from initialization sections, and results part way through the second iteration. The intervening pages are omitted, and then the iteration at which convergence occurred, the tenth, is shown in Figure 6 along with the results of the straightforward integration of the equations of motion using the initial values as determined by the QUASLN program.

Most of the output is self explanatory. Note that the boundary conditions and all results are given in the rotating barycenter coordinate system and that the actual time scale of the integration is given in the line entitled (LORMU). There remains in the output a number of results used for check-out purposes. The data headed by "IN XIEM PATCHED CONIC VALUES ARE" is printed by the patched conic routine (XIEM). The Jacobian, the derivatives, and a few other items are printed by JACOB the first time it is called.

The output from an iteration begins with the STORAGE INTERVAL list. The combination coefficients are the c_1, c_2, c_3 of Equation 15 and are obtained by inversion of the matrix given. Only the twelve values indicated pertain to this problem. The convergence parameters are the components of the vector, ϵ (not ρ), (Equation 16). There follows the trajectory obtained as the result of the iteration, the quantities being time, Jacobi constant $\dot{x}$, $\dot{y}$, $\dot{z}$, x, y, z. The final quantity on each line is the earth-moon distance, but is is removed for all three dimensional cases as this is by tearing the wide margin from the output papers. Computation times from the beginning of the case are given at two points in each iteration.

The final output pages are given in Figure 6 which begins near the end iteration 9. All of the output from iteration 10 is included and it can be seen that there is very little change in the impulse between these two iterations. It will be observed that the integrated trajectory does not reach the final boundary point at the final time. The reason for the miss is that the convergence requirement is not sufficiently tight, even though the impulse appears to have converged well enough for many practical purposes.

In the further use of this program it will undoubtedly be wise to modify and to simplify the output so as to obtain whatever is the desired information without the clutter of extraneous material. It is hoped that such changes can be managed easily through the additional data transfer and control capabilities supplied as shown in Table 5.

```
INPUT FOR 03/14/68            TEST CASE D TO B, MOON TO NEAR EARTH  (3 DIMEN.--XIEM, CONIC)

  CARD  COLS.                     VALUE                                     VARIABLE
   2     12                          2                       UNITS    1= LUNAR, 2= METRIC, 3= ALTERNATE
   2     24                          1                       TIME     1= REAL,  2= ARTIFICIAL
   2  36,48                      -2  1                       COORD. CENTER    1=EARTH, 2=MOON, 3=BARY
   2  60,72                       1  1                       ROTAT. SYSTEM    1=STAND., 2=SYS1, 3=SYS2
                                                         *** ORBITAL ELEMENTS IN--5 = SYS1, 6 = SYS2
   3     12                          1                       LUNAR ORBIT      1=CIRCULAR, 2=ELLIPTICAL
   3     24                          1                       ANOMALY FLAG     1=TRUE, 2=MEAN
   4    1-36    3.00000D 03  -4.00000D 03   1.00000D 01       INITIAL POSITION VECTOR
   4   37-72   -1.73200D 04   1.00000D 04  3.00000D 01          FINAL POSITION VECTOR
   5    1-36   -7.90000D-01  -5.92000D-01   0.0              INITIAL VELOCITY VECTOR
   5   37-72   -2.23000D 00  -3.96000D 00   0.0                FINAL VELOCITY VECTOR
   6    1-12                0.0                              INITIAL TIME (HRS.)
   6   13-24                7.20000D 01                        FINAL TIME (HRS.)
   6   37-48                0.0                             *MOON'S POSITION, NODE
   6   49-60                0.0                             *MOON'S POSITION, PERIGEE
   6   61-72                0.0                             *MOON'S POSITION, ANOMALY
   6   25-36                4.000912D 04                   **EPOCH (MJD)

* FOR ARTIFICAL TIME ONLY
**FOR REAL TIME ONLY
*** P AND V VECTORS CONTAIN A,F,I,SOMEGA,COMEGA,PHI


 REAL TIME CASE.  REFERENCE EPOCH/DEPARTURE TIME/ARRIVAL TIME
JULIAN DATE 24 40009.1250 IS    JUNE  1, 1968  15.000 HOURS U.T.  OR 1900 +    68.4178645 YEARS
JULIAN DATE 24 40009.1250 IS    JUNE  1, 1968  15.000 HOURS U.T.  OR 1900 +    68.4178645 YEARS
JULIAN DATE 24 40012.1250 IS    JUNE  4, 1968  15.000 HOURS U.T.  OR 1900 +    68.4260780 YEARS


  CONSTANTS ARE IN  METRIC  UNITS, THEY ARE

  ECEN =   5.49005E-02    ECON =   1.05649E 00    XNO =   1.31764E 01    XIE  =   2.34446E 01
   XIL =   5.14540E 00     XLE =   6.68040E 00   TCON =   4.34836E 00    DD   =   3.84748E 05
    MU =   1.21286E-02    RMJD =   3.95005E 04   RNOD =   6.93226E 01    DNOD =  -5.29539E-02
  RPER =   5.65280E 01    DPER =   1.64358E-01    RM  =   4.11600E 01    DOTM =   1.30650E 01
  GXM1 =   3.98611E 05    GXM2 =   4.89394E 03    B1  =   4.66644E 03    B2   =   3.80081E 05


SPARE(1-12)=DATA FOR XIN
 0.0            0.0            -0.150000E 02    0.0             0.900000E 00    0.0
```

Figure 5. Example of Output - Initial Pages

```
  0.0             0.206200E 01   -0.100000E 01    0.0             0.0             0.0


                     THE BOUNDARY CONDITIONS ARE
            TIME           M.ANOM.            TRUNOM             THETA                 E-M DIST.
INIT. 0.0        HRS.   2.063418D 02 DEG.  2.063418D 02 DEG.  3.464664D 02 DEG.    3.847479D 05
FINAL 7.20000D 01       2.458710D 02       2.458710D 02       3.859956D 02          3.847479D 05


INIT. POSITION VECTOR =      3.830814D 05    -4.000000D 03     1.000000D 01
FINAL POS.       VEC.       -2.198644D 04     1.000000D 04     3.000000D 01

INIT. VELOCITY           -8.006468D-01    -5.999851D-01     0.0
FINAL  VEL.              -2.203383D 00    -3.913899D 00     0.0


           THE CONTROLS ARE
AT STEP           1          16         31         71         86        101         0          0          0          0
T(HRS.)        0.0         1.00       5.00      68.00      71.00      72.00       0.0        0.0        0.0        0.0
(LORMU) 0.0         3.600D 03 1.800D 04 2.448D 05 2.556D 05 2.592D 05 0.0        0.0        0.0        0.0

INTEGERS NOSTEP, NINTP1, PRINT ,MAX.ITR,IMAX, NDIF,NO.DIM., JPRI,KFLAG, ISET, NC,  NCM,  NLS,  NKS.
           2       6       0      12     101      3     3      2     1     1     0     0     0     0

12 RHOS,10XI, XERR
  1.000000E-05   1.000000E-05   1.000000E-05   9.999999E-04   9.999999E-04   9.999999E-04
  0.0            0.0            0.0            0.0            0.0            0.0
  2.000000D-02   2.000000D-02   2.000000D-02   2.000000D-03   2.000000D-03   2.000000D-03
  0.0            0.0            0.0            0.0            1.029512E-84
PROGRAM ENTERS QASLN
INITIAL TIME HISTORY IS OBTAINED FROM XIEM (JOINED CONIC)


 IN XIEM PATCHED CONIC VALUES ARE
SP,EE,SA,HP,XNE,AM1/APH1,AA1,X,Y,AA2,EM/ RM2,RM3,SF2,SF3,SPM/ SAM,PH3,XITM,SGNE,SIGN, I2,J   /
    3.7945795D 04     9.2898908D-01      2.7701847D 05      3.3641789D 05      4.8818665D-06      -3.6067819D-03
   -5.2361148D-01    -2.6179939D-01      3.4369599D 05      8.6267172D 03      0.0                 8.2248985D-01
    5.0000000D 03     3.7394129D 04     -9.2729522D-01     -3.3743867D 00      7.4674695D 03
    2.3082622D 04    -3.3743867D 00 -1.00E 00 -1.00E 00 -1.00E 00    1   39
AM2,AM3,CHF2,CHF3,XNM, AND IEND ARE
-0.59004115D-01 -0.43989664D 01  0.92134425D 06  0.18935431D-79  0.68496879D-04      1
```

Figure 5. Example of Output - Initial Pages (Continued)

```
STORAGE INTERVAL =        240.00000          1          16


THE JACOBIAN AND THE DERIVATIVES AT T=0 ARE
   0.0            -5.323414D-06    0.0              1.000000D 00    0.0              0.0
   5.323414D-06    0.0             0.0              0.0             1.000000D 00     0.0
   0.0             0.0             0.0              0.0             0.0              1.000000D 00
   3.152689D-09   -5.637783D-08    1.409446D-10     0.0             0.0              0.0
  -5.637783D-08    3.601913D-08   -1.879254D-10     0.0             0.0              0.0
   1.409446D-10   -1.879254D-10   -3.915765D-08     0.0             0.0              0.0
  -1.254157D-04    1.611173D-04   -3.915812D-07    -8.478042D-01   -1.507491D 00     7.716049D-05
   1.212856D-02    3.847479D 05    3.877479D 05     3.877685D 05    5.000010D 03     3.986114D 05
   4.893939D 03    2.661707D-06    9.582146D-03     3.600000D 03    1.000000D 00

              3               2                1               1

          0.567 SEC.

STORAGE INTERVAL =        960.00000         16          31

STORAGE INTERVAL =       5670.00000         31          71

STORAGE INTERVAL =        720.00000         71          86

STORAGE INTERVAL =        240.00000         86         101
   NUMBER OF ORTHOGONALIZATIONS =     0

          8.900 SECONDS


THE MATRIX IS
   7.9673034D 03    3.7852765D 03    2.7835956D 00    6.3799433D-83    2.5098038D-01   -5.9035800D 57
   6.2207066D 03    1.0699940D 04    8.6286402D 00    1.3599484D-79    2.5098038D-01   -4.8043976D 57
  -1.7804214D 00    1.6109878D 00    7.5505651D 02    2.8674827D-79    2.5098038D-01   -4.8047807D 57
  -4.9947385D 04    3.7774791D 04    1.0705483D 02    4.2168840D-79    2.5098038D-01   -4.8289175D 57
   4.8915847D-79    2.5098038D-01   -4.8291091D 57    5.5662854D-79    2.5098038D-01   -4.8293006D 57
   6.2409860D-79    2.5098038D-01   -4.8294922D 57    6.9156867D-79    2.5098038D-01   -9.4917028D 58
   7.4322544D-79    2.5098038D-01   -9.4920093D 58    8.1069551D-79    2.5098038D-01   -9.5701666D 58
   8.8132823D-79    2.5098038D-01   -9.5704731D 58    9.4879830D-79    2.5098038D-01   -9.5707796D 58
  -4.9947385D 04    3.7774791D 04    1.0705483D 02   -2.3812377D 00   -5.6708192D 00    7.7160494D-05
  -5.6375012D 01   -1.0306876D 03   -2.1243030D 20   -4.9047158D 18   -2.0755260D 20    1.0112093D 36
   4.4182857D-29    1.1132949D 37   -1.9216800D 02    4.3438034D 45
```

Figure 5. Example of Output - Initial Pages (Continued)

```
COMBINATICN COEFF
-1.25D 01 7.94D 00 9.70D-C2
              9.067 SECONDS
CONVERGENCE PARAMETERS----RHO
1.84D 00 1.56D 00 1.68D-02 2.93D 04 7.40D 04 1.31D 02
```

Figure 5. Example of Output - Initial Pages (Continued)

```
    CUTPUT FROM QUASLIN, JSW =  1
0.0                 2.030571960D 00 -1.0972D 00 -1.3487D 00  2.0171D-03  3.8308D 05 -4.0000D 03  1.0000D 01  3
2.4000000D 02       2.031612745D 00 -1.1247D 00 -1.3092D 00  1.9245D-03  3.8281D 05 -4.3190D 03  1.0473D 01  3
4.8000000D 02       2.037058253D 00 -1.1468D 00 -1.2692D 00  1.8377D-03  3.8254D 05 -4.6284D 03  1.0924D 01  3
7.2000000D 02       2.045902936D 00 -1.1631D 00 -1.2299D 00  1.7597D-03  3.8226D 05 -4.9282D 03  1.1356D 01  3
9.6000000D 02       2.056100265D 00 -1.1744D 00 -1.1926D 00  1.6912D-03  3.8198D 05 -5.2189D 03  1.1770D 01  3
1.2000000D 03       2.065764994D 00 -1.1815D 00 -1.1579D 00  1.6318D-03  3.8170D 05 -5.5009D 03  1.2168D 01  3
1.4400000D 03       2.073752277D 00 -1.1854D 00 -1.1260D 00  1.5805D-03  3.8142D 05 -5.7749D 03  1.2554D 01  3
1.6800000D 03       2.079569252D 00 -1.1870D 00 -1.0969D 00  1.5361D-03  3.8113D 05 -6.0416D 03  1.2927D 01  3
1.9200000D 03       2.083156590D 00 -1.1867D 00 -1.0705D 00  1.4976D-03  3.8085D 05 -6.3016D 03  1.3291D 01  3
2.1600000D 03       2.084696257D 00 -1.1852D 00 -1.0465D 00  1.4640D-03  3.8056D 05 -6.5556D 03  1.3647D 01  3
2.4000000D 03       2.084480033D 00 -1.1828D 00 -1.0246D 00  1.4345D-03  3.8028D 05 -6.8041D 03  1.3994D 01  3
2.6400000D 03       2.082830260D 00 -1.1798D 00 -1.0047D 00  1.4084D-03  3.8000D 05 -7.0476D 03  1.4335D 01  3
2.8800000D 03       2.080055974D 00 -1.1764D 00 -9.8647D-01  1.3852D-03  3.7971D 05 -7.2865D 03  1.4671D 01  3
3.1200000D 03       2.076432756D 00 -1.1727D 00 -9.6975D-01  1.3645D-03  3.7943D 05 -7.5212D 03  1.5001D 01  3
3.3600000D 03       2.072195148D 00 -1.1688D 00 -9.5435D-01  1.3459D-03  3.7915D 05 -7.7521D 03  1.5326D 01  3
3.6000000D 03       2.067569347D 00 -1.1649D 00 -9.4011D-01  1.3290D-03  3.7887D 05 -7.9794D 03  1.5647D 01  3
4.5600000D 03       2.051311062D 00 -1.1496D 00 -8.8982D-01  1.2701D-03  3.7776D 05 -8.8569D 03  1.6893D 01  3
5.5200000D 03       2.032104625D 00 -1.1350D 00 -8.5309D-01  1.2313D-03  3.7666D 05 -9.6925D 03  1.8092D 01  3
6.4800000D 03       2.015820234D 00 -1.1221D 00 -8.2343D-01  1.2015D-03  3.7558D 05 -1.0497D 04  1.9260D 01  3
7.4400000D 03       2.002715836D 00 -1.1107D 00 -7.9868D-01  1.1779D-03  3.7451D 05 -1.1275D 04  2.0401D 01  3
8.4000000D 03       1.992464116D 00 -1.1009D 00 -7.7746D-01  1.1584D-03  3.7345D 05 -1.2031D 04  2.1522D 01  3
9.3600000D 03       1.[illegible]46168980D 00 -1.0922D 00 -7.5886D-01  1.1420D-03  3.7239D 05 -1.2769D 04  2.2626D 01  3
1.0320000D 04       1.978742442D 00 -1.0647D 00 -7.4226D-01  1.1278D-03  3.7135D 05 -1.3489D 04  2.3716D 01  3
1.1280000D 04       1.974470201D 00 -1.0780D 00 -7.2721D-01  1.1155D-03  3.7031D 05 -1.4194D 04  2.4792D 01  3
1.2240000D 04       1.971496044D 00 -1.0722D 00 -7.1340D-01  1.1045D-03  3.6928D 05 -1.4886D 04  2.5858D 01  3
1.3200000D 04       1.969575167D 00 -1.0671D 00 -7.0058D-01  1.0945D-03  3.6825D 05 -1.5564D 04  2.6913D 01  3
1.4160000D 04       1.968511915D 00 -1.0626D 00 -6.8859D-01  1.0856D-03  3.6723D 05 -1.6231D 04  2.7960D 01  3
1.5120000D 04       1.968149976D 00 -1.0586D 00 -6.7727D-01  1.0773D-03  3.6621D 05 -1.6887D 04  2.8998D 01  3
1.6080000D 04       1.968364225D 00 -1.0550D 00 -6.6651D-01  1.0696D-03  3.6520D 05 -1.7532D 04  3.0028D 01  3
1.7040000D 04       1.969054045D 00 -1.0519D 00 -6.5623D-01  1.0625D-03  3.6419D 05 -1.8166D 04  3.1052D 01  3
1.8000000D 04       1.970150525D 00 -1.0492D 00 -6.4636D-01  1.0557D-03  3.6318D 05 -1.8792D 04  3.2068D 01  3
2.3670000D 04       1.983326163D 00 -1.0390D 00 -5.9187D-01  1.0192D-03  3.5726D 05 -2.2299D 04  3.7948D 01  3
2.9340000D 04       1.999520897D 00 -1.0349D 00 -5.4495D-01  9.9170D-04  3.5138D 05 -2.5519D 04  4.3645D 01  3
3.5010000D 04       2.017345198D 00 -1.0350D 00 -5.0053D-01  9.6693D-04  3.4552D 05 -2.8482D 04  4.9197D 01  3
4.0680000D 04       2.035824828D 00 -1.0379D 00 -4.5689D-01  9.4311D-04  3.3964D 05 -3.1196D 04  5.4612D 01  3
4.6350000D 04       2.054599342D 00 -1.0427D 00 -4.1281D-01  9.1908D-04  3.3374D 05 -3.3663D 04  5.9892D 01  3
5.2020000D 04       2.073708959D 00 -1.0490D 00 -3.6706D-01  8.9384D-04  3.2781D 05 -3.5875D 04  6.5032D 01  3
5.7690000D 04       2.093572975D 00 -1.0563D 00 -3.1799D-01  8.6630D-04  3.2185D 05 -3.7819D 04  7.0024D 01  3
6.3360000D 04       2.115256625D 00 -1.0640D 00 -2.6280D-01  8.3490D-04  3.1584D 05 -3.9469D 04  7.4849D 01  3
6.9030000D 04       2.136040838D 00 -1.0710D 00 -2.0771D-01  8.0455D-04  3.0978D 05 -4.0800D 04  7.9494D 01  3
7.4700000D 04       2.151735403D 00 -1.0783D 00 -1.6035D-01  7.8086D-04  3.0369D 05 -4.1839D 04  8.3985D 01  3
8.0370000D 04       2.163669871D 00 -1.0864D 00 -1.1700D-01  7.6112D-04  2.9755D 05 -4.2624D 04  8.8355D 01  3
```

Figure 5. Example of Output - Initial Pages (Continued)

8.6040000D 04	2.172598970D 00	-1.0956D 00	-7.6179D-02	7.4404D-04	2.9137D 05	-4.3170D 04	9.2621D 01
9.1710000D 04	2.179132298D 00	-1.1060D 00	-3.6901D-02	7.2869D-04	2.8513D 05	-4.3490D 04	9.6795D 01
9.7380000D 04	2.183733155D 0C	-1.1175D 00	1.4587D-03	7.1437D-04	2.7882D 05	-4.3590D 04	1.0089D 02
1.0305000D 05	2.186746248D 00	-1.1302D 00	3.9322D-02	7.0060D-04	2.7245D 05	-4.3474D 04	1.0490D 02
1.0872000D 05	2.188426720D 00	-1.1441D 00	7.6982D-02	6.8696D-04	2.6600D 05	-4.3145D 04	1.0883D 02
1.1439000D 05	2.188963211D 00	-1.1593D 00	1.1465D-01	6.7314D-04	2.5948D 05	-4.2601D 04	1.1269D 02
1.2006000D 05	2.188495438D 00	-1.1757D 00	1.5248D-01	6.5884D-04	2.5286D 05	-4.1844D 04	1.1646D 02
1.2573000D 05	2.187126540D 00	-1.1936D 00	1.9059D-01	6.4380D-04	2.4614D 05	-4.0872D 04	1.2016D 02
1.3140000D 05	2.184933465D 00	-1.2131D 00	2.2908D-01	6.2776D-04	2.3932D 05	-3.9682D 04	1.2376D 02
1.3707000D 05	2.181973588D 00	-1.2343D 00	2.6801D-01	6.1044D-04	2.3238D 05	-3.8273D 04	1.2727D 02
1.4274000D 05	2.178291574D 00	-1.2574D C0	3.0744D-01	5.9156D-04	2.2532D 05	-3.6642D 04	1.3068D 02
1.4841000D 05	2.173923912D 00	-1.2829D 00	3.4741D-01	5.7078D-04	2.1812D 05	-3.4786D 04	1.3398D 02
1.5408000D 05	2.168904090D 00	-1.3108D 00	3.8795D-01	5.4771D-04	2.1076D 05	-3.2701D 04	1.3715D 02
1.5975000D 05	2.163269646D 00	-1.3418D 00	4.2906D-01	5.2192D-04	2.0325D 05	-3.0385D 04	1.4018D 02
1.6542000D 05	2.157065669D 00	-1.3763D 00	4.7074D-01	4.9284D-04	1.9554D 05	-2.7835D 04	1.4306D 02
1.7109000D 05	2.150356867D 00	-1.4149D 00	5.1295D-01	4.5980D-04	1.8763D 05	-2.5046D 04	1.4577D 02
1.7676000D 05	2.143240947D 00	-1.4583D 00	5.5560D-01	4.2194D-04	1.7949D 05	-2.2017D 04	1.4827D 02
1.8243000D 05	2.135866696D 00	-1.5076D 00	5.9855D-01	3.7814D-04	1.7108D 05	-1.8745D 04	1.5054D 02
1.8810000D 05	2.128466604D 00	-1.5640D 00	6.4158D-01	3.2697D-04	1.6238D 05	-1.5229D 04	1.5254D 02
1.9377000D 05	2.121409335D 00	-1.6289D 00	6.8433D-01	2.6647D-04	1.5333D 05	-1.1470D 04	1.5423D 02
1.9944000D 05	2.115289549D 00	-1.7046D 00	7.2620D-01	1.9398D-04	1.4389D 05	-7.4708D 03	1.5554D 02
2.0511000D 05	2.111077549D 00	-1.7937D 00	7.6629D-01	1.0571D-04	1.3398D 05	-3.2384D 03	1.5640D 02
2.1078000D 05	2.110411741D 00	-1.9001D 00	8.0306D-01	-3.8733D-06	1.2351D 05	1.2126D 03	1.5670D 02
2.1645000D 05	2.116159429D 00	-2.0290D 00	8.3397D-01	-1.4323D-04	1.1239D 05	5.8571D 03	1.5630D 02
2.2212000D 05	2.133556715D 00	-2.1884D 00	8.5455D-01	-3.2591D-04	1.0045D 05	1.0650D 04	1.5500D 02
2.2779000D 05	2.172797366 00	-2.3903D 00	8.5653D-01	-5.7513D-04	8.7491D 04	1.5513D 04	1.5248D 02
2.3346000D 05	2.255427861D 00	-2.6539D 00	8.2340D-01	-9.3415D-04	7.3227D 04	2.0298D 04	1.4827D 02
2.3913000D 05	2.432104914D 00	-3.0121D 00	7.1836D-01	-1.4935D-03	5.7219D 04	2.4717D 04	1.4152D 02
2.4480000D 05	2.875664773D 00	-3.5158D 00	4.4955D-01	-2.4530D-03	3.8804D 04	2.8150D 04	1.3063D 02
2.4552000D 05	2.899781997D 00	-3.6050D 00	3.8320D-01	-2.6761D-03	3.6242D 04	2.8451D 04	1.2879D 02
2.4624000D 05	2.999992490D 00	-3.6881D 00	3.1871D-01	-2.8704D-03	3.3615D 04	2.8703D 04	1.2679D 02
2.4696000D 05	3.115189703D 00	-3.7750D 00	2.4555D-01	-3.0838D-03	3.0929D 04	2.8907D 04	1.2464D 02
2.4768000D 05	3.246145184D 00	-3.8661D 00	1.6195D-01	-3.3210D-03	2.8178D 04	2.9054D 04	1.2234D 02
2.4840000D 05	3.394976535D 00	-3.9614D 00	6.6123D-02	-3.5860D-03	2.5361D 04	2.9137D 04	1.1986D 02
2.4912000D 05	3.564202303D 00	-4.0606D 00	-4.4118D-02	-3.8837D-03	2.2473D 04	2.9146D 04	1.1717D 02
2.4984000D 05	3.756389076D 00	-4.1634D 00	-1.7142D-01	-4.2200D-03	1.9513D 04	2.9070D 04	1.1425D 02
2.5056000D 05	3.974031928D 00	-4.2691D 00	-3.1900D-01	-4.6023D-03	1.6477D 04	2.8894D 04	1.1108D 02
2.5128000D 05	4.219108320D 00	-4.3761D 00	-4.9077D-01	-5.0397D-03	1.3365D 04	2.8604D 04	1.0761D 02
2.5200000D 05	4.492325643D 00	-4.4825D 00	-6.9144D-01	-5.5436D-03	1.0176D 04	2.8181D 04	1.0381D 02
2.5272000D 05	4.791868160D 00	-4.5844D 00	-9.2663D-01	-6.1280D-03	6.9113D 03	2.7601D 04	9.9612D 01
2.5344000D 05	5.111547894D 00	-4.6765D 00	-1.2028D 00	-6.8098D-03	3.5766D 03	2.6837D 04	9.4961D 01
2.5416000D 05	5.438238479D 00	-4.7502D 00	-1.5270D 00	-7.6095D-03	1.8157D 02	2.5857D 04	8.9778D 01
2.5488000D 05	5.749328589D 00	-4.7926D 00	-1.9060D 00	-8.5497D-03	-3.2562D 03	2.4625D 04	8.3970D 01

Figure 5. Example of Output - Initial Pages (Continued)

```
  2.5560C00D 05   6.0103891650 00 -4.7863D 00 -2.3428D 00 -9.6444D-03 -6.7083D 03   2.3099D 04   7.7432D 01   3
  2.5584000D 05   6.091805048D 00 -4.7667D 00 -2.5041D 00 -1.0065D-02 -7.8549D 03   2.2518D 04   7.5067D 01   3
  2.5608000D 05   6.151390666D 00 -4.7401D 00 -2.6694D 00 -1.0491D-02 -8.9958D 03   2.1897D 04   7.2600D 01   3
  2.5632000D 05   6.198475683D 00 -4.7033D 00 -2.8402D 00 -1.0935D-02 -1.0129D 04   2.1236D 04   7.0030D 01   3
  2.5656000D 05   6.233314541D 00 -4.6550D 00 -3.0162D 00 -1.1398D-02 -1.1252D 04   2.0533D 04   6.7350D 01   3
  2.5680000D 05   6.255776536D 00 -4.5941D 00 -3.1962D 00 -1.1879D-02 -1.2363D 04   1.9788D 04   6.4557D 01   3
  2.5704000D 05   6.266798733D 00 -4.5196D 00 -3.3791D 00 -1.2376D-02 -1.3457D 04   1.8999D 04   6.1647D 01   3
  2.5728000D 05   6.267505387D 00 -4.4305D 00 -3.5635D 00 -1.2885D-02 -1.4531D 04   1.8166D 04   5.8616D 01   3
  2.5752000D 05   6.260241495D 00 -4.3262D 00 -3.7476D 00 -1.3403D-02 -1.5582D 04   1.7288D 04   5.5462D 01   3
  2.5776000D 05   6.247642804D 00 -4.2062D 00 -3.9293D 00 -1.3925D-02 -1.6606D 04   1.6367D 04   5.2182D 01   3
  2.5800000D 05   6.232764600D 00 -4.0707D 00 -4.1065D 00 -1.4444D-02 -1.7600D 04   1.5403D 04   4.8778D 01   3
  2.5824000D 05   6.218437734D 00 -3.9200D 00 -4.2768D 00 -1.4954D-02 -1.8559D 04   1.4396D 04   4.5250D 01   3
  2.5848000D 05   6.207120648D 00 -3.7552D 00 -4.4381D 00 -1.5445D-02 -1.9480D 04   1.3350D 04   4.1602D 01   3
  2.5872000D 05   6.200083594D 00 -3.5775D 00 -4.5882D 00 -1.5909D-02 -2.0360D 04   1.2267D 04   3.7839D 01   3
  2.5896000D 05   6.197158067D 00 -3.3890D 00 -4.7254D 00 -1.6338D-02 -2.1197D 04   1.1149D 04   3.3968D 01   3
  2.5920000D 05   6.196592862D 00 -3.1917D 00 -4.8483D 00 -1.6722D-02 -2.1986D 04   1.0000D 04   3.0000D 01   3

            THE IMPULSES ARE (X,Y,Z,COMP., MAGN.)
-2.965528489618D-01 -7.486711637442D-01  2.017126331741D-03  8.052677644412D-01
-9.883575011919D-01 -9.344395007731D-01 -1.672164152200D-02  1.360260028106D 00

 TOTAL IMPULSE IS        2.165527792547D 00

 STORAGE INTERVAL =        240.00000         1          16

 STORAGE INTERVAL =        960.00000        16          31

 STORAGE INTERVAL =       5670.00000        31          71

 STORAGE INTERVAL =        720.00000        71          86

 STORAGE INTERVAL =        240.00000        86         101
   NUMBER OF ORTHOGONALIZATIONS =      0

          17.983 SECONDS

THE MATRIX IS
  9.7489699D 03    1.6527119D 04    6.5459453D 01    6.3799433D-83    2.5098038D-01   -5.9035800D 57
  1.4103474D 04    2.9986178D 04    1.1650106D 02    1.3599484D-79    2.5098038D-01   -4.8043976D 57
 -8.8369474D 00   -1.2856591D 01   -1.2409626D 02    2.8674827D-79    2.5098038D-01   -4.8047807D 57
  1.4770059D 04   -2.5742676D 04   -3.8584617D 01    4.2168840D-79    2.5098038D-01   -4.8289175D 57
```

Figure 5. Example of Output - Initial Pages (Continued)

```
 4.89158470-79    2.50980380-01   -4.8291091D 57    5.5662854D-79    2.50980380-01   -4.8293006D 57
 6.24098600-79    2.50980380-01   -4.8294922D 57    6.9156867D-79    2.50980380-01   -9.4917028D 58
 7.43225440-79    2.50980380-01   -9.4920093D 58    8.1069551D-79    2.50980380-01   -9.5701666D 58
 8.81328230-79    2.50980380-01   -9.5704731D 58    9.4879830D-79    2.50980380-01   -9.5707796D 58
 1.4770059D 04   -2.5742676D 04   -3.8584617D 01   -3.1917404D 00   -4.8483387D 00   -1.6721642D-02
-5.6375012D C1   -1.0306876D 03   -2.1243030D 20   -4.9047158D 18   -2.0755260D 20    1.0112093D 36
 4.41828570-29    1.1132949D 37   -1.9216800D 02    4.3438034D 45

COMBINATICN COEFF

1.36D 01-8.36D 00-3.58D-01

           18.150 SECONDS

CONVERGENCE PARAMETERS----RHO

1.19D 00 1.12D 00 7.74D-02 8.56D 03 2.37D 04 8.34D 02
```

Figure 5. Example of Output - Initial Pages (Continued)

```
2.5560000D 05  2.040368224D 00 -5.7671D 00 -1.4778D 00  3.6790D-02 -2.9774D 03  2.1217D 04 -1.2749D 02  3
2.5584000D 05  2.031884771D 00 -5.7796D 00 -1.6873D 00  3.8059D-02 -4.3632D 03  2.0837D 04 -1.1851D 02  3
2.5608000D 05  2.031710094D 00 -5.7776D 00 -1.9045D 00  3.9300D-02 -5.7504D 03  2.0406D 04 -1.0923D 02  3
2.5632000D 05  2.031656140D 00 -5.7596D 00 -2.1294D 00  4.0503D-02 -7.1352D 03  1.9922D 04 -9.9651D 01  3
2.5656000D 05  2.031622029D 00 -5.7241D 00 -2.3602D 00  4.1650D-02 -8.5136D 03  1.9384D 04 -8.9792D 01  3
2.5680000D 05  2.031589498D 00 -5.6699D 00 -2.5945D 00  4.2722D-02 -9.8813D 03  1.8789D 04 -7.9665D 01  3
2.5704000D 05  2.031560303D 00 -5.5960D 00 -2.8296D 00  4.3699D-02 -1.1234D 04  1.8138D 04 -6.9293D 01  3
2.5728000D 05  2.031536224D 00 -5.5021D 00 -3.0625D 00  4.4563D-02 -1.2566D 04  1.7431D 04 -5.8699D 01  3
2.5752000D 05  2.031518859D 00 -5.3884D 00 -3.2903D 00  4.5297D-02 -1.3873D 04  1.6669D 04 -4.7913D 01  3
2.5776000D 05  2.031509388D 00 -5.2558D 00 -3.5096D 00  4.5887D-02 -1.5151D 04  1.5852D 04 -3.6968D 01  3
2.5800000D 05  2.031508384D 00 -5.1058D 00 -3.7177D 00  4.6325D-02 -1.6394D 04  1.4985D 04 -2.5899D 01  3
2.5824000D 05  2.031515714D 00 -4.9404D 00 -3.9118D 00  4.6606D-02 -1.7600D 04  1.4069D 04 -1.4744D 01  3
2.5848000D 05  2.031530565D 00 -4.7622D 00 -4.0901D 00  4.6730D-02 -1.8765D 04  1.3108D 04 -3.5409D 00  3
2.5872000D 05  2.031551589D 00 -4.5738D 00 -4.2508D 00  4.6703D-02 -1.9885D 04  1.2107D 04  7.6740D 00  3
2.5896000D 05  2.031577115D 00 -4.3781D 00 -4.3931D 00  4.6534D-02 -2.0960D 04  1.1070D 04  1.8865D 01  3
2.5920000D 05  2.031576209D 00 -4.1781D 00 -4.5167D 00  4.6235D-02 -2.1986D 04  1.0000D 04  3.0000D 01  3

          THE IMPULSES ARE (X,Y,Z,COMP., MAGN.)
-4.1051430764440-02 -9.108659063363D-01 -2.440158714949D-03  9.117937670702D-01
-1.974707071069D 00 -6.027685434037D-01  4.623536229927D-02  2.065172061155D 00

 TOTAL IMPULSE IS       2.976965828225D 00

 STORAGE INTERVAL =       240.00000       1      16        Beginning of Tenth Iteration

 STORAGE INTERVAL =       960.00000      16      31

 STORAGE INTERVAL =      5670.00000      31      71

 STORAGE INTERVAL =       720.00000      71      86

 STORAGE INTERVAL =       240.00000      86     101
   NUMBER OF ORTHOGONALIZATIONS =     0

        90.717 SECONDS

THE MATRIX IS
  1.1911460D 04    1.4442516D 04   -1.4002979D 02    6.3799433D-83    2.5098038D-01   -5.9035800D 57
  1.5433046D 04    2.3837377D 04   -2.1292682D 02    1.3599484D-79    2.5098038D-01   -4.8043976D 57
 -2.7557960D 01   -4.7607312D 01   -1.9057247D 02    2.8674827D-79    2.5098038D-01   -4.8047807D 57
 -5.8471687D-07   -1.1314814D-06    1.5195499D-05    4.2168840D-79    2.5098038D-01   -4.8289175D 57
```

Figure 6. Example of Output - Final Pages

```
 4.89158470-79    2.50980380-01   -4.82910910 57    5.56628540-79    2.50980380-01   -4.82930060 57
 6.24098600-79    2.50980380-01   -4.82949220 57    6.91568670-79    2.50980380-01   -9.49170280 58
 7.43225440-79    2.50980380-01   -9.49200930 58    8.10695510-79    2.50980380-01   -9.57016660 58
 8.81328230-79    2.50980380-01   -9.57047310 58    9.48798300-79    2.50980380-01   -9.57077960 58
-5.84716870-07   -1.13148140-06    1.51954990-05   -4.17809000 00   -4.51666780 00    4.62353620-02
-5.63750120 01   -1.03068760 03   -2.12430300 20   -4.90471580 18   -2.07552600 20    1.01120930 36
 4.41828570-29    1.11329490 37   -1.92168000 02    4.34380340 45

COMBINATION COEFF

1.59D-10-3.03D-10-7.95D-08

            90.883 SECONDS

CONVERGENCE PARAMETERS----RHO

1.13D-10 1.49D-10 2.59D-08 8.10D-07 1.45D-06 2.04D-04

        PROCESS CONVERGED, ITER.NO.=   10
```

Figure 6. Example of Output - Final Pages (Continued)

```
     OUTPUT FROM QUASLIN, JSW =   2
0.0               2.062164195D 00 -8.4170D-01 -1.5109D 00 -2.4402D-03  3.8308D 05 -4.0000D 03  1.0000D 01  3
2.4000000D 02     2.062164195D 00 -8.6947D-01 -1.4725D 00 -2.5268D-03  3.8288D 05 -4.3580D 03  9.4037D 00  3
4.8000000D 02     2.062167174D 00 -8.9290D-01 -1.4350D 00 -2.5999D-03  3.8266D 05 -4.7069D 03  8.7882D 00  3
7.2000000D 02     2.062168888D 00 -9.1249D-01 -1.3989D 00 -2.6608D-03  3.8245D 05 -5.0469D 03  8.1567D 00  3
9.6000000D 02     2.062170198D 00 -9.2871D-01 -1.3643D 00 -2.7110D-03  3.8223D 05 -5.3784D 03  7.5119D 00  3
1.2000000D 03     2.062171184D 00 -9.4206D-01 -1.3314D 00 -2.7521D-03  3.8200D 05 -5.7019D 03  6.8561D 00  3
1.4400000D 03     2.062171914D 00 -9.5294D-01 -1.3004D 00 -2.7854D-03  3.8177D 05 -6.0177D 03  6.1915D 00  3
1.6800000D 03     2.062172447D 00 -9.6176D-01 -1.2711D 00 -2.8121D-03  3.8154D 05 -6.3262D 03  5.5197D 00  3
1.9200000D 03     2.062172831D 00 -9.6884D-01 -1.2436D 00 -2.8331D-03  3.8131D 05 -6.6280D 03  4.8421D 00  3
2.1600000D 03     2.062173101D 00 -9.7446D-01 -1.2178D 00 -2.8495D-03  3.8108D 05 -6.9233D 03  4.1601D 00  3
2.4000000D 03     2.062173287D 00 -9.7886D-01 -1.1936D 00 -2.8619D-03  3.8085D 05 -7.2126D 03  3.4747D 00  3
2.6400000D 03     2.062173409D 00 -9.8227D-01 -1.1709D 00 -2.8711D-03  3.8061D 05 -7.4964D 03  2.7867D 00  3
2.8800000D 03     2.062173485D 00 -9.8483D-01 -1.1496D 00 -2.8775D-03  3.8037D 05 -7.7748D 03  2.0968D 00  3
3.1200000D 03     2.062173527D 00 -9.8670D-01 -1.1296D 00 -2.8817D-03  3.8014D 05 -8.0483D 03  1.4056D 00  3
3.3600000D 03     2.062173544D 00 -9.8800D-01 -1.1108D 00 -2.8840D-03  3.7990D 05 -8.3171D 03  7.1372D-01  3
3.6000000D 03     2.062173526D 00 -9.8883D-01 -1.0931D 00 -2.8847D-03  3.7966D 05 -8.5815D 03  2.1453D-02  3
4.5600000D 03     2.062129022D 00 -9.8865D-01 -1.0322D 00 -2.8759D-03  3.7871D 05 -9.6004D 03 -2.7449D 00  3
5.5200000D 03     2.062123272D 00 -9.8587D-01 -9.8304D-01 -2.8581D-03  3.7777D 05 -1.0567D 04 -5.4976D 00  3
6.4800000D 03     2.062121052D 00 -9.8199D-01 -9.4275D-01 -2.8363D-03  3.7682D 05 -1.1491D 04 -8.2312D 00  3
7.4400000D 03     2.062119286D 00 -9.7785D-01 -9.0902D-01 -2.8133D-03  3.7588D 05 -1.2379D 04 -1.0943D 01  3
8.4000000D 03     2.062118048D 00 -9.7386D-01 -8.8024D-01 -2.7904D-03  3.7494D 05 -1.3238D 04 -1.3633D 01  3
9.3600000D 03     2.062117126D 00 -9.7022D-01 -8.5530D-01 -2.7685D-03  3.7401D 05 -1.4070D 04 -1.6301D 01  3
1.0320000D 04     2.062116417D 00 -9.6700D-01 -8.3335D-01 -2.7477D-03  3.7308D 05 -1.4881D 04 -1.8949D 01  3
1.1280000D 04     2.062115857D 00 -9.6423D-01 -8.1379D-01 -2.7282D-03  3.7215D 05 -1.5671D 04 -2.1577D 01  3
1.2240000D 04     2.062115410D 00 -9.6188D-01 -7.9617D-01 -2.7100D-03  3.7123D 05 -1.6444D 04 -2.4187D 01  3
1.3200000D 04     2.062115047D 00 -9.5994D-01 -7.8012D-01 -2.6929D-03  3.7031D 05 -1.7200D 04 -2.6780D 01  3
1.4160000D 04     2.062114751D 00 -9.5837D-01 -7.6539D-01 -2.6770D-03  3.6939D 05 -1.7942D 04 -2.9358D 01  3
1.5120000D 04     2.062114506D 00 -9.5715D-01 -7.5175D-01 -2.6622D-03  3.6847D 05 -1.8670D 04 -3.1921D 01  3
1.6080000D 04     2.062114303D 00 -9.5624D-01 -7.3903D-01 -2.6483D-03  3.6755D 05 -1.9386D 04 -3.4470D 01  3
1.7040000D 04     2.062114132D 00 -9.5562D-01 -7.2710D-01 -2.6353D-03  3.6663D 05 -2.0089D 04 -3.7006D 01  3
1.8000000D 04     2.062114174D 00 -9.5526D-01 -7.1585D-01 -2.6232D-03  3.6571D 05 -2.0782D 04 -3.9530D 01  3
2.3670000D 04     2.060123681D 00 -9.5762D-01 -6.6008D-01 -2.5650D-03  3.6029D 05 -2.4675D 04 -5.4228D 01  3
2.9340000D 04     2.060065241D 00 -9.6462D-01 -6.1325D-01 -2.5213D-03  3.5485D 05 -2.8283D 04 -6.8643D 01  3
3.5010000D 04     2.060068753D 00 -9.7462D-01 -5.7157D-01 -2.4870D-03  3.4935D 05 -3.1640D 04 -8.2838D 01  3
4.0680000D 04     2.060068560D 00 -9.8656D-01 -5.3277D-01 -2.4586D-03  3.4379D 05 -3.4770D 04 -9.6857D 01  3
4.6350000D 04     2.060068170D 00 -9.9982D-01 -4.9564D-01 -2.4339D-03  3.3816D 05 -3.7685D 04 -1.1073D 02  3
5.2020000D 04     2.060067806D 00 -1.0140D 00 -4.5943D-01 -2.4117D-03  3.3245D 05 -4.0392D 04 -1.2446D 02  3
5.7690000D 04     2.060067518D 00 -1.0289D 00 -4.2368D-01 -2.3909D-03  3.2666D 05 -4.2895D 04 -1.3808D 02  3
6.3360000D 04     2.060067306D 00 -1.0442D 00 -3.8808D-01 -2.3709D-03  3.2078D 05 -4.5197D 04 -1.5158D 02  3
6.9030000D 04     2.060067158D 00 -1.0600D 00 -3.5240D-01 -2.3511D-03  3.1482D 05 -4.7296D 04 -1.6496D 02  3
7.4700000D 04     2.060067064D 00 -1.0761D 00 -3.1648D-01 -2.3312D-03  3.0876D 05 -4.9192D 04 -1.7824D 02  3
8.0370000D 04     2.060067015D 00 -1.0926D 00 -2.8019D-01 -2.3106D-03  3.0261D 05 -5.0884D 04 -1.9140D 02  3
```

Figure 6. Example of Output - Final Pages (Continued)

8.6040000D 04	2.060067005D 00	-1.1093D 00	-2.4342D-01	-2.2891D-03	2.9637D 05	-5.2369D 04	-2.0444D 02
9.1710000D 04	2.060067031D 00	-1.1263D 00	-2.0608D-01	-2.2663D-03	2.9003D 05	-5.3644D 04	-2.1735D 02
9.7380000D 04	2.060067091D 00	-1.1437D 00	-1.6810D-01	-2.2419D-03	2.8360D 05	-5.4705D 04	-2.3013D 02
1.0305000D 05	2.060067186D 00	-1.1614D 00	-1.2940D-01	-2.2156D-03	2.7706D 05	-5.5548D 04	-2.4277D 02
1.0872000D 05	2.060067318D 00	-1.1795D 00	-8.9903D-02	-2.1871D-03	2.7043D 05	-5.6171D 04	-2.5526D 02
1.1439000D 05	2.060067490D 00	-1.1980D 00	-4.9547D-02	-2.1558D-03	2.6369D 05	-5.6566D 04	-2.6757D C2
1.2006000D 05	2.060067709D 00	-1.2171D 00	-8.2566D-03	-2.1214D-03	2.5684D 05	-5.6731D 04	-2.7970D 02
1.2573000D 05	2.060067982D 00	-1.2369D 00	3.4041D-02	-2.0834D-03	2.4988D 05	-5.6658D 04	-2.9162D 02
1.3140000D 05	2.060068320D 00	-1.2574D 00	7.7427D-02	-2.0412D-03	2.4281D 05	-5.6343D 04	-3.0331D 02
1.3707000D 05	2.060068737D 00	-1.2789D 00	1.2199D-01	-1.9942D-03	2.3562D 05	-5.5778D 04	-3.1476D 02
1.4274000D 05	2.060069250D 00	-1.3014D 00	1.6781D-01	-1.9414D-03	2.2831D 05	-5.4957D 04	-3.2592D 02
1.4841000D 05	2.060069885D 00	-1.3252D 00	2.1501D-01	-1.8821D-03	2.2086D 05	-5.3872D 04	-3.3676D 02
1.5408000D 05	2.060070673D 00	-1.3507D 00	2.6369D-01	-1.8150D-03	2.1328D 05	-5.2516D 04	-3.4725D 02
1.5975000D 05	2.060071656D 00	-1.3780D 00	3.1398D-01	-1.7387D-03	2.0554D 05	-5.0879D 04	-3.5733D 02
1.6542000D 05	2.060072894D 00	-1.4077D 00	3.6604D-01	-1.6514D-03	1.9764D 05	-4.8952D 04	-3.6694D 02
1.7109000D 05	2.060074467D 00	-1.4401D 00	4.2003D-01	-1.5509D-03	1.8957D 05	-4.6724D 04	-3.7603D 02
1.7676000D 05	2.060076487D 00	-1.4760D 00	4.7612D-01	-1.4344D-03	1.8131D 05	-4.4185D 04	-3.8450D 02
1.8243000D 05	2.060079117D 00	-1.5162D 00	5.3454D-01	-1.2981D-03	1.7283D 05	-4.1321D 04	-3.9226D 02
1.8810000D 05	2.060082591D 00	-1.5616D 00	5.9552D-01	-1.1372D-03	1.6410D 05	-3.8118D 04	-3.9917D 02
1.9377000D 05	2.060087262D 00	-1.6137D 00	6.5930D-01	-9.4511D-04	1.5511D 05	-3.4562D 04	-4.0509D 02
1.9944000D 05	2.060093668D 00	-1.6743D 00	7.2617D-01	-7.1255D-04	1.4579D 05	-3.0636D 04	-4.0981D 02
2.0511000D 05	2.060102663D 00	-1.7461D 00	7.9638D-01	-4.2644D-04	1.3610D 05	-2.6321D 04	-4.1307D 02
2.1078000D 05	2.060115641D 00	-1.8325D 00	8.7012D-01	-6.7385D-05	1.2596D 05	-2.1598D 04	-4.1451D 02
2.1645000D 05	2.060134972D 00	-1.9391D 00	9.4734D-01	3.9455D-04	1.1528D 05	-1.6447D 04	-4.1364D 02
2.2212000D 05	2.060164897D 00	-2.0743D 00	1.0274D 00	1.0081D-03	1.0392D 05	-1.0850D 04	-4.0975D 02
2.2779000D 05	2.060213611D 00	-2.2515D 00	1.1084D 00	1.8579D-03	9.1677D 04	-4.7945D 03	-4.0177D 02
2.3346000D 05	2.060300213D 00	-2.4950D 00	1.1839D 00	3.1057D-03	7.8260D 04	1.7095D 03	-3.8795D 02
2.3913000D 05	2.060500808D 00	-2.8515D 00	1.2355D 00	5.0992D-03	6.3174D 04	8.5889D 03	-3.6518D 02
2.4480000D 05	2.060429955D 00	-3.4237D 00	1.1973D 00	8.7376D-03	4.5527D 04	1.5564D 04	-3.2714D 02
2.4552000D 05	2.041547898D 00	-3.5223D 00	1.1836D 00	9.3798D-03	4.3027D 04	1.6421D 04	-3.2061D 02
2.4624000D 05	2.041263253D 00	-3.6282D 00	1.1573D 00	1.0136D-02	4.0453D 04	1.7264D 04	-3.1360D 02
2.4696000D 05	2.041283863D 00	-3.7429D 00	1.1233D 00	1.0980D-02	3.7800D 04	1.8085D 04	-3.0600D 02
2.4768000D 05	2.041284451D 00	-3.8673D 00	1.0799D 00	1.1927D-02	3.5061D 04	1.8879D 04	-2.9776D 02
2.4840000D 05	2.041283833D 00	-4.0025D 00	1.0249D 00	1.2995D-02	3.2229D 04	1.9638D 04	-2.8880D 02
2.4912000D 05	2.041281330D 00	-4.1495D 00	9.5533D-01	1.4205D-02	2.9295D 04	2.0352D 04	-2.7901D 02
2.4984000D 05	2.041275529D 00	-4.3092D 00	8.6729D-01	1.5585D-02	2.6250D 04	2.1009D 04	-2.6830D 02
2.5056000D 05	2.041263994D 00	-4.4824D 00	7.5581D-01	1.7165D-02	2.3086D 04	2.1595D 04	-2.5652D 02
2.5128000D 05	2.041242461D 00	-4.6694D 00	6.1436D-01	1.8984D-02	1.9792D 04	2.2090D 04	-2.4352D 02
2.5200000D 05	2.041203296D 00	-4.8690D 00	4.3448D-01	2.1085D-02	1.6359D 04	2.2470D 04	-2.2912D 02
2.5272000D 05	2.041132463D 00	-5.0784D 00	2.0536D-01	2.3512D-02	1.2778D 04	2.2704D 04	-2.1308D 02
2.5344000D 05	2.041003433D 00	-5.2908D 00	-8.6313D-02	2.6307D-02	9.0454D 03	2.2751D 04	-1.9517D 02
2.5416000D 05	2.040765366D 00	-5.4932D 00	-4.5565D-01	2.9489D-02	5.1620D 03	2.2561D 04	-1.7511D 02
2.5488000D 05	2.040323873D 00	-5.6634D 00	-9.1701D-01	3.3027D-02	1.1428D 03	2.2073D 04	-1.5262D 02

Figure 6. Example of Output - Final Pages (Continued)

```
2.5560000D 05  2.040368224D 00 -5.7671D 00 -1.4778D 00  3.6790D-02 -2.9774D 03  2.1217D 04 -1.2749D 02
2.5584000D 05  2.031884771D 00 -5.7796D 00 -1.6873D 00  3.8059D-02 -4.3632D 03  2.0837D 04 -1.1851D 02
2.5608000D 05  2.031710094D 00 -5.7776D 00 -1.9045D 00  3.9300D-02 -5.7504D 03  2.0406D 04 -1.0923D 02
2.5632000D 05  2.031656140D 00 -5.7596D 00 -2.1294D 00  4.0503D-02 -7.1352D 03  1.9922D 04 -9.9651D 01
2.5656000D 05  2.031622028D 00 -5.7241D 00 -2.3602D 00  4.1650D-02 -8.5136D 03  1.9384D 04 -8.9792D 01
2.5680000D 05  2.031589498D 00 -5.6699D 00 -2.5945D 00  4.2722D-02 -9.8813D 03  1.8789D 04 -7.9665D 01
2.5704000D 05  2.031560303D 00 -5.5960D 00 -2.8296D 00  4.3699D-02 -1.1234D 04  1.8138D 04 -6.9293D 01
2.5728000D 05  2.031536224D 00 -5.5021D 00 -3.0625D 00  4.4563D-02 -1.2566D 04  1.7431D 04 -5.8699D 01
2.5752000D 05  2.031518859D 00 -5.3884D 00 -3.2903D 00  4.5297D-02 -1.3873D 04  1.6669D 04 -4.7913D 01
2.5776000D 05  2.031509388D 00 -5.2558D 00 -3.5096D 00  4.5887D-02 -1.5151D 04  1.5852D 04 -3.6968D 01
2.5800000D 05  2.031508384D 00 -5.1058D 00 -3.7177D 00  4.6325D-02 -1.6394D 04  1.4985D 04 -2.5899D 01
2.5824000D 05  2.031515714D 00 -4.9404D 00 -3.9118D 00  4.6606D-02 -1.7600D 04  1.4069D 04 -1.4744D 01
2.5848000D 05  2.031530565D 00 -4.7622D 00 -4.0901D 00  4.6730D-02 -1.8765D 04  1.3108D 04 -3.5409D 00
2.5872000D 05  2.031551588D 00 -4.5738D 00 -4.2508D 00  4.6703D-02 -1.9885D 04  1.2107D 04  7.6740D 00
2.5896000D 05  2.031577115D 00 -4.3781D 00 -4.3931D 00  4.6534D-02 -2.0960D 04  1.1070D 04  1.8865D 01
2.5920000D 05  2.031576209D 00 -4.1781D 00 -4.5167D 00  4.6235D-02 -2.1986D 04  1.0000D 04  3.0000D 01


           THE IMPULSES ARE (X,Y,Z,COMP., MAGN.)
-4.105143076125D-02 -9.108659063423D-01 -2.440160305245D-03  9.117937670804D-01
-1.974707070956D 00 -6.027685432548D-01  4.623538817041D-02  2.065172061583D 00

 TOTAL IMPULSE IS       2.976965828663D 00
```

Figure 6. Example of Output - Final Pages (Continued)

INTEGRATION OF EQUATIONS WITH INITIAL CONDITIONS DETERMINED BY QASLN

TIME	JAC. INT.	VELOCITIES(KM/SEC OR LD/LT)			POSITIONS(KM OR LD)		
2.400D 02	2.06216127D 00	-8.69474D-01	-1.47247D 00	-2.52684D-03	3.828760D 05	-4.357986D 03	9.403672D 00
4.800D 02	2.06216405D 00	-8.92905D-01	-1.43501D 00	-2.59988D-03	3.826644D 05	-4.706361D 03	8.788207D 00
7.200D 02	2.06216410D 00	-9.12486D-01	-1.39886D 00	-2.66077D-03	3.824477D 05	-5.046896D 03	8.156701D 00
9.600D 02	2.06216409D 00	-9.28713D-01	-1.36429D 00	-2.71104D-03	3.822267D 05	-5.378440D 03	7.511885D 00
1.200D 03	2.06216405D 00	-9.42057D-01	-1.33145D 00	-2.75215D-03	3.820022D 05	-5.701894D 03	6.856133D 00
1.440D 03	2.06216400D 00	-9.52946D-01	-1.30040D 00	-2.78543D-03	3.817747D 05	-6.017679D 03	6.191479D 00
1.680D 03	2.06216394D 00	-9.61763D-01	-1.27114D 00	-2.81208D-03	3.815449D 05	-6.326229D 03	5.519656D 00
1.920D 03	2.06216389D 00	-9.68839D-01	-1.24364D 00	-2.83313D-03	3.813132D 05	-6.627968D 03	4.842128D 00
2.160D 03	2.06216385D 00	-9.74459D-01	-1.21782D 00	-2.84950D-03	3.810800D 05	-6.923311D 03	4.160126D 00
2.400D 03	2.06216381D 00	-9.78868D-01	-1.19360D 00	-2.86194D-03	3.808456D 05	-7.212651D 03	3.474683D 00
2.640D 03	2.06216378D 00	-9.82268D-01	-1.17088D 00	-2.87110D-03	3.806102D 05	-7.496361D 03	2.786659D 00
2.880D 03	2.06216376D 00	-9.84833D-01	-1.14957D 00	-2.87754D-03	3.803741D 05	-7.774788D 03	2.096773D 00
3.120D 03	2.06216374D 00	-9.86705D-01	-1.12956D 00	-2.88171D-03	3.801375D 05	-8.048258D 03	1.405623D 00
3.360D 03	2.06216372D 00	-9.88007D-01	-1.11075D 00	-2.88399D-03	3.799006D 05	-8.317072D 03	7.137049D-01
3.600D 03	2.06216371D 00	-9.88837D-01	-1.09307D 00	-2.88472D-03	3.796633D 05	-8.581509D 03	2.143145D-02
4.560D 03	2.06214991D 00	-9.89225D-01	-1.03188D 00	-2.87679D-03	3.787137D 05	-9.600355D 03	-2.745228D 00
5.520D 03	2.06216374D 00	-9.86149D-01	-9.82757D-01	-2.85903D-03	3.777655D 05	-1.056656D 04	-5.498906D 00
6.480D 03	2.06216251D 00	-9.82261D-01	-9.42461D-01	-2.83720D-03	3.768206D 05	-1.149007D 04	-8.233266D 00
7.440D 03	2.06215205D 00	-9.78117D-01	-9.08722D-01	-2.81416D-03	3.758796D 05	-1.237818D 04	-1.094594D 01
8.400D 03	2.06216191D 00	-9.74128D-01	-8.79945D-01	-2.79132D-03	3.749426D 05	-1.323640D 04	-1.363652D 01
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1.032D 04	2.06216191D 00	-9.67261D-01	-8.33044D-01	-2.74857D-03	3.730791D 05	-1.487895D 04	-1.895406D 01
1.128D 04	2.06216194D 00	-9.64479D-01	-8.13484D-01	-2.72905D-03	3.721519D 05	-1.566911D 04	-2.158322D 01
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1.320D 04	2.06216200D 00	-9.60179D-01	-7.79812D-01	-2.69377D-03	3.703045D 05	-1.719766D 04	-2.678835D 01
1.416D 04	2.06216203D 00	-9.58609D-01	-7.65075D-01	-2.67787D-03	3.693835D 05	-1.793911D 04	-2.936665D 01
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1.608D 04	2.06216206D 00	-9.56473D-01	-7.38717D-01	-2.64915D-03	3.675453D 05	-1.938216D 04	-3.447997D 01
1.704D 04	2.06216207D 00	-9.55850D-01	-7.26786D-01	-2.63615D-03	3.666274D 05	-2.008554D 04	-3.701685D 01
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5.202D 04	2.06217441D 00	-1.01362D 00	-4.58072D-01	-2.41138D-03	3.324534D 05	-4.034458D 04	-1.244696D 02
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6.336D 04	2.06217472D 00	-1.04375D 00	-3.86745D-01	-2.37057D-03	3.207900D 05	-4.513412D 04	-1.515803D 02
6.903D 04	2.06217479D 00	-1.05949D 00	-3.51082D-01	-2.35078D-03	3.148275D 05	-4.722594D 04	-1.649653D 02
7.470D 04	2.06217483D 00	-1.07558D 00	-3.15180D-01	-2.33078D-03	3.087747D 05	-4.911494D 04	-1.782377D 02
8.037D 04	2.06217485D 00	-1.09197D 00	-2.78908D-01	-2.31019D-03	3.026298D 05	-5.079938D 04	-1.913952D 02

Figure 6. Example of Output - Final Pages (Continued)

8.604D 04	2.06217486D 00	-1.10866D 00	-2.42162D-01	-2.28867D-03	2.963912D 05	-5.227686D 04	-2.044335D 02
9.171D 04	2.06217487D 00	-1.12565D 00	-2.04851D-01	-2.26589D-03	2.900570D 05	-5.354443D 04	-2.173464D 02
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1.030D 05	2.06217487D 00	-1.16061D 00	-1.28221D-01	-2.21521D-03	2.770953D 05	-5.543566D 04	-2.427615D 02
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1.144D 05	2.06217486D 00	-1.19718D 00	-4.84349D-02	-2.15536D-03	2.637284D 05	-5.644054D 04	-2.675528D 02
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1.711D 05	2.06217414D 00	-1.43891D 00	4.20566D-01	-1.55075D-03	1.896740D 05	-4.654934D 04	-3.759905D 02
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1.881D 05	2.06217264D 00	-1.56025D 00	5.95722D-01	-1.13758D-03	1.642269D 05	-3.793675D 04	-3.991373D 02
1.938D 05	2.06217149D 00	-1.61229D 00	6.59362D-01	-9.45744D-04	1.552363D 05	-3.437997D 04	-4.050601D 02
1.994D 05	2.06216958D 00	-1.67283D 00	7.26058D-01	-7.13602D-04	1.459275D 05	-3.045380D 04	-4.097861D 02
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2.108D 05	2.06216016D 00	-1.83074D 00	8.69538D-01	-6.99620D-05	1.261180D 05	-2.141990D 04	-4.145041D 02
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2.549D 05	2.06222161D 00	-5.62533D 00	-8.54083D-01	3.23443D-02	1.719199D 03	2.226418D 04	-1.557761D 02

Figure 6. Example of Output - Final Pages (Continued)

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2.592D 05  2.06211313D 00 -4.26713D 00 -4.45266D 00  4.61082D-02 -2.150871D 04  1.059107D 04  2.427062D 01

CONVERGENCE PARAMETERS----RHO

8.90D-02 1.10D-01 7.83D-04 6.13D 02 5.91D 02 5.73D 00


    FOR THE INTEGRATED TRAJECTORY


               THE IMPULSES ARE (X,Y,Z,COMP., MAGN.)
-4.105143076125D-02 -9.108659063423D-01 -2.440160305245D-03  9.117937670804D-01
-2.063745437951D 00 -5.387615226184D-01  4.610816173199D-02  2.133409284103D 00

  TOTAL IMPULSE IS        3.045203051183D 00

    TIME AT END OF CASE =  93.517  SECONDS          ISTOP =    1
DVAR(3,4) ARE
  0.0             0.0             0.0             9.70000000D 02  0.0             0.0
  0.0             0.0             0.0             0.0             0.0             0.0
VBL(18) ARE
  3.00000000D 03 -4.00000000D 03  1.00000000D 01 -1.73200000D 04  1.00000000D 04  3.00000000D 01
 -7.90000000D-01 -5.92000000D-01  0.0            -2.23000000D 00 -3.96000000D 00  0.0
  0.0             7.20000000D 01  0.0             0.0             0.0             4.00091250D 04
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Figure 6. Example of Output - Final Pages (Continued)

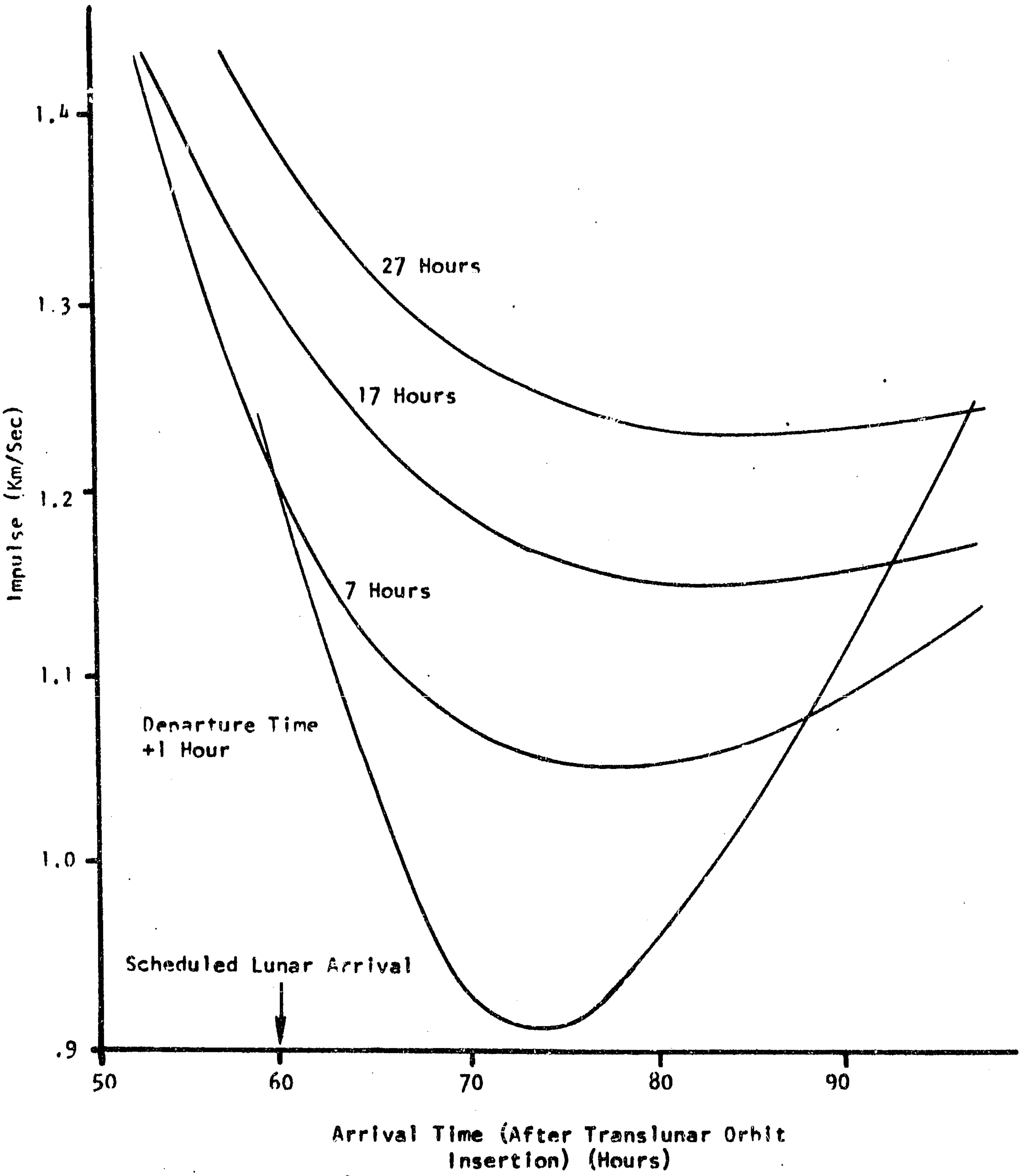

Figure 7. Abort Impulse to Lagrange Point, L1

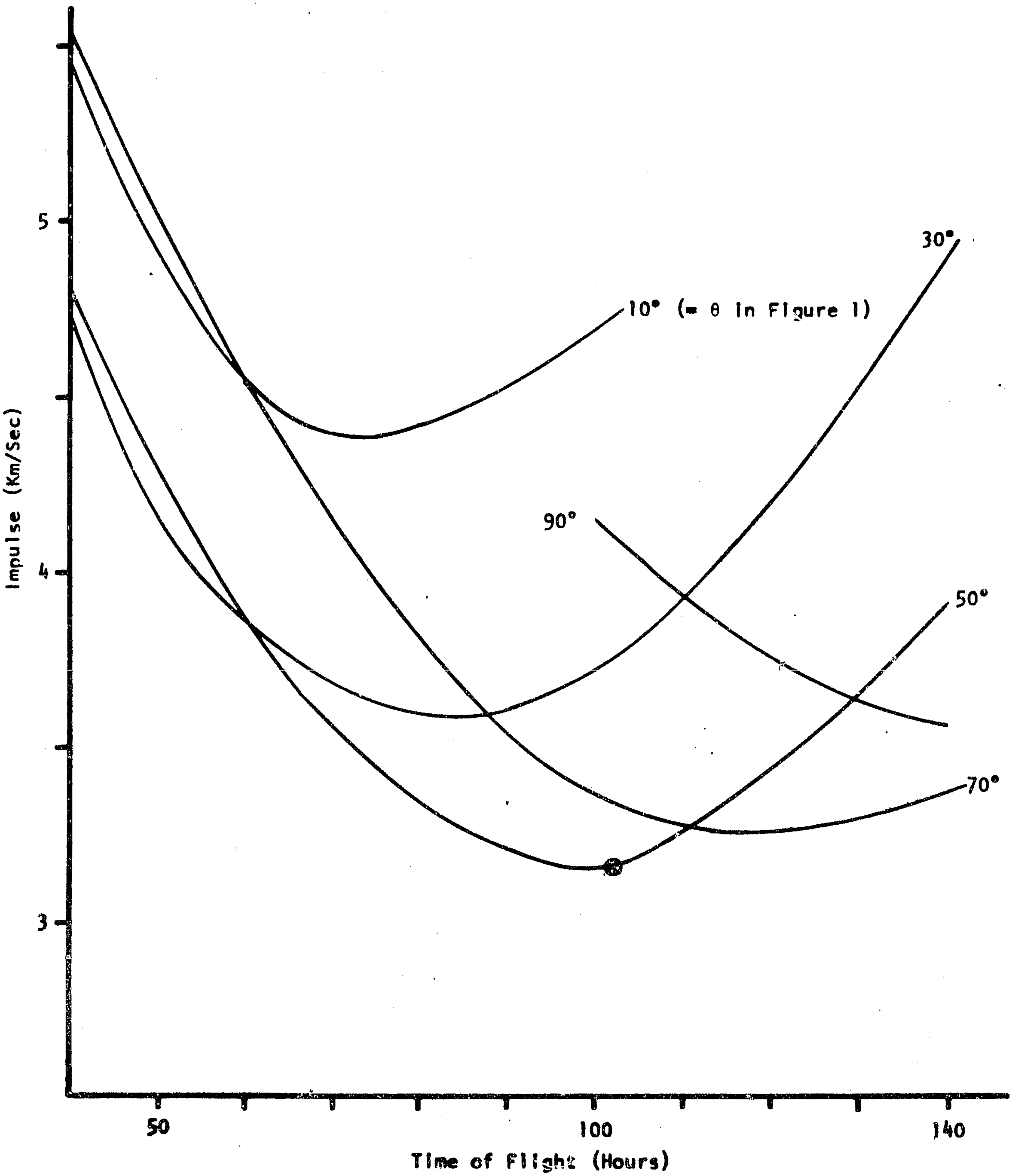

Figure 8. Impulsive Transfers to L1 From a Circular Earth Orbit

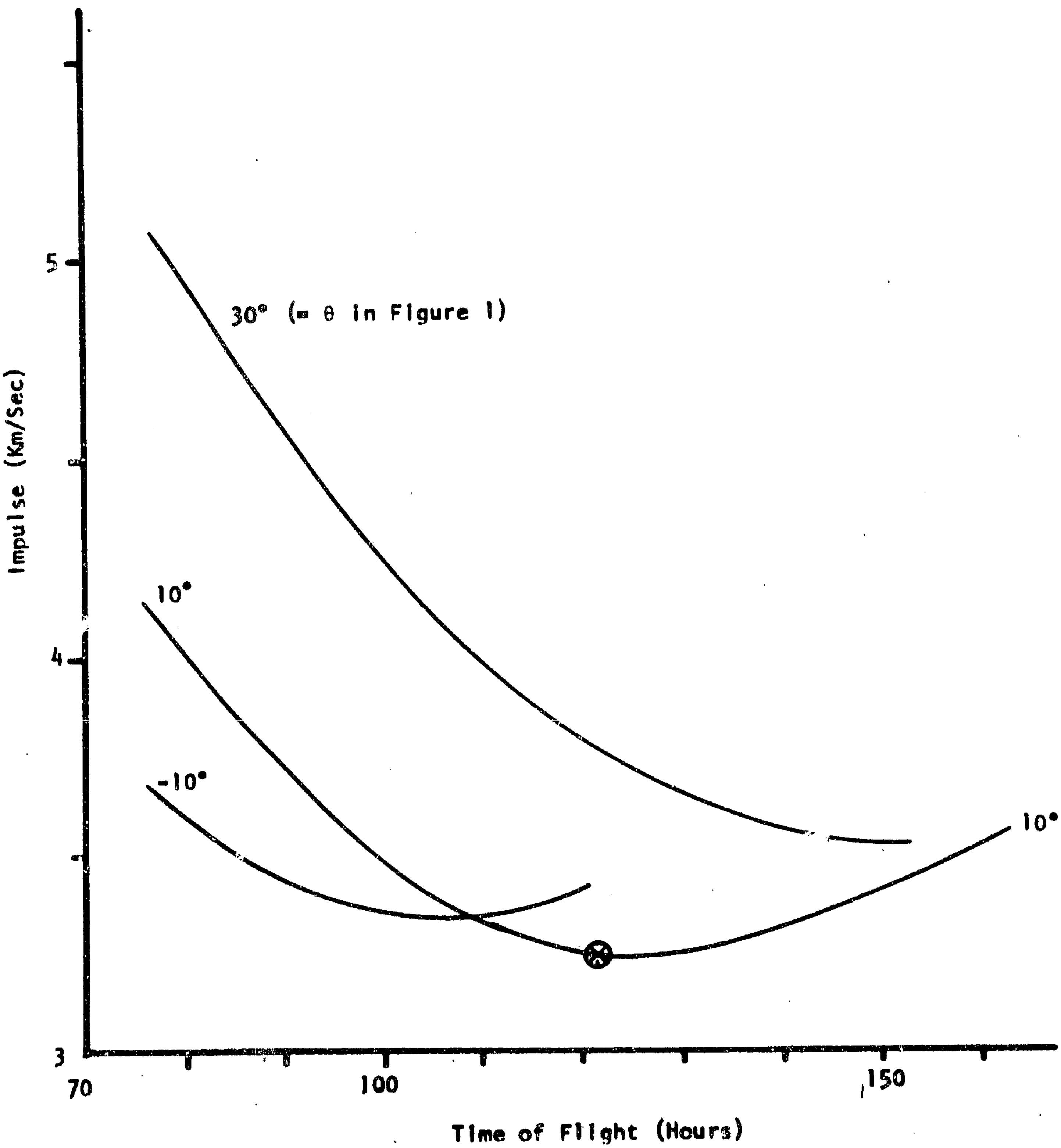

Figure 9. Impulsive Transfers to L5 From a Circular Earth Orbit

IV. SOME PRELIMINARY RESULTS

The computer program has been designed with a great deal of flexibility as regards input information form and trajectory type. Most of the input possibilities have been tested for proper boundary point computation and for first time history generation which are straightforward computations. Convergence of the quasilinearization routine upon a trajectory of the desired type does not always occur. Some trajectories are so sensitive that the gross errors of the initial time history computation cannot be corrected and the process will find a different shape trajectory or will diverge. Thus initial trials of planar moon to earth trajectories with ICØRC(1) = -2, and with ICØRC(2) = ±1 have failed to converge on the desired trajectory. It is not known whether this can be corrected with a better choice of parameters for the initial time history estimation. Initial trials for all other angular momentum possibilities have converged on the desired trajectory.

Three dimensional cases are generally slower to converge than two dimensional cases, for example, the two dimensional case with the same x, y coordinate boundary points as that of the sample output case given in Figures 5 and 6 converged in 8 iterations (compared to 10 for the 3-D case). More experience will be required to determine if an improved z component initial time history is needed and what form will be effective if it is. Three examples of the utility of this program for impulsive transfer studies are illustrated in Figures 7, 8, and 9. The first concerns transfers from a translunar trajectory to the Lagrange point L1 (between the earth and the moon) and the second and third show transfers from a 7000 km circular earth orbit to L1 and to L5 (the equilateral point with the negative value of y).

For the first illustration we suppose that during the course of a certain 60 hour flight to the moon it is desired to change the objective and to fly to the region of the Lagrange point between the earth and moon and to stop there as seen in the rotating coordinate system. The problem is to determine what impulses are required and to find minimum impulse cases if they exist. Results of the computation are summarized in Figure 7. The 60 hour earth-moon trajectory which departed from a point 6546.31 km from the earth (x = -10,065.24 km, y = -3702.31 km, θ = 34.4410°) and arrived at the point 1866.00 km from the moon on the side away from the earth was first determined. Then for a series of departure points from this trajectory (1 hour, 7 hours, 17 hours, and 27 hours after translunar insertion), the trajectories to the Lagrange point were determined for a series of flight times, and the impulses computed. The computer time involved was 43 seconds for the earth to moon trajectory and an average of 12.24 seconds/case for the 36 trajectories to the L1 point (not including loading time for the program). It can be seen (Figure 7) that the impulse required depends on the time of departure with the earliest decision to utilize the abort the best and that for each departure point there exists a minimum impulse.

The second and third illustrations are similar to the first one except that the initial orbit was a circular orbit at a radius of 7000 km about the earth and the departure points were specified by a series of values of the location angle (θ of B_o in Figure 1). Again the final condition was to be at rest in the rotating system at one of the Lagrange points. The results of the computations are shown in Figures 8 and 9. It is clear that a minimum impulse has been found for each maneuver. These are indicated by the symbol and the transfer characteristics are given in Table 6.

Table 6. Optimum Impulsive Transfers Circular Earth Orbit (7000 km) to Lagrange Points

Lagrange Point	Impulse (In Meters/Sec.)	Departure Point (θ in Figure 1)	Time of Flight (Hours)
L1	3159	56.7	103
L5	3228	5.5	122

V. CONCLUSION

A double precision computer program which uses quasilinearization to find impulsive transfers between given terminals in a given time in the reduced three-body problem has been shown to give accurate solutions with considerable speed. It accepts a wide variety of input trajectory plans and coordinate systems. It is designed to be used in impulsive transfer studies in this system of bodies and it has been shown to yield meaningful results in an application to the problem of reaching Lagrange points with two-impulse transfers from some given trajectory.

REFERENCES

1. McGill, R. and Kenneth, P., "Solution of Variational Problems by Means of a Generalized Newton-Raphson Operator," AIAA J., pp. 1761-1766 (October 1964).

2. Radbill, John R. and Gary A. McCue, "Generalized Quasilinearization Routines for Solving Nonlinear Boundary-Value Problems," North American Rockwell Corporation, SD 68-1055 (15 December 1968).