NASA TECHNICAL NOTE

NASA TN D-5217

c. 1

NASA TN D-5217

ANALYSIS OF ELECTRON ANALOG SIMULATION OF ION CONTAINMENT IN MAGNETIC FIELDS

by Nelson L. Milder and Harold R. Kaufman

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MAY 1969

ANALYSIS OF ELECTRON ANALOG SIMULATION OF ION CONTAINMENT IN MAGNETIC FIELDS

By Nelson L. Milder and Harold R. Kaufman

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

ABSTRACT

The π-theorem of dimensional analysis has been used to obtain similarity parameters characterizing the interaction of a charged-particle gas with a magnetic field. These parameters are used to derive scaling relations for simulating ion-containment systems with electron-containment systems (electron analogs). The scaling conditions are used to correlate the results of an electron-containment experiment to a similar ion-containment experiment, both reported in the literature. The analysis is extended to include a discussion of the possibility of collisionless plasma simulation. Plasma-containment simulation is considered from the viewpoint of the stability of static systems. The theoretical possibility of flute instability simulation is discussed.

ANALYSIS OF ELECTRON ANALOG SIMULATION OF ION CONTAINMENT IN MAGNETIC FIELDS

by Nelson L. Milder and Harold R. Kaufman

Lewis Research Center

SUMMARY

A physical model of a one-component charged fluid was used to develop similarity parameters. These parameters, obtained by means of the π-theorem of dimensional analysis, are assumed to characterize the macroscopic motion and containment of the fluid in a magnetic field. The similarity described is between an ion-containment system and an electron analog of such a system.

Scaling relations are derived from the similarity parameters. They are used to scale ion-containment systems to their electron analogs. These relations are expressed in terms of the electron to ion mass ratio m_e/m_i and the electron to ion experiment geometric length ratio l_e/l_i.

Mass scaling has not been explicitly introduced in previous analyses. However, the results reported herein are in general agreement with two cases reported in the literature. These are adiabatic particle motion in a magnetic field and the motion of a collisionless (Vlasov) charged-particle gas in a magnetic field. The scaling relations were used to correlate the results of an electron-model ion-prototype experiment reported in the literature. The scaling relations were found to yield good correlation. This indicates that, under controlled design simulation, dimensional analysis may well be applicable to interpretation of experimental results.

An extension of the one-component results to the description of a two-component plasma is discussed. The similarity described is between an ion plasma (ion kinetic energy much greater than electron kinetic energy) and its electron analog (electron kinetic energy much greater than ion kinetic energy). Here, the simulation of a hot collisionless ion plasma by means of a hot collisionless electron plasma is examined. It is demonstrated that simulation is possible for static systems. The stability of static collisionless plasmas is considered, and it is shown that simulation of at least one type of instability associated with such systems - the flute instability - is theoretically possible. This result is in agreement with recent experiments reported in the literature.

INTRODUCTION

This paper shows how dimensional analysis can be used in interpreting electron-containment experiments as analogs of ion-containment experiments. Such analyses can aid our understanding of charged-particle containment phenomena. It extends the application of limited experimental results to physically different systems.

Electrons have been used for some time to simulate phenomena associated with ion motion in magnetic fields. The lower electron inertia and lower magnetic-field strength requirements lead to simplifications in the design of the experiments and to substantial reduction in costs. As early as 1908, Birkeland studied charged-particle interaction with the Earth's magnetic field by injecting electrons toward a magnetized sphere (ref. 1). More recently, the effectiveness of various magnetic-trap configurations has been studied by means of electron injection into model field configurations (refs. 2 to 4). Interest in containing high-temperature ion plasmas for production of thermonuclear fusion power has led to experiments such as the Table Top IV (ref. 5). In the experiment described in reference 5, a hot electron plasma (electron temperature much larger than ion temperature) is trapped in an open-ended magnetic well.

Similarity parameters and scaling relations for plasma devices have been obtained by Janes (ref. 6) using the kinetic and Maxwell equations. Schindler (ref. 7) has used a similar approach to obtain similarity laws for the interaction of the solar wind with the magnetosphere. In the present report, the techniques of the π-theorem of dimensional analysis (refs. 8 and 9) are used to establish parameters of similarity between experiments in which electrons are used to simulate ion motion and the actual ion experiments. Such electron experiments can be referred to as electron analogs.

In the π-theorem approach, theoretical and experimental knowledge is used to select the pertinent variables. The π-theorem, together with these pertinent variables, permits derivation of dimensionless variables. These dimensionless variables can then be used to simulate an ion device with a much more tractable electron device. This technique is advantageous because (1) understanding an ion-plasma device can be attained without either the construction of the ion device or an explicit mathematical solution of its operation and (2) the performance of related systems (ion and electron) is readily correlated, aiding the systematic understanding of such related devices.

Herein, simple physical models are used to derive similarity parameters and scaling relations. The useful properties of these models are that (1) the dimensionless parameters can be checked with known equations (In this way the analog concept can be verified.), (2) the models can be used to interpret and correlate the results of charged-particle containment experiments reported in the literature (e.g., ref. 10), and (3) the models can be used to demonstrate the possibility of constructing plasma analogs in a limited sense.

A physical model of a one-component charged-particle fluid is presented and analysed. The model is used to obtain a set of physical variables from which appropriate dimensionless parameters are derived by means of the π-theorem of dimensional analysis. These parameters are used to establish a set of similarity conditions applicable to designing electron analogs of ion trapping experiments. The dimensionless, or similarity, parameters are interpreted in terms of quantities characterizing physical systems. The resultant similarity conditions are applied to three one-component physical systems. The analysis is then used to correlate the data of similar electron- and ion-containment experiments reported in the literature. Finally, the possibility of using dimensional analysis techniques in plasma simulation is discussed. Although explicit application of the technique to the problem of plasma containment is not included in this report, the possible simulation of the flute instability is considered.

PHYSICAL MODEL

Single-Species Charged Gas

Consider the nonrelativistic steady-state motion of a gas of charged particles in an arbitrary magnetic field. The set of equations governing the motion are the set of Maxwell equations,

$$\nabla \times \vec{E} = 0 \tag{1}$$

$$\nabla \cdot \vec{E} = \frac{nq}{\epsilon_0} \tag{2}$$

$$\nabla \cdot \vec{B} = 0 \tag{3}$$

$$\nabla \times \vec{B} = \mu_0 nq\vec{v} \tag{4}$$

and the momentum equation (neglecting gravitational body forces).

$$nm(\vec{v} \cdot \nabla)\vec{v} = nq(\vec{E} + \vec{v} \times \vec{B}) - \nabla p \tag{5}$$

(All symbols are given in appendix A; SI units (rationalized MKS system) are used throughout this report.) The kinetic stress tensor reduces to a scalar pressure $p = 1/3\ nmu^2$ when collisions are sufficient to produce a Maxwellian particle distribution with mean random speed much greater than the drift speed ($|\vec{u}| >> |\vec{v}|$). The total particle velocity $\vec{V}$ is given by the sum of the random velocity and the net drift velocity

$$\vec{V} = \vec{u} + \vec{v} \tag{6}$$

Here, $|\vec{u}| = \left(\overline{u^2}\right)^{1/2}$ is used for convenience, since the small constant factor discrepancy between rms and mean speed is unimportant in present dimensional considerations. It is now possible to use equations (1) to (5) to select the pertinent variables that describe the physical system. These variables and their dimensions are as follows (The fundamental dimensions are mass M, length L, time T, and charge Q.):

u	random or rms particle speed, L/T
v	drift speed of gas, L/T
B	magnetic-flux density, M/QT
l	characteristic length, L
m	mass of charged particle, M
n	density of charged particles, $1/L^3$
q	electric charge, Q
ϵ_0	free space permittivity, T^2Q^2/ML^3
μ_0	free space permeability, ML/Q^2

Here the quantity l is used to represent all geometric lengths. The justification for this will be considered when conditions of similarity are discussed. The electric field is assumed to arise solely from the charge distribution and is therefore not included as a separate parameter.

Application of π-Theorem

The preceding set of nine quantities describe the physical system by means of functional relations of the form

$$\varphi(u, v, B, l, n, m, q, \epsilon_0, \mu_0) = 0 \tag{7}$$

There are four fundamental dimensions in these nine quantities so that, according to the π-theorem, five dimensionless parameters can be formed. Functionally, one has

$$f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0 \tag{8}$$

The techniques of the π-theorem (refs. 8 and 9) are used to obtain the five dimensionless parameters. Making the choice of m, q, ϵ_0, and μ_0 for the primary quantities, the equations to be solved are

$$\pi_1 = u^{a_1} m^{b_1} q^{c_1} \epsilon_0^{d_1} \mu_0^{e_1} \tag{9}$$

$$\pi_2 = B^{a_2} m^{b_2} q^{c_2} \epsilon_0^{d_2} \mu_0^{e_2} \tag{10}$$

$$\pi_3 = l^{a_3} m^{b_3} q^{c_3} \epsilon_0^{d_3} \mu_0^{e_3} \tag{11}$$

$$\pi_4 = n^{a_4} m^{b_4} q^{c_4} \epsilon_0^{d_4} \mu_0^{e_4} \tag{12}$$

$$\pi_5 = v^{a_5} m^{b_5} q^{c_5} \epsilon_0^{d_5} \mu_0^{e_5} \tag{13}$$

The method of solution is to substitute the dimensions of the variables shown and to determine the values for all the exponents such that the right sides of relations (9) to (13) are dimensionless (refs. 8 and 9). In this manner, the resulting dimensionless parameters are

$$\pi_1 = \epsilon_0 \mu_0 u^2 \tag{14}$$

$$\pi_2 = \frac{\epsilon_0 \mu_0^3 B^2 q^6}{m^4} \tag{15}$$

$$\pi_3 = \frac{m l}{\mu_0 q^2} \tag{16}$$

$$\pi_4 = \frac{\mu_0^3 q^6 n}{m^3} \tag{17}$$

$$\pi_5 = \epsilon_0 \mu_0 v^2 \tag{18}$$

These parameters may be used in the form shown, or they may be rearranged to obtain new dimensionless parameters. The only limitations on the use of new parameters for the problem under consideration is that they be five in number and that they should be expressible as separate and distinct combinations of π_1, π_2, π_3, π_4, and π_5. The separate and distinct combinations chosen for this report, in order to facilitate physical interpretation, are

$$P_1 = \left(\frac{\pi_1}{\pi_5}\right)^{1/2} = \frac{u}{v} \tag{19}$$

$$P_2 = \frac{\pi_1^{1/2}}{\pi_2^{1/2}\pi_3} = \frac{mu}{B l q} \tag{20}$$

$$P_3 = \frac{\pi_1 \pi_4}{\pi_2} = \frac{\mu_0 mnu^2}{B^2} \tag{21}$$

$$P_4 = \frac{\pi_3 \pi_4}{\pi_1^2} = \frac{l nq^4}{\epsilon_0^2 m^2 u^4} \tag{22}$$

$$P_5 = \frac{\pi_3^2 \pi_4^2}{\pi_2} = \frac{\mu_0 l^2 n^2 q^2}{\epsilon_0 B^2} \tag{23}$$

Physical Interpretation

An understanding of the physical significance of the dimensionless parameters (eqs. (19) to (23)) is helpful in evaluating their uses and limitations. The first parameter P_1 is the ratio between the drift and rms speeds.

$$P_1 = \frac{\text{(Drift speed)}}{\text{(rms speed)}}$$

The grouping mu/Bq in the second dimensionless parameter (eq. (20)) is effectively (to within a constant of order one) the cyclotron radius of the charged particle. The remaining variable in P_2 is the characteristic length of the device. The dimensionless parameter P_2 can therefore be expressed as the physical ratio

$$P_2 = \frac{\text{(Cyclotron radius)}}{\text{(Characteristic length)}}$$

which is the well known adiabatic ratio.

The kinetic pressure of the charged-particle gas can be written as $mnu^2/3$. The transverse stress of a magnetic field is $B^2/2\mu_0$, so that the third dimensionless parameter P_3 can be expressed as the physical ratio

$$P_3 = \frac{3}{2} \frac{\text{(Kinetic pressure)}}{\text{(Magnetic stress)}}$$

This parameter (without the numerical factor) is generally denoted as β in the literature.

The physical meaning of the fourth dimensionless parameter requires a more extended analysis. This analysis is included in appendix B and shows that P_4 is a measure of the transverse diffusional effect of Coulomb collisions. To within a logarithmic factor, P_4 is the ratio of characteristic length to Coulomb mean free path.

$$P_4 \simeq \frac{\text{(Characteristic length)}}{\text{(Coulomb mean free path)}}$$

In the fifth dimensionless parameter (eq. (23)) the grouping $B^2/2\mu_0$ is the magnetic-field stress. The remaining grouping $n^2q^2l^2/\epsilon_0$ can be interpreted to within a constant K (which depends on the spatial shape of the charge distribution) as the electric field stress. The ratio of the two groupings of variables thus gives

$$P_5 = \left(\frac{1}{2} K\right) \frac{\text{(Electric stress)}}{\text{(Magnetic stress)}}$$

Similarity Conditions

This report defines conditions of similarity between an electron analog of an ion-containment system. To summarize the physical interpretation of the dimensionless parameters, similarity would be established between an ion device and its electron analog if five physical ratios were equal for the two devices. These ratios are

$$P_1 = \frac{\text{(Drift speed)}}{\text{(rms speed)}}$$

$$P_2 = \frac{\text{(Cyclotron radius)}}{\text{(Characteristic length)}}$$

$$P_3 = \frac{3}{2}\frac{\text{(Kinetic pressure)}}{\text{(Magnetic stress)}}$$

$$P_4 = \frac{\text{(Characteristic length)}}{\text{(Coulomb mean free path)}}$$

$$P_5 = \frac{K}{2}\frac{\text{(Electric stress)}}{\text{(Magnetic stress)}}$$

The ratios of geometric lengths can be included as additional dimensionless parameters in an analysis of this type. The approach used herein for establishing similarity, however, is that all geometric lengths must scale by the same factor. For example, if the length of an electron analog device is half that of the ion prototype, then all other electron analog dimensions should also be one-half of the corresponding ion-prototype dimensions. For this reason, only one length scaling ratio need be considered between ion and analog devices.

Application of Similarity Conditions to Single-Species Gas

The similarity discussed in the preceding section can be used to set conditions for designing electron analog experiments. Three possible applications are considered in which the similarity conditions can be specialized to yield appropriate scaling relations. The procedure for deriving scaling relations is presented in appendix C. In each case the applicability of relations (19) to (23) is discussed.

High-density charged-particle gas trapping in magnetic field. - In this case the complete set of equations (19) to (23) are needed. The appropriate scaling relations are (appendix C)

$$\frac{n_e}{n_i} = Z^6 M^3 \tag{24}$$

$$\frac{B_e}{B_i} = Z^3 M^2 \tag{25}$$

$$\frac{v_e}{v_i} = \frac{u_e}{u_i} = 1 \tag{26}$$

$$\frac{l_i}{l_e} = Z^2 M \tag{27}$$

Here M is the ratio of electron mass to ion mass $m_e/m_i \approx 1/2000$. It is apparent that, for this case and for reasonable ion model dimensions, analog dimensions become so large that experiments are impractical. Physically, the difficulty arises from considering only a single-species self-interacting gas of charged particles. For this case, large electric stresses can develop as a result of space charge buildup, and the parameter P_5 of relation (23) is particularly significant.

Low-density, low-temperature case. - In terms of the physical model discussed previously, this case corresponds to the situation in which $n \rightarrow 0$ and $u \rightarrow 0$. The Maxwell equations then are not involved in the calculations and the equation of motion of the charged particle becomes

$$m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}) \tag{28}$$

Studies involving the injection of electrons into magnetic fields have been used to evaluate the trapping characteristics of various field configurations (refs. 2 to 4). Studies have also been used as a basis for checking the validity of the adiabatic theory of particle motion in magnetic fields (refs. 11 to 13). In terms of the present simulation analysis, these electron-injection experiments can be related, through scaling, to corresponding ion injection and trapping experiments. This is done by specializing parameters (19) to (23) to this situation.

$$P_1 \rightarrow 0 \tag{29}$$

$$P_3 = P_4 = P_5 = 0 \tag{30}$$

The remaining parameter P_2 corresponds to the scale factor derived by Northrop (ref. 14). By setting $P_{2i} = P_{2e}$ and using $P_{1i} = P_{1e}$, the following relation is obtained:

$$\frac{v_e}{v_i} = \frac{m_i B_e l_e}{m_e B_i l_i Z} = \frac{u_e}{u_i} \tag{31}$$

where

$$q_i = Ze$$

Thus, for example, one can independently scale mass, length, and magnetic-field strength. The velocity scaling will then depend on these three scaling factors.

Low-density, collisionless case. - This case corresponds to the situation in which the Coulomb mean free path exceeds all characteristic lengths of the system. The equations governing this case are the set of Maxwell equations and the Vlasov equation. The scaling laws have been derived by Schindler (ref. 7) using the complete set of equations. Scaling relations can be obtained from the present analysis, again, by specializing the similarity parameters (19) to (23). For this case $P_4 = 0$ and only P_1, P_2, P_3, and P_5 need be kept constant for scaling. The scaling relations are

$$\frac{v_e}{v_i} = \frac{u_e}{u_i} = 1 \tag{32}$$

$$\frac{n_e}{n_i} = Z^2 M L^{-2} \tag{33}$$

$$\frac{B_e}{B_i} = Z M L^{-1} \tag{34}$$

These results are in agreement with those of reference 7.

CORRELATION WITH EXPERIMENT

It is of interest to attempt to correlate this similarity analysis with experiment. The experiment must permit the correlation of data obtained from a scaled electron analog of an ion-injection-trapping machine to the data obtained from the actual ion prototype.

Such an experiment has been reported in reference 10. Here, an electron-injection experiment was studied preliminary to the performance of an ion-injection experiment. Both experiments consisted of the injection of charged particles parallel to the axis of a cusped magnetic-field configuration and measuring the trapping characteristics of the configuration.

The electron model was geometrically scaled by a factor of two over the ion prototype. A 185-eV electron beam was injected into cusped fields with maxima at about 90×10^{-4} and 135×10^{-4} tesla. For the case of 90×10^{-4} tesla, the nonadiabatic confinement time (time required for an electron to escape as a result of magnetic-field gradients only) was found to be 0.245 microsecond. Prolonged containment was attained by electrostatically closing the cusp leaks, so that electron-containment times for $B_{max} = 135\times10^{-4}$ tesla ranging from about 1.43 to 5 microseconds were realized, depending on background gas pressure. The electrostatic field was designed so as not to penetrate into the trapping region.

The ion prototype experiment used proton injection into similar cusped fields with maxima ranging from 1.3 to 1.7 tesla. Beam injection energy was 17 keV. The energy distribution of trapped ions was not determined; however, estimates of the mean energy were of the order of several keV. Containment times depended on background pressure and ranged from 12 to about 60 microseconds. Extrapolation of the data to zero background pressure yielded a nonadiabatic trapping time of about 4 microseconds.

To examine these results within the framework of the present analysis, it is necessary to use the similarity parameters (19) to (23). Intuitively, one might initially suspect that Coulomb collisions in this experimental environment might be negligible (i.e., $P_4 = 0$) since charged-particle densities are relatively low. However, the scaling relations derivable from the condition $P_4 \approx 0$ do not result in correlation between the electron and ion experiments. This lack of correlation is consistent with the fact that Coulomb collisions, although a small effect, can be the dominant source of charged-particle diffusion. In order to correlate these experiments by means of the present analysis, it is necessary to assume that the parameter P_5 vanishes. This parameter is the ratio between electric- and magnetic-field stresses, and can be shown to be essentially the ratio $(v/c)^2$. Thus requiring P_5 to vanish, in effect, implies that the condition $v \ll c$ holds, which is the case for most plasma experiments.

The scaling relations for this case are obtained by a procedure analogous to that given in appendix C.

$$\frac{n_i}{n_e} = Z^{-2}M^{-1}L^2 \tag{35}$$

$$\frac{B_i}{B_e} = Z^{-1/2}M^{-3/4}L^{5/4} \tag{36}$$

$$\frac{\mathscr{E}_i}{\mathscr{E}_e} = ZM^{-1/2}L^{1/2} \tag{37}$$

In steady state, time scales as l/v, so that

$$\frac{t_i}{t_e} = Z^{-1/2}M^{-1/4}L^{-5/4} \tag{38}$$

For values of $L = 1/2$ and $M^{-1} = 1836$, the scaling relations become $(Z = 1)$

$$\frac{n_i}{n_e} = 459$$

$$\frac{\mathscr{E}_i}{\mathscr{E}_e} = 30$$

$$\frac{B_i}{B_e} = 118$$

$$\frac{t_i}{t_e} = 15.6$$

where $\mathscr{E}_i$ and $\mathscr{E}_e$ are ion and electron kinetic energies, respectively. Table I compares calculations using these relations with the data of reference 10. The data obtained from the electron analog experiment were used to calculate corresponding ion prototype values. The calculated values are then compared with measurements reported from the ion injection experiment. Particle containment times were measured in reference 10 for both the electron- and ion-trapping experiments. The nonadiabatic escape time given for the ion-trapping experiment was obtained by extrapolating the experimental containment time curve (ref. 10) to zero background pressure.

TABLE I. - COMPARISON OF CALCULATION TO EXPERIMENTS OF REFERENCE 10

Item	Containment number density, m^{-3}	Mean particle energy, eV	Magnetic flux density, T	Containment times	
Electron analog (experimental data, ref. 10)	$>10^{12}$	185	90×10^{-4} to 135×10^{-4}	Measured with leaks open (nonadiabatic escape), 0.245 μsec	Measured with leaks closed (diffusional escape), 1.43 to 5 μsec
Ion prototype (experimental data, ref. 10)	5×10^{14} to 5×10^{15}	$>10^{3}$ to 17×10^{3}	1.3 to 1.7	Extrapolated to zero pressure (nonadiabatic escape), 4 μsec	Measured as function of pressure (differential escape), 20 to 50 μsec
Ion prototype (calculated from electron analog data)	$>4.6\times10^{14}$	5.6×10^{3}	1.1 to 1.6	Nonadiabatic escape, 3.8 μsec	Diffusional escape, 22 to 78 μsec

Experimentally, the containment time increased with background pressure. Within the framework of the present analysis, this time varies as l/v. Momentum-transfer collisions between charged particles and background particles would effectively decrease v, thus increasing t. Such collisions result in the diffusional losses observed in the experiments.

The good agreement between calculated and measured ion-prototype variables would seem to indicate that correlation between an electron analog of an ion-trapping experiment using dimensional analysis techniques is indeed possible.

PLASMA SIMULATION

So far we have considered systems containing positively or negatively charged particles only. It may be even more interesting to determine whether these similarity parameters can be used to scale plasma systems; if so, what are the limitations of such a simulation?

For purposes of discussion, one can distinguish between an ion plasma and an electron plasma. The ion plasma is characterized by ion energy much greater than electron energy; that is, electrons serve as space-charge neutralizing background particles. For an electron plasma, the situation is reversed (electron energy is much greater than ion energy). Herein, the conditions for which an electron plasma can be regarded as the analog of an ion plasma will be determined.

In the following analysis, two-component plasmas are considered. The two species are regarded as interacting fluids, and the appropriate hydrodynamic equations are investigated. A complete set of equations describing these systems are as follows:

Momentum equations:

$$n_{\alpha s} m_{\alpha s}\left(\frac{\partial \vec{v}_{\alpha s}}{\partial t_s} + \vec{v}_{\alpha s} \cdot \nabla_s \vec{v}_{\alpha s}\right) = n_{\alpha s} q_{\alpha s}(\vec{E} + \vec{V}_{\alpha s} \times \vec{B}_s) - \nabla_s p_{\alpha s} - \vec{P}_{\alpha\beta, s} \tag{39}$$

Continuity equations:

$$\nabla_s \cdot (n_{\alpha s} \vec{v}_{\alpha s}) + \frac{\partial n_{\alpha s}}{\partial t_s} = 0 \tag{40}$$

Maxwell equations:

$$\nabla_s \cdot \vec{E}_s = \frac{1}{\epsilon_0} \sum_\alpha n_{\alpha s} q_{\alpha s} \tag{41}$$

$$\nabla_s \cdot \vec{B}_s = 0 \tag{42}$$

$$\nabla_s \times \vec{E}_s = -\frac{\partial \vec{B}_s}{\partial t_s} \tag{43}$$

$$\nabla_s \times \vec{B}_s = \mu_0 \sum_\alpha n_{\alpha s} q_{\alpha s} \vec{v}_{\alpha s} + \frac{1}{c^2} \frac{\partial \vec{E}_s}{\partial t_s} \tag{44}$$

where

$$\vec{B}_s = \mu_0 \vec{H}_s \qquad \vec{D}_s = \epsilon_0 \vec{E}_s$$

Equations of state:

$$p_{\alpha s} = \frac{1}{3} n_{\alpha s} m_{\alpha s} u_{\alpha s}^2 \tag{45}$$

In these equations the subscripts α and β denote either ions or electrons ($\alpha \neq \beta$) and s denotes the system (ion plasma, s = 1; electron plasma, s = 2).

Now, consider the following problem. Under what conditions can the previously derived similarity parameters be used to transform system $s = 1$ equations into system $s = 2$ equations, leaving the form of the resulting equations invariant? The analysis will again be restricted to steady-state systems. This eliminates all $\partial/\partial t$ terms in the equations. Two sets of physical variables must be considered for each plasma system. If α denotes the primary species and β the background species, then the two-species variables are $n_{\alpha s}$, $n_{\beta s}$, $\vec{v}_{\alpha s}$, $\vec{v}_{\beta s}$, $u_{\alpha s}$, $u_{\beta s}$, $m_{\alpha s}$, $m_{\beta s}$. ($P_{\alpha\beta, s}$ is considered separately.) In order to bring the problem into the realm of the one-component similarity parameters (19) to (23), equations (39) to (43) must be appreciably simplified. To begin with, consider the term $P_{\alpha\beta, s}$. This term represents the momentum transferred by collisions per unit time between the primary charged species α and the background species β in system s. From conservation of momentum, we have that $P_{\alpha\beta, s} = - P_{\beta\alpha, s}$. In the present case it will be assumed that collisional mean free paths are sufficiently long to permit neglect of these momentum-transfer terms. Thus the analysis which follows holds for collisionless plasmas. Additional assumptions required are as follows:

(1) Electron and ion number densities are equal for both $s = 1$ and $s = 2$.

(2) $u_{\beta s}/u_{\alpha s} << m_e/m_i$

(3) $\vec{v}_{\alpha s} >> \vec{v}_{\beta s}$

(4) Second order terms in the equation of motion for the background species are negligible.

(5) $q_{\alpha s} = - q_{\beta s}$ (e.g., electrons and protons)

Equations (39) to (45) then reduce to the following set:

Momentum equations:

$$n_{\alpha s} m_{\alpha s} \vec{v}_{\alpha s} \cdot \nabla_s \vec{v}_{\alpha s} = n_{\alpha s} q_{\alpha s} (\vec{E}_s + \vec{v}_{\alpha s} \times \vec{B}_s) - \nabla_s p_{\alpha s} \tag{46}$$

$$0 = n_{\beta s} q_{\beta s} (\vec{E}_s + \vec{v}_{\beta s} \times \vec{B}_s) - \nabla_s p_{\beta s} \tag{47}$$

Continuity equations:

$$\nabla_s \cdot (n_{\alpha s} \vec{v}_{\alpha s}) = \nabla_s \cdot (n_{\beta s} \vec{v}_{\beta s}) = 0 \tag{48}$$

Maxwell equations:

$$\nabla_s \cdot \vec{E}_s = 0 \tag{49}$$

$$\nabla_s \cdot \vec{B}_s = 0 \tag{50}$$

$$\nabla_s \times \vec{E}_s = 0 \tag{51}$$

$$\nabla_s \times \vec{B}_s = \mu_0 n_{\alpha s} \vec{v}_{\alpha s} q_{\alpha s} \tag{52}$$

Equations of state:

$$\left.\begin{aligned} p_{\alpha s} &= \frac{1}{3} m_{\alpha s} n_{\alpha s} u_{\alpha s}^2 \\ \text{and} \qquad & \\ p_{\beta s} &= \frac{1}{3} m_{\beta s} n_{\beta s} u_{\beta s}^2 \end{aligned}\right\} \tag{53}$$

From assumption (2), and equation (53) the partial pressures $p_{\alpha s}$ and $p_{\beta s}$ are such that $p_{\alpha s} >> p_{\beta s}$. Adding the momentum equations (46) and (47) yields the total momentum conservation equation

$$n_s m_{\alpha s}(\vec{v}_{\alpha s} \cdot \nabla_s \vec{v}_{\alpha s}) = (\vec{v}_{\alpha s} - \vec{v}_{\beta s}) \times \vec{B}_s - \nabla_s p_s \tag{54}$$

The only Maxwell equation involved is

$$\nabla_s \times \vec{B}_s = \mu_0 n_s q_s(\vec{v}_{\alpha s} - \vec{v}_{\beta s}) \tag{55}$$

Note that the change of variables $p_s = p_{\alpha s} + p_{\beta s} \simeq p_{\alpha s}$, $n_s = n_{\alpha s} = n_{\beta s}$, and $q_s = q_{\alpha s} = - q_{\beta s}$ has been made. From assumption (3), we have that $(\vec{v}_{\alpha s} - \vec{v}_{\beta s}) \approx \vec{v}_{\alpha s}$ in equations (54) and (55). In this manner the complete set of plasma equations can be reduced to one involving single-particle variables only, and the similarity parameters (19) to (23) are thus applicable to such systems. For nonrelativistic particle speeds, P_5 can be neglected (see the argument given in the Experimental Correlation section). The scaling relations appropriate to this plasma system are as follows:

$$\frac{n_1}{n_2} = Z^{-2}M^{-1}L^2 \tag{56}$$

$$\frac{B_1}{B_2} = Z^{-1/2}M^{-3/4}L^{5/4} \tag{57}$$

$$\frac{|\vec{v}_{i1} - \vec{v}_{e1}|}{|\vec{v}_{e2} - \vec{v}_{i2}|} \approx \frac{v_{i1}}{v_{e2}} = Z^{1/2}M^{1/4}L^{1/4} = \frac{u_{i1}}{u_{e2}} \tag{58}$$

where $s = 1$ denotes the ion plasma and $s = 2$, the electron analog,

$$M = \frac{m_e}{m_i}$$

and

$$L = \frac{l_2}{l_1}$$

Using the steady-state time scaling $t \sim l/v$ results in

$$\frac{t_1}{t_2} = Z^{-1/2}M^{-1/4}L^{-5/4} \tag{59}$$

By solving for all the quantities with subscript 1 in terms of subscript 2 quantities and substituting into equations (54) and (55) for $s = 1$ (noting that $\nabla_1 = L\nabla_2$), the appropriate $s = 2$ equations are obtained. They are identical in form to the $s = 1$ equations. Thus the total momentum, continuity, and Maxwell equations for the described plasma systems are invariant under the proposed transformations (and assumptions).

Possible Simulation of Plasma-Containment Systems

Of great interest would be a rigorous demonstration of the possibility of using hot electron plasma containment to simulate hot ion-containment systems. Difficulties are encountered from the fact that many observed and predicted instabilities associated with such systems result from locally, or microscopically, developed growth mechanisms which cannot be simply correlated with macroscopic variables. For the dimensional analysis used in this report, the complexity of the relation between instability growth mechanisms and physical variables need not be known exactly. However, it is essential to know all the variables on which these mechanisms depend in order to obtain appropriate similarity parameters. There does exist a class of instabilities for which the nine quantities of equation (7) are applicable, and thus the scaling parameters derived herein can be used to describe their simulation.

Simulation of Flute Instability

One type of hydromagnetic instability which is known to depend, in part, on the magnetic-field configuration is the interchange, or flute, instability. This instability has been extensively considered in the literature (refs. 15 to 17). An energy principle has been used (ref. 17) to derive the following stability criterion for open-ended mirror systems.

$$\int \frac{d\ell}{rRB^2} > 0 \tag{60}$$

Here, the integration is performed along a magnetic-field line. The radius of curvature R of a field line is defined as positive at the mirrors. The radius r is measured from the symmetry axis. Consider this integral in an analog experiment. The scaling relation (57) can be used to derive the following relation between model and prototype

$$\int \frac{d\ell_e}{r_e R_e B_e^2} = \left(\frac{L}{M}\right)^{3/2} \int \frac{d\ell_i}{r_i R_i B_i^2} \tag{61}$$

Since the scale factor $(L/M)^{3/2}$ is always positive, it is evident that, if the left side of equation (61) is positive, the integral on the right side will also be positive. Thus, simulation of flute stabilization should be theoretically possible for mirror systems. This conclusion is in qualitative agreement with results obtained from recent Table Top IV experiments (ref. 5). Therein a linear quadrupole magnetic field superimposed on an axial field stabilized a hot electron plasma against the flute. Although scaling was not rigorously satisfied, the authors concluded that their results (including a measured instability) may have been the electron analog of hot ion plasma, quadrupole-stabilized systems such as DCX, OBRA, PHOENIX, and ALICE.

CONCLUSIONS

A physical model of a one-component charged fluid was used to develop similarity parameters. These parameters, obtained by means of the π-theorem of dimensional analysis, are assumed to characterize the macroscopic motion and containment of the fluid in a magnetic field. The similarity described is between an ion-containment system and an electron analog of such a system.

Scaling relations are derived from the similarity parameters. They are used to scale ion-containment systems to their electron analogs. These relations are expressed in terms of the electron to ion mass ratio m_e/m_i and the electron to ion experiment geometric length ratio l_e/l_i.

Mass scaling (e. g., replacing protons by electrons) has not been explicitly introduced in previous analyses. However, the results reported herein are in general agreement with two cases reported in the literature. These are adiabatic particle motion in a magnetic field and the motion of a collisionless (Vlasov) charged-particle gas in a magnetic field. The scaling relations were used to correlate the results of an electron model-ion prototype experiment reported in the literature. The scaling relations were found to yield good correlation. This indicates that, under controlled design simulation, dimensional analysis may well be applicable to interpretation of experimental results.

An extension of the one-component results to the description of a two-component plasma is discussed. The similarity described is between an ion plasma (ion kinetic energy much greater than electron kinetic energy) and its electron analog (electron kinetic energy much greater than ion kinetic energy). Here, the simulation of a hot collisionless ion plasma by means of a hot collisionless electron plasma is examined. It is demonstrated that simulation is possible for static systems.

The stability of static collisionless plasmas in a magnetic field is also considered, and it is shown that simulation of one type of instability associated with such systems - the flute instability - is theoretically possible. This result is in agreement with recent experiments reported in the literature.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 27, 1969
120-26-02-01-22.

APPENDIX A

SYMBOLS

$a_{j=1,5}$	exponents in relations (9) to (13)
B	magnetic-flux density
b	impact parameter in appendix B
$b_{j=1,5}$	exponents in relations (9) to (13)
b_0	distance of closest approach, $q^2/4\pi\epsilon_0 m v^2$
c	speed of light
$c_{j=1,5}$	exponents in relations (9) to (13)
$\vec{D}$	displacement current
$d_{j=1,5}$	exponents in relations (9) to (13)
$\vec{E}$	electric field
$\mathscr{E}$	particle kinetic energy in eq. (37)
$e_{j=1,5}$	exponents in relations (9) to (13)
$\vec{H}$	magnetic-field intensity
K	geometrical shape factor in P_5
L	length ratio
ℓ	length measured along field line
l	characteristic length of system
l_E	effective cut-off length for Coulomb interactions
M	mass ratio
m	charged-particle mass
n	charged-particle density
P	momentum transfer term
$P_{j=1,5}$	dimensionless parameters defined by relations (19) to (23)
p	pressure
q	electric charge
R	radius of curvature of a field line

r	radius measured from symmetry axis in eq. (60)
t	time
u	rms speed
$\vec{u}$	rms velocity
$\vec{V}$	total particle velocity
v	drift speed or particle speed
$\vec{v}$	drift velocity
$\Delta v_{\perp}$	total velocity change averaged over many encounters
$(\Delta v_{\perp}/v)^2$	dimensionless quantity proportional to diffusion coefficient (herein referred to as a scattering parameter)
$\delta v_{\perp}$	incremental change in velocity normal to initial velocity direction resulting from a single binary encounter
w	mean relative speed of incident test particle in appendix B
Z	electronic charge number
ϵ_0	free-space permittivity
λ_c	Coulomb mean free path, $4\pi\epsilon_0^2(mv^2)^2/q^2 \ln \Lambda$
μ_0	free-space permeability
$\pi_{j=1,5}$	dimensionless parameters defined by relations (14) to (18)
φ	generalized function defined by eq. (7)

Subscripts:

e	electrons in analog
i	ions in simulated system
s	plasma system
α	electrons or ions
β	electrons or ions ($\alpha \neq \beta$)

APPENDIX B

THE PHYSICAL INTERPRETATION OF P_4

Consider a charged particle with mean relative speed w traversing a gas of like particles of uniform density n. The mean kinetic energy of the gas is assumed to be much less than the kinetic energy of the incident particle, so that the mean relative speed w is very nearly the incident particle speed v. As a result of Coulomb collisions with the background particles, the incident particle will suffer a cumulative large angle deflection after traversing a characteristic length l of the gas. The total deflection is measured by the quantity $(\Delta v_\perp/v)^2$, where $\Delta v_\perp$ is the total deflection normal to the incident direction. Herein the quantity $(\Delta v_\perp/v)^2$ is defined as a scattering parameter which can be related to a diffusion coefficient.

For a binary encounter between two charged particles, the incremental change in velocity normal to the incident direction is given by

$$\frac{\delta v_\perp}{v} = \frac{q^2}{2\pi\epsilon_0 m b v^2} \tag{B1}$$

where b is the impact parameter. Squaring equation (B1) yields

$$\left(\frac{\delta v_\perp}{v}\right)^2 = \frac{q^4}{4\pi^2\epsilon_0^2 m^2 b^2 v^4} \tag{B2}$$

Denote by n the background density resulting from some space charge distribution established in a time of the order of l/v. Then the scattering parameter is given by

$$\left(\frac{\Delta v_\perp}{v}\right)^2 = \int_{b_0}^{l_E} \left(\frac{\delta v_\perp}{v}\right)^2 n\, d\tau \tag{B3}$$

The volume element $d\tau$ is that swept out by the test particle as it passes through the scatterers of uniform density n. Thus,

$$d\tau = 2\pi l b\, db$$

It is known from energy considerations that the lower limit on the integral must approximate the distance of closest approach

$$b_0 = \frac{q^2}{4\pi\epsilon_0 m v^2}$$

The upper limit l_E denotes an effective cut-off length for Coulomb interactions. Generally, this length is taken to be the Debye length (refs. 18 and 19). This is rigorously true for a plasma in which thermal equilibrium is maintained. For the present problem, in which there are charges of only one sign, there is no cut-off distance short of the extent of the charged-particle gas l. It is therefore assumed that $l_E \simeq l$ in equation (B3), so that substituting equation (B2) into (B3) and performing the integration yields, for the scattering parameter,

$$\left(\frac{\Delta v_\perp}{v}\right)^2 = \frac{q^4 n l}{2\pi\epsilon_0^2 m^2 v^4} \ln\left(\frac{l}{b_0}\right) \tag{B4}$$

This relation can be written in terms of the dimensionless parameters P_1, P_3, P_4, and P_5 given by relations (19) to (23). Substituting for b_0 and multiplying the numerator and denominator of the logarithmic factor by $(nl)^{1/2}$ yields the result

$$\left(\frac{\Delta v_\perp}{v}\right)^2 = \frac{q^4 n l}{2\pi\epsilon_0^2 m^2 v^4} \ln\left[\frac{4\pi\epsilon_0 m v^2 (nl^3)^{1/2}}{q^2 n^{1/2} l^{1/2}}\right] \tag{B5}$$

so that in terms of the dimensionless parameters

$$\left(\frac{\Delta v_\perp}{v}\right)^2 = \frac{1}{2\pi} P_4 \ln\left[\frac{4\pi}{P_4}\left(\frac{P_5}{P_1^2 P_3}\right)\right] \tag{B6}$$

Because P_1, P_3, P_4, and P_5 are independent quantities, P_4 can be altered without affecting the ratio $P_5/P_1^2 P_3$ in the logarithm. Thus the scattering parameter has the form

$$\left(\frac{\Delta v_\perp}{v}\right)^2 = \frac{1}{2\pi} P_4 \ln\left(\frac{C}{P_4}\right) \tag{B7}$$

if $C = 4\pi(P_5/P_1^2 P_3)$ is held constant.

The scattering parameter (B7) can be shown to be equal to the ratio of the characteristic length l to a mean free path. The mean free path is defined as that corresponding to a 90^{o} deflection of the incident electron. The total deflection of the particle after traversing a characteristic length l of gas is given by $(\Delta v_{\perp})^{2}$. The number of collisions required to deflect the particle through a right angle is the number of deflections $\Delta v_{\perp}$ required to change all the particles' motion into transverse motion. This number is $(v/\Delta v_{\perp})^{2}$. The characteristic time for traversing a distance l is l/v. Thus the mean free path for 90^{o} scattering is

$$\lambda_c = v\left(\frac{v}{\Delta v_{\perp}}\right)^2 \frac{l}{v} = \frac{2\pi\epsilon_0^2 m^2 v^4}{q^4 n \ln \dfrac{l_E}{b_o}} \tag{B8}$$

It is thus apparent that

$$\left(\frac{\Delta v_{\perp}}{v}\right)^2 = \frac{l}{\lambda_c} \tag{B9}$$

and thus the scattering parameter and the ratio l/λ_c have the same functional dependence on P_4.

APPENDIX C

DERIVATION OF THE SCALING RELATIONS

In the following calculations, we use the similarity parameters (19) to (23) to obtain scaling laws. These scaling laws are applied specifically to electron simulation of ion motion in a magnetic field. The subscripts on the physical quantities distinguish between electron and ion quantities. Thus m_e is the electron mass, and m_i is the ion mass.

Similarity between an electron analog and an ion prototype is rigorously established if

$$P_{ej} = P_{ij} \qquad j = 1, 2, 3, 4, 5 \tag{C1}$$

Using equations (19) to (23), we obtain from (C1)

$$\frac{m_e u_e}{B_e l_e q_e} = \frac{m_i u_i}{B_i l_i q_i} \tag{C2}$$

$$\frac{\mu_0 m_e n_e u_e^2}{B_e^2} = \frac{\mu_0 m_i n_i u_i^2}{B_i^2} \tag{C3}$$

$$\frac{v_e}{u_e} = \frac{v_i}{u_i} \tag{C4}$$

$$\frac{l_e n_e q_e^4}{\epsilon_0^2 m_e^2 u_e^4} = \frac{l_i n_i q_i^4}{\epsilon_0^2 m_i^2 u_i^4} \tag{C5}$$

$$\frac{\mu_0 l_e^2 n_e^2 q_e^2}{\epsilon_0 B_e^2} = \frac{\mu_0 l_i^2 n_i^2 q_i^2}{\epsilon_0 B_i^2} \tag{C6}$$

Now, $q_e = e$ and $q_i = Ze$. Also, ϵ_0 and μ_0 are the same for both electron and ion systems. Equations (C2) to (C6) then yield

$$\frac{m_e u_e}{B_e l_e} = \frac{m_i u_i}{B_i l_i Z} \tag{C7}$$

$$\frac{m_e n_e u_e^2}{B_e^2} = \frac{m_i n_i u_i^2}{B_i^2} \tag{C8}$$

$$\frac{l_e n_e}{m_e^2 u_e^4} = \frac{l_i n_i Z^4}{m_i^2 u_i^4} \tag{C9}$$

$$\frac{l_e^2 n_e^2}{B_e^2} = \frac{l_i^2 n_i^2 Z^2}{B_i^2} \tag{C10}$$

From equation (C9),

$$\frac{n_e}{n_i} = Z^4 \left(\frac{m_e}{m_i}\right)^2 \left(\frac{u_e}{u_i}\right)^4 \frac{l_i}{l_e} \tag{C11}$$

Solution of equation (C7) for $(u_e/u_i)^2$ gives

$$\left(\frac{u_e}{u_i}\right)^2 = \frac{1}{Z^2} \left(\frac{m_i}{m_e}\right)^2 \left(\frac{B_e}{B_i}\right)^2 \left(\frac{l_e}{l_i}\right)^2 \tag{C12}$$

From equation (C8),

$$\left(\frac{B_e}{B_i}\right)^2 = \left(\frac{m_e}{m_i}\right)\left(\frac{n_e}{n_i}\right)\left(\frac{u_e}{u_i}\right)^2 \tag{C13}$$

Substitute equation (C12) into equation (C13) to obtain

$$\frac{n_e}{n_i} = Z^2 \left(\frac{m_e}{m_i}\right)\left(\frac{l_i}{l_e}\right)^2 \tag{C14}$$

From equations (C10) and (C14),

$$\frac{B_e}{B_i} = Z \left(\frac{m_e}{m_i}\right)\left(\frac{l_i}{l_e}\right) \tag{C15}$$

Substituting equation (C15) into equation (C12) leads to the result

$$\frac{u_e}{u_i} = 1 \tag{C16}$$

Substituting equations (C14) and (C16) into equation (C11) yields, finally,

$$\frac{l_i}{l_e} = Z^2 \frac{m_e}{m_i} \tag{C17}$$

Equations (C14) to (C17) constitute the complete set of scaling laws satisfying the similarity conditions (C1). If the mass ratio m_e/m_i is defined by the scaling factor M, then equations (C14) to (C17) can be written as follows:

$$\frac{n_e}{n_i} = Z^6 M^3 \tag{C18}$$

$$\frac{B_e}{B_i} = Z^3 M^2 \tag{C19}$$

$$\frac{u_e}{u_i} = 1 = \frac{v_e}{v_i} \tag{C20}$$

$$\frac{l_i}{l_e} = Z^2 M \tag{C21}$$

For those cases in which the assumption of space charge neutralization is valid, similarity conditions $P_{5i} = P_{5e}$ can be neglected. Proceeding in a manner analogous to that used with relations (C7) to (C9), one obtains the following scaling relations

$$\frac{n_e}{n_i} = Z^2 \left(\frac{m_e}{m_i}\right)\left(\frac{l_i}{l_e}\right)^2 \tag{C22}$$

$$\left(\frac{v_e}{v_i}\right)^2 = \left(\frac{u_e}{u_i}\right)^2 = \frac{1}{Z}\left(\frac{l_i}{l_e}\right)^{1/2}\left(\frac{m_i}{m_e}\right)^{1/2} \tag{C23}$$

$$\frac{B_e}{B_i} = Z^{1/2}\left(\frac{l_i}{l_e}\right)^{5/4}\left(\frac{m_e}{m_i}\right)^{3/4} \tag{C24}$$

Introducing the scaling factors M and L results in relations (C22) to (C24) becoming relations (35) to (37) of the text (Equation (37) is obtained by multiplying equation (C23) by M and inverting).

REFERENCES

1. Birkeland, K.: The Norwegian Aurora Polaris Expedition. H. Aschehoug and Co. Publ., Christiana (Oslo) 1908, 1913.
2. Akshanov, B. S.; Marinin, V. G.; Strel'tsov, A. I.; and Sinel'nikov, K. D.: Injection of Charged Particles into a Mirror System. Soviet Phys. - Tech. Phys., vol. 10, no. 12, June 1966, pp. 1710-1711.
3. Akshanov, B. S.; Volkolupov, Yu. Ya.; and Sinel'nikov, K. D.: Injection and Trapping of Charged Particles in a Magnetic Mirror. Soviet Phys. - Tech. Phys., vol. 11, no. 4, Oct. 1966, pp. 446-451.
4. Akshanov, B. S.; Volkolupov, Yu. Ya.; and Sinel'nikov, K. D.: Capture of Charged Particles in Pulsed Injection into a Trap with Stationary Fields. Soviet Phys. - Tech. Phys., vol. 11, no. 4, Oct. 1966, pp. 452-455.
5. Perkins, W. A.; and Barr, W. L.: Observation of Velocity - Distribution Instability. Plasma Physics and Controlled Nuclear Fusion Research. Vol. 2. International Atomic Energy Agency, 1966, pp. 115-134.
6. Janes, G. S.: Scaling Relations for Plasma Devices. Res. Rep. 80, AVCO-Everett Research Lab. (AFOSR-TN-60-367), Dec. 1959.
7. Schindler, K.: On the Similarity Laws for the Interaction of the Solar Wind with the Magnetosphere. Rep. ESRO-SN-16, European Space Research Inst., Italy, Dec. 1966.
8. Bridgman, P. W.: Dimensional Analysis. Yale University Press, 1943.
9. Streeter, Victor L.: Fluid Mechanics. Fourth ed., McGraw-Hill Book Co., Inc., 1966, Ch. 4.
10. de Kluiver, H.; Brandt, H. B.; Strijland, W.; and Huizenga, W.: Injection Experiments in Jutphaas. Plasma Physics and Controlled Nuclear Fusion Research. Vol. 2. International Atomic Energy Agency, 1966, pp. 177-189.
11. Gibson, G.; Jordan, W. C.; and Lauer, E. J.: Particle Behavior in Static, Axially Symmetric, and Magnetic Mirror and Cusp Geometries. Phys. Fluids, vol. 6, no. 1, Jan. 1963, pp. 116-133.
12. Roth, J. Reece: Nonadiabatic Particle Losses in Axisymmetric and Multipolar Magnetic Fields. NASA TN D-3164, 1965.
13. White, Elmer: Analysis of a 4-Bar Mirror Cusp Magnetic Field by Means of an Electron Analog Device. Rep. AFAPL-TR-67-79, Air Force Aero Propulsion Lab., Sept. 1967. (Available from DDC as AD-663425.)

14. Northrop, Theodore G.: The Guiding Center Approximation to Charged Particle Motion. Ann. Phys. (N.Y.), vol. 15, July 1961, pp. 79-101.

15. Morton, A. H.: Plasma in Mirror Machines. Rep. CLM-R-38, Culham Laboratory, England, June 1964.

16. Lehnert, B.: Experimental Evidence of Plasma Instabilities. Plasma Phys., vol. 9, no. 3, 1967, pp. 301-337.

17. Rosenbluth, M. N.; and Longmire, C. L.: Stability of Plasmas Confined by Magnetic Fields. Ann. Phys. (N.Y.), vol. 1, no. 2, May 1957, pp. 120-140.

18. Rosenbluth, Marshall N.; MacDonald, William M.; and Judd, David L.: Fokker-Planck Equation for an Inverse-Square Force. Phys. Rev., vol. 107, no. 1, July 1, 1957, pp. 1-6.

19. Spitzer, Lyman, Jr.: Physics of Fully Ionized Gases. Interscience Publishers, 1956.

NASA-Langley, 1969 —— 25 E-4927

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546

OFFICIAL BUSINESS

FIRST CLASS MAIL

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546