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PREPROCESSING OF MINITRACK DATA

by Edward R. Watkins, Jr.

Goddard Space Flight Center
Greenbelt, Md.

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ABSTRACT

The principles embodied in the Minitrack System are summarized so that the reader may better understand the reduction process. The use of computers for tracking-data processing necessitates the transition of principles into algorithms; where applicable, the development of algorithms is included. The current procedures for preparing Minitrack data messages, and the current computer processing procedures, are presented with emphasis on a description of the IBM 360 computer program designed and developed on the basis of past experience with Minitrack data reduction.

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INTRODUCTION

The 136-MHz Minitrack interferometer system is a radio phase-angle measuring system that supplies necessary information for accurately determining the angular position and orbital path of a satellite relative to a fixed geographical position. A general physical picture of the Minitrack station is given in Figure 1. The 13 antennas are set up to form two equal length perpendicular radio baselines with a north-south and east-west orientation. The phase angle is determined by antenna pairs that measure the phase difference of a carrier wave (cw) signal as it is received by each of the antennas in a pair. This cw signal can be any one of 1000 frequencies ranging from 136.000 MHz to 136.999 MHz. While the antenna field consists of 13 antennas, the phase measurement is made using only nine of the antennas, depending on whether the orbital path of the satellite is polar or equatorial; the grouping is designated in the same manner (i.e., polar or equatorial). The pairing of these antennas is discussed below.

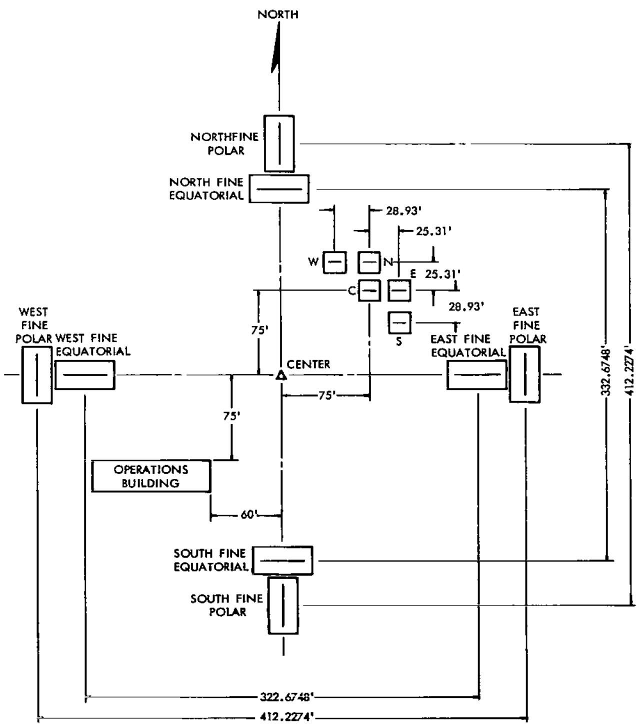

Figure 1—Relative location and function of antennas in a Minitrack antenna field.

The center or origin of the Minitrack is that point where the two radio baselines intersect. The zenith line of the system is the line which extends directly upward from the origin. The 13 antennas employed include eight precise phase measurement antennas and five ambiguity antennas. The precise phase measurement antennas are the eight outermost antennas located so that there are two antennas at each end of the two baselines. These are known as the north, east, south, and west polar fine antennas and the north, east, south, and west equatorial fine antennas. The antennas are paired to give fan-shaped beams approximately 76 × 11 degrees along each baseline. The five remaining antennas are known as the north, east, south, west, and common ambiguity antennas. Their beam pattern is also fan-shaped with beam widths of 106 degrees in the north-south direction and 78 degrees in the east-west direction.

Considering first the equatorial antenna system, in the north-south direction six antennas form three pairs to produce three north-south phase measurement signals—fine, medium and coarse. The north and south equatorial fine antennas produce a phase measurement called the north-south fine; the east and south ambiguity antennas produce the north-south medium phase measurement; the north and common ambiguity antennas produce the north-south coarse phase measurement. The six antennas producing the three east-west equatorial phase measurements are the east and west equatorial fine antennas (east-west fine phase), the north and west ambiguity antennas (east-west medium phase), and the east and common ambiguity antennas (east-west coarse phase). If the polar antenna system is used, the medium and coarse phase measurements are produced in the same manner as the equatorial medium and coarse phase measurements. However, the east-west fine and the north-south fine use the east, west, north, and south polar fine antennas. Table 1 gives a listing of each antenna pair and phase relationships.

Table 1

Antenna Pairs and Phase Relationships.

Equatorial orbit antenna pairs	Signal	Polar orbit antenna pairs
North fine equatorial South fine equatorial	North-south fine	North fine polar South fine polar
East ambiguity South ambiguity	North-south medium	East ambiguity South ambiguity
North ambiguity Common ambiguity	North-south coarse	North ambiguity Common ambiguity
East fine equatorial West fine equatorial	East-west fine	East-west polar West-west polar
North ambiguity West ambiguity	East-west medium	North ambiguity West ambiguity
East ambiguity Common ambiguity	East-west coarse	East ambiguity Common ambiguity

As mentioned earlier, the purpose of the Minitrack system is to obtain the angular position of the satellite with respect to a known geographical location. This is done by measuring the difference in radio path lengths from the satellite to each of a pair of antennas. To illustrate this, assuming a satellite is transmitting a cw on the nominal frequency of 136.000 MHz, Figure 2 shows antennas A and B separated by a distance of N wavelengths. The cw signal from satellite S arrives simultaneously at point P and antenna A. Thus PB is the difference between radio paths SA and SB (equal to M wavelengths). When satellite S is at a very high altitude (100 miles or more), angle APB can be considered a right angle.

The direction angle, B, of the satellite is given by

$$\text{cosine } \angle B = \frac{PB}{BA} \quad \text{or} \quad \frac{M\lambda}{N\lambda} .$$

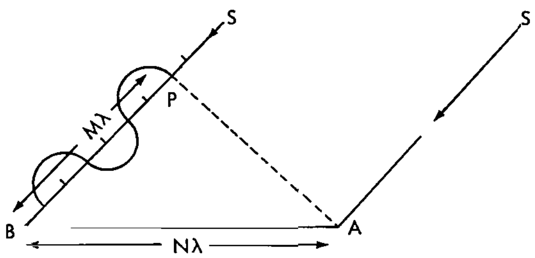

Figure 2—Phase difference due to signal arriving at antennas A and B at slightly different times.

Knowing two direction angles to the satellite, we can determine its true angular position γ, with respect to the Minitrack station by use of the familiar geometric identity.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 , \tag{1}$$

where

γ = direction of the satellite measured from the zenith

α = direction of the satellite measured with respect to the east-west baseline

β = direction of the satellite measured with respect to the north-south baseline.

Since the system is not capable of tracking a satellite below the horizon, the magnitude of γ will never exceed 90 degrees. It was also assumed that the *absolute phase difference* (total number of wavelengths between two signals) could be measured. However, in actual practice only the *relative phase difference* (the measured difference in phase between these two signals from 0 to 1λ) can be measured. Because the length of the baseline (46 or 57 wavelengths) is greater than the maximum relative phase difference, the direction angle of the satellite can be calculated to a fine degree of accuracy. But since only the relative phase is known, readings above 1λ repeat as though they were referred to zero, depending on the length of path PB; therefore, ambiguous readings arise whenever the path length difference (SB - SA) exceeds 1 wavelength. Since the length of the baseline between two fine antennas is either 46 or 57 wavelengths, it is theoretically possible to obtain either 92 or 114 indications of equal relative phase difference in one pass.

The ambiguity antennas resolve the problem of finding the total number of wavelengths farther ($M\lambda$) that the cw must travel between the antennas. The ideal arrangement would be to have two ambiguity antennas separated by 1/2 wavelength, but mechanical interference problems prevent this. Instead, two antenna pairs are separated by distances of 4 and 3.5 wavelengths respectively, thus allowing 0.5 wavelength separation to be mathematically determined from the relation

$$F - T = H ,$$

where F, T, and H are the relative phase differences on the 4, 3.5, and 0.5 wavelength baselines, respectively.

In the Minitrack system the most important sources of error requiring an overall system calibration are those related to the antennas and radio frequency feed lines as follows: (1) baselines not perpendicular; (2) baselines not horizontal; (3) incorrect baseline lengths; (4) uncertainty of location of antenna centers; (5) variations of antenna center locations as functions of arrival angles of radio waves; and (6) actual radio length of radio frequency feed lines not known.

When a Minitrack station is first established, and also at periodic intervals thereafter, the system is calibrated to ensure exactness of the direction cosines with respect to a known geographical location.* To do this, the Minitrack system must be boresighted on the zenith and the relationship of its measured direction cosines compared with direction cosines of a calibration source within the system antenna pattern of ±50 degrees from zenith. An optical technique is used whereby an airplane carrying a Minitrack transmitter and flashing light is photographed against the star background. The aircraft and its transmitter cut a plane that perpendicularly bisects the antenna baseline and provides a calibration constant known as K_c. K_c is the phase difference between the cw signals being emitted from a point on a line exactly perpendicular to the baseline and equidistant from the two antennas. If the system is ideal, then K_c should be zero; K_c is obtained for each of the six antenna pairs in use.

In addition to this external calibration process, an internal system calibration check is also made by inputting a common signal to all of the preamplifier units. By introducing a radio signal at a point in the system where the radio frequency feed lines enter the electronic phase measurement equipment, it is possible to calibrate any factors other than those which might be introduced by the antenna and feed lines. Such a calibration process is done at the same time as the aforementioned external calibration process and gives a calibration constant K_{s1} which represents phase shift introduced by electrically unstable components. This same internal calibration check is then performed immediately before a satellite pass, and the corresponding figure is designated as K_{s2}. This calibration figure is obtained for the electronic subsystems connected to each of the six antenna pairs being used during the pass.

The difference $(K_{s2} - K_{s1})$ accounts for any internal changes such as aging and maintenance that have occurred since the last periodic calibration. Hence we have $(K_c - K_{s1}) + K_{s2} = Z$, the calibrated constant for zenith that must be included in determining the true relative phase measurements. The value of $(K_c - K_{s1})$ represents the phase shift caused by antenna and feed lines. $(K_c - K_{s1})$ is virtually constant for periods of up to 6 months.

Each raw Minitrack phase reading must be calibrated before it can be used in ambiguity resolution. This is done in the following manner:

$$a = a_r - Z , \qquad (2)$$

where $0 \leq a \leq 0.999$ is the calibrated phase and $0 \leq a_r \leq 0.999$ is the raw relative phase.†

*Berbert, J. H., Oosterhout, J. D., Engels, P. D., and Habib, E. J., "Minitrack Calibration System," Photographic Science and Engineering, 7(2), 78-83, March-April 1963.
†Terms are defined in Appendix C.

The following information is required to resolve ambiguities on the east-west (or north-south) channels of the 136-MHz system:

(1) One value of "a" from each antenna pair, i.e., fine, medium, or coarse

(2) The length of the baseline from which each reading was taken, i.e., coarse baseline is 3.5 wavelengths, medium baseline is 4.0 wavelengths, fine baseline is 46 (or 57) wavelengths. (For this purpose a code is given in the data format specifying whether the equatorial or the polar system was used.)

The first step in the process is to obtain the absolute phase difference for the artificial 0.5 wavelength baseline. This is done as follows:

$$\bar{a}_{0.5} = a_4 - a_{3.5} , \tag{3}$$

where $-0.500 < \bar{a}_{0.5} < +0.500$ (subscripts denote baseline length; a bar denotes absolute phase). As the satellite moves from one horizon to the other, $\bar{a}_{0.5}$ changes from -0.500λ to $+0.500\lambda$. From the $\bar{a}_{0.5}$ reading, the direction angle for the corresponding axis can be uniquely determined. Unfortunately, this measurement is not precise enough to be used directly in obtaining the position of the satellite. However, it can be used to help determine the absolute phase differences for the medium (4λ) baseline and the coarse (3.5λ) baseline. This is done by applying a plane geometry theorem. In Figure 3, which shows signals arriving simultaneously at baselines of different lengths, the triangles ABC and abc are similar. Hence

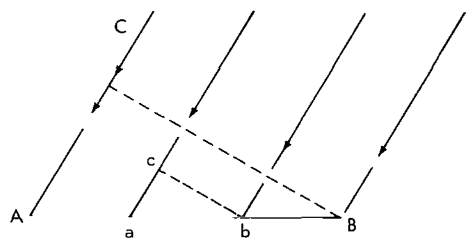

Figure 3—Signals arriving simultaneously at baselines AB and ab.

$$\overline{AC} : \overline{ac} = \overline{AB} : \overline{ab} ,$$

or

$$\overline{AC} = \overline{ac} \times (\overline{AB}/\overline{ab}) , \tag{4}$$

where $\overline{AC}$ and $\overline{ac}$ are the absolute phase measurements corresponding to baselines $\overline{AB}$ and $\overline{ab}$, respectively.

Using Equation 4 the absolute phase for the 3.5 or 4.0λ baseline can be computed from $\bar{a}_{0.5}$ and the respective baseline ratios of 7.0 and 8.0. For example,

$$\bar{a}_4' = 8.0 \times \bar{a}_{0.5} ,$$

where the superscript, ′, denotes a value that is an estimate. It should be noted that the value $\bar{a}_4'$ may differ from the value $0 \geq a_4 \geq 0.999$ by an integer number of wavelengths and possibly an error such that $-0.500 < \text{error} < +0.500$. For clarity $\bar{a}_4'$ may be expressed as

$$\bar{a}_4' = a_4 + n + \left(\bar{a}_4' - a_4\right) ,$$

but by definition

$$\bar{a}_4 = a_4 + n ;$$

hence

$$\bar{a}_4' = \bar{a}_4 + \left(\bar{a}_4' - a_4\right) ,$$

and

$$\bar{a}_4 = \bar{a}_4' - \left(\bar{a}_4' - a_4\right) , \tag{5}$$

where $-0.500 < \left(\bar{a}_4' - a_4\right) < +0.500$ is the phase error mentioned above. Similar reasoning yields the equations for determining $\bar{a}_{3.5}'$:

$$\bar{a}_{3.5}' = 7.0 \times \bar{a}_{0.5} ;$$

$$\bar{a}_{3.5} = \bar{a}_{3.5}' - \left(\bar{a}_{3.5}' - a_{3.5}\right) . \tag{6}$$

Either of the values $\bar{a}_4$ or $\bar{a}_{3.5}$ could be used at this point to compute rough direction angles, but the accuracy would be poor. However, $\bar{a}_4$ and $\bar{a}_{3.5}$ are sufficiently accurate to be used in determination of absolute phase on the fine baseline. The same geometry illustrated in Figure 3 can be used for this purpose, but the ratio $\overline{AB}/ab$ is so large that any error in $a_{3.5}$ or a_4 will be greatly magnified. Consequently, it is desirable to have a baseline with a length intermediate to the fine and medium or coarse. This is achieved by adding $\bar{a}_{3.5}$ to $\bar{a}_4$ to obtain $\bar{a}_{7.5}$, a value associated with a mathematical 7.5 wavelength baseline.

The combination of $\bar{a}_{3.5}$ and $\bar{a}_4$ into $\bar{a}_{7.5}$ tends to minimize random errors from poor readings of $a_{3.5}$ and a_4. With $\bar{a}_{7.5}$, the $\bar{a}_F$, where B_F is the length of the fine baseline (46 or 57), can be computed from

$$\bar{a}_F' = \left(B_F/7.5\right) \times \bar{a}_{7.5} ,$$

$$\bar{a}_F = \bar{a}_F' - \left(\bar{a}_F' - a_F\right) , \quad \left|\bar{a}_F' - a_F\right| < 0.500 . \tag{7}$$

Having computed the absolute phase difference for the fine baseline, the direction cosine is

$$\cos\alpha = \bar{a}_F/B_{FC} , \tag{8}$$

where B_{FC}, the corrected fine baseline, is computed for each satellite such that for the equatorial baseline

$$B_{FC} = 46 \times f_t/136.0 ,$$

and for the polar

$$B_{FC} = 57 \times f_t/136.0 ,$$

with f_t being the exact tracking frequency in MHz. B_{FC} accounts for baseline lengthening caused by higher frequencies.

For continuity and reference the entire set of equations is listed here in the order in which they are used:

Step 1
$$a_{3.5} = {}_{3.5}a_r - (K_c - K_{s1} + K_{s2})_{3.5} ;$$
$$a_4 = {}_4a_r - (K_c - K_{s1} + K_{s2})_4 ;$$
$$a_F = {}_Fa_r - (K_c - K_{s1} + K_{s2})_F .$$

Step 2
$$\bar{a}_{0.5} = a_4 - a_{3.5} , \quad |\bar{a}_{0.5}| < +0.500 .$$

Step 3
$$\bar{a}_4' = 8.0 \times \bar{a}_{0.5} ;$$
$$\bar{a}_4 = \bar{a}_4' - (\bar{a}_4' - a_4), \quad |\bar{a}_4' - a_4| < 0.500 .$$

Step 4
$$\bar{a}_{3.5}' = 7.0 \times \bar{a}_{0.5} ;$$
$$\bar{a}_{3.5} = \bar{a}_{3.5}' - (\bar{a}_{3.5}' - a_{3.5}), \quad |\bar{a}_{3.5}' - a_{3.5}| < 0.500 .$$

Step 5
$$\bar{a}_{7.5} = \bar{a}_{3.5} + \bar{a}_4 .$$

Step 6
$$\bar{a}_F' = (B_F/7.5) \times \bar{a}_{7.5} ;$$
$$\bar{a}_F = a_F - (\bar{a}_F' - a_F), \quad |\bar{a}_F' - a_F| < 0.500 .$$

Step 7
$$\cos\alpha = \bar{a}_F/B_{FC} .$$

The steps outlined above are carried out for the east-west data to obtain $\cos\alpha$ and for the north-south data to obtain $\cos\beta$.

DATA REDUCTION PROCEDURE

Message Composition and Editing

Each message consists of three distinct parts: (1) satellite identification line, (2) internal calibration frame, and (3) up to 31 data frames. The satellite identification line is manually prepared at the site; the calibration frame is manually edited from a Digital Recording System tape made during pass set-up time, and data frames are manually edited from a Digital Recording System tape taken during the pass. From these three parts a single teletype tape is made and transmitted to Goddard Space Flight Center.

The Digital Recording System can be set by station personnel to record at five different frame rates. Since the frame format is unalterable, variation in recording rates is accomplished by suppression of unwanted frames. Selectable rates are:

1. 1 frame each second
2. 1 frame each 2 seconds
3. 1 frame each 10 seconds
4. 1 frame each 20 seconds
5. 1 frame each 60 seconds

Frame rates are scheduled so that 30 frames provide maximum tracking coverage. A sample message is provided in Appendix B; Appendix A contains a format explanation for each Digital Recording System frame.

The satellite identification line is always preceded by an ampersand character. A table of acceptable satellite codes is searched to determine if the satellite identification is valid; if not valid, the message is not processed. A frequency code and date are included.

The internal calibration frame is first checked for properly located periods (.). All other characters are scanned and tested for legal numeric values (0 through 9). To signify a valid internal calibration frame, the five signal strength indicators are tested and must all register "9." The data frames are also checked for properly located periods. All other characters are scanned and tested for legal numeric values. Any frame which does not meet these tests is deleted from further processing. If there are less than five acceptable data frames, or if more than five consecutive data frames are deleted, the message is not processed.

Ambiguity Cable Length Inequality Correction

The cables to the fine antennas are of equal length, but the cables for the ambiguity channels connecting the paired antennas to their respective coaxial switches near the preamplifiers are of different lengths at some stations.

The values in Table 2 indicate the excess of one cable over another in the antenna pairs. The letters E, W, etc. (east, west, etc.) indicate the leg of the baseline that has the excess cable.

Table 2

Cable Length Inequalities for North, South, East West, and Common Ambiguity Antennas.

Station	North and West	East and Common	East and South	North and Common
BPOINT	29'E*	25'E	1'S	28'N
FTMYRS	28'E	25'E	00	26'N
QUITOE	00	00	00	00
LIMAPU	29'E	52'E	00	00
SNTAGO	00	00	00	00
NEWFLD	29'E	25'E	00	28'N
COLLEGE	30'W	55'W	00	00
GFORKS	29'E	25'E	00	28'N
WNKFLD	29'E	25'E	00	28'N
JOBURG	29'W	00	29'S	00
MOJAVE	31'E	15'E	00	28'N
OOMERA	29'E	25'E	00	28'N

*The letters E, W, etc. indicate the leg of the baseline having excess cable. ULASKA, MADGAR, ORORAL do not require correction.

These corrections can be made by altering the calibration constant $K_c - K_s$ in accordance with the tracking frequency used and the cable length discrepancy for the baseline in question. The appropriate formula is

$$\text{Correction in wavelengths} = \frac{\text{cable inequality}}{\text{velocity of propagation}} \times 136.5 \times 10^6\,\text{Hz} - \text{Frequency (Hz)} .$$

It is assumed that the station was calibrated at the 136.5 MHz frequency, and the correction is added to $K_c - K_s$; cable inequality is positive where the excess is in the north or east antenna connection and negative where the excess is to the west or south. Since the cable length inequalities given above are stated in feet, the velocity of propagation should be taken as 0.846×10^6 ft/sec.

Frame Compression and Counter Delay Correction

In order to put the fine and ambiguity samples into a one-to-one correspondence, the five fine samples are reduced to one by the following method.

$$\text{DIF1} = (a_2 - a_1),*$$

$$\text{DIF2} = (a_3 - a_2),*$$

*For the purpose of computing phase differences, it is assumed that the absolute phase difference between consecutive raw readouts within a frame will be less than 0.500 since readouts are only 0.2 second apart. This assumption is valid for earth-orbiting satellites at altitudes above 120 km.

$$\text{DIF3} = (a_4 - a_3),*$$

$$\text{DIF4} = (a_5 - a_4),*$$

$$a_m = a_3 + (9(\text{DIF3} - \text{DIF2}) - 3(\text{DIF4} - \text{DIF1}))/35 ,$$

where a_1, a_2, a_3, a_4, a_5 are five consecutive raw phase readouts from the same frame, and a_m is the fitted mid value. The compression formula is applied to the east-west and north-south readings.

The phase readout digitizing equipment in the Minitrack system introduces an error in the resulting raw readouts as a result of the counting time it requires. Since the counter frequency is 100 kHz, there is a time error of 0.01 millisecond for each count of phase. To correct the phase readouts, the average rate of phase change is used to correct the third fine sample of each frame. The expressions used are

$$\text{rate} = (5/4)(\text{DIF1} + \text{DIF2} + \text{DIF3} + \text{DIF4}) ,$$

and

$$a_r = a_m - (0.01)(\text{rate})(a_3) ,$$

where a_r is the smoothed phase readout from compression, and DIF1, DIF2, DIF3, DIF4 are the phase differences computed in conjunction with frame compression. The counter correction is applied to both east-west and north-south readings in the same manner.

Linearize Ambiguity Phase Readings

Phase readouts are presented in a modulo 1000 number system. In order to apply standard polynomial curve fitting techniques, the readouts must be converted to a nonmodular number set. The numerical technique for doing this is relatively simple if it can be assumed that the maximum difference between consecutive readings is less than 500. When a difference exceeds 500 it can only mean that a new cycle has begun. Linearity is forced by adding (or subtracting) increments of 1000. For example, given consecutive readouts a, b,

Step 1	$x = \|b - a\|$;
Step 2	if $x < 500$, then there is no change,
Step 3	if $x > 500$, then $b < a \Rightarrow b = b + 1000$,
Step 4	or $b > a \Rightarrow b = b - 1000$.

Steps 1 through 4 are repeated until the condition in Step 2 is satisfied.

*For the purpose of computing phase differences, it is assumed that the absolute phase difference between consecutive raw readouts within a frame will be less than 0.500 since readouts are only 0.2 second apart. This assumption is valid for earth-orbiting satellites at altitudes above 120 km.

Polynomial Fit to Ambiguity Data

A quadratic least squares fit is applied to the ambiguity data to reduce the effects of noise, to obtain a rate for linearizing fine data, and to provide a convenient method for making time adjustments. Frame time is the independent variable. The smoothing technique employs sigma-rejection for screening out spurious data points. A sigma multiplier of 2.0 provides the best results. The smoothing procedure can be outlined as follows:

Step 1	determine coefficients for smoothing equation,
Step 2	compute standard deviation of fit,
Step 3	sigma test each data point,
Step 4	save acceptable data for next iteration,
Step 5	test for convergence,
Step 6	repeat Steps 1 through 5 until convergence is met.

The result is a set of polynomial coefficients A, B, and C to represent the ambiguity data forming the ambiguity polynomial as follows:

$$\mathrm{Amb}(t) = A + Bt + Ct^2 ,$$

where t is the time measured in seconds from time T, and

T = frame time - 0.15 seconds for east-west medium,

T = frame time +0.05 seconds for east-west coarse,

T = frame time +0.25 seconds for north-south medium,

T = frame time +0.45 seconds for north-south coarse.

The values -0.15, 0.05, 0.25 and 0.45 are due to the Digital Recording System format and a 2-Hz bandwidth filter in the system.

Linearize Fine Phase Readings

The problem of linearizing fine phase readings is complicated because the difference between consecutive readings often exceeds 500. This is remedied by obtaining an estimated rate from the ambiguity polynomials such that

$$\text{equatorial fine rate} = 46 \times \left(B_c/3.5 + B_m/4\right)/2 ,$$

and

$$\text{polar fine rate} = 57 \times \left(B_c/3.5 + B_m/4\right)/2 ,$$

where B_c and B_m are the coarse and medium phase rates, respectively.

For example, given consecutive readouts a, b,

Step 1 $X = |b - (a + \text{rate})|$,

Step 2 if $X < 500$, then there is no required adjustment,

Step 3 if $X > 500$, then $b < (a + \text{rate}) \Rightarrow b = b + 1000$,

Step 4 or $b > (a + \text{rate}) \Rightarrow b = b - 1000$.

Steps 1 through 4 are repeated until the condition in Step 2 is satisfied.

Polynomial Fit to Fine Data

Fine data are fitted to a cubic polynomial using the method of least squares. The procedure is identical to that used for ambiguity data with the exception that the degree of the polynomial is one higher. The result of the fitting process is a set of polynomial coefficients A, B, C, and D to represent the data as follows:

$$\text{Fine}(t) = A + Bt + Ct^2 + Dt^3,$$

where t is seconds from T with

T = (frame time + 0.4 + T_{wwv} - KF),

T_{wwv} = timing station propagation delay constant, and

KF = filter delay constant due to filter in electronic system (Table 3).

Table 3

Propagation Times and 10-Hz Filter Delays.

Tracking station	Time station	Propagation delay (msec)	KF_{EW} east-west (msec)	KF_{NS} north-south (msec)
FTMYRS	WWV*	9.65	36	36
QUITOE	WWV	19.07	36	37
LIMAPU	WWV	23.00	38	38
SNTAGO	WWV	31.64	37	37
NEWFLD	WWV	15.29	36	36
WNKFLD	WWV	26.12	36	37
JOBURG	WWV	53.67	32	33
ALASKA	WWV	13.49	38	37
ORORAL	WWVH	30.10	36	36
MADGAR	WWV	57.72	37	38

*WWV located at Fort Collins, Colorado.

Time Adjustments and Zenith Calibration

The smoothing polynomials are used to remove time biases and adjust the data to a common time as required before ambiguity resolution. The common time is arbitrarily chosen to be the east-west fine midframe time. The following steps accomplish this:

1. define $T_{BE} = -(0.4 + T_{wwv} - KF_{EW})$ for east-west fine bias;

2. then $a_s = A + BT_{BE} + CT_{BE}^2 + DT_{BE}^3$;

3. define $T_{BN} = -(0.4 + T_{wwv} - KF_{NS})$ for north-south fine bias,

   T_e = east-west fine frame time,

   T_n = north-south fine frame time;

4. then $a_s = A + B(T_e - T_n + T_{BN}) + C(T_e - T_n + T_{BN})^2 + D(T_e - T_n + T_{BN})^3$;

5. define T_m = medium frame time,

   T_C = coarse frame time;

6. then for medium channels

   $$a_s = A + B(T_e - T_m) + C(T_e - T_m)^2,$$

7. and for the coarse channels

   $$a_s = A + B(T_e - T_c) + C(T_e - T_c)^2 ;$$

   KF_{EW} = east-west fine filter delay time;

   KF_{NS} = north-south fine filter delay time.

Zero set calibrations are applied to the a_s values given above for each of the six data channels. At the same time any integer biases caused by the linearizing processors are removed. The resulting values do not represent the calibrated phase readings. The general form for this correction is:

$$a = \left[a_s - (K_c - K_{s1} + K_{s2})\right] ;$$

where a is appropriate calibrated phase, a_s is appropriate time adjusted phase, $K_c - K_{s1}$ is appropriate zero set calibration constant, K_{s2} is appropriate prepass internal calibration, and the symbols [ ] represent "fractional part of." Zero set calibration constants are dependent upon the specific baseline, i.e., equatorial fine, polar fine, medium, coarse for both east-west and north-south patterns.

Ambiguity Resolution

The ambiguity inherent in the fine data is resolved through use of the proportionality factors common to the antenna configuration. Data are measured on three baselines, and data for two additional baselines can be constructed from these to improve the resolution process. The constructed baselines are 0.5 wavelength and 7.5 wavelengths obtained by the two algebraic combinations of the data from the medium and coarse baselines. The following steps for resolution apply to both east-west and north-south data as a result of symmetry:

1. $\bar{a}_{0.5} = \{[a_4 - a_{3.5}]\}$
2. $\bar{a}'_{3.5} = 7 \times \bar{a}_{0.5}$
3. $\bar{a}_{3.5} = \bar{a}'_{3.5} - \{[\bar{a}'_{3.5} - a_{3.5}]\}$
4. $\bar{a}'_{4.0} = 8 \times \bar{a}_{0.5}$
5. $\bar{a}_{4.0} = \bar{a}'_{4.0} - \{[\bar{a}'_{4.0} - a_{4.0}]\}$
6. $\bar{a}_{7.5} = \bar{a}_{3.5} + \bar{a}_{4.0}$
7. $\bar{a}_F' = \bar{a}_{7.5} \times B_F/7.5$
8. $\bar{a}_F = \bar{a}_F' - \{[\bar{a}_F' - a_F]\}$

where

B_F = length of appropriate fine baseline (i.e., 46.0 = equatorial; 57.0 = polar),

$[X]$ symbols represent "fractional part of x,"

$\{y - x\}$ symbols represent minimum phase difference (i.e., $-0.5 < \{y - x\} \leq +0.5$).

Antenna Field Correction

A special polynomial whose coefficients are determined in the calibration process for each station is used to correct the fine resolution phase for various predictable antenna field distortions. If $\bar{a}_e$ is the east-west fine resolution and $\bar{a}_n$ is the north-south fine resolution, then for

Equatorial data correction

$$\begin{aligned}\bar{a}_e \text{ corrected} = {} & C(E,0) + C(E,1) \times \bar{a}_e + C(E,2) \times \bar{a}_n + C(E,3) \times \bar{a}_e \times \bar{a}_n \\ & + C(E,4) \times \bar{a}_e^2 + C(E,5) \times \bar{a}_n^2 + C(E,6) \times \bar{a}_n^3 \\ & + C(E,7) \times \sin(2\pi\bar{a}_e) + C(E,8) \times \cos(2\pi\bar{a}_e)\end{aligned}$$

$$\begin{aligned}\bar{a}_n \text{ corrected} = {} & D(E,0) + D(E,1) \times \bar{a}_e + D(E,2) \times \bar{a}_n + D(E,3) \times \bar{a}_e \times \bar{a}_n \\ & + D(E,4) \times \bar{a}_e^2 + D(E,5) \times \bar{a}_n^2 + D(E,6) \times \bar{a}_n^3 \\ & + D(E,7) \times \sin(2\pi\bar{a}_n) + D(E,8) \times \cos(2\pi\bar{a}_n)\end{aligned}$$

Polar data correction

$$\bar{a}_e \text{ corrected} = C(P,0) + C(P,1) \times \bar{a}_e + C(P,2) \times \bar{a}_n + C(P,3) \times \bar{a}_e \times \bar{a}_n$$
$$+ C(P,4) \times \bar{a}_e^2 + C(P,5) \times \bar{a}_n^2 + C(P,6) \times \bar{a}_e^3$$
$$+ C(P,7) \times \sin(2\pi\bar{a}_e) + C(P,8) \times \cos(2\pi\bar{a}_e)$$

$$\bar{a}_n \text{ corrected} = D(P,0) \times D(P,1) \times \bar{a}_e + D(P,2) \times \bar{a}_n + D(P,3) \times \bar{a}_e \times \bar{a}_n$$
$$+ D(P,4) \times \bar{a}_e^2 + D(P,5) \times \bar{a}_n^2 + D(P,6) \times \bar{a}_e^3$$
$$+ D(P,7) \times \sin(2\pi\bar{a}_n) + D(P,8) \times \cos(2\pi\bar{a}_n)$$

Conversion to Direction Cosines

The corrected absolute phase difference is converted to direction cosine by dividing by the absolute phase difference corresponding to the distance between the measuring antennas. The antennas are positioned so as to be separated by 46 cycles and 57 cycles of 136.000 MHz for the respective equatorial and polar configurations. For satellite frequencies, f_t, the corrected baseline is $B_{FC} = 46\, f/136.0$ for equatorial and $B_{FC} = 57\, f/136.0$ for polar. If a_e is the corrected east-west fine absolute phase difference then

$$\ell \equiv \cos\alpha = \bar{a}_e/B_{FC}\,.$$

If a_n is the corrected north-south fine phase difference then

$$m \equiv \cos\beta = \bar{a}_n/B_{FC}\,.$$

Goddard Space Flight Center
National Aeronautics and Space Administration
Greenbelt, Maryland, September 11, 1968
311-07-21-01-51

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Appendix A

Digital Recording System Frame Format

Character(s)	Contents
1, 2	second (time) of frame start
3, 4	hundreds and tens digits of east-west medium phase
5	period "." separator
6 to 8	first east-west fine phase
9	signal strength indicator (AGC)
10 to 12	first north-south fine phase
13	period "." separator
14, 15	minute of frame start
16, 17	hundreds and tens digits of east-west coarse phase
18	period "." separator
19, 21	second east-west fine phase
22	signal strength indicator (AGC)
23 to 25	second north-south fine phase
26	period "." separator
27, 28	hour of frame start
29, 30	hundreds and tens digits of north-south medium phase
31	period "." separator
32 to 34	third east-west fine phase
35	signal strength indicator (AGC)
36 to 38	third north-south fine phase
39	period "." separator
40 to 42	day of year for frame start
43, 44	hundreds and tens digits of north-south coarse phase
45	period "." separator
46 to 48	fourth east-west fine phase

Character(s)	Contents	
49	signal strength indicator (AGC)	
50 to 52	fourth north-south fine phase	
53	period "." separator	
54*	equatorial/polar antenna indicator	
	1 = equatorial	
	2 = polar	
55, 56	station number	
	03 = FTMYRS	15 = WNKFLD
	05 = QUITOE	16 = JOBURG
	06 = LIMAPU	19 = ALASKA
	08 = SNTAGO	21 = ORORAL
	12 = NEWFLD	23 = MADGAR
57	period "." separator	
58 to 60	fifth east-west fine phase	
61	signal strength indicator (AGC)	
62 to 64	fifth north-south fine phase	
65	period "." separator	

*This position of the internal calibration frame is used to manually insert a filter indicator whenever it is used in a standard data message.

0 = 2 cps filter in fine channels
2 = 10 cps filter in fine channels
3 = narrow band tracking filter system.

Appendix B

Sample Minitrack Data Message

```
GPU083C
RR GPUT
DE LWNK 061
03/1312Z

&6406401 1 690103
4350.2639114.3580.2639114.1230.2639114.00380.2639114.215.2639114.
1456.3071750.4503.3231786.1217.3121830.00334.3251881.215.3391904.
1662.3591152.4505.3581199.1222.3761243.00335.3771280.215.3731324.
1861.4081559.4504.4241597.1217.4221636.00339.4252685.215.4321729.
2061.4741972.4505.4672022.1220.4761060.00342.4812083.215.4881142.
2263.5342388.4506.5391436.1222.5301463.00347.5411519.215.5472546.
2463.5911798.4507.5912840.1227.6082886.00348.6062920.215.6102969.
2664.6502208.4507.6491255.1228.6522305.00350.6721342.215.6792385.
2863.7162633.4505.7212679.1232.7312717.00351.7372762.215.7452802.
3064.7882057.4508.7992104.1236.8062143.00354.8202186.215.8192239.
3265.8572483.4507.8672528.1238.8342574.00358.8842604.215.8961652.
3467.9482918.4507.9481950.1238.9612006.00359.9592031.215.9802078.
3668.0162345.4508.0281380.1241.0372431.00366.0482469.215.0582517.
3867.1022771.4509.1162812.1247.1242870.00365.1312904.215.1412946.
4068.1892200.4507.2011247.1249.2102297.00366.2152342.215.2252384.
4269.2831634.4510.2992684.1251.2932731.00371.3131768.215.3051830.
4469.3721086.4511.3871115.1256.3921171.00370.4031207.215.4121258.
4670.4741514.4511.4821560.1259.4921607.00376.4941661.215.5021704.
4870.5701966.4511.5801993.1261.5871055.00376.5981087.215.6031146.
5071.6691395.4513.6841450.1265.6931503.00379.6941544.215.7081598.
5271.7781848.4512.7841909.1268.7951947.00383.8001990.215.8281045.
5472.8891289.4514.8961351.1271.9131397.00385.9151439.215.9310481.
5672.9841751.4515.0080801.1273.0231826.00387.0161901.215.0611942.
5874.1140218.4516.1190238.1278.1370292.00390.1691338.215.1500394.
0074.2341658.4615.2440702.1281.2700739.00393.2860777.215.2850835.
0274.3590109.4616.3740175.1283.3990211.00397.3930246.215.4070299.
0475.4750568.4619.5000613.1287.5170665.00300.4830695.215.5330745.
0676.6200008.4619.6460072.1290.6330121.00302.6540176.215.6470231.
0879.7680522.4620.7230527.1294.7930567.00305.7900650.215.3420678.
1078.8850954.4620.8280014.1298.9310085.00308.9620126.215.9890143.
1280.1020389.4621.0980426.1200.0230478.00309.0300507.215.0990595.

6406401
03/1321Z JAN LWNK
```

Appendix C

Glossary of Terms

K_c - The relative phase difference between a CW signal emitted from a point in space on a line perpendicular to the baseline and equidistant from the two antennas, values for K_c are determined at the time of optical calibration and used in data reduction.

K_{s1} - The portion of K_c due to electronic components exclusive of antennas and feed lines, values for K_{s1} are determined electronically at the time of calibration and used in conjunction with K_c.

K_{s2} - This value is determined in the same fashion as K_{s1} only prior to each tracking operation.

a_r - Raw phase measurement in readout units, i.e., a fraction of a cycle with three decimal place accuracy.

a - The calibrated phase measurement, determined by applying the appropriate values of K_c, K_{s1}, and K_{s2} as follows

$$a = a_r - (K_c - K_{s1}) - K_{s2}$$

(To distinguish between measurements from different baselines, numerical subscripts are used for all but the fine data which is denoted by an "F" subscript.)

$\bar{a}$ - Absolute or total difference in radio path length

$\bar{a}'$ - Estimated total difference in radio path length

B_F - Radio length of a fine baseline at 136.000 MHz; for equatorial, $B_F = 46.000$, for polar, $B_F = 57.000$

B_{FC} - Radio length of fine baseline at spacecraft frequency

f_t - Frequency of space craft transmitter in MHz

KF_{EW}, KF_{NS} - Time delay associated with the 10-Hz filters in the electronics of the Minitrack system; the east west fine and North south fine subsystems each have a separate 10-Hz filter.

T_{WWV} - Time delay between the transmission of a timing signal and its receipt at the station

WWV, WWVH - Names of time standard broadcast stations; WWV at Ft. Collins, Colorado, WWVH in Hawaii

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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