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MACROSCALE TURBULENCE NEAR THE
POTENTIAL CORE OF A SUBSONIC JET

By

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SUMMARY

Large scale turbulence was investigated beside the potential core of a 4-inch, Mach 0.1, cold air jet. The outputs from an array of eight hot-film anemometers were rapidly multiplexed and stored digitally on tape so that phase relationships were essentially preserved. These data and a modified Taylor's hypothesis were then used to provide plots of velocity and vorticity as functions of time in the turbulent shear region at $x/d = 3$. The sensor array was located at a position near the potential core and a position near the outer jet boundary. Photographs from a qualitative visualization study were used, together with the anemometer data, in formulating a model for the flow processes in this region. Conclusions resulting from the use of hot-film sensors in this program and from a literature search for information concerning the overall accuracy of the hot-wire anemometer are also presented.

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LIST OF SYMBOLS

A	A constant in the basic calibration relationship for the hot-film anemometers
a	An empirically obtained parameter in the expression for the yaw sensitivity of hot-wire anemometers
B	A constant in the basic calibration relationship for the hot-film anemometers
b	A limiting value of the radial coordinate r at which turbulent jet flow is negligible
b_0	A value of b corresponding to some arbitrary axial position x_0
d	The diameter of a hot wire or hot film sensor
E	The voltage output of a hot wire or film corresponding to the velocity $U + u$ for an unyawed wire
$\bar{E}$	The mean voltage output of a hot wire or film corresponding to the velocity U for an unyawed wire
E_0	Technically, the value of voltage corresponding to zero velocity; practically, a calibration constant (because of departures from the calibration relationship at low velocities)
e	The fluctuating voltage corresponding to the fluctuating velocity u for an unyawed wire
f	Frequency
$f.c.$	A subscript indicating a measured velocity value for free convection
g_k	A discrete low pass filter function (Equation 32)
h	A constant in the basic calibration relationship for the hot-film anemometers
i, j, k	Unit vectors
K	A constant introduced in Equation 25
l	The length of a hot wire or hot film sensor (elsewhere: a length indicating the size of a typical eddy, or a typical sensor array dimension, when so defined)

LIST OF SYMBOLS (Continued)

M	The molecular mass of <u>air</u> (in this case)
P	The static pressure
P_0	The total pressure
P_A	Atmospheric pressure
P_S	The static pressure minus atmospheric pressure
P_T	The total pressure minus atmospheric pressure
P_r	The Prandtl number
$\hat{q}$	A time series filtered by use of g_k
q	An unfiltered time series
R	The universal gas constant
Re	The Reynolds number
r	A coordinate perpendicular to the symmetry axis of the jet and in the same direction as the $-v$ component of velocity (r equals zero at the jet axis)
s	The sampling rate
T	The fluctuating component of the equilibrium wire temperature (in this case, approximately the free stream temperature fluctuation)
T_0	The stagnation or total temperature
T_R	The temperature in degrees Rankine
t_k	The time increment in a time series
$U, \bar{U}$	The velocity in the axial direction (the overbar indicates a mean value)
$\bar{U}_a$	The mean velocity on the jet axis
U_p	The velocity in the potential core

LIST OF SYMBOLS (Continued)

u, v, w	Fluctuating velocity components in the $\vec{i}$, $\vec{j}$, and $\vec{k}$ directions, respectively
u_t	Velocity fluctuations in fully turbulent fluid (excluding patches of irrotational fluid)
$\vec{v}$	The fluctuating velocity vector (including mean values)
x	A coordinate for displacement along the symmetry axis of the jet
x_0	An arbitrary position on the axial coordinate, x
<u>Greek Symbols</u>	
α	An angle parameter describing the spread of a jet
α_1, α_2	Constants used in relating Prandtl number to temperature
β_1, β_2	Constants used in relating kinematic viscosity to temperature
γ	The intermittency factor which describes the fraction of time that a fluid is fully turbulent (in Section 5: the specific heat ratio)
γ_1, γ_2	Constants used in relating the thermal conductivity to temperature
ϵ	The turbulent transport coefficient
θ	The angle between the wire axis and the mean flow direction
$\theta_a, \bar{\theta}_a$	The wire equilibrium temperature including fluctuations; the superscript indicates a mean value only
θ_f	Film temperature equal to $(\theta_a + \theta_f)/2$
θ_w	The wire overheat temperature
λ	The thermal conductivity
ν	The kinematic viscosity
ξ, η, ζ	Fluctuating vorticity components in the $\vec{i}$, $\vec{j}$, and $\vec{k}$ directions, respectively
$\rho_{oil}, \rho_{mercury}$	Manometric densities

LIST OF SYMBOLS (Continued)

$\vec{\tau}$	A unit vector indicating the orientation of an anemometer sensor in the x-y plane
ϕ	The angle between the wire axis and the instantaneous velocity vector

1.0 INTRODUCTION

Lighthill's theory of aerodynamic noise generation (Reference 1) relates fluctuating velocity components in a turbulent flow to the characteristics of the radiated sound. The application of such a theory requires a detailed knowledge of the turbulent flow structure. Statistical descriptions of turbulence together with Lighthill's theory permit the estimation of some acoustic parameters. However, statistical treatments fall short of a complete specification of a turbulent flow field. Turbulence lends itself to statistical treatment because of its random character. However, several investigators (References 2, 3, 4 and 5) have proposed that the large scale structure of free turbulence has a less random character than the associated small-scale eddies. This would seem to imply that the larger turbulent eddies may be described by a more deterministic theoretical framework. Such a specification of macro-scale turbulence could be of considerable value in identifying noise source locations in turbulent jet or rocket exhausts.

This report summarizes recent experimental results in a continuing program to better define the large-scale mixing process in the turbulent shear region of a cold, subsonic, circular jet. Previous work in this program is outlined in Reference 6, which presents details on the development and testing of computer programs and describes the basic test apparatus. The apparatus described in Reference 6 has been extended and improved to eliminate numerous problems which were previously caused by structural vibration and temperature effects.

Two sets of fluctuating velocity and vorticity components were measured for the turbulent mixing region beside the potential core at three diameters from the jet exit plane. The measuring system consisted of eight constant temperature hot film anemometers which were operated simultaneously. The fluctuating outputs of these anemometers were rapidly multiplexed and stored digitally on tape so that phase relationships between channels were essentially preserved.

New, more informative data presentation techniques have been developed for the fluctuating components of velocity and vorticity. Also the mean flow field has been carefully mapped from the jet exit plane to a point about 16 diameters downstream of the jet exit plane. Equations are presented which have been derived from a phenomenological model of the mixing process. These equations describe the mean flow field very well throughout the mapped region. Reference 7 contains details of this tentative phenomenological model.

2.0 APPARATUS

The basic experimental apparatus used in the current program has been discussed in Reference 6. Nevertheless a brief discussion of this equipment is in order in view of the improvements which have been made since the last report. Conclusions have also been reached concerning the desirability of the hot film sensors for this particular application.

2.1 The Basic System

The experimental apparatus and the data acquisition system for the present study was basically the same as described in Reference 6. Figure 1 illustrates the general features of this open return system. It consisted of a centrifugal blower, muffler, settling tank and nozzle, which were contained within a closed room. The instrumentation made possible the simultaneous operation of eight constant temperature hot film anemometers. The data was acquired in "real time" with phase relationships between channels preserved by use of an "on-line" digital computer. Figure 2 shows the multi-channel impedance circuit, which matched the anemometer outputs to the computer and also separated the dc and ac components of the anemometer signals. Thus, the dc components, which were used to compute mean velocities, were read in the laboratory while the ac components of the signals were multiplexed and stored digitally on tape by the computer. These results were then combined with calibration data to compute the fluctuating velocity and vorticity components.

Calibration was accomplished by comparing anemometer and pitot-static tube measurements when both instruments were aligned in the potential core of the jet. The temperature of the flow was carefully regulated in order to prevent false calibration information from being obtained as a result of temperature rise in the flow. Room temperature changes of 20°F have been observed during prolonged jet operation without cooling.

Figure 3 shows the complete system from a position just behind the traversing table. Figure 4 gives the dimensional information on the jet nozzle used in acquiring the data for this report.

2.2 The Traversing Table

Details of the traversing table are shown in Figure 5. The table was designed to have a large transverse excursion so that it could be used with a large jet nozzle (i.e., four inch exit diameter). The use of a large nozzle allows better resolution by the anemometer array. The table was constructed of steel stock and mounted on a tubular bed. The height above the floor was established by the use of shims. Isolation pads were used to prevent floor-induced vibration of the structure. The massive nature of the table prevented inadvertent shifting of the orientation, once the table had been properly aligned, and also helped in providing a stable platform for the anemometers.

Threaded stock having ten turns to the inch was used to drive the anemometer mount in the axial and transverse directions. The table on which the anemometers were mounted could be driven by hand or by use of a 1/2 inch hand drill. The latter method permitted a fast traverse. When used with a slide-wire position indicator and an x-y plotter, it was an extremely efficient system for obtaining mean velocity and turbulent intensity profiles.

2.3 Air Temperature Control

Because the action of the blower was to heat the air and because careful hot-film anemometer calibration required fairly long jet operation (approximately 15 minutes), it was necessary to control the temperature of the jet during this period. This temperature control feature assured that the anemometer measurements were actually made at the calibration temperature.

Details of the cooling system which was developed to stabilize the jet temperature are shown in Figures 6, 7 and 8. Initially only a radiator was mounted over the jet intake and carbon dioxide gas was vented through it to the outside. However, it was found that formation of frost on the radiator restricted the flow. Therefore, a coil of copper tubing (Figure 6) was placed just ahead of the radiator to circumvent this problem. Cold carbon dioxide could be circulated through either the coil, the radiator, or both. The flow of this coolant was controlled by two solenoid valves. The valves could be actuated manually or by the regulator system. The regulator system sought to minimize the temperature difference between the total temperature of the jet and the room temperature. This was possible to within $\pm 1^\circ\text{F}$.

In operation, the following procedure was observed. The jet was run for several minutes, with the room sealed and carbon dioxide circulating through the copper coil. This had a dehumidifying effect. After conditions had stabilized carbon dioxide was allowed to circulate through the radiator also, and the regulator was adjusted for the desired operating temperature. Temperatures were monitored on a chart recorder and only data which were obtained within the $\pm 1^\circ\text{F}$ tolerance were retained. This system and procedure permitted consistent temperature control with no apparent change in flow due to frost formation.

2.4 The Hot Film Sensors

The sensors used in acquiring data for this report were of the hot-film variety (see Figure 9). The resistance element was a platinum film of less than 1000 Angstrom units thickness, which was deposited on a glass rod having a circular cross section. The platinum was covered by a protective film of quartz. The working surface was about 0.06 of an inch long and the rod diameter was about 0.001 inch. The hot-film sensors are mounted on somewhat more flexible supports than typical hot-wire sensors in order to withstand dynamic loads. Mean flow measurements were made with a hot-wire anemometer which was constructed at Wyle Laboratories. This instrument is shown in Figure 10. The sensor was designed to be compatible with the hot-film anemometer electronics.

In the course of this investigation it was noted that the hot-film sensors failed much more frequently than the tungsten wire anemometers. Also, they repeatedly arrived in damaged condition. This apparent fragility appears to be inconsistent with the relatively large diameter of the hot-film sensors. A possible explanation is that glass, while exhibiting strength under compression, has relatively poor tensile strength. This factor together with the nonrigid supports may result in a high sensitivity to vibration. Vibrations set up by mechanical shocks or high velocity flows may therefore be injurious to this type of sensor.

If such vibration is indeed a problem, it must also affect the output of the anemometer system. For this reason, all measurements were made around Mach 0.1, in order to preclude the possibility of sensor damage or erroneous data. In any subsequent work with these sensors at higher Mach numbers, this problem must be seriously considered.

3.0 SOME CHARACTERISTICS OF SUBSONIC JET FLOW

The following sections define and discuss some of the parameters commonly used to describe free shear flows. Emphasis is placed on parameters which characterize the different flow regions in a subsonic jet. Figure 11 was obtained from the mean velocity data in this report. It may be useful to refer to this figure to clarify terminology or concepts in the subsequent discussions.

3.1 Mean Flow Properties

The mean velocity on the axis of a subsonic jet is known to vary in a hyperbolic manner (i.e., $U_a \propto x^{-1}$, where x increases in a downstream direction) for distances greater than 10 diameters from the jet exit plane. From the tip of the potential core of the jet to 10 diameters the change in velocity with distance obeys a different relationship. Figure 12 compares axial mean velocity data acquired during this program with data from other sources. The agreement seems to be quite good and the general features of the axial mean velocity variations are as described.

The flow regions of the jet are sometimes defined in terms of the axial velocity variation. Along the symmetry axis of the jet from zero to about 5 diameters from the exit plane the velocity is fairly constant and this region is termed the "initial" or "potential" region. From 5 to about 10 diameters the flow is "transitional". From 10 to about 30 diameters, the flow obeys the hyperbolic relationship quite well.

Reference 7 presents an equation which agrees with the trend shown by the axial mean velocity data. This equation is,

$$\frac{\bar{U}_a(x)}{\bar{U}_a(x_o)} = 0.425 \left\{ \left(\frac{b_o}{b} \right)_{0.56}^3 + 1.65 \left(\frac{b_o}{b} \right)_{0.56} \left[1.60 - \left(\frac{b_o}{b} \right)_{0.56}^4 \right]^{\frac{1}{2}} \right\} \quad (1)$$

where
$$\left(\frac{b_o}{b} \right)_{0.56} = \left[1 + \frac{(x - x_o)}{b_o} \tan \alpha \right]^{-1}$$

The value of b_o is obtained from the $\bar{U}(x,r) = 0.56 \bar{U}_a(x)$ plot in Figure 11. The value of b_o corresponds with the initial value of x_o , for which $\bar{U}(x)/\bar{U}_a(x_o)$ has a maximum value of unity. In Reference 7, x_o was taken to be at 5.4 diameters from

the jet exit plane. Figure 11 gives α as equal to 3.2° . Equation (1) is based upon a tentative phenomenological model having a rather intuitive basis.

The transverse or diametric profiles of the coaxial component of mean velocity are shown in Figures 13, 14 and 15. These profiles were obtained by traversing the jet rapidly using a hot wire anemometer. Initial traverses using a pitot-static tube confirmed the validity of data acquired in this manner. Traverses were made at 2.5, 3, 4, 5, 7, 9, 11, 13, 15 and 16.48 diameters from the jet exit plane. The wire axis was horizontal, and parallel to the direction of traverse. The diametric traverse direction was parallel to the jet exit plane.

The diametric profiles were quite symmetric about the jet axis. The degree of similarity between the profiles located from 7 to 16.45 diameters from the jet exit plane is shown in Figure 14. It may be seen in this figure that the 15 and 16.48 diameter profiles deviate slightly from the other data for $U(r, x) < 0.5 U_a(x)$. This may be caused by upstream impingement of the expanding jet upon the traversing bed or simply be due to error in fairing a line through anemometer data having considerable scatter. The scatter which was considerable for $U(r, x) < 0.5 U_a(x)$ is probably due to intermittency effects. It increased in relative severity for larger distances from the jet exit plane.

The validity of points corresponding to velocities less than 6 feet per second is questionable for axial and transverse data in Figures 12, 13, 14 and 15. It is doubtful that calibrations of the anemometer having points ranging from 20 to 200 feet per second can be legitimately extrapolated to velocities below about 6 feet per second.

The phenomenological model which was used in obtaining an expression for the axial mean velocity variation was also applied with some success to the diametric profiles. A relatively simple expression was obtained for these profiles. The following expression is an approximate form of a more exact expression which is shown in Reference 7.

$$\frac{\bar{U}(r, x)}{\bar{U}_a(x)} = 0.56 + 0.62 \left(1 - \frac{r}{r_{0.56}}\right) \left[1 + \left(1 - \frac{r}{r_{0.56}}\right)^2\right]^{-\frac{1}{2}}, \quad (2)$$

$$r_{0.56} \equiv b_{0.56} \equiv (b_o)_{0.56} + (x - x_o) \tan \alpha$$

r is the diametric distance from the jet axis, x is the distance along the jet axis and b_o may be obtained from Figure 11 and the desired value for x_o , as in the axial case, which is described by Equation (1).

Figure 11 shows that the expansion of the jet is linear from about 7 diameters to the

axial limit of the apparatus at 16.48 diameters. This is precisely the range over which good similarity between diametric profiles is also observed as may be seen in Figure 14. Corrsin (Reference 8) has noted a comparable lower limit on x/d for similarity of the diametric profiles. This lower similarity limit quoted by Corrsin is at 8 exit diameters from the jet orifice.

3.2 Turbulent Intensity

A convenient and generally accepted definition for the intensity of turbulent velocity fluctuations in a direction parallel to the jet axis, is

$$\frac{1}{U_p} \sqrt{u^2(r, x, t)}$$

where u is the fluctuating velocity component parallel to the jet axis, U_p is the jet exit velocity and t is time. The definitions of the intensities of the other fluctuating velocity components are similar.

The data of Laurence (Reference 9) and Corrsin (Reference 8) indicates that the maximum turbulent intensity, for the u component, occurs near the points of inflection in the mean velocity profiles, from the jet exit plane to the tip of the potential core. As the distance from the jet exit plane increases further, the position of maximum intensity moves out slightly from the jet axis and the intensity maximum decreases. At about twenty jet diameters from the exit plane, there is no longer a local minimum of turbulent intensity on the jet axis. Instead, the turbulent intensity profile has a more bell-shaped symmetry about the axis. This profile becomes flatter as the distance from the jet exit plane increases further.

Corrsin has noted that although similarity in the diametric mean velocity profiles seems to occur as near to the jet exit plane as eight diameters, the local on-axis minimum in the related turbulent intensity profiles does not disappear before twenty diameters. Thus, complete kinematic similarity does not occur before twenty diameters is reached.

3.3 Intermittency

The relative amount of time spent by a probe in a turbulent fluid was first measured by Townsend (Reference 10) and called the intermittency factor. Townsend measured the intermittency factor, γ , in two ways. One method involved determining the flattening factor or "kurtosis" of the intermittent signal. The other method involved measuring the mean square output of on-off signals triggered by the passage of intermittent bursts of turbulence through a gate (Reference 11). Corrsin and Kistler

(Reference 12) have used a modification of the second method. The relative "on time" was measured by counting the number of events in a high-frequency pulse train which was modulated by the on-off signal. This technique was used to get better results at low values of γ .

Corrsin (Reference 8) has also observed that, for a "fully developed turbulent jet", a completely turbulent flow exists only in the core region out to the radius at which the velocity is about one-half the velocity at the axis of symmetry. He found that an intermittent transition region existed beyond the boundary of the fully turbulent region, which becomes more laminar and less turbulent, as observations are made at larger distances from the axis.

In this case, "jet" or "fully developed turbulent jet" indicate the region over which complete kinematic similarity is observed. This region is downstream of $x/d = 20$. Corrsin and Kistler (Reference 12) have made measurements which are consistent with this observed trend for $x/d > 20$.

Upstream of $x/d = 20$ there seems to have been few measurements of intermittency factor. Bradshaw, Ferris and Johnson (Reference 5) have investigated the diametric intermittency profile for $x/d = 2$. Figure 12 compares this profile with diametric mean velocity profiles, which were obtained for this report, for flow upstream of $x/d = 5$. This figure shows the intermittency factor rising to a maximum of unity at a diametric point, r , determined by $U(r, x) \approx 0.5 U_p$ and decreasing on either side of this point.

Based on the data of Corrsin and Kistler and Bradshaw, et al, it seems that there may be a general correspondence between the trends observed in the diametric turbulent intensity profiles and in the diametric intermittency profiles throughout the jet. This is reasonable since the larger the relative amount of time spent by a probe in turbulent fluid, the larger should be the turbulent intensity. The degree to which intermittency and turbulent intensity may be regarded as independent phenomena depends upon the means by which intermittency is measured and the criteria which are used to distinguish laminar from turbulent fluid.

It would seem to be worthwhile in any subsequent work to investigate the relationship between the diametric intermittency factor profiles and the turbulent intensity profiles for $5 < x/d < 20$ in order to understand the mechanisms which give rise to the observed intermittency phenomenon. Such information could be very helpful in understanding the jet mixing process in the sound producing region of a jet.

4.0 ERROR ANALYSIS

Despite careful calibration, there are a number of ways in which errors can arise in experimental investigations using the hot-wire principle. Some of these errors arise from the idealizations which are most convenient for determining the components of a turbulent velocity vector from yawed sensors. Others are complex functions of the sensor material, sensor size, yaw angle; and the composition and design of the sensor supports. Recent evidence has been presented that the orientation of the body of a probe relative to the sensor, when the sensor direction is held constant is also a factor which affects the errors inherent in hot-wire measurements.

4.1 The Effect of Yaw on Hot-Wire Measurements

The manner in which the angle between the wire axis and the mean flow direction, θ , affects hot wire measurement has been investigated by a number of people. Initial observations by Schubauer and Klebanoff (Reference 13) led to the conclusion that the rate of heat loss per unit length of a finite wire depended only upon the normal component of velocity, as is known to be the case for infinite cylinders, (Sears, Reference 14). Hinze (Reference 15), however, suggests that an expression of the form,

$$U^2(\theta) = U^2(0) [\sin^2 \theta + a^2 \cos^2 \theta] \quad (3)$$

would be more precise where the value of "a" lies between 0.1 and 0.3 depending on the magnitude of the velocity (i.e., decreasing with increasing velocity). As may be seen from Equation 3, "a" is not appreciable for values of θ close to 90 degrees but becomes significant for values of θ greater than 30 degrees. The law for very long cylinders is formed if "a" equals zero so that one would expect "a" to lose importance as the length of a cylinder increases.

Webster (Reference 16) has made a number of measurements which confirm that "a" is finite and he suggests that a good average value for "a" would be 0.2.

These measurements were made with Wollaston wire. Lengths ranged from 0.625×10^{-3} ft to 1.056×10^{-2} ft and length to diameter ratios from 86/1 to 1456/1. Measurements were made over a velocity range from 10 to 23 ft/sec.

More recently Champagne, Sleicher and Wehrman (Reference 17) have used Equation 3 to correlate data from platinum wires for several values of Reynolds number, length to diameter ratio (l/d) and overheat ratio with varying support configurations. The constant "a" was found to be primarily dependent upon l/d . For platinum wires, "a" is approximately 0.20 for $l/d = 200$. It decreases with increasing l/d and becomes effectively zero at $l/d = 600$.

Davies and Fisher (Reference 18) have investigated heat loss from yawed wires. They infer that the temperature maximum may be closer to the downwind end of a yawed hot wire so that a larger proportion of the support heat-loss occurs at the downwind support. This, they suggest, would make platinum a more suitable wire material than tungsten by virtue of its lower thermal conductivity. However, Champagne, et al. (Reference 17) conclude, on the basis of rather extensive infrared measurements of the temperature distributions along a number of wires, that the end conduction losses for normal wires and inclined wires are identical, although the temperature distribution in the inclined case is asymmetrical. From this they conclude that the higher heat loss observed in the case of the yawed wire results from an increase in convection heat loss and this increase is attributed to a component of velocity directed parallel to the wire axis.

Champagne and Sleicher (Reference 19) proceed from these conclusions and, using Equation 3, estimate the error resulting from the assumption of a simple $U \sin \theta$ dependence of velocity upon yaw angle. They conclude that the error in selected turbulence quantities depends upon the quantity measured, the method of hot-wire operation and l/d . For an l/d of 20, they estimate errors as high as 17 percent.

The significance of these investigations to this particular program are difficult to determine without a thorough study of the hot-film anemometer. In this program l/d ratios of 60 are typical. The manufacturer states that, because of the thinness of the platinum film (on the order of 1000 Å), the glass rods upon which the films are deposited effectively determine the heat conduction to the supports. Glass typically has a thermal conductivity which is a factor of 10^2 smaller than either tungsten or platinum. The diameter of the glass rod is at least a factor of 10 larger than a typical wire. This means that the area across which heat is conducted will be larger by a factor of 10^2 . This suggests that the l/d criterion used by Champagne, et al., for hot wires may not be a valid indication of the error resulting from the asymmetry of the temperature distribution along a hot-film sensor. In other words, in terms of conduction losses to the supports, an l/d ratio of 60 for a hot-film sensor may correspond more directly to an l/d ratio of 600 for a hot wire with the same overheat ratio and length. The situation becomes even more complex when the probability is introduced that a larger temperature gradient exists along the radius of the glass rod in a hot-film sensor, than along the radius of a hot wire of the same diameter.

Because of the uncertainties involved in selecting a value of "a" and because of the above reasoning, which suggests that an effective l/d ratio of 600, or more, may exist in the case of the hot-film sensors used in this program, it was decided to let "a" equal zero for analysis purposes. However, a more thorough investigation of the effect of axially parallel velocity components upon hot-film sensors is clearly needed. A more detailed review of previous work on the yaw sensitivity of hot wires has been given by Champagne, et al. (Reference 17).

4.2 Orientation of Probe with Constant Sensor Direction

Sandborn (Reference 20) has critically considered the accuracy of the hot-wire technique and has concluded that a 20 percent inconsistency exists in the measurement of normal components of turbulence using a yawed hot wire. He gives no explanation for this inconsistency but does eliminate a number of possible explanations from the contention. One possible explanation could be the effect observed by Champagne and Sleicher (Reference 19), which is discussed in Section 4.1. Another possible explanation lies in the work of Hoole and Calvert (Reference 21) who note that the calibration of a hot wire may vary by as much as 20 percent, depending upon the incidence of the probe body relative to the stream direction. They find that this effect is significant, not only where the variation of mean flow direction from place to place may be accounted for by calibration, but also in unsteady flow where calibration is not effective, and a correction must be applied to remove the inconsistency. This effect is reported to be quite independent of errors due to finite wire length or angle between the mean flow direction and the wire axis (yaw angle). Figure 16, taken from Reference 21, shows the reported probe-orientation dependence of sensor output for a single wire normal to the flow direction.

The results of Sandborn and of Hoole and Calvert are suggestive when considered together; however, the mechanism by which the orientation of the probe body might affect the hot-wire calibration and fluctuating velocity measurements is apparently still not known. More work is necessary to more clearly define the problem. Since the heat transfer to the supports for hot-wire and hot-film sensors of comparable size is quite different, future work aimed at isolating the cause of this effect should include both types of anemometers. It would also appear to be instructive to mount wire supports of various lengths at various angles from the probe body for comparison purposes. Finally, a flow visualization study of a typical probe configuration with the probe body at various angles of incidence to the mean flow direction, and sensor direction constant, would appear to be helpful.

4.3 Errors Resulting from Linearization of the Basic Voltage-Velocity Equation

In Appendix A, a convenient operational form of the voltage-velocity relationship for a hot wire is derived. This equation, for a wire oriented in a direction normal to the mean flow direction, is:

$$E^2 = A + B (U)^h \quad (4)$$

Kramers' analysis of Ulsamer's data suggests that this expression is at least valid for a Reynolds number range of $9.5 < Re(\theta_f) < 1400$ if the wire is in an air environment and $h \cong 0.5$. Comparison of data from several sources, however, indicates that Equation 4 is fairly precise down to Reynolds numbers on the order of $Re(\theta_f) \cong 4$. Since $Re(\theta_f) = 0.379 U$ for the hot-film sensors used in this report, this implies the approximate validity of Equation 4 down to about 10 fps for constant values of A, B and h.

It is clear from data of Davis and Fisher (Reference 18), Collis and Williams (Reference 22) and Almquist and Legath (Reference 23) that a simple extrapolation of Equation 4, where A, B and h are constant, to lower Reynolds numbers is not valid. Two effects seem to cause deviations from the heat loss relationship, which applies at higher Reynolds numbers. Collis and Williams note an effect in the region $Re(\theta_f) \cong 4$ which they attribute to the development of standing vortices. A similar trend is observed in the data of Davies and Fisher and Almquist and Legath. At still lower values of Reynolds number, mixed forced and free convection effects are observed. The anemometer probe thus reads a finite velocity at zero Reynolds number because of free convection. Using the rule proposed by Collis and Williams for estimating the velocity at which the onset of free convection occurs (as presented in Appendix B) one obtains a velocity of 0.1 fps for the hot-film anemometer. This rule is quite approximate, however, and was obtained for an essentially infinite cylinder having an axis which is normal to the mean flow direction. The significance of the stated velocity should therefore not be overemphasized.

For simplicity, the assumption has been made in this investigation that "A" could be determined by simply taking a value of E^2 corresponding to the free convection velocity, $U_{f.c.}$, where $U_{f.c.} \cong 0$. Low velocity data of Davies and Fisher indicate that the error in measured velocity resulting from this calibration assumption will probably be quite small. The facts that the average value of h obtained using this value of "A" varies little from the value proposed by Kramers (i.e., 0.49 versus 0.50) and that $(E_0, U_{f.c.})$ is only one of six calibration points assure that the uncertainty introduced by this assumption lies within the already existing uncertainty of 20 percent, which Sandbarn observed in the measurement of normal components of velocity with yawed wires. In any subsequent, more precise measurements which do not extend to very low velocities, "A" should be determined by the extrapolation of a curve fitting the higher velocity points. Since it was assumed that $E_0^2 = A$ at $U_{f.c.}$, Equation 4 becomes

$$E^2 = E_0^2 + B(U)^h \quad (5)$$

where E_0 is the voltage value for free convection and B and h are determined by means of a least-square curve fit of five velocity points over a range of about 20 to 190 fps. The wires were calibrated in a yawed position after the array had been formed. Yaw dependence must therefore be included in Equation 5. Also, the standard technique for using Equation 5 in experimental investigations of turbulence is linearization by assuming that $E = \bar{E} + e$ and $U = (\bar{U} + u) \sin \theta \pm v \cos \theta + w$, for a sensor in the x-y plane. By substituting, it may be shown that

$$\bar{E}^2 = E_0^2 + B(\bar{U} \sin \theta)^h, \quad (6)$$

and

$$e = \frac{B h (\bar{U} \sin \theta)^{(h-1)}}{2 \bar{E}} (u \sin \theta \pm v \cos \theta + w) , \quad (7)$$

where

$$\frac{B h (\bar{U} \sin \theta)^{h-1}}{2 \bar{E}} = \frac{h B^{1/h} (\bar{E}^2 - E_0^2)^{h-1/h}}{2 \bar{E}} = \frac{\partial(\bar{E} - E_0)}{\partial(\bar{U} \sin \theta)} . \quad (8)$$

The assumption that this linearization is valid has been checked for a wire normal to the flow, by Sandborn (Reference 20). This assumption was examined by comparing graphic evaluation of the data with the approximate tangent to the curve at the point in question. Nonlinearity accounts for a 1 percent error in the turbulent intensity for intensity levels up to 0.4 according to his analysis.

4.4 Errors Resulting from Yawed Wire Configurations and Calibration

The common assumption in the use of yawed cylinders is that the sensor is sensitive to the normal velocity component in a plane determined by the cylinder axis and the mean flow direction. Potter (Reference 24) has discussed deviations from this idealization. For completeness sake, it would seem to be worthwhile to consider them here, also.

Figure 17 demonstrates a general relationship between wire and mean velocity vector. The mean velocity vector $\bar{U}$ taken to be along the x axis except for deviations caused by turbulent fluctuations, u , v , and w . The two axes correspond in origin and direction and are only separated for the sake of clarity.

Two unit vectors, $\vec{\tau}$ and $\vec{v}$, may be defined which indicate the direction in space of the wire axis and the velocity vector, respectively. Since the wire is taken to be in the x-y plane, it is simply represented by

$$\vec{\tau} = \cos \theta \vec{i} + \sin \theta \vec{j} , \quad (9)$$

while v is given by

$$\vec{v} = \frac{(\bar{U} + u)}{|\vec{U}|} \vec{i} + \frac{v}{|\vec{U}|} \vec{j} + \frac{w}{|\vec{U}|} \vec{k} , \quad (10)$$

where

$$|\vec{U}| = [(\bar{U} + u)^2 + v^2 + w^2]^{1/2} \quad (11)$$

In order to determine the effect of a velocity fluctuation on a sensor output, one can investigate the effect of the fluctuation on the angle, θ , between the wire axis and the velocity vector. It would be most convenient to have an expression in terms of $\sin \theta$ because of the known relationship between yaw angle and velocity for steady flows.

Since,

$$\vec{T} \cdot \vec{V} = \cos \phi, \quad (12)$$

$$\sin \phi = \left\{ 1 - \frac{\left[\left(1 + \frac{u}{\bar{U}}\right) \cos \theta + \frac{v}{\bar{U}} \sin \theta \right]^2}{\left(1 + \frac{u}{\bar{U}}\right)^2 + \left(\frac{v}{\bar{U}}\right)^2 + \left(\frac{w}{\bar{U}}\right)^2} \right\}^{1/2} \quad (13)$$

For $u = v = w \ll \bar{U}$,

$$\sin \theta = \sin \phi \quad (14)$$

In general, Equation 13 predicts that $\sin \theta$ is unequal to $\sin \phi$. For a known, unsteady flow environment, Equation 13 can be used to indicate the sort of uncertainty which is to be expected from calibrations of a sensor in turbulent or unsteady flows.

A less severe approximation (i.e., that $u/\bar{U}$, $v/\bar{U}$ and $w/\bar{U} \gg (u/\bar{U})^2$, $(v/\bar{U})^2$ and $(w/\bar{U})^2$, respectively) produces a relation which is independent of w and consistent with the idea that the inclined wire is only sensitive to motion in a plane determined by the mean velocity vector and the wire axis. This approximate form is

$$\sin \phi \cong \left\{ \sin^2 \theta - \frac{\frac{v}{\bar{U}} \sin 2\theta}{\left(1 + \frac{2u}{\bar{U}}\right)} \right\}^{1/2} \quad (15)$$

Comparison of Equations 13 and 15 in a known turbulent environment can therefore be used to estimate the magnitude of the error due to w . Apparently, for turbulent intensities below 0.2 in magnitude, the error in Equation 15, resulting from approximation, remains acceptable.

In this program a fluctuating signal was run through a transformer in order that the fluctuating signals could be separated from the dc voltage level. The dc voltage level provided an indication of the mean velocity. However, an ideal fluctuation-free dc level was not attained. Laurence (Reference 9) has noted a similar problem.

The fluctuation of the needle on the dc voltmeter apparently corresponds to low-frequency turbulent fluctuations and transitions between laminar and turbulent flow (intermittency effects). Considerable care was therefore necessary in obtaining an average value of dc voltage. An adequate time was allowed to assure that the mean voltage value was truly representative of the mean flow and not a part of some slowly changing flow condition.

4.5 Error Resulting from the Indistinguishability of Velocity and Temperature Fluctuations

In order to obtain an estimate of the sort of error which might be expected to occur as a result of temperature fluctuations, due to the turbulent mixing of a hot or cold jet with a surrounding, stationary medium, an analysis has been carried out based on the constant-temperature, hot-wire anemometer relationship derived in Appendix A. An effort was made to choose parameters which were typical of the flow parameters which were studied and of the hot-film anemometers which were used in obtaining data for this report. The basic expression from Appendix A is

$$\frac{\sigma E^2}{R} = \pi \ell \lambda \left[0.42 \text{Pr}^{0.2} + 0.57 \text{Pr}^{0.33} \text{Re}^{0.5} \right] (\theta_w - \theta_a) \quad (16)$$

By assuming Pr , ν and λ (as defined in Appendix A) to vary linearly with temperature over a film temperature range of 684°R to 828°R and assuming a wire temperature, θ_w , of 910°R , the following relationships may be obtained:

$$\text{Pr} = \alpha_1 + \alpha_2 \theta_a \quad (17)$$

$$\nu = \beta_1 + \beta_2 \theta_a \text{ (ft}^2\text{/sec)} \quad (18)$$

$$\lambda = \gamma_1 + \gamma_2 \theta_a \text{ (lb/sec-}^\circ\text{R)} \quad (19)$$

where

$$\alpha_1 = 0.708, \quad \alpha_2 = -3.33 \times 10^{-5};$$

$$\beta_1 = 1.06 \times 10^{-4}, \quad \beta_2 = 3.36 \times 10^{-7}; \text{ and}$$

$$\gamma_1 = 2.94 \times 10^{-3}, \quad \gamma_2 = 2.47 \times 10^{-6};$$

These expressions are accurate enough for the purposes of this analysis. They are based upon data from Reference 25.

Linearization of θ_a , E , and U is accomplished by the following substitutions which assume a mean and fluctuating component for each parameter.

$$U = \bar{U} + u \quad (20)$$

$$E = \bar{E} + e, \text{ and} \quad (21)$$

$$\theta_a = \bar{\theta}_a + T \quad (22)$$

A convenient expression for the error in the measurement of velocity fluctuations, due to temperature fluctuations, may be obtained from the following expression,

$$\delta u = \left(\frac{\partial u}{\partial T} \right)_{e=0} \delta T + \left(\frac{\partial u}{\partial e} \right)_{T=0} \delta e \quad (23)$$

Since only the voltage-velocity relationship is of interest, the error due to temperature fluctuations may be defined by

$$\frac{\delta u(T)}{\bar{U}} \equiv \frac{1}{\bar{U}} \left(\frac{\partial u}{\partial T} \right)_{e=0} \delta T \quad (24)$$

After substitution of Equations 17 through 22 into Equation 16, differentiation produces the following expression.

$$\left(\frac{\partial u}{\partial T} \right)_{e=0} = \frac{3.51}{d^{1/2}} \frac{(\beta_1 + \beta_2 \bar{\theta}_a)^{0.5}}{(\alpha_1 + \alpha_2 \bar{\theta}_a)^{0.33}} \left\{ \frac{K \bar{E}^2 \left[0.5 \left(\frac{\beta_1}{\beta_2} + \bar{\theta}_a \right)^{-1} - 0.33 \left(\frac{\alpha_1}{\alpha_2} + \bar{\theta}_a \right)^{-1} - \left(\frac{\gamma_1}{\gamma_2} + \bar{\theta}_a \right)^{-1} - A(1 - A \bar{\theta}_a)^{-1} \right]}{(\gamma_1 + \gamma_2 \bar{\theta}_a)(1 - A \bar{\theta}_a)} \right. \\ \left. - 0.42 (\alpha_1 + \alpha_2 \bar{\theta}_a)^{0.2} \left[\left(\frac{\beta_1}{\beta_2} + \bar{\theta}_a \right)^{-1} + 0.13 \left(\frac{\alpha_1}{\alpha_2} + \bar{\theta}_a \right)^{-1} \right] \right\} \quad (25)$$

where

$$K \bar{E}^2 = (\gamma_1 + \gamma_2 \bar{\theta}_a)(1 - A \bar{\theta}_a) \left[0.42 (\alpha_1 + \alpha_2 \bar{\theta}_a)^{0.2} + 0.57 \frac{(\alpha_1 + \alpha_2 \bar{\theta}_a)^{0.33}}{(\beta_1 + \beta_2 \bar{\theta}_a)^{0.5}} \right]$$

and

$$A = 1.10 \times 10^{-3} (\text{°R})^{-1} .$$

Substitution of Equation 25 into Equation 24 permits the temperature fluctuation error to be calculated, as a function of $\bar{U}$ and $\bar{\theta}_a$, for a cylindrical sensor with a length of 5.00×10^{-3} ft and diameter of 8.33×10^{-5} ft. A constant temperature fluctuation level of $\delta T = 3.40^\circ\text{R}$ was assumed to exist. This value was obtained as a worst condition by assuming a 20°R temperature difference across the jet and a temperature fluctuation level of $0.17 (20^\circ\text{R})$. This latter factor was taken from the work of Corrsin and Uberoi (Reference 26).

Figure 18 shows the results of this analysis. The results are not entirely unexpected. The error due to a constant level of temperature fluctuation increases slightly with the mean equilibrium temperature, $\bar{\theta}_a$, and decreases with increasing mean velocity $\bar{U}$.

4.6 The Frequency Limitation Imposed by a Sensor Array of Finite Size

In estimating the upper frequency which can be reliably measured by the array of hot-film anemometers used for this program, a simple rule based on dimensional reasoning was used.

This upper frequency limit was estimated by

$$f = \frac{U}{l} = \frac{80 \text{ fps}}{3.33 \times 10^{-2} \text{ ft}} = 2400 \text{ Hz}$$

Here, f is the frequency, U is the mean velocity at the sensor array and l is a typical array dimension. Since each probe is sampled 10,400 times per second, and the average measured velocities for the array did not exceed 80 fps, finite array size could play a rather significant role in imposing an upper frequency limitation.

Power spectral density plots, Figures 19a through 19p, of the fluctuating anemometer output voltage, reveal that noise may also become a significant factor above about 2000 Hz. The array size thus appears to be quite compatible with the quality of the data. Because of the unreliable nature of the data above 2000 Hz, frequencies above this limit were removed by a filtering process. This upper limit corresponds to an upper limit in Strouhal number of about 6, based upon jet exit velocity and diameter. Some noise above 2000 Hz is still apparent in the data. However, these small fluctuations may be ignored because of the filtering operation.

5.0 EXPERIMENTAL PROCEDURE

The objective of this program is measurement of vorticity and velocity fluctuations in the noise-producing region of a circular jet. Before such measurements could be undertaken, knowledge of the location of the jet axis and of the jet dimensions was needed to assure that a typical situation existed. The acquisition of such information constituted a calibration of the traversing gear, in that it provided reference positions from which the location of the probes in the jet flow could be determined. This mapping of the flow field was accomplished by traversing a hot-wire anemometer parallel to the jet exit plane along a horizontal line. This wire was calibrated according to a standard procedure after each period of operation, which always consisted of less than 30 minutes.

5.1 General Calibration Procedure

Prior to a run of any sort, the following steps were taken. The electronic equipment was switched on "standby" and approximately two hours of warm-up time was allowed. The air conditioning system for the room was also set for around 70°F and the room temperature was allowed to stabilize during this period. After at least two hours, the voltage readings for the anemometer units at zero flow velocity, were measured. To do this, the sensors were shielded from convection within the room by placing them within a special container. After the E_0 readings were recorded, the jet was switched on. The jet together with the cooling unit was used as a dehumidifying device. This application of cooling unit and jet is discussed in Section 2.3. After five minutes the temperature regulation system was turned on and adjusted until the jet total temperature was automatically held within $\pm 1^\circ\text{F}$ of the temperature at which the E_0 values were measured. Time was then allowed for the room temperature to return to this value.

Calibration was undertaken with a pitot-static tube and a total-temperature thermocouple located in the laminar region of the jet potential core. This laminar region was considered to be less than one diameter from the jet exit plane. Jet flow-visualization photographs obtained by Bradshaw, et al. (Reference 5) and Davies and Fisher (Reference 27) confirm the laminar character of the flow in the region near the jet exit plane and provide a rough idea of its extent. The hot-film anemometer sensor, or sensors, were located immediately beside the pressure and temperature measurements. The minute velocity gradient in this region made error due to small separations in instrumentation negligible.

The Saint-Venant and Wantzel equation, which relates velocity to total ("0" subscript) and static properties, is

$$U^2 = \frac{2\gamma}{\gamma-1} \frac{P_0}{\rho_0} \left[1 - \left(\frac{P}{P_0} \right)^{\gamma-1/\gamma} \right] \quad (26)$$

Using a measuring system consisting of a thermocouple, which measures total temperature, and two manometers, referenced to atmospheric pressure, makes it more convenient to put this expression in the following operational form by use of the ideal gas law.

$$U^2 = \frac{2\gamma}{\gamma-1} \frac{RT_0}{M} \left[1 - \left(\frac{P_A + P_S}{P_A + P_T} \right)^{\gamma-1/\gamma} \right] \quad (27)$$

where R is the universal gas constant, M is the molecular mass, γ is the specific heat ratio, T_0 is the total temperature, P_A is atmospheric pressure and P_S and P_T are the excesses of the static and total pressure above atmospheric pressure, respectively.

The velocities in the potential core region were estimated, by use of Figure 20 and the total pressure measurements, for the immediate purposes of defining the velocity range and determining the spacing of the calibration points. Figure 20 was plotted from the Bernoulli relationship for incompressible flows shown in Equation 28. This equation may be obtained as a special case of Equation 26. If $U^2/C_0^2 \ll 1$, where C_0 is the velocity of sound under stagnation conditions, Equation 26 reduces to

$$U^2 = \frac{2(P_0 - P)}{\rho} \quad (28)$$

since for low velocities $\rho \cong \rho_0$. Comparison of Equations 28 and 26 show that Equation 28 is adequate for velocity estimation purposes to at least 200 fps. Equation 27 was used in all calibration work to avoid slight errors due to compressibility effects.

Since the manometer indicating atmospheric pressure was located in a different room from the jet, it seemed a wise precaution to correct for the temperature difference between the two locations. Density-temperature information for mercury was obtained from Reference 28, while that for the manometer oil (Meriam D-2673, red oil) was furnished by the manufacturer. The following density-temperature relations were used for a temperature range of 60-130°F,

$$P_{\text{mercury}} = - [1.355 (10)^{-3}] T_R + 14.262 \quad (29)$$

where

$$P_{\text{oil}} = - [3.773 (10)^{-4}] T_K + 1.023 \quad (30)$$

where T_R is in $^{\circ}\text{R}$ and p is in grams per cubic centimeter.

The atmospheric pressure in the laboratory in inches of oil was therefore determined by

$$P_A(\text{oil}) = \frac{P_{\text{mercury}}}{P_{\text{oil}}} P_A(\text{mercury}) \quad (31)$$

This equation produced a temperature corrected value of P_A which had units that were consistent with P_S and P_T . This made possible the direct use of $P_A(\text{oil})$ in Equation 27.

Possible humidity errors in mean velocity measurements, appearing as departures from the assumed constant values for molecular mass and specific heat ratio, are quite obviously minor. This point has been discussed in Reference 6 and it has been shown that the error is less than 0.5 percent at 100 fps. The additional effort needed to incorporate a correction for this effect into the computer data processing routine was slight. A routine measurement of relative humidity during calibration and acquisition of data also required little effort. It was therefore decided to make the correction on a routine basis. In this way, at least, conditions in the laboratory during any run would be more accurately specified as a result of the addition of the humidity measurement to the schedule.

The calibration plots consisted of values of $(E^2 - E_0^2)$ plotted versus U on log-log paper. Examples are shown in Figures 21 and 22. An eight point calibration was used in obtaining data on the mean velocity profiles, however, this was reduced to a five point calibration for the fluctuating data acquisitions, because of the length of time required to calibrate eight wires simultaneously. The manner in which the computer processed calibration data is discussed in Section 6.

5.2 Determining the Jet Axis

The traversing equipment was aligned and leveled by determining the central minimum in the rms voltage output from an anemometer, which was traversed approximately across the diameter of the jet flow field at various axial positions. Threaded driving shafts on the traversing mechanism having 10 turns to the inch for horizontal movement and 20 turns to the inch for vertical movement were used to specify probe position in the jet flow. Mechanical turn counters were used on the horizontal driving shafts in order to provide a direct indication of position. The vertical position was specified once and the table leveled accordingly. After this was accomplished only horizontal traverses were necessary. For both the vertical and horizontal diametric traverses, a plot was made of the number of transverse turns (corresponding to the rms voltage minimum) versus the axial position. These two plots were used to specify the location of the jet axis throughout the flow field. Because the central minimum, in curves of rms output versus diametric position, disappears for $x/d > 20$, this method of defining the jet axis only offers advantages over the measurement of mean velocity profiles up to $x/d = 20$. However, since the traversing gear only allowed a convenient investigation of the flow field to about 16.5 diameters from the jet axis plane, no change in procedure was necessary in this respect.

5.3 Acquiring Data on Turbulent Velocity Fluctuations

The calibration procedure, in the case of the computer acquisition of fluctuating data, was generally the same as that used to acquire the mean flow data with the following minor exceptions.

The angle between the cylindrical sensor and the probe body was measured using an optical comparator. This device was very sensitive and consisted of a mount, azimuth ring and optical system which projected an enlarged silhouette of the probe upon a translucent screen. These angles are given for the hot-film sensors in Figure 23. The four cross-wire probes were mounted within a large circular ring and adjusted relative to one another until they formed a fairly symmetric array and were parallel to the jet axis. A sighting system which mounted on the traversing table was helpful axial alignment of the probes. Once the final configuration was reached, the probes were locked into position and taped to prevent relative vibration of the probe bodies. Measurements were then made of the dimensions of the array. The ring structure in which the probes were mounted may be seen in Figures 4 and 5. The ring structure, which is designed for use close to the four-inch jet, is of sufficient size that flow cannot impinge upon it for an axial sensor position of $x/d = 3$.

The computer acquisition of the data lasted about five seconds. Each probe (one set of crossed sensors) was sampled at a rate of 10,400 samples per second, and the computer proceeded around the array in a clockwise manner as one faces the sensor elements. Figure 23 gives a schematic illustration of the sensor array together with pertinent dimensions and details. During the sampling period, dc values of voltage were recorded in the laboratory for later use in determining, from calibration information, the mean flow velocity at each wire and the sensitivities of the anemometers to fluctuating velocities.

6.0 DATA PROCESSING

6.1 Computer Programs

The program "MILPOT" shown in Figure 24 is used in conjunction with subroutines "CALIB," Figure 25, and "FILTER," Figure 26, to convert anemometer data into the fluctuating velocity and vorticity components characterizing turbulence. Other auxiliary routines are not shown. These are: a subroutine for removing digital data from tape for processing, and two subroutines for graphical representation of the data. A flow diagram of the programs is shown in Figure 27.

The calibration subroutine is based upon Equation 6. A linear relationship between values of $\ln(U \sin \theta)$ and $\ln(\bar{E} - \bar{E}_0^2)$ is assumed. The constants h and B are then determined from the slope and intercept of this line. Equation 8 is next used with h , B , E_0 and a local value of E to determine the sensitivity, S , of a wire to velocity fluctuations at a local value of mean velocity. This procedure may be somewhat questionable at higher frequencies where a sensitivity based strictly upon a mean flow calibration may not apply. However, for frequencies below 2 kHz this procedure is acceptable.

Table 1 gives values of h , B and E_0 . Sensitivities, S_1 and S_2 , for Position 1 and 2 in the flow, respectively, are also shown.

TABLE 1
CALIBRATION CONSTANTS AND SENSITIVITIES
FOR THE HOT FILM SENSORS

Sensor	h	B	E_0	S_1	S_2
4-1	0.4612	0.3699	1.39	4.781×10^{-3}	1.340×10^{-2}
4-2	0.4879	0.8512	4.00	8.018×10^{-3}	1.941×10^{-2}
5-1	0.5102	0.9670	5.02	1.027×10^{-2}	2.342×10^{-2}
5-2	0.4745	0.9232	3.17	9.498×10^{-3}	2.840×10^{-2}
2-1	0.4958	0.9426	3.84	8.599×10^{-3}	2.102×10^{-2}
2-2	0.5212	0.6381	3.35	7.779×10^{-3}	1.955×10^{-2}
1-1	0.4848	1.0421	4.66	8.127×10^{-3}	1.851×10^{-2}
1-2	0.4995	1.1329	6.35	8.547×10^{-3}	1.864×10^{-2}

The equations which were used in the calculation of the components of the fluctuating velocity vector are developed in Appendix C and presented in Figure 23. The time series of velocity component values, shown graphically for Positions 1 and 2 in Figures 28 and 30 were obtained by averaging arithmetically over the array as shown by the equations in Figure 23. This operation gave rise to a sampling rate of 10,400 velocity values per second, although the actual probe-by-probe multiplexing rate was a factor of four higher. The products uv and vw shown in Figures 28 and 30 are obtained from these averaged values of u , v and w .

Vorticity components were determined from the equations shown in Appendix D. The ξ component was determined directly from the measurements while the η and ζ vorticity components were obtained by application of Taylor's hypothesis as indicated in Appendix D. Time differentiation was accomplished by a three-point technique. A matching set of v and w values for the calculation of a vorticity vector was obtained over a time interval of 2.88×10^{-4} seconds and at a rate of 10,400 sets per second.

The turbulent intensities shown in Figure 15 were obtained from an average of 400 values of u , v and w which in turn are obtained from arithmetic averages over the array, as discussed above. The 400 values correspond roughly to a 0.038 second average. The turbulent intensity values may therefore be slightly in error because of some very low frequency effect. This effect would have to be below about 30 Hz to escape the averaging process.

6.2 Graphical Presentation of Data and Filtering

The plots in Figures 29 and 31 were obtained by filtering the time series for the array velocity and vorticity components to eliminate frequencies above 250 Hz. This frequency limit corresponds to an upper limit in Strouhal number of 0.77 if Strouhal number is calculated from jet exit velocity and diameter. The cutoff frequency was selected because of the character of the power spectral density plots, for the fluctuating voltage output of the hot-film sensors. The plots for Position 1, near the potential core, in the jet flow show a definite maximum at, or slightly below, 10^2 Hz. It was also found, by trial and error, that plots including higher frequencies produced graphs with an increasingly random positioning of points, as the upper frequency limit increased, although similar average trends were still observable. It was therefore decided that the essential character of the macroscale of turbulence could best be studied by eliminating the frequencies above 250 Hz and plotting the data.

The data in Figures 28 and 30 contain frequencies up to 2 kHz. All data was pre-filtered at 2 kHz for reasons discussed in Section 4.6. Where necessary, a dotted line on the power spectral density plots indicates the 2 kHz cutoff point.

The numerical filtering was accomplished by use of a discrete low-pass filter function (Reference 29) of the form

$$g_k = \sqrt{\frac{1}{2\pi}} \left[1 + \cos \left(\frac{2\pi f_c t_k}{a} \right) \right] \left[\frac{\sin 2\pi f_c t_k}{t_k} \right] \quad (32)$$

where

f_c = the filter cutoff frequency,

t_k = k/s , the incremental time,

k = $0, 1, 2, \dots, as/2f_c$,

a = a parameter which determines the number of zero crossings of g_k for constant f_c , and

s = the sampling rate.

Each time series, q , was operated on using Equation 37 to obtain the filtered time series $\hat{q}$. The following equation was used:

$$\hat{q} = \frac{1}{\sqrt{2\pi}} \sum_{k=-as/2f_c}^{as/2f_c} \frac{1}{s} [q_{(i+k)} \times g_k] \quad (33)$$

6.3 Trends Observed in the Power Spectral Density Plots of the Fluctuating Anemometer Output Voltage

Because the data which was acquired in this program was stored digitally on tape, after being multiplexed, it would have been extremely difficult to evaluate without the use of some sort of preliminary diagnostic program. A power spectral density program, which was available, was therefore used for data evaluation. Figure 19 shows plots of the power spectral densities for the fluctuating output voltages of the anemometers. Since these plots were only intended to be used for diagnostic purposes, a common scaling factor was used for all sensors. Certain trends are noted in these plots, which are identical with those observed in spectra by Bradshaw, Ferris and Johnson (Reference 5) for the same jet regions. Bradshaw, et al., state that for the "inner region" (Position 1) all three spectra, u , v , and w , have marked peaks; and in the "outer region" (Position 2) all three spectral fall monotonically with increasing frequency. The location of the peaks for Position 1, also occur in practically the same spectral region as those in power spectral density plots for a comparable jet region presented by Laurence (Reference 9).

There was some scatter in the power spectral density data. Therefore, a solid line was faired through the approximate center of scatter and dotted envelopes were used to indicate the maximum excursions.

The error introduced into these power spectral density plots by the data processing has been estimated. Since one hundred statistical degrees of freedom were used in the calculations, the normalized standard error for these calculations is therefore about 14 percent. That is, the probability that a given power spectral density point lies within ± 14 percent of the true value is 68.26 percent. The normalized standard error is the ratio of the standard deviation of the computed power spectral density to the true power spectral density expressed in percent.

A tendency toward rather large spikes in the power spectral density plots was noted at, or slightly below 100 Hz. This tendency was very noticable beside the potential core, at Position 1, but is not so apparent at Position 2 near the jet boundary. These spikes may be the result of preferential energy distribution in various frequency bands due to the large eddy structure. The fact that Bradshaw, et al., has identified the existence of a spectrum peak with large eddy structures in this frequency domain tends to support this point of view.

7.0 REVIEW OF PREVIOUS OBSERVATIONS

One of the earliest models for mixing by free turbulent flows is that of Prandtl (Reference 30). For a flow in a round jet, this model predicts that the rate of transfer of any quantity, ψ , across a unit area parallel to the jet exit plane equals $-\epsilon (d\bar{\psi}/dr)$, where ϵ is called the turbulent transport coefficient, r is a radial coordinate perpendicular to the jet axis, and the distribution, $\bar{\psi}$, expresses the mean value of ψ as a function of r . Thus, fluctuating motions are assumed to be responsible for the transport of ψ , which are small in scale compared with the scale of the distribution, $\bar{\psi}$.

At first it was assumed that

$$\epsilon \propto \ell v \quad (39)$$

where ℓ and v are characteristic of the size and velocity, respectively, of a typical eddy, and that

$$v \propto \ell \left| \frac{d\bar{U}}{dr} \right| \quad (40)$$

where $\bar{U}$ is a mean velocity, so that

$$\epsilon \propto \ell^2 \left| \frac{d\bar{U}}{dr} \right| \quad (41)$$

Later Prandtl (Reference 31) and Görtler (Reference 32) suggested the replacement of Equation 41 by the postulate

$$\epsilon \propto b \left| \bar{U}_{\max} - \bar{U}_{\min} \right| \quad (42)$$

for the case of free turbulence. The vector $(\bar{U}_{\max} - \bar{U}_{\min})$, is the total change in mean velocity over the distance "b". In the case of a round jet, "b" is the perpendicular distance from the jet axis to an outer boundary, where turbulence is negligible. Equation 42 accounts for the action of "large eddies" in transfer by free turbulent flows. As Batchelor (Reference 32) has noted, this later modification is inconsistent with the basic idea that turbulent transfer is proportional to the local gradient (which implies that small eddies are responsible for such transfer).

Theodorsen (Reference 4) has proposed a model for the "large eddy" structure of a three-dimensional boundary layer based upon theoretical considerations. He also notes that slight modifications would make the same considerations applicable to jets and wakes. A handle-like, wall-bound "horseshoe" or "tornado" of turbulence (Figure 32) is proposed. The word "tornado" indicating a similarity in creative mechanisms.

These "horseshoe" structures are stated to appear in a plane normal to the wall around a region of low velocity near the boundary. (The vortex strength is thus proportional to the velocity difference across this region.) Next they are pulled downstream by the drag force of the main stream and away from the wall by a cross force, which is normal to the wall. The energy extracted from the main stream is almost equal to the energy absorbed by the vortex structures. A typical horseshoe vortex is oriented so that energy transfer from the main stream is optimum. This orientation is at 45 degrees to the direction of mean flow. Beyond an angle of 70 degrees, the rate of energy transfer becomes smaller and an old vortex structure is replaced by a new one; the old being spun around the center of the new. The fine structure resulting from the old vortices is responsible for most of the dissipation. This fine structure may be observed as secondary fluctuations superimposed upon primary fluctuations. Since these secondary fluctuations dissipate efficiently, the primary structures, on-average, have a much longer lifetime. The concentrated flow of a transferable quantity through the center of each "horseshoe" is proposed as an explanation for the skewness observed in shear flows.

Townsend (Reference 3) has proposed an idealized model for turbulent flow processes which he observes to provide a "coherent qualitative account of the whole turbulent motion that is physically reasonable." Basic to this model is the assumption that the rate of spread of turbulent fluid is controlled by the process of entrainment of irrotational fluid and by a system of "large eddies," which contort the boundary between the turbulent and irrotational fluid. This boundary is assumed to correspond to a sharp change in the character of the observed velocity fluctuations. Actually, velocity fluctuations propagate into the irrotational fluid to a distance roughly equal to the scale of the energy-containing, turbulent eddies. Convolution of this boundary by the "large eddies" allows entrainment to proceed over a considerably larger surface than would otherwise be possible.

The large eddies are considered to have a scale about equal to the width of a region for which the velocity gradient is of the same sign. They are basically simple, with central vorticity along the principal axis of positive mean strain rate. They are elongated in the direction of flow and centered near the plane of maximum shear rate. It is also noted that these "large eddies" are transient, with new ones arising as the old ones disappear.

The turbulent intensity is nearly uniform over the rotational fluid, according to Townsend, except at the boundary, where the process of entrainment is proceeding and except near a position of zero shear rate (for instance, near the symmetry axis of a round jet) where the rate of turbulent energy production is low. As defined, the turbulent intensity is a time average. According to this model, the contribution to this average by the constant-intensity turbulent fluid is diminished by the probability that the fluid is not turbulent at the point of observation at any moment.

Townsend has formalized this concept by the introduction of an "intermittency factor" in the following manner:

$$\overline{u^2} = \gamma \overline{u_t^2} \quad (34)$$

The parameter u_t is the actual fluctuating velocity component in the turbulent fluid, γ is the intermittency factor and u is the measured component of fluctuating velocity. According to this model $\overline{u_t^2}$ is constant so that the turbulent intensity profile should be similar to the profile for the square root of the intermittency factor. This point is consistent with the observation concerning the similarity in intermittency factor and turbulent intensity profiles, which was made in Section 3.3. However, Townsend (Reference 3) and Kibens and Kovasnay (Reference 34) note that Equation 34 is an idealization and indicate that care must be used in applying it. In particular, the contribution to the turbulent intensity by the large eddies and by significant velocity fluctuations in the irrotational fluid cause it to lack precision.

Grant (Reference 2) has formulated a model for large eddy structure in the turbulent wake of an infinite cylinder. This model is based upon visualization experiments and extensive correlation measurements. It postulates the existence of two typical large scale structures.

The first kind of typical structure is a pair of counter-rotating vortices whose circulation is in the "plane of the wake." (This plane is defined by the cylinder axis and the mean velocity vector.) The circulation is about axes inclined in the downstream direction at an angle of 45 degrees to the plane of the wake. The direction of rotation of these vortices is such that they can draw energy from the velocity gradient (see Figure 33).

The second kind of typical structure is a series of re-entrant mixing jets. These jets project turbulent fluid outward from near the core of the wake. Their roots are out of phase in the plane of the wake along a line parallel with the cylinder axis. A single jet was noted to trip off a roughly periodic series, the members of which are irregular in velocity and extent of travel. This phenomenon was presumed to be the result of stress relief by secondary flows.

The relation between the "vortex-pair" eddies and the re-entrant "mixing jets" was not specified with certainty. An origin of the eddies near the vortex street of the cylinder was considered most probable. However, Grant also notes that the vortices could form about a fast tongue of irrotational fluid, which is drawn into the wake as a part of the mixing jet action.

Recently, Lumley (Reference 35) proposed a technique for formulating an objective definition of what is meant by a "large eddy." His formulation involves decomposing the velocity covariance into a sum of its eigenfunctions and interpreting the dominant

eigenfunctions as large eddies. This technique has been applied by Payne (Reference 36) to the correlation measurements of Grant. The resulting structure of the dominant eigenfunction was qualitatively similar to Grant's "vortex-pair" and re-entrant "mixing jet" model. Payne concluded that although the dominant eigenfunctions may differ in detail from previous models, they do exhibit the behavior and scales usually associated with large eddies.

The shear region beside the potential core of a two-inch diameter, Mach 0.3, cold air jet was investigated extensively by Bradshaw, Ferriss and Johnson (Reference 5) by means of correlation measurements. It was concluded on the basis of these measurements that this region is characterized by a system of "large eddies" covering a narrow band of wavelengths and taking the form of mixing jets. These eddies cross the shear region at an angle of 45 degrees in the v - w plane (see Figure 34) so that the v and w components of these eddies are either in phase or in antiphase. They also undergo large changes in the u -component of velocity relative to the local mean velocity. After reference to Schlieren photographs, Bradshaw, et al., conclude that the maximum rate of extension of the vortex lines occurs at an angle of -45 degrees in the u - v plane. It is stated with certainty that the mixing jets are asymmetrical in the v - w plane. No strong evidence was found by these investigators to specify whether the mixing jets were inward or outward going, although they conclude that the observed growth rate of the jet makes outward going mixing jets more plausible. From the decay of the v and w spectrum peaks, Bradshaw, et al., conclude that the jets die out in an early stage of development, presumably at around $x/d = 7$. It is also stated that these mixing jets are the dominant cause of the near-field pressure fluctuations of a jet.

Corrsin (Reference 8) has noted an extremely rapid spread of turbulence into the potential core of a jet. A turbulence level was noted at an axial distance of only one diameter from the jet exit plane, whereas the on-axis velocity remained essentially constant to about 4.5 diameters. In particular, Corrsin confirmed a regular type of fluctuation, first observed in a three-inch jet by Thiele in 1940, and presented oscillograph records of these fluctuations at two positions in the jet. These positions were 2 diameters from the exit plane. Measurements were made both on the jet axis and at a distance of $d/2$ from the jet axis, where " d " is the exit diameter. It was noted that the region at greater distances from the jet axis apparently constituted a turbulent boundary region through which the regular pulsations would not propagate.

Both Bradshaw, Ferriss and Johnson (Reference 5) and Davies and Fisher (Reference 27) have observed that the fluctuating u -components of velocity contain occasional "spikes" which are predominantly negative-going for radial position, $r < d/2$, and positive-going for $r > d/2$. Davies and Fisher further observe that, at $r = d/2$, the u -fluctuations are completely symmetric about the zero velocity point and they present velocity distributions and oscillograph traces confirming these trends. Corrsin and Kistler (Reference 12) have made similar observations for a turbulent boundary layer. They attribute the "one sidedness" of the spikes observed in the $u(t)$ oscillograms to the fact that, on the average, turbulent fluid is moving more slowly than nonturbulent fluid

passing the same probe position. This they explain by the fact that the turbulent fluid originates near the wall at a lower value of mean velocity, while the laminar fluid comes from the faster moving free stream.

The reasoning of Corrsin and Kistler may be carried over to the case of a jet if one identifies the potential core and the external medium as sources of laminar fluid and considers the relative velocities of the predominantly laminar and predominantly turbulent fluid in Region 1 and Region 2 (Region 1, $d/2 > r > 0$; Region 2, $b > r > d/2$, where b is the jet boundary). The case of Region 1 is very similar to that of the boundary layer with a faster moving, more predominantly laminar potential core playing the role of the free stream and Region 1 identified with the turbulent boundary layer. On the average, the turbulent fluid will be moving more slowly than the laminar, irrotational, fluid, and $u(t)$ measured at fixed point in the shear region, should have occasional excursions which are negative and, on the average, larger than the positive excursions.

In the case of Region 2, the predominantly turbulent region is moving more rapidly than the predominantly laminar region so that one would expect a reversal of the trends observed in Region 1, with $u(t)$ showing occasional, large, positive spikes. Since $r = d/2$ corresponds to the approximate center of the shear region, it is quite reasonable that at this radial position, a greater randomness should prevail and the u -fluctuations should, on-average, be more symmetrically spaced about $u = 0$. The Kurtosis or "flatness factor" data of Bradshaw, Ferriss and Johnson (Reference 5), confirms the relative lack of occasional sharp $u(t)$ "spikes" near $r = d/2$.

According to the theoretical analysis of Stewart (Reference 37) and the observations of Corrsin and Kistler (Reference 12) the mean flow at the outermost radial limit of the jet is directed inward toward the jet axis. Stewart shows that the direction of the mean velocity component at the outermost boundary is practically normal to the axis, for a circular free jet. This point is consistent with the entrainment and mixing of irrotational fluid.

8.0 RESULTS

8.1 General Discussion

Figures 28 through 31 give the results of the present hot-film anemometer investigation. Figures 28 and 30 both give the results as a direct function of time. Figure 28 corresponds to a measurement at Position 1 near the potential core, while Figure 30 corresponds to Position 2, which is nearer to the outer jet boundary (see Figure 15). Both figures give plots of the three velocity components u , v and w , two velocity products, $\text{SGN}(uv) \sqrt{uv}$ and $\text{SGN}(vw) \sqrt{vw}$, and three vorticity components ξ , η , ζ . Coordinate sign conventions are shown in Figure 34. The u and ξ components are positive in the downstream direction, v and η are positive radially inward toward the symmetry axis of the jet, and w and ζ complete a right-hand coordinate system.

The velocity values presented in Figures 28 through 31 correspond to averages over the array, as discussed in Section 6.1 and illustrated in Figure 23. The basic array gives four instantaneous estimates of u and two each of v and w ; the averages of these estimates are plotted in each case. The velocity product terms are products of these array averages. The vorticity was calculated as discussed in Appendix D, using a modified Taylor's hypothesis. Because of the finite array size, as discussed in Section 4.6, all the plots presented in Figures 28 and 30 were filtered to reject frequencies above 2 kHz (see Figure 19). This action was taken to eliminate the contribution of turbulent eddies smaller than a typical array dimension.

Figures 29 and 31 give two-dimensional cross plots of the data presented in Figures 28 and 30, respectively. Plots of u versus v , v versus w , ξ versus η and η versus ζ are given in each of these two figures and designated by a, b, c and d, respectively. As mentioned in Section 6.2, the results showed an increase in isotropy as the cutoff frequency was raised during trial runs of the low pass filtering routine. As a result of these trial studies, the results now shown in Figures 29 and 31 have been subjected to further low pass filtering, so that all frequencies above 250 Hz are rejected. (Inspection of the peaks in the power spectral density data of Figure 19 also supported the selection, in this case, of 250 Hz as the optimum cutoff frequency.) This additional filtering reveals more clearly the non-isotropic nature of the macroscale processes. These processes shall later be discussed in more detail. Note, also, that both Figures 29 and 31 correspond to a limited part of the data presented in Figures 28 and 30. Only data between approximately the 10 and the 24 millisecond marks (see Figures 28 and 30) are shown in the two-dimensional plots. The index numbers in Figures 29 and 31 refer to a minimum, maximum or zero point in the ordinate or abscissa of these plots. These points are also shown on Figures 28 and 30 to facilitate comparison.

In interpreting these data, a few points should be kept in mind. First of all, it should be noted that Figures 28 and 30 represent a sample length of about 38 milliseconds, while Figures 29 and 31 correspond to a filtered 15 millisecond segment of the data

shown in Figures 28 and 30. Samples of such short duration could cause an unrealistic mean value for a time series, resulting from a low frequency trend or a transient and atypical flow condition. Secondly, the small size of the room may have contributed to experimentally undesirable regions of recirculation near the jet flow boundary.

With these points in mind it, at least, appears safe to state that these data should be representative of velocity fluctuations above about 30 Hz and below 2 kHz. Confidence in these data in this frequency domain is sufficient to permit conclusions to be drawn concerning the large eddies, since these eddies apparently dominate a narrow band of frequencies between 80 Hz and 200 Hz. The study of individual and concurrent events in these time series, within this frequency band, appears to offer support to many existing ideas concerning the behavior of large eddy structures in shear flows, although the data are noted to differ in some respects, which can be explained in terms of the problems mentioned in the previous paragraph.

In the subsequent discussion, the terms "typical," "trend," or "average" must not be considered to infer a negligible standard deviation about a mean condition. As may be seen from Figures 19, 28 and 30, there is nothing to support an extreme degree of organization below 2 kHz except possibly within a narrow band of frequencies between 80 and 200 Hz.

8.2 Discussion of Trends at Position 1

A tendency for the u-component to alternate between positively and negatively directed surges within a narrow band of frequencies between 80 and 200 Hz is apparent in Figure 28. The negatively directed surges seem to be the stronger of the two. A periodicity is also evident in the v-component, which is similar in frequency content (80 - 200 Hz), except perhaps between 10 and 20 milliseconds. The time series for the v and w components appear to have small positive mean values. A less regular alternation is evident in the case of the w-component than in the u and v series.

The velocity product series, $\text{SGN}(u v) \sqrt{|u v|}$, consists of large negative surges and a negative mean value. Comparison of the $\text{SGN}(u v) \sqrt{|u v|}$, u and v traces suggests that u and v are essentially out of phase by 180 degrees, (where v is positive when directed inward toward the jet axis). The series $\text{SGN}(v w) \sqrt{|v w|}$ has a negligible mean value. There is also a trend toward periodicity within the 80 to 200 Hz frequency domain for this series except from 5 to 22 milliseconds, where more random behavior is evident. Comparison of the $\text{SGN}(v w) \sqrt{|v w|}$ series with the v and w series indicates that from 5 to 22 milliseconds, v and w are close to 90 degrees out of phase, while over the rest of the trace the phase difference between v and w alternates between 0 and 180 degrees. This alternation is within the characteristic frequency domain of 80 to 200 Hz.

The ξ component of vorticity exhibits a negligible mean value and a lack of characteristic frequencies (80 to 200 Hz) except possibly in the last 15 milliseconds of the series. The η component seems to show more evidence of characteristic frequencies with a negligible mean value for the series. The ζ component also exhibits a negligible mean value. Within the characteristic frequency domain, η changes sharply from being in phase with the ζ -component to being 180 degrees out of phase. These phase changes seem to be coincident with a simultaneous decay of all the vorticity components to near zero and with large negative surges in the v -component of velocity.

The filtered data for Position 1 (low-pass with cutoff at 250 Hz) shown in the two-dimensional plots of Figure 29 also include some interesting trends. Figure 29 also include some interesting trends. Figure 29a shows negative u values closely corresponding to positive v , while positive u values are largely in the neighborhood of small negative or zero values of v . Figure 29b shows a tendency of the velocity vector in the u - v plane to spiral about a line making an angle of 45 degrees in the v - w plane. The nature of this spiral suggest that v and w are out of phase by an angle which changes from near 0 degree to near 90 degrees over the sample. The velocity and vorticity cross plots are not entirely typical since the region over which the filtered data were obtained corresponds almost entirely to the η and ζ components being in phase. It is evident that the phase difference between η and ζ (between 80 and 200 Hz) is closer to 180 degrees over much of the rest of the series. The point 9 η in Figures 29c and 29d is also of interest. At this point all vorticity components decay to near zero, with the ζ component remaining the largest. This point corresponds to a strong negative surge in v which is concurrent with a phase switch between η and ζ .

8.3 Discussion of Trends at Position 2

The u -component behavior in Figure 30 is similar to that in Figure 28. Periodicity is present in a narrow band of frequencies between 80 and 200 Hz. This trace also exhibits positive excursions which are larger than those in the negative direction. The apparent presence of frequencies below 80 Hz makes it difficult to establish any average value over this trace. The v -component series exhibits a positive mean value and alternates between positively and negatively directed surges in the characteristic frequency domain (80 - 200 Hz). The w -component series seems to exhibit a negative mean. The w and v components alternate, in the characteristic frequency domain, between rather brief periods during which they are in phase and longer periods over which they are in antiphase.

The velocity-product series, $\text{SGN}(uv) \sqrt{|uv|}$ displays largely negative values with occasional positive excursions, which indicate a more predominately 180-degree phase difference over the series. The $\text{SGN}(vw) \sqrt{|vw|}$ series exhibits similar behavior, which also indicates that the phase difference between v and w is substantially 180 degrees. However, characteristic periodicity (between 80 and 200 Hz) is evident in $\text{SGN}(vw) \sqrt{|vw|}$, which indicates a tendency of the v - w phase difference to alternate between brief periods of being in phase and longer periods of being more strongly 180 degrees out of phase (an observation confirmed in the discussion of the individual series above).

The vorticity components in Figure 30 exhibit some rather unexpected trends which indicate the presence of very low frequencies. The ξ -series exhibits a rather strong positive bias, while the η component undergoes a transition at about the 17 millisecond mark from a negative to a positive bias. The ζ -series shows less very low frequency content and exhibits characteristic periodicity (between 80 and 200 Hz) over most of the trace.

Figure 31a shows the influence of a large positive surge in u which is accompanied by essentially positive values of v . Figure 31b shows an essentially 180-degree phase difference between v and w . Centered in this sample is a rather rapid phase-antiphase switching. The vorticity plots in Figures 31c and 31d exhibit the results of a large negatively directed surge in u corresponding to generally positive v values between $4v$ and $5v$. This surge drives the η and ζ components to near zero and the ξ component to large positive values.

8.4 Discussion and Interpretation of Concurrent Events

According to the discussion of Section 7, a stationary probe should be able to determine whether surges in the u -component of velocity are caused by fluid arriving from an interior or exterior region of a jet. A positive surge, for instance, can be said to originate in the faster moving fluid between the probe and the potential core, while a negative surge can be identified with the slower moving fluid between the probe and the outer jet boundary. In addition, an outward moving mass of fluid should have a negative value for the v -component and an inward moving mass of fluid should correspond to a positive value of v . A significant jet of fluctuation-free irrotational fluid from the potential core or from outside the jet should also correspond to the vorticity components approaching zero.

Working from the above premises, some interesting conclusions can be drawn from Figures 28 and 30 (corresponding to Positions 1 and 2, respectively). One notices immediately that at both positions the same concurrent trends in u , v and ζ are observed. It is also significant that, for both positions, the u -component alternates from positively to negatively directed surges over a narrow range of frequencies (between 80 and 200 Hz), which seem to be characteristics of macroscale effects.

According to the data and the previous discussion, it appears that motion in the u - v plane is typified by a regular sign alternation (at frequencies between 80 and 200 Hz) in the u and v components of velocity and the ζ component of vorticity. Such alternations would appear to be measurable by a stationary probe at practically any point in the shear region.

This suggests that a jet of fluid which moves outward from near the potential core rotates in a manner that permits it to draw rotational energy from the velocity gradient. An inward moving jet of fluid appears alternately with the outward one. This inward jet rotates in a direction in the u - v plane, which suggests that it must lose rotational energy upon interacting with the velocity gradient. It seems from concurrent

w-component surges that the inward moving fluid has smaller values of w near the outer region of the jet (Position 2), but that w surges become more significant by the time the inner region (Position 1) is reached.

Bradshaw, Ferriss and Johnson (Reference 5) have noted that $\overline{vw}$ should be zero for a sufficiently long time-average because of the basic jet symmetry. Two possibilities were presented by which this could be true. The first is for v and w to differ in phase by 90 degrees. The second is that periods during which the v and w series are in phase or in antiphase follow one another so that the $\overline{vw}$ contribution is small. To determine which of these alternatives was true, the quantity $\overline{v^2 w^2} / \overline{v^2} \cdot \overline{w^2}$ was measured by Bradshaw, et al., using an unlinearized, unmatched, cross-wire configuration. In order to restrict these measurements to the large scale turbulence, the data from these wires were filtered in third octave bands using a center frequency determined from the v component spectrum peak. From these measurements, it was concluded that the v and w components of the large eddies switch from being either in phase or in antiphase so that the average effect is that $\overline{vw}$ equals zero.

In previous presentation of observed trends it was noted that the η and ξ components shifted from being in phase to being 180 degrees out of phase in a rather abrupt manner. This switching seemed to be accompanied by a strong negative surge in the v -component, which was concurrent with the vorticity components decaying to near zero. Careful investigation of the $\text{SGN}(vw) \sqrt{|vw|}$ time series leads to the conclusion that the period over which η and ξ are in phase corresponds to v and w being in quadrature. This may be seen in the period between the 5 and 22 millisecond marks in Figure 28. The fact that the v and w series are in quadrature is confirmed by the lack of characteristic frequencies (between 80 and 200 Hz) or a significant non-zero mean in $\text{SGN}(vw) \sqrt{|vw|}$ over this period.

Over the period from 22 to 38 milliseconds, η and ξ are close to being 180 degrees out of phase. This region corresponds to $\text{SGN}(vw) \sqrt{|vw|}$ displaying a rather weak but noticable periodic variation in sign, which seems to indicate a switching of v and w from being in phase to being in antiphase, as was observed by Bradshaw, et al. Apparently, both a helical motion of the velocity vector in the v - w plane and a regular switching back and forth between motion along two lines in the v - w plane can be observed in the inner region of the jet (Position 1).

The data for the outer region of the jet (Position 2) are also consistent, in some respects, with the observations of Bradshaw, et al. The v and w components regularly alternate between brief periods of being in phase and longer periods of being more strongly in antiphase, over much of these two series. For this reason, the $\text{SGN}(vw) \sqrt{|vw|}$ trace has a rather large negative mean. Possible exceptions to this trend may be seen in the last eight and first four milliseconds of the $\text{SGN}(vw) \sqrt{|vw|}$ series, where an absence of periodic variations in the frequency range of 80 Hz to 200 Hz indicates that the macroscale features of the v and w series may be in quadrature.

It is interesting that the ξ -component of vorticity displays a marked positive bias and that the η vorticity component undergoes a change at 12 milliseconds from a negative to a positive bias. These points seem inconsistent with the symmetry of the jet unless the existence of very low frequency effects is considered. A sufficiently long sample of data has apparently not been taken in this region to resolve certain very low frequency effects, which are probably introduced by recirculation in the laboratory.

9.0 A MODEL FOR EDDY STRUCTURE

The purpose of the model, which is proposed in this section, is to provide a simple picture, which correlates the more typical and more obvious features of large scale phenomena in the turbulent shear region beside the potential core of a subsonic jet. In this context, the measurements of Bradshaw, et al., indicate that the macroscale features of the v and w time series have a tendency to switch from being in phase to being in antiphase at a fixed point in the shear region. This switching of phase satisfies the symmetry condition that $\overline{vw}$ equal zero. Over a segment of the current data for Position 1, the v and w time series have also been noted to be in quadrature. Despite this, the measurements of Bradshaw, et al., seem to show that this 90-degree phase difference between v and w is not a typical condition over a prolonged period. Thus, in formulating this model, data from Position 1 will be used in which the large-scale switching of phase between v and w was observed (see Figure 28, from 22 through 38 milliseconds).

Low frequency effects due to recirculation have been previously noted to affect the data at Position 2. This fact makes it necessary to limit conclusions to concurrent surges between 30 Hz and 2 kHz for this position. For these data, the essential features of macroscale turbulence have been identified as occupying a narrow band of frequencies between 80 and 200 Hz, so that this limitation poses no serious problem.

9.1 Typical Concurrent Phenomena

The following observations, based upon the current data, are considered to be the most typical of large scale phenomena in the turbulent shear region beside the potential core of a subsonic jet. These observations are for concurrent events in a narrow band of frequencies between 80 and 200 Hz.

Position 1 ($d/2 > r > 0$, at $x/d = 3$)

1. Events Concurrent with Negative u -Component Surges
 - (a) Positively directed v -component surges
 - (b) Positively directed w -component surges
 - (c) Positively directed ζ -component surges
 - (d) Negatively directed η -component surges
2. Events Concurrent with Positive u -Component Surges
 - (a) The v -component having small values near zero
 - (b) Negatively directed w -component surges

- (c) Negatively directed ξ -component surges
- (d) Positively directed η -component surges
- 3. The most typical behavior of v and w component time series is a regular switching from being in phase to being in antiphase.
- 4. The most typical phase difference between the η and ξ time series is 180 degrees.

Position 2 ($b > r > d/2$, at $x/d = 3$)

- 1. Events Concurrent with Negative u -Component Surges
 - (a) Positively directed v -component surges
 - (b) Positively directed ξ -component surges
- 2. Events Concurrent with a Positive u -Component Surges
 - (a) A negatively directed v -component surge
 - (b) A negatively directed ξ -component surge
- 3. The v and w time series typically alternate between being in phase and in antiphase.

The periodic alternation of the u component from a positively directed surge to a negatively directed surge should be mentioned. This phenomenon is observed at both Position 1 and 2. The same narrow band of frequencies characterizes this phenomenon at both Position 1 and 2. These frequencies lie between 80 and 200 Hz, which is the same range quoted for the list of concurrent phenomena presented above.

9.2 The Initiation of Typical Large Scale Behavior

On the basis of the limited sample lengths shown in this report, certain concurrent events at Positions 1 and 2 in the shear region seem to immediately precede strongly "typical" large scale behavior. "Typical behavior" is defined in terms of the concurrent events which are summarized in Section 9.1. Position 1 is characterized by a strong surge in the v -component of velocity, which is coincident with a weaker positive u -component surge and a decay of the vorticity components to near zero (see 4.5 and 22 milliseconds in Figure 28). The concurrent events for Position 2 are a strong, positive, u -component surge, which occurs with a weaker negative v component surge (see 18 milliseconds in Figure 30). No obvious simultaneous trend in the vorticity components is apparent at Position 2, unless perhaps it may be described as a complex higher frequency interrelationship during which the ξ -component moves fairly steadily in the negative direction.

The same basic phase relationship is observed between u and v at the initiation of "typical" behavior at the two positions. The relative magnitude of the u and v surges is the only difference. It therefore seems that events at Positions 1 and 2 could be related by a common phenomenon. That is, the events may be related by an energetic jet of irrotational fluid, which transports larger u -component values from the potential core into a region of characteristically lower u -component values near the jet boundary. While this occurs, the radial component of momentum steadily decreases until, near the boundary, it equals zero. From the current data, this action would seem to trigger an almost simultaneous series of alternately inward and outward fluid motions upstream of the initial outward jet, similar to that observed by Grant (Reference 2) for the wake of an infinite cylinder.

This picture, however, is not supported by sufficient evidence to make it more than a tentative conclusion, which must await a longer sample of data for verification.

9.3 A General Model for Typical Large Scale Behavior

In developing the current model no attempt will be made to include the w -component behavior completely. For this reason, the model will primarily represent motion in a plane determined by the jet axis of symmetry and a radial coordinate perpendicular to this axis. The primary reason for this is that the w -component of the motion in the outer region of the jet (Position 2) shows trends which are almost certainly not typical. There is a much more negative mean value for the w series than should result from a proper average. This point is clear from the facts that $\text{SGN}(vw)/\sqrt{|vw|}$ and ξ have quite substantial mean values when a non-zero mean is inconsistent with the basic symmetry of the jet. The v -component of velocity demonstrates a positive mean value, which is not unreasonable when the observations of Corrsin and Kistler (Reference 12) and Stewart (Reference 37) are considered. (As mentioned in Section 7, the work of these investigators indicates a primarily inward directed mean velocity at the outermost edge of the jet flow. This mean velocity is in the positive v direction so that it is not surprising to find positive v -components near the jet boundary that are larger and more persistent.) Since the trend in the v -component is reasonable, it seems more likely that the w -component is exhibiting the effect of some transient or low-frequency flow, possibly due to recirculation in the laboratory. This point seems further confirmed by the fact that the ξ and η (w dependent) components of vorticity demonstrate inconsistent trends while ζ (w independent) does not.

The behavior of the w -component near the inner region of the jet is more typical and satisfies the symmetry condition that $\overline{vw}$ equal zero. It is interesting that at Position 1 the largest w excursions are positive and correspond to the larger negative u excursions. This point suggests that inward fluid motion is complex and contributes most significantly to w -component surges. The periodic switching in sign of the η -component of vorticity also testifies to a complex u - w interaction at Position 1. In fact, the 180-degree phase difference between η and ζ over the more typical segments of the data for Position 1 indicate that the large scale vorticity, which is convected past a stationary probe is primarily along an axis that is oriented at -45 degrees in the v - w plane. The direction

of this rotation alternates regularly at the probe. The ξ -component of vorticity also exhibits a periodicity, but this is weaker and less persistent. Such behavior could be attributed to the axes of rotation actually being tilted out of the v - w plane at some angle in the u - w plane.

Restricting comment to the u and v components of velocity and the ξ component of vorticity leads to a simplified picture, which only describes motion in a diametric plane of the jet. A stationary probe typically registers the same concurrent u , v and ξ phenomena in both regions (Positions 1 and 2) of the jet. This behavior is summarized in Section 9.1. Relating a negatively directed u -component surge to an inward moving body of fluid and a positively directed u -component surge to an outward moving body of fluid, one sees that the corresponding ξ -component associated with the inward and outward fluid motion is different in each case. The outward moving fluid rotates in a sense which permits it to draw energy from the velocity gradient. This suggests that the outward moving fluid will have a more energetic rotational component. It further suggests that the inward-directed fluid, which rotates in the opposite sense, cannot sustain itself in the presence of the existing velocity gradient. The rotational motion of the inward directed fluid is therefore more likely to be induced or parasitic. That is, it draws energy from the rotational motion associated with the outward moving fluid.

The fact that the surges in the u -components of velocity for the two positions alternate in direction in a fairly periodic manner at a stationary probe position, suggests that regions of in-flow alternate with regions of out-flow along the axial coordinate of the jet. This further suggests that the regions of in-flow act on buffers between the more energetic eddying motions, associated with fluid out-flow, and prevent the mutual destruction of these energetic flow structures upon contact. In order for this to be true, the fluid in the induced eddies would have to be consistently renewed as the shear region broadened, and as their mass was depleted by an eating process of the more energetic eddies. Thus, such parasitic fluid rotation appears to be consistent with the apparent mass entrainment associated with the less energetic eddies.

Figure 35 illustrates this type of situation. As may be seen in this figure, the stretching of vortex filaments between two energetic eddies could account for the maximum of turbulent intensity observed near the center of the shear region and possibly be a turbulence generating mechanism provided that sufficiently large velocity fluctuations existed in the entrained irrotational fluid. Figure 35 indicates that this model is similar in some respects to the two-dimensional "mixing jet" model of Grant (Figure 33) except that the roots of the jets are located at the same point on the jet axis.

Figures 36 through 39 present the results of a qualitative investigation of flow near the potential core of a submerged, round jet of liquid. The technique of flow birefringence was used to obtain these photographs. Reference 38 contains details of this flow visualization technique and some examples of its application in a qualitative investigation of some specialized cases of separated flows.

Three important conclusions about large eddy structure may be drawn from these photographs. The first is that the eddy structure is symmetric about the jet axis in the neighborhood of the potential core. The second is that it is generated in a fairly periodic manner. The third is that it disintegrates into smaller eddies at about 7 diameters from the jet exit plane. This latter point appears to reiterate an observation of Bradshaw, et al., which was based upon the decay of the v and w spectrum peaks.

A more detailed interpretation of these photographs will not be attempted, since the observed patterns are the result of a 5×10^{-4} second flash of white light passed through a jet of non-Newtonian fluid, which was located between a linear polarizer-analyzer system having optical axes at 90 degrees to each other. The non-Newtonian character of the fluid should not make the large eddy structures in the liquid jet significantly different from those of an air jet because the visualization liquid is known to be approximately Newtonian for low velocity gradients. Also, the length of the potential core is typical of air jets, which suggests similar large eddy structures exist, for the two cases, beside the potential core. (This point relies upon the hypothesis of Townsend that large eddy structures determine the width of a shear region.)

The velocities shown in Figures 36 through 39 increase with figure number. The jet orifice was located in a plane wall and flow resistance devices were used to eliminate surges resulting from the impeller of the centrifugal pump, which generated the flow.

10.0 CONCLUSIONS AND RECOMMENDATIONS

10.1 Hot-Wire Anemometry

- (1) There is presently a 20 percent uncertainty in the measurement of normal components of velocity with yawed wires. This observation is based both upon existing literature (Reference 20) and observations made during this program.
 - (a) Some of this uncertainty may be removed by using an equation of the sort proposed by Hinze for the yaw sensitivity of a hot wire (Equation 3), as suggested by Webster (Reference 16) and Champagne, et al. (References 17 and 19). This expression contains an empirically obtained parameter which is a function of the ℓ/d ratio of the sensor and the sensor material, according to recent work by Champagne, et al.
 - (b) For a sensor with its axis aligned so that it makes a constant angle with the mean velocity vector, Hoole and Calvert (Reference 21) have observed that the sensor output is proportional to the orientation of the probe body with respect to the mean velocity vector. This bears further investigation since this effect suggests another, as yet undetermined, source of error.
- (2) The error in velocity fluctuations due to a constant level of temperature fluctuation:
 - (a) Increases slightly with the mean temperature of the medium in which measurements are made (if the mean velocity is constant),
 - (b) Decreases slowly with increasing velocity at constant mean temperature, and
 - (c) At low velocities it increases sharply with decreasing velocity at constant mean temperature (see Figure 18).
- (3) The Hot Film Anemometer
 - (a) A comparative study of hot wire and hot film anemometers would be useful in order to specify more clearly the relative strengths and weaknesses of each.
 - (b) Such a study should include:
 - (i) The comparative measurement of temperature profiles along wire and film sensors of equivalent and typical geometries for controlled overheat ratios. (For such measurement, the infrared technique of Champagne, et al. (Reference 17) would appear to offer promise.)

- (ii) Evaluation of the empirical constant in the yaw sensitivity equation (Equation 3) as a function of ℓ/d ratio for hot-film sensors of standard design.
- (iii) Shock and vibration testing to provide valuable information on the relative practical merits of the two sensor types; and possibly
- (iv) Use of the two types of sensor to provide information effect of probe body orientation (see 1b) reported by Hoole and Calvert (Reference 21).

10.2 Investigation of Large Scale Turbulence

- (1) The shear region beside the potential core of a low speed (Mach 0.1) subsonic jet is dominated by a system of large scale fluid motions covering a narrow band of frequencies. (At $x/d = 3$ this band lies between 80 and 200 Hz.)
- (2) For $d/2 > r > 0$, the large scale vortical motion, which is convected past a stationary sensor array is about an axis that is at -45 degrees in the v - w plane and (judging from visualization studies) and at -45 degrees in the u - v plane.
- (3) Considering motion in the u - v plane (neglecting w behavior) suggests:
 - (a) A series of axially symmetric eddies, which are arranged along the symmetry axis of the jet so that each is rotating in a direction that is opposite to the direction of its neighbors.
 - (b) That this system is composed of two distinct types of eddy,
 - (i) An energetic eddy which can draw its rotational energy from the mean velocity gradient and which is largely responsible for outward fluid transport.
 - (ii) An induced eddy which draws its rotational energy from its more energetic neighbors and is largely responsible for the mass entrainment process.
 - (c) That the eddy system may be triggered by a strong outward burst of fluid from near the potential core of the jet, which sets up a disturbance that propagates rapidly upstream and initiates the eddying motions.
 - (d) That a diametric section of the jet may have the appearance of Figure 35.
- (4) The w -component of velocity seems to be stronger in the inner region of the jet than in the outer region. It also seems to be more strongly associated with an inward directed surge of fluid. This suggests a swirling motion about the axis which increases in velocity as the entrainment process proceeds and which may

depend, in sign of rotation, upon currents in outer region of the jet. Such behavior suggests vorticity conservation during an axially symmetric entrainment process.

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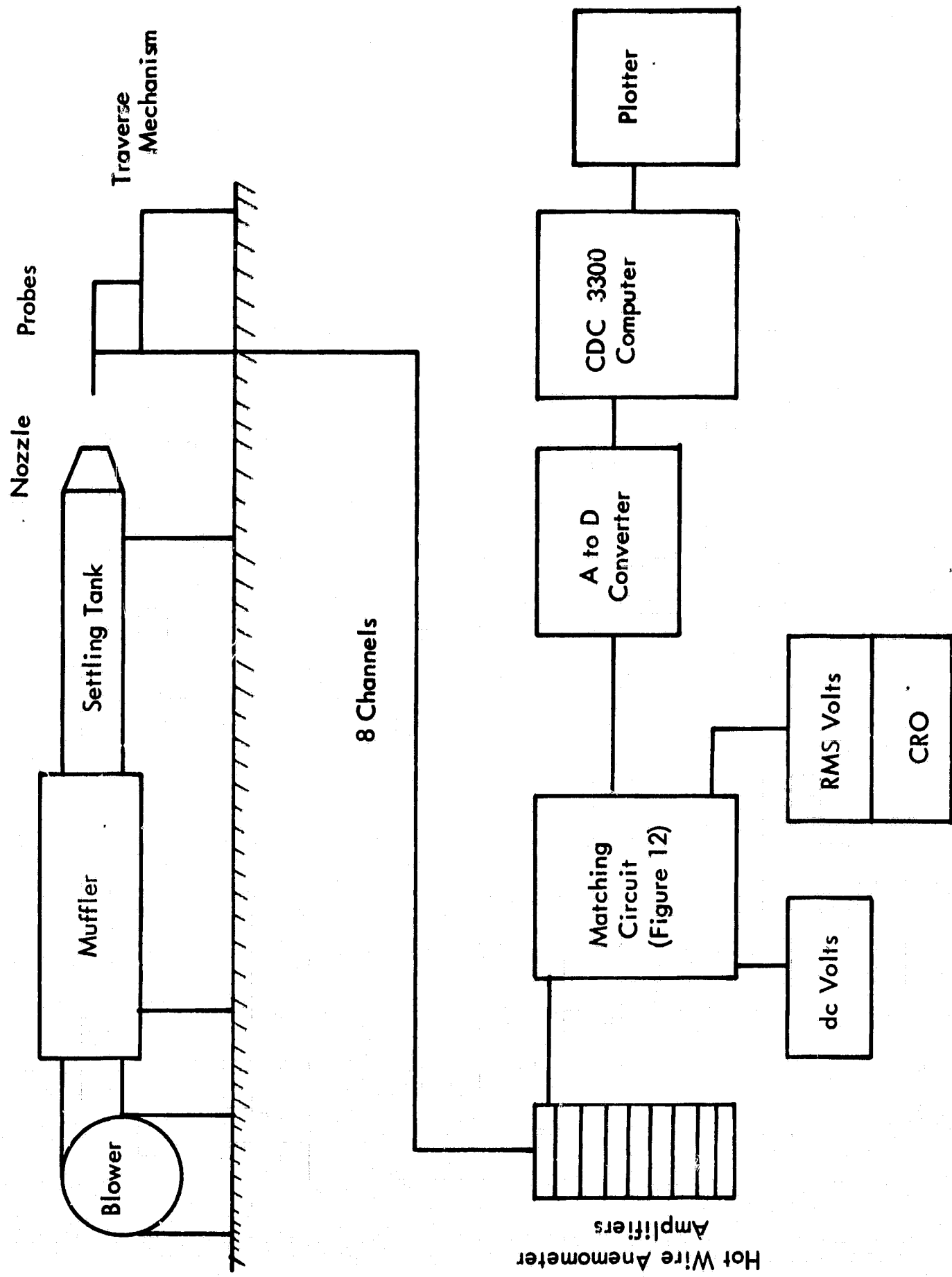


Figure 1. Rig and Data Acquisition System

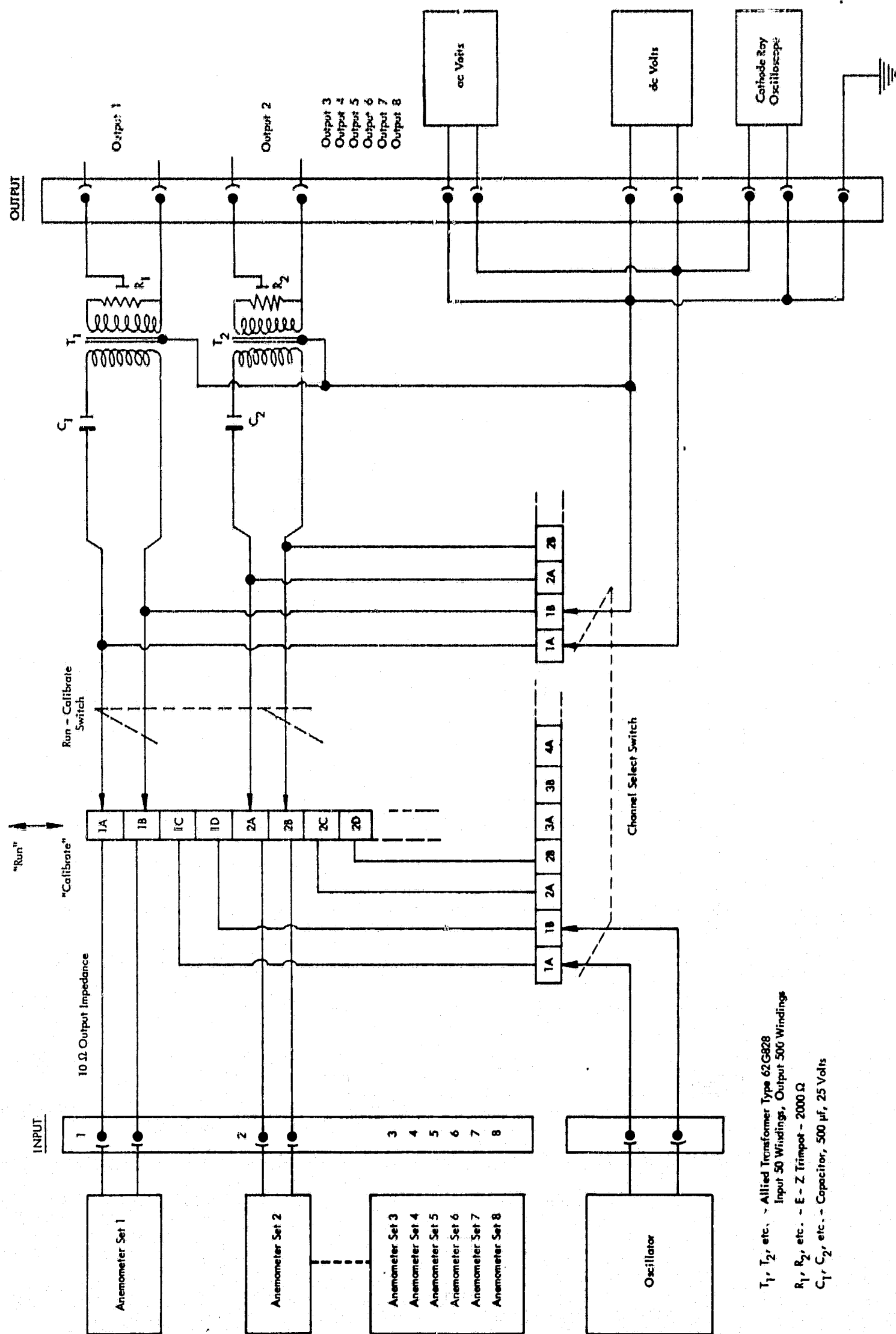


Figure 2. Multi-Channel Impedance Matching Circuit for the Anemometer System-Computer Interference

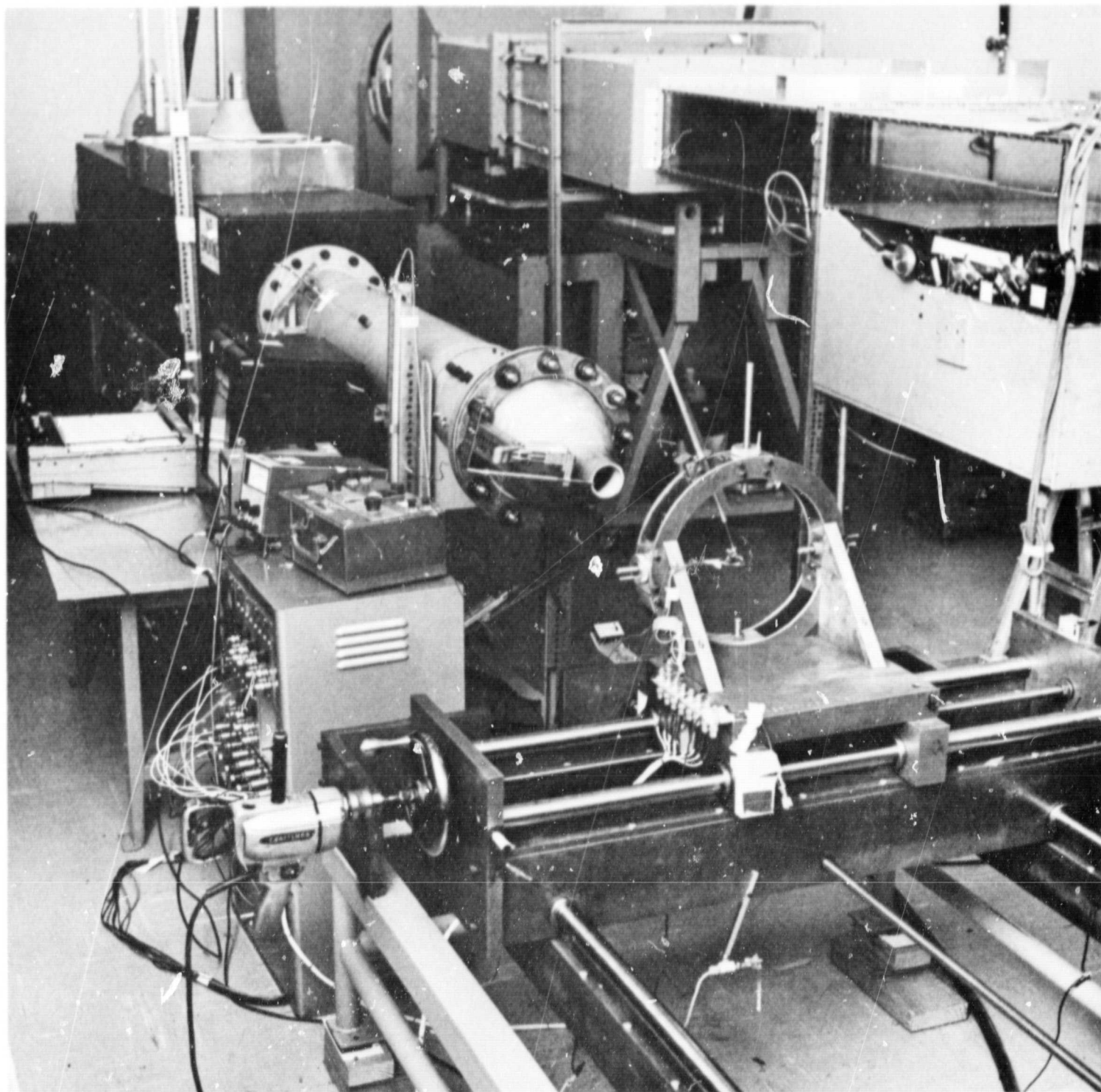


Figure 3. Traversing Table, Jet and Anemometer Mounting Ring

4" Exit Diameter Nozzle

DIMENSIONS

y	x
inch	inch rad.
0	5.125
1	5.06
2	4.91
3	4.64
4	4.18
5	3.49
6	2.94
7	2.58
8	2.31
9	2.11
10	2.00

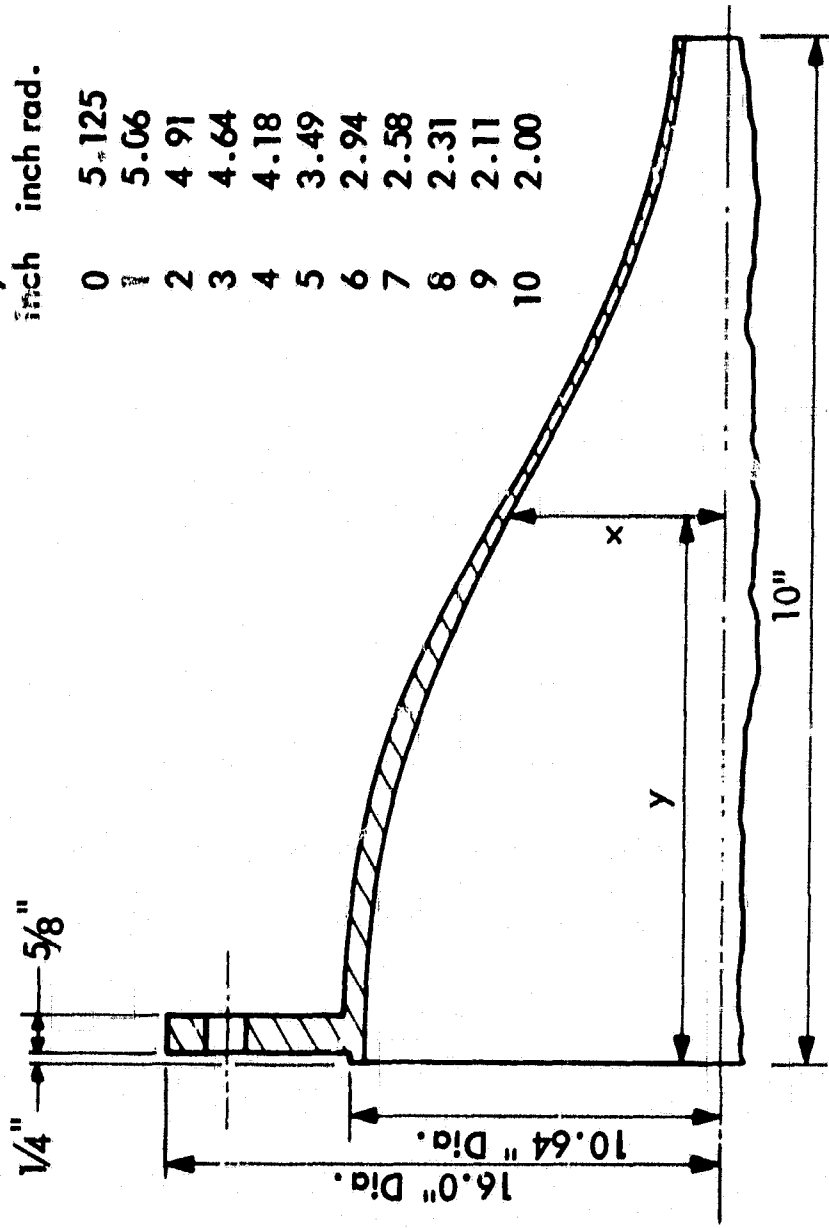


Figure 4. Dimensions of 4" Exit Diameter Nozzle

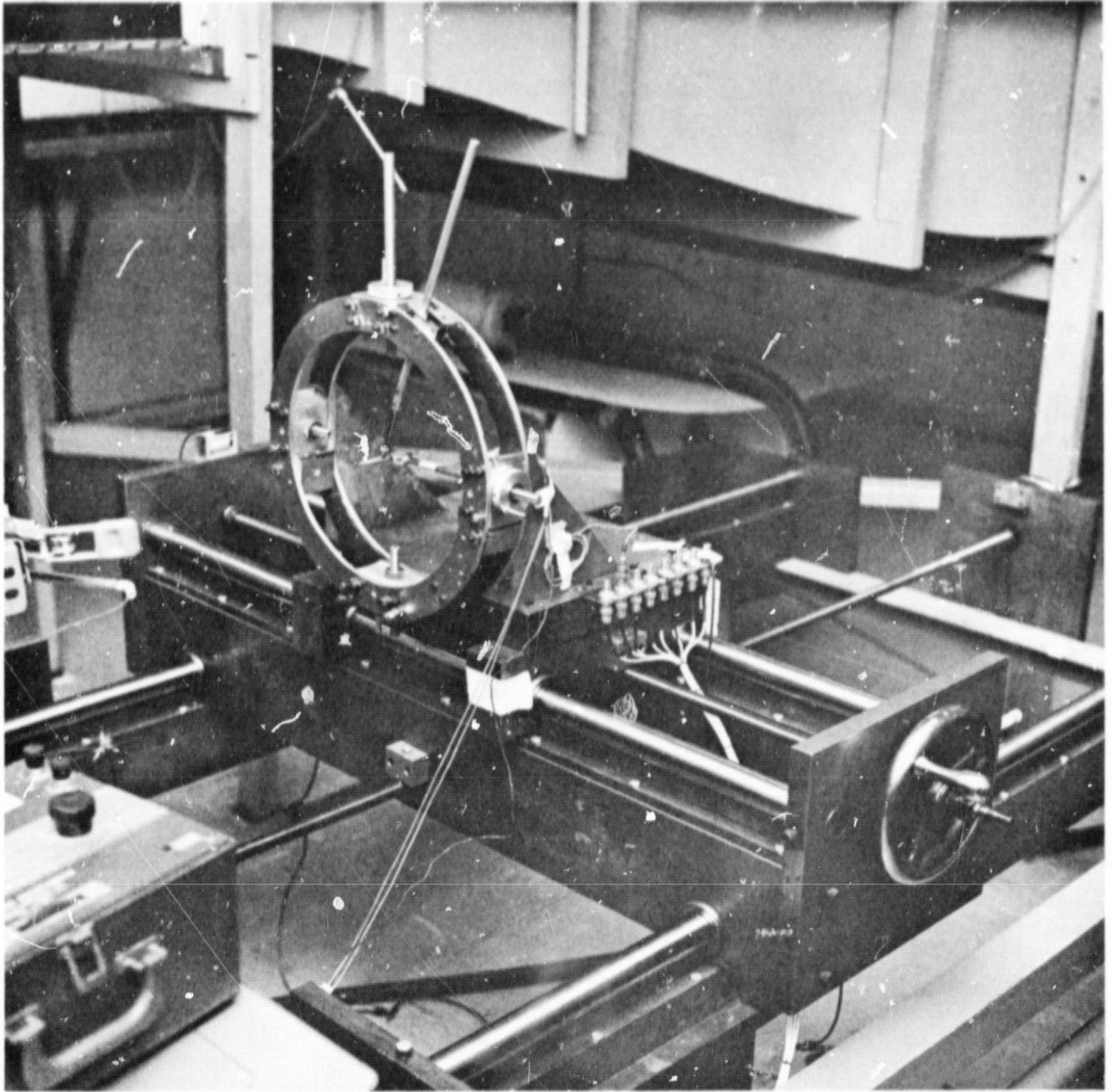


Figure 5. Traversing Table and Anemometer Mounting Ring



Figure 6. Dehumidifying Coil and Duct

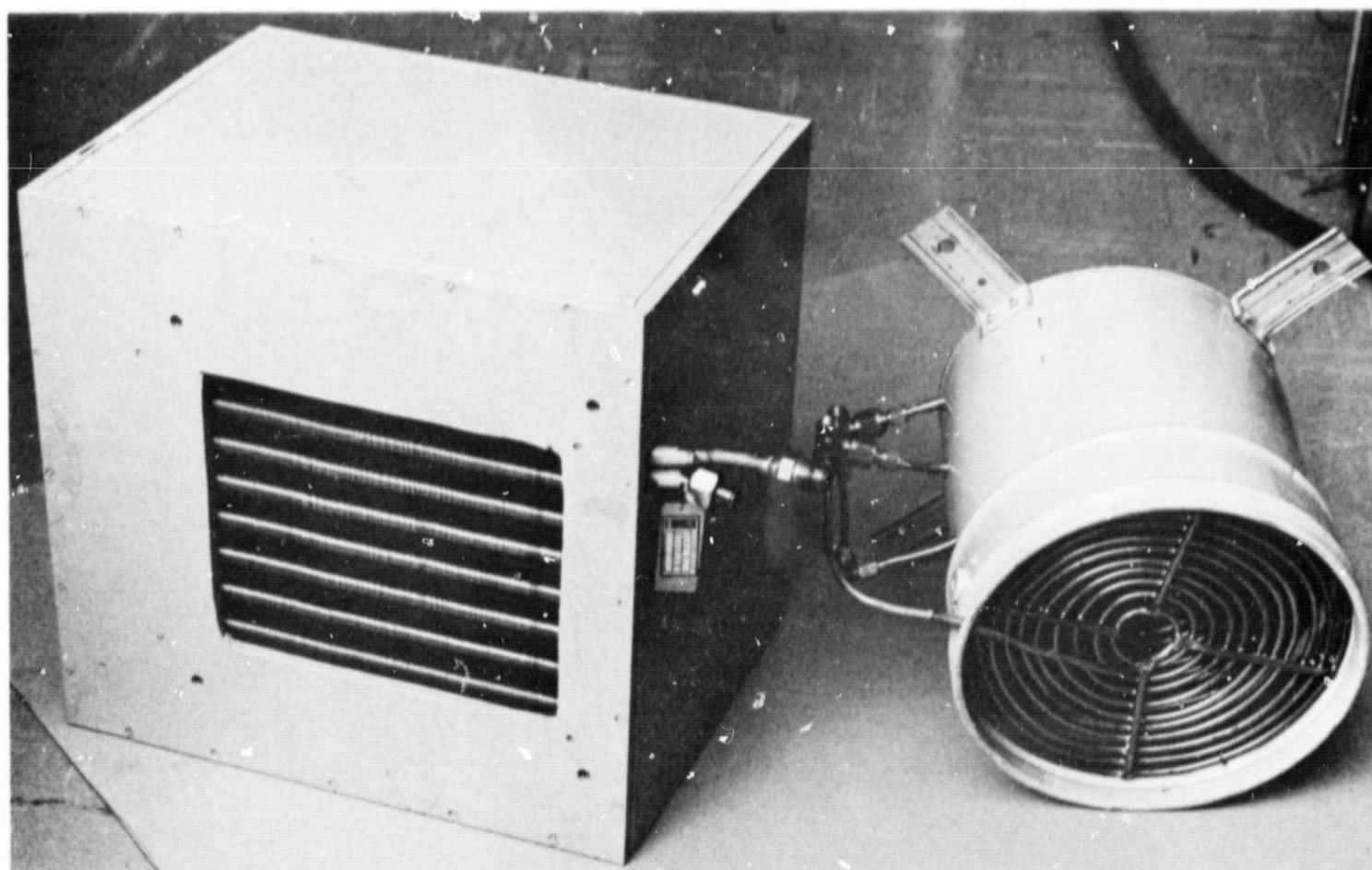


Figure 7. Radiator and Dehumidifying Coil

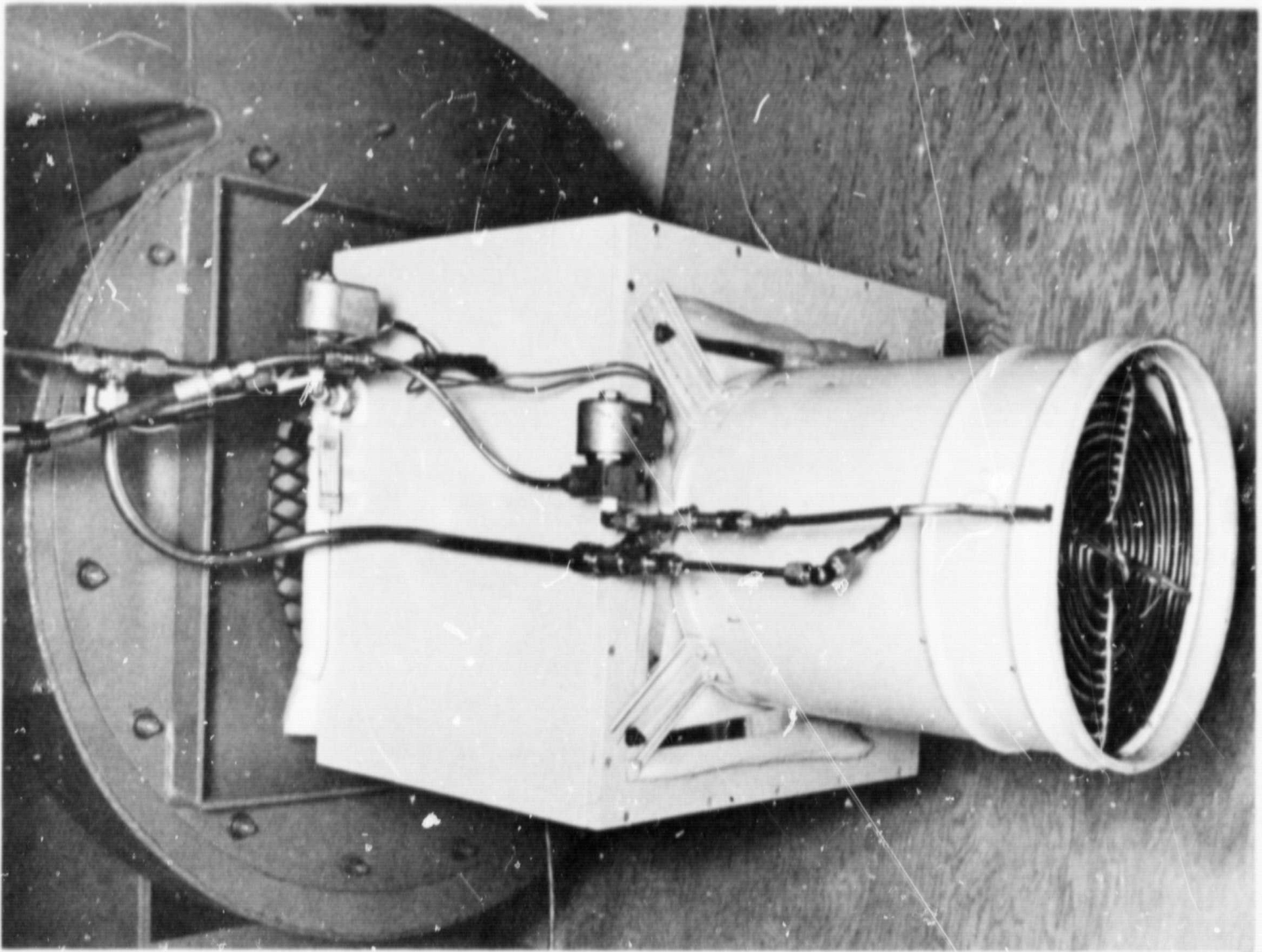


Figure 8a. Final Assembly of Radiators and Valves on Centrifugal Blower Intake

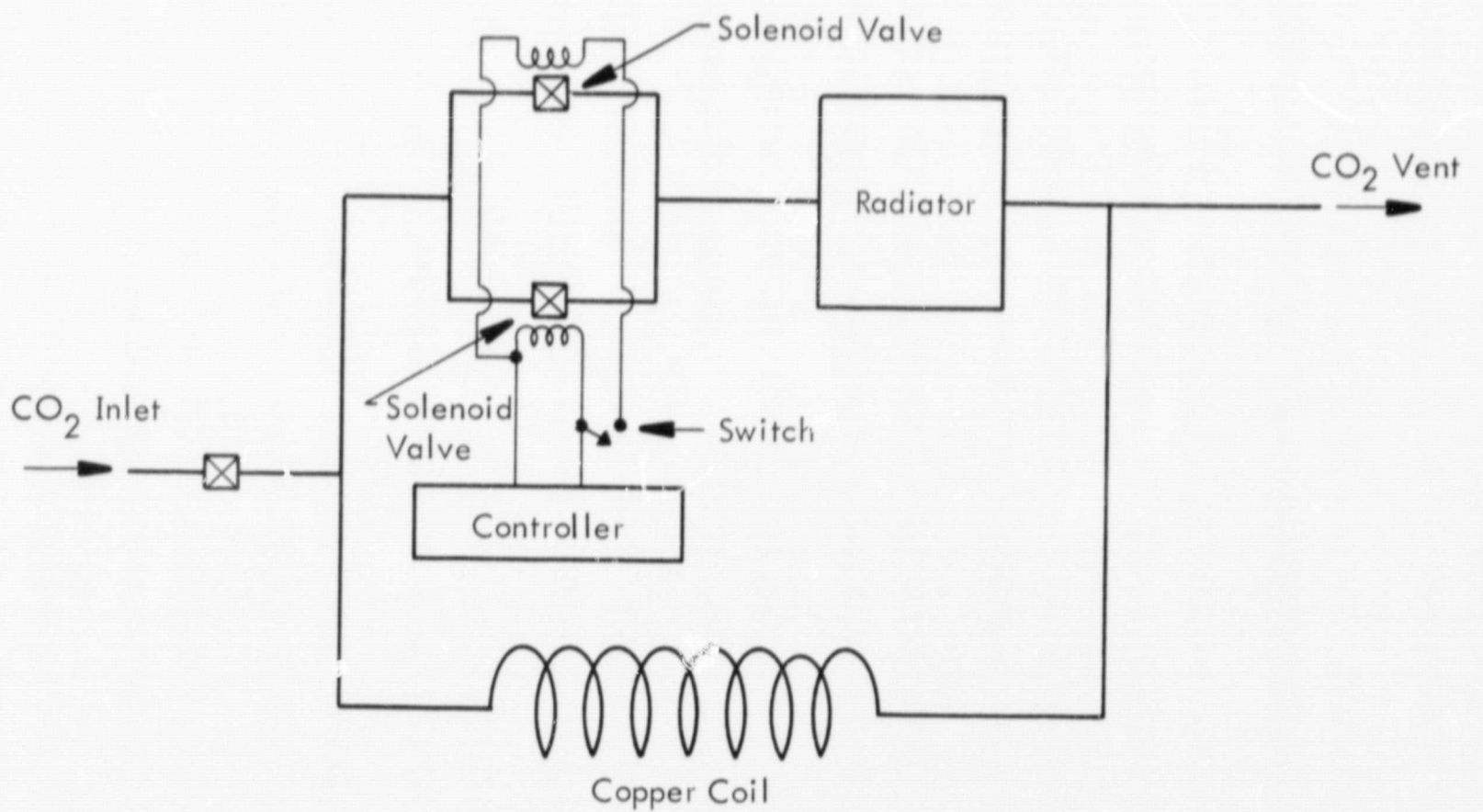


Figure 8b. Schematic Diagram of Temperature Regulation System



Figure 9. An Eight-Wire Array of Hot Film Sensors

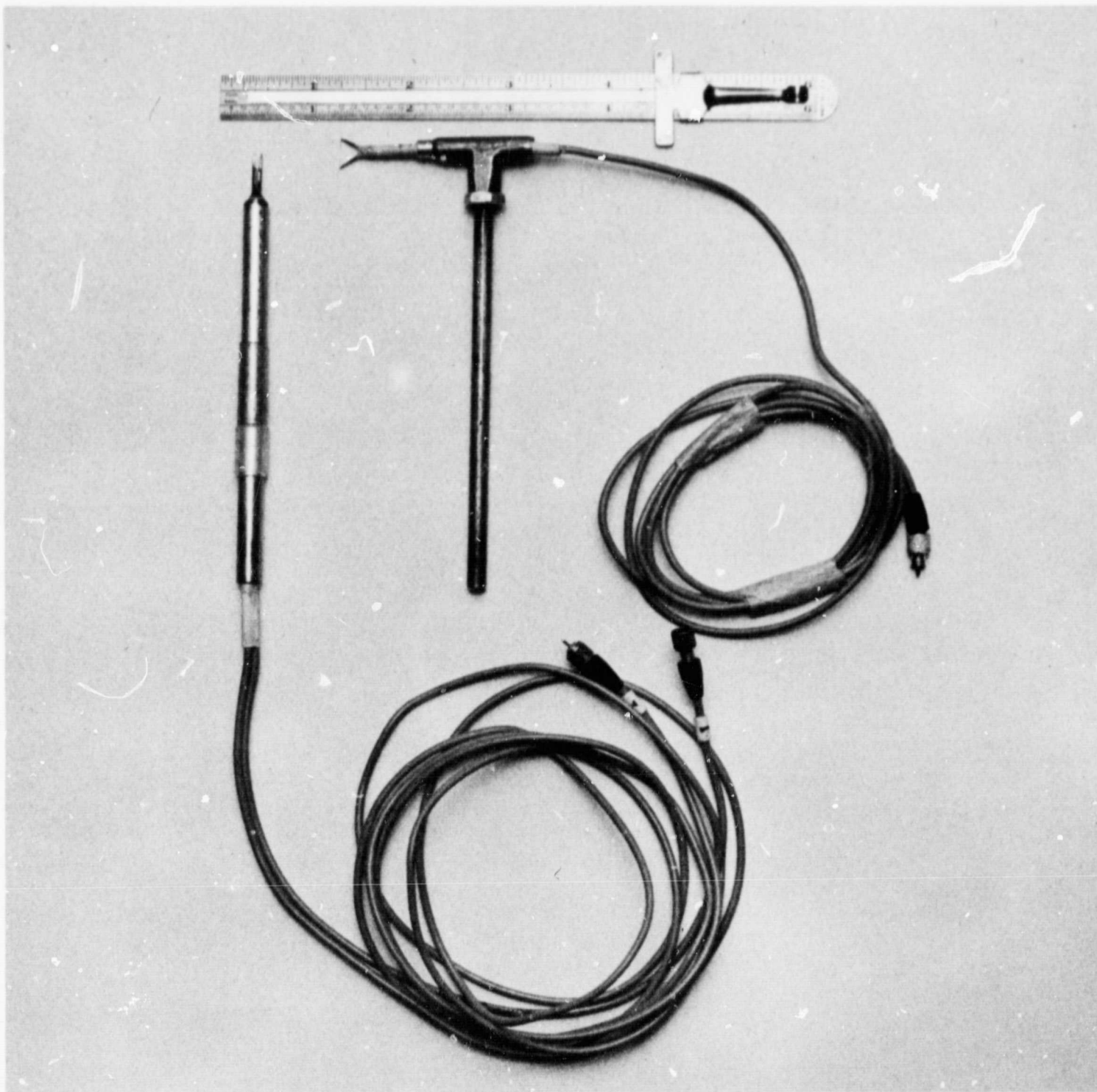


Figure 10. Two Hot Wire Anemometer Probes Constructed by Wyle Laboratories.
The probe on the right was used extensively in mean flow measurements.

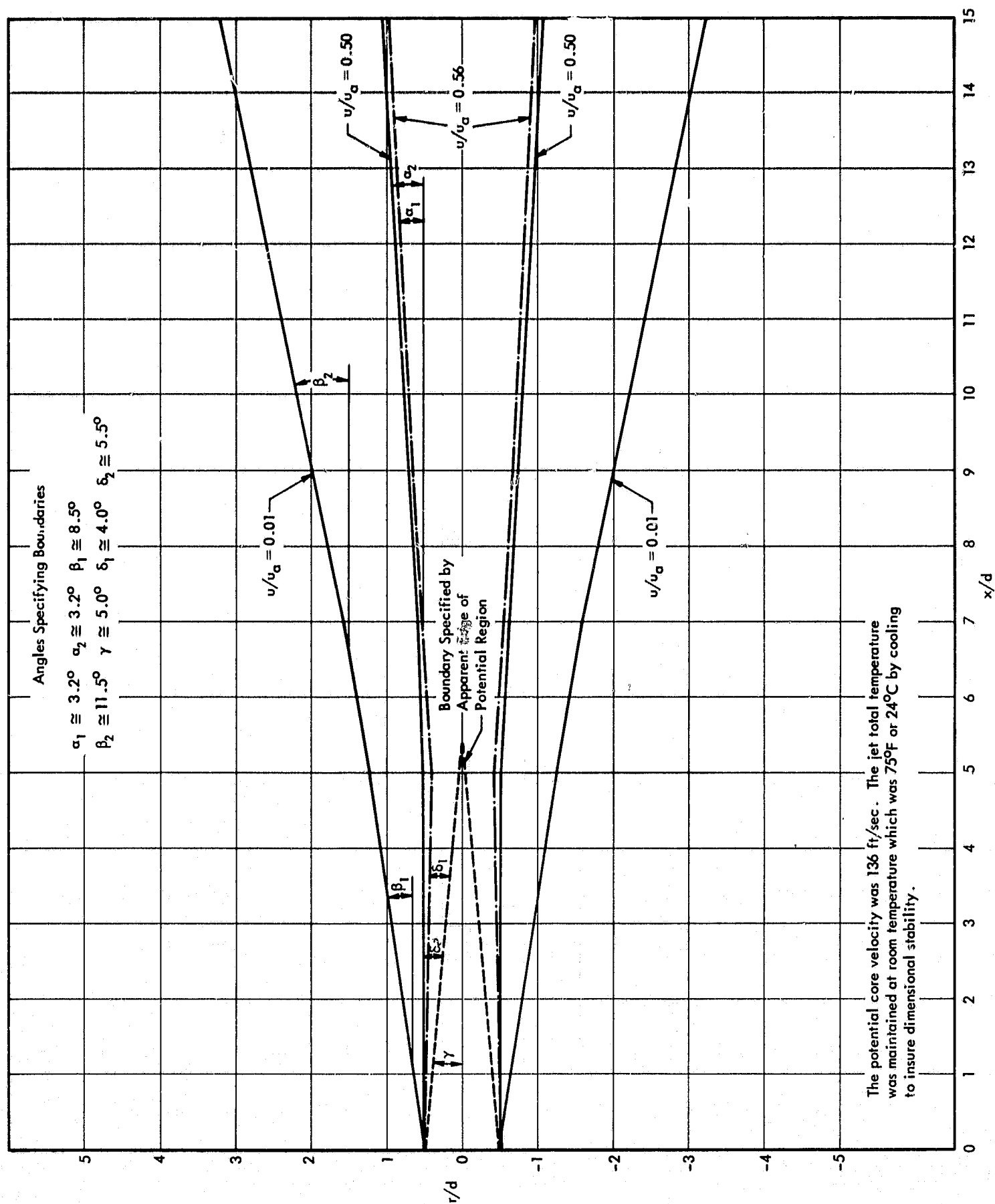


Figure 11. The Geometry of a Subsonic Jet. In this figure, U and U_a lie on a line perpendicular to the jet axis. U_a is the axial mean velocity and U is the mean velocity at some off-axis point.

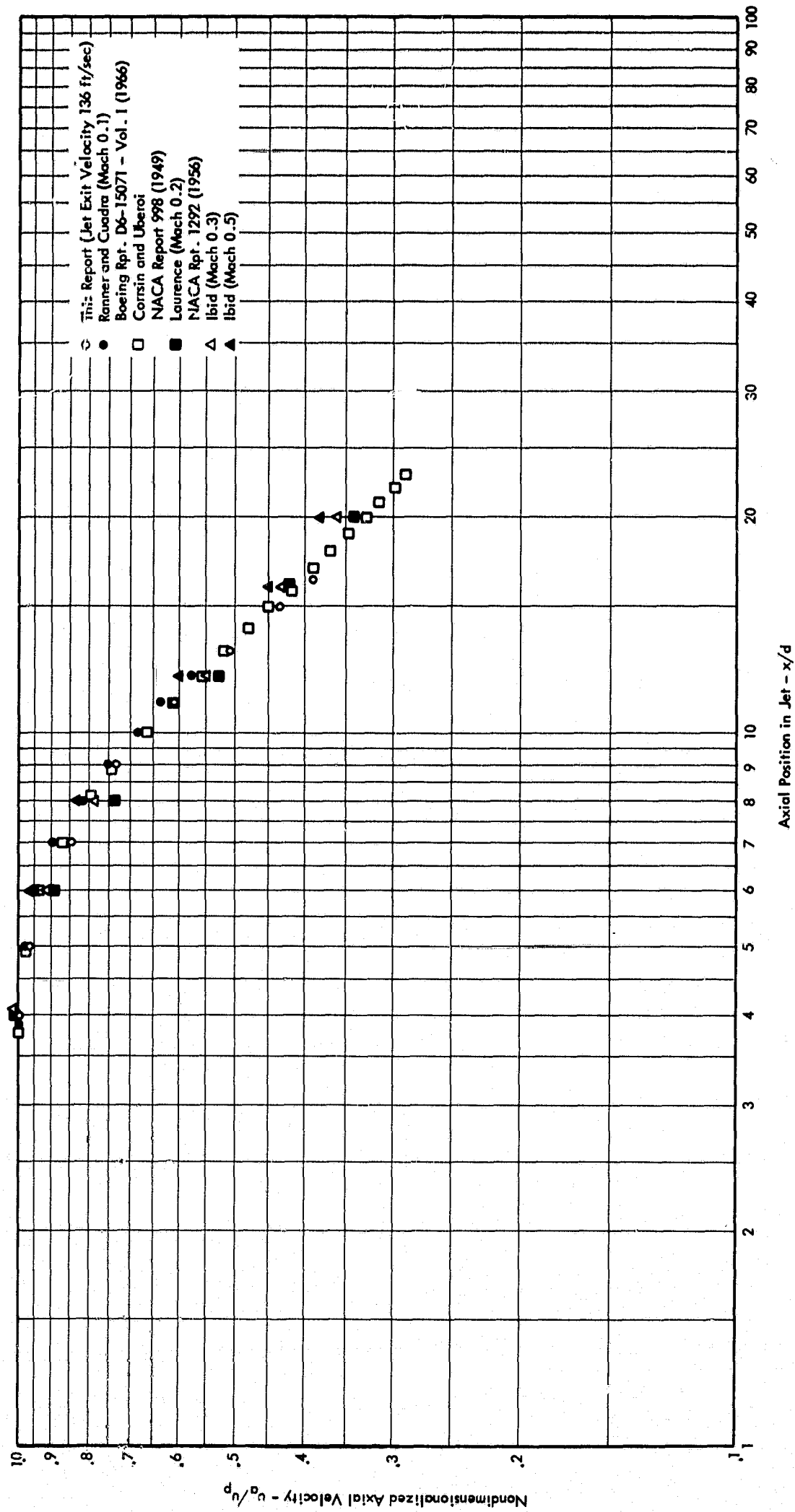


Figure 12. The Velocity on the Axis as a Function of Axial Position. In this figure U_a is an axial mean velocity and U_p is the mean velocity in the potential core.

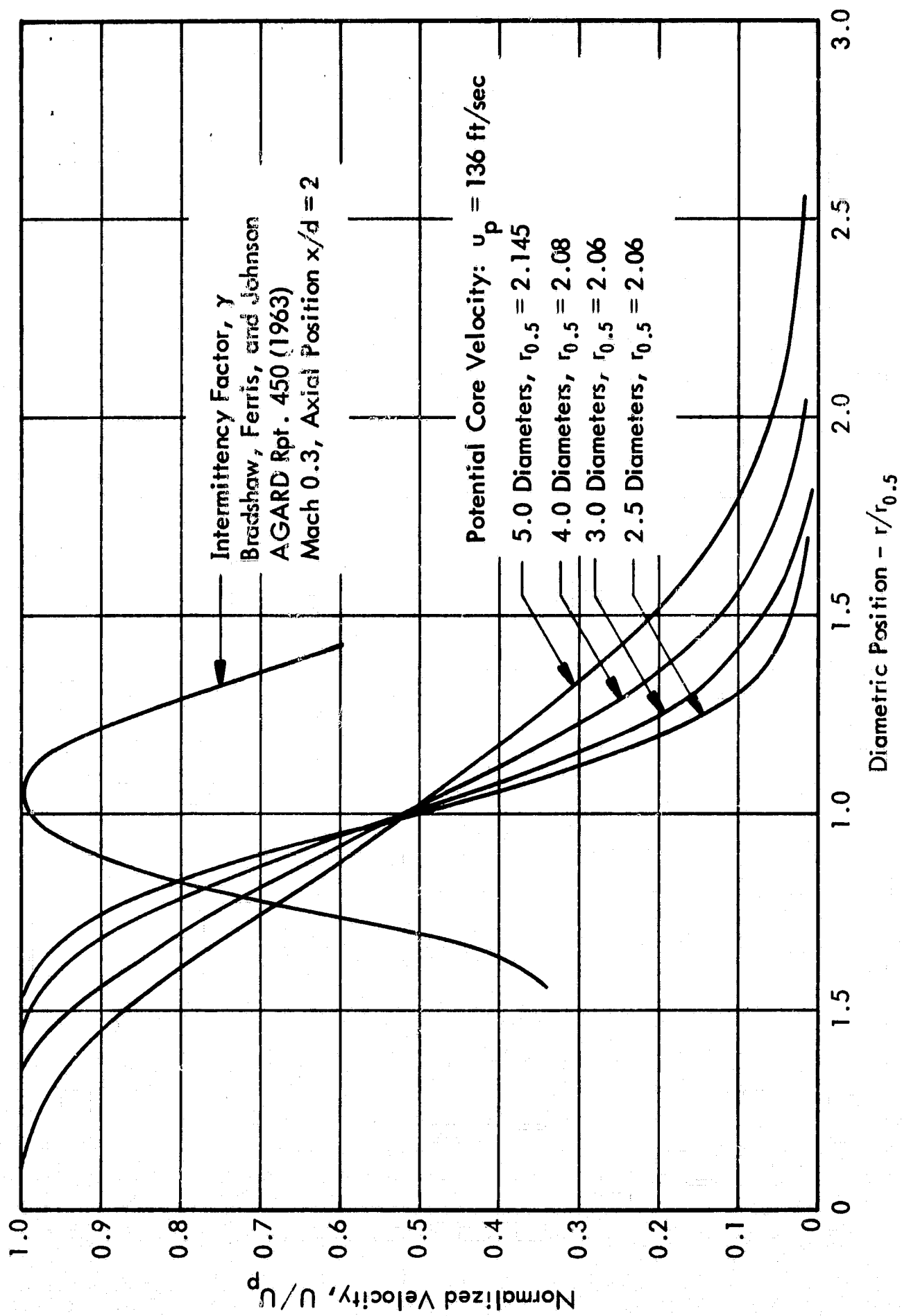


Figure 13. Mean Velocity Profiles Beside the Potential Core. The validity of the data below 4 or 5 ft/sec is questionable because the extrapolation of the hot-wire calibration below these points may not be justified.

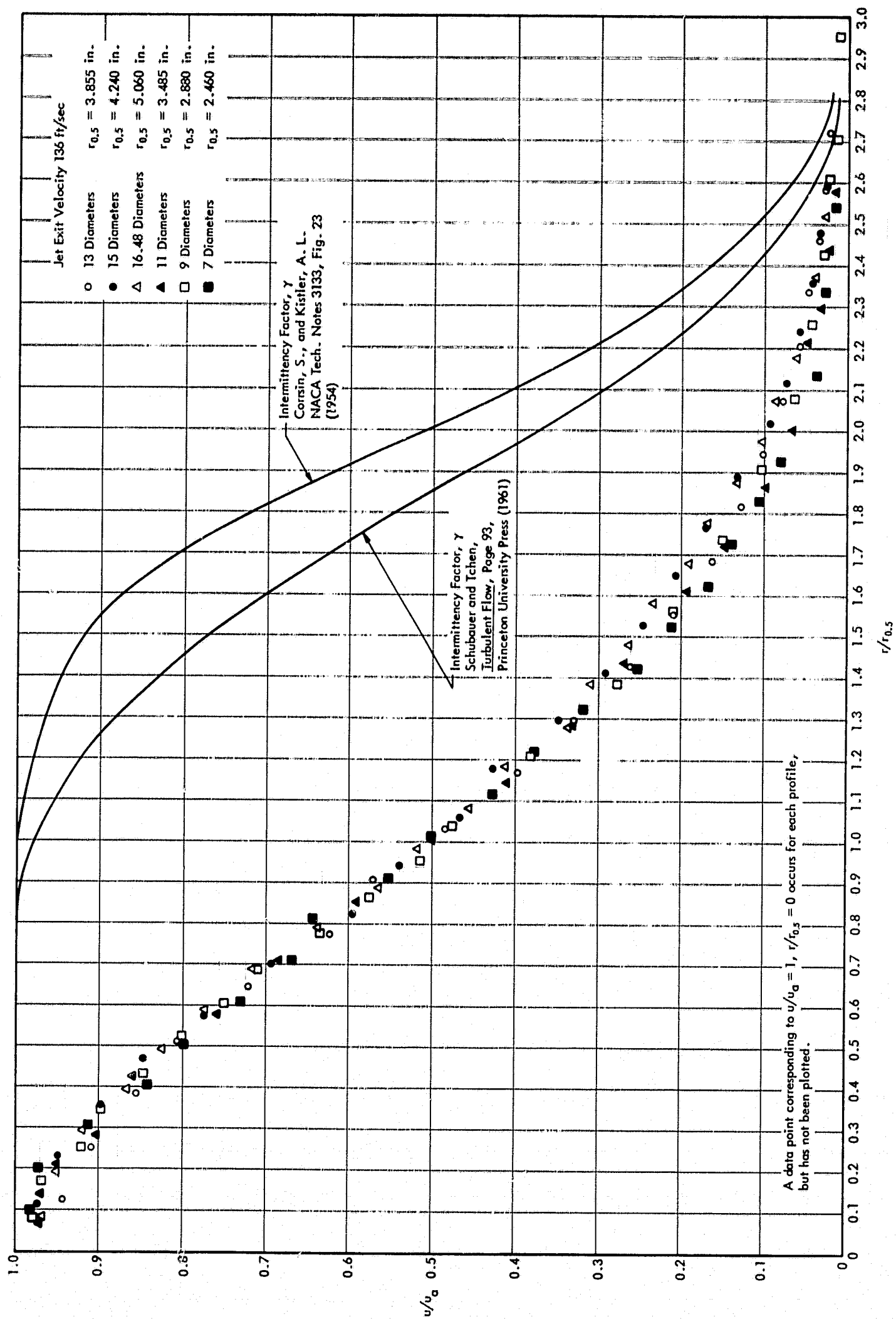


Figure 14. Similarity Plot of Diametric Mean Velocity Profiles. In this figure, U and U_a lie on a line perpendicular to the jet axis. U_a is the axial mean velocity and U is the mean velocity at some off-axis point.

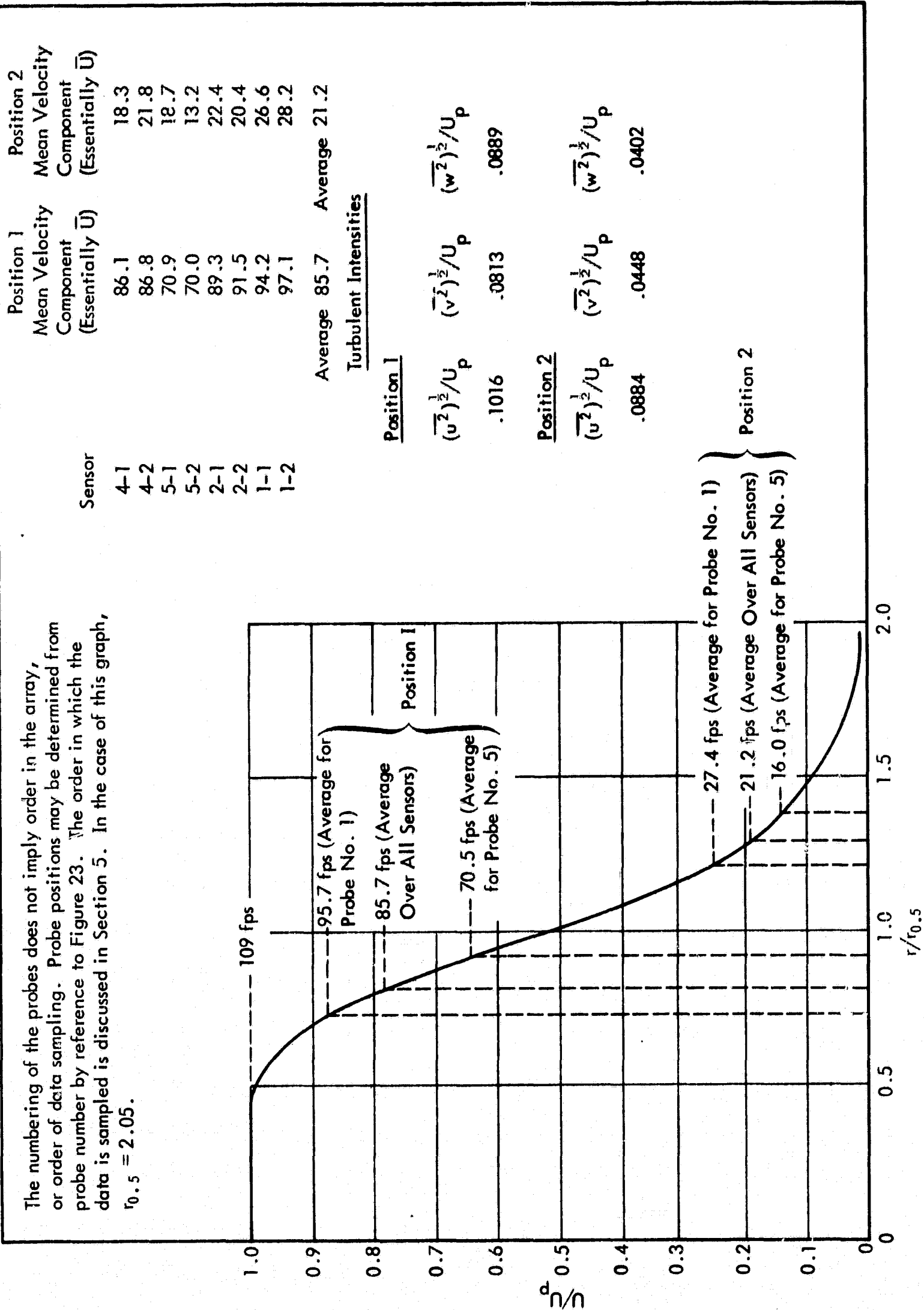


Figure 15. Mean Velocity Profile at Three Diameters from the Jet Orifice. (Extent of the sensor array, mean velocities and turbulent intensities are shown.)

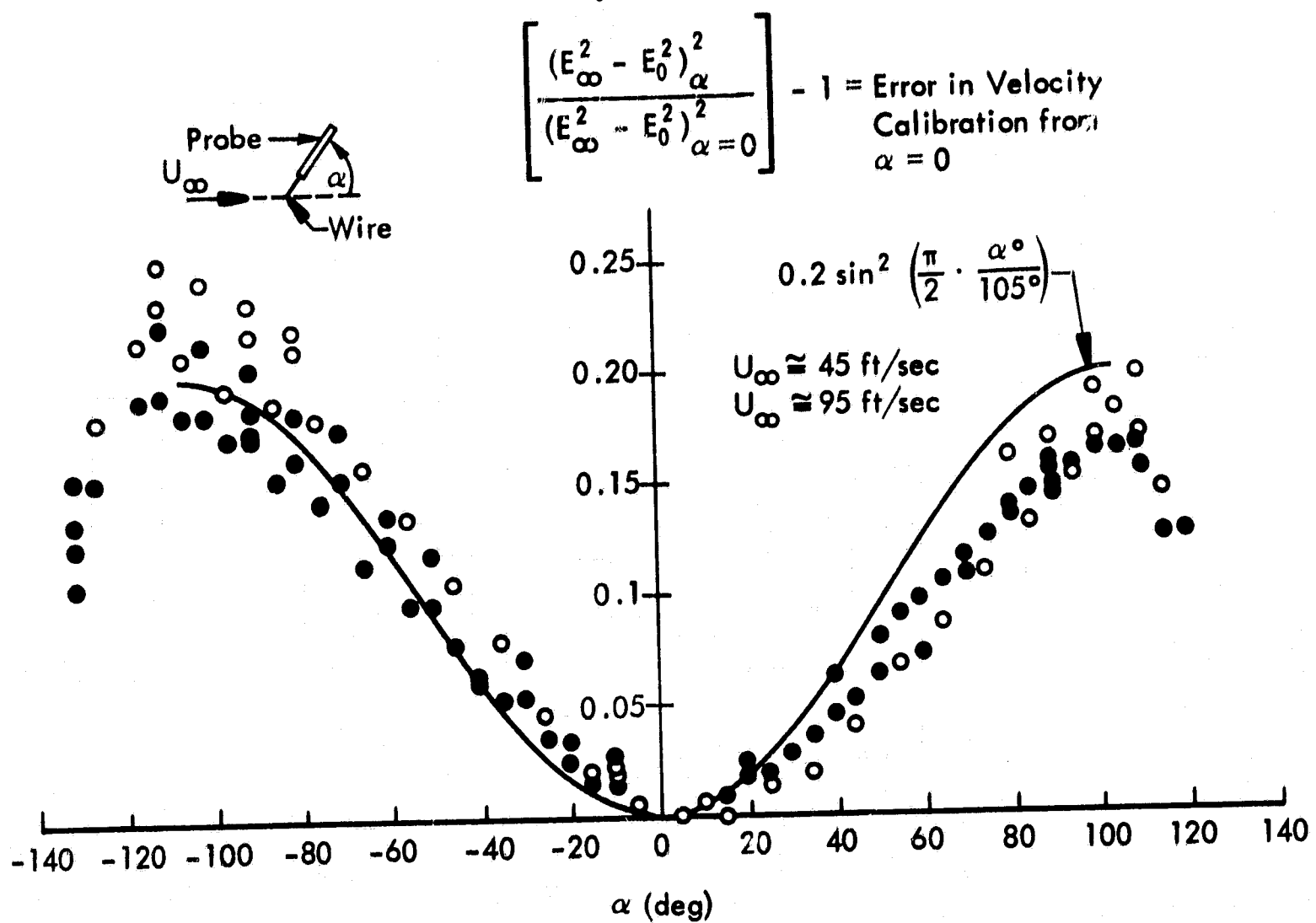


Figure 16. Data of Hoole and Calvert Indicating an Observed Relationship Between Orientation of the Probe Body to the Mean Flow Direction and Sensor Output

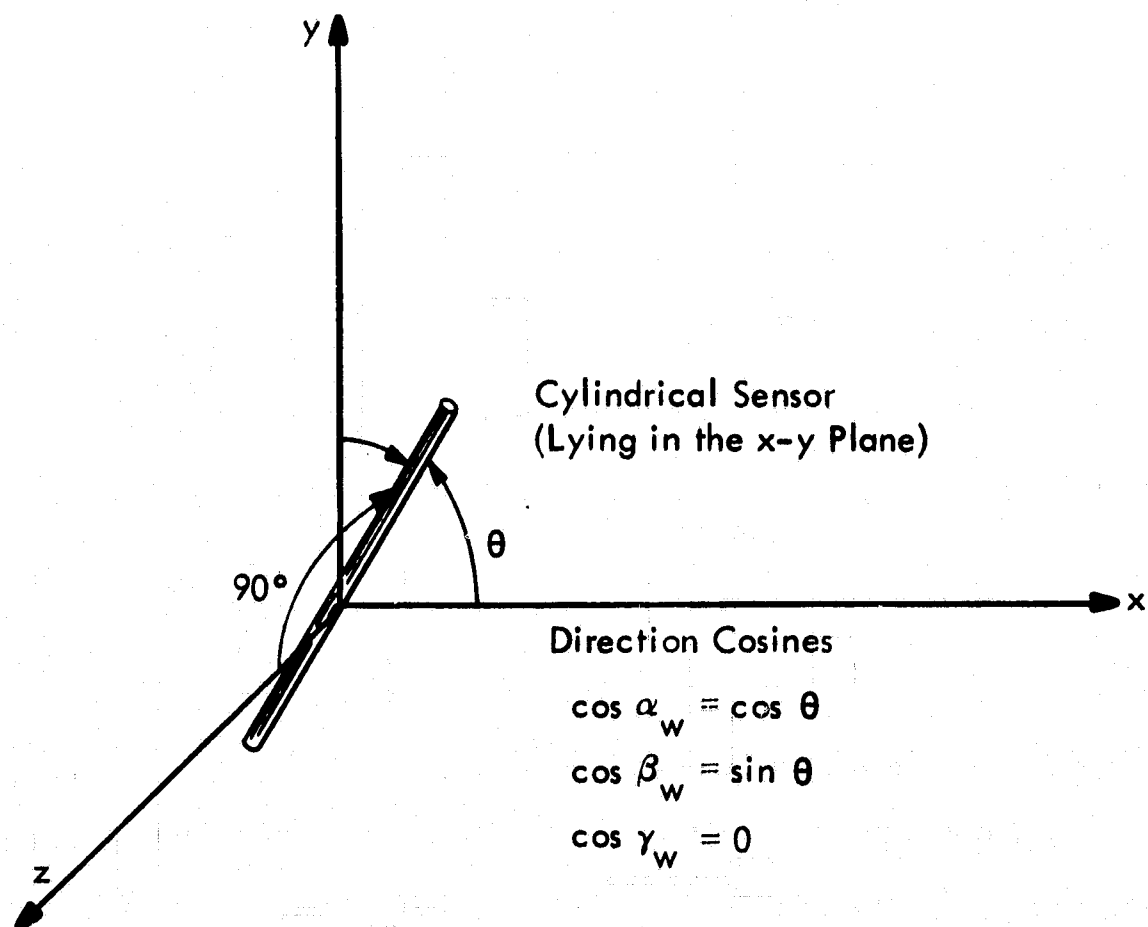
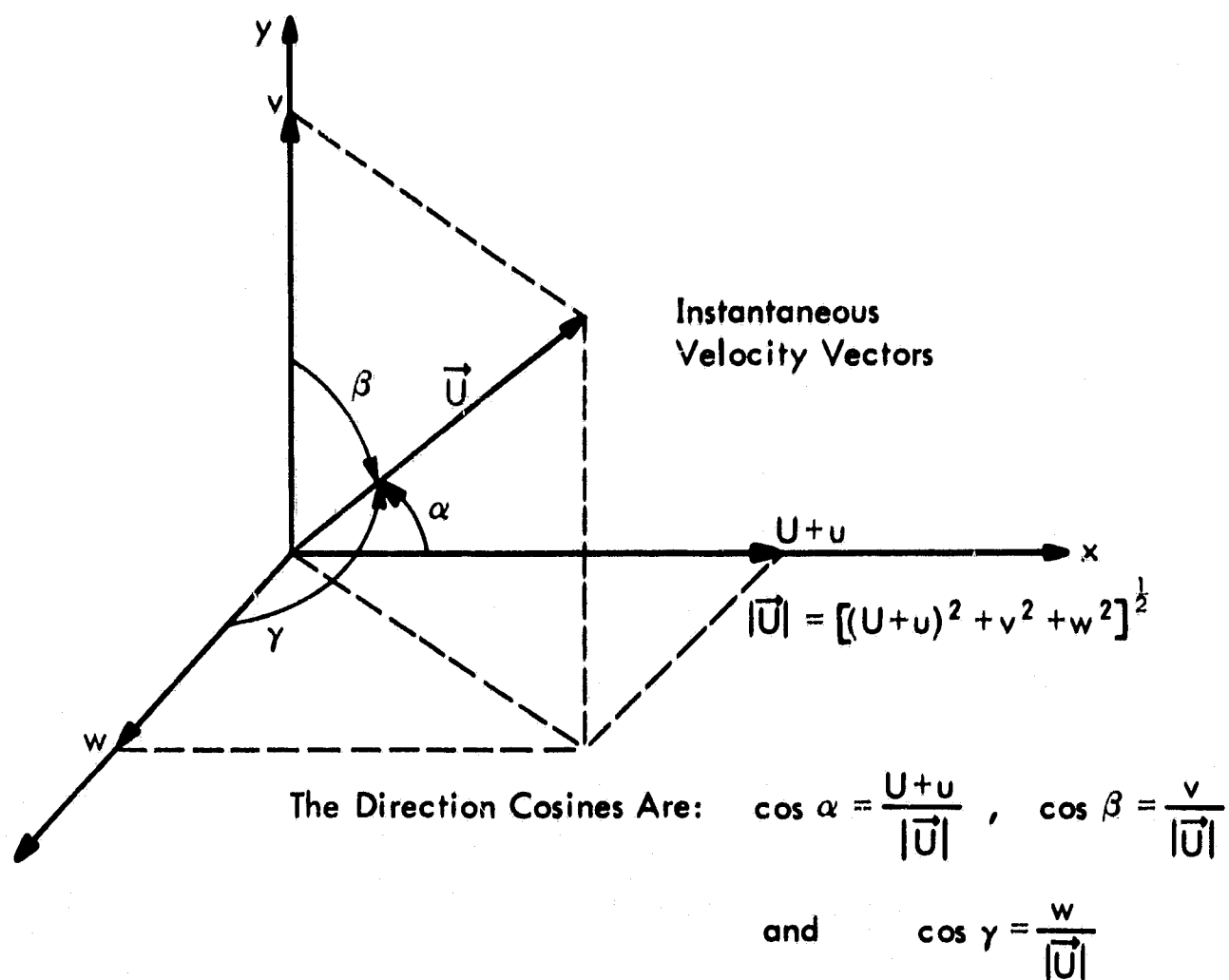
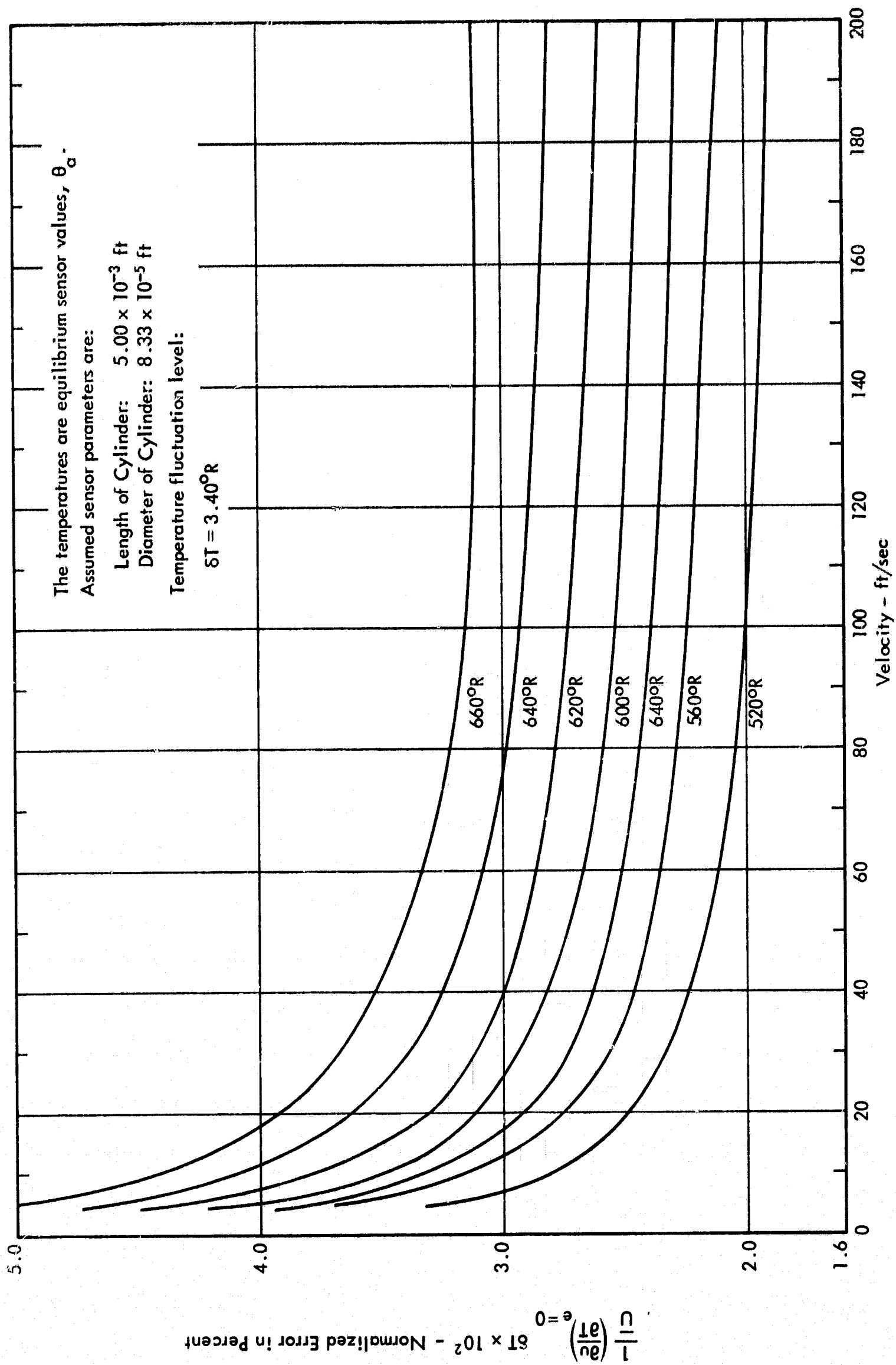
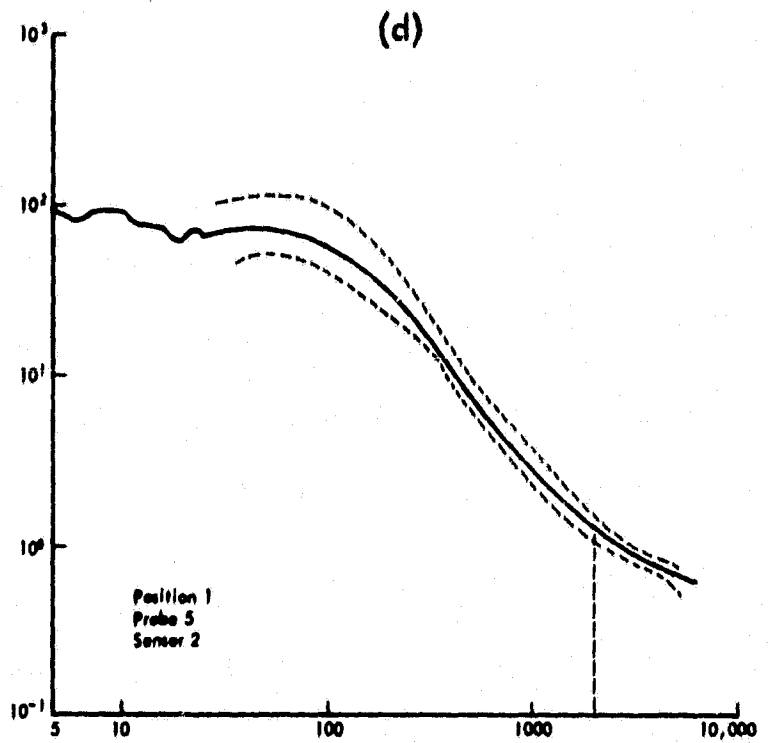
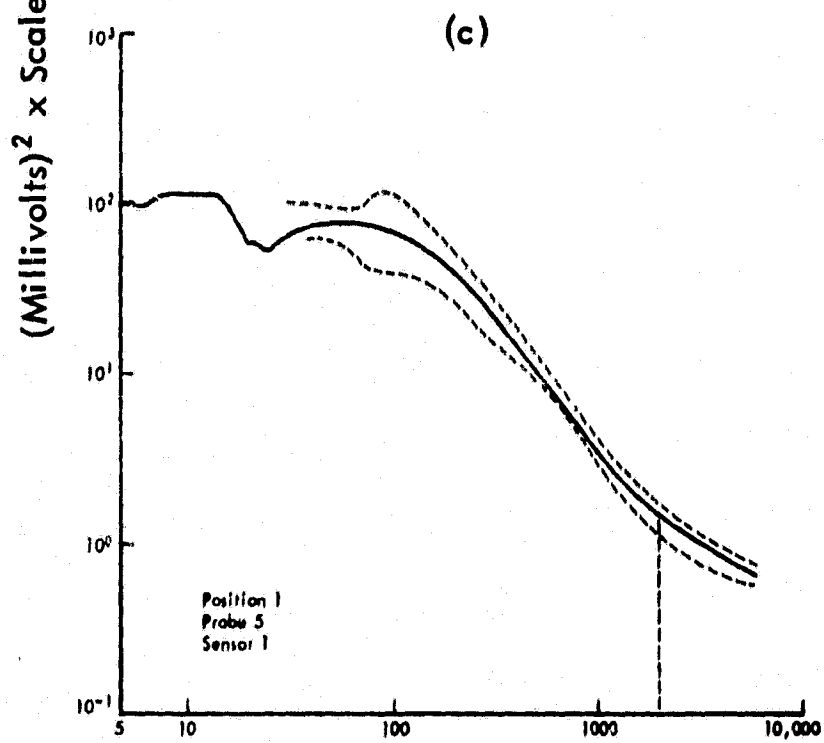
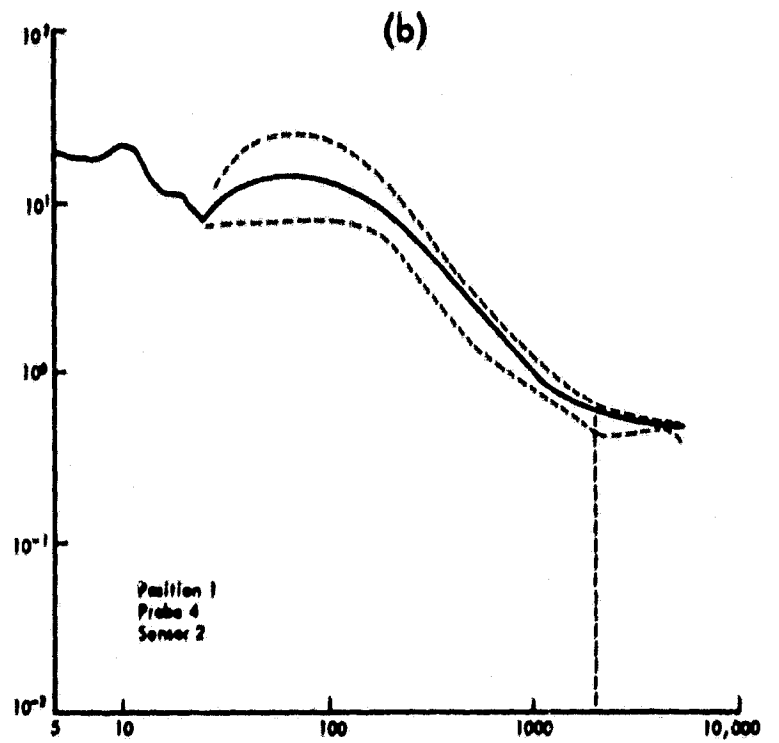
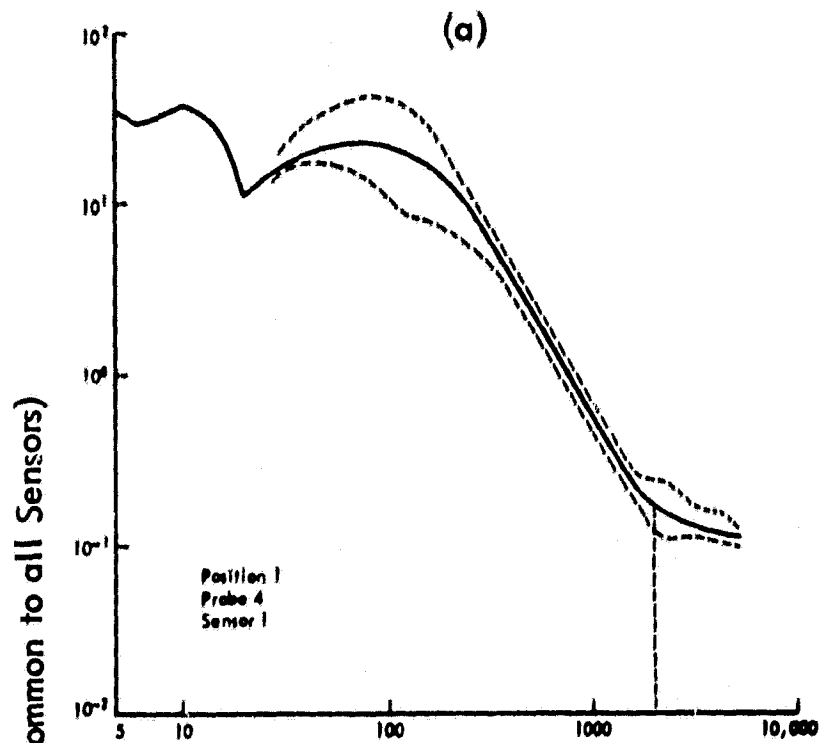


Figure 17. General Coordinates for the Instantaneous Mean Velocity Vector and Cylindrical Sensor Element



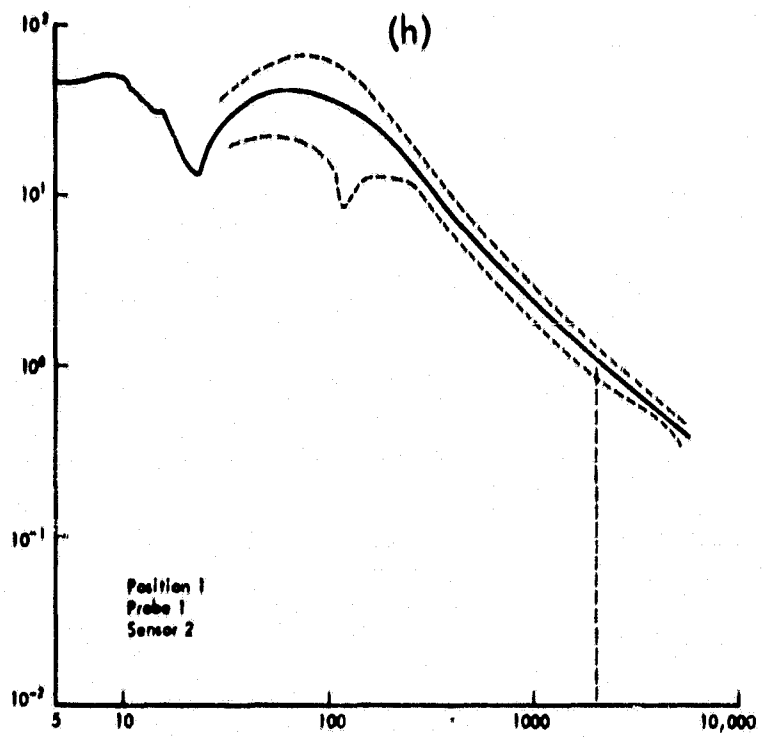
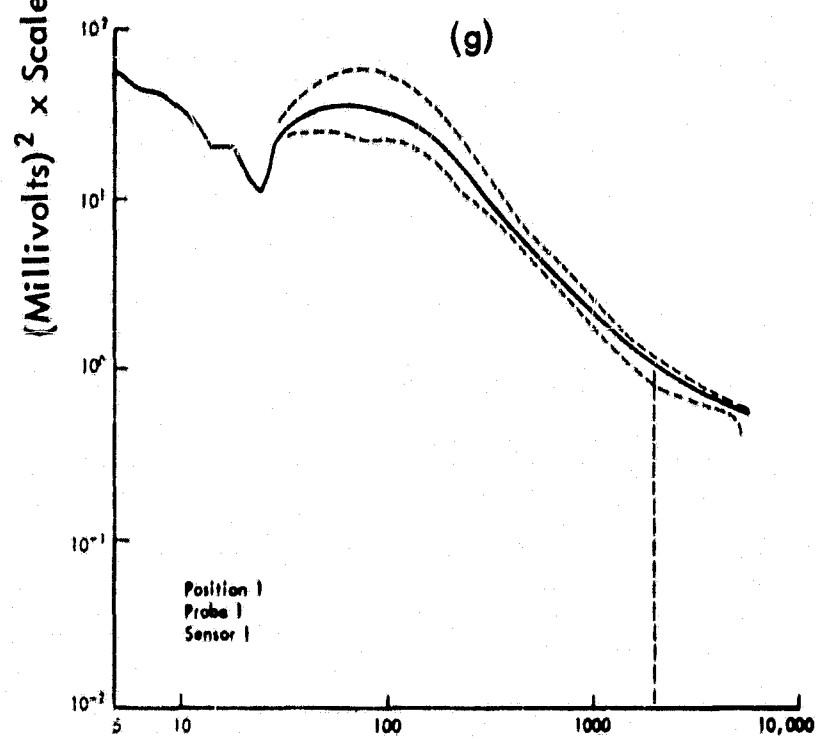
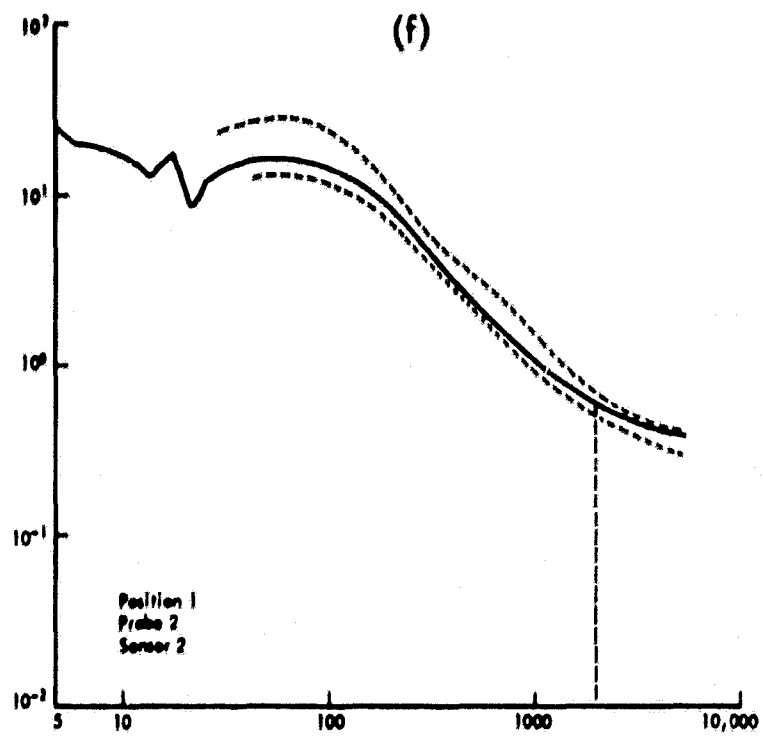
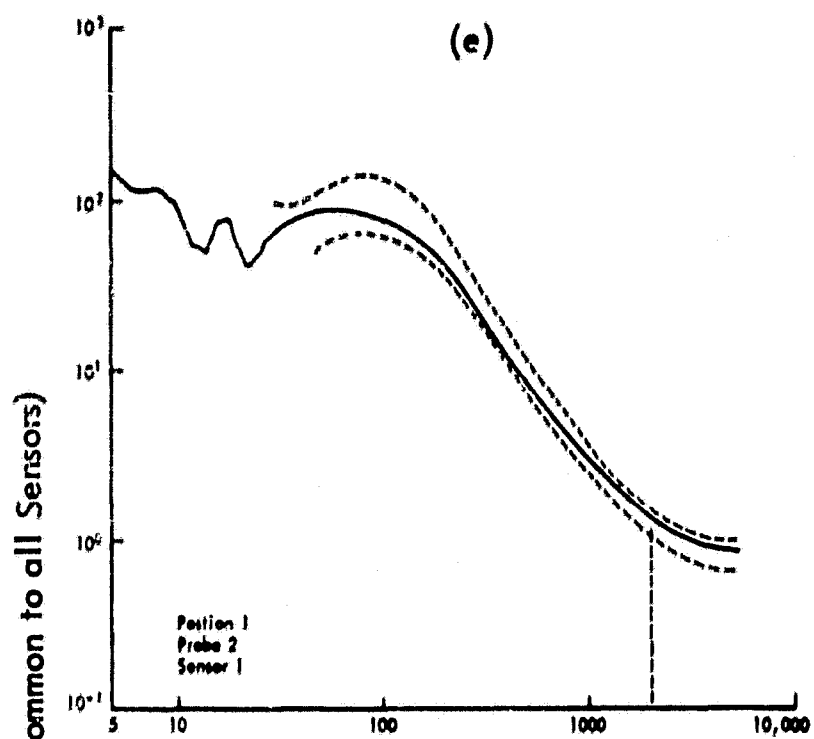
Normalized Error in Fluctuating Velocity Due to Temperature Fluctuations



Frequency - Hz

Frequency - Hz

Figure 19. Power Spectral Density Plots for Fluctuating Hot-Film Anemometer Output. Vertical dotted line at 2 kHz indicates the upper frequency limit of data used in computations. A solid line is faired through the center of scatter. The dotted envelopes indicate maximum excursions.



Frequency - Hz

Frequency - Hz

Figure 19. (Continued)

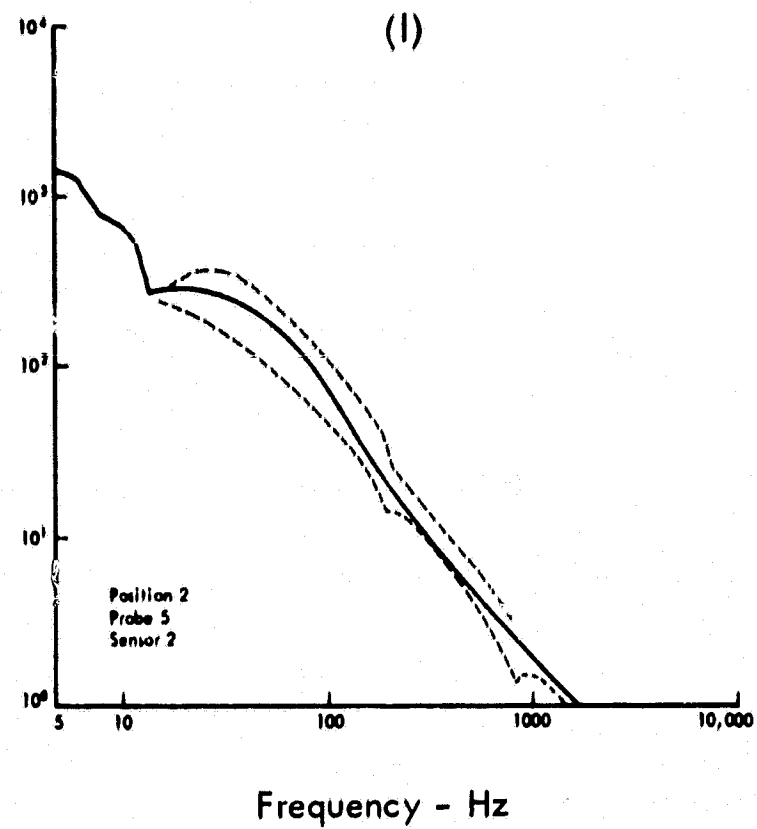
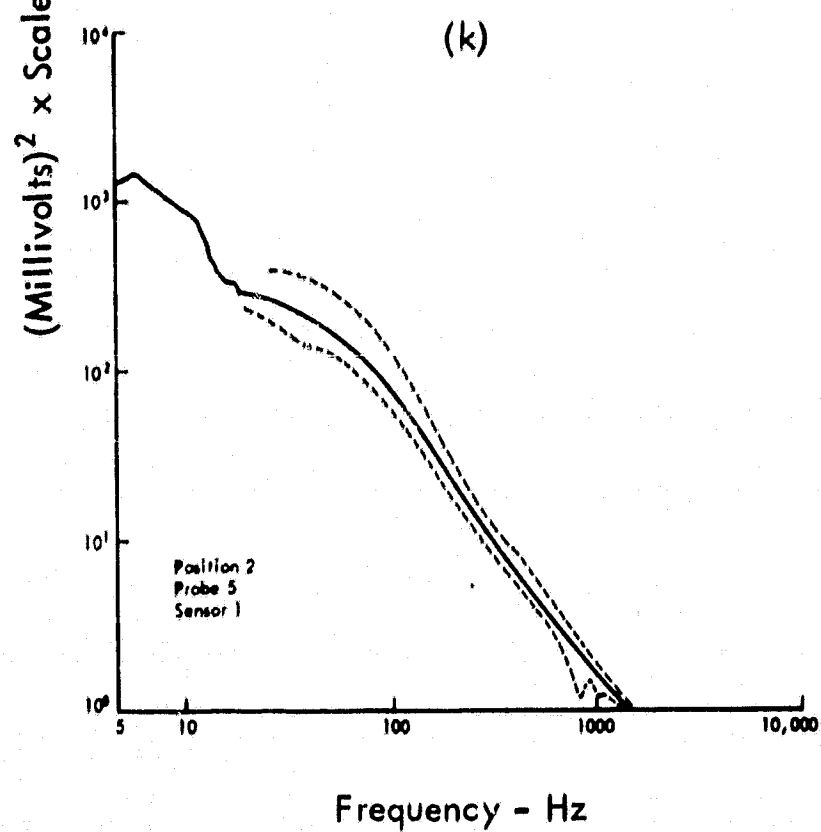
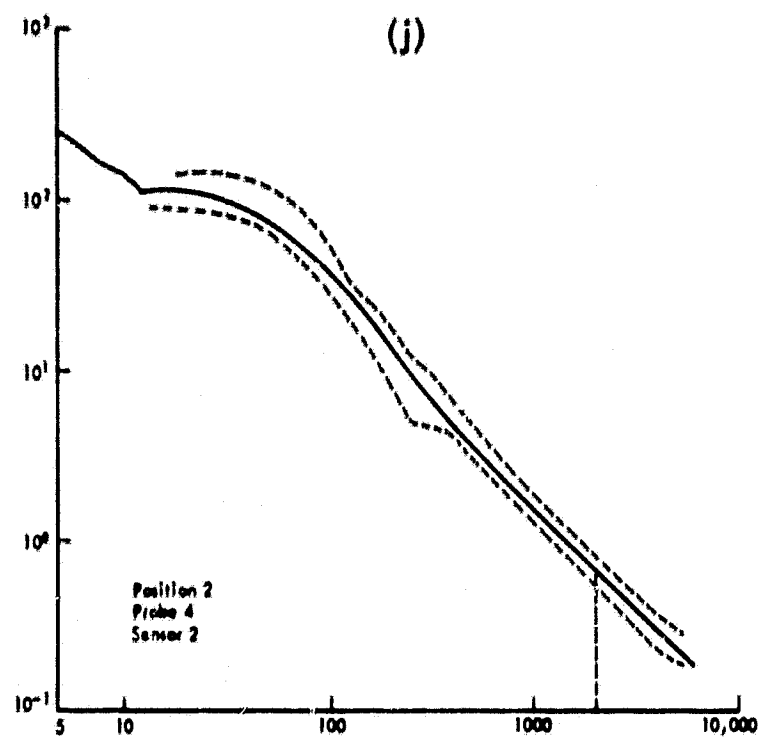
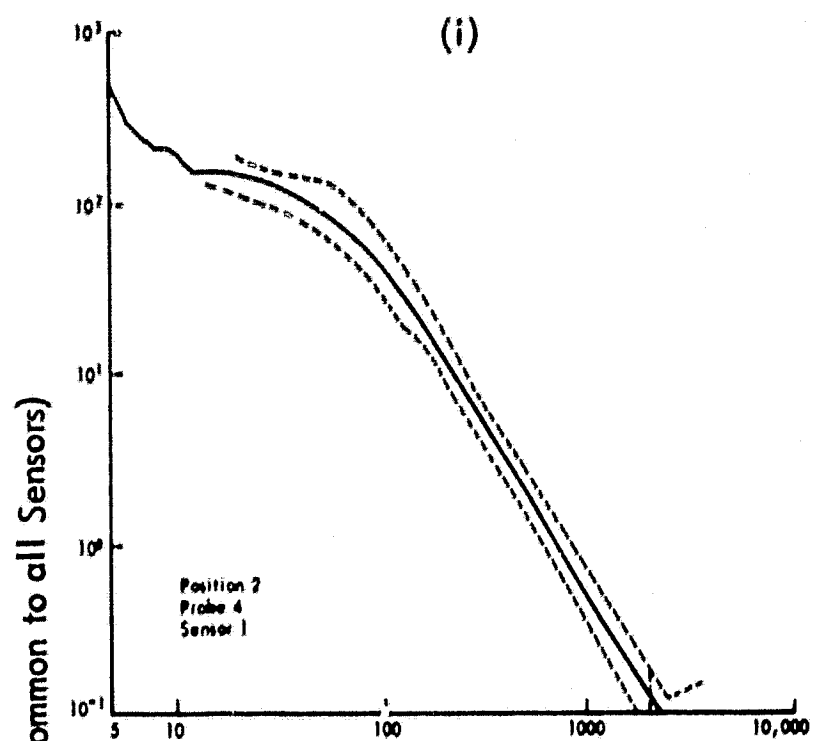
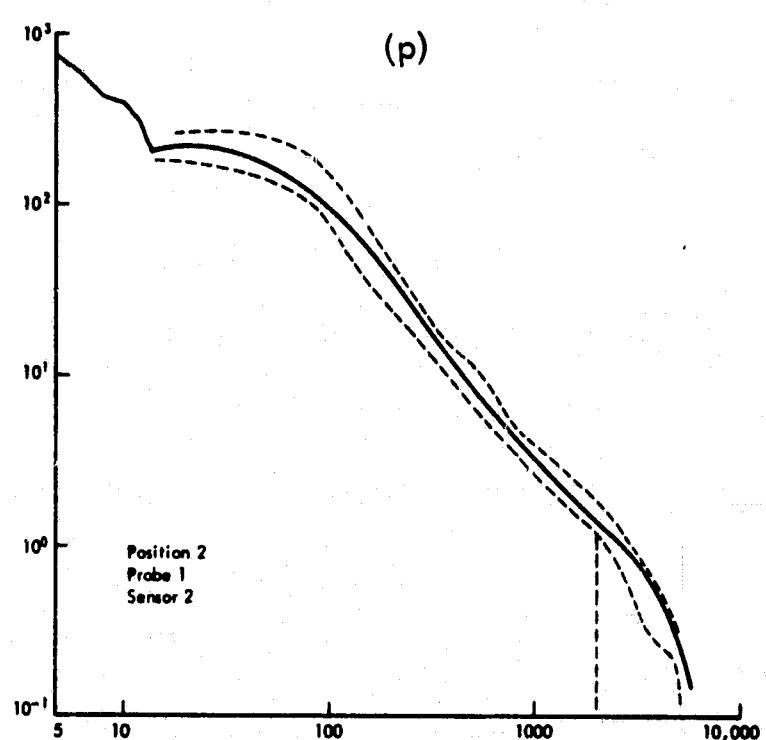
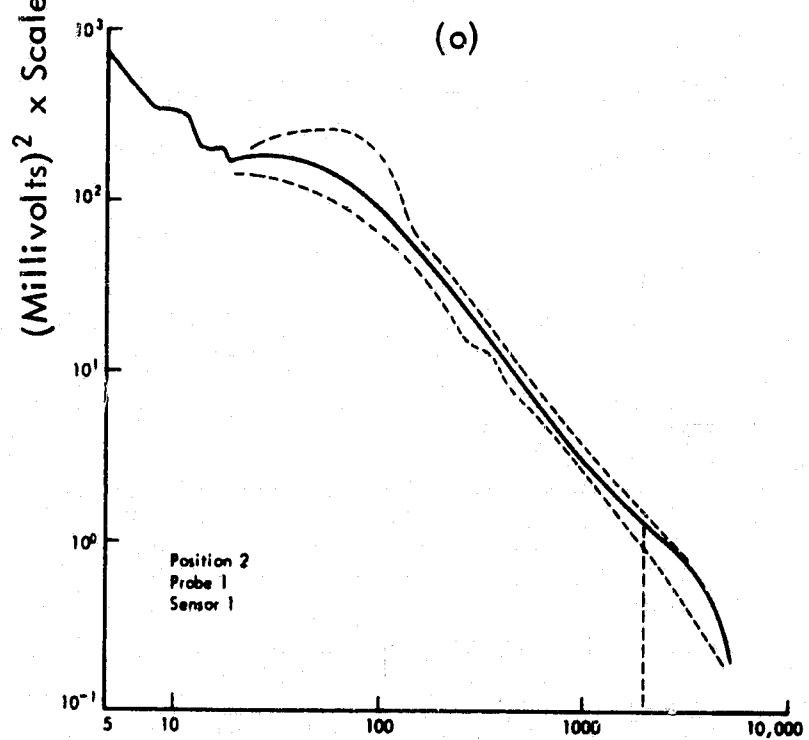
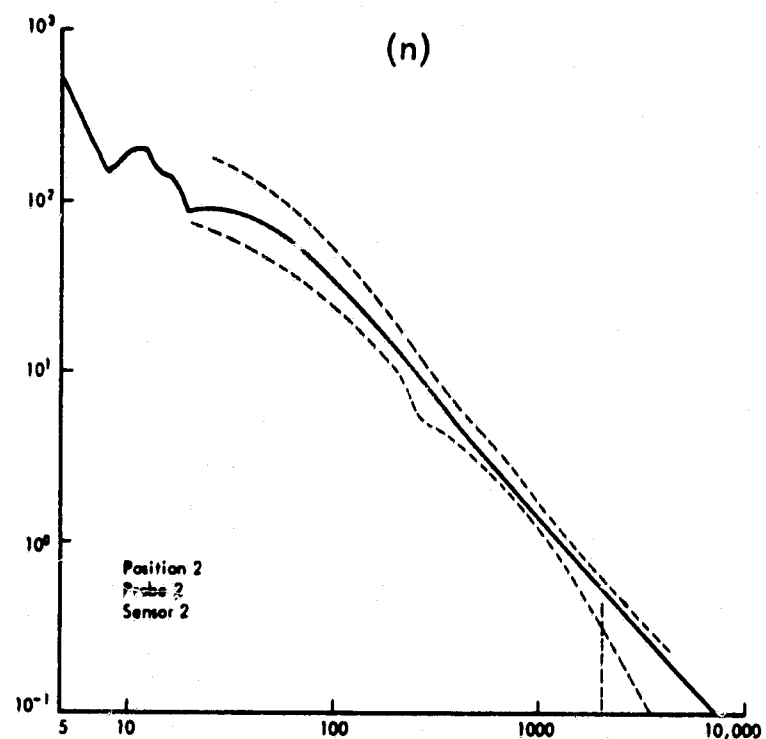
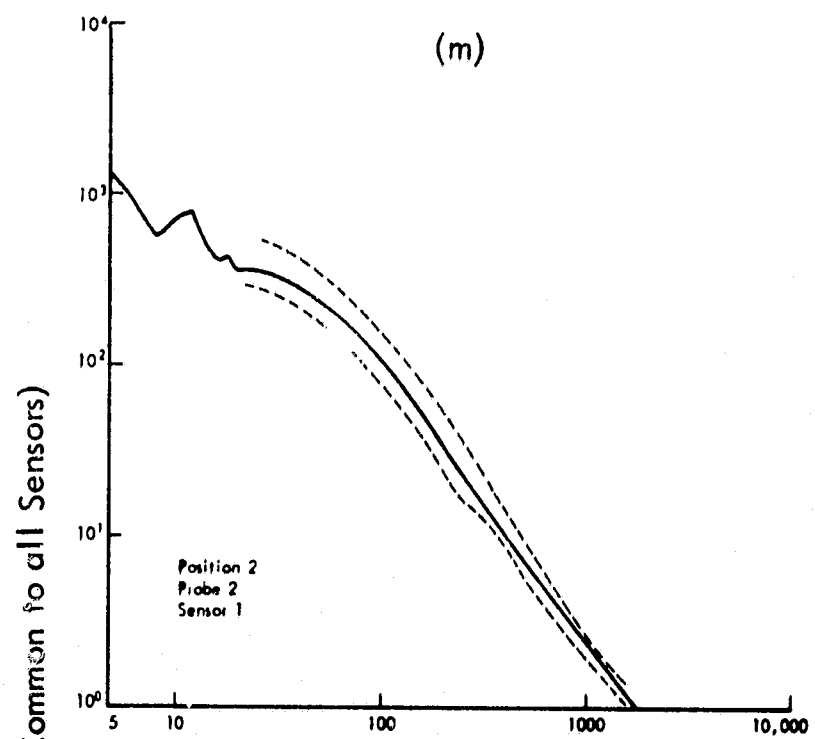


Figure 19. (Continued)



Frequency - Hz

Frequency - Hz

Figure 19. (Continued)

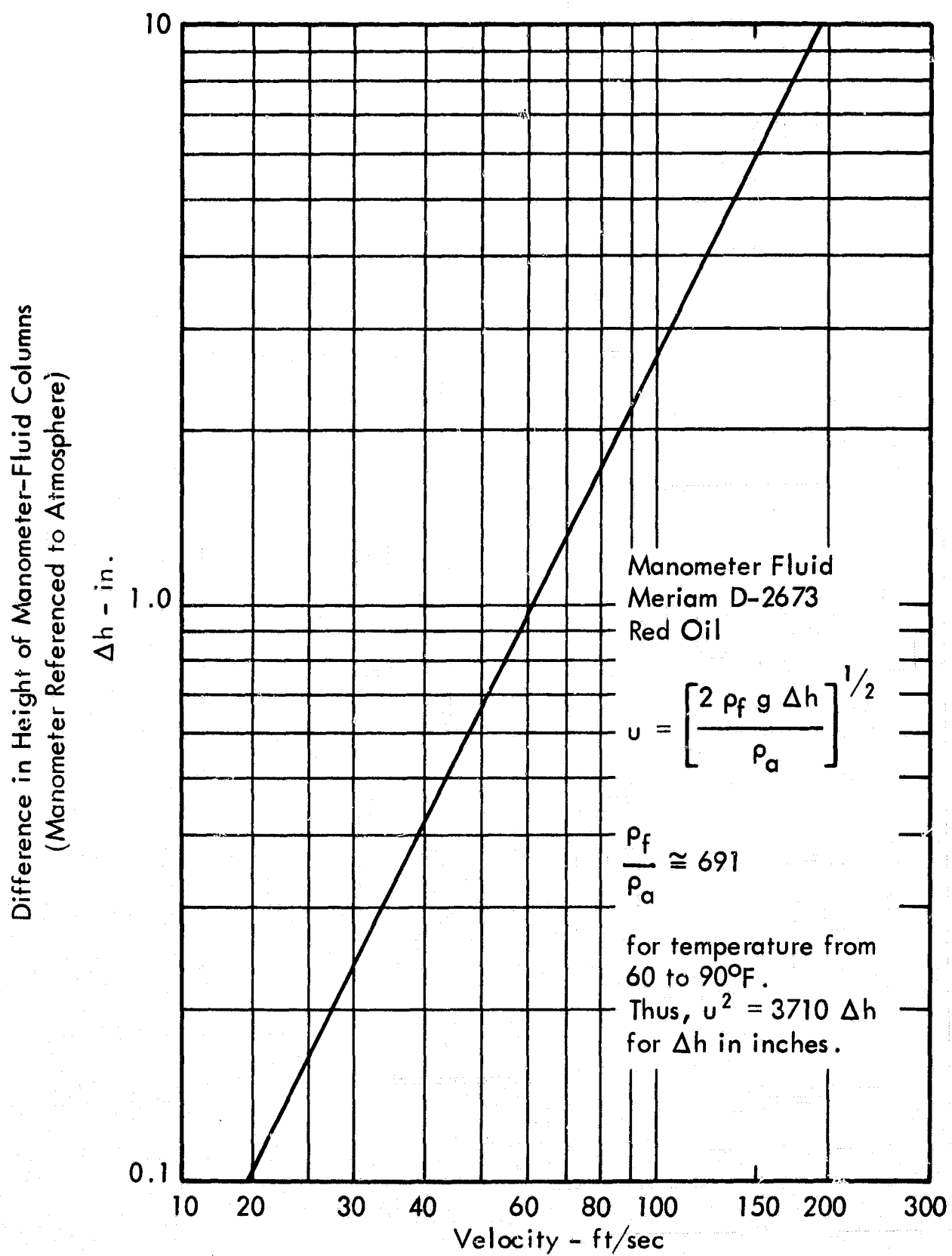


Figure 20. Approximate Pressure-Velocity Relationship for Pitot Tube

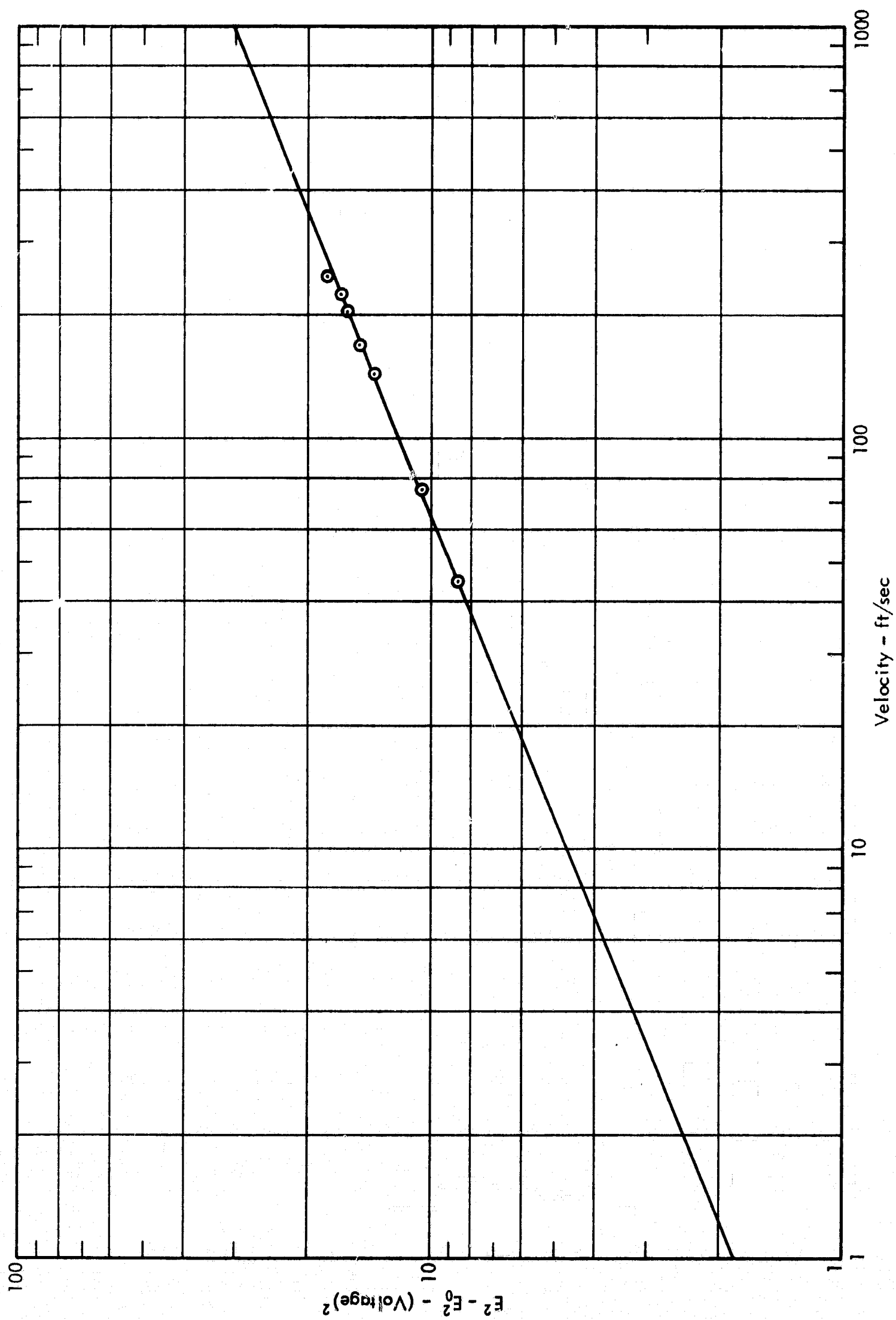


Figure 21. Typical Calibration Plot for Mean Flow Measurements

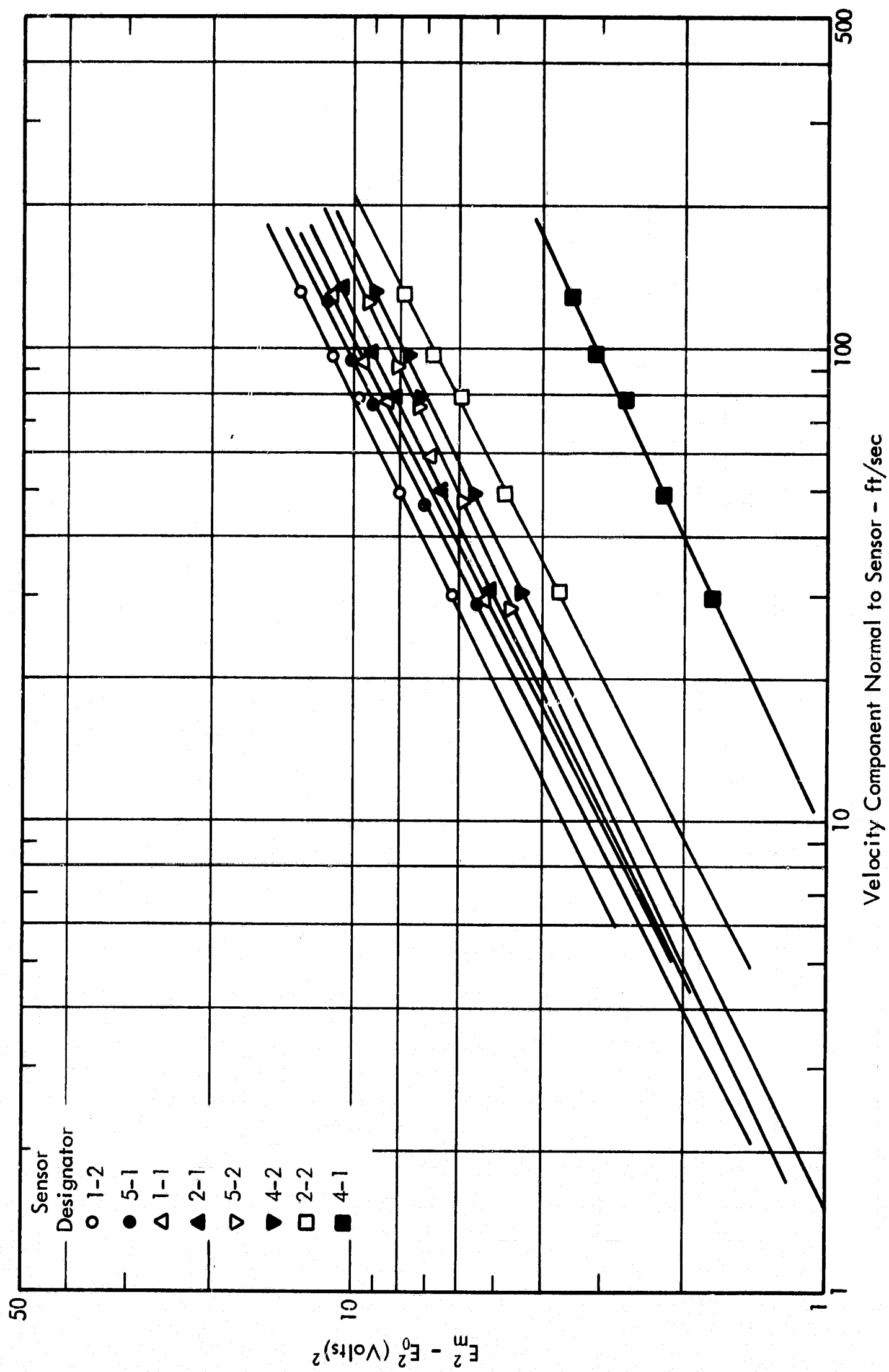
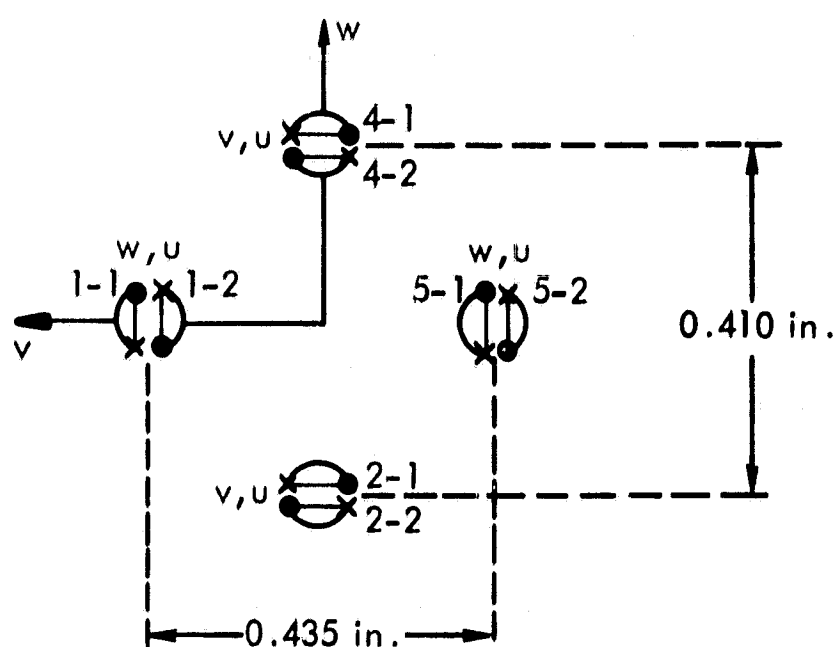


Figure 22. Calibration Plot for Measurement of Velocity Fluctuations



Angles Between Sensors and Axis of Probe Bodies

Sensor No.	θ
4-1	$44^\circ 26'$
4-2	$43^\circ 6'$
5-1	$46^\circ 0'$
5-2	$44^\circ 0'$
2-1	$45^\circ 20'$
2-2	$42^\circ 26'$
1-1	$45^\circ 25'$
1-2	$45^\circ 30'$

A cross, X, indicates the sensor support located beneath the plane of the page, while a dot, ●, indicates the sensor support in the plane of the page. The u component is positive when directed into the page. The plane of the array is perpendicular to the jet axis and the sensors are facing in the upstream direction.

Equations used in calculating u, v, and w are:

$$u = \frac{\frac{e_1}{S_1} \cos \theta_2 + \frac{e_2}{S_2} \cos \theta_1}{\sin (\theta_1 + \theta_2)} \quad (\text{Probes 4, 5, 2, 1})$$

$$v = \frac{\frac{e_2}{S_2} \sin \theta_1 - \frac{e_1}{S_1} \sin \theta_2}{\sin (\theta_1 + \theta_2)} \quad (\text{Probes 4 and 2}); \quad w = \frac{\frac{e_1}{S_1} \sin \theta_2 - \frac{e_2}{S_2} \sin \theta_1}{\sin (\theta_1 + \theta_2)} \quad (\text{Probes 5 and 1})$$

An arithmetic average was used to obtain array values of the velocity components:

$$u \text{ component: } [(u)_4 + (u)_5 + (u)_2 + (u)_1] / 4$$

$$v \text{ component: } [(v)_4 + (v)_2] / 2$$

$$w \text{ component: } [(w)_5 + (w)_1] / 2$$

The numeric subscript indicates the probe, which measured the velocity component.

Figure 23. Sensor Positions, Yaw Angle; Array Dimensions and Orientation

```

PROGRAM MILPOT
COMMON G(500) ,MAX,KC2
COMMON DUP(400),VVP(400),WWP(400),UVP(400),VWP(400),XI(400),
,ETA(400),ZETA(400)
COMMON DU,VV,WW,E
DIMENSION E0(4,2),EM(4,2),H(4,2),Z(4,2),U( 4),V( 4),
CE(424,4,2)
C,V1(4),U1(4)
C,THETA(4,2)
DIMENSION XXX(400),LABEL(5),IX(1000),ID(2) ,S(4,2) ,ICARD(20) ,
1UN(4,2)
PI2=6.2831852
DO 51 K=1,24
T=FLCAT(K-12)/41/00, *4,
51 G(K)=(1,+COS(400,*PI2*T))*SIN(2000,*PI2*T)/T
SVV=SWW=SUU=UT=0.
READ(60,2)ICARD
WRITE(61,3)ICARD
READ(60,22)((E0(I,J),J=1,2),I=1,4)
READ(60,22)((EM(I,J),J=1,2),I=1,4)
READ(60,22)((THETA(I,J),J=1,2),I=1,4)
CALL CALIB(A,B,H)
99 FORMAT(5E15.5)
21 FORMAT(13E6,0)
22 FORMAT( 8E6,0)
DO 1 I=1,4
DO 1 J=1,2
THETA(I,J)=THETA(I,J)*.0174533
CALL CAL (H(I,J),Z(I,J),THETA(I,J))
S(I,J)=H(I,J)*Z(I,J)**(1./H(I,J))*((EM(I,J)**2-E0(I,J)**2)**(1,-1,
C/H(I,J)))/(2.*EM(I,J))
UM=EXP((ALOG(EM(I,J)**2-E0(I,J)**2)-ALOG(Z(I,J)))/H(I,J))
C/SIN(THETA(I,J))
UN(I,J)=UM
UT=UT+UM
1 CONTINUE
WRITE(61,1003)S
KC2=24
US=UT/8.
WRITE(61,1003)UN,US
US=109.
1003 FORMAT(9E15,5)
3 FORMAT(1H1,26X,20A4/)
2 FORMAT(20A4)
MAX=424
DO 201 J=1,2
K3=L+2
DO 200 I=1,4
24 READ (60,500) ID
40 CALL TAPE (K3,1,LABEL(1),LABEL(5),11)
25 IF (LABEL(1).EQ.ID(1).AND.LABEL(5).EQ.ID(2)) 15,40

```

Figure 24. Program "MILPOT"

```

015 CALL TAPE 'K3,1,LABEL(1),LABEL(5),11)
15 CALL TAPE( K3,1,IX(1),IX(1000),11 )
50 WRITE (61,501)ID
   N=0
   DO 60 K=1,MAX
   N=N+1
60 E(N,1,J) =FLOAT(IX(K+100)) *.001

      GO TO 350
300 WRITE (61,502)
   PAUSE 5 GO TO 24
320 WRITE (61,503)
   PAUSE 4 GO TO 24
   CALL FILTER(E(1,1,J))
350 CONTINUE
200 CONTINUE
201 CONTINUE
   MAX =MAX-24
   MAX1=MAX-1
   REWIND 3
   REWIND 4
500 FORMAT (A4,A4)
501 FORMAT (10X,2A4,2X,25HIS THE CHANNEL BEING USED)
502 FORMAT (10X,15HREAD TAPE HEAD)
503 FORMAT (10X,12HPARITY ERROR)
   DO 10 K=1,MAX
   DO 11 I=1,4
   P2=(E(K,I,2))/S(I,2)
   P1=(E(K,I,1))/S(I,1)
   P3=SIN(THETA(I,1)+THETA(I,2))
   V(I)=(P1*SIN(THETA(I,2))-P2*SIN(THETA(I,1)))/P3
   U(I)=(P1*COS(THETA(I,2))+P2*COS(THETA(I,1)))/P3
11 CONTINUE
   VV=(V(1)+V(3))/2.
   WW=(V(2)+V(4))/2.
   UU=(U(1)+U(2)+U(3)+U(4))/4.
   UV=UU*VV
   VW=VV*WW
   UUP(K)=UU /US *2.
   VVP(K)=VV /US *2.
   WWP(K)=WW /US *2.
   AVW=SGRT(ABSF(VW))
   AUV=SGRT(ABSF(UV))
   VWP(K)=SIGNF(AVW,VW)/US *2.
   UVP(K)=SIGNF(AUV,UV)/US *2.
1012 FORMAT(1X,3F20.5,2F10.5)
1014 FORMAT(1X,6F10.5)

```

Figure 24. (Continued)

```

SUU=SUU+UU    **2
SVV=SVV+VV    **2
SWW=SWW+WW    **2
RC    =SGRT (UU    **2+VV    **2+WW    **2)
THETA    =ATAN (VV    /UU    )
PHI    =ATAN (WW    /SGRT (UU    **2+VV    **2))
I=1,/41700.
DELY= .435    /12.
DELZ= .410 /12.
10  CONTINUE
SUU=SGRT(SUU/400.)
SVV=SGRT(SVV/400.)
SWW=SGRT(SWW/400.)
SUU=SUU/US
SVV=SVV/US
SWW=SWW/US
WRITE(61,1003)SUU,SVV,SWW
DO 90 K=2,MAX1
DO 91 I=1,4
P2=(E(K-1,I,2))/S(I,2)
P1=(E(K-1,I,1))/S(I,1)
P3=SIN(THETA(I,1)+THETA(I,2))
P21=E(K+1,I,2)/S(I,2)

P11=E(K+1,I,1)/S(I,1)
V(I)=(P1*SIN(THETA(I,2))-P2*SIN(THETA(I,1)))/P3
V1(I)=(P11*SIN(THETA(I,2))-P21*SIN(THETA(I,1)))/P3
U1(I)=(P11*COS(THETA(I,2))+P21*COS(THETA(I,1)))/P3
U(I)=(P1*COS(THETA(I,2))+P2*COS(THETA(I,1)))/P3
91  CONTINUE
V1(1)=-V1(1)
V1(3)=-V1(3)
VR1=(V( 4)-V( 2))/DELY-(V( 1)-V( 3))/DELZ
VR2=(U( 1)-U( 3))/DELZ-(V( 1(2))-V( 2)+V( 1(4))-V(4))/16./US/T
VR3=(V( 1(1))-V( 1)+V( 1(3))-V(3))/16./US/T+(U( 2)-U( 4))/DELY
XI(K)=VR1    *DELZ/US
ETA(K)=VR2    *DELZ/US
ZETA(K)=VR3    *DELZ/US
90  CONTINUE
CALL PL01( 1.,12.,-3)
CALL PL1(UUP(1),MAX)
CALL PL1(VVP(1),MAX)
CALL PL1(WWP(1),MAX)
CALL PL1(UVP(1),MAX)
CALL PL1(VWP(1),MAX)
CALL PL1(XI(1),MAX)
CALL PL1(ETA(1),MAX)
CALL PL1(ZETA(1),MAX)

```

Figure 24. (Continued)

```

KC2=2.*PI*(5.*41700./8./250.)
DO 20 IP=1,KC2
I=FLOAT(IP-KC2/2)/41700.*4.
20  G(IP)=(1.+COS(PI*(I*250./5.)))*SIN(250.*PI*(I*1)/1
    CALL FILTER(UUP)
    CALL FILTER(VVP)
    CALL FILTER(WWP)
    CALL FILTER(UVP)
    CALL FILTER(WVW)
    CALL FILTER(XI)
    CALL FILTER(ETA)
    CALL FILTER(ZETA)
    MAX=150
        CALL PL(UUP,VVP,MAX)
        CALL PL(VVP,WWP,MAX)
        CALL PL(XI,ETA,MAX)
    CALL PL(ETA,ZETA,MAX)
    WRITE(59,1002) & PAUSE
    PAUSE 1
1000 FORMAT(1H1,7X2HUU,16X2HVV,16X2HWW,16X2HRO,15X,4H1H1A,15X3HPH1/)
1001 FORMAT(6E18,8)
1002 FORMAT(5X32HMOVE PEN TO CENTER PAGE, PUSH GO/)
END

```

Figure 24. (Continued)

```

SUBROUTINE CALIB(H,Z,THETA)
DIMENSION ICARD(20),U(20)
DIMENSION PO(8),TO(8),PSTAT(8),TR(8),PSAT(8)
DIMENSION EM(20),EMEU(20),EMEODS(20)
    READ(60,24) HR,PA,TC
    READ(60,24)(TO(J),J=1,8)
    READ(60,24)(PO(J),J=1,8)
    READ(60,24)(TR(J),J=1,8)
    READ(60,24)(PSTAT(J),J=1,8)
4   FORMAT(1H1,26X,20A4/)
    DO 987 J=1,8
        TR(J)=TR(J)+459.688
        TO(J)=TO(J)+459.688
987  CONTINUE
24  FORMAT(8F5,0)
    NOR=5

```

Figure 25. Subroutine "CALIB"

```

25  FORMAT(13F5,0)
    ICAR=4H9999
    SU=FLCATF(VJR)
    RETURN
ENTRY CAL
10  CONTINUE
    SX=SY=SXY=SXX=0.
    REAC(60,1)ICARD
    IF(ICARD.EJ,ICAR:99,91
90  WRITE(61,300)
    RETURN
91  CONTINUE
    READ(60,24)                (EM(I),I=1,5),EO
    WRITE(61,4)ICARD
    DO 2 I=1,5
    J=I
    PSAT(J)=(1.714E-05)*IR(J)**2-(1.755E-02)*IR(J)+4.4068
    ALCO2   =(-8.7217E-09)*IR(J)**3+(1.4518E-05)*IR(J)**2+(-8.4119E
C-03)*IR(J)+2.9773
    ALO2    =(-4.386E-08)*IR(J)**2+(4.4585E-05)*IR(J)+1.3887
    XAL     =29.92*HR*PSAT(J)/PA
    BMA     =(28.016*.7809+32,*.2095+39.944*.0093+44.011*.0003)*(1.-XA
CL      )+XAL      *18.016
    ALA     =1.+1./((1.-XAL      )*(.7809/.4+.2095/(ALO2      -1.)+.0093/.67
C+.0003/(ALCO2      -1.))+XAL      /.3)

    ROHG=(TC-70.)*(13.4625-13.5438)/(130.-70.)+13.5438
    ROIL  =(TR(J)-120.-459.688)*(1.8325-.8042)/(45.-120.)+.8042
    P     =2.*PSAT(J)+ROHG/ROIL      *PA
    PG    =P  O(J) +ROHG/ROIL      *PA
    U(I)=2.*ALA      /(ALA      -1.)+49720.*10      /BMA      *(1.-(P      /PG      ))*
C*(1.-1./ALA      )
    U(I)=SQRTF(U(I))
    U(I)=U(I)*SINF(THETA)
    EMEC(I)=EM(      I)-EO
    EMECDS(I)=EM(      I)**2-EO      **2
    UPJ=ALOG(U(I))
    EPJ=ALOG(EMECDS(I))
C  WRITE(61,31)UPJ,EPJ
    SX=SX+UPJ
    SY=SY+EPJ
    SXY=SXY+UPJ*EPJ
    SXX=SXX+UPJ*UPJ

```

Figure 25. (Continued)

```

2      CONTINUE
      H=(SX*SY-SY*SXY)/(SX*SA-SU*SXX)
      Z=(SY-SX*H)/SU
      Z=EXP(Z)
C      WRITE(61,6)
      6      FORMAT(10X,5HH      ,10X,5HZ      )
      300    FORMAT (1X,7/10X      ,/10X,10HEND OF RUN)
      1      FORMAT(20A4)
C      WRITE(61,31)H,Z
      31     FORMAT(2E15,6)
      RETURN
      END

```

Figure 25. (Continued)

```

      SUBROUTINE FILTER(U)
      COMMON C(500) ,MAX,KC2
      DIMENSION U(624),V(600)
      N=MAX-KC2
      PI2=6.2831852
      DO 1 J=1,N
      SUM=0.
      DO 2 K=1,KC2
      JT=K+J
      2      SUM=SUM+C(K)*U(JT)
      1      V(J)=SUM/PI2/41700.      *4.
      DO 3 K=1,N
      J=K+50
      J = K
      3      U(K)=V(J)
      RETURN
      END

```

Figure 26. Subroutine "FILTER"

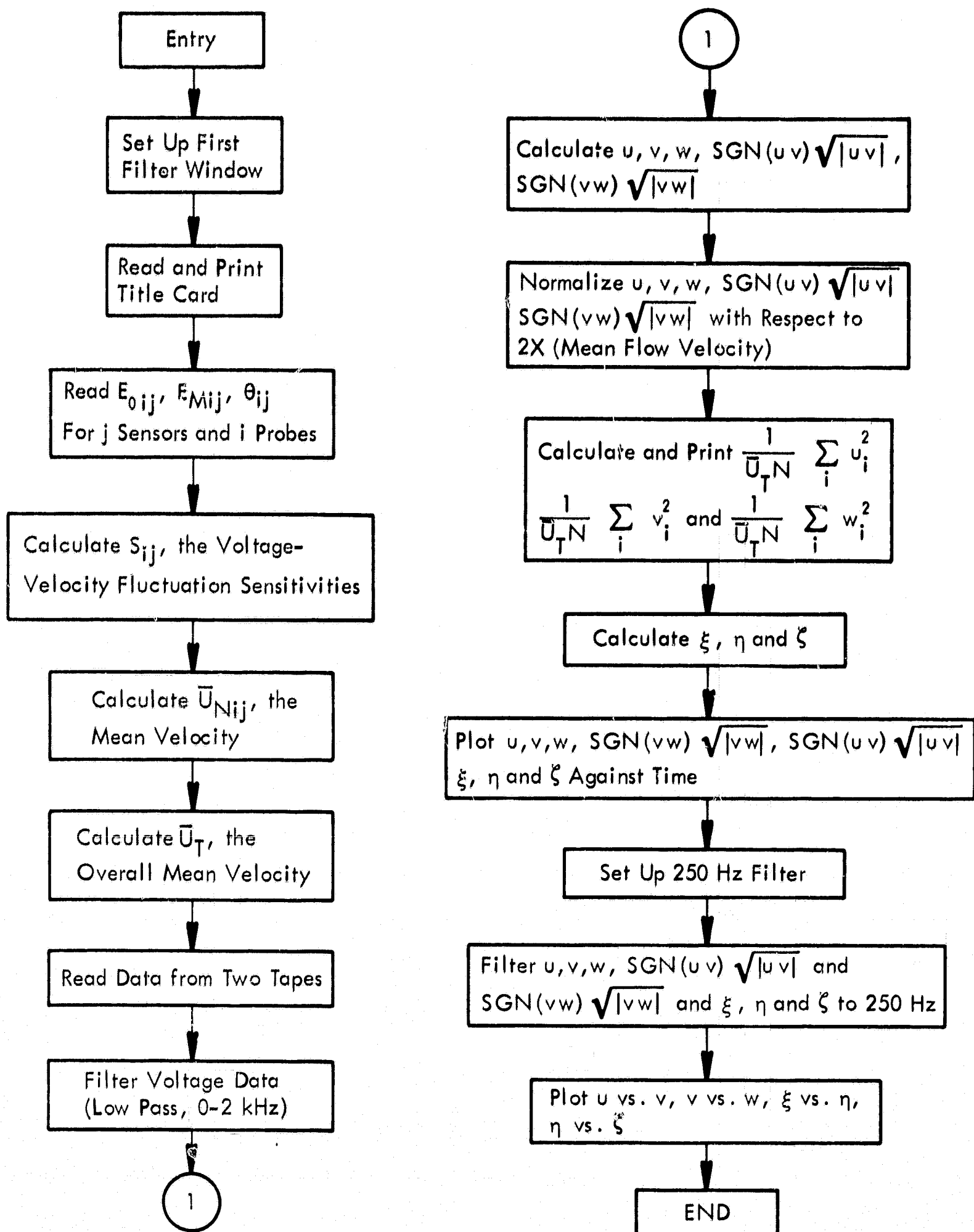


Figure 27. Flow Diagram of Computer Operations for Position 1 and Position 2

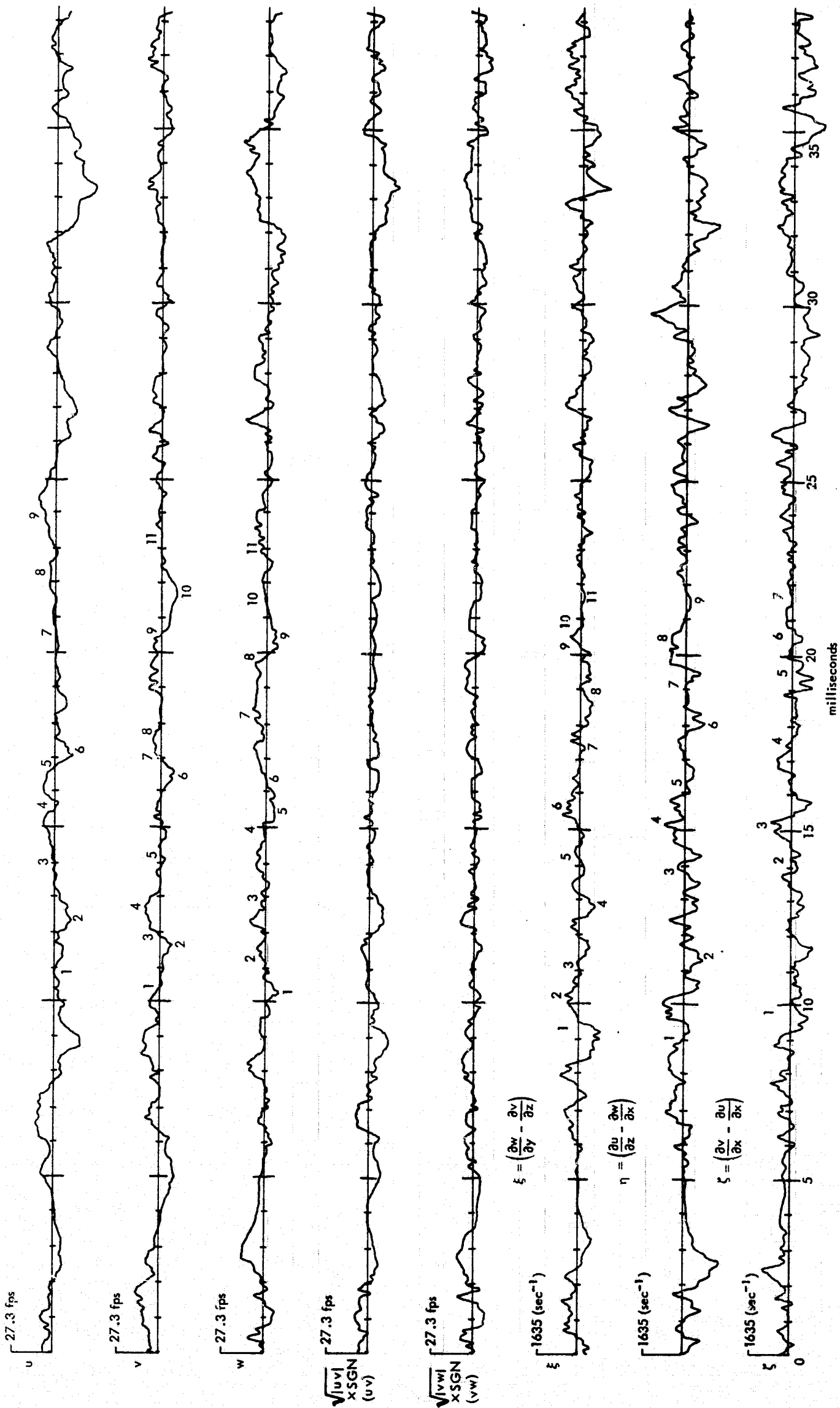


Figure 28. Fluctuating Values of Velocity and Vorticity for Position 1 at Three Diameters from the Jet Exit Plane (Low-Pass Filtering at 2 kHz)

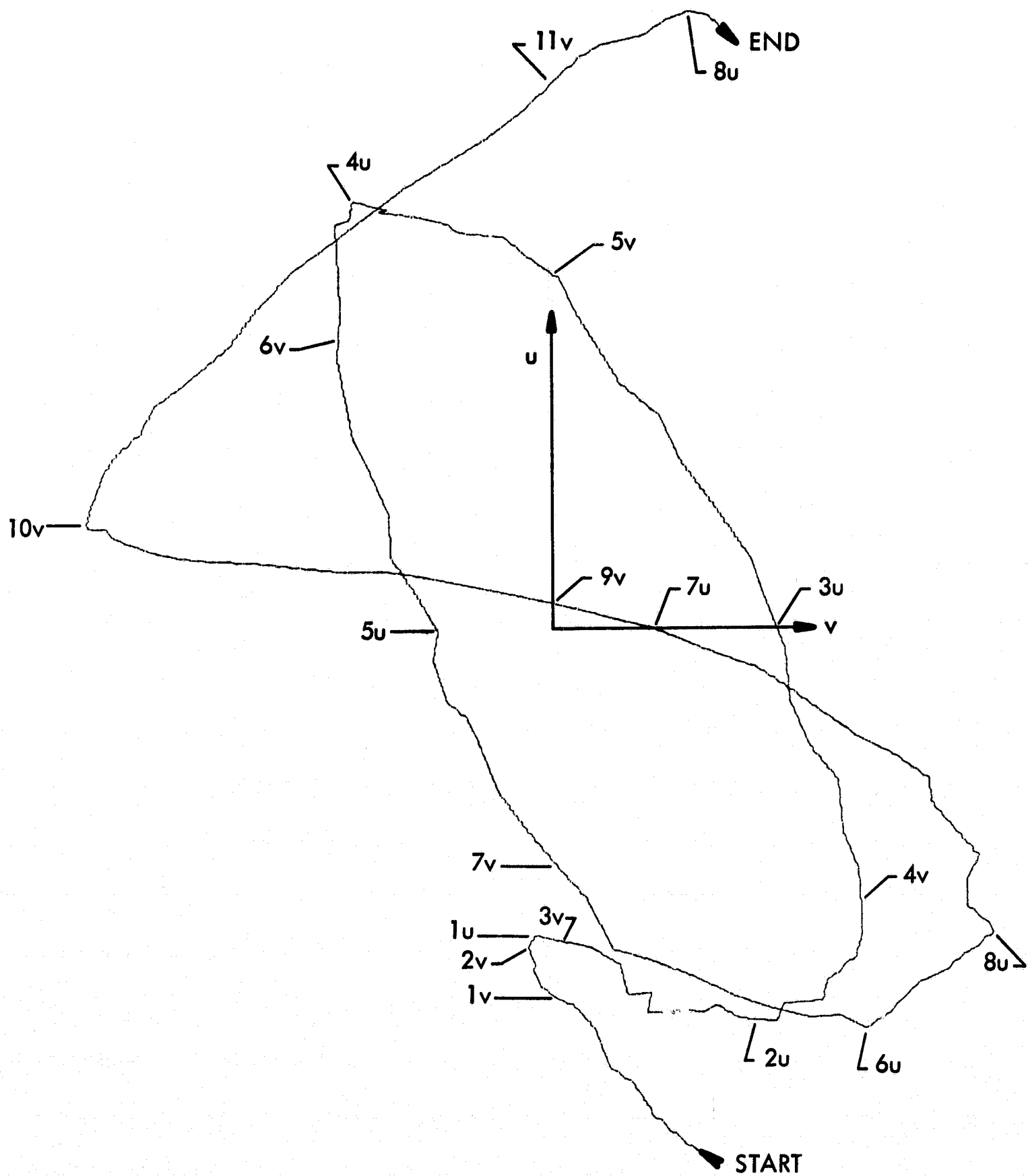


Figure 29a. Simultaneous Values of u and v with Low-Pass Filtering at 250 Hz, at Position 1

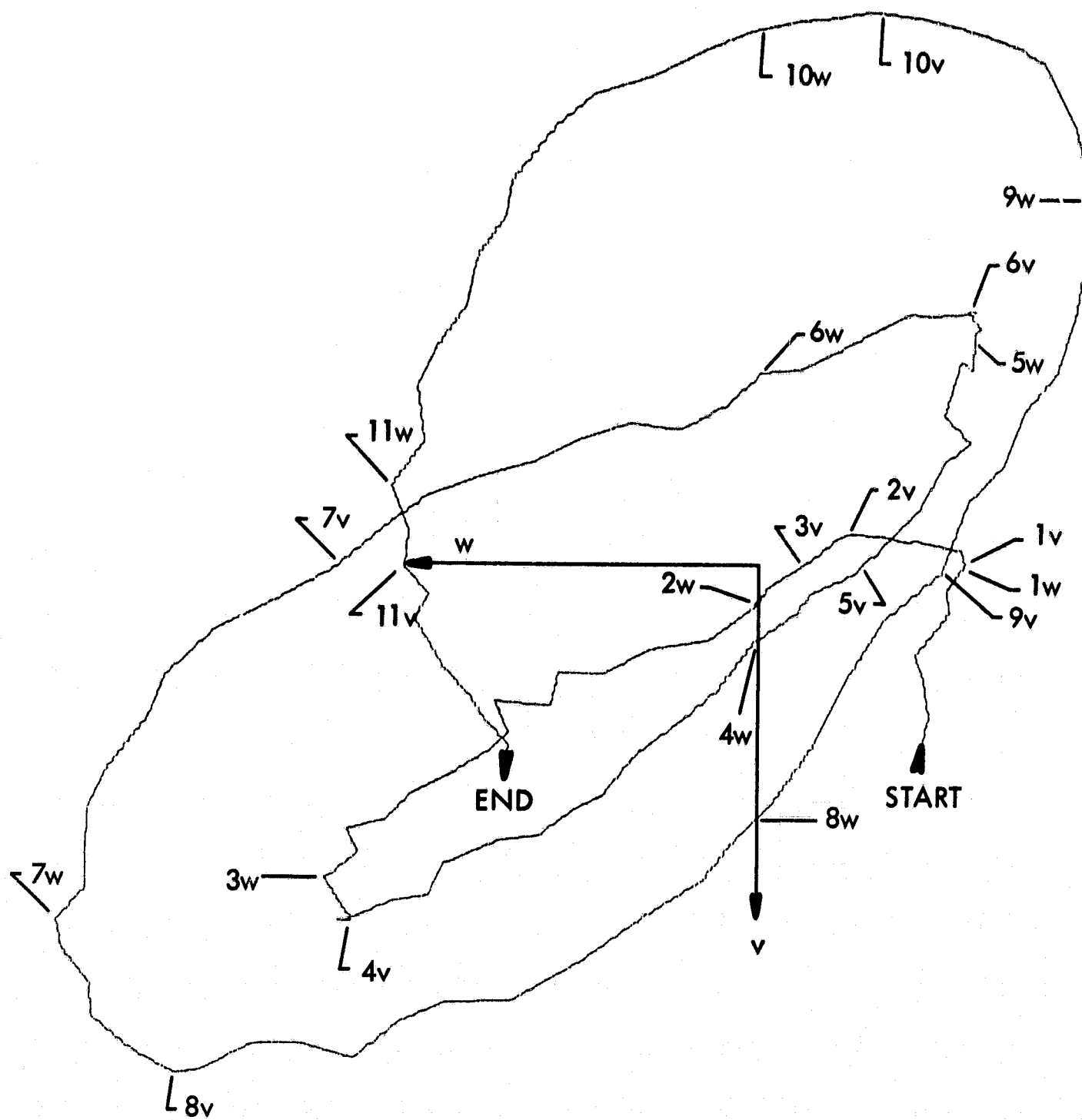


Figure 29b. Simultaneous Values of v and w with Low-Pass Filtering at 250 Hz, at Position 1

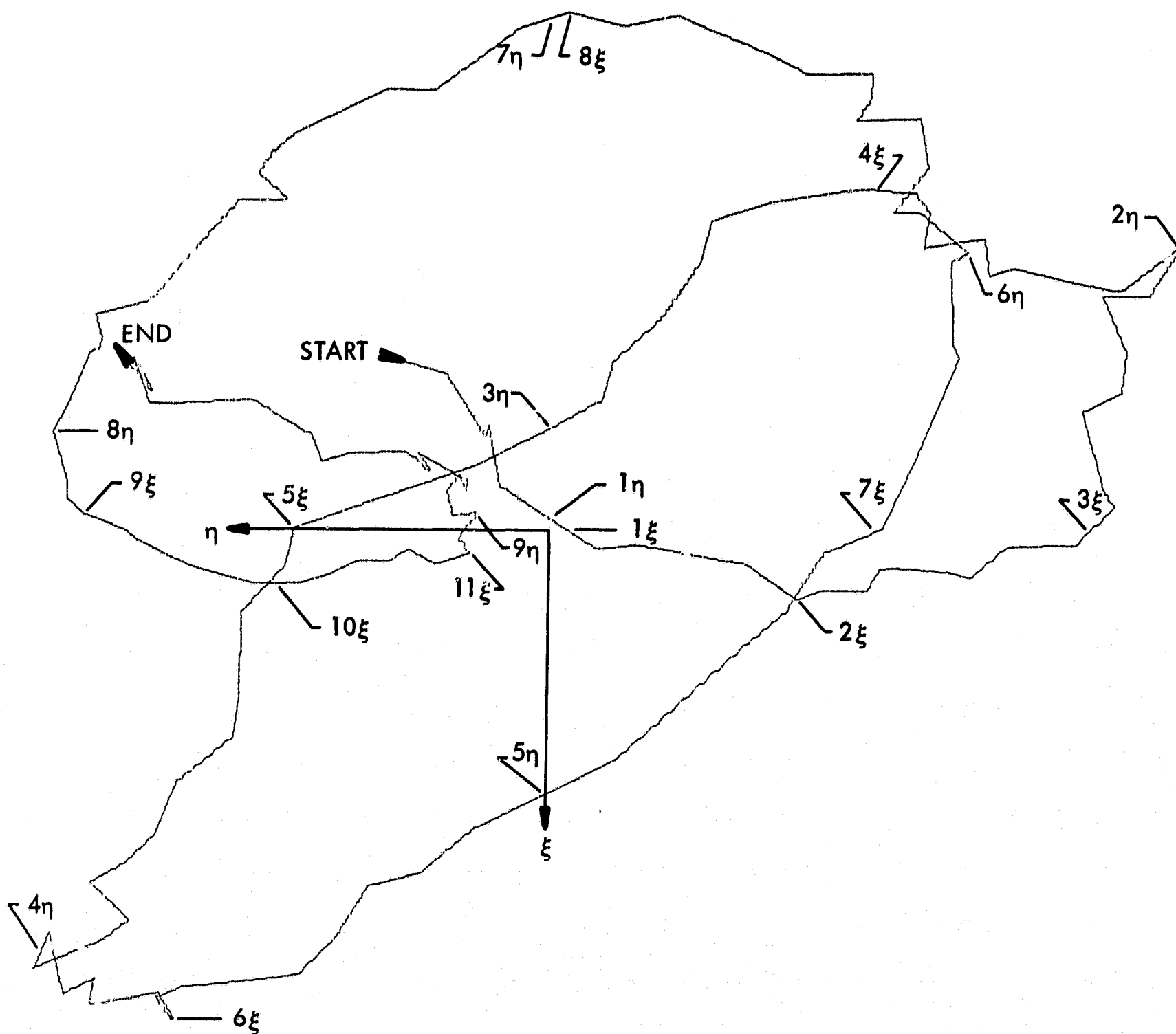


Figure 29c . Simultaneous Values of ξ and η with Low-Pass Filtering at 250 Hz, at Position 1

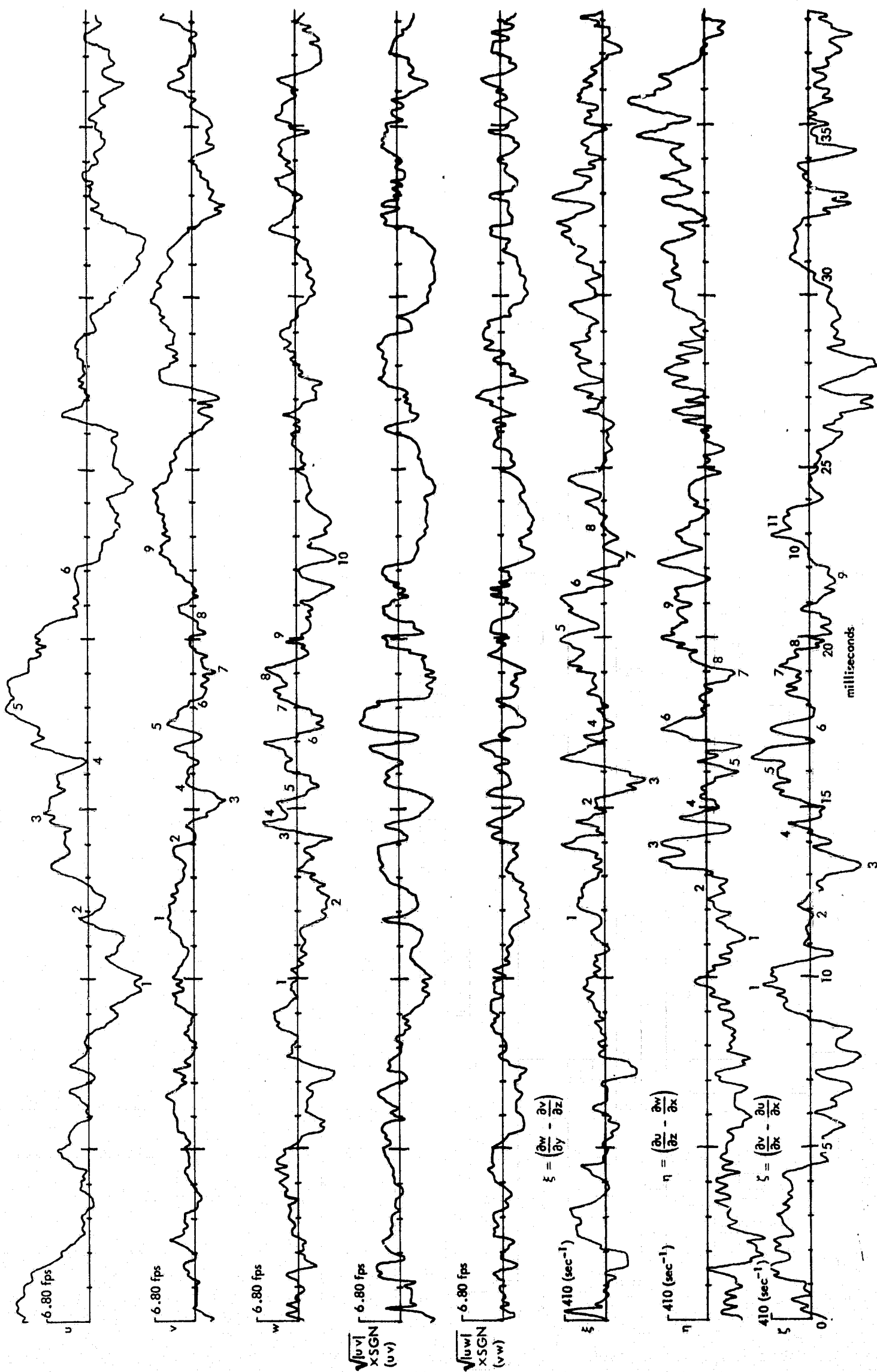


Figure 30. Fluctuating Values of Velocity and Vorticity for Position 2 at Three Diameters from the Jet Exit Plane (Low-Pass Filtering at 2 kHz)

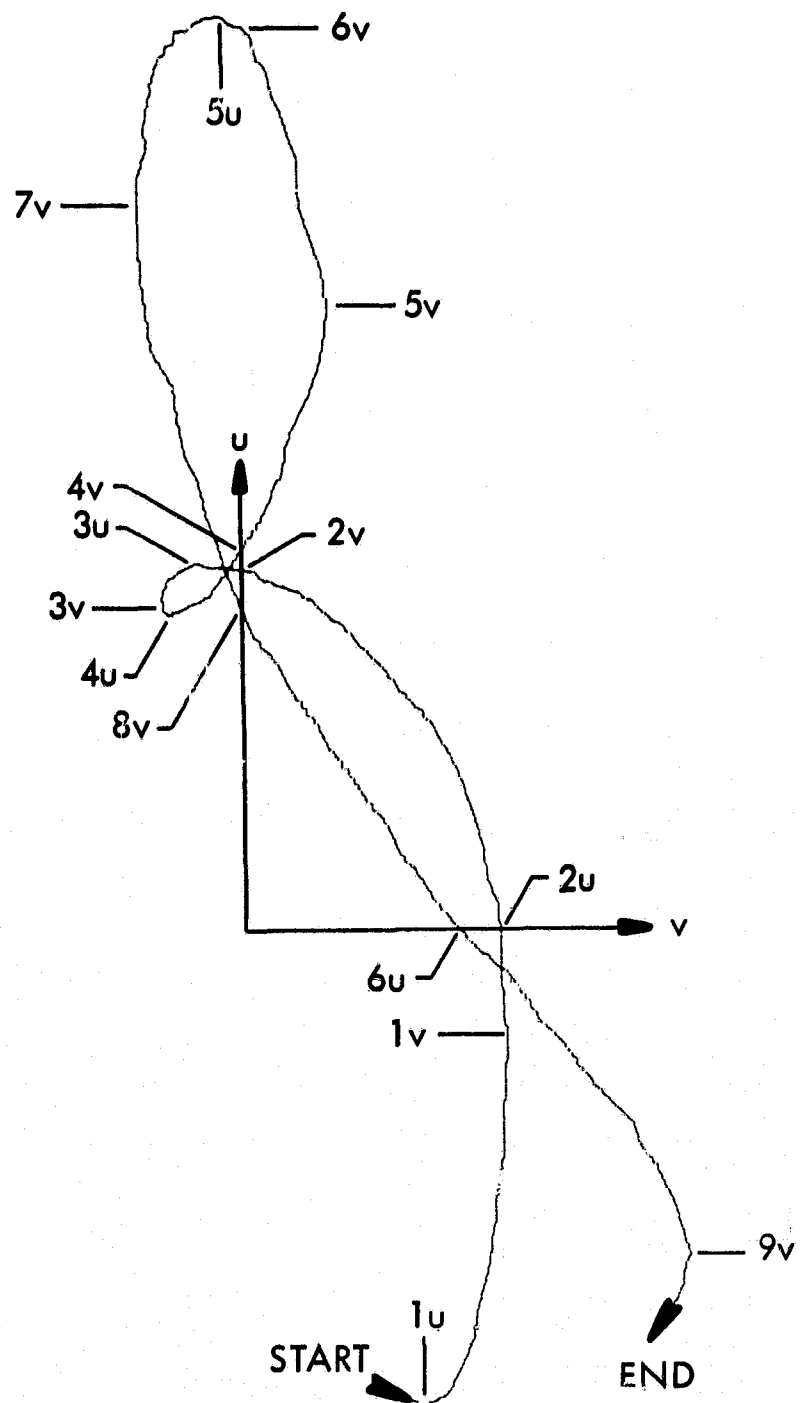


Figure 31a. Simultaneous Values of u and v with Low-Pass Filtering at 250 Hz, at Position 2

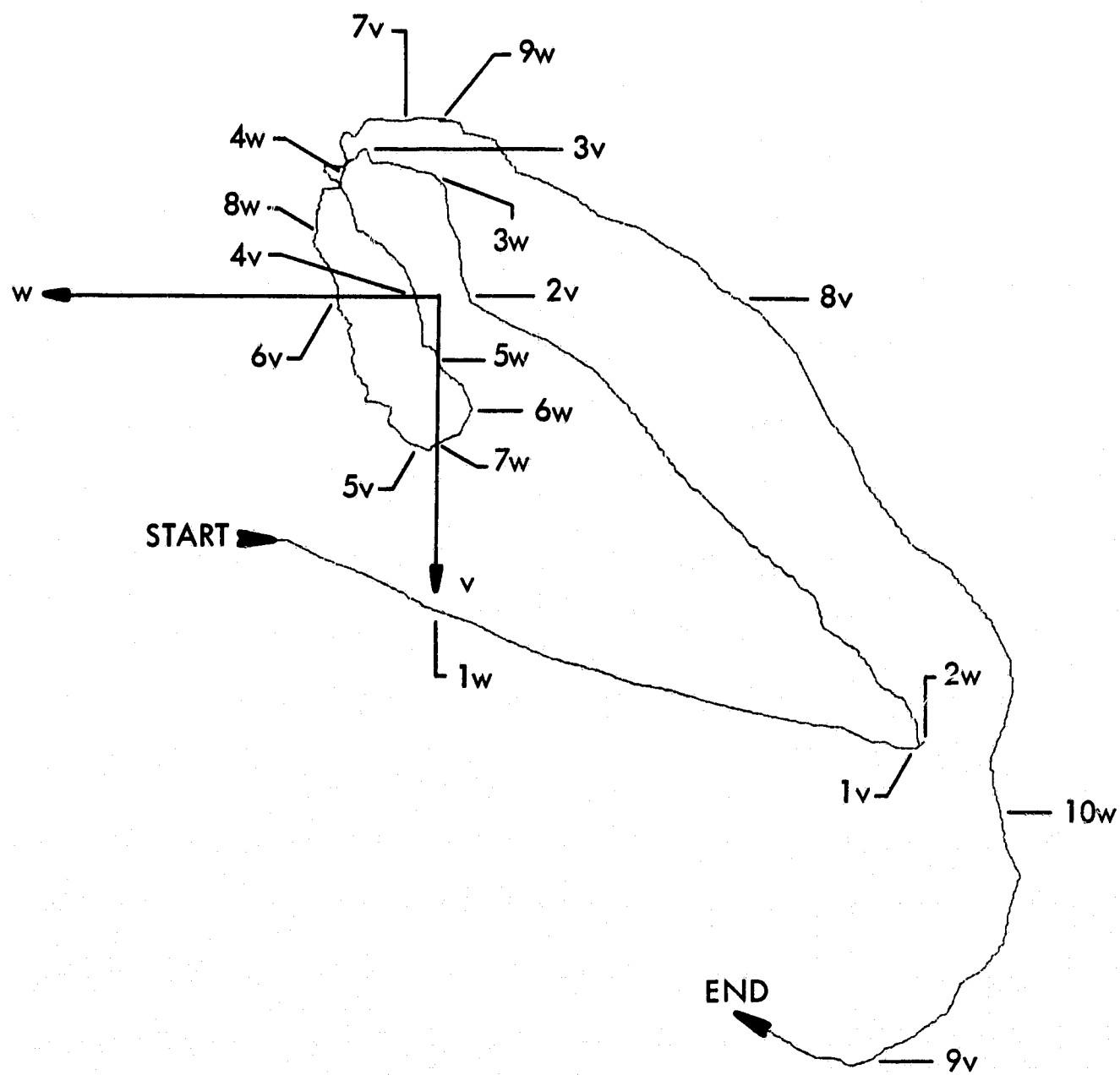


Figure 31b. Simultaneous Values of v and w with Low-Pass Filtering at 250 Hz, at Position 2

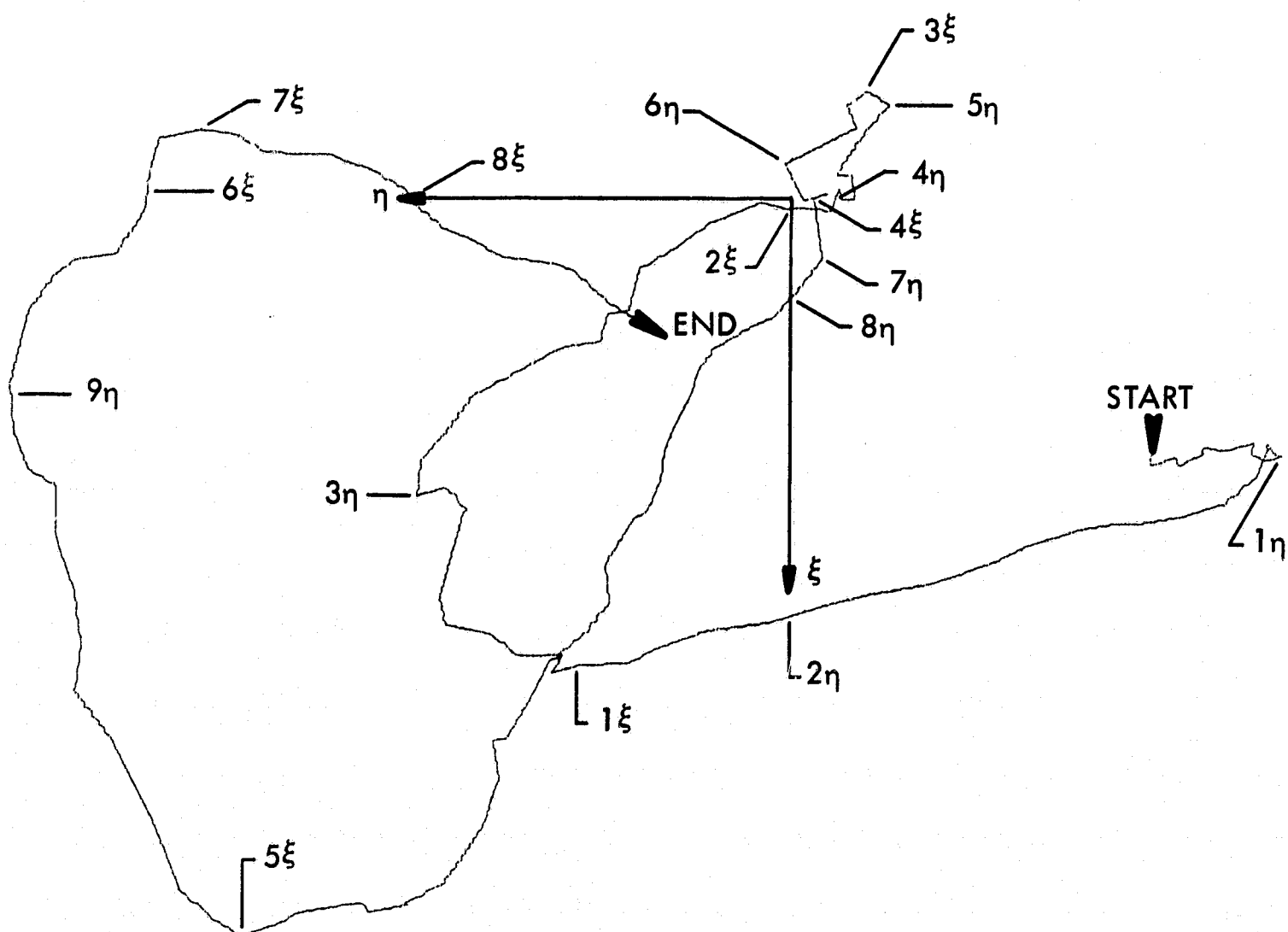


Figure 31c. Simultaneous Values of ξ and η with Low-Pass Filtering at 250 Hz, at Position 2

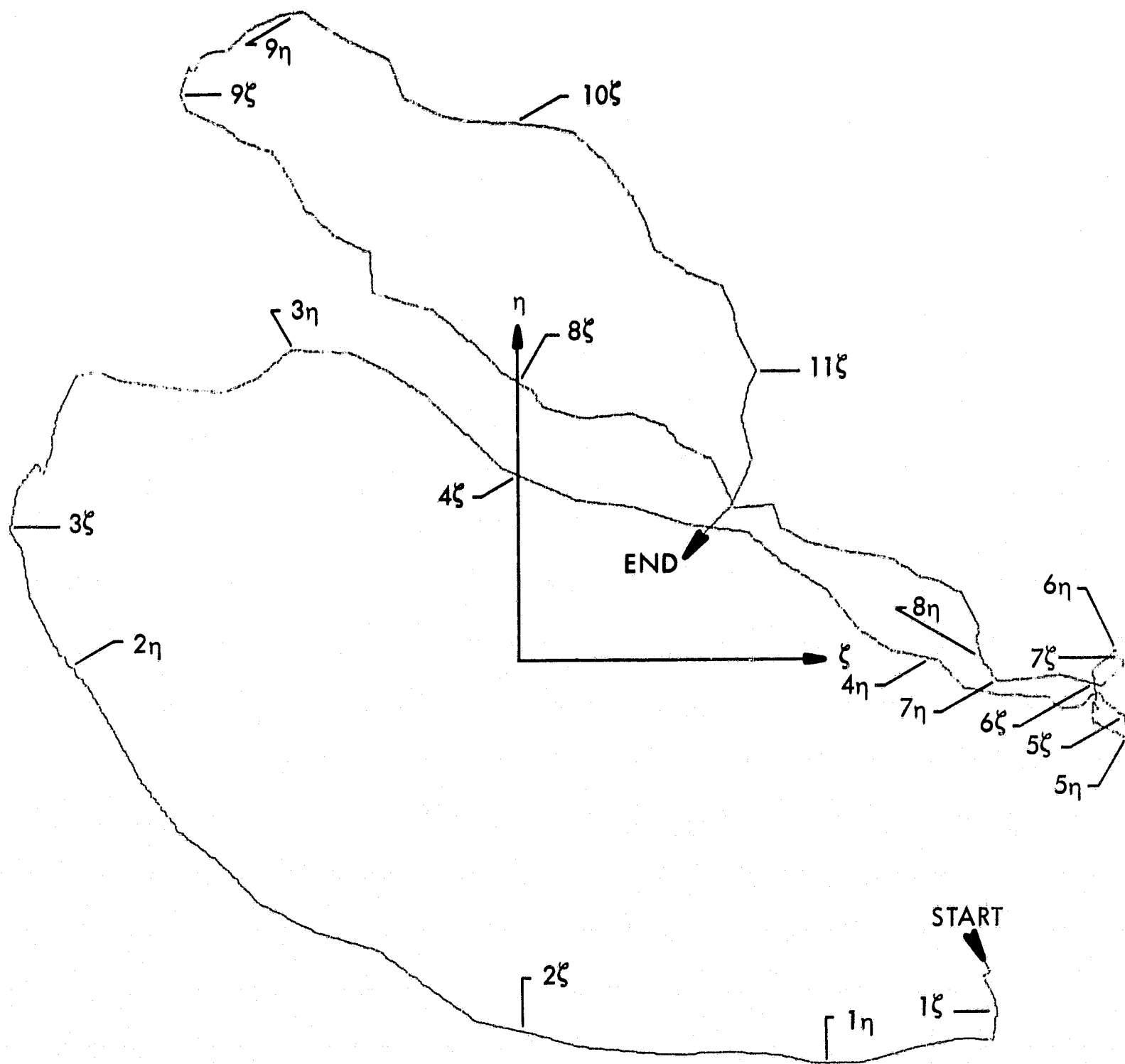


Figure 31d. Simultaneous Values of η and ξ with Low-Pass Filtering at 250 Hz, at Position 2

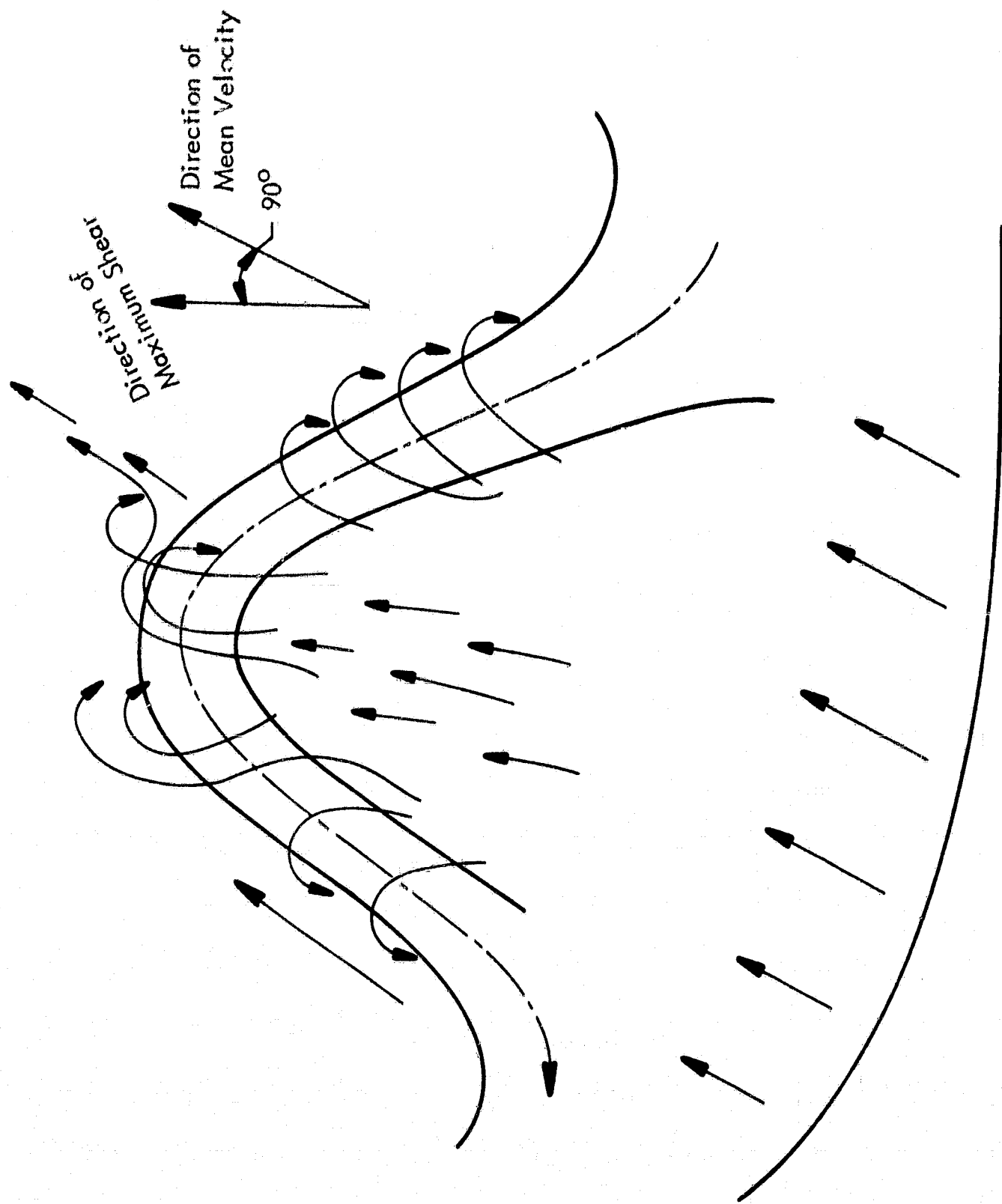
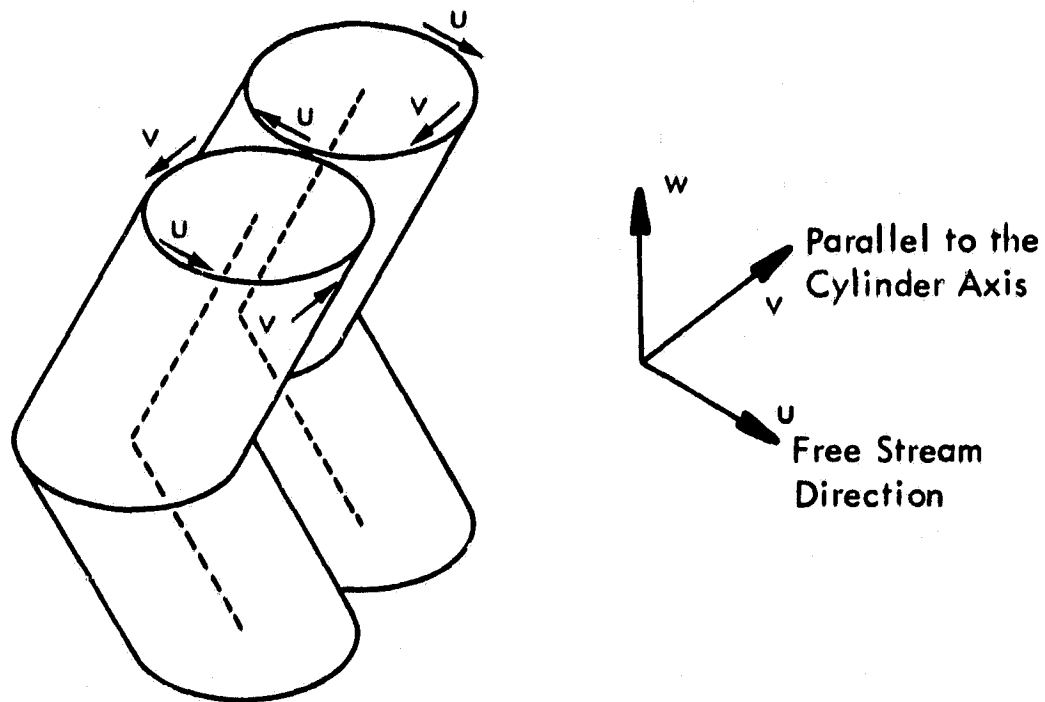
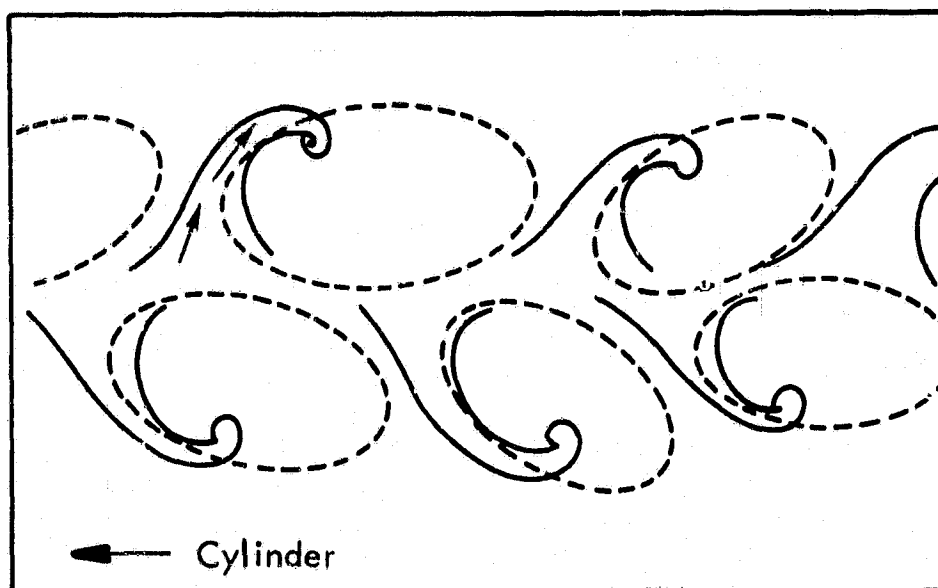


Figure 32. The "Horseshoe Vortex" Model of Theodorsen -- A Proposed "Primary Structure" for Wall-Bound Turbulence



Schematic Model of the Vortex Pair Eddy



Sketch of a Jet System in a Late Stage of Development

Figure 33. Grant's Vortex Pair and Re-Entrant Jet Model

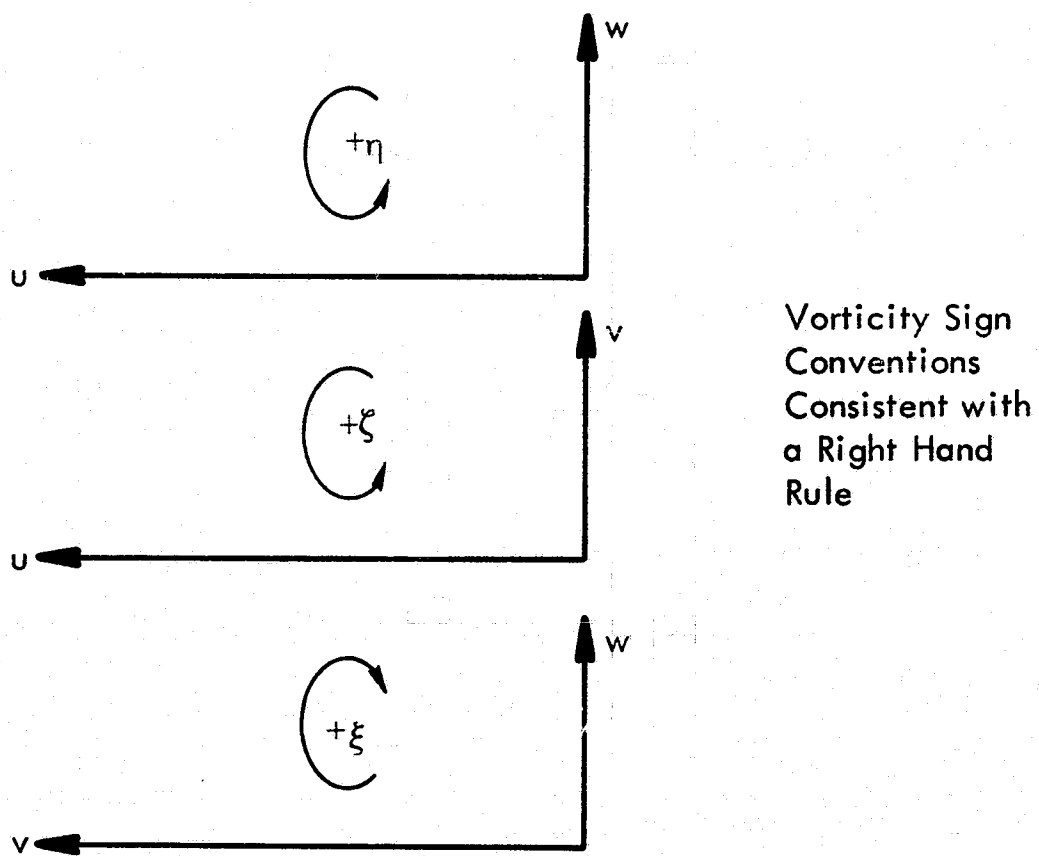
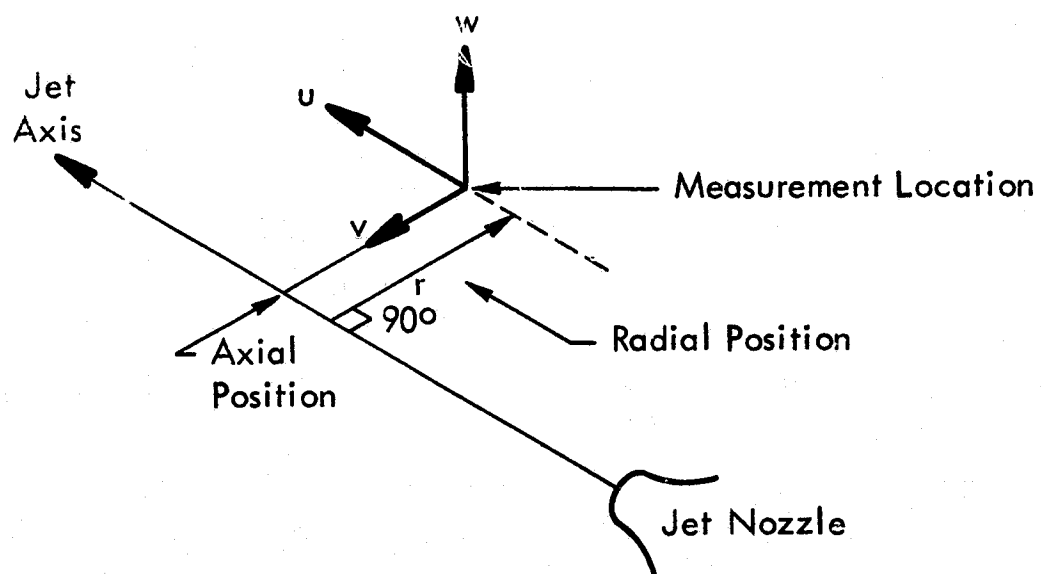


Figure 34. Coordinates and Sign Conventions for Velocity and Vorticity Measurements

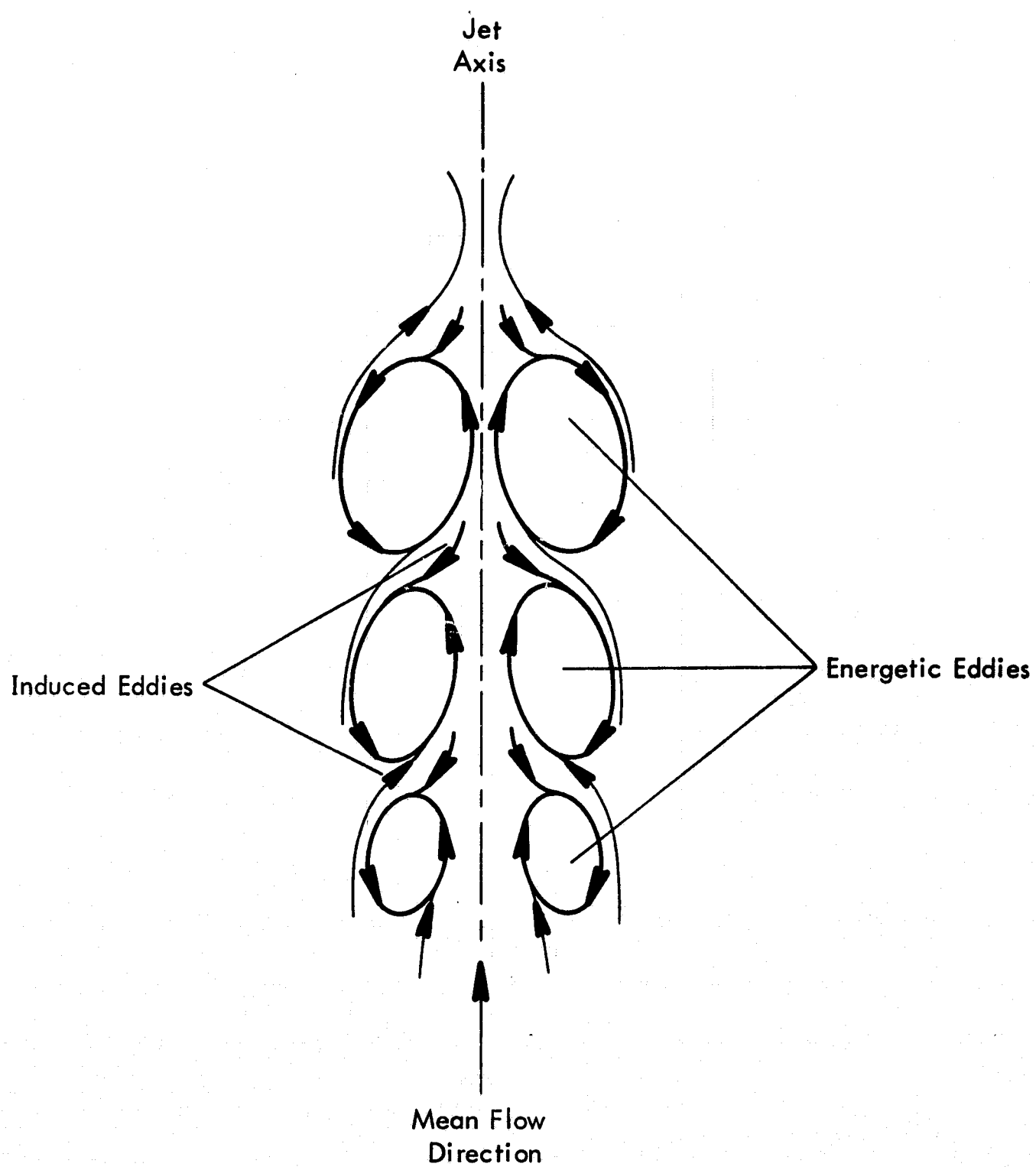


Figure 35. A Diametric Section of Eddy Structure Beside the Potential Core of a Subsonic Jet. This structure is suggested by the data of Figures 28 through 31 and visualization photographs shown in Figures 36 through 39.



Figure 36. Visualization of a Submerged Liquid Jet Using Flow Birefringence

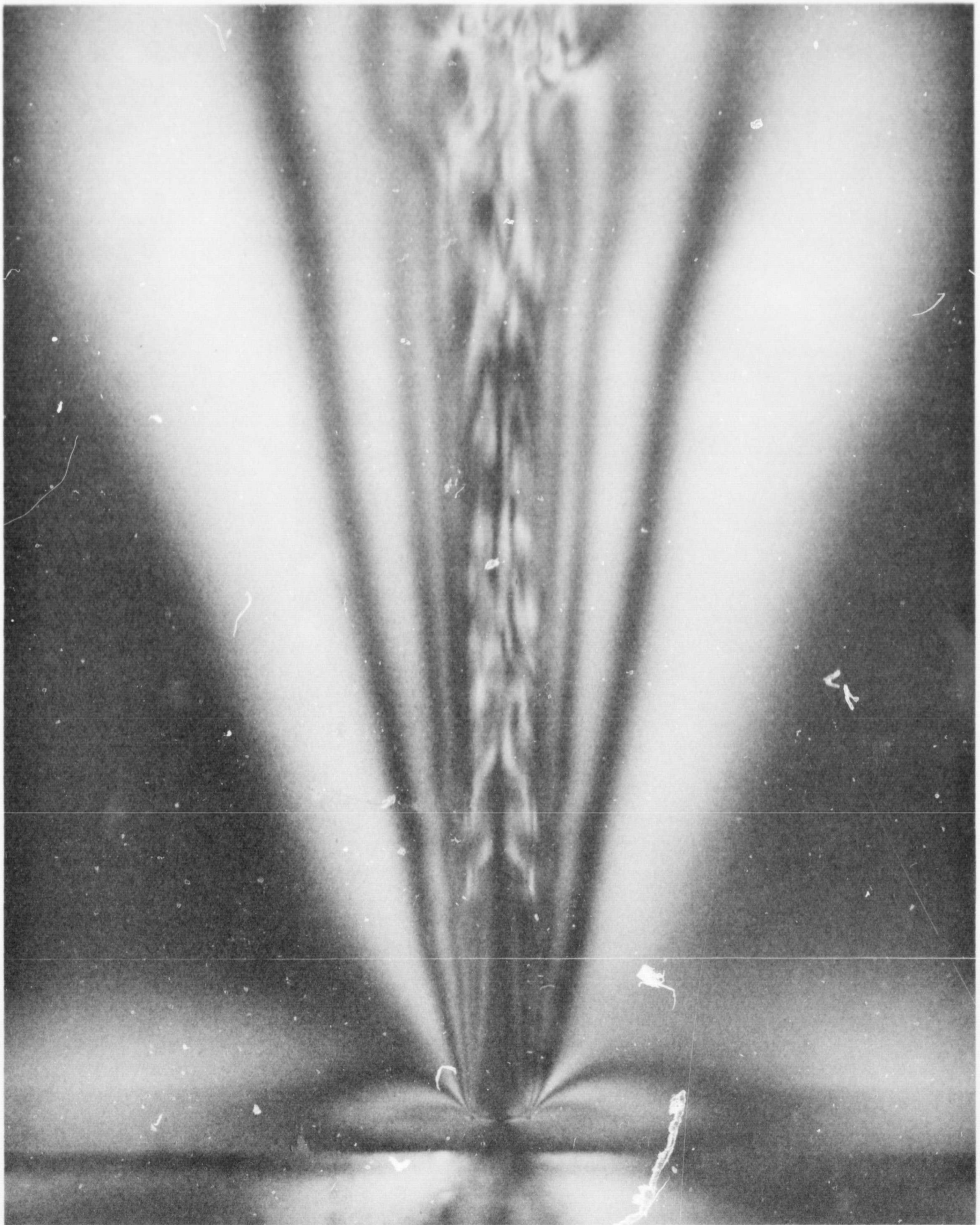


Figure 37. Visualization of a Submerged Liquid Jet Using Flow Birefringence

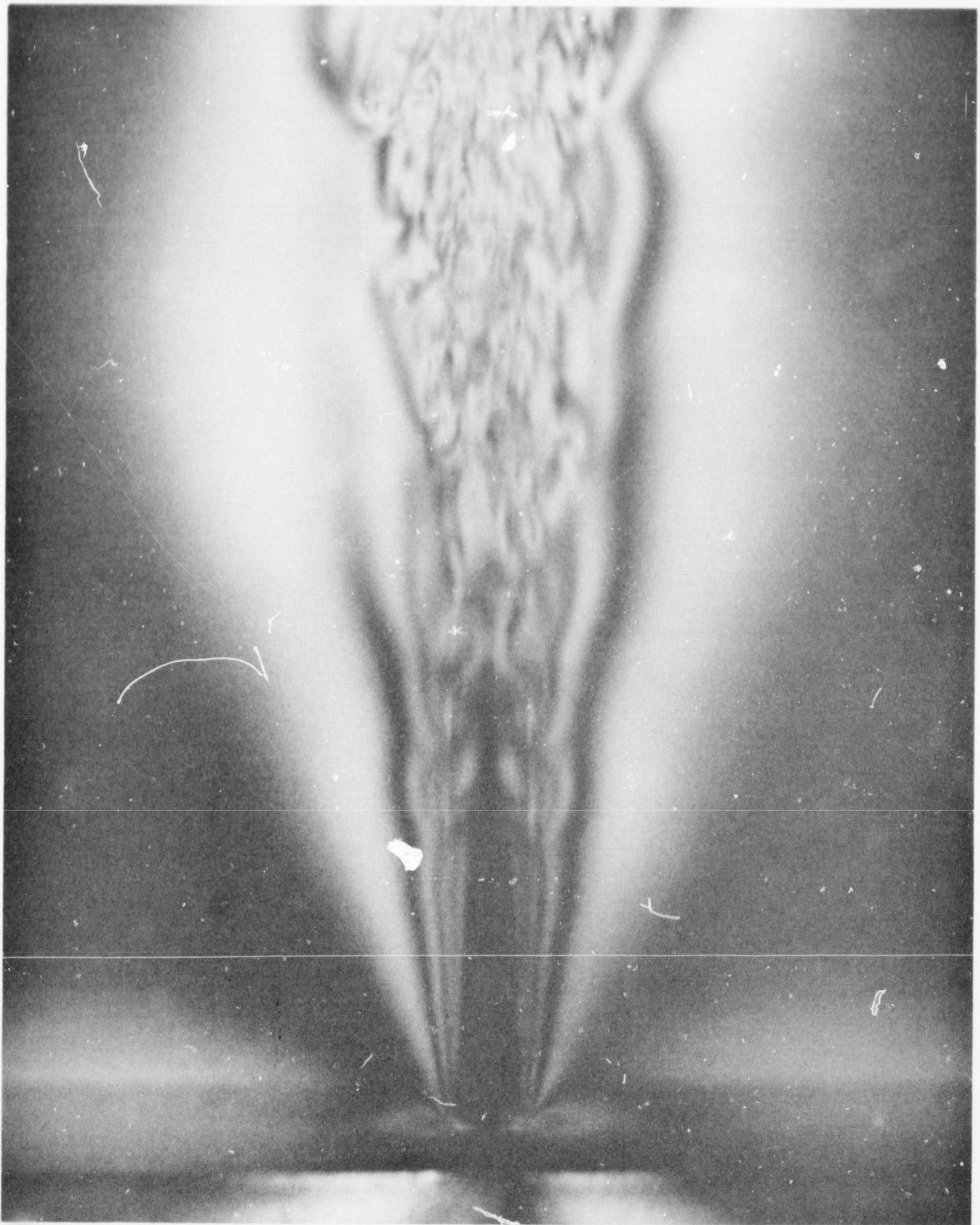


Figure 38. Visualization of a Submerged Liquid Jet Using Flow Birefringence

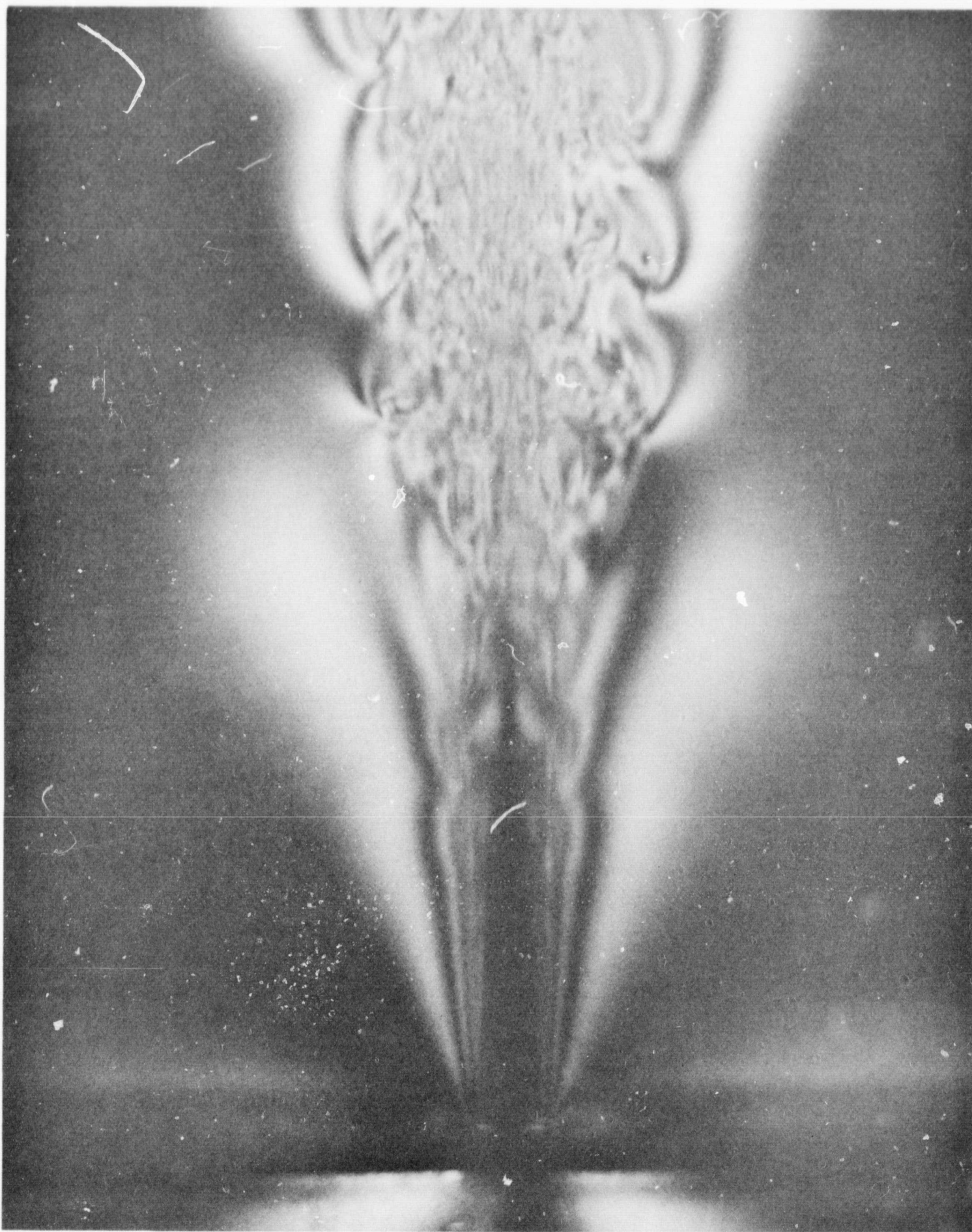


Figure 39. Visualization of a Submerged Liquid Jet Using Flow Birefringence

APPENDIX A

A REVIEW OF ELEMENTARY THEORY RELATING TO THE OPERATION OF HOT-WIRE ANEMOMETERS

The heat loss characteristics of a hot-wire anemometer can be obtained by identifying and combining additively the power sources and sinks. This is equivalent to applying the second law of thermodynamics to the solution of the problem.

If E is the internal energy of the wire, Q is the heat added to the wire and $i^2 R$ is the electrical power used in heating the wire, then

$$\frac{dE}{dt} = M C_v \frac{d\theta}{dt} = i^2 R - \left(\frac{dQ}{dt} \right)_{\text{surroundings}} - \left(\frac{dQ}{dt} \right)_{\text{supports}} \quad (\text{A1})$$

where

$$M = \frac{\rho_w \pi^2 d_w}{4} l_w$$

The constants ρ_w , d_w and l_w are the density, diameter and length of the wire, respectively.

Neglecting conduction to the sensor supports, which is a much better assumption for the case of a hot film sensor than for a hot wire of the same size, Equation A1 reduces to the expression

$$\rho_w \frac{\pi^2 d_w}{4} l_w C_v \frac{dT}{dt} = i^2 R - \left(\frac{dQ}{dt} \right)_{\text{surroundings}} \quad (\text{A2})$$

In order to proceed further, an assumption is needed concerning the rate of heat loss to the environment surrounding the wires. Radiation is generally overlooked as a minor factor at usual operating temperatures and subsonic velocities. The dominant heat loss mechanism over a wide range of Reynolds number appears to be forced convection. As discussed in Appendix B, free convection becomes of importance at low Reynolds number. The above Reynolds number which free convection may be ignored can be estimated by the equation

$$Re = (Gr)^{1/3} \quad (\text{A3})$$

where Gr is the Grashof number as defined in Appendix B.

Newton's cooling law can be used to express the rate of energy loss due to forced convection.

$$\left(\frac{dQ}{dt}\right)_{\text{convection}} = H (\pi d_w l_w) (\theta_w - \theta_a) \quad (\text{A4})$$

where θ_a is the equilibrium wire temperature, or the temperature which the unheated wire would have in the flow; and θ_w is the temperature of the heated wire. The parameter H is called the film coefficient. It may be evaluated by use of a relationship of the type

$$Nu = f(Re, Pr), \quad (\text{A5})$$

where

$$Nu = \frac{H d_w}{\lambda} \quad \text{is the Nusselt number}$$

$$Re = \frac{U d_w}{\nu} \quad \text{is the Reynolds number, and}$$

$$Pr = \frac{C_p \mu}{\lambda} \quad \text{is the Prandtl number.}$$

The parameters μ , λ , ν , C_p and U are the viscosity, thermal conductivity, kinematic viscosity, specific heat at constant pressure and flow velocity, respectively.

In evaluating Nu , Re and Pr there is frequently discussion concerning the temperature at which values should be taken. A conventional approach is to use a film temperature, θ_f , defined by

$$\theta_f \equiv \frac{\theta_w + \theta_a}{2} \quad (\text{A6})$$

A form of the heat loss relationship assumed for this report is developed for a constant film temperature in Appendix B. The final form is

$$Nu = A' + B' Re^h \quad (\text{A7})$$

where A' , B' and h are constants. According to Kramers' evaluation of Ulsamer's data for air (see Appendix B), this expression is valid for constant film temperature over a range of Reynolds number given by

$$9.5 < Re(\theta_f) < 1400 \quad (A8)$$

Substitution of Equation A7 into A4 produces an expression of the form

$$\frac{\sigma E^2}{R} = \pi \ell \lambda \left[A' + B' Re^h \right] (\theta_w - \theta_a) \quad (A9)$$

where σ indicates the need for a constant to relate units of power in the MKS system to units of power in the English system. The parameter E is the voltage across the sensor and R is the sensor resistance.

A more convenient operational form of Equation A9 may now be obtained by assuming constant temperature sensor operation, and regrouping constants. That expression is

$$E^2 = A + B U^h . \quad (A10)$$

APPENDIX B

HEAT LOSS RELATIONSHIPS FOR INFINITE WIRES NORMAL TO THE MEAN FLOW DIRECTION

Empirical Investigations of the Relationship Between Nusselt and Reynolds Numbers

In 1932 Ulsamer (Reference B1) observed that, in the case of an infinite cylinder of circular cross section,

$$Nu = 0.91 (Pr)^{0.31} (Re)^{0.385} \quad \text{for} \quad 0.1 < Re < 50 \quad (B1)$$

and

$$Nu = 0.60 (Pr)^{0.31} (Re)^{0.50} \quad \text{for} \quad 50 < Re < 10,000$$

where Nu , Pr and Re are calculated at the "film temperature," θ_f .

More recently, a similar trend has been noted by Collis and Williams (Reference B2), who also use the film temperature and choose a general expression of the form

$$Nu \left(\frac{\theta_f}{\theta_\infty} \right)^{0.17} = A' + B' Re^h, \quad (B2)$$

where

	$0.02 < Re < 44$	$44 < Re < 140$
h	0.45	0.51
A'	0.24	0
B'	0.56	0.48

The parameter θ_∞ is the free stream temperature.

Davies and Fisher (Reference B3), however, find the factor $(\theta_f/\theta_\infty)^{-0.17}$ unnecessary provided that the fluid properties are evaluated for conditions at the surface of the cylinder, in the absence of heat transfer. They propose relations of the form

$$Nu = \frac{1.4 Pr}{\gamma \pi} Re^{0.50}, \quad \text{for} \quad 40 < Re < 1000$$

and

$$Nu = \frac{2.6 Pr}{\gamma \pi} Re^{0.33}, \quad \text{for } 0 < Re < 50, \quad (B3)$$

where the aforementioned temperature convention is employed. These relations were developed from skin friction measurements, for cylinders, made by Thom (Reference B4), Tritton (Reference B5), Relf (Reference B6) and Lamb (Reference B7).

Equations B1, B2 and B3, obtained in three independent investigations confirm a transition in the nature of the flow around a cylinder for $Re(\theta_\infty) \cong 40$. This transition is attributed by Collis and Williams to the development or decay of a vortex street in the wake of the cylinder. A second, and less well defined, discontinuity in the power law relationships which define the heat transfer process has been noted by Hilpert (Reference B8) to occur for $Re(\theta_f) \cong 4$. Collis and Williams attribute this effect to a more gradual process involving the possible formation of standing vortices.

Kramers (Reference B9) and Davies and Fisher seem to indicate that the partitioning of the heat transfer relationship in the manner of Equations B1, B2 and B3 is somewhat artificial. In fact Kramers finds that an expression of the form

$$Nu = 0.42 (Pr)^{0.20} + 0.57 (Pr)^{0.33} (Re)^{0.50} \quad (B4)$$

is a fair approximation for air when $9.5 < Re(\theta_f) < 1400$. This conclusion is based upon careful examination of the data of Ulsamer. It thus seems justifiable in working with instrumentation and flows bridging the transition region of $Re \cong 40$ to assume, for constant film temperature, a relationship of the form

$$Nu = A' + B' Re^h \quad (B5)$$

where A' , B' and h are constant values which may be determined from calibration data.

There is considerable scatter in the Nusselt number measurements of various observers. Davies and Fisher conclude that this scatter results from experimental error in the estimation of the surface temperature of a cylinder. They state that the reason for this error has been the use of the sensor, itself, as a resistance thermometer.

Champagne, Sleicher and Wehrman (Reference B10) have recently used an infrared optical technique to determine the surface temperature of a platinum hot wire. This technique was sensitive enough to provide a good indication of the temperature distribution along the wire. It may, therefore, be a possible method of improving the quality of Nusselt number data.

This method relied on a calibration which was obtained by focusing the infrared detector upon a platinum sheet. The temperature of the sheet was determined by a platinum and platinum-rhodium thermocouple, which was spot welded to the sheet. The validity of this method therefore depended upon the assumption that the emissivities of the wires and the sheet were the same. Flow contamination by dust particles was prevented by an electrostatic precipitator. This device assured that there was no change in emissivity due to particles adhering to a wire. It also prevented damage or alteration of the wires due to particle collisions.

Mixed Free and Forced Convection Effects Upon Hot Wire Measurements

At the lower end of the Reynolds number range, free convection effects begin to predominate. Collis and Williams (Reference B2) have formulated some tentative conclusions regarding the Reynolds number at which free convection effects become significant for a given sensor size and temperature in a given fluid environment.

In the region in which free convection effects begin to influence a hot wire, the heat loss equation may be considered to also be dependent upon Grashof number.

$$G \equiv \frac{g d^3 (\theta_w - \theta_\infty)}{v^2 \theta_\infty} \quad (B6)$$

where g is the acceleration of gravity and the other parameters are defined in Appendix A. Data of Collis and Williams indicates that for increasing Reynolds numbers, the effect of buoyancy on forced convection rapidly becomes negligible. A very approximate rule which they derive indicates that buoyancy effects are small provided that

$$Re(\theta_\infty) > Gr(\theta_\infty)^{1/3} \quad (B7)$$

where (θ_∞) indicates a functional dependence upon the free stream temperature.

For a more limited range of Reynolds numbers, that is, for $Re < 0.1$ a more accurate criterion was derived. That is,

$$Re(\theta_\infty) = 1.85 Gr(\theta_\infty)^{0.39} \left[\frac{\theta_f}{\theta_\infty} \right]^{0.76} \quad (B8)$$

It follows that the lowest unambiguous velocity which may be measured by a hot wire of large aspect ratio (l/d) is given by

$$V_{\min} \cong \text{const.} \times d^{1/5} (\theta_w - \theta_\infty)^{2/5} \left(\frac{\theta_f}{\theta_\infty} \right)^{3/4} \quad (\text{B9})$$

Apparently this minimum velocity increases with the increase in θ_f/θ_∞ and is weakly dependent upon wire diameter.

APPENDIX B

REFERENCES

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APPENDIX C

EQUATIONS GOVERNING PARTICULAR SENSOR ARRAYS

The basic assumption, here, is that discussed in Sections 4.2 and 4.4. That is, the cylindrical sensors are only sensitive to velocity fluctuations which occur in a plane defined by the axis of the cylinder and the mean flow direction. Also, the sensor is only sensitive to the velocity component normal to the cylinder.

Two types of sensor configurations shall be considered. The first case is that of two sensor axes inclined in opposite directions to the mean flow direction and forming planes with the mean velocity vector which are mutually parallel. The separation of these two planes is very small compared with the scale of the measured velocity fluctuation. The second configuration consists of three mutually perpendicular sensors arranged so that the vertex of the array points in the mean flow direction and the three sensors make equal angles with the mean velocity vector.

Determining Velocity Components Using Two Crossed Wires

Figure C1 illustrates the angles and velocity components for the two crossed wires. The wires are separated for clarity.

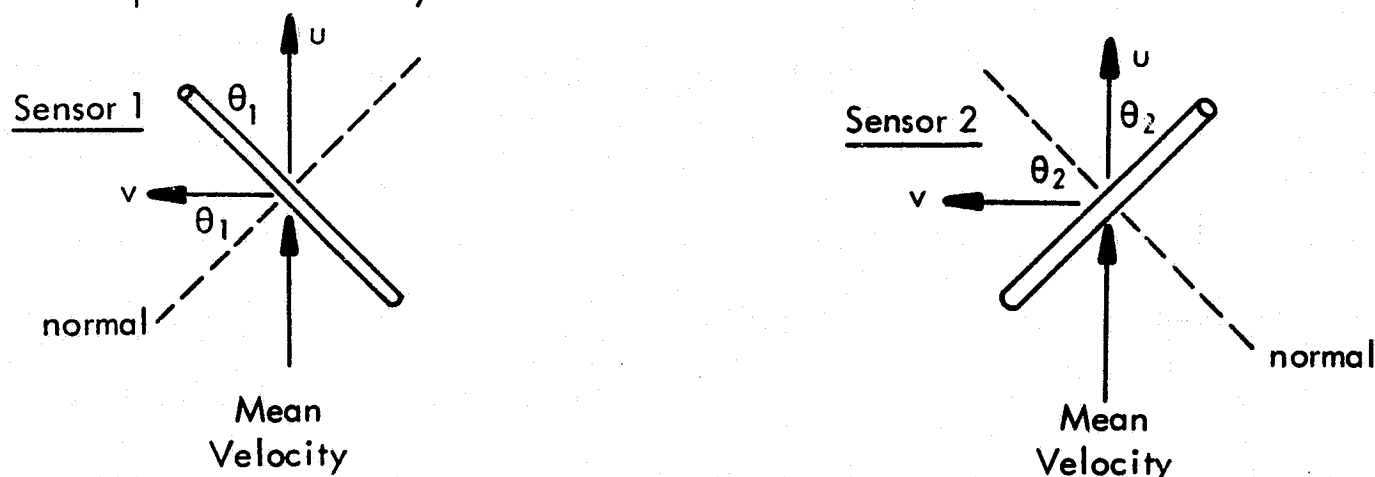


Figure C1

It is apparent from Equation 7 that

$$\frac{e_1}{S_1} = u \sin \theta_1 - v \cos \theta_1 \quad (C1)$$

$$\frac{e_2}{S_2} = u \sin \theta_2 + v \cos \theta_2 \quad (C2)$$

where S is defined for a particular sensor by

$$S \equiv \frac{\partial(\bar{E} - E_0)}{\partial(\bar{U} \sin \theta)} = \frac{h B^{1/h} (\bar{E}^2 - E_0^2)^{h-1/h}}{2 \bar{E}} \quad (C3)$$

Equations C1 and C2 may now be solved for u and v in terms of the e_1/S_1 and e_2/S_2 . The resulting expressions are

$$u = \frac{\frac{e_1}{S_1} \cos \theta_2 + \frac{e_2}{S_2} \cos \theta_1}{\sin(\theta_1 + \theta_2)} \quad (C4)$$

$$v = \frac{\frac{e_2}{S_2} \sin \theta_1 - \frac{e_1}{S_1} \sin \theta_2}{\sin(\theta_1 + \theta_2)} \quad (C5)$$

Of course, identical reasoning applies to the use of crossed wires in measuring the u and w components of velocity.

The Measurement of Velocity Components Using a Three-Sensor Configuration

If the geometry of a three-wire probe is that shown in Figure C2, with the vertex of the three mutually perpendicular sensors directed out of the plane of the page and the mean velocity vector directed inward, it is not difficult to show that the angle between each sensor axis and the mean velocity vector is about 35.3 degrees. This angle shall subsequently be designated by ϕ .

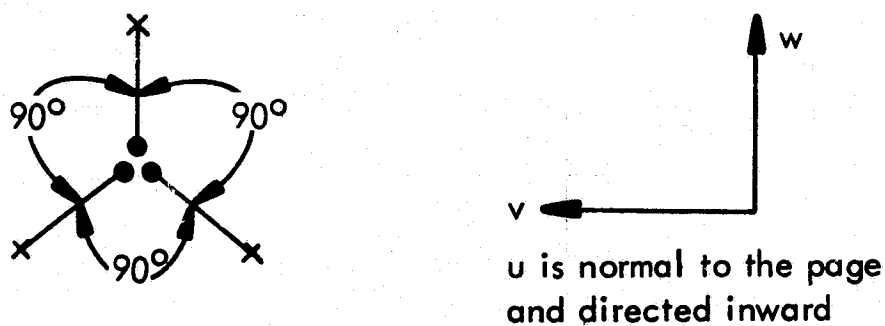


Figure C2

There are therefore three simultaneous equations which may be solved for u , v and w . These equations are:

$$\frac{e_1}{S_1} = Au + 0 - Bw$$

$$\frac{e_2}{S_2} = Au + CAv + BDw$$

$$\frac{e_3}{S_3} = Au - CAv + BDw$$

where $A = \sin \phi$, $B = \cos \phi$, $C = \cos 30^\circ$, and $D = \sin 30^\circ$.

Thus,

$$u = \frac{\begin{vmatrix} e_1/S_1 & 0 & -B \\ e_2/S_2 & CA & BD \\ e_3/S_3 & -CA & BD \end{vmatrix}}{\Delta}$$

$$v = \frac{\begin{vmatrix} A & e_1/S_1 & -B \\ A & e_2/S_2 & BD \\ A & e_3/S_3 & BD \end{vmatrix}}{\Delta}$$

$$w = \frac{\begin{vmatrix} A & 0 & e_1/S_1 \\ A & CA & e_2/S_2 \\ A & -CA & e_3/S_3 \end{vmatrix}}{\Delta}$$

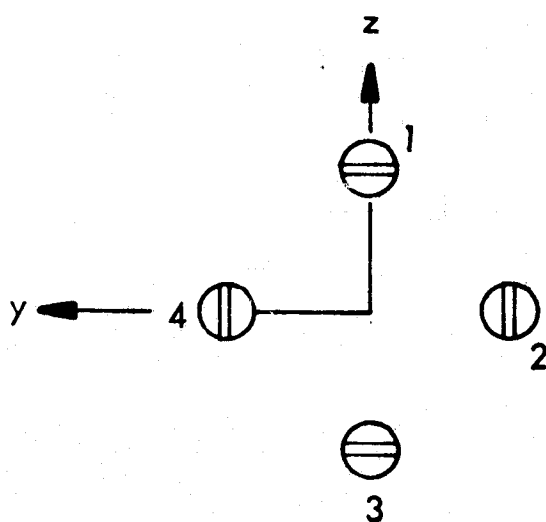
where

$$\Delta = \begin{vmatrix} A & 0 & -B \\ A & CA & BD \\ A & -CA & BD \end{vmatrix}$$

APPENDIX D

CALCULATION OF VORTICITY VECTOR

The probe arrangement of four cross-wire pairs in a square allows the vorticity vector for the flow to be calculated. The computer program outputs the four values of the u velocity fluctuations and the two values of the v and w velocity fluctuations at the four corners of the array. With the wires arranged as shown in the sketch below, and the velocity components numbered according to the wires, the vorticity vector can be determined.



Sketch of wires and probes looking down-stream onto the array; x is normal to the page and directed inward.

For example the velocity component w_2 is the component measured by probe 2, in the direction z .

The vorticity vector $\underline{\Gamma}$ is defined by

$$\begin{aligned} \underline{\Gamma} &= (\xi, \eta, \zeta) \\ \text{where } \xi &= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \\ \eta &= \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \\ \zeta &= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \end{aligned} \quad (D1)$$

For the evaluation we will use

$$\xi = \frac{w_4 - w_2}{\Delta y} - \frac{(v_1 - v_3)}{\Delta z}$$

$$\begin{aligned}
 \eta &= \frac{(u_1 - u_3)}{\Delta z} - \frac{1}{2U} \left(\frac{dw_2}{dt} + \frac{dw_4}{dt} \right) \\
 \xi &= \frac{1}{2U} \left(\frac{dv_1}{dt} + \frac{dv_3}{dt} \right) - \frac{(u_4 - u_2)}{\Delta y}
 \end{aligned}
 \tag{D2}$$

to give the required vorticity components at each instant of time.

Δy and Δz , the separations of the probes, and U , the mean velocity, will be input constants to the program, and obtained directly from measurements of the probe and the mean velocity calibration.

dw/dt may be evaluated by any desired differentiation technique of the digital data.

The method of Equation (D2) is based on the assumption of Taylor's Hypothesis to determine the rate of change of the velocity components in the x direction, the direction of the mean flow. Because of the problem of probe interference, it will not be possible to arrange a series of wires downstream of the array to measure the space rate of change of the velocity components. It is assumed that the turbulence is convected unchanged over the short distance of separation in the x axis equal to the probe separation in the other two axes. Because the time for the flow pattern to be swept this distance downstream is very short, the turbulence pattern will only change slightly in the small time inferred. The validity of this approach has been confirmed in several experimental investigations, notably that of Taylor himself (Reference D1), in his study of the relationship between correlation and spectra.

APPENDIX D

REFERENCES

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