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THEORETICAL TRANSFER FUNCTIONS
FOR THE SATURN V

by
Martin T. Solfer, Ph.D.
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ABSTRACT

Transfer functions describing the yaw-plane response of the Saturn V vehicle when subjected to wind penetration during its boost phase are developed. It is assumed that the vehicle is rigid in the longitudinal direction. Bending, propellant slosh, and rigid body motions are accounted for in the lateral direction. It is further assumed that the deformations are small, and that the yaw-plane motion is uncoupled from the pitch-plane motion.

The equations of motion are obtained from the system's Lagrangian, in conjunction with a dissipation function. These equations are then combined with the control network equations, and the desired solution is obtained in the complex plane by invoking the Laplace transform.

A computer program was developed, and the desired transfer functions were evaluated for four flight configurations during the boost phase. The results are presented in the form of amplitude and phase angle transfer functions; twenty-one of each, for each flight time, as functions of frequency.

A comparison between the theoretical results obtained and flight test data is presented.

TABLE OF CONTENTS

	<u>Page No.</u>
I. INTRODUCTION	1
II. DERIVATION OF EQUATIONS	3
1. Equations of Motion	3
2. Generalized Forces and Moments	10
3. Control Equations	14
III. METHOD OF SOLUTION	16
IV. NUMERICAL INPUT	20
V. DISCUSSION OF RESULTS AND COMPARISON WITH FLIGHT TEST DATA	23
VI. REFERENCES	25
APPENDIX A	26
APPENDIX B	33
FIGURES	

I. INTRODUCTION

The purpose of this study is the determination of the yaw-plane response of the Saturn V vehicle when subjected to wind penetration during the boost phase.

Since the vehicle is symmetric about its longitudinal axis, it is assumed that the yaw-plane and pitch-plane motions are uncoupled. It is also assumed, in accordance with the scope of work associated with this project, that the vehicle is rigid in the longitudinal direction. It is conceivable that adaption of the latter assumption may render erroneous results for some flight configurations. Thrust perturbations may result in a "Pogo" type behavior in the longitudinal direction which could subsequently couple with the lateral response. Further investigations would be necessary to clarify this issue. However, for the analysis presented here, the assumption of longitudinal rigidity is maintained.

The linear ordinary differential equations describing the yaw-plane vehicle response, are derived from the system's Lagrangian, in conjunction with a dissipation function. The Lagrangian is obtained by allowing for bending deformation, propellant slosh, and rigid body motion in the lateral direction. The dissipation function is an artificial function introduced to account for structural and slosh damping. The generalized forces and moments acting on the vehicle are obtained from the thrust and aerodynamic loading; assumed "quasi-steady" in nature. Wind penetration, in the form of a sinusoidal spacial distribution, is also accounted for. The equations of motion, and expressions for the generalized forces and moments, are derived in Sections II-1, and II-2, respectively.

Since the Saturn V vehicle contains a closed loop autopilot control network, the equations derived in Sections II-1 and II-2 are not self-contained. To complete the formulation, it is necessary to include the behavior of the control network whose function is to maintain stabilized flight. The equations describing the control network are presented in Section II-3.

The desired vehicle response is amplitude and phase angle transfer functions relating response parameters to the wind penetration input as a function of frequency. These transfer functions are most readily obtained, from the equations developed in Section II, by invoking the Laplace transform. Section III is devoted to this aspect of the analysis. Additional response parameters of interest are also presented in this section, as well as the technique employed to eliminate the unknown phase angle associated with the wind penetration input.

Solution of the system of simultaneous linear algebraic equations presented in Section III, is obtained, in the complex plane, using the IBM 7094 computer in conjunction with the Stromberg-Carlson SC4020 plotter, for four flight times during the boost phase. The flight times selected were 61, 78, 97, and 120 seconds after S-IC lift-off. The computer program employed is not presented here. However, a flow chart describing the program is presented in Figure 4. The sources for the input data used are presented in Section IV.

The results of this analysis are presented in Figures 5a through 88b for the four flight times of interest. There are forty-two figures associated with each flight time; twenty-one amplitude and twenty-one phase angle transfer functions. Each figure contains two curves, one corresponding to maximum slosh damping, the other to minimum slosh damping. A discussion of the results obtained, and a comparison with flight test data is presented in Section V.

The work presented here is essentially an outgrowth of a wind penetration effects study presented by J. G. Papadopoulos⁽¹⁾.

II. DERIVATION OF EQUATIONS

1. Equations of Motion

The linear ordinary differential equations describing the yaw-plane vehicle response, are derived from the system's Lagrangian in conjunction with a dissipation function. The vehicle is assumed rigid in the longitudinal direction. Bending deformation, propellant slosh, and rigid body motion are accounted for in the lateral direction.

The deformed longitudinal axis of the vehicle, and the coordinate systems employed are shown in Figure 1. The (x, y) coordinate system is fixed in space with its origin located at the instantaneous vehicle c.g. prior to the onset of yaw-plane perturbations. The $(\bar{x}, \bar{y})$ coordinate system is also fixed in space, but with its origin located at the instantaneous vehicle c.g. in its perturbed configuration. The $(\bar{x}, \bar{y})$ system is obtainable from the (x, y) system through pure translations. The (ξ, η) coordinate system is fixed in the vehicle, with its origin located at the vehicle c.g., and the ξ - axis oriented parallel to the undeformed longitudinal axis of the vehicle. The (ξ, η) system is obtainable from the $(\bar{x}, \bar{y})$ system through a pure rotation.

The displacement, $y(x, t)$, of any point on the deformed vehicle axis, with respect to the (x, y) coordinate system, is given by

$$y(x, t) = y_{c.g.} + \bar{x} \tan \theta + \eta(\xi, t) / \cos \theta \quad (1)$$

and the local slope is given by

$$\frac{\partial y(x, t)}{\partial x} = \left(\frac{\partial \bar{x}}{\partial x} \right) \tan \theta + \frac{\partial \eta(\xi, t)}{\partial \xi} / \cos \theta \quad (2)$$

where θ denotes the rigid body rotation as indicated in Figure 1, and $\eta(\xi, t)$ denotes the lateral bending displacement of the vehicle measured normal to the ξ - axis.

Restricting the analysis to small rotations, it is justified to assume that

$$\sin \theta \approx \theta \quad \text{and} \quad \cos \theta \approx 1.$$

Restricting the analysis to small displacements, and noting that

$$x = \bar{x} + x_{c.g.}$$

and that $\tan \theta \approx \theta$, for θ small, equations (1) and (2) may be rewritten in the form

$$y(\bar{x}, t) = y_{c.g.} + \bar{x} \theta(t) + \eta(\bar{x}, t), \quad (3)$$

and

$$y'(\bar{x}, t) = \theta(t) + \eta'(\bar{x}, t), \quad (4)$$

where prime denotes differentiation with respect to the spacial argument, $\bar{x}$.

The quantity $\eta(\bar{x}, t)$ denotes the lateral bending deformation of the vehicle axis, which, under the assumption of linear elasticity, may be written in the form

$$\eta(\bar{x}, t) = \sum_{i=1}^n \varphi_i(\bar{x}) q_i(t), \quad (5)$$

where $\varphi_i(\bar{x})$ denotes the i th bending mode shape, and $q_i(t)$ denotes the associated i th generalized coordinate. The φ_i 's are known quantities; obtained as eigenfunctions of a free vibration analysis. The q_i 's are unknown, and are obtained as a result of this analysis. The index n denotes the number of modes considered.

Substituting equation (5) into equations (3) and (4) yields

$$y(\bar{x}, t) = y_{c.g.} + \bar{x} \theta(t) + \sum_{i=1}^n \varphi_i(\bar{x}) q_i(t), \quad (6)$$

and

$$y'(\bar{x}, t) = \theta(t) + \sum_{i=1}^n \varphi_i'(\bar{x}) q_i(t) \quad (7)$$

The system's Lagrangian is defined as

$$L = T - V,$$

where T denotes the total kinetic energy of the system, and V denotes total strain energy stored in the system during deformation.

Determination of T

The total kinetic energy, T , is composed of the kinetic energy of the dry vehicle less engines, T_M , the kinetic energy of the sloshing propellants, T_S , and the kinetic energy of the four swivel and one stationary engine, T_E . Hence, T may be written in the form,

$$T = T_M + T_S + T_E \quad (9)$$

with

$$T_E = (4 T_E^{sw} + T_E^{st}) \quad (10)$$

where T_E^{sw} and T_E^{st} denote respectively, the kinetic energies of a swivel and stationary engine.

From Figure 1, and equations (6) and (7), it is found that

$$\begin{aligned} T_M = & \frac{1}{2} \int_{\ell} \rho(\bar{x}) \left[\dot{y}_{c.g.} + \bar{x} \dot{\theta} + \sum_{i=1}^n \varphi_i \dot{q}_i \right]^2 d\bar{x} + \\ & + \frac{1}{2} \int_{\ell} I(\bar{x}) \left[\dot{\theta} + \sum_{i=1}^n \varphi_i' \dot{q}_i \right]^2 d\bar{x} \quad (11) \end{aligned}$$

where $\rho(\bar{x})$ and $I(\bar{x})$ denote respectively, the mass per unit length and the mass moment of inertia per unit length of the dry vehicle without engines. The index ℓ on the integration symbol denotes integration over the entire vehicle length. The dot over a variable denotes differentiation with respect to time.

A slosh model for the j th propellant tank is given in reference (2), and is presented here in Figure 2. The model consists of a slosh mass, m_{s_j} , connected to a rigid tank through the linear spring of stiffness, k_{s_j} , and dash pot with damping coefficient c_{s_j} , and is located a distance h_{s_j} above the j th propellant c.g. The nonsloshing portion of the j th propellant is represented by the stationary mass, m_{o_j} , of inertia I_{o_j} , and is located a distance h_{o_j} below the j th propellant c.g.

Equations (6) and (7), with $\bar{x} = \bar{x}_{o_j}$ (vector parallel to $\bar{x}$ - axis from vehicle c.g. to m_{o_j}), give respectively, the displacement and rotation of m_{o_j} with respect to the inertial (x, y) reference frame. The displacement of the slosh mass m_{s_j} is obtained by setting $\bar{x} = \bar{x}_{s_j}$ (vector parallel to $\bar{x}$ - axis from vehicle c.g. to m_{s_j}) in equation (6), and adding the component of the slosh displacement in the y direction, $\xi_{s_j} \cos \theta \approx \xi_{s_j}$. Hence, the kinetic energy of k propellants is

$$\begin{aligned} T_s = & \frac{1}{2} \sum_{j=1}^k m_{o_j} \left[\dot{y}_{c.g.} + \bar{x}_{o_j} \dot{\theta} + \sum_{i=1}^n \varphi_i(\bar{x}_{o_j}) \dot{q}_i \right]^2 + \\ & + \frac{1}{2} \sum_{j=1}^k I_{o_j} \left[\dot{\theta} + \sum_{i=1}^n \varphi'_i(\bar{x}_{o_j}) \dot{q}_i \right]^2 + \\ & + \frac{1}{2} \sum_{j=1}^k m_{s_j} \left[\dot{y}_{c.g.} + \bar{x}_{s_j} \dot{\theta} + \sum_{i=1}^n \varphi_i(\bar{x}_{s_j}) \dot{q}_i + \dot{\xi}_{s_j} \right]^2 . \end{aligned} \quad (12)$$

The geometry associated with a swivel engine is shown in Figure 3. The angle $\bar{\psi}$ denotes the engine rotation with respect to a line parallel to the $\bar{x}$ - axis, whereas, the angle β denotes the net engine rotation. The quantity $\bar{x}_E$ denotes the distance (absolute value) from the vehicle c.g. to the engine swivel point, measured parallel to the $\bar{x}$ - axis. The quantity l_E denotes the distance from the engine swivel point to the engine c.g. From Figure 3 and equation (6), the displacement of the swivel engine c.g., $y_E(t)$, with respect

to the (x, y) reference frame, is found to be

$$y_E = y_{c.g.} - \bar{x}_E \theta + \sum_{i=1}^n \varphi_i(\bar{x}_E) q_i - l_E \bar{\psi} \quad , \quad (13)$$

where

$$\bar{\psi} = \beta + \theta + \sum_{i=1}^n \varphi_i'(\bar{x}_E) q_i \quad . \quad (14)$$

Hence, the kinetic energy of a single swivel engine is given by

$$\begin{aligned} T_E^{sw} = & \frac{1}{2} m_E \left[\dot{y}_{c.g.} - (\bar{x}_E + l_E) \dot{\theta} - l_E \dot{\beta} + \right. \\ & \left. + \sum_{i=1}^n \left\{ \varphi_i(\bar{x}_E) - l_E \varphi_i'(\bar{x}_E) \right\} \dot{q}_i \right]^2 + \\ & + \frac{1}{2} I_E \left[\dot{\beta} + \dot{\theta} + \sum_{i=1}^n \varphi_i'(\bar{x}_E) \dot{q}_i \right]^2 \quad , \quad (15) \end{aligned}$$

where m_E and I_E denote respectively, engine mass and inertia about its own c.g. The kinetic energy of the stationary engine is obtainable from the kinetic energy of a swivel engine by setting $\dot{\beta} = 0$. That is

$$T_E^{st} = T_E^{sw} \Big|_{\dot{\beta}=0} \quad . \quad (16)$$

Substituting equations (11), (12), (15) and (16) into equation (9), and noting equation (10), yields the expression for the total kinetic energy, T , of the system under consideration.

Determination of V

The total strain energy, V , stored in the system during deformation may be written in the form

$$V = V_b + V_s \quad , \quad (17)$$

where V_b denotes the strain energy due to bending, and V_s the strain energy associated with propellant slosh.

The strain energy stored in the i th bending mode, V_b^i , may be shown to be

$$V_b^i = \frac{1}{2} \omega_i^2 \mu_i q_i^2, \quad (18)$$

where ω_i and μ_i denote respectively, the i th mode frequency and generalized mass. Hence,

$$V_b = \sum_{i=1}^n V_b^i = \frac{1}{2} \sum_{i=1}^n \omega_i^2 \mu_i q_i^2. \quad (19)$$

Similarly, the strain energy stored in the k propellants is given by

$$V_s = \frac{1}{2} \sum_{j=1}^k \omega_{s_j}^2 m_{s_j} \xi_{s_j}^2, \quad (20)$$

where ω_{s_j} and m_{s_j} denote respectively, the frequency and mass of the j th propellant slosh model.

Substituting equations (19) and (20) into equation (17) yields the expression for the total strain energy stored in the system during deformation, and hence, establishes the expression for the Lagrangian, L , in accordance with equation (8).

The Lagrangian, as presented, represents a purely conservative system. In order to account for energy dissipation associated with structural and slosh damping, an artificial dissipation function, D , is introduced in the form

$$D = D_b + D_s, \quad (21)$$

where D_b and D_s denote respectively, dissipation functions associated with bending deformation and slosh motion, and may be written in the form

$$D_b = \sum_{i=1}^n \omega_i \mu_i \zeta_i \dot{q}_i^2, \quad (22)$$

and

$$D_s = \sum_{j=1}^k \omega_{s_j} m_{s_j} \zeta_{s_j} \dot{\xi}_{s_j}^2, \quad (23)$$

where ζ_1 and ζ_{s_j} denote respectively, damping ratios for the i th bending mode and j th propellant.

If Δ_1 represents the i th generalized coordinate for the system under consideration, then Lagrange's equations of motion, for L and D as given above, reduce to the form

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\Delta}_1} \right) + \frac{\partial D}{\partial \dot{\Delta}_1} + \frac{\partial V}{\partial \Delta_1} = Q_1, \quad (24)$$

where Q_1 denotes the generalized force associated with the i th generalized coordinate, Δ_1 .

The details of the differentiations indicated in equation (24) are presented in Appendix A. The results are presented here.

Translation Equation:

$$M \ddot{y}_{c.g.} + \sum_{j=1}^k m_{s_j} \ddot{\xi}_{s_j} - 4 m_E \ell_E \ddot{\beta} = F_y \quad (25)$$

Rotation Equation:

$$I \ddot{\theta} + \sum_{j=1}^k m_{s_j} \bar{x}_{s_j} \ddot{\xi}_{s_j} + 4 [m_E \ell_E (\bar{x}_E + \ell_E) + I_E] \ddot{\beta} = M_z \quad (26)$$

Bending Equations:

$$\begin{aligned} \mu_k \left\{ \ddot{q}_k + (2 \omega_k \zeta_k) \dot{q}_k + \omega_k^2 q_k \right\} + \sum_{j=1}^k m_{s_j} \varphi_k (\bar{x}_{s_j}) \ddot{\xi}_{s_j} + \\ - 4 [m_E \ell_E \{ \varphi_k (\bar{x}_E) - \ell_E \varphi'_k (\bar{x}_E) \} - I_E \varphi'_k (\bar{x}_E)] \ddot{\beta} = F_{q_k} \end{aligned} \quad (27)$$

$$k = 1, 2, \dots, n$$

Slosh Equations:

$$\begin{aligned}
 m_{s_l} \left\{ \ddot{\xi}_{s_l} + \left(2 \omega_{s_l} \zeta_{s_l} \right) \dot{\xi}_{s_l} + \omega_{s_l}^2 \xi_{s_l} \right\} + m_{s_l} \ddot{y}_{c.g.} + \\
 + m_{s_l} \bar{x}_{s_l} \ddot{\theta} + m_{s_l} \sum_{i=1}^n \varphi_i \left(\bar{x}_{s_l} \right) \ddot{q}_i = F_{\xi_{s_l}}
 \end{aligned} \tag{28}$$

$$l = 1, 2, \dots, k$$

where M and I denote respectively, the total vehicle mass and inertia about its c.g. The quantities on the right hand side of equations (25) through (28) correspond to generalized forces and moments to be determined next.

2. Generalized Forces and Moments

The external forces and moments acting on the vehicle are thrust forces (stationary and controlled), and aerodynamic forces (including wind penetration). So that

$$\left. \begin{aligned}
 F_Y &= F_Y^T + F_Y^A \\
 M_{\bar{Z}} &= M_{\bar{Z}}^T + M_{\bar{Z}}^A \\
 F_{q_k} &= F_{q_k}^T + F_{q_k}^A
 \end{aligned} \right\} \tag{29}$$

where $()^T$ and $()^A$ denote respectively, generalized forces and/or moments due to thrust and aerodynamics. However, since there are no external forces applied directly to the k propellants,

$$F_{\xi_{s_l}} \equiv 0, \text{ for all } l. \tag{30}$$

Thrust Forces and Moments

The generalized thrust force in the y -direction, F_Y^T , is equal to the thrust component in that direction, which, for small deformations is given

by (see Figure 3)

$$F_Y^T \approx T_s \left[\theta + \sum_{i=1}^n \varphi_i'(\bar{x}_E) q_i \right] + T_c \left[\beta + \theta + \sum_{i=1}^n \varphi_i'(\bar{x}_E) q_i \right]. \quad (31)$$

The generalized thrust moment, M_Z^T , is equal to the net moment of the thrust forces about the $\bar{z}$ -axis (axis normal to the $\bar{x} - \bar{y}$ plane and emanating from the vehicle c.g.). For small deformations

$$\begin{aligned} M_Z^T \approx & -T_s \left[\sum_{i=1}^n \left\{ \varphi_i(\bar{x}_E) + \bar{x}_E \varphi_i'(\bar{x}_E) \right\} q_i \right] + \\ & -T_c \left[\sum_{i=1}^n \left\{ \varphi_i(\bar{x}_E) + \bar{x}_E \varphi_i'(\bar{x}_E) \right\} q_i + \bar{x}_E \beta \right]. \end{aligned} \quad (32)$$

The generalized bending force due to thrust is given by

$$F_{q_k}^T = F_Y^T \frac{\partial y_E}{\partial q_k}; \quad k = 1, 2, \dots, n \quad (33)$$

where F_Y^T is given in equation (31), and from equations (13) and (14)

$$\frac{\partial y_E}{\partial q_k} = \left[\varphi_k(\bar{x}_E) - \ell_E \varphi_k'(\bar{x}_E) \right]. \quad (34)$$

Aerodynamic Forces and Moments

Under the assumption of quasi-steady aerodynamics, the generalized aerodynamic forces and moments are functions of the local angle of attack only. That is,

$$F_Y^A = \bar{q} A \int_{\ell} \left(\frac{\partial c_{\ell}}{\partial \alpha} \right) \alpha(\bar{x}, t) d\bar{x}, \quad (35)$$

$$M_Z^A = \bar{q} A \int_{\ell} \left(\frac{\partial c_{\ell}}{\partial \alpha} \right) \bar{x} \alpha(\bar{x}, t) d\bar{x}, \quad (36)$$

and

$$F_{q_k}^A = \bar{q} A \int_{\ell} \left(\frac{\partial c_{\ell}}{\partial \alpha} \right) \varphi_k(\bar{x}) \alpha(\bar{x}, t) d\bar{x} , \quad (37)$$

where $\bar{q}$, A , c_{ℓ} , and α denote respectively, dynamic pressure, reference area, local normal force coefficient per unit length, and local angle of attack.

The local angle of attack, $\alpha(\bar{x}, t)$, is approximated, for small angles, by the sum of the local slope with respect to the freestream flow, V , and the slope effects due to transverse flow (wind penetration), $V_{w\bar{x}}$, and lateral vehicle motion, $\dot{y}(\bar{x}, t)$. That is,

$$\begin{aligned} \alpha(\bar{x}, t) \approx \frac{\partial y}{\partial \bar{x}} + \frac{1}{V} \left[V_{w\bar{x}} - \dot{y} \right] &= \theta + \sum_{i=1}^n \varphi_i'(\bar{x}) q_i + \\ &+ \frac{1}{V} \left[V_{w\bar{x}} - \dot{y}_{c.g.} - \bar{x} \dot{\theta} + \sum_{i=1}^n \varphi_i(\bar{x}) \dot{q}_i \right] . \end{aligned} \quad (38)$$

The wind penetration, $V_{w\bar{x}}$, is assumed to have a sinusoidal spacial distribution of frequency ω , phase angle ψ , and amplitude V_w , (see Ref. (1)). That is,

$$V_{w\bar{x}} = V_w \sin \left[\frac{\omega \bar{x}}{V} + \psi \right] . \quad (39)$$

For convenience, equation (39) will be used in the expanded form

$$V_{w\bar{x}} = V_w \left[\sin \psi \cos \left(\frac{\omega \bar{x}}{V} \right) + \cos \psi \sin \left(\frac{\omega \bar{x}}{V} \right) \right] , \quad (40)$$

Substituting equations (38) and (40) into equations (35) through (37) yields the generalized aerodynamic forces and moments as follows:

$$\begin{aligned}
F_Y^A = & \bar{q}A \left(\theta - \frac{\dot{Y}_{C.G.}}{V} \right) \int_{\ell} c_{\ell, \alpha} d\bar{x} - \frac{\bar{q}A \dot{\theta}}{V} \int_{\ell} c_{\ell, \alpha} \bar{x} d\bar{x} + \\
& + \bar{q}A \sum_{i=1}^n q_i \int_{\ell} c_{\ell, \alpha} \varphi_i'(\bar{x}) d\bar{x} - \frac{\bar{q}A}{V} \sum_{i=1}^n \dot{q}_i \int_{\ell} c_{\ell, \alpha} \varphi_i(\bar{x}) d\bar{x} + \\
& + \bar{q}A \frac{V_w}{V} \left\{ \cos \psi \int_{\ell} c_{\ell, \alpha} \sin \left(\frac{\omega \bar{x}}{V} \right) d\bar{x} + \sin \psi \int_{\ell} c_{\ell, \alpha} \cos \left(\frac{\omega \bar{x}}{V} \right) d\bar{x} \right\},
\end{aligned} \tag{41}$$

$$\begin{aligned}
M_Z^A = & \bar{q}A \left(\theta - \frac{\dot{Y}_{C.G.}}{V} \right) \int_{\ell} c_{\ell, \alpha} \bar{x} d\bar{x} - \frac{\bar{q}A \dot{\theta}}{V} \int_{\ell} c_{\ell, \alpha} \bar{x}^2 d\bar{x} + \\
& + \bar{q}A \sum_{i=1}^n q_i \int_{\ell} c_{\ell, \alpha} \bar{x} \varphi_i'(\bar{x}) d\bar{x} - \frac{\bar{q}A}{V} \sum_{i=1}^n \dot{q}_i \int_{\ell} c_{\ell, \alpha} \bar{x} \varphi_i(\bar{x}) d\bar{x} + \\
& + \bar{q}A \frac{V_w}{V} \left\{ \cos \psi \int_{\ell} c_{\ell, \alpha} \bar{x} \sin \left(\frac{\omega \bar{x}}{V} \right) d\bar{x} + \sin \psi \int_{\ell} c_{\ell, \alpha} (\bar{x}) \cos \left(\frac{\omega \bar{x}}{V} \right) d\bar{x} \right\},
\end{aligned} \tag{42}$$

and

$$\begin{aligned}
F_{q_k}^A = & \bar{q}A \left(\theta - \frac{\dot{Y}_{C.G.}}{V} \right) \int_{\ell} c_{\ell, \alpha} \varphi_k(\bar{x}) d\bar{x} - \frac{\bar{q}A \dot{\theta}}{V} \int_{\ell} c_{\ell, \alpha} \bar{x} \varphi_k(\bar{x}) d\bar{x} + \\
& + \bar{q}A \sum_{i=1}^n q_i \int_{\ell} c_{\ell, \alpha} \varphi_i'(\bar{x}) \varphi_k(\bar{x}) d\bar{x} - \frac{\bar{q}A}{V} \sum_{i=1}^n \dot{q}_i \int_{\ell} c_{\ell, \alpha} \varphi_i(\bar{x}) \varphi_k(\bar{x}) d\bar{x} + \\
& + \bar{q}A \frac{V_w}{V} \left\{ \cos \psi \int_{\ell} c_{\ell, \alpha} \varphi_k(\bar{x}) \sin \left(\frac{\omega \bar{x}}{V} \right) d\bar{x} + \right. \\
& \left. + \sin \psi \int_{\ell} c_{\ell, \alpha} \varphi_k(\bar{x}) \cos \left(\frac{\omega \bar{x}}{V} \right) d\bar{x} \right\}.
\end{aligned} \tag{43}$$

where,

$$c_{l, \alpha} \equiv \frac{\partial c_l}{\partial \alpha} \quad .$$

The equations of motion, equations (25) through (28), in conjunction with the defining equations for the generalized forces and moments (equations (31) through (34) and (41) through (43), constitute a set of $(n + k + 2)$ simultaneous ordinary linear differential equations in $(n + k + 3)$ unknowns, and, is consequently, unsolvable. In order to properly specify the problem, the swivel engine rotation, β , must be related to the vehicle deformation. This is achieved, in the vehicle, through a closed loop autopilot control network whose function is to maintain stabilized flight. The equations describing this network will be considered next.

3. Control Equations

The "chain of command" leading to the engine rotation β is, in reverse, as follows:

The swivel engines undergo the rotation β as a result of the engine actuator imposing a control rotation β_c . These rotations are related through the actuator transfer function G_A , by the equation

$$\beta = G_A \beta_c \quad . \quad (44)$$

The actuator, in turn, receives its command to impose the control rotation, β_c , from the control network, which, for the Saturn V, consists of an attitude gyro with transfer function G_θ and gain a_0 , and a rate gyro with transfer function $G_{\dot{\theta}}$ and gain a_1 , mounted on a stabilized platform. If θ_1 and $\dot{\theta}_1$ denote respectively, indicated attitude and rate, then β_c is related to θ_1 and $\dot{\theta}_1$ through the control equation

$$\beta_c = a_0 G_\theta \theta_1 + a_1 G_{\dot{\theta}} \dot{\theta}_1 \quad . \quad (45)$$

The indicated attitude and rate appearing in equation (45) are related to the vehicle deformation through equation (7), as follows:

$$\theta_1 = y'(\bar{x}, t) \Big|_{\bar{x} = \bar{x}_\theta} = \theta + \sum_{i=1}^n \varphi_i'(\bar{x}_\theta) q_i \quad (46)$$

and

$$\dot{\theta}_1 = \frac{\partial y'(\bar{x}, t)}{\partial t} \Big|_{\bar{x} = \bar{x}_\theta} \quad (47)$$

where $\bar{x}_\theta$ denotes the vectorial distance (measured parallel to the $\bar{x}$ -axis) from the vehicle c.g. to the stabilized platform. Combining equations (45), (46) and (47) with equation (44), results in the necessary additional equation relating β to the vehicle deformation, namely,

$$\begin{aligned} \beta = & a_0 G_\theta G_A \left[\theta + \sum_{i=1}^n \varphi_i'(\bar{x}_\theta) q_i \right] + \\ & + a_1 G_\theta G_A \left[\dot{\theta} + \sum_{i=1}^n \varphi_i'(\bar{x}_\theta) \dot{q}_i \right] . \end{aligned} \quad (48)$$

Hence, adding equation (48) to the equations of motion presented above results in a solvable set of $(n + k + 3)$ equations in $(n + k + 3)$ unknowns. Imposing zero initial conditions (i.e., at $t = 0$ the motion is unperturbed) renders the problem completely specified.

III. METHOD OF SOLUTION

Allowing for four bending modes ($n = 4$), and six slosh modes ($k = 6$), the equations of motion (equations (25) - (28)), in conjunction with equation (48), reduce to a set of thirteen simultaneous ordinary linear differential equations. Invoking the Laplace transform, and noting the zero initial conditions, the equations were transformed to thirteen simultaneous linear algebraic equations containing the transform parameter S . These equations may be presented in matrix form by the single equation

$$A(S) X = B(\omega, \psi) , \quad (49)$$

where the elements of the coefficient matrix $A(S)$, and column matrices X and B are given in Appendix B.

Since the input (wind penetration) is harmonic of frequency ω , replacing S by $i\omega$; $i = \sqrt{-1}$, in equation (49), transforms it to

$$A(i\omega) T = B(\omega, \psi) \quad (50)$$

where the elements of the A matrix are as before but with S replaced by $i\omega$, the elements of the B matrix are unchanged, and the elements of the T matrix, t_k ; $k = 1, 2, \dots, 13$, are:

$$\left. \begin{aligned} t_1 &= y_{c.g.} / V_w \\ t_2 &= \theta / V_w \\ t_3 &= \beta / V_w \\ t_k &= \alpha_{(k-3)} / V_w ; \quad k = 4, 5, 6, 7 \\ t_k &= \xi_{s(k-7)} / V_w ; \quad k = 8, 9, \dots, 13. \end{aligned} \right\} \quad (51)$$

The t_k 's defined in equation (51) are readily obtainable from equation (50) by a matrix inversion and multiplication sequence. The resulting expressions are complex with magnitudes, $T_k \equiv \pm |t_k|$; $k = 1, 2, \dots, 13$. Only the (+) sign is retained. However, the influence of the (-) sign should be noted in the phase angle computation.

The T_k 's are functions of the frequency, ω , and the, as yet, undetermined phase angle ψ . That is,

$$T_k = T_k(\omega, \psi) ; k = 1, 2, \dots, 13. \quad (52)$$

Determination of ψ

Since the value of ψ is not known a priori, it will be chosen so as to render each T_k a maximum for all ω . To accomplish this, we note that from the form of the input (B matrix), T_k must be harmonic in ψ . Therefore, T_k may be written in the form

$$T_k(\omega, \psi) = X_k(\omega) \sin(\psi + \phi_k) \quad (53)$$

$$k = 1, 2, \dots, 13$$

where the X_k 's are the desired maximum response amplitudes, and the ϕ_k 's are the desired phase angles between response and input. To obtain the X_k 's and ϕ_k 's, at least two values of $T_k(\omega, \psi)$ must be specified for each k . In particular, if we know the values of T_k for $\psi = 0^\circ$ and $\psi = 90^\circ$, then, from equation (53), we obtain

$$\left. \begin{aligned} X_k(\omega) &= \sqrt{T_k^2(\omega, 0^\circ) + T_k^2(\omega, 90^\circ)} \\ \text{and} \\ \phi_k(\omega) &= \tan^{-1} \left\{ \frac{T_k(\omega, 0^\circ)}{T_k(\omega, 90^\circ)} \right\} \\ k &= 1, 2, \dots, 13. \end{aligned} \right\} \quad (54)$$

Hence, the solution procedure is to evaluate each $T_k(\omega, \psi)$, as given in equation (52), for $\psi = 0^\circ$ and $\psi = 90^\circ$, and then use equation (54) to obtain the desired transfer functions in the form of amplitudes and phase angles. It should be noted that since T_k is always taken positive, $0 \leq \phi_k \leq \pi/2$ rad. This fact must be taken into account in the comparison with flight test data.

Additional Response Parameters

In addition to the thirteen response parameters specified by equation (51), there are eight other parameters which are of interest for the Saturn V. These additional parameters are:

$$\left. \begin{aligned} t_{14} &= \ddot{y}_{c.g.}/V_w \\ t_{15} &= \dot{y}_{c.g.}/V_w \\ t_{16} &= \ddot{\theta}/V_w \\ t_{17} &= \dot{\theta}/V_w \\ t_{18} &= \beta_c/V_w \\ t_{19} &= \theta_1/V_w \\ t_{20} &= \dot{\theta}_1/V_w \\ t_{21} &= \alpha/V_w \end{aligned} \right\} \quad (55)$$

The Laplace transform of these parameters, in the form $\mathcal{L}[()]/\mathcal{L}[V_w]$, are related to the Laplace transform of previously defined parameters either directly, as for example

$$\mathcal{L}[\ddot{y}_{c.g.}]/\mathcal{L}[V_w] = S^2 x_1$$

or indirectly, through a previously defined equation, as for example (see equation (38))

$$\begin{aligned} \mathcal{L}[\alpha]/\mathcal{L}[V_w] &= -\frac{S}{V} x_1 + \left[1 - \frac{S \bar{x}_\alpha}{V}\right] x_2 + \\ &+ \sum_{i=4}^7 \left[\varphi'_{(i-3)}(\bar{x}_\alpha) - \frac{S}{V} \varphi_{(i-3)}(\bar{x}_\alpha) \right] x_i + \frac{1}{V} \sin \left[\frac{\omega \bar{x}_\alpha}{V} + \psi \right]. \end{aligned}$$

Hence, the additional parameters defined by equation (55) may be obtained either by returning to equation (49) and increasing its order to a system of

twenty-one simultaneous equations, or by first solving the system as given in equation (49), and then, using the results obtained, evaluate the parameters of equation (55). Employing either approach, the procedure outlined above is followed so that the desired results (equation (54)) would consist of twenty-one X_k 's and twenty-one Φ_k 's.

IV. NUMERICAL INPUT

Vehicle response, in the form of amplitude and phase angle transfer functions (in accordance with equation (54), was obtained for four flight times during the boost phase of the Saturn V. The flight times considered are $t = 61, 78, 97$, and 120 seconds after S-IC lift-off. The input data corresponding to these flight times was obtained either directly, or indirectly (through interpolation), from references (3) through (12).

Dynamic characteristics of the vehicle (i.e., bending modes, frequencies, etc.) were obtained from reference (3). Bending mode data was directly available for $t = 78$ and 120 seconds after S-IC lift-off. However, it was necessary to interpolate between available data to obtain bending data at $t = 61$ and 97 seconds. Linear interpolation was used.

Flight trajectory data (i.e., Mach number, vehicle mass, thrust, etc., versus flight time), was obtained from references (4) and (5).

Aerodynamic data, in the form of distributions of the normal force coefficient per unit angle of attack, was obtained from reference (6) for $t = 61, 78$, and 97 seconds. For $t = 120$ seconds, the data presented in reference (7) was used to generate the required distribution of the normal force coefficient per unit angle of attack.

Damping data for vehicle bending was obtained from reference (8). Damping data associated with propellant slosh was obtained from reference (9). Two extremes of slosh damping were used; one set corresponding to maximum slosh amplitudes, the other to minimum slosh amplitudes.

Data for the engine actuator transfer function was obtained from reference (10). Data for the attitude and rate sensor gains and transfer functions was obtained from reference (11).

Data for the five S-IC stage F-1 engines was obtained from reference (12).

A computer program was written in FORTRAN IV to solve the equations developed in Section III. The program is not presented here. However, a flow chart describing the program is presented in Figure 4 with accompanying

definitions given below.

- Box 1 - Read all data.
- Box 2 - Indices for storing data corresponding to both slosh and omega.
- Box 3 - Starting point for omega.
- Box 4 - Test to see if omega has exceeded omega maximum.
- Box 5 - Set psi to zero the first time through and the index to one for storing data.
- Box 6 - The coefficients for the A-Matrix are computed.
- Box 7 - The coefficients for the B-Matrix are computed.
- Box 8 - The inverse of A is obtained and multiplied by B to solve for X.
- Box 9 - Eight more equations are solved using the solution from Box 8.
- Box 10 - The absolute value of these numbers are stored as functions of equation and psi.
- Box 11 - The psi index is checked to see if this is the second pass through.
- Box 12 - Set $\psi = 90^\circ$ and the psi index to 2 and return to Box 7.
- Box 13 - The final solution is computed for a given omega. This solution includes both the X's and $\bar{x}$'s.
- Box 14 - The omega index is increased by one and omega is increased by delta omega. A return is made to Box 4.
- Box 15 - A test is made to see if the equations have been solved for two sloshes.
- Box 16 - The slosh index is set to 2, and a return is made to Box 3.
- Box 17 - Box 17 is used to print all data except input data.
- Box 18 - All plots are generated in this section of the program. A return is made to Box 1.

Production runs for the four flight times of interest were performed at the George C. Marshall Space Flight Center Computing Facility using the IBM 7094 computer and a Stromberg-Carlson SC4020 plotter. The results of these runs (amplitude and phase angle transfer functions), are presented in Figures 5a through 25b for $t = 61$ seconds, Figures 26a through 46b for $t = 78$ seconds, Figures 47a through 67b for $t = 97$ seconds, and Figures 68a through 88b for $t = 120$ seconds after lift-off.

All phase angle transfer functions ϕ_k 's, are plotted in degrees. The amplitude transfer functions presented in equations (51) and (55) are in the following units:

$$t_1, t_k; k = 4, 5, \dots, 13 : \text{ft}/(\text{ft}/\text{sec})$$

$$t_{15} : (\text{ft}/\text{sec})/(\text{ft}/\text{sec})$$

$$t_{14} : (\text{ft}/\text{sec}^2)/(\text{ft}/\text{sec})$$

$$t_2, t_3, t_{18}, t_{19}, t_{21} : \text{rad}/(\text{ft}/\text{sec})$$

$$t_{17}, t_{20} : (\text{rad}/\text{sec})/(\text{ft}/\text{sec})$$

$$t_{16} : (\text{rad}/\text{sec}^2)/(\text{ft}/\text{sec})$$

V. DISCUSSION OF RESULTS AND COMPARISON WITH FLIGHT TEST DATA

The results of the theoretical study are presented, for four flight times of interest ($t = 61, 78, 97$, and 120 seconds after S-IC lift-off), in Figures 5a through 88b. Flight test data was obtained for the flight mechanic parameters during a Saturn 502 flight. This data was then analyzed using a Fourier Frequency Program for the four flight times used in the theoretical transfer function calculations. A 10 second time slice was used, resulting in output frequencies in increments of 0.1 cps. Since the time history track for the wind was not available through direct measurement, flight mechanic parameters (i.e., angle of attack, vehicle attitude, etc.) were combined with a simplified vehicle model to generate the wind profile. Using this profile and the flight mechanic parameters of interest, flight test transfer functions were computed. Since the flight data was only available every 0.1 seconds, the flight test transfer functions would not be expected to be too reliable above say 2 cps.

For the purpose of presentation and comparison, flight test transfer functions are presented from 0.1 cps to 2.0 cps, for the amplitude transfer functions $X(14)$, $X(18)$, and $X(20)$. These results are presented for each flight time, as over-lay curves on the transparent pages preceeding Figures 18a, 22a, and 24a, for $t = 61$ seconds; Figures 39a, 43a, and 45a, for $t = 78$ seconds; Figures 60a, 64a, and 66a, for $t = 97$ seconds; and Figures 81a, 85a, and 87a, for $t = 120$ seconds.

It is readily seen that for the flight times $t = 61, 78$, and 97 seconds after S-IC lift-off, the flight test transfer functions are reproduced fairly well by the theoretical results. Agreement is attained both order of magnitude wise as well as trend wise. It is expected that had a more sophisticated model been used for the purpose of establishing the wind profile in the test results, even greater agreement between theory and flight test would be demonstrated.

The above observations are however, not valid for the 120 second flight time case. In this case, $X(20)$ demonstrates the best trend wise and order of magnitude wise comparisons. $X(14)$ demonstrates a fair trend

comparison, but poor order of magnitude comparison. X(18) demonstrates poor trend as well as poor order of magnitude comparisons.

Based on these comparisons it seems reasonable to assume that in the neighborhood of 120 seconds after lift-off, a physical phenomenon occurs (such as the "Pogo" type behavior discussed in the Introduction) which did not manifest itself previously. Should this be the case, there are several possibilities:

1. The phenomenon is such that it does not effect the theory significantly. However, it does effect the instrument readings, and consequently, the flight test results.
2. The phenomenon is such that it does not effect the instrument readings, and consequently, the flight test transfer functions are fairly accurate.
3. The phenomenon is such that it renders both the theory and instrument readings invalid.

Should the first possibility be the case, then the instrumentation would have to be re-examined, while the theory could be relied upon. Should the second possibility be the case, then the theory would have to be modified to account for the physics of the situation. Should the third, and most likely, possibility be the case then both theory and instrumentation would have to be re-examined and modified. In any event, the results presented for the 120 second case should be viewed with caution. Any final conclusions about this case should only be drawn after sufficient additional studies have been made.

VI. REFERENCES

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APPENDIX A

The details of the differentiations indicated in equation (24) are presented here.

From equations (9) and (10)

$$T = T_M + T_S + 4T_E^{sw} + T_E^{st}$$

The quantities,

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}_{c.g.}} \right), \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right), \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_1} \right), \text{ and}$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\xi}_{s_j}} \right) \text{ are sought.}$$

Determination of $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}_{c.g.}} \right)$:

From equation (11), obtain

$$\frac{d}{dt} \left(\frac{\partial T_M}{\partial \dot{y}_{c.g.}} \right) = m_M \ddot{y}_{c.g.} + S_M \ddot{\theta} + \sum_{i=1}^n a_i \ddot{q}_i \quad (A-1)$$

where,

$$m_M \equiv \int_{\ell} \rho(\bar{x}) d\bar{x}$$

$$S_M \equiv \int_{\ell} \rho(\bar{x}) \bar{x} d\bar{x}$$

$$a_i \equiv \int_{\ell} \rho(\bar{x}) \varphi_i(\bar{x}) d\bar{x}$$

From equation (12), obtain

$$\begin{aligned}
\frac{d}{dt} \left(\frac{\partial T_s}{\partial \dot{y}_{c.g.}} \right) &= \sum_{j=1}^k (m_{o_j} + m_{s_j}) \ddot{y}_{c.g.} + \sum_{j=1}^k (m_{o_j} \bar{x}_{o_j} + m_{s_j} \bar{x}_{s_j}) \ddot{\theta} + \\
&+ \sum_{j=1}^k m_{s_j} \ddot{\xi}_{s_j} + \sum_{j=1}^k \sum_{i=1}^n [m_{o_j} \varphi_i (\bar{x}_{o_j}) + \\
&+ m_{s_j} \varphi_i (\bar{x}_{s_j})] \ddot{q}_i
\end{aligned} \tag{A-2}$$

From equation (15), obtain

$$\begin{aligned}
\frac{d}{dt} \left(\frac{\partial T_E^{sw}}{\partial \dot{y}_{c.g.}} \right) &= m_E \left\{ \ddot{y}_{c.g.} - (\bar{x}_E + l_E) \ddot{\theta} + \right. \\
&- l_E \ddot{\beta} + \sum_{i=1}^n [\varphi_i (\bar{x}_E) - l_E \varphi'_i (\bar{x}_E)] \ddot{q}_i \left. \right\}
\end{aligned} \tag{A-3}$$

and, consequently,

$$\frac{d}{dt} \left(\frac{\partial T_E^{st}}{\partial \dot{y}_{c.g.}} \right) = \frac{d}{dt} \left(\frac{\partial T_E^{sw}}{\partial \dot{y}_{c.g.}} \right) \Big|_{\ddot{\beta} = 0} \tag{A-4}$$

Summing equations (A-1) through (A-3), and four times equation (A-4), we obtain,

$$\begin{aligned}
\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}_{c.g.}} \right) &= \left[m_M + 5m_E + \sum_{j=1}^k (m_{o_j} + m_{s_j}) \right] \ddot{y}_{c.g.} + \\
&+ \left[S_M + \sum_{j=1}^k (m_{o_j} \bar{x}_{o_j} + m_{s_j} \bar{x}_{s_j}) - 5m_E (\bar{x}_E + l_E) \right] \ddot{\theta} +
\end{aligned}$$

$$\begin{aligned}
& + \left[\sum_{j=1}^k m_{s_j} \right] \ddot{\xi}_{s_j} - \left[4m_E \ell_E \right] \ddot{\beta} + \\
& + \sum_{i=1}^n \left[a_i + \sum_{j=1}^k \left\{ m_{o_j} \varphi_i(\bar{x}_{o_j}) + m_{s_j} \varphi_i(\bar{x}_{s_j}) \right\} + \right. \\
& \left. + 5m_E \left\{ \varphi_i(\bar{x}_E) - \ell_E \varphi_i'(\bar{x}_E) \right\} \right] \ddot{q}_i
\end{aligned} \tag{A-5}$$

The coefficient of $\ddot{y}_{c.g.}$ is the total vehicle mass, M . The coefficient of $\ddot{\theta}$ is identically zero since it is the static moment about the origin of the coordinate system; which was taken at the vehicle c.g. The coefficient of $\ddot{q}_1$ is also identically zero since linear momentum is conserved during a free vibration (see reference (13)). Hence, equation (A-5) reduces to

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}_{c.g.}} \right) = M \ddot{y}_{c.g.} + \sum_{j=1}^k m_{s_j} \ddot{\xi}_{s_j} - 4m_E \ell_E \ddot{\beta} \tag{A-6}$$

Determination of $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right)$:

From equation (11), obtain

$$\begin{aligned}
\frac{d}{dt} \left(\frac{\partial T_M}{\partial \dot{\theta}} \right) &= S_M \ddot{y}_{c.g.} + \left[I_M + \int_{\ell} I(\bar{x}) d\bar{x} \right] \ddot{\theta} + \\
&+ \sum_{i=1}^n \left[b_i + \int_{\ell} I(\bar{x}) \varphi_i'(\bar{x}) d\bar{x} \right] \ddot{q}_i
\end{aligned} \tag{A-7}$$

where,

$$I_M \equiv \int_{\ell} \rho(\bar{x}) \bar{x}^2 d\bar{x}$$

$$b_i \equiv \int_{\ell} \rho(\bar{x}) \bar{x} \varphi_i'(\bar{x}) d\bar{x}$$

From equation (12), obtain

$$\begin{aligned}
\frac{d}{dt} \left(\frac{\partial T_s}{\partial \dot{\theta}} \right) &= \sum_{j=1}^k \left[m_{O_j} \bar{x}_{O_j} + m_{S_j} \bar{x}_{S_j} \right] \ddot{y}_{c.g.} + \\
&+ \sum_{j=1}^k \left[m_{O_j} \bar{x}_{O_j}^2 + m_{S_j} \bar{x}_{S_j}^2 + I_{O_j} \right] \ddot{\theta} + \\
&+ \sum_{j=1}^k \left[m_{S_j} \bar{x}_{S_j} \right] \ddot{\xi}_{S_j} + \\
&+ \sum_{j=1}^k \sum_{i=1}^n \left[I_{O_j} \varphi_i'(\bar{x}_{O_j}) + m_{O_j} \bar{x}_{O_j} \varphi_i(\bar{x}_{O_j}) + \right. \\
&\quad \left. + m_{S_j} \bar{x}_{S_j} \varphi_i(\bar{x}_{S_j}) \right] \ddot{q}_i
\end{aligned} \tag{A-8}$$

From equation (15), obtain

$$\begin{aligned}
\frac{d}{dt} \left(\frac{\partial T_E^{sw}}{\partial \dot{\theta}} \right) &= - \left[m_E (\bar{x}_E + l_E) \right] \ddot{y}_{c.g.} + \\
&+ \left[I_E + m_E (\bar{x}_E + l_E)^2 \right] \ddot{\theta} + \\
&+ \left[I_E + m_E l_E (\bar{x}_E + l_E) \right] \ddot{\beta} + \\
&+ \sum_{i=1}^n \left[I_E \varphi_i'(\bar{x}_E) - m_E (\bar{x}_E + l_E) \{ \varphi_i(\bar{x}_E) - l_E \varphi_i'(\bar{x}_E) \} \right] \ddot{q}_i
\end{aligned} \tag{A-9}$$

and note that equation (A-4) holds with $\dot{y}_{c.g.}$ replaced by $\dot{\theta}$.

Summing the above, we note that the coefficients of $\ddot{y}_{c.g.}$ and $\ddot{q}_1$ are identically zero. The former is zero due to the choice of the coordinate system, the latter is zero since angular momentum is conserved during a free vibration (see reference (13)). The coefficient of $\ddot{\theta}$ is the total vehicle inertia, I . Hence,

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right) &= I \ddot{\theta} + \sum_{j=1}^k m_{s_j} \bar{x}_{s_j} \ddot{\xi}_{s_j} + \\ &+ 4 \left[I_E + m_E \ell_E (\bar{x}_E + \ell_E) \right] \ddot{\beta} \end{aligned} \quad (A-10)$$

Determination of $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_1} \right)$:

In a similar manner, it was found that

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_1} \right) &= \mu_1 \ddot{q}_1 + \sum_{j=1}^k m_{s_j} \varphi_1 (\bar{x}_{s_j}) \ddot{\xi}_{s_j} + \\ &+ 4 \left[I_E \varphi_1' (\bar{x}_E) - m_E \ell_E \{ \varphi_1 (\bar{x}_E) - \ell_E \varphi_1' (\bar{x}_E) \} \right] \ddot{\beta} \end{aligned} \quad (A-11)$$

In this derivation, the coefficients of $\ddot{y}_{c.g.}$ and $\ddot{\theta}$ were identically zero by the conservation of linear and angular momentum respectively. The coefficient of $\ddot{q}_1$ was found to be equal to the generalized mass associated with the i th bending mode, μ_1 .

Determination of $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\xi}_{s_1}} \right) :$

From equations (11) and (15), it is readily seen that

$$\frac{\partial T_M}{\partial \dot{\xi}_{s_1}} = \frac{\partial T_E^{sw}}{\partial \dot{\xi}_{s_1}} = 0$$

and, consequently,

$$\frac{\partial T_E^{st}}{\partial \dot{\xi}_{s_1}} = 0.$$

Hence,

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\xi}_{s_1}} \right) &= \frac{d}{dt} \left(\frac{\partial T_s}{\partial \dot{\xi}_{s_1}} \right) = m_{s_1} \ddot{y}_{c.g.} + m_{s_1} \bar{x}_{s_1} \ddot{\theta} + \\ &+ m_{s_1} \ddot{\xi}_{s_1} + m_{s_1} \sum_{j=1}^n \varphi_j (\bar{x}_{s_1}) \ddot{q}_j \end{aligned} \quad (A-12)$$

From equation (17)

$$V = V_b + V_s$$

and from equations (19) and (20) it is seen that the only nonzero derivatives are:

$$\frac{\partial V}{\partial q_1} = \mu_1 \omega_1^2 q_1$$

and

(A-13)

$$\frac{\partial V}{\partial \xi_{s_1}} = m_{s_1} \omega_{s_1}^2 \xi_{s_1}.$$

Similarly, for the dissipation function, D , defined in equation (21), it is found that the only nonzero derivatives are:

$$\frac{\partial D}{\partial \dot{q}_1} = 2 \mu_1 \omega_1 \zeta_1 \dot{q}_1$$

and

(A-14)

$$\frac{\partial D}{\partial \dot{\xi}_{s_1}} = 2 m_{s_1} \omega_{s_1} \zeta_{s_1} \dot{\xi}_{s_1} .$$

Combining the equations presented above in accordance with equation (24), results in the expressions given on the left hand sides of equations (25) through (28).

APPENDIX B

The elements, a_{ij} , of the A matrix appearing in equation (49) are as follows:

$$a_{11} = \left[MS^2 + \frac{\bar{q}AS}{V} \int_{\ell} c_{\ell, \alpha} d\bar{x} \right]$$

$$a_{12} = - \left\{ (T_s + T_c) + \bar{q}A \left[\int_{\ell} c_{\ell, \alpha} d\bar{x} - \frac{S}{V} \int_{\ell} c_{\ell, \alpha} \bar{x} d\bar{x} \right] \right\}$$

$$a_{13} = - \left[4m_E \ell_E S^2 + T_c \right]$$

$$a_{1k} = \left\{ (T_s + T_c) \varphi'_{(k-3)}(\bar{x}_E) + \bar{q}A \left[\int_{\ell} c_{\ell, \alpha} \varphi'_{(k-3)}(\bar{x}) d\bar{x} + \right. \right. \\ \left. \left. - \frac{S}{V} \int_{\ell} c_{\ell, \alpha} \varphi_{(k-3)}(\bar{x}) d\bar{x} \right] \right\} ; \quad k = 4, 5, 6, 7$$

$$a_{1k} = m_{s_{(k-7)}} S^2 ; \quad k = 8, 9, \dots, 13$$

$$a_{21} = \frac{\bar{q}AS}{V} \int_{\ell} c_{\ell, \alpha} \bar{x} d\bar{x}$$

$$a_{22} = \left\{ IS^2 - \bar{q}A \left[\int_{\ell} c_{\ell, \alpha} \bar{x} d\bar{x} - \frac{S}{V} \int_{\ell} c_{\ell, \alpha} \bar{x}^2 d\bar{x} \right] \right\}$$

$$a_{23} = \left\{ 4S^2 \left[m_E \ell_E (\bar{x}_E + \ell_E) + I_E \right] + T_c \bar{x}_E \right\}$$

$$a_{2k} = \left\{ (T_s + T_c) \left[\varphi_{(k-3)}(\bar{x}_E) + \bar{x}_E \varphi'_{(k-3)}(\bar{x}_E) \right] + \right.$$

$$- \bar{q} A \left[\int_{\ell} c_{\ell, \alpha} \bar{x} \varphi'_{(k-3)}(\bar{x}) d\bar{x} - \frac{S}{V} \int c_{\ell, \alpha} \bar{x} \varphi_{(k-3)}(\bar{x}) d\bar{x} \right] \} ;$$

$$k = 4, 5, 6, 7$$

$$a_{2k} = m_{s_{(k-7)}} \bar{x}_{s_{(k-7)}} S^2 ; \quad k = 8, 9, \dots, 13$$

$$a_{31} = 0$$

$$a_{32} = -G_A \{ a_0 G_{\theta} + a_1 G_{\dot{\theta}} S \}$$

$$a_{33} = +1$$

$$a_{3k} = a_{32} \varphi'_{(k-3)}(\bar{x}_{\theta}) ; \quad k = 4, 5, 6, 7$$

$$a_{3k} = 0 ; \quad k \geq 8$$

The engine actuator transfer function, G_A , is given in reference (10), in the form

$$G_A = \sum_{i=0}^1 A_i S^i / \sum_{i=0}^7 B_i S^i$$

where the A_i 's and B_i 's are constants for a given flight time. The transfer functions, G_{θ} and $G_{\dot{\theta}}$, for the control network, are given in reference (11), in the form

$$G_{\theta} = \sum_{i=0}^3 A_i S^i / \sum_{i=0}^4 B_i S^i$$

and

$$G_{\dot{\theta}} = \sum_{i=0}^6 A_i S^i / \sum_{i=0}^8 B_i S^i$$

where the A_1 's and B_1 's are constant for a given flight time.

$$a_{k1} = \frac{\overline{q}AS}{V} \int_{\ell} c_{\ell, \alpha} \varphi_{(k-3)}(\overline{x}) d\overline{x}$$

$$a_{k2} = - \left\{ (T_s + T_c) \left[\varphi_{(k-3)}(\overline{x}_E) - \ell_E \varphi'_{(k-3)}(\overline{x}_E) \right] + \right. \\ \left. + \overline{q}A \left[\int_{\ell} c_{\ell, \alpha} \varphi_{(k-3)}(\overline{x}) d\overline{x} - \frac{S}{V} \int_{\ell} c_{\ell, \alpha} \overline{x} \varphi_{(k-3)}(\overline{x}) d\overline{x} \right] \right\}.$$

$$a_{k3} = -4 \left\{ m_E \ell_E \left[\varphi_{(k-3)}(\overline{x}_E) - \ell_E \varphi'_{(k-3)}(\overline{x}_E) \right] - I_E \varphi'_{(k-3)}(\overline{x}_E) \right\} S^2 \\ + T_c \left[\varphi_{(k-3)}(\overline{x}_E) - \ell_E \varphi'_{(k-3)}(\overline{x}_E) \right].$$

$$a_{kj} = \left\{ \mu_{(k-3)} \left[S^2 + 2 \left(w_{(k-3)} \zeta_{(k-3)} \right) S + w_{(k-3)}^2 \right] \delta_k^j + \right. \\ \left. - (T_s + T_c) \left[\varphi_{(k-3)}(\overline{x}_E) - \ell_E \varphi'_{(k-3)}(\overline{x}_E) \right] \varphi'_{(j-3)}(\overline{x}_E) + \right. \\ \left. - \overline{q}A \left[\int_{\ell} c_{\ell, \alpha} \varphi_{(k-3)}(\overline{x}) \varphi'_{(j-3)}(\overline{x}) d\overline{x} + \right. \right. \\ \left. \left. + \frac{S}{V} \int_{\ell} c_{\ell, \alpha} \varphi_{(k-3)}(\overline{x}) \varphi_{(j-3)}(\overline{x}) d\overline{x} \right] \right\} ; j = 4, 5, 6, 7$$

$$a_{kj} = m_{s(j-7)} \varphi_{(k-3)}(\overline{x}_{s(j-7)}) S^2 ; j = 8, 9, \dots, 13$$

where,

$$\delta_k^j = \begin{cases} 1 & ; \text{ for } j = k \\ 0 & ; \text{ for } j \neq k \end{cases}$$

and k takes on the values 4, 5, 6, and 7.

$$a_{11} = S^2$$

$$a_{12} = \bar{x}_{s(1-7)} S^2$$

$$a_{13} = 0$$

$$a_{1k} = \varphi_{(k-3)}(\bar{x}_{s(1-7)}) S^2 ; \quad k = 4, 5, 6, 7$$

$$a_{1k} = \left[S^2 + 2 \left(\omega_{s(k-7)} \zeta_{s(k-7)} \right) S + \omega_{s(k-7)}^2 \right] \delta_1^k ; \quad k = 8, 9, \dots, 13$$

where δ_1^k is the Kronecker delta defined above, and 1 takes on the values 8, 9, . . . , 13.

The elements, b_1 , of the B matrix appearing in equation (49) are:

$$b_1 = \frac{\bar{q}A}{V} \left\{ \cos \psi \int_{\ell} c_{\ell, \alpha} \sin \frac{\omega \bar{x}}{V} d\bar{x} + \sin \psi \int_{\ell} c_{\ell, \alpha} \cos \frac{\omega \bar{x}}{V} d\bar{x} \right\}$$

$$b_2 = \frac{\bar{q}A}{V} \left\{ \cos \psi \int_{\ell} c_{\ell, \alpha} \bar{x} \sin \frac{\omega \bar{x}}{V} d\bar{x} + \sin \psi \int_{\ell} c_{\ell, \alpha} \bar{x} \cos \frac{\omega \bar{x}}{V} d\bar{x} \right\}$$

$$b_3 = 0$$

$$b_k = \frac{\bar{q}A}{V} \left\{ \cos \psi \int_{\ell} c_{\ell, \alpha} \varphi_{(k-3)}(\bar{x}) \sin \frac{\omega \bar{x}}{V} d\bar{x} + \right. \\ \left. + \sin \psi \int_{\ell} c_{\ell, \alpha} \varphi_{(k-3)}(\bar{x}) \cos \frac{\omega \bar{x}}{V} d\bar{x} \right\} ; k=4, 5, 6, 7$$

$$b_k = 0 \text{ for } k \geq 8$$

The elements, x_i , of the X matrix appearing in equation (49) are:

$$x_1 = \mathcal{L}[y_{c.g.}] / \mathcal{L}[V_w]$$

$$x_2 = \mathcal{L}[\theta] / \mathcal{L}[V_w]$$

$$x_3 = \mathcal{L}[\beta] / \mathcal{L}[V_w]$$

$$x_i = \mathcal{L}[q_{(i-3)}] / \mathcal{L}[V_w] ; i = 4, 5, 6, 7$$

$$x_i = \mathcal{L}[\xi_{s_{(i-7)}}] / \mathcal{L}[V_w] ; i = 8, 9, \dots, 13$$

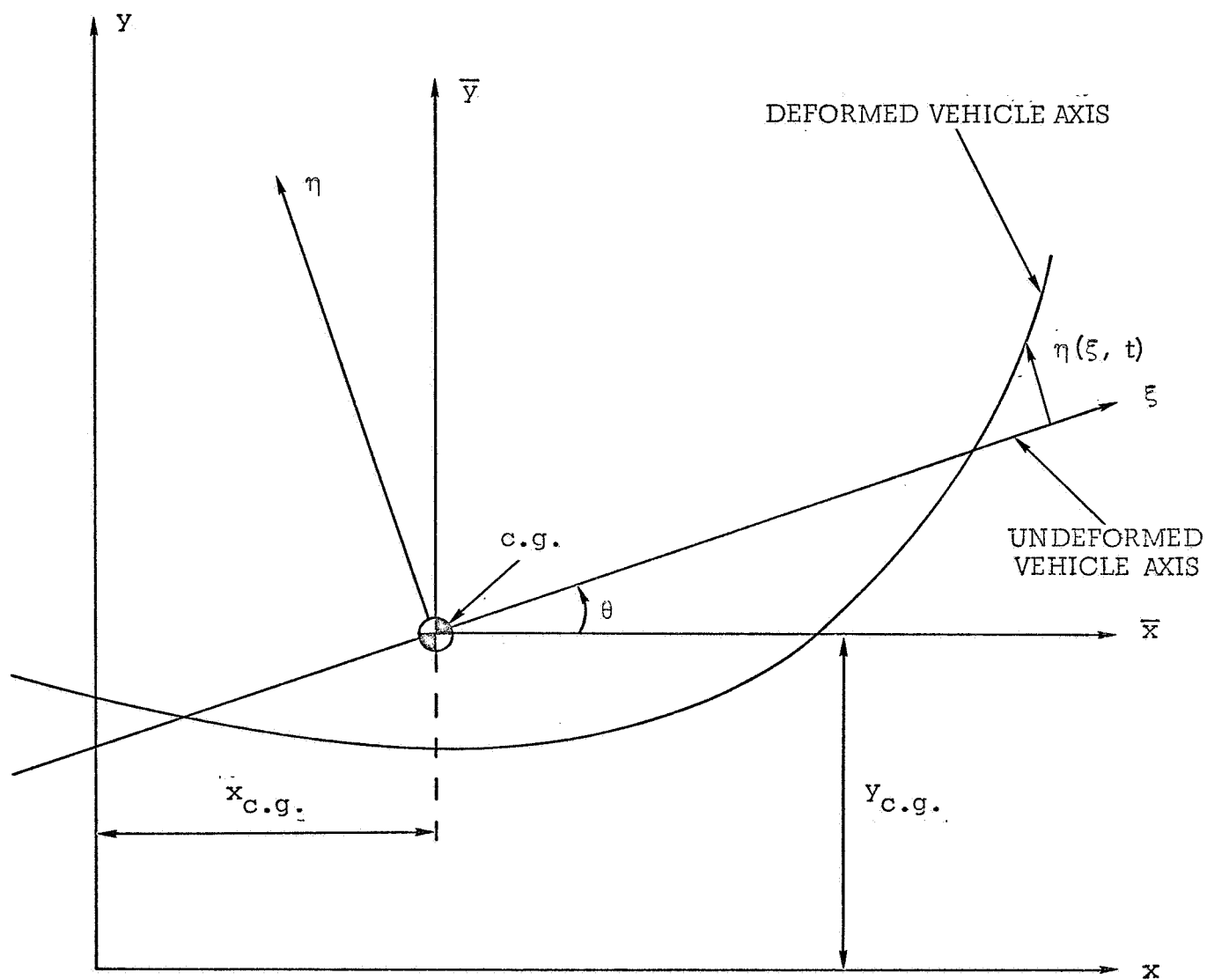


FIGURE 1

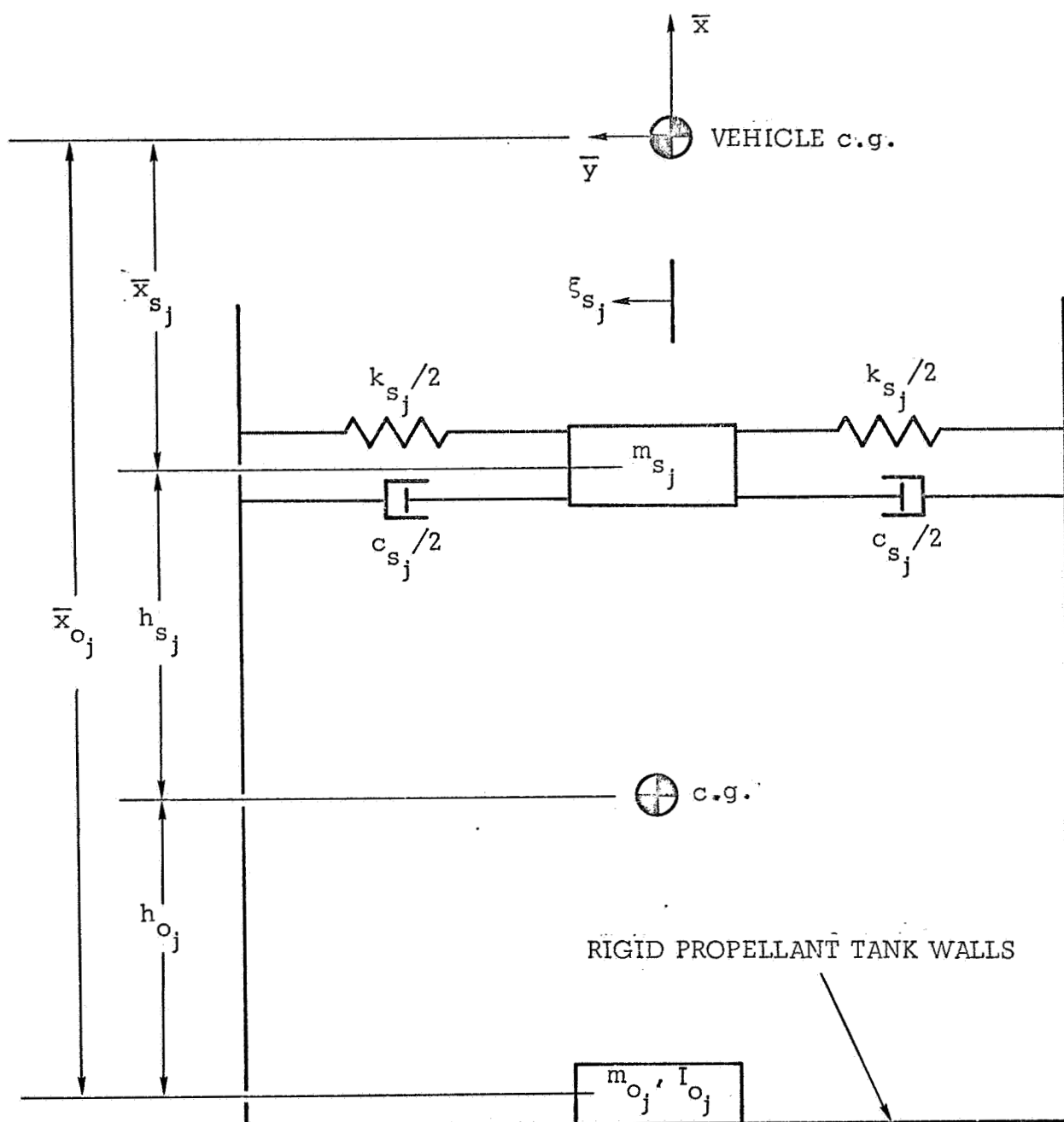


FIGURE 2

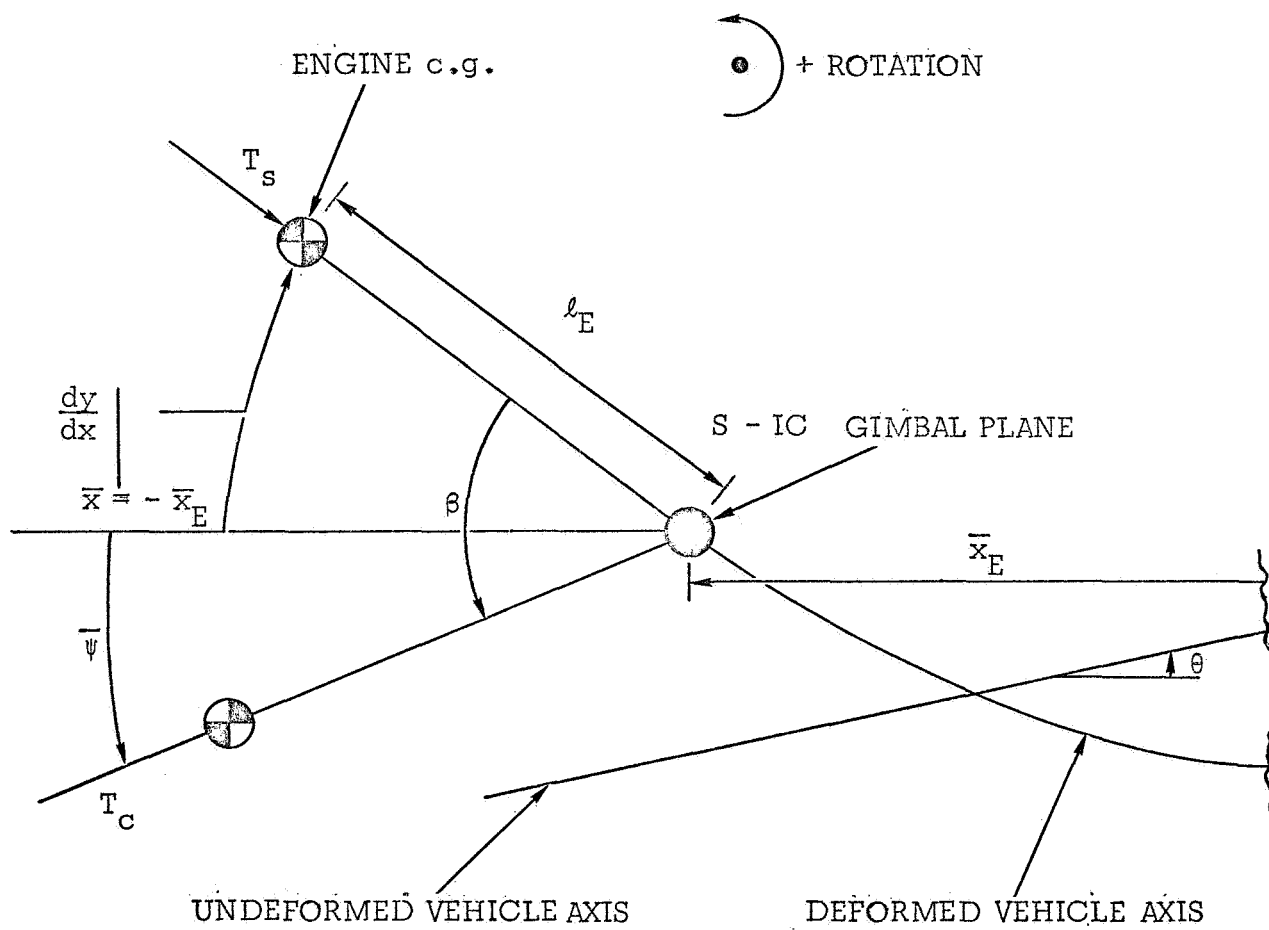


FIGURE 3

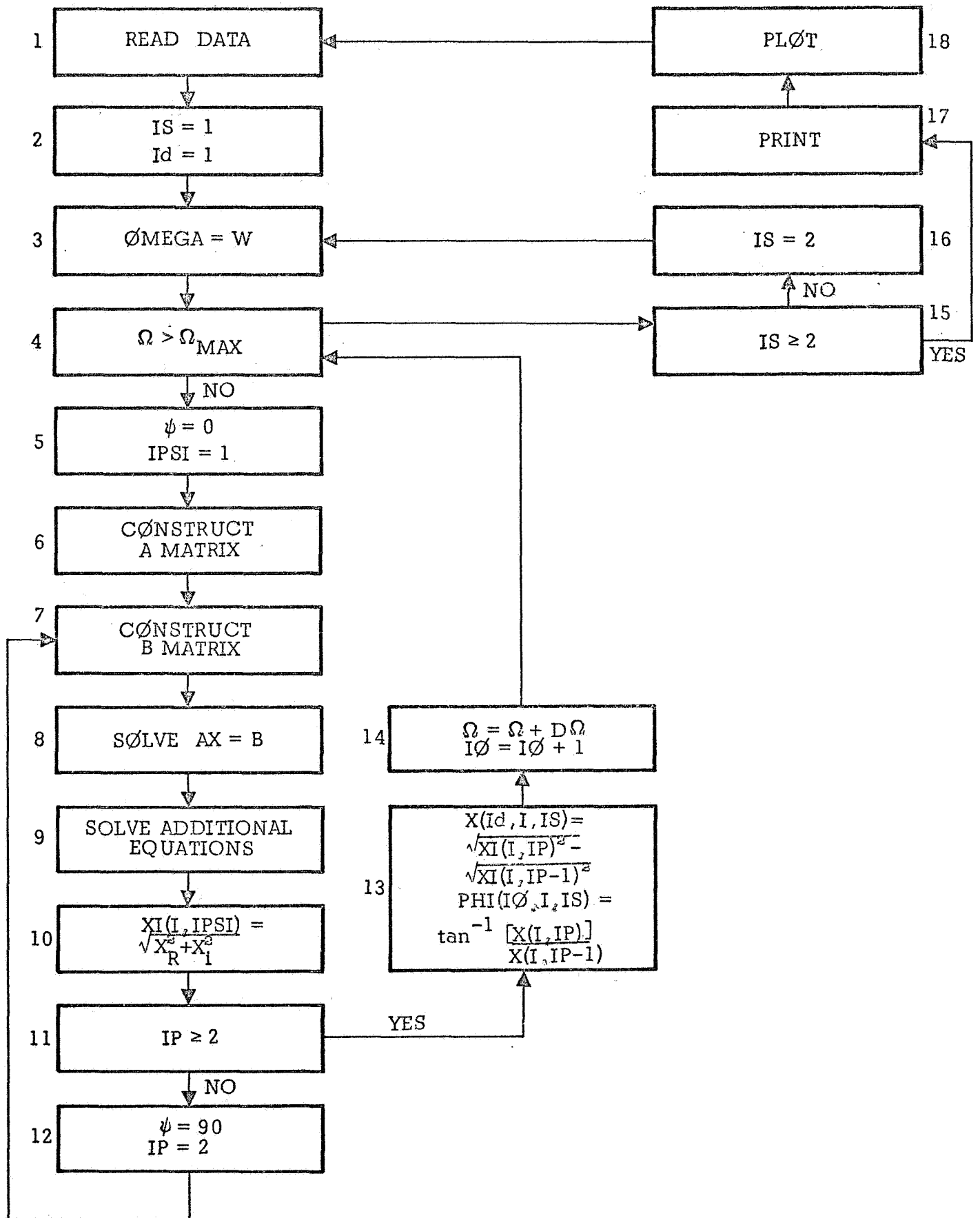
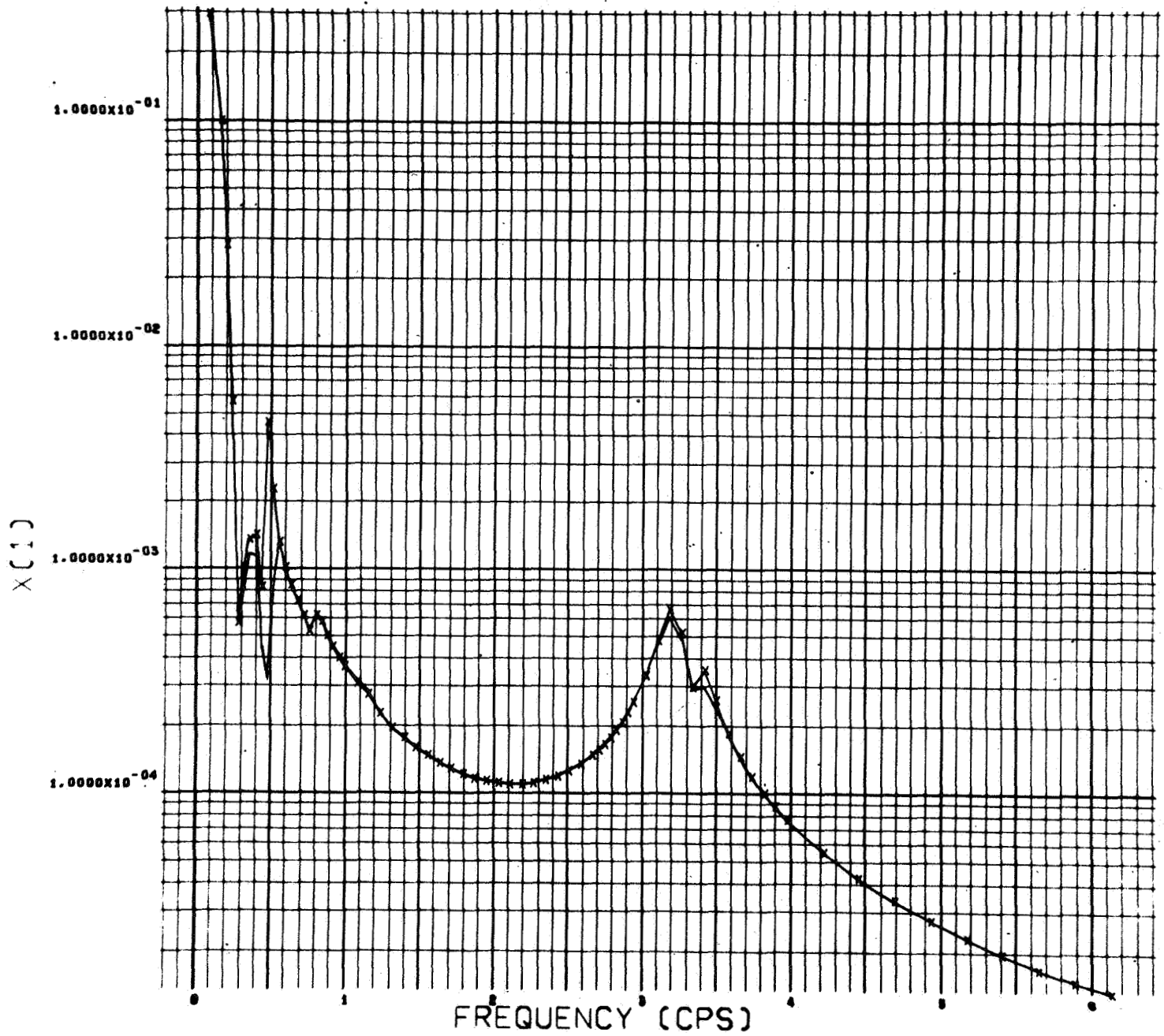


FIGURE 4

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490370
148 148



t = 61 seconds

Figure 5a

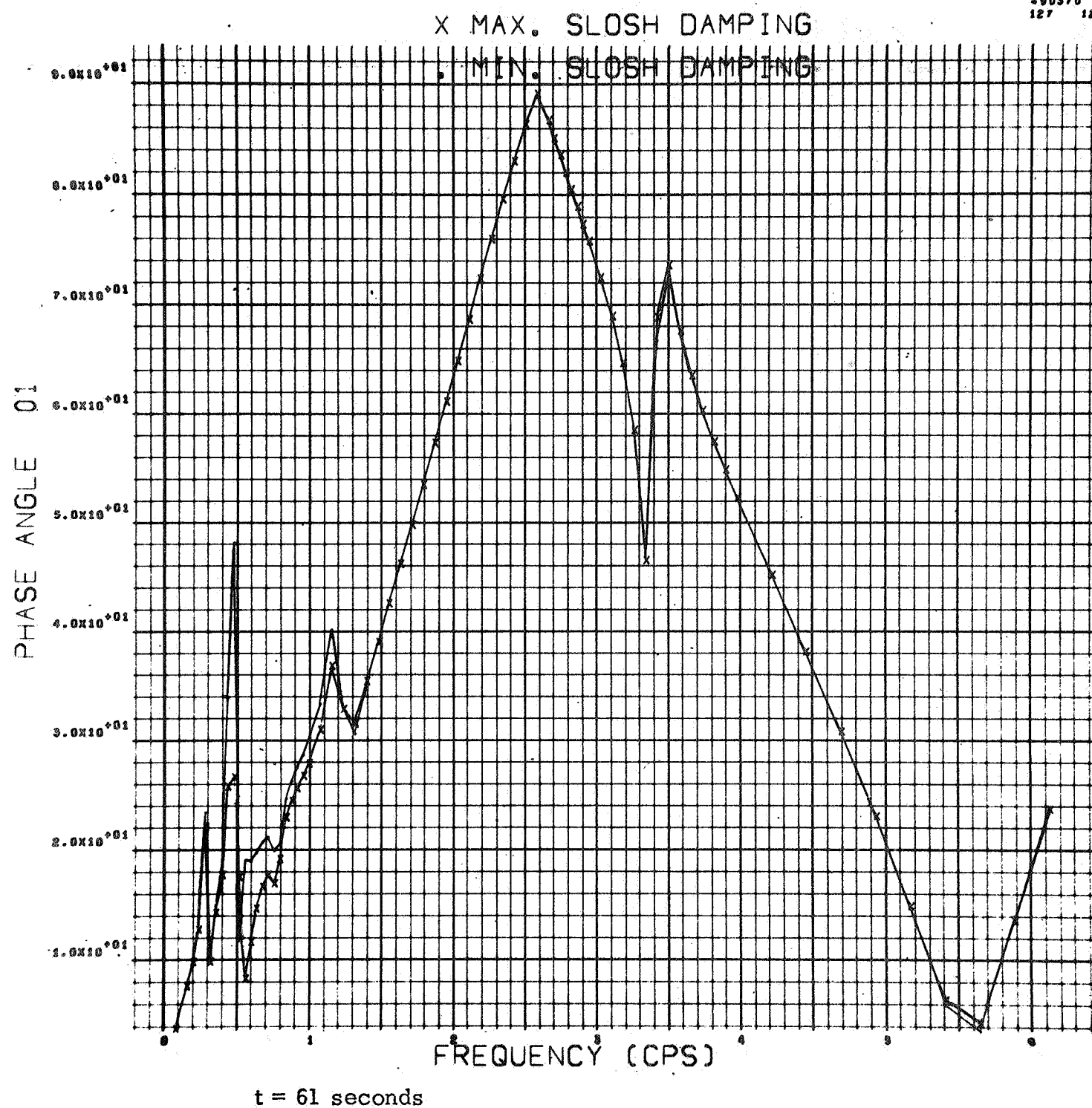
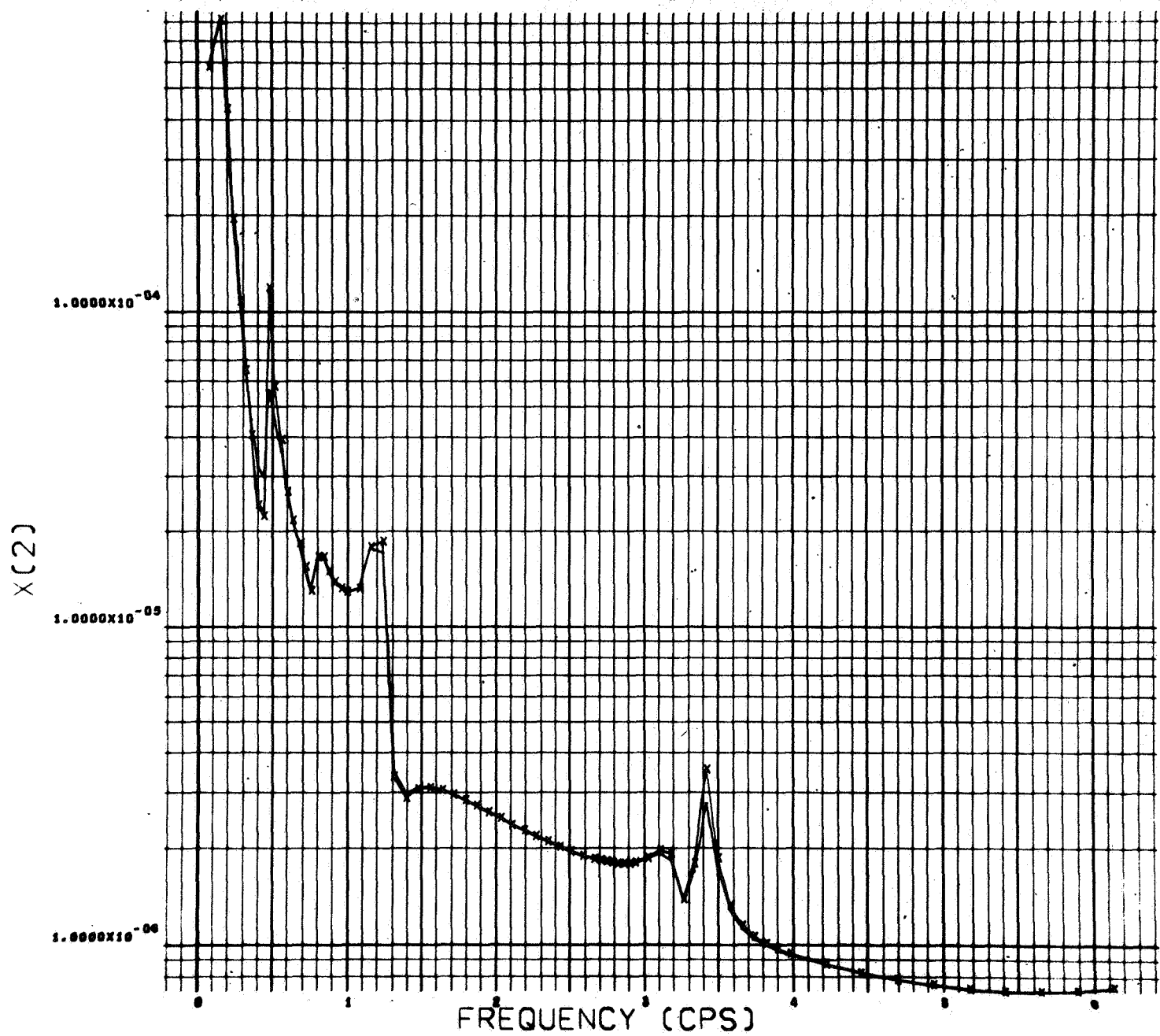


Figure 5b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING



t = 61 seconds

Figure 6a

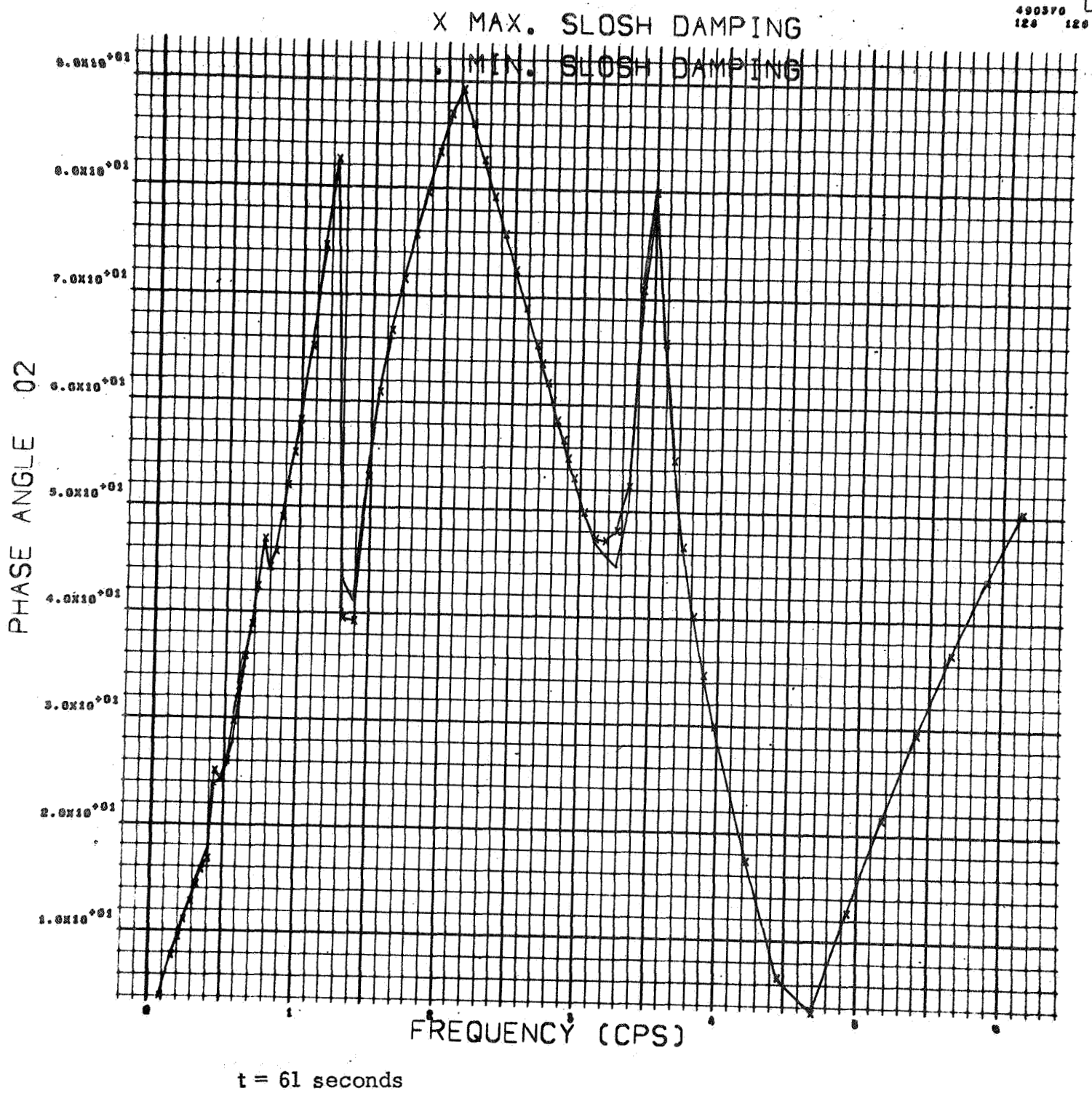
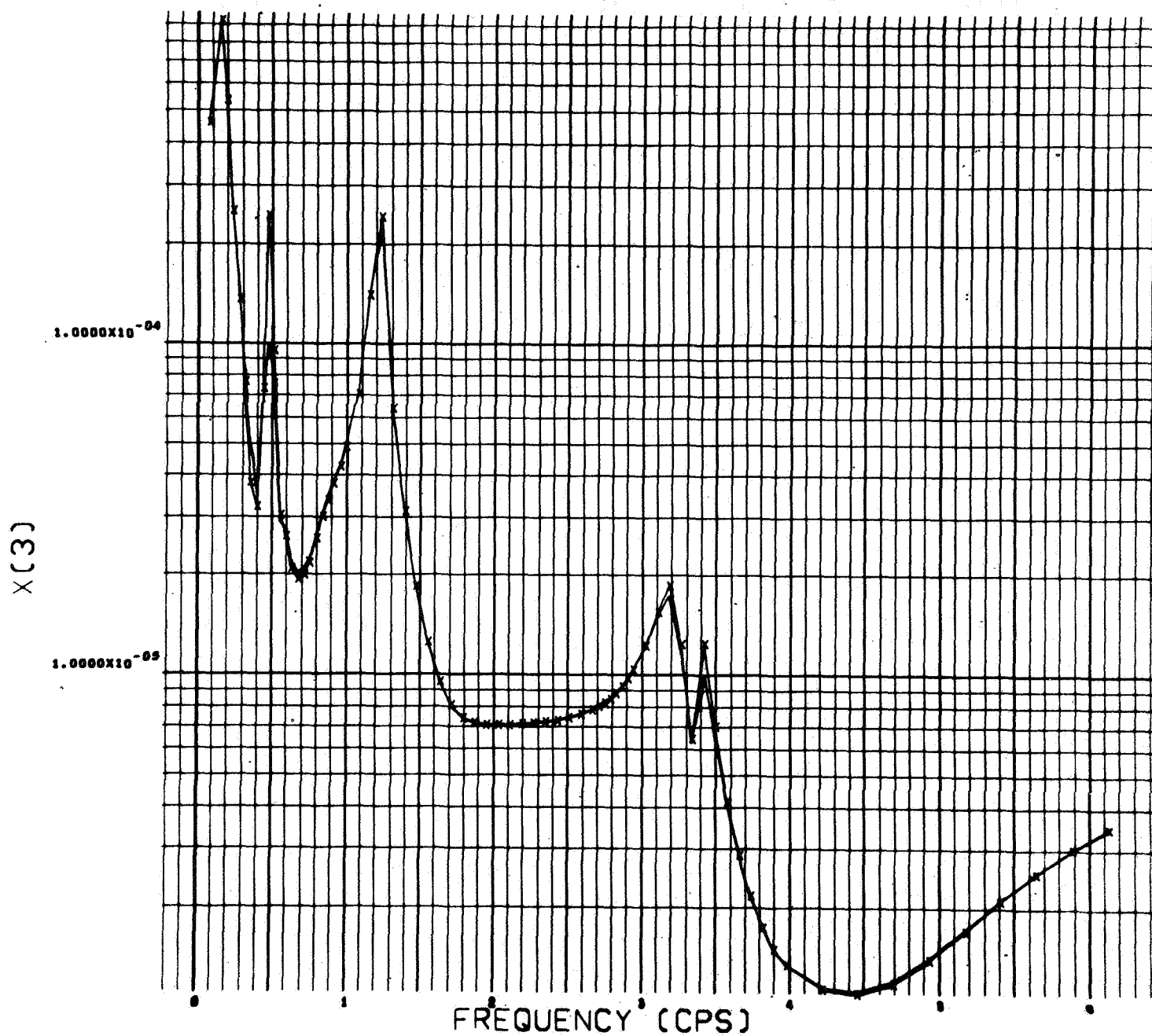


Figure 6b

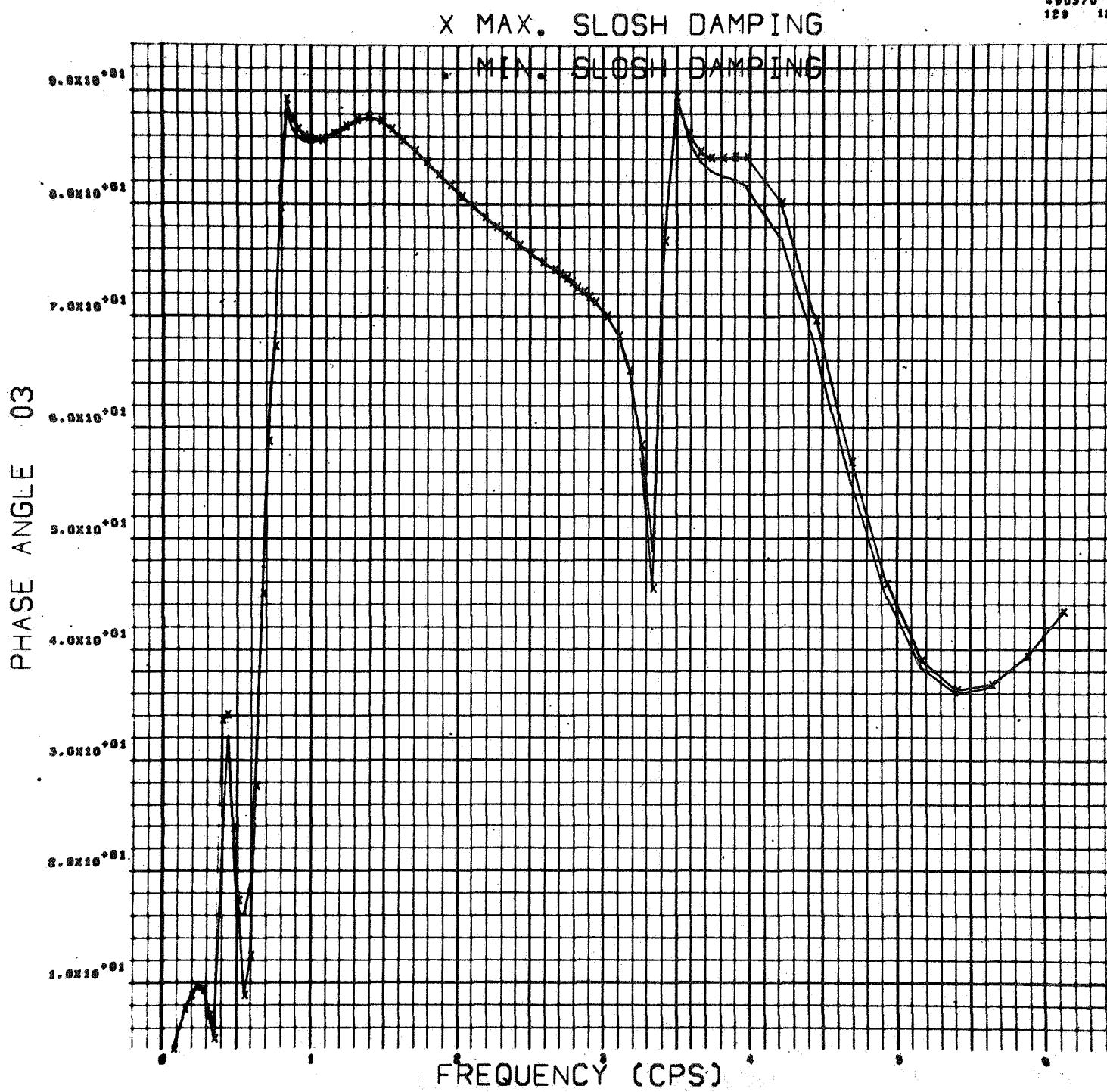
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490370 L
150 150



t = 61 seconds

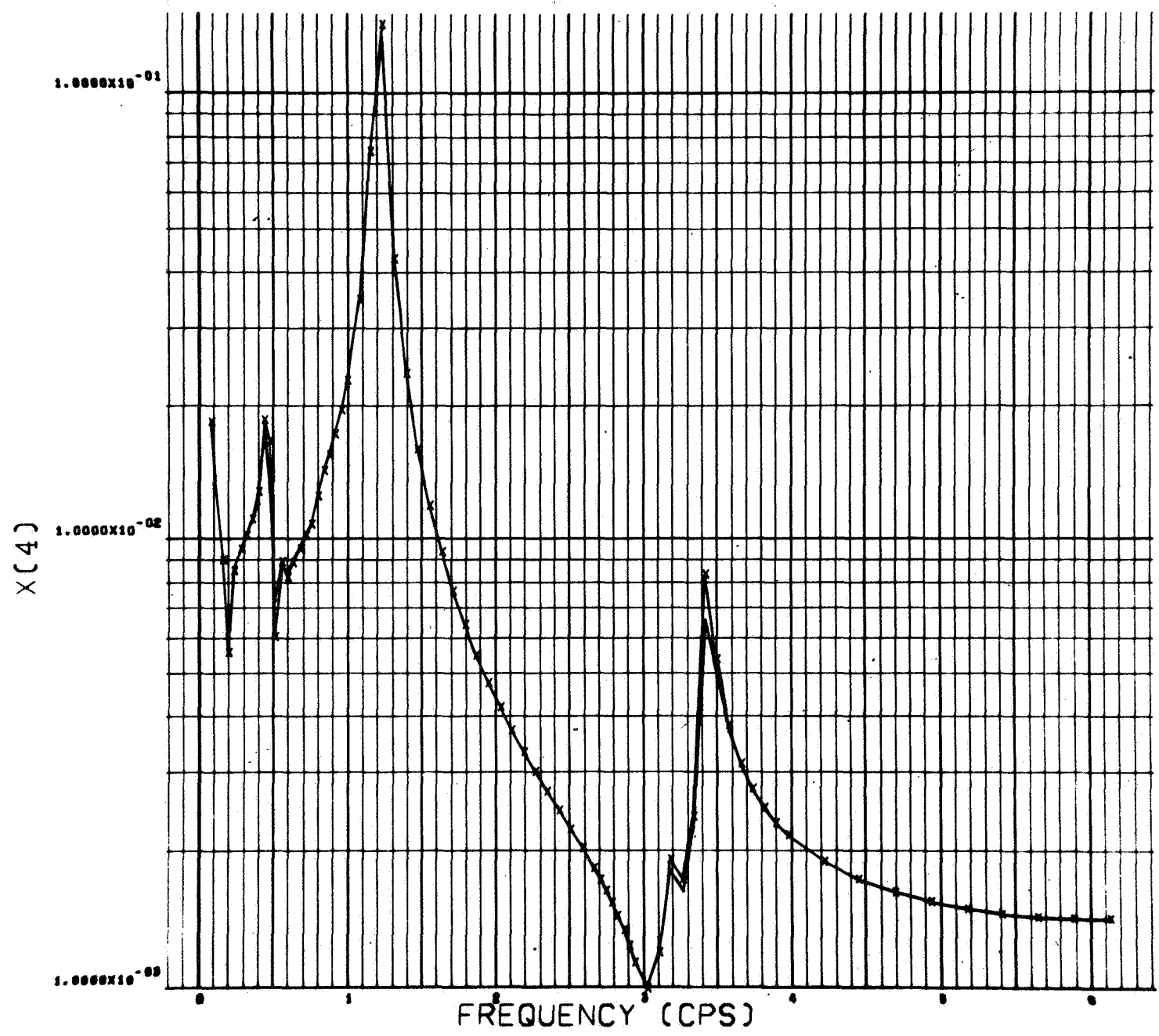
Figure 7a



t = 61 seconds

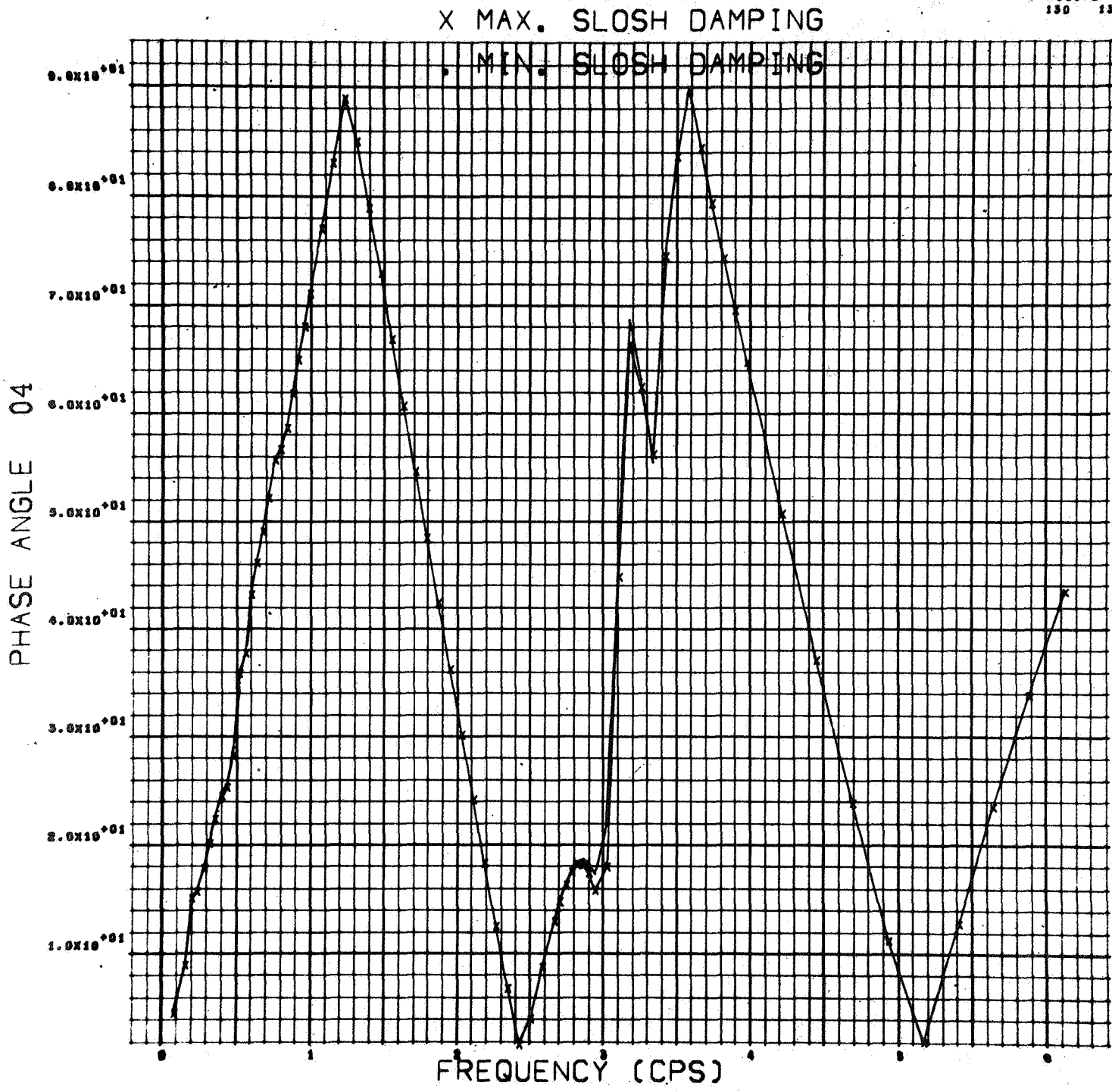
Figure 7b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



t = 61 seconds

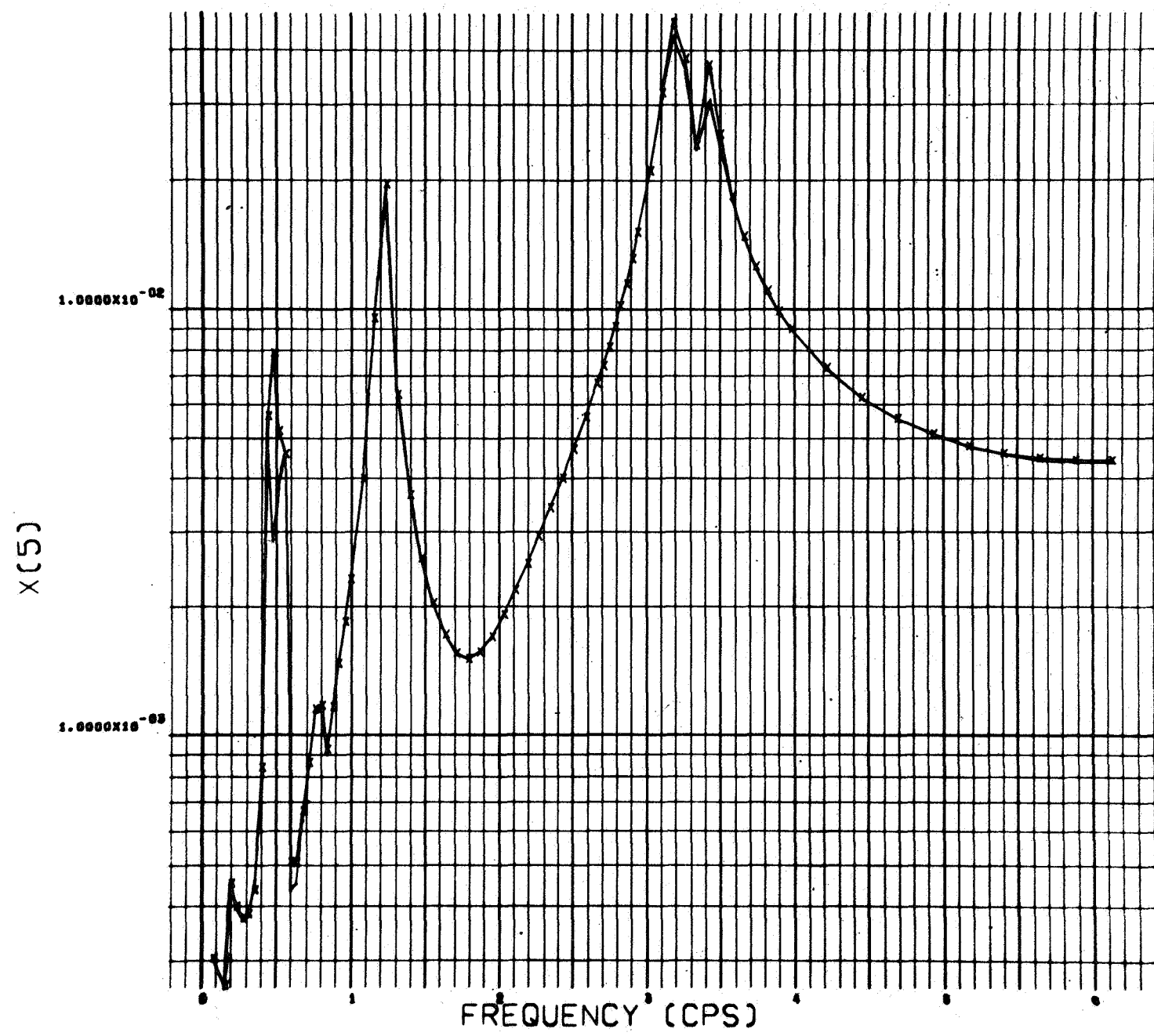
Figure 8a



t = 61 seconds

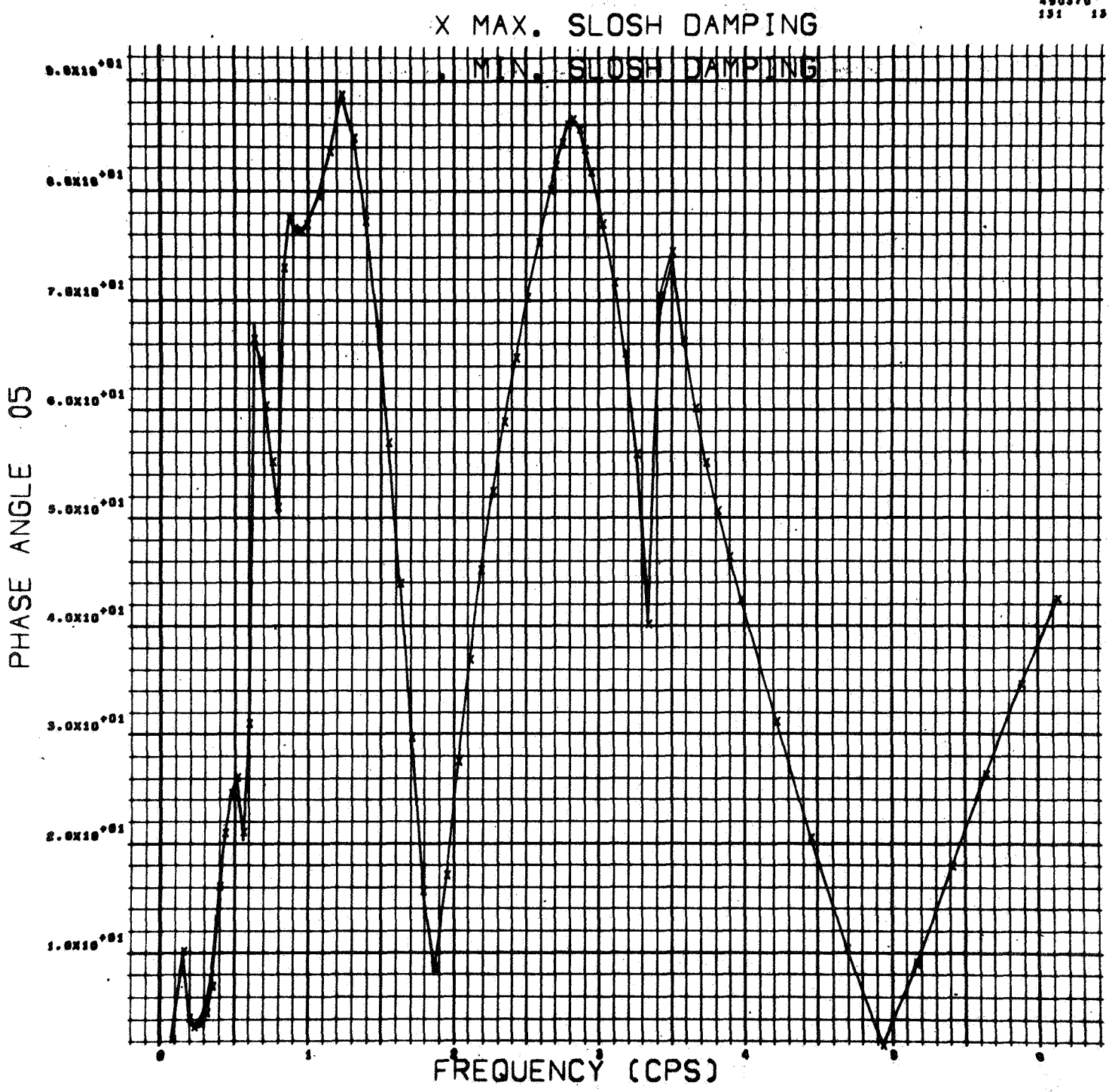
Figure 8b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING



t = 61 seconds

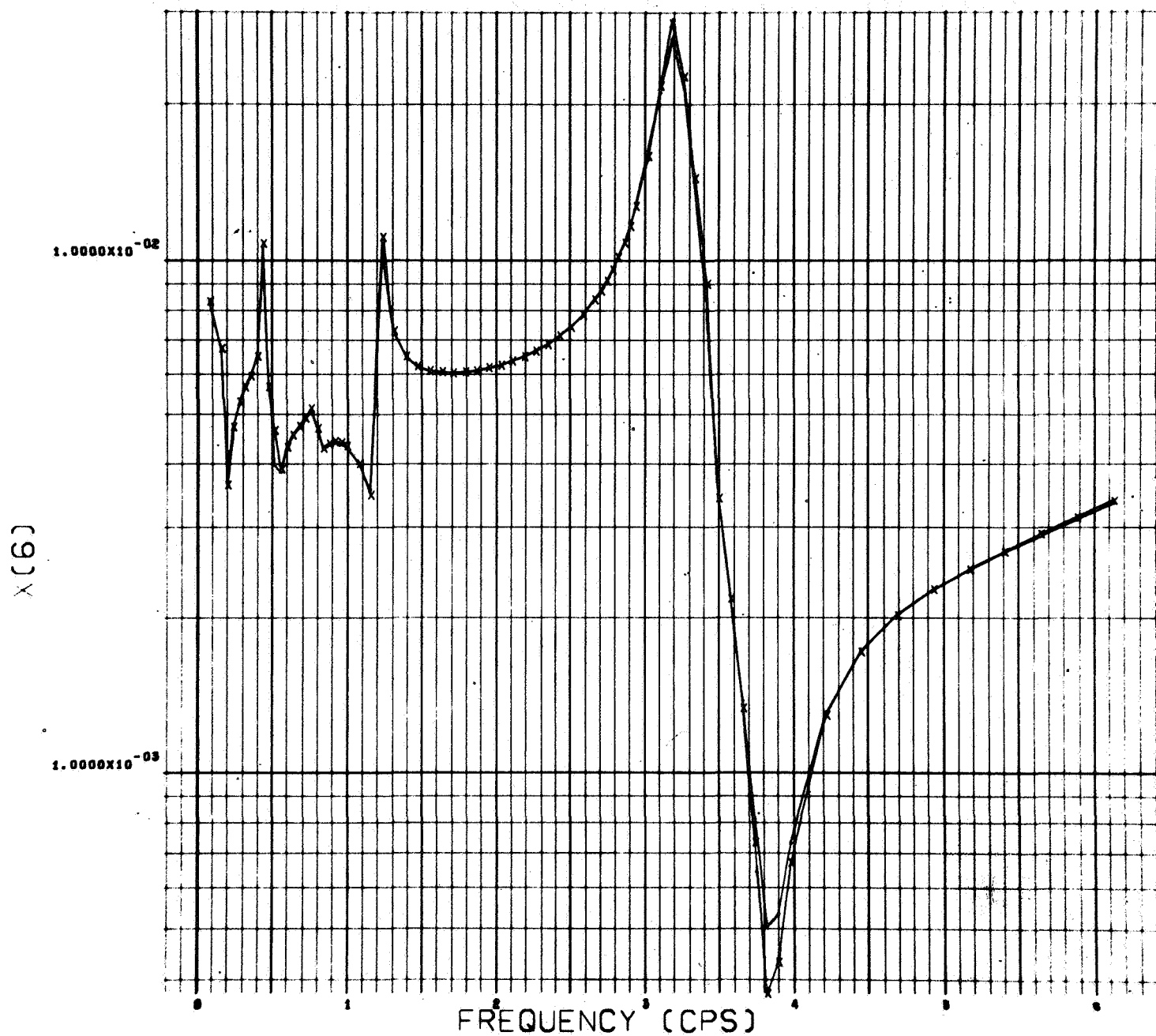
Figure 9a



t = 61 seconds

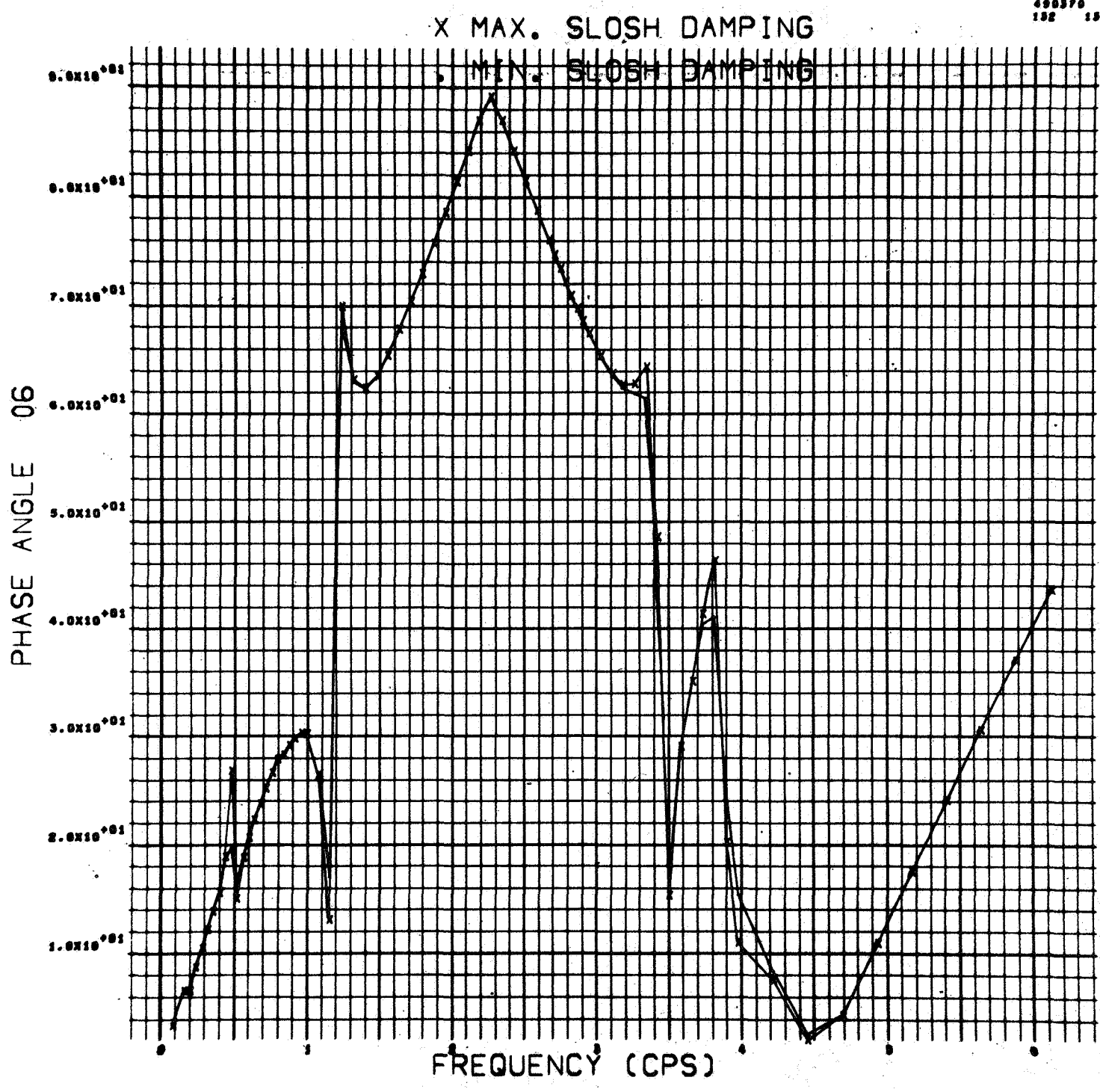
Figure 9b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 61 seconds

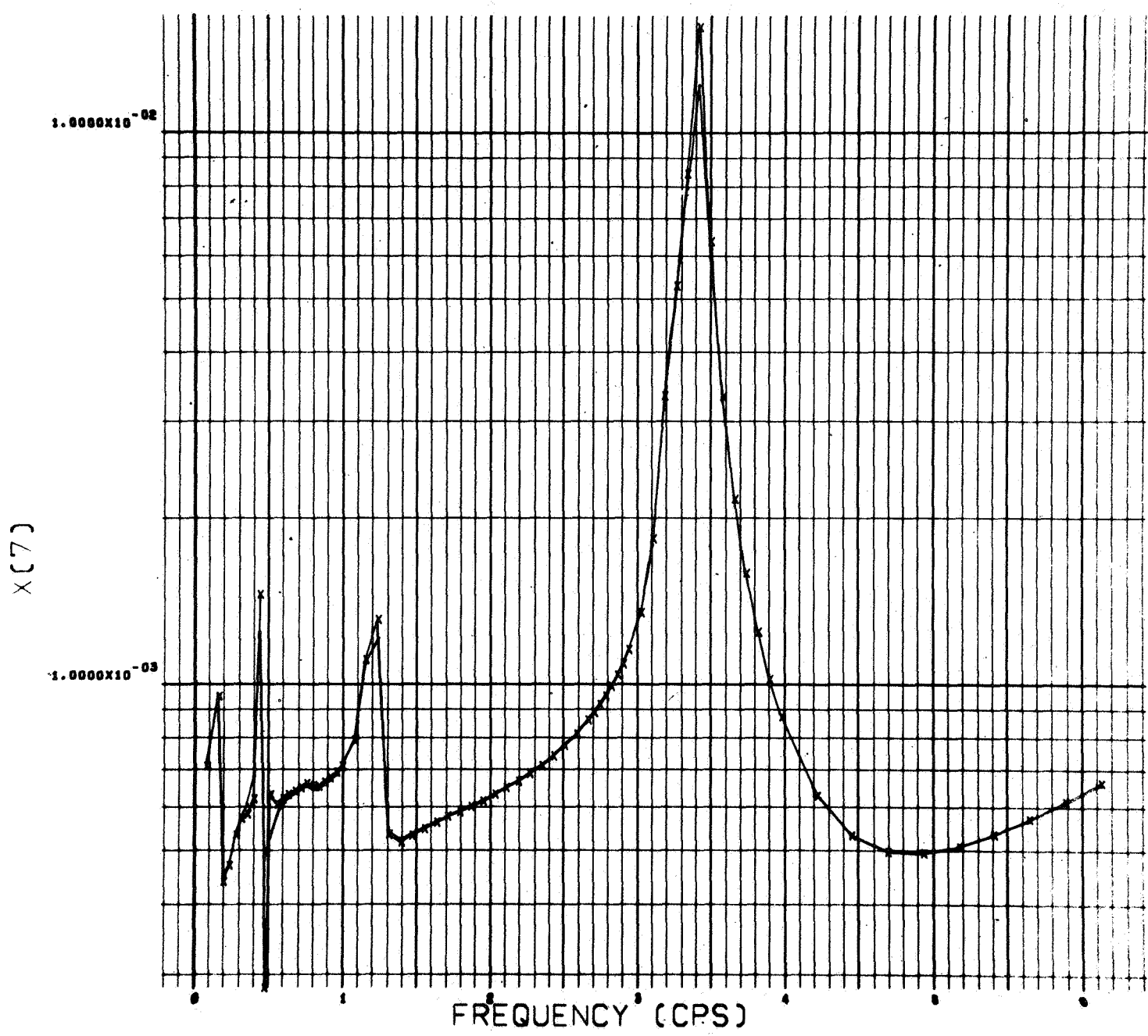
Figure 10a



t = 61 seconds

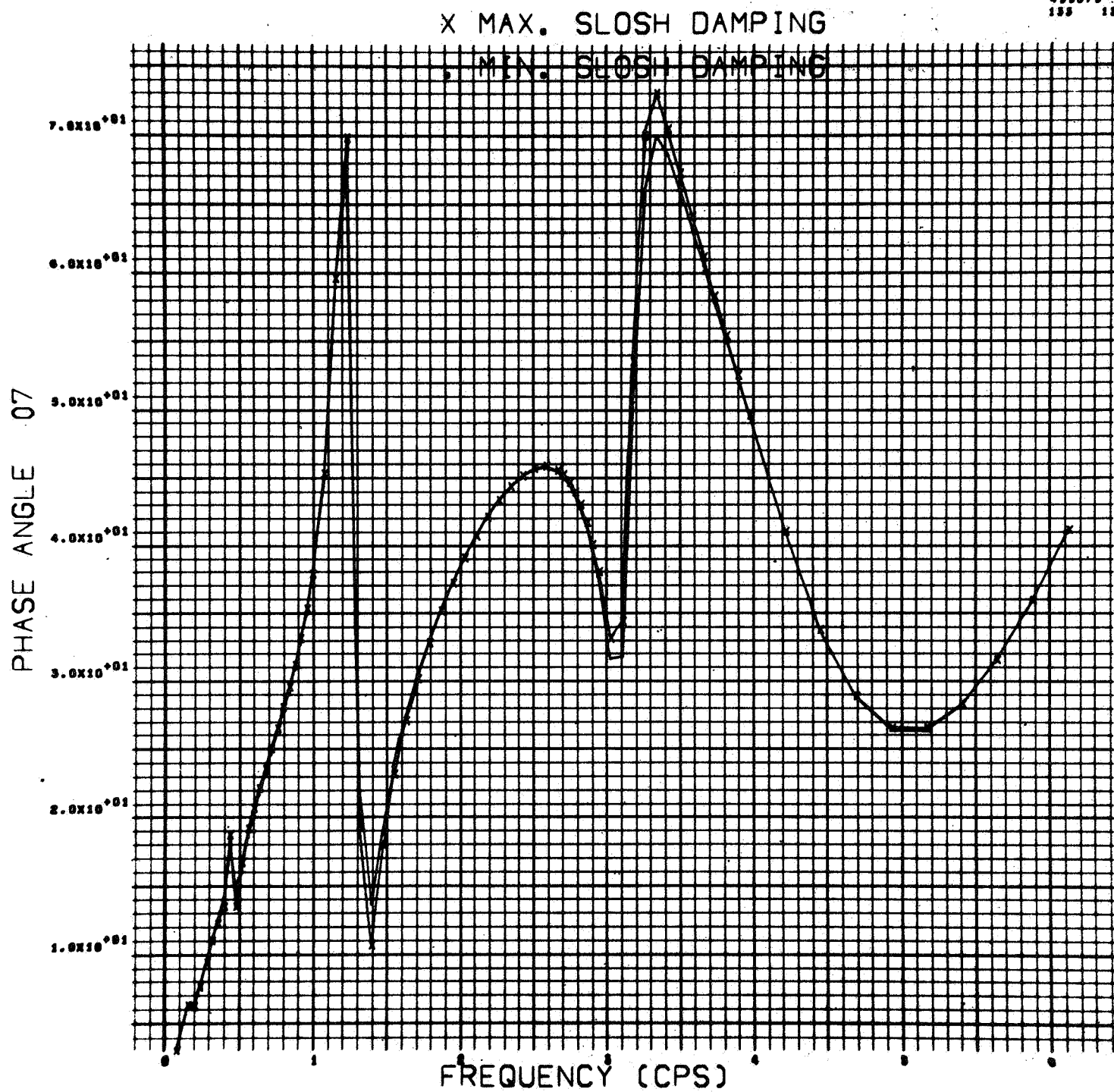
Figure 10b

X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING



t = 61 seconds

Figure 11a



t = 61 seconds

Figure 11b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

400376 L
100 100

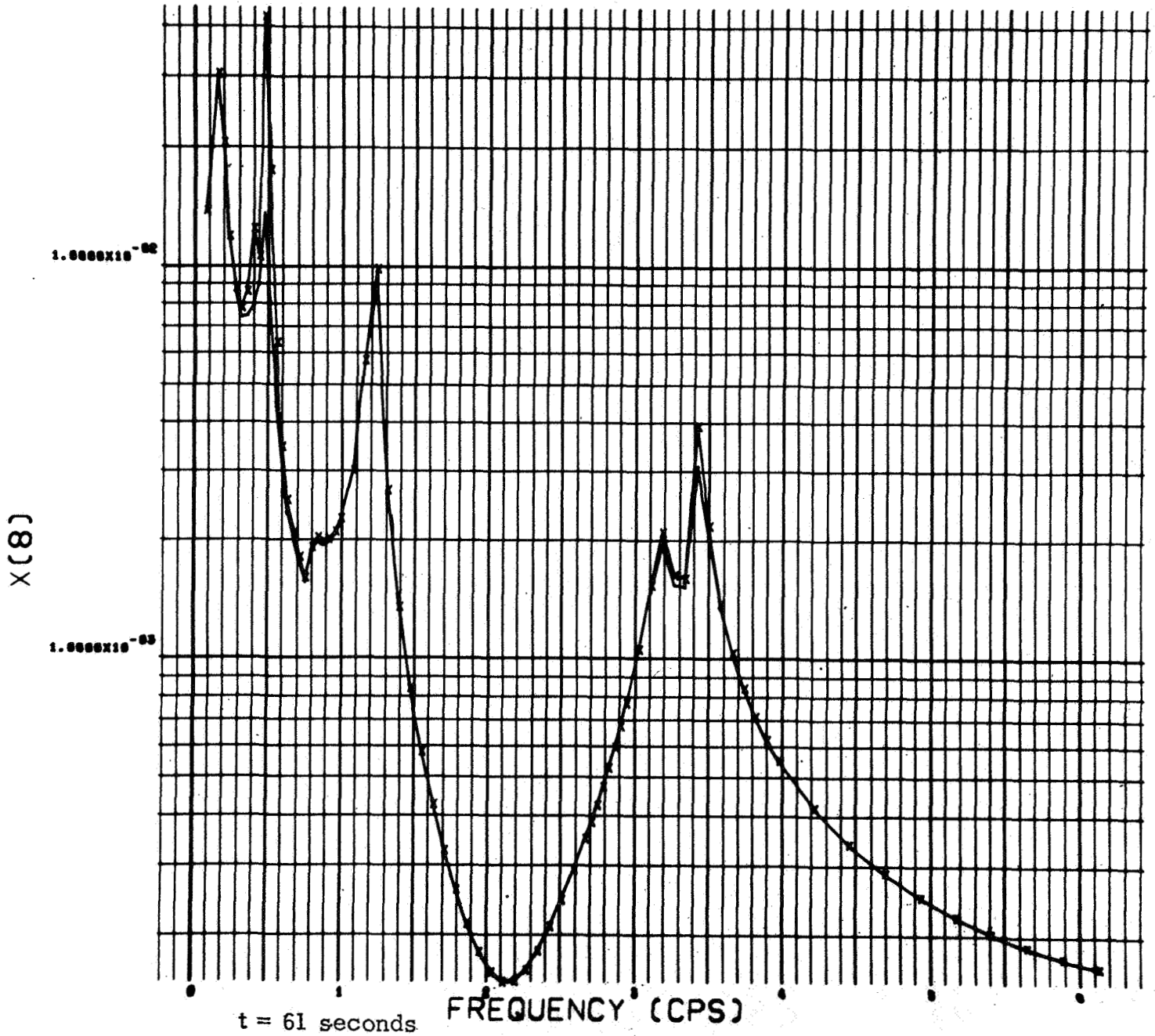
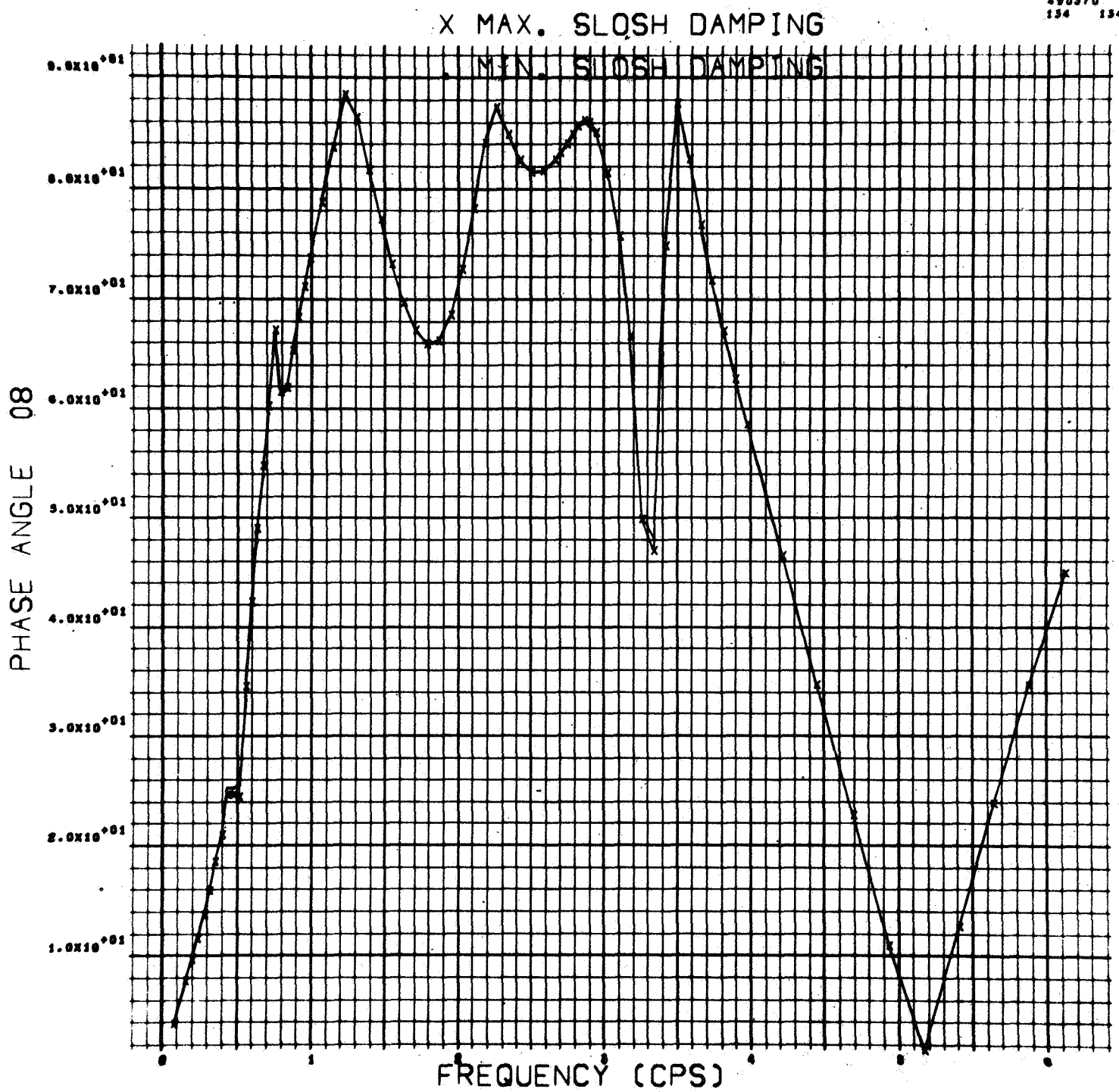


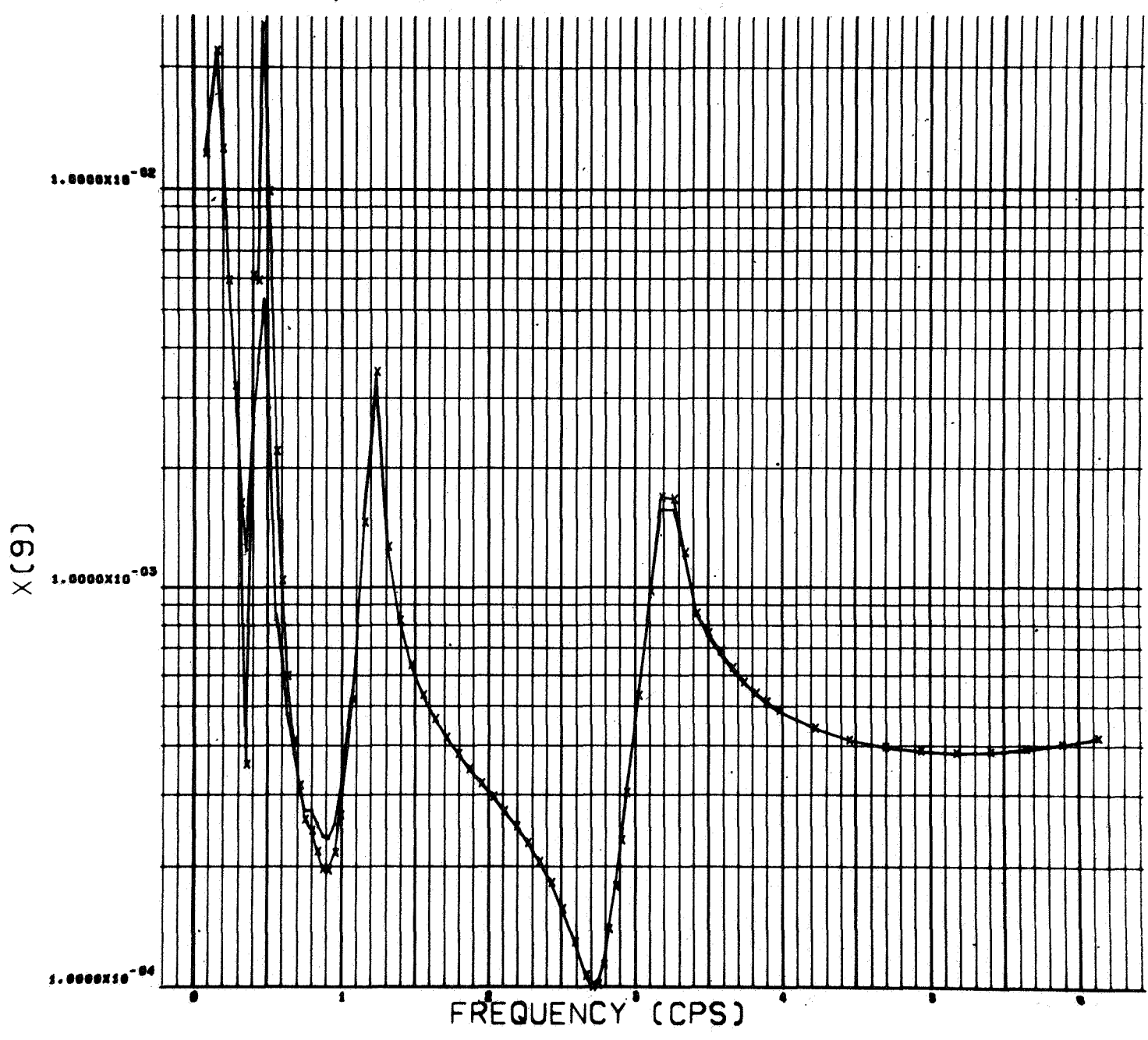
Figure 12a



t = 61 seconds

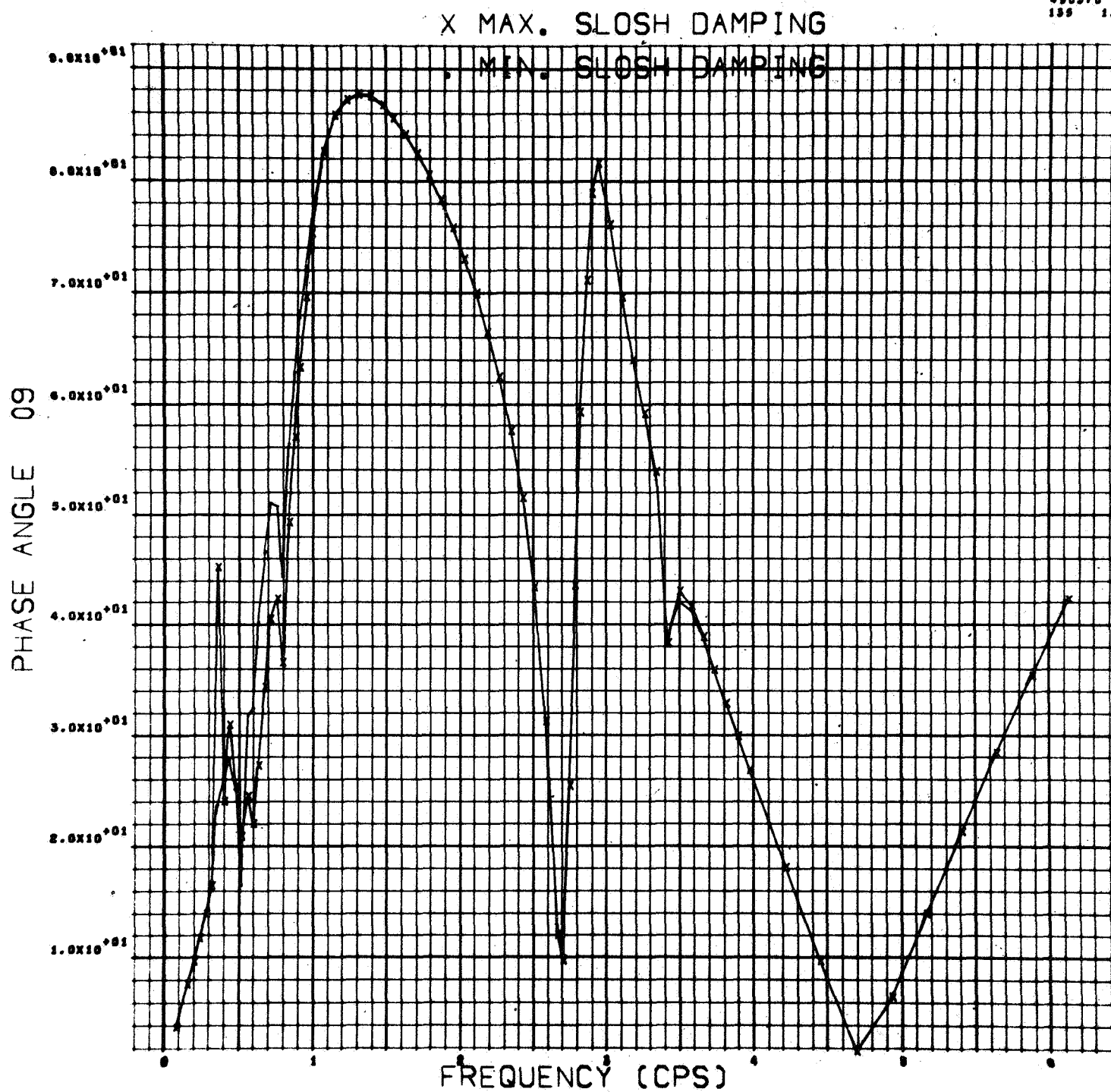
Figure 12b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING



t = 61 seconds

Figure 13a



t = 61 seconds

Figure 13b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

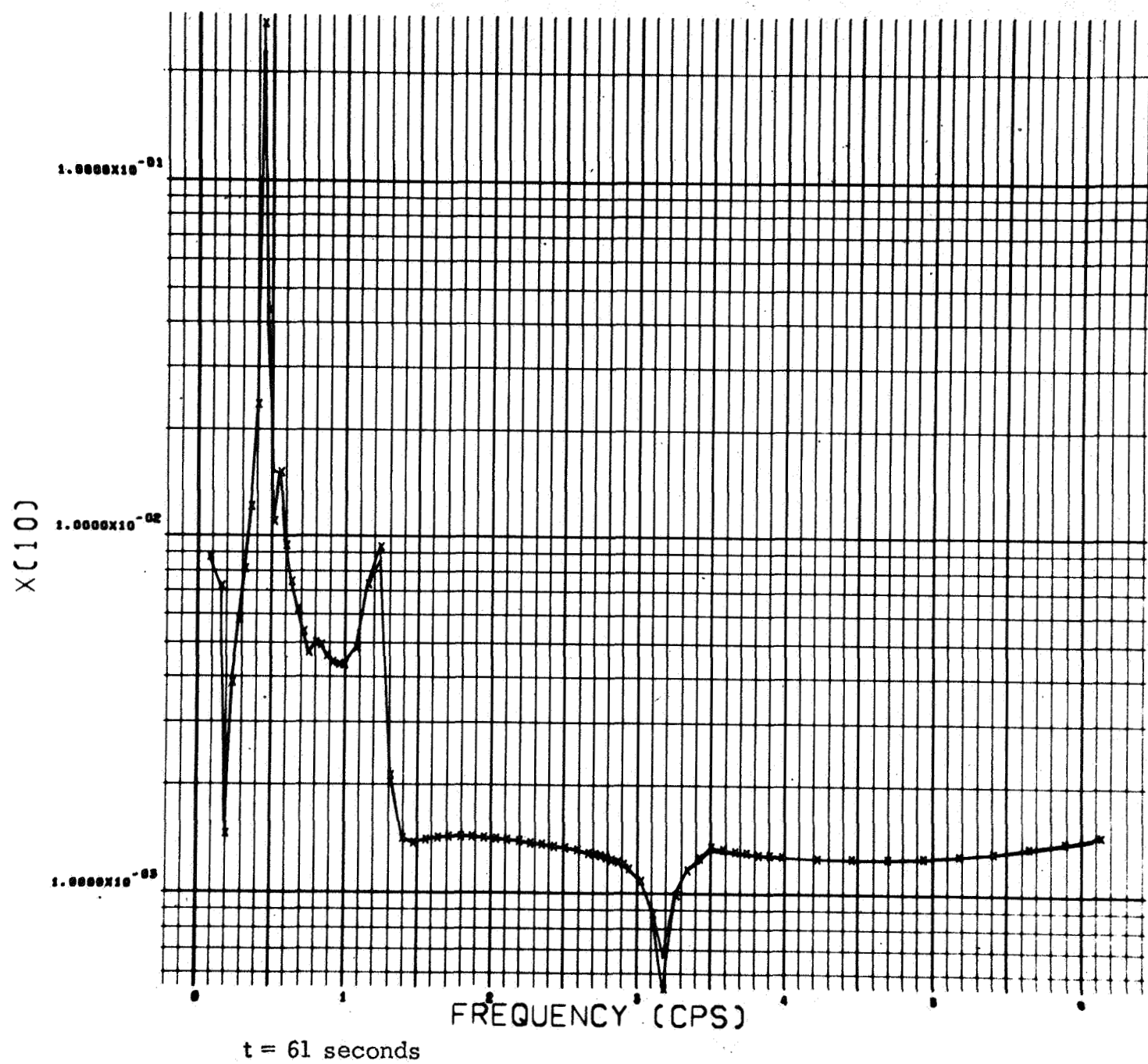
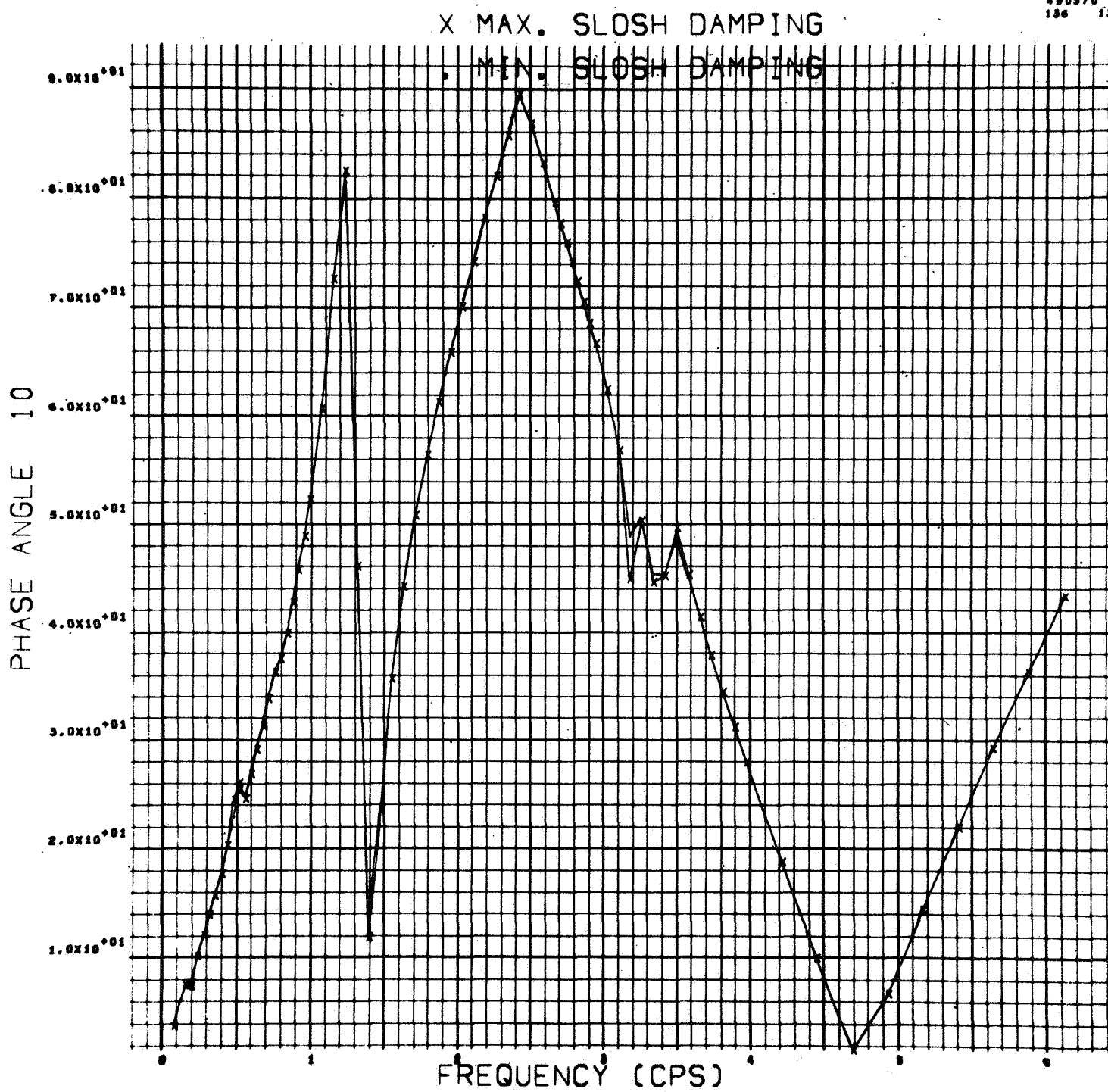


Figure 14a



t = 61 seconds

Figure 14b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING

400370
 198 198

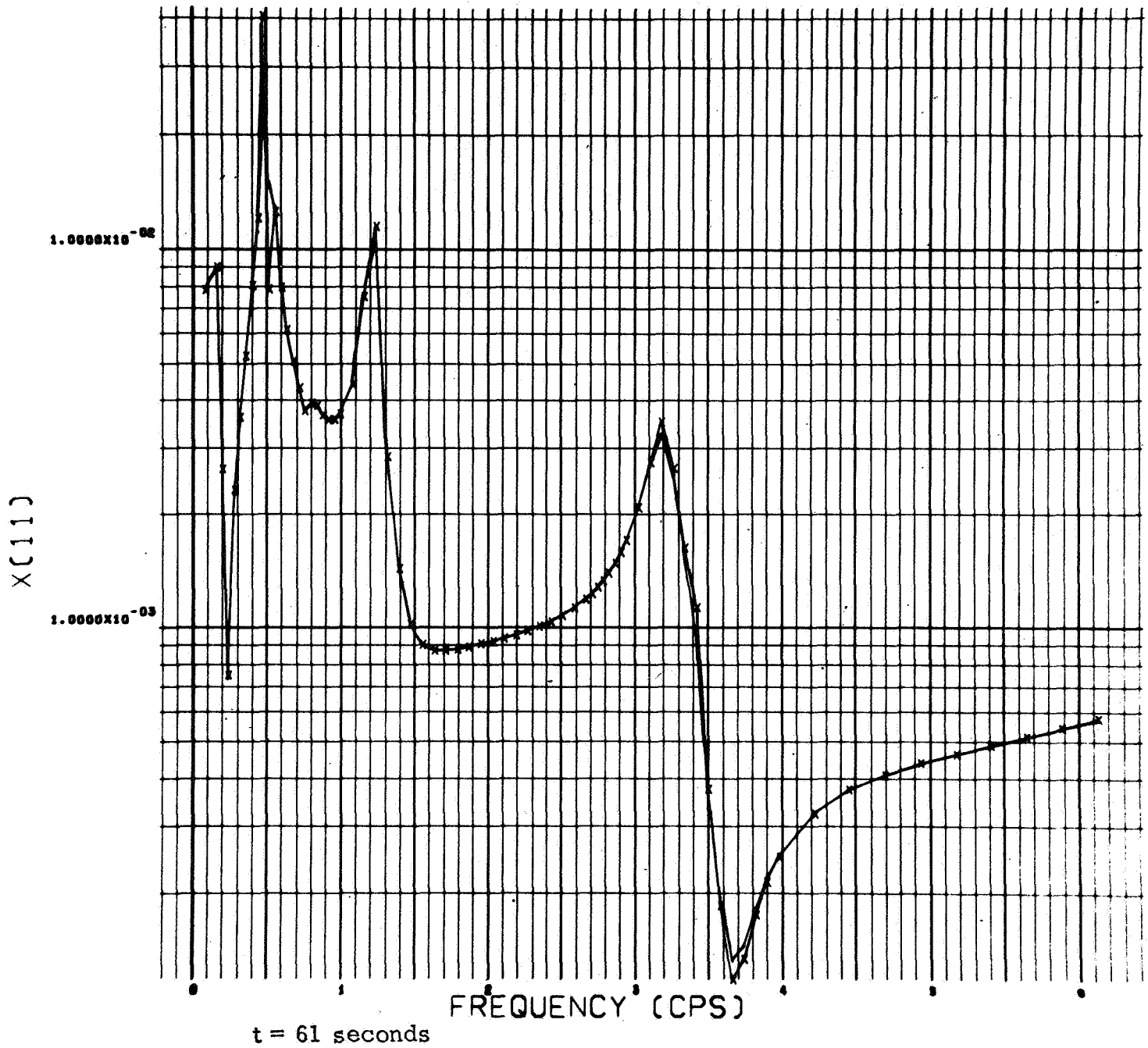
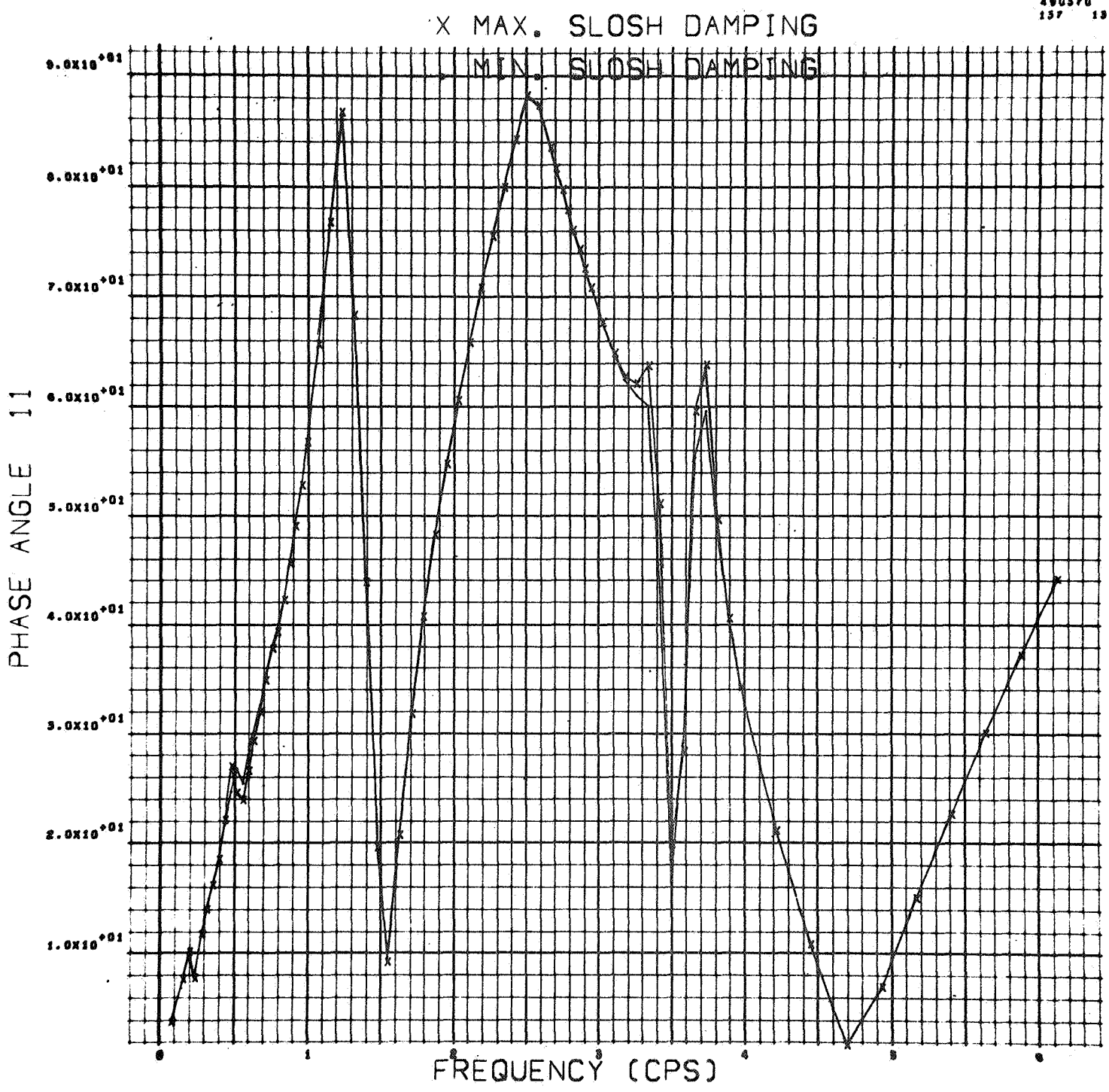


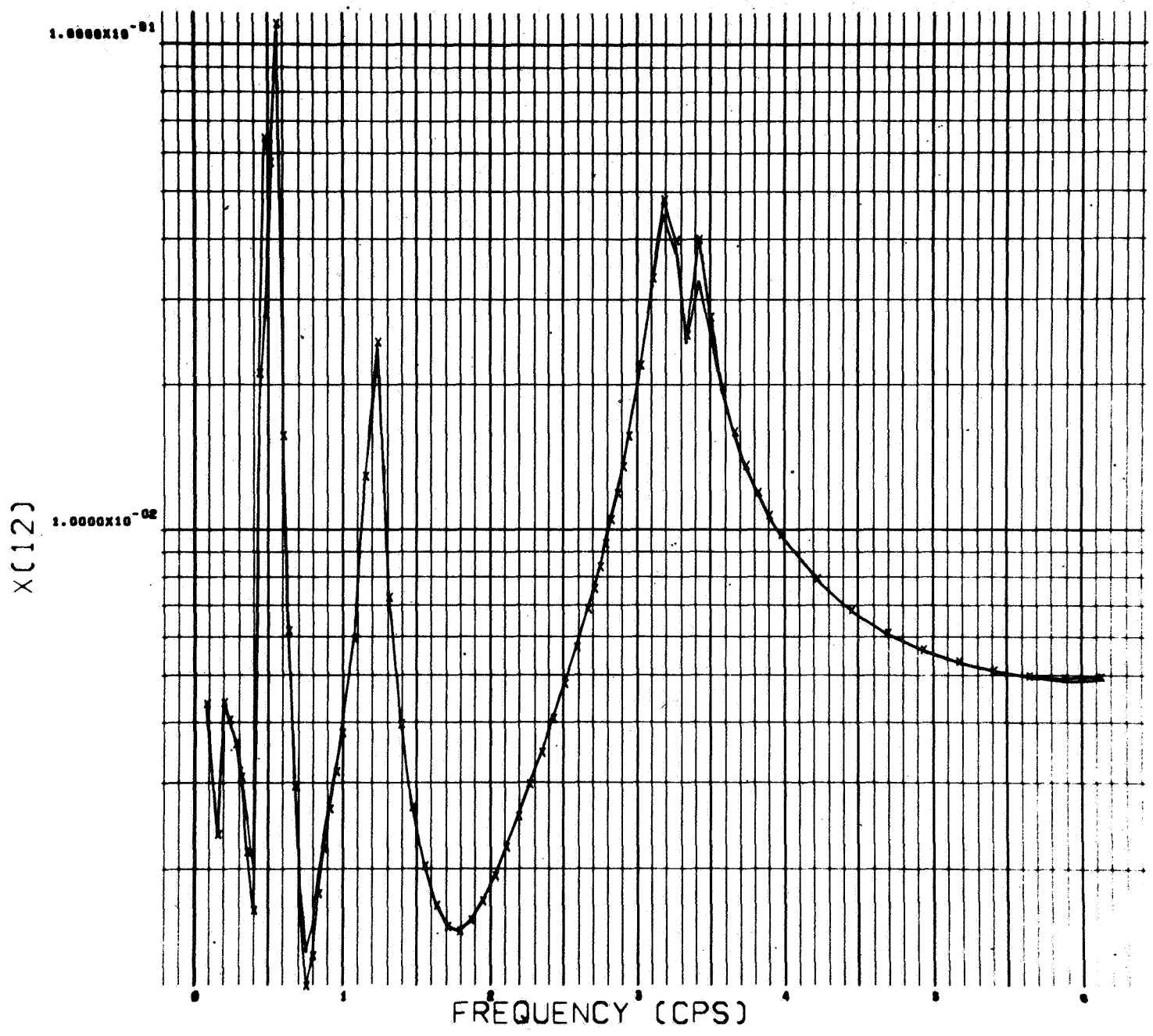
Figure 15a



t = 61 seconds

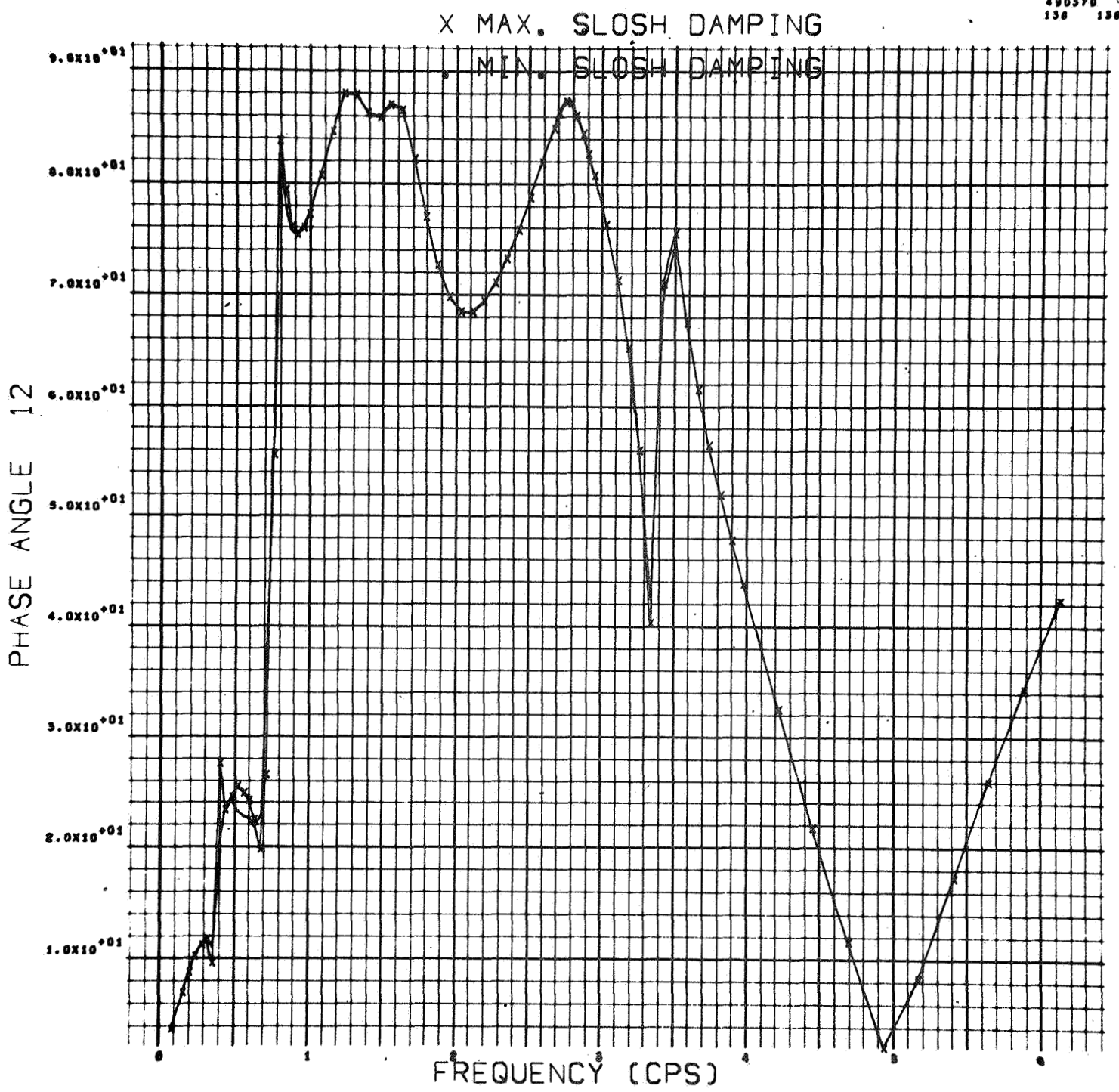
Figure 15b

X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING



t = 61 seconds

Figure 16a



t = 61 seconds

Figure 16b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

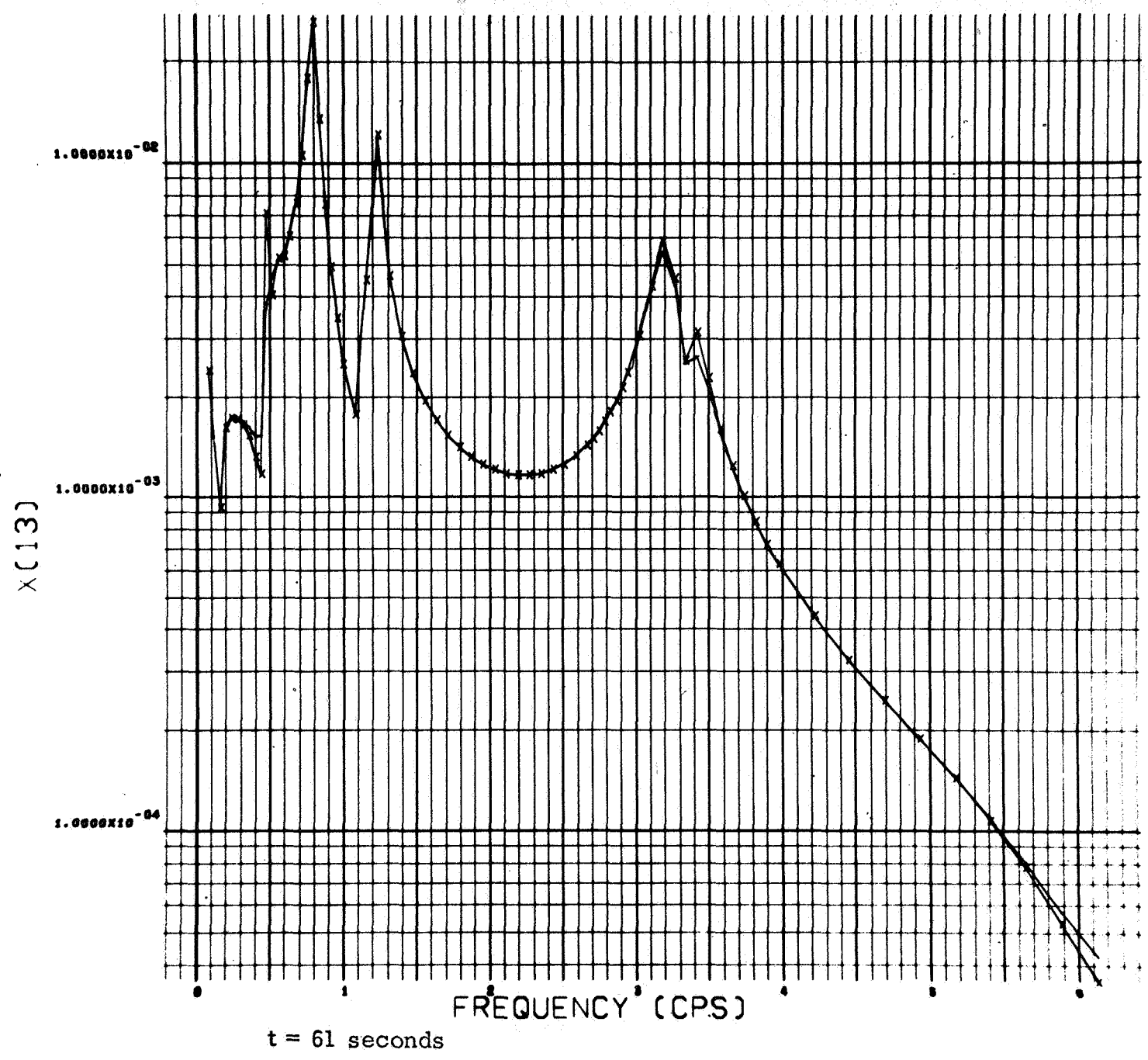
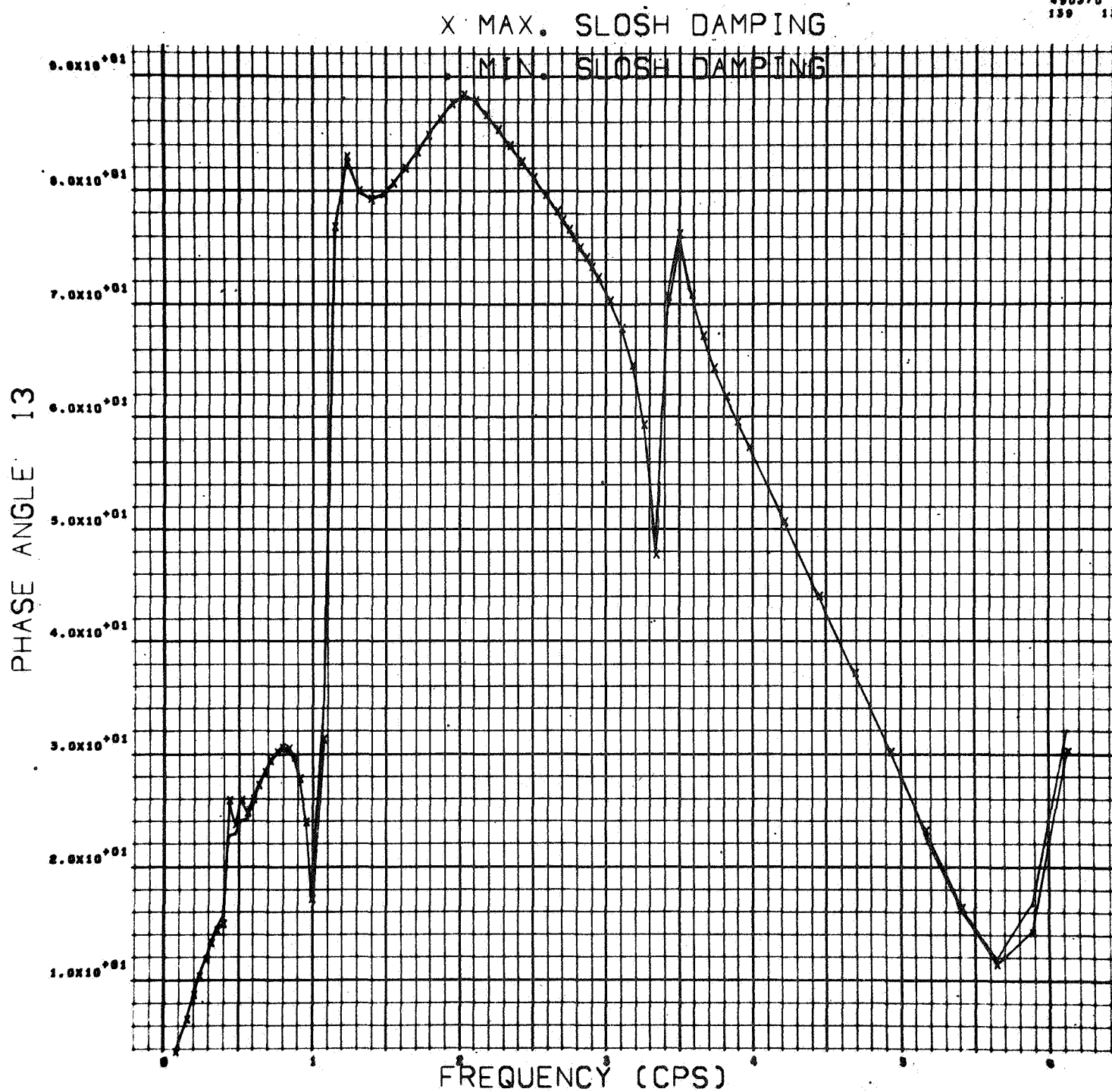
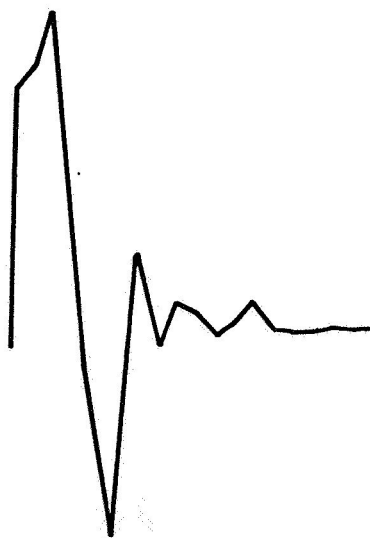


Figure 17a



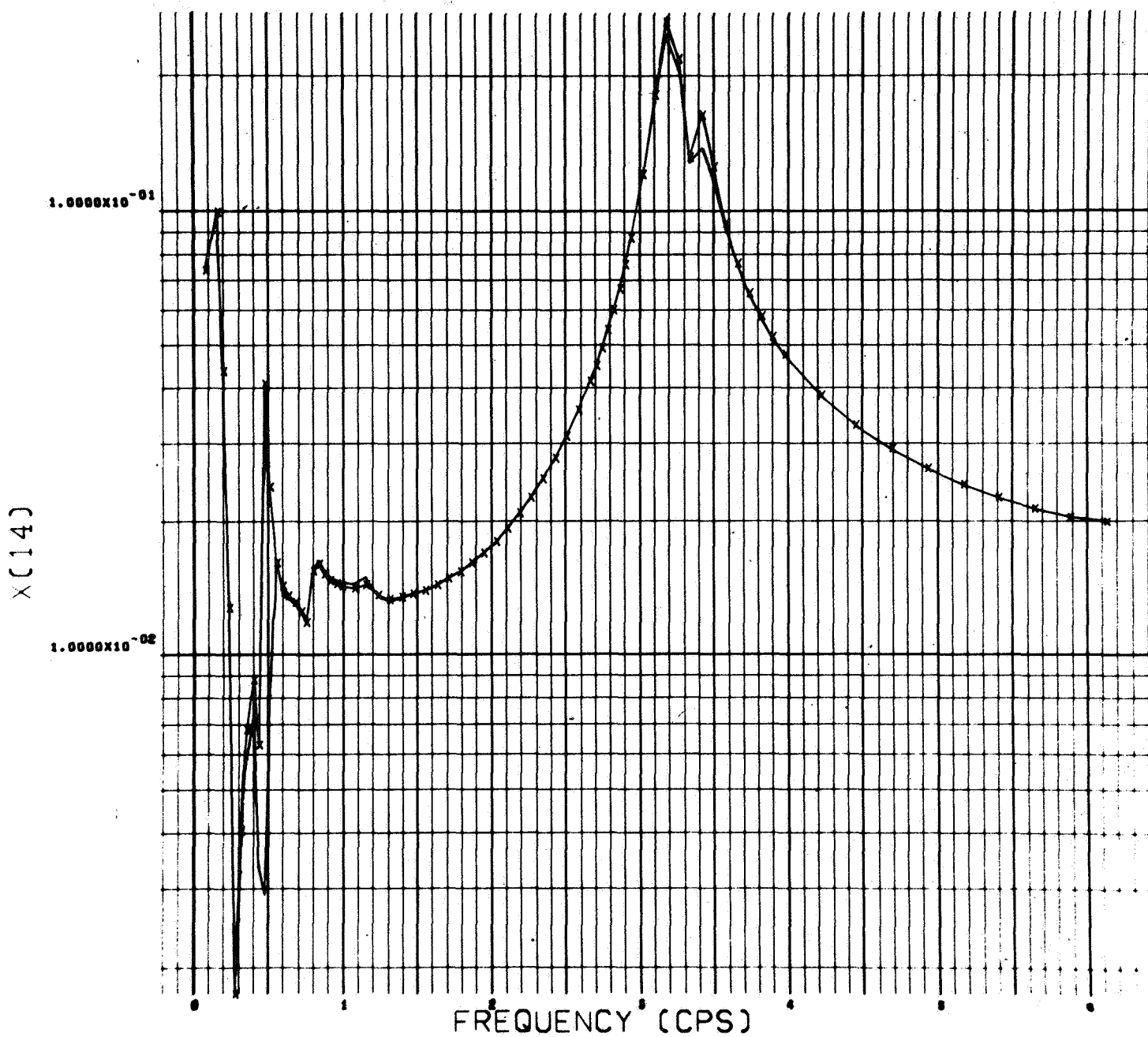
t = 61 seconds

Figure 17b



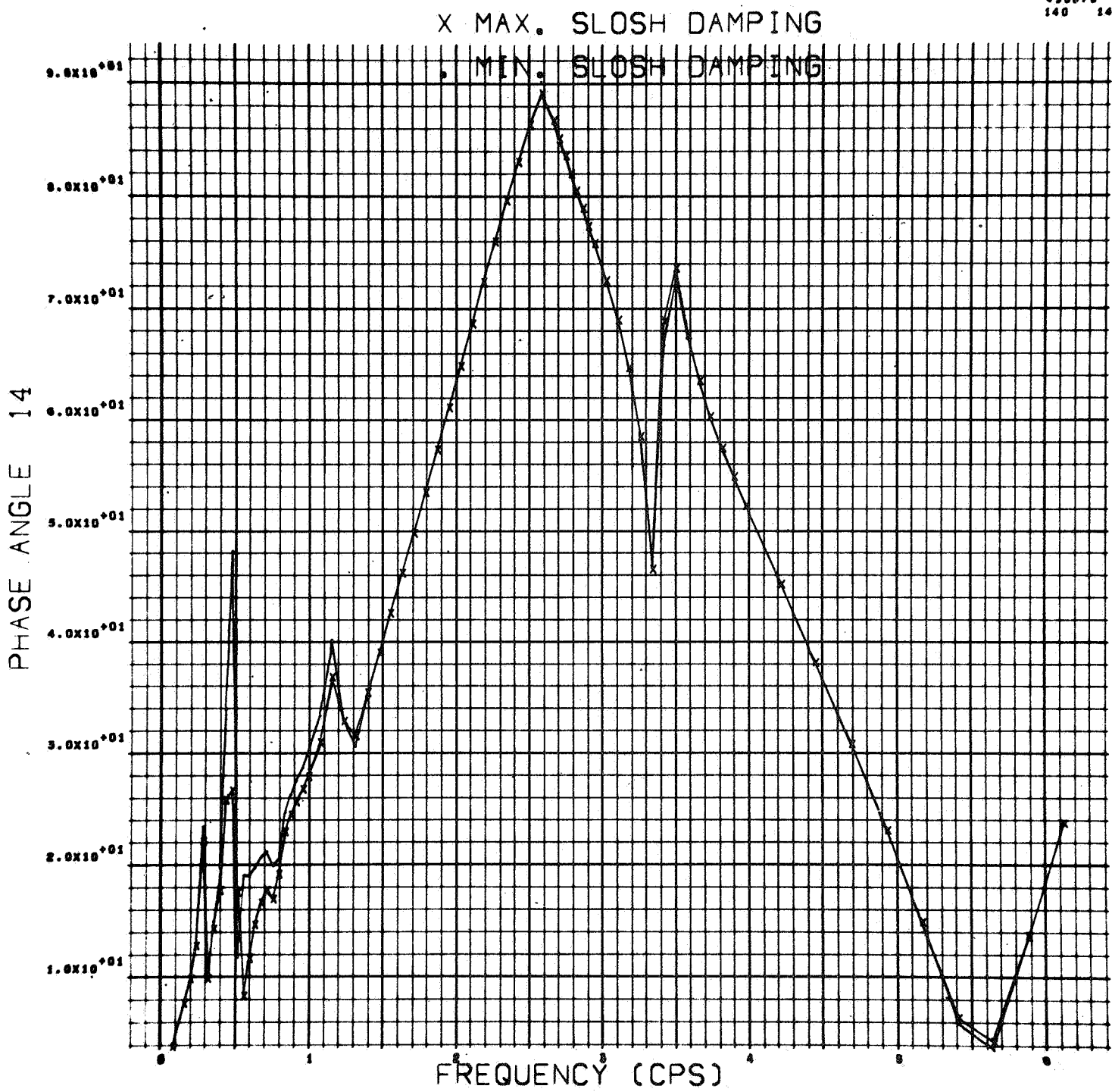
Flight Test Data
X(14) , t = 61 seconds

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING



t = 61 seconds

Figure 18a

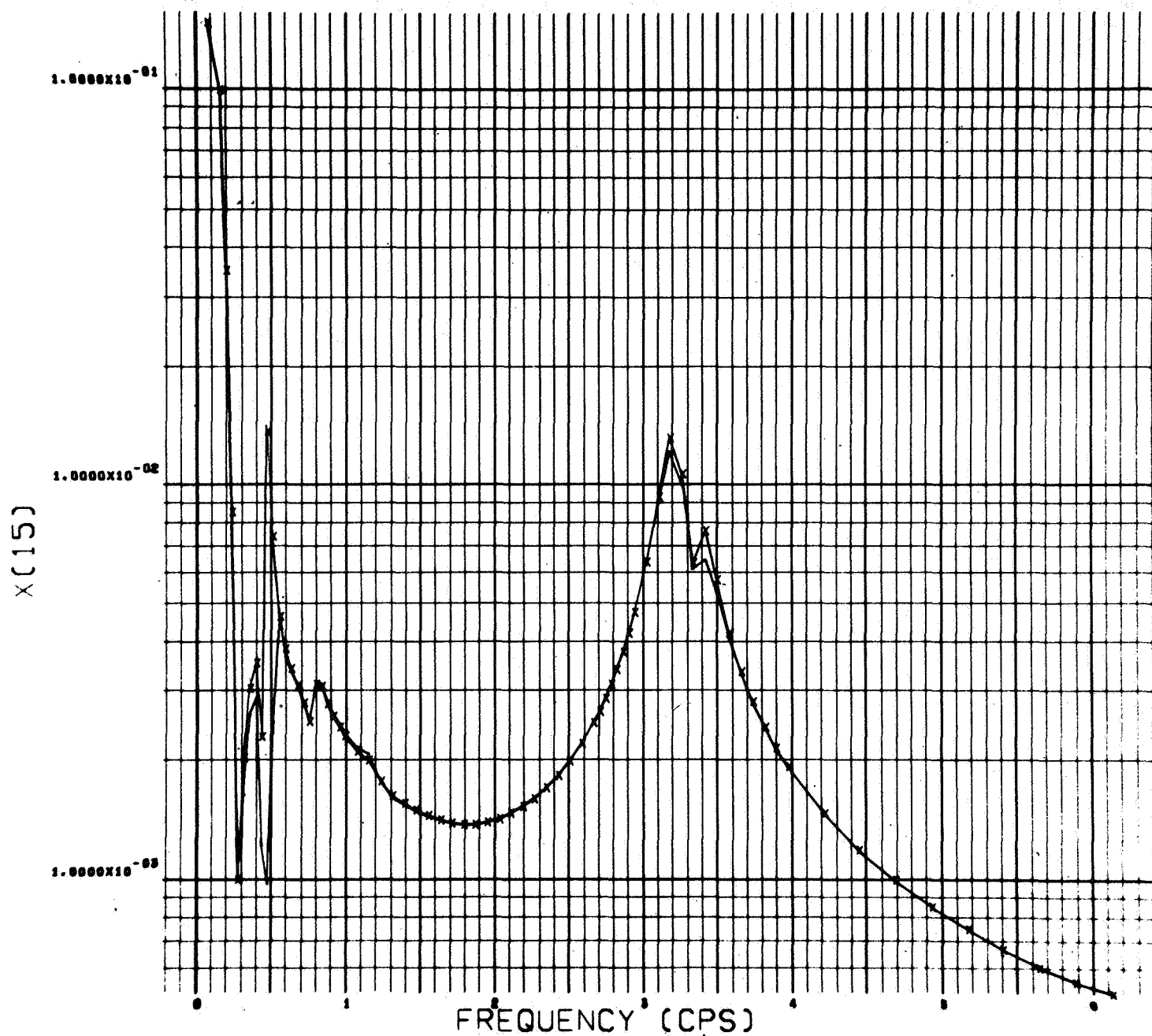


t = 61 seconds

Figure 18b

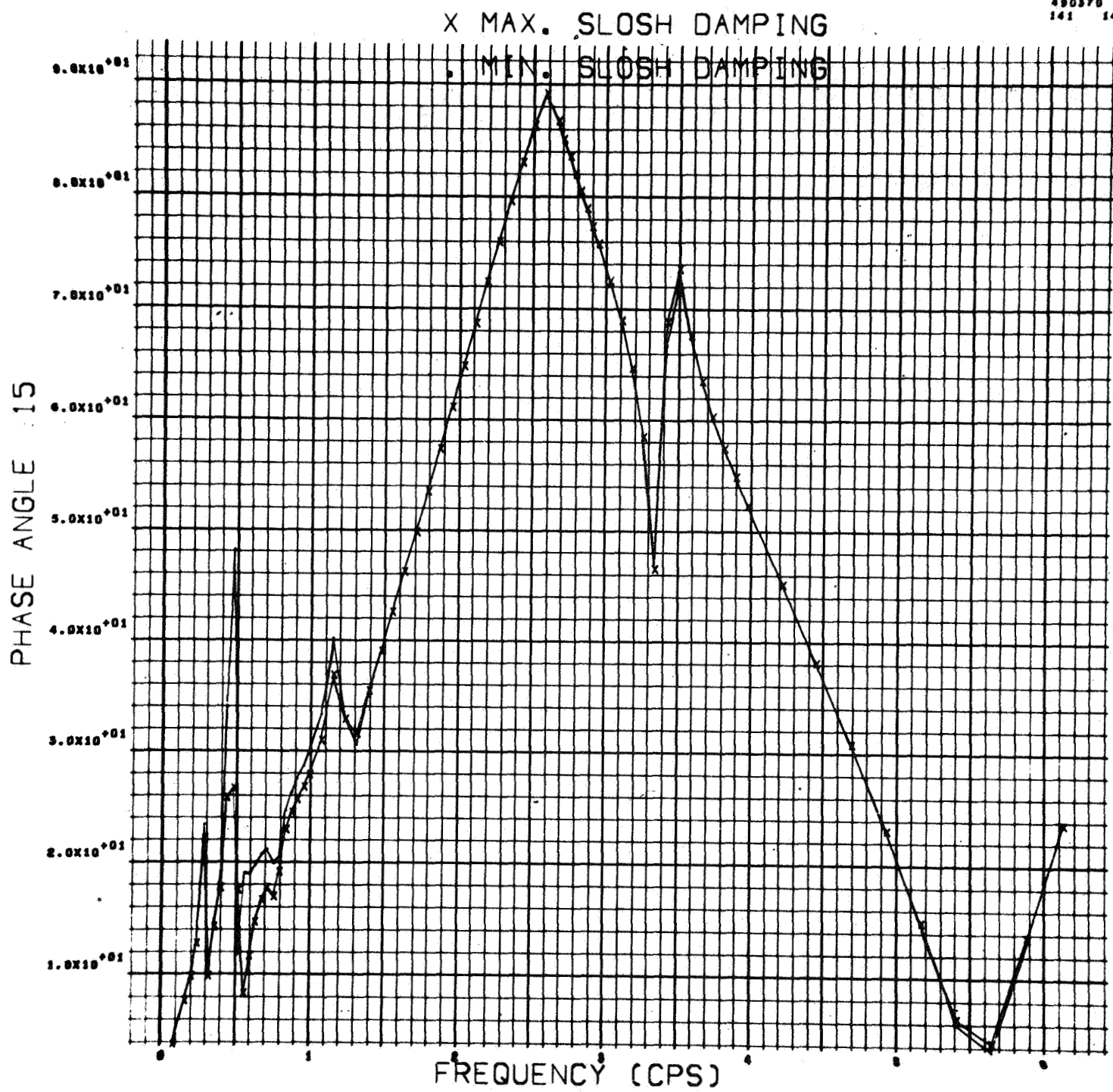
X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

490370 L
162 162



t = 61 seconds

Figure 19a



t = 61.seconds

Figure 19b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

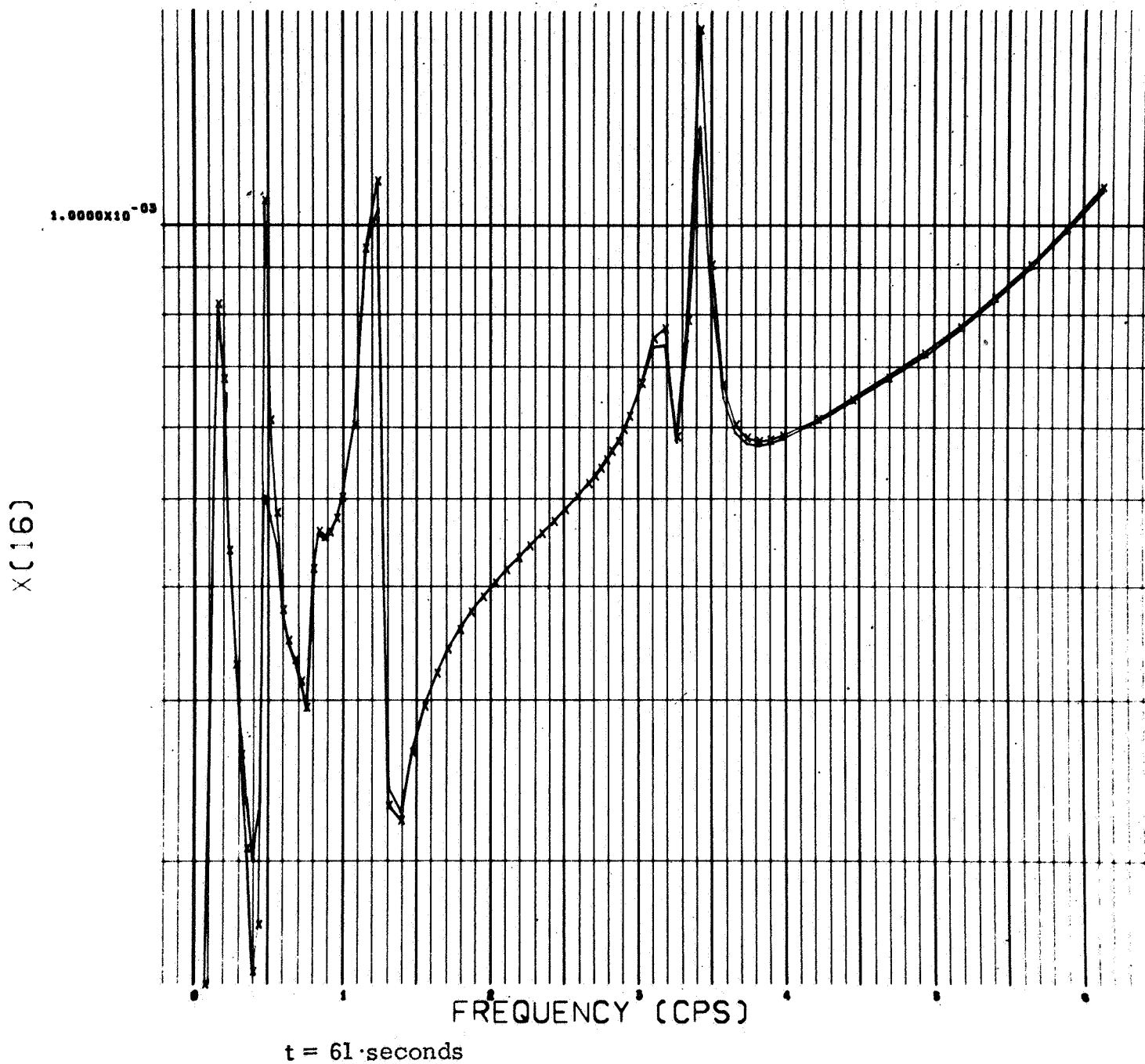


Figure 20a

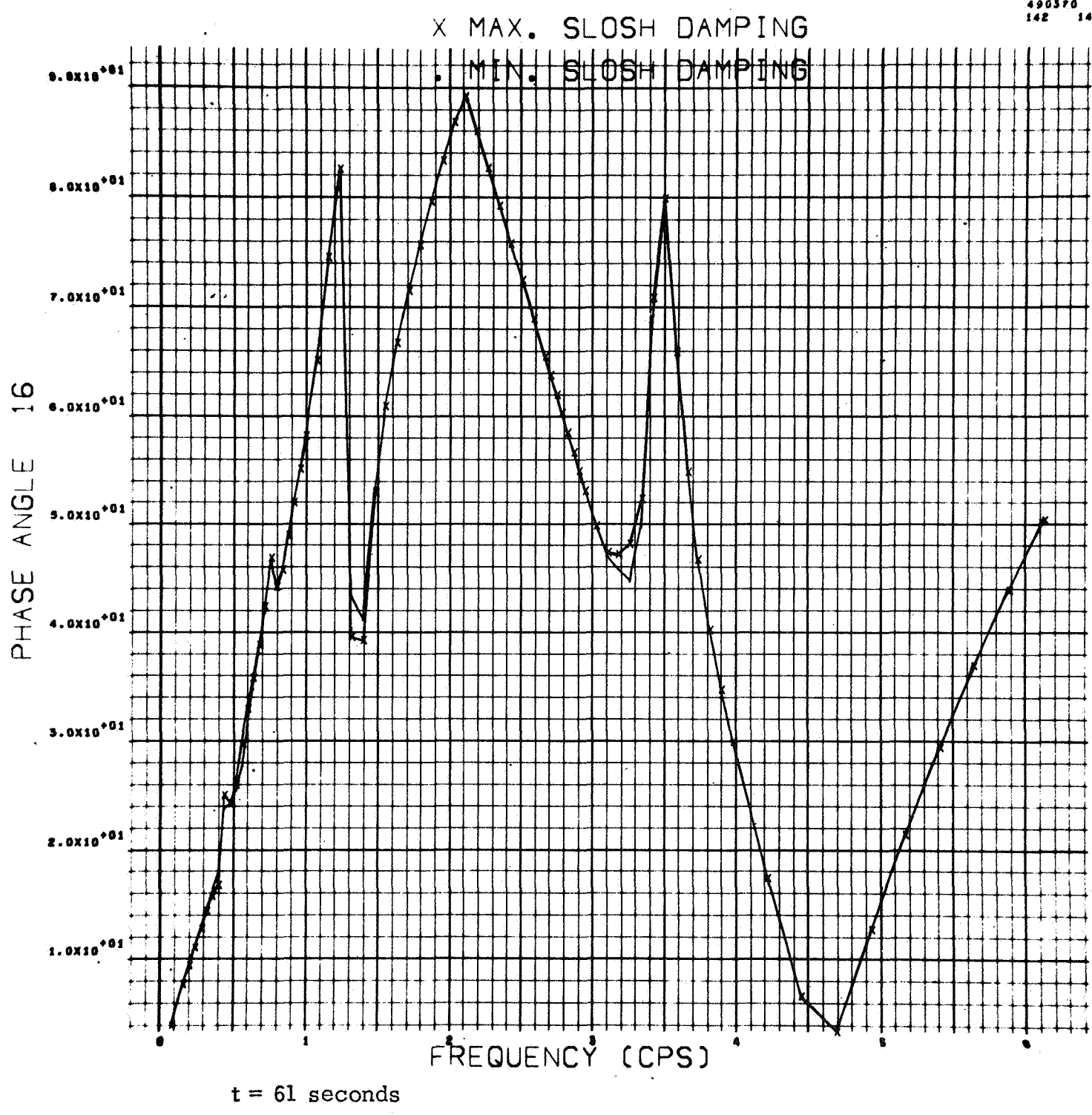
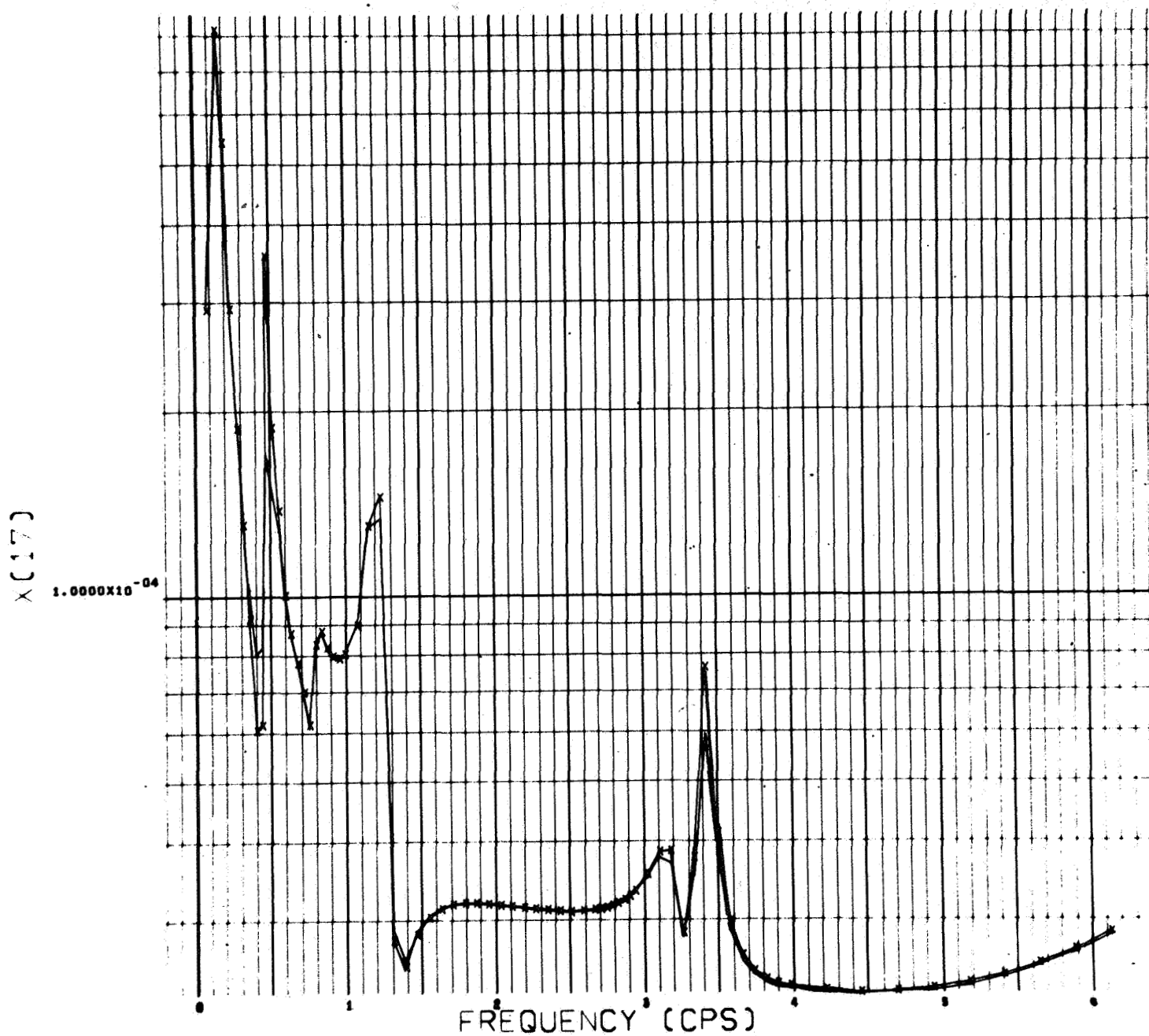


Figure 20b

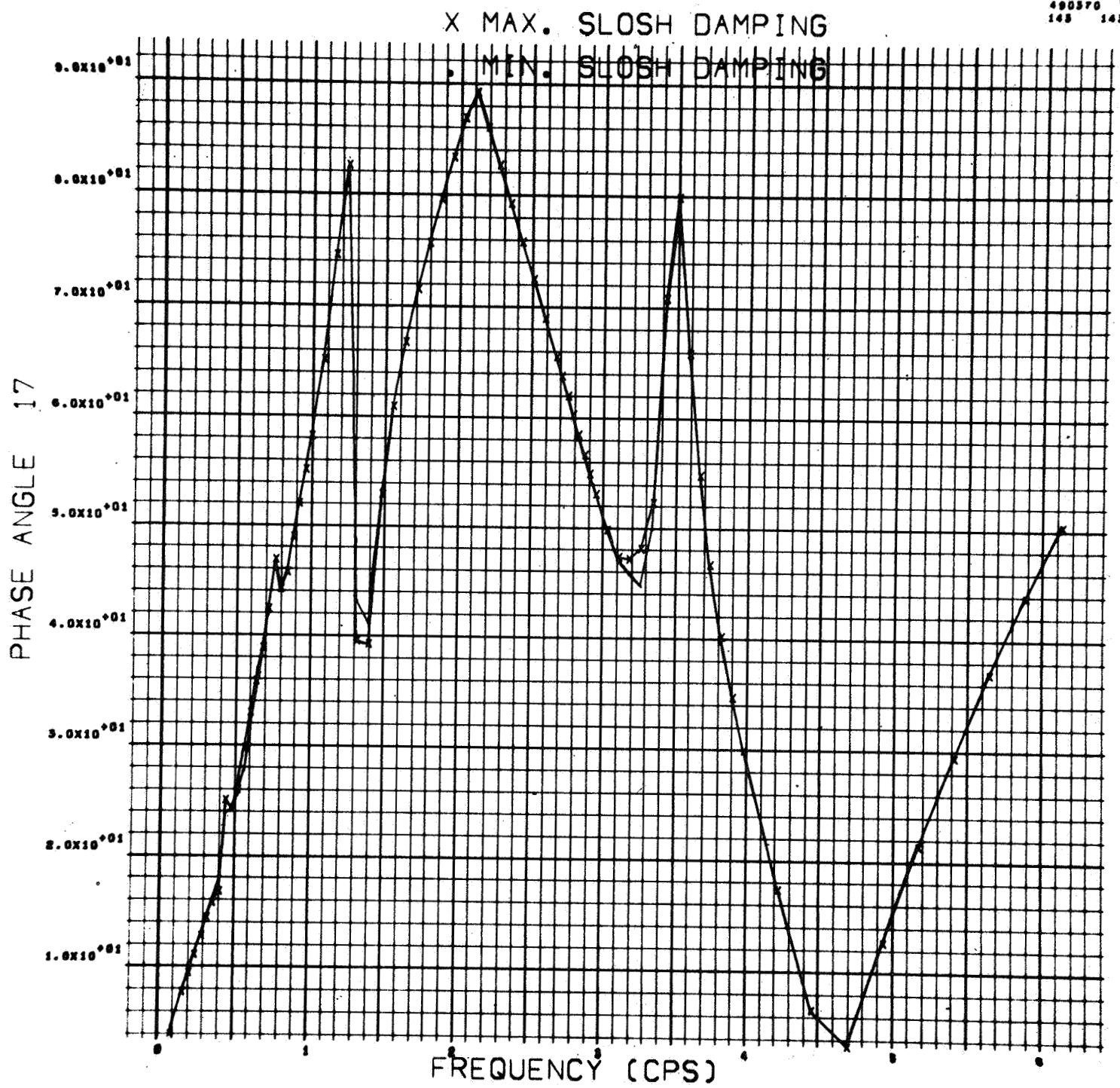
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490570
164 164



t = 61 seconds

Figure 21a



t = 61 seconds

Figure 21b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

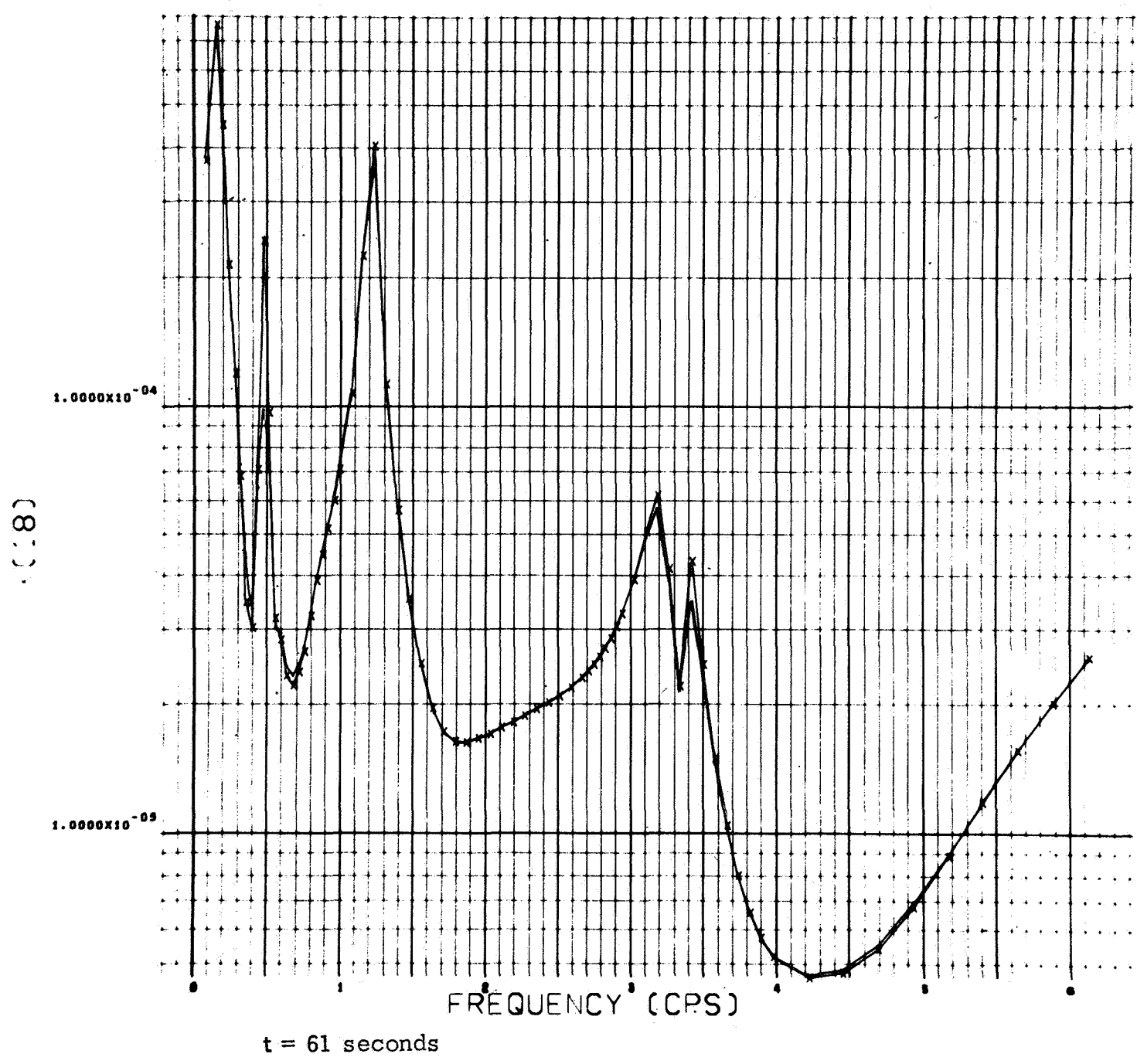
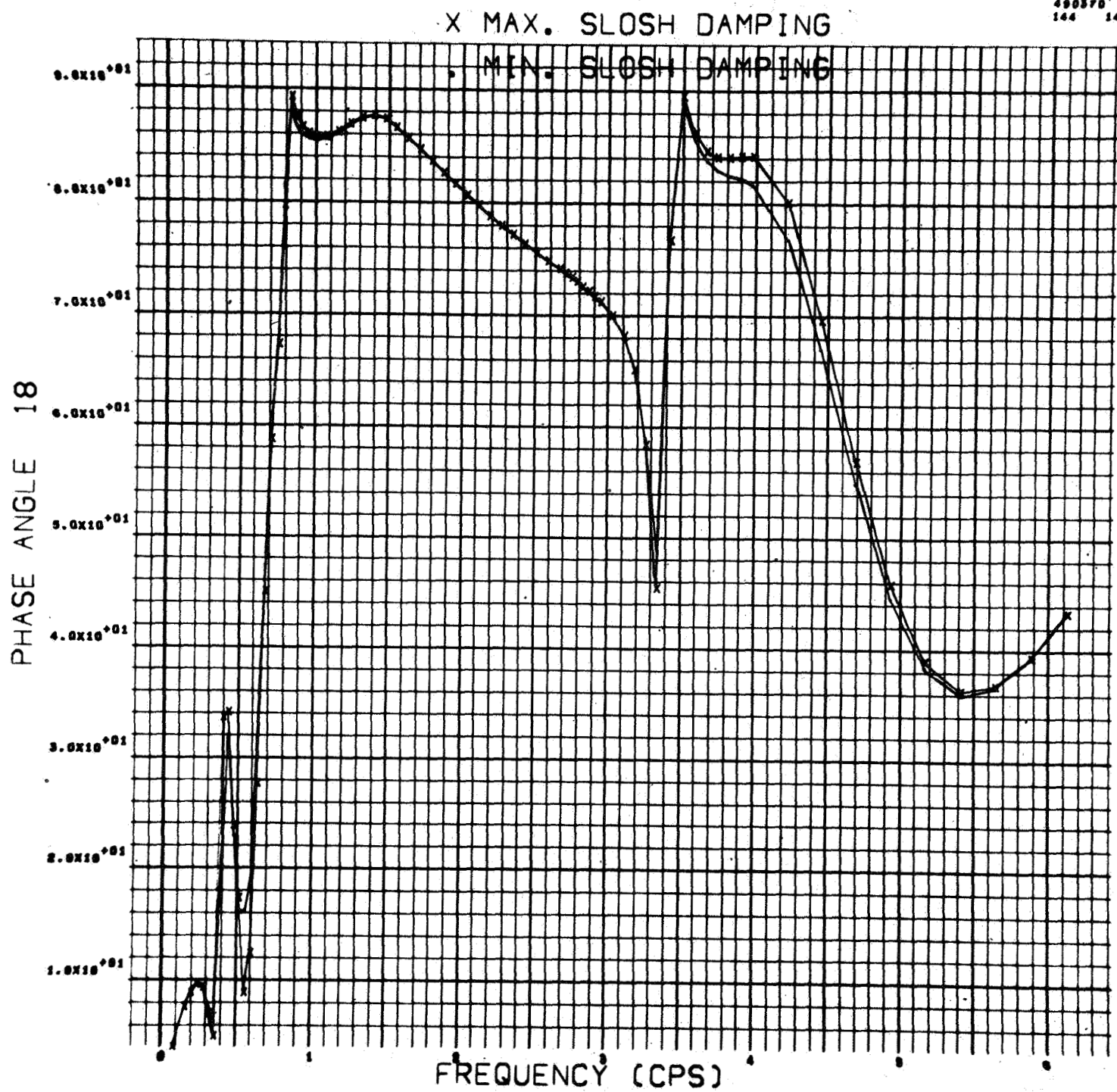


Figure 22a



t = 61 seconds

Figure 22b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING

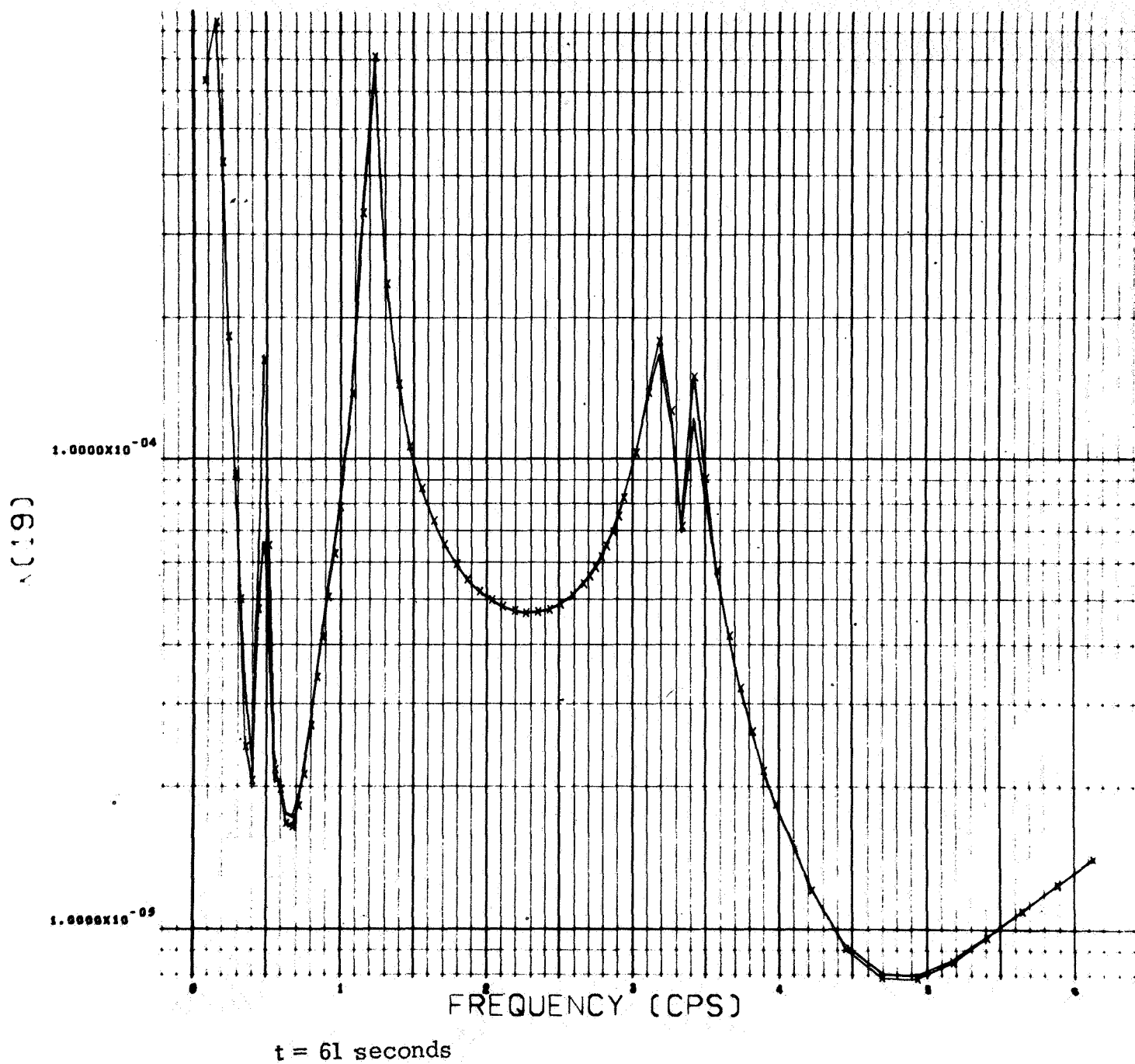
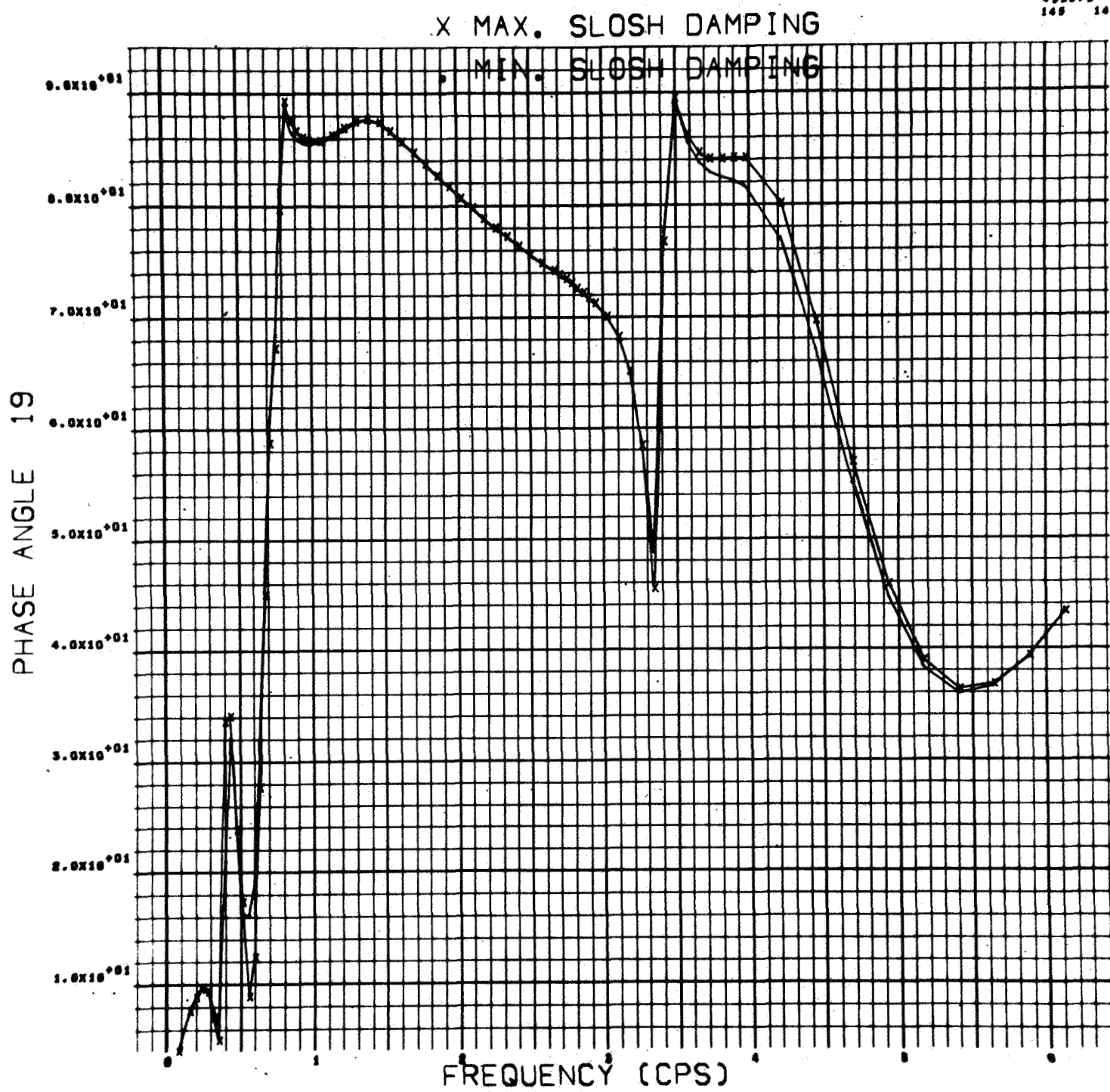


Figure 23a



t = 61 seconds

Figure 23b



Flight Test Data
 $X(20)$, $t = 61$ seconds

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

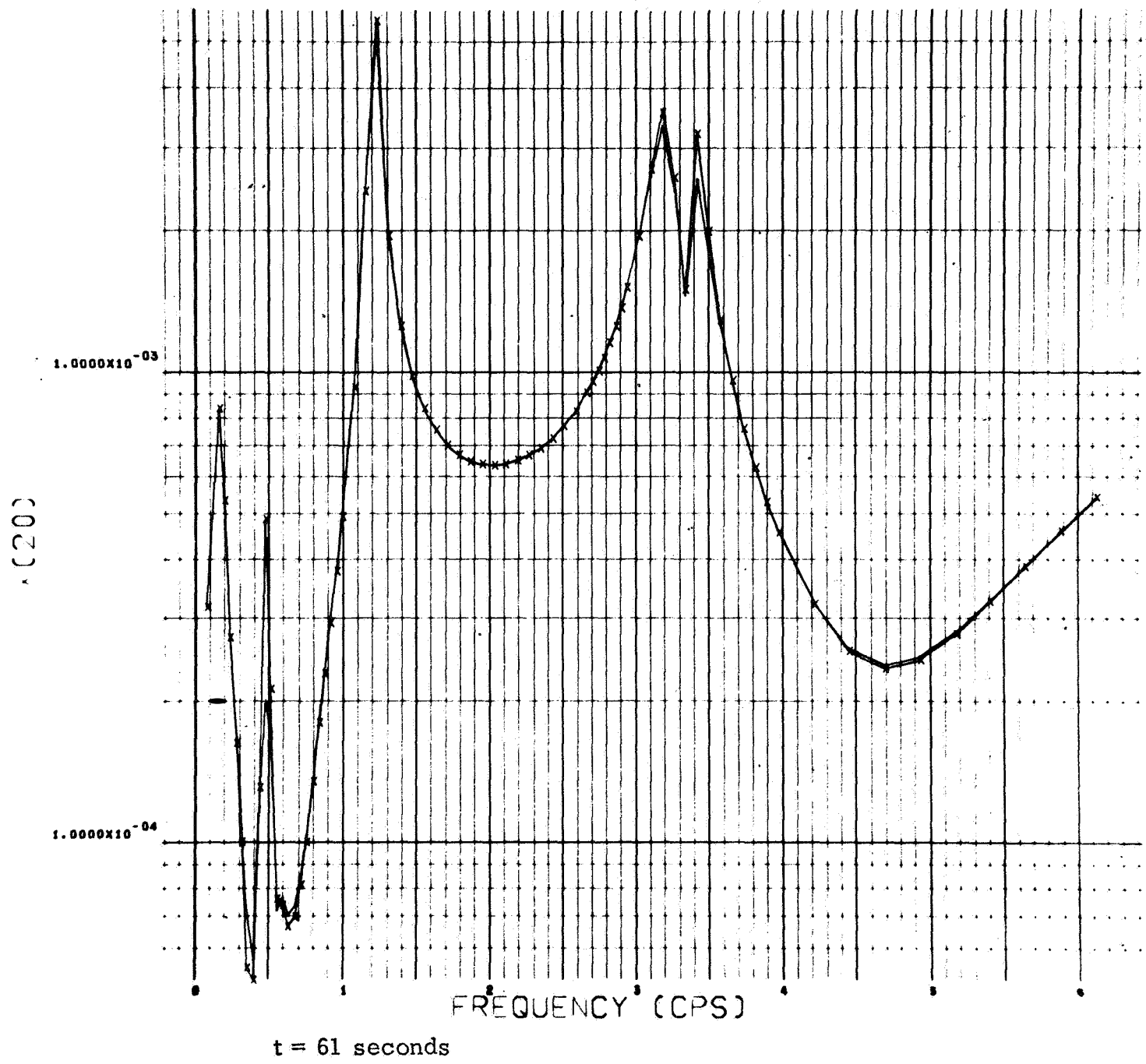


Figure 24a

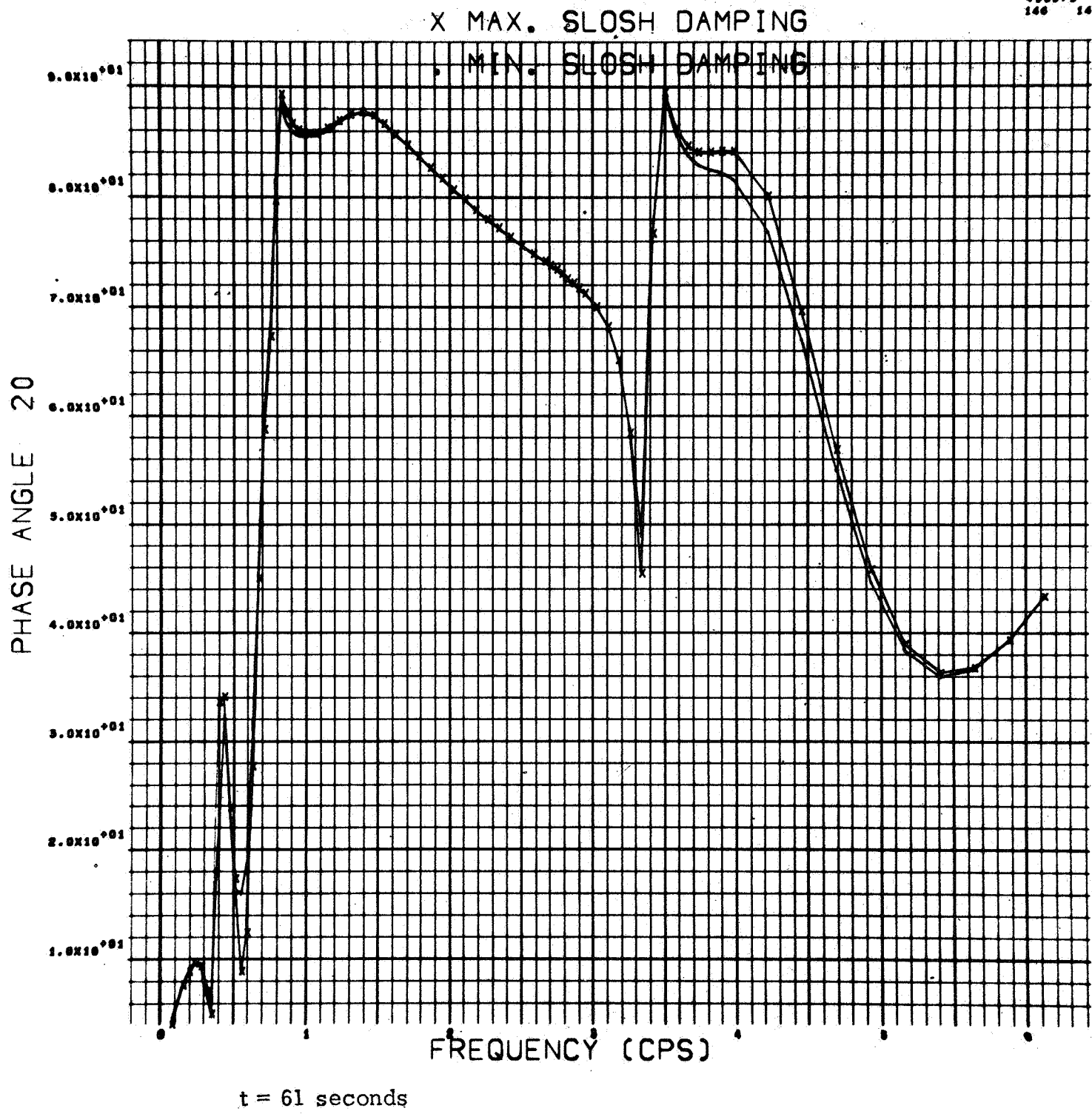


Figure 24b

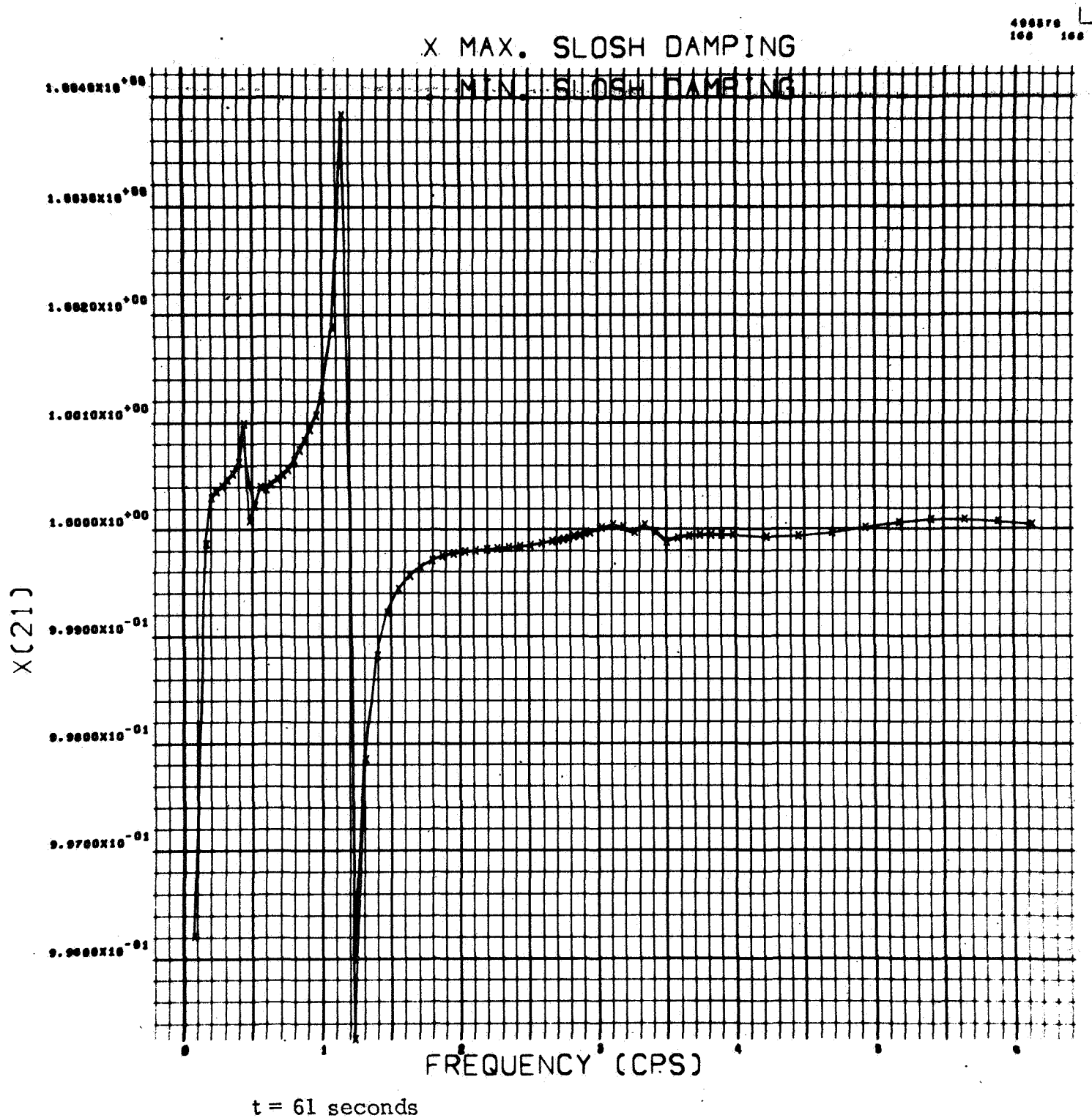


Figure 25a

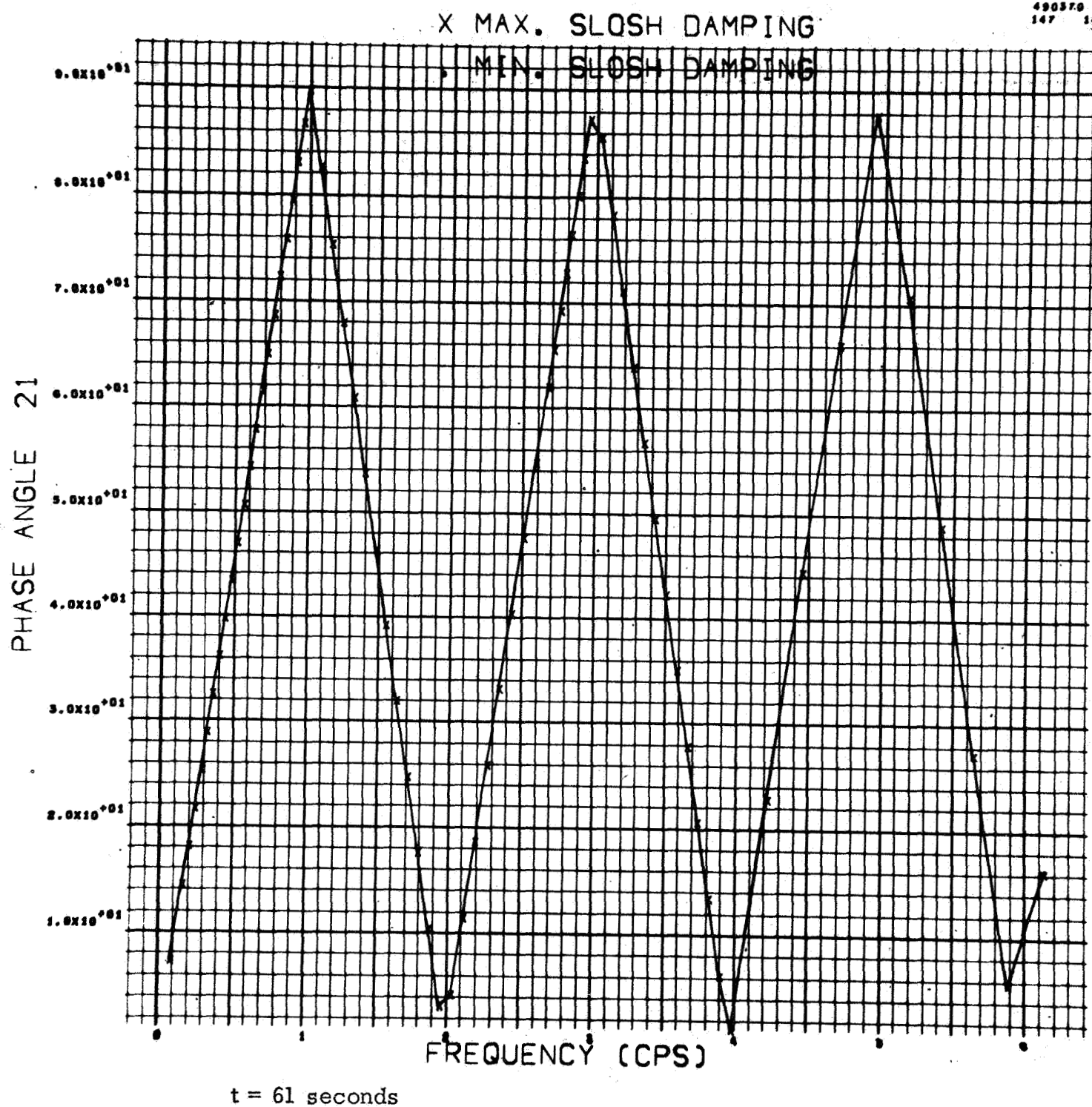
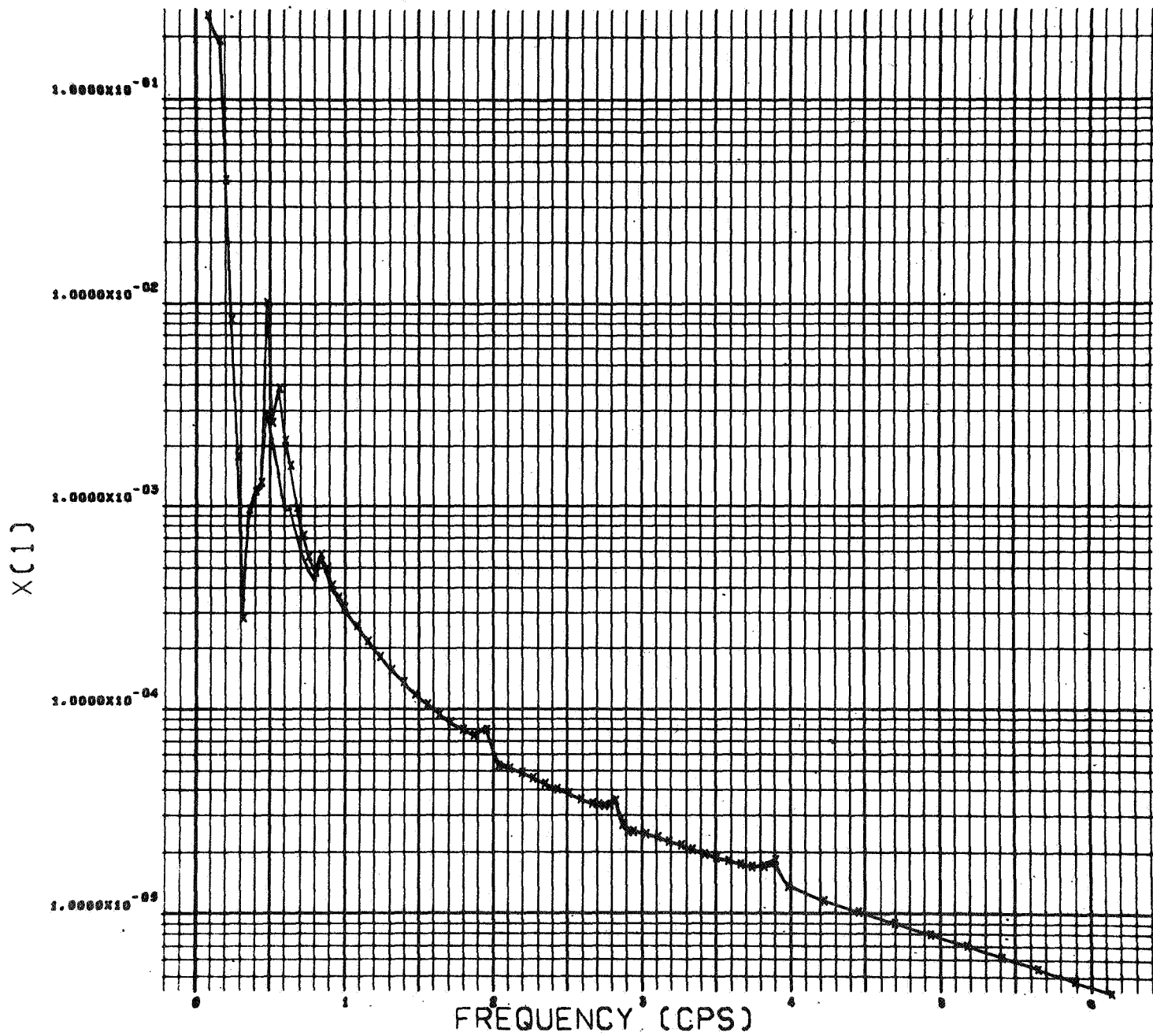


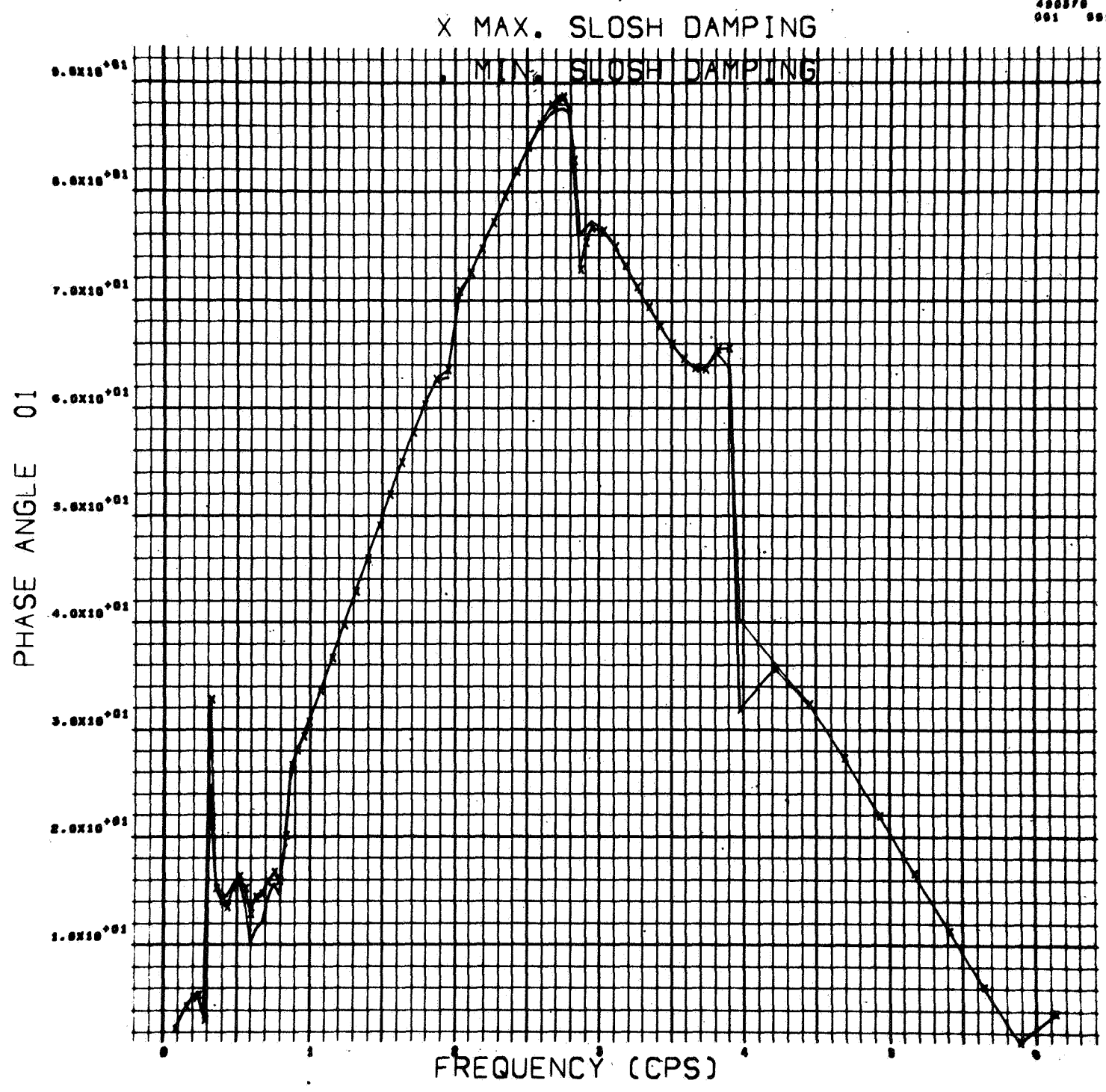
Figure 25b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 78 seconds

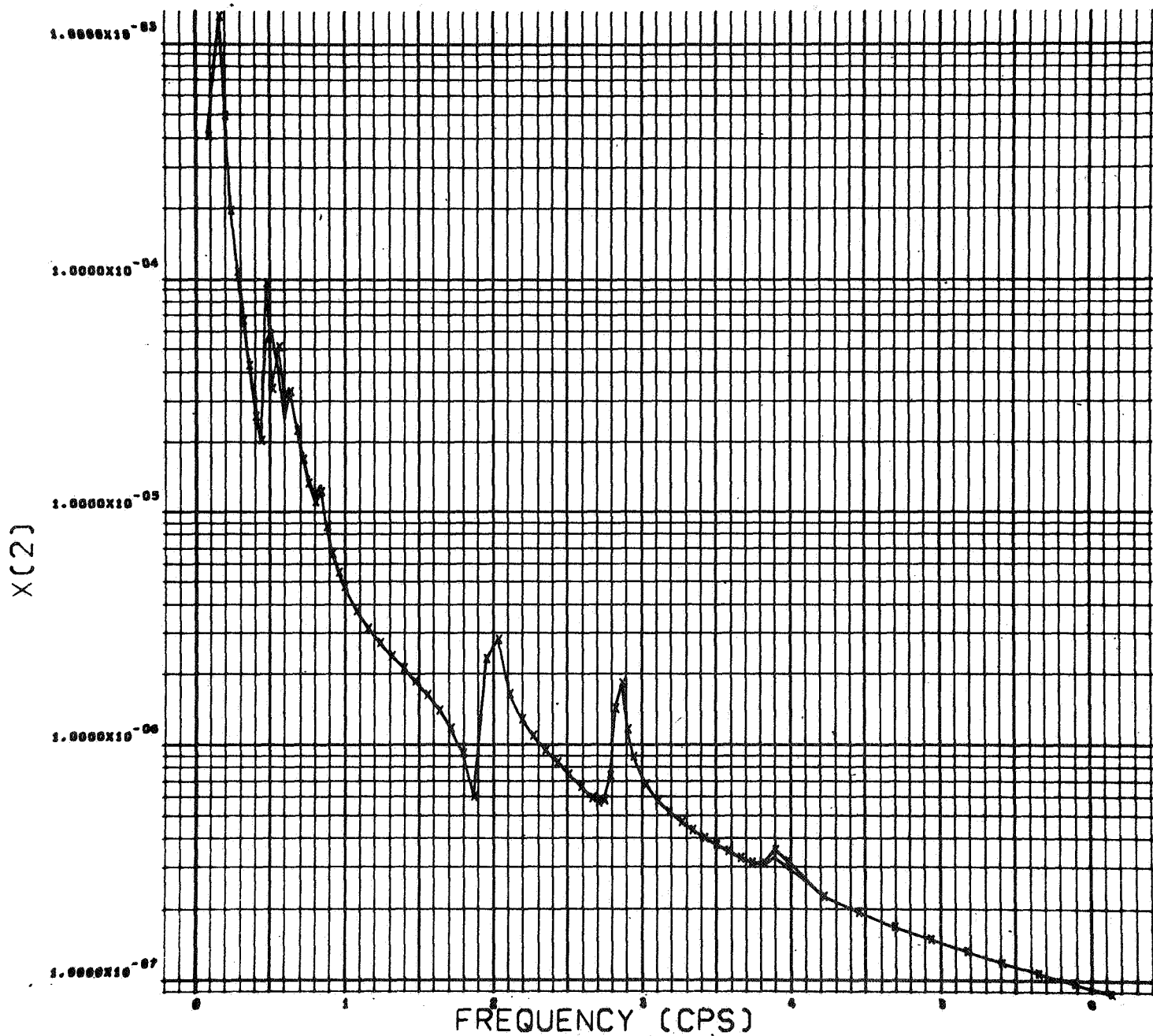
Figure 26a



t = 78 seconds

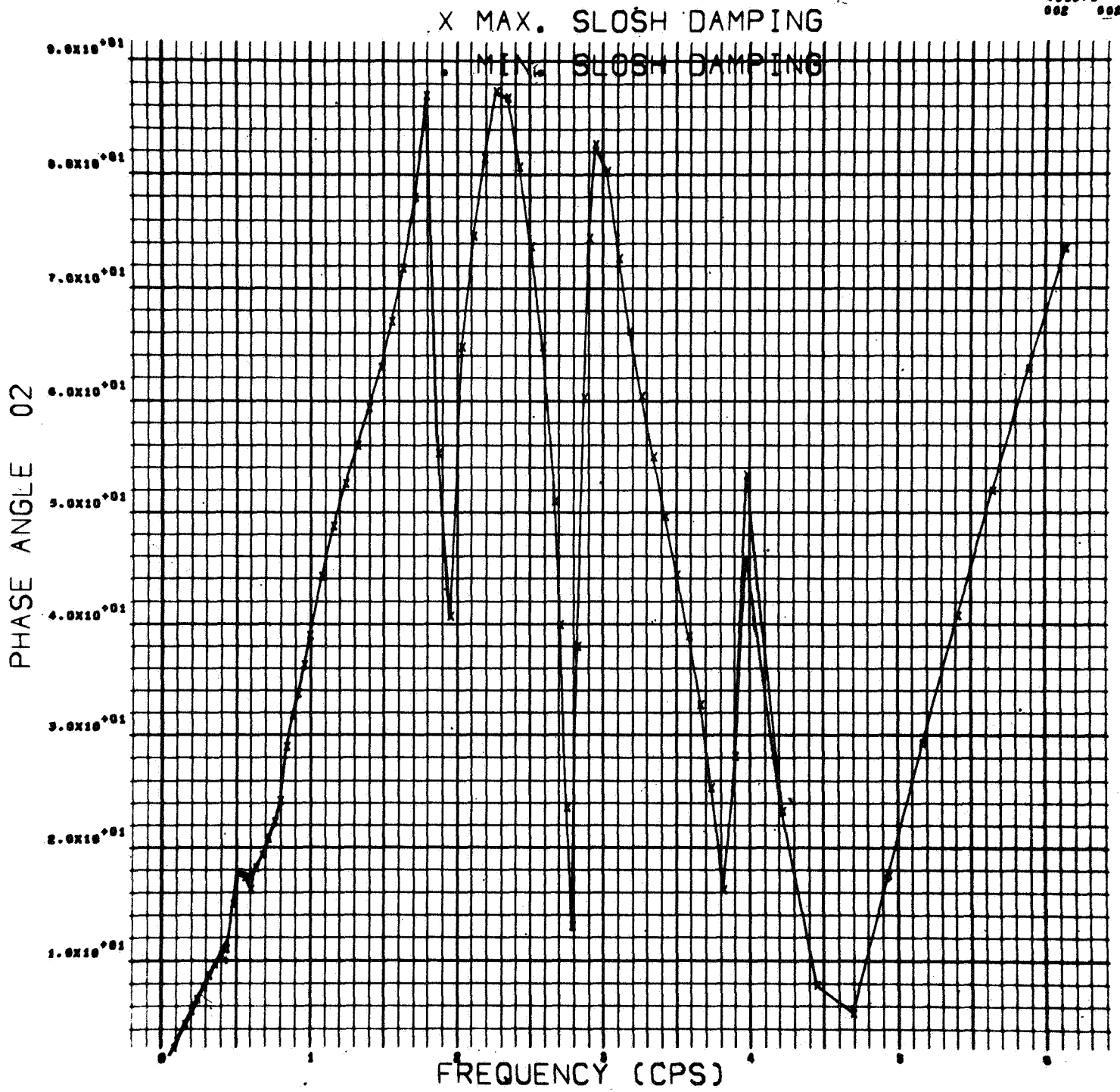
Figure 26b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 78 seconds

Figure 27a

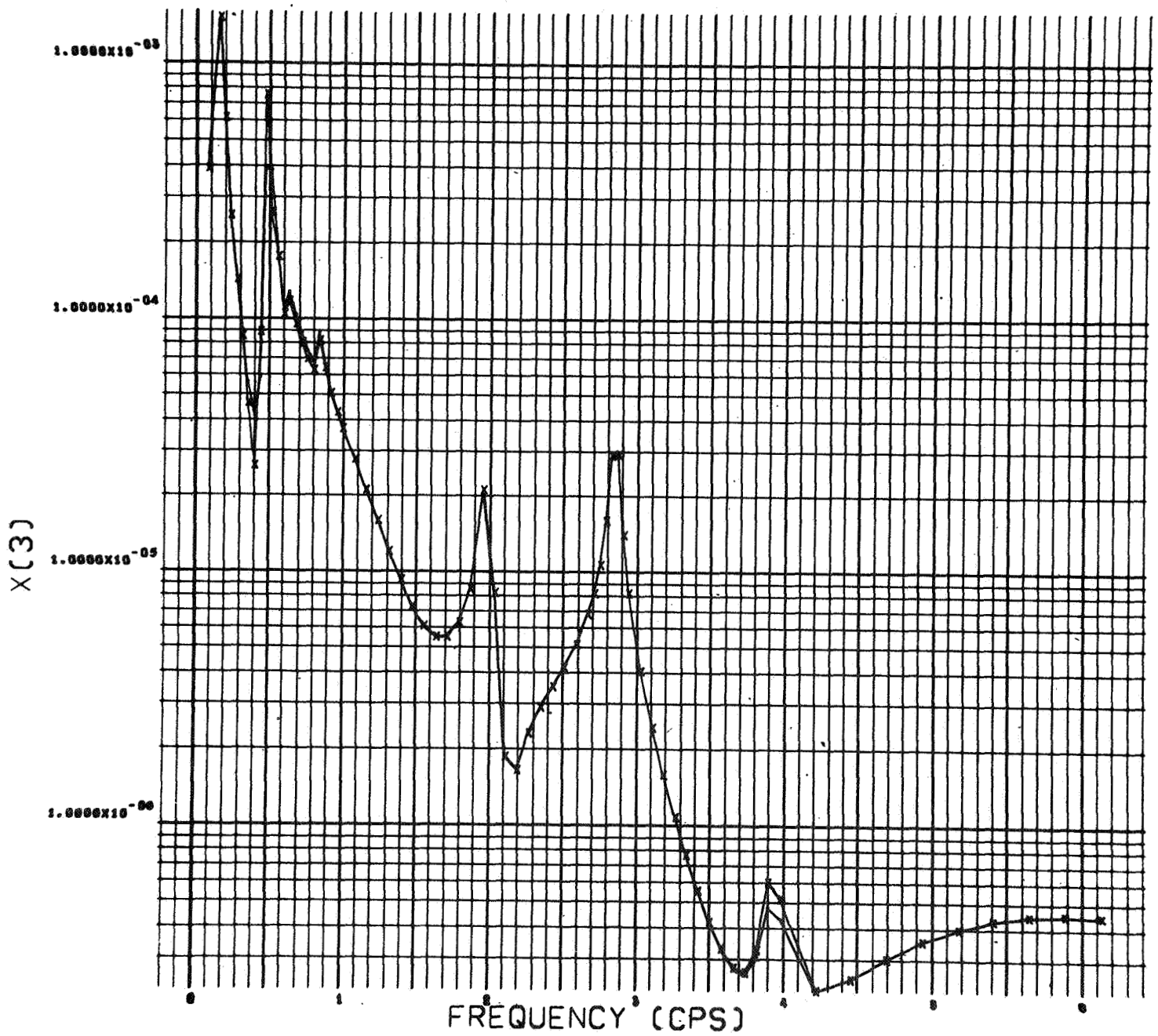


t = 78 seconds

Figure 27b

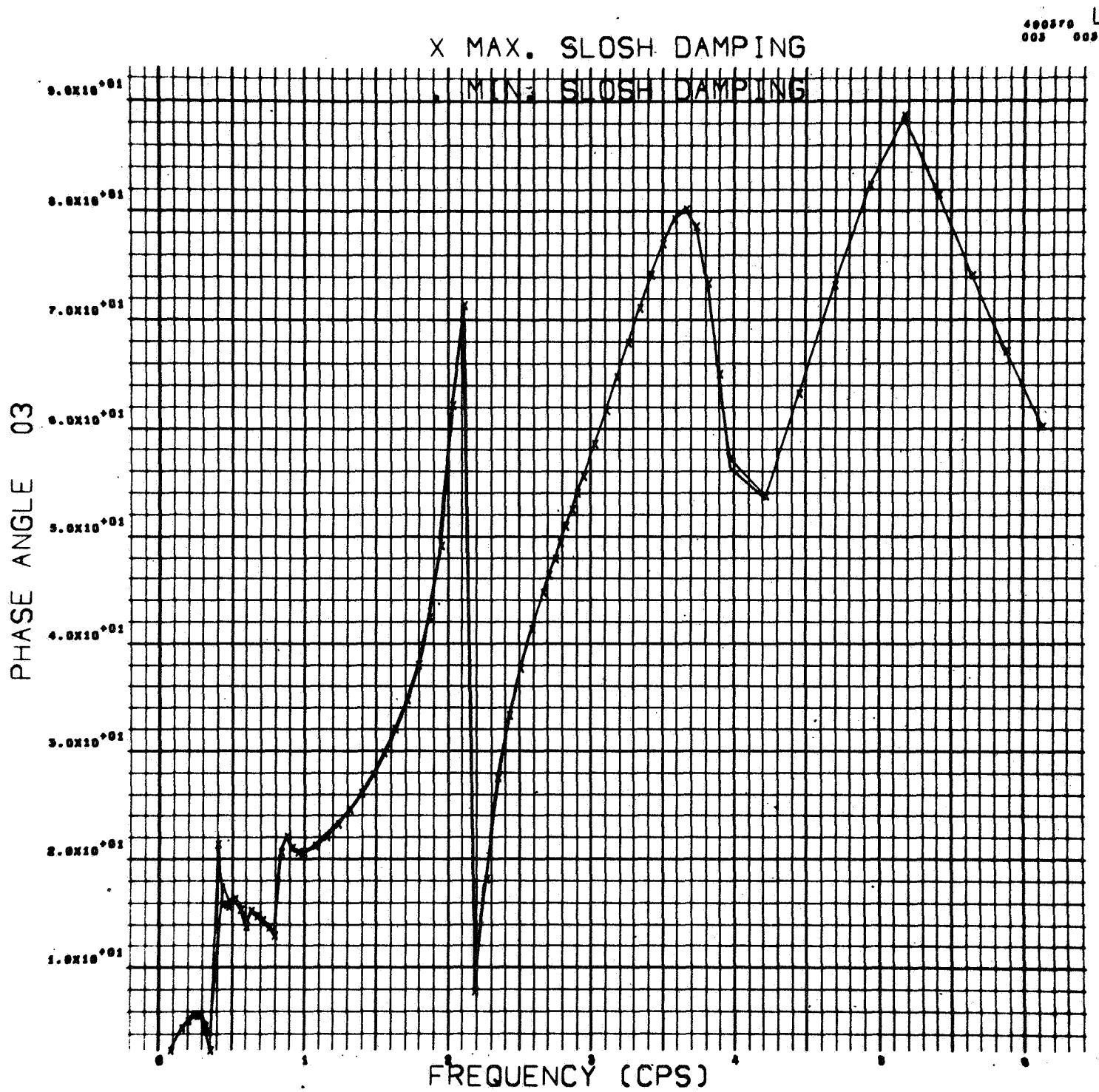
X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

490370 L
024 024



t = 78 seconds

Figure 28a

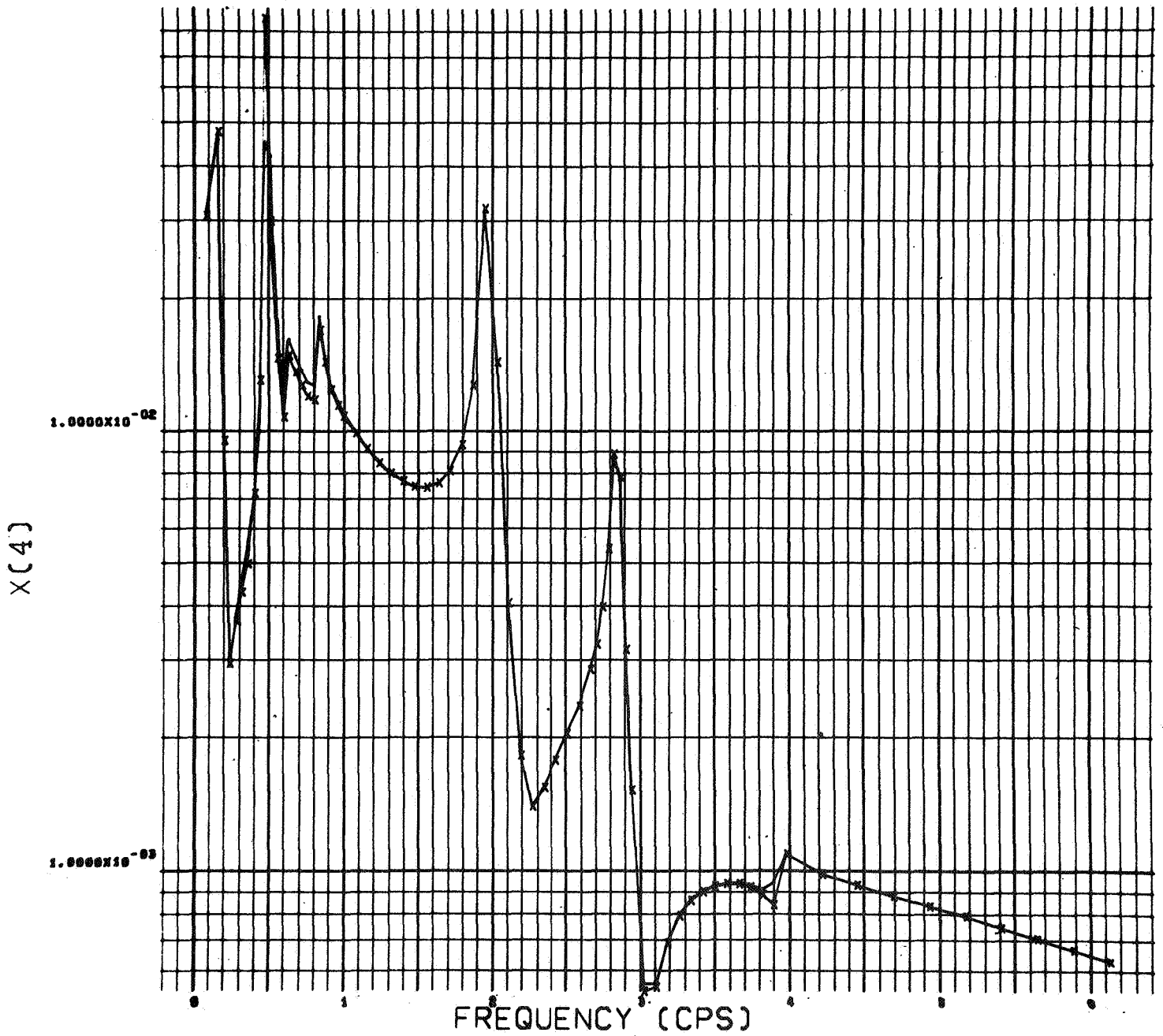


t = 78 seconds

Figure 28b

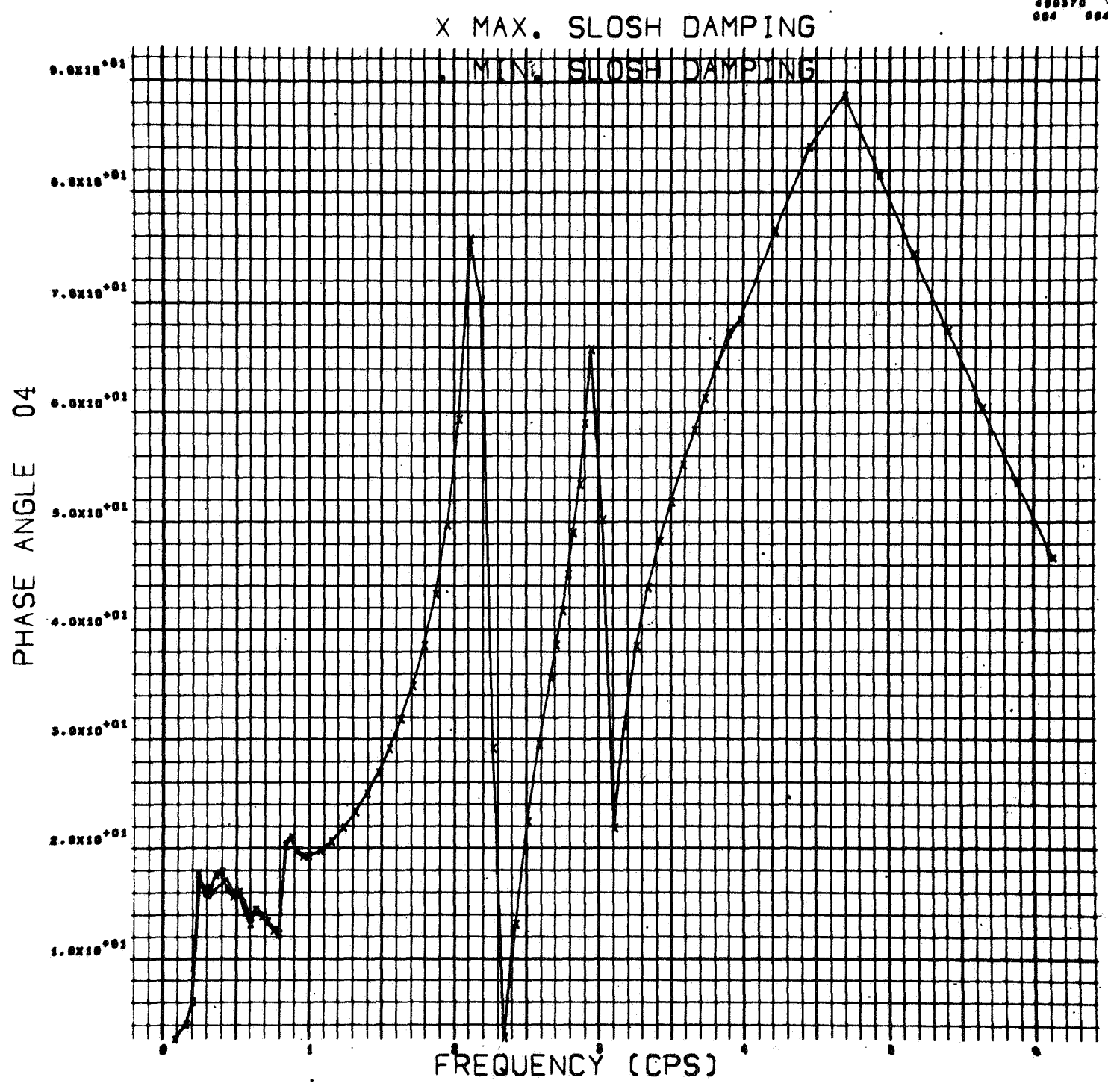
X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

490376 L
029 029



t = 78 seconds

Figure 29a

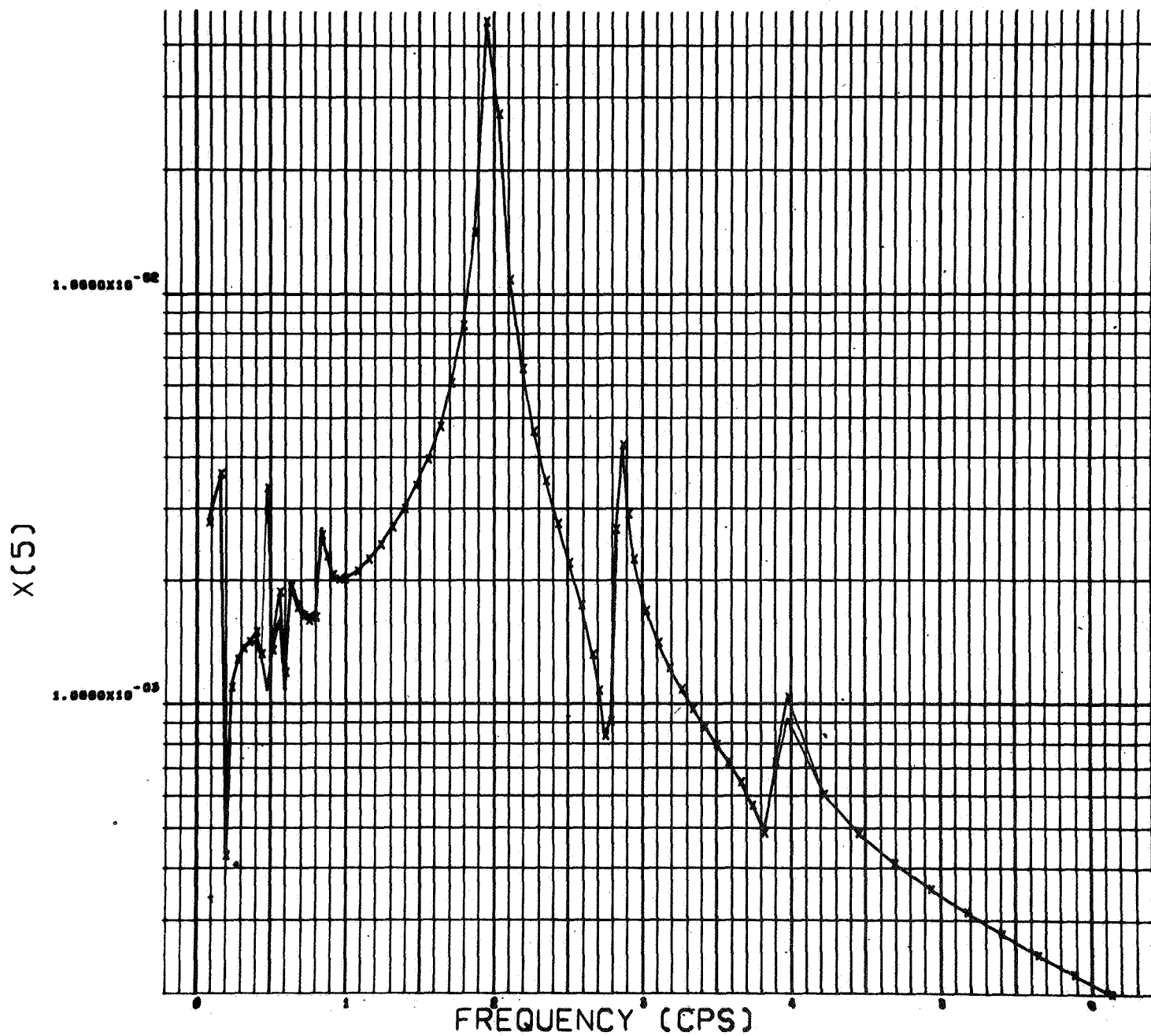


t = 78 seconds

Figure 29b

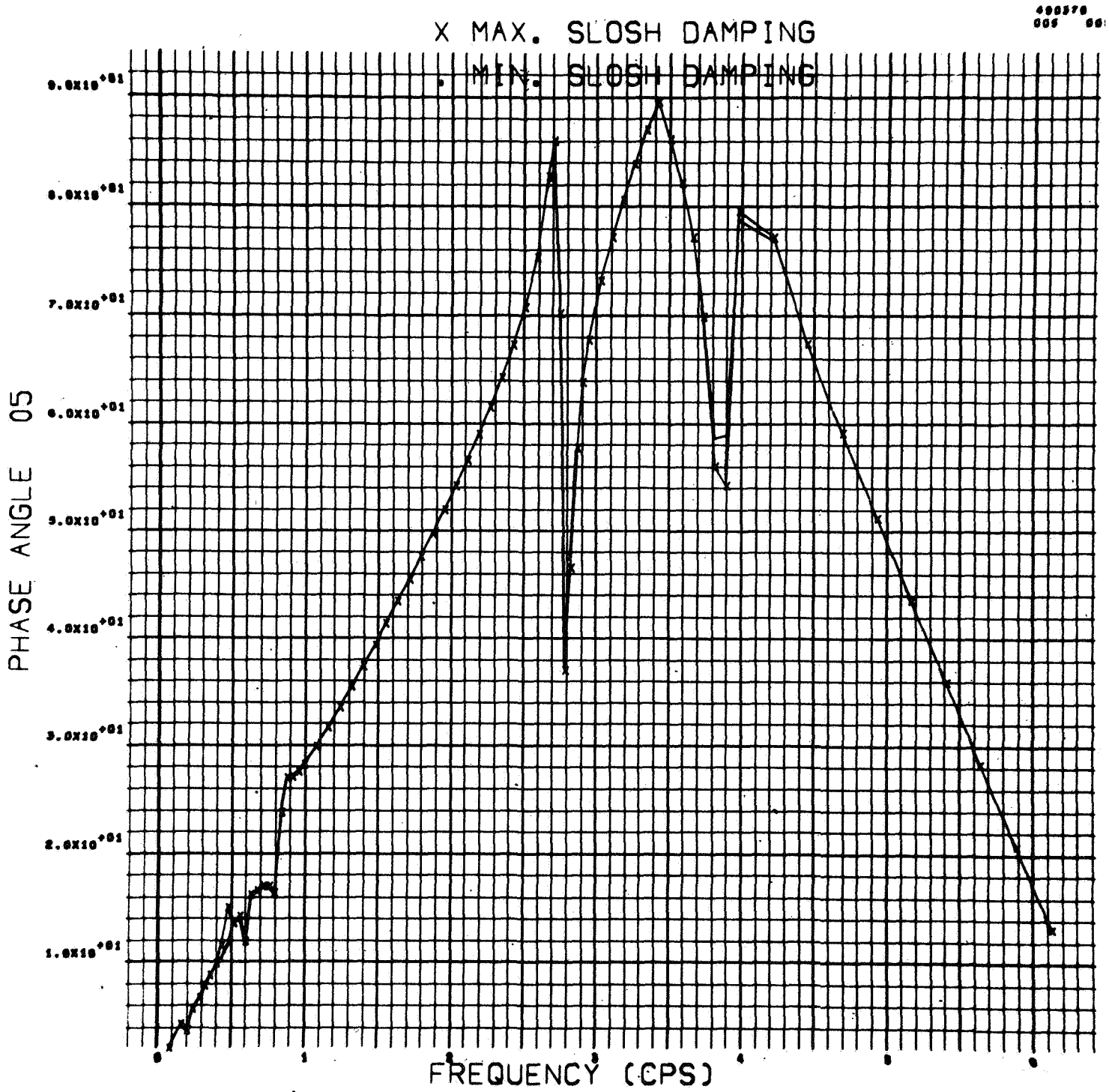
X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

400570 L
020 020



t = 78 seconds

Figure 30a

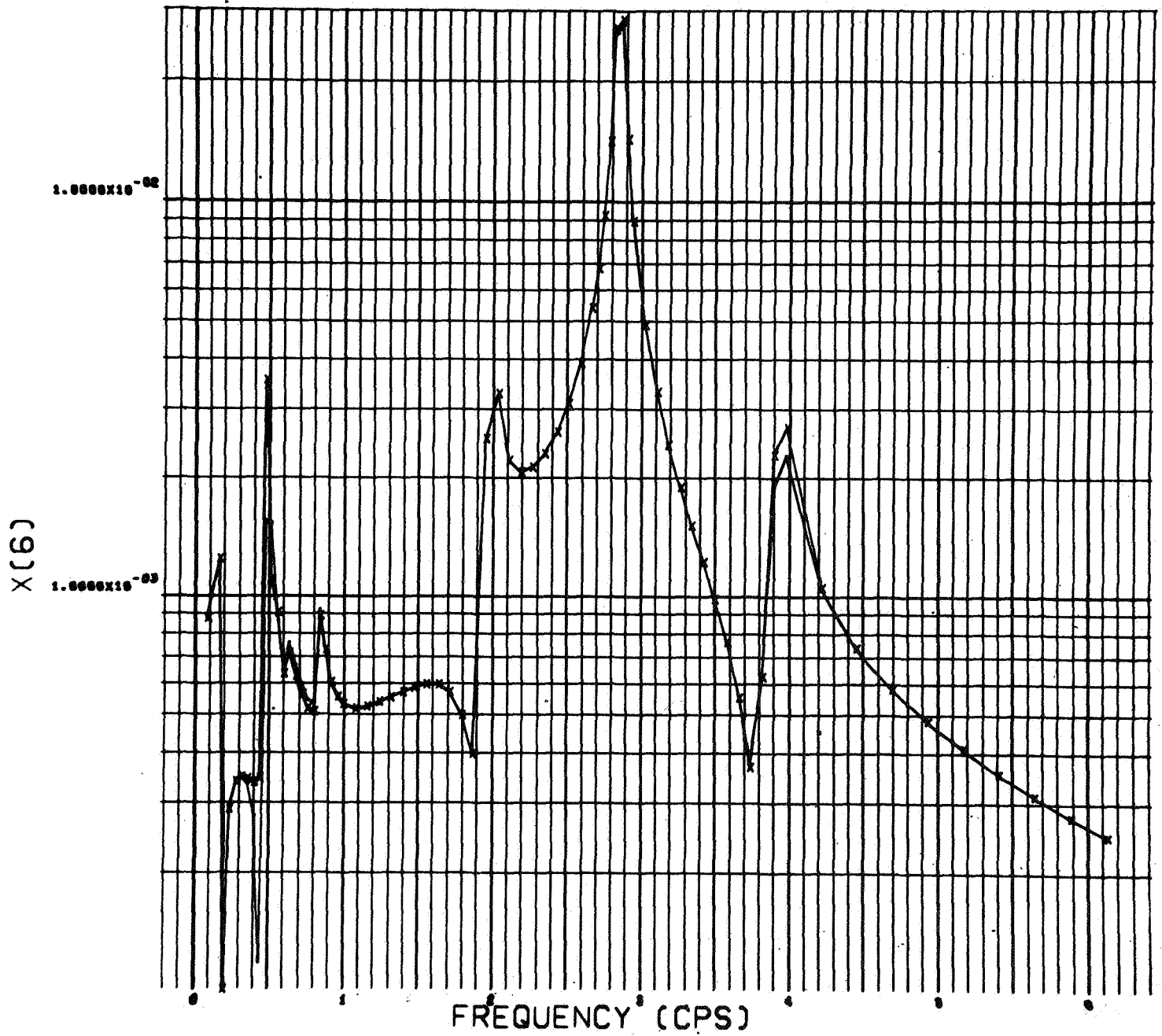


t = 78 seconds

Figure 30b

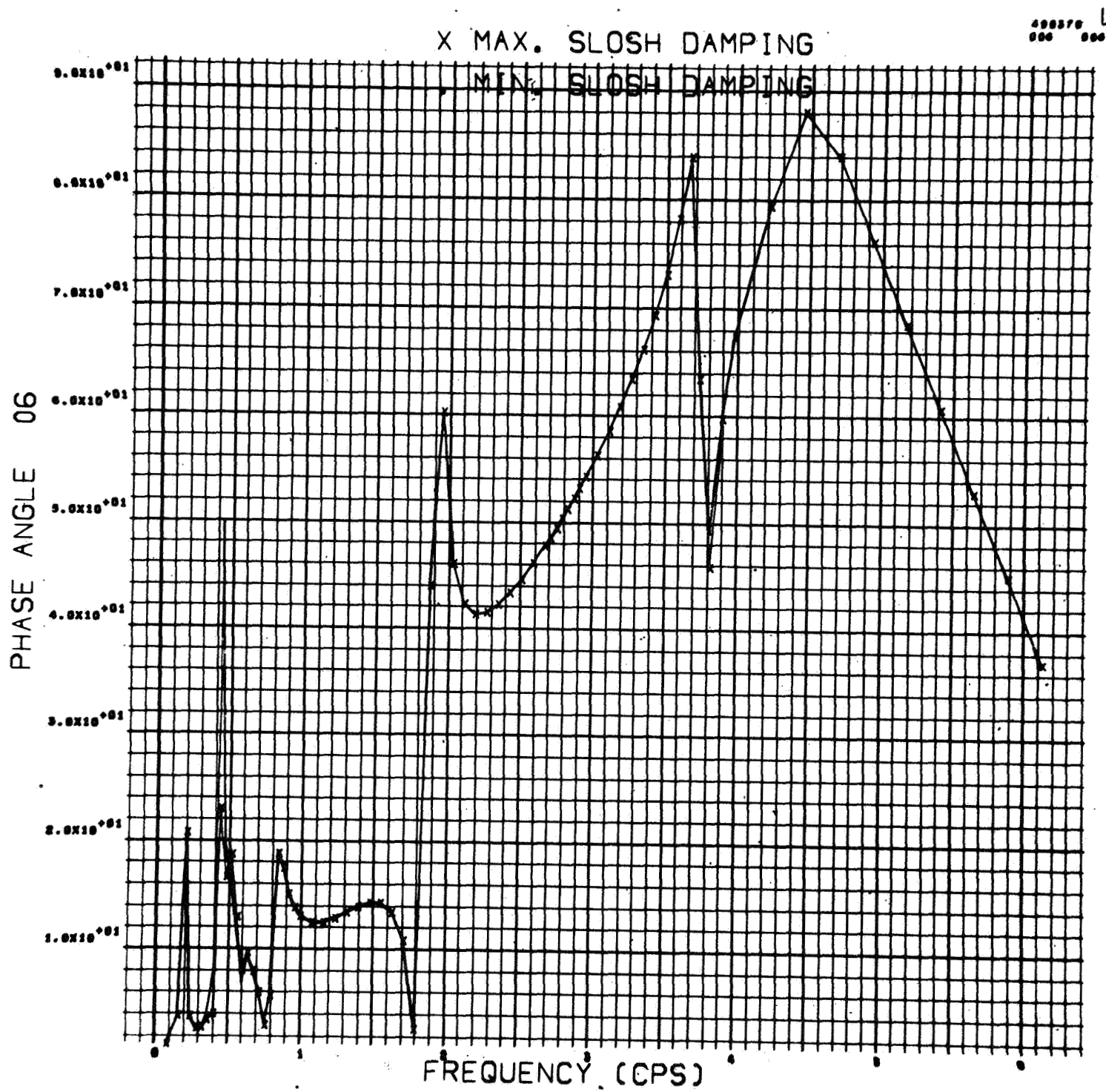
X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING

400070
027 02



t = 78 seconds

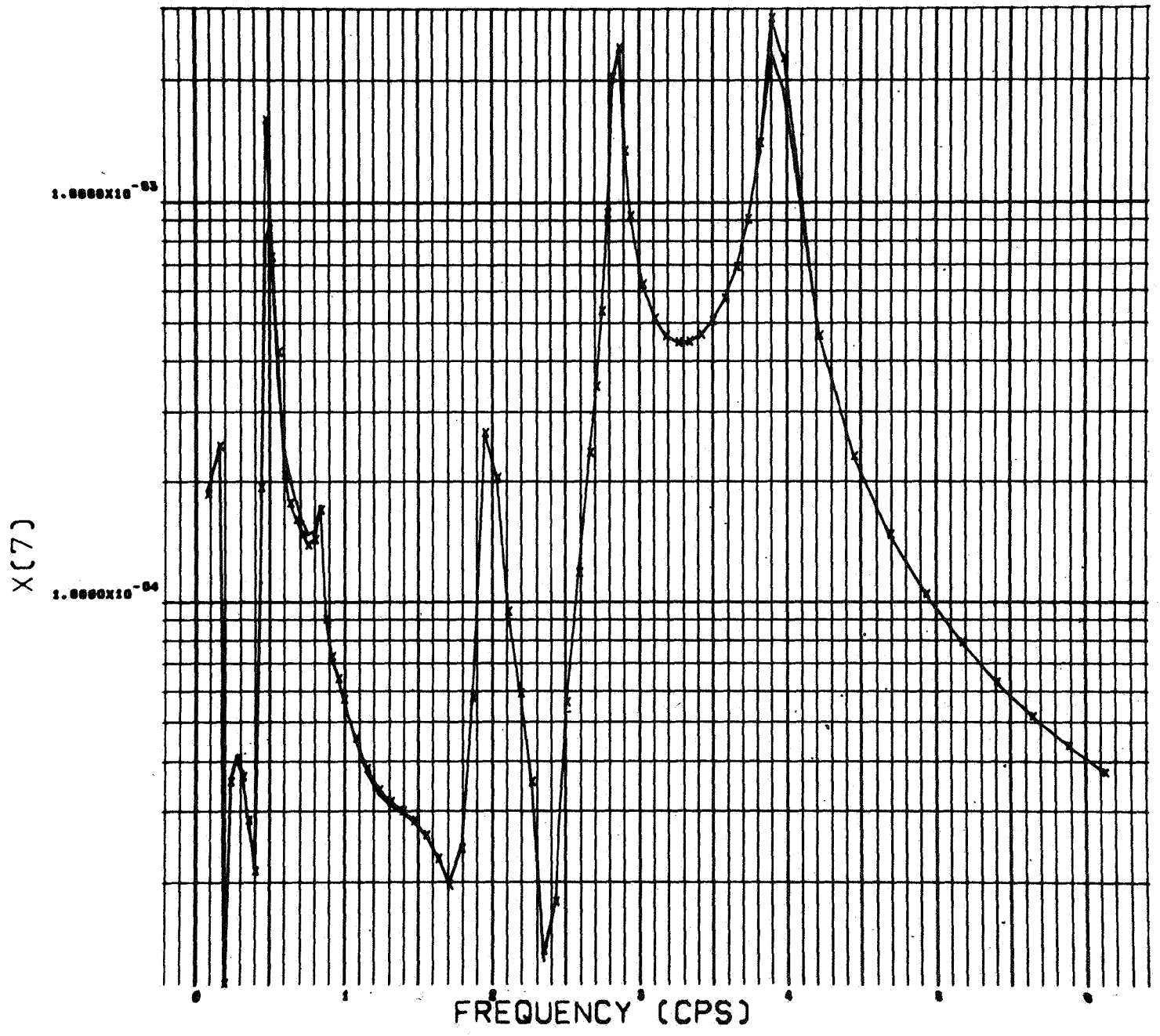
Figure 31a



t = 78 seconds

Figure 31b

X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING



t = 78 seconds

Figure 32a

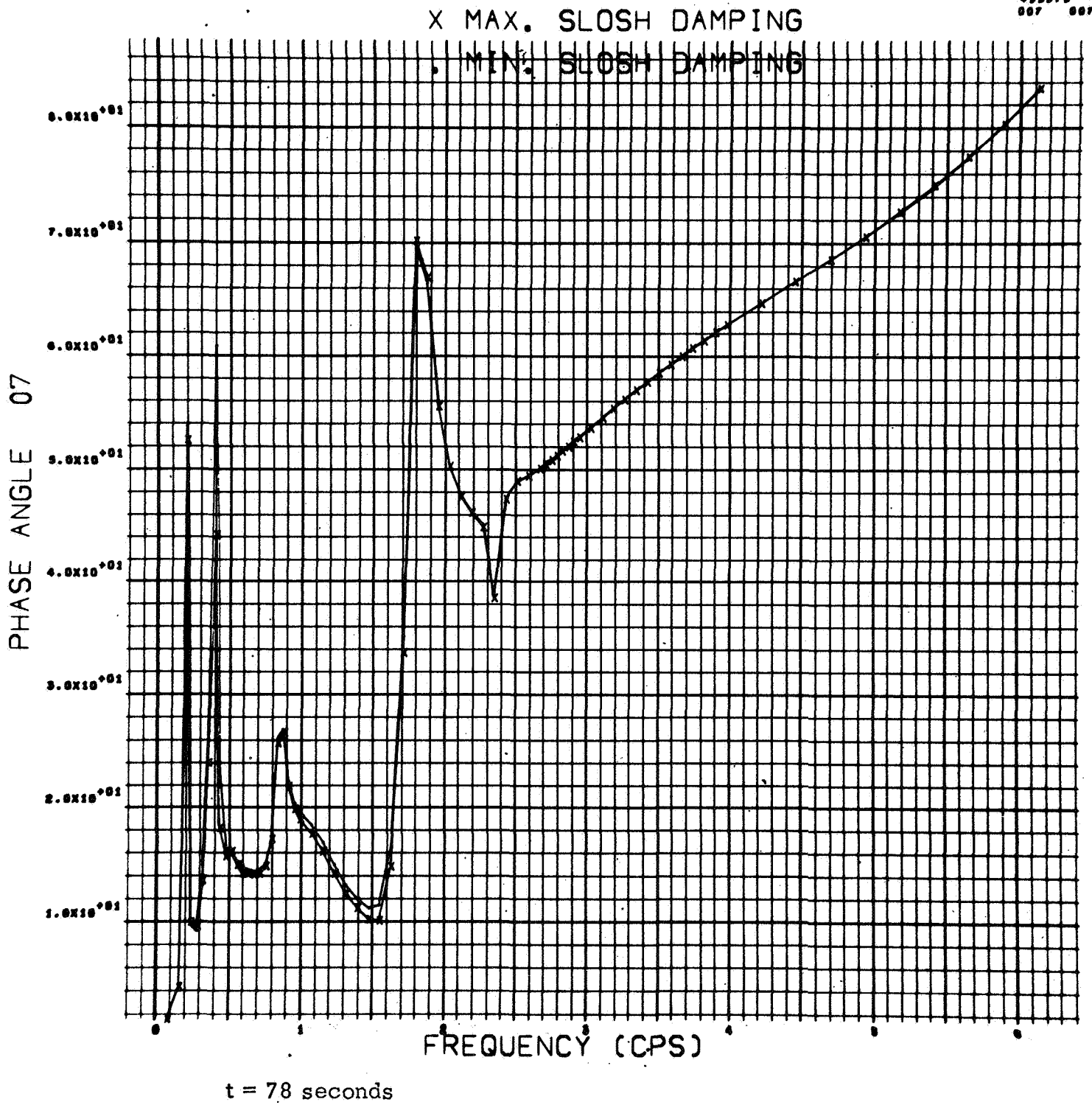
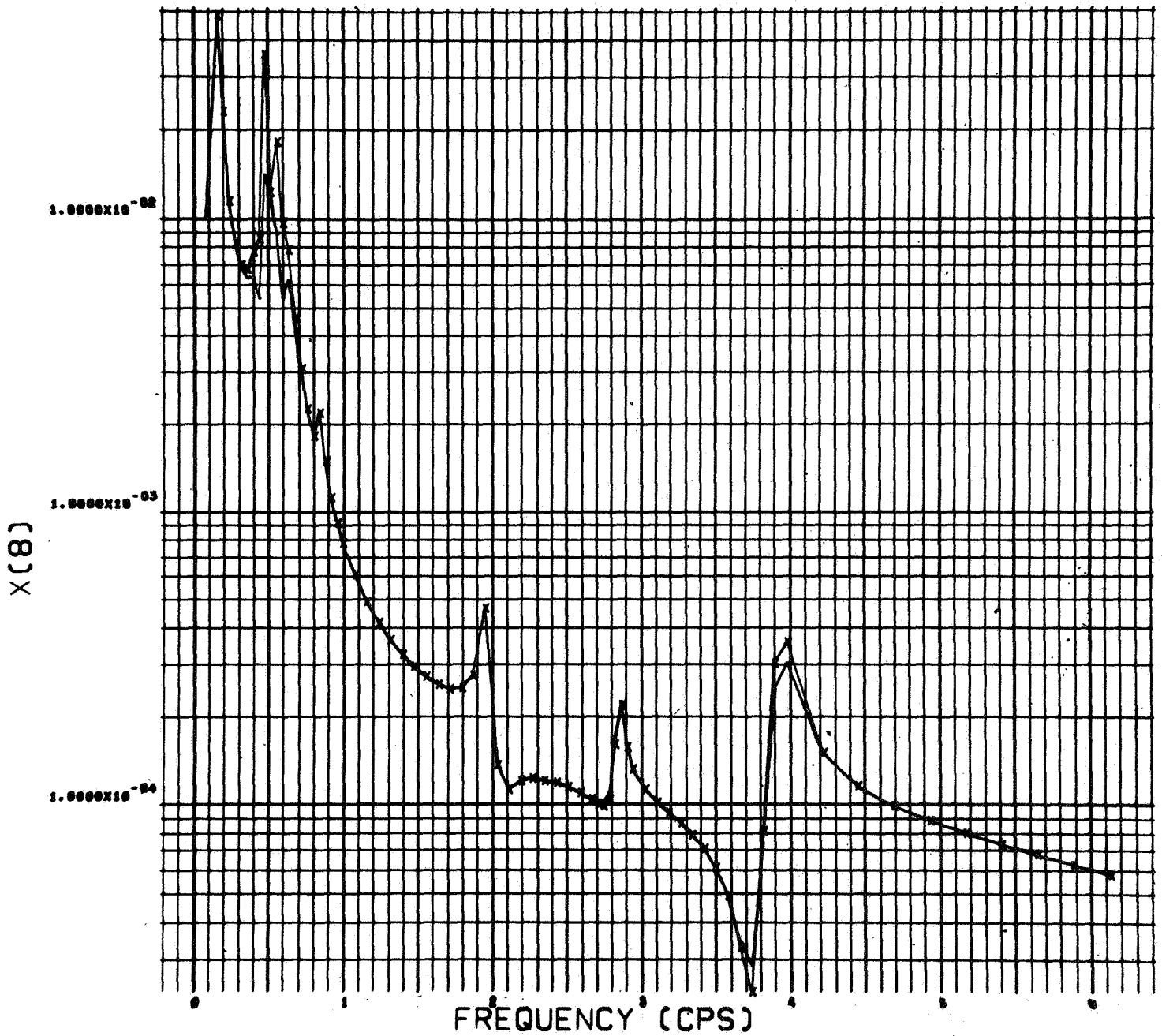


Figure 32b

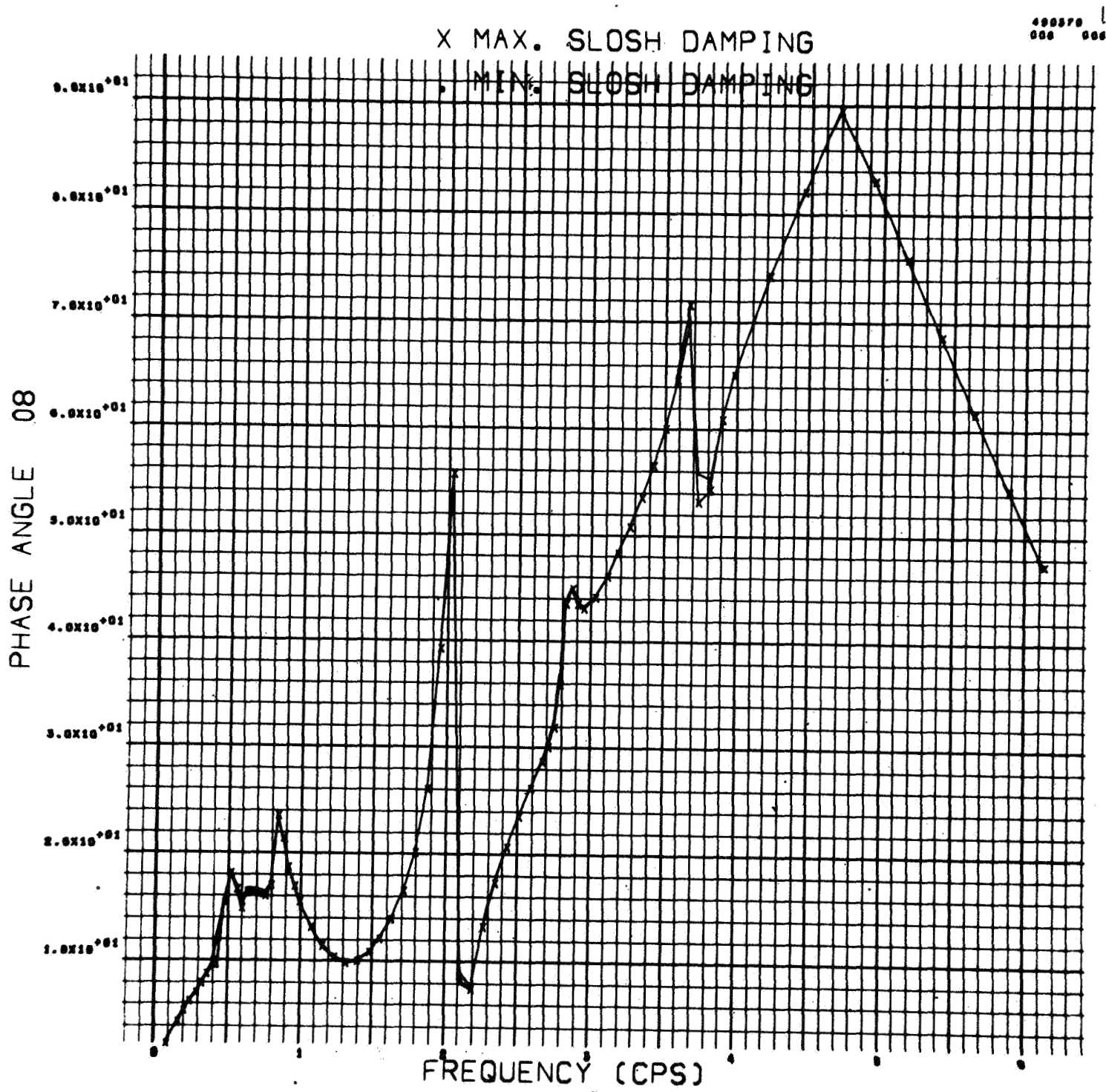
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490378 L
029 029



t = 78 seconds

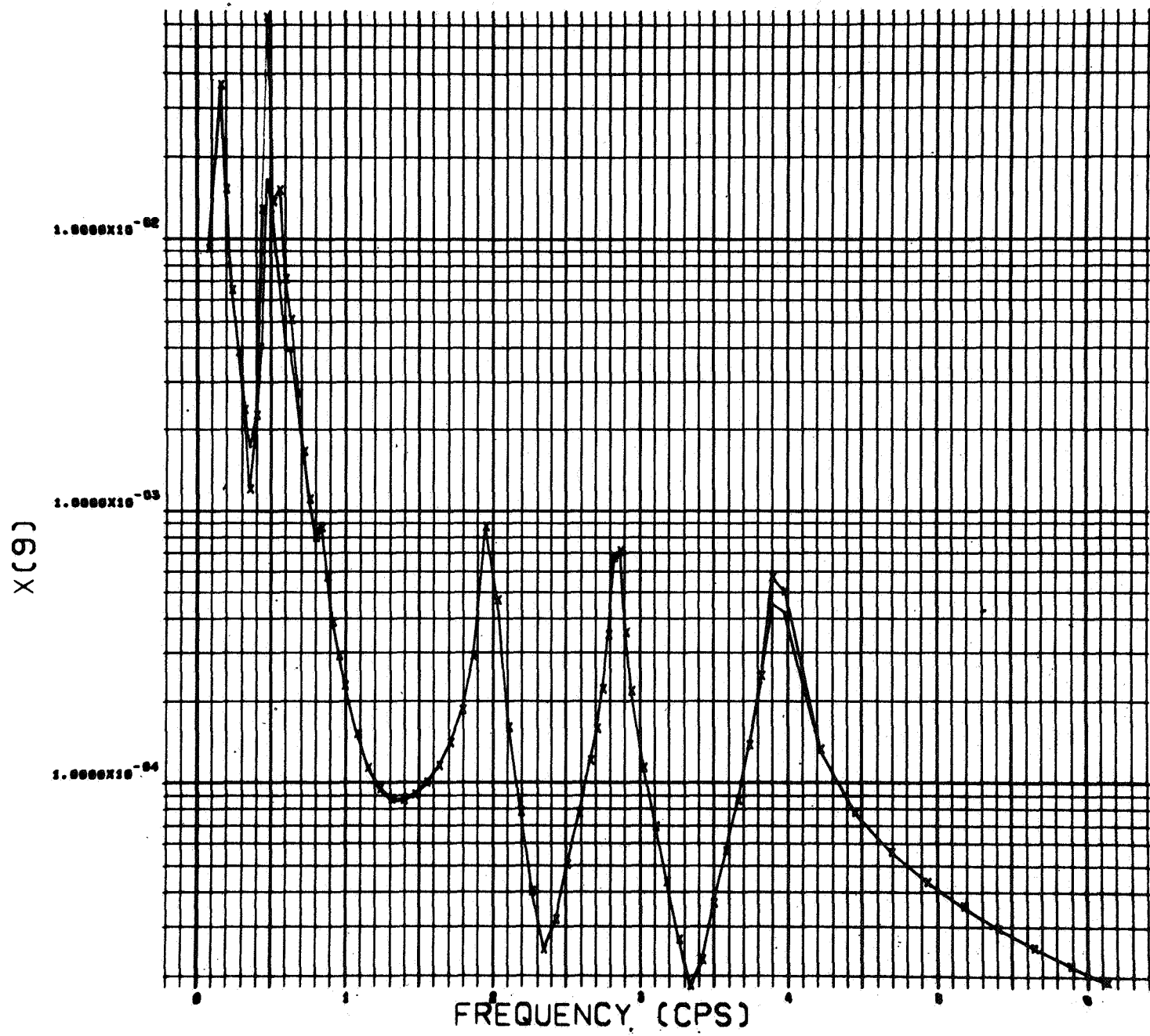
Figure 33a



t = 78 seconds

Figure 33b

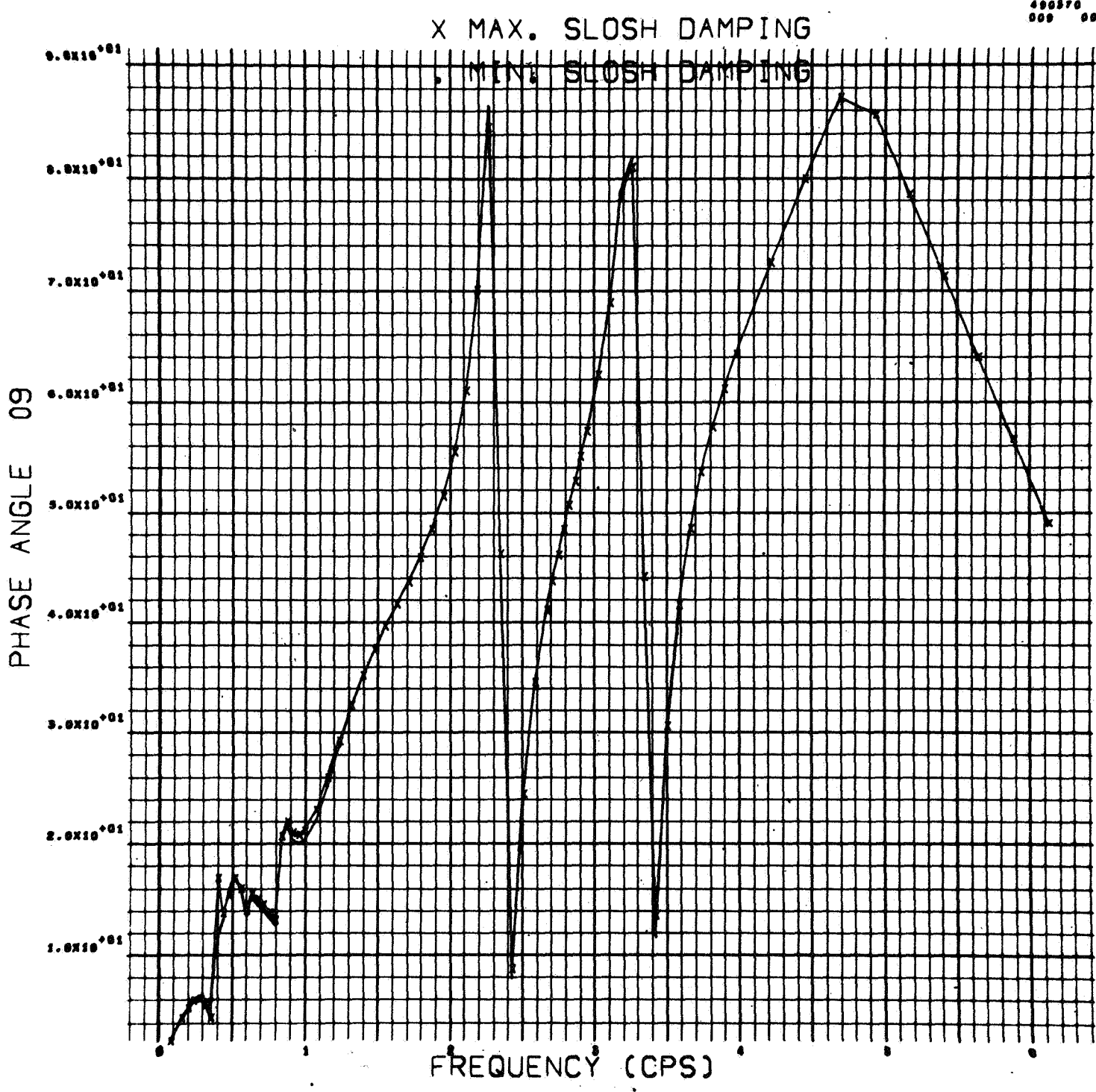
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING



t = 78 seconds

Figure 34a

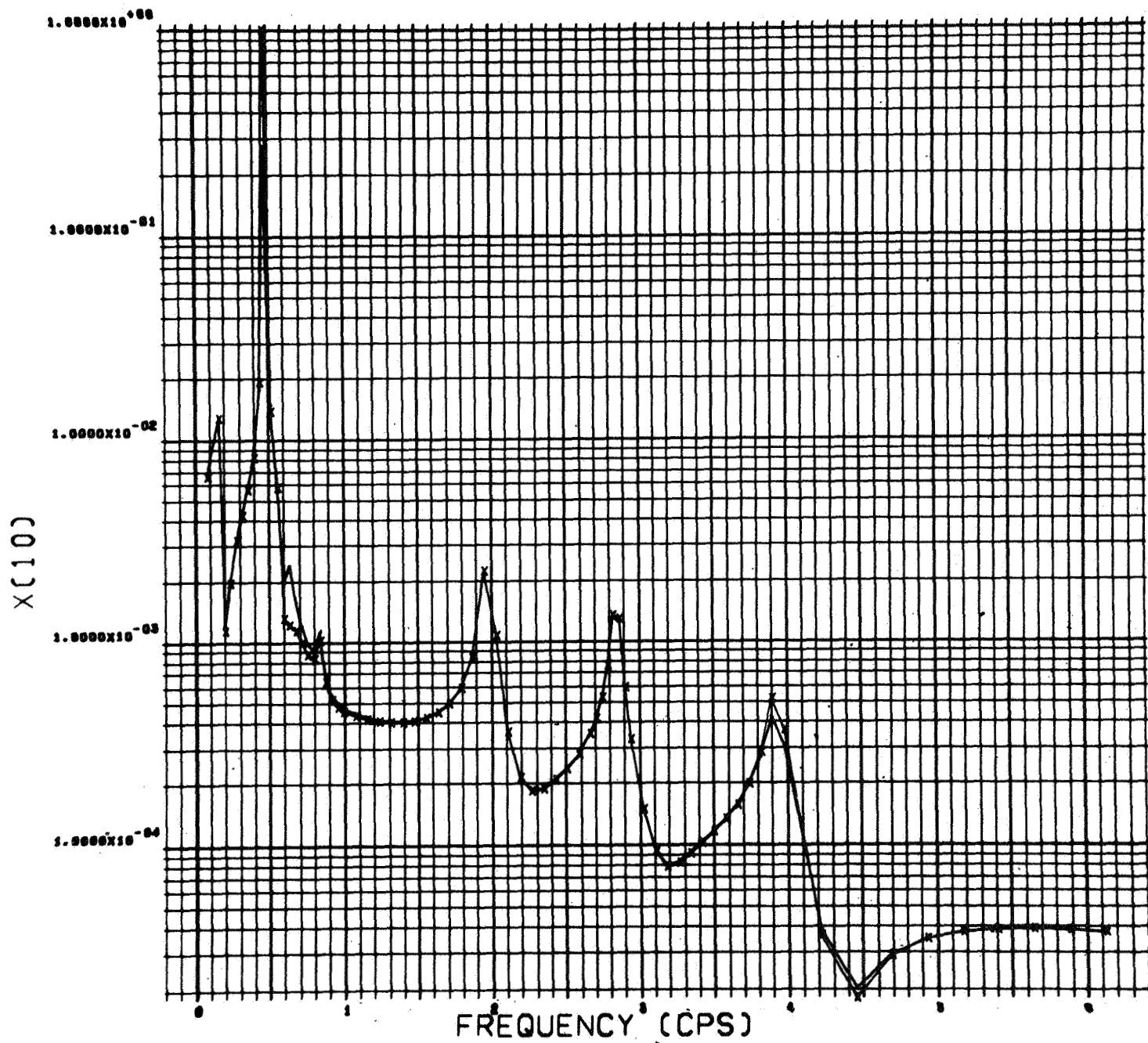
490370 L
000 000



t = 78 seconds

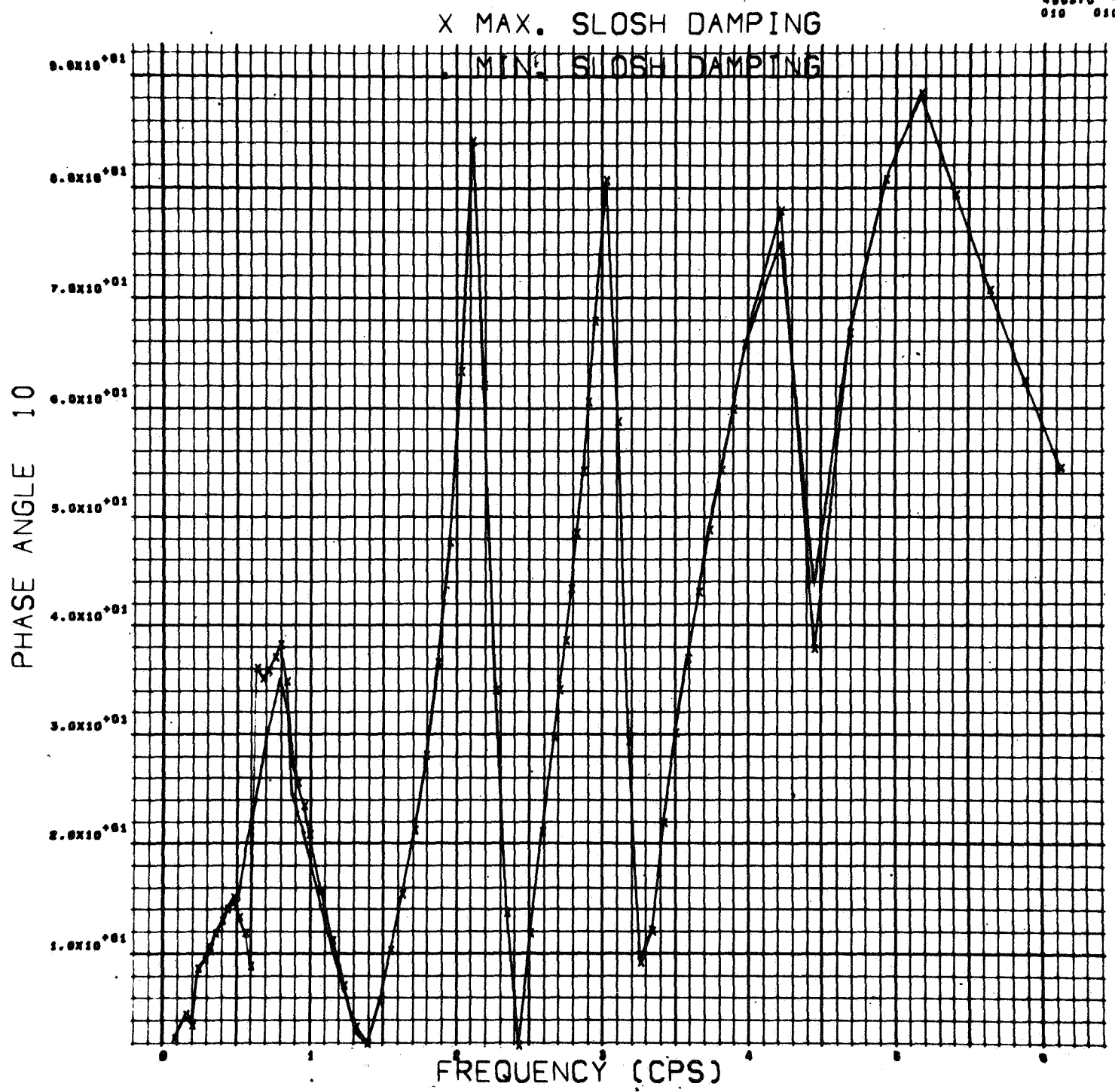
Figure 34b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 78 seconds

Figure 35a

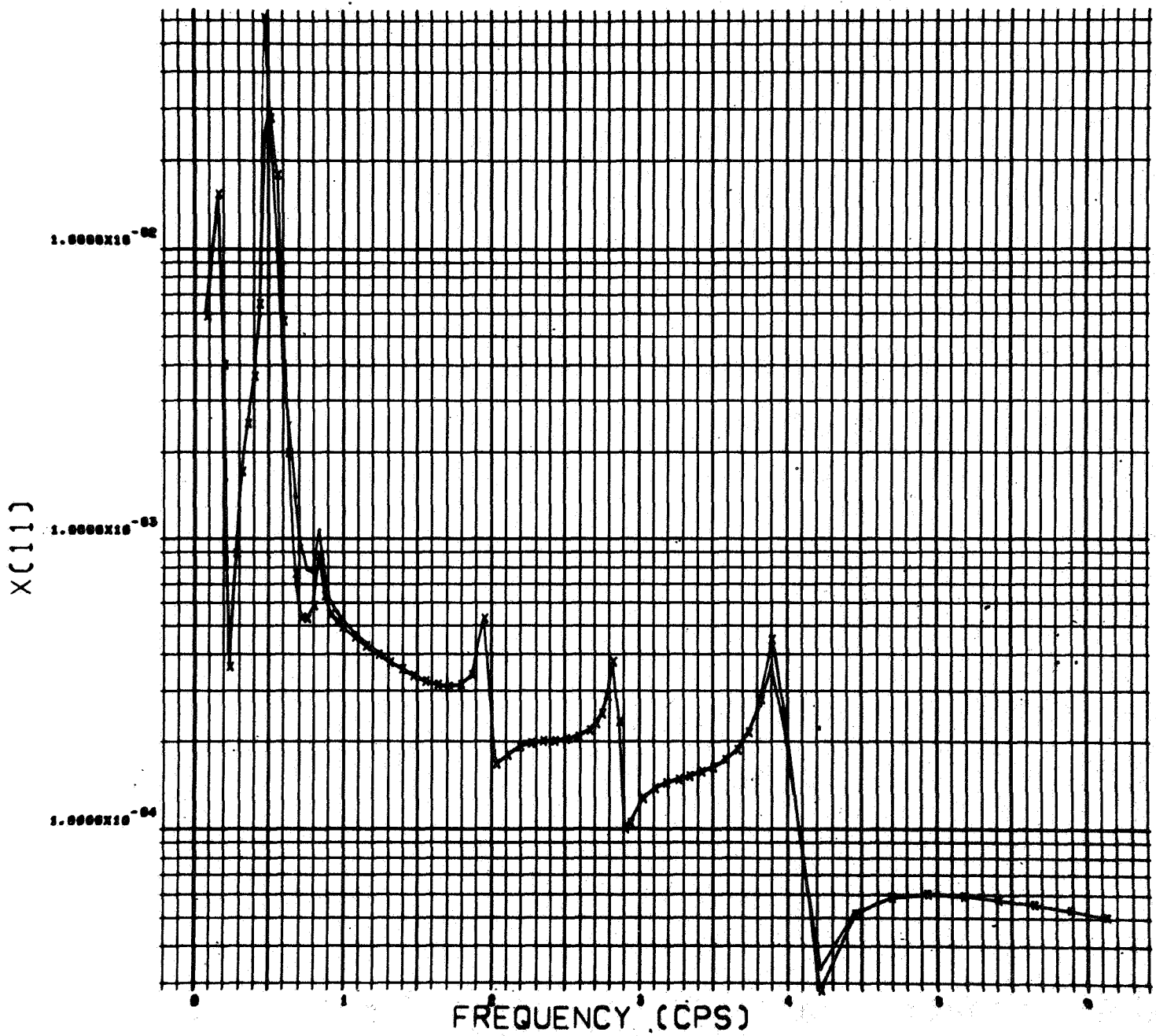


t = 78 seconds

Figure 35b

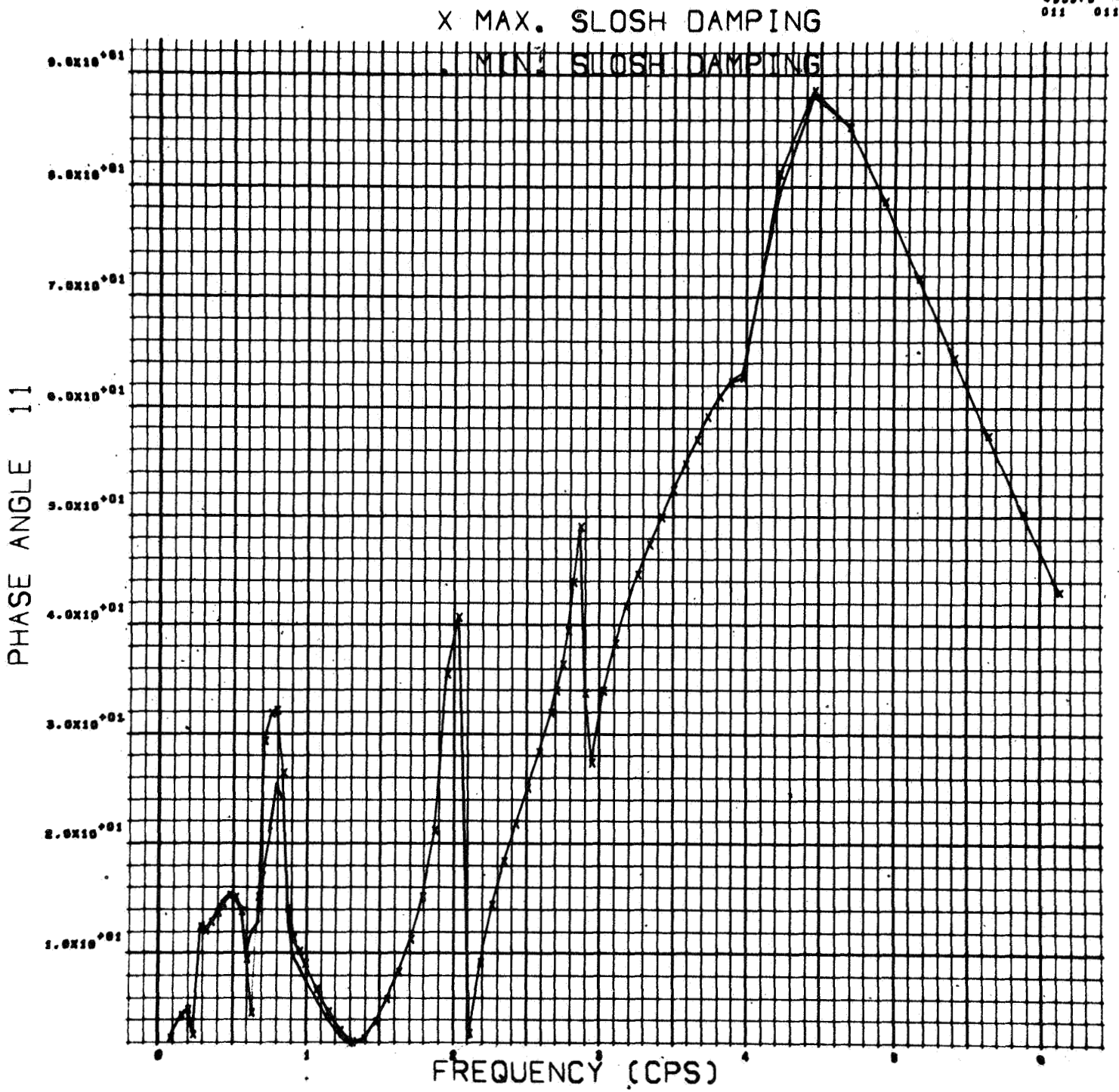
X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

400370 L
002 002



t = 78 seconds

Figure 36a

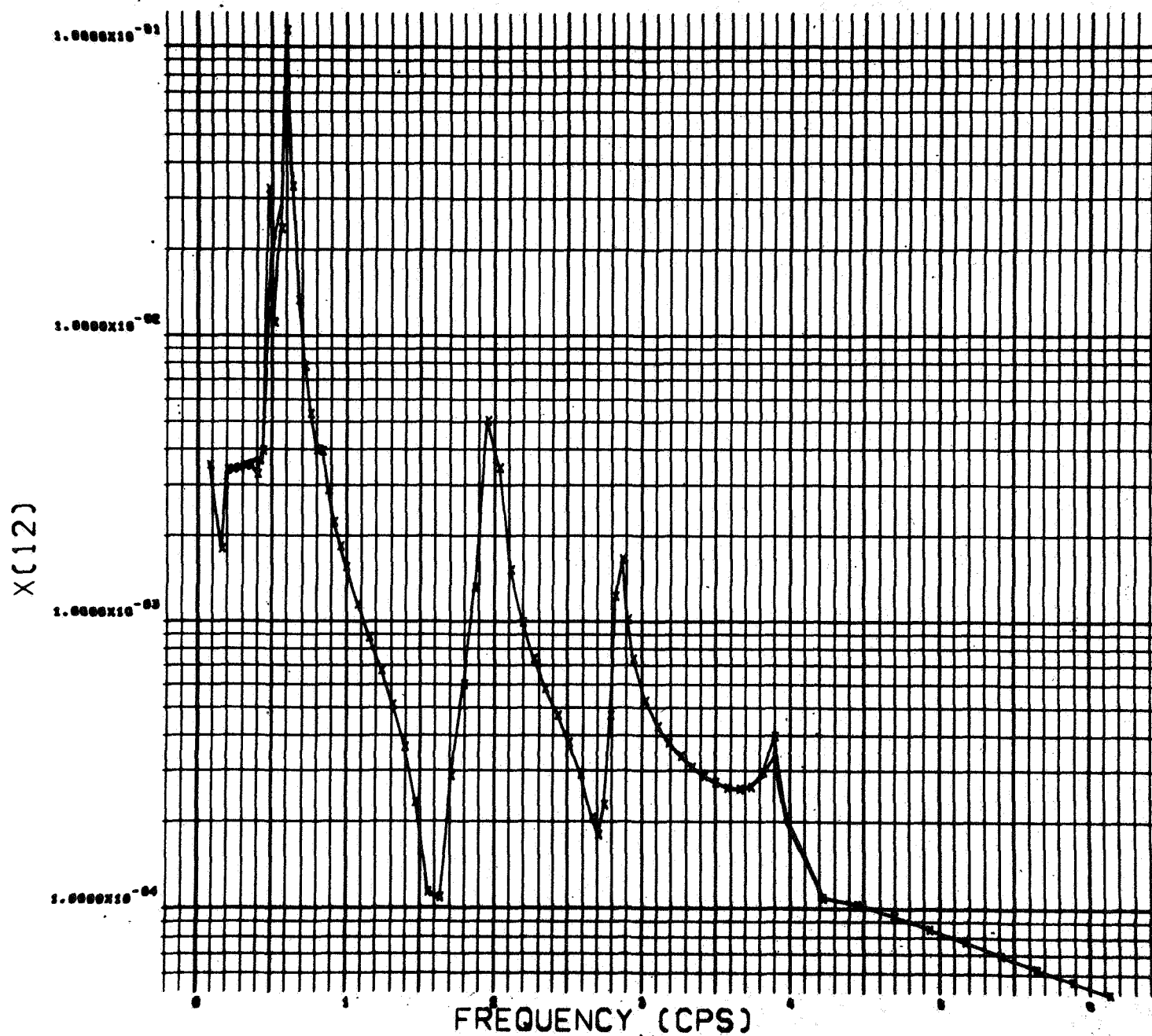


t = 78 seconds

Figure 36b

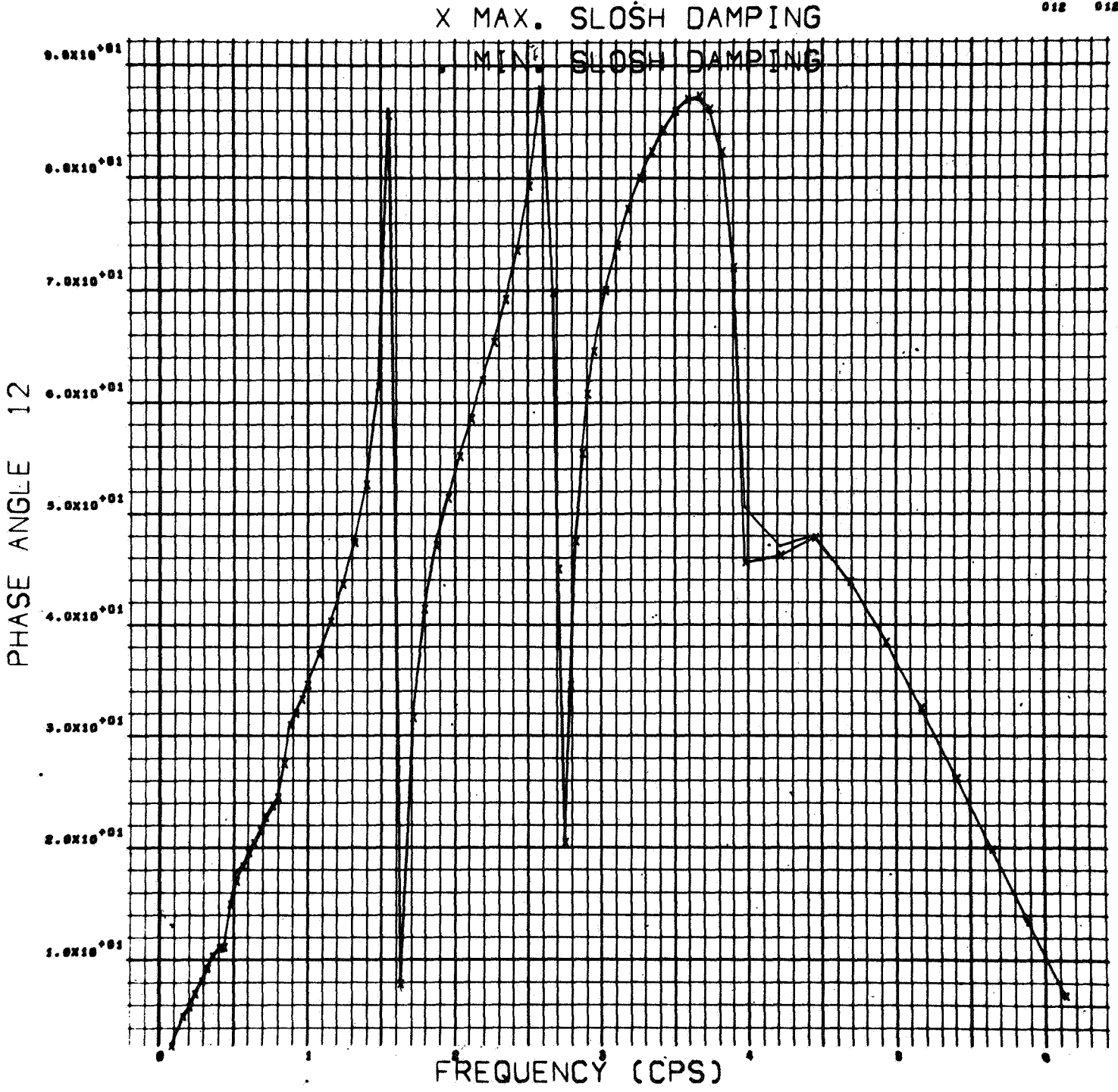
X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

400570
000 001



t = 78 seconds

Figure 37a

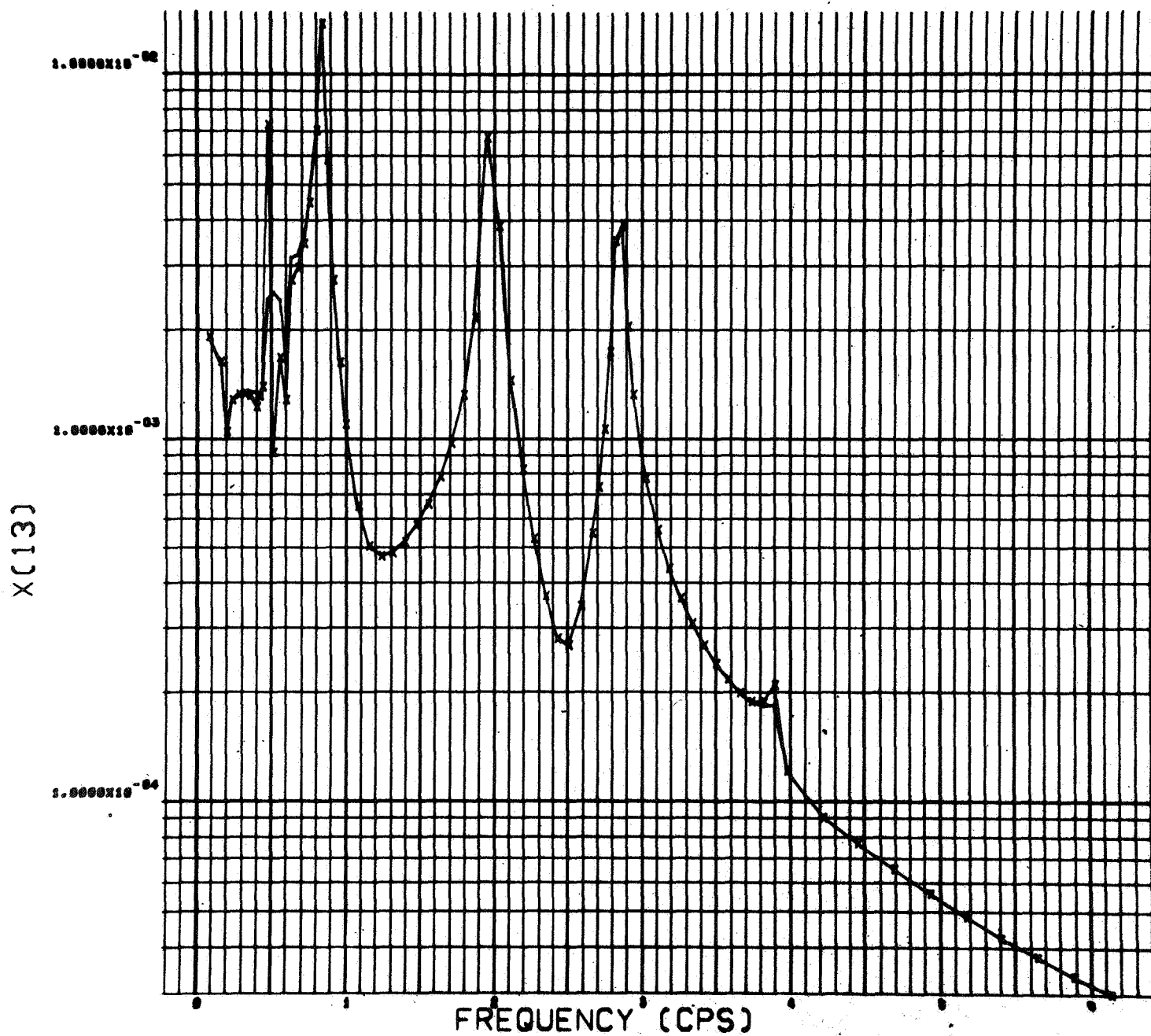


t = 78 seconds

Figure 37b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

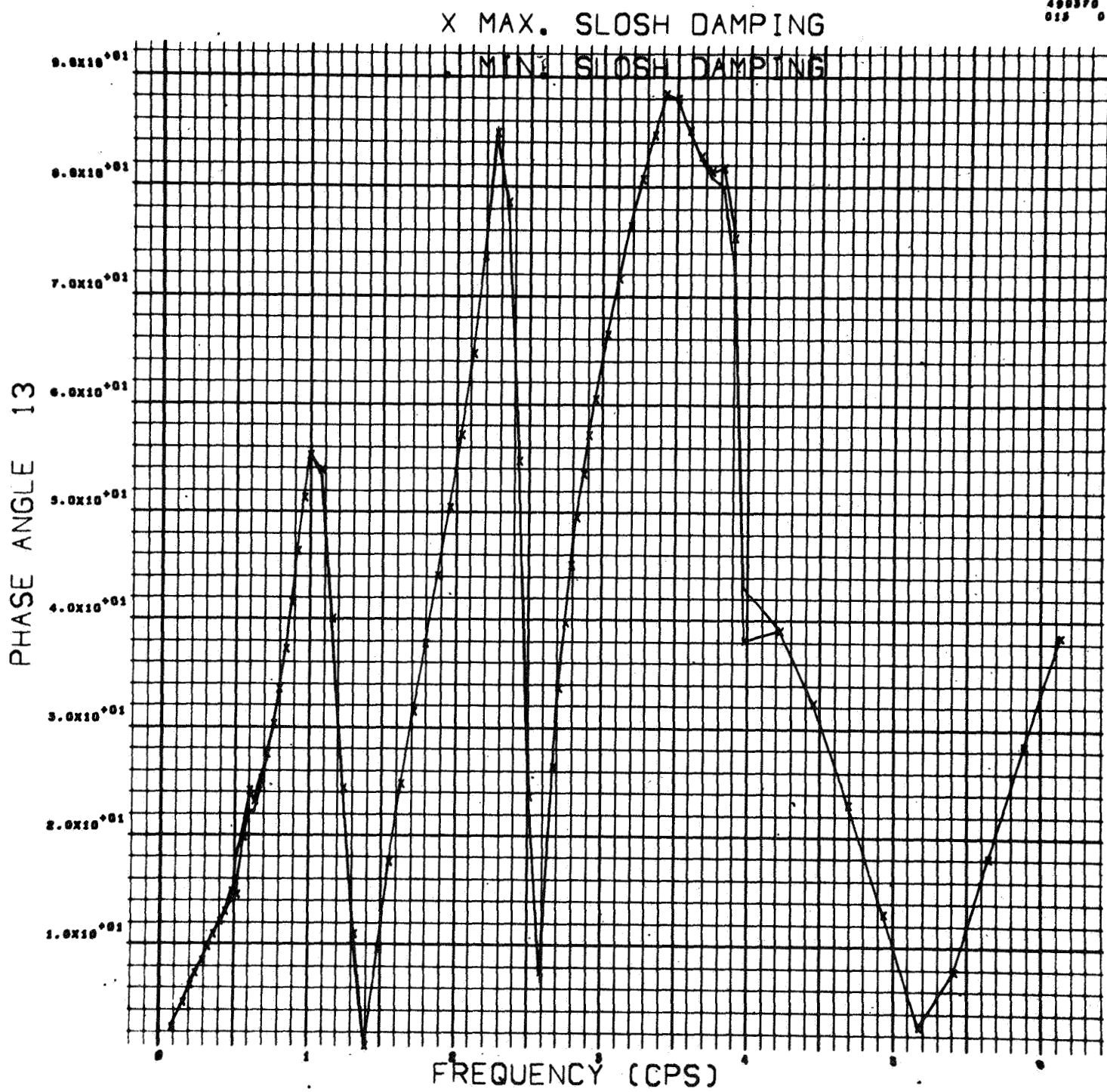
490570 L
034 034



t = 78 seconds

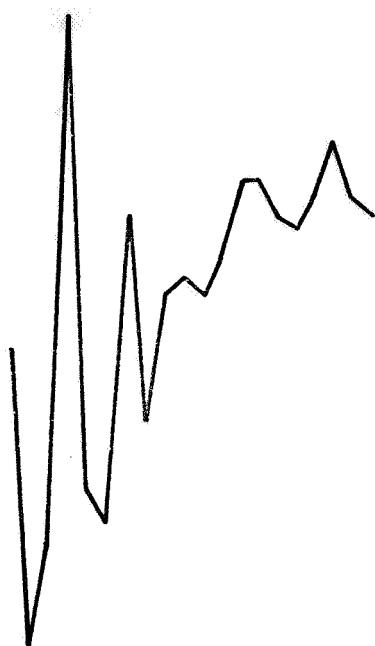
Figure 38a.

498376 L
013 013



t = 78 seconds

Figure 38b



Flight Test Data
X(14) , t = 78 seconds

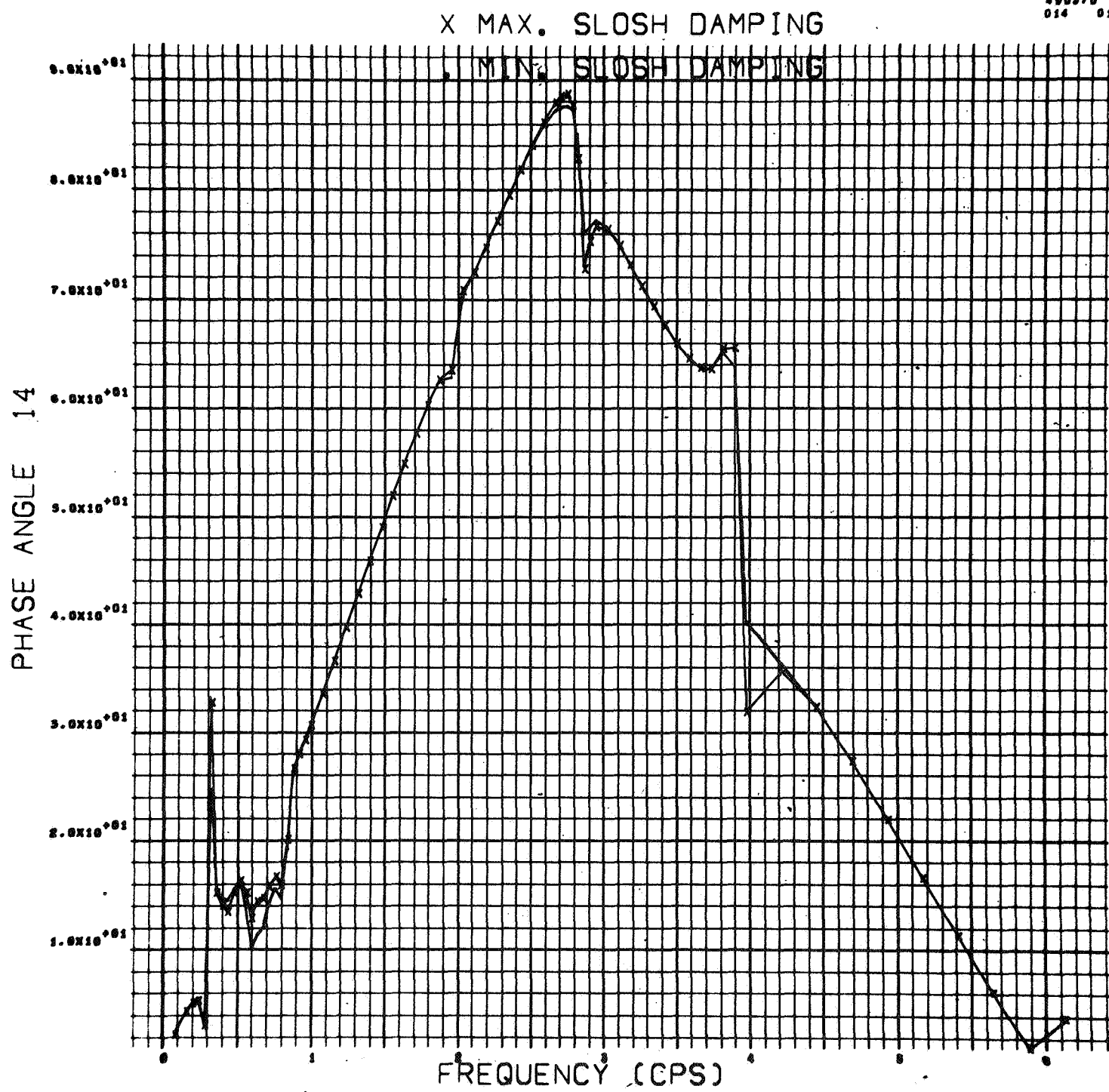
0000000000



Figure 39a



Flight Test Data
X(18) , t = 61 seconds



t = 78 seconds

Figure 39b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

400370
000 001

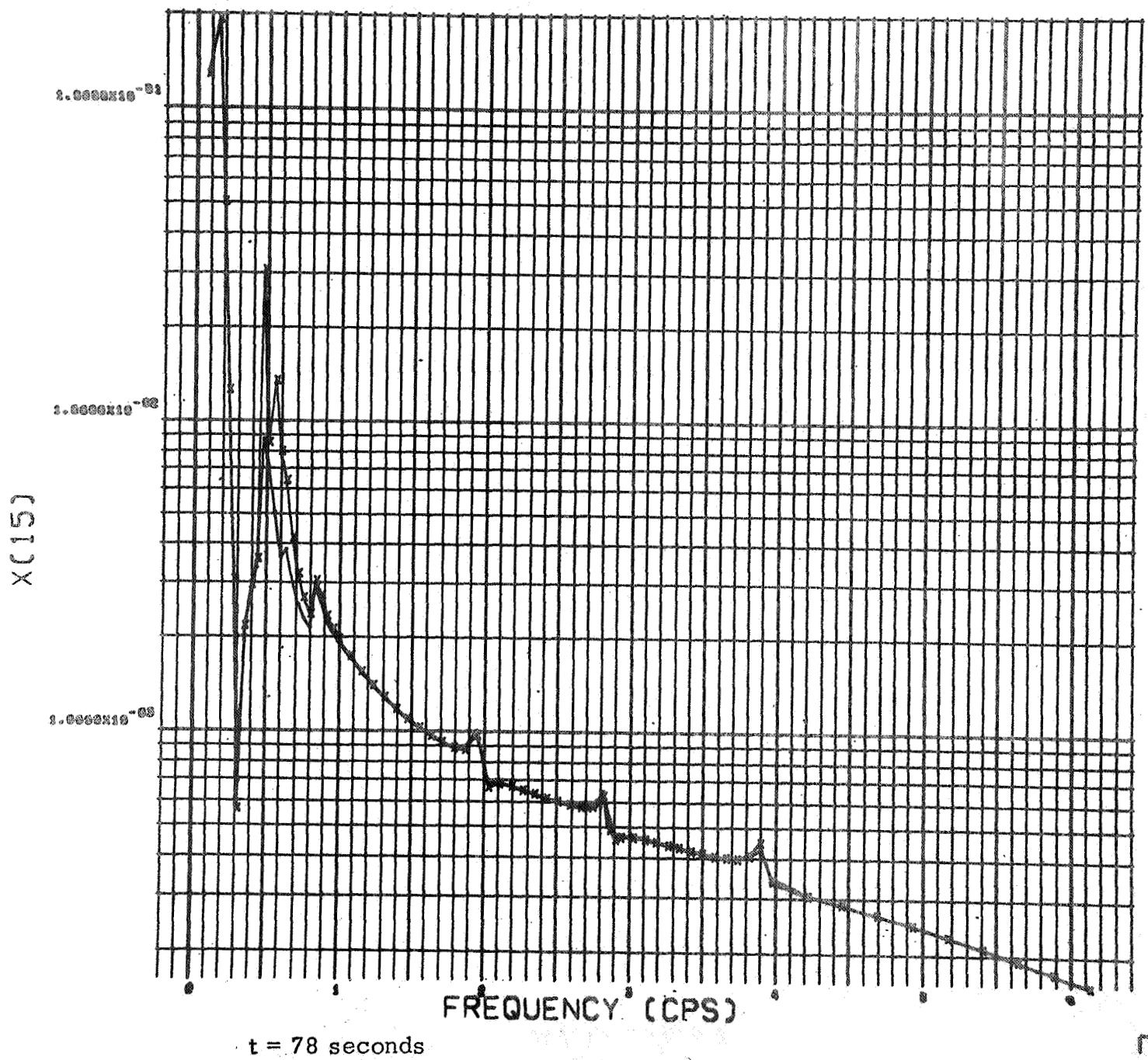
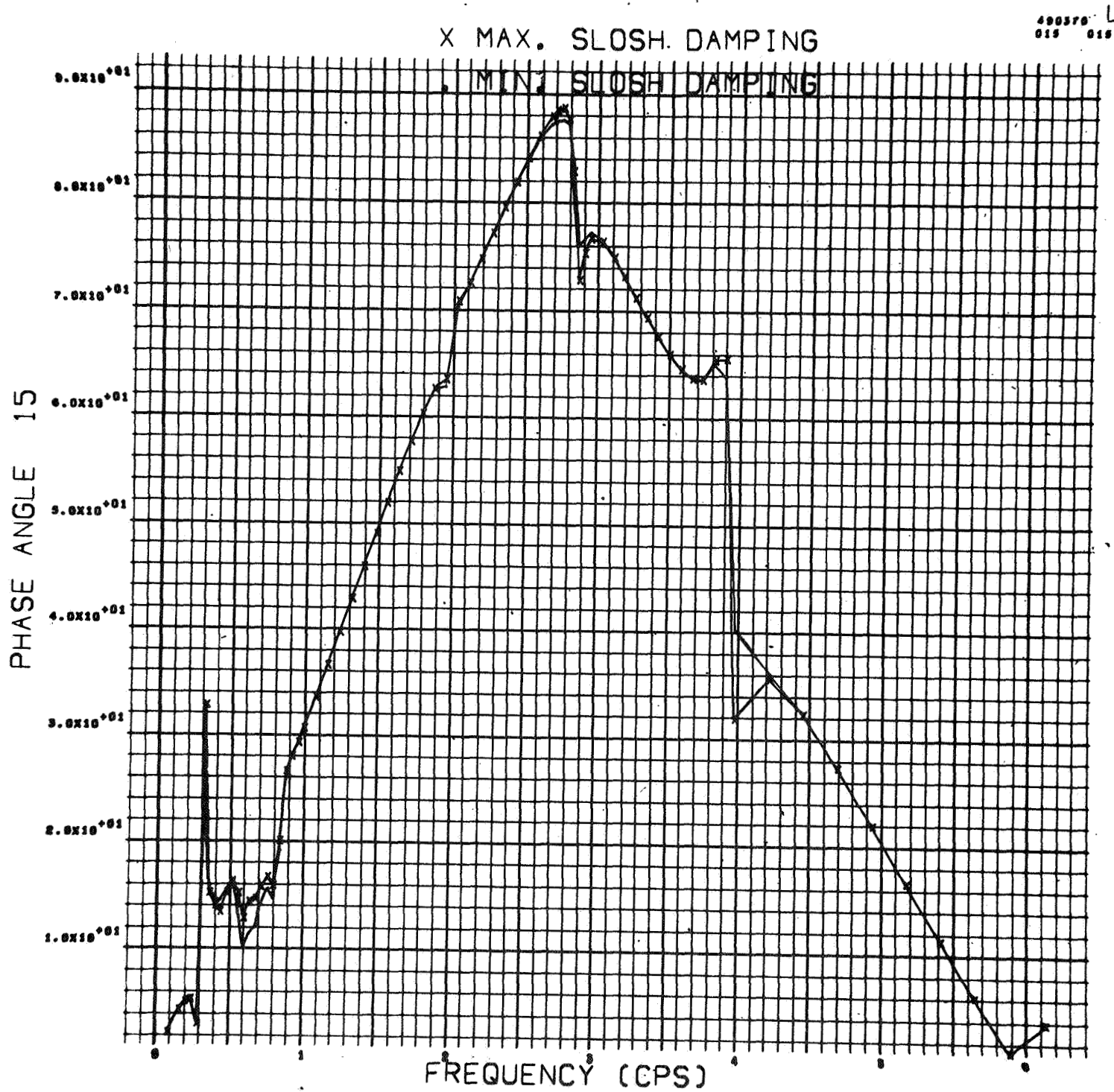


Figure 40a

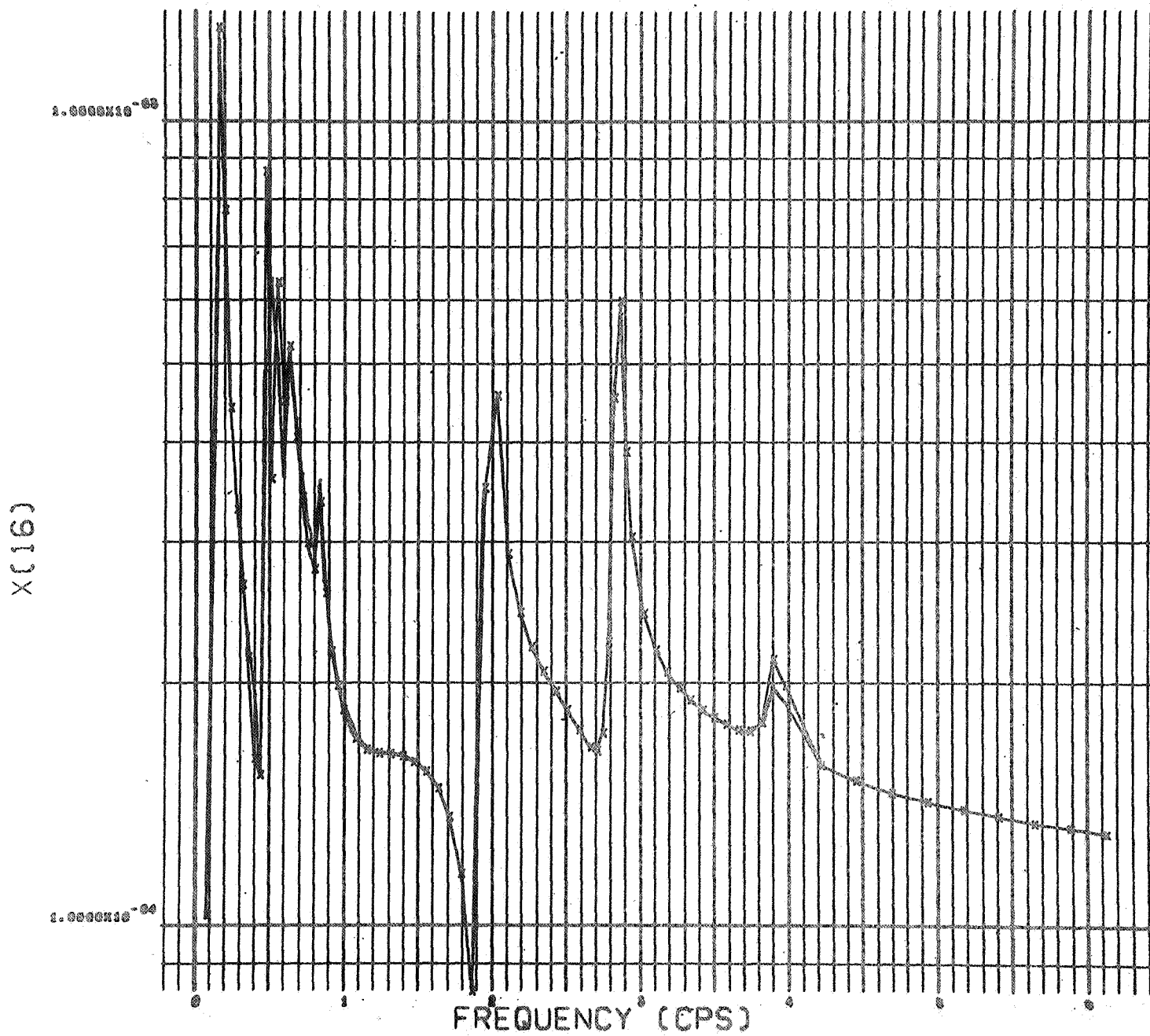


t = 78 seconds

Figure 40b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

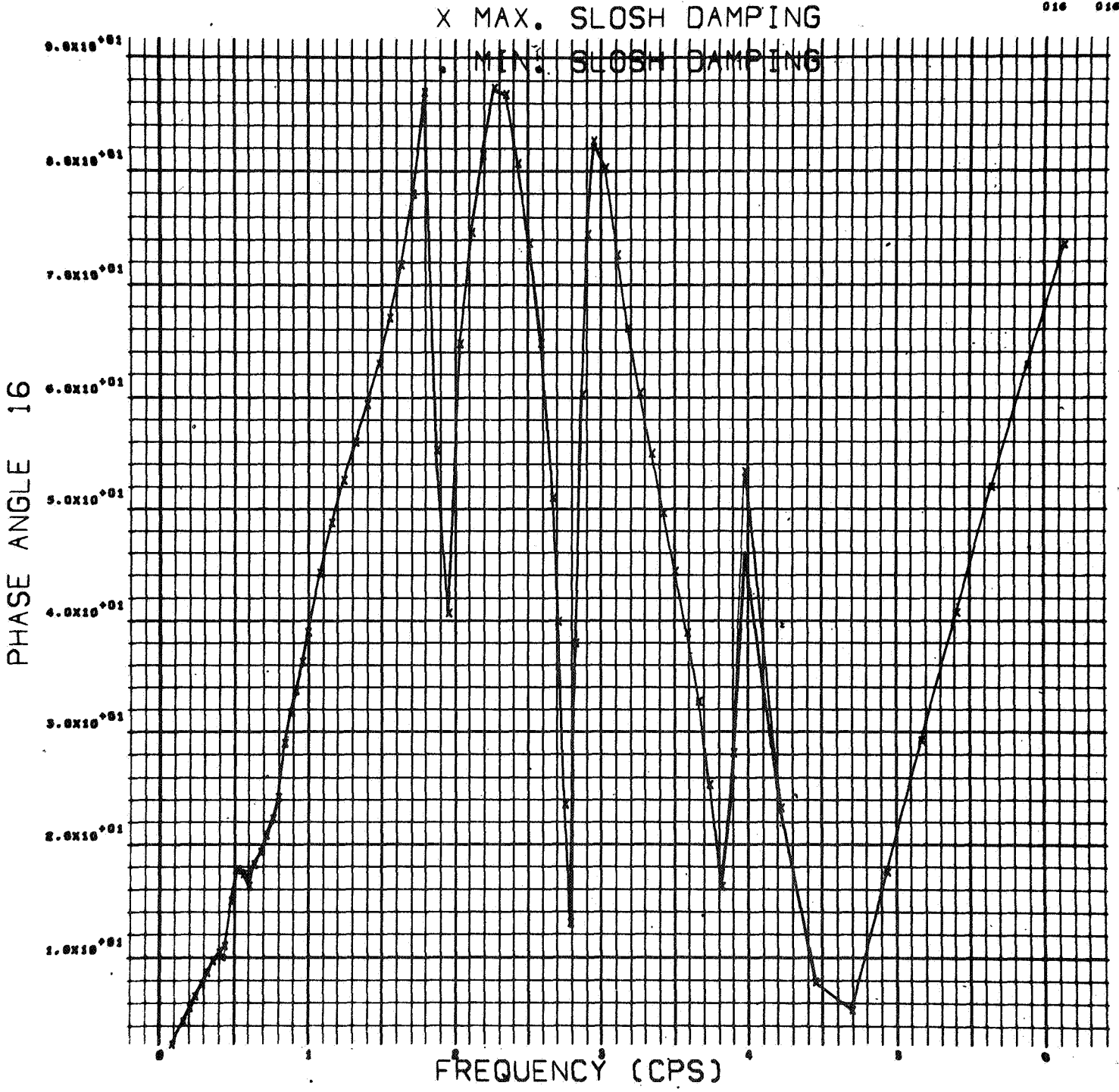
400070 1
007 007



t = 78 seconds.

Figure 41a

490576 L
010 010



t = 78 seconds

Figure 41b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

000070 L
000 000

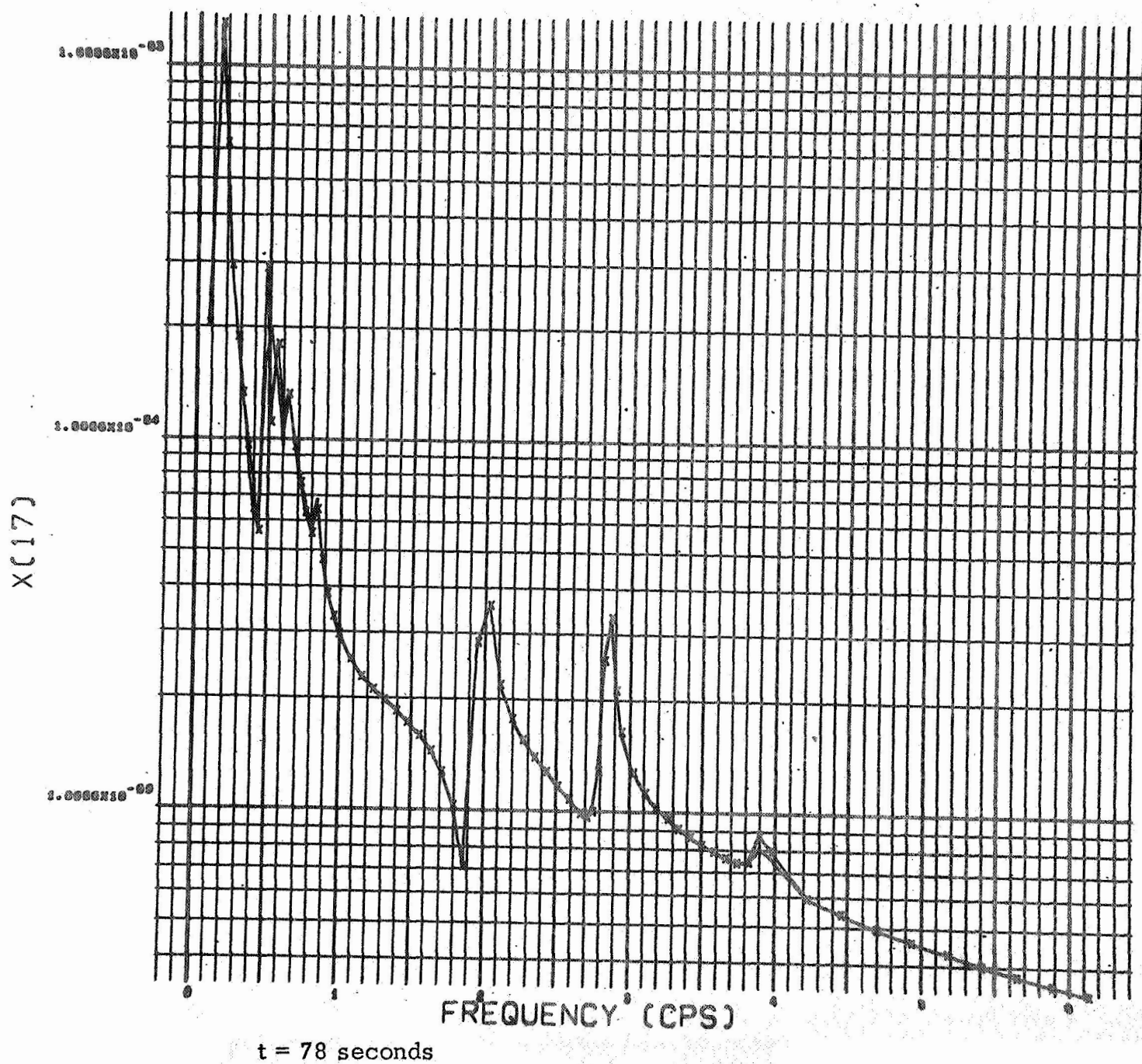
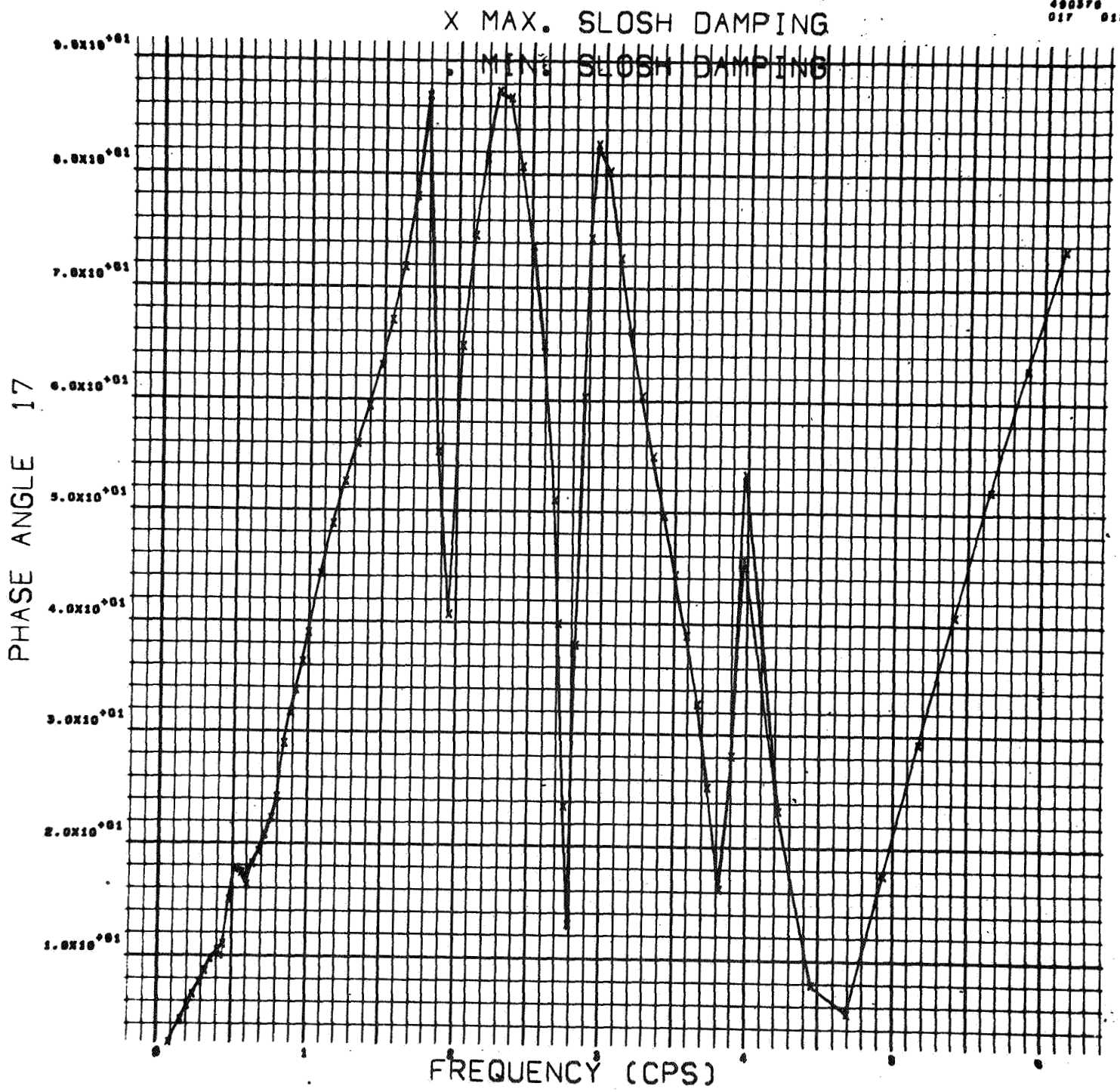
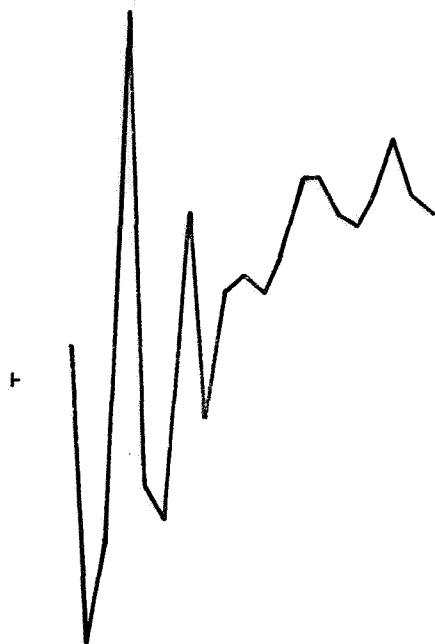


Figure 42a

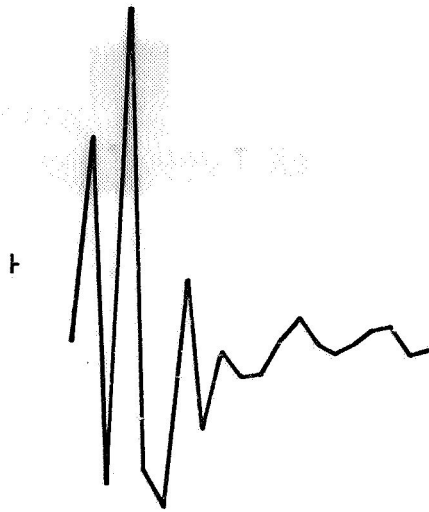


t = 78 seconds

Figure 42b



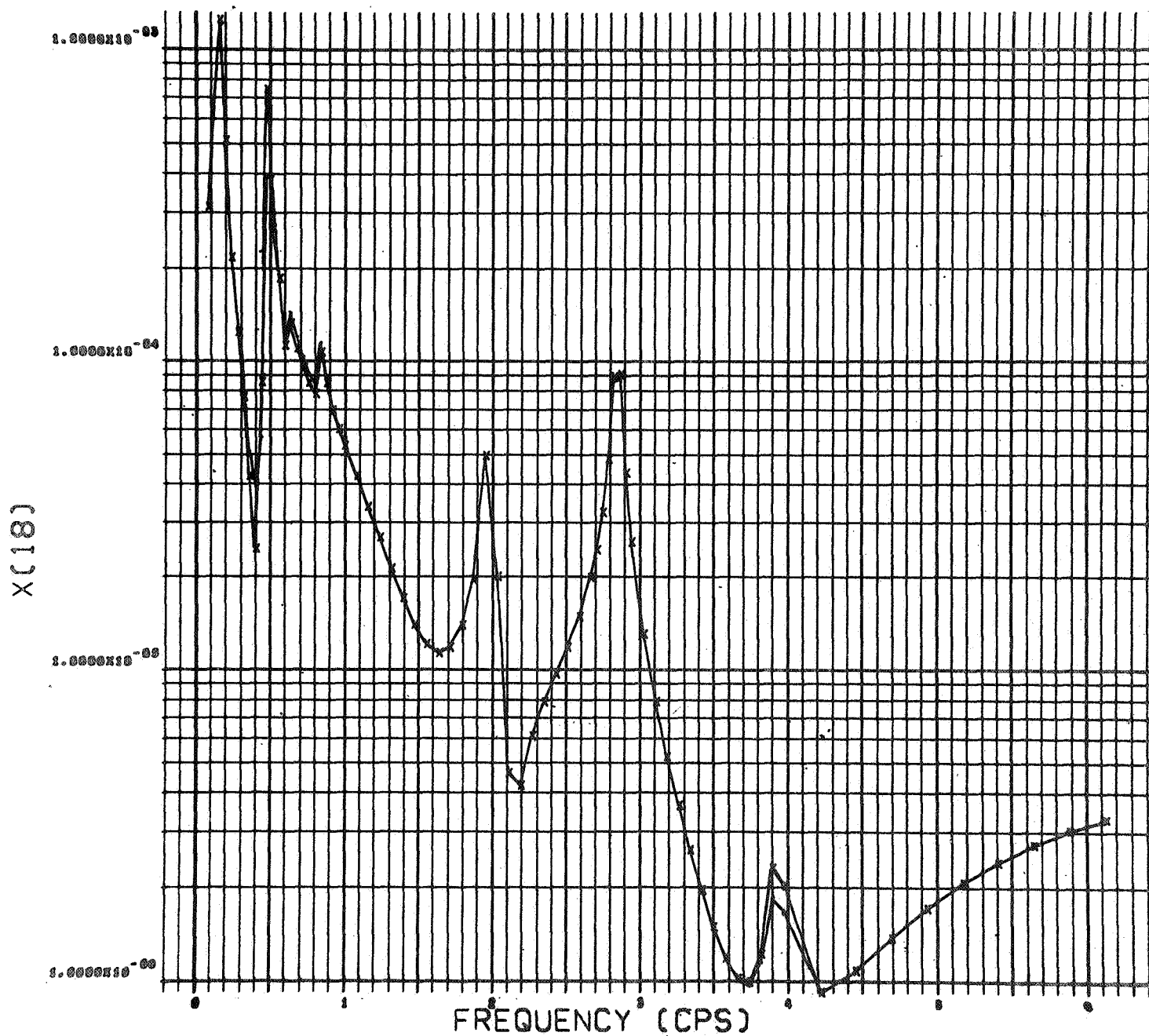
Flight Test Data
X(14) , t = 78 seconds



Flight Test Data
X(18), t = 78 seconds

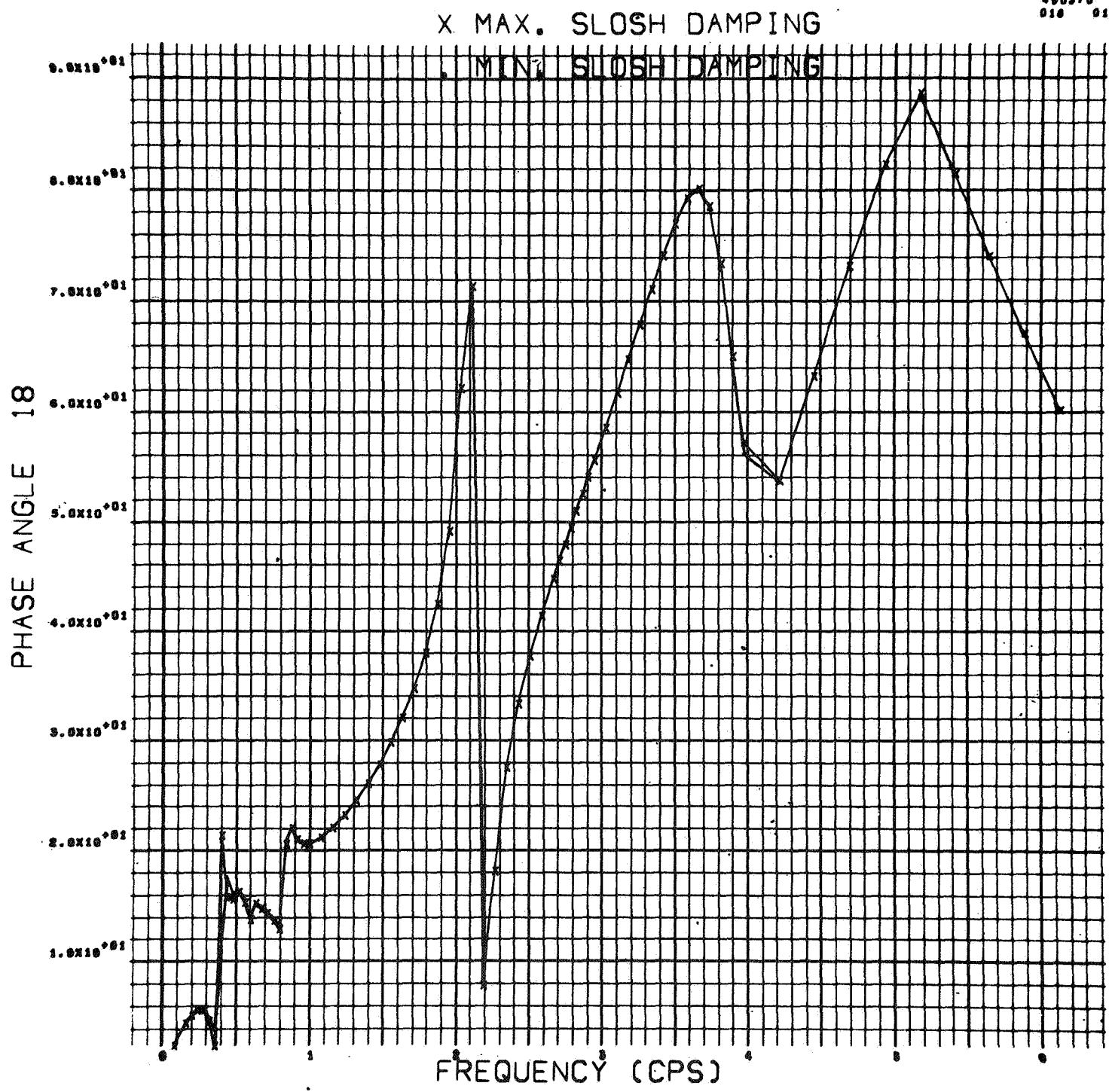
X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING

488370
 029 029



t = 78 seconds

Figure 43a

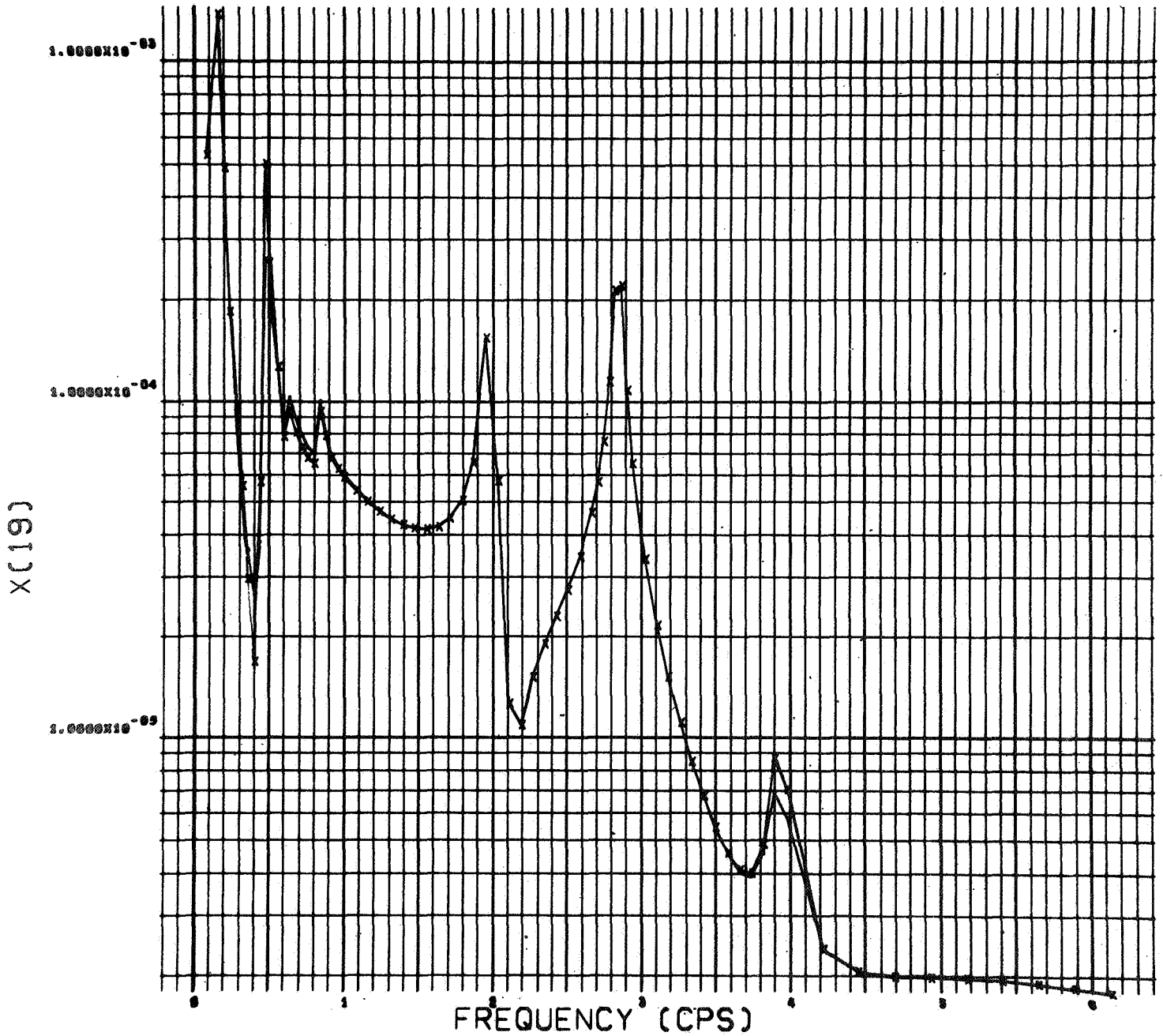


t = 78 seconds

Figure 43b

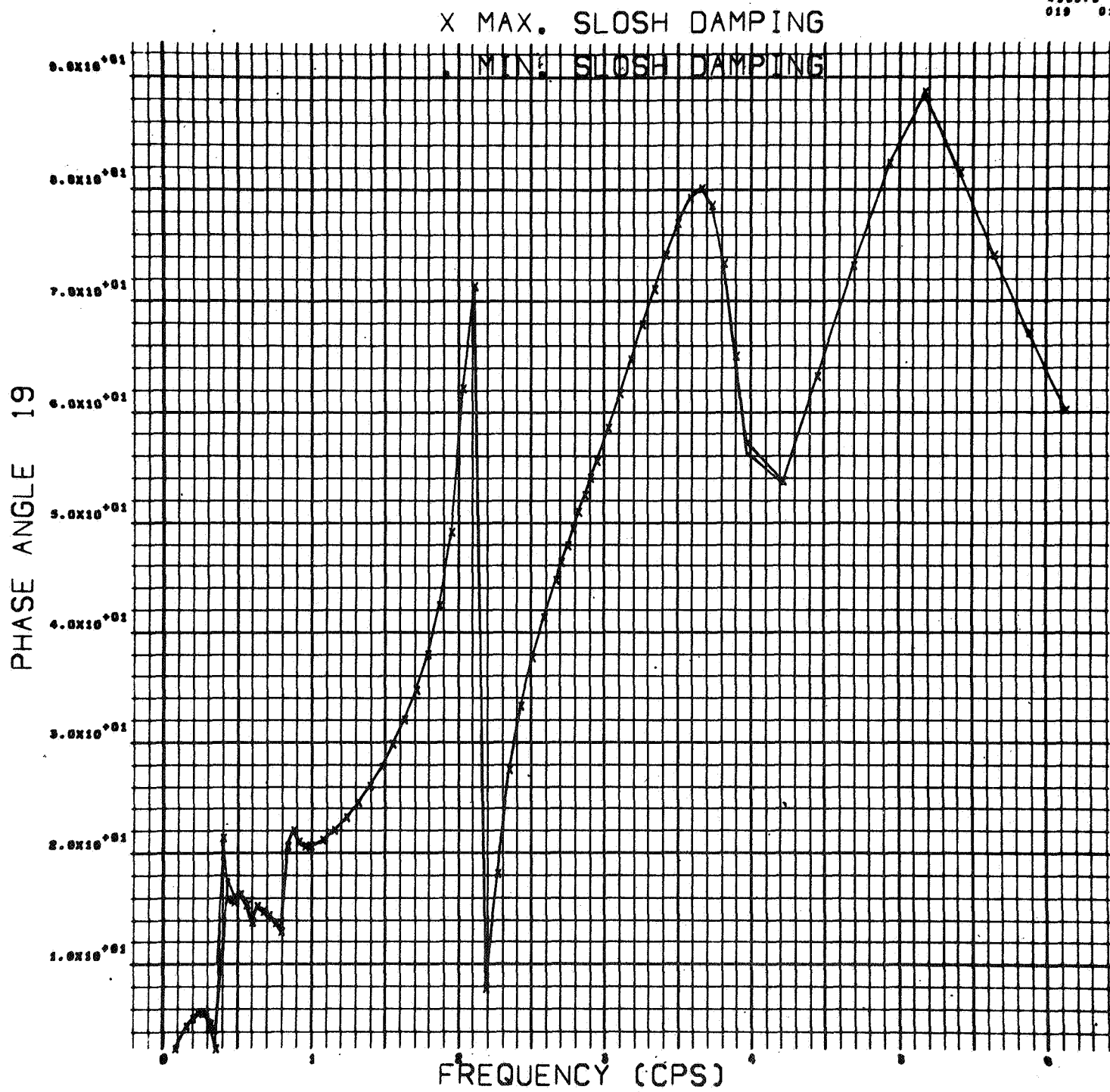
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490370 L
040 040



t = 78 seconds

Figure 44a



t = 78 seconds

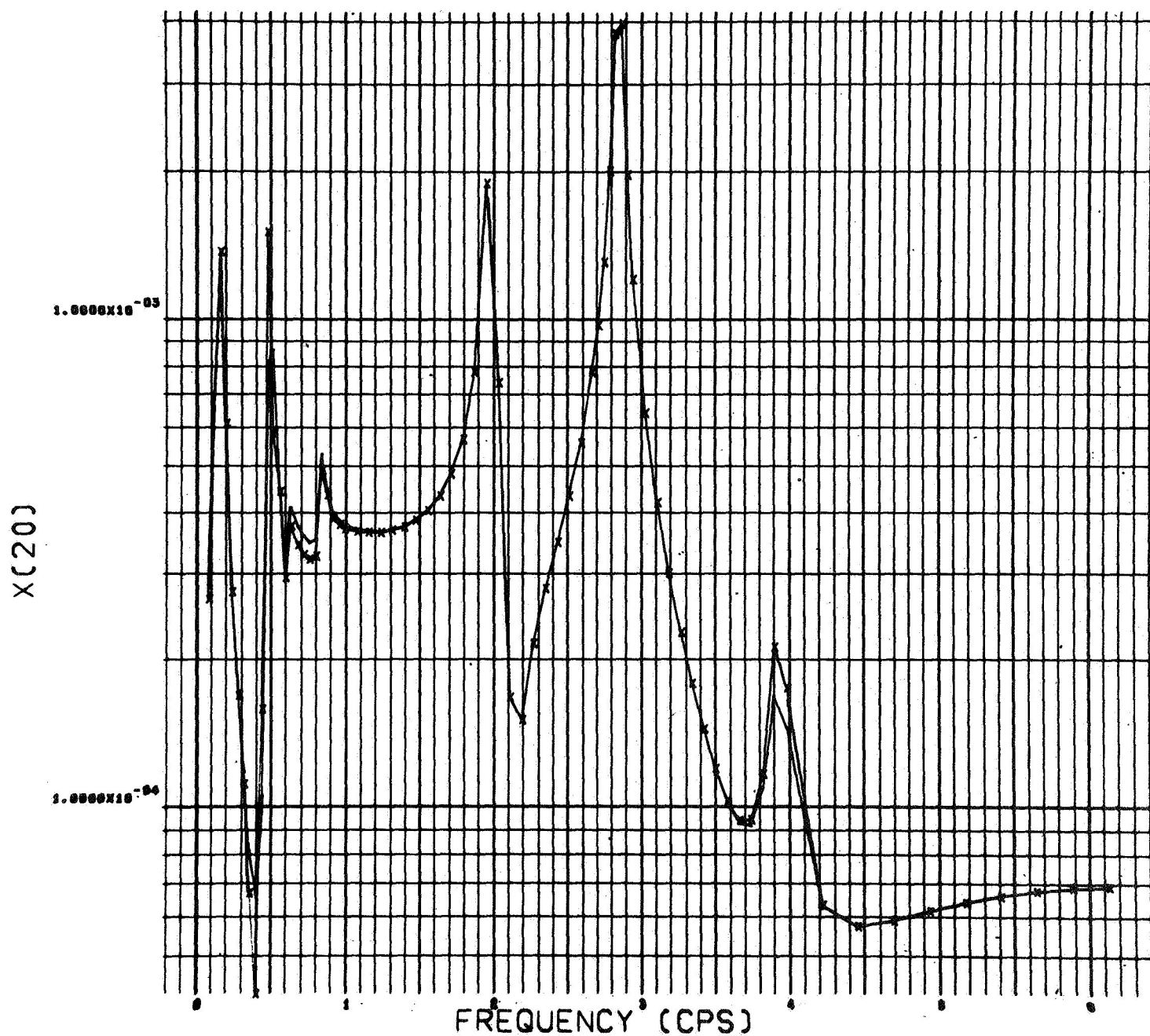
Figure 44b



Flight Test Data
 $X(20)$, $t = 78$ seconds

X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING

490370 L
041 041



t = 78 seconds

Figure 45a

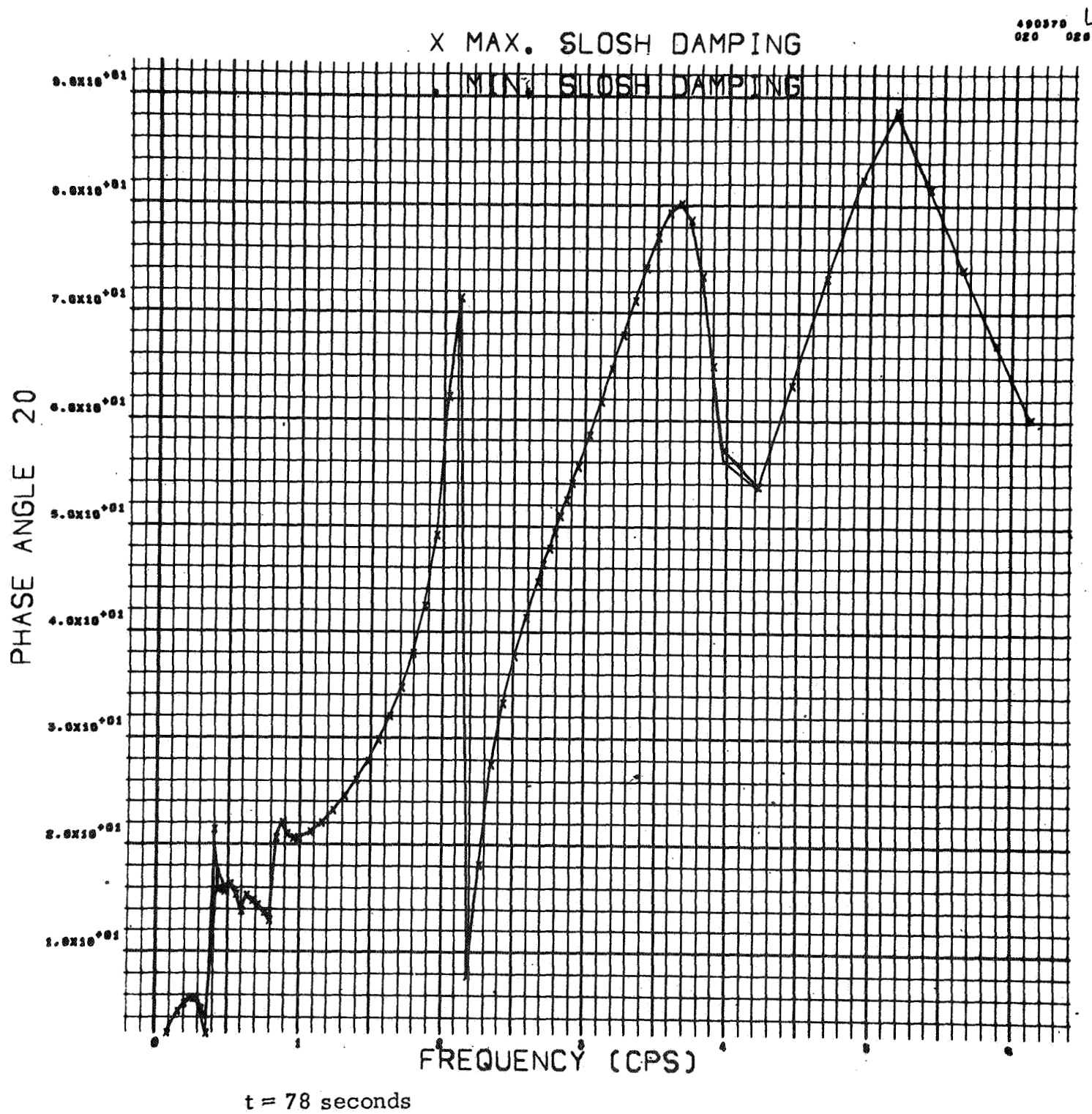
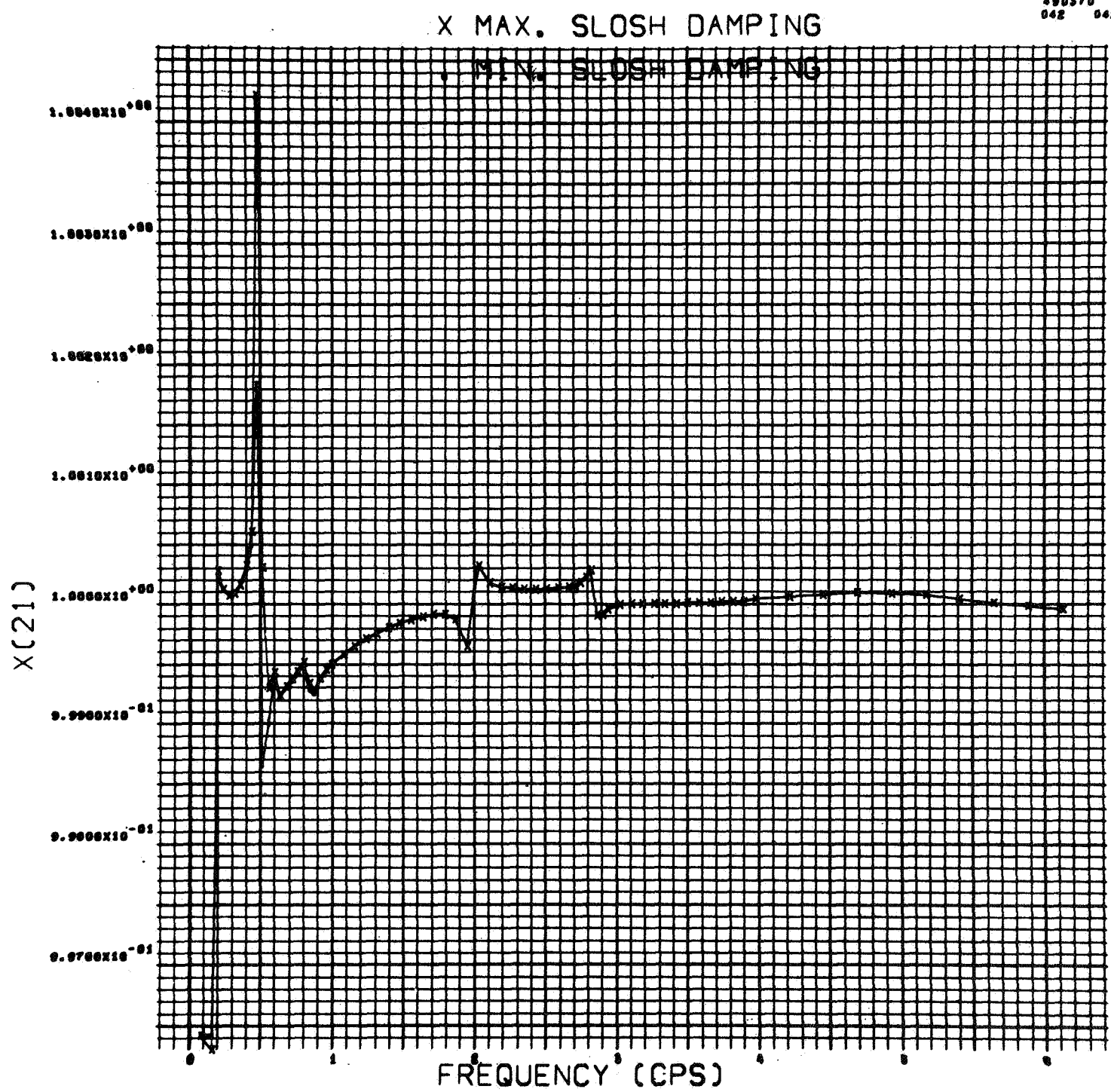


Figure 45b



t = 78 seconds

Figure 46a

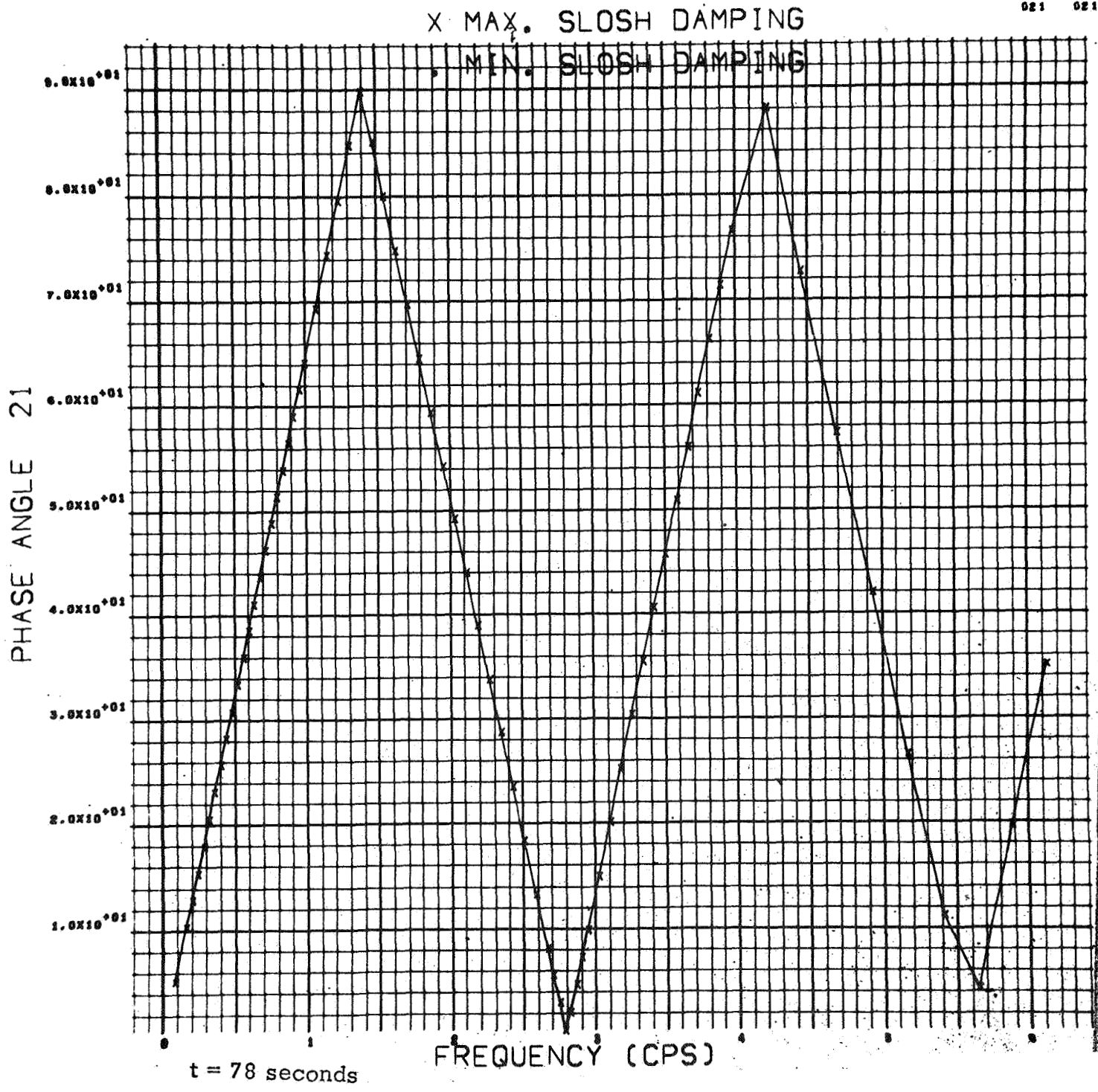
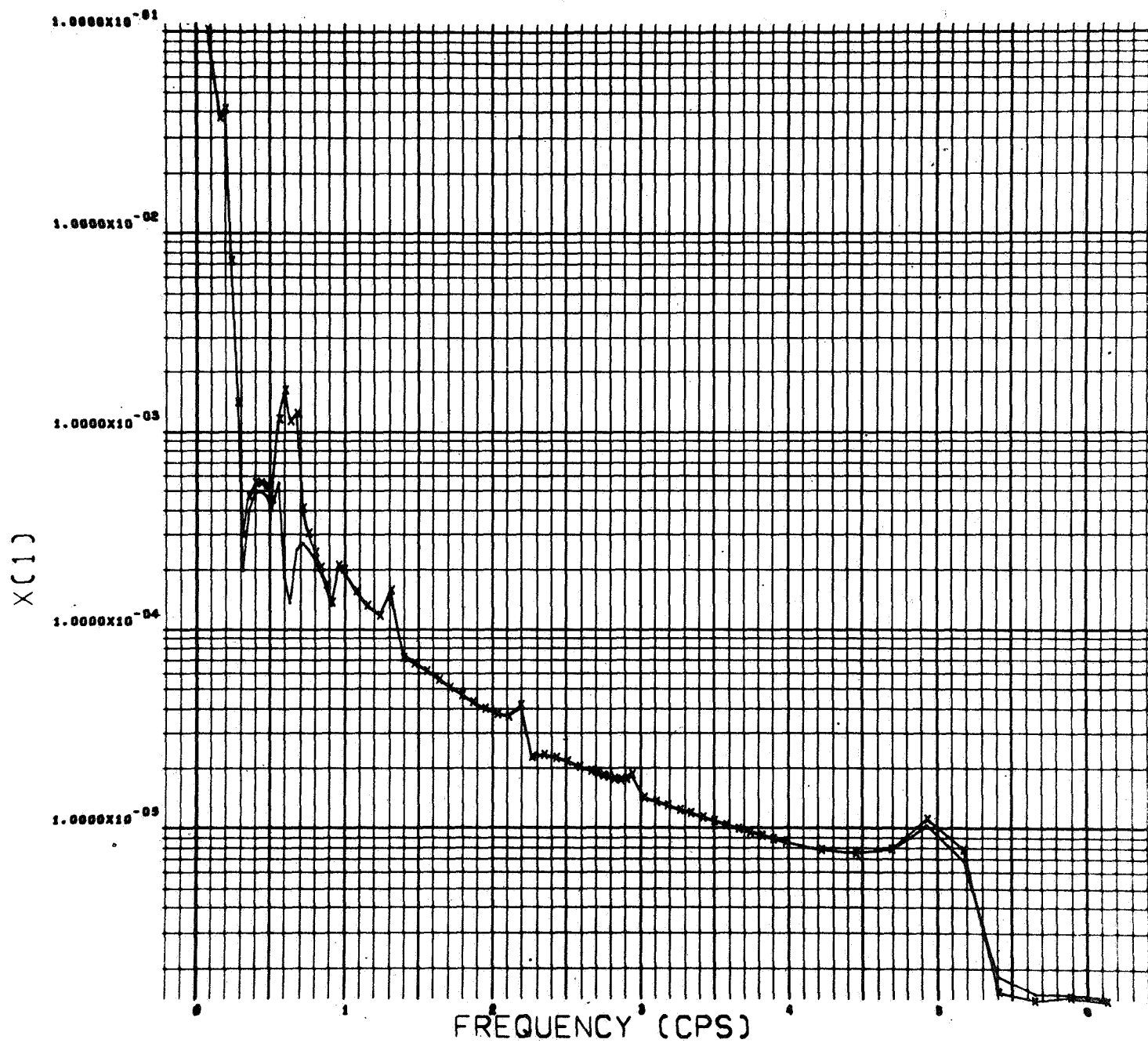


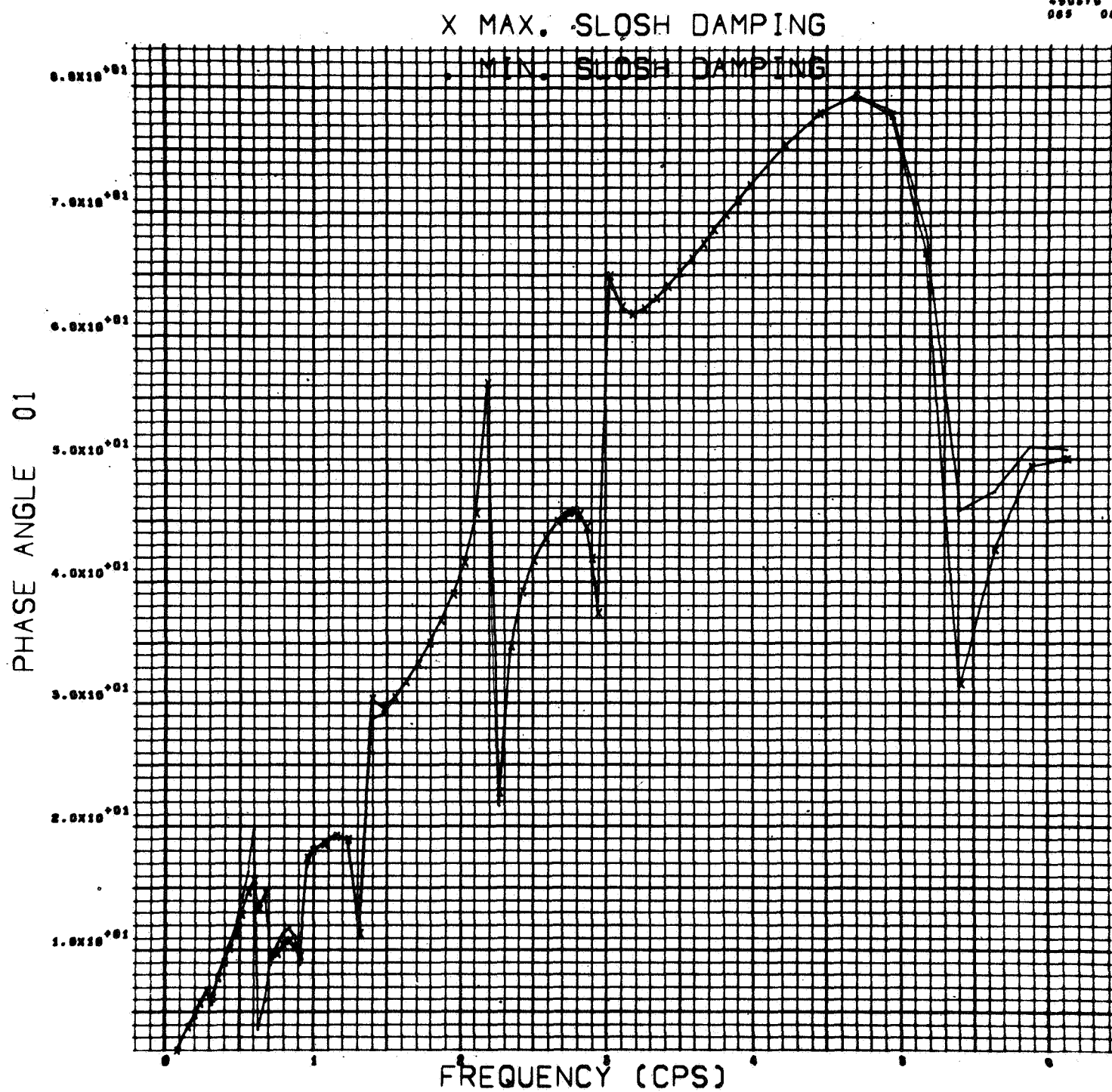
Figure 46b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



t = 97 seconds

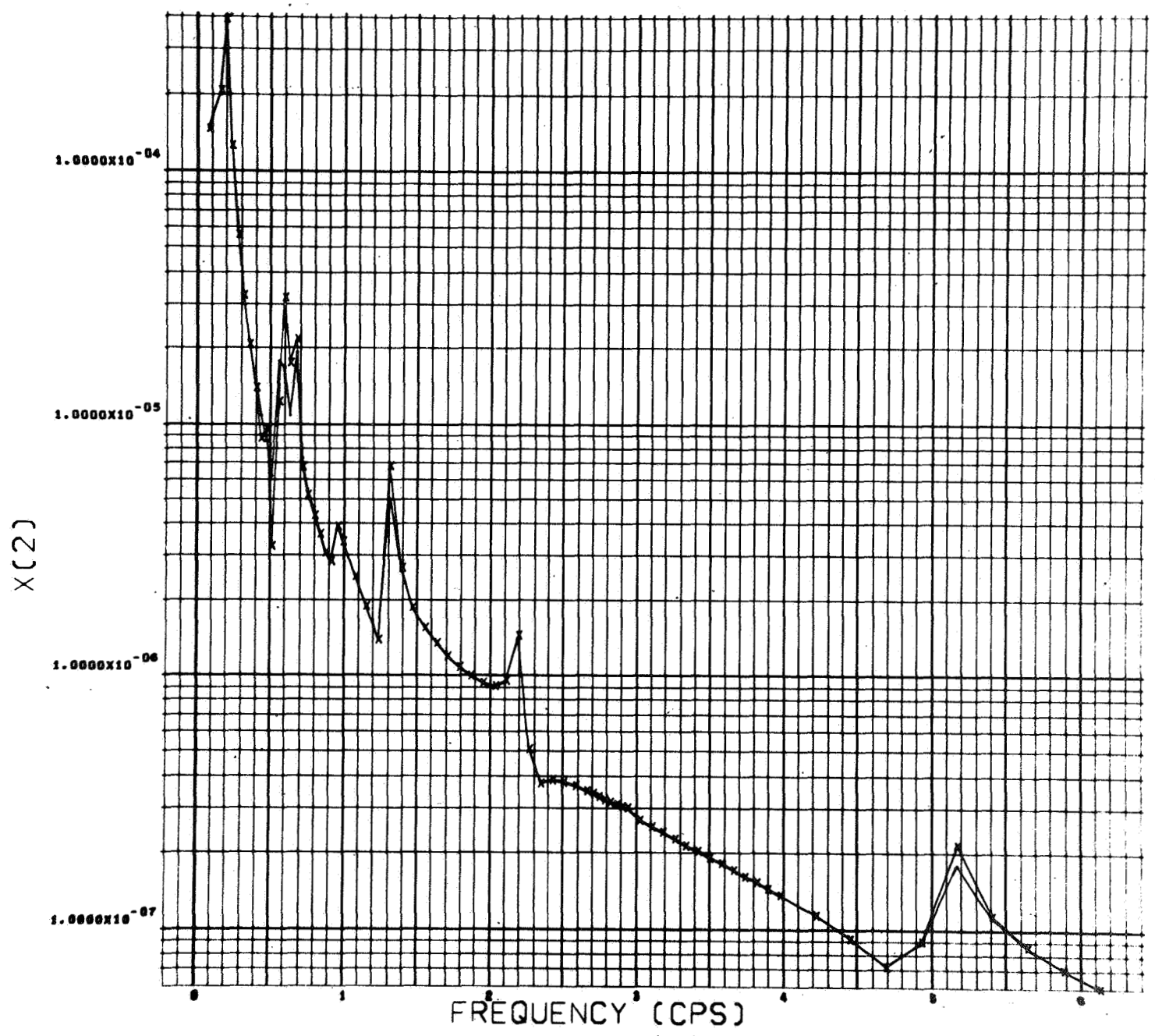
Figure 47a



t = 97 seconds

Figure 47b

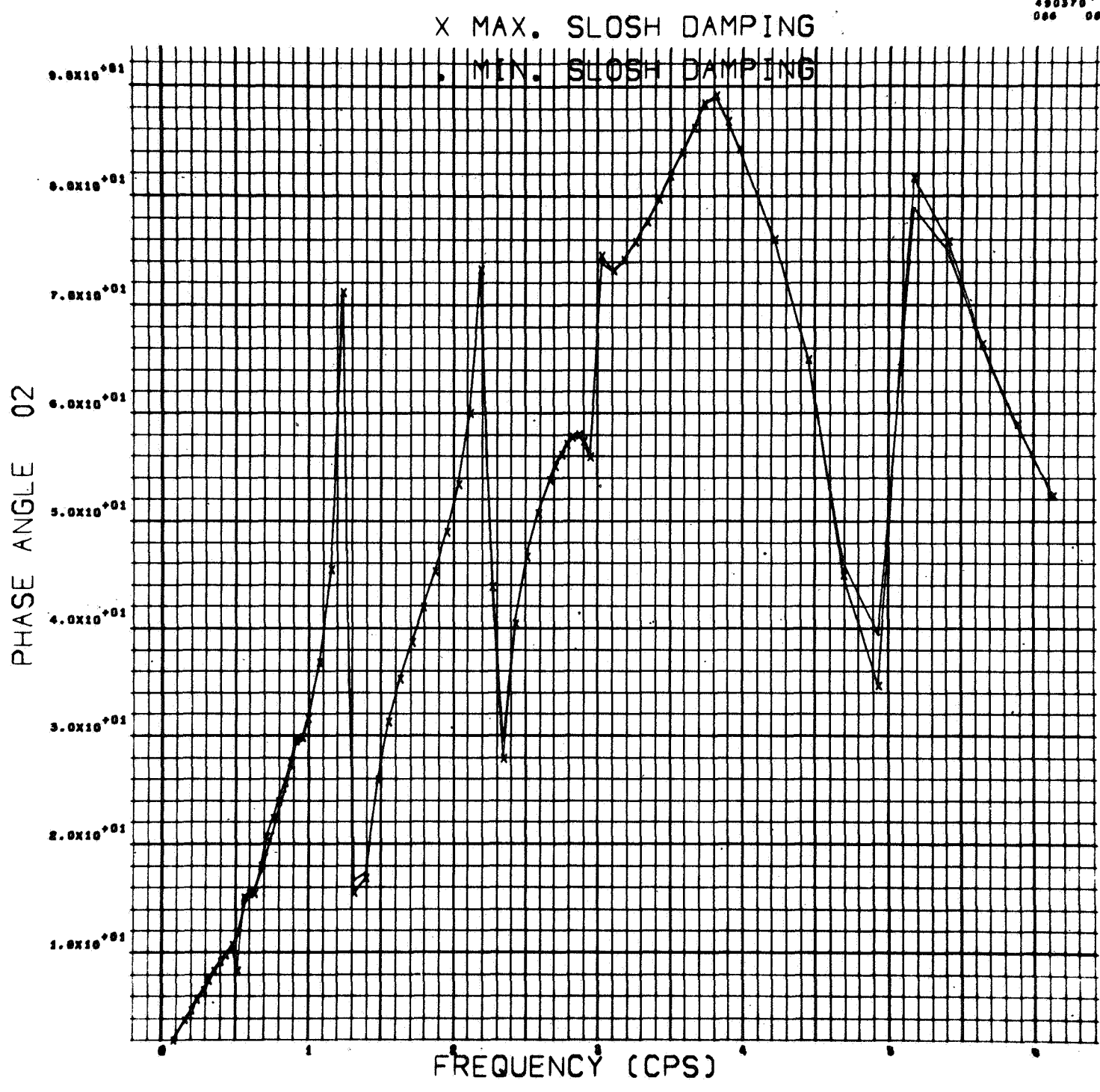
X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



t = 97 seconds

Figure 48a

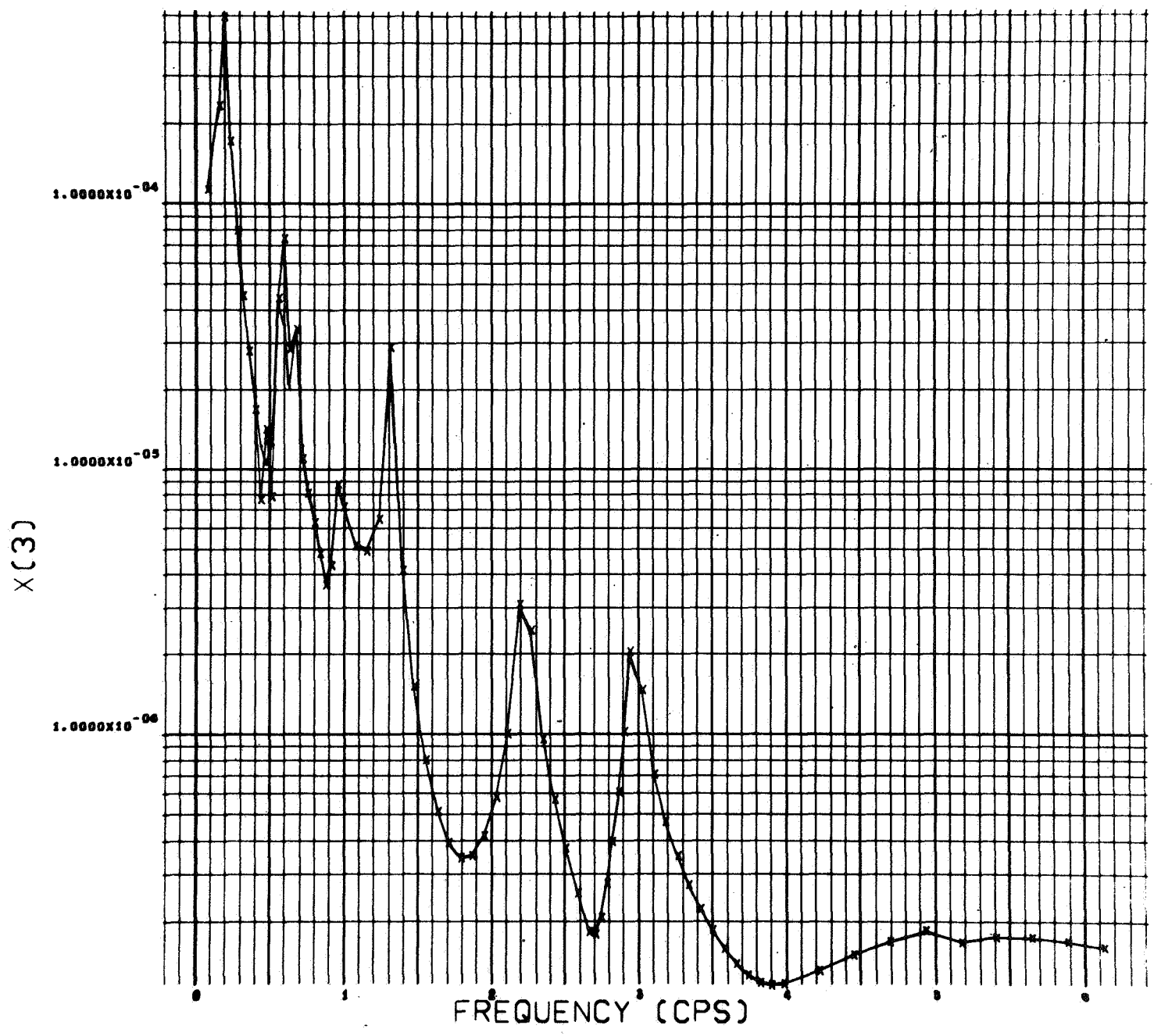
490370 L
000 000



t = 97 seconds

Figure 48b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING



t = 97 seconds

Figure 49a

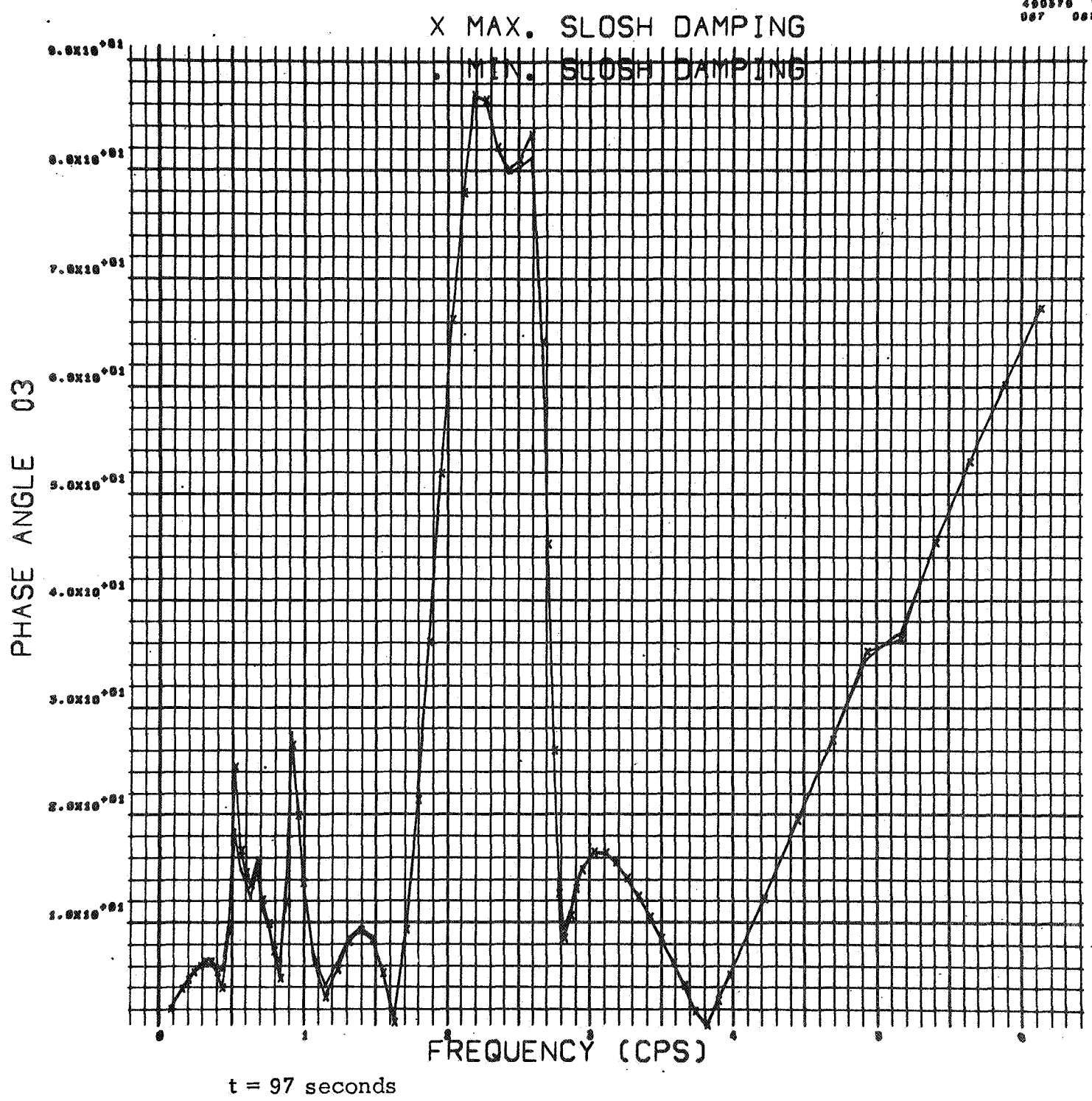
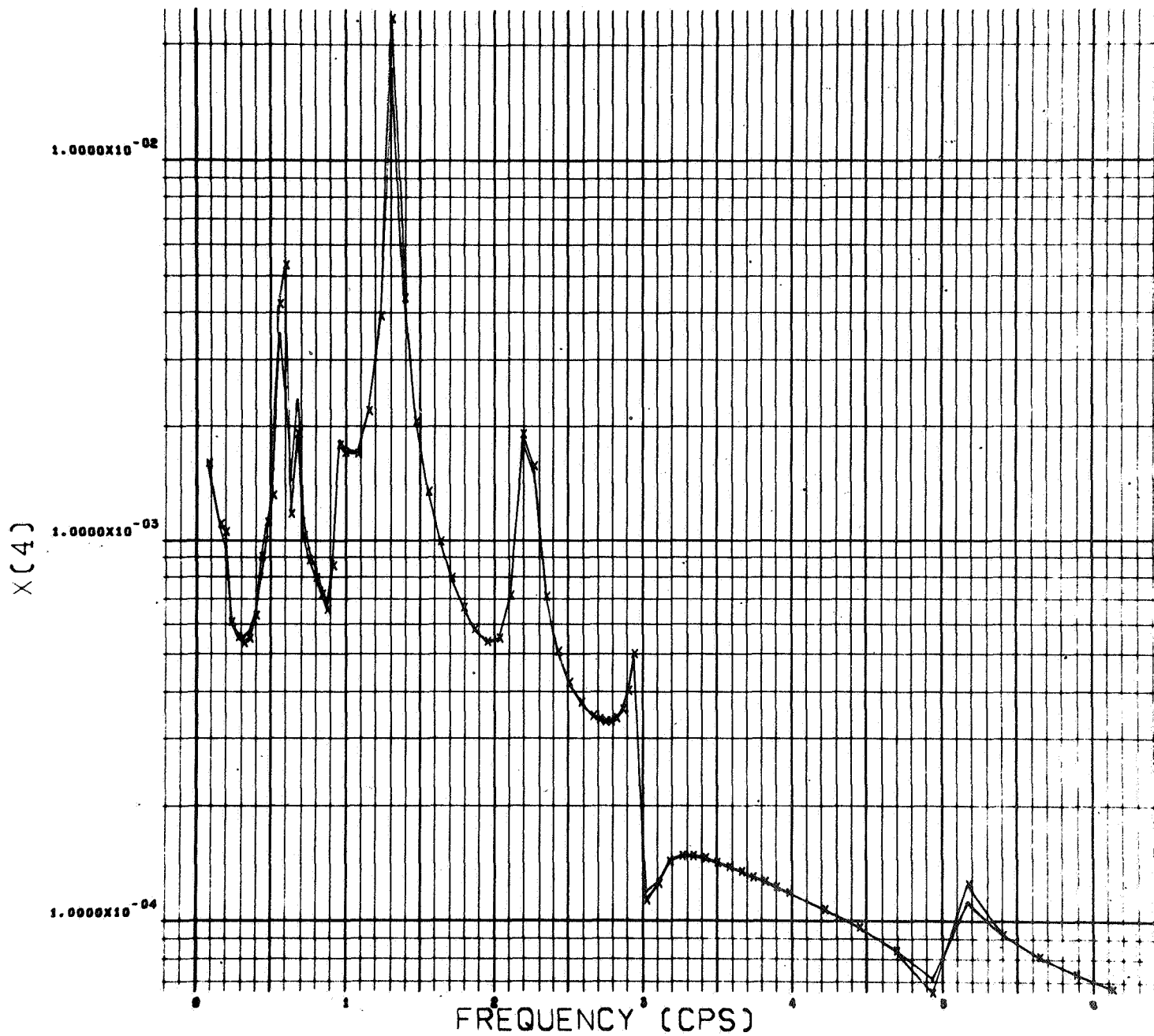


Figure 49b

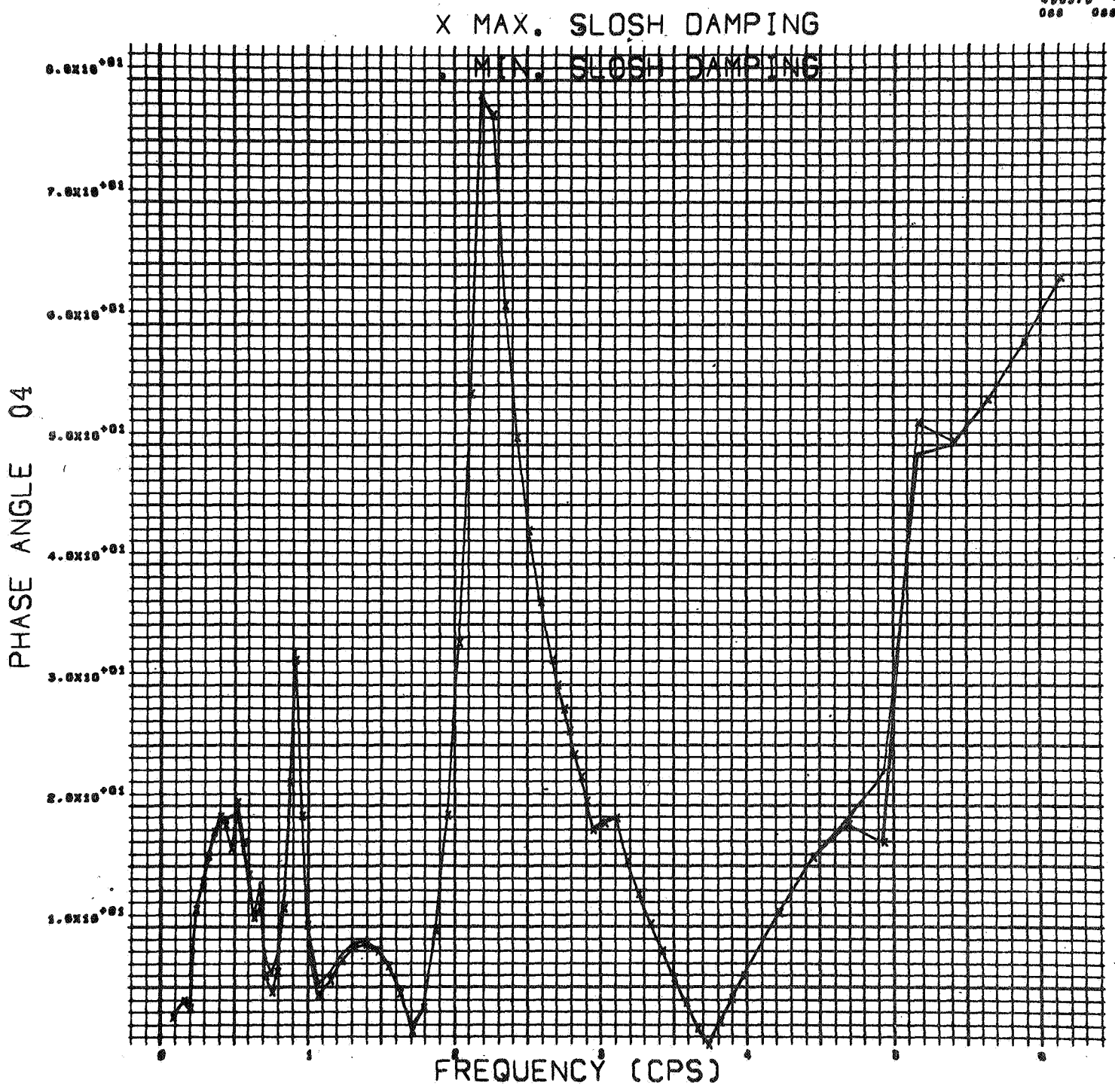
X MAX. SLOSH DAMPING

. MIN. SLOSH DAMPING



t = 97 seconds

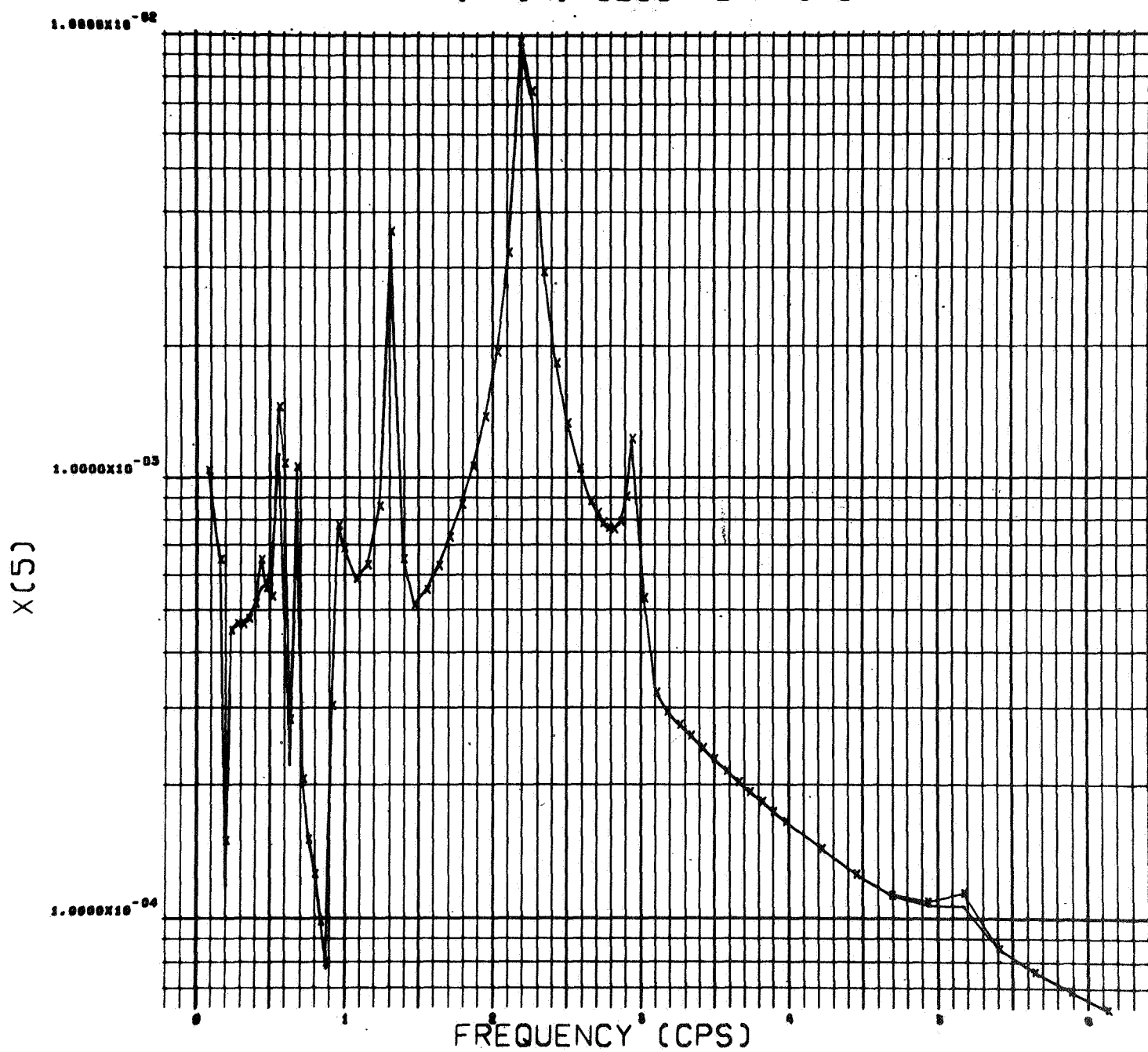
Figure 50a



t = 97 seconds

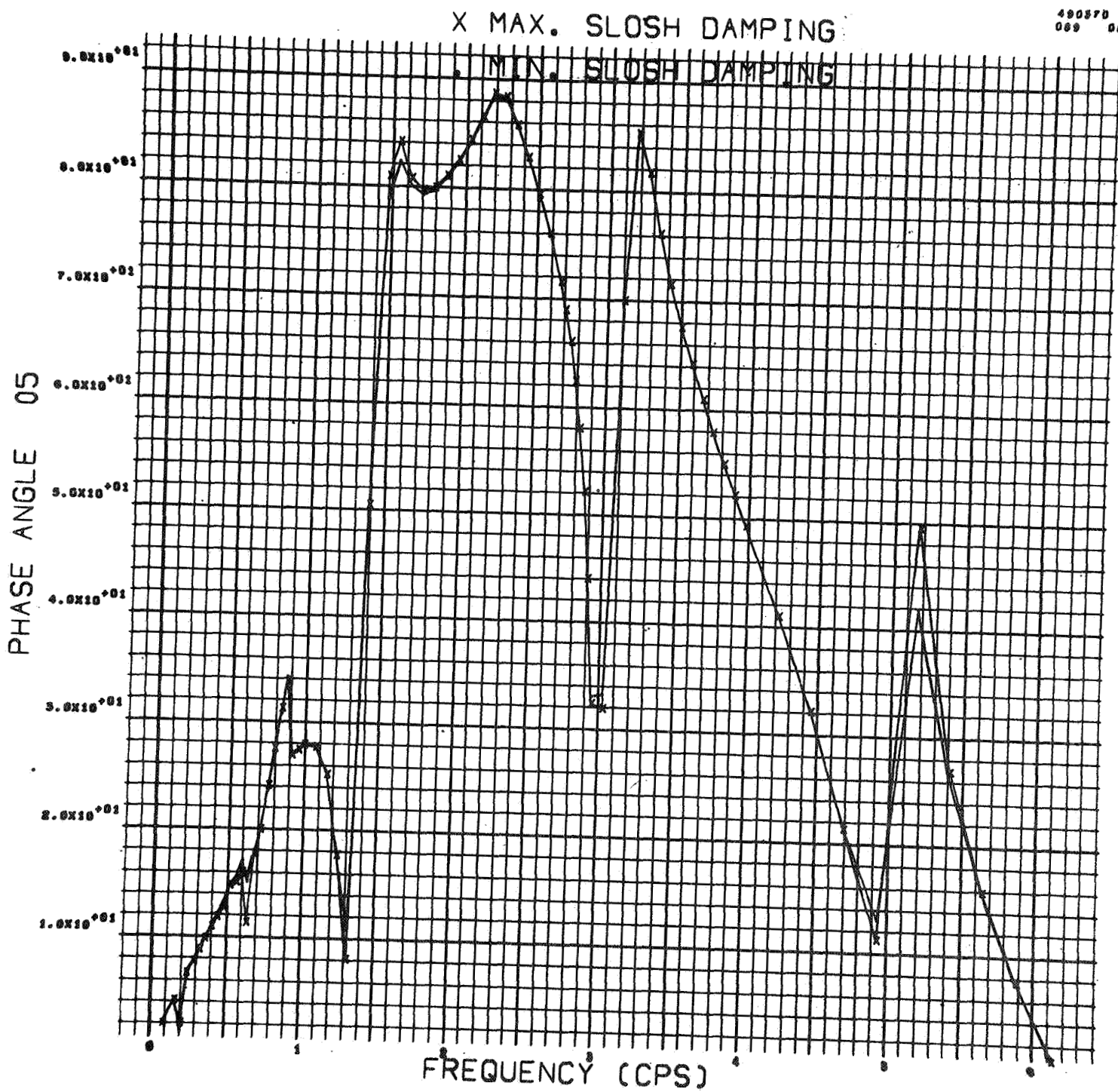
Figure 50b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



t = 97 seconds

Figure 51a

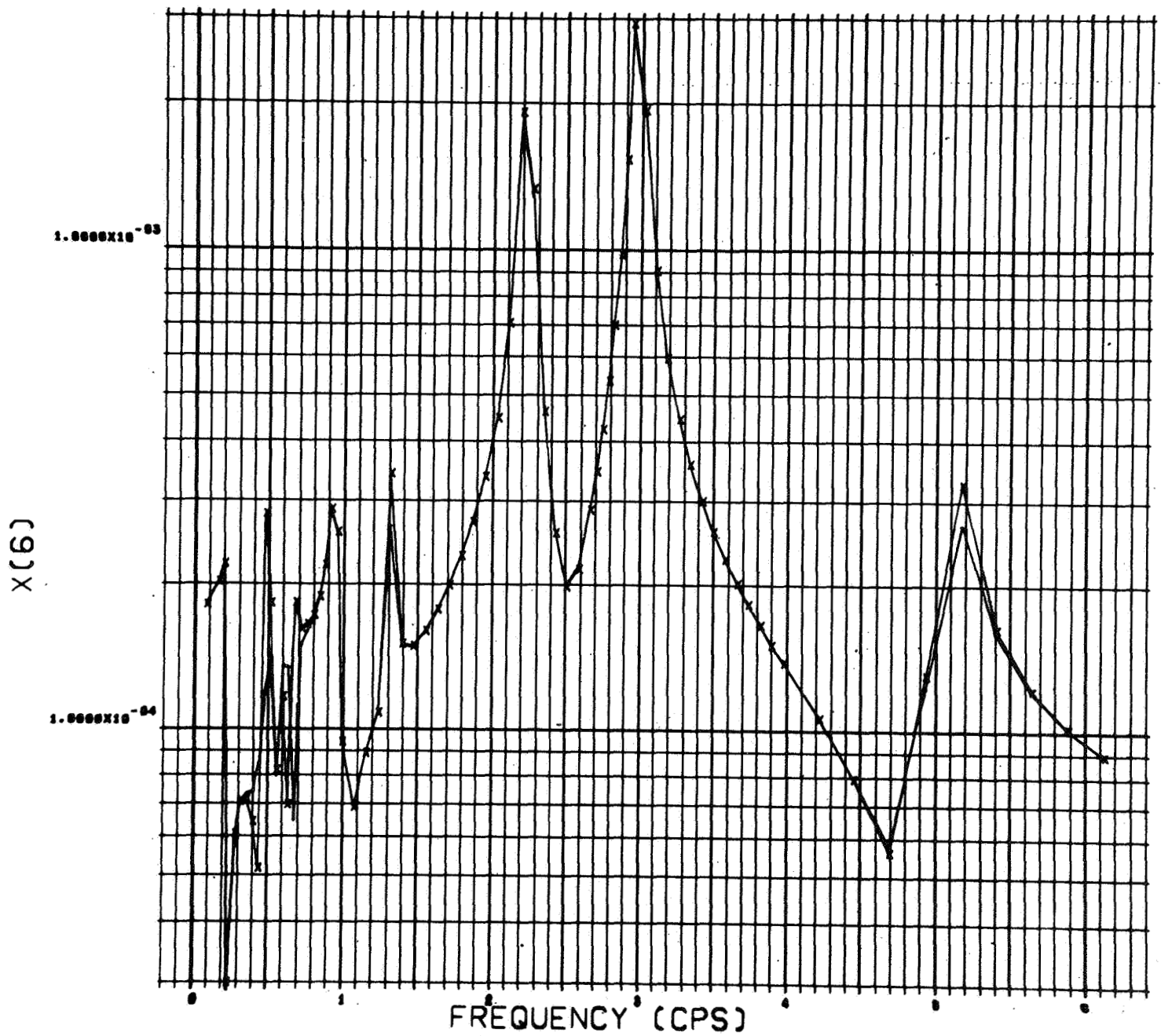


t = 97 seconds

Figure 51b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

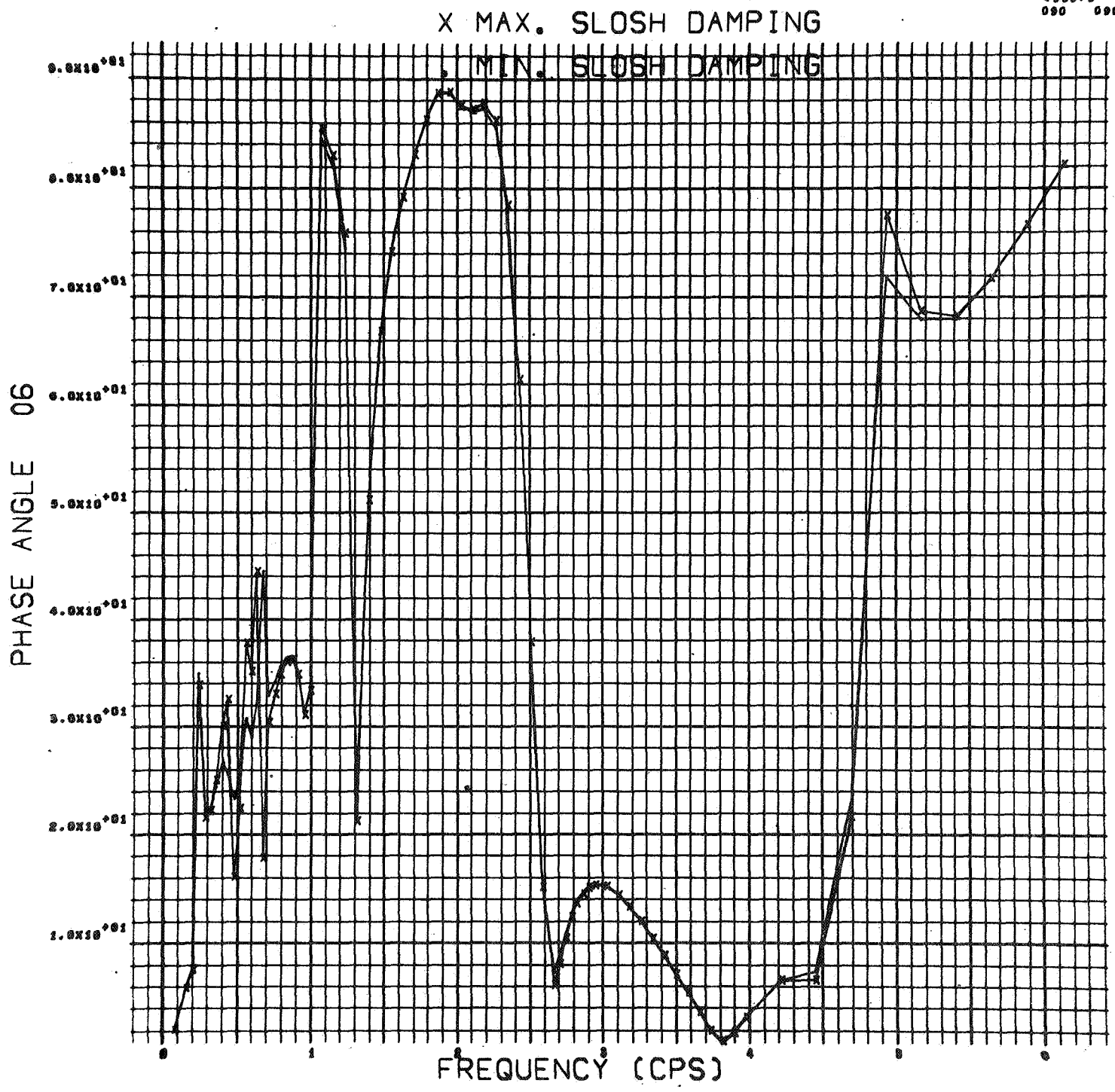
498570
111 111



t = 97 seconds

Figure 52a

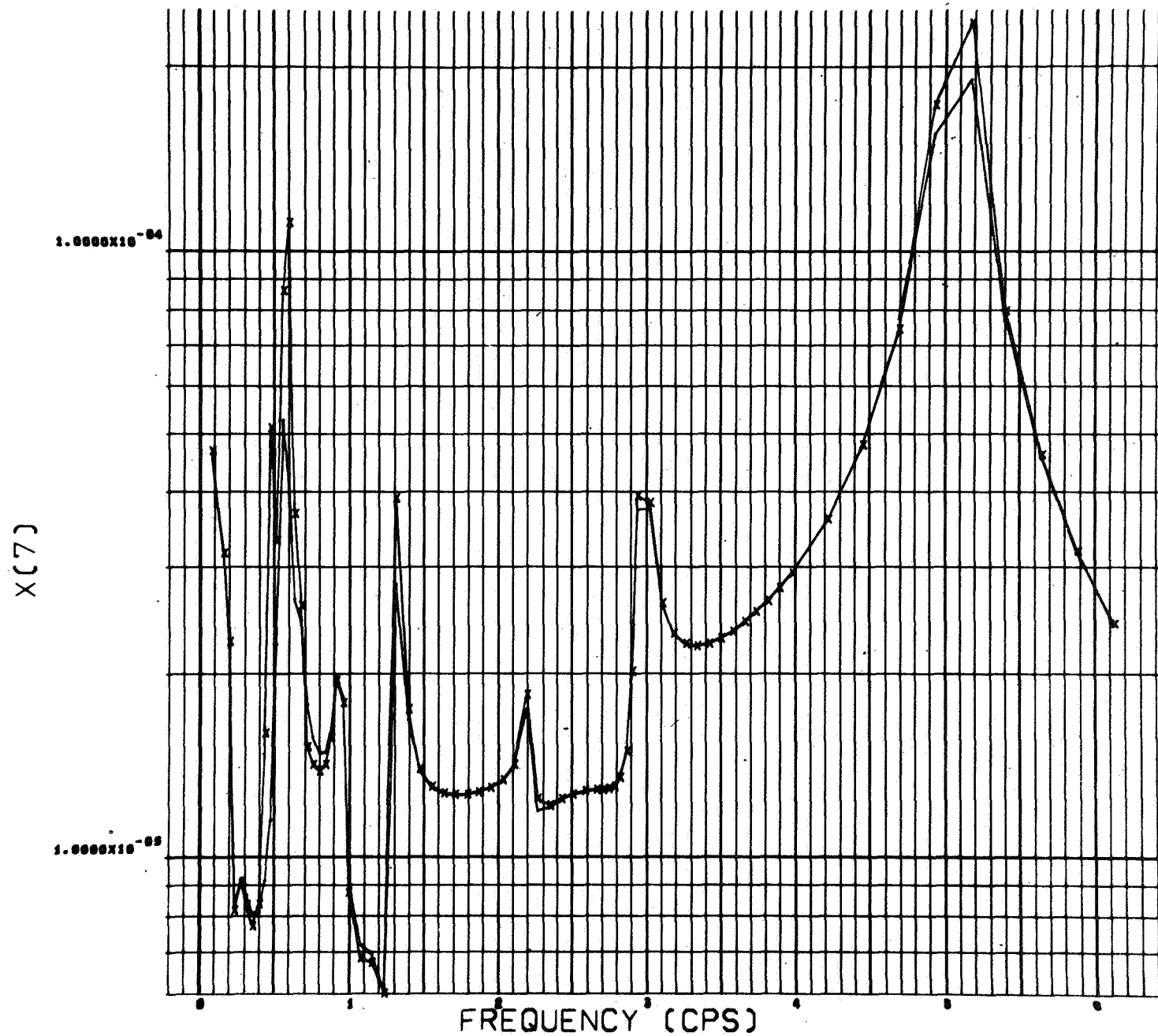
490370 L
090 090



t = 97 seconds

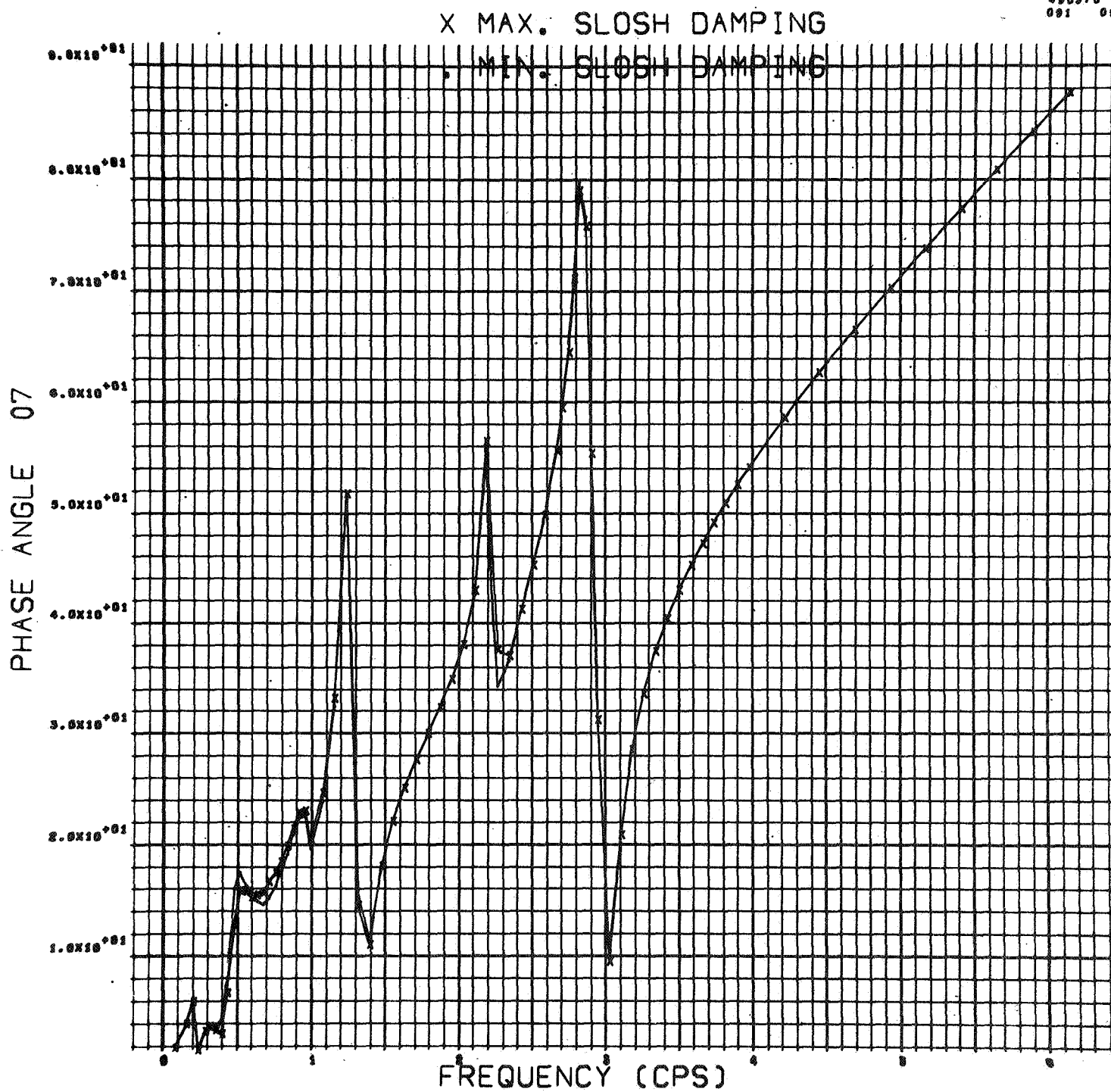
Figure 52b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 97 seconds

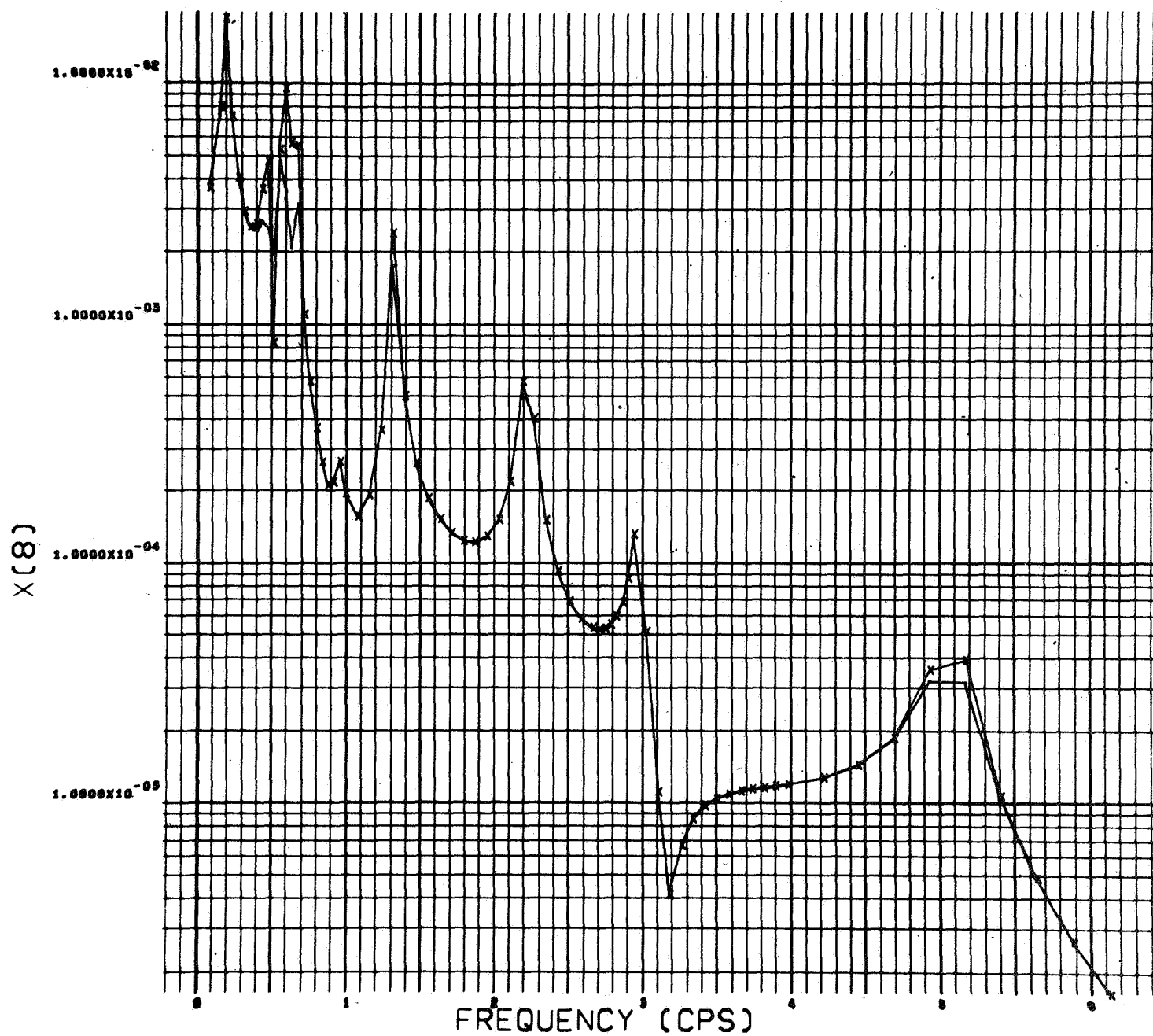
Figure 53a



t = 97 seconds

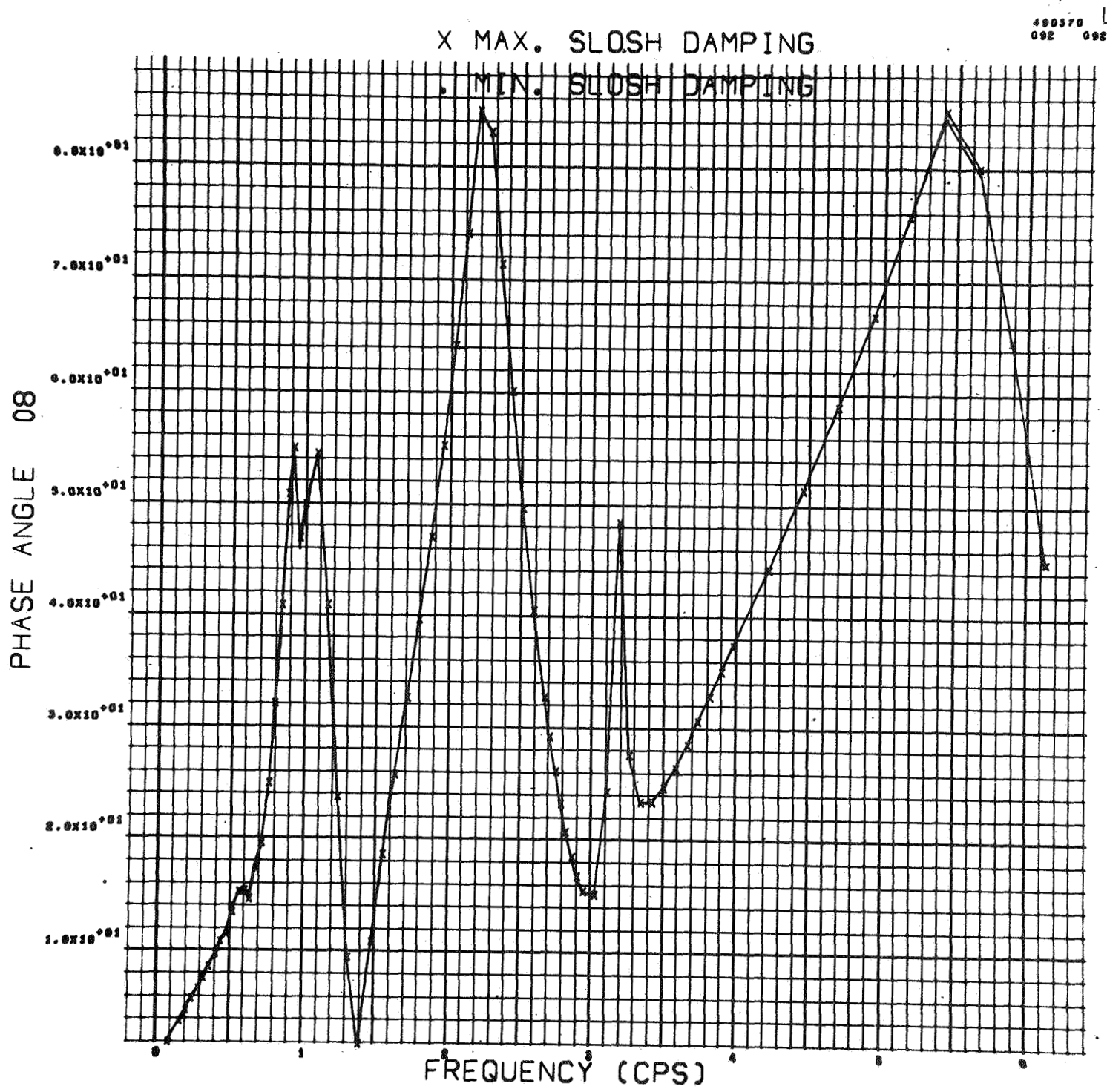
Figure 53b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 97 seconds

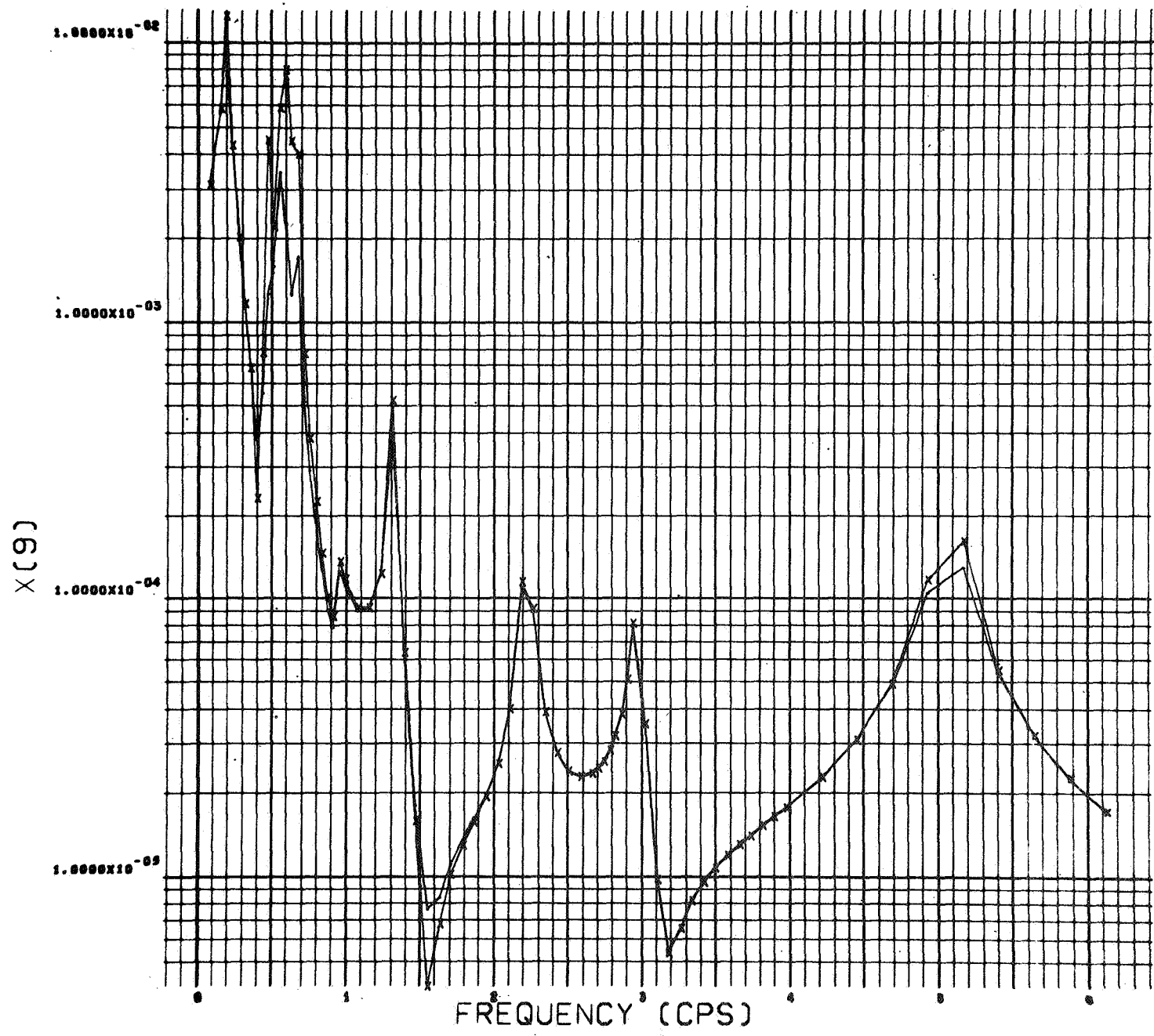
Figure 54a



t = 97 seconds

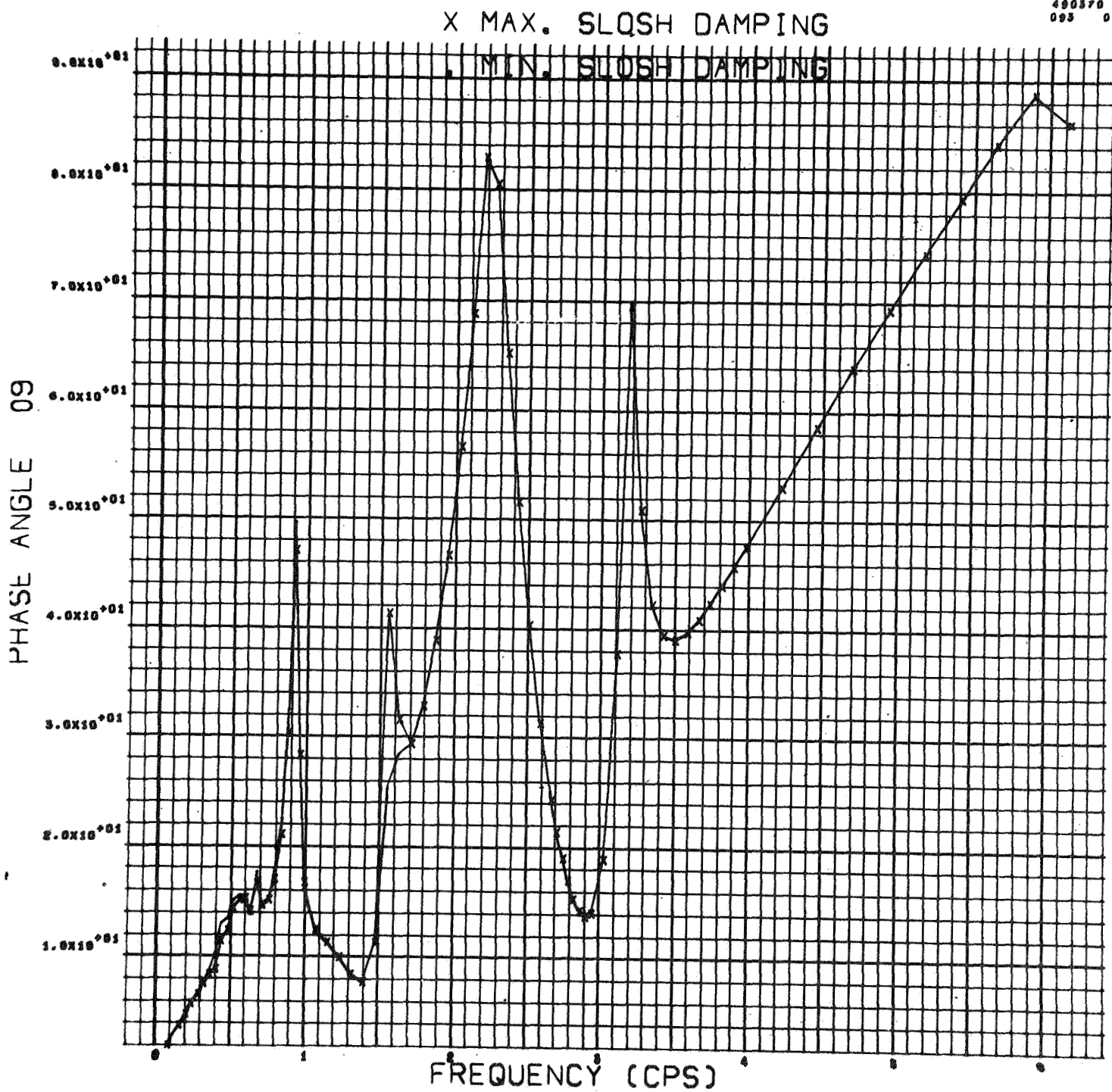
Figure 54b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 97 seconds

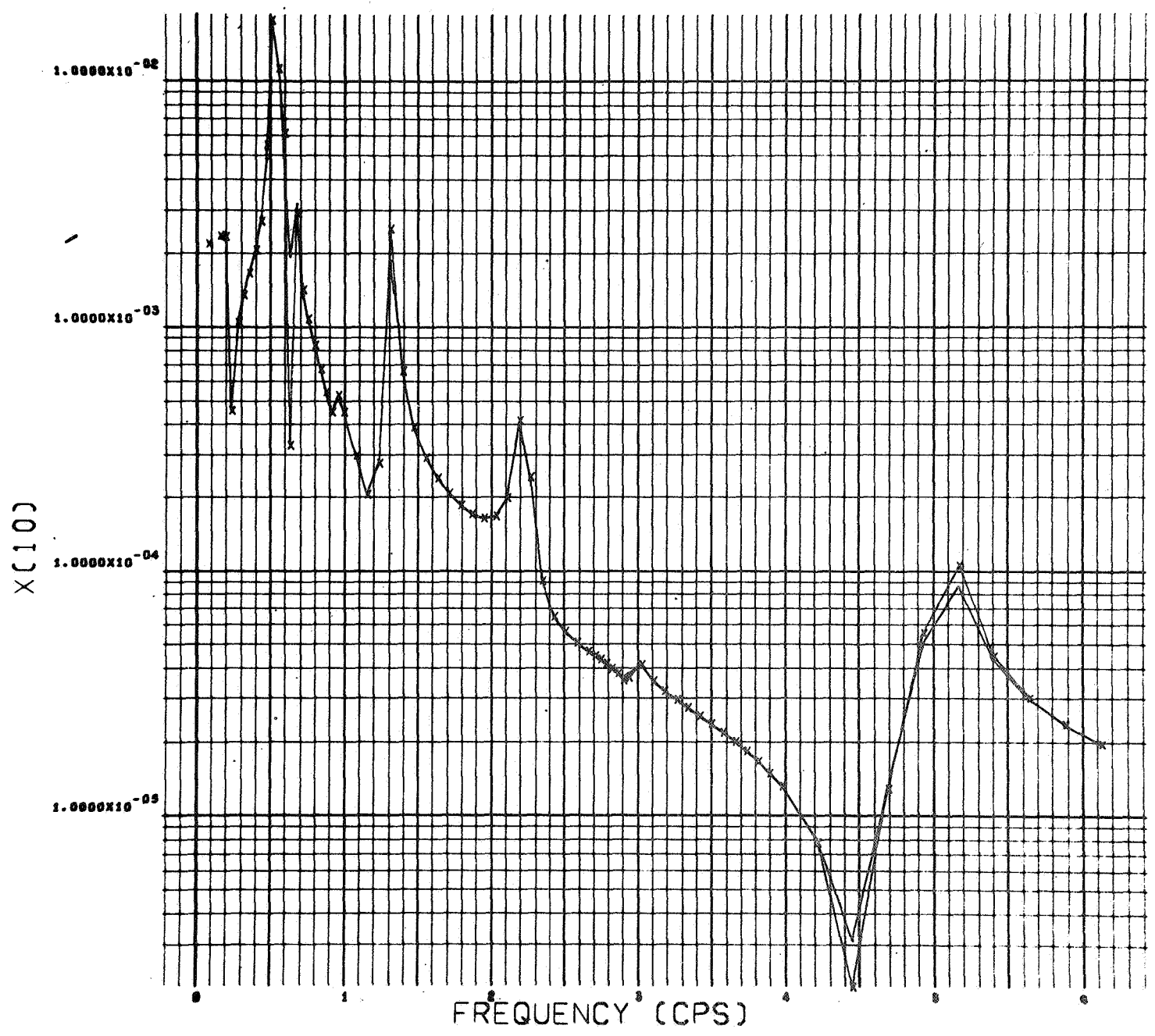
Figure 55a



t = 97 seconds

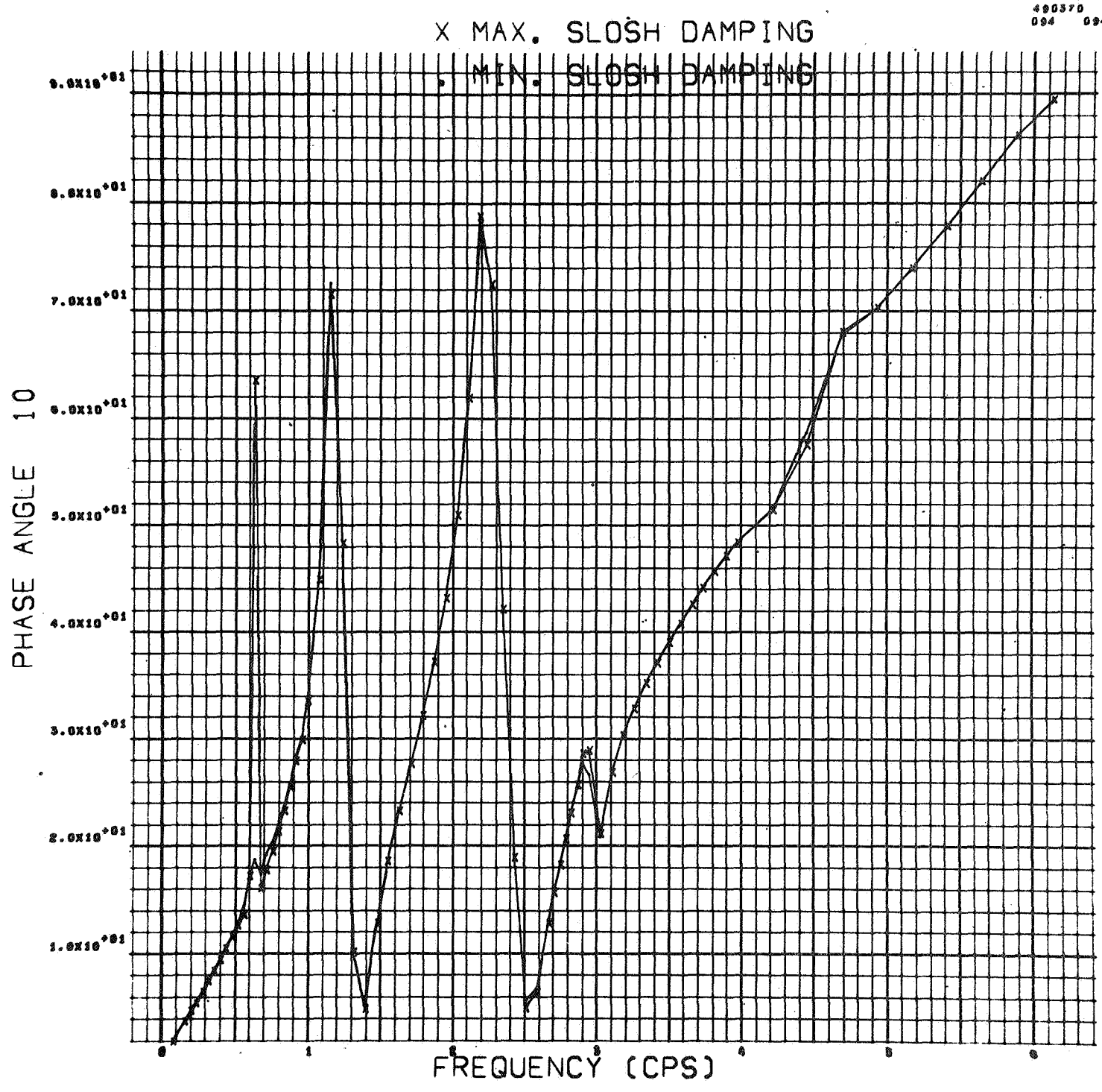
Figure 55b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



t = 97 seconds

Figure 56a

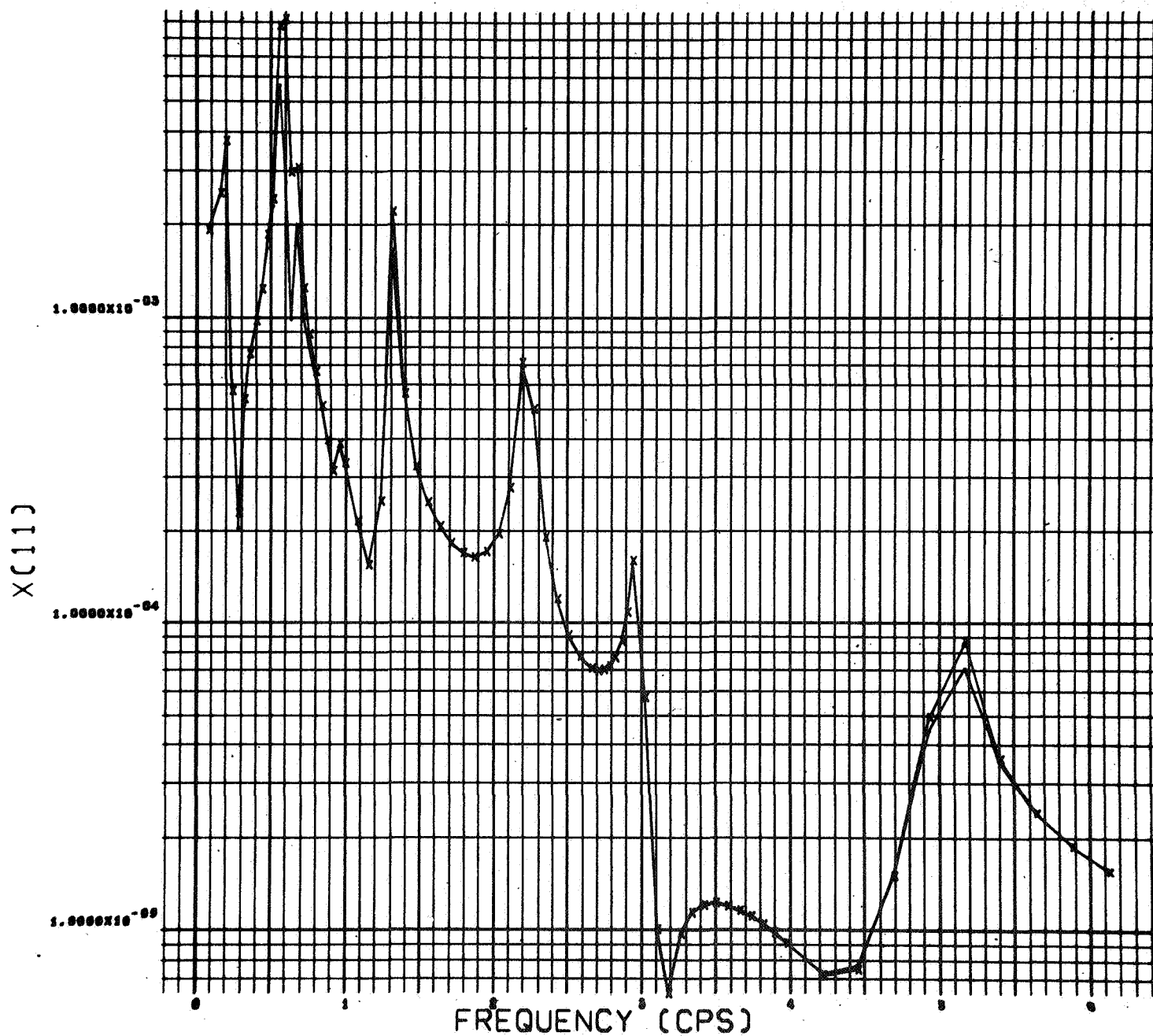


t = 97 seconds

Figure 56b

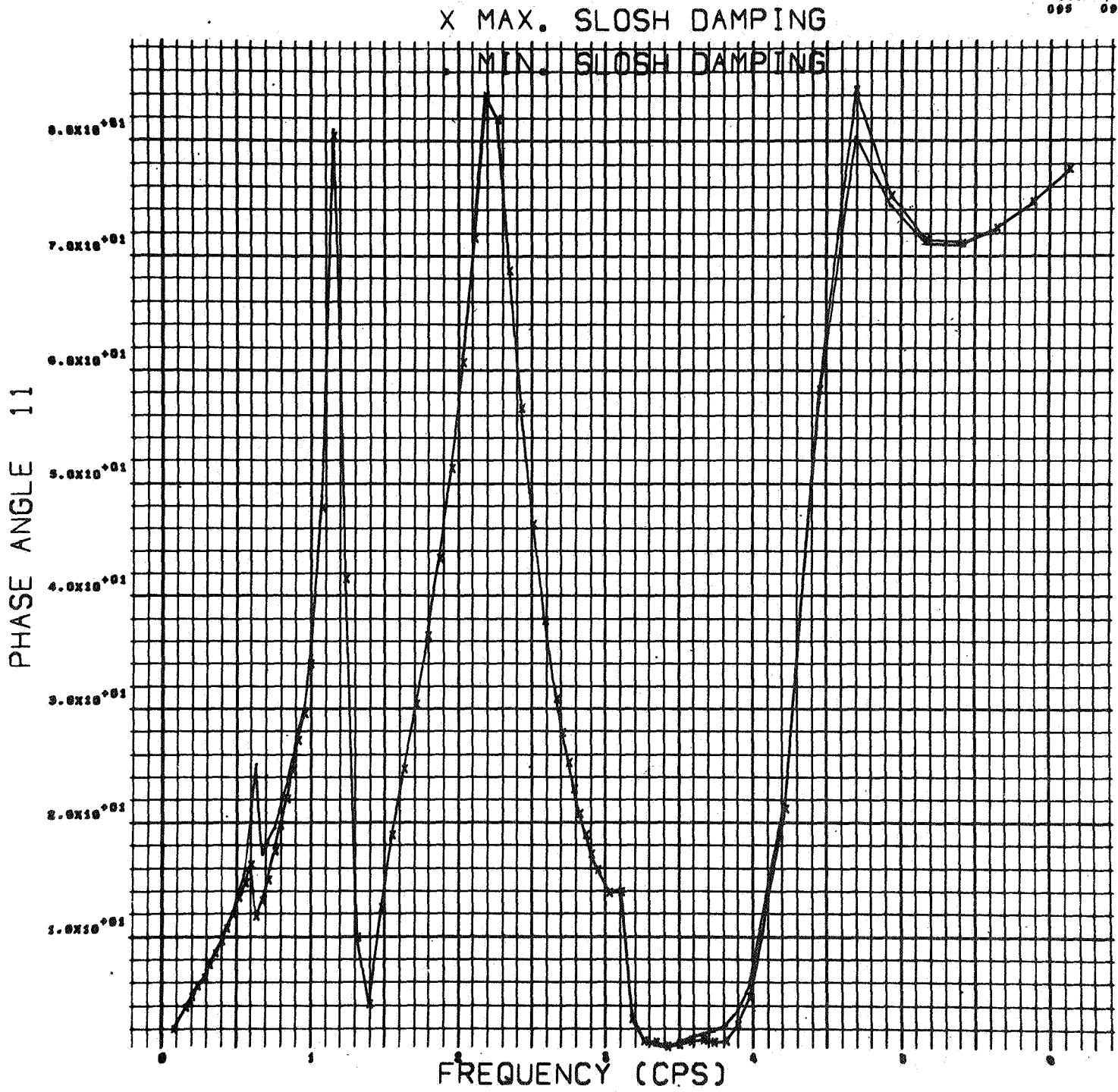
X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING

400370 L
110 110



t = 97 seconds

Figure 57a

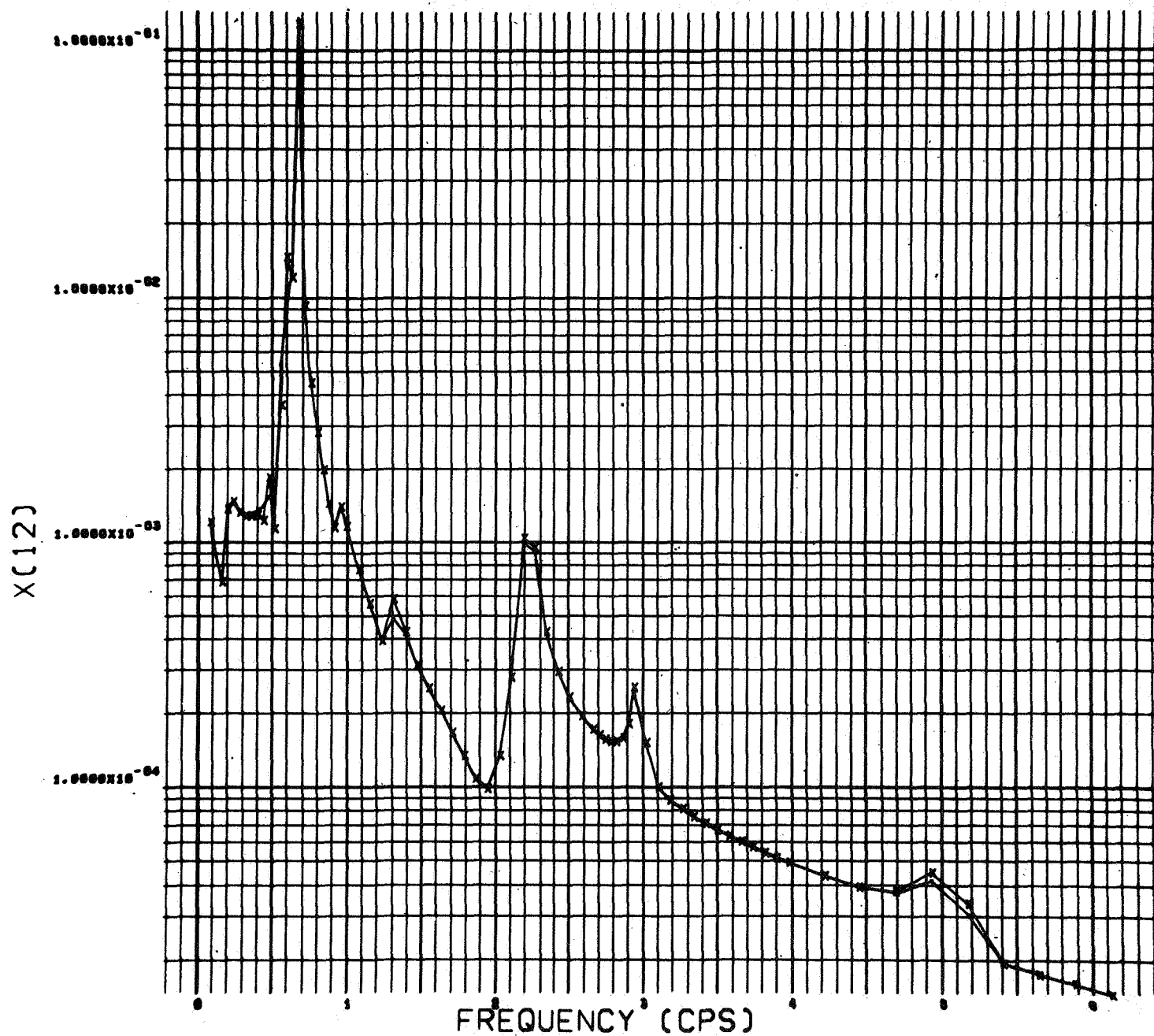


t = 97 seconds

Figure 57b

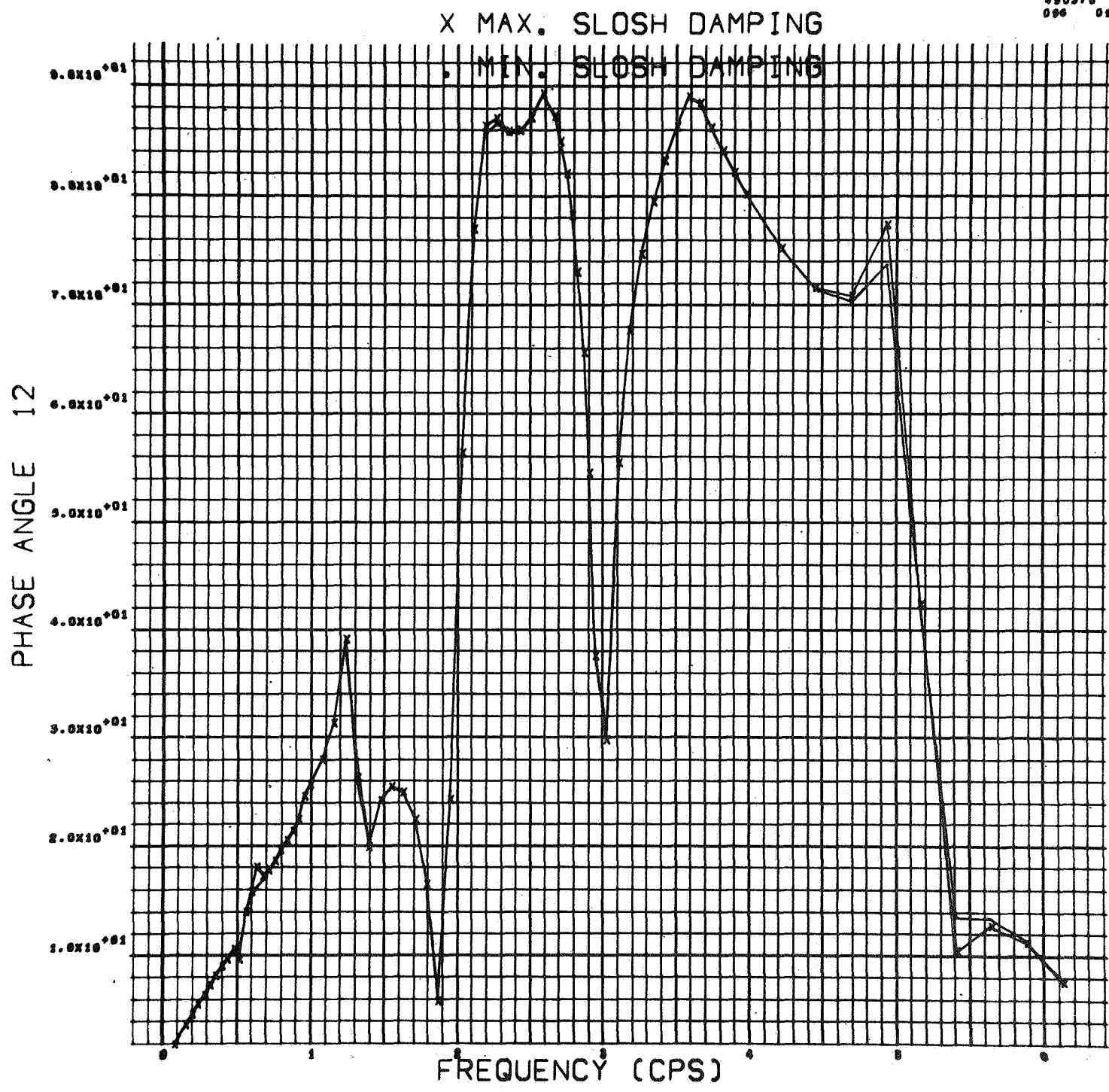
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490370
117 111



t = 97 seconds

Figure 58a

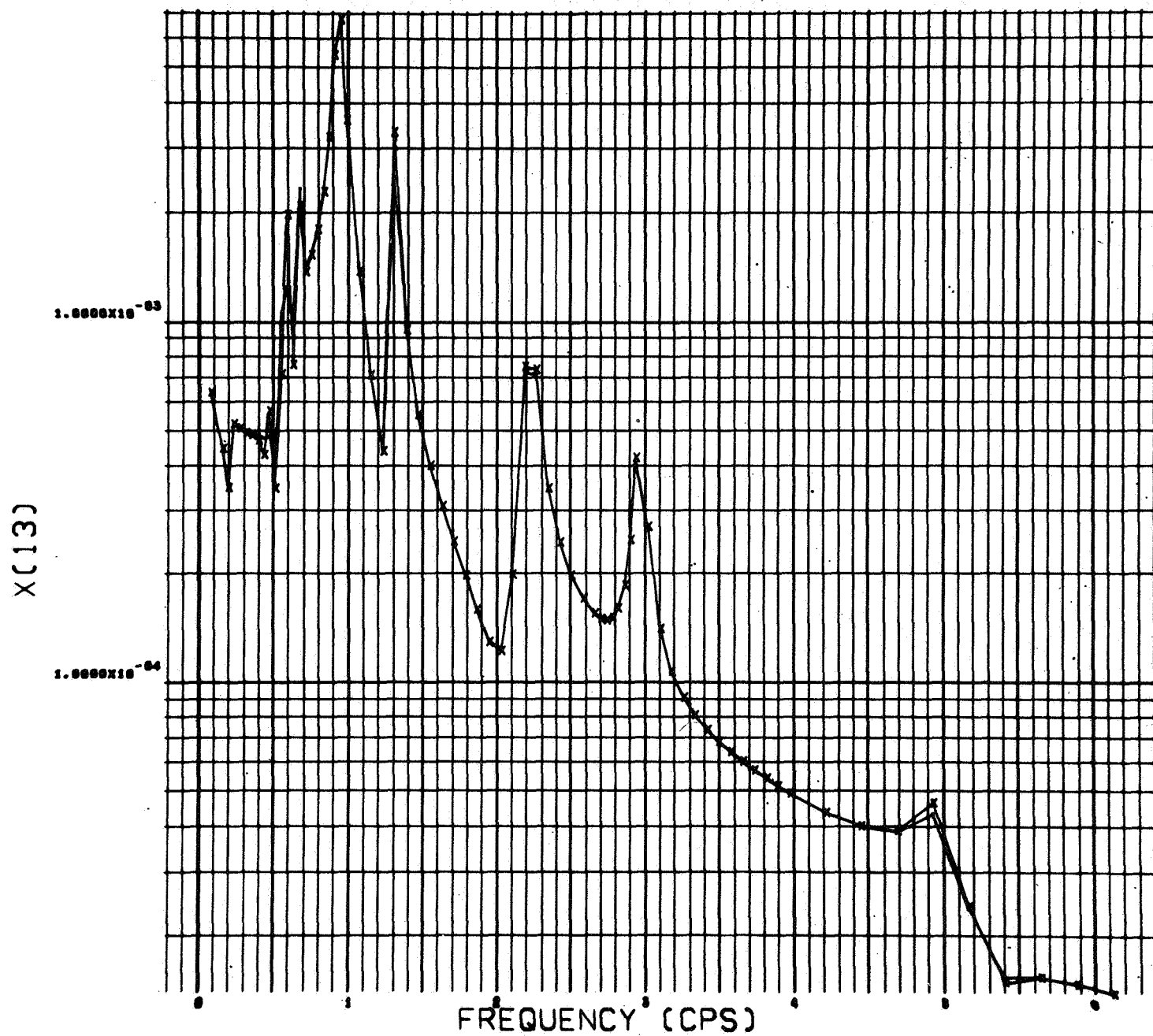


t = 97 seconds

Figure 58b

X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING

490370 L
110 120



t = 97 seconds

Figure 59a

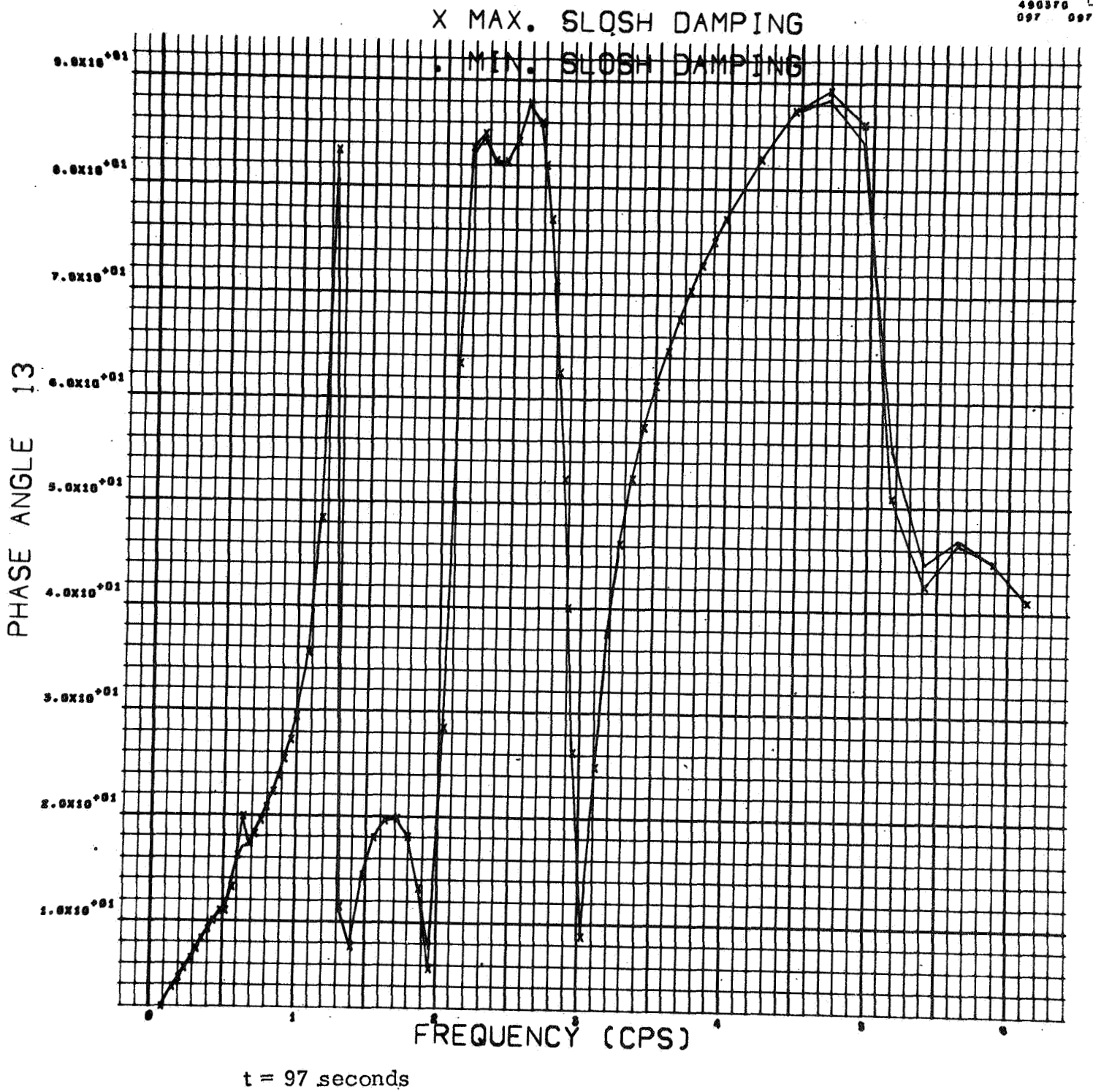
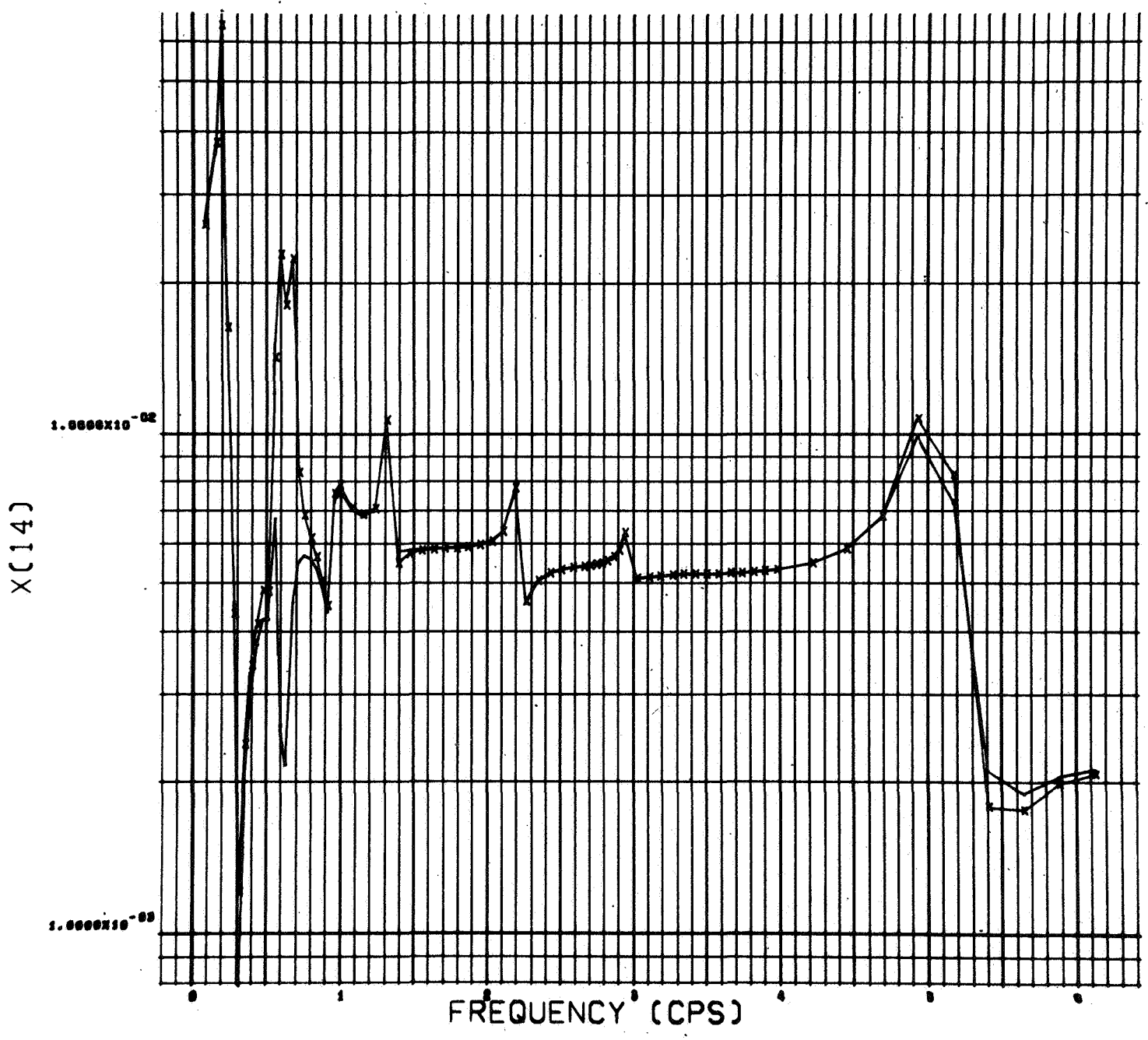


Figure 59b



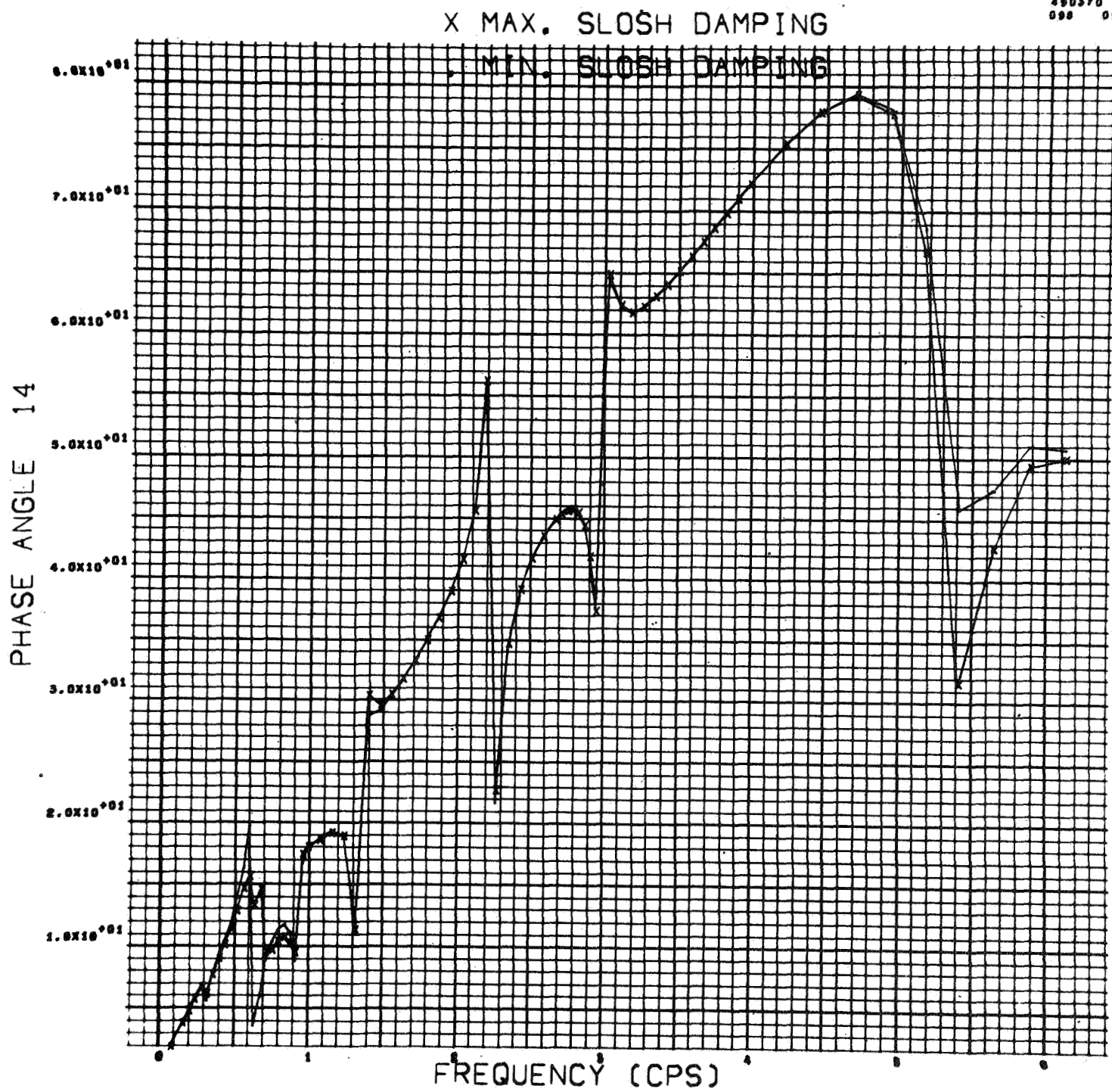
Flight Test Data
X(14), t = 97 seconds

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



t = 97 seconds

Figure 60a

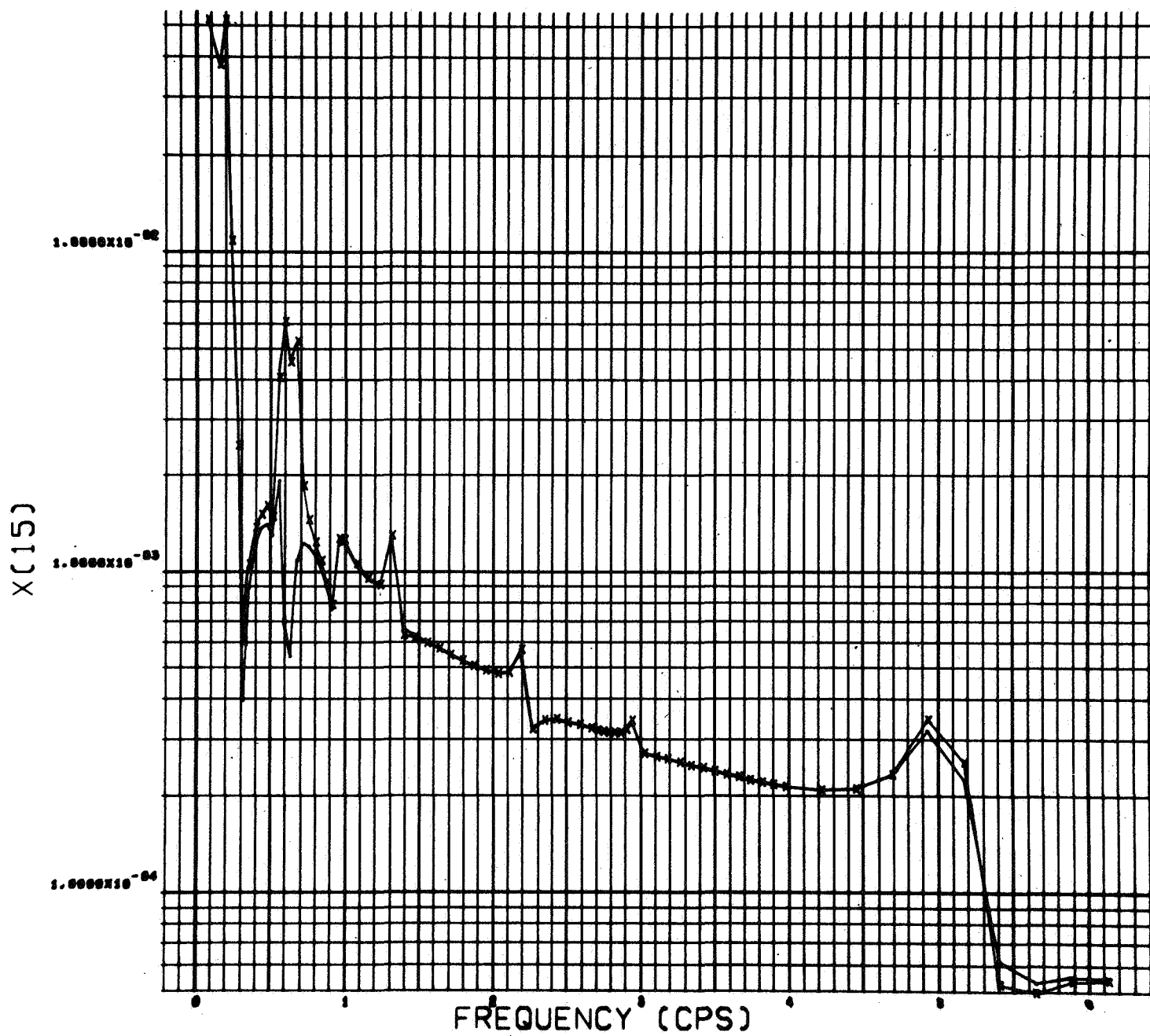


t = 97 seconds

Figure 60b

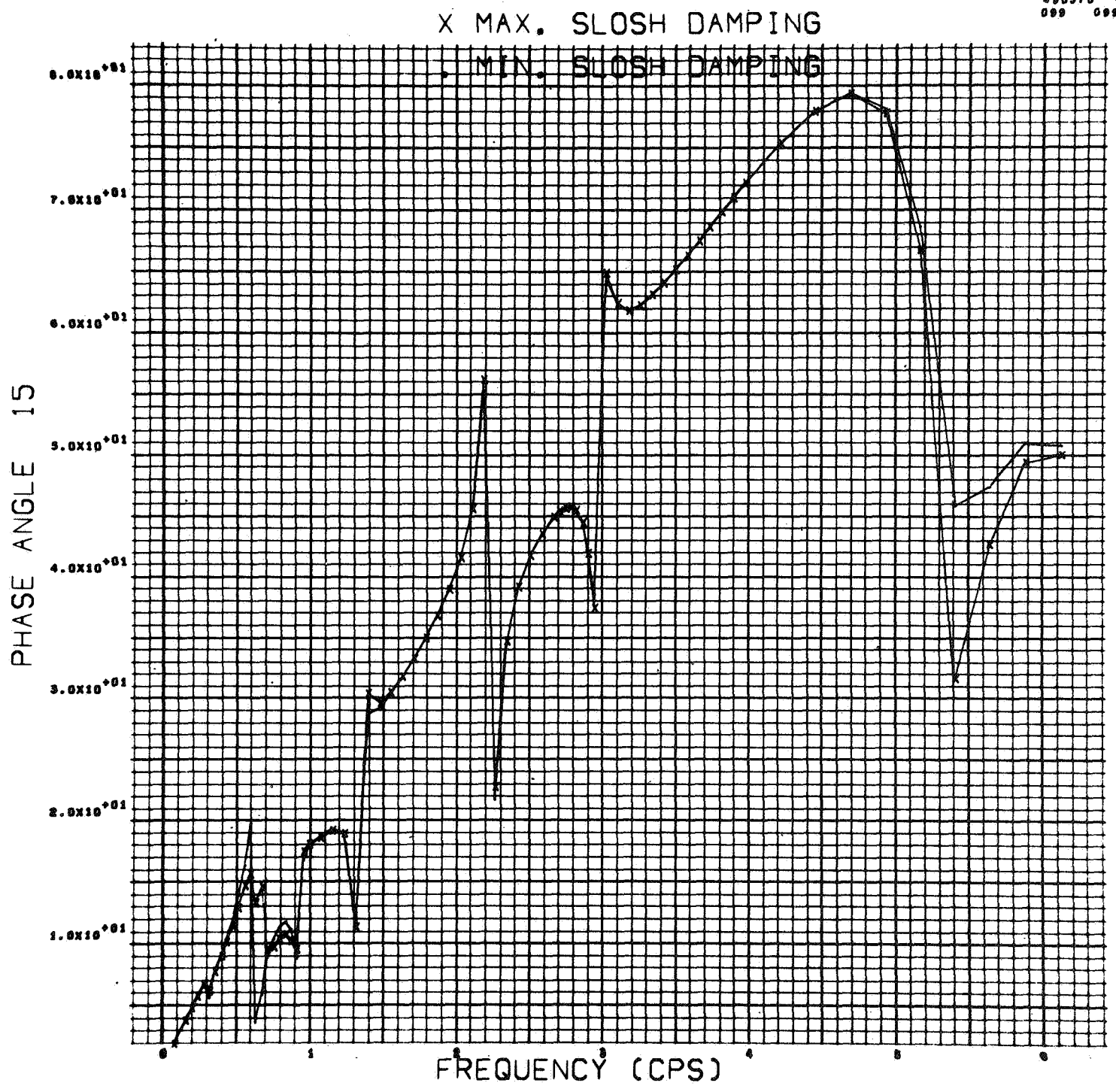
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

400370 L
120 120



t = 97 seconds

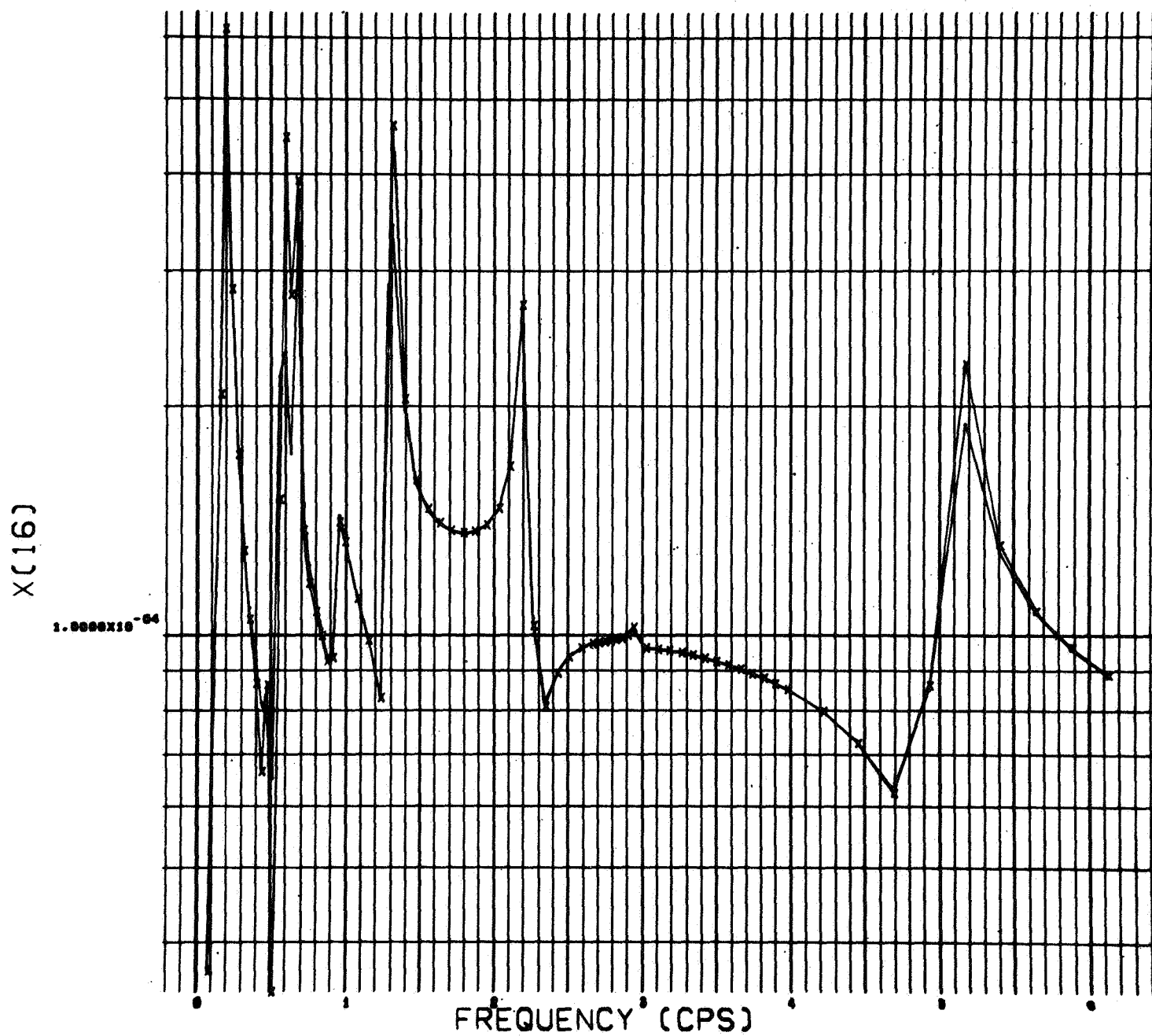
Figure 61a



t = 97 seconds

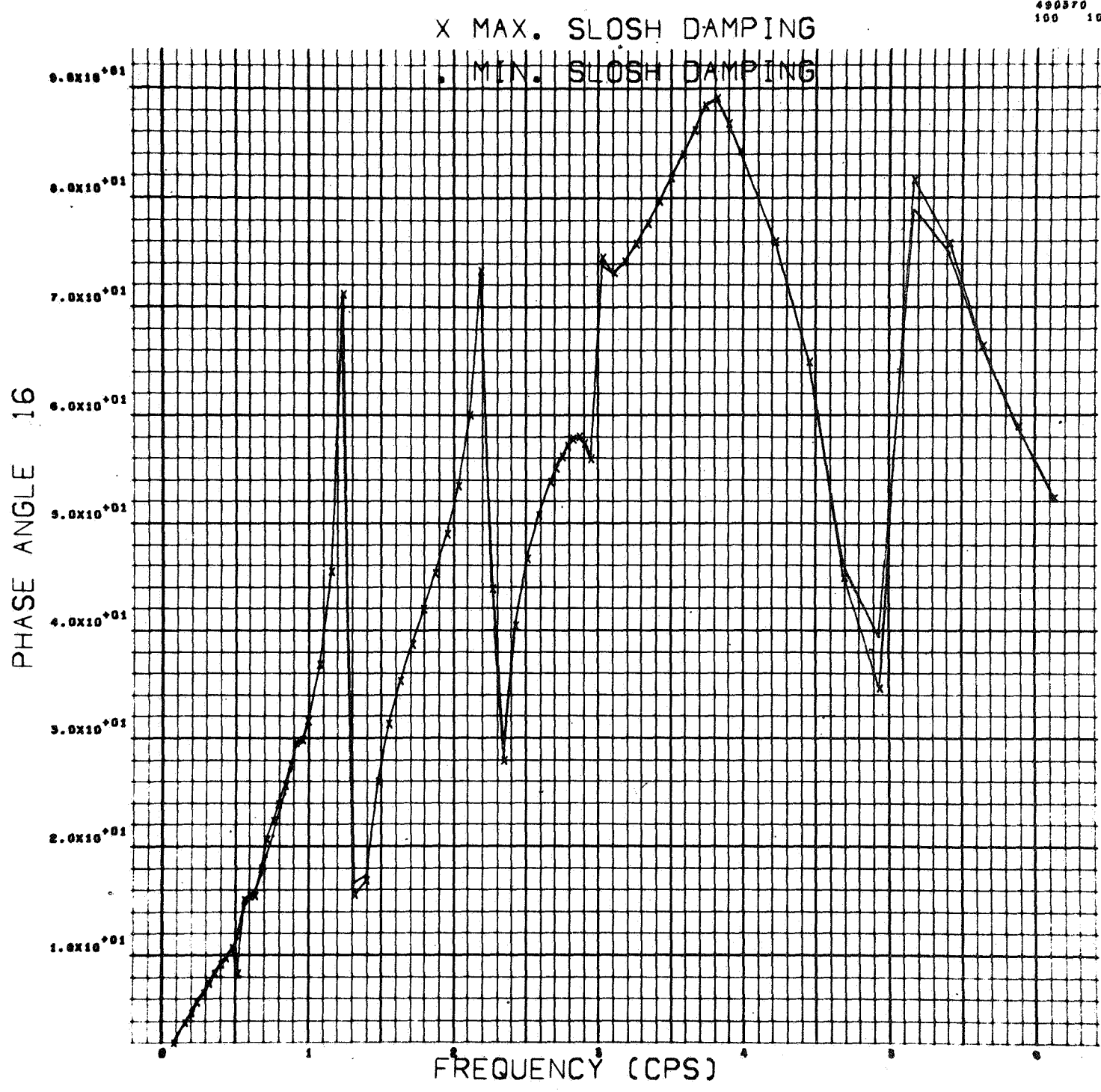
Figure 61b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 97 seconds

Figure 62a

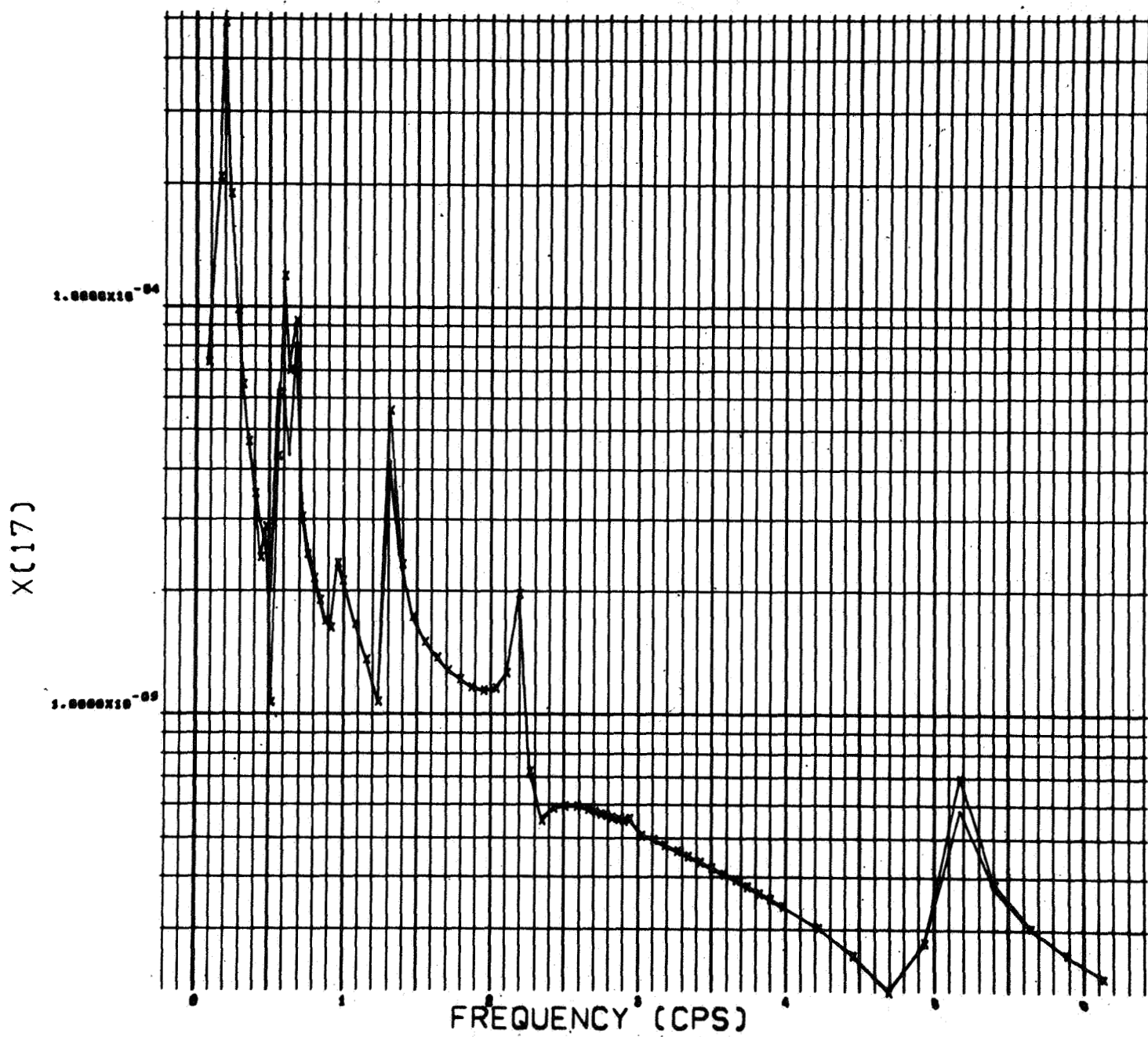


t = 97 seconds.

Figure 62b

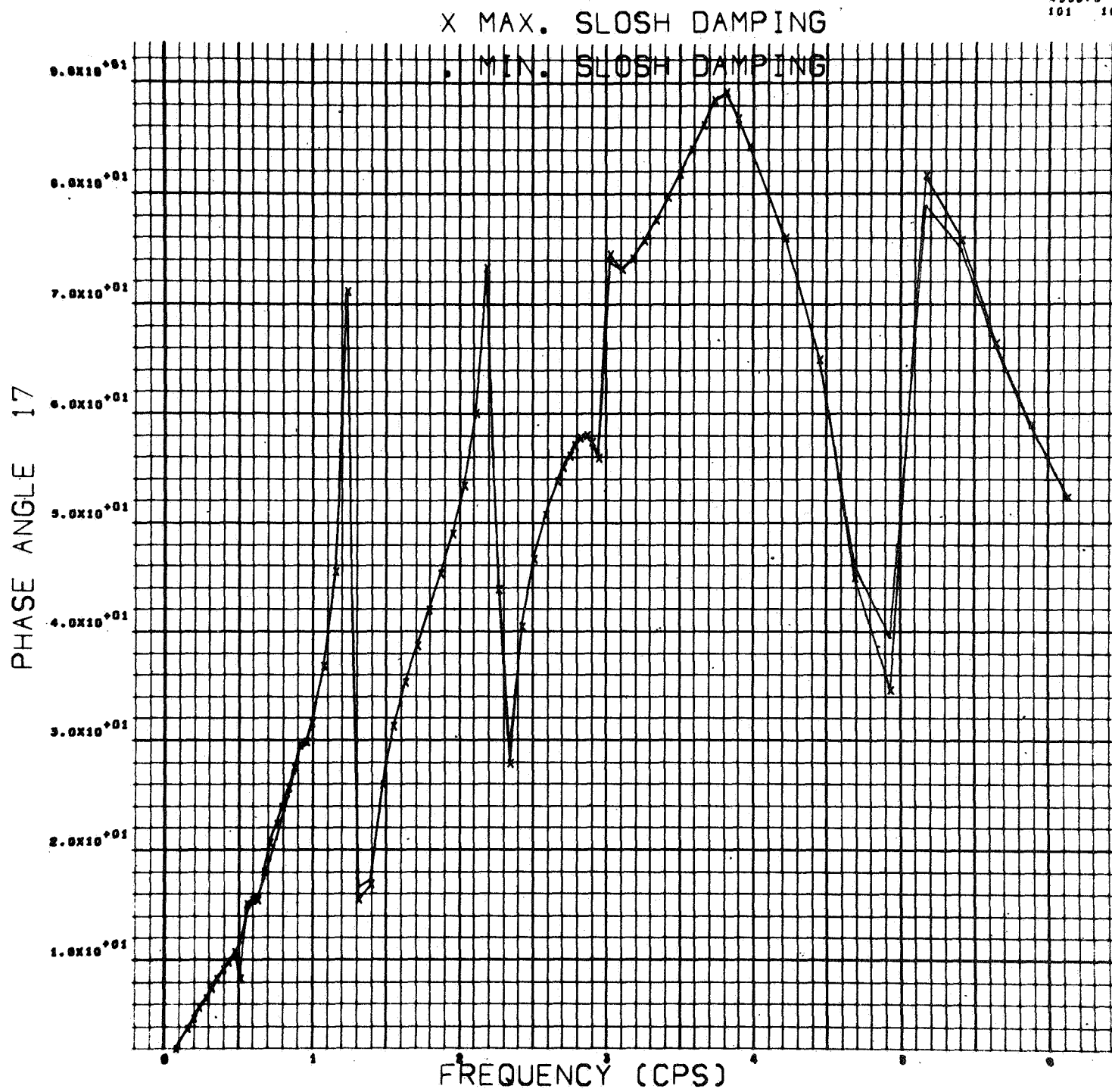
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490370 1
122 122



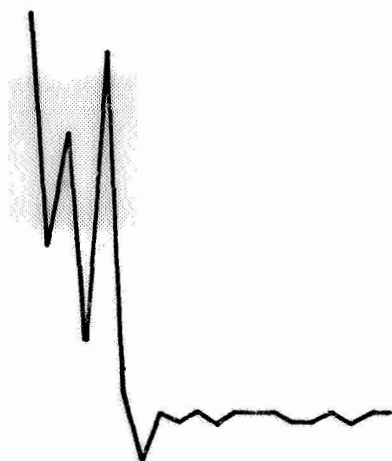
t = 97 seconds

Figure 63a



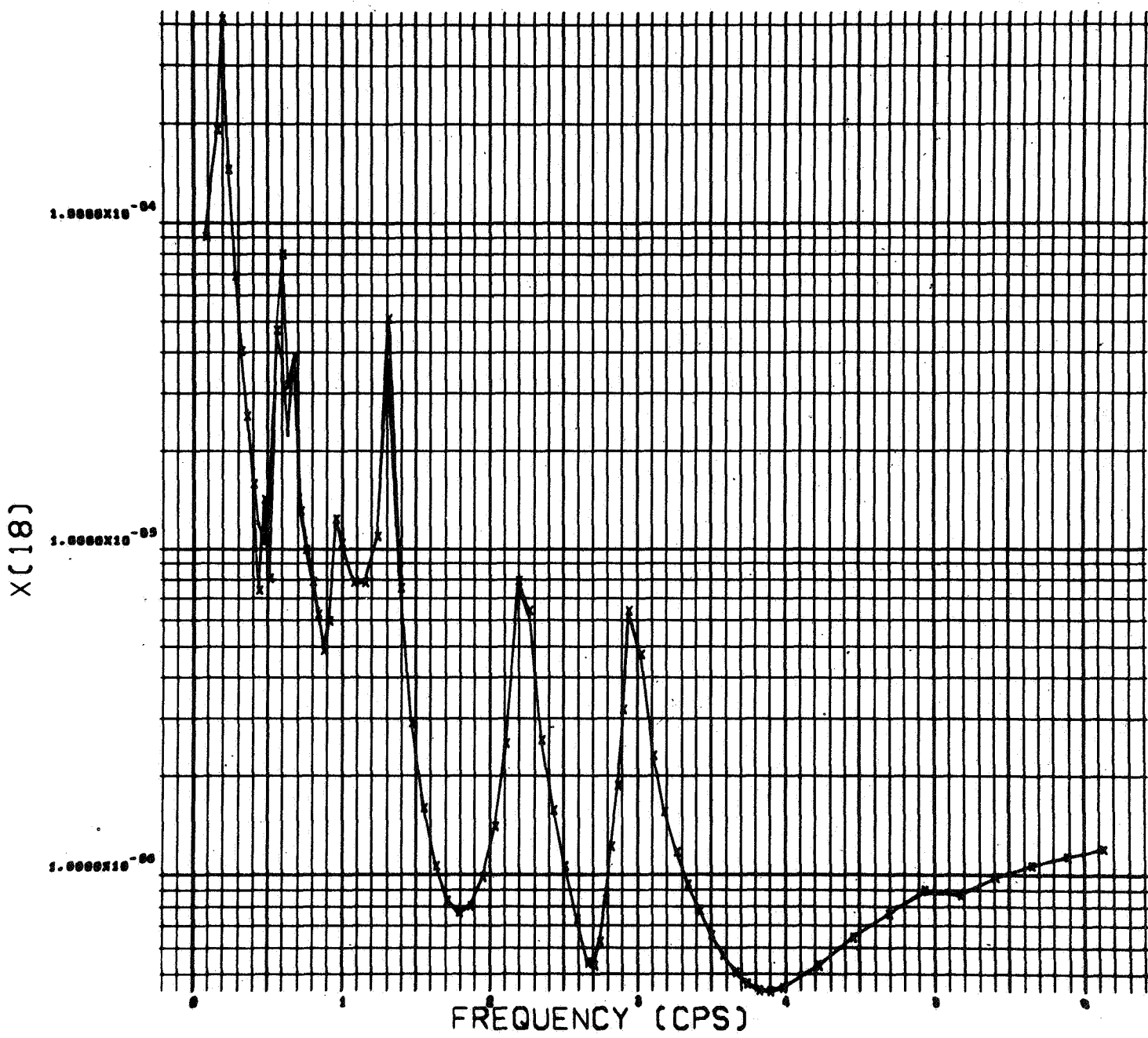
t = 97 seconds

Figure 63b



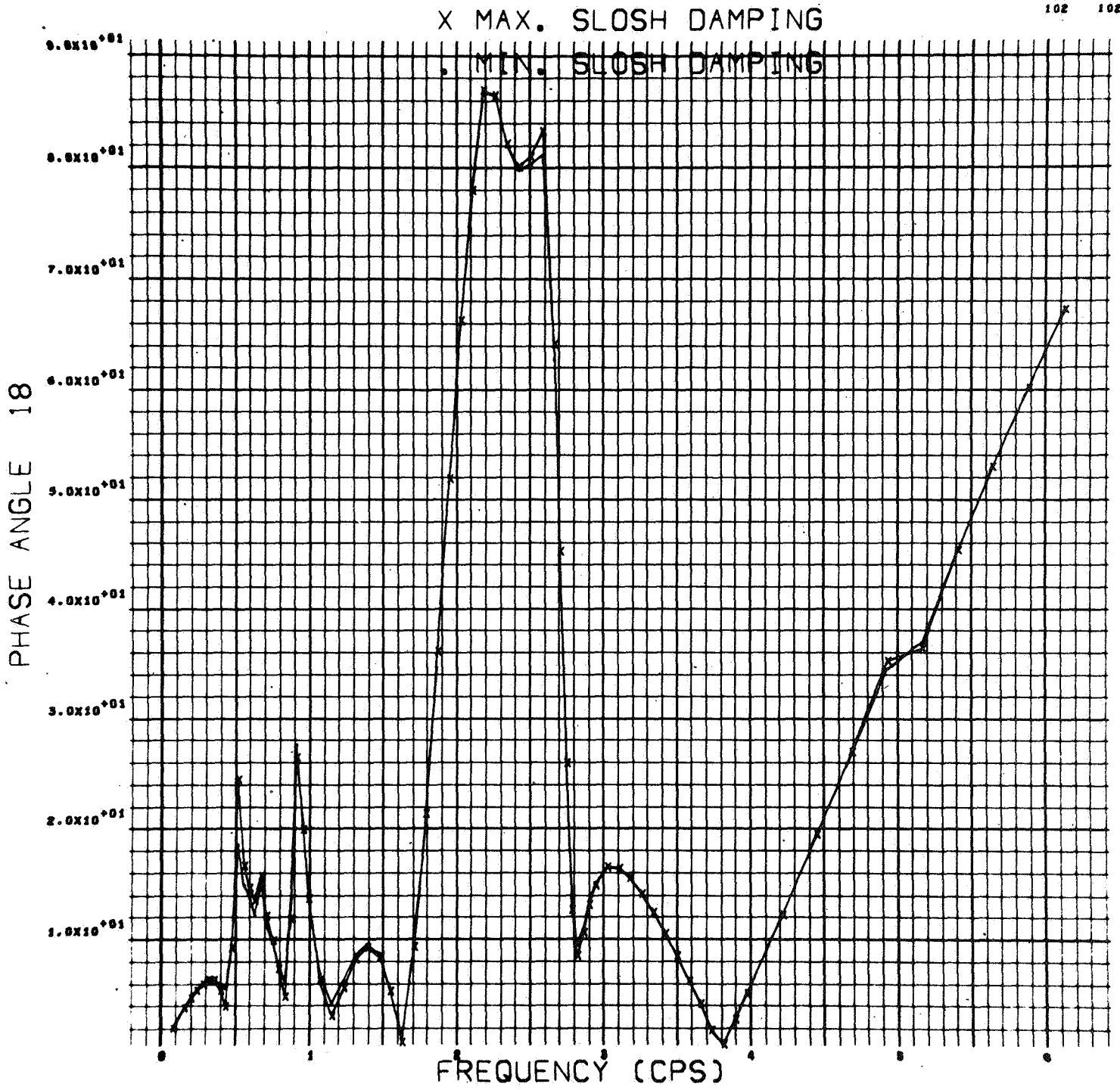
Flight Test Data
X(18), t = 97 seconds

X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING



t = 97 seconds

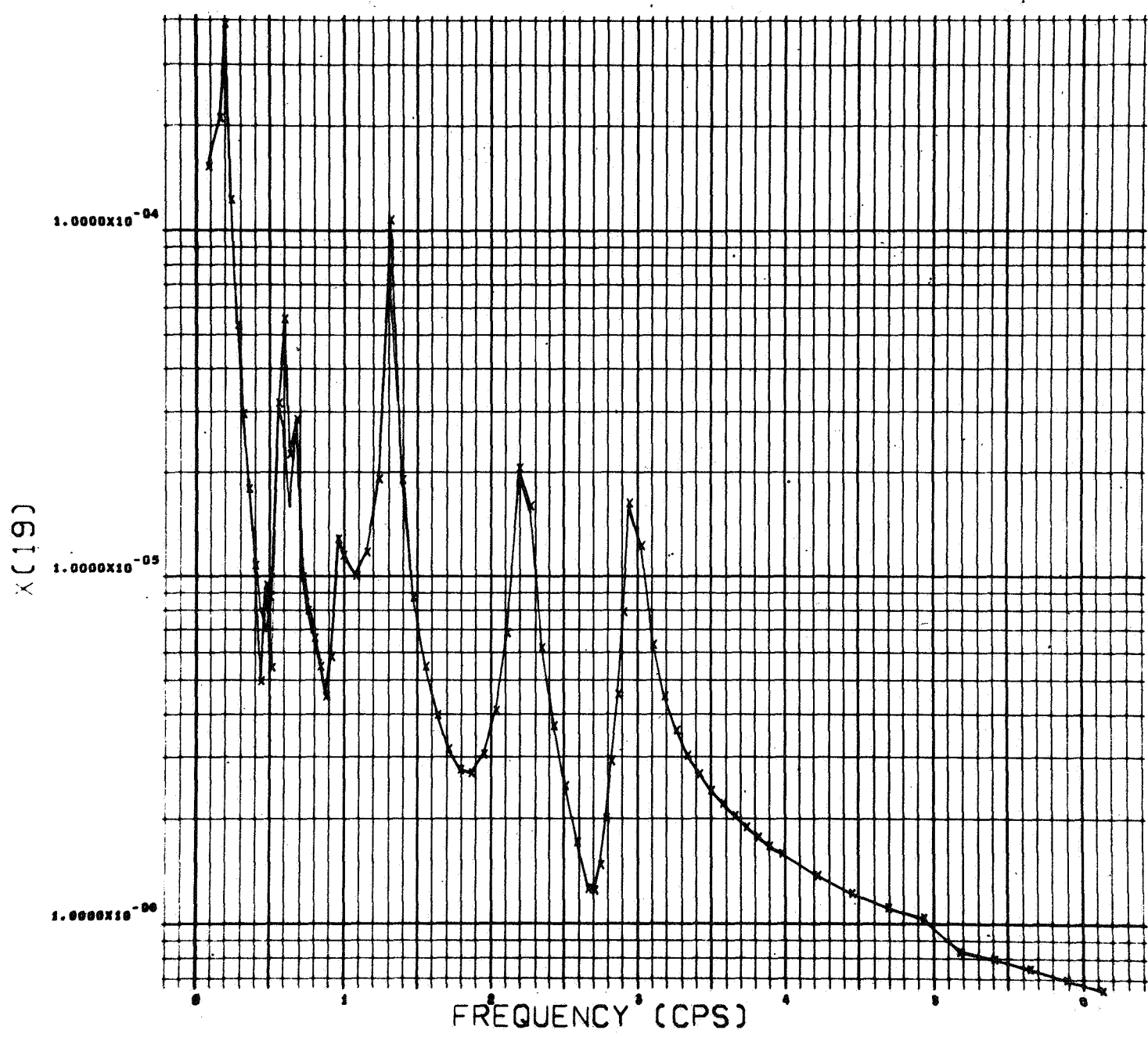
Figure 64a



t = 97 seconds

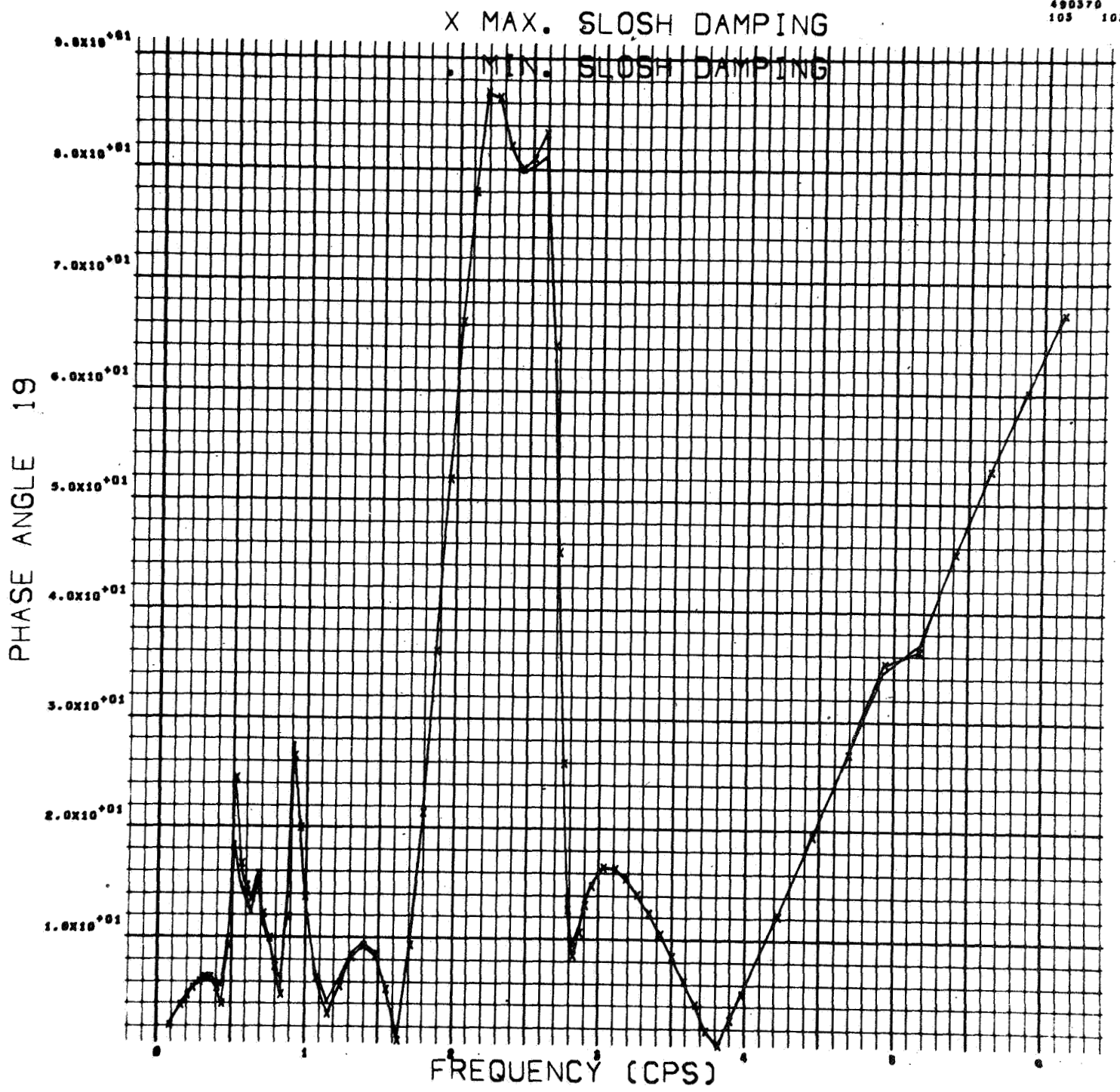
Figure 64b

X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING



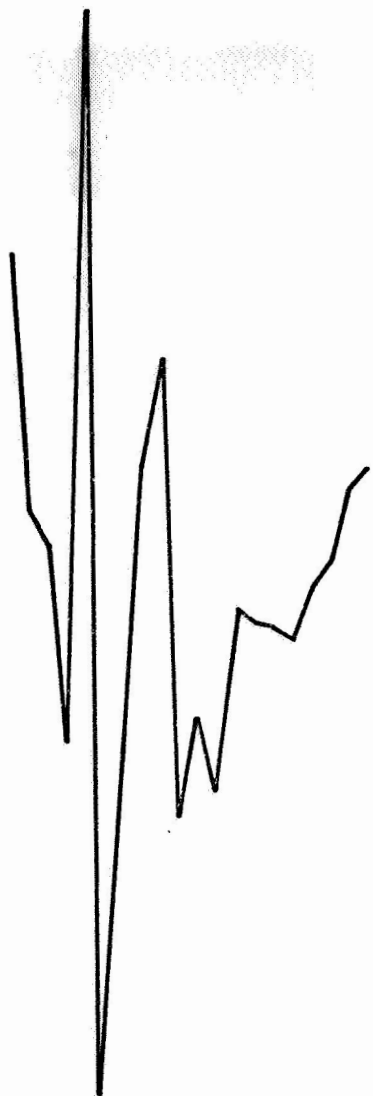
t = 97 seconds

Figure 65a



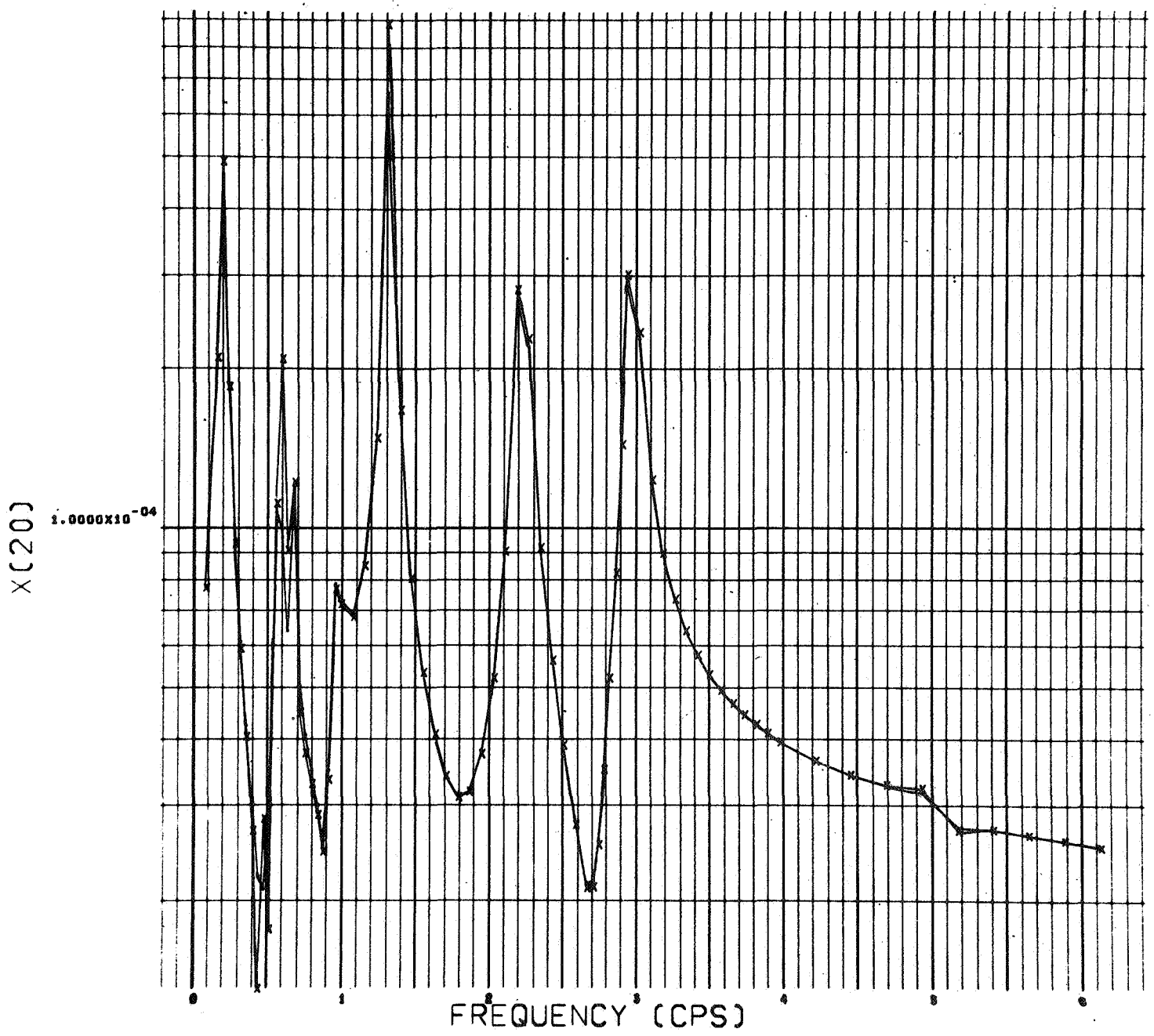
t = 97 seconds

Figure 65b



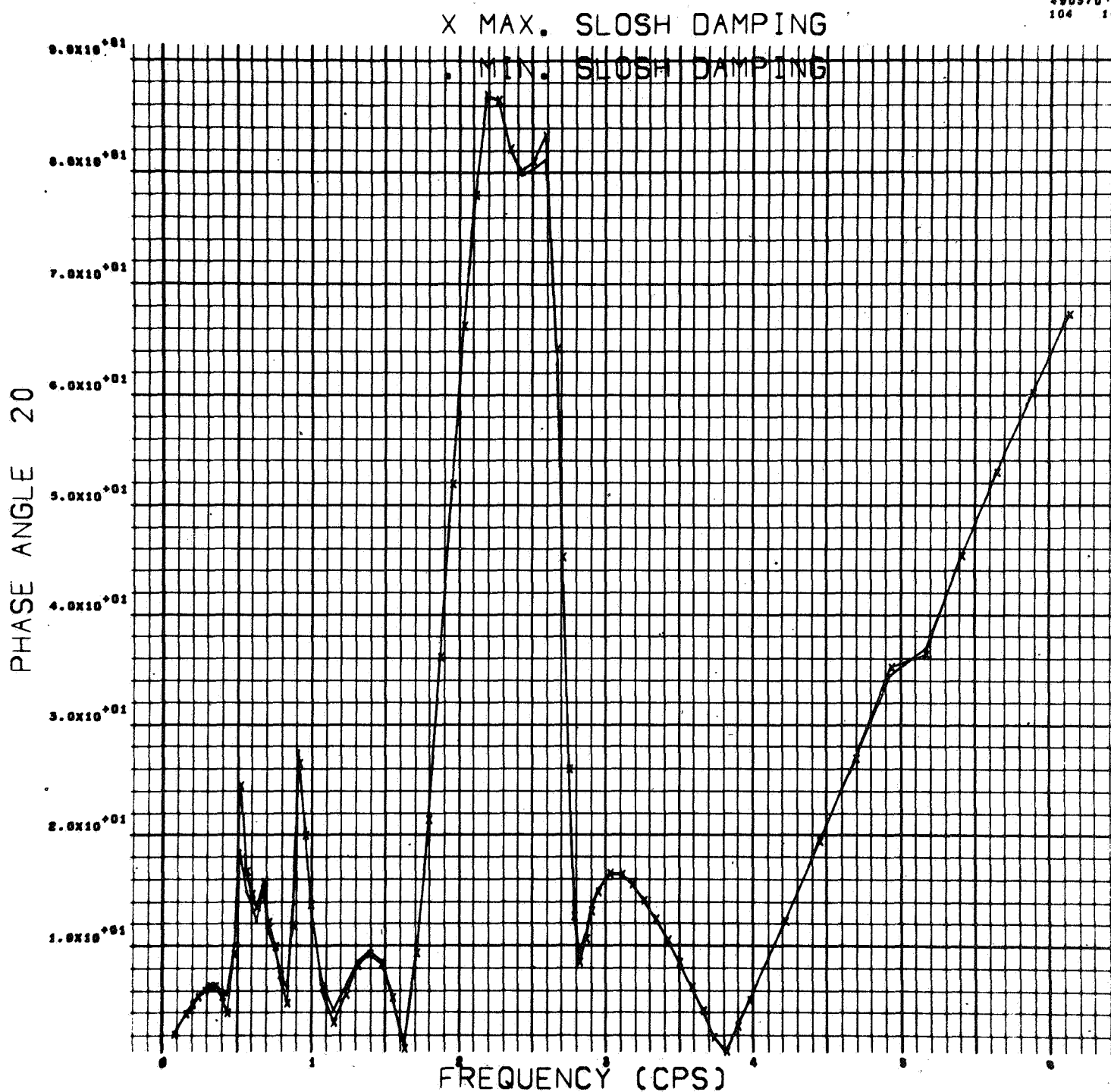
Flight Test Data
 $X(20)$, $t = 97$ seconds

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



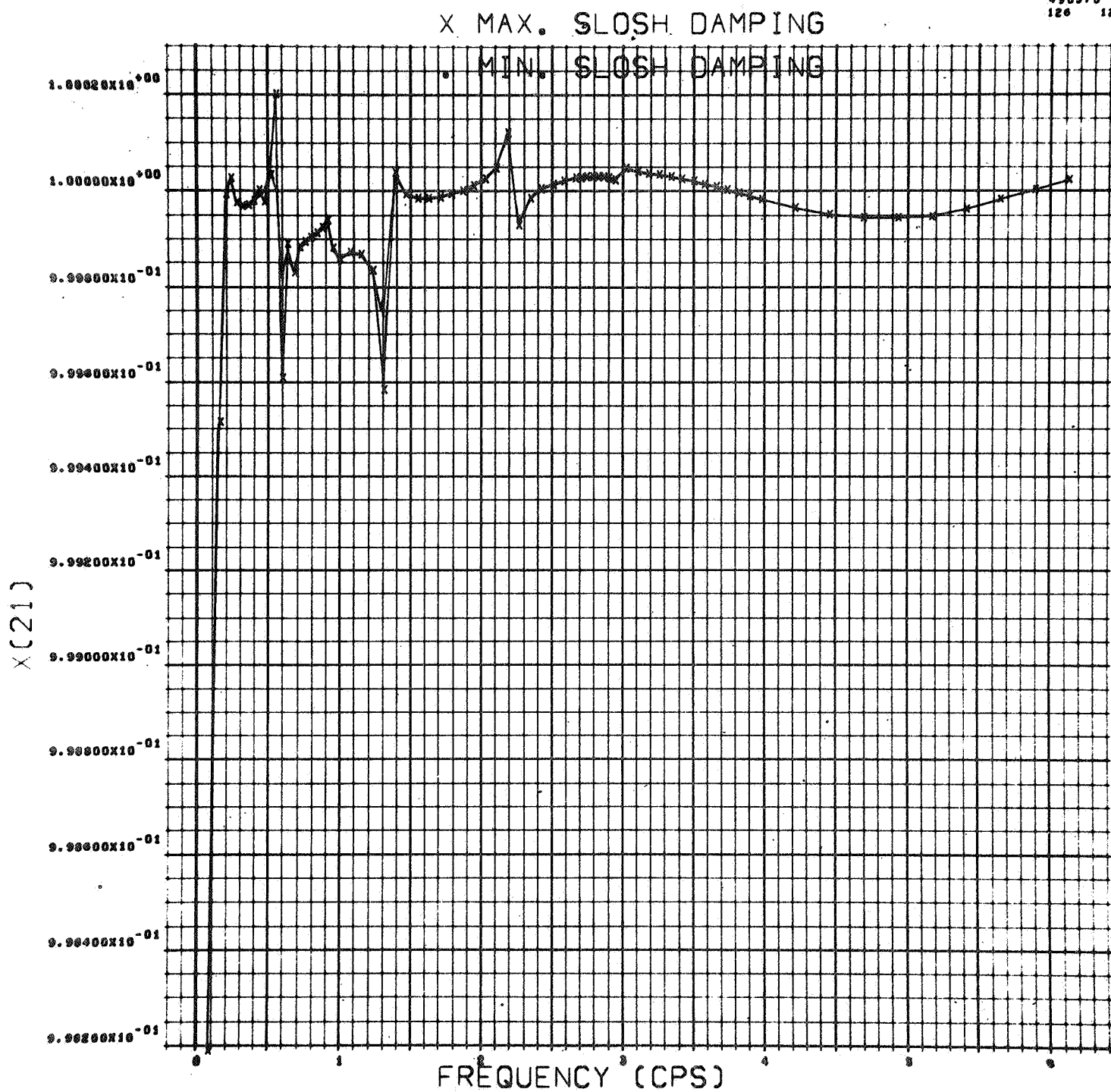
t = 97 seconds

Figure 66a



t = 97 seconds

Figure 66b



t = 97 seconds

Figure 67a

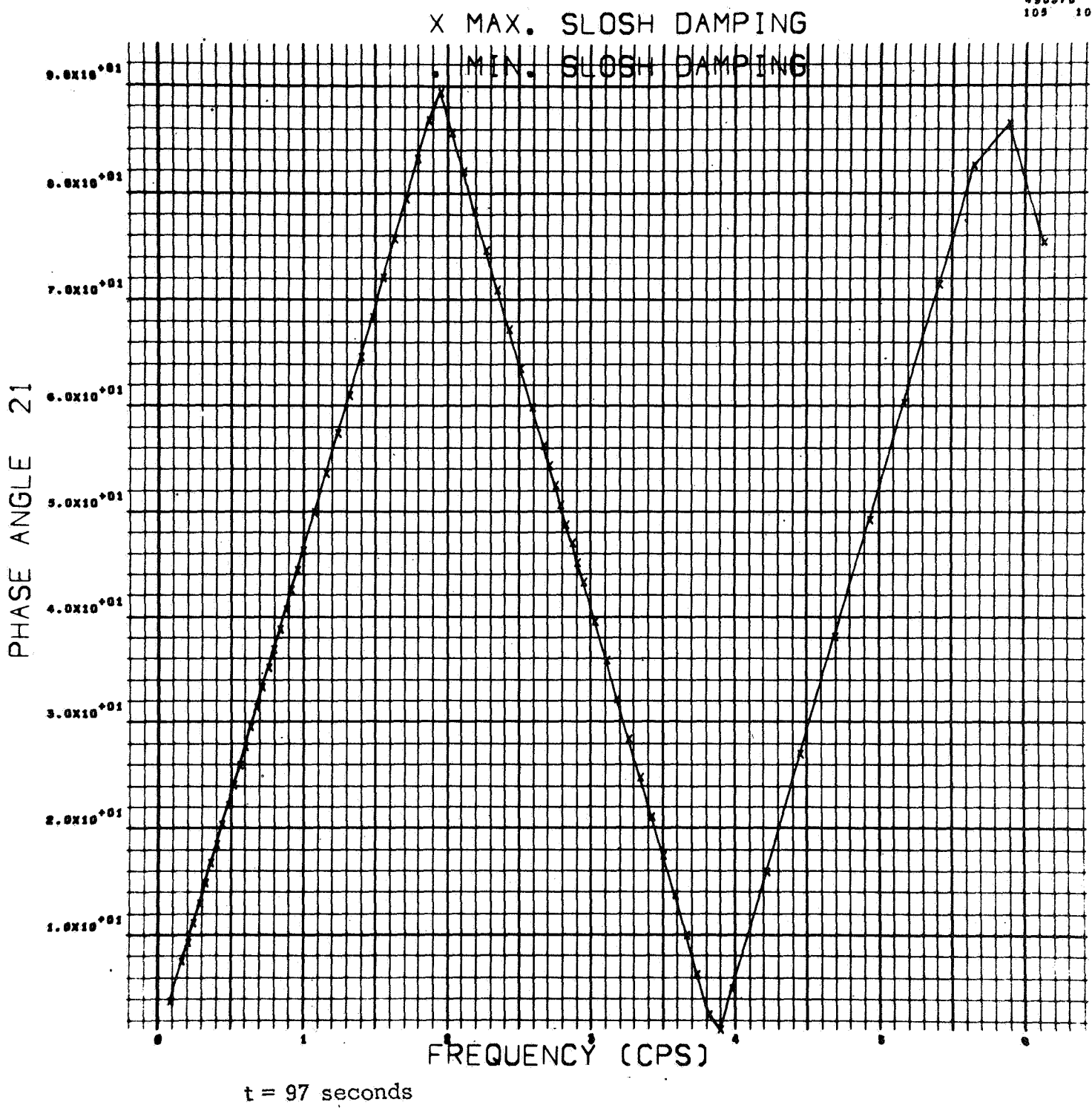
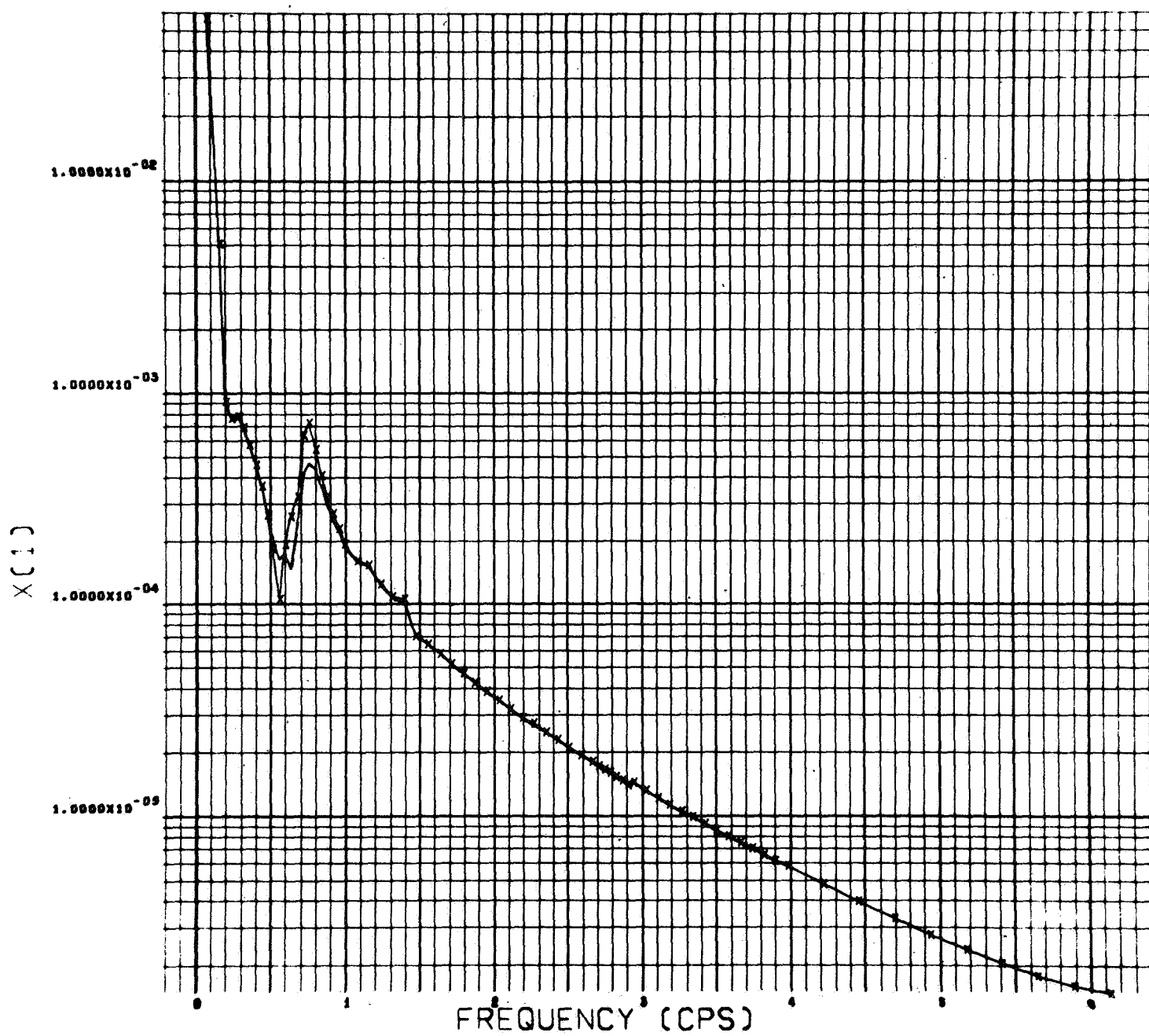


Figure 67b

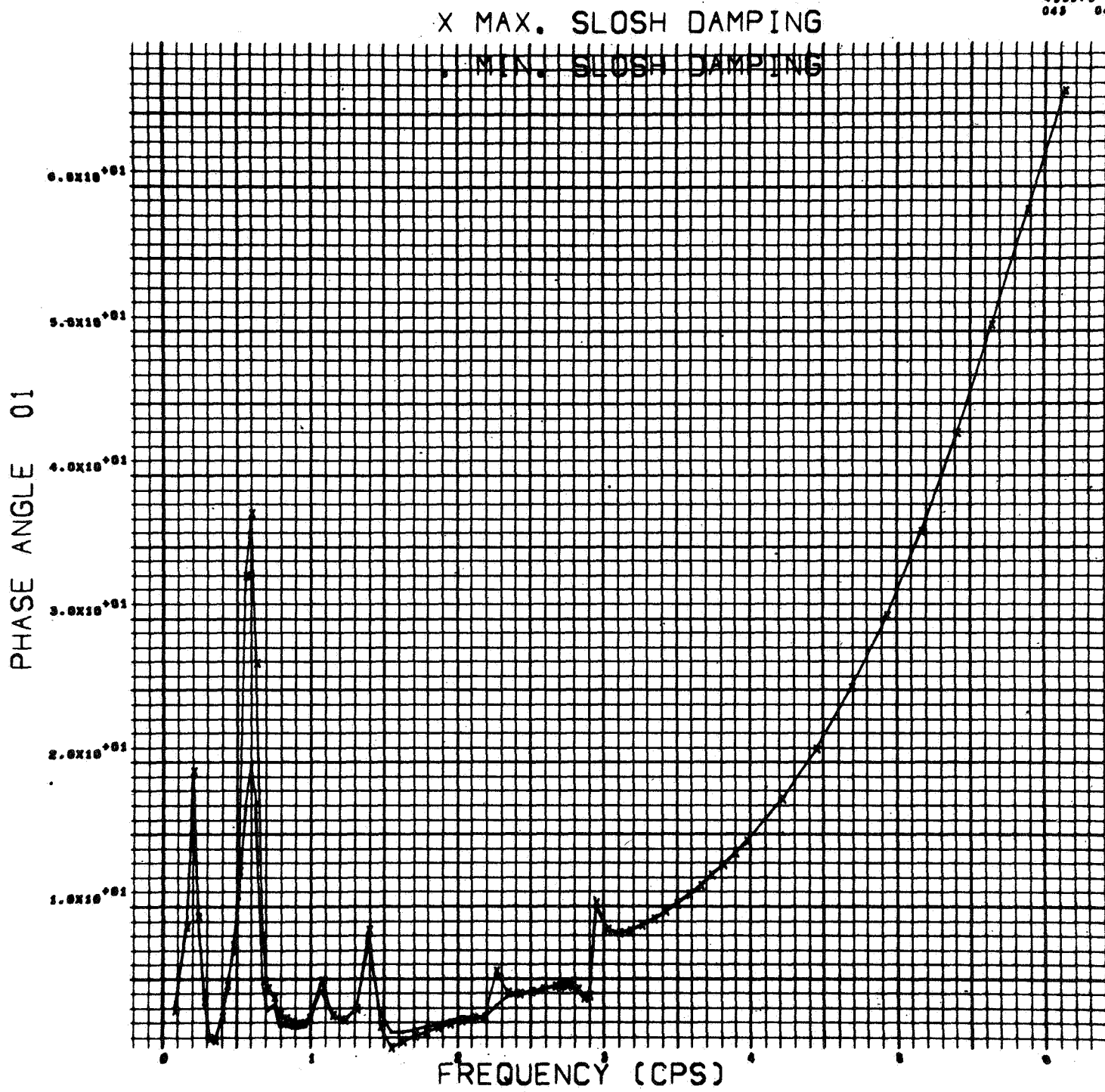
X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 120 seconds

Figure 68a

490570 L
043 043

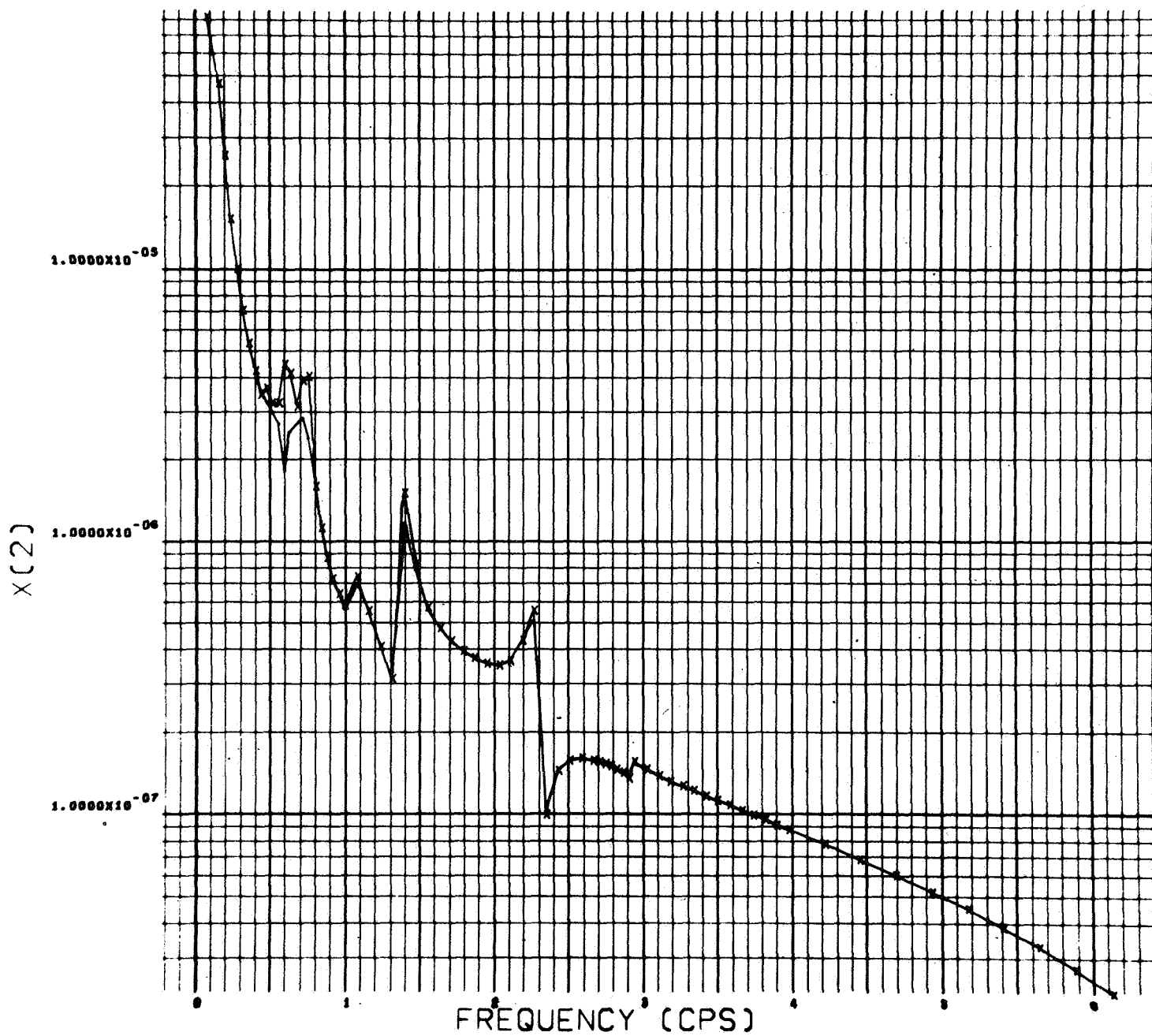


t = 120 seconds

Figure 68b

X MAX. SLOSH DAMPING

. MIN. SLOSH DAMPING



t = 120 seconds

Figure 69a

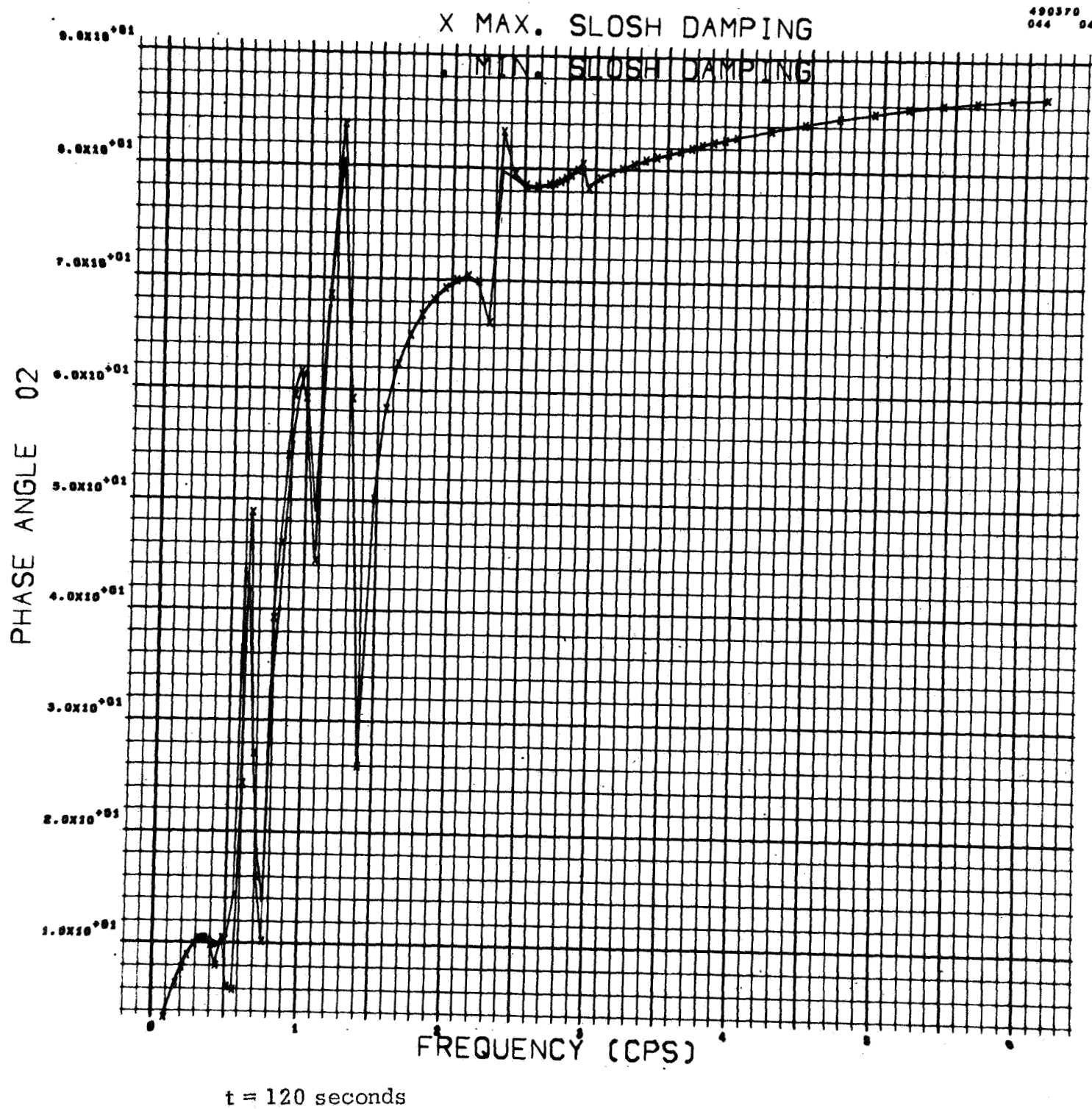
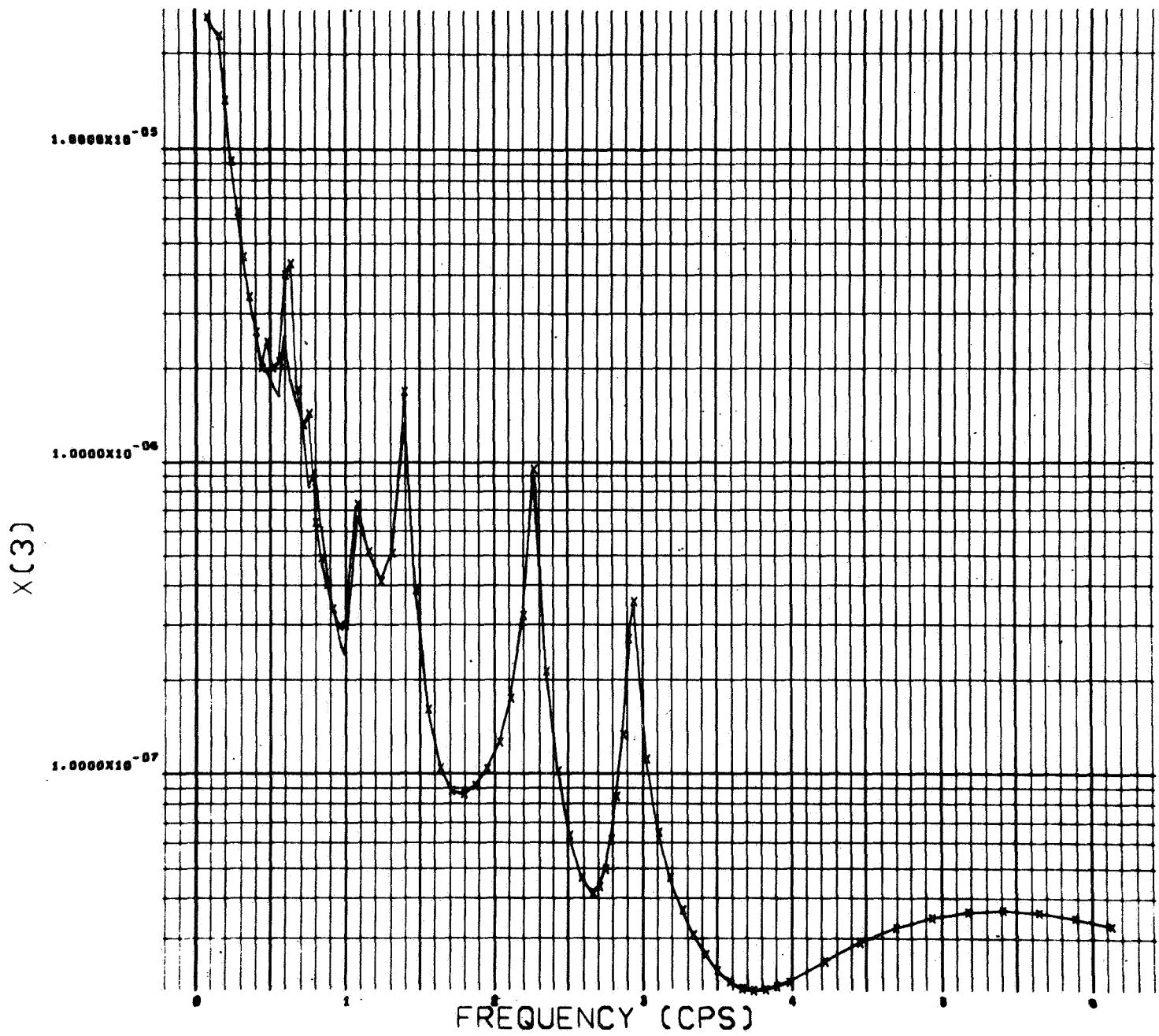


Figure 69b

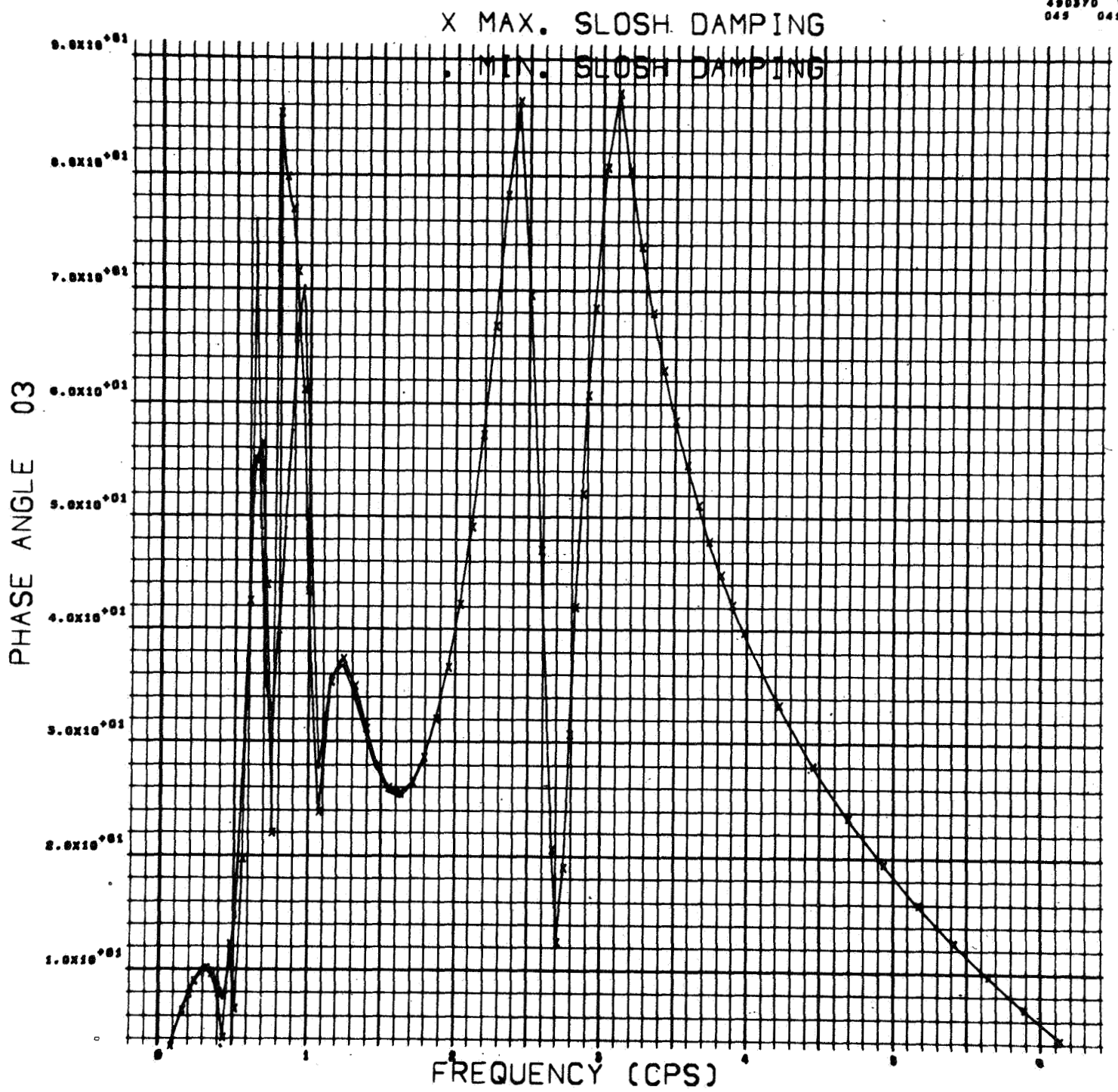
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490370
000 000



t = 120 seconds

Figure 70a

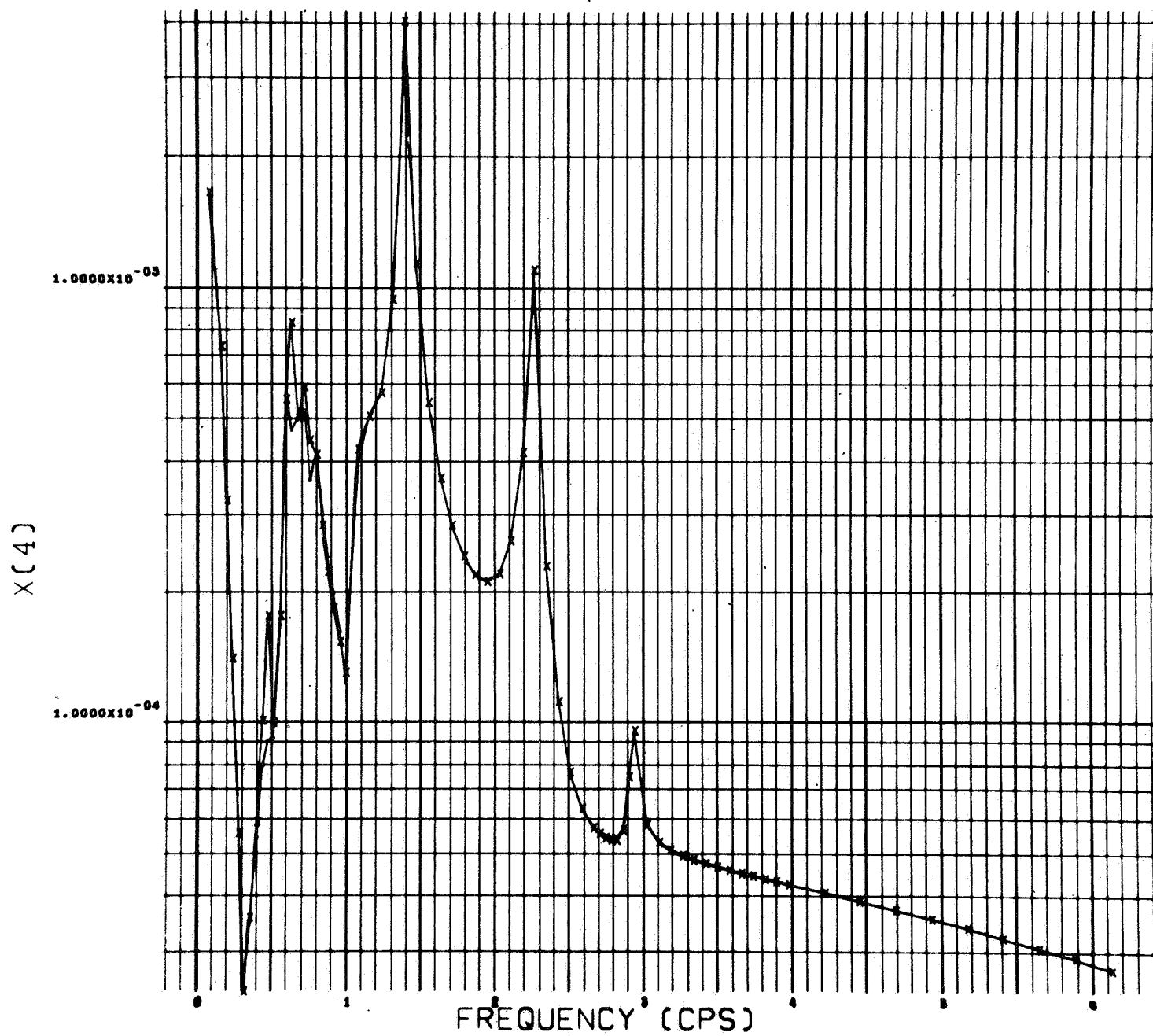


t = 120 seconds

Figure 70b

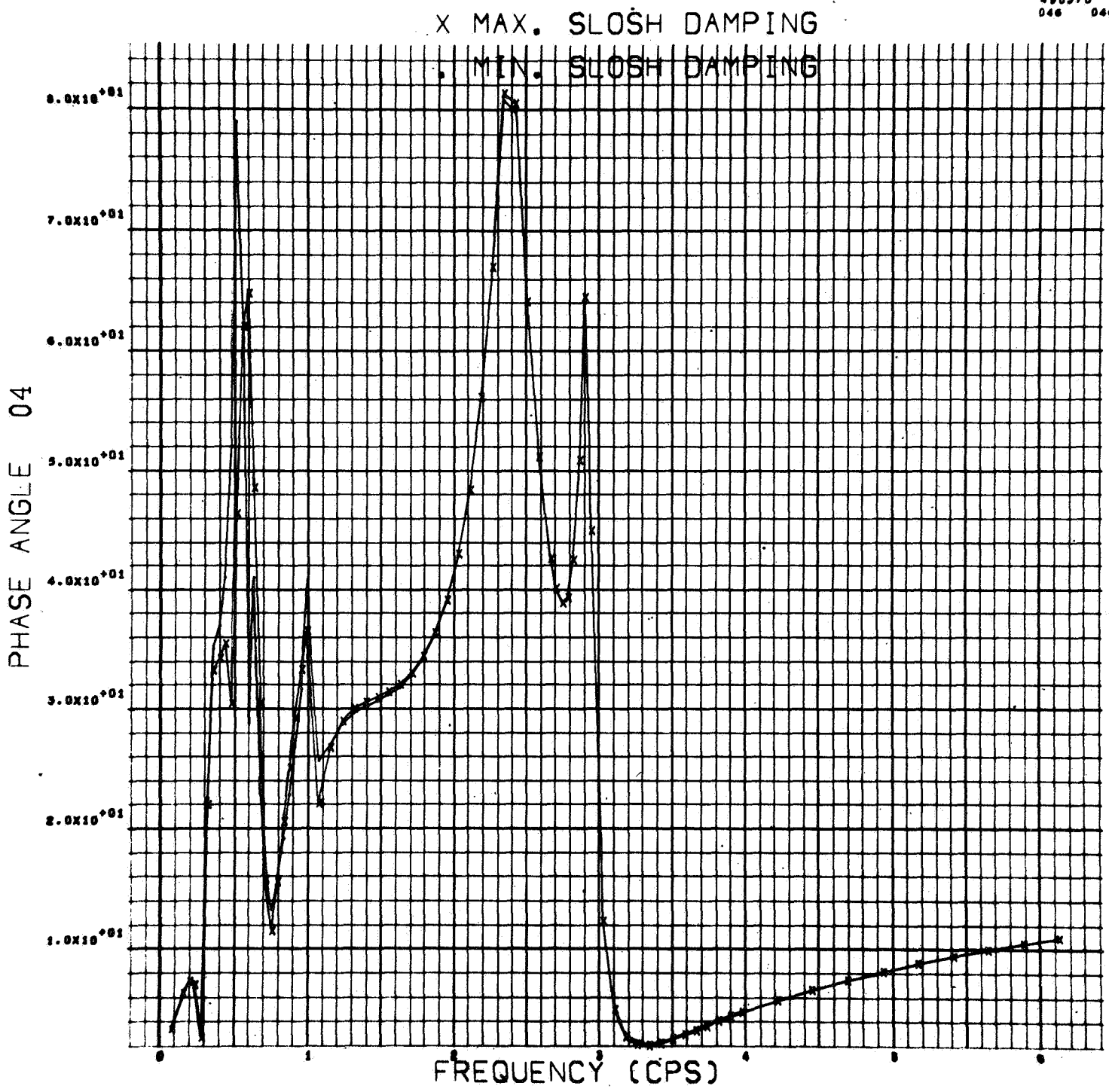
X MAX. SLOSH DAMPING

. MIN. SLOSH DAMPING



t = 120 seconds

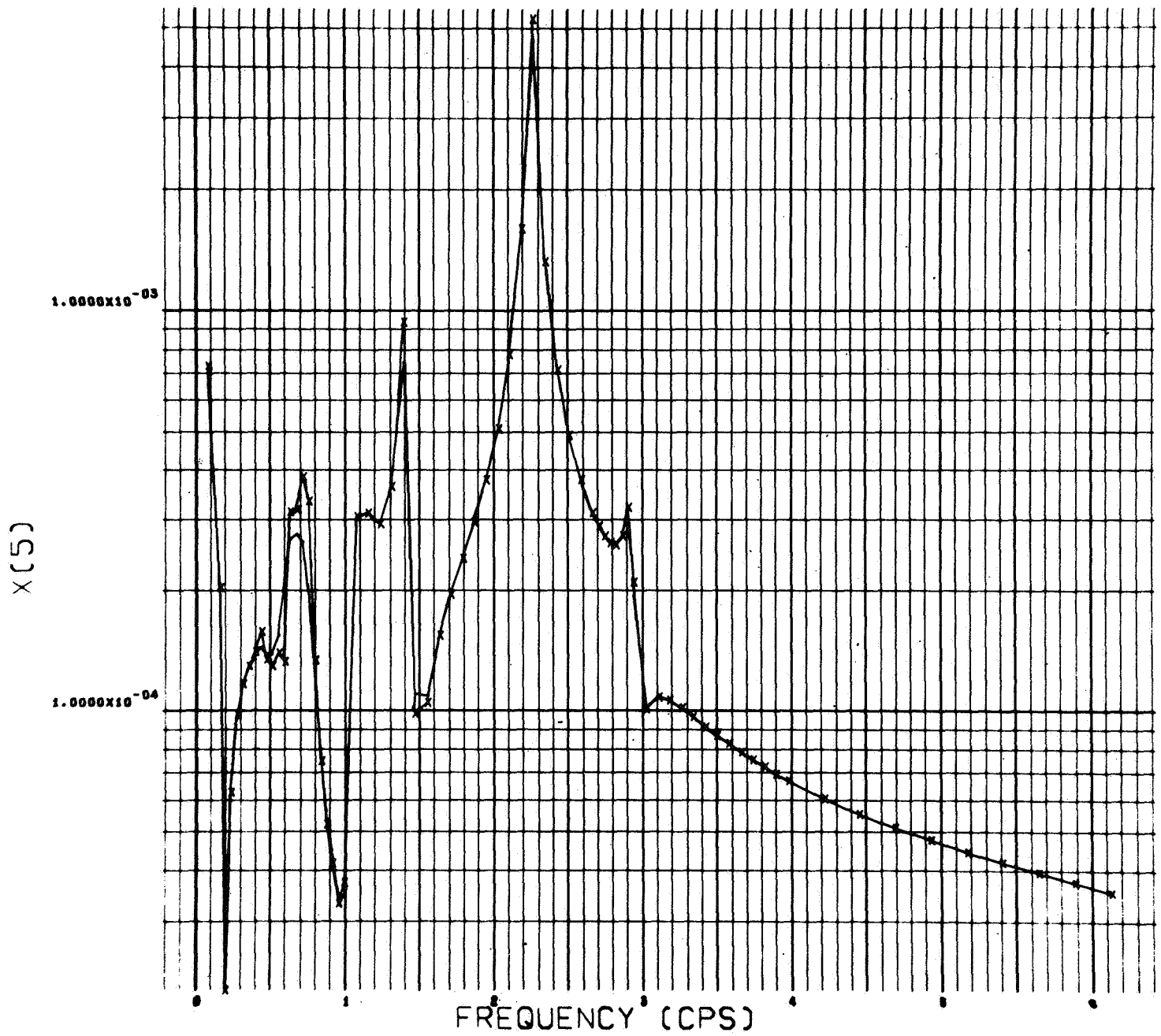
Figure 71a



t = 120 seconds

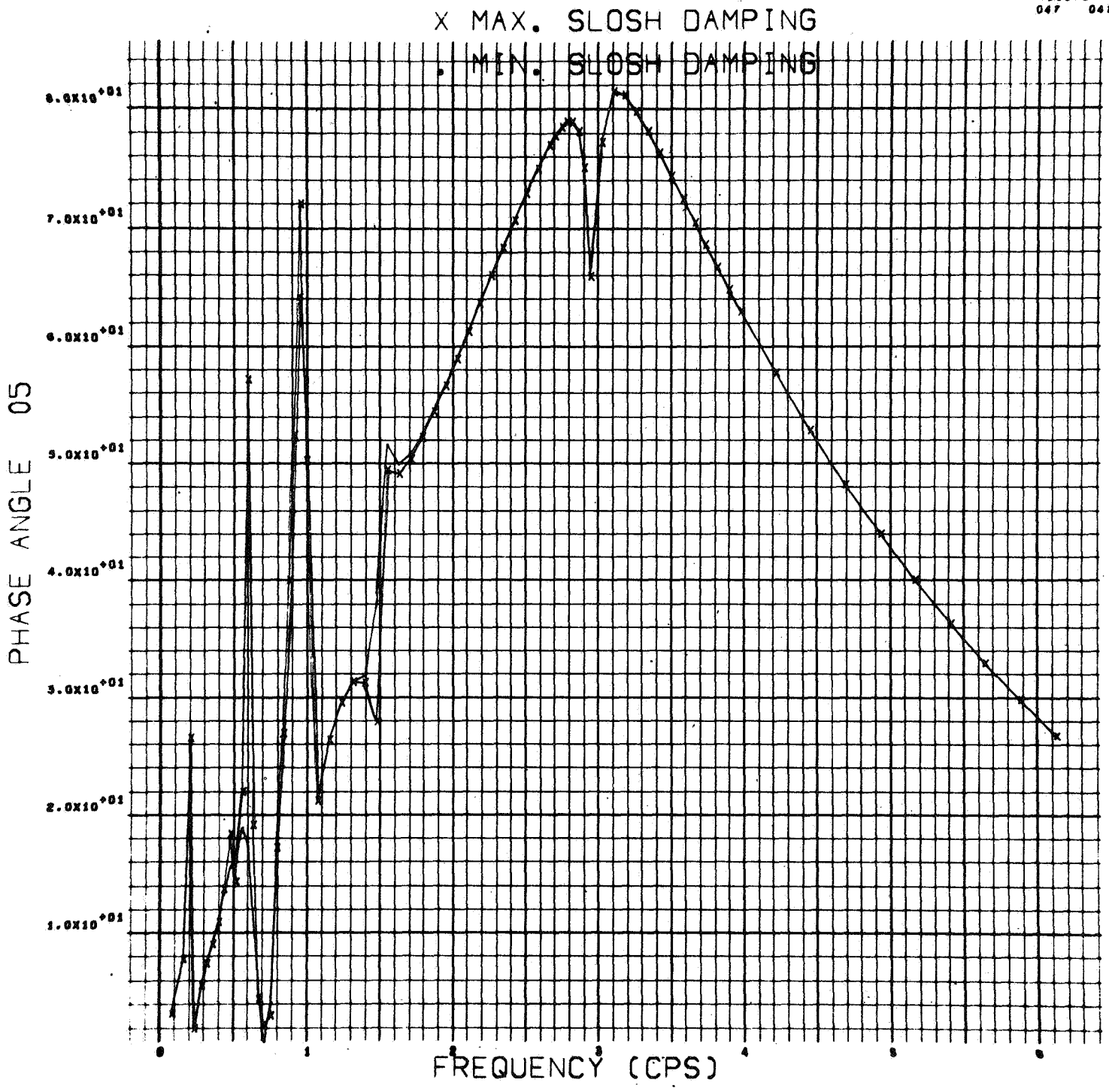
Figure 71b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 120 seconds

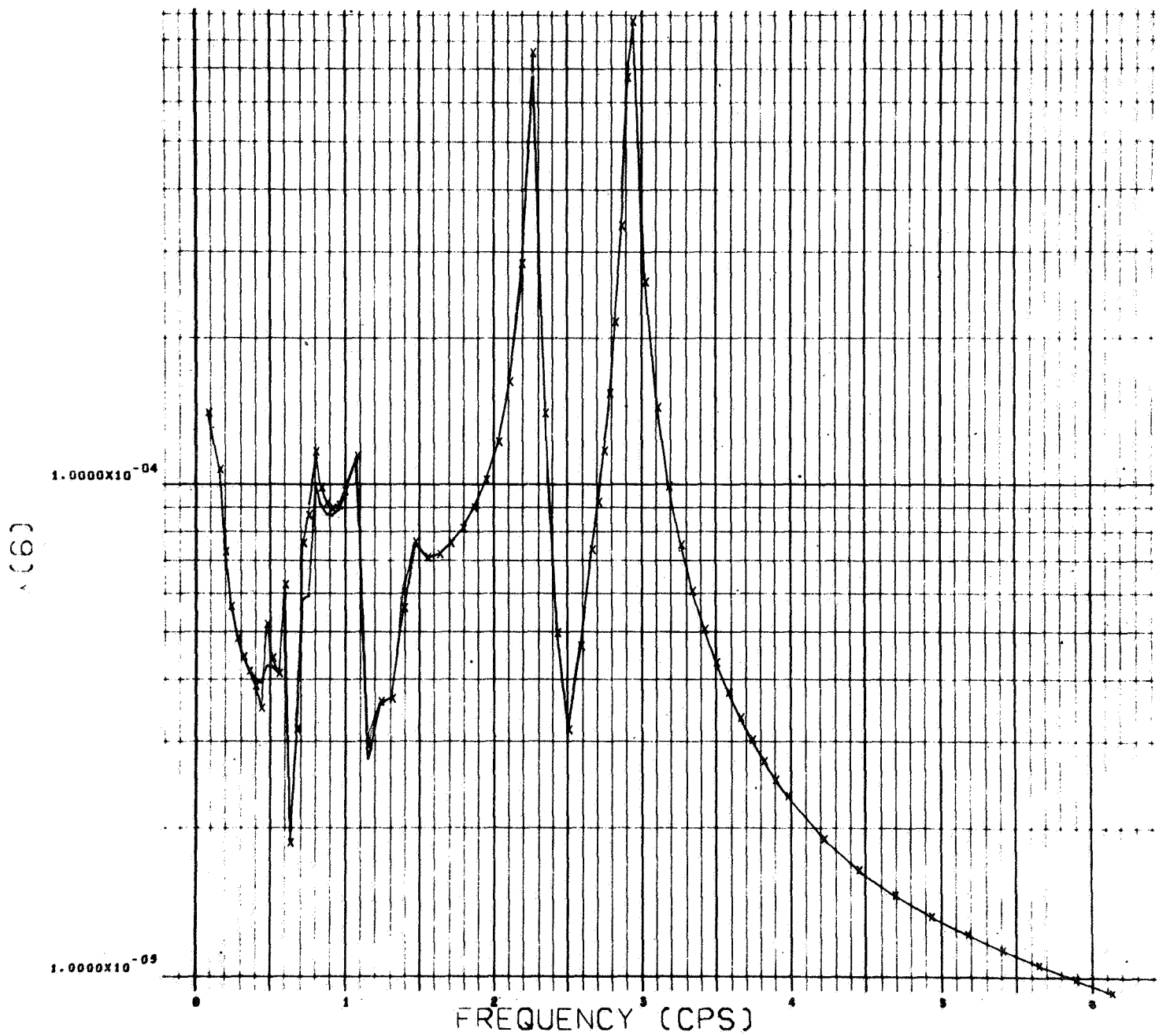
Figure 72a



t = 120 seconds

Figure 72b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



t = 120 seconds

Figure 73a

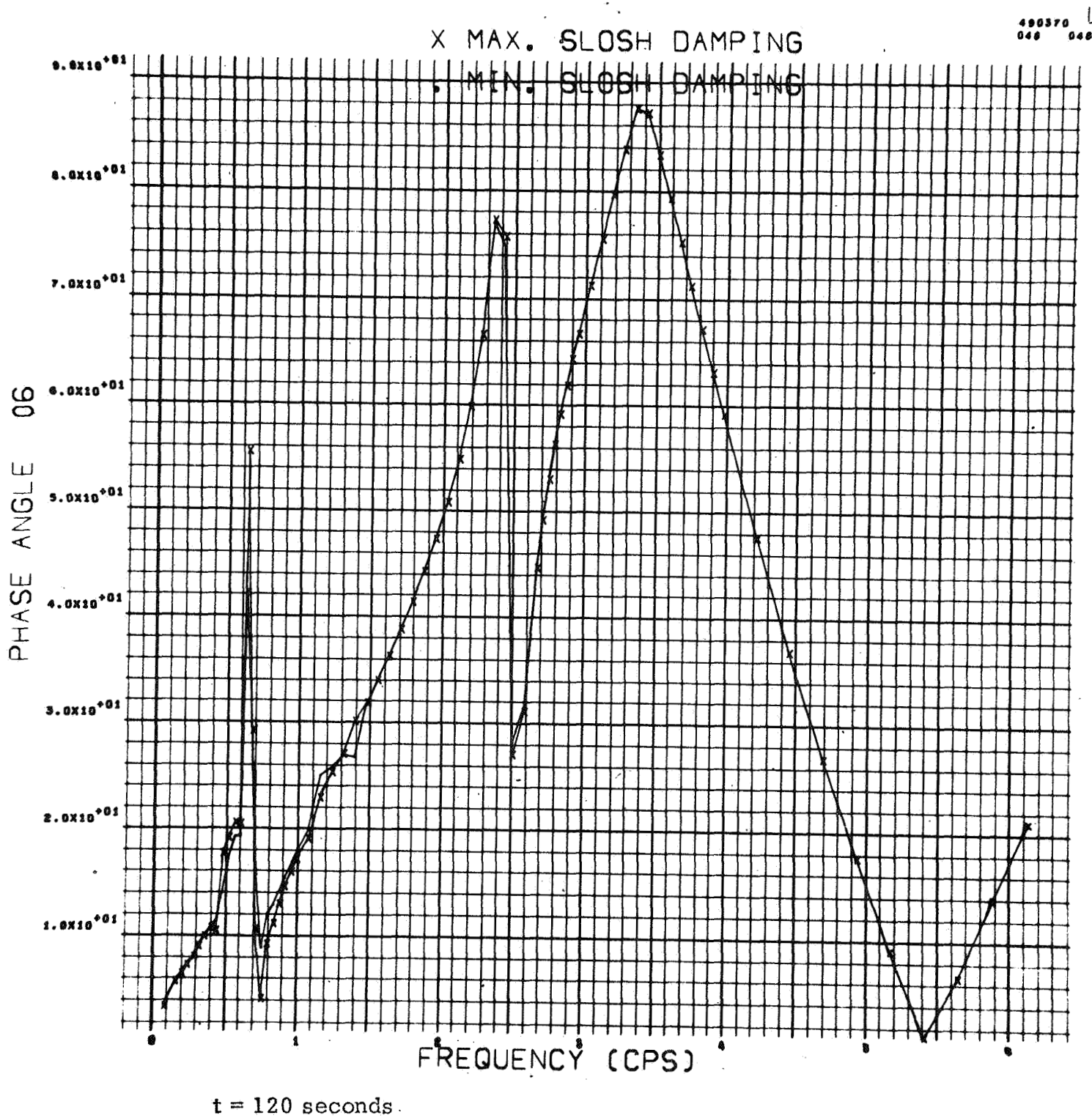
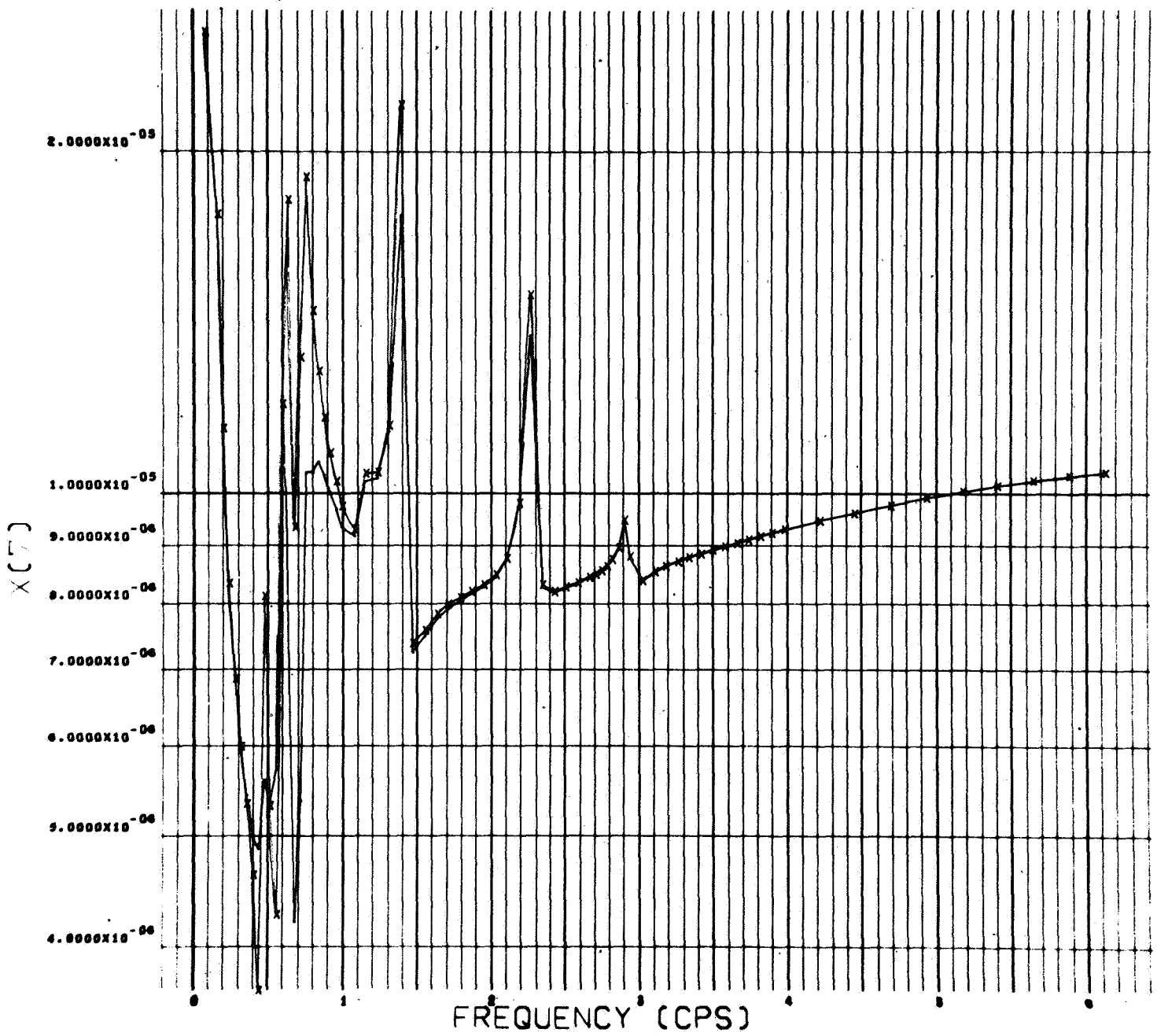


Figure 73b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490370
070 070



t = 120 seconds

Figure 74a

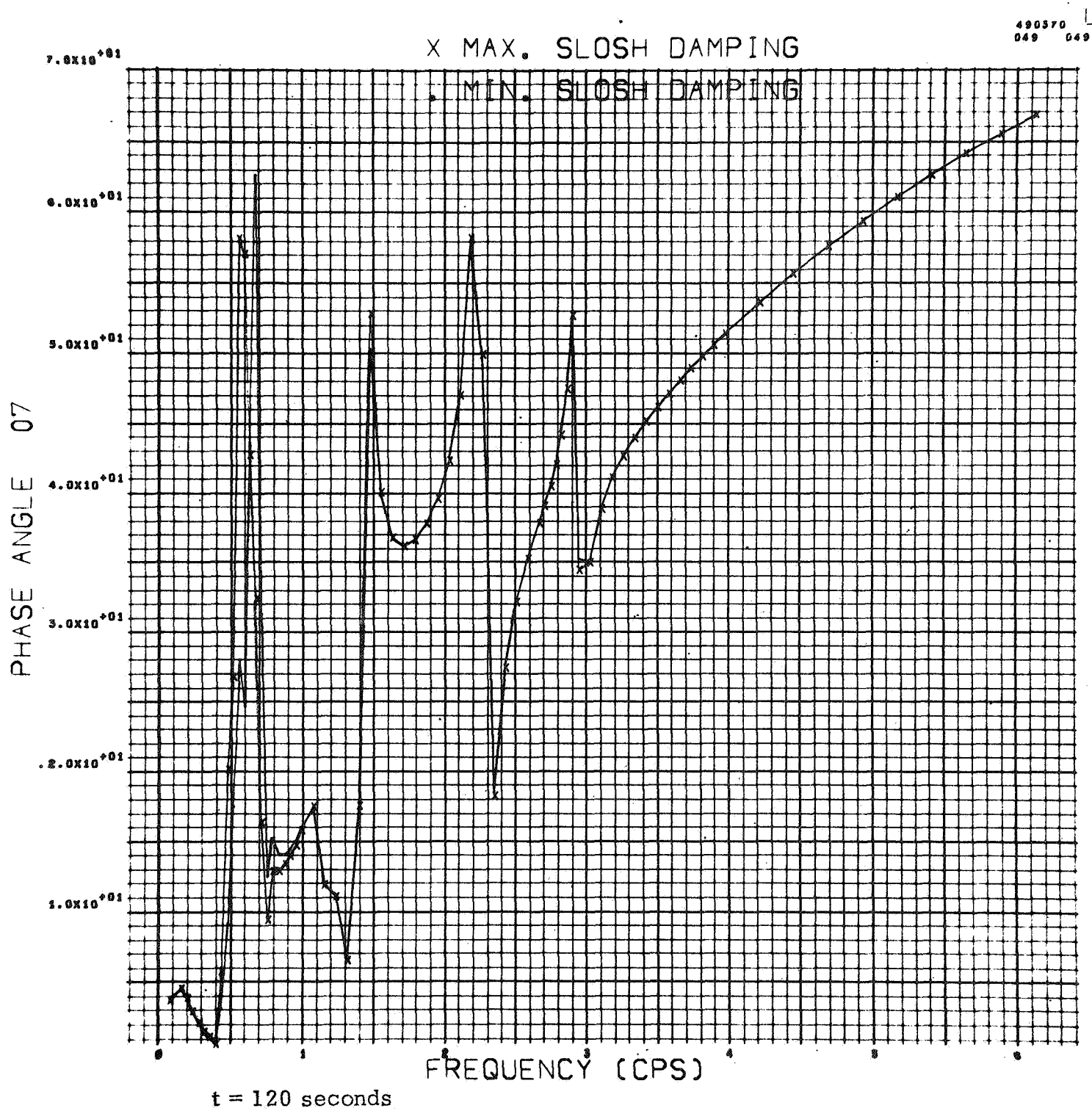
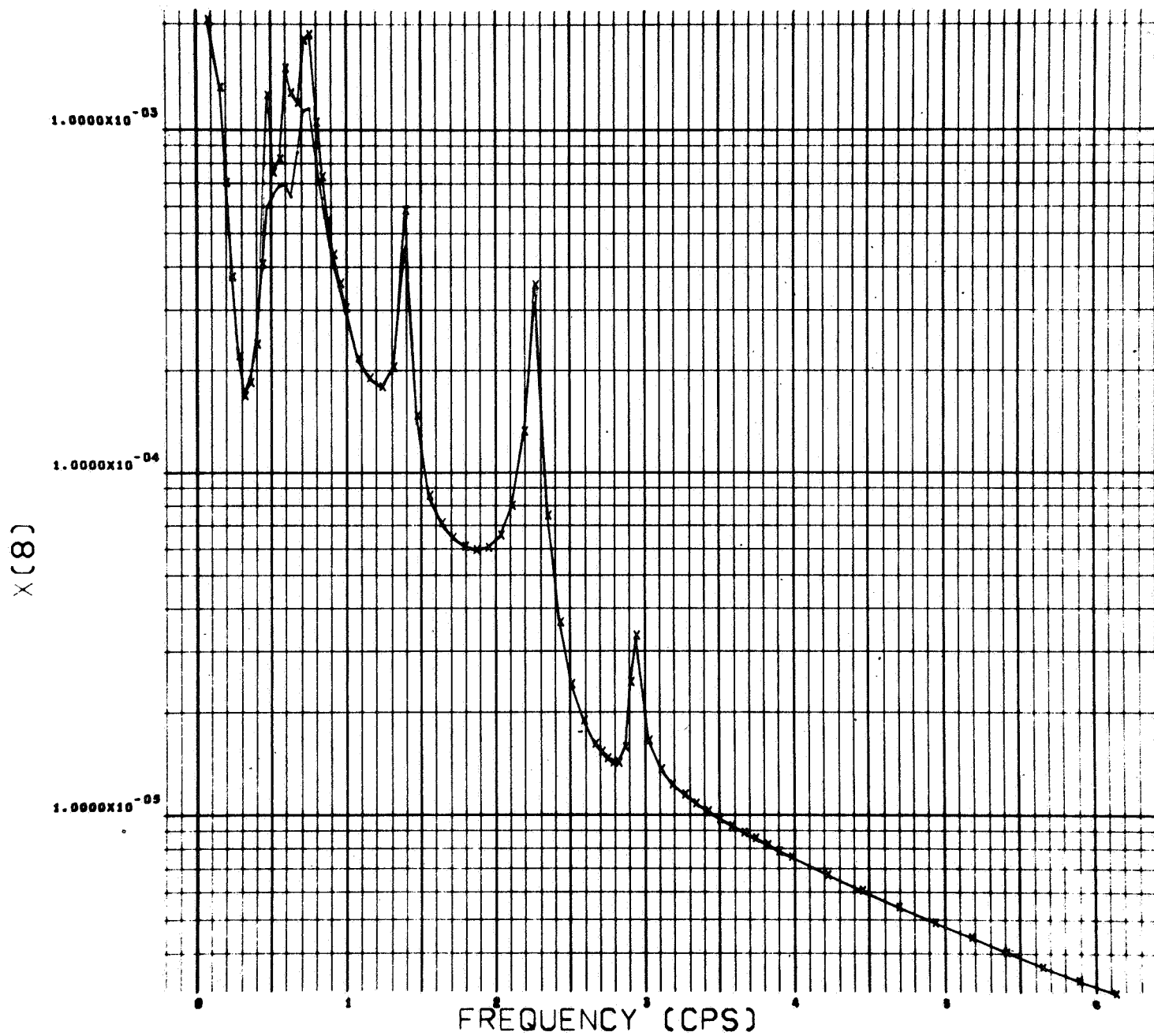


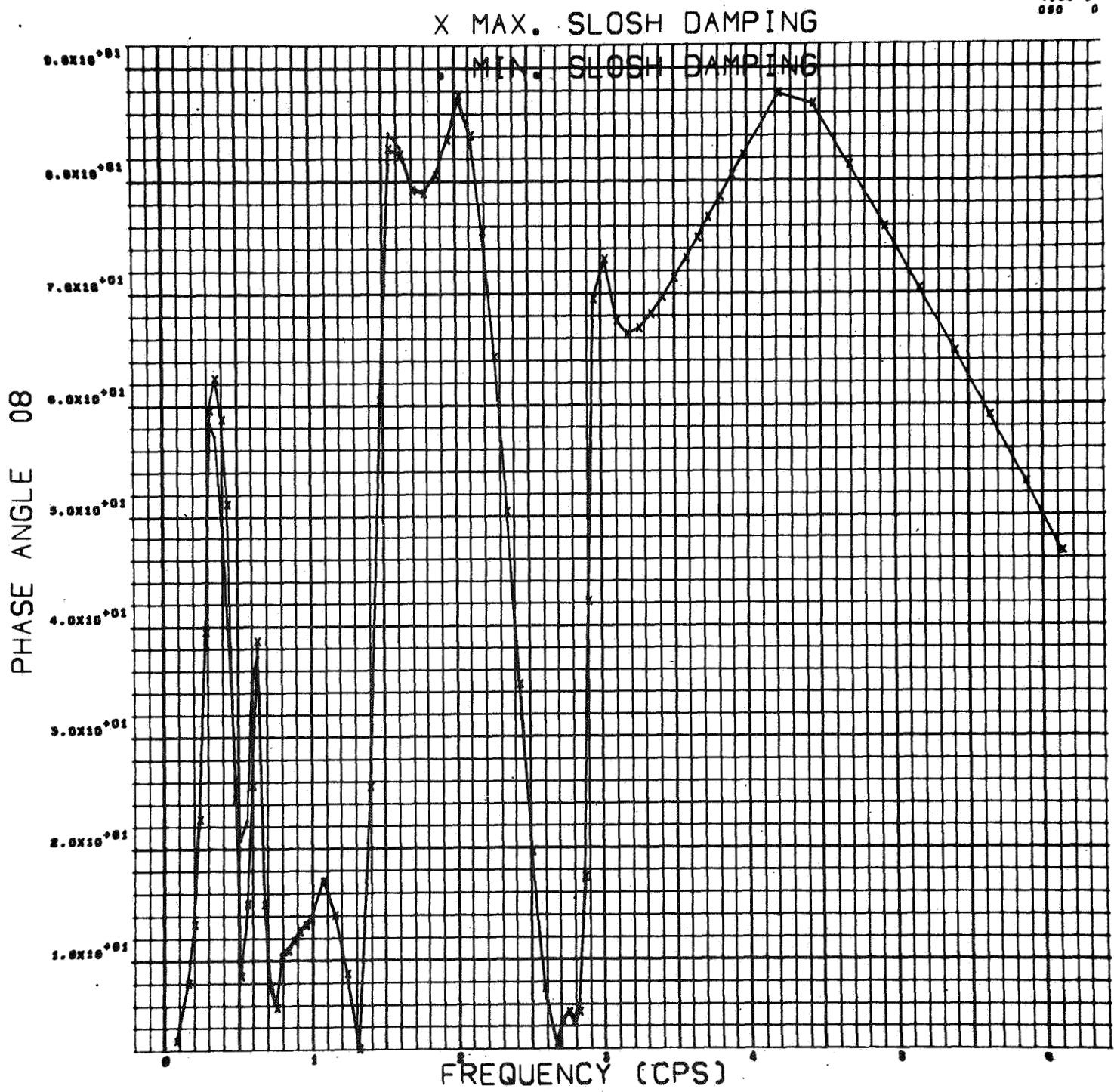
Figure 74b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 120 seconds

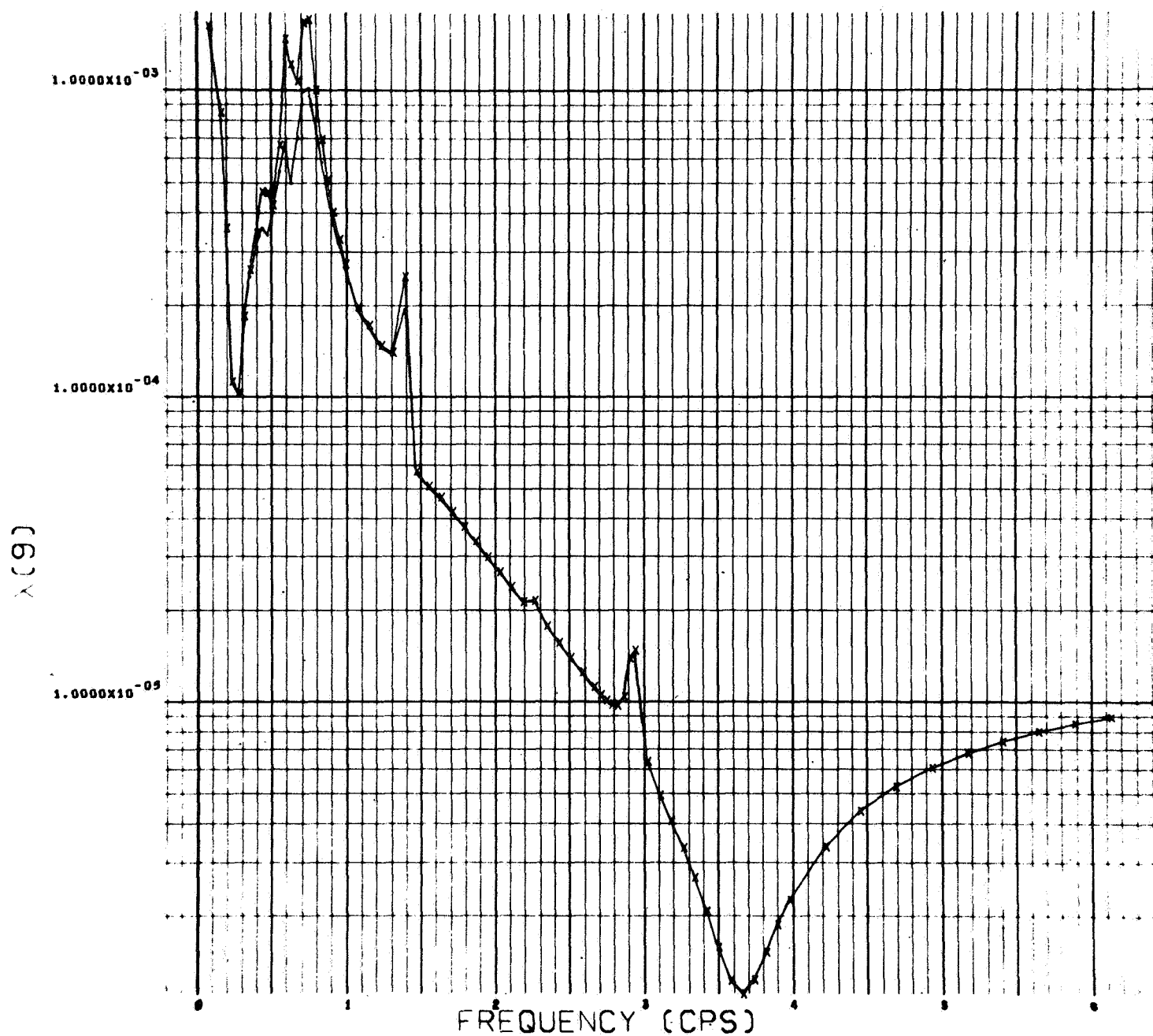
Figure 75a



t = 120 seconds

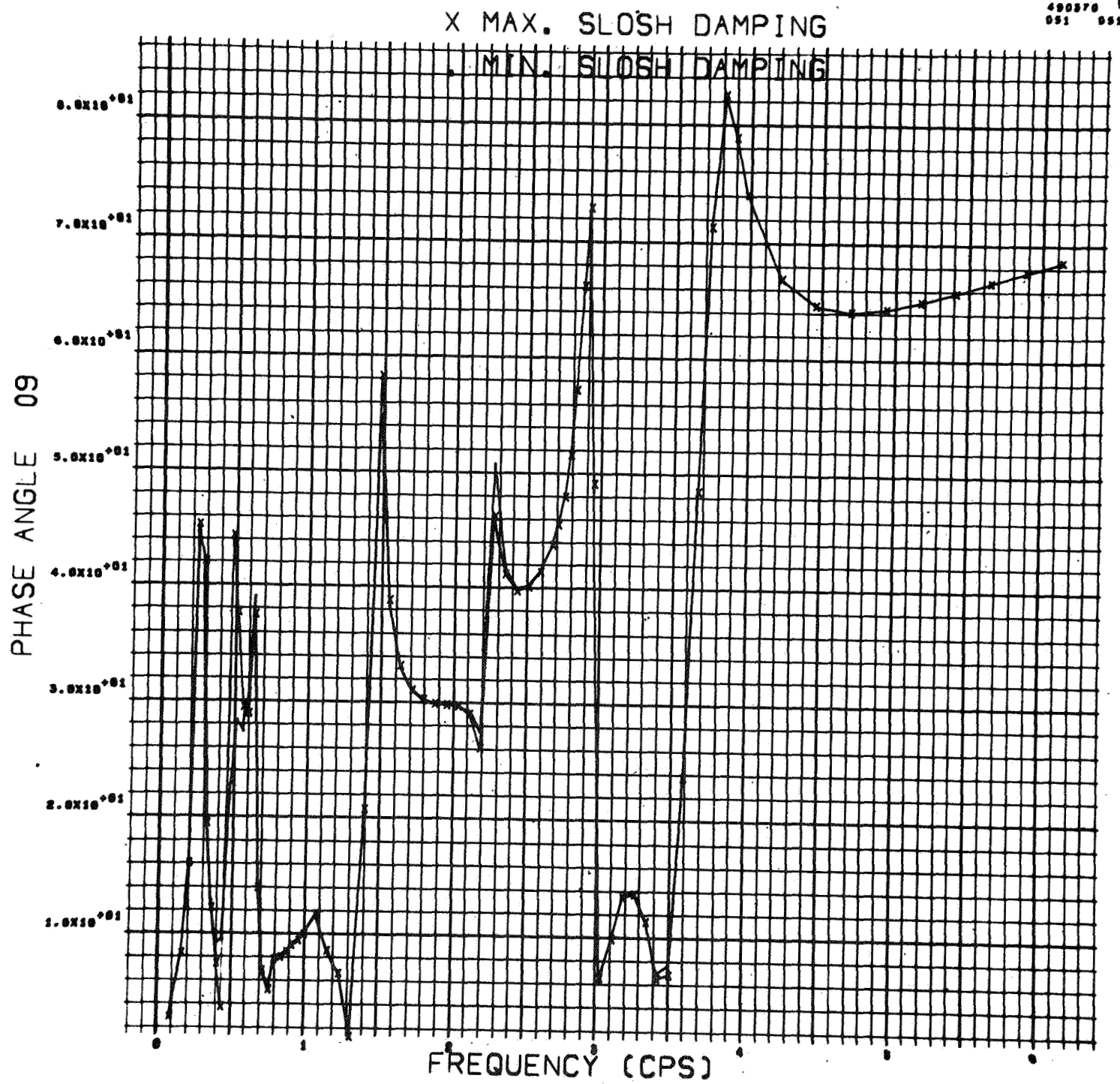
Figure 75b

x MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 120 seconds

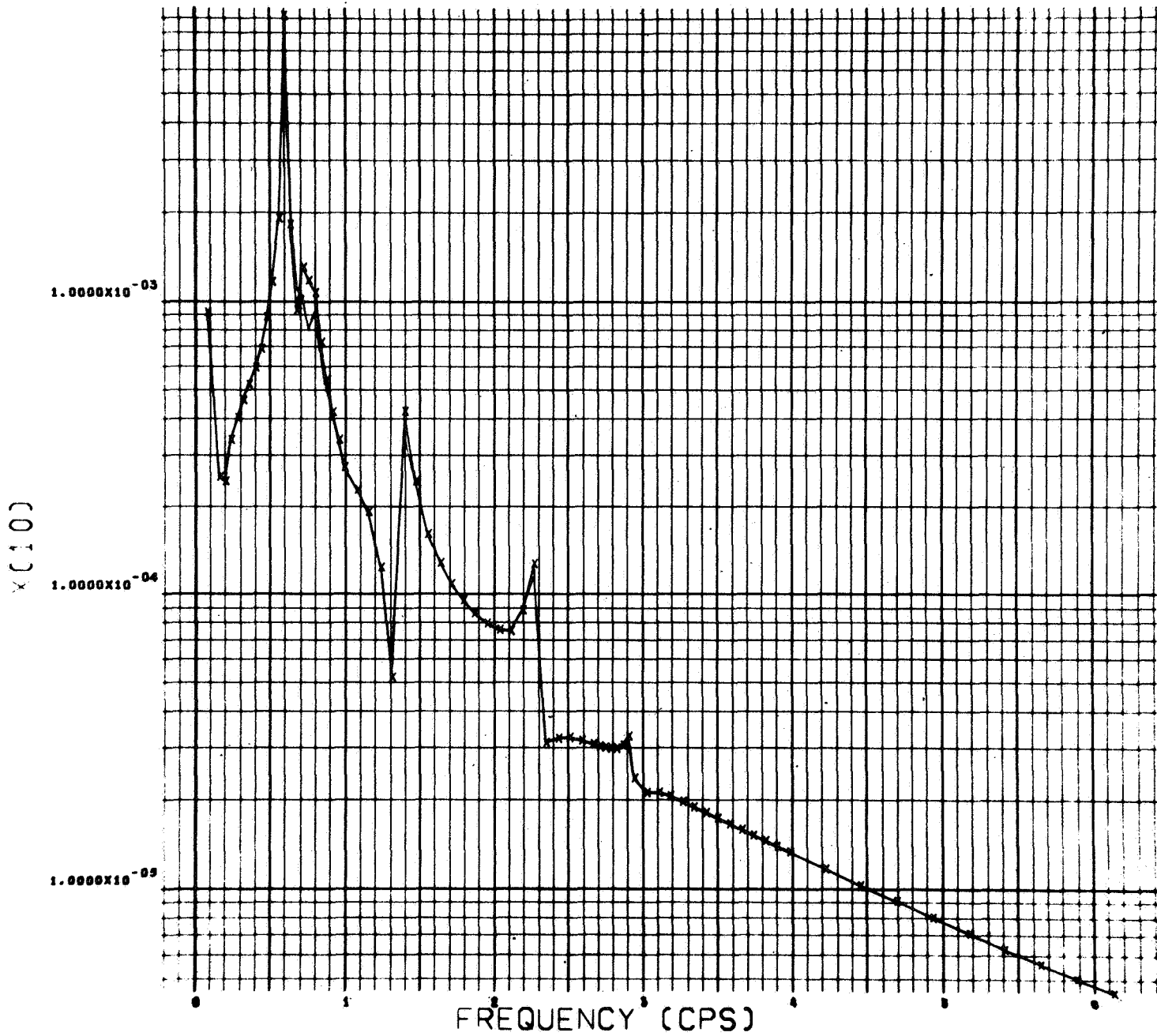
Figure 76a



t = 120 seconds

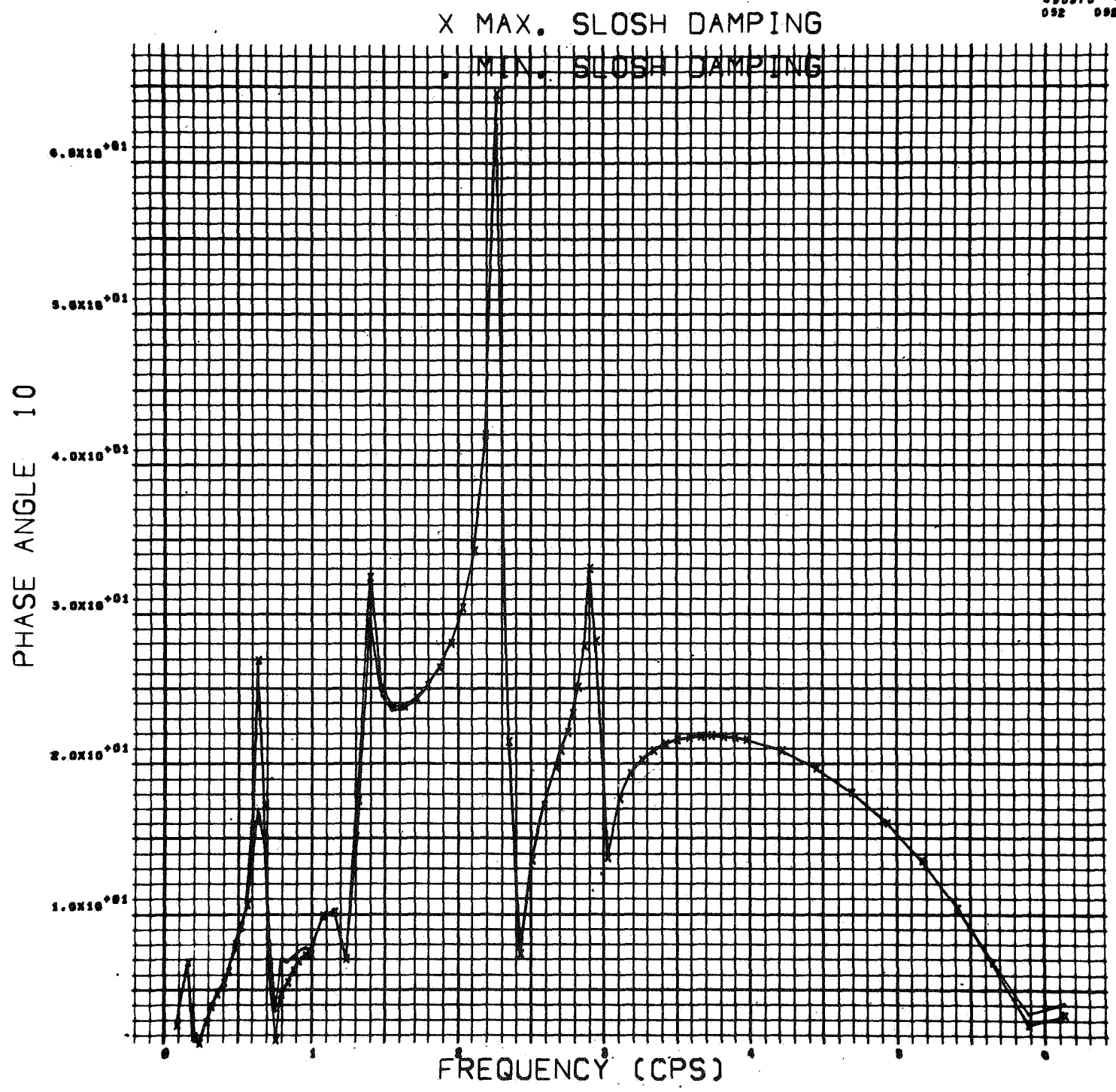
Figure 76b

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING



t - 120 seconds

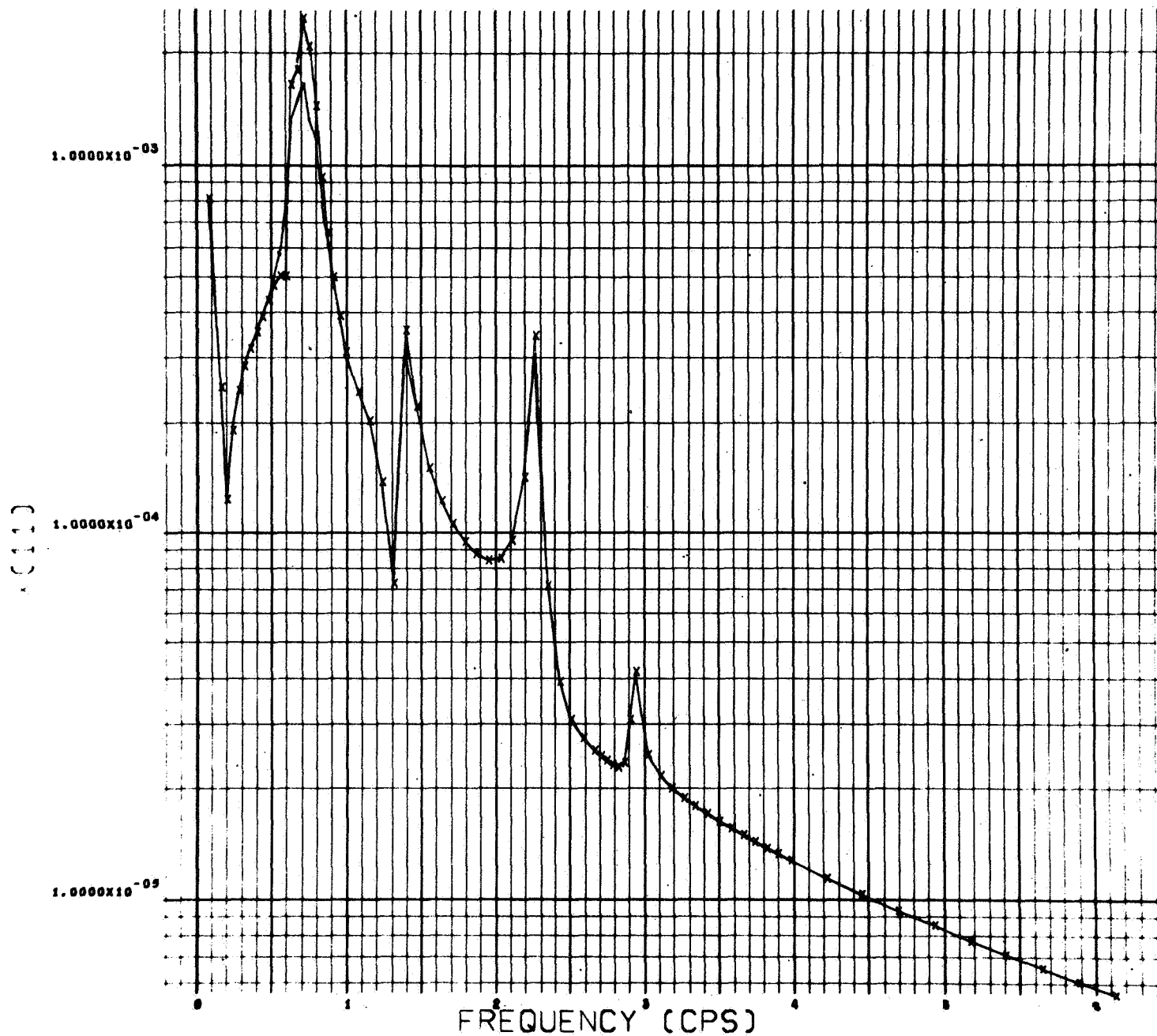
Figure 77a



t = 120 seconds

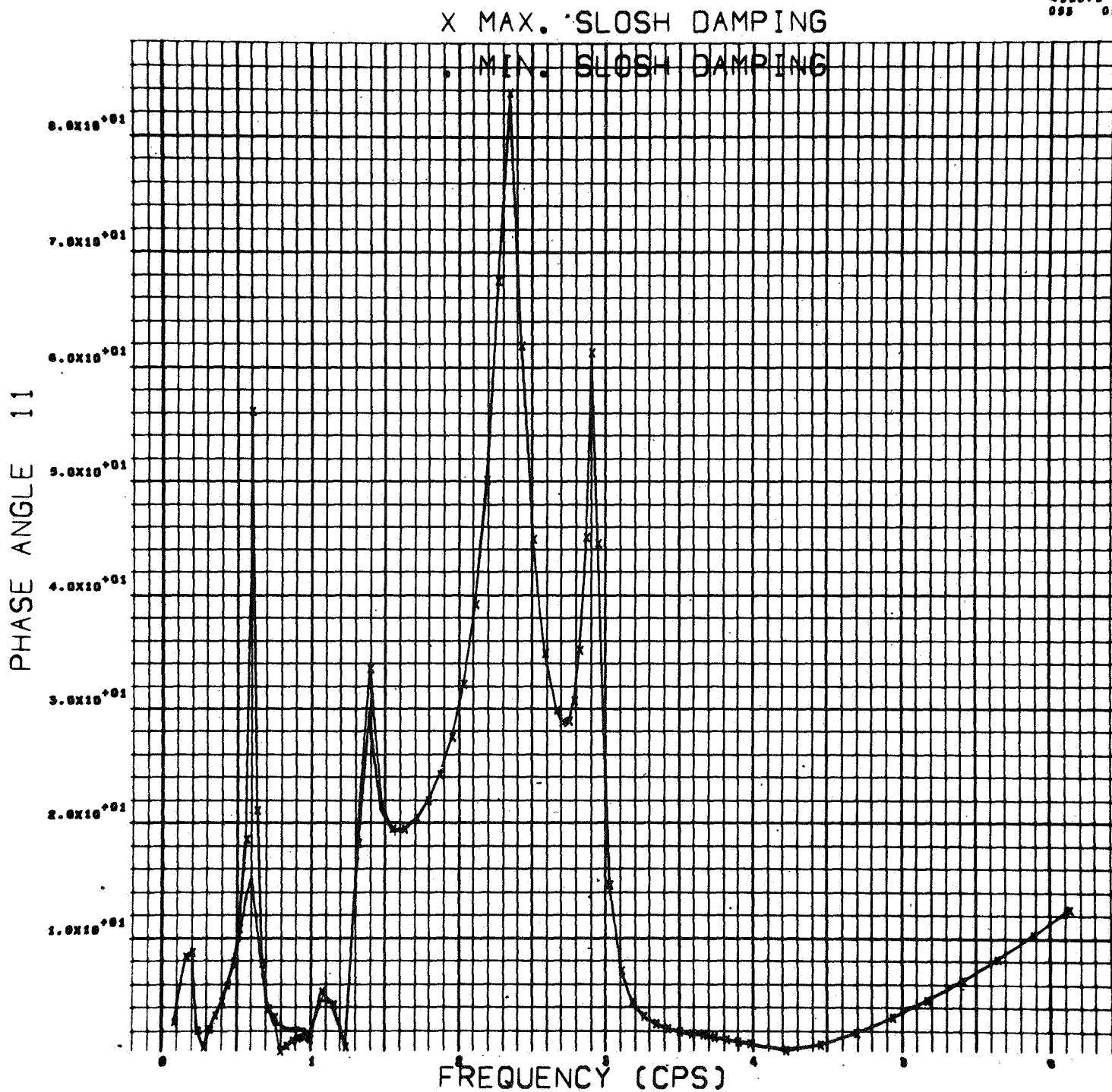
Figure 77b

X MAX. SLOSH DAMPING
 . MIN. SLOSH DAMPING



t = 120 seconds

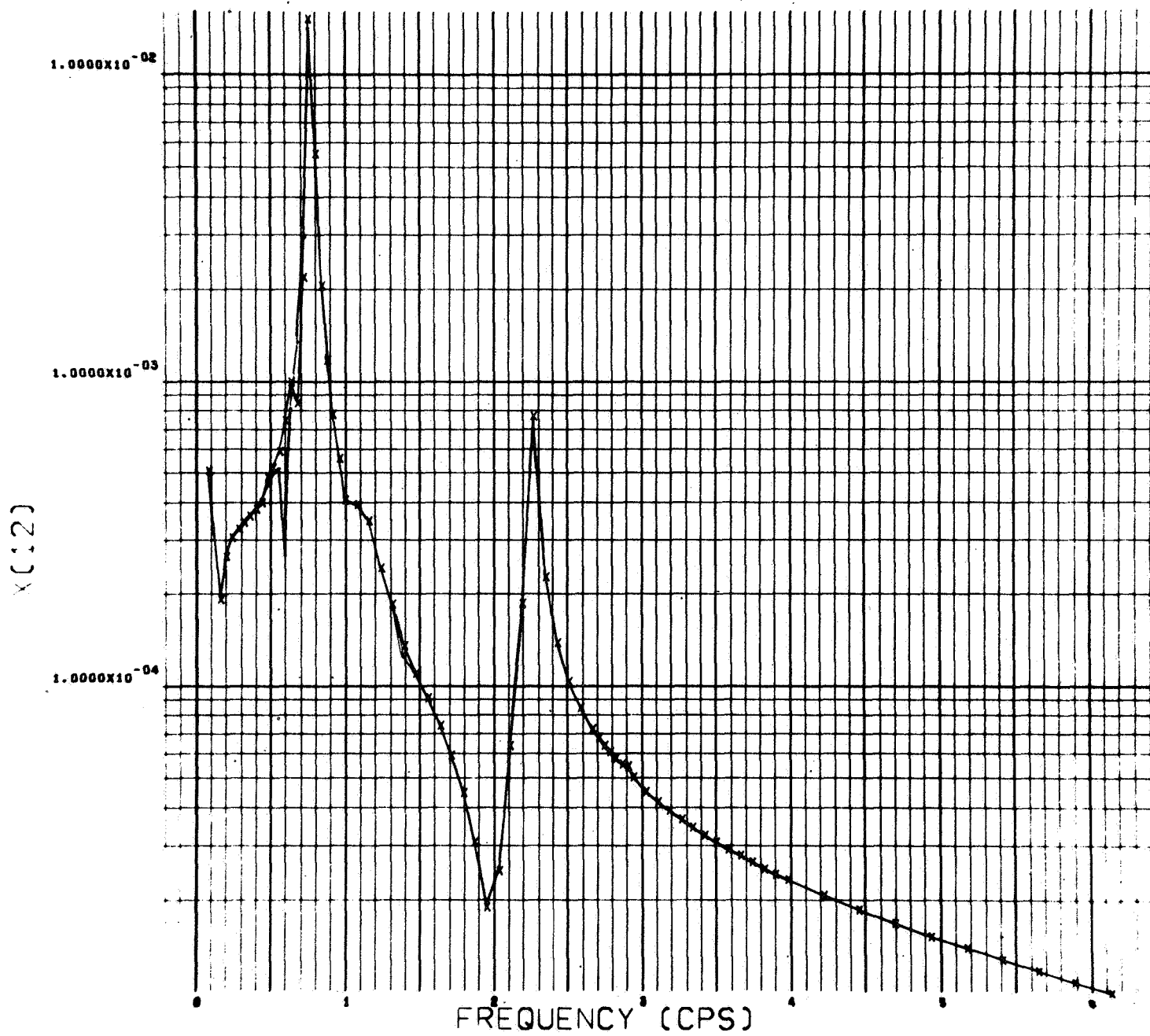
Figure 78a



t = 120 seconds

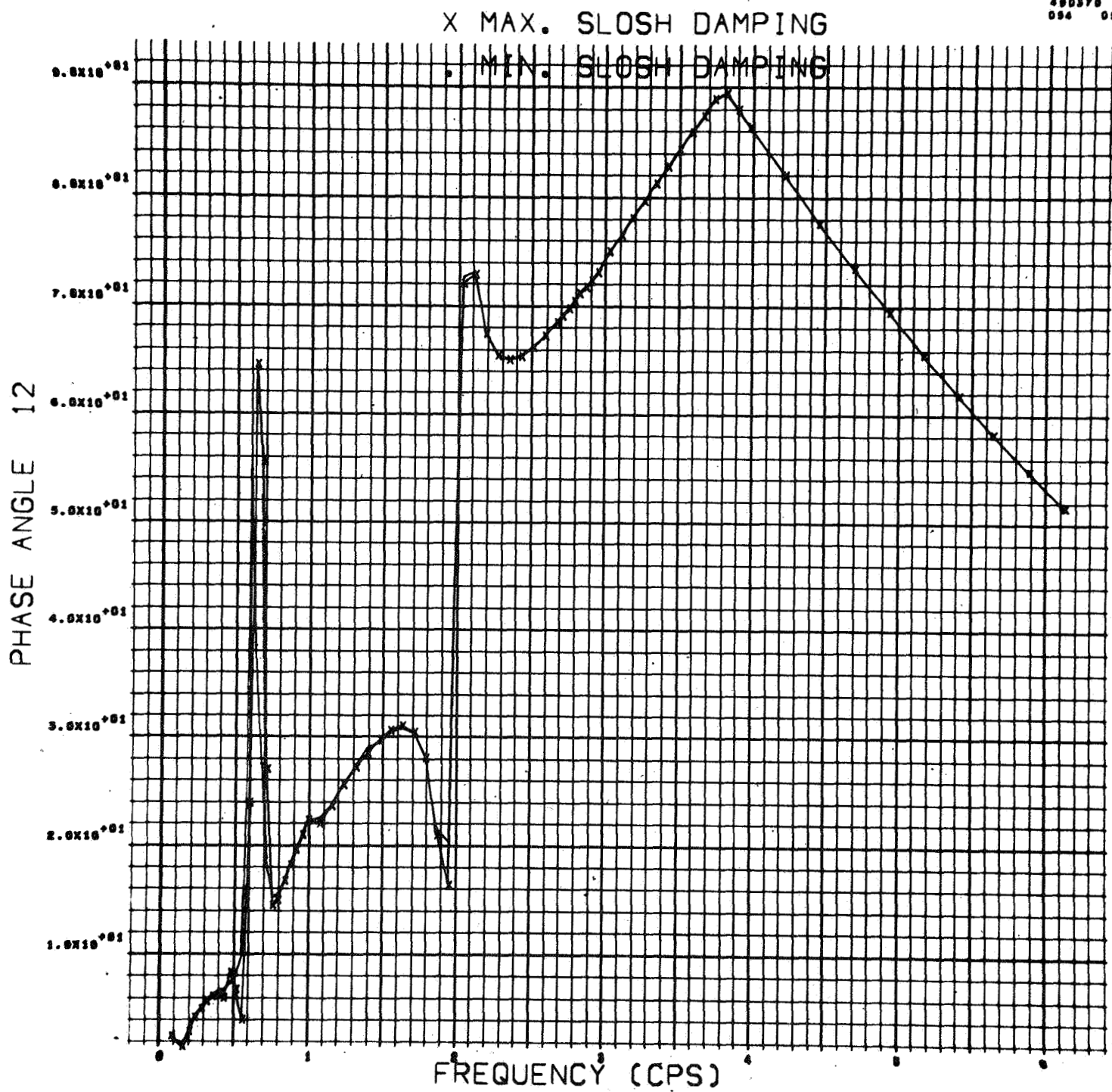
Figure 78b

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



t = 120 seconds

Figure 79a

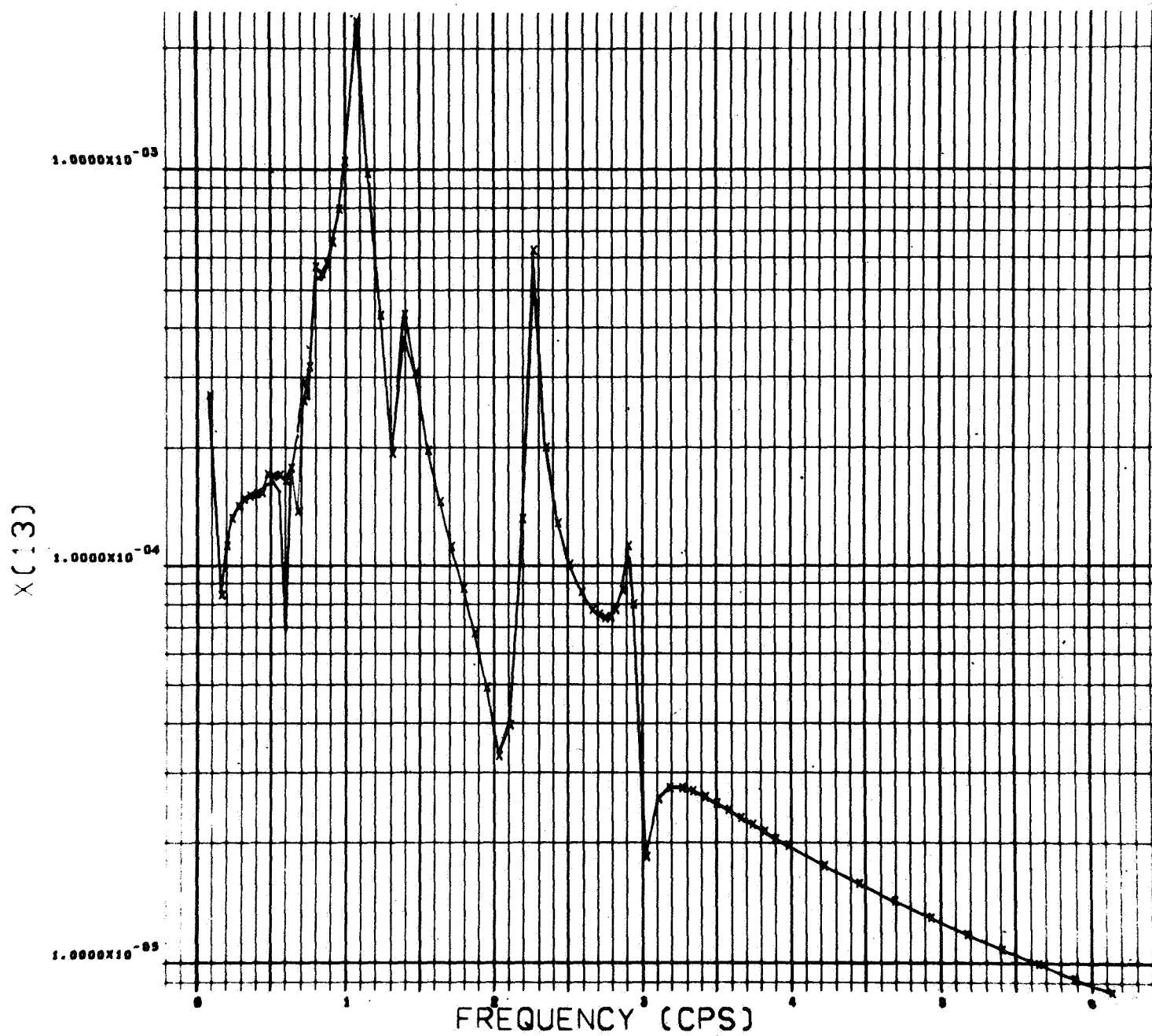


t = 120 seconds

Figure 79b

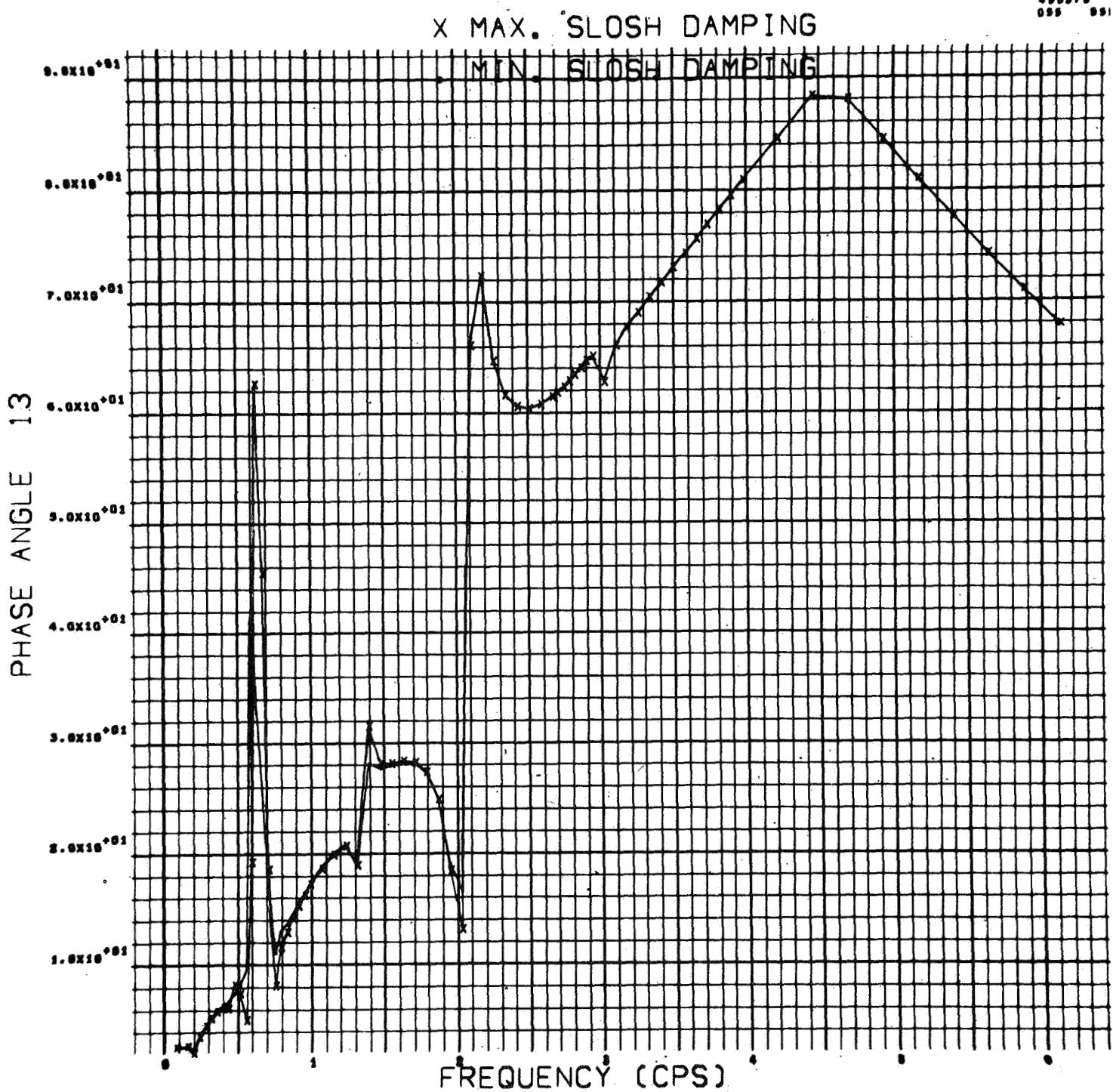
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

490370
076 076



t = 120 seconds

Figure 80a



t = 120 seconds

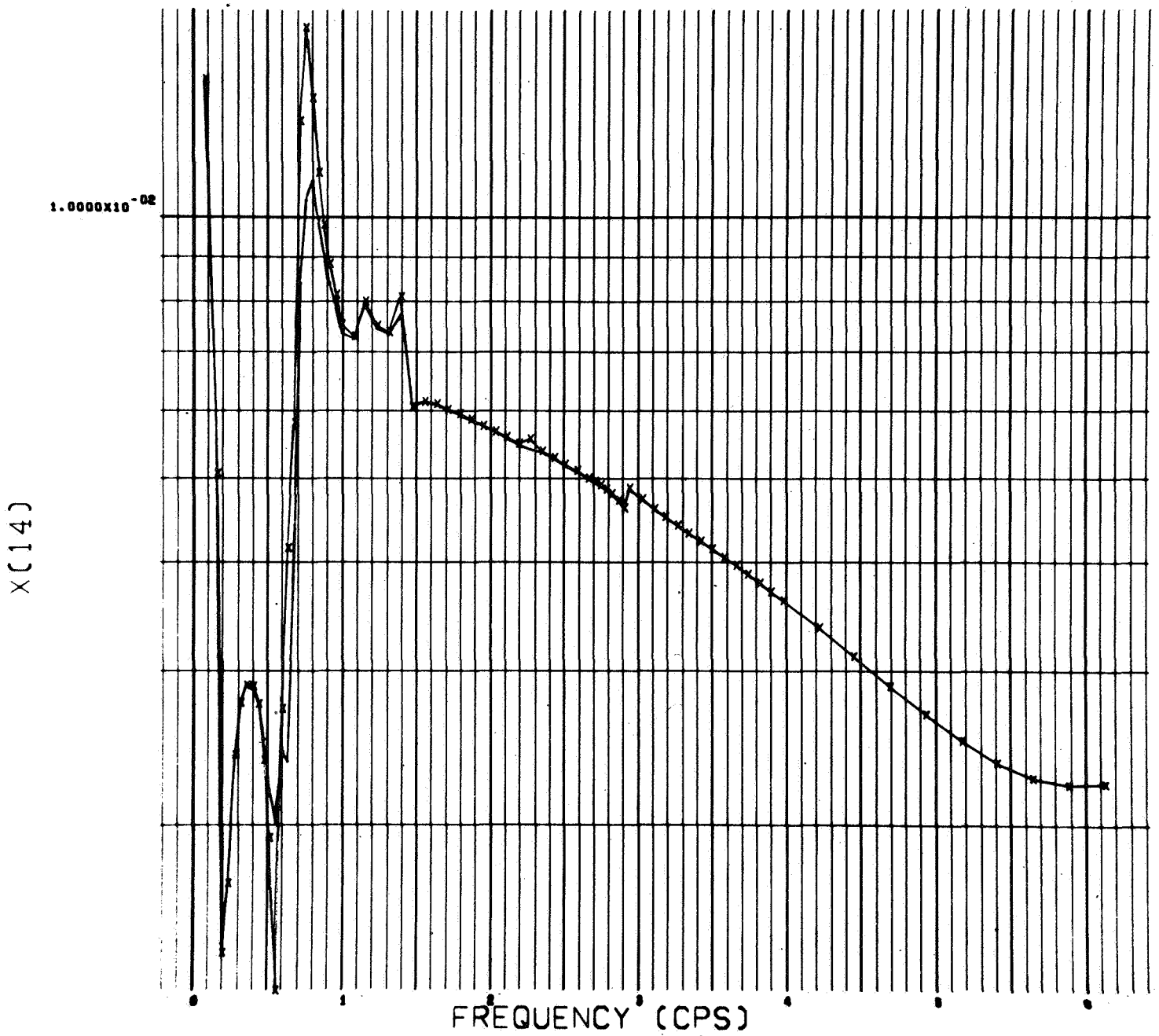
Figure 80b



Flight Test Data
X(14), t = 120 seconds

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

400370 L
077 077



t = 120 seconds

Figure 81a

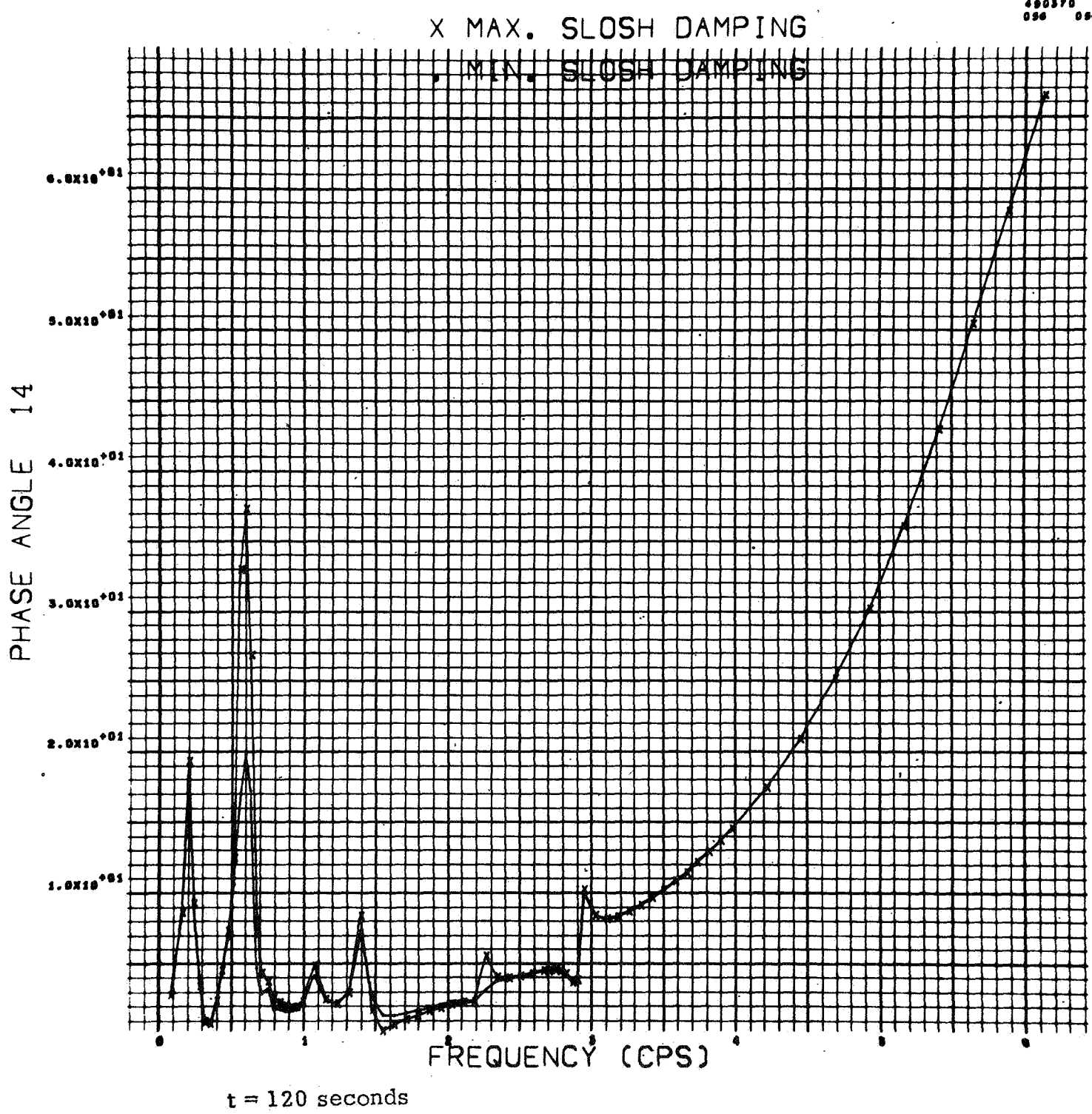
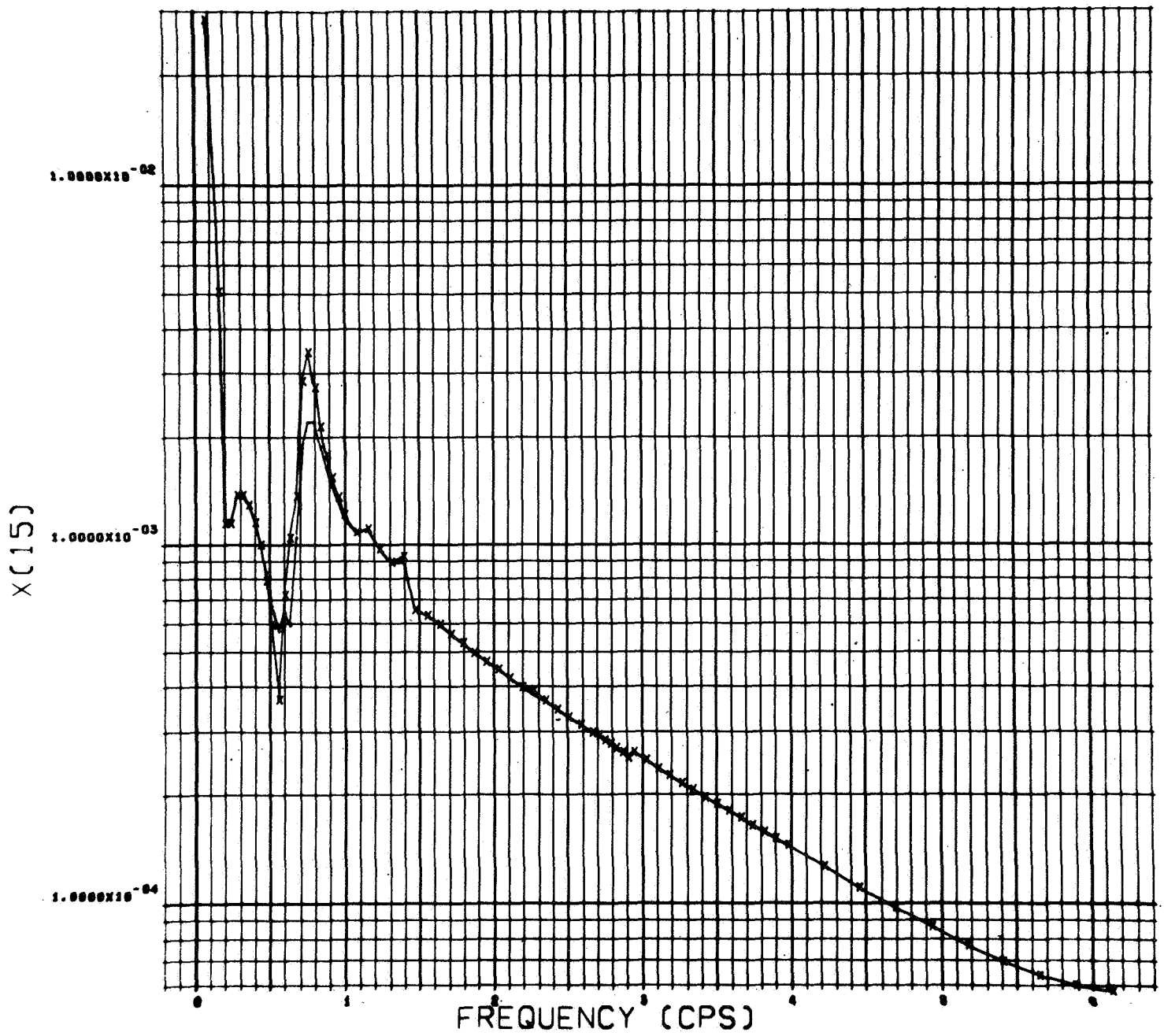


Figure 81b

X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING



t = 120 seconds

Figure 82a

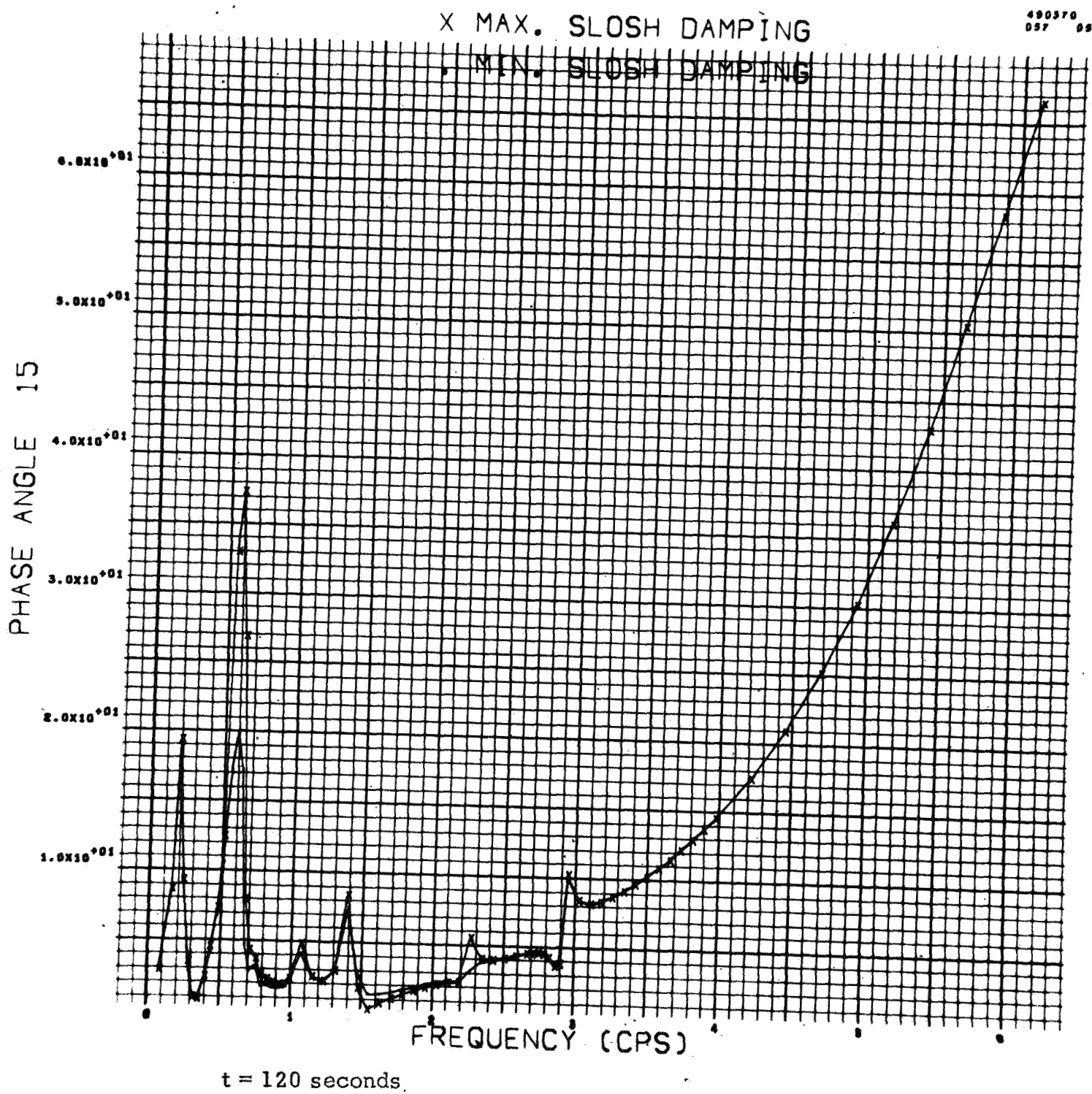
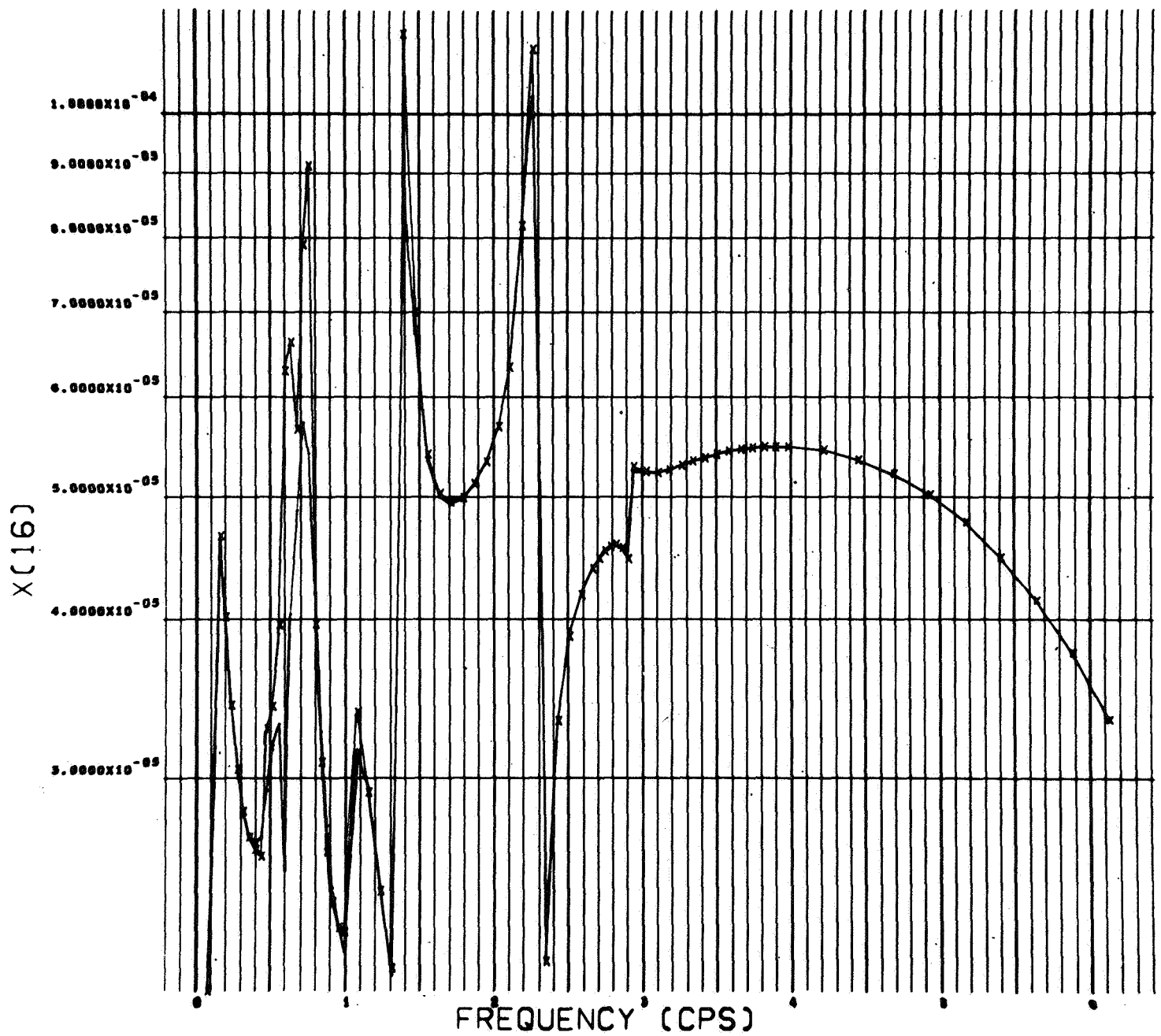


Figure 82b

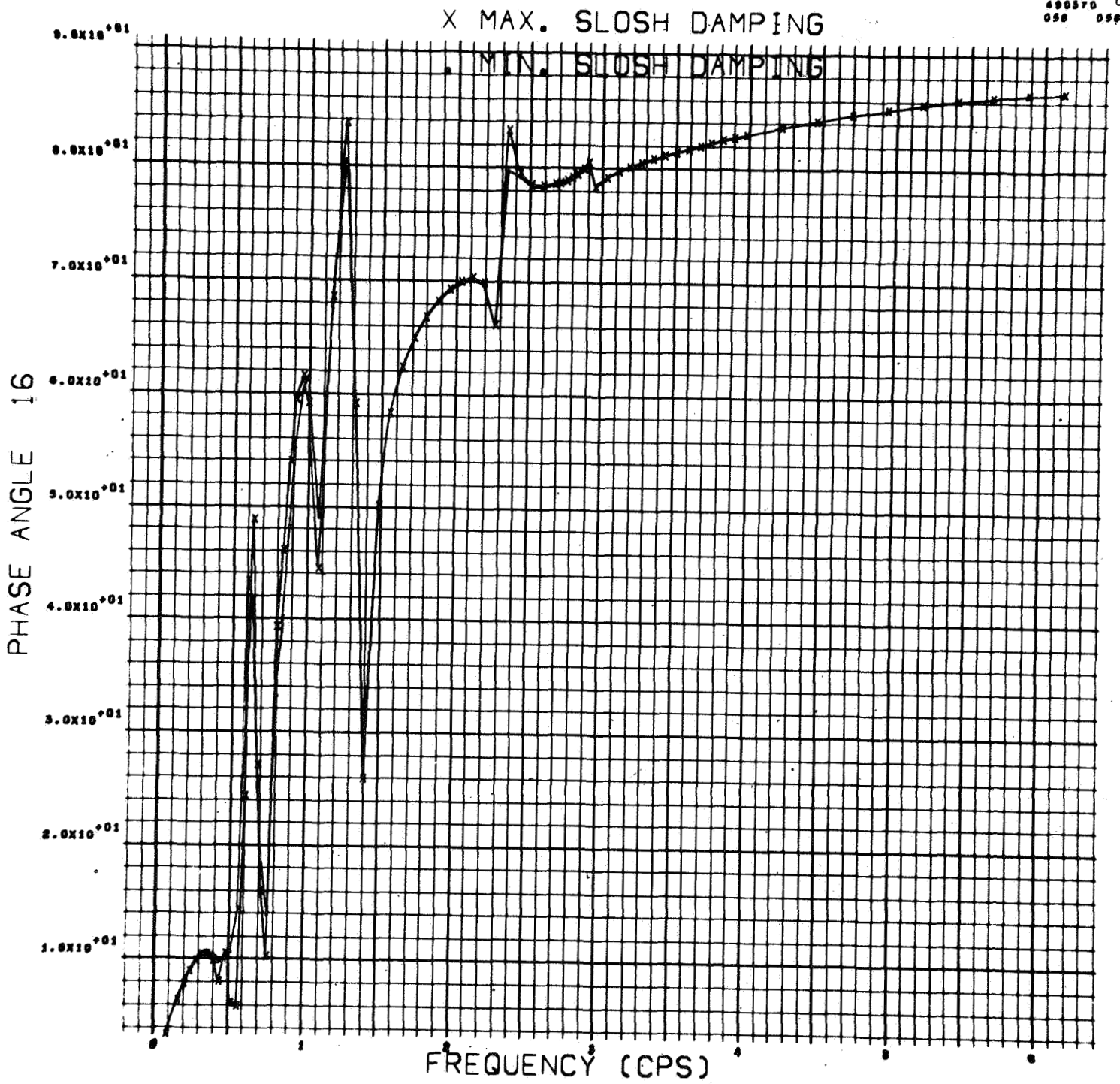
X MAX. SLOSH DAMPING
. MIN. SLOSH DAMPING

498878
079 071



t = 120 seconds

Figure 83a

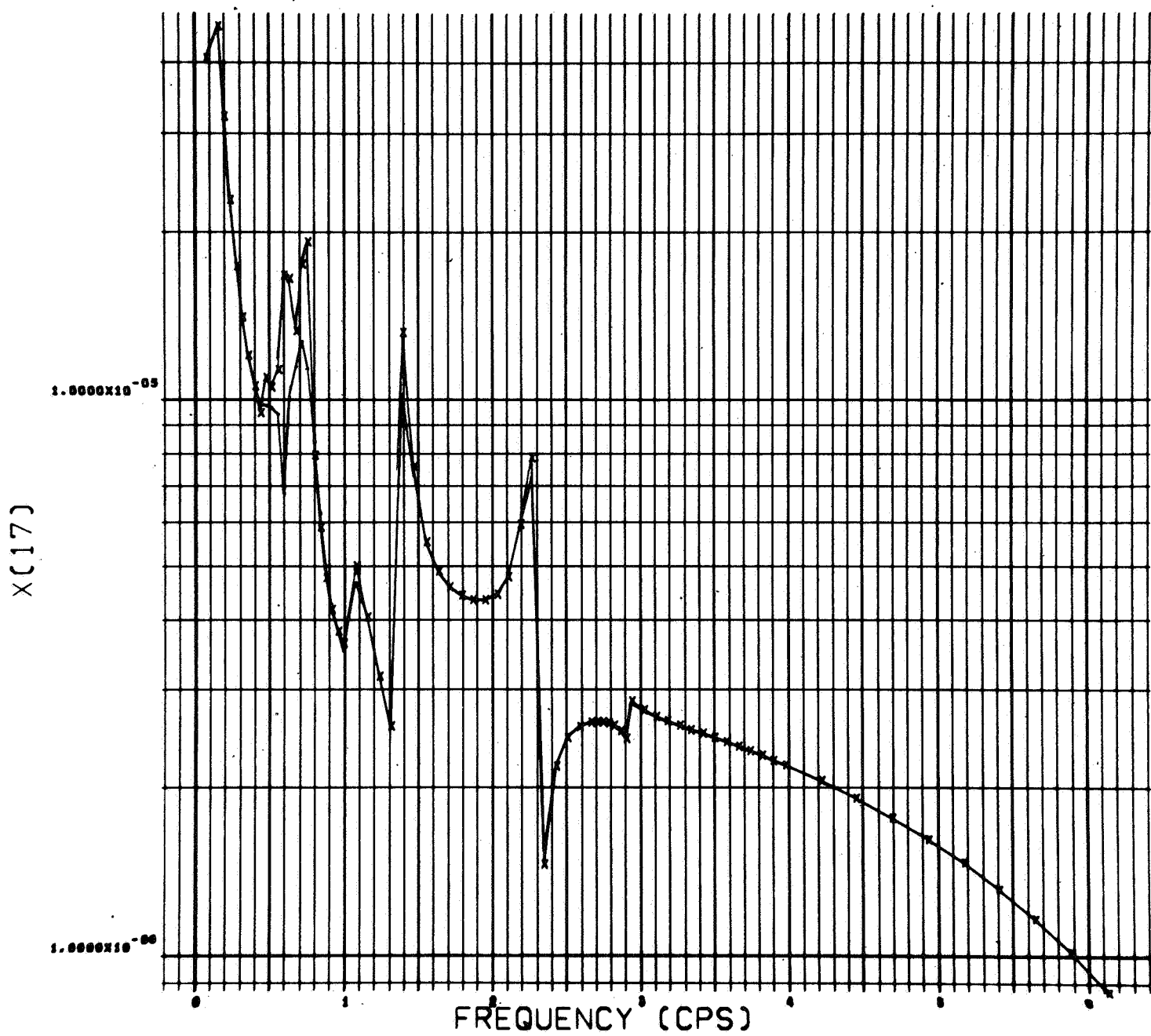


t = 120 seconds

Figure 83b.

X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

498378 L
000 000



t = 120 seconds

Figure 84a

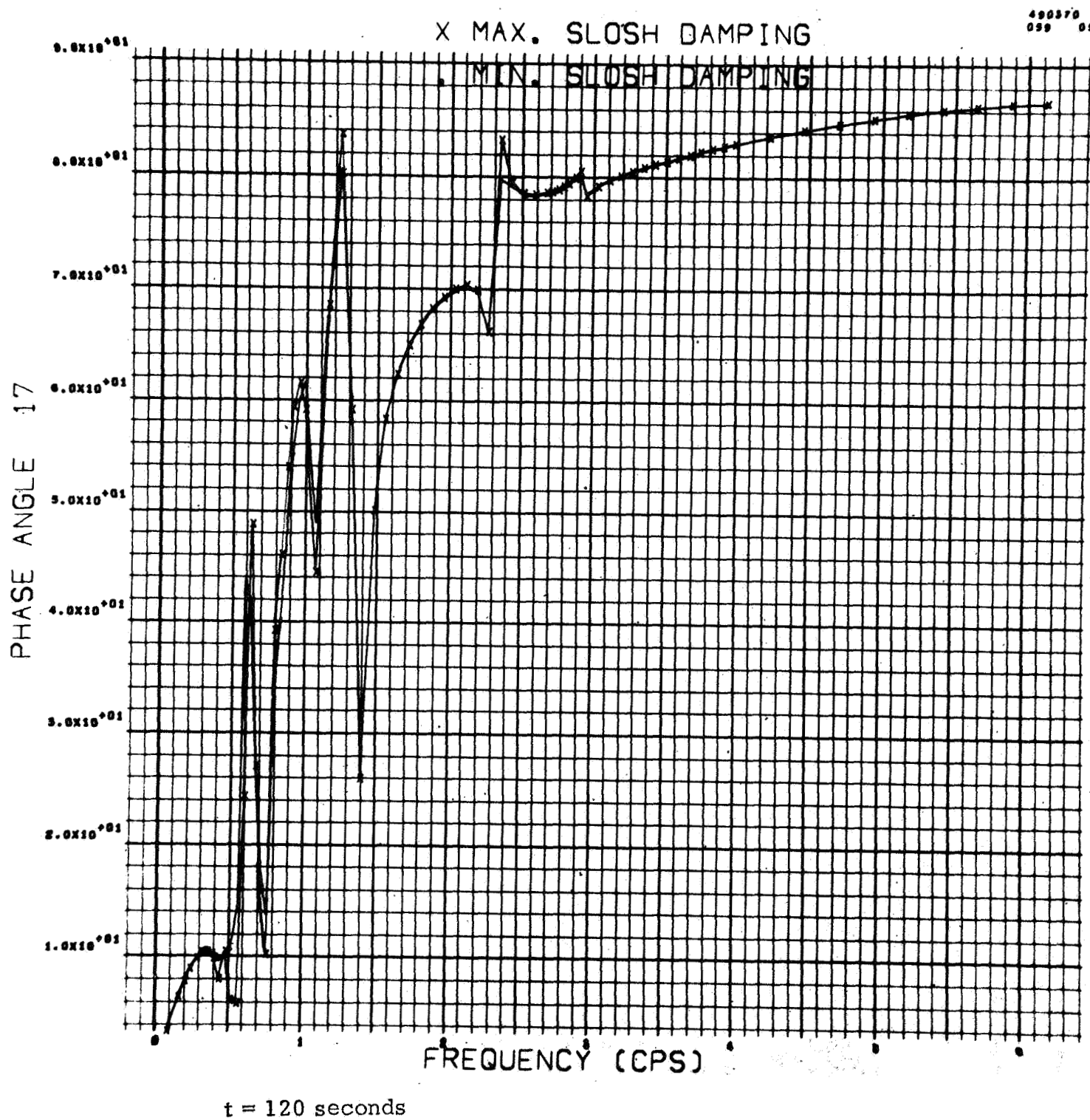
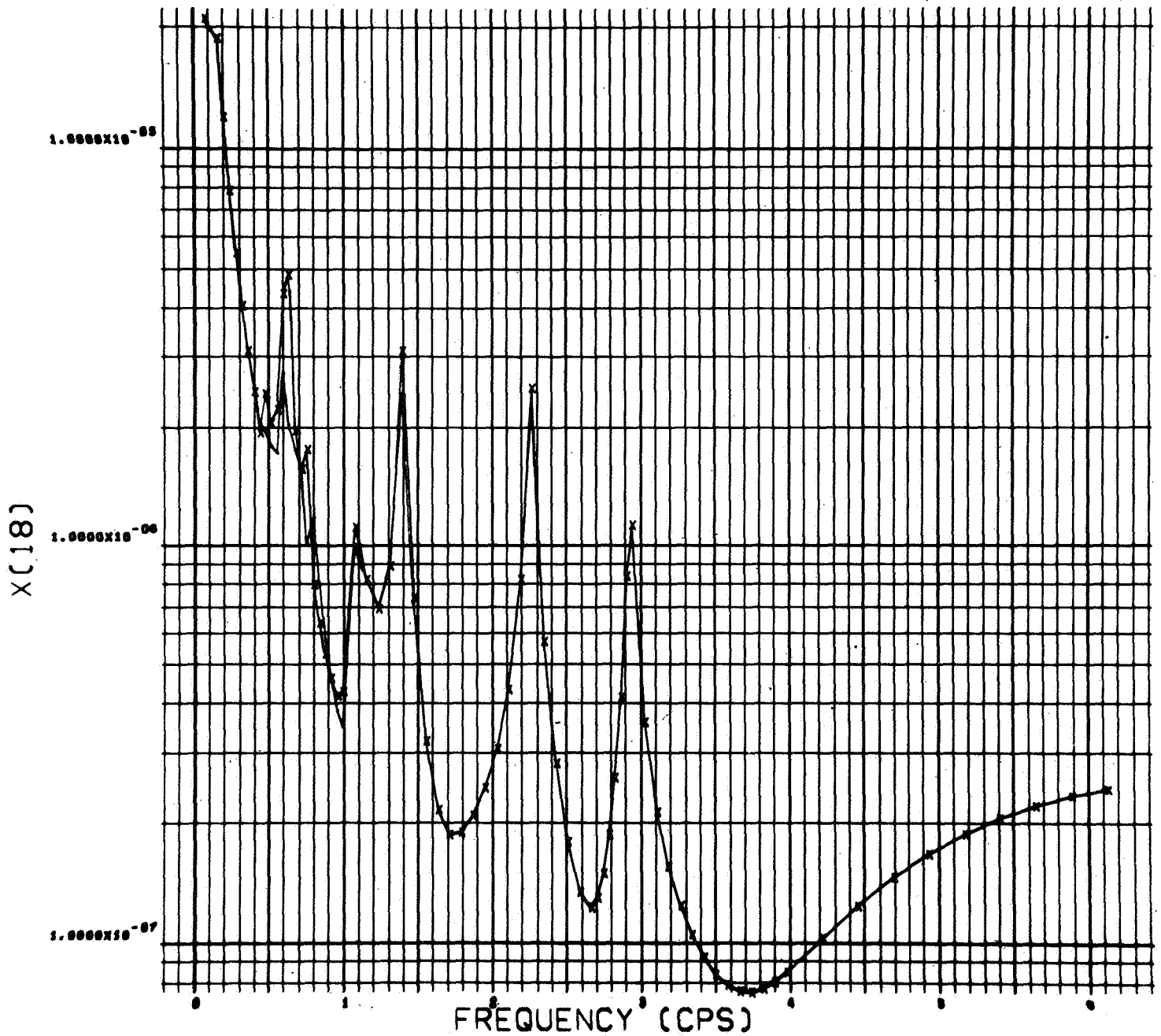


Figure 84b

Flight Test Data
X(18), t = 120 seconds

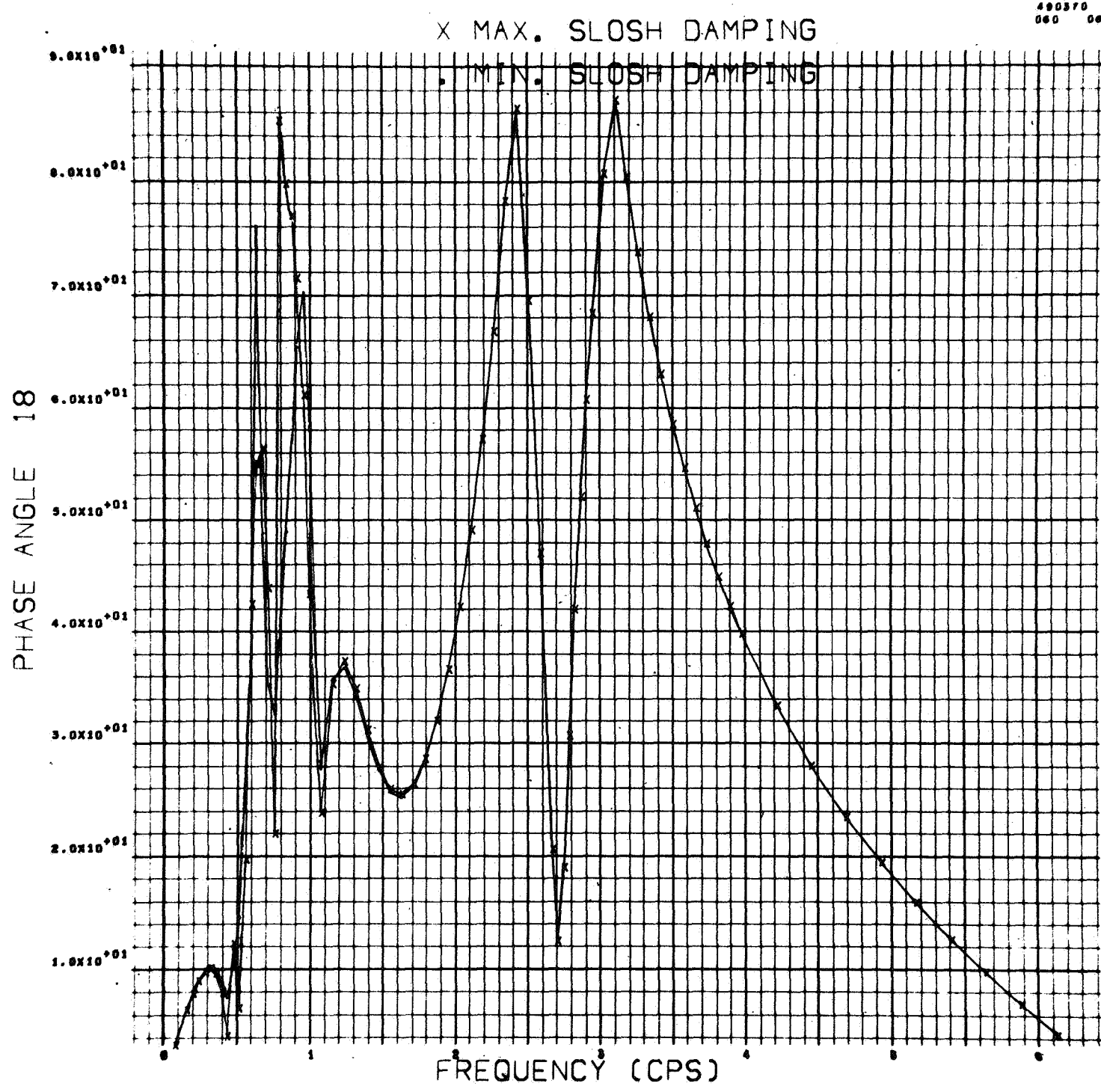
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

400370 L
001 001



t = 120 seconds

Figure 85a

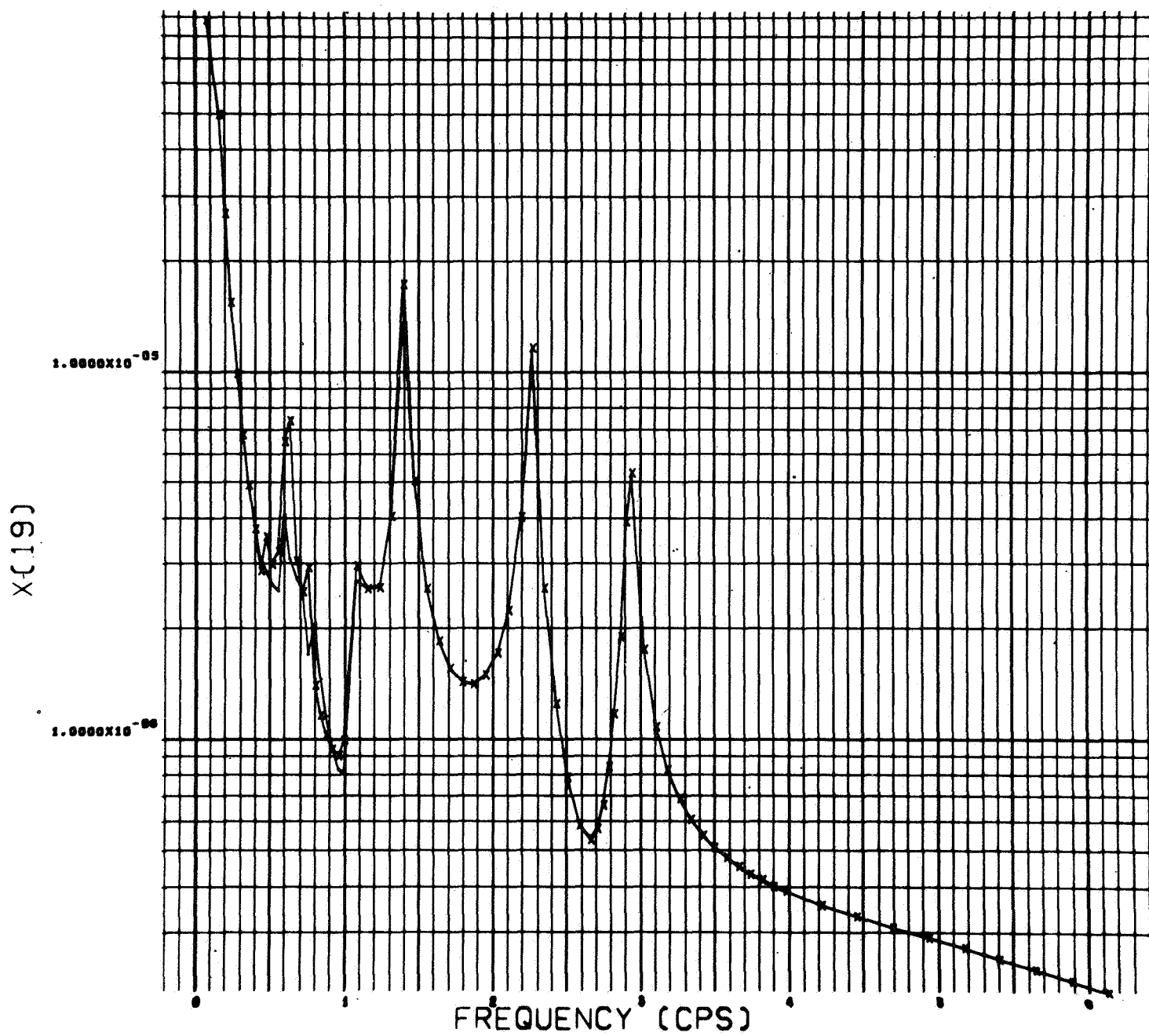


t = 120 seconds

Figure 85b

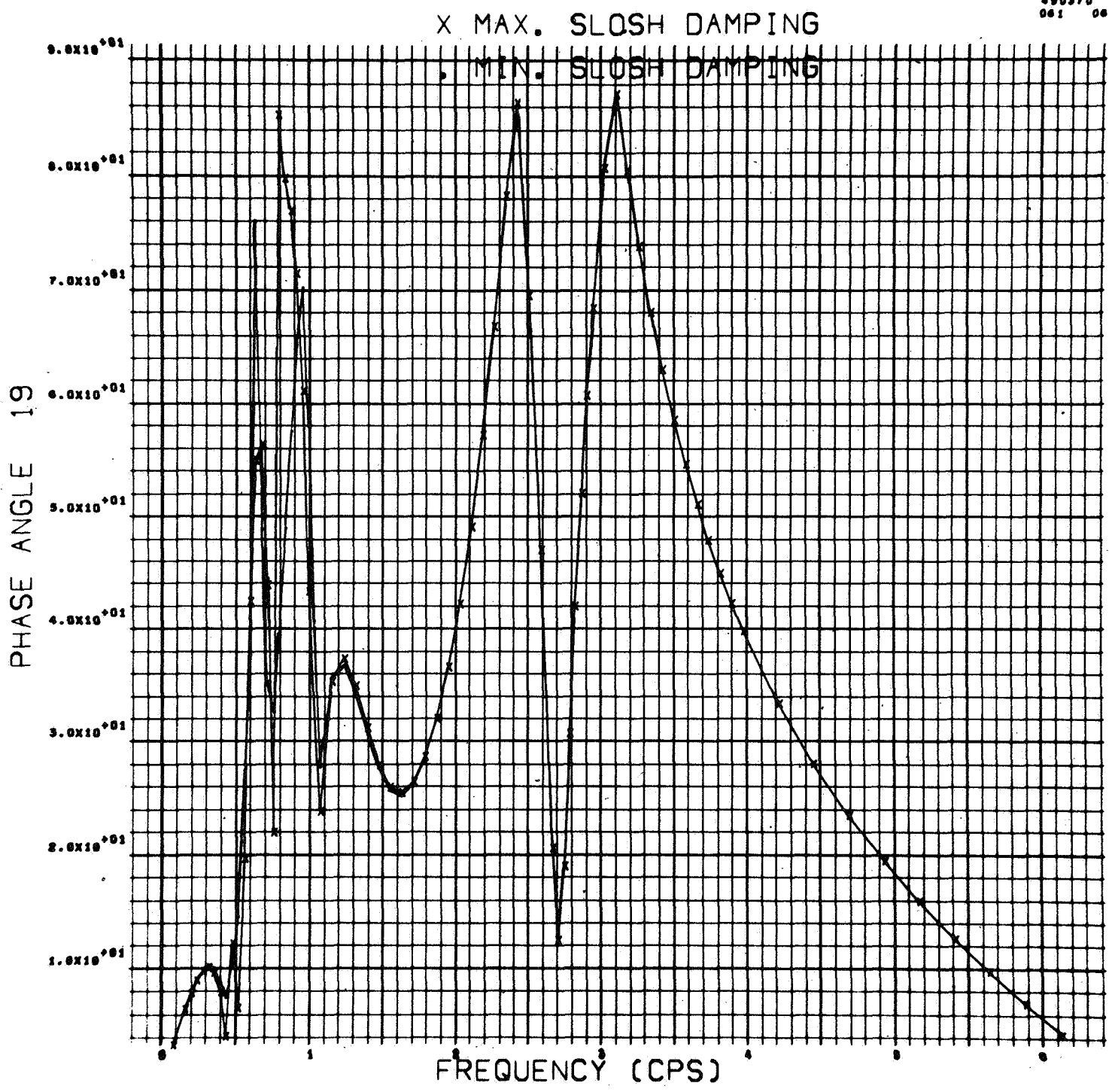
X MAX. SLOSH DAMPING
· MIN. SLOSH DAMPING

498576 1
002 002



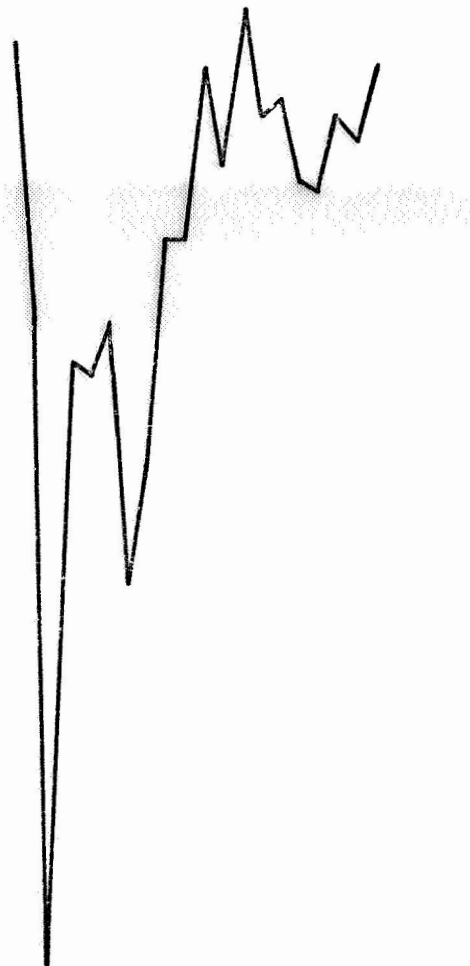
t = 120 seconds

Figure 86a



t = 120 seconds.

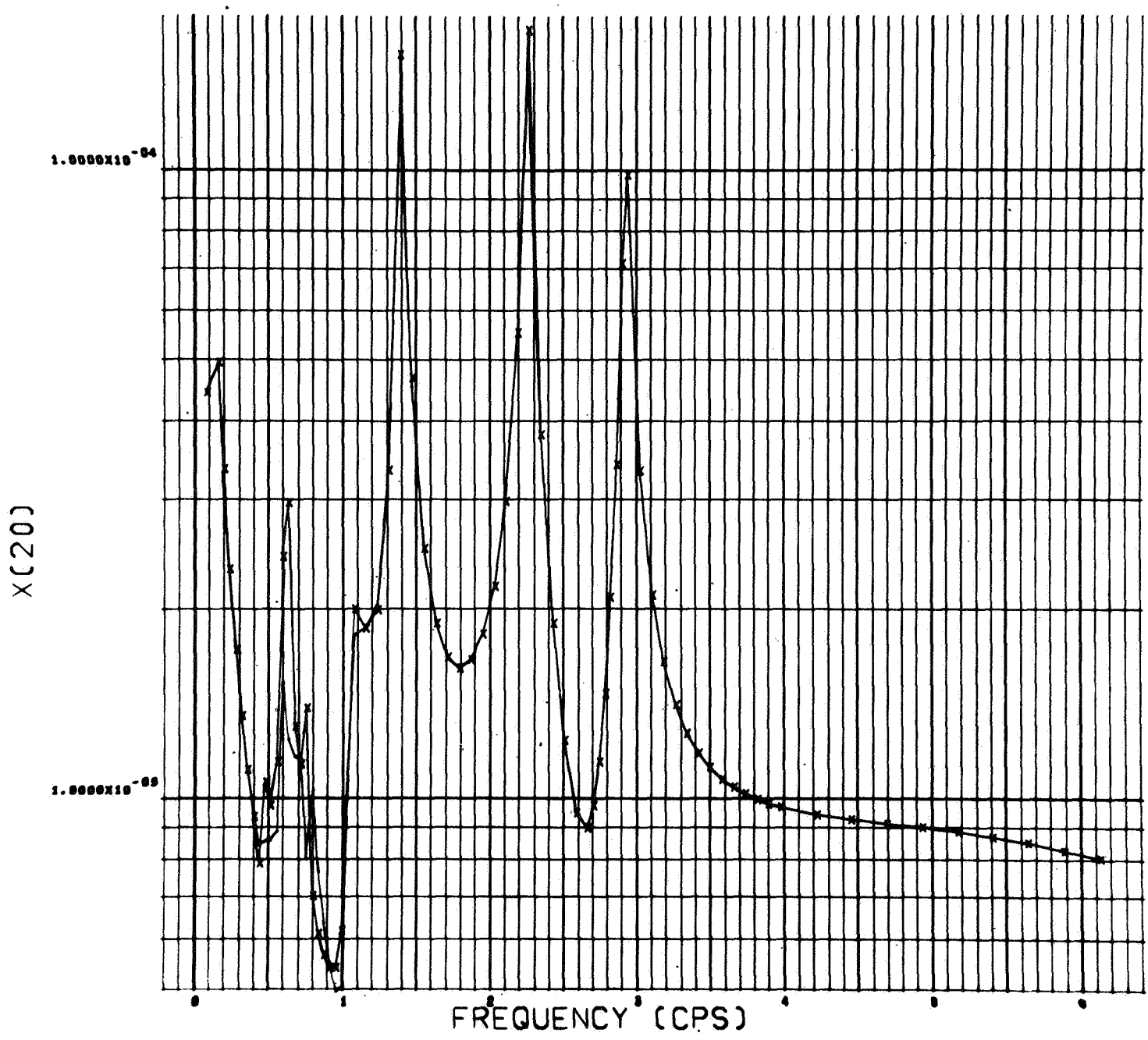
Figure 86b



Flight Test Data
 $X(20)$, $t = 120$ seconds

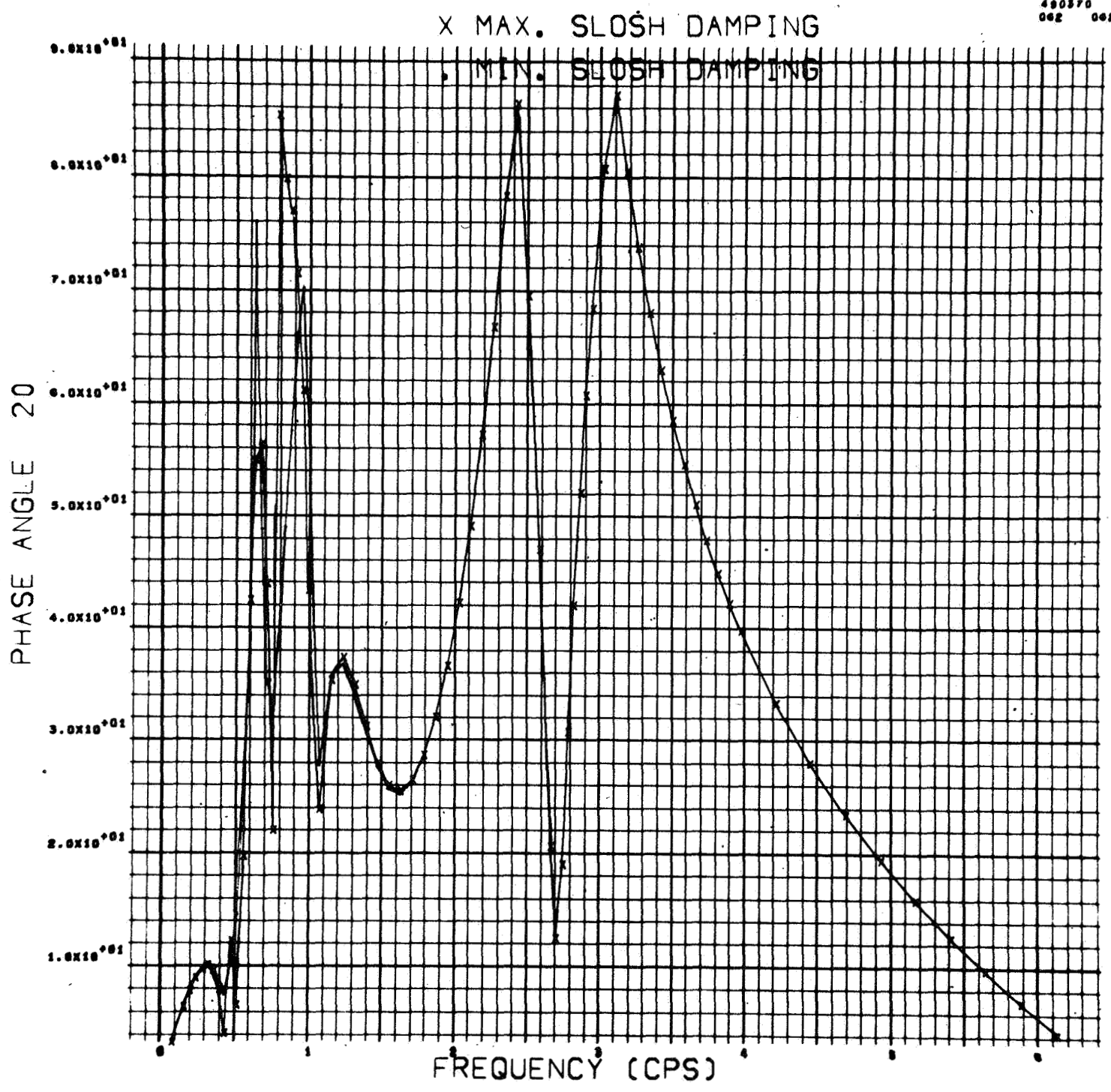
490370 L
003 003

X MAX. SLOSH DAMPING
• MIN. SLOSH DAMPING



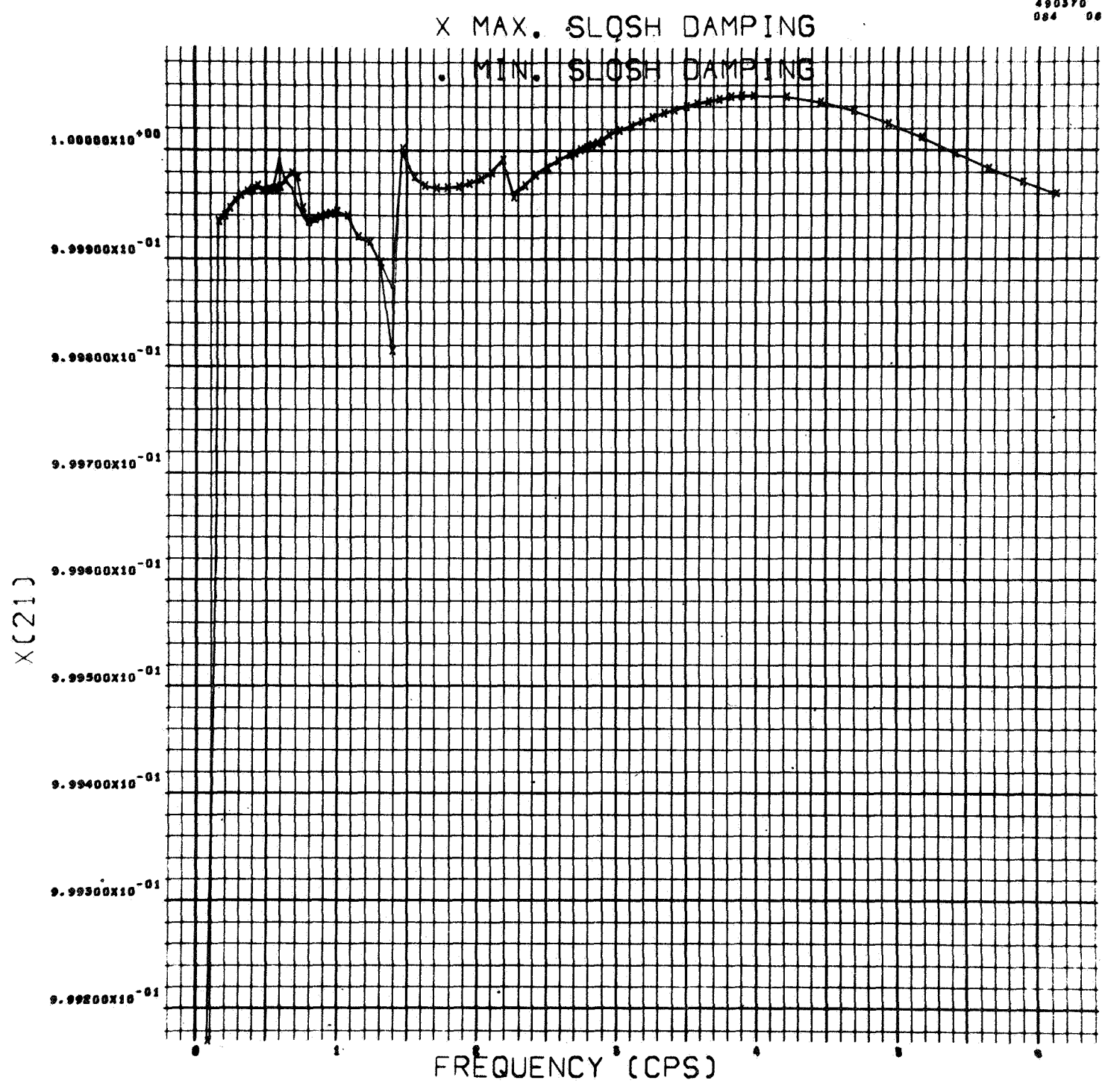
t = 120 seconds

Figure 87a



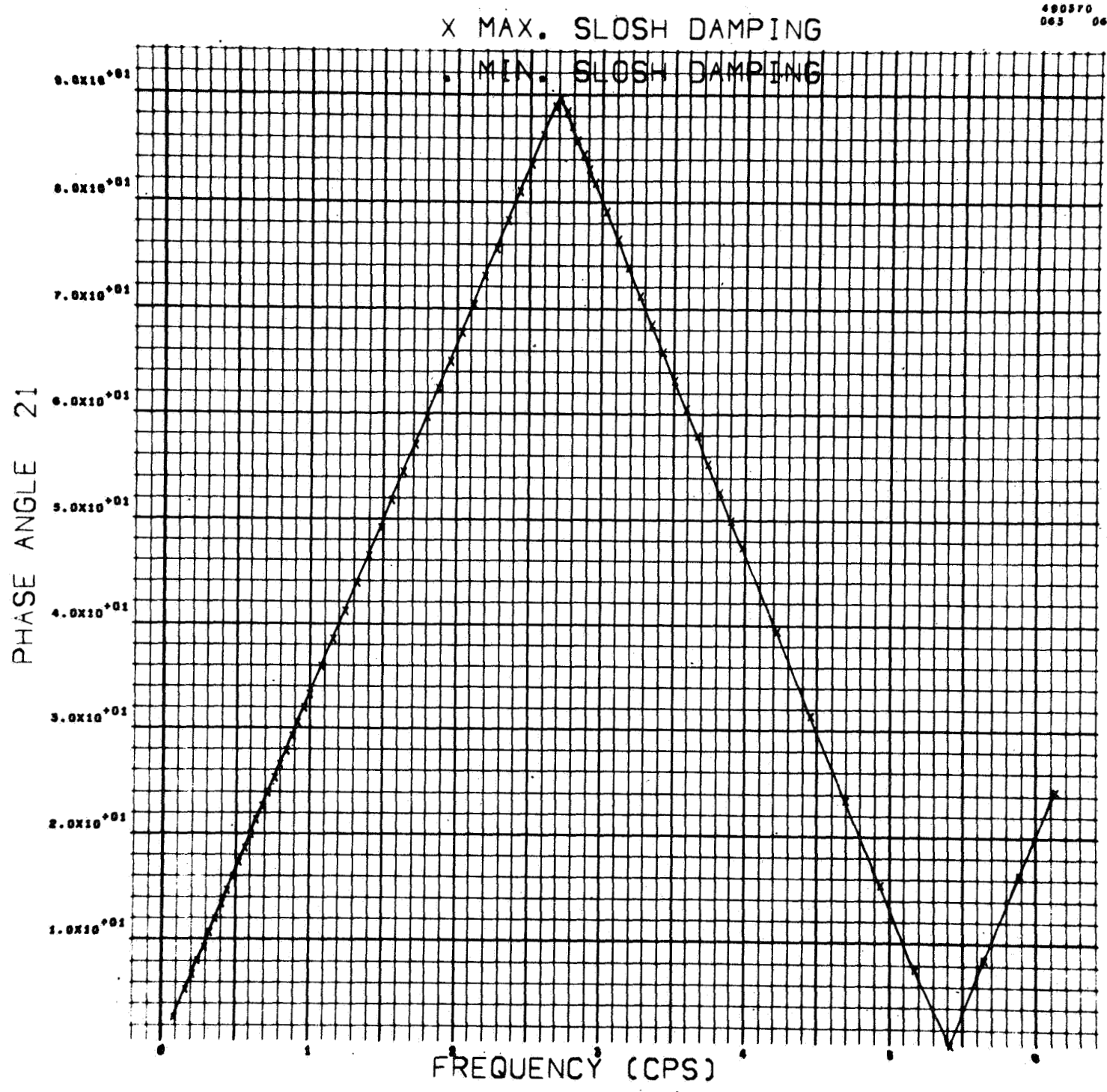
t = 120 seconds

Figure 87b



t = 120 seconds

Figure 88a



t = 120 seconds

Figure 88b