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**REPORT NO. 28**

**AUTOMATED OPTIMUM DESIGN OF STRUCTURES  
UNDER DYNAMIC RESPONSE RESTRICTIONS**

**BY**

**MAHESH P. KAPOOR**

**JANUARY 1969**

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Report No. 28

by

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SOLID MECHANICS,  
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## FOREWORD

This report is a reproduction of a thesis submitted to Case Western Reserve University in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

The study reported herein was carried out under the supervision of Professor R. L. Fox, as part of a continuing effort in the application of structural synthesis to aerospace vehicle structures.



Automated Optimum Design of Structures  
Under Dynamic Response Restrictions

by

Mahesh P. Kapoor

ABSTRACT

A method to use dynamics technology within the structural synthesis framework is presented. The object being the weight minimization of linear planar discrete structures consisting of straight one dimensional members. The restrictions imposed upon the dynamic behavior involve limitations on dynamic displacements, stresses, and the range in which the natural frequencies of the structure are allowed to fall. The dynamic response is assumed to be linear, undamped and the result of known externally applied exciting forces at the joints or the support point displacements. The shock spectral approach is used with the modal superposition method to obtain a conservative estimate of the response quantities (dynamic displacements and stresses) for the multi-degree-of-freedom systems. The eigensolution is obtained by minimizing the Rayleigh quotient employing the conjugate gradient method. This approach extends the important computer storage advantages of the conjugate gradient method by eliminating the need for assembled stiffness and mass matrices of the structure. The method of

feasible directions has been used to optimize the weight. This necessitated use of information regarding the dependence of the behavior quantities upon the design variables. These relations have been obtained in closed form, thus providing the normals to the constraint surfaces in the design space. Several applications are presented which demonstrate the feasibility for automated optimum design of structures where dynamic behavior is to be considered.

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## INTRODUCTION

The importance of dynamic response in current structural problems has prompted an increased attention towards the synthesis (automated optimum design) of structures for which the dynamic response will be a controlling criterion. The present work demonstrates the feasibility of using a dynamics technology within the structural synthesis framework. The restrictions imposed on the dynamic behavior involve limitations on dynamic displacements, stresses, and the range in which the natural frequencies of the structure will be allowed to fall.

As a first phase, the automated minimum weight design of the general planar truss-frame system has been attempted. The dynamic response is assumed to be linear, undamped and the result of known externally applied exciting forces at the joints or support point displacements.

The structural design problems that arise within several of the more common design philosophies take the form of mathematical programming problems. The design criterion used in the present work is described in Chapter 1 and the problem of minimum weight design of general planar truss-frame system has been cast as a mathematical programming problem.

The modeling of the structure is the topic of Chapter 2, where it is shown that a general planar beam element should be used to model the planar truss-frame structure for the discrete

element dynamic analysis.

Usually a dynamic analysis is considered solved if the time response of the system is obtained. However, in most cases, only certain aspects of the response history are significant to the design problem. For example, certain maxima of the response quantities may be of interest for the design processes. Moreover, in a systematic method of synthesis the judgements regarding the quantitative maxima of the response quantities are to be formalized into explicit decisions. These are described in Chapter 3 to obtain the upper bound displacement and stresses occurring on the structure due to any deterministic forcing function.

A minimization method is described in Chapter 4 to obtain the natural modes of vibration of the structure and their associated natural frequencies. This is an iterative method which can be applied directly to a general eigenproblem arising in structural dynamics without any preliminary modification to formulate it to a special form. Frequently, in applying the discrete element analysis to the study of the vibration characteristics of a structure, the order of the stiffness and mass matrices is so high that it is impractical or prohibitively expensive to obtain the complete eigensolution. On the other hand, to carry out a reasonably accurate dynamic analysis of the structure, it is often possible to get along with only a partial eigensolution. It is this class of problems for which the method described in Chapter 4 is especially useful.

To accomplish synthesis we must ultimately utilize the information regarding the dependence of the behavior quantities upon the design variables. Thus, useful information is contained in the quantities representing the rates of change of eigenvalue (square of the natural frequency) and the eigenvector (natural mode of vibration) with respect to the design parameters. Exact expressions for these quantities are derived in Chapter 5.

In the computer design methods (for example, the method of feasible directions), there is a fundamental requirement for information relating changes in the dynamic response quantities to changes in the design variables, in order to satisfy continuously in the design space the limitations on the upper bound displacement and upper bound stress at any point on the structure. In Chapter 6, the expressions for the rates of change of the upper bound response quantities (displacements and stresses) with respect to design parameters are derived.

Chapter 7 deals with the optimization technique. Reasons for using the method of feasible directions are discussed in this chapter and the method has been described in some detail. Chapter 8 extends the application of the analysis to the automated optimum design of the general planar truss-frame structures. Results for the numerical examples are given in Chapter 8.

As is often true with efforts of this kind, certain difficulties inherent to the subject matter were encountered and are discussed in Chapter 9. The need for rapid reanalysis of modified



trial designs has been recognized to design complex structures satisfying dynamic response restrictions under the structural synthesis framework. A method is indicated in this chapter to approximate the eigenvalue analysis of new designs. This chapter concludes with the recommendations for further research intended to extend the scope of structural synthesis under dynamic response restrictions.

Use of illustrate examples has been made to elucidate the ideas wherever necessary. All equations, figures and tables are numbered consecutively in each chapter [that is, Eq. (4.40) refers to the 40th equation of Chapter 4].

## CHAPTER 1

### FORMULATION OF THE OPTIMIZATION PROBLEM

When a means for predicting the behavior of any design within a particular design concept is available, limitations on the performance and other external constraints on the design can be stated, and an acceptance criterion can be established, it is possible to cast the design modification problem in the form of a mathematical programming problem.

A mathematical programming problem is one in which a multi-variable function  $f(\vec{D})$ , where  $\vec{D}$  is a  $n$  dimensional vector consisting of  $d_j$ ,  $j = 1, 2, \dots, n$ , is minimized (or maximized) subject to given constraints  $g_i(\vec{X}) \{ \leq, =, \geq \} b_i$ ,  $i = 1, 2, \dots, m \leq n$  where  $b_i$  are known constants. There are usually inequality constraints on some or all of  $d_j$ . The function  $f(\vec{D})$  is called the objective function and its choice is governed by the nature of the problem.

In this chapter, the design criteria used in the present work are described. The reasons for choosing the weight as the objective function has been discussed and the problem of minimum weight design of the truss-frame structure has been cast as a mathematical programming problem.

#### 1.2 Design Criteria

Characterization of a design philosophy involves many

considerations.<sup>1</sup> In the present work, the problem of designing the planar truss-frame system has been considered to be deterministic as against probability based. In other words, all the quantities involved in the analysis and the design of the structural system are considered to be deterministic. The adequate performance of the structural system has been sought by trying to avoid failure modes such as initial yielding and excessive deflection, and by striving to limit the first few natural frequencies of the structure to be within a specified range. The dynamic response is assumed to be linear, undamped, and the result of known externally applied exciting forces at the joints or the support point displacements.

The quantifiable portion of the analysis is based on the design criterion and accordingly the information regarding the natural frequencies of the structure and the upper bound on displacements and stresses at any point on the structure due to the given dynamic loading will be made available in subsequent chapters for use within the structural optimization procedure.

### 1.3 Acceptance Criterion or Objective Function

The nature of the structural design problem is such that there will usually be many designs that perform the specified functional purposes adequately. A basis for choice between alternate acceptable designs should be selected in order to cast

a structural design problem as a mathematical programming problem. The objective of structural design optimization is frequently taken to be the weight minimization. In addition to being readily quantifiable, weight is often the most important property in aerospace structural design applications. Structural weight saved can be converted directly into increased payload. The minimum weight of the truss-frame structure has been, therefore, considered as the objective.

#### 1.4 Optimization Problem

The design criteria and the objective function being known, the structural optimization problem for the truss-frame structure can be cast as a mathematical programming problem if we decide the shape of members and the design parameters. It is decided to use the tubular members of uniform cross section. This choice is the simplest and exhibits characteristics similar to those of other special shapes used in practical problems. In order to further simplify the problems, we will take only the mean diameters of the tubular members as design variables and keep the wall thickness constant. The choice of mean diameter of the members as design variables is based on the fact that it is the important parameter which governs both the area of cross section of the member and its moment of inertia.

Therefore, the structural optimization problem is:

Minimize

$$W(\vec{D}) = \sum_{j=1}^m \pi d_j t_j l_j \rho_j \quad (1.1)$$

subject to

$$\begin{aligned} |Y(\vec{D}, \vec{X}, t)| &\leq \gamma(u) \\ | \sigma(\vec{D}, \vec{X}, t) | &\leq \sigma(u) \quad t > 0 \end{aligned} \quad (1.2)$$

$$\lambda_i^{(L)} \leq \lambda_i \leq \lambda_i^{(u)} \quad i = 1, \dots, p \quad (1.3)$$

and

$$d_j^{(L)} \leq d_j \leq d_j^{(u)} \quad j = 1, \dots, m \quad (1.4)$$

where

$W$  represents the weight of the structure

$\vec{D}$  is the vector of design variables

$d_j$ ,  $j = 1, \dots, m$  is the mean diameter of the tubular member  $j$  of the truss-frame structure

$t_j$  is the wall thickness of the  $j^{\text{th}}$  member

$\rho_j$  is the weight density of the  $j^{\text{th}}$  member

$l_j$  is the length of the  $j^{\text{th}}$  member

$|Y(\vec{D}, \vec{X}, t)|$  represents the displacement at any point on the structure as a function of space variables  $\vec{X}$  and time  $t$

$\gamma(u)$  is the upper limit on the displacement

$|\sigma(\vec{D}, \vec{X}, t)|$  represents the stress at any point on the structure as a function of space variables  $\vec{X}$  and time  $t$

$\sigma^{(u)}$  is the upper limit on the stress, taken to be the yield point of the material. (In the present work, the material has been considered to have the same yield stress in tension as well as in compression)

$\lambda_i$  represents the  $i^{\text{th}}$  eigenvalue of the structure (eigenvalues are the square of natural frequencies)

$\lambda_i^{(u)}$  is the upper limit on the  $i^{\text{th}}$  eigenvalue  
 $\lambda_i^{(L)}$  is the lower limit on the  $i^{\text{th}}$  eigenvalue

The behavior quantities  $Y$ ,  $\sigma$ , and  $\lambda_i$  are dependent on the design variables and these constraint equations, Eqs. (1.2) and (1.3) are called "behavior constraints". The "side constraints", Eqs. (1.4), impose the limits on the size of the design variables.

The objective function, Eq. (1.1), is a linear function of the design variables. Side constraints, Eqs. (1.4), are also linear inequalities. However, the behavior constraints, Eqs. (1.2) and (1.3) are, in general, nonlinear and hence the mathematical programming problem formulated above is a non-linear programming problem. When the mean diameter of each tubular member is selected to be a design variable, then the design vector  $\vec{D}$  consists of  $m$  components  $d_j$ ,  $j = 1, \dots, m$ . Consider, for example, the simple

planar structure of Fig. 1.1 consisting of five tubular members. If the mean diameter of all the members are allowed to change independently in the design space, then the design variable vector is given as

$$\vec{D} \equiv \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix}$$

where  $d_1$  to  $d_5$  are respectively the mean diameter of members 1 to 5 marked on Fig. 1.1.

However, it is quite likely that several members of the structure are desired to be of the same diameter continuously in the design space. This imposes additional constraints on the optimization problem and can be handled rather easily as described below:

Consider that the structure of Fig. 1.1 is required to satisfy the condition

$$d_1 = d_3$$

and

$$d_4 = d_5$$

Therefore, the optimization problem has only three design

variables. Let these be designated as  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$ . Then the design variable vector is given as

$$\vec{D} = \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{pmatrix}$$

where

$$\delta_1 = d_1 = d_3$$

$$\delta_2 = d_2$$

and

$$\delta_3 = d_4 = d_5$$

We write for each design variable a horizontal array of numbers corresponding to the diameter of the members it governs as given below:

| Design Variable | Member | Numbers |
|-----------------|--------|---------|
| 1               | 1      | 3       |
| 2               | 2      | 0       |
| 3               | 4      | 5       |

Logical operations through the above tableau, henceforth called as Design Variable Correlation Table, enables us to obtain the design variable vector from the diameter of the members and



vice versa.

When the number of design variables are less than the number of members, the side constraint inequalities, Eqs. (1.4), modify to

$$\delta_j^{(L)} \leq \delta_j \leq \delta_j^{(u)} \quad , j = 1, \dots, r < m$$

The upper bound on the design variables is governed by the overall dimensions of the structure, while the lower bound on the design variable is governed by the wall thickness so that the structure is comprised of thin-wall tubes.

The involvement of time parameter in the behavior variables  $Y$  and  $\sigma$ , Eqs. (1.2), adds another dimension of difficulty to the design problem. Not only must the solution of a system of differential equations be obtained but since the behavior so determined will be a function of time, all limitations imposed on it must be satisfied continuously in time. In order to obtain a solution to the optimization problem through the commonly known algorithms of mathematical programming, the time parameter will be eliminated in the present work by estimating an upper bound to the behavior quantities (dynamic displacements and stresses). Let us designate  $|y_m(\vec{D}, \vec{X})|$  as upper bound displacement and  $|\sigma_m(\vec{D}, \vec{X})|$  as upper bound stress over the entire time period  $t$ ,  $t > 0$ , then we have behavior constraints as

$$|y_m(\vec{D}, \vec{X})| \leq y^{(u)}$$

and

$$\left| \sigma_m(\vec{D}, \vec{X}) \right| \leq \sigma(u)$$

A method to handle the behavior constraint inequalities in the form of Eqs. (1.2) is briefly indicated in Chapter 9. The reasons for avoiding its use in the present work are indicated in Chapter 7.

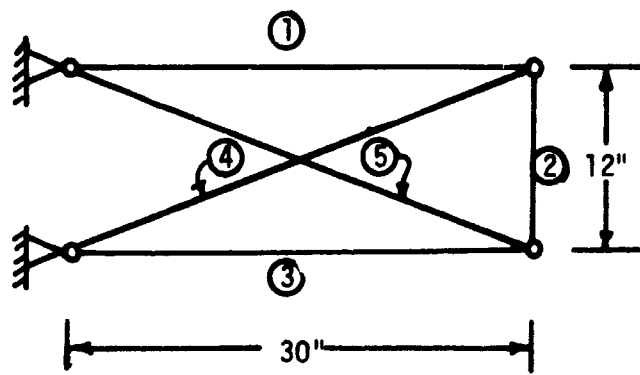


Fig. 1.1 Planar Structure

## CHAPTER 2

### MODELING OF THE STRUCTURE

A general planar truss-frame structure is idealized as a system of straight members lying in a plane and interconnected at joints. The members have their axes of symmetry in the plane and the translation of the members are in the plane of the structure. All applied forces (static or dynamic) are assumed to act in the plane of the structure. The static analysis of a structure having hinged joints when subjected only to the joint loads results in axial forces of tension or compression in the members. However, it is expected that the dynamic analysis of a similar structure with the dynamic loads at the joints or support point displacements may result in general in a bending couple, a shearing force and an axial force in the plane of the structure at any cross section of its members. A simple study confirms this surmise and is discussed below:

A planar truss-frame structure consisting of the tubular members is shown in Fig. (2.1). The discrete element method of structural analysis is used in the present work. The matrix formulation of the general structural dynamic response problem results in the equation

$$[M]\ddot{\vec{Y}} + [K]\dot{\vec{Y}} = \vec{F}(t) \quad (2.1)$$

where  $[M]$  and  $[K]$  are respectively the master mass matrix and

the master stiffness matrix of the structure and their order  $n$  corresponds to the elastic degrees of freedom of the system. The vectors,  $\vec{Y}$ ,  $\ddot{\vec{Y}}$ , and  $F(t)$  represent the absolute displacement, acceleration and load respectively. A frequent intermediate step in the dynamic analysis is the solution of the eigenproblem

$$[K]\vec{X} = \lambda[M]\vec{X} \quad (2.2)$$

which requires the determination of the scalar quantities  $\lambda$  (eigenvalues) and the corresponding nontrivial solutions  $\vec{X}$  (eigenvectors) for the given  $n \times n$  matrices  $[K]$  and  $[M]$ . The matrices  $[K]$  and  $[M]$  are assumed to result from the discrete element idealization of the actual structure.

The discrete element method of structural analysis consists of representing a structural system as an assembly of elements, generating a stiffness and a mass matrix of the element and assembling the element stiffness and mass matrices to obtain the  $[K]$  and  $[M]$  matrices of the structure. It is discussed in Chapter 4 that the need for the assembled stiffness and mass matrices of the structure can actually be eliminated, and the dynamic response problem can be solved by using (1) the element stiffness and mass matrices, (2) a definite method of eigensolution, and (3) some computational coding devices. However, for the purposes of this chapter, it is assumed that the solution is obtained by a conventional approach. The element stiffness and mass matrices are derived from the local assumed displacement states (usually

polynomials) and they are arranged, for convenience, so that the undetermined coefficients correspond to the physical degrees of freedom at certain points (usually the ends) of the element. Thus, the discrete element analysis may be thought of as an assumed mode analysis in which the assumed modes, instead of having a uniform definition over the entire structure, have a local definition for each element producing a sort patchwork assumed mode.

It has been pointed out in Ref. 2 that in selecting the assumed displacement states on which to base the element stiffness matrix derivation, it is necessary to require, not only that displacements satisfy compatibility within the subelement (intraelement compatibility) but also it be possible to insure overall compatibility for an assemblage of elements. By using the Hermite interpolation formula (see Appendix I), it is possible to generate displacement state having the property that the displacements at an end depend only on the degrees of freedom at that end. This facilitates the insuring of interelement compatibility. Moreover, since the displacement state generated by the use of Hermite interpolation formula is a continuous function, the intraelement compatibility is also satisfied.

Furthermore, in doing natural frequency studies of structures using discrete element analysis, it is extremely advantageous to use mass matrices consistent with the assumed modes of the element. This point has been well made in Ref. (3) where it is shown that a consistent mass matrix provides the exact inertial force

associated with each assumed degree of freedom rather than some arbitrary mass lumping. When compatible elements are used in conjunction with their consistent mass matrices to predict the natural frequencies of a structure, the frequencies so obtained will always be upper bounds to the lowest frequency. Moreover, these frequencies are usually quite accurate even with relatively crude modeling of the structure.

It is for these reasons that element stiffness matrix and the companion consistent mass matrix for straight elements will be derived using the assumed modes obtained from the Hermite interpolation formula. The concept of Hermite polynomials is well documented in the literature<sup>4,5</sup>, however, for the sake of completeness, it is described in some detail in Appendix I.

The structure of Fig. 2.1 will be called a "planar truss" when it is modeled of the "planar truss element", Fig. 2.2, and will be termed as a "planar frame" only for the purposes of this chapter when a "general planar beam element", Fig. 2.4, is used to model it.

## 2.2 Derivation of element stiffness and mass matrices

### (a) Planar Truss Element

A planar truss element is shown in Fig. 2.2 and has four degrees of freedom. It provides elastic resistance only to the longitudinal elongation or contraction and obeys the laws of linear elasticity under deformation. Furthermore, in certain modes of vibration of the structure, the element undergoes a

rigid body motion. Thus, any point on the element in general undergoes an axial deformation and a displacement normal to the axis of the element. A geometrically admissible assumed mode for the planar truss element satisfies: (1) the imposed displacement boundary conditions, and (2) the displacement continuity within and between adjacent elements which can be expressed as:

$$u(x) = u_1 H_{01}^{(0)}(x) + u_2 H_{02}^{(0)}(x) \quad (2.3)$$

and

$$w(x) = u_3 H_{01}^{(0)}(x) + u_4 H_{02}^{(0)}(x) \quad (2.4)$$

where  $u(x)$  is the displacement function along the axis of the element,  $u_1$  and  $u_2$  are the longitudinal displacements of the element at its two ends,  $w(x)$  is the displacement function normal to the axis of the element,  $u_3$  and  $u_4$  are the displacements normal to the axis of the element at its ends,  $x$  is measured along the length of the element and its origin is taken to be on the left end marked '1' on Fig. 2.2.  $H_{01}^{(0)}(x)$  and  $H_{02}^{(0)}(x)$  is a pair of Hermite polynomials derived in Appendix I and if  $\ell$  is the length of the element, these polynomials are

$$\begin{aligned} H_{01}^{(0)}(x) &= -\frac{(x-\ell)}{\ell} \\ H_{02}^{(0)}(x) &= \frac{x}{\ell} \end{aligned} \quad (2.5)$$



These are called Lagrange interpolation formulas and are sketched in Fig. I.1 of Appendix I.

Substituting the expressions of Eq. (2.5) into Eqs. (2.3) and (2.4) we obtain

$$u(x) = u_1 + \frac{u_2 - u_1}{\ell} x \quad (2.6)$$

and

$$w(x) = u_3 + \frac{u_4 - u_3}{\ell} x \quad (2.7)$$

The element stiffness and mass matrices of the planar truss element are derived from the assumed modes of Eqs. (2.6) and (2.7).

#### Element stiffness matrix:

We want to express the potential energy of the element in the form

$$\pi_p^{(E)} = \frac{1}{2} \vec{u}^T [k] \vec{u} \quad (2.8)$$

where  $[k]$  is the stiffness matrix of the element and  $\vec{u}$  is the generalized displacement vector of the element in the element coordinate system which is given as:

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \quad (2.9)$$

The exact potential energy of the element is given by

$$\pi_p^{(E)} = \frac{1}{2} \int_0^l AE \left( \frac{\partial u}{\partial x} \right)^2 dx + \frac{1}{2} \int_0^l EI \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dx \quad (2.10)$$

where  $A$  is the cross sectional area of the element,  $I$  is its moment of inertia and  $E$  is the Young's modulus of elasticity of its material. Taking  $A$  and  $E$  to be constant and using the expressions  $u(x)$  and  $w(x)$  of Eqs. (2.6) and (2.7) we obtain from Eq. (2.10)

$$\pi_p^{(E)} = \frac{1}{2} AE \int_0^l \left( \frac{u_2 - u_1}{l} \right)^2 dx \quad (2.11)$$

Integrating the right hand side of Eq. (2.11) we obtain

$$\pi_p^{(E)} = \frac{1}{2} AE \frac{(u_1^2 - 2u_1u_2 + u_2^2)}{l} \quad (2.12)$$

which can be written in matrix form as

$$\pi_p^{(E)} = \frac{1}{2} (u_1 \ u_2) \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (2.13)$$

$$= \frac{1}{2} \vec{u}^T \frac{AE}{l} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vec{u} \quad (2.14)$$

Comparing Eqs. (2.14) and (2.8) we obtain the stiffness matrix of

the truss element as

$$[k]_{\text{truss}} = \frac{AE}{\ell} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.15)$$

where the value  $k_{ij}$  represents the elastic restraining force acting at the coordinate  $i$  associated with a unit displacement of coordinate  $j$ . Thus, this represents the stiffness coupling between coordinates  $i$  and  $j$  which explains as to why the last two columns and the last two rows of the element stiffness matrix, Eq. (2.15), are zero.

#### Element mass matrix:

Assuming the generalized displacements  $u_1, u_2, u_3$ , and  $u_4$  to be time dependent, we want to express the kinetic energy of the element in the form

$$T(E) = \frac{1}{2} \dot{\mathbf{u}}^T [m] \dot{\mathbf{u}} \quad (2.16)$$

where  $[m]$  is the element mass matrix and the vector  $\dot{\mathbf{u}}$  is given by

$$\dot{\mathbf{u}} = \begin{pmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \\ \dot{u}_4 \end{pmatrix} \quad (2.17)$$

where  $\dot{u}_i = \frac{\partial u_i}{\partial t}$

The exact kinetic energy of the element is given by

$$T(E) = \frac{1}{2} \int_0^{\ell} \rho A \left[ \left( \frac{\partial u}{\partial t} \right)^2 + \left( \frac{\partial w}{\partial t} \right)^2 \right] dx \quad (2.18)$$

Differentiating Eqs. (2.6) and (2.7) we obtain

$$\frac{\partial u}{\partial t} = \dot{u}_1 + \frac{\dot{u}_2 - \dot{u}_1}{\ell} x \quad (2.19)$$

and

$$\frac{\partial w}{\partial t} = \dot{u}_3 + \frac{\dot{u}_4 - \dot{u}_3}{\ell} x$$

Substituting Eqs. (2.19) into Eq. (2.18) and considering the cross sectional area of the element to be constant and the mass per unit length,  $\rho$ , to be distributed uniformly, we obtain

$$T(E) = \frac{1}{2} \rho A \int_0^{\ell} \left[ \left( \dot{u}_1 + \frac{\dot{u}_2 - \dot{u}_1}{\ell} x \right)^2 + \left( \dot{u}_3 + \frac{\dot{u}_4 - \dot{u}_3}{\ell} x \right)^2 \right] dx \quad (2.20)$$

Integrating the right hand side and simplifying the results we obtain

$$T(E) = \frac{1}{2} \rho A \ell \left[ \frac{1}{3} \dot{u}_1^2 + \frac{1}{3} \dot{u}_2 \dot{u}_1 + \frac{1}{3} \dot{u}_2^2 + \frac{1}{3} \dot{u}_3^2 + \frac{1}{3} \dot{u}_3 \dot{u}_4 + \frac{1}{3} \dot{u}_4^2 \right] \quad (2.21)$$

Writing Eq. (2.21) in the matrix form we obtain

$$T^{(E)} = \frac{1}{2} (\dot{u}_1 \ \dot{u}_2 \ \dot{u}_3 \ \dot{u}_4) \rho A \ell \begin{bmatrix} 1/3 & 1/6 & 0 & 0 \\ 1/6 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 1/6 \\ 0 & 0 & 1/6 & 1/3 \end{bmatrix} \begin{pmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \\ \dot{u}_4 \end{pmatrix} \quad (2.22)$$

Comparing Eqs. (2.22) and (2.16) we obtain the mass matrix of the truss element as

$$[m]_{\text{truss}} = \begin{bmatrix} 1/3 & 1/6 & 0 & 0 \\ 1/6 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 1/6 \\ 0 & 0 & 1/6 & 1/3 \end{bmatrix} \quad (2.23)$$

where the value  $m_{ij}$  represents the mass inertia force acting at the coordinate  $i$  associated with the unit acceleration of coordinate  $j$ . This quantity can be referred to as the mass coupling between coordinates  $i$  and  $j$ .

#### (b) General Planar Beam Element

A general planar beam element is shown in Fig. (2.4) and has six degrees of freedom. It provides elastic resistance to the axial elongation or contraction, to the shear force and to the bending couple at any cross section of the element. Let  $u_1, \dots, u_6$  be the deformations of the element at the degrees of

freedom defined at its ends. A geometrically admissible displacement function for this element is required to satisfy (1) the imposed displacement boundary conditions, (2) the displacement continuity within and between adjacent elements, and (3) continuous first derivatives of the transverse displacement within the elements and on the common ends of adjacent elements. Such a displacement state can be written as

$$u(x) = u_1 H_{01}^{(0)}(x) + u_2 H_{02}^{(0)}(x) \quad (2.24)$$

and

$$w(x) = u_3 H_{01}^{(1)}(x) + u_4 H_{02}^{(1)}(x) + u_5 H_{11}^{(1)}(x) + u_6 H_{12}^{(1)}(x) \quad (2.25)$$

where  $u(x)$  represents the axial deformation of the beam element and is similar to Eq. (2.3). The displacement function  $w(x)$ , Eq. (2.25), represents the transverse displacement of the beam element where the first two terms correspond to the transverse displacement induced by end displacements and the last two terms represent the transverse displacement due to end rotations.

The set of Hermite polynomials derived in Appendix I and sketched in Fig. I.2 are

$$H_{01}^{(0)}(x) = -\frac{(x - \ell)}{\ell}$$

$$H_{02}^{(0)}(x) = \frac{x}{\ell}$$

$$\begin{aligned}
 H_{01}^{(1)}(x) &= \frac{1}{\ell^3} (2x^3 - 3\ell x^2 + \ell^3) \\
 H_{02}^{(1)}(x) &= -\frac{1}{\ell^3} (2x^3 - 3\ell x^2) \\
 H_{11}^{(1)}(x) &= \frac{1}{\ell^2} (x^3 - 2\ell x^2 + \ell^2 x) \\
 H_{12}^{(1)}(x) &= \frac{1}{\ell^2} (x^3 - \ell x^2)
 \end{aligned} \tag{2.26}$$

where  $\ell$  is the length of the element and  $x$  is measured along the length of the element, origin being on the left end marked '1' on Fig. (2.4).

Using the displacement functions, Eqs. (2.24) and (2.25), and by following a treatment parallel to the one described in Section 2.2(a) of this chapter, we obtain the stiffness and mass matrices of the general planar beam element as:

$$[k]_{\text{beam}} = \frac{E}{\ell^3} \begin{bmatrix} A\ell^2 & -A\ell^2 & 0 & 0 & 0 & 0 \\ -A\ell^2 & A\ell^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12I & -12I & -6I\ell & -6I\ell \\ 0 & 0 & -12I & 12I & 6I\ell & 6I\ell \\ 0 & 0 & -6I\ell & 6I\ell & 4I\ell^2 & 2I\ell^2 \\ 0 & 0 & -6I\ell & 6I\ell & 2I\ell^2 & 4I\ell^2 \end{bmatrix} \tag{2.27}$$

$$[m]_{\text{beam}} = \frac{\rho A \ell}{42\eta} \begin{bmatrix} 140 & 70 & 0 & 0 & 0 & 0 \\ 70 & 140 & 0 & 0 & 0 & 0 \\ 0 & 0 & 156 & 54 & -22\ell & 13\ell \\ 0 & 0 & 54 & 156 & -13\ell & 22\ell \\ 0 & 0 & -22\ell & -13\ell & 4\ell^2 & -3\ell^2 \\ 0 & 0 & 13\ell & 22\ell & -3\ell^2 & 4\ell^2 \end{bmatrix} \quad (2.28)$$

### 2.3 Numerical Results:

If each member of the structure, Fig. (2.1) is modeled of one discrete element, the number of degrees of freedom of the truss, Fig. (2.3), is four and that of the frame, Fig. (2.5), is fourteen. The method to obtain the master stiffness and mass matrices of a structure from its element stiffness and mass matrices is well-documented in the literature<sup>6,7,8,9</sup>. Having known the master stiffness and mass matrices, the eigenvalues (square of the natural frequencies) and the associated eigenvectors (natural mode shapes of free vibration) of the truss and the frame were obtained by solving Eq. (2.2). The eigen-solution of these simple structures were obtained by the use of Householder transformation method. The first (lowest) natural frequency and its corresponding mode shape is given in Table 2.1. The moment of inertia of the members was increased by increasing their mean diameter, thus making them stiffer to bending resistance, while their area of cross section was maintained constant



by proportionately decreasing the wall thickness. With these stiffer members, the eigensolution of the truss would obviously remain unaffected but that of the frame would change. The first (lowest) natural frequency and the corresponding mode shape of the frame with members four, sixteen and sixty-four times stiffer in bending than the original structure are also tabulated in Table 2.1. Interestingly enough, it is observed that the dynamic behavior of the frame approaches to that of the truss as its members are made stiffer. In other words, the lowest natural frequency and the corresponding mode shape of the frame converges to that of the truss, with the components corresponding to the rotational degrees of freedom of the frame becoming very small, as the members of the frame are made stiffer. This is seen from Fig. 2.6 and Table 2.1.

This observation has lead to the conclusion that in order to perform the dynamic analysis of a planar truss-frame structure, it should be modeled using a "general planar beam element" such as that shown in Fig. 2.4. This modeling would obviously be suitable for a similar structure having rigid joints and for any general dynamic loading on the structure.

Another interesting observation was made regarding the interaction of the individual member frequency with the overall frequency of the structure. The long diagonal tubes of the structure of Fig. 2.1 having a length of 32.32 inches has an exact natural frequency of 89.0 cycles/sec. for the case of

$I = 0.0427 \text{ in}^4$  and considering the tube to be simply supported. However, for the assumed mode shape to be a parabola, the same member has a frequency of 98.7 cycles/sec. This figure closely resembles to the second lowest natural frequency (98.5 cycles/sec.) of the structure, Fig. 2.5, when all its members have a moment of inertia of  $I = 0.0427 \text{ in}^4$ . This reveals that the second natural frequency of the structure is predominately the lowest natural frequency of the long member. Similar observations were made on other frame structures whose members had the moment of inertia of  $4I$ ,  $16I$  and  $64I$ ,  $I = 0.0427 \text{ in}^4$  as shown in Table 2.2.

The second lowest natural frequency of the frame structure, as seen from Table 2.2, is slightly lower in each case, than the natural frequency of the corresponding long member considered simply supported. This is due to the fact that one end of the long member is on the elastic support as against being hinged.

#### 2.4 Conclusion

In order to perform the dynamic analysis of a planar truss-frame structure, it should be modeled using a general planar beam element such as that in Fig. 2.4, having six degrees-of-freedom.

| Type of Structure                           | Truss | Frame |     |     |     |
|---------------------------------------------|-------|-------|-----|-----|-----|
| Area of Cross Section<br>(in <sup>2</sup> ) | A*    | A     | A   | A   | A   |
| Moment of Inertia<br>(in <sup>4</sup> )     | I**   | I     | 4I  | 16I | 64I |
| Lowest Frequency<br>Cycles/Sec.             | 300   | 97.5  | 178 | 260 | 290 |

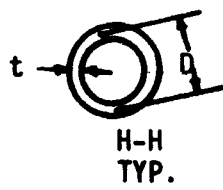
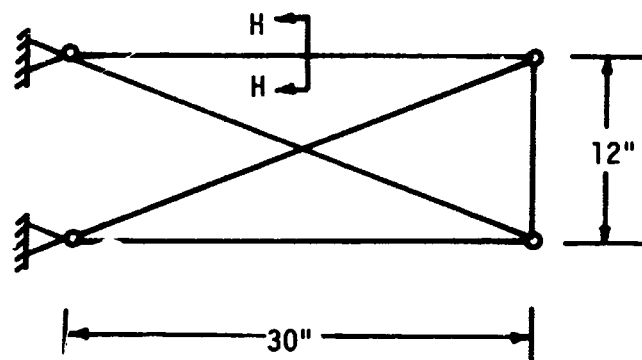
| Mode Shapes Corresponding to Lowest Frequency |        |        |        |        |        |
|-----------------------------------------------|--------|--------|--------|--------|--------|
| Degree of Freedom                             | Truss  | Frame  |        |        |        |
|                                               | I      | I      | 4I     | 16I    | 64I    |
| 1                                             | -0.184 | -0.191 | -0.188 | -0.185 | -0.184 |
| 2                                             | 1.0    | 1.0    | 1.0    | 1.0    | 1.0    |
| 3                                             | 0.184  | 0.191  | 0.188  | 0.185  | 0.184  |
| 4                                             | 1.0    | 1.0    | 1.0    | 1.0    | 1.0    |
| 5                                             | -----  | -0.222 | -0.175 | -0.078 | -0.043 |
| 6                                             | -----  | 0.162  | 0.116  | 0.012  | -0.023 |
| 7                                             | -----  | -0.031 | -0.031 | -0.031 | -0.031 |
| 8                                             | -----  | -0.032 | -0.031 | -0.031 | -0.031 |
| 9                                             | -----  | -0.228 | -0.175 | -0.075 | -0.043 |
| 10                                            | -----  | 0.172  | 0.116  | 0.012  | 0.023  |
| 11                                            | -----  | -2.518 | -0.485 | -0.094 | -0.043 |
| 12                                            | -----  | 2.465  | 0.433  | 0.037  | -0.017 |
| 13                                            | -----  | -2.596 | -0.486 | -0.094 | -0.043 |
| 14                                            | -----  | 2.545  | 0.433  | 0.037  | -0.017 |

Table 2.1 Lowest Frequency and Corresponding Mode Shape  
of Truss and Frame.

$$*A = \pi D t = 0.503 \text{ in}^2 \quad **I = \frac{\pi D t (n^2 + t^2)}{8} = 0.0427 \text{ in}^4$$

| Moment of Inertia<br>of<br>The Members | Second Lowest Natural<br>Frequency of the<br>Frame Structure<br>Cycles/Sec. | Lowest Natural Frequency<br>of the Long Member<br>Considered<br>Simply Supported<br>Cycles/Sec. |
|----------------------------------------|-----------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| I                                      | 98.5                                                                        | 98.7                                                                                            |
| 4I                                     | 192.0                                                                       | 197.4                                                                                           |
| 16I                                    | 382.0                                                                       | 394.8                                                                                           |
| 64I                                    | 754.0                                                                       | 789.6                                                                                           |

Table 2.2 Comparison of Second Lowest Natural Frequency of the Frame Structure and Lowest Natural Frequency of its Long Member Considered Simply Supported.



$$E = 30 \times 10^6 \text{ lbs/in.}^2$$

$$\rho = 0.28 \text{ lbs/in.}^3$$

$$A = \pi D t = 0.503 \text{ in.}^2$$

Fig. 2.1 Planar Structure



Fig. 2.2 Planar Truss Element

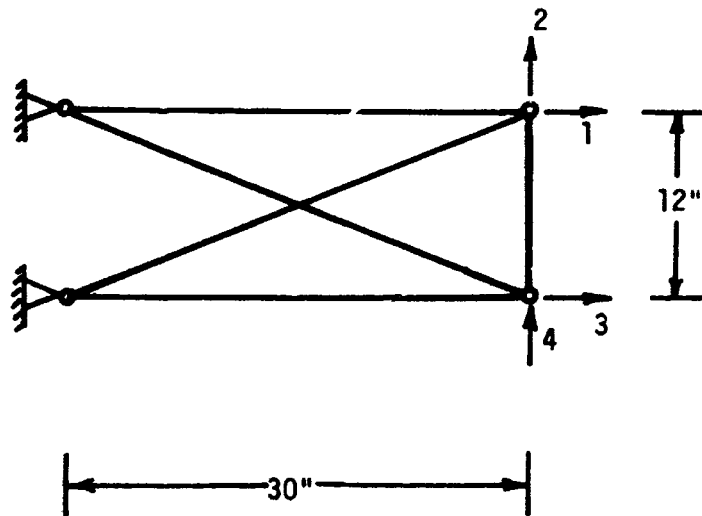


Fig. 2.3 Planar Truss; 4 D.O.F.

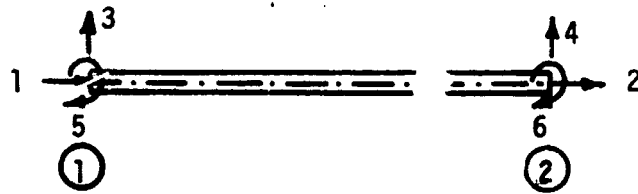


Fig. 2.4 General Planar Beam Element

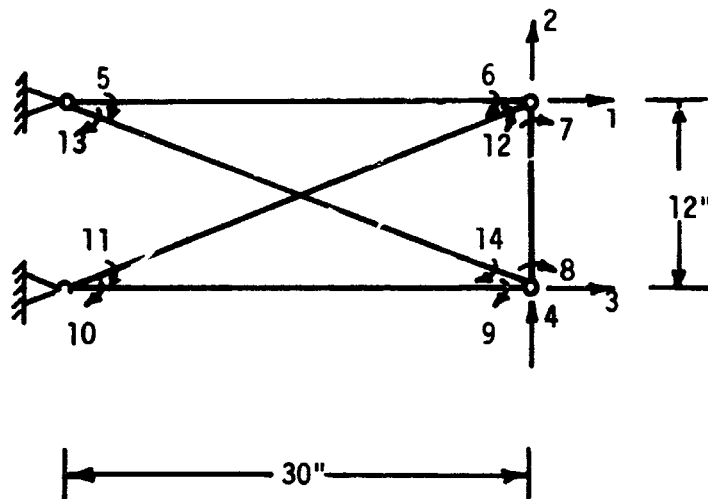


Fig. 2.5 Planar Frame; 14 D.O.F.

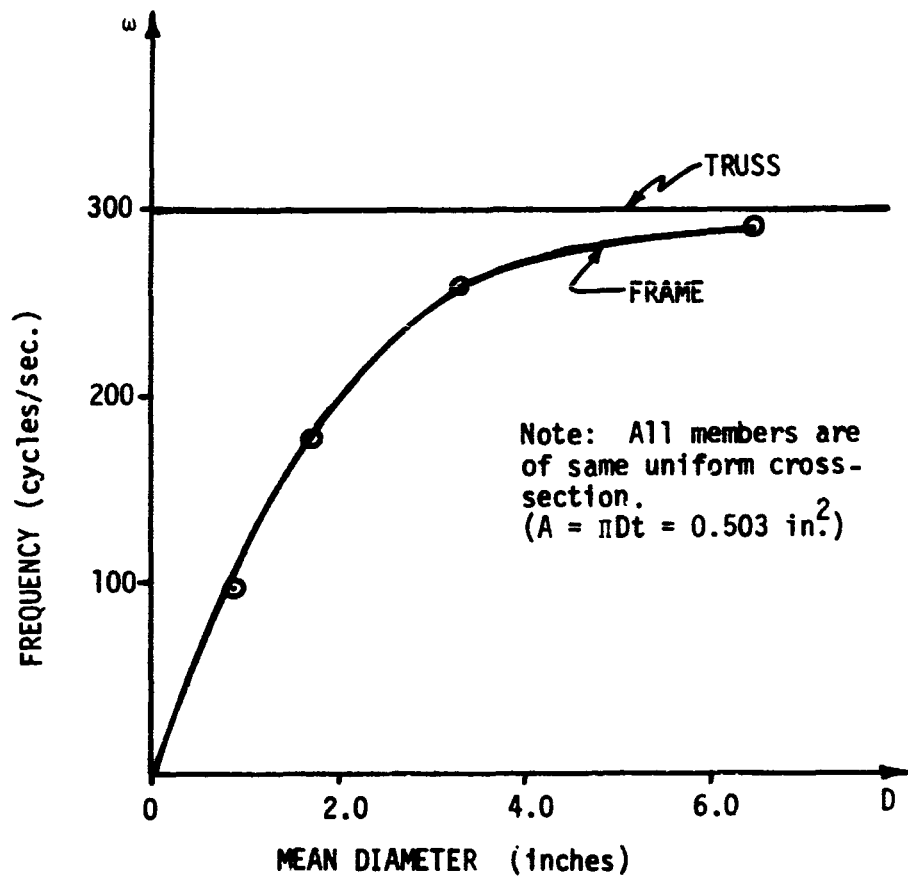


Fig. 2.6 Lowest Natural Frequency of Planar Truss (Fig. 2.3) and General Planar Frame (Fig. 2.5).



## CHAPTER 3

### DYNAMIC ANALYSIS

The conventional usage of the term "Dynamic Analysis" refers to obtaining the time response of a system subject to an external excitation. However, in most cases only certain aspects of the response history are significant to the design processes. For example, only maximum possible displacement and maximum possible stresses on the structure under externally applied dynamic loads may be of interest if the design philosophy specifies only limitations on the displacement and stress at any point on the structure. Furthermore, when the design process is an automated optimum design procedure, the judgements regarding the quantitative maxima of the response quantities (displacements and stresses) are to be formalized explicitly. In this chapter, the term "Dynamic Analysis" refers to obtaining the explicit expression for an upper bound of the displacements and the stresses occurring on the structures during the time history of an externally applied dynamic load, presuming that its "spectrum" is already known. The spectrum<sup>10,11</sup> of a given forcing function represents the peak response quantity of a single-degree-of-freedom oscillator as a function of the natural frequency of the structure. The subject of the spectrum will be elaborated later in this chapter.

#### 3.2 Analysis:

Engineering judgements are made to simplify the structure and

to make approximations in order to develop an adequate and computationally efficient method of analysis. This is a prerequisite to undertaking a rational design optimization. One such approximation is to idealize an infinite degree-of-freedom continuous structure as a finite degree of freedom discretized system, where the configuration of the structure is described by a set of generalized coordinates. Performing a discrete element analysis of the structure, the matrix formulation of the general dynamic response problem for the undamped structure is

$$[M]\ddot{\vec{Y}} + [K]\vec{Y} = \vec{F}(t) \quad (3.1)$$

where the matrices  $[M]$  and  $[K]$  are respectively the master mass and the stiffness matrices of the structure and their order,  $n$ , corresponds to the degrees of freedom of the discretized system. The vectors  $\ddot{\vec{Y}}$ ,  $\vec{Y}$ , and  $\vec{F}(t)$  represent the acceleration, the displacement and the generalized forces.

The set of equations, Eqs. (3.1) consist of a set of coupled second order ordinary differential equations in the generalized coordinates. These can be uncoupled<sup>6,12</sup> through "modal matrix" transformation, which expresses the degrees of freedom in terms of a different system of coordinates. The form of the uncoupled equations resemble the equation of motion of a single degree-of-freedom system. The solution of  $n$  independent differential equations is relatively easier than a set of  $n$  coupled differential equations and hence the modal transformation is very useful.

The  $n \times n$  modal matrix is the matrix of the natural modes of the structure which are obtained from the study of free-vibration of the structure. The equations of motion for the free vibration of the structure are obtained from Eqs. (3.1) by setting the right hand side as zero vector, i.e.

$$[M]\ddot{\vec{Y}} + [K]\vec{Y} = \vec{0} \quad (3.2)$$

The set of equations (3.2) can be reduced to an eigenproblem

$$[K]\vec{X} = \lambda[M]\vec{X} \quad (3.3)$$

the solution to which gives the scalar quantity  $\lambda$  (eigenvalue representing square of the natural frequency) and its corresponding nontrivial vector  $\vec{X}$  (eigenvector or natural modes). If the eigenproblem, Eq. (3.3) has distinct eigenvalues, then the system will have  $n$  distinct natural modes of vibration<sup>13</sup>.

There are various methods<sup>13</sup> to obtain the eigensolution of the general eigenproblem given by Eq. (3.3). Another iterative method developed by the author especially suited for the purposes of the present work is presented in Chapter 4.

Once the modal matrix is known, the transformation can be carried out by expressing

$$\vec{Y} = [X] \vec{q}(t) = \sum_{i=1}^n \vec{X}_i q_i(t) \quad (3.4)$$

where the  $n \times n$  matrix

$$[X] \equiv [\vec{X}_1, \vec{X}_2, \dots, \vec{X}_n] \quad (3.5)$$

is the modal matrix whose columns  $\vec{X}_i$ ,  $i = 1, \dots, n$  represent the natural modes of the system, and the  $n \times 1$  vector,  $\vec{q}(t)$  represents the time dependent generalized coordinates of the structure.

Substituting Eq. (3.4) into Eq. (3.1) and premultiplying the result by  $[X]^T$ , we obtain

$$[X]^T [M] [X] \ddot{\vec{q}} + [X]^T [K] [X] \vec{q} = [X]^T \vec{F}(t) \quad (3.6)$$

The eigenvectors for any two natural modes satisfy a pair of orthogonality relations viz.,

$$\vec{X}_i^T [M] \vec{X}_j = 0$$

$$\text{and} \quad i \neq j \quad (3.7)$$

$$\vec{X}_i^T [K] \vec{X}_j = 0$$

Therefore, the coefficient matrices on the left hand side of Eq. (3.6) reduce to the diagonal matrices, thus providing a system of uncoupled equations. The  $i$ th equation of the uncoupled system, Eqs. (3.6) can be written as

$$\vec{X}_i^T [M] \vec{X}_i \ddot{q}_i + \vec{X}_i^T [K] \vec{X}_i q_i = \vec{X}_i^T \vec{F}(t) \quad (3.8)$$

Dividing Eq. (3.8) by the scalar quantity  $\vec{X}_i^T [M] \vec{X}_i$  and using the definition of the Rayleigh quotient

$$R(\vec{X}_i) \equiv \frac{\vec{X}_i^T [K] \vec{X}_i}{\vec{X}_i^T [M] \vec{X}_i} = \lambda_i \quad (3.9)$$

we obtain

$$\ddot{q}_i + \lambda_i q_i = \frac{\vec{X}_i^T \vec{F}(t)}{\vec{X}_i^T [M] \vec{X}_i} \quad (3.10)$$

If the eigenvectors are made M-orthonormal, i.e.,

$$\vec{X}_i^T [M] \vec{X}_i = 1 \quad (3.11)$$

then we obtain

$$\ddot{q}_i + \lambda_i q_i = \vec{X}_i^T \vec{F}(t) \quad (3.12)$$

The form of Eq. (3.12) is identical to the equation of motion of a single degree-of-freedom system. Thus, the modal matrix transformation, Eq. (3.4), makes it possible to represent the dynamic response of a n-degree-of-freedom system by the linear superposition of the dynamic response of n single degree-of-freedom systems.

Using Eqs. (3.9) to (3.12), the set of Eqs. (3.8) can be rewritten as

$$\ddot{\vec{q}} + [\lambda] \vec{q} = [X]^T \vec{F}(t) \quad (3.13)$$

where the diagonal matrix  $[\lambda]$  has on its  $i$ th diagonal the scalar quantity  $\lambda_i$  which is given by Eq. (3.9).

### 3.3 Dynamic Loading

In actual practice the dynamic loading can be classified as of three general types: (1) Transient loading, (2) Periodic loading and (3) Non-periodic loading.

The first occurs for a finite duration. Usually it occurs only once. However, if it repeats, it is presumed that the structure had reached the quiescent state before the repeated loading occurred. It, therefore, produces transient motions in the responding system. The familiar example of this type can be a blast pulse.

The periodic loading is cyclic in nature and has a definite time period. The responding system will be found to vibrate in a periodic state, if a sufficiently large number of cycles have occurred. The familiar example of this type of loading is that caused by vibrating motors mounted on structures.

The third type of loading is random in character and the information about the forcing function would have to be stated in probabilistic or statistical terms. The familiar examples of this kind of loading are the gusts in the atmosphere or the waves in a confused sea. The problem is stochastic in nature and is beyond the scope of the present work where only deterministic forcing functions are considered.

The periodic forcing functions can be represented by Fourier Approximation, the coefficients of the Fourier Series

can be obtained by numerical integration and the resulting "peak" response can be easily obtained from the transmissibility curves. The peak response is defined to mean the maximum displacement without regard for sign that may occur during the period of motion.

The information regarding the peak response of a single degree-of-freedom system subject to a transient load is contained in what is commonly known as "Shock Spectrum" which are readily available for variety of shock pulses in standard textbooks.<sup>10</sup> The increased need to design complex structures subjected to shock environments has made the study of shock and structural response an important class of problems in itself. It is for this reason that the transient loading and its corresponding shock spectral information has been considered in the present work. It may be well to emphasize that the conceptual part of the dynamic analysis of a structure subjected to the periodic forcing function is analogous to the method of obtaining the shock spectrum of a transient load and hence the fundamental ideas of the present work will apply equally to transient and periodic forcing functions.

A unidirectional shock load resulting from the ground acceleration of a one-half cycle sine pulse as shown in Fig. 3.1 is considered in the present work. The shock spectrum corresponding to this shock pulse will be made available and the numerical results for this dynamic load on various structures will be

studied.

### 3.4 Shock Spectrum:

A shock spectrum of a suddenly applied force is the peak response of a single degree-of-freedom oscillator to the force, stated as a function of the frequency of the oscillator. It can be mathematically formulated as follows:

Let a single degree-of-freedom mass spring oscillator shown in Fig. 3.2 be subjected to a ground acceleration  $\ddot{\xi}_g(t)$ . Let the spring be linear of stiffness  $k$ . Let  $\xi$  be the displacement of the mass  $m$  relative to the ground. Then, the equation of motion is

$$m\ddot{\xi} + k\xi = -m\ddot{\xi}_g(t) \quad (3.14)$$

Denoting

$$\frac{k}{m} = \omega^2 = \lambda \quad (3.15)$$

where  $\omega$  is the natural frequency of the oscillator and  $\lambda$  is the eigenvalue, we obtain from Eq. (3.14)

$$\ddot{\xi} + \lambda\xi = -\ddot{\xi}_g(t) \equiv f(t) \quad (3.16)$$

The complete solution to Eq. (3.16) can be written as

$$\xi(t, \omega) = \frac{1}{\omega} \int_0^t f(\eta) \sin \omega (t - \eta) d\eta \quad (3.17)$$

provided the system is at rest at zero time. The right hand side



of Eq. (3.17) is Duhamel's integral<sup>10</sup>. The relative displacement  $\xi(t, \omega)$ , is a function of time variable and the frequency of the oscillator. Let  $\xi_p(\omega)$  be the maximum of  $\xi(t, \omega)$ , then the peak displacement can be expressed as

$$\xi_p(\omega) = \max_{t>0} |\xi(t, \omega)| \quad (3.18)$$

The shock spectrum of the "shock" due to the ground acceleration  $\ddot{\xi}_g(t)$  is defined by Eq. (3.18). It is to be noted that the shock spectrum gives the peak displacement as a positive quantity, irrespective of the direction of the force applied or the sign of  $\xi$ .

We can obtain the shock spectrum for any other forcing function directly applied to the mass in a manner similar to the one described above. However, in such cases, the behavior quantity of interest would be the absolute displacement.

The shock spectral curves representing the variation of  $\xi_p$  versus  $\omega$  can be plotted from Eq. (3.18). Usually the ordinate,  $\xi_p$ , and the abscissa,  $\omega$ , are respectively normalized with respect to the characteristic displacement,  $\xi_s$ , and the frequency,  $\omega_f$ , of the forcing function. Such curves representing the variation of  $\xi_p/\xi_s$  versus  $\omega/\omega_f$  are available in Ref. 10 for a variety of transient forcing functions. The shock spectral curve for one-half cycle sine pulse is shown in Chapter 4 of Ref. 10.

### 3.5 Peak Response of Multi-Degree-Of-Freedom System

Let  $\hat{e}_g$  be the direction in which the ground acceleration  $\ddot{\xi}_g(t)$  be applied on the multi-degree-of-freedom system shown in Fig. 3.3. Therefore,

$$\hat{e}_g = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

where  $\theta$  is the angle the direction vector  $\hat{e}_g$  makes with the horizontal. The direction of the ground acceleration  $\hat{e}_g$  can be represented as a vector in the generalized coordinates of the system by the use of a tableau called "SR" table described in Appendix III and some logical operations. Thus we obtain

$$\hat{e} \xleftarrow{\text{SR}} \hat{e}_g$$

and therefore the vector of generalized forces

$$\vec{F}(t) = -[M] \hat{e} \ddot{\xi}_g(t)$$

From Eq. (3.1) we have, therefore

$$[M]\ddot{\vec{Y}} + [K]\vec{Y} = -[M] \hat{e} \ddot{\xi}_g(t)$$

where  $\vec{Y}$  represents the vector of displacements relative to ground.

Furthermore, we obtain from Eq. (3.10)

$$\ddot{q}_i + \lambda_i q_i = - \frac{\vec{x}_i^T [M] \hat{e}}{\vec{x}_i^T [M] \vec{x}_i} \ddot{\xi}_g(t)$$

If the eigenvectors are made M-orthonormal, i.e.

$$\vec{X}_i^T [M] \vec{X}_i = 1$$

then we obtain from Eq. (3.12)

$$\ddot{q}_i + \lambda_i q_i = -\vec{X}_i^T [M] \hat{e} \ddot{\xi}_g(t)$$

Denoting

$$c_i = \vec{X}_i^T [M] \hat{e} \quad (3.19)$$

we obtain

$$\ddot{q}_i + \lambda_i q_i = -c_i \ddot{\xi}_g(t) \equiv c_i f(t) \quad (3.20)$$

Comparing Eqs. (3.16) and (3.20), we obtain the peak response of Eq. (3.20) as

$$\begin{aligned} q_{ip} &= \max_{t>0} |q_i(t)| \\ &= |c_i| \varepsilon_p(\omega_i) \end{aligned} \quad (3.21)$$

The peak response,  $q_{ip}$ , for any transient forcing function applied at the joints can be obtained in a similar manner, if we know the shock spectral information for that forcing function.

The response of an n-degree-of-freedom system is given by Eq. (3.4) as

$$\vec{Y}(t) = [X] \vec{q}(t) \equiv \sum_{i=1}^n \vec{X}_i q_i(t)$$

Expressing the  $j^{\text{th}}$  component of the response vector,  $\vec{Y}(t)$  as  $y_j(t)$ , we have

$$y_j(t) = \sum_{i=1}^n x_{ji} q_i(t) \quad (3.22)$$

where  $x_{ji}$  is the  $j^{\text{th}}$  component of the  $i^{\text{th}}$  eigenvector.

The peak value of the  $j^{\text{th}}$  component, expressed as  $y_{jp}$  will then be

$$y_{jp} = \max_{t>0} \left| \sum_{i=1}^n x_{ji} q_i(t) \right| \quad (3.23)$$

The components  $y_{jp}$ ,  $j = 1, 2, \dots, n$  are the elements of the peak response vector  $\vec{Y}_p$ . It may be appropriate to mention here that a reasonably accurate response analysis of the  $n$ -degree-of-freedom structure can be performed by taking the response of the system as a linear superposition of only  $s$ , ( $s < n$ ), modal vectors.

This set of  $s$  modal vectors can be any subset of the  $n$  natural modes of the structure. Usually the first  $s$  natural modes from the lowest end of the spectrum are taken for the approximate analysis, which is based on the surmise that higher modes are not excited under the forced vibration of the actual structure. Thus, one can get away with only the partial eigensolution of the eigenproblem, Eq. (3.3), in order to perform the approximate analysis; the approximated peak response, therefore, is obtained from

$$y_{jp} \approx \max_{t>0} \left| \sum_{i=1}^s x_{ji} q_i(t) \right| \quad (3.24)$$

### 3.6 Upper Bound of the Peak Response

For design problems, one needs to know an estimate of the enveloping maximum, i.e., the maximum of the peak response which is to be compared with the limiting values of the system. An obvious upper bound to the  $j^{\text{th}}$  component of the peak response vector,  $\vec{Y}_p$ , can be written as

$$\begin{aligned} y_{jm} &\leq \sum_{i=1}^n |x_{ji}| q_{ip} \\ &= \sum_{i=1}^n |x_{ji}| |c_i| \varepsilon_p(\omega_i) \end{aligned} \quad (3.25)$$

where  $y_{jm}$ ,  $j = 1, 2, \dots, n$  are the elements of the vector  $\vec{Y}_m$ , henceforth, referred to as the upper bound displacement vector. The positive scalar quantity  $[|c_i| \varepsilon_p(\omega_i)]$  represents the peak value of the  $i^{\text{th}}$  generalized coordinate of the system. Henceforth, we will refer to this quantity as the peak modal participation coefficient.

The expression of Eq. (3.25) is based on the assumption that all the modal contributions reach a maximum at the same time and in the same direction. The amount of overestimation of the components of upper bound displacement vector varies with the type of shock and the structure. In many cases where exact solutions have been compared with the upper bound values, the differences have seldom exceeded 15%.<sup>14</sup> If only  $s < n$  of the eigenvectors are used for the response analysis, then an approximate estimate of the upper bound displacement is given as:

$$y_{jm} \approx \sum_{i=1}^S |x_{ji}| |c_i| \varepsilon_p(\omega_i) \quad (3.26)$$

### 3.7 The Maximum Stress and its Upper Bound

The stress at any point on a linear elastic structure composed of general planar beam elements is obtained by superposing the stress due to the axial deformation and the stress due to the transverse displacement. The maximum stress will be the summation of the absolute values of the two stresses. Therefore,

$$\sigma_{ab}(x) = |\sigma^{(A)}(x)| + |\sigma^{(B)}(x)| \quad (3.27)$$

where

$$\sigma^{(A)}(x) = E \frac{\partial u}{\partial x} \quad (3.28a)$$

represents the axial stress, and

$$\sigma^{(B)}(x) = Eh \frac{\partial^2 w}{\partial x^2} \quad (3.28b)$$

represents the bending stress. The expressions  $\partial u / \partial x$  and  $\partial^2 w / \partial x^2$  are obtained by differentiating  $u(x)$  and  $w(x)$  respectively. The function  $u(x)$  represents the axial deformation given by Eq. (2.24), and the function  $w(x)$  represents the transverse deformation given by Eq. (2.25).  $E$  denotes the Young's Modulus of Elasticity and  $h$  represents the distance of the extreme fiber

of the beam element from its neutral axis.

Differentiating Eq. (2.24) once and Eq. (2.25) twice with respect to  $x$ , we obtain

$$\frac{\partial u}{\partial x} = u_1 H_{01}^{(0)'}(x) + u_2 H_{02}^{(0)'}(x)$$

and

$$\frac{\partial^2 w}{\partial x^2} = u_3 H_{01}^{(1)''}(x) + u_4 H_{02}^{(1)''}(x) + u_5 H_{11}^{(1)''}(x) + u_6 H_{12}^{(1)''}(x) \quad (3.29)$$

where  $H_{01}^{(0)'}(x)$  and  $H_{01}^{(1)''}(x)$  represent the first and the second derivative of the Hermite polynomials  $H_{01}^{(0)}(x)$  and  $H_{01}^{(1)}(x)$  respectively. These polynomials are derived in Appendix I and their derivatives are given below

$$\begin{aligned} H_{01}^{(0)'}(x) &= -\frac{1}{\ell} \\ H_{02}^{(0)'}(x) &= \frac{1}{\ell} \\ H_{01}^{(1)''}(x) &= \frac{1}{\ell^3} (12x - 6\ell) \\ H_{02}^{(1)''}(x) &= -\frac{1}{\ell^3} (12x - 6\ell) \\ H_{11}^{(1)''}(x) &= \frac{1}{\ell^2} (6x - 4\ell) \\ H_{12}^{(1)''}(x) &= \frac{1}{\ell^2} (6x - 2\ell) \end{aligned} \quad (3.30)$$

Let

$$\vec{f}^{(u)}(x) = \begin{pmatrix} H_{01}^{(0)'}(x) \\ H_{02}^{(0)'}(x) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (3.31)$$

and

$$\vec{f}^{(w)}(x) = \begin{pmatrix} 0 \\ 0 \\ H_{01}^{(1)''}(x) \\ H_{02}^{(1)''}(x) \\ H_{11}^{(1)''}(x) \\ H_{12}^{(1)''}(x) \end{pmatrix} \quad (3.32)$$

then from Eq. (3.29) we obtain

$$\frac{\partial u}{\partial x} = \vec{f}^{(u)T}(x) \vec{u} \quad (3.33)$$

and



$$\frac{\partial^2 w}{\partial x^2} = f(w)^T(x) \vec{u} \quad (3.34)$$

where the vector  $\vec{u}$  is defined as

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} \quad (3.35)$$

The components of the vector  $\vec{u}$ ,  $u_i$ ,  $i = 1, \dots, 6$ , correspond to the deformations at the degrees of freedom 1 to 6 respectively in the local coordinate system of the general planar beam element shown in Fig. 3.4.

Substituting Eqs. (3.33) and (3.34) into Eq. (3.28a) and (3.28b) we obtain

$$\sigma^{(A)}(x) = E \vec{f}(u)^T(x) \vec{u} \quad (3.36a)$$

$$\sigma^{(B)}(x) = E \vec{f}(w)^T(x) \vec{u} \quad (3.36b)$$

The element displacement vector  $\vec{u}$  in its local coordinate system is obtained from its corresponding vector in the reference coordinate system by the use of the direction cosine matrix  $[T]$  given by

$$[T] = \begin{bmatrix} \ell & 0 & m & 0 & 0 & 0 \\ 0 & \ell & 0 & m & 0 & 0 \\ -m & 0 & \ell & 0 & 0 & 0 \\ 0 & -m & 0 & \ell & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.37)$$

where  $\ell \equiv \cos \theta$ ,  $m \equiv \sin \theta$ , and  $\theta$  is the angle shown in Fig. 3.4. Thus

$$\vec{u}_i = [T]_i \vec{V}_i \quad (3.38)$$

where the components of  $\vec{V}_i$  represent the displacement at the degrees-of-freedom of the  $i^{\text{th}}$  element in the reference coordinate system.

The displacement vector of the  $i^{\text{th}}$  element in the reference coordinate system,  $\vec{V}_i$ , can be obtained from the corresponding generalized displacement vector of the system by a simple logical operation explained in Appendix III. Therefore,

$$\vec{V}_i \xleftarrow{SR_i} \vec{Y}(t) \quad (3.39)$$

where  $\vec{Y}(t)$  is the generalized displacement vector in the reference coordinate system. If we write the operation of Eq. (3.39) as

$$\vec{V}_i = [SR_i] \vec{Y}(t) \quad (3.40)$$

where it is to be emphasized that the right hand side of Eq. (3.40) does not represent the matrix-vector multiplication but represents only a logical operation by the  $i^{\text{th}}$  row of the SR-table on the vector  $\vec{Y}(t)$ . Then, from Eqs. (3.27), (3.28a), (3.28b), (3.36a), (3.36b) and (3.38) we get

$$\sigma_{ab}^i(x,t) = E \left| \vec{r}^{(u)T}(x) [T]_i \vec{V}_i \right| + E |h| \left| \vec{r}^{(w)T}(x) [T]_i \vec{V}_i \right| \quad (3.41)$$

where the notation  $\sigma_{ab}^i(x,t)$  is meant to denote the maximum stress  $\sigma$  at any point  $x$  on the  $i^{\text{th}}$  element at any time  $t$  due to the generalized displacement vector  $\vec{Y}(t)$  of the structure.

In order to compute the maximum stress in time at any point on the structure from Eq. (3.41), we need to know its displacement mode. We have available to us two choices: (1) to use the upper bound displacement vector whose components are given by Eq. (3.25), or (2) to use the natural modes of the structure,  $\vec{X}_j$ ,  $j = 1, 2, \dots, n$  (for the approximate analysis to use only  $s$  of these  $n$  eigenvectors from the lowest end of the spectrum) and then superpose the resulting stresses corresponding to each natural mode in proportion to the peak modal participation coefficients.

The use of the upper bound displacement vector seems unreasonable because its components represent an upper bound to the corresponding peak displacement and is merely an approximation (though reasonable) to the actual displacement of the structure.

Moreover, its use may lead to a conservative estimate of stress at one point on the structure and simultaneously may result into a nonconservative estimate of the stress at some other point on the structure.

The use of natural modes as indicated in choice (2) seems more logical, since the structure in any case vibrates in its natural modes. We have, therefore, the upper bound stress at any point,  $x$ , of the  $i^{\text{th}}$  element as

$$\sigma_m^i(x) \leq \sum_{j=1}^n \sigma_{ab}^{ji}(x) |c_j| \varepsilon_p(\omega_j) \quad (3.42)$$

where  $\sigma_{ab}^{ji}(x)$  is given from Eq. (3.41) as

$$\sigma_{ab}^{ji}(x) = E \left| \vec{r}^{(u)T}(x) [T]_i \{\vec{X}_j\}_i \right| + E|h| \left| \vec{r}^{(w)T}(x) [T]_i \{\vec{X}_j\}_i \right| \quad (3.43)$$

where

$$\{\vec{X}_j\}_i \equiv SR_i \vec{X}_j$$

or

(3.44)

$$\{\vec{X}_j\}_i \xleftarrow{SR_i} \vec{X}_j$$

Once again, if only  $s < n$  of the eigenvectors are used for the response analysis, then an approximate estimate of the upper bound stress is given by

$$\sigma_m^i(x) \approx \sum_{j=1}^S \sigma_{ab}^{ji}(x) |c_j| \varepsilon_p(\omega_j) \quad (3.45)$$

### 3.8 Remarks

Interesting observations have been made from the numerical results obtained on the general planar truss-frame structures. These are as follows:

The higher eigenvalues correspond to the mode shapes where the overall structure frequency is dominated by the individual frequency of the members of the structure. Thus, the higher mode shapes of the overall structure are predominately the higher modes of vibration of the individual members. The contribution of these modes to the upper bound displacement vector,  $\vec{Y}_m$ , of the structure is, therefore, small. On the other hand, the stresses at any point on the structure corresponding to higher mode shapes are higher because of the increased curvature which the elements experience. Hence, the number of eigenvectors,  $s$ , required to approximate the upper bound displacement is considerably less as compared to the number required to approximate, to the same degree, the upper bound stress.

In Chapter 8 of Ref. 6, it is mentioned that the computation of the stresses from the spatial derivatives of the natural mode functions,  $u(x)$  and  $w(x)$ , may differ greatly from the exact values depending upon the selected interpolating polynomials.

This method, of course, would be perfectly acceptable when the mode shapes are the correct mode shape functions. However, the author has observed that by breaking the individual members into smaller discrete elements, a reasonably accurate estimate of the stresses at any point on the member can be made by the use of spatial derivatives even with the assumed mode technique. This is because a refined modeling enables the assumed modes of the compatible elements to fit closer to the exact mode function of the structure.

If we decide to investigate the upper bound displacement only at the nodes, then the upper bound response quantities (displacement and stress) need be computed at the two ends of each element used to model the planar truss-frame structure. The upper bound stress occurs at any of the two ends of the element, since the expression of the upper bound stress is a linear function of  $x$  as seen from Eqs. (3.30), (3.42) and (3.43). A fictitious node (common joint between two adjacent elements) can always be created at any intermediate point on the member of the structure which seems to be a viable candidate for investigation. For example, the middle point of the members of the truss-frame structure need be investigated for upper bound displacement because of the flapping of the member when the structural frequency is predominately the member frequency.

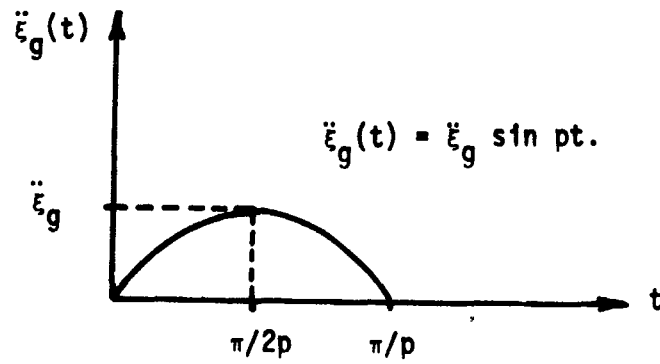


Fig. 3.1 Idealized Ground Acceleration; One Half-Cycle Sine Pulse

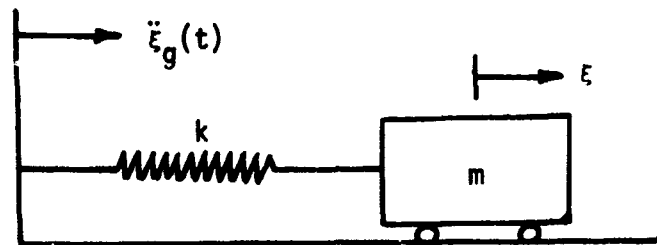


Fig. 3.2 Single Degree-Of-Freedom Mass Spring Oscillator

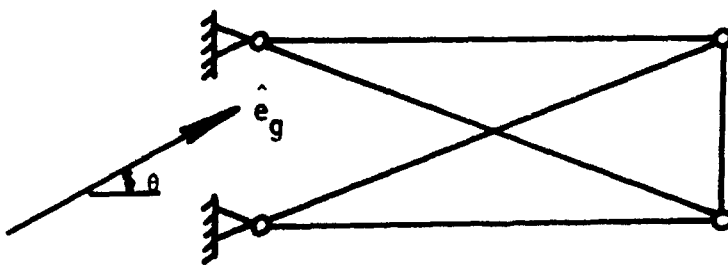


Fig. 3.3 Direction of Shock Pulse



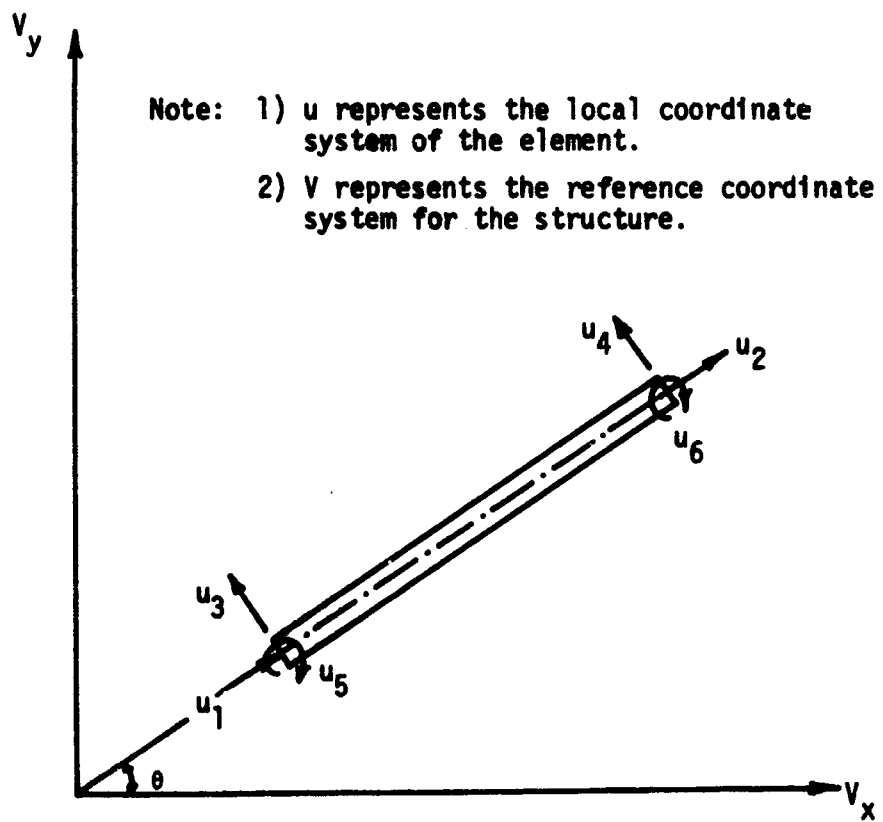


Fig. 3.4 General Planar Beam Element

## CHAPTER 4

### EIGENVALUE ANALYSIS

In Chapter 3 we recognized the need to solve an eigenproblem

$$[K]\vec{X} = \lambda[M]\vec{X} \quad (4.1)$$

in order to obtain the dynamic response of structures through modal analysis. The eigenvalue analysis refers to determining the scalar quantities  $\lambda$  (eigenvalues) and the corresponding nontrivial solutions  $\vec{X}$  (eigenvectors) for the given  $n \times n$  matrices  $[K]$  and  $[M]$ , respectively representing the master stiffness and mass matrices of the structure.

Frequently, the order of  $[K]$  and  $[M]$  matrices is so high that it is impractical or prohibitively expensive to obtain the complete eigensolution. On the other hand, as indicated in Chapter 3, it is possible to get along with only a partial eigensolution in order to carry out a reasonably accurate dynamic analysis of the structure. It is, therefore, natural to look for a method which is especially useful for this class of problems.

There are two general types of methods for the eigensolution of Eq. (4.1): transformation methods and iterative methods. The transformation methods such as the Jacobi, Givens and Householder schemes<sup>13</sup> are almost always preferable when a complete eigensolution is required. On the other hand, the labor savings

involved in obtaining only a partial solution by these methods can be a small fraction of the total. Furthermore, the transformation methods accomplish the solution by operating on the matrices of the system which necessitates the storage of large matrices. With the emergence of the "consistent mass matrix"<sup>3</sup>, another difficulty develops because of the necessity to transform Eq. (4.1), the general eigenproblem, into a special eigenproblem:

$$[D]\vec{Z} = \lambda\vec{Z} \quad (4.2)$$

as required by the transformation methods. Even if  $[K]$  or  $[M]$  happen to be sparse or banded, this step generally produces a dynamic matrix  $[D]$  with more extensive storage requirements than either  $[K]$  or  $[M]$ .

The direct iterative methods, on the other hand, can avoid the necessity of storing the entire matrix by using modern matrix interpretive methods, yet they are disadvantaged in the general eigenproblem, Eq. (4.1), and generally require preliminary modification similar to the transformation methods. In addition, these methods are plagued by convergence difficulties and are computationally expensive for the intermediate eigenvalues and eigenvectors.

In Ref. 15, an iterative method is presented which can be applied directly to the eigenproblem, Eq. (4.1). It uses the well known property of the Rayleigh quotient

$$R(\vec{X}) = \frac{\vec{X}^T [K] \vec{X}}{\vec{X}^T [M] \vec{X}} \quad (4.3)$$

that it equals the eigenvalue when the eigenvector is substituted into it and that it is stationary in the neighborhood of an eigenvector. The Rayleigh quotient is minimized to obtain the lowest eigenvalue and the associated eigenvector. The minimization is done numerically using the conjugate gradient method.

This approach has been extended by the author to obtain the intermediate eigenvalues and associated eigenvectors without a lessening of the storage and efficiency advantages.

The extension to intermediate eigenvalues is accomplished by using a gradient projection scheme for constraining the minimization search to the subspace M-orthogonal to the previously determined eigenvectors.

The advantage of the formulation of Eq. (4.3) is that both the numerator and denominator, as well as all of the other quantities required by the iteration procedure for all of the eigenvalues desired, can be computed without having the assembled  $[K]$  and  $[M]$  matrices at hand. This is accomplished by noting that the numerator is twice the strain energy for a given  $\vec{X}$  (the displacement vector), and that the denominator is twice the maximum kinetic energy of the structure and that these can be computed by summing the potential and kinetic energies of the individual elements of the discretized structural model.

#### 4.2 Formulation of the Problem

The eigenproblem, Eq. (4.1) can be rewritten as

$$[K - \lambda M]\vec{X} = 0 \quad (4.4)$$

If  $\vec{X}$  is a solution to Eq. (4.4), then  $b\vec{X}$  is also a solution for any nonzero value of the scalar  $b$ , thus the eigenvector corresponding to any eigenvalue  $\lambda$  is arbitrary to the extent of a scalar multiplier. In other words, the Rayleigh quotient defined in Eq. (4.3) has no unique minimum, but takes on the same value at every point along any line in the  $n$ -dimensional space passing through the origin. Furthermore, the quotient is not defined at the origin. Consequently, the minimization of the Rayleigh quotient is not quite as simple as that of a function with a well defined minimum.

The redundant degree of freedom, which prevents us from determining the absolute magnitudes of the components of the eigenvector, can be eliminated by an arbitrary normalization. The simplest normalization for the present purpose is to set any nonzero component of the eigenvector equal to one.

The Rayleigh quotient, Eq. (4.3), equals the eigenvalue when the eigenvector is substituted in it. Moreover, it is stationary in the neighborhood of an eigenvector and its value is bounded by the lowest and highest eigenvalues of the physical system. Thus, the minimization of the Rayleigh quotient will yield the lowest eigenvalue.

The minimization problem to find the lowest eigenvalue can thus be stated as:

$$\text{Find } \vec{X} = \vec{X}_1 \text{ such that}$$

$$R(\vec{X}_1) \equiv \frac{\vec{X}_1^T [K] \vec{X}_1}{\vec{X}_1^T [M] \vec{X}_1} \quad (4.5)$$

is minimum, subject to,

$$x_{q1} \equiv \vec{X}_1^T \vec{e}_q = 1 \quad (4.6)$$

where  $x_{q1}$  is the normalizing or reference component and  $\vec{e}_q$  is a vector with its  $q^{\text{th}}$  component as one and zero elsewhere (i.e.  $\vec{e}_q$  is a unit coordinate vector for the  $q^{\text{th}}$  coordinate)

An illustrative example with a geometrical interpretation might be convenient to elucidate the underlying idea. Consider the three degrees of freedom system depicted in Fig. 4.1. The tubular member AB of mean diameter  $D = 0.8''$  and the wall thickness  $t = 0.2''$  is held fixed at the end A and is hinged at the end B. A model consisting of two standard beam elements was used and the vertical displacement of the middle point C of the beam and the rotation at the points C and B are taken to be the three degrees of freedom.

The stiffness and the mass matrices of the structure are given as:

$$[K] = \begin{bmatrix} 0.729 \times 10^4 & 0.0 & -0.295 \times 10^5 \\ 0.0 & 0.635 \times 10^6 & 0.159 \times 10^6 \\ -0.295 \times 10^5 & 0.159 \times 10^6 & 0.317 \times 10^6 \end{bmatrix}$$

$$[M] = \begin{bmatrix} 0.438 \times 10^{-2} & 0.0 & 0.295 \times 10^{-2} \\ 0.0 & 0.293 \times 10^{-1} & -0.110 \times 10^{-1} \\ 0.295 \times 10^{-2} & -0.110 \times 10^{-1} & 0.147 \times 10^{-1} \end{bmatrix}$$

Let the vector space, comprised of the degrees of freedom be represented as

$$\vec{X} \equiv \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

and if we pick the normalizing component as  $x_1$ , then the normalized vector has only two unknown components:

$$\vec{X} \equiv \begin{pmatrix} 1 \\ x_2/x_1 \\ x_3/x_1 \end{pmatrix} \equiv \begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$$

where  $a = \frac{x_2}{x_1}$  and  $b = \frac{x_3}{x_1}$ .

Note that in this example we have chosen  $\vec{e}_q$  vector as

$$\vec{e}_q = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

From the expressions of  $[K]$ ,  $[M]$  and  $\vec{X}$  given, we can write the Rayleigh quotient,  $R$ , explicitly as:

$$R(\vec{X}) = \frac{.73 - 6.0b + 63.5a^2 + 31.7ab + 31.7b^2}{.44 + 0.6b + 2.9a^2 - 2.2ab + 1.5b^2} \times 10^6$$

A plot of  $R(\vec{X})$  for different values of  $a$  and  $b$  is shown in Fig. 4.2, which represents the contour map of the values of the Rayleigh quotient corresponding to the normalized modes of the system. As is seen in Fig. 4.2, the Rayleigh quotient takes on the minimum value at the point 1, a maximum value at the point 3 and an intermediate value at the saddle point 2.

The first three eigenvalues  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  and their associated eigenvectors  $\vec{X}_1$ ,  $\vec{X}_2$ , and  $\vec{X}_3$  are given by

$$\lambda_1 = 0.780 \times 10^6, \lambda_2 = 0.1099 \times 10^8, \lambda_3 = 0.780 \times 10^8 (\text{rad./sec})^2$$

and

$$\vec{X}_1 = \begin{pmatrix} 1.0 \\ -0.033 \\ 0.122 \end{pmatrix}, \quad \vec{X}_2 = \begin{pmatrix} 1.0 \\ 0.592 \\ -0.662 \end{pmatrix}, \quad \vec{X}_3 = \begin{pmatrix} 1.0 \\ -0.793 \\ -1.29 \end{pmatrix}$$

The minimization problem posed in Eqs. (4.5), (4.6) will yield  $\lambda_1$  and  $\vec{X}_1$ .

In problems of structural dynamics, the eigenvectors represent the mode shapes and the choice of a nonzero component ordinarily presents no serious problem. The mechanization of this aspect of the method is briefly described in the discussion



of Example (1), Section 4.7.

Once the lowest eigenvalue  $\lambda_1 \equiv R(\vec{X}_1)$  is known, the next higher or (second) eigenvalue and its associated eigenvector can be determined by posing a new minimization problem:

Find  $\vec{X} = \vec{X}_2$  such that

$$R(\vec{X}_2) = \frac{\vec{X}_2^T [K] \vec{X}_2}{\vec{X}_2^T [M] \vec{X}_2} \quad (4.7)$$

is minimum, subject to

$$\vec{X}_2^T \vec{e}_r = 1 \quad (4.8)$$

and

$$\vec{X}_2^T [M] \vec{X}_1 = 0 \quad (4.9)$$

In the subspace defined by the constraints, Eqs. (4.8) and (4.9), the Rayleigh quotient takes on a unique minimum (assuming distinct eigenvalues) at the eigenvector associated with the second lowest eigenvalue. The constraint Eq. (4.8) is of the type already discussed and Eq. (4.9) represents the imposition of the M-orthogonality condition between  $\vec{X}_1$  and  $\vec{X}_2$ . Geometrically speaking, these constraints merely restrict the portion of vector space in which the search for the second eigenvector is carried out and in this restricted space  $R$  has a minimum corresponding to  $\lambda_2$ .

The determination of the third and subsequent eigenvalues together with their associated eigenvectors up to the complete eigensolution is accomplished by solving a sequence of problems similar to the one presented by Eqs. (4.7) to (4.9). The only change is that each time one additional equation of constraint has to be imposed on the minimization problem to satisfy the condition that the eigenvector currently being sought is M-orthogonal to all of the previously determined eigenvectors. The problem of determining the  $\ell$ th eigenvalue ( $2 \leq \ell \leq n$ ) can thus be written

Find  $\vec{X} = \vec{X}_\ell$  such that

$$R(\vec{X}_\ell) = \frac{\vec{X}_\ell^T [K] \vec{X}_\ell}{\vec{X}_\ell^T [M] \vec{X}_\ell} \quad (4.10)$$

subject to

$$\vec{X}_\ell^T \vec{e}_j = 1 \quad (4.11)$$

and

$$\vec{X}_\ell^T [M] \vec{X}_i = 0 \quad , \quad i = 1, 2, \dots, \ell-1 \quad (4.12)$$

where  $\vec{X}_i$  ,  $i = 1, 2, \dots, \ell-1$  are assumed to be known when the  $\ell$ th eigenvector is being sought.

Denoting

$$[M] \vec{X}_i \equiv \vec{V}_i \quad (4.13)$$

the constraint equation (4.12) can be written as

$$\vec{X}_\ell^T \vec{V}_i = 0 \quad , \quad i = 1, 2, \dots, \ell-1 \quad (4.12a)$$

### 4.3 Minimization Algorithm

The methods of minimizing a function of several variables can, in general, be classified as gradient methods and non-gradient methods. The gradient methods use the local information about the rate of change of the function with respect to the changes in the variables and require the evaluation of the gradient vector, in this case  $\nabla R$ . These methods are inherently the more powerful as more information about the function is used and are preferred over the nongradient methods.

The Rayleigh quotient as a function of the  $n$  variables  $(x_1, x_2, \dots, x_n) \equiv \vec{X}$  defined in Eq. (4.3) is differentiable, and its gradient vector

$$\begin{aligned} \nabla R \equiv \vec{g} &= \frac{2[\mathbf{K}]\vec{X}}{\vec{X}^T[\mathbf{M}]\vec{X}} - \frac{(\vec{X}^T[\mathbf{K}]\vec{X})}{(\vec{X}^T[\mathbf{M}]\vec{X})^2} \times 2[\mathbf{M}]\vec{X} \\ &= \frac{2([\mathbf{K}]\vec{X} - R[\mathbf{M}]\vec{X})}{(\vec{X}^T[\mathbf{M}]\vec{X})} \end{aligned} \quad (4.14)$$

is easily computed. Therefore, it is logical to carry out the minimization of the Rayleigh quotient by one of the gradient methods.

Through the middle 1950's, one of the most popular gradient methods was the method of steepest descent. This method chooses

each direction of search to be the negative of the gradient vector. Though used with moderate success on a variety of problems, it often turned out to be hopelessly slow because of the fact that successive moves are perpendicular to each other and the method gradually settles into a steady n-dimensional zig-zag for functions having any significant eccentricity. The convergence difficulties of the steepest descent method have been largely eliminated by a modification of the basic iteration which has been called the conjugate gradient method.<sup>16</sup> This method has the property that, for a quadratic function of  $n$  variables, it will converge in  $n$  steps, apart from round off errors. For general functions, as the iterate approaches the minimum, the function is usually more nearly approximated by a quadratic and so convergence accelerates toward the solution.

The method of Davidon<sup>17</sup> (1959) which was amended by Fletcher and Powell<sup>18</sup> is also a gradient technique which has the property of quadratic convergence. However, the simplicity of the conjugate gradient method and its modest demands on storage, compounded by the successful experience of Fletcher and Bradbury<sup>15</sup>, dictates the use of conjugate gradient method to minimize the Rayleigh quotient.

The algorithm can be written as:

$$\vec{X}_0 = \text{arbitrary} \quad (a)$$

$$\vec{G}_0 = \nabla R(\vec{X}_0) \quad (b)$$

$$\vec{S}_0 = -\vec{G}_0 \quad (c)$$

$$\vec{X}_{i+1} = \vec{X}_i + \alpha_i^* \vec{S}_i \quad (d)$$

$$\vec{G}_{i+1} = \nabla R(\vec{X}_{i+1}) \quad (e) \quad (4.15)$$

$$B_i = |\vec{G}_{i+1}|^2 / |\vec{G}_i|^2 \quad (f)$$

$$\vec{S}_{i+1} = -\vec{G}_{i+1} + B_i \vec{S}_i \quad (g)$$

where the step length  $\alpha_i^*$  is the value of  $\alpha$  which minimizes  $R(\vec{X}_i + \alpha \vec{S}_i)$ . From Eq. (4.15) we note that  $\vec{S}_{i+1}$  is a linear combination of  $\vec{G}_{i+1}$  and  $\vec{S}_0, \vec{S}_1, \dots, \vec{S}_i$  and hence it is a linear combination of  $\vec{G}_0, \vec{G}_1, \dots, \vec{G}_{i+1}$ . The algorithm is based on a Gram-Schmidt orthogonalization of the  $\vec{G}_i$  and the derivation is documented in the literature.<sup>19</sup>

The method described in Eqs. (4.15) is applicable in principle to any unconstrained minimization problem. It will be noted that the constraint Eq. (4.6) is trivially satisfied by setting the  $q^{\text{th}}$  component of the starting point to be unity and setting the corresponding component of the gradient vector to be zero throughout the search space. Thus, the problem of minimizing the Rayleigh quotient function to find the lowest eigenvalue is similar to an unconstrained minimization problem and the Fletcher-Reeves algorithm can be directly applied.

However, the use of conjugate gradient method for finding the intermediate eigenvalue is possible only when the minimization of the Rayleigh quotient is restricted on to a subspace of  $\vec{X}$

in which the constraint Eqs. (4.11) and (4.12) are continuously satisfied. In order to insure that the search is carried out in the desired subspace of  $\vec{X}$ , it is necessary: (1) to start the iteration with a point in that subspace; and (2) to project the gradient vector  $\vec{g}$ , Eq. (4.14) on to that subspace. Both of these requirements necessitate the use of some matrices which project the arbitrary starting point and the gradient vector  $\vec{g}$  on to the subspace of constraints. Henceforth, such a matrix will be called the "projection" matrix and the way it is generated is discussed in the subsequent section.

#### 4.4 Projection Matrix:

Let  $[P]$  be a matrix which has the property that for any vector  $\vec{W}$ , the vector

$$\vec{W}_p \equiv [P]\vec{W} \quad (4.16)$$

satisfies

$$\vec{W}_p^T \vec{Z}_i = 0 \quad , \quad i = 1, 2, \dots, q \quad (4.17)$$

where  $\vec{Z}_i$ ,  $i = 1, 2, \dots, q$  are  $q$  linearly independent vectors.

Note that Eq. (4.17) can also be written in matrix form as:

$$\begin{matrix} [N]^T \vec{W}_p = 0 \\ (q \times n)(n \times 1) \end{matrix} \quad (4.17a)$$

where

$$\begin{matrix} [N] & \equiv & [\vec{Z}_1, \vec{Z}_2, \dots, \vec{Z}_q] \\ (n \times q) \end{matrix} \quad (4.18)$$

In other words, the matrix operator  $[P]$  eliminates from the vector  $\vec{W}$  the non-orthogonal components, thus giving the vector  $\vec{W}_p$  which is orthogonal to the subspace spanned by the vectors  $\vec{Z}_i$ ,  $i = 1, 2, \dots, q$ . This idea can thus be expressed differently as:

$$\vec{W}_p = \vec{W} - \sum_{i=1}^q u_i \vec{Z}_i \quad (4.19)$$

or, in matrix form as:

$$\begin{matrix} \vec{W}_p = \vec{W} - [N] \vec{U} \\ (n \times q)(q \times 1) \end{matrix} \quad (4.19a)$$

where the components of vector  $\vec{U}$  are  $u_i$ ,  $i = 1, 2, \dots, q$ .

Premultiplying Eq. (4.19a) by  $[N]^T$  we obtain from Eq. (4.17a)

$$[N]^T \vec{W}_p = [N]^T \vec{W} - ([N]^T [N]) \vec{U} = 0 \quad (4.20)$$

Therefore

$$\vec{U} = ([N]^T [N])^{-1} [N]^T \vec{W} \quad (4.21)$$

From Eqs. (4.21) and (4.19a) we obtain

$$\begin{aligned} \vec{W}_p &= \vec{W} - [N] ([N]^T [N])^{-1} [N]^T \vec{W} \\ &= \{ [I] - [N] ([N]^T [N])^{-1} [N]^T \} \vec{W} \end{aligned} \quad (4.22)$$

where  $[I]$  is the identity matrix.

Note that  $([N]^T[N])$  will be a  $(q \times q)$  symmetric matrix, and is nonsingular since  $[N]$  is a  $(n \times q)$  matrix composed of  $q$  linearly independent columns. Hence  $([N]^T[N])^{-1}$  exists.

Comparing Eqs. (4.16) and (4.17) we obtain the projection matrix as:

$$[P] = [I] - [N] ([N]^T[N])^{-1} [N]^T \quad (4.23)$$

In particular, for determining the  $\ell$ th eigensolution, a projection matrix

$$[P_\ell] = \left\{ [I] - [N_\ell] ([N_\ell]^T[N_\ell])^{-1} [N_\ell]^T \right\} \quad (4.24)$$

$(n \times n) \qquad (n \times \ell) \qquad (\ell \times \ell) \qquad (\ell \times n)$

where

$$[N_\ell] = [\vec{e}_j, \vec{V}_1, \vec{V}_2, \dots, \vec{V}_{\ell-1}] \quad (4.25)$$

will project the gradient vector  $\vec{g}$ , Eq. (4.14) on to the subspace of constraints defined by Eqs. (4.11) and (4.12). Note that the column vectors of the matrix  $[N_\ell]$  are linearly independent. Thus

$$\vec{g}_p = [P_\ell] \vec{g} \quad (4.26)$$

A question which immediately warrants attention is, since we use  $\vec{g}_p$  instead of  $\vec{g}$ , will the conjugate gradient method actually converge? In other words, is there a function of which



$\vec{g}_p$  is the gradient and whose function value equals  $R$  in the subspace defined by Eqs. (4.11) and (4.12)? To see that the answer is affirmative, it is only necessary to construct the Lagrange function for the constrained minimization problem, Eqs. (4.10) to (4.12). Define a "Lagrange-Rayleigh" function for the  $\ell$ th eigensolution as:

$$L_\ell = R(\vec{X}) - u_1(\vec{X}^T \vec{e}_j - 1) - \sum_{i=2}^{\ell} u_i \vec{X}^T \vec{V}_{i-1} \quad (4.27)$$

where  $u_i$ ,  $i = 1, 2, \dots, \ell$  are the Lagrange multipliers. Note that the stationary condition over the variables  $\vec{X}$  is

$$\nabla L_\ell = \nabla R - u_1 \vec{e}_j - \sum_{i=2}^{\ell} u_i \vec{V}_{i-1} = 0 \quad (4.28)$$

or in matrix form

$$\nabla L_\ell = \vec{g} - [N_\ell] \vec{U} = 0 \quad (4.29)$$

and if the Lagrange multiplier vector  $\vec{U}$  is defined by Eq. (4.6) as

$$\vec{U} = \left( [N_\ell]^T [N_\ell] \right)^{-1} [N_\ell]^T \vec{g} \quad (4.30)$$

we obtain

$$\nabla L_\ell = \vec{g} - [N_\ell] \left( [N_\ell]^T [N_\ell] \right)^{-1} [N_\ell]^T \vec{g} \equiv [P_\ell] \vec{g} = \vec{g}_p \quad (4.31)$$

where  $[P_\ell]$  is given by Eq. (4.24). Furthermore,  $L_\ell = R$  in the proper subspace, because the second and third terms of

Eq. (4.27) are identically zero by virtue of constraint Eqs. (4.11) and (4.12).

Thus, the "Lagrange-Rayleigh" function has the same value as the Rayleigh quotient in the proper subspace and the projection matrix  $[P_\ell]$  projects the gradient vector  $\vec{g}$  on to this subspace giving thereby the gradient of the "Lagrange-Rayleigh" function.

#### 4.5 Recursion Procedure

It is always possible to form the  $(\ell \times \ell)$  matrix  $([N_\ell]^T [N_\ell])$  and then invert it to obtain the projection matrix  $[P_\ell]$ , Eq. (4.24) when the search for the  $\ell^{\text{th}}$  eigenvalue and its corresponding eigenvector is made. However, it is desirable to avoid this computation, as the value of  $\ell$  will be large when higher eigenvalues are searched.

It will be noticed from Eq. (4.25) that the size of the rectangular matrix  $[N_\ell]$  increases by one column, every time a new eigenvalue and its associated eigenvector is searched. This suggests that some sort of a recursion procedure should be used which permits the insertion of the vector  $\vec{V}_{\ell-1}$  on to the set

$$[N_{\ell-1}] = [\vec{e}_j, \vec{V}_1, \vec{V}_2, \dots, \vec{V}_{\ell-2}] \quad (4.32)$$

and uses  $([N_{\ell-1}]^T [N_{\ell-1}])^{-1}$ , which is presumed to be known. Such a recursion procedure to obtain  $([N_\ell]^T [N_\ell])^{-1}$  from  $([N_{\ell-1}]^T [N_{\ell-1}])^{-1}$  is described in Ref. 20. It is based on

the method for the inverse of a matrix in terms of the inverses of certain of its partitions. For purposes of the present work, let

$$[Q_{\ell-1}] \equiv \begin{pmatrix} [N_{\ell-1}]^T [N_{\ell-1}] \end{pmatrix} \\ (\ell-1) \times (\ell-1)$$

$$= \begin{bmatrix} \vec{e}_j^T \vec{e}_j & & & & \\ & \vec{e}_j^T \vec{v}_1 & \vec{v}_1^T \vec{v}_1 & & \\ & \vec{e}_j^T \vec{v}_2 & \vec{v}_1^T \vec{v}_2 & \vec{v}_2^T \vec{v}_2 & \\ & \vdots & \vdots & \vdots & \\ & \vec{e}_j^T \vec{v}_{\ell-2} & \vec{v}_1^T \vec{v}_{\ell-2} & \dots & \vec{v}_{\ell-2}^T \vec{v}_{\ell-2} \end{bmatrix} \quad \text{SYM}$$

(4.33)

Therefore,

$$[Q_\ell] = \begin{pmatrix} [N_\ell]^T [N_\ell] \end{pmatrix}$$

( $\ell \times \ell$ )

$$= \left[ \begin{array}{c|c} & \begin{matrix} \vec{e}_j^T \vec{v}_{\ell-1} \\ \vec{v}_1^T \vec{v}_{\ell-1} \\ \cdot \\ \cdot \\ \cdot \end{matrix} \\ \hline \begin{matrix} [Q_{\ell-1}] \\ (\ell-1) \times (\ell-1) \end{matrix} & \end{array} \right]$$

$$= \left[ \begin{array}{c|c} [A_1] & \vec{A}_2 \\ \hline \vec{A}_2^T & A_3 \end{array} \right] \quad (4.34)$$

where  $[A_1] = [Q_{\ell-1}]$  and  $\vec{A}_2^T = [\vec{e}_j^T \vec{v}_{\ell-1}, \vec{v}_1^T \vec{v}_{\ell-1}, \dots, \vec{v}_{\ell-2}^T \vec{v}_{\ell-1}]$ .

We presume that  $[A_1]^{-1}$  is known, then

$$[Q_\ell]^{-1} = \left( [N_\ell]^T [N_\ell] \right)^{-1} = \left[ \begin{array}{c|c} [B_1] & \vec{B}_2 \\ \hline \vec{B}_2^T & B_3 \end{array} \right] \quad (4.35)$$

can be obtained from

$$\begin{aligned}
 [B_1] &= [A_1]^{-1} + \frac{1}{s} [A_1]^{-1} \vec{A}_2 \vec{A}_2^T [A_1]^{-1} \\
 \vec{B}_2 &= -\frac{1}{s} [A_1]^{-1} \vec{A}_2
 \end{aligned}
 \tag{4.36}$$

and

$$B_3 = \frac{1}{s}$$

where the scalar quantity  $s$  is given by

$$s = A_3 - \vec{A}_2^T [A_1]^{-1} \vec{A}_2 \tag{4.37}$$

A procedure to obtain  $[B_1]$ ,  $\vec{B}_2$ , and  $B_3$  which is even more efficient than the direct formulas shown above and which uses only the old projection matrix,  $[P_{\ell-1}]$ , the old inverse,  $[Q_{\ell-1}]^{-1}$  and the vector  $\vec{V}_{\ell-1}$  is described in Appendix II.

#### 4.6 Step by Step Procedure

##### (a) Choice of a starting point:

The iterative methods should have a good starting point, otherwise unnecessary time is wasted inside the minimization procedure to minimize the function. However, there seems to be no simple method for finding a good starting value for any iterative method. In Ref. 15, a choice of the starting point is made by choosing a coordinate vector on the basis that the first step of the minimization procedure makes the fastest descent towards the minimum. It is a unit vector  $\vec{e}_1$

(the  $j^{\text{th}}$  element of  $\vec{e}_i$  is  $\delta_{ij}$ ) lying along the coordinate axes. Such a starting point gives an awfully distorted mode shape in the dynamics problem and was not found to be the best choice in this context.

Contrary to expectation, a set of random vectors proved to be superior to the selected unit vectors and in this work they were used as starting points. The method showed reasonably good convergence from these points. Once the solutions converged, a knowledge of the mode shapes of the structural system was obtained. Experiments were conducted in which several of the design variables of the structure were changed and the eigensolution of the modified system was obtained by using the mode shapes of the original design as the starting points. This showed rapid convergence. It was also observed that the mode shapes change positions, in the sense that their appearance in the spectrum of eigenvalues shifts from design to design. Consequently, whichever was the  $\ell^{\text{th}}$  mode shape in the original design may not correspond to the  $\ell^{\text{th}}$  mode shape for the new design. If we denote  $\vec{x}_i^{(0)}$ ,  $i = 1, 2, \dots, n$  as the  $n$  eigenvectors of the original design, then there is one "best" starting point, say  $\vec{x}_p^{(0)}$ , amongst  $\vec{x}_i^{(0)}$ ,  $i = 1, 2, \dots, n$  which corresponds to the lowest value of the Rayleigh quotient in the proper space in order to obtain the lowest eigenvalue of the new design. It was observed that the convergence to the lowest eigenvalue and its associated eigenvector of the new design was relatively faster by using  $\vec{x}_p^{(0)}$  as the starting

point as against  $\vec{x}_1^{(0)}$  (assuming of course that  $p \neq 1$ ). This technique was extended to pick up the best starting point from amongst the potential candidates  $[\vec{x}_i^{(0)}, i, 1, 2, \dots, p-1, p+1, \dots, n]$  to start the iteration in order to obtain the second lowest eigensolution of the new design. Notice that the number of potential candidates for starting points to obtain the intermediate eigensolution upto the complete eigensolution of the new design successively reduce as the order of the eigensolution increases. This is because we eliminate those  $\lambda-1$  eigenvectors of the old design which have been used as starting points for the already determined  $\lambda-1$  eigensolutions of the new design. This technique of picking up the best starting point greatly accelerated the convergence.

The starting point for the search of the lowest eigenvalue needs to satisfy only one constraint, Eq. (4.6), which is trivially satisfied by dividing through by the  $q^{\text{th}}$  component. However, the minimization algorithm, Eq. (4.15) generates a sequence of vectors which, in the limit, tend directionally to the minimum eigenvalue on the search space. Thus the  $q^{\text{th}}$  component of the vectors so generated have to be maintained as unity, so as to satisfy the constraint Eq. (4.6) continuously in the space. This is achieved by setting the  $q^{\text{th}}$  component of the gradient vector at the particular point equal to zero. In other words, no "move" is made in the  $q^{\text{th}}$  direction of the search space.

The starting point for the search of second eigenvalue has to

satisfy an additional constraint, Eq. (4.9), and this can be easily satisfied by Schmidt orthogonalization

Let  $\vec{\tilde{x}}(0)$  be some initial estimate. Therefore,

$$\vec{\tilde{x}}_2(0) = \vec{\tilde{x}}_2(0) - (\vec{\tilde{x}}_2(0)^T \vec{V}_1) \vec{V}_1 \quad (4.38)$$

satisfies Eq. (4.9) if  $\vec{V}_1^T \vec{V}_1 = 1$ . Note that  $\vec{V}_1 = [M] \vec{\tilde{x}}_1$ , Eq. (4.13) where  $\vec{\tilde{x}}_1$  is the eigenvector corresponding to the first (lowest) eigenvalue.

In order to get the starting point for the search of subsequent eigenvalues, the initial estimate is passed through a projection matrix as discussed in Section 4.4 where the projection matrix is given by Eq. (4.24) but the matrix  $[\tilde{N}_\ell]$  where

$$\begin{matrix} [\tilde{N}_\ell] & = & [\vec{V}_1, \vec{V}_2, \dots, \vec{V}_{\ell-1}] \\ (n \times \ell - 1) \end{matrix} \quad (4.39)$$

is to be used instead of matrix  $[N_\ell]$ , Eq. (4.25).

(b) Function Evaluation:

The value of the function to be minimized is required at the end of each iteration cycle for almost all of the iteration methods since the convergence criterion is based on the function value. Moreover, for the particular problem of minimizing the Rayleigh quotient, the function value is required at each cycle in order to evaluate the gradient vector, Eq. (4.14). It is, therefore, necessary to have an efficient routine for function evaluation, in order to avoid the time which would be otherwise



wasted inside the minimization procedure. The value of the Rayleigh quotient can be computed without having the assembled  $[K]$  and  $[M]$  matrices at hand. This is accomplished by noting that the numerator is twice the strain energy for a given  $\vec{X}$  (the generalized displacements), and that the denominator is twice the maximum kinetic energy of the structure and these can be computed by summing the potential and kinetic energies of the individual elements of the discretized structural model. In this sense, the approach is an extension of energy search method documented in the literature.<sup>21,22</sup> Thus,  $R(\vec{X})$  can be written as

$$R(\vec{X}) = \frac{\sum_{i=1}^r \vec{Y}_i^T [k_i] \vec{Y}_i}{\sum_{i=1}^r \vec{Y}_i^T [m_i] \vec{Y}_i} \quad (4.40)$$

where  $r$  is the number of discrete elements,  $[k_i]$  and  $[m_i]$  are, respectively, the stiffness and mass matrices of the  $i$ th element of the discretized structure and  $\vec{Y}_i$  is the displacement vector of the  $i$ th element corresponding to the generalized displacement vector  $\vec{X}$ . To obtain the vector  $\vec{Y}_i$ ,  $i = 1, 2, \dots, r$  from the vector  $\vec{X}$  is rather easy and is a matter of logical operations. A variety of methods exist for such logical operations. One such scheme is described in Ref. 7 and an equivalent technique is discussed in detail in Appendix III.

For the purposes of the present work, a separate subroutine

was written in Fortran V to decompose the vector  $\vec{X}$  to the vectors  $\vec{Y}_i$ , to evaluate the vectors  $[k_i]\vec{Y}_i$  and  $[m_i]\vec{Y}_i$  and then to assemble back the product  $[K]\vec{X} = \vec{A}$  and  $[M]\vec{X} = \vec{B}$ .

The assembly of the vectors  $\vec{A}$  and  $\vec{B}$  is accomplished through a logical operation on the vectors  $[k_i]\vec{Y}_i$  and  $[m_i]\vec{Y}_i$  which is merely the inverse of the operation described in Appendix III.

As will be seen later, this subroutine was used over and over again to evaluate certain other quantities required in the iteration process, other than the function evaluation which is now obtained by two vector multiplications  $\vec{X}^T\vec{A}$  and  $\vec{X}^T\vec{B}$  and one division in order to get:

$$R(\vec{X}) = \frac{\vec{X}^T\vec{A}}{\vec{X}^T\vec{B}}$$

Needless to emphasize the advantage gained by getting along without the assembly of  $[K]$  and  $[M]$  matrices. Frequently, the order of  $[K]$  and  $[M]$  matrices encountered for large structures is so high that it is impractical or prohibitively expensive to study their vibration characteristics without making approximations. The size of element stiffness and mass matrices are relatively much smaller than the size of the assembled stiffness and mass matrices of large complex structures. Furthermore, advantage can also be taken of similar elements. For example, in a structural system of a large number of elements, if only three types of elements are used, then we need to store only the stiffness and mass matrices corresponding to these three elements, rather than

for all of the  $r$  elements.

(c) Gradient Evaluation:

The minimization algorithm, Eq. (4.15) requires the evaluation of the gradient vector at each cycle of the iteration. The gradient vector,  $\vec{g}$ , of the Rayleigh quotient function is given by Eq. (4.14). For determining the lowest (first) eigensolution, the component of the gradient vector corresponding to the normalizing component of the eigenvector is set equal to zero, in order to continuously satisfy the constraint imposed due to the normalization of the eigenvector. While determining the intermediate eigensolution, the projection matrix  $[P_g]$ , Eq. (4.24) is used to project this gradient vector,  $\vec{g}$ , on to the proper subspace of search,  $M$ -orthogonal to the previously determined eigenvectors. The component of the gradient vector,  $\vec{g}_p$ , corresponding to the normalizing component of the eigenvector turns out to be zero automatically, but a small number often appears due to roundoff errors and this is simply removed by setting that component equal to zero.

(d) Evaluation of Step Length

Once a direction of move  $\vec{S}_i$  has been decided, we must determine  $\alpha_i^*$  so that the function is minimized in that direction. Thus the problem of finding the step length is essentially the linear search problem which requires the determination of the  $\alpha_i^*$  along  $\vec{S}_i$  through  $\vec{X}_i$  at which the value of the Rayleigh

quotient function

$$R(\vec{X}_i + \alpha_i \vec{S}_i) = \frac{(\vec{X}_i + \alpha_i \vec{S}_i)^T [K] (\vec{X}_i + \alpha_i \vec{S}_i)}{(\vec{X}_i + \alpha_i \vec{S}_i)^T [M] (\vec{X}_i + \alpha_i \vec{S}_i)} \quad (4.41)$$

is a minimum, i.e.,

$$\left. \frac{dR}{d\alpha_i} \right|_{\alpha_i = \alpha_i^*} = R'(\alpha_i^*) = 0 \quad (4.42)$$

In the general problem, no expression is available to determine  $\alpha_i^*$ , so an interpolation approach<sup>16,17,18</sup> is adopted. In the particular problem of the Rayleigh quotient, however, an explicit expression in  $\alpha_i$  can be generated<sup>15</sup> from Eqs. (4.41) and (4.42) which has the form

$$u\alpha_i^2 + v\alpha_i + w = 0 \quad (4.43)$$

where

$$\begin{aligned} u &= (\vec{S}_i^T [K] \vec{S}_i)(\vec{X}_i^T [M] \vec{S}_i) - (\vec{X}_i^T [K] \vec{S}_i)(\vec{S}_i^T [M] \vec{S}_i) \\ v &= (\vec{S}_i^T [K] \vec{S}_i)(\vec{X}_i^T [M] \vec{X}_i) - (\vec{X}_i^T [K] \vec{X}_i)(\vec{S}_i^T [M] \vec{S}_i) \\ w &= (\vec{X}_i^T [K] \vec{S}_i)(\vec{X}_i^T [M] \vec{X}_i) - (\vec{X}_i^T [K] \vec{X}_i)(\vec{X}_i^T [M] \vec{S}_i) \end{aligned} \quad (4.44)$$

The two roots of Eq. (4.43) correspond to the maximal and minimal points of the Rayleigh quotient in the direction  $\vec{S}_i$  through  $\vec{X}_i$  as shown in Fig. 4.3. The minimal function value corresponds to  $\alpha_i^*$ .

Note that the matrix products required in Eq. (4.44) to evaluate the coefficients of the quadratic, Eq. (4.43), are easily obtained through the subroutine discussed in Section 4.6(b).

#### 4.7 Illustrative Examples:

In all the examples presented in this section, (1) a general planar beam element having six degrees of freedom was used to model the various structures, (2) the distributed mass of the system was used to evaluate the components of the mass matrix (the "consistent" mass matrix), (3) the Univac 1108 digital computer was used to obtain the numerical results and (4) except where noted, random vectors were taken as starting points for the minimization algorithm to obtain the eigensolution.

##### Example (1):

As a simple application, the cylindrical cantilever rod shown in Fig. 4.4(a) is considered. An attempt was made to obtain the complete eigensolution of this simple structure by the successive minimization of the Rayleigh quotient in the appropriate subspace.

Based on the surmise that the lowest eigenvalue would correspond to the first cantilever mode, the degree of freedom associated with the transverse deflection of the cantilever at the free end ( $x_8$ ) was made to equal one and the others were taken as zero in order to start the search for the lowest eigenvalue. (This obviously makes  $x_8$  the normalizing component). A

capability was built in the computer program to change the normalizing component whenever the magnitude of any other component of the vector in the appropriate subspace exceeded five times the magnitude of the current normalizing component (which in any case is one). Every change of the normalizing component necessitated the restart of the minimization algorithm due to the change in the subspace of search.

Curiously enough the complete eigensolution so obtained gave the first six eigenvalues and their associated eigenvectors and did not pick up the remaining three eigenvalues. This is because the three axial degrees of freedom are uncoupled from the six translational and rotational degrees of freedom of the cantilever beam and with the particular choice of starting point, the method could not enter into the subspace in which the eigenvalues associated with the axial modes of vibration are located. If instead of  $\vec{e}_8$ , the starting points are taken as  $\vec{e}_7$ , only the eigenvalues associated with the three axial modes of vibration are obtained.

Furthermore, if the same structure is oriented differently as shown in Fig. 4.4(b) where the translational, rotational and axial degrees of freedom are all coupled, the complete eigensolution, i.e. all the nine eigenvalues and the corresponding eigenvectors, were obtained from the starting point of  $\vec{e}_8$ .

The first three eigenvalues of the model are:  $.2466 \times 10^8$ ,  $.1161 \times 10^9$  and  $.9747 \times 10^9$  which correspond to the frequencies

of  $.4966 \times 10^4$  ,  $.1079 \times 10^5$  and  $.3122 \times 10^5$  radians/sec. These compare favorably with the exact values of  $.4945 \times 10^4$  ,  $.1065 \times 10^5$  and  $.3100 \times 10^5$  radians/sec., and respectively correspond to the first cantilever beam mode, the first axial mode and the second cantilever beam mode.

Example (2):

Consider as another illustrative example the planar frame shown in Fig. 4.5(a) consisting of tubular members pinned together at the nodes. Each member was modeled with a single beam element. Thus, the structure has fourteen degrees of freedom. A complete eigensolution (i.e. all the fourteen eigenvalues and eigenvectors) of the system was obtained by successive minimization of the Rayleigh quotient. The time taken was 4.17 seconds.

Example (3):

The partial eigensolution of a 56 degrees of freedom system shown in Fig. 4.5(b) was obtained by minimizing the Rayleigh quotient. Each member of the four bay planar frame, pinned at the nodes, was modeled with a single beam element. The determination of ten eigenvalues and eigenvectors took 77.91 seconds.

Example (4):

A frame structure having 134 degrees of freedom shown in Fig. 4.6 was analyzed to obtain the first five eigenvalues and the associated eigenvectors. Each member was pinned at its nodes

and modeled with only one element. The number of iterations required to converge to the solution up to the eighth decimal place, the time taken to obtain each eigenvalue and the numerical result obtained are given in Table 4.1. The eigenvectors so obtained were used as the starting points for the minimization algorithm to obtain the eigensolution of a changed design of the same structure. The change in the original design was brought about by increasing the mean diameter of members 1 to 20 by 10% and by decreasing the mean diameter of members 21 to 29 by 10%. The results for the changed design are also given in Table 4.1. The total time taken to obtain the partial (first five) eigensolution of the original design was 46.95 seconds while that for the changed design was 20.30 seconds. Thus, the method converges rapidly for the changed design. The reason is that the eigenvectors of the original design provide the reasonable initial estimates of the eigenvectors for the changed design and thus are good starting points for the minimization algorithm.

It would be observed from Table 4.1 that the first two eigenvalues are well separated while the next three are closely spaced. A study of the associated eigenvectors explains this behavior. The first two eigenvalues, respectively, correspond to the mode shapes schematically represented in Fig. 4.7 and 4.8. The next three closely spaced eigenvalues correspond to the mode shapes where the overall structure frequency is dominated by the individual frequency of any of the long members of the structure marked 34, 35, 44, and 45. This interaction of the



individual member frequency with the overall structure frequency is responsible for the three closely spaced eigenvalues.

The same example was rerun with a relaxed convergence criterion (this time the solution was required to converge to only the sixth decimal place) and the results obtained are given in Table 4.2. The total time taken for the partial eigensolution of the original design is 24.34 seconds while that for the changed design is 7.97 seconds. Thus, there is a considerable saving of time in choosing a less stringent convergence criterion. However, although the error does not seem to propagate badly from eigenvalue to eigenvalue, the eigenvectors so obtained are not as accurate as those obtained by the more stringent convergence criterion.

Example (5):

The number of degrees of freedom of the system shown in Fig. 4.6 turned out to be 281 when each of its members was modeled by two beam elements. The results for the first five eigenvalues of this refined model are given in Table 4.3. As anticipated, the refined modeling gave a better correlation between the closely spaced eigenvalues of the overall structure and the eigenvalue corresponding to the lowest natural frequency of the individual members having a length of 21.21" (any of the ones marked 34, 35, 44, and 45 on Fig. 4.6). The lowest natural frequency of the simply supported tubular beam of mean diameter 1.6", wall thickness 0.1" and a length of 21.21" is

$.251 \times 10^4$  radians/sec , which correspond to an eigenvalue of  $.6300 \times 10^7$  . Thus, the closely spaced eigenvalues are the result of the interaction between the individual member frequency and the overall structure frequency.

#### 4.8 Conclusion

The minimization of the Rayleigh quotient to obtain a partial or complete solution of the general eigenproblem does not require any preliminary modification to put it into the form of a special eigenvalue problem. Furthermore, this formulation extends the important computer storage advantages of the conjugate gradient method by eliminating the need for assembled stiffness and mass matrices. The method has been found to be effective to obtain the partial eigensolution and is thus useful for reasonably accurate dynamic analysis of complex structures. Because the method is iterative and converges quite rapidly when reasonable initial estimates of the eigenvector are available, it lends itself well to embedment within structural optimization procedures when dynamic behavior is to be considered. This is because as the optimum structural design is evolved, the eigensolution generally is expected to change only incrementally from design to design. Hence the previous solutions provide good initial estimates of the eigenvector.

| No. of<br>Eigenvalue | Original Design |                 |                         | Changed Design |                 |                         |
|----------------------|-----------------|-----------------|-------------------------|----------------|-----------------|-------------------------|
|                      | Iterations      | Time<br>(secs.) | Value                   | Iterations     | Time<br>(secs.) | Value                   |
| 1                    | 59              | 5.3             | $.62309556 \times 10^6$ | 26             | 2.22            | $.64067844 \times 10^6$ |
| 2                    | 56              | 4.27            | $.58762266 \times 10^7$ | 40             | 3.04            | $.60087324 \times 10^7$ |
| 3                    | 205             | 19.53           | $.75253340 \times 10^7$ | 73             | 6.11            | $.75745767 \times 10^7$ |
| 4                    | 100             | 8.36            | $.77330454 \times 10^7$ | 46             | 4.50            | $.75729213 \times 10^7$ |
| 5                    | 105             | 9.49            | $.79875840 \times 10^7$ | 43             | 4.43            | $.75753798 \times 10^7$ |

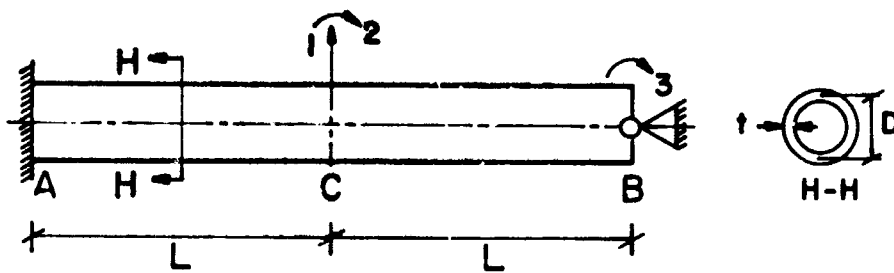
Table 4.1 - First Five Eigenvalues of the Original  
and the Changed Design. Eigenvalues  
Required to Converge Up to the Eighth  
Decimal Place.

| No. of<br>Eigenvalue | Original Design |                 |                           | Changed Design |                 |                           |
|----------------------|-----------------|-----------------|---------------------------|----------------|-----------------|---------------------------|
|                      | Iterations      | Time<br>(secs.) | Value                     | Iterations     | Time<br>(secs.) | Value                     |
| 1                    | 45              | 3.49            | .623109 x 10 <sup>6</sup> | 6              | 0.56            | .640686 x 10 <sup>6</sup> |
| 2                    | 45              | 3.51            | .587632 x 10 <sup>7</sup> | 9              | 1.14            | .600891 x 10 <sup>7</sup> |
| 3                    | 50              | 4.15            | .752796 x 10 <sup>7</sup> | 22             | 2.13            | .757763 x 10 <sup>7</sup> |
| 4                    | 55              | 5.55            | .773182 x 10 <sup>7</sup> | 27             | 2.11            | .757269 x 10 <sup>7</sup> |
| 5                    | 76              | 7.64            | .798727 x 10 <sup>7</sup> | 28             | 2.03            | .757271 x 10 <sup>7</sup> |

Table 4.2 - First Five Eigenvalues of the Original  
and the Changed Design. Eigenvalues  
Required to Converge Upto the Sixth  
Decimal Place.

| No. | Iterations | Time<br>(secs.) | Value               |
|-----|------------|-----------------|---------------------|
| 1   | 417        | 71.78           | $.6207 \times 10^6$ |
| 2   | 301        | 55.57           | $.5291 \times 10^7$ |
| 3   | 614        | 117.22          | $.6295 \times 10^7$ |
| 4   | 222        | 42.48           | $.6371 \times 10^7$ |
| 5   | 297        | 59.12           | $.6643 \times 10^7$ |

Table 4.3 - Results of 281 Degrees of  
Freedom System of Fig. 4.6.  
Eigenvalues Required to  
Converge Upto the Eighth  
Decimal Place.



Mean diameter ( $D$ ) = 0.8"

Wall thickness ( $t$ ) = 0.2"

$L = 16.16"$

$E = 30 \times 10^6 \text{ lbs./in.}^2$

$\rho = 0.28 \text{ lbs./in.}^3$

Fig. 4.1 Tubular Beam Fixed at the End A and Hinged at the End B.

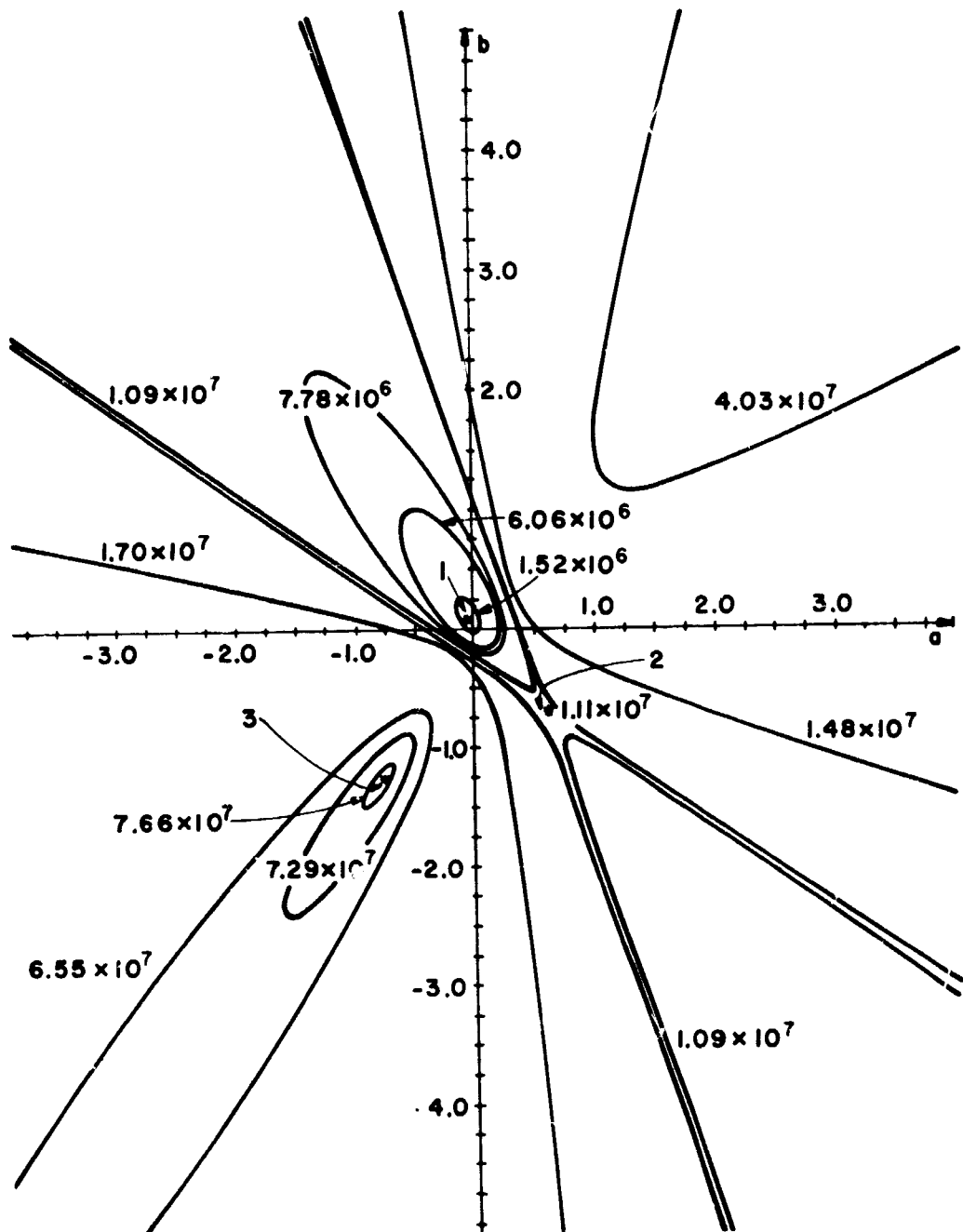


Fig. 4.2 Contour Map of Rayleigh Quotient.

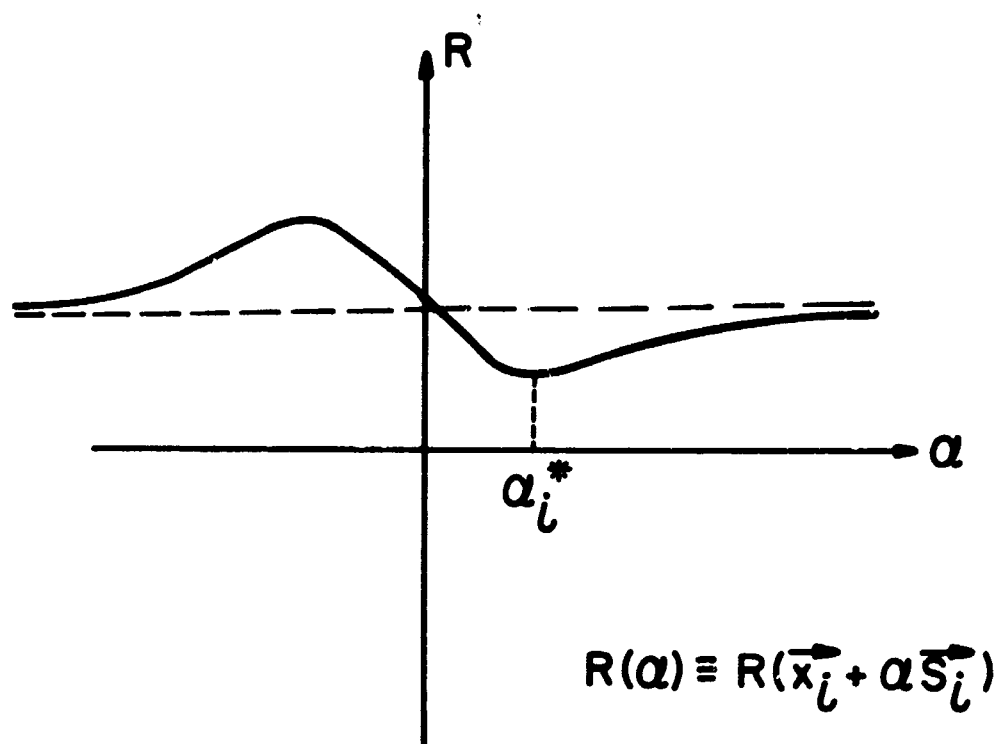
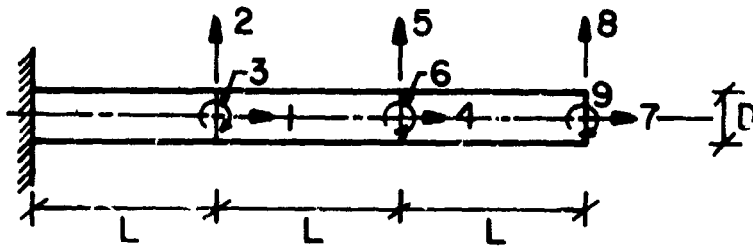
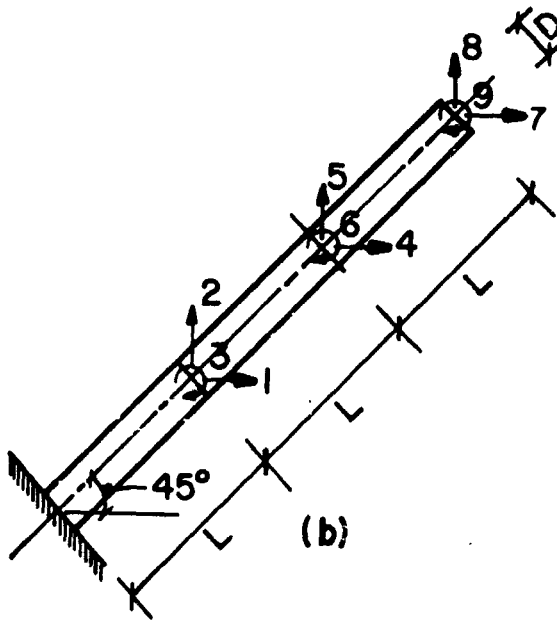


Fig. 4.3 Representative Variation of Rayleigh Quotient Along a Line.





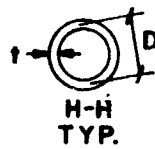
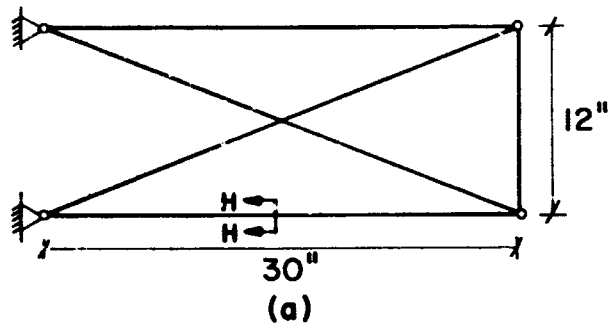
(a)



(b)

$$\begin{aligned}
 L &= 1.0'' \\
 D &= 0.25'' \\
 E &= 30 \times 10^6 \text{ lbs./in.}^2 \\
 \rho &= 0.28 \text{ lbs./in.}^3
 \end{aligned}$$

Fig. 4.4 Cantilever Rod; 9 Degrees-of-Freedom



Mean Dia. (D) = 1.6"  
 Wall Thickness (t) = 0.1"  
 $E = 30 \times 10^6 \text{ lbs./in}^2$   
 $\rho = 0.28 \text{ lbs./in}^3$

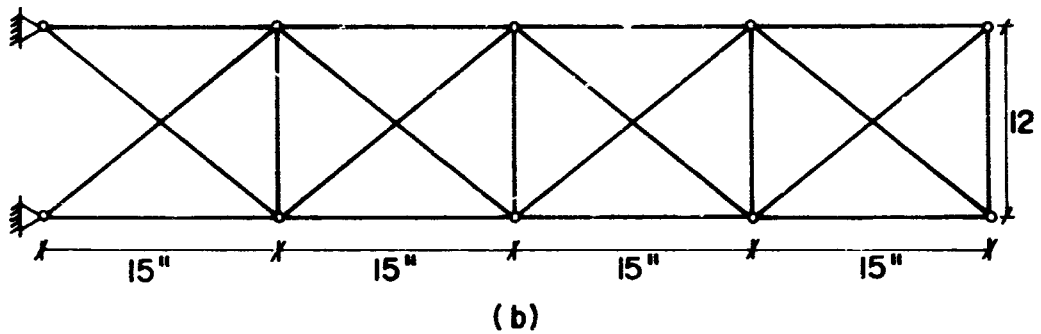


Fig. 4.5 Planar Frames: (a) Fourteen Degrees of Freedom;  
 (b) Fifty-six Degrees of Freedom.

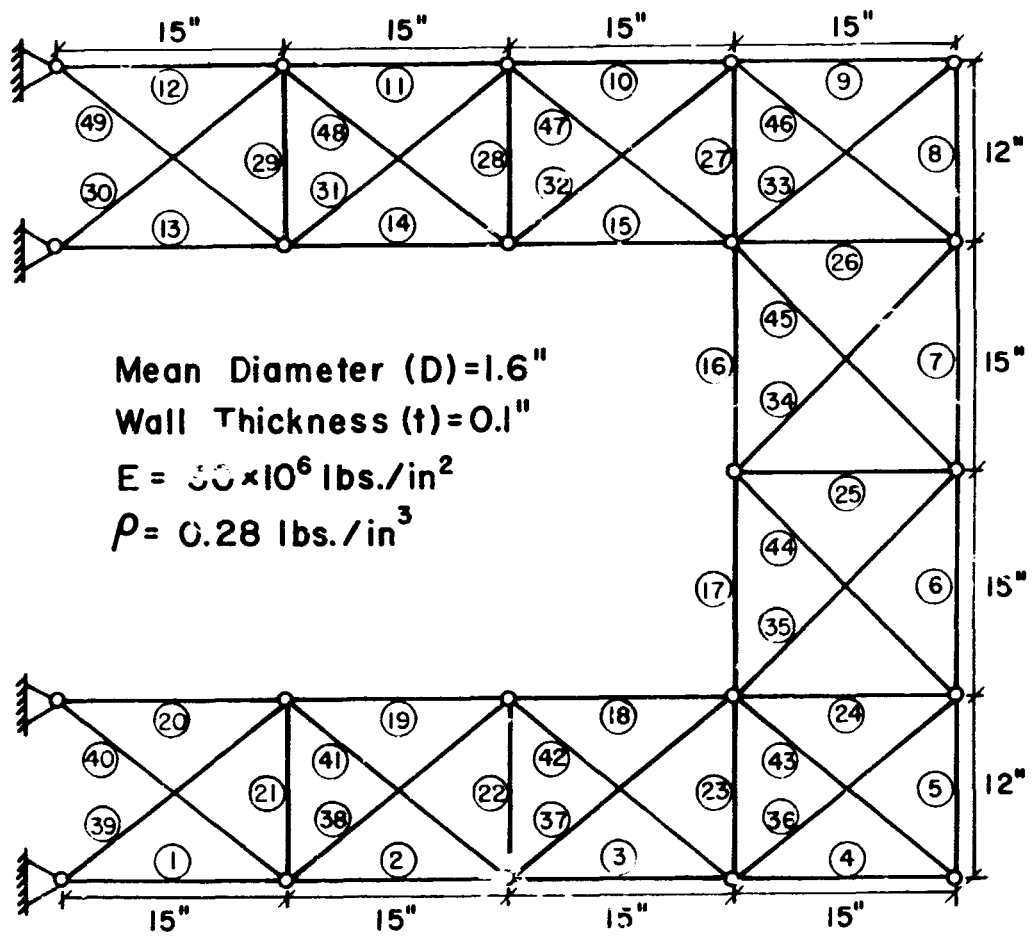


Fig. 4.6 Planar Frame (134 Degrees of Freedom, If Each Member is Model'd of One Element).

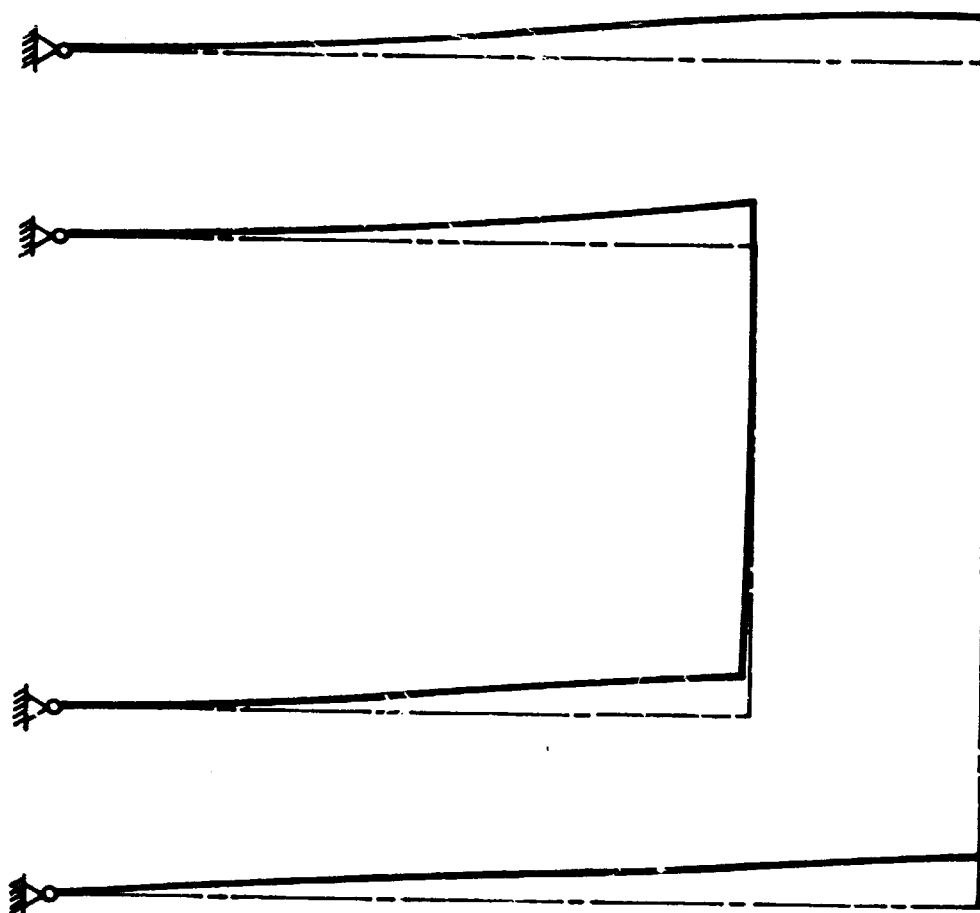


Fig. 4.7 Mode Shape Corresponding to (Lowest) Eigenvalue No. 1;  
(134 Degrees of Freedom System).

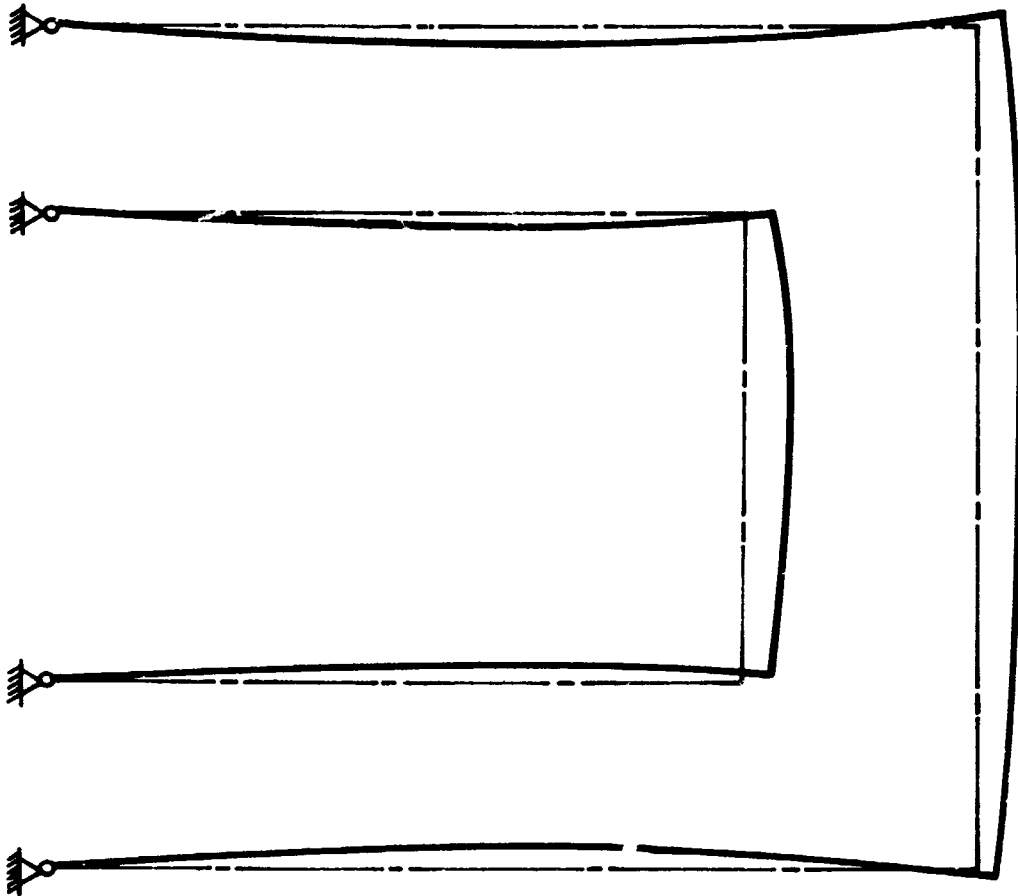


Fig. 4.8 Mode Shape Corresponding to Eigenvalue No. 2;  
(134 Degrees of Freedom System)

## CHAPTER 5

### RATES OF CHANGE OF EIGENVALUES AND EIGENVECTORS

There is a fundamental requirement for information relating changes in the dynamic response quantities to changes in the design variables for certain computer design methods, for example, the method of feasible directions. These methods use the gradients to the objective function and to the constraint surfaces, in order to obtain a useable feasible direction.<sup>†</sup> The dynamic response quantities depend on the eigenvalues and associated eigenvectors, as formulated in Chapter 3. Furthermore, eigenvalues are also considered to be the constraint surfaces in the present work. Therefore, useful information is contained in the quantities representing rates of change of eigenvalues and eigenvectors with respect to design parameters. Exact expressions for these quantities are derived in this chapter for a general eigenvalue problem

$$[K]\vec{X} = \lambda[M]\vec{X} \quad (5.1)$$

where the scalar quantities  $\lambda$  (eigenvalues) and the corresponding nontrivial solutions  $\vec{X}$  (eigenvectors) are presumed to be known. In the structural application, the master stiffness matrix,  $[K]$  and the master mass matrix,  $[M]$ , are symmetric, their order,

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<sup>†</sup>See Chapter 9 .

n , corresponds to the elastic degrees of freedom of the system and their elements are dependent on the design parameters.

## 5.2 Rate of Change of an Eigenvalue

The eigenvalue problem, Eq. (5.1), can be rewritten as

$$[K - \lambda M]\vec{X} = 0 \quad (5.2)$$

Let

$$[F_i] \equiv [F(\lambda_i, \vec{\delta})] \equiv [K - \lambda_i M] \quad (5.3)$$

where  $\lambda_i$  is  $i^{\text{th}}$  eigenvalue and therefore

$$[F_i]\vec{X}_i = \vec{0} \quad (5.4)$$

Premultiplication of Eq. (5.4) by  $\vec{X}_i^T$  gives

$$\vec{X}_i^T [F_i] \vec{X}_i = 0 \quad (5.5)$$

Differentiating Eq. (5.5) with respect to  $\delta_j$  , we get

$$\frac{\partial \vec{X}_i^T}{\partial \delta_j} [F_i] \vec{X}_i + \vec{X}_i^T \left[ \frac{\partial F_i}{\partial \delta_j} \right] \vec{X}_i + \vec{X}_i^T [F_i] \frac{\partial \vec{X}_i}{\partial \delta_j} = 0 \quad (5.6)$$

The first and the third term of Eq. (5.6) are zero, by virtue of Eq. (5.4) and therefore

$$\vec{X}_i^T \left[ \frac{\partial F_i}{\partial \delta_j} \right] \vec{X}_i = 0 \quad (5.7)$$

Differentiating Eq. (5.3) we obtain

$$\left[ \frac{\partial F_i}{\partial \delta_j} \right] \equiv [F_{i,j}] = \left[ [K,j] - \lambda_i [M,j] - \lambda_{i,j} [M] \right] \quad (5.8)$$

where  $[K,j]$  and  $[M,j]$  denote the matrices formed by differentiating the elements of  $[K]$  and  $[M]$  matrices, respectively, with respect to  $\delta_j$ . These matrices are presumably readily computed and they are generally quite sparse. Substituting Eq. (5.8) into Eq. (5.7) gives

$$\vec{X}_i^T ([K,j] - \lambda_i [M,j]) \vec{X}_i - \lambda_{i,j} \vec{X}_i^T [M] \vec{X}_i = 0 \quad (5.9)$$

or, to obtain the desired expression

$$\lambda_{i,j} = \frac{\vec{X}_i^T ([K,j] - \lambda_i [M,j]) \vec{X}_i}{\vec{X}_i^T [M] \vec{X}_i} \quad (5.10)$$

If the eigenvectors are taken to be M-orthonormal, i.e.

$$\vec{X}_i^T [M] \vec{X}_i = 1 \quad (5.11)$$

then, Eq. (5.10) simplifies to

$$\lambda_{i,j} = \vec{X}_i^T ([K,j] - \lambda_i [M,j]) \vec{X}_i \quad (5.12)$$

It is to be noted that the expression of Eq. (5.12) involves only the eigenvalue and the eigenvector under consideration and thus, a complete solution of the eigenproblem is not needed to obtain these derivatives. Expressions similar to Eq. (5.12) are available in Refs. 23 and 24. Furthermore, in Ref. 25, an



identical expression has been used for the elementary error analysis of the eigenproblem arising in structural dynamics.

### 5.3 Rate of Change of an Eigenvector

#### (a) Formulation 1

Consider the vector Eq. (5.4). Note that the matrix  $[F_i]$  is singular and is of rank  $(n-1)$  if the system of equations represented by Eqs. (5.2) has distinct eigenvalues.<sup>13</sup> Let us define

$$\vec{X}_i^T[M] \equiv \vec{V}_i^T \quad (5.13)$$

Then, Eq. (5.11) which represents that the eigenvectors are M-orthonormal can be written as:

$$\vec{V}_i^T \vec{X}_i = 1 \quad (5.14)$$

It is easy to show that Eq. (5.14) is linearly independent of the equations represented by Eqs. (5.4).

Let

$$[F_i] \equiv \begin{bmatrix} \vec{f}_i^{(1)T} \\ \hline \vec{f}_i^{(2)T} \\ \hline \cdot \\ \cdot \\ \hline \vec{f}_i^{(n)T} \end{bmatrix} \quad (5.15)$$

where  $\vec{f}_i^{(k)T}$ ,  $k = 1, 2, \dots, n$ , represent the row vectors of the matrix  $[F_i]$ . Then, from Eq. (5.4) we obtain

$$\vec{f}_i^{(k)T} \vec{X}_i = 0, \quad k = 1, 2, \dots, n \quad (5.16)$$

i.e.  $\vec{f}_i^{(k)T}$ ,  $k = 1, 2, \dots, n$  are orthogonal to the  $i^{\text{th}}$  eigenvector,  $\vec{X}_i$ .

Let  $\vec{V}_i^T$  be a linear combination of the row vectors  $\vec{f}_i^{(k)T}$ ,  $k = 1, 2, \dots, n$ . Therefore,

$$\vec{V}_i^T = a_1 \vec{f}_i^{(1)T} + a_2 \vec{f}_i^{(2)T} + \dots + a_n \vec{f}_i^{(n)T} \quad (5.17)$$

Postmultiplying Eq. (5.17) by  $\vec{X}_i$  we obtain

$$\vec{V}_i^T \vec{X}_i = a_1 \vec{f}_i^{(1)T} \vec{X}_i + a_2 \vec{f}_i^{(2)T} \vec{X}_i + \dots + a_n \vec{f}_i^{(n)T} \vec{X}_i \quad (5.18)$$

The right hand side of Eq. (5.18) is zero by virtue of Eq. (5.16) and this violates Eq. (5.14). Hence, we obtain that  $\vec{V}_i^T$  is linearly independent of the rows of matrix  $[F_i]$ .

Taking the set of Eqs. (5.4) and (5.11) together, we obtain a consistent but overdetermined system which can be written in partitioned form as

$$\begin{bmatrix} [F_i] \\ \vec{V}_i^T \\ \vec{X}_i^T [M] \end{bmatrix} \vec{X}_i = \begin{bmatrix} \vec{0} \\ -\vec{1} \end{bmatrix} \quad (5.19)$$

$(n+1) \times n \quad n \times 1 \qquad (n+1) \times 1$

An expression for  $\vec{X}_{i,j}$  can be obtained by differentiating

Eq. (5.19). For ease of algebra, let us go back to Eqs. (5.4) and (5.11) which constitute the set of Eqs. (5.19).

Differentiating the vector equation (5.4) we obtain

$$[F_i]\vec{X}_{i,j} + [F_{i,j}]\vec{X}_i = 0$$

or

$$[F_i]\vec{X}_{i,j} = -[F_{i,j}]\vec{X}_i \quad (5.20)$$

where  $[F_{i,j}]$  is given by Eq. (5.8).

Also differentiating Eq. (5.11) and collecting the terms we obtain the scalar equation

$$2\vec{X}_i^T[M]\vec{X}_{i,j} + \vec{X}_i^T[M,j]\vec{X}_i = 0$$

or

$$2\vec{X}_i^T[M]\vec{X}_{i,j} = -\vec{X}_i^T[M,j]\vec{X}_i \quad (5.21)$$

Taking Eqs. (5.20) and (5.21) together, we obtain

$$\begin{bmatrix} [F_i] \\ -\frac{\vec{X}_i^T[M]}{2\vec{X}_i^T[M]} \end{bmatrix} \vec{X}_{i,j} = - \begin{bmatrix} [F_{i,j}] \\ \vec{X}_i^T[M,j] \end{bmatrix} \vec{X}_i \quad (5.22)$$

Premultiplying Eq. (5.22) by

$$\begin{bmatrix} F_i \\ \vec{X}_i^T[M] \end{bmatrix}^T$$

we have

$$([F_i][F_i] + 2[M]\vec{X}_i\vec{X}_i^T[M])\vec{X}_{i,j} = -([F_i][F_{i,j}] + [M]\vec{X}_i\vec{X}_i^T[M,j])\vec{X}_i \quad (5.23)$$

or to obtain the desired expression

$$\vec{X}_{i,j} = -([F_i][F_i] + 2[M]\vec{X}_i\vec{X}_i^T[M])^{-1}([F_i][F_{i,j}] + [M]\vec{X}_i\vec{X}_i^T[M,j])\vec{X}_i \quad (5.24)$$

Once again the expression of Eq. (5.24) to find the rate of change of an eigenvector with respect to any design parameter involves only the eigenvalue and eigenvector under consideration and other known quantities. However, it does require the inverse of a  $n \times n$  matrix. Moreover, while checking the computational validity of the expression for  $\vec{X}_{i,j}$ , Eq. (5.24), it was observed that the terms of the matrix of which the inverse is needed are predominately those of the matrix product  $[F_i][F_i]$ . In other words, the order of magnitude of the elements of the second matrix expression  $2[M]\vec{X}_i\vec{X}_i^T[M]$  was relatively smaller than those of the matrix  $[F_i][F_i]$ . This resulted in an almost singular matrix, since  $[F_i]$  is a singular matrix. To circumvent this trouble, the last equation of the set of Eqs. (5.19) was multiplied by a large scalar quantity,  $b$ , and this "beefed up" the elements of the second matrix expression to  $2b[M]\vec{X}_i\vec{X}_i^T[M]$  and hence the final expression is

$$\vec{X}_{i,j} = -([F_i][F_i] + 2b[M]\vec{X}_i\vec{X}_i^T[M])^{-1}([F_i][F_{i,j}] + b[M]\vec{X}_i\vec{X}_i^T[M,j])\vec{X}_i \quad (5.25)$$

(b) Formulation 2

Since the eigenvectors form a complete set of vectors, any n-component vector can be represented as a linear combination of these and, in particular,  $\vec{X}_{i,j}$  can be represented as

$$\vec{X}_{i,j} = \sum_{k=1}^n a_{ijk} \vec{X}_k \quad (5.26)$$

Substituting Eq. (5.26) into Eq. (5.20) yields

$$[F_i] \sum_{k=1}^n a_{ijk} \vec{X}_k = -[F_{i,j}] \vec{X}_i \quad (5.27)$$

and premultiplying both sides of Eq. (5.27) by  $\vec{X}_\ell^T$ ,  $\ell \neq i$  gives

$$\sum_{k=1}^n a_{ijk} \vec{X}_\ell^T [F_i] \vec{X}_k = -\vec{X}_\ell^T [F_{i,j}] \vec{X}_i \quad (5.28)$$

which reduces to

$$a_{ij\ell} (\lambda_\ell - \lambda_i) = -\vec{X}_\ell^T [K_j - \lambda_i M_j] \vec{X}_i \quad (5.29)$$

since the M-orthonormal vectors are also K-orthogonal. Therefore,

$$a_{ij\ell} = \frac{\vec{X}_\ell^T ([K_j] - \lambda_i [M_j]) \vec{X}_i}{(\lambda_i - \lambda_\ell)}, \quad \ell \neq i \quad (5.30)$$

In order to get  $a_{iji}$ , we substitute Eq. (5.26) into Eq. (5.21)

which gives

$$2\vec{X}_i^T [M] \sum_{k=1}^n a_{ijk} \vec{X}_k = -\vec{X}_i^T [M_j] \vec{X}_i \quad (5.31)$$

Using the M-orthonormal relationship of the eigenvectors we get

$$a_{iji} = - \frac{\vec{x}_i^T [M, j] \vec{x}_i}{2} \quad (5.32)$$

It is to be noted that in contrast with Eq. (5.24), the expression of Eq. (5.26), where the coefficients are given by Eqs. (5.30) and (5.32), involves the complete solution of an eigenproblem, but does not require the inversion of an additional  $n \times n$  matrix. On the other hand, as indicated in Chapter 3 for the approximate dynamic analysis, it should be possible to approximate the derivative of the eigenvector in Eq. (5.26) by a partial sum

$$\vec{x}_{i,j} \approx \sum_{k=1}^s a_{ijk} \vec{x}_k, \quad s < n \quad (5.33)$$

This possibility has not been completely explored in the present work, but the results obtained by taking the first  $s$  of the  $n$  natural modes from the lowest end of the spectrum has indicated that the derivative of the eigenvector so obtained at least points in a direction approximately similar to the gradient vector obtained from Eq. (5.26). It is natural to speculate that a different subset of  $s$  eigenvectors might result in a better approximation.

#### 5.4 Numerical Results:

As a simple application, the cylindrical cantilever beam shown in Fig. (5.1) is considered. It is modeled with three discrete elements and hence the dynamic analysis is approximated

by six degrees of freedom. The design variables are the diameters of each element. The results obtained for the rate of change of eigenvalues are given in Table 5.1. The results for the rate of change of the first eigenvector (corresponding to the lowest eigenvalue) with respect to the design parameters are shown in Table 5.2.

Experience with a variety of problems has indicated that the expressions for the partial derivatives of eigenvalues and the components of an eigenvector are computationally reasonable. It will be seen in Chapter 6 that these quantities enter into the expressions for the derivative of the dynamic response quantities (displacements and stresses). Furthermore, it is indicated in Chapter 9 that this information can be useful in approximating the analysis of a new design which is not substantially different from the old one.

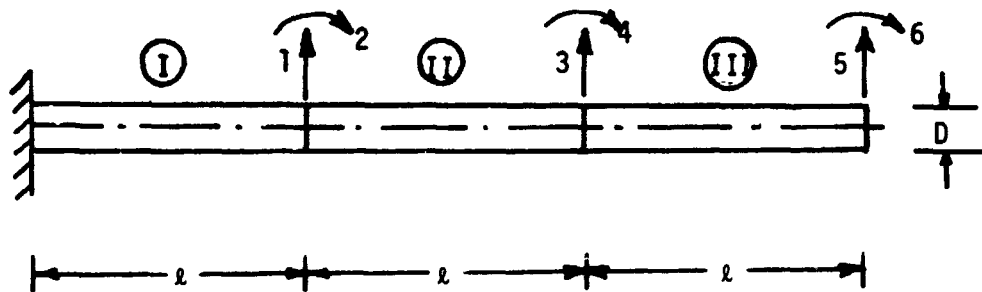
| Number | Eigenvalue             | $\lambda_{i,I}$         | $\lambda_{i,II}$       | $\lambda_{i,III}$      |
|--------|------------------------|-------------------------|------------------------|------------------------|
| i      | $\lambda_i$            |                         |                        |                        |
| 1      | $.2466 \times 10^8$    | $.3209 \times 10^9$     | $.3456 \times 10^8$    | $-.1582 \times 10^9$   |
| 2      | $.9747 \times 10^9$    | $.3860 \times 10^{10}$  | $.4352 \times 10^{10}$ | $-.4144 \times 10^9$   |
| 3      | $.7782 \times 10^{10}$ | $.2350 \times 10^{11}$  | $.1709 \times 10^{11}$ | $.2167 \times 10^{11}$ |
| 4      | $.3947 \times 10^{11}$ | $.1073 \times 10^{12}$  | $.7894 \times 10^{11}$ | $.1295 \times 10^{12}$ |
| 5      | $.5556 \times 10^{12}$ | $-.4899 \times 10^{11}$ | $.3229 \times 10^{12}$ | $.4171 \times 10^{13}$ |
| 6      | $.1398 \times 10^{12}$ | $.1411 \times 10^{12}$  | $.7540 \times 10^{12}$ | $.2233 \times 10^{12}$ |

Table 5.1 Eigenvalues and Rate of Change of Eigenvalues  
With Respect to  $D_I$ ,  $D_{II}$ , and  $D_{III}$ .



| Design Variable Number | Components of $x_{1,j} \times 10^{-2}$ |            |            |            |            |            |
|------------------------|----------------------------------------|------------|------------|------------|------------|------------|
| (j)                    | $x_{11,j}$                             | $x_{12,j}$ | $x_{13,j}$ | $x_{14,j}$ | $x_{15,j}$ | $x_{16,j}$ |
| I                      | -0.962                                 | 1.745      | -0.668     | -1.708     | 1.478      | -2.298     |
| II                     | 0.762                                  | -1.324     | -0.183     | 2.558      | -2.734     | 2.550      |
| III                    | -1.082                                 | 1.913      | -3.384     | 2.528      | -6.486     | 3.300      |

Table 5.2 - Rate of Change of Eigenvector No. 1  
With Respect to  $D_I$ ,  $D_{II}$  and  $D_{III}$ .



$$l = 1''$$

$$D_I = 0.25'' = D_{II} = D_{III}$$

$$\rho = 0.28 \text{ lbs./in.}^3$$

$$E = 30 \times 10^6 \text{ lbs./in.}^2$$

Fig. 5.1 Cantilever Rod; 6 Degrees-of-Freedom

## CHAPTER 6

### DERIVATIVES OF RESPONSE QUANTITIES

In automated design methods (for example, the method of feasible directions used in the present work) information relating changes in the dynamic response quantities to changes in the design variables is required, in order to satisfy, continuously in the design space, the limitations imposed on the response quantities. The restrictions imposed on the dynamic behavior in the present work involve limitations on upper bound displacements, upper bound stresses and the range in which the natural frequencies of the structure will be allowed to fall. Thus, we need to derive either approximate or exact expressions for the derivatives of upper bound dynamic displacements and upper bound stresses on the structure. We have already derived the exact expression for the derivative of eigenvalues, Eq. (5.12). We also have the expressions of upper bound displacements, Eq. (3.25), and of the upper bound stresses, Eqs. (3.43) and (3.44), and since the quantities involved in these expressions are continuous functions in the design space, an exact expression for the derivatives of these response quantities seems tractable.

#### 6.2 Derivative of Upper-Bound Dynamic Displacement

The  $j^{\text{th}}$  component of the upper bound displacement vector derived in Chapter 3, Eq. (3.25), is given by

$$y_{jm} \leq \sum_{i=1}^n |x_{ji}| |c_i| \varepsilon_p(\omega_i), \quad j = 1, 2, \dots, n \quad (6.1)$$

where the scalar quantity  $|c_i| \varepsilon_p(\omega_i)$  is the  $i^{\text{th}}$  peak modal participation coefficient and  $x_{ji}$  represents the  $j^{\text{th}}$  component of the  $i^{\text{th}}$  eigenvector.

The expression of Eq. (6.1) can be rewritten as

$$\begin{aligned} y_{jm} &\leq \sum_{i=1}^n |x_{ji} c_i \varepsilon_p| \\ &= \sum_{i=1}^n \text{sign}(x_{ji} c_i \varepsilon_p) x_{ji} c_i \varepsilon_p \end{aligned} \quad (6.2)$$

where

$$\text{sign}(z) = \begin{cases} 1 & z > 0 \\ 0 & z = 0 \\ -1 & z < 0 \end{cases}$$

Notice that though the function  $y_{jm}$  is continuous, its derivative may not be defined at the point where one of the summands is zero, and in such an eventuality, the expression of the derivative which follows would break down. However, in the present work no trouble was encountered.

Differentiating Eq. (6.2) we get

$$\begin{aligned} \frac{\partial y_{jm}}{\partial \delta_k} &= \sum_{i=1}^n \text{sign}(x_{ji} c_i \varepsilon_p) \left[ \frac{\partial x_{ji}}{\partial \delta_k} c_i \varepsilon_p \right. \\ &\quad \left. + x_{ji} \frac{\partial c_i}{\partial \delta_k} \varepsilon_p + x_{ji} c_i \frac{\partial \varepsilon_p}{\partial \delta_k} \right] \end{aligned} \quad (6.3)$$

where  $\frac{\partial x_{ji}}{\partial \delta_k} = x_{ji,k}$  is the change in the  $j^{\text{th}}$  component of the  $i^{\text{th}}$  eigenvector with respect to the change in the  $k^{\text{th}}$  design variable and is given in Chapter

From Eq. (3.19) we have expression for  $c_i$  as

$$c_i = \vec{X}_i^T [M] \hat{e} \quad (6.4)$$

Differentiating Eq. (6.4) we obtain

$$\frac{\partial c_i}{\partial \delta_k} = \frac{\partial \vec{X}_i}{\partial \delta_k} [M] \hat{e} + \vec{X}_i^T \left[ \frac{\partial M}{\partial \delta_k} \right] \hat{e} \quad (6.5)$$

where  $\frac{\partial \vec{X}_i}{\partial \delta_k} \equiv \vec{X}_{i,k}$  is given in Chapter 5 and  $\left[ \frac{\partial M}{\partial \delta_k} \right] \equiv [M, k]$

denotes the matrix formed by differentiating the elements of matrix  $[M]$  with respect to  $\delta_k$ .

It has been seen that the shock spectrum gives the peak response quantity  $\xi_p$  as a function of  $\omega$ . Hence the quantity  $\frac{\partial \xi_p}{\partial \omega}$  can be obtained from the shock spectrum. Notice that  $\xi_p$  may not be a continuously differentiable function of  $\omega$ . This would lead to trouble, if we happen to hit the points of sharp kinks on the shock spectral curve. However, in the present work, no such trouble was encountered. Using the chain rule we obtain

$$\left. \frac{\partial \xi_p}{\partial \delta_k} \right|_{\omega=\omega_i} = \left. \frac{\partial \xi_p}{\partial \omega} \right|_{\omega=\omega_i} \cdot \left. \frac{\partial \omega}{\partial \delta_k} \right|_{\omega=\omega_i} \quad (6.6)$$

Denoting

$$\left. \frac{\partial \xi_p}{\partial \omega} \right|_{\omega=\omega_i} \equiv \frac{\partial \xi_p}{\partial \omega_i} \quad \text{and} \quad \left. \frac{\partial \omega}{\partial \delta_k} \right|_{\omega=\omega_i} \equiv \frac{\partial \omega_i}{\partial \delta_k}$$

we obtain from Eq. (6.6)

$$\frac{\partial \xi_p}{\partial \delta_k} = \frac{\partial \xi_p}{\partial \omega_i} \cdot \frac{\partial \omega_i}{\partial \delta_k} \quad (6.7)$$

Since

$$\omega_i = \sqrt{\lambda_i}$$

we obtain

$$\frac{\partial \omega_i}{\partial \delta_k} = \frac{1}{2} \cdot \frac{1}{\sqrt{\lambda_i}} \cdot \frac{\partial \lambda_i}{\partial \delta_k} \quad (6.8)$$

Substituting the expression of Eq. (6.8) into Eq. (6.7) we obtain

$$\frac{\partial \xi_p}{\partial \delta_k} = \frac{1}{2} \cdot \frac{1}{\sqrt{\lambda_i}} \cdot \frac{\partial \xi_p}{\partial \omega_i} \cdot \frac{\partial \lambda_i}{\partial \delta_k} \quad (6.9)$$

The expression for  $\frac{\partial \lambda_i}{\partial \delta_k} \equiv \lambda_{i,k}$  is given by Eq. (5.12). Thus,

we see that the expression for  $\frac{\partial y_{jm}}{\partial \delta_k} \equiv y_{jm,k}$  involves all the known quantities and hence the derivative of the upper bound displacement can be computed.

### 6.3 Derivative of Upper-Bound Stress

The upper bound stress on the  $i$ th element derived in

Chapter 3 is given by Eqs. (3.42) and (3.43) as

$$\sigma_m^i(x) \leq \sum_{j=1}^n \sigma_{ab}^{ji}(x) |c_j| \varepsilon_p(\omega_j) \quad (6.10)$$

where

$$\begin{aligned} \sigma_{ab}^{ji}(x) &= E \left| \vec{r}^{(u)T}(x) [T]_i \{\vec{\lambda}_j\}_i \right| + \\ &\quad E |h| \left| \vec{r}^{(w)T}(x) [T]_i \{\vec{\lambda}_j\}_i \right| \\ &= \left| \sigma^{ji(A)}(x) \right| + \left| \sigma^{ji(B)}(x) \right| \end{aligned} \quad (6.11)$$

The first term on the right hand side of Eq. (6.11) represents the absolute value of the axial stress and the second term represents the absolute value of the bending stress at any point  $x$  on the  $i$ th element due to the displacement corresponding to the  $j$ th natural mode shape (eigenvector).

We have, therefore,

$$\begin{aligned} \sigma_m^i(x) &\leq \sum_{j=1}^n \left[ \left| \sigma^{ji(A)}(x) \right| + \left| \sigma^{ji(B)}(x) \right| \right] |c_j| \varepsilon_p(\omega_j) \\ &= \sum_{j=1}^n \left[ \left| \sigma^{ji(A)}(x) c_j \varepsilon_p \right| + \left| \sigma^{ji(B)}(x) c_j \varepsilon_p \right| \right] \end{aligned} \quad (6.12)$$

Differentiating Eq. (6.12) with respect to  $\delta_k$  we obtain

$$\begin{aligned}
\frac{\partial \sigma_m^i(x)}{\partial \delta_k} = & \sum_{j=1}^n \left| \text{sign}(\sigma^{ji(A)}(x) c_j \varepsilon_p) \left\{ \frac{\partial \sigma^{ji(A)}(x)}{\partial \delta_k} c_j \varepsilon_p \right. \right. \\
& + \sigma^{ji(A)}(x) \frac{\partial c_j}{\partial \delta_k} \varepsilon_p + \sigma^{ji(A)}(x) c_j \frac{\partial \varepsilon_p}{\partial \delta_k} \Big\} \\
& + \text{sign}(\sigma^{ji(B)}(x) c_j \varepsilon_p) \left\{ \frac{\partial \sigma^{ji(B)}(x)}{\partial \delta_k} c_j \varepsilon_p \right. \\
& + \sigma^{ji(B)}(x) \frac{\partial c_j}{\partial \delta_k} \varepsilon_p + \sigma^{ji(B)}(x) c_j \frac{\partial \varepsilon_p}{\partial \delta_k} \Big\} \Big|
\end{aligned} \quad (6.13)$$

where

$$\frac{\partial \sigma^{ji(A)}(x)}{\partial \delta_k} = E \vec{r}(u)^T(x) [T]_i \frac{\partial \{\vec{X}_j\}_i}{\partial \delta_k}$$

and

$$\begin{aligned}
\frac{\partial \sigma^{ji(B)}(x)}{\partial \delta_k} = & \left| E h \vec{r}(w)^T(x) [T]_i \frac{\partial \{\vec{X}_j\}_i}{\partial \delta_k} \right. \\
& \left. + E \frac{\partial h}{\partial \delta_k} \vec{r}(w)^T(x) [T]_i \{\vec{X}_j\}_i \right|
\end{aligned} \quad (6.14)$$

It should be remembered that

$$\{\vec{X}_j\}_i \xleftarrow{SR_i} \vec{X}_j$$

and similarly

$$\frac{\partial \{\vec{X}_j\}_i}{\partial \delta_k} \xleftarrow{SR_i} \frac{\partial \vec{X}_j}{\partial \delta_k} \quad (6.15)$$



All the quantities on the right hand side of Eq. (6.13) are known and thus the derivative of the upper bound stress at any point on an element of the structure can be computed.

## CHAPTER 7

### OPTIMIZATION ALGORITHM<sup>†</sup>

The problem of minimum weight design of truss-frame structures has been stated as a nonlinear programming problem in Chapter 1. The method of analysis leading us to the desired quantifiable information of the dynamic behavior on which restrictions are to be imposed has been developed in Chapters 3 and 4. The selection of a solution procedure and its description is the topic of this chapter.

There are many algorithms available to seek the solution of a nonlinear programming problem. These can be classified as: (1) Direct Methods, and (2) Indirect Methods.

Direct Methods are the ones where the constraint equations are explicitly handled in the algorithm while indirect methods cast a constrained minimization problem as a sequence of unconstrained minimization problem. It should be pointed out that the equivalent unconstrained minimization formulations of the constrained minimization problems generate functions which are sometimes difficult to minimize. Relative minima present in the basic problem statement are in some cases compounded in the substitute problem. Usually the convexity of the functions involved in the basic problem statement cannot be assured and strict equivalence of the substitute problem cannot be guaranteed. Furthermore, such substitute formulations usually require the minimization of a sequence of unconstrained minimization

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<sup>†</sup>The discussion in this chapter closely follows that in a book on optimization methods by Professor R. L. Fox, to be published by the Addison Wesley Co. The material is included here for completeness.

minimization problems, each of which quite often require a large number of function and gradient evaluations. Unless the equivalent unconstrained minimization problem generates a function whose value can be evaluated rather easily and so also the gradient vector can be explicitly computed, such formulations turn out to be computationally very inefficient.

One approach to automated optimum design problems with difficult analyses is to adopt a method which conserves the time spent doing the analysis. In other words, when the analysis consumes the majority of the computational effort, it is an indication that redesign should be done with a great deal of care. It is for this reason that an optimization technique from the Direct Methods category was chosen for the purposes of the present work. These are the methods in which, at least in principle, each step which is taken and which will require a reanalysis of the structure is made only after a great deal of care is taken to ensure that effort expended in analysis will be worthwhile.

Of the direct methods of attack on general nonlinear inequality constrained optimization problems a significant class is referred to as the Methods of Feasible Directions. Two well known procedures which embody the philosophy of the method of feasible directions are (1) Rosen's procedure<sup>20,26</sup> and (2) Zoutendijk's procedure<sup>27</sup>. Rosen's gradient projection algorithm is perhaps one of the most efficient when all the constraints of the nonlinear programming problem are linear although less so in the case where the

constraints are nonlinear. Hence Zoutendijk's method of feasible direction has been used to optimize the truss-frame structures considered in the present work.

## 7.2 Methods of Feasible Directions

These methods consist of an iteration of the form

$$\vec{D}_{q+1} = \vec{D}_q + \alpha \vec{S}_q \quad (7.1)$$

where the direction of move  $\vec{S}_q$  and the "step length",  $\alpha$ , are always chosen so that  $\vec{D}_{q+1}$  is in the feasible domain. They are called feasible direction methods because of the properties of the direction  $\vec{S}_q$  that (1) a small move in the direction  $\vec{S}_q$  violates no constraint, and (2) the objective function improves in the direction  $\vec{S}_q$ .

A direction  $\vec{S}_q$  satisfying (1) is called a feasible direction while a direction satisfying both (1) and (2) is called a useable feasible direction.

Let all the inequality constraints be cast in the form  $g_j \leq 0$ ,  $j = 1, 2, \dots, q$  where  $q$  are the total inequality constraints on the minimization problem, then for problems with sufficiently smooth "inward curving" constraint surfaces, (1) is satisfied if

$$\vec{S}^T \nabla g_j < 0 \quad (7.2)$$

or if a constraint is linear or "outward curving"

$$\vec{S}^T \nabla g_j \leq 0 \quad (7.3)$$

The significance of this interpretation of (1) is that the vector  $\vec{S}$  must make an obtuse angle with all constraint normals except that, for the linear or outward curving ones, the angle may go to  $90^\circ$  as shown in Fig. 7.1(b). Any move vector,  $\vec{S}$ , satisfying the strict inequality lies, at least partly, in the feasible region of the space. In other words, there is some  $\alpha > 0$  for which  $\vec{D}_{q+1}$  is in the feasible domain.

If  $W(\vec{D})$  is the objective function, then a useable feasible direction,  $\vec{S}$ , has to satisfy

$$\vec{S}^T \nabla W < 0 \quad (7.4)$$

in addition to the strict inequality of Eq. (7.2) or (7.3). A vector,  $\vec{S}$ , satisfying the strict inequality of Eq. (7.4) is guaranteed to produce, for some  $\alpha > 0$ , a design  $\vec{D}_{q+1}$  which reduces the weight  $W$ .

Methods of feasible directions are those which produce an improving succession of feasible  $\vec{D}_q$  vectors by moving in a succession of useable feasible directions. There are two general parts to these algorithms. First, a useable feasible vector must be determined for each step of the iteration and second, the step size must be determined.

### 7.3 Finding a Direction

Assuming that a feasible point  $\vec{D}_q$  has been attained at

which one or more constraints  $g_j = 0$  ,  $j \in J$  where  $J$  is a set composed of the indices of the active constraints and all other constraints are negative, finding a direction requires to seek a vector  $\vec{S}$  such that

$$\nabla g_j^T \vec{S} < 0 \quad , \quad j \in J \quad (7.5)$$

and

$$\nabla W^T \vec{S} < 0 \quad (7.6)$$

If there were no constraints active, (i.e. if  $J$  is empty), then  $\vec{S}$  could be the same as  $-\nabla W$  and the move would be in the direction of steepest descent. When, however, some constraints are active, the determination of a suitable  $\vec{S}$  requires the establishment of an algorithm. There are many ways of choosing such a direction vector  $\vec{S}$  and hence many different methods of feasible directions. In the present work, one of Zoutendijk's methods of obtaining the direction vector has been used and will now be discussed.

Find the direction vector  $\vec{S}$  and the scalar quantity  $\beta$  such that

$\beta$  is a maximum

and

$$\nabla g_j^T \vec{S} + \theta_j \beta \leq 0 \quad , \quad j \in J \quad (7.7)$$

$$\nabla W^T \vec{S} + \beta \leq 0 \quad (7.8)$$

and

some measure of the length of  $\vec{S}$  is bounded

In the present work the direction vector  $\vec{S}$  is normalized by imposing bounds on the components of  $\vec{S}$

$$-1 \leq s_i \leq 1, \quad i = 1, \dots, m \quad (7.9)$$

The scalar quantity  $\theta_j$  in Eq. (7.7) are arbitrary non-negative constants (taken as zero or +1, in the present work, depending on whether the  $j^{\text{th}}$  constraint is linear or nonlinear). Clearly, if  $\beta_{\max} > 0$ , then the strict inequalities of Eqs. (7.5) and (7.6) are satisfied.

There are other possible normalizations<sup>27</sup> and the direction finding problem will be a linear or nonlinear programming problem depending on the way the length of the direction vector  $\vec{S}$  is fixed. For example, if we would have decided to use the Euclidean norm as the length of  $\vec{S}$ , then in place of Eq. (7.9) we would require

$$\vec{S}^T \vec{S} \leq 1$$

This obviously introduces a nonlinear (quadratic) constraint and the direction finding problem becomes a nonlinear programming problem. It is shown by Zoutendijk that this nonlinear programming problem may be modified and treated by

linear programming. However, the use of Eq. (7.9) as the normalization of vector  $\vec{S}$  is simple and leads to the direction finding problem as an optimization problem of linear programming, the solution to which is obtained by Simplex<sup>28</sup> algorithm. The normalization of Eq. (7.9) biases the direction vector  $\vec{S}$  towards a corner of the unit hypercube, (i.e. there is a tendency to produce the vector whose components are +1 or -1). However, this was not considered to be a serious disadvantage.

It would be observed from the above that Zoutendijk's procedure of finding the useable feasible direction requires to solve a secondary optimization problem at each iteration of the primary optimization problem. However, the secondary optimization problem is quite easy to solve and the results are usually worth the extra effort.

#### 7.4 Step Size:

Assuming that a useable feasible vector has been obtained, the objective of selecting the step length,  $\alpha$ , is to reduce the objective function  $W$ , as much as possible, while remaining in the feasible domain. The outcome of such a step length can be:

1. The point  $\vec{D}_{q+1}$  is bounded with respect to a subset of  $g_j$  which may contain some of the same constraints that  $\vec{D}_q$  was bound against. (Fig. 7.2).

or



2. The point  $\vec{D}_{q+1}$  may be an unconstrained minimum with respect to  $\alpha$ . (Fig. 7.3).

The only possible outcome in our case is (1) since a linear objective function has no unconstrained minima in any direction. In other words, the objective function is decreasing at all points along  $\vec{S}$ . Thus, we can seek a step size as large as possible without violating any constraint.

The actual determination of step length is a trial and error technique which involves complicated logical decisions. The general idea is: (1) to take a trial step,  $\alpha_t$ , (to reduce the objective function by a given amount, usually 5 or 10 percent) and (2) to check the constraints at the trial point; (2a) if they are in violation, interpolate the worst (i.e. the largest  $g_j$ ) violated constraint (approximating the constraint to be a linear or quadratic function of  $\alpha$ ) which will send us back to a feasible point; (2b) if the constraints are not bounded or violated, take another step in the same direction; and (2c) if they are bounded, determine a new direction.

#### 7.5 The Initial Trial Step Size:

The selection of an initial step size,  $\alpha_t$ , is to reduce the objective function a given amount (usually 5 to 10%). To do this, we express the linear objective function  $W(\vec{D}_q + \alpha\vec{S})$  by a linear function

$$W(\alpha) = a + b\alpha \quad (7.10)$$

and given the information

$$W_q = W(0) = W(\vec{D}_q) \quad (7.11)$$

$$W'_q = \left. \frac{dW}{d\alpha} \right|_{\alpha=0} = \vec{S}^T \nabla W$$

we have

$$W(\alpha) = W_q + \vec{S}^T \nabla W \alpha \quad (7.12)$$

In order to attain a reduction in  $W$  of  $\Delta W$ , we select  $\alpha_t$  according to

$$W_q + \vec{S}^T \nabla W \alpha_t = W_q - \Delta W \quad (7.13)$$

or

$$\alpha_t = - \frac{\Delta W}{\vec{S}^T \nabla W} \quad (7.14)$$

$\alpha_t$  would be positive since  $\vec{S}^T \nabla W$  is negative from satisfaction of the strict inequality, Eq. (7.6). Note that  $\alpha_t$  will be large if  $\vec{S}$  is nearly perpendicular to  $\nabla W$  and smaller as it is more nearly parallel to  $\nabla W$ .

## 7.6 Correcting the Step:

Given the trial value of  $\alpha_t$ , the next operation is

tentatively to take the step

$$\vec{D}_t = \vec{D}_q + \alpha_t \vec{S}_q$$

and check the constraints at the new point.

There are three possible results of this test:

- (a) the point is feasible and not bound
- (b) the point is feasible and bound
- (c) the point is infeasible

If (a) is the outcome, then another step may be taken in the same direction. (Fig. 7.4).

Result (b), as mentioned before, is the desired one and calls for a new solution to the direction finding problem. It is not, however, a straight-forward proposition to determine that the point is indeed bound because of the computational problem of obtaining a point where one or more constraints,  $g_j$ , are exactly equal to zero. Therefore, some sort of margin is required.

In order to provide some margin and still keep the point to be strictly feasible, a point is considered to be bound, in the present work, if

$$-\epsilon \leq g_j \leq 0 \quad (7.15)$$

In a sense, the criterion of Eq. (7.15) gives the constraint some "thickness" and makes it an easier target. This thickness of the constraint would be, henceforth, referred to as constraint

tolerance.

If (c) is the result of a trial step, then presumably the step was too long and it must be shortened to attain a bound point. A simple and effective means of modifying the step is to interpolate the value of the offended constraint. Often more than one constraint will be in violation and in such a case the interpolation is carried out with respect to the worst violated constraint, say  $g_k$ . Denote

$$g_k^{(0)} \equiv g_k(\vec{D}_q) < 0$$

and

(7.16)

$$g_k^{(N)} \equiv g_k(\vec{D}_q + \alpha_t \vec{S}) > 0$$

then, an interpolation to obtain a bound  $g_k$  can be accomplished by assuming  $g_k$  to be (1) a linear function or (2) a quadratic function of  $\alpha$ . The quadratic interpolation is used when we obtain in the design cycle a worst violated constraint which was bounded in the last feasible design. The need for quadratic interpolation arises from the fact that the corrected step size may turn out to be negative by the use of linear interpolation when the worst violated constraint in any cycle of iteration happened to be bounded in the last feasible design as shown in Fig. 7.5. The negative step length results into a move in the direction opposite to the useable feasible direction which we obviously do not want.

In order to carry out the interpolation by assuming a linear function, the values of the offended constraint are required at the old (feasible) point and the new (violated) point which are given by Eqs. (7.16). However, one more piece of information is needed to perform the quadratic interpolation and this can be taken as the slope of the constraint  $g_k$  at the old point.

Linear Interpolation:

A linear interpolation to obtain a bound  $g_k$  can be derived from assuming a linear function

$$g_k(\alpha) \approx a + b\alpha \quad (7.17)$$

from which

$$g_k^{(0)} \equiv g_k(0) = a \quad (7.18)$$

and

$$g_k^{(N)} \equiv g_k(\alpha_t) = a + b\alpha_t$$

Therefore

$$b = \frac{g_k^{(N)} - g_k^{(0)}}{\alpha_t} \quad (7.19)$$

and the desired  $\alpha_I$  is obtained from

$$g_k(\alpha_I) = g_k^{(0)} + \frac{g_k^{(N)} - g_k^{(0)}}{\alpha_t} \alpha_I = 0 \quad (7.20)$$

If the center of the tolerance zone,  $g_k = -\frac{\epsilon}{2}$ , is the worst get rather than  $g_k = 0$ , we have from Eq. (7.20)

$$g_k^{(0)} + \frac{g_k^{(N)} - g_k^{(0)}}{\alpha_t} \alpha_I = -\frac{\epsilon}{2} \quad (7.21)$$

where from

$$\alpha_I = -\frac{g_k^{(0)} + \epsilon/2}{g_k^{(N)} - g_k^{(0)}} \alpha_t \quad (7.22)$$

Note that the denominator of the expression on the right hand side of Eq. (7.22) is always positive from Eqs. (7.16). Hence the value of interpolated step length,  $\alpha_I$ , would be positive or negative depending on the relative magnitude of  $g_k^{(0)}$  and  $\epsilon/2$ . For example,

$$|g_k^{(0)}| < \left|\frac{\epsilon}{2}\right|$$

will result in the negative value of  $\alpha_I$ . This situation is depicted in Fig. 7.5. It should be noticed that the negative step length would be possible only when a bounded constraint of a feasible design is the worst violated constraint in the subsequent design cycle. A plot of the value of the offended constraint in the direction of  $\vec{S}$  approximated as a linear function of  $\alpha$  resulting into the case  $\alpha_I < 0$  is shown in Fig. 7.6. We notice from this figure that the approximation of the function,  $g_k(\alpha)$  by a quadratic between the points  $g_k^{(0)}$

and  $g_k^{(N)}$  will give us two positive roots of  $\alpha_I$  provided the slope at the point  $g_k^{(0)}$  is negative, i.e.

$$g_k^{(0)'} \equiv \left. \frac{d}{d\alpha} g_k(\vec{D}_q + \alpha \vec{S}_q) \right|_{\alpha=0} < 0$$

which means

$$g_k^{(0)'} = \vec{S}_q^T \nabla g_k^{(0)} < 0$$

and this in fact is the definition of the feasible direction,  $\vec{S}_q$ , Eq. (7.2).

#### Quadratic Interpolation:

A quadratic interpolation to obtain a bound  $g_k$  can be derived from assuming a quadratic function

$$g_k(\alpha) \approx c + b\alpha + a\alpha^2 \quad (7.23)$$

from which

$$g_k^{(0)} \equiv g_k(0) = c \quad (7.24)$$

$$g_k^{(0)'} \equiv \vec{S}_q^T \nabla g_k^{(0)} = b \quad (7.25)$$

and

$$g_k^{(N)} \equiv g_k(\alpha_t) = c + b\alpha_t + a\alpha_t^2$$

Therefore

$$a = \frac{g_k^{(N)} - g_k^{(0)} - g_k^{(0)'} \alpha_t}{\alpha_t^2} \quad (7.26)$$

and the desired  $\alpha_I$  is obtained from

$$g_k(\alpha_I) = c + b\alpha_I + a\alpha_I^2 = -\frac{\epsilon}{2} \quad (7.27)$$

where the center of the tolerance zone is the target.

We obtain from Eq. (7.27)

$$\alpha_I = \frac{-b \pm \sqrt{b^2 - 4a(c + \epsilon/2)}}{2a} \quad (7.28)$$

We are interested in the largest positive value, as is seen from Fig. 7.7; therefore,

$$\alpha_I = \frac{-b + \sqrt{b^2 - 4a(c + \epsilon/2)}}{2a} \quad (7.29)$$

where the quantities  $a$ ,  $b$ , and  $c$  are respectively given by Eqs. (7.26), (7.25), and (7.24).

It is to be noted that  $c$  and  $b$  are always negative, and since  $\alpha_t$  is a positive quantity, we have the value of  $a$ , Eq. (7.26), to be positive. However, if the slope is very small at the old (bounded) point such that the discriminant of Eq. (7.28) becomes a negative quantity, then the quadratic interpolation also fails. Geometrically, this means that the



approximating quadratic function does not cross the line represented by

$$g_k(\alpha) = -\frac{\epsilon}{2}$$

This situation is represented in Fig. 7.8. However, by reducing the tolerance, i.e. by setting

$$-\frac{\epsilon}{2} = g_k^{(0)}$$

we obtain

$$\alpha_I = -\frac{b}{a} \quad (7.30)$$

which is a positive step length in the direction of  $\vec{S}_q$ , as shown in Fig. 7.9 yielding the desired design point.

### 7.7 Constraint Tolerance, the Zigzag and the Termination Criterion

The practical necessity of using constraint margins also relieves us from a phenomenon commonly called, zigzagging. In order to visualize this problem let us consider a three dimensional design space with two constraints as shown in Fig. 7.10. At point A, constraint 1 is active but not constraint 2 and vice versa at point B. The "downhill" direction with respect to the objective function is shown by the arrow. If the direction finding problem is given only the gradients to the exactly active constraints at A or B, then the useable feasible direction which results may point directly at the "almost active" constraint. This produces

a path as shown in Fig. 7.11. One means of avoiding this situation is to use a substantial tolerance in the determination of the boundedness of a constraint. Points A or B, by this approach, are considered bound on both  $g_1$  and  $g_2$  and the only useable feasible directions left are those which are directed more or less along the intersection of the constraints; thus relieving us from the zigzag path.

A point ultimately is reached where the gradient  $\nabla W$  and the constraint normals whose indices are included in  $J$  so delimit the direction finding problem that it has only trivial ( $\vec{S} = \vec{0}$  ;  $\beta = 0$ ) solutions. It has been shown<sup>27</sup> that under normal circumstances this indicates that the Kuhn-Tucker conditions are satisfied and the design is optimal. However, since the constraint have some constraint margins, the constraint indices included in the set  $J$  are not only those of the bounded constraints and hence the trivial solution ( $\vec{S} = \vec{0}$  ;  $\beta = 0$ ) may be a false indication. What is done, therefore, is to reduce the constraint tolerance  $\epsilon$  , say in half, when no useable feasible direction can be found, and the direction finding problem is reattempted. This process is reiterated several times. If, after  $\epsilon$  is reduced to some level of  $\epsilon_f$  , a direction still can not be found, the point is assumed to be an approximation to a local minimum.

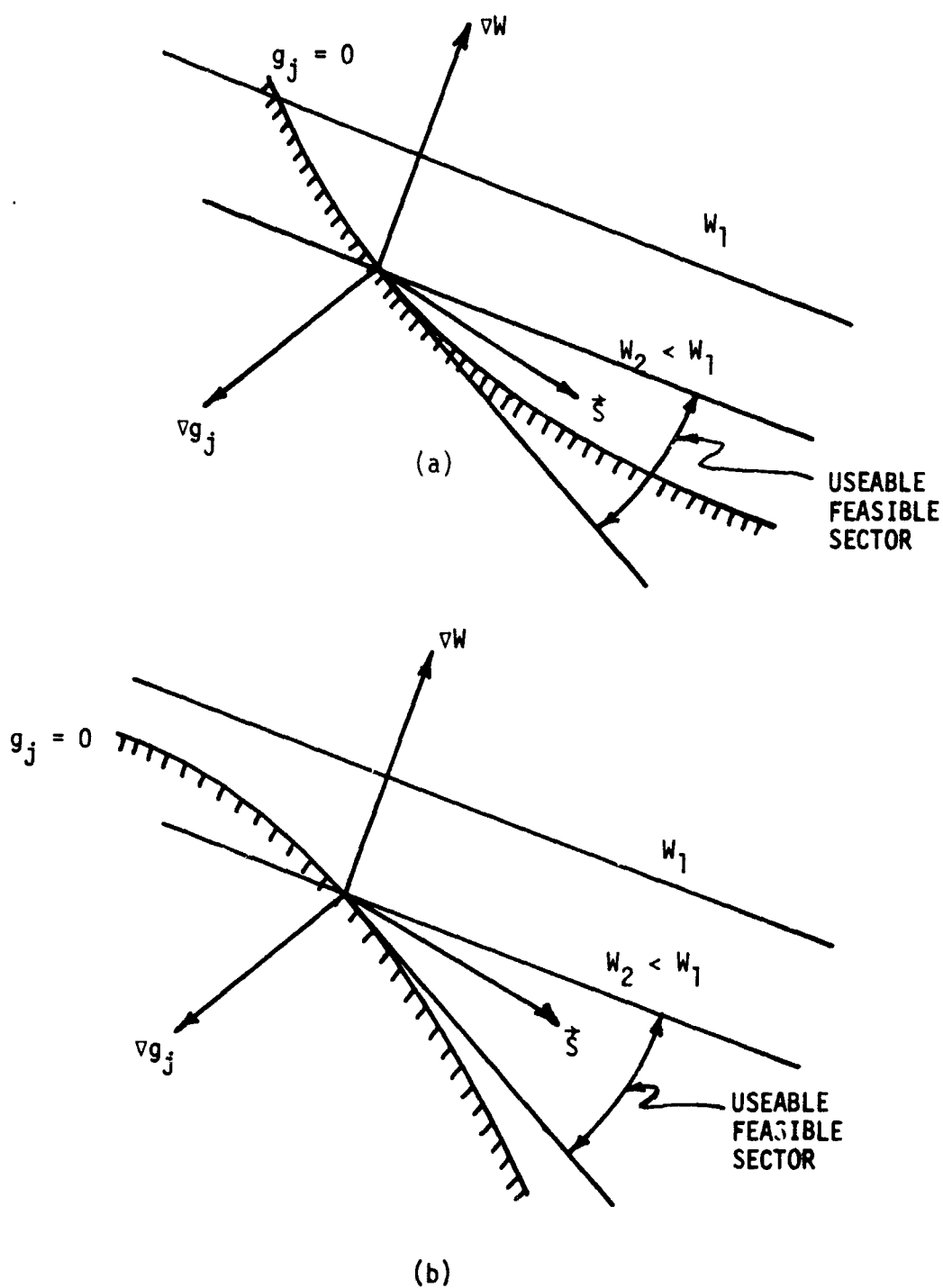


Fig. 7.1 Schematic of Useable Feasible Sector

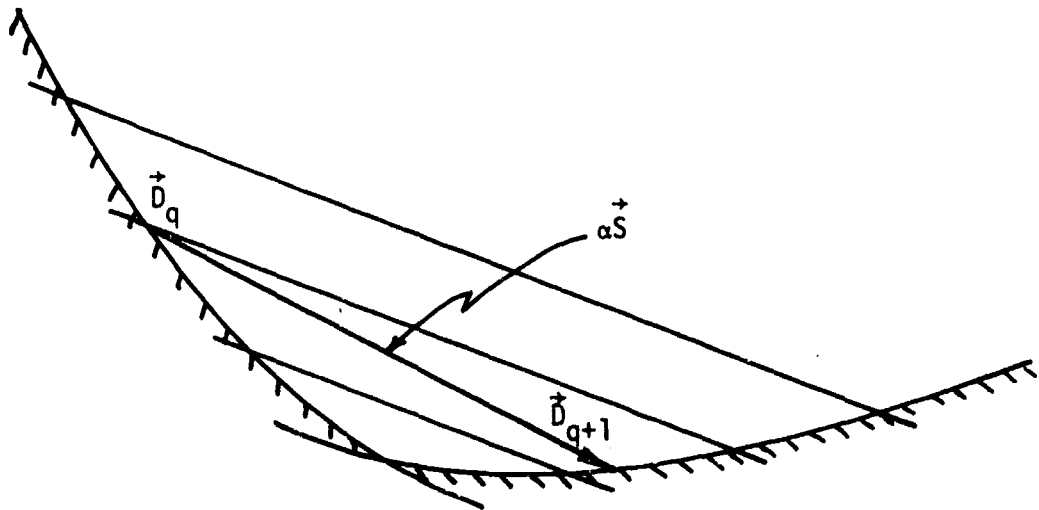


Fig. 7.2 Linear Objective Function

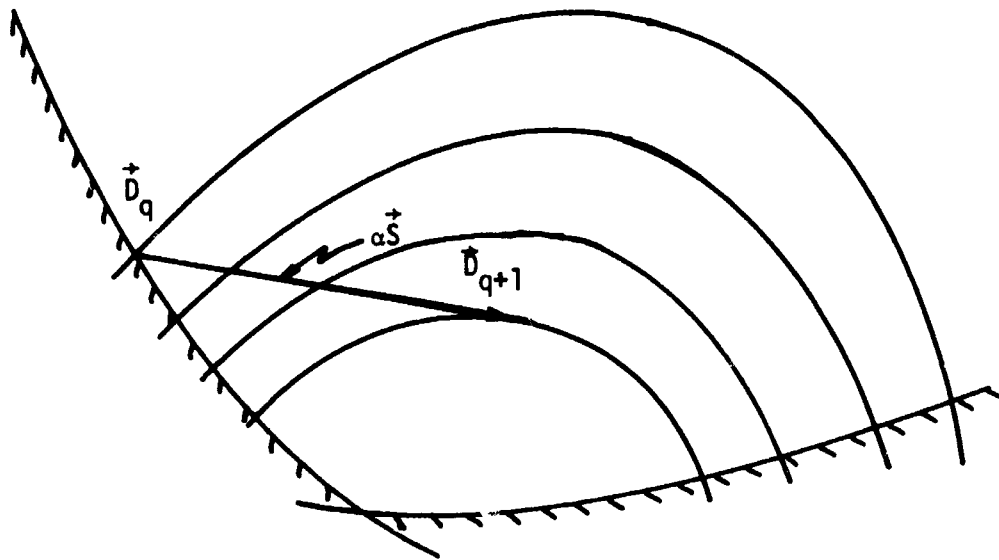


Fig. 7.3 Non-linear Objective Function

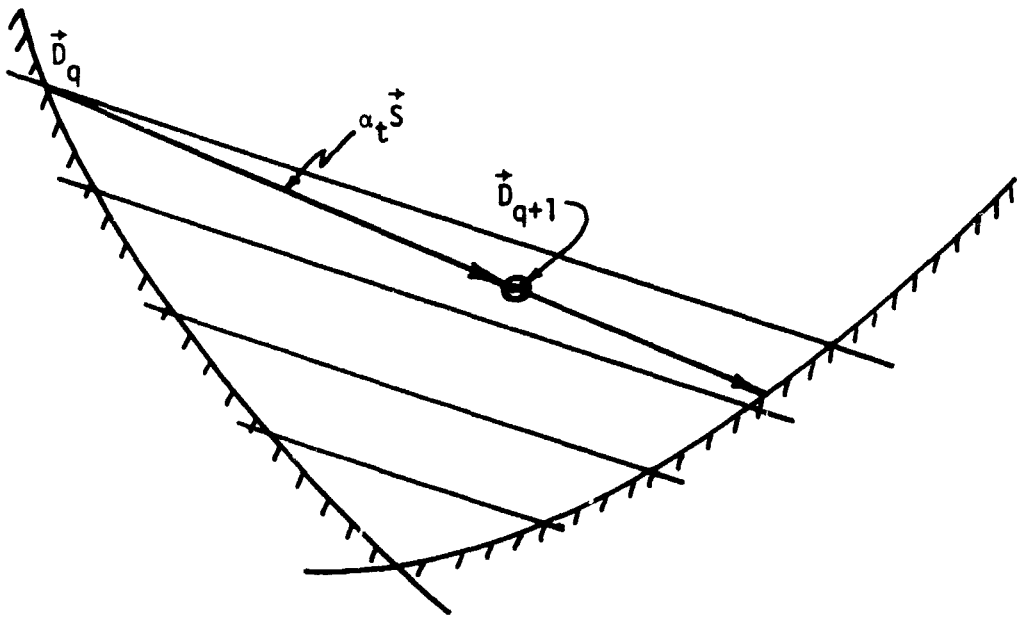


Fig. 7.4 Continuing Feasible Direction Step

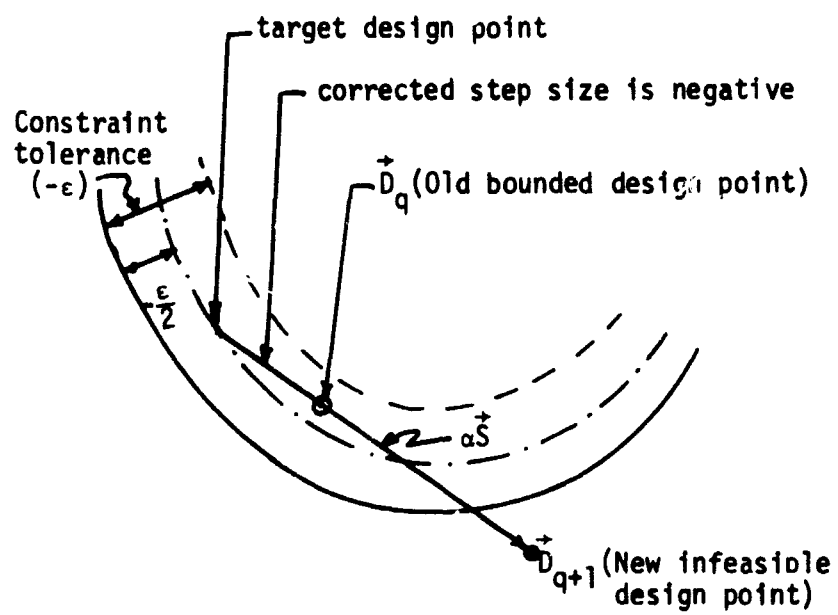


Fig. 7.5 Linear Interpolation Leading to Negative Corrected Step Size.

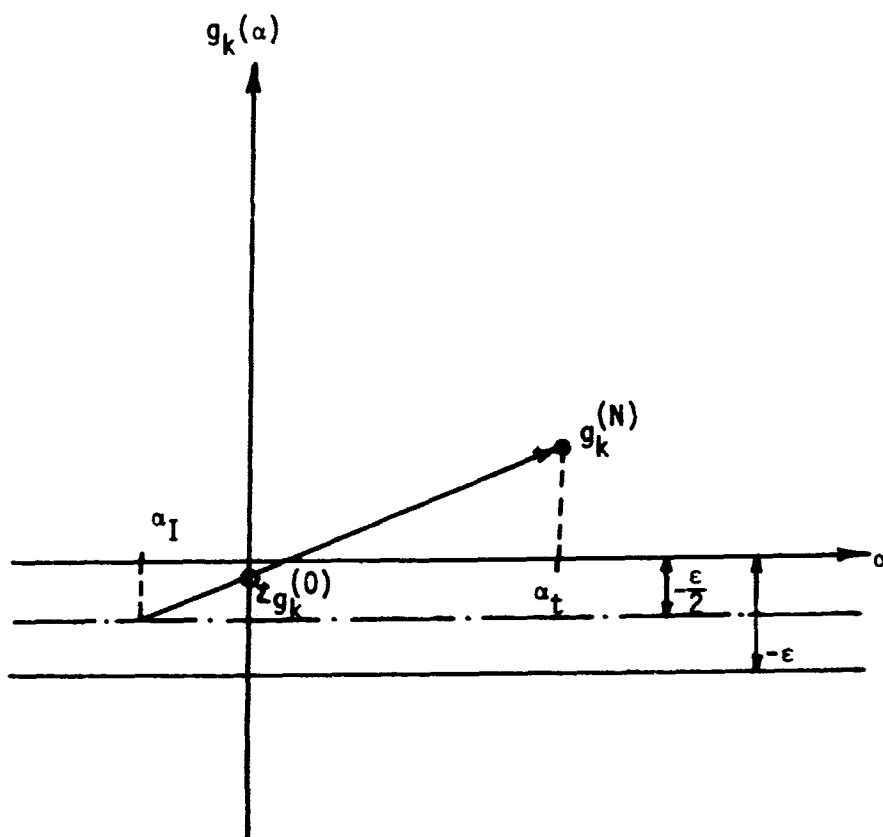


Fig. 7.6 Linear Interpolation Leading to Negative Corrected Step Size.



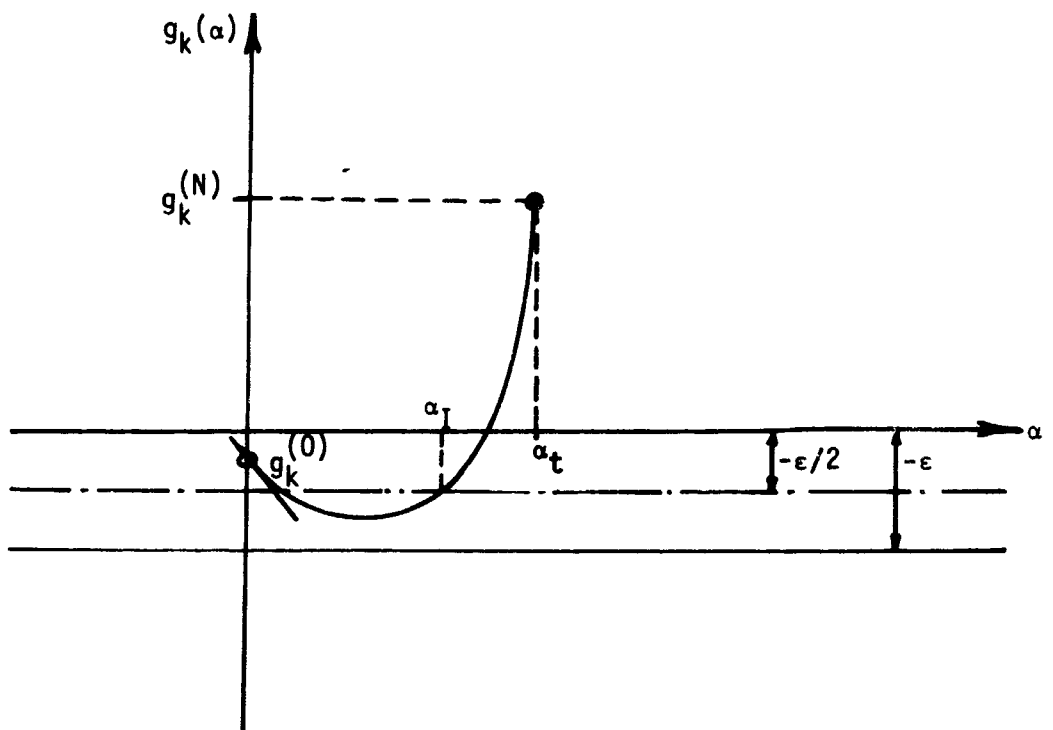


Fig. 7.7 Quadratic Interpolation

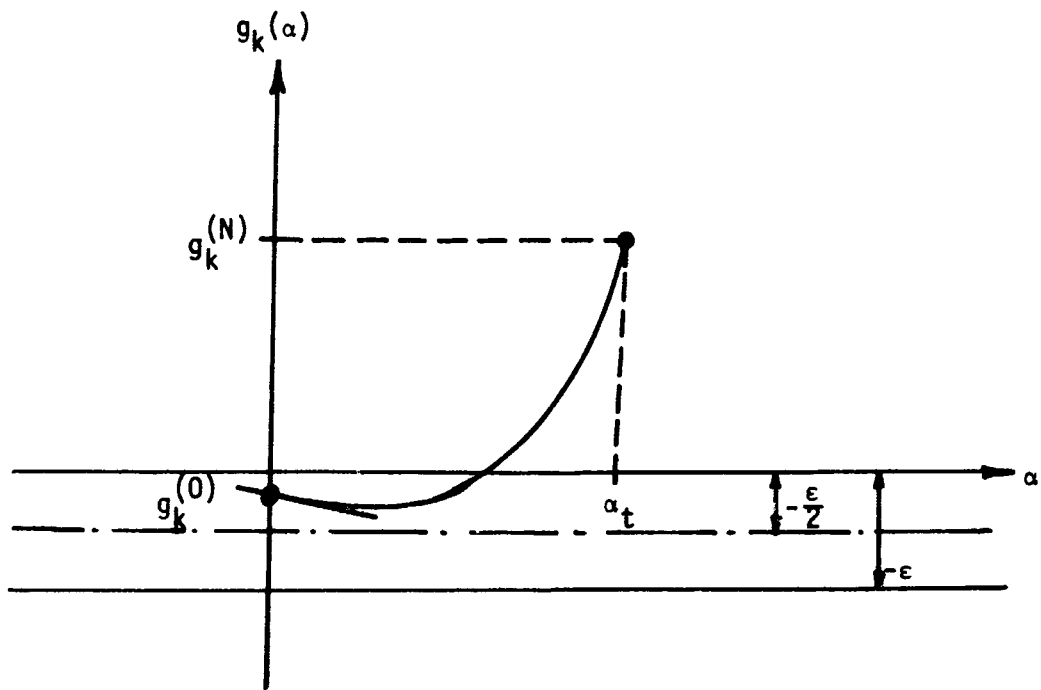


Fig. 7.8 Slope of the Old Bounded Design Point too Small for Successful Quadratic Interpolation with the Selected Value of the Constraint Tolerance.

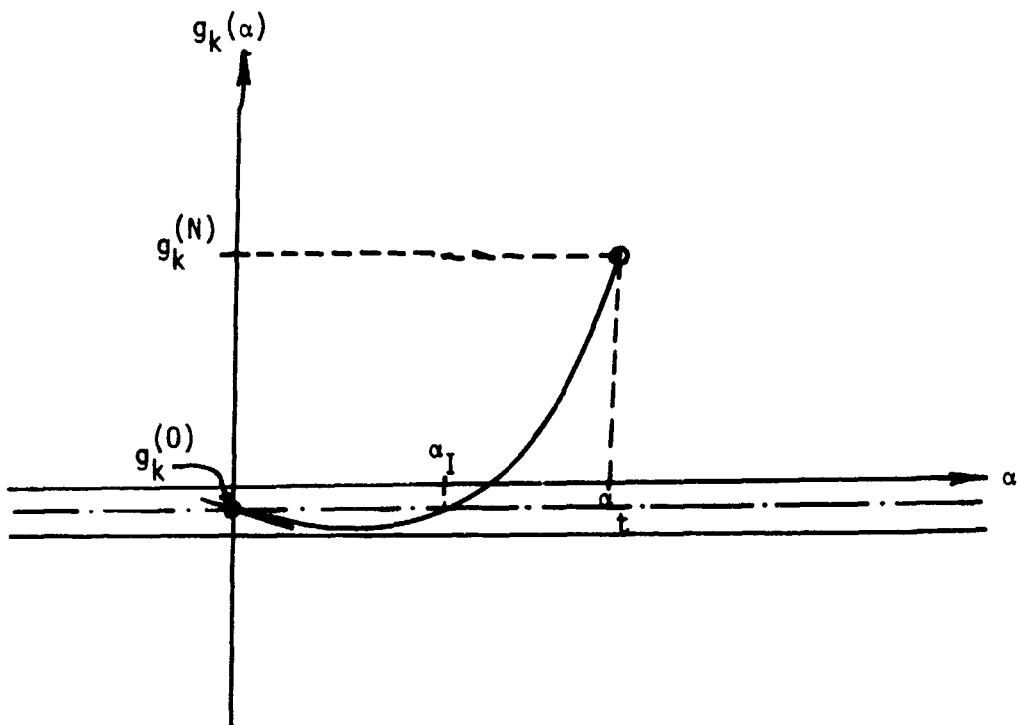


Fig. 7.9 Constraint Tolerance Reduced for Successful Quadratic Interpolation.

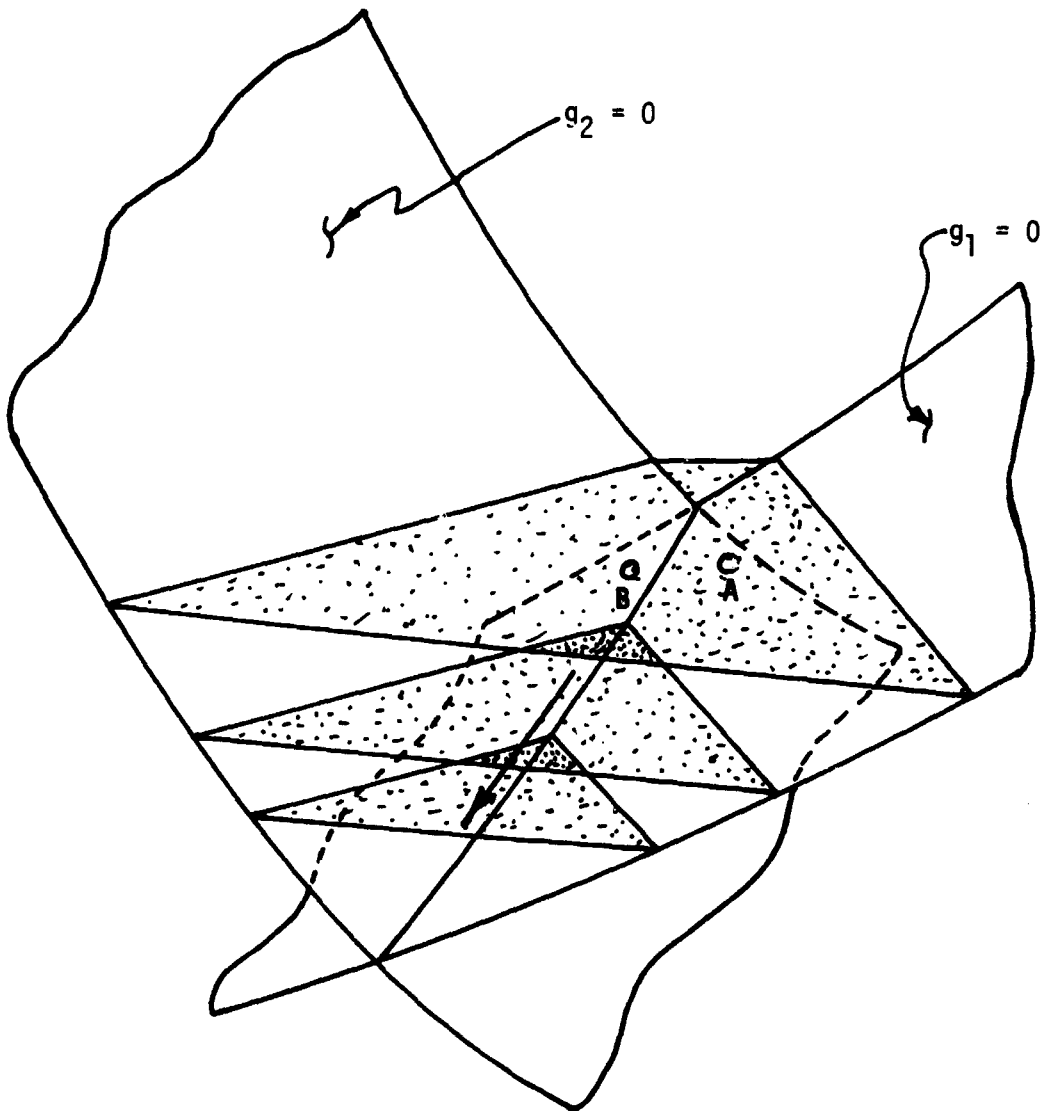


Fig. 7.10 Three Dimensional Design Space. At Point A, Constraint 1 is active and at Point B, Constraint 2 is active

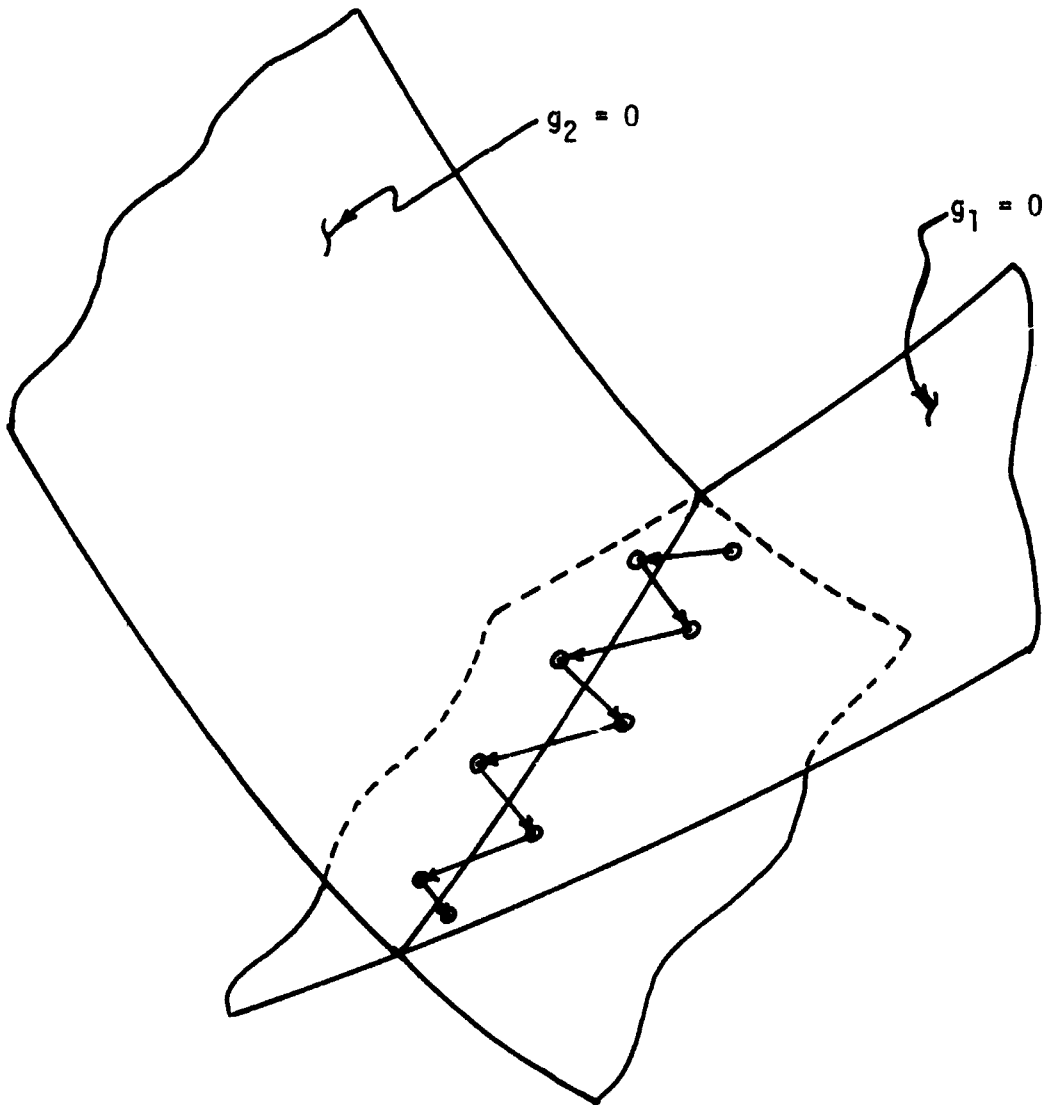


Fig. 7.11 Zigzagging

## CHAPTER 8

### ILLUSTRATIVE EXAMPLES

The applicability of the method of analysis and the technique of minimum weight design, discussed in the previous chapters, is illustrated through several numerical examples. In all the examples presented in this chapter, (1) a general planar beam element having six degrees-of-freedom was used to model the various planar truss-frame structures, (2) the distributed mass of the system was used to evaluate the components of the mass matrix (the "consistent" mass matrix), (3) the Univac 1108 digital computer was used to obtain the numerical results, (4) random vectors were used as starting points for the minimization algorithm to obtain the eigensolution of the initial design and (5) a ground shock in the form of one-half cycle sine pulse, Fig. 3.1, was used as the forcing function.

The response quantity of interest in a ground type forcing function applied to a single degree-of-freedom system is often the displacement of the mass  $m$  relative to the ground, called the relative displacement. The shock spectrum of a one half-cycle sine pulse representing the peak relative displacement is shown in Chapter 4 of Ref. 10 (denoted by  $X_D$  in Fig. 4.3, P. 163). An idealized form of this spectrum shown in Fig. 8.1 has been used in the present work. It has been obtained by fitting piecewise smooth curves to the actual spectrum. This not only

facilitates the supply of shock spectral information to the computer, but also enables us to obtain directly the derivative of the response quantity with changes in the design variables—information required in the present work as indicated by Eq. (6.9).

All iterative procedures need some kind of termination criterion. The method described in Chapter 7 indicates that the optimal design corresponds to a trivial solution of the direction finding problem ( $\vec{S} = \vec{0}$ ,  $\beta = 0$  where  $\vec{S}$  is the feasible direction vector and  $\beta$  is a scalar quantity). However, computationally it is difficult to obtain exactly zero values. Therefore, in the present work, a design is considered optimal and the program is allowed to stop whenever the step length in any feasible direction,  $\vec{S}$ , is less than .0001 or the value of the scalar quantity  $\beta$  is .01.

#### Example (1):

As a simple application, consider the planar truss-frame shown in Fig. 8.2 consisting of tubular members pinned together at the nodes. Members '1', '3', '4', and '5' were modeled with two general planar beam elements while the shortest member, marked '2', was modeled with a single beam element. Thus, the structure has 26 degrees-of-freedom. The approximate dynamic analysis was performed by considering the first sixteen eigenvectors from the lowest end of the spectrum. The problem of optimum design consists of obtaining the diameters  $d_1$ ,  $d_2$ ,  $d_3$ ,  $d_4$ , and  $d_5$  which give the lowest weight of the structure. The design variable

$\delta_1$  corresponds to members 1 and 3,  $\delta_2$  corresponds to member 2 and  $\delta_3$  corresponds to members 4 and 5. Thus, the problem has three design variables. A starting design with all members of mean diameter 1.6" resulted into a weight of 19.23 lbs. The limits on the behavior constraints and the side constraints were as follows:

$$y_{jm} \leq 0.2" \quad , \quad j = 1, 2, \dots, 12$$

$$\sigma_m^i \leq 45,000 \text{ psi} \quad , \quad i = 1, 2, \dots, 9$$

$$25 \text{ cs/sec.} \leq \omega_1 \leq 90 \text{ cs/sec.}$$

$$140 \text{ cs/sec.} \leq \omega_k \quad , \quad k = 2, 3, 4, \text{ and } 5$$

$$0.8" \leq \delta_\ell \leq 2.5" \quad \ell = 1, 2, 3$$

A constraint tolerance of 10% was allowed. The proposed least weight design turned out to have a weight of 16.76 lbs. with

$$d_1 = d_3 = 1.51"$$

$$d_2 = 0.86"$$

$$d_4 = d_5 = 1.39"$$

and five active constraints. The vertical member, marked '2', in Fig. 8.2 is at its lower bound. The second and the third



eigenvalues of the structure,  $\lambda_2$  and  $\lambda_3$ , are at their lower bounds and the middle point of member, marked '4', has a stress of 40.6 kips/in<sup>2</sup>. All the feasible designs were observed to have the second eigenvalue,  $\lambda_2$ , constraint to be active.

Example (2):

The two bay planar truss-frame of Fig. 8.3 consisting of tubular members pinned together at the nodes has 58 degrees-of-freedom if each member is modeled with two general planar beam elements. The system has 10 design variables as each member is allowed to change independently in the design space. The first fourteen eigenvectors (starting from the lowest end of the spectrum) were considered to approximate the dynamic analysis. The limits on the behavior constraints and the side constraints were as follows:

$$y_{jm} \leq 0.2" \quad , \quad j = 1, \dots, 28$$

$$\sigma_m^i \leq 70,000 \text{ psi} \quad , \quad i = 1, \dots, 20$$

$$25 \text{ cs/sec.} \leq \omega_1 \leq 90 \text{ cs/sec.}$$

$$140 \text{ cs/sec.} \leq \omega_k \quad , \quad k = 2, 3, 4, \text{ and } 5$$

$$0.8" \leq \delta_k \leq 4.0" \quad , \quad k = 1, 2, \dots, 10$$

The first bounded design with member sizes

$$d_1 = d_2 = d_4 = 2.15"$$

$$d_3 = 2.42"$$

$$d_5 = d_6 = 2.49"$$

$$d_7 = d_8 = d_9 = d_{10} = 1.91"$$

had a weight of 29.85 lbs. and the second eigenvalue constraint,  $\lambda_2$ , was bounded. The proposed least weight design has a weight of 19.03 lbs. with

$$d_1 = 0.93"$$

$$d_2 = 2.52"$$

$$d_3 = 1.17"$$

$$d_4 = 0.84"$$

$$d_5 = 1.78"$$

$$d_6 = 0.88"$$

$$d_7 = 2.49"$$

$$d_8 = 0.84"$$

$$d_9 = 1.16"$$

$$d_{10} = 0.86"$$

and six active constraints. Members '4', '6', '8', and '10' in Fig. 8.3 are at the lower bounds and two eigenvalue constraints,  $\lambda_2$  and  $\lambda_3$  are at the lower bounds. An interesting observation is that the members marked '4', '6', and '10' are at the lower bounds and if we eliminate these members from the structure, the resulting

structure is still statically stable. Hence, a modified structure as shown in Fig. 8.4 was considered.

### Example 3(a)

The structure of Fig. 8.4 consisting of tubular members pinned together at the nodes has 41 degrees-of-freedom if each of its members is modeled with two general planar beam elements. The forcing function, the lumped masses and the limitations on the behavior constraints and side constraints were taken to be the same as those on the structure of Fig. 8.3, Example 2. The first eleven eigenvectors (starting from the lowest end of the spectrum) were considered to approximate the dynamic analysis. An initial design with all members having a mean diameter of 2.5" had a weight of 25.21 lbs. A single move in the direction of steepest descent and a step length corresponding to 10% decrease in weight resulted in a bounded design, Fig. 8.5, having a weight of 22.69 lbs. The second eigenvalue constraint,  $\lambda_2$ , is active. The proposed least weight design is shown in Fig. 8.6. It has a weight of 16.46 lbs. and has three active constraints. The member '1' in Fig. 8.4 is at its lower bound and the eigenvalue constraints  $\lambda_2$  and  $\lambda_3$  are bounded.

A plot of the cumulative number of analyses performed to obtain the bounded designs and their weight is shown in Fig. 8.7. The complete analysis of the initial design, using random vectors as starting points for the eigenvalue analysis, took about 40

seconds for this system of 41 degrees-of-freedom and 7 variables in the design space. The time taken for the analysis of intermediate designs varied from 15 to 25 seconds. Out of the total weight reduction of 27.4%, 27.2% of the weight was reduced in the first fifteen analyses as seen from Fig. 8.7. This took approximately 310 seconds including the analysis of the initial design.

The value of the scalar quantity,  $\beta$ , at the fifteenth analysis is .015 and at the thirty-sixth analysis is .002. The almost flat plateau and the slow rate of decrease of weight between the fifteenth and the thirty-sixth analyses is considered to be a manifestation that the optimum design has been attained.

The first six frequencies (starting from the lowest end of the eigenvalue spectrum) of the proposed optimum design are marked on the idealized shock spectrum, Fig. 8.8.

In order to test whether the least weight design of Fig. 8.6 corresponds to a local minimum or the absolute minimum in the design space, the example was rerun with a different initial starting design as follows:

$$d_1 = d_2 = d_3 = 3.0''$$

$$d_4 = 2.0''$$

$$d_5 = d_6 = d_7 = 1.5''$$

A plot of cumulative number of analyses performed to obtain the bounded designs versus the weight of the bounded designs is

shown in Fig. 8.9. This plot is similar to that of Fig. 8.7 and the least weight design obtained (weight = 16.43 lbs.) is also similar to that of Fig. 8.6. Merely on the basis of two trial starting designs, it is hard to say that the minimum obtained is the absolute minimum over the design space; however, finding the similar least weight design by starting from two different initial designs is at least a pointer in that direction.

### Example 3(b)

All the examples considered so far show that the proposed least weight designs have active eigenvalue constraints. Hence, the structure of Fig. 8.4 was reconsidered, this time with the reduced lower limits on the frequency constraints. Furthermore, the lower limit on the size of design variables and upper limit on maximum stress was also reduced. The new limitations imposed on the optimization problem were:

$$y_{jm} \leq 0.2" \quad , \quad j = 1, 2, \dots, 20$$

$$\sigma_m^i \leq 50,000 \text{ psi} \quad , \quad i = 1, \dots, 14$$

$$25 \text{ cs/sec.} \leq \omega_1$$

$$0.6 \leq \delta_\ell \leq 4.0" \quad , \quad \ell = 1, \dots, 7$$

A starting design, Fig. 8.10, with all members having a mean diameter of 2.5" and a weight of 25.27 lbs. happened to be bounded with the maximum stress at the middle point of member marked '3'

on Fig. 8.4 at its upper bound. The proposed least weight design, Fig. 8.11, has a weight of 10.60 lbs. Two members marked '4' and '5' on Fig. 8.4 are at the lower bounds in this design and the members marked '1', '3', '6', and '7' are fully stressed since the maximum stresses at the middle and the end points of these members are at their upper bounds. Furthermore, the maximum stress at the middle and the end points of member marked '2' is almost at its upper bound. Thus, we observe that this proposed optimum design has a character where either the members are fully stressed or are at their lower bounds. A plot of cumulative number of analyses performed to obtain the bounded designs versus the weight of the bounded designs is shown in Fig. 8.12. The time taken for the analysis of the initial design was about 40 seconds while that for the intermediate analysis varied from 10 to 15 seconds. The proposed least weight design has a value of  $\beta$  equal to .012.

The first seven frequencies (starting from the lowest end of the eigenvalue spectrum) of the proposed optimum design are marked on the idealized shock spectrum, Fig. 8.13.

The mode shapes corresponding to the first six eigenvalues (starting from the lowest end of the eigenvalue spectrum) of the proposed optimum design are shown in Figs. 8.14(a) to 8.14(f). The peak modal participation coefficients,  $|c_i \xi_p(\omega_i)|$ , and the axial and bending stresses in each member of the structure contributed by the respective mode shapes are also shown on

Figs. 8.14.

Example 4(a)

Consider as another illustrative example a three storied planar frame, Fig. 8.15, consisting of tubular members rigidly joined at the ends. Assuming the columns and the beam members to be inextensional, the structure turns out to have 15 degrees-of-freedom if the column members are modeled with one beam element and the beam members are modeled with two beam elements. A uniformly distributed mass of 0.25 lbs./inch is considered to be superimposed on the beam members of the structure. Only the dynamic analysis of the structure is considered taking the complete eigensolution of the problem. The limitations imposed on the behavior constraints are

$$y_{jm} \leq 10.0" \quad , \quad j = 1, \dots, 6$$

$$\sigma_m^i \leq 95,000 \text{ psi} \quad , \quad i = 1, \dots, 12$$

$$0.48 \text{ cs/sec.} \leq \omega_1$$

$$9.0" \leq \delta_\ell \leq 36.0" \quad , \quad \ell = 1, \dots, 6$$

The six design variables correspond to the mean diameters of the columns and beam members of each floor.

An initial starting design shown in Fig. 8.16 turned out to be bounded with a weight of 457094 lbs. The frequency constraint is active. The proposed least weight design, Fig. 8.17, has a

weight of 220886 lbs. and has five members marked '2', '5', '7', '8' and '9' at their lower bounds. The frequency constraint is active.

A plot of cumulative number of analyses performed to obtain the bounded designs versus the weight of the bounded designs is shown in Fig. 8.18. Out of a total of 52% reduction in weight, 51.8% reduction was accomplished in only 13 analyses as seen from Fig. 8.18. The value of scalar quantity  $\beta$  was equal to .006 in the thirteenth analysis. The exit criterion to stop the program on the value of the scalar quantity  $\beta$  was changed from .01 to .001 and the problem was rerun. The value of  $\beta$  obtained at the 39<sup>th</sup> analysis is .0012. Once again the almost flat plateau on Fig. 8.18 indicating the slow rate of decrease of weight is taken to be a manifestation that the optimum design has been attained. The analysis of the initial design took about 25 seconds while that of the intermediate designs took 10 to 12 seconds. The tighter convergence criterion was used in the eigenvalue analysis where the function value was made to converge upto eighth decimal place as against sixth decimal place in the previous examples.

The first eight frequencies (starting from the lowest end of the eigenvalue spectrum) of the proposed optimum design are marked on the idealized shock spectrum, Fig. 8.19 .



Example 4(b)

The structure of Fig. 8.15, Example 4(a), was redesigned with relaxed limitations on the frequency constraints. The restrictions imposed on the behavior and side constraints are

$$y_{jm} \leq 10.0" \quad , \quad j = 1, \dots, 6$$

$$\sigma_m^i \leq 55,000 \text{ psi} \quad , \quad i = 1, \dots, 12$$

$$0.1 \text{ cs/sec.} \leq \omega_1$$

$$9.0" \leq \delta_\ell \leq 36.0" \quad , \quad \ell = 1, \dots, 6$$

A bounded design with a weight of 242919 lbs. shown in Fig. 8.20 was used as a starting design with members marked '5' and '8' on Fig. 8.15 at their lower bounds and the stresses in the columns at the ground point were at the upper bounds. The proposed least weight design, Fig. 8.21, has a weight of 217871 lbs. and has all the beam members at their lower bounds and the stresses in the columns marked '7' and '9' on Fig. 8.15 are at the upper bounds. A plot of cumulative number of analyses performed to obtain the bounded designs versus the weight of the bounded designs is shown in Fig. 8.22. The value of the scalar quantity,  $\beta$ , at the proposed optimal design is .039 but the step length in the subsequent feasible direction was very small (below .0001) and hence the program was terminated. Out of the total decrease of 10.3% weight, 10.25% of the weight was reduced in the first three

analyses. The character of the proposed optimal design, Fig. 8.21, gives rise to some doubts. The size of the column members marked '4' and '6' on Fig. 8.15 might be expected to be smaller in size than those marked '1' and '3'. The maximum stress on the column members marked '4' and '6' is only 21 kips/in<sup>2</sup> at the proposed optimal design. This low value of maximum stress would obviously go up, if the member size were decreased. The higher value of  $\beta$  ( $\beta = .039$ ) all along the almost flat plateau on Fig. 8.22 also leads one to suspect that the proposed optimal design may not be the minimum weight design and that there might be a numerical or other computational problem associated with the direction finding problem or the computation of the constraint surface normals. The problem when started from a different initial design did not show any better results. Further investigation is needed to get a deeper insight into the nature of this problem.

The first seven frequencies (starting from the lowest end of the eigenvalue spectrum) of the proposed optimum design are marked on the idealized shock spectrum, Fig. 8.23.

The mode shape corresponding to the lowest eigenvalue (frequency of .43 cs./sec.) is shown in Fig. 8.24.

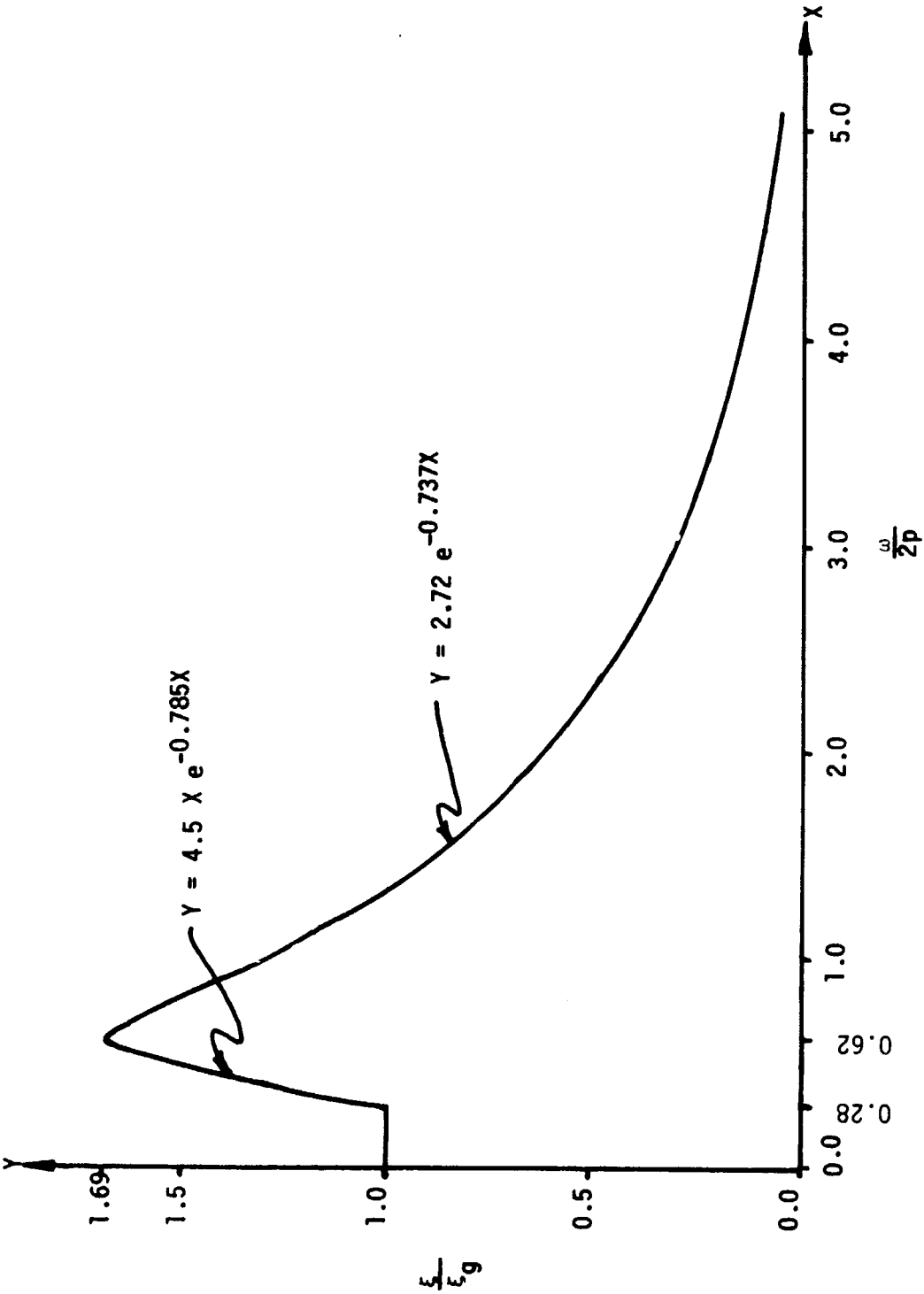
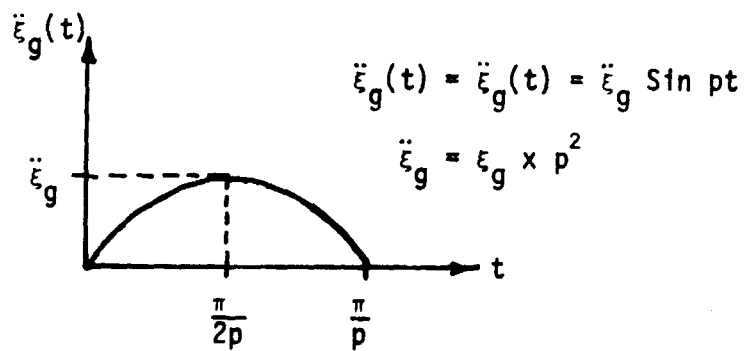
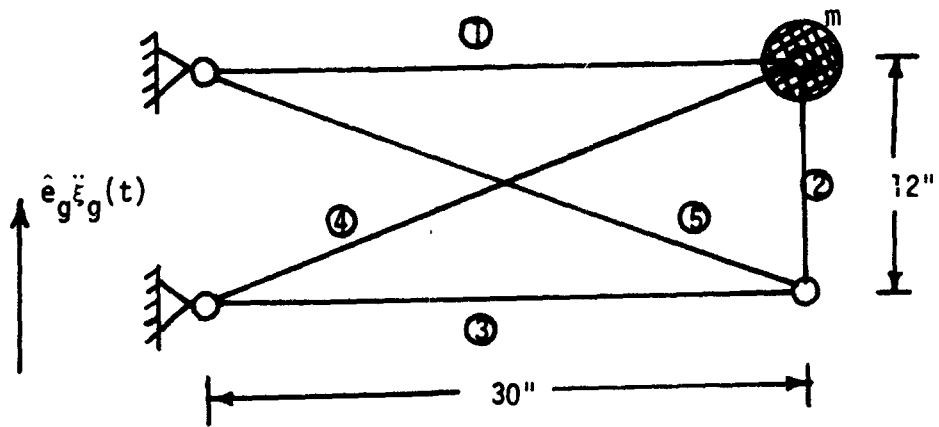
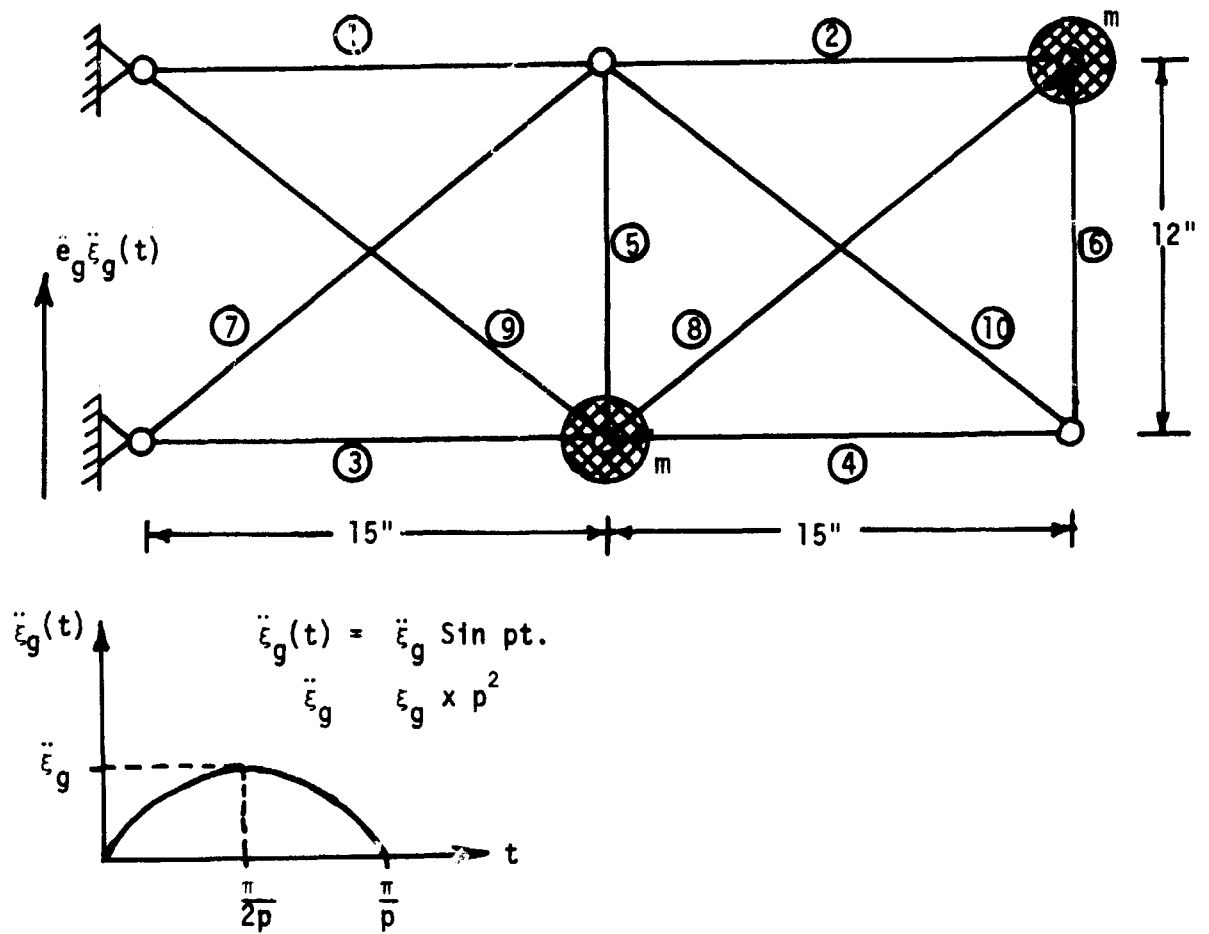


Fig. 8.1 Idealized Shock Spectrum For The Shock Pulse of Fig. 3.1



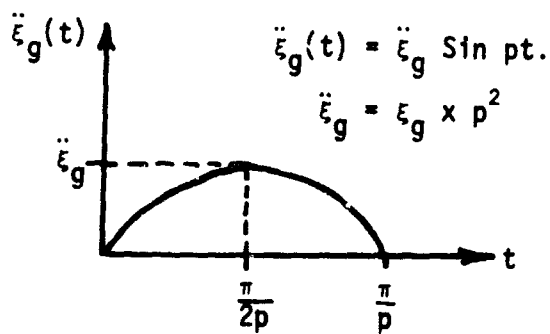
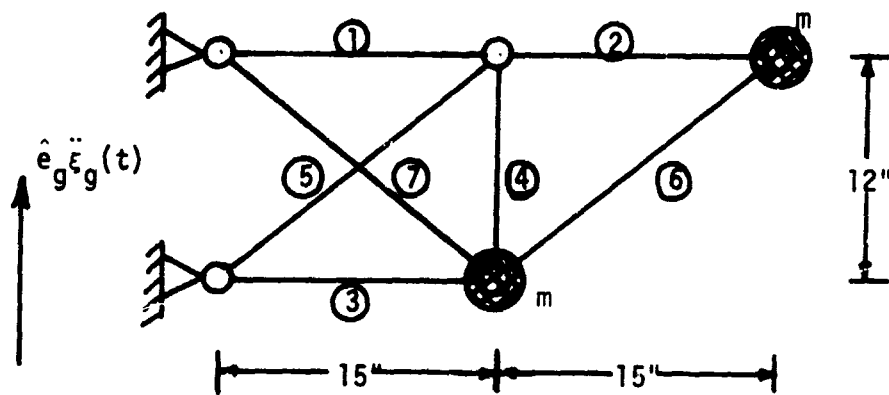
Wall thickness(t) = 0.1"  
 $m = 386 \text{ lbs}$   
 $E = 30 \times 10^6 \text{ lbs/in}^2$   
 $\rho = 0.28 \text{ lbs/in}^3$   
 $\xi_g = 0.05"$   
 $p = 500 \text{ rad./sec.}$

Fig. 8.2 Planar Truss-Frame



Wall Thickness(t) = 0.1"  
 $m = 386 \text{ lbs.}$   
 $E = 30 \times 10^6 \text{ lbs/in}^2$   
 $\rho = 0.28 \text{ lbs/in}^3$   
 $\xi_g = 0.05"$   
 $p = 250 \text{ rad/sec.}$

Fig. 8.3 Two Bay Planar Truss-Frame



Wall thickness(t) = 0.1"  
 $m = 386 \text{ lbs.}$   
 $E = 30 \times 10^6 \text{ lbs./in}^2$   
 $\rho = 0.28 \text{ lbs./in}^3$   
 $\xi_g = 0.05''$   
 $p = 250 \text{ rad./sec.}$

Fig. 8.4 Planar Truss-Frame; 41 D.O.F.

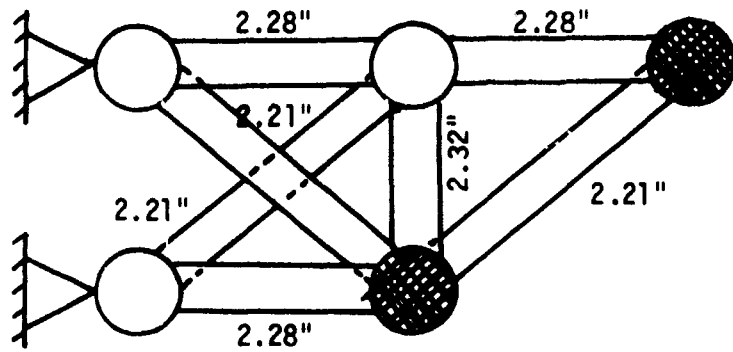


Fig. 8.5 First Bounded Design  
(Weight = 22.69 lbs.)

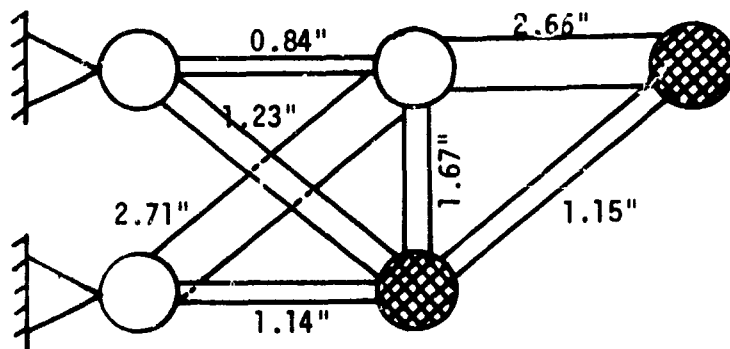


Fig. 8.6 Least Weight Design  
(Weight 16.46 lbs.)

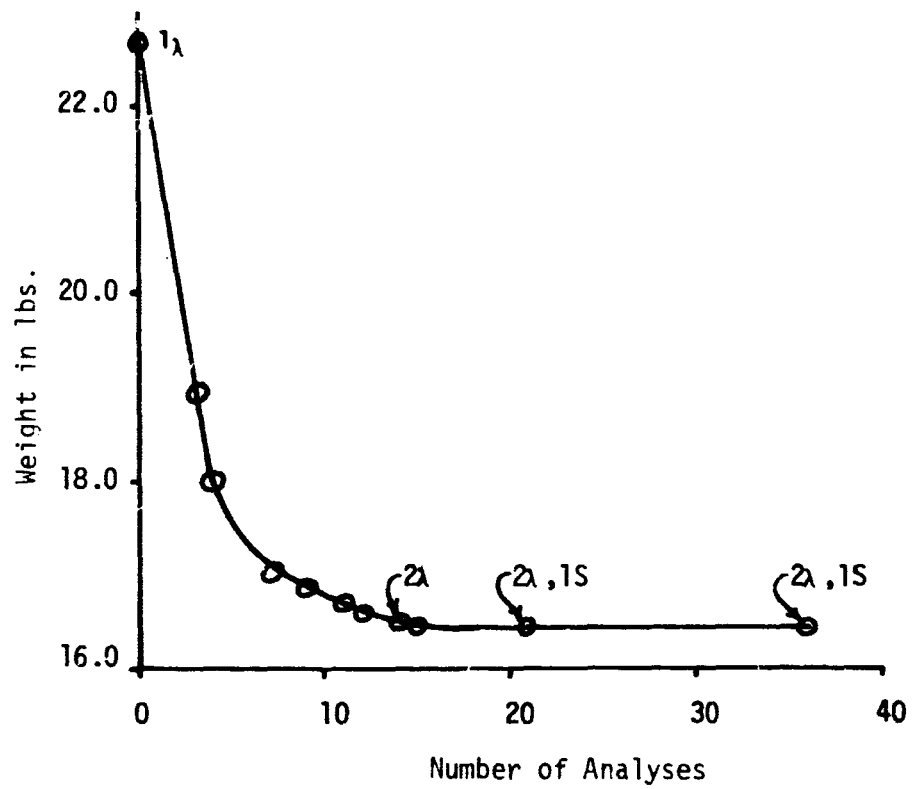


Fig. 8.7 Cumulative Number of Analyses Performed To Obtain the Bounded Designs Versus The Weight Of The Bounded Designs



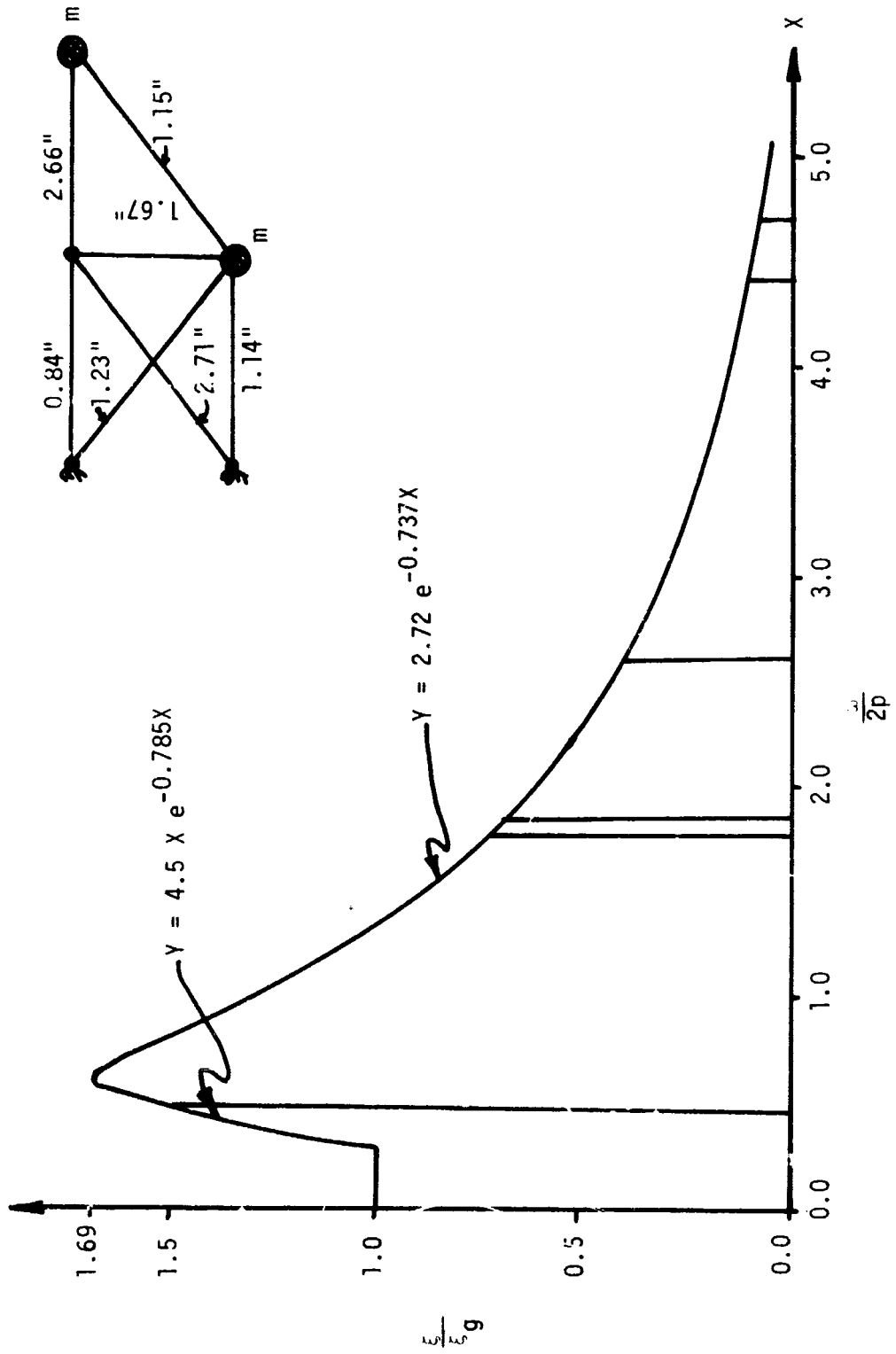


Fig. 8.8 First Six Frequencies of the Least Weight Design

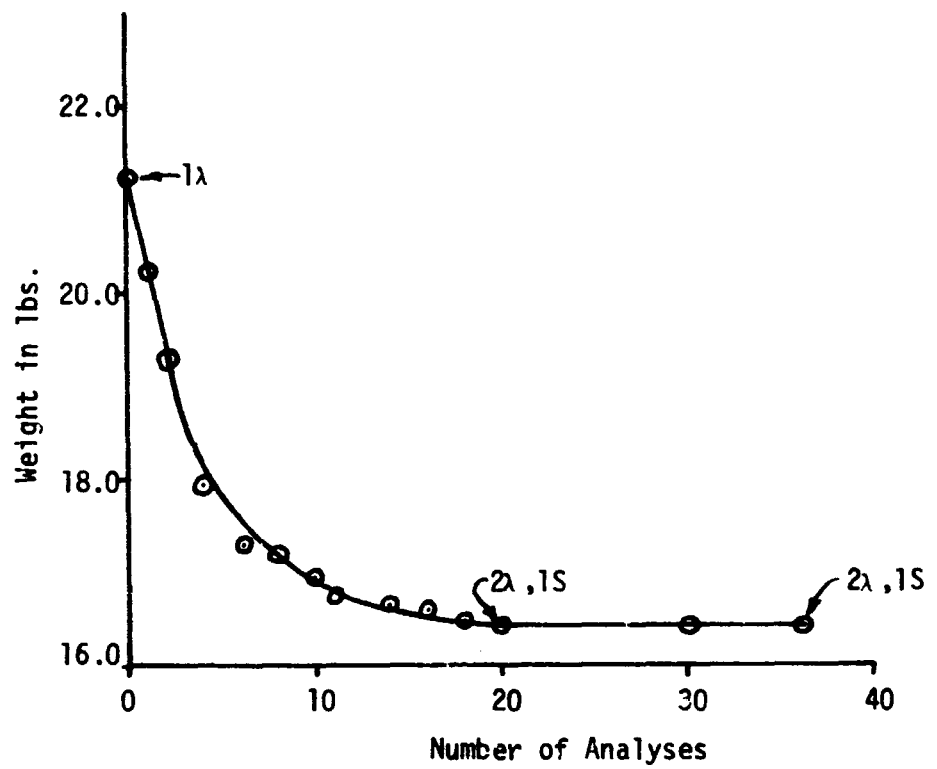


Fig. 8.9 Cumulative Number of Analyses Performed To Obtain The Bounded Designs Versus The Weight Of The Bounded Designs

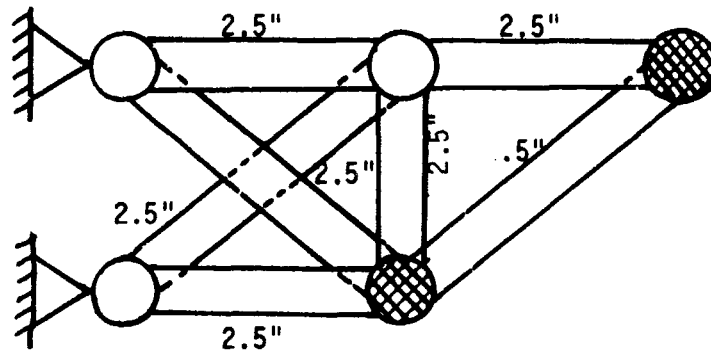


Fig.8.10 First Bounded Design  
(Weight = 25.21 lbs.)

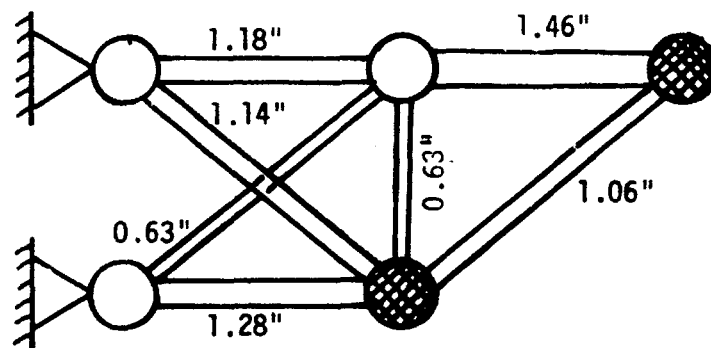


Fig.8.11 Least Weight Design  
(Weight = 10.60 lbs.)

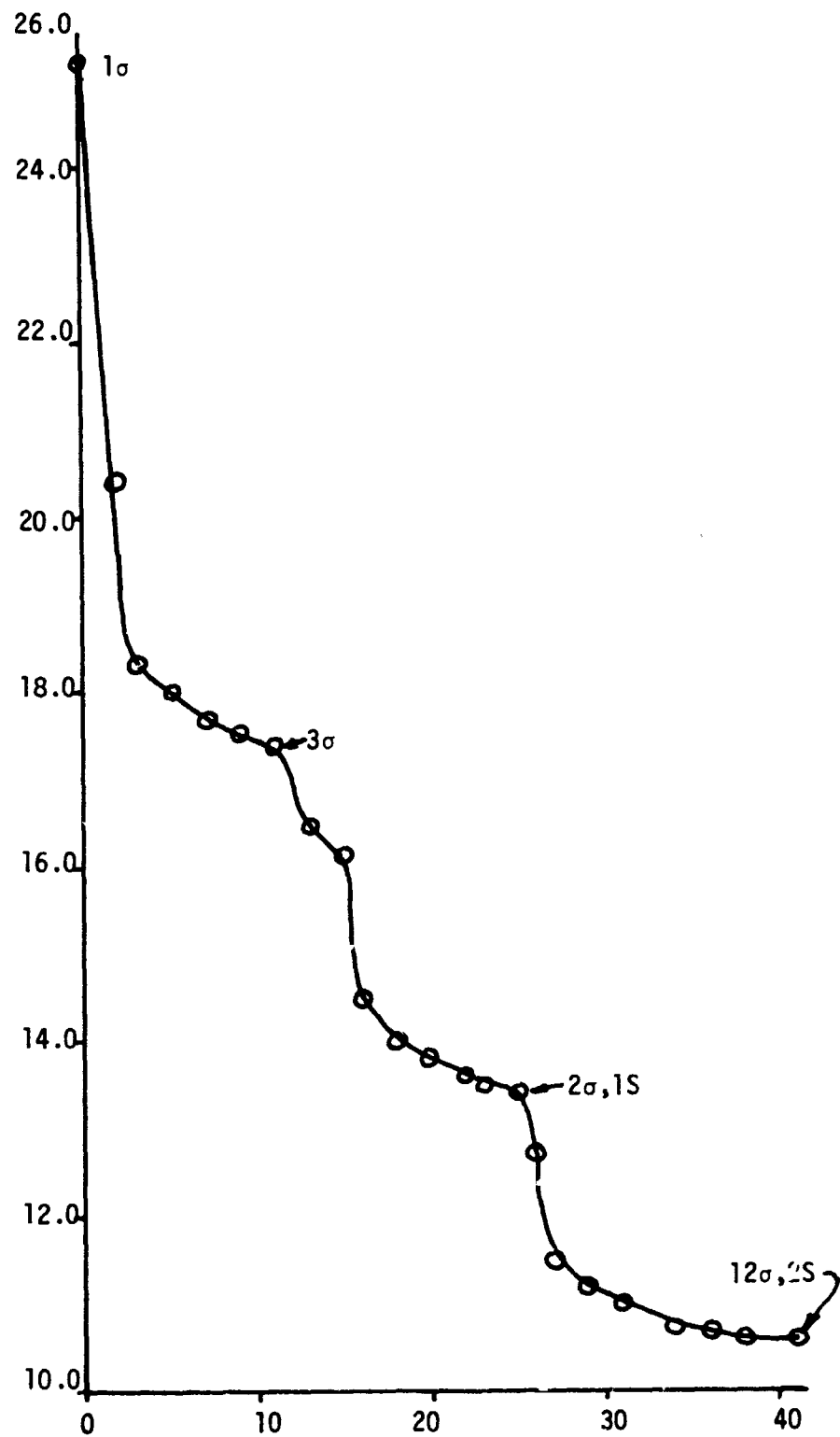


Fig. 8.12 Cumulative Number of Analyses Performed To Obtain the Bounded Designs Versus the Weight of the Bounded Designs

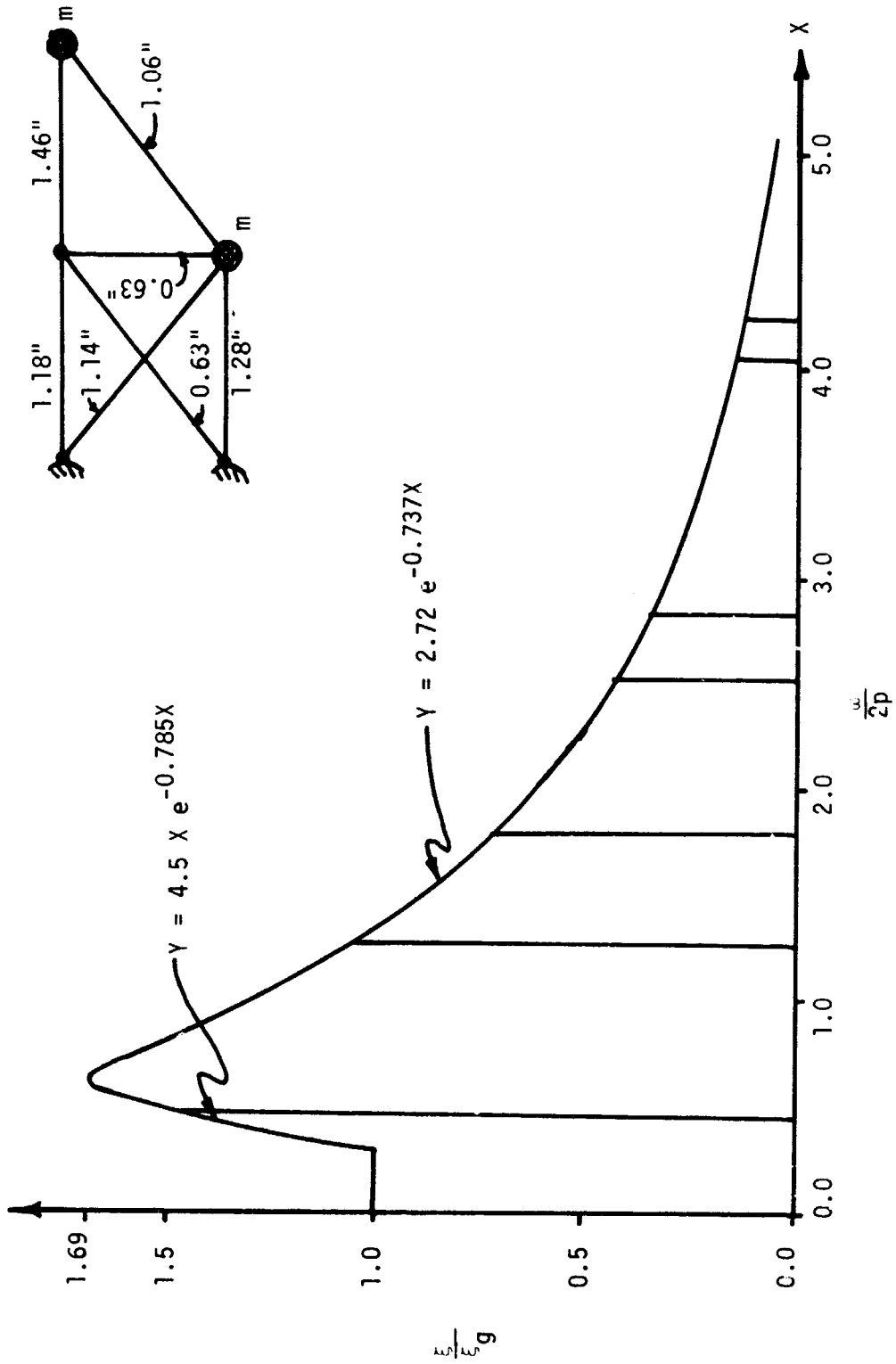


Fig. 8.13 First seven Frequencies of the Least Weight Design

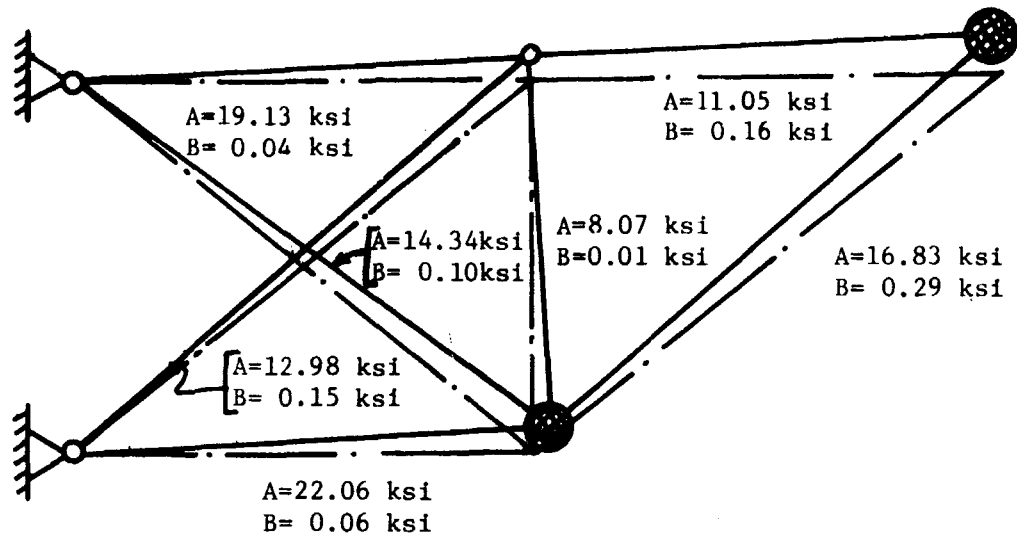


Fig. 8.14(a) Mode shape no. 1, Frequency = 33.6 cycles/sec.,  
Peak Modal Participation Coefficient = 0.086

[A denotes Axial Stress; B denotes Bending Stress.]

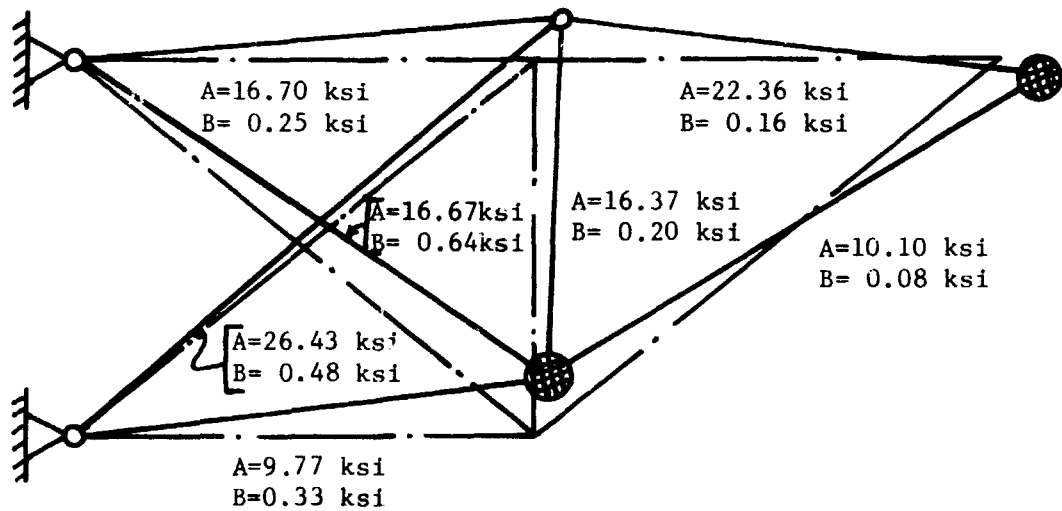


Fig. 8.14(b) Mode shape no. 2, Frequency = 99.0 cycles/sec.,  
Peak Modal Participation Coefficient = 0.031

(NOT TO SCALE)

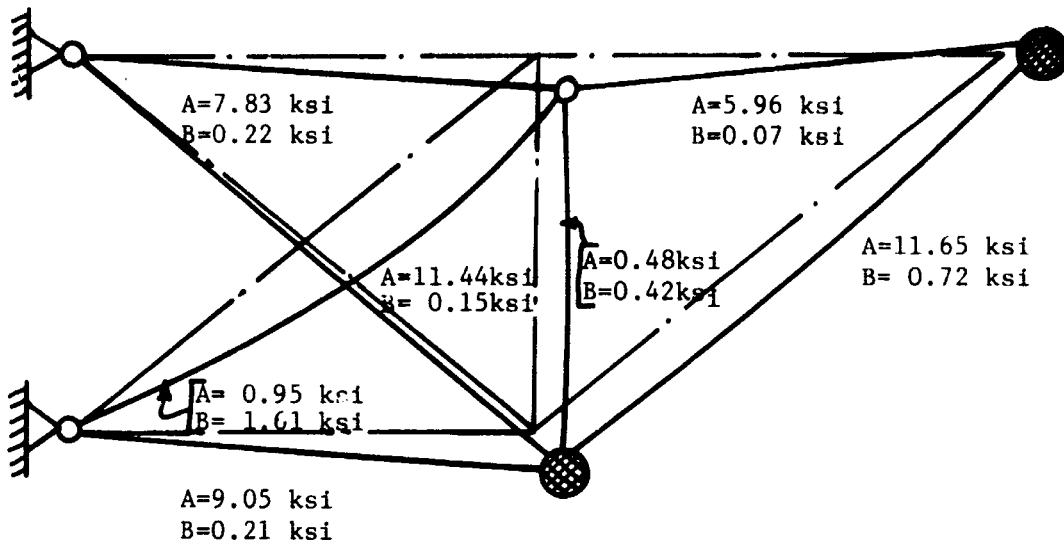


Fig. 8.14(c) Mode shape no. 3, Frequency = 145 cycles/sec.,  
Peak Modal Participation Coefficient = 0.011

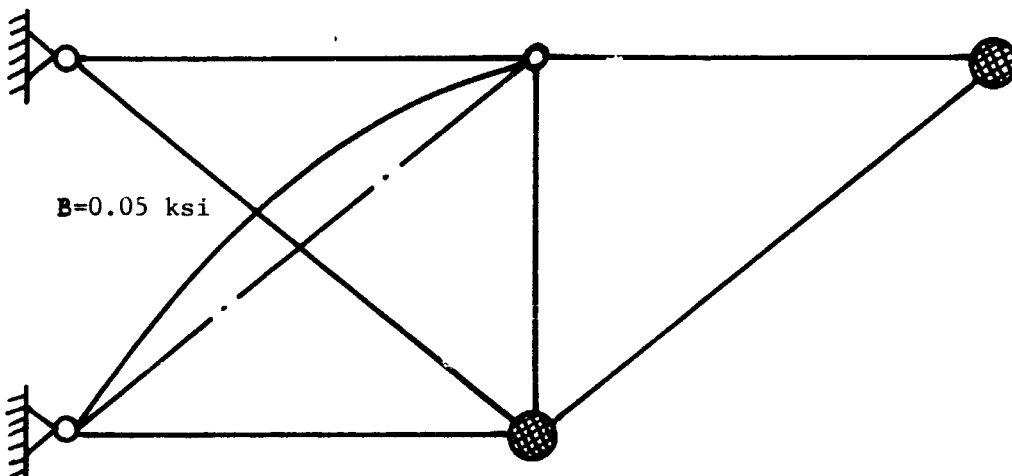


Fig. 8.14(d) Mode shape no. 4, Frequency = 196 cycles/sec.,  
Peak Modal Participation Coefficient = 0.000+

(NOT TO SCALE)

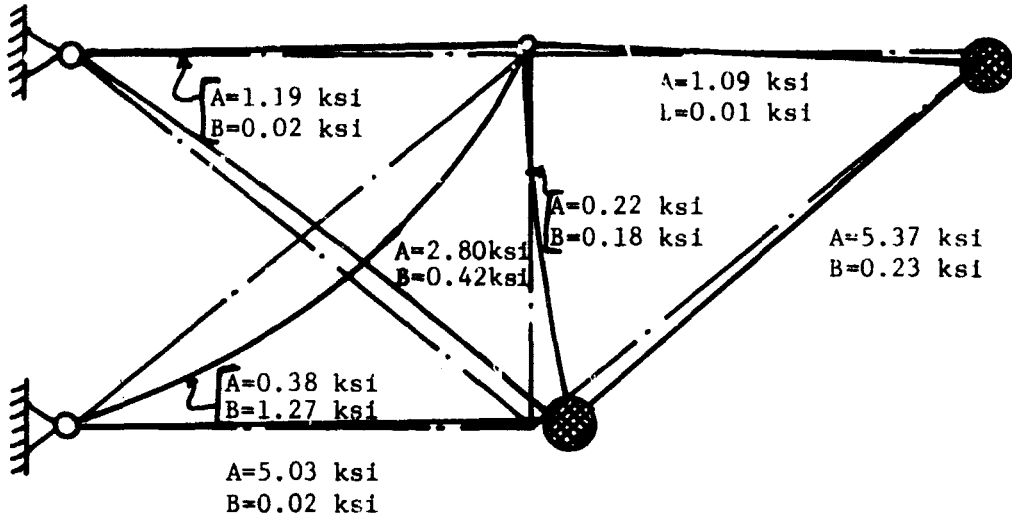


Fig. 8.14(e) Mode shape no. 5, Frequency = 205 cycles/sec.,  
Peak Modal Participation Coefficient = 0.003

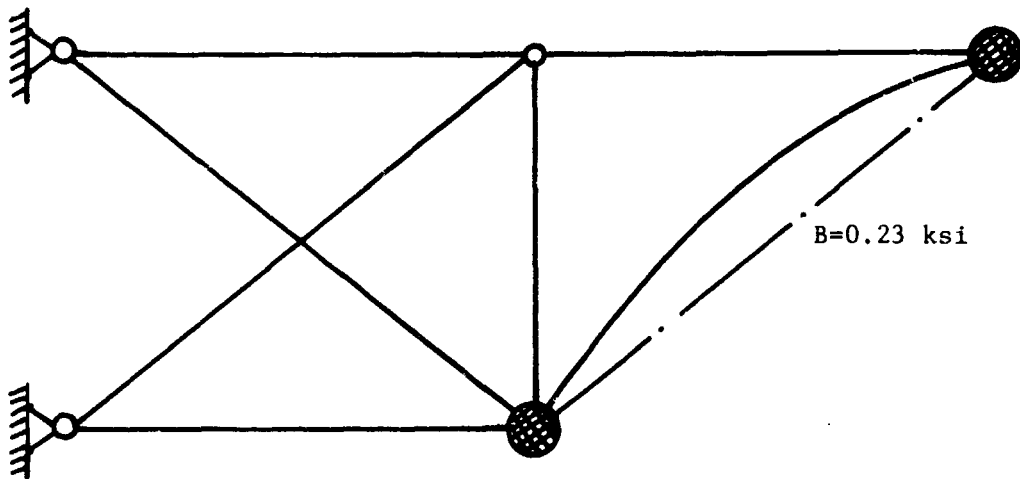
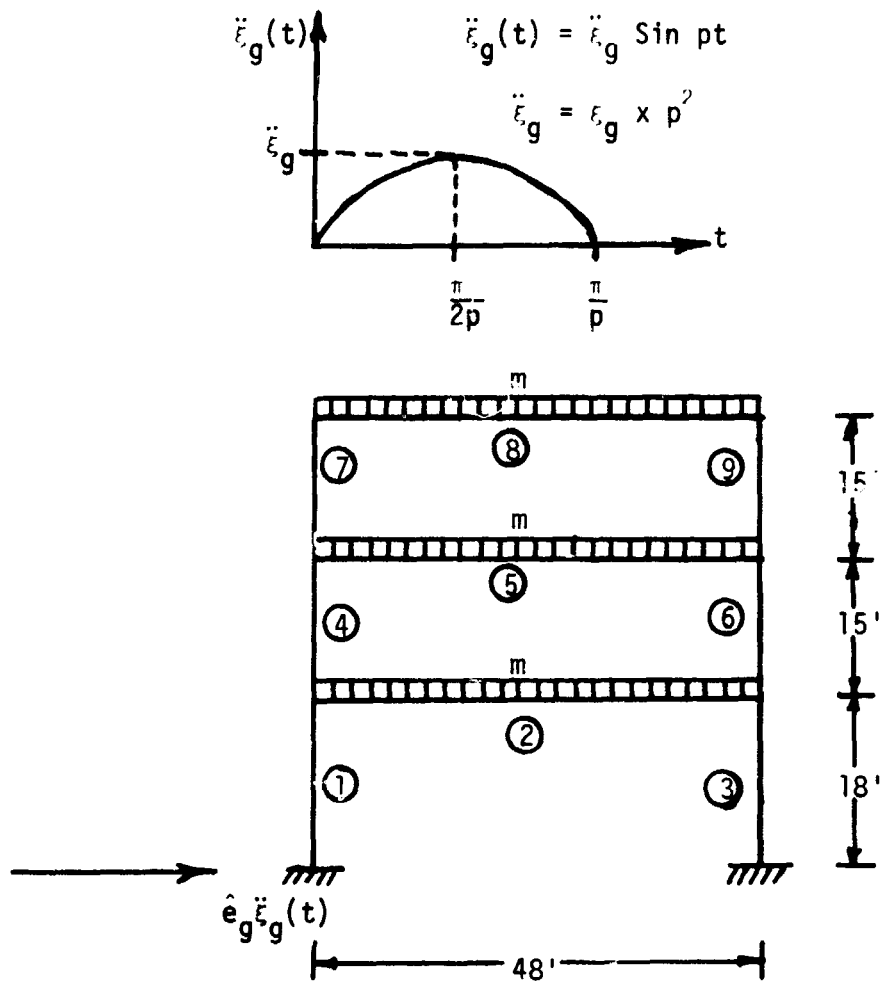


Fig. 8.14(f) Mode shape no. 6, Frequency = 328 cycles/sec.,  
Peak Modal Participation Coefficient = 0.000<sup>+</sup>

(NOT TO SCALE)





Wall Thickness(t) = 0.5"  
 $m = 0.25 \text{ lbs/}$   
 $E = 30 \times 10^6 \text{ lb/in}^2$   
 $\rho = 0.28 \text{ lbs/in}^3$   
 $\xi_g = 3.5''$   
 $p = 3 \text{ rad./sec}$

Fig. 8.15 Planar Frame; 15 D.O.F.

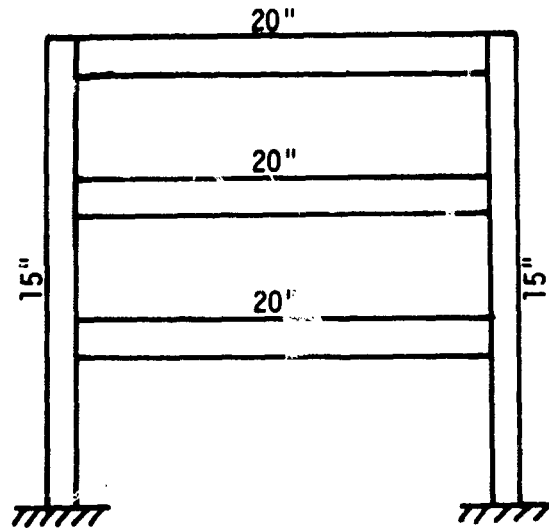


Fig. 8.16 First Bounded Design  
(Wt. = 457094 lbs)

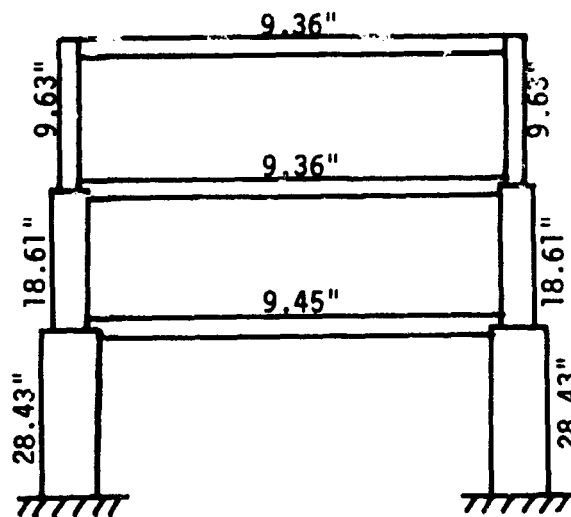


Fig. 8.17 Least Weight Design  
(Wt. = 220886 lbs)

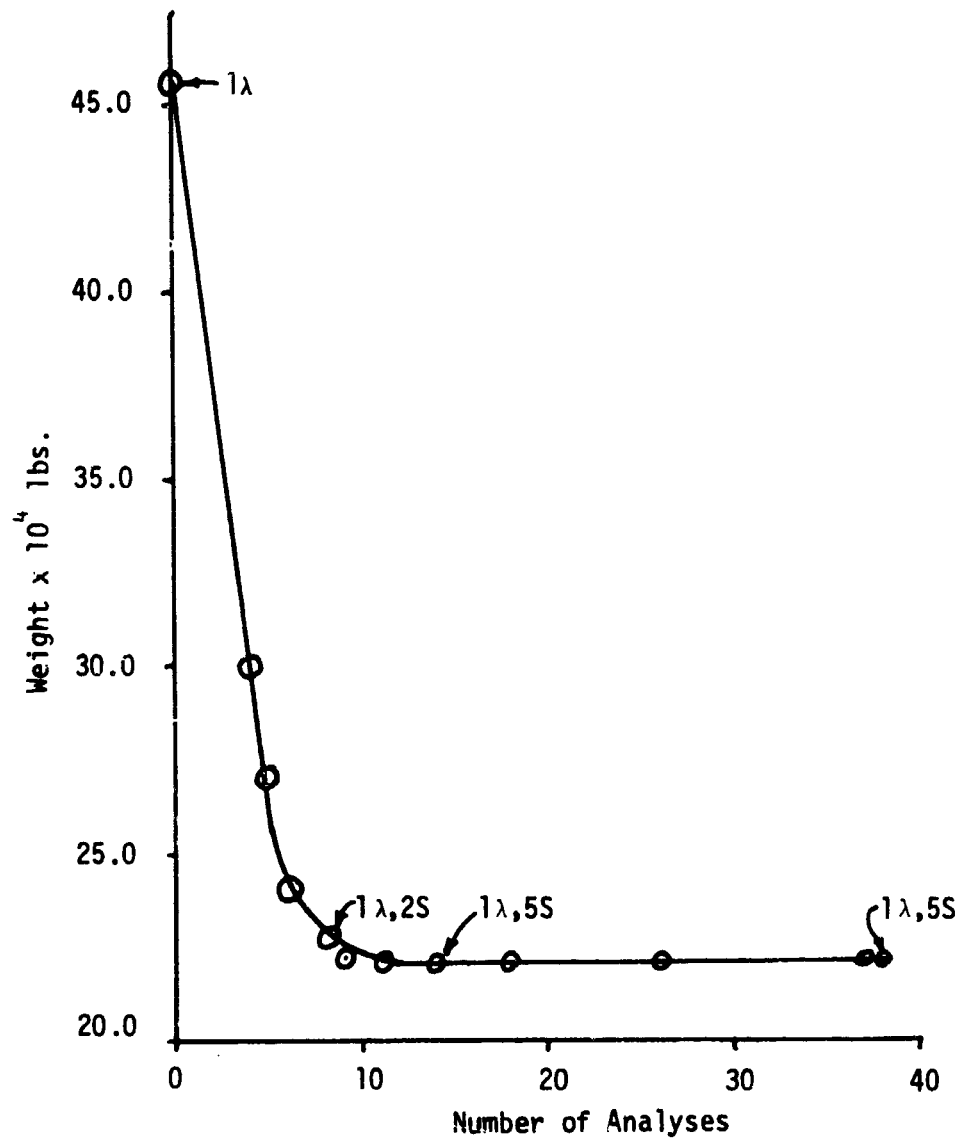


Fig. 8.18 Cumulative Number of Analyses Performed To Obtain The Bounded Designs Versus The Weight Of The Bounded Designs

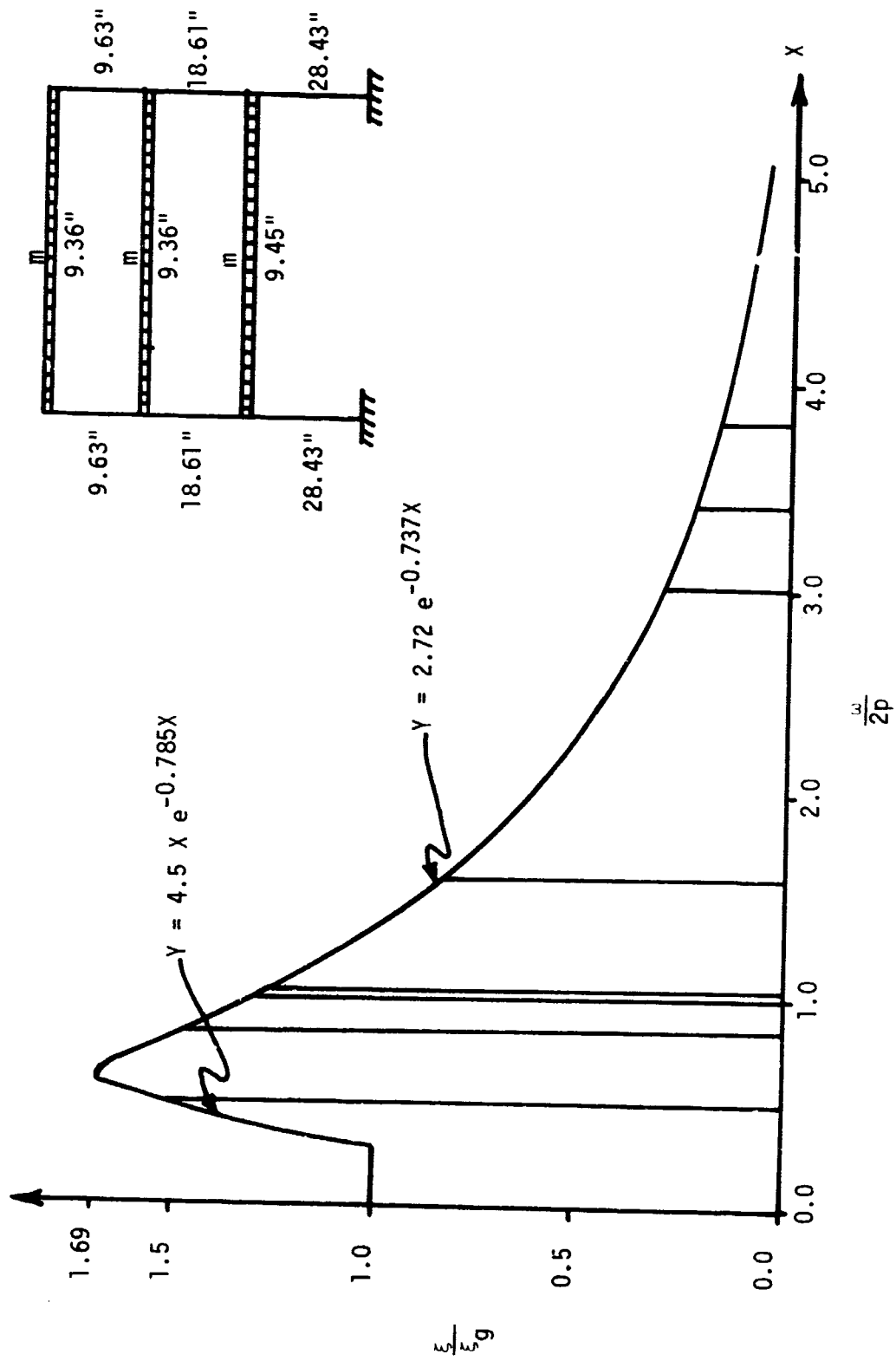


Fig. 8.19 First Eight Frequencies of the Least Weight Design

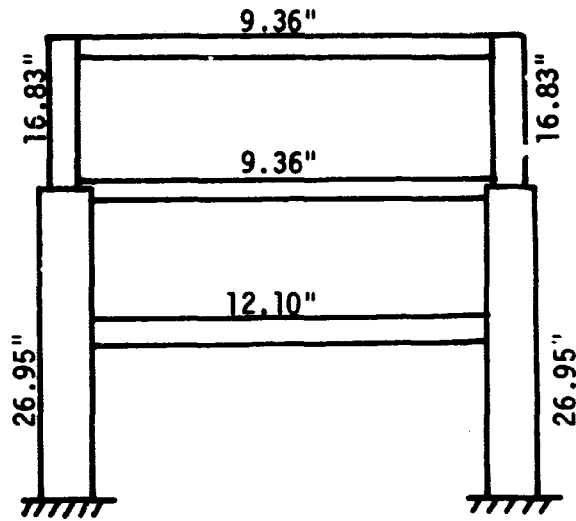


Fig. 8.20 First Bounded Design  
(Wt. = 242919 lbs.)

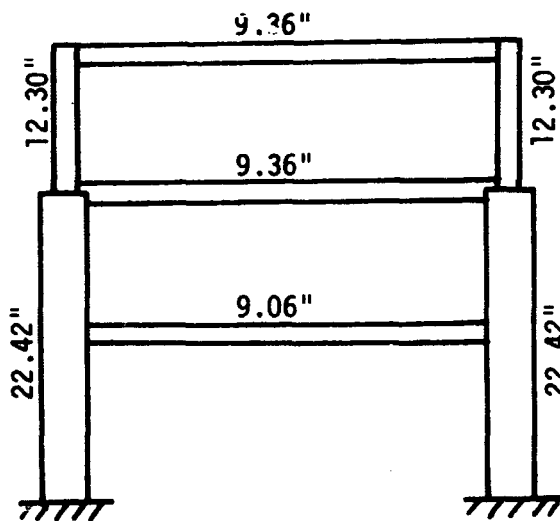


Fig. 8.21 Least Weight Design  
(Wt. = 217871 lbs.)

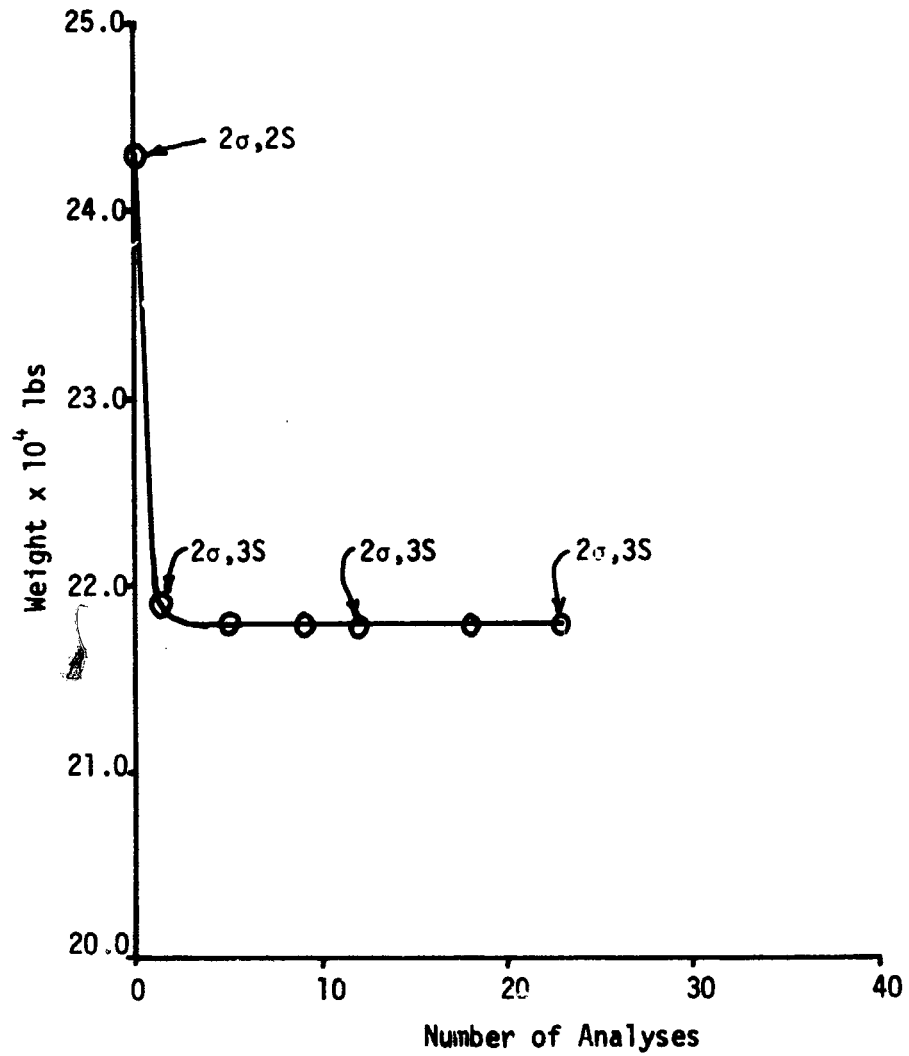


Fig. 8.22 Cumulative Number of Analyses Performed To Obtain The Bounded Designs Versus The Weight of the Bounded Designs

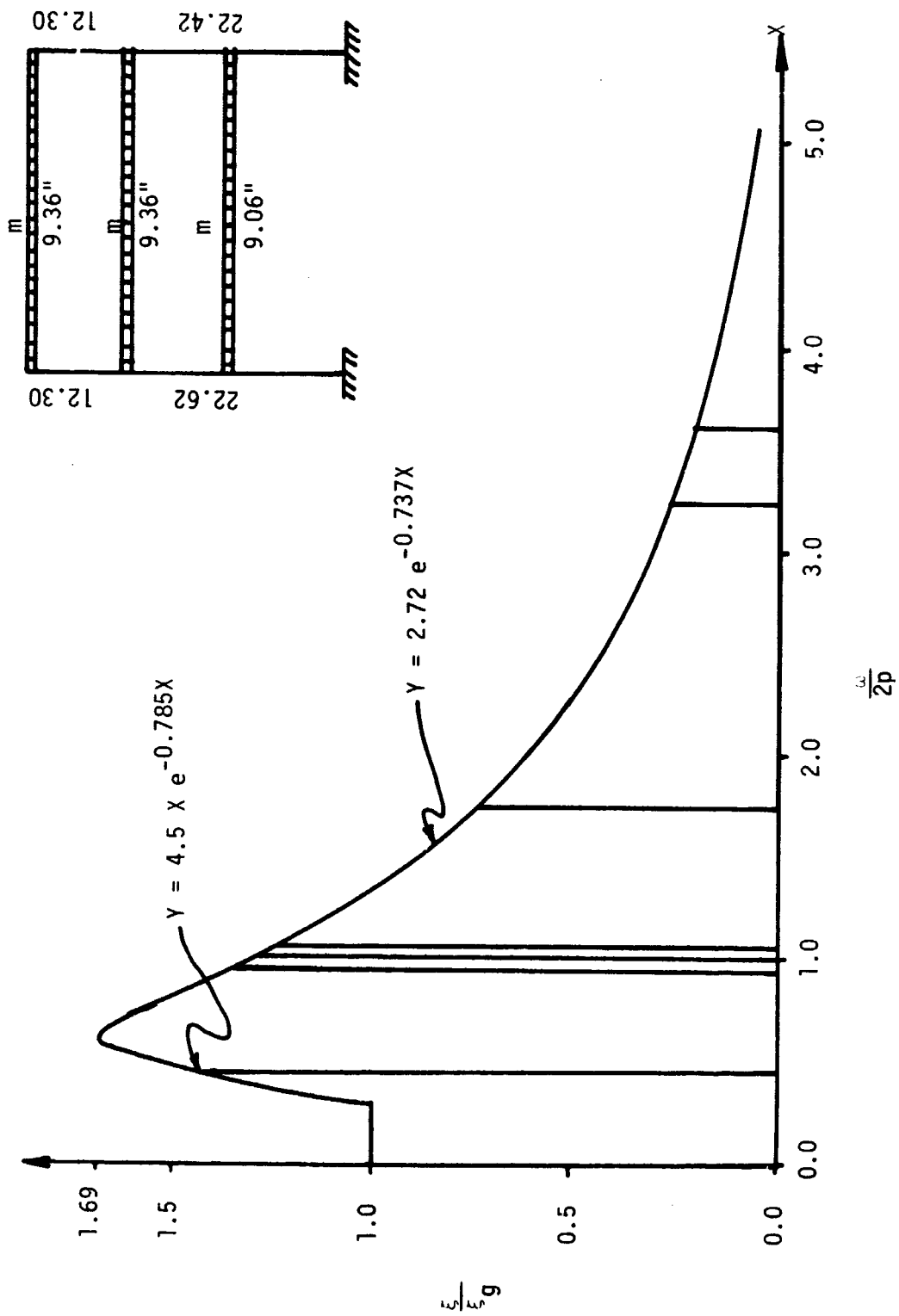


Fig. 8.23 First Seven Frequencies of the Least Weight Design

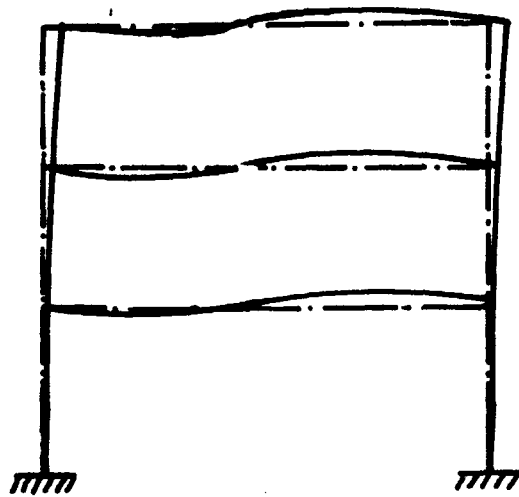


Fig. 8.24 Mode Shape Corresponding  
To Lowest Eigenvalue No. 1  
For The Least Weight Design  
(Fig. 8.21 )

(NOT TO SCALE)



## CHAPTER 9

### SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The present effort demonstrates the feasibility of automated optimum design for structural systems consisting of linear, planar, straight, one-dimensional members. The analysis method presented represents an application of the discrete element displacement method to the prediction of upper bound displacements and stresses at any point of an elastic planar truss-frame structure within the framework of a dynamics technology. The shock spectral approach has been used with the modal superposition method to obtain a conservative estimate of the response quantities (displacements and stresses) in multi-degree-of-freedom linear system. The eigensolution is obtained by minimizing the Rayleigh quotient employing the conjugate gradient method.

As should be true with the development of any synthesis program, the analysis and the optimization of the planar truss-frame structure have been viewed simultaneously. An efficient synthesis program can be developed only if advantage is taken of the characteristics of the problem at hand. It has been realized, at least in principle, that in the automated optimum design of structures, with difficult analysis, a reanalysis of the intermediate designs should be performed only when it is expected that effort expended in analysis will be worthwhile. Since the analysis consumes the majority of the computational effort in the present

work, it is an indication that redesign should be done with a great deal of care. It is because of this that a direct attack on the optimization problem has been attempted by using the method of feasible directions, using the constraints explicitly. This necessitated information regarding the dependence of the behavior quantities upon the design variables, which has been obtained in closed form thus providing the normals to the constraint surfaces in the design space.

The results from the present study lead to certain conclusions which are grouped in Section 9.2. A set of recommendations to extend the scope of structural synthesis under dynamic response restrictions is presented in Section 9.3.

## 9.2 Conclusions

(1) In order to perform the dynamic analysis of a planar truss-frame system, the structure should be modeled using general planar beam elements having six degrees-of-freedom, three (the axial deformation, the transverse displacement and the rotation) at each end.

(2) A reasonably accurate dynamic analysis of the structure can be performed by considering only a partial eigensolution starting from the lowest end of the eigenvalue spectrum.

(3) A refined modeling enables the assumed modes of the compatible elements to fit closer to the exact mode function of the structure and hence, by refined modeling, a reasonably accurate estimate of the stresses at any point on the members of the

truss-frame system can be made by the use of spatial derivatives.

(4) The minimization of Rayleigh quotient is an efficient method of obtaining the partial eigensolution within the structural synthesis framework where dynamic behavior is considered. This is because the method is iterative and converges quite rapidly when reasonable initial estimates of the eigenvector are available. As the optimum structural design is evolved, the eigensolution generally changes only incrementally from design to design. Hence, the previous solutions provide good initial estimates of the eigenvector.

(5) The interaction between the individual member frequency and the overall structure frequency results in closely spaced eigenvalues for symmetric truss-frame systems. In other words, certain modes of the truss-frame structure are predominately the vibration modes of the long members of the structure.

(6) The mode shapes change positions, in the sense that their appearance in the spectrum of eigenvalues shifts from design to design, and hence, even though the index of the frequency (or the eigenvalue) constraint may remain the same due to the sequence in the spectrum of the eigenvalues, yet the behavior constraint corresponding to it may be quite different.

### 9.3 Recommendations

(1) A need for rapid reanalysis of modified trial designs was recognized. Since the eigenvalue analysis consumes the majority of the computational effort of analysis of the structure, the author explored the possibility of approximating the eigenvalue analysis of new designs. This method uses the expressions for the rates of

change of eigenvalues and eigenvectors with respect to the design parameters of the actual structure and is discussed below:

### Approximate Eigenvalue Analysis

Suppose  $\lambda_i$  is the eigenvalue corresponding to the  $i^{\text{th}}$  natural frequency of a design  $\vec{\delta}$  where  $\vec{\delta}$  denotes the  $r$ -component vector of design variables for the structure. Suppose further that the design variables undergo a change of  $\Delta\vec{\delta} = (\Delta\delta_1, \Delta\delta_2, \dots, \Delta\delta_r)$ . An approximation to the  $i^{\text{th}}$  eigenvalue of the new design  $\vec{\delta}^*$  is given by

$$\lambda_i^* \equiv \lambda_i + \Delta\vec{\delta}^T \nabla \lambda_i \equiv \lambda_i^{(E)} \quad (9.1)$$

where  $\nabla \lambda_i$  represents the vector  $(\lambda_{i,1}, \lambda_{i,2}, \dots, \lambda_{i,r})$  and  $\lambda_{i,j}$  denotes  $\partial \lambda_i / \partial \delta_j$ , Eq. (5.12). Furthermore, if  $\vec{X}_i$  is the associated eigenvector, then

$$\vec{X}_i^* \approx \vec{X}_i + [\nabla \vec{X}_i] \Delta\vec{\delta} \equiv \vec{X}_i^{(E)} \quad (9.2)$$

where the  $n \times r$  matrix

$$[\nabla \vec{X}_i] \equiv [\vec{X}_{i,1}, \vec{X}_{i,2}, \dots, \vec{X}_{i,r}]$$

and  $\vec{X}_{i,j}$  denotes  $\partial \vec{X}_i / \partial \delta_j$ , Eq. (5.33). Using the Rayleigh quotient, another approximation to  $\lambda_i^*$  is given by

$$\lambda_i^* \approx \frac{\vec{X}_i^{(E)T} [K^*] \vec{X}_i^{(E)}}{\vec{X}_i^{(E)T} [M^*] \vec{X}_i^{(E)}} \equiv \lambda_i^{(R)} \quad (9.3)$$

where  $[K^*] \equiv [K(\vec{\delta}^*)]$  and  $[M^*] \equiv [M(\vec{\delta}^*)]$ .

Though the method of approximate eigenvalue analysis shows promise<sup>29</sup>, yet it was not used in the present work pending more careful study and the logical decisions as to when the approximate analysis should be performed in the synthesis program. A rapid reanalysis is the key factor in speeding the synthesis and hence additional research on developing and improving the efficiency of analysis is in order.

(2) It has been observed in the course of reanalysis that certain eigensolutions take more iterations to converge than others. This needs some further investigation.

(3) It has been observed, at times, that the analysis of a similar design is not exactly similar. This could be because of the use of a less stringent convergence criterion on the convergence of the Rayleigh quotient function. There is a considerable saving of time in obtaining the eigensolution by choosing a less stringent convergence criterion. However, although the error does not seem to propagate from eigenvalue to eigenvalue, the eigenvectors so obtained are not as accurate as those obtained by the more stringent convergence criterion. Since the estimates of upper bound displacements and stresses are dependent on eigenvectors it is expected that the use of stringent convergence criterion might result in a similar analysis. It may also be a sort of numerical problem. The number of operations involved to carry out the analysis is rather large and round off errors may be accumulating leading to slightly dissimilar analysis results

for similar designs.

(4) The overestimation of the response quantities (upper bound displacements and stresses) discussed in Chapter 3 can be avoided by taking a different approach to analysis and synthesis. The analysis can be accomplished by obtaining the solution of the system of differential equations as a function of time. The involvement of time parameter in the behavior variables would introduce another dimension of difficulty in the design problem, since all limitations imposed on behavior functions must be satisfied continuously in time. This approach to the dynamic analysis of the structure would require a different method to attack the optimization problem. It seems that the extension of the SUMT method<sup>30</sup> by using a generalized form of the Fiacco-McCormick penalty function in an integral form<sup>31</sup> might make consideration of limitations on transient behavior tractable. In this optimization approach, the objective function is augmented with a penalty term which is small at points away from the constraints in the feasible region, but which "blows up" as the constraints are approached. The function which is sequentially minimized is given by

$$\phi(\vec{D}, r) = W(\vec{D}) + r \left[ \sum_{j=1}^s \int_T \frac{dt}{g_j(\vec{D}, t)} + \sum_{k=s+1}^m \frac{1}{g_k(\vec{D})} \right] \quad (9.4)$$

where  $W$  is the weight function which is to be minimized over all  $\vec{D}$  satisfying  $g_j(\vec{D}, t) \leq 0$ ,  $j = 1, 2, \dots, s$ ;  $0 < t$  and

$g_k(\vec{D}) \leq 0$  ,  $k = s+1 \dots, m$  . The penalty parameter,  $r$  , is made successively smaller in order to obtain the constrained minimum of  $W$  .

The approach suggested in the preceeding paragraph will be altogether new to handle the automated optimum design of structures under dynamic response restrictions. Further research in the direction pointed out and comparison with the present work might lead the way towards the selection of the optimization technique and the development of the method of analysis for structures where dynamic behavior is to be considered.

(5) In order to improve the design philosophy used in the present work, the following failure modes should be guarded:

- (a) Failure modes due to the static loads. (static displacement, static stress and elastic instability modes).
- (b) The fatigue failure.

(6) Investigations for multiple dynamic load conditions is another area of research in the automated optimum design of structures under dynamic response restrictions.

(7) An obvious extension of the present work is to apply the procedure of analysis and automated optimum design discussed in the present work to space frame structures. This would require decisions regarding the estimation of upper bound stress at any point on a member of a space frame which in general would be

subjected to stresses due to an axial deformation, bending of the member in the two planes and a torsion.

(8) Further research effort could explore the application of the procedure to the plate and shell structures. This would require considerations about the estimation of upper bound stress and the failure criterion. Furthermore, the number of degrees-of-freedom associated with plate and shell elements are more than those associated with beam elements and this will place additional burden on the storage capacity of existing digital computers.



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## APPENDIX I

### THE HERMITE INTERPOLATION FORMULA

An approximation polynomial which is equal to the function it approximates at a number of specified points is called an interpolation formula. A generalization of the interpolation formula is obtained by requiring agreement with not only the function value,  $f(x)$ , but also with the first  $N$  derivatives of  $f(x)$  at any number of discrete set of distinct points  $x_i$ ,  $i = 1, 2, \dots, n+1$ . When  $N = 0$ , i.e. when only the function values are required to agree at each point of interpolation, the interpolation polynomial is called Lagrange interpolation formula, whereas for the case of  $N = 1$ , i.e., when the function and its first derivative are to be assigned at each point of interpolation, the procedure is called the Hermite or Osculatory interpolation. If higher derivatives of  $f(x)$  are assigned, i.e. when  $N > 1$  we obtain Hyperosculatory interpolation formulae.

Let us consider the planar truss element and the general planar beam element, respectively shown in Figs. 2.2 and 2.4. We need an interpolation formula which will enable us to generate a displacement state for these elements having the property that the displacements at the end depend only on the degrees of freedom at that end. We would see that by using Hermite interpolation it is possible to generate such a displacement state. Moreover, such interpolated displacement functions turn out to be the exact

solutions to the homogeneous differential equation of the statics problem for the element under consideration.

Let us denote  $H_{kj}^{(N)}(x)$  to be the Hermite interpolation polynomial where  $N$  is the number of derivatives the set has to interpolate,  $k$  is an index varying from 0 to  $N$  and  $j$  corresponds to the station index, i.e., the  $j^{\text{th}}$  point of the discrete set of distinct points of interpolation. In our case, the points of interpolation for both the truss and beam elements are only two — the left and the right ends of the elements. The left end would be taken to correspond to  $j = 1$  ( $x_1 = 0$ ) and the right end would be  $j = 2$  ( $x_2 = \ell$ ), where  $\ell$  is the length of the elements. These polynomials have the properties:

(1):

$$H_{0j}^{(N)}(x_i) = \delta_{ij}$$

$$d H_{0j}^{(N)}(x)/dx \Big|_{x=x_i} = 0$$

$$d^2 H_{0j}^{(N)}(x)/dx^2 \Big|_{x=x_i} = 0$$

.

.

.

$$d^N H_{0j}^{(N)}(x)/dx^N \Big|_{x=x_i} = 0$$

(2):

$$H_{1j}^{(N)}(x_i) = 0$$

$$d H_{1j}^{(N)}(x)/dx \Big|_{x=x_i} = \delta_{ij}$$

$$d^2 H_{1j}^{(N)}(x)/dx^2 \Big|_{x=x_i} = 0 \quad (I.1)$$

.

.

.

$$d^N H_{1j}^{(N)}(x)/dx^N \Big|_{x=x_i} = 0$$

.

.

.

(N+1):

$$H_{Nj}^{(N)}(x_i) = 0$$

.

.

.

$$d^N H_{Nj}^{(N)}(x)/dx^N \Big|_{x=x_i} = \delta_{ij}$$

where  $\delta_{ij}$  is the Kronecker delta, i.e.,

$$\delta_{ij} = 0 \quad , \quad i \neq j$$

and

$$\delta_{ij} = 1 \quad , \quad i = j$$

### I.2 Planar Truss Element:

A planar truss element is shown in Fig. 2.2 and has four degrees-of-freedom. Any point on the element, in general, undergoes an axial deformation and a displacement normal to the axis of the element. Let a displacement state be defined as:

$$u(x) = u_1 P_1(x) + u_2 P_2(x)$$

and

(I.2)

$$w(x) = u_3 P_3(x) + u_4 P_4(x)$$

where

$u(x)$  is the displacement function along the axis of the element,  $u_1$  and  $u_2$  are the longitudinal displacements of the element at its two ends,  $w(x)$  is the displacement function normal to the axis of the element,  $u_3$  and  $u_4$  are the displacements normal to the axis of the element at its ends,  $x$  is measured along the length of the element and its origin is taken to be on the left end marked '1' on Fig. 2.2. The polynomials  $P_1(x)$  ,  $P_2(x)$  ,  $P_3(x)$  and  $P_4(x)$  have the property

$$\begin{aligned}
P_1(x) \Big|_{x=0} &= 1, & P_1(x) \Big|_{x=\ell} &= 0 \\
P_2(x) \Big|_{x=0} &= 0, & P_2(x) \Big|_{x=\ell} &= 1 \\
P_3(x) \Big|_{x=0} &= 1, & P_3(x) \Big|_{x=\ell} &= 0 \\
P_4(x) \Big|_{x=0} &= 0, & P_4(x) \Big|_{x=\ell} &= 1
\end{aligned} \tag{I.3}$$

This structure corresponds to Hermite Polynomial for the case of  $N = 0$ . We have, therefore,

$$P_1(x) = H_{01}^{(0)}(x) = P_3(x)$$

and (I.4)

$$P_2(x) = H_{02}^{(0)}(x) = P_4(x)$$

From Eqs. (I.2) and (I.4) we obtain

$$u(x) = u_1 H_{01}^{(0)}(x) + u_2 H_{02}^{(0)}(x) \tag{I.5}$$

$$w(x) = u_3 H_{01}^{(0)}(x) + u_4 H_{02}^{(0)}(x)$$

The polynomial  $H_{01}^{(0)}(x)$  has to satisfy the conditions



$$H_{01}^{(0)}(x_1) = 1 \quad , \quad H_{01}^{(0)}(x_2) = 0 \quad (I.6)$$

A polynomial of degree 1 will have the required number of undetermined coefficients to satisfy the two conditions of Eqs. (I.6). Therefore, let

$$H_{01}^{(0)}(x) = a + bx \quad (I.7)$$

From Eqs. (I.6) and (I.7) we obtain

$$a = 1$$

and

$$b = -\frac{1}{\ell}$$

Therefore

$$H_{01}^{(0)}(x) = -\frac{(x-\ell)}{\ell} \quad (I.8)$$

Similarly we can obtain

$$H_{02}^{(0)}(x) = \frac{x}{\ell} \quad (I.9)$$

These polynomials are sketched in Fig. I.1.

### I.3 General Planar Beam Element

A general planar beam element is shown in Fig. 2.4 and has six degrees-of-freedom. The axial degrees of freedom  $u_1$  and  $u_2$  are uncoupled to the transverse and the rotational degrees of

freedom  $u_3$  to  $u_6$ . Let a displacement statement be defined as

$$u(x) = u_1 P_1(x) + u_2 P_2(x)$$

and (I.10)

$$w(x) = u_3 P_3(x) + u_4 P_4(x) + u_5 P_5(x) + u_6 P_6(x)$$

where  $u(x)$  is the displacement function along the axis of the element,  $u_1$  and  $u_2$  are the longitudinal displacements of the element at its two ends,  $w(x)$  is the deformation function normal to the axis of the element,  $u_3$  and  $u_4$  are the displacements normal to the axis of the element at its ends,  $u_5$  and  $u_6$  are the rotations at the two ends of the element,  $x$  is measured along the length of the element and its origin is taken to be on the left end marked '1' on Fig. 2.4. The polynomials  $P_1(x)$  to  $P_6(x)$  have the property

$$P_1(x) \Big|_{x=0} = 1, \quad P_1(x) \Big|_{x=\ell} = 0$$

$$P_2(x) \Big|_{x=0} = 0, \quad P_2(x) \Big|_{x=\ell} = 1$$

$$P_3(x) \Big|_{x=0} = 1, \quad P_3(x) \Big|_{x=\ell} = 0$$

$$P_3'(x) \Big|_{x=0} = 0, \quad P_3'(x) \Big|_{x=\ell} = 0$$

(I.11)

$$\begin{aligned}
P_4(x) \Big|_{x=0} &= 0 & , & & P_4(x) \Big|_{x=\ell} &= 1 \\
P_4'(x) \Big|_{x=0} &= 0 & , & & P_4'(x) \Big|_{x=\ell} &= 0 \\
P_5(x) \Big|_{x=0} &= 0 & , & & P_5(x) \Big|_{x=\ell} &= 0 \\
P_5'(x) \Big|_{x=0} &= 1 & , & & P_5'(x) \Big|_{x=\ell} &= 0 \\
P_6(x) \Big|_{x=0} &= 0 & , & & P_6(x) \Big|_{x=\ell} &= 0 \\
P_6'(x) \Big|_{x=0} &= 0 & , & & P_6'(x) \Big|_{x=\ell} &= 1
\end{aligned} \tag{I.11}$$

where the prime denotes first derivative with respect to  $x$  .

Comparing this structure with that of Eqs. (I.2) we obtain

$$P_1(x) = H_{01}^{(0)}(x)$$

$$P_2(x) = H_{02}^{(0)}(x)$$

$$P_3(x) = H_{01}^{(1)}(x)$$

$$P_4(x) = H_{02}^{(1)}(x)$$

(I.12)

$$P_5(x) = H_{11}^{(1)}(x)$$

and

$$P_6(x) = H_{12}^{(1)}(x) \quad (I.12)$$

From Eqs. (I.10) and (I.12) we obtain

$$u(x) = u_1 H_{01}^{(0)}(x) + u_2 H_{02}^{(0)}(x)$$

and

(I.13)

$$w(x) = u_3 H_{01}^{(1)}(x) + u_4 H_{02}^{(1)}(x) + u_5 H_{11}^{(1)}(x) + u_6 H_{12}^{(1)}(x)$$

Axial displacement being independent of the transverse deformation, we notice that the axial displacement function for the planar truss element and the general planar beam element is identical and therefore, the polynomials  $H_{01}^{(0)}(x)$  and  $H_{02}^{(0)}(x)$  of Eq. (I.12) are respectively given by Eqs. (I.8) and (I.9).

The polynomial  $H_{01}^{(1)}(x)$  has to satisfy four conditions

$$\begin{aligned} H_{01}^{(1)}(x_1) &= 1, & H_{01}^{(1)}(x_2) &= 0 \\ \left. \frac{d H_{01}^{(1)}(x)}{dx} \right|_{x=x_1} &= 0, & \left. \frac{d H_{01}^{(1)}(x)}{dx} \right|_{x=x_2} &= 0 \end{aligned} \quad (I.14)$$

A polynomial of degree 3 will have the required number of undetermined coefficients to satisfy the conditions of Eq. (I.14).

Therefore, let

$$H_{01}^{(1)}(x) = a + bx + cx^2 + dx^3 \quad (I.15)$$

From Eqs. (I.14) and (I.15), we obtain

$$a = 1$$

$$b = 0$$

$$c = -\frac{3}{\ell^2}$$

and

$$d = \frac{2}{\ell^3}$$

Therefore,

$$H_{01}^{(1)}(x) = \frac{1}{\ell^3} (2x^3 - 3\ell x^2 + \ell^3) \quad (I.16)$$

Similarly, we can obtain

$$H_{02}^{(1)}(x) = -\frac{1}{\ell^3} (2x^3 - 3\ell x^2) \quad (I.17)$$

$$H_{11}^{(1)}(x) = \frac{1}{\ell^2} (x^3 - 2\ell x^2 + \ell^2 x) \quad (I.18)$$

and

$$H_{12}^{(1)}(x) = \frac{1}{\ell^2} (x^3 - \ell x^2) \quad (I.19)$$

The polynomials  $H_{01}^{(1)}(x)$  ,  $H_{02}^{(1)}(x)$  ,  $H_{11}^{(1)}(x)$  and  $H_{12}^{(1)}(x)$  are pictured in Fig. (I.2).

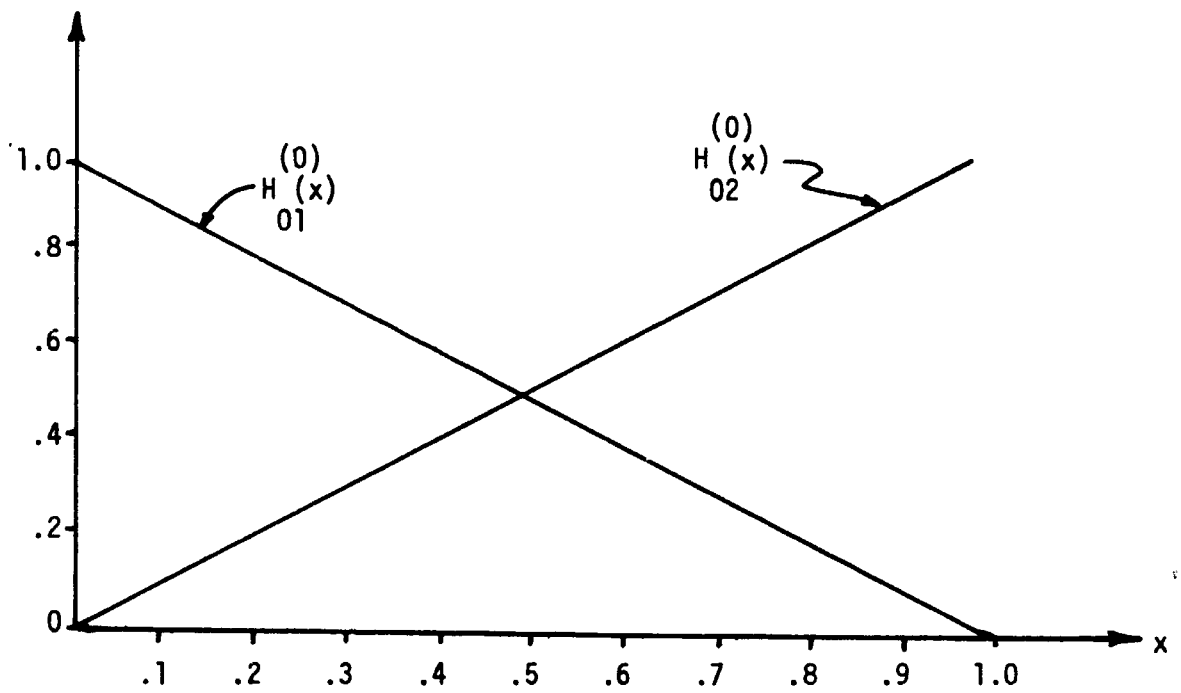


Fig. I.1 Lagrange Polynomials.

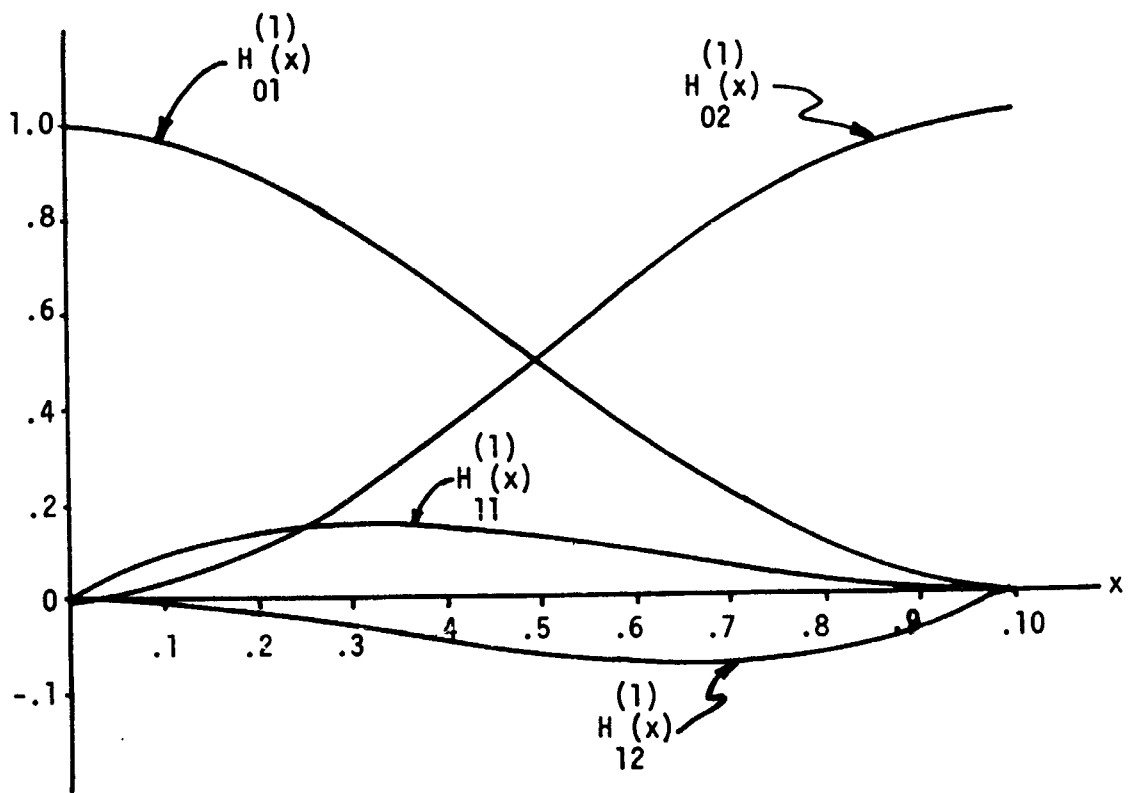


Fig. I.2 Osculatory Polynomials.



## APPENDIX II

### PROCEDURE TO OBTAIN $[Q_\ell]^{-1}$ FROM $[Q_{\ell-1}]^{-1}$

As discussed in Chapter 4,  $[Q_\ell]^{-1}$  is given by:

$$[Q_\ell]^{-1} = ([N_\ell]^T [N_\ell])^{-1} \equiv \left[ \begin{array}{c|c} [B_1] & \vec{B}_2 \\ \hline \vec{B}_2^T & B_3 \end{array} \right] \quad (\text{II.1})$$

where

$$\begin{aligned} [B_1] &= [A_1]^{-1} + \frac{1}{s} \left( [A_1]^{-1} \vec{A}_2 \vec{A}_2^T [A_1]^{-1} \right) \\ \vec{B}_2 &= -\frac{1}{s} \left( [A_1]^{-1} \vec{A}_2 \right) \\ \vec{B}_3 &= \frac{1}{s} \end{aligned} \quad (\text{II.2})$$

and

$$s = A_3 - \vec{A}_2^T [A_1]^{-1} \vec{A}_2 \quad (\text{II.3})$$

Furthermore,

$$\begin{aligned} [A_1] &= [Q_{\ell-1}] \equiv [N_{\ell-1}]^T [N_{\ell-1}] \\ \vec{A}_2^T &= [\vec{e}_j^T \vec{v}_{\ell-1}, \vec{v}_1^T \vec{v}_{\ell-1}, \dots, \vec{v}_{\ell-2}^T \vec{v}_{\ell-1}] \end{aligned}$$

and

$$A_3 = \vec{V}_{\ell-1}^T \vec{V}_{\ell-1} \quad (\text{II.4})$$

It is presumed that

$$[A_1]^{-1} = [Q_{\ell-1}]^{-1} \equiv \left( [N_{\ell-1}]^T [N_{\ell-1}] \right)^{-1} \quad (\text{II.5})$$

is known.

We can write

$$\vec{A}_2 = [N_{\ell-1}]^T \vec{V}_{\ell-1} \quad (\text{II.6})$$

where

$$[N_{\ell-1}] = [\vec{e}_j, \vec{V}_1, \vec{V}_2, \dots, \vec{V}_{\ell-2}]$$

Therefore, from Eqs. (II.3), (II.4), (II.5), and (II.6), we obtain

$$\begin{aligned} s &= \vec{V}_{\ell-1}^T \vec{V}_{\ell-1} - \vec{V}_{\ell-1}^T [N_{\ell-1}] \left( [N_{\ell-1}]^T [N_{\ell-1}] \right)^{-1} [N_{\ell-1}]^T \vec{V}_{\ell-1} \\ &= \vec{V}_{\ell-1}^T [P_{\ell-1}] \vec{V}_{\ell-1} \end{aligned} \quad (\text{II.7})$$

where

$$[P_{\ell-1}] \equiv \left\{ [I] - [N_{\ell-1}] \left( [N_{\ell-1}]^T [N_{\ell-1}] \right)^{-1} [N_{\ell-1}]^T \right\} \quad (\text{II.8})$$

is the "projection" matrix.

The projection matrix, Eq. (II.8), has a neat property that

the vectors  $[P_{\ell-1}]\vec{W}$  and  $\{[I] - [P_{\ell-1}]\}\vec{W}$  are orthogonal. This can be seen by mere multiplication of the two vectors:

$$\begin{aligned} \vec{W}^T [P_{\ell-1}] \{[I] - [P_{\ell-1}]\} \vec{W} &= \vec{W}^T \{[I] - [N_{\ell-1}] ([N_{\ell-1}]^T [N_{\ell-1}])^{-1} \times \\ &\quad [N_{\ell-1}]^T\} \{[N_{\ell-1}] ([N_{\ell-1}]^T [N_{\ell-1}])^{-1} \times \\ &\quad [N_{\ell-1}]^T\} \vec{W} \\ &= \vec{W}^T \{[N_{\ell-1}] ([N_{\ell-1}]^T [N_{\ell-1}])^{-1} [N_{\ell-1}]^T \\ &\quad - [N_{\ell-1}] ([N_{\ell-1}]^T [N_{\ell-1}])^{-1} [N_{\ell-1}]^T\} \vec{W} \end{aligned} \quad (II.9)$$

since  $([N_{\ell-1}]^T [N_{\ell-1}])^{-1} ([N_{\ell-1}]^T [N_{\ell-1}]) \equiv [I]$ .

The term within the brackets  $\{ \}$  on the right hand side of Eq. (II.9) is identically zero and hence we obtain

$$\vec{W}^T [P_{\ell-1}] \{[I] - [P_{\ell-1}]\} \vec{W} = 0 \quad (II.10)$$

Eq. (II.10) can be rewritten as

$$\vec{W}^T [P_{\ell-1}] \vec{W} = \vec{W}^T [P_{\ell-1}] [P_{\ell-1}] \vec{W} = |[P_{\ell-1}] \vec{W}|^2 \quad (II.11)$$

Therefore, we obtain from Eq. (II.7)

$$s = |[P_{\ell-1}] \vec{V}_{\ell-1}|^2 \quad (II.12)$$

Note that the projection matrix  $[P_{\ell-1}]$  is already known and thus to obtain  $s$  from Eq. (II.12) is computationally more

efficient than that from Eq. (II.3).

If we denote

$$\begin{aligned}\vec{r}_{\ell-1} &\equiv [A_1]^{-1} \vec{A}_2 \\ &= \left( [N_{\ell-1}]^T [N_{\ell-1}] \right)^{-1} [N_{\ell-1}]^T \vec{V}_{\ell-1}\end{aligned}\quad (\text{II.13})$$

then from Eqs. (II.8) and (II.13) we obtain

$$[P_{\ell-1}] \vec{V}_{\ell-1} = \vec{V}_{\ell-1} - [N_{\ell-1}] \vec{r}_{\ell-1} \quad (\text{II.14})$$

Furthermore, from Eqs. (II.5), (II.13), (II.14) and (II.2) we obtain

$$\begin{aligned}[B_1] &= \left( [N_{\ell-1}]^T [N_{\ell-1}] \right)^{-1} + \frac{1}{s} \vec{r}_{\ell-1} \vec{r}_{\ell-1}^T \\ \vec{B}_2 &= -\frac{1}{s} \vec{r}_{\ell-1}\end{aligned}\quad (\text{II.15})$$

and

$$B_3 = \frac{1}{s}$$

Thus the procedure to obtain  $[Q_\ell]^{-1}$  from  $[Q_{\ell-1}]^{-1}$  can be summarized as:

1. Compute the vector  $\vec{r}_{\ell-1}$  from Eq. (II.13) as two matrix vector multiplications.

2. Compute the scalar,  $s$ , from Eq. (II.14) and (II.12).
3. Compute the matrix  $[B_1]$ , the vector  $\vec{E}_2$  and the scalar  $B_3$  from Eq. (II.15).
4. Form the desired  $[Q_\rho]^{-1}$  from Eq. (II.1)

### APPENDIX III

#### TRANSFORMATION BETWEEN REFERENCE AND LOCAL COORDINATES

Let  $SR$  be an  $rx(t+1)$  table whose first column contains the element numbers arranged from 1 to  $r$  and whose rows contain in the remaining  $t$  columns the corresponding local consistent coordinates of the elements, replaced one to one by the number of the corresponding degree-of-freedom of the structure in the reference coordinate system. The  $i^{th}$  row of the  $SR$  table is used to obtain the  $(tx1)$  vector  $\vec{Y}_i$ ,  $i = 1, 2, \dots, r$  from the  $(nx1)$  vector  $\vec{X}$ .  $\vec{Y}_i$  is the displacement vector of the  $i^{th}$  element corresponding to the generalized displacement vector  $\vec{X}$  of the system. As an illustrative example, consider the simple cantilever beam shown in Fig. III.1, which is modeled with three general planar beam elements. Thus, the number of elements,  $r$ , is 3, the local degrees of freedom of an element in the reference coordinate system,  $t$ , is 6, and the number of degrees of freedom of the system,  $n$ , is 9. The  $SR$  table for this system is:

| Element<br>No. | Degrees of Freedom              |   |                                      |   |                    |   |
|----------------|---------------------------------|---|--------------------------------------|---|--------------------|---|
|                | Axial<br>Displacement<br>At End |   | Transverse<br>Displacement<br>At End |   | Rotation<br>of End |   |
|                | P                               | Q | P                                    | Q | P                  | Q |
| 1              | 0                               | 1 | 0                                    | 2 | 0                  | 3 |
| 2              | 1                               | 4 | 2                                    | 5 | 3                  | 6 |
| 3              | 4                               | 7 | 5                                    | 8 | 6                  | 9 |

Let the generalized displacement vector be:

$$\vec{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix}$$

Then

$$\vec{Y}_1 = \begin{pmatrix} 0 \\ x_1 \\ 0 \\ x_2 \\ 0 \\ x_3 \end{pmatrix}, \quad \vec{Y}_2 = \begin{pmatrix} x_1 \\ x_4 \\ x_2 \\ x_5 \\ x_3 \\ x_6 \end{pmatrix}, \quad \vec{Y}_3 = \begin{pmatrix} x_4 \\ x_7 \\ x_5 \\ x_8 \\ x_6 \\ x_9 \end{pmatrix}$$

Thus we can say that

$$\vec{Y}_i \xleftarrow{SR_i} \vec{X}$$

i.e., the vector  $\vec{X}$  is transformed to the vector  $\vec{Y}_i$  through the  $i^{\text{th}}$  row of the SR tableau, where the nonzero components of the vector  $\vec{Y}_i$  are the components of the vector  $\vec{X}$  associated with the  $i^{\text{th}}$  element.



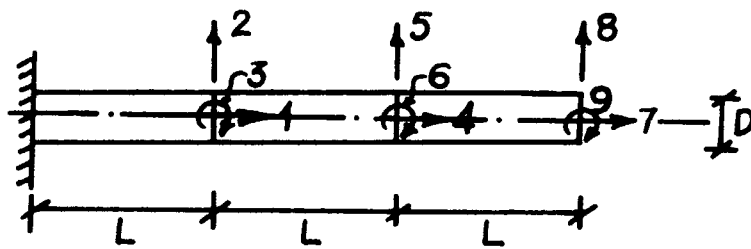


Fig. III.1 Cantilever Beam

## APPENDIX IV

### COMPUTER PROGRAM

This appendix consists of four major sections. Section IV.2 contains a description of each of the eleven Fortran Subprograms which make up the complete program for the automated minimum weight design of planar truss-frame system. A flow diagram of MAIN program and the listing of each subprogram is also included.

Section IV.3 contains a description of the necessary input data.

Section IV.4 describes the output data.

Section IV.5 contains a listing of the input data deck and representative portions of the printed output for a typical illustrative example.

#### IV.2 The Program

The complete program for automated minimum weight design of the planar truss-frame system consists of (A) a main program which reads the input data, checks the side constraints for the initial design, evaluates the element stiffness and mass matrices, controls the flow through various subroutines, checks the convergence, and prints the output and (B) the other ten subroutines as described below:

1. (a) Name of Subroutine: FBMIN

(b) Argument List: M, RDF, NDFS, SR, KR, MR

M: Total number of elements.

RDF: Number of degrees-of-freedom of an element in system coordinates.

NDFS: Number of degrees-of-freedom of system.

SR: Assembly control table.

KR: Element stiffness matrix in system coordinates.

MR: Element mass matrix in system coordinates.

(c) Common Variables: SDIFF, AA, BB, NUMER, DENOM, VEC,  
BX, MIN, MINO, ITER, OMEGA, VALU,  
VECO, ID

SDIFF: Scalar; Denotes convergence of Rayleigh quotient function; [MAIN]<sup>†</sup>

AA: Vector; Denotes  $[K]\vec{X}$ , where  $[K]$  is master stiffness matrix and  $\vec{X}$  is generalized displacement vector; [FANDG, KXMX, ANAL, DRESP, MAIN]

BB: Vector; Denotes  $[M]\vec{X}$ , where  $[M]$  is master mass matrix and  $\vec{X}$  is generalized displacement vector; [FANDG, KXMX, ANAL, DRESP, MAIN]

NUMER : Scalar; Denotes  $\vec{X}^T[K]\vec{X}$ ; [FANDG]

---

<sup>†</sup>The names of subprograms within brackets are those to which the indicated variable is common.

- DENOM: Scalar; Denotes  $\vec{X}^T[M]\vec{X}$  ; [FANDG]
- VEC: Counter; Denotes the number of eigenvalues-eigenvectors being searched; [FANDG, PROD, INGEN]
- BX: Matrix; The  $i^{\text{th}}$  column of BX represents the vector  $[M]\vec{X}_i$  where  $[M]$  is master mass matrix and  $\vec{X}_i$  represents the  $i^{\text{th}}$  eigenvector; [FANDG]
- MIN: Counter; Denotes the current component number of the eigenvector which is set to one; [FANDG]
- MINO: Counter; Denotes the old component number of the eigenvector which was set to one; [FANDG]
- ITER: Counter; Denotes the number of iteration of the conjugate gradient method; [FANDG]
- OMEGA: Vector; The components of OMEGA represent the frequencies (Square root of eigenvalues); [ANAL, DRESP, MAIN]
- VALU: Vector; The components of VALU represent the eigenvalues; [DEIG, MAIN]
- VECO: Matrix; The rows of VECO represent the eigenvectors; [ANAL, DEIG, DRESP, MAIN]
- ID: Counter; Denotes the number of analysis,[MAIN]
- (d) Purpose: To compute the eigenvalues and associated eigenvectors. This subroutine contains three subroutines:

- (1) 'FANDG' - to compute the function (Rayleigh quotient) value and to evaluate the gradient vector to the Rayleigh quotient at any point.
- (2) 'PROD' - to compute the product  $([N][N^T N]^{-1}[N]^T)\vec{W}$  where

$$[N] \equiv BX$$

and  $\vec{W}$  is any vector in system coordinates.

- (3) 'INGEN' - to obtain  $\left([N_\ell]^T[N_\ell]\right)^{-1}$  from  $\left([N_{\ell-1}]^T[N_{\ell-1}]\right)^{-1}$ .

2. (a) Name of Subroutine: KXXM

(b) Argument List: NDFS, SR, X, KR, MR, LBB

NDFS: Number of degrees-of-freedom of an element in system coordinates.

SR: Assembly control table.

X: Vector in system coordinates.

KR: Element stiffness matrix in system coordinates.

MR: Element mass matrix in system coordinates.

LBB: Logical; LBB = •TRUE• provides the normalized vector  $BB \equiv [M]\vec{X}$  (Euclidean norm of the vector is equal to one).

(c) Common Variables: AA, BB, Y

AA: Vector; Denotes  $[K]\vec{X}$ , where  $[K]$  is master stiffness matrix and  $\vec{X}$  is generalized

displacement vector; [FBMIN, FANDG, ANAL, DRESP, MAIN]

BB: Vector; Denotes  $[M]\vec{X}$ , where  $[M]$  is master mass matrix and  $\vec{X}$  is generalized displacement vector; [FBMIN, FANDG, ANAL, DRESP, MAIN]

Y: Matrix; The  $i^{\text{th}}$  row of the matrix denotes the displacement vector of the  $i^{\text{th}}$  element corresponding to any generalized displacement vector  $\vec{X}$  in system coordinates; [DRESP]

(d) Purpose: To compute the vectors  $[K]\vec{X}$  and  $[M]\vec{X}$  where  $[K]$  and  $[M]$  are respectively the master stiffness and mass matrices of the structure and  $\vec{X}$  is any vector in system coordinates. Note that master stiffness and mass matrices are not assembled and the desired vectors are obtained by the use of only element stiffness and mass matrices.

3. (a) Name of Subroutine: SHOCK

(b) Argument List: NVEC, X, Y, DY

NVEC: Number of eigenvectors used for modal transformation.

X: X-coordinate of the spectrum.

Y: Y-coordinate of the spectrum.

DY: Rate of change of the spectral curve.

(c) Common Variables: PF, XG

PF: Scalar; Denotes the frequency of the forcing function in radians/sec.; [MAIN]

XG: Scalar; Denotes the maximum ground displacement due to the forcing function; [MAIN]

(d) Purpose: To compute the peak response and the rate of change of peak response with respect to the frequency of a single degree-of-freedom mass-spring oscillator from the idealized shock spectral curve for a one-half cycle sine wave acceleration pulse.

4. (a) Name of Subroutine: POLYN

(b) Argument List: RDF, TRR, TL, NP, PX, HPY, HPW

RDF: Number of degrees-of-freedom of an element in system coordinates.

TRR: Transformation matrix which transforms a displacement vector of an element in the system coordinates to the displacement vector of the element in the element coordinate system.

TL: Length of element.

NP: Number of points at which the upper bound displacements and stresses are computed.

PX: Vector; The components of which represent the distance of the points along the length of an element at which upper bound displacements and stresses are computed.

HPY: Vector representing  $\vec{r}^{(u)T}(x)$  [T] where  $\vec{r}^{(\cdot)}(x)$  is given by Eq. (3.31) and [T] represents the transformation matrix TRR.

HPW: Vector representing  $\vec{r}^{(w)T}(x)$  [T] where  $\vec{r}^{(w)}(x)$  is given by Eq. (3.32) and [T] represents the transformation matrix TRR.

(c) Common Variables: There are no common variables in this subprogram.

(d) Purpose: To compute the components of the vectors  $\vec{r}^{(u)}(x)$ , Eq. (3.31), and  $\vec{r}^{(w)}(x)$ , Eq. (3.32), and to evaluate the vectors  $\vec{r}^{(u)T}(x)$  [T] and  $\vec{r}^{(w)T}(x)$  [T] where the transformation matrix  $[T] \equiv TRR$  is given by Eq. (3.37).

5. (a) Name of Subroutine: ANAL

(b) Argument List: M, RDF, SR, EG, XG, PF

M: Total number of elements.

RDF: Number of degrees-of-freedom of an element in system coordinates.

SR: Assembly control table.



EG: Vector containing one or zero as its components and which when multiplied by scalar function,  $f(t)$ , gives the generalized force vector.

XG: Maximum displacement of the ground in inches corresponding to the acceleration pulse.

PF: Frequency of the forcing function in radians/sec.

(c) Common Variables: KR, MR, TR, AA, BB, D, E, L, YC,  
DYC, C, PROD, PY, PW, VECO, OMEGA,  
YM, SIGMA

KR: Element stiffness matrix in reference coordinate system; [FANDG, DRESP, MAIN]

MR: Element mass matrix in reference coordinate system; [FANDG, DRESP, MAIN]

TR: Transformation matrix which transforms a displacement vector of an element in the system coordinates to the displacement vector of the element in the element coordinate system; [DEIG, MAIN]

AA: Vector; Denotes  $[K]\vec{X}$ , where  $[K]$  is master stiffness matrix and  $\vec{X}$  is generalized displacement vector; [FBMIN, FANDG, KXMX, DRESP, MAIN]

BB: Vector; Denotes  $[M]\vec{X}$ , where  $[M]$  is master mass matrix and  $\vec{X}$  is generalized displacement

- vector; [FBMIN, FANDG, KXMX, DRESP, MAIN]
- D: Vector; The components of vector D represent mean diameter of tubular elements; [DEIG, DRESP, MAIN]
- E: Vector; The components of vector E represent Young's Modulus of Elasticity of the material of the elements; [DEIG, DRESP, MAIN]
- L: Vector; The components of vector L represent the lengths of the elements; [DEIG, MAIN]
- YC: Vector; The components of vector YC represent the peak response quantity of a single degree-of-freedom system corresponding to its frequency  $\omega_i$  — these are obtained from the shock spectrum, [DRESP]
- DYC: Vector; The components of vector DYC represent the rate of change of peak response quantity of a single degree-of-freedom system with respect to its frequency; [DRESP]
- C: Vector; The  $i^{\text{th}}$  component of vector C represents  $-\vec{X}_i^T [M] \vec{e}$  where  $\vec{X}_i$  is the  $i^{\text{th}}$  eigenvector,  $[M]$  is master mass matrix and  $\vec{e}$  is the vector EG such that  $\vec{F}(t) = \vec{e} f(t)$ ; [DRESP]
- PROD: Vector; the  $i^{\text{th}}$  component of vector PROD represents  $i^{\text{th}}$  peak modal participation

coefficient; [DRESP]

PY: Matrix; The rows of matrix PY represent  $\hat{r}^{(u)T}(x) [T]_j$ , Eqs. (3.31) and (3.37); [DRESP]

PW: Matrix; The rows of matrix PW represent  $\hat{r}^{(w)T}(x) [T]_j$ , Eqs. (3.32) and (3.37); [DRESP]

VECO: Matrix; the rows of VECO represent the eigenvectors; [FBMIN, DEIG, DRESP, MAIN]

OMEGA: Vector; The components of OMEGA represent the frequencies (square root of eigenvalues); [FBMIN, DRESP, MAIN]

YM: Vector; The components of YM represent upper bound displacements at the degrees of freedom in the direction of system coordinate axes; [MAIN]

SIGMA: Matrix; the  $i^{th}$  row represents the upper bound stresses at the two ends of the  $i^{th}$  element; [MAIN]

- (d) Purpose: To compute the upper bound displacements and upper bound stresses at the two ends of the general planar beam elements used to model the planar truss-frame structure.

6. (a) Name of Subroutine: MANDCH

(b) Argument List: S, PCTR, WT, GW, NDV, SCON, BSC, NBSC, BSCL, DELTA

- S: The move vector.
- PCTR: Percentage reduction in weight desired in the direction of vector S.
- WT: Weight of the structure.
- GW: Gradient to the weight function (A constant vector in the present work).
- NDV: Number of design variables.
- SCON: Upper and lower bounds on 'side constraints'.
- BSC: Vector whose components represent the indices of bounded side constraints.
- NBSC: Number of bounded side constraints.
- BSCL: Logical, BSCL = 'TRUE' means that some side constraints are bounded.
- DELTA: Constraint tolerance.
- (c) Common Variables: DV, ALPHA, OD
- DV: Vector of current design variables; [ADJUST, MAIN]
- ALPHA: Scalar, Denotes step length; [MAIN]
- OD: Vector of old design variables; [ADJUST]
- (d) Purpose: To make a move in the direction of S (steepest descent or feasible direction), to check the side constraints of the new design and adjust by linear interpolation the violated side constraints.

7. (a) Name of Subroutine: ADJUST

(b) Argument List: S, WCV, OWCV, ALPHA, BSCL, BSC, NBSC,  
DELTA, SCON, NDV, LQ, SLOPE

S: The move vector.

WCV: Value of worst violated constraint.

OWCV: Value of the same constraint which is worst violated in the current design at the old design point where it was bounded.

ALPHA: Step length.

BSCL: Logical, BSCL = -TRUE- means that some side constraints are bounded.

BSC: Vector whose components represent the indices of bounded side constraints.

NBSC: Number of bounded side constraints.

DELTA: Constraint tolerance.

SCON: Upper and lower bounds on side constraints.

NDV: Number of design variables.

LQ: Logical, LQ = -TRUE- means adjustment is accomplished through quadratic interpolation.

SLOPE: Denotes the slope to the constraint at the old bounded design point which is worst violated in the current design.

(c) Common Variables: DV, EXIT, OD

DV: Vector of current design variables; [MANDCH, MAIN]

EXIT: Logical, EXIT = TRUE terminates the run whenever negative value of step length is encountered; [MAIN]

OD: Vector of old design variables; [MANDCH]

- (d) Purpose: To adjust the worst violated behavior constraint by linear or quadratic interpolation and check the side constraints of the new design.

8. (a) Name of Subroutine: DEIG

(b) Argument List: NDFS, SR, NDV, MMAX, IDVCR

NDFS: Number of degrees-of-freedom of the system.

SR: Assembly control table.

NDV: Number of design variables.

MMAX: Maximum number of elements governed by a design variable.

IDVCR: Design variable correlation table.

(c) Common Variables: E, L, T, RO, B1, TR, VALU, VECO, DTS, CLAMB, CVEC

D: Vector; The components of vector D represent the mean diameter of tubular elements; [ANAL, DRESP, MAIN]

E: Vector; The components of vector E represent Young's Modulus of Elasticity of the material of the elements; [ANAL, DRESP, MAIN]

- L: Vector; The components of vector L represent the lengths of the elements; [ANAL, MAIN]
- T: Vector; The components of vector T represent the wall thickness of elements; [MAIN]
- RO: Vector; The components of vector RO represent the weight density of the material; [MAIN]
- B1: Logical;  $B_1 = \text{TRUE}$  means structure comprises of truss elements; [MAIN]
- TR: Transformation matrix which transforms a displacement vector of an element in the system coordinates to the displacement vector of the element in the element coordinate system; [ANAL, MAIN]
- VALU: Vector; The components of VALU represent the eigenvalues; [FBMIN, MAIN]
- VECO: Matrix; The rows of VECO represent the eigenvectors; [FBMIN, ANAL, DRESP, MAIN]
- DTS: Matrix; represents  $[M, j]$  which denotes a matrix whose elements are obtained by differentiating the elements of mass matrix with respect to the diameter  $d_j$ ; [DRESP]
- CLAMB: Matrix; The  $i^{\text{th}}$  row of the matrix CLAMB represents the derivative of  $i^{\text{th}}$  eigenvalue with respect to design variables; [DRESP, MAIN]

CVEC: Matrix; The  $i^{\text{th}}$  row of the matrix represents the derivative of the  $i^{\text{th}}$  eigenvector with respect to design variables; [DRESP]

- (d) Purpose: To compute the derivative of the eigenvalues and eigenvectors with respect to design parameters.

9. (a) Name of Subroutine: DRESP

(b) Argument List: M, RDF, NDFS, SR, EG

M: Total number of elements.

RDF: Number of degrees-of-freedom of an element in system coordinates.

NDFS: Number of degrees-of-freedom of the system.

SR: Assembly control table.

EG: Vector containing one or zero as its components and which when multiplied by scalar function,  $f(t)$ , gives the generalized force vector.

(c) Common Variables: KR, MR, VECO, OMEGA, D, E, AA, BB, YC, DYC, C, PROD, DTS, CLAMB, CVEC, PY, PW, IDVCR, Y, DY, DSIGMA

KR: Element stiffness matrix in reference coordinate system; [FANDG, ANAL, MAIN]

MR: Element mass matrix in reference coordinate system; [FANDG, ANAL, MAIN]



- VECO: Matrix; The rows of VECO represent the eigenvectors; [FBMIN, ANAL, DEIG, MAIN]
- OMEGA: Vector; The components of OMEGA represent the frequencies (square root of eigenvalues); [FBMIN, ANAL, MAIN]
- D: Vector; The components of vector D represent the mean diameter of tubular elements; [ANAL, DEIG, MAIN]
- E: Vector; The components of vector E represent Young's Modulus of Elasticity of the material of the elements; [ANAL, DEIG, MAIN]
- AA: Vector; Denotes  $[K]\vec{X}$ , where  $[K]$  is master stiffness matrix and  $\vec{X}$  is generalized displacement vector; [FBMIN, FANDG, KXMX, ANAL, MAIN]
- BB: Vector; Denotes  $[M]\vec{X}$ , where  $[M]$  is master mass matrix and  $\vec{X}$  is generalized displacement vector; [FBMIN, FANDG, KXMX, ANAL, MAIN]
- YC: Vector; The components of vector YC represent the peak response quantity of a single degree-of-freedom system corresponding to its frequency  $\omega_i$  — these are obtained from the shock spectrum; [ANAL]
- DYC: Vector; The components of vector DYC represent the rate of change of peak response quantity

of a single degree-of-freedom system with respect to its frequencies; [ANAL]

C: Vector; the  $i^{\text{th}}$  component of vector C represents  $-\vec{X}_i^T [M] \vec{e}$  where  $\vec{X}_i$  is the  $i^{\text{th}}$  eigenvector,  $[M]$  is master mass matrix, and  $\vec{e}$  is the vector EG such that

$$\vec{F}(t) = \vec{e} f(t) ; \text{ [ANAL]}$$

PROD: Vector; The  $i^{\text{th}}$  component of vector PROD represents  $i^{\text{th}}$  peak modal participation coefficient; [ANAL]

DTS: Matrix; represents  $[M, j]$  which denotes a matrix whose elements are obtained by differentiating the elements of mass matrix with respect to the diameter  $d_j$ ; [DEIG]

CLAMB: Matrix, The  $i^{\text{th}}$  row of the matrix CLAMB represents the derivative of  $i^{\text{th}}$  eigenvalue with respect to design variables; [DEIG, MAIN]

CVEC: Matrix, The  $i^{\text{th}}$  row of the matrix CVEC represents the derivative of the  $i^{\text{th}}$  eigenvector with respect to design variables; [DEIG]

PY: Matrix; the rows of matrix PY represent  $\vec{F}^{(u)T}(x) [T]_i$ , Eqs. (3.31) and (3.37); [ANAL]

PW: Matrix; The rows of matrix PW represent  $\vec{F}^{(w)T}(x) [T]_i$ , Eqs. (3.32) and (3.37); [ANAL]

IDVCR: Design variable correlation table; [MAIN]

Y: Matrix, The  $i^{\text{th}}$  row of the matrix denotes the displacement vector of the  $i^{\text{th}}$  element corresponding to any generalized displacement vector  $\vec{X}$  in system coordinates; [KXMX]

DY: Matrix; The  $i^{\text{th}}$  row of the matrix represents the derivative of  $i^{\text{th}}$  upper bound displacement with respect to design variables; [MAIN]

DSIGMA: Matrix; the  $i^{\text{th}}$  row of the matrix represents the derivative of upper bound stress in the element  $i$  with respect to design variables; [MAIN]

(d) Purpose: To compute the derivative of the response quantities (upper bound displacements and stresses) with respect to design parameters.

10. (a) Name of Subroutine: FEASIB

(b) Argument List: NAC, NDV, NC, GW, THETA, SIGMA, S

NAC: Number of active constraints.

NDV: Number of design variables.

NC: Matrix; the rows of matrix NC represent the constraint normals.

GW: Gradient to the weight function.

THETA: A vector of positive numbers: In the present work, the components of THETA are 0 or 1.

SIGMA: An extra variable

S: Feasible direction vector

(c) Common Variables: There are no common variables in this subprogram.

(d) Purpose: To compute a useable feasible direction of move in the design space. This subroutine also contains an internal subroutine called 'SIMPLE' to solve the linear programming problem by Simplex Algorithm.

A flow diagram of the MAIN program is sketched in the subsequent pages.

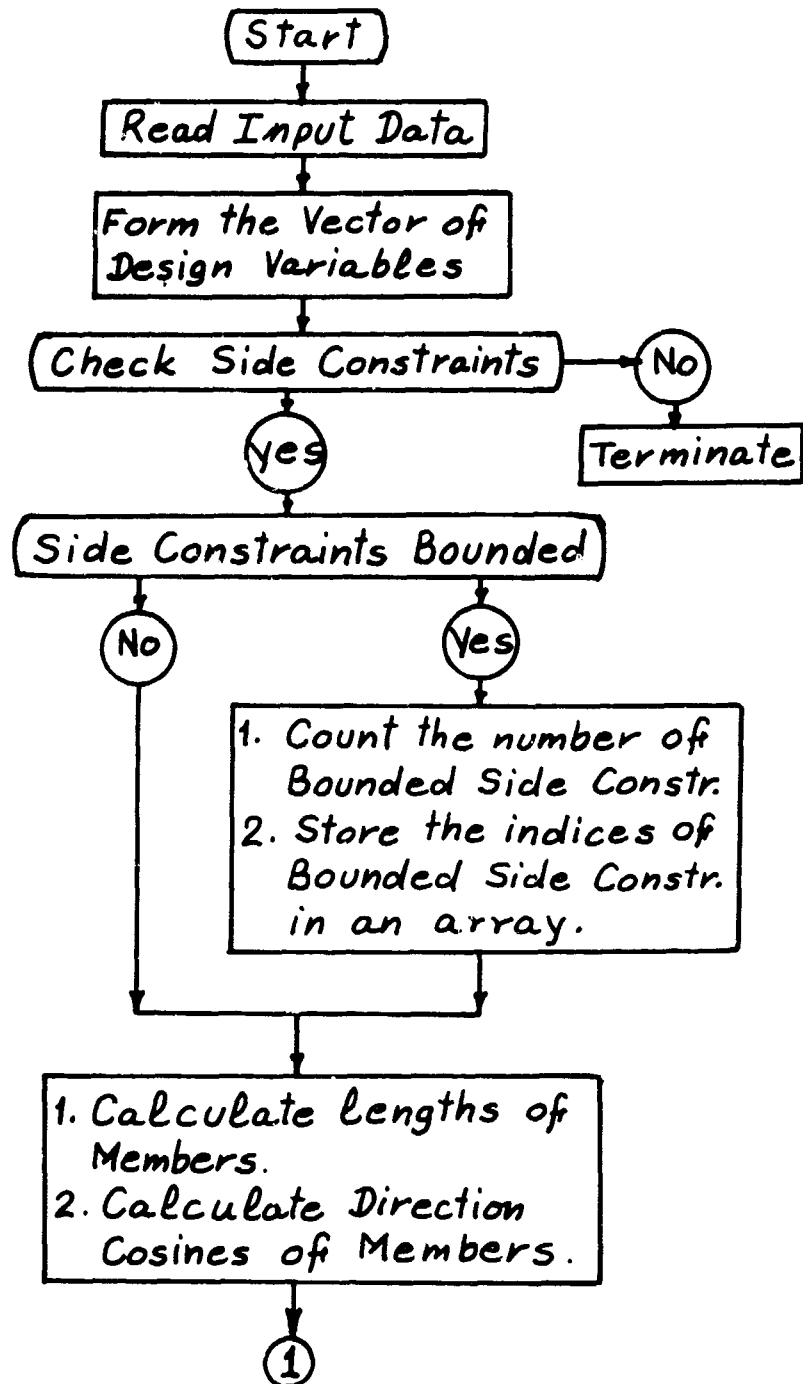
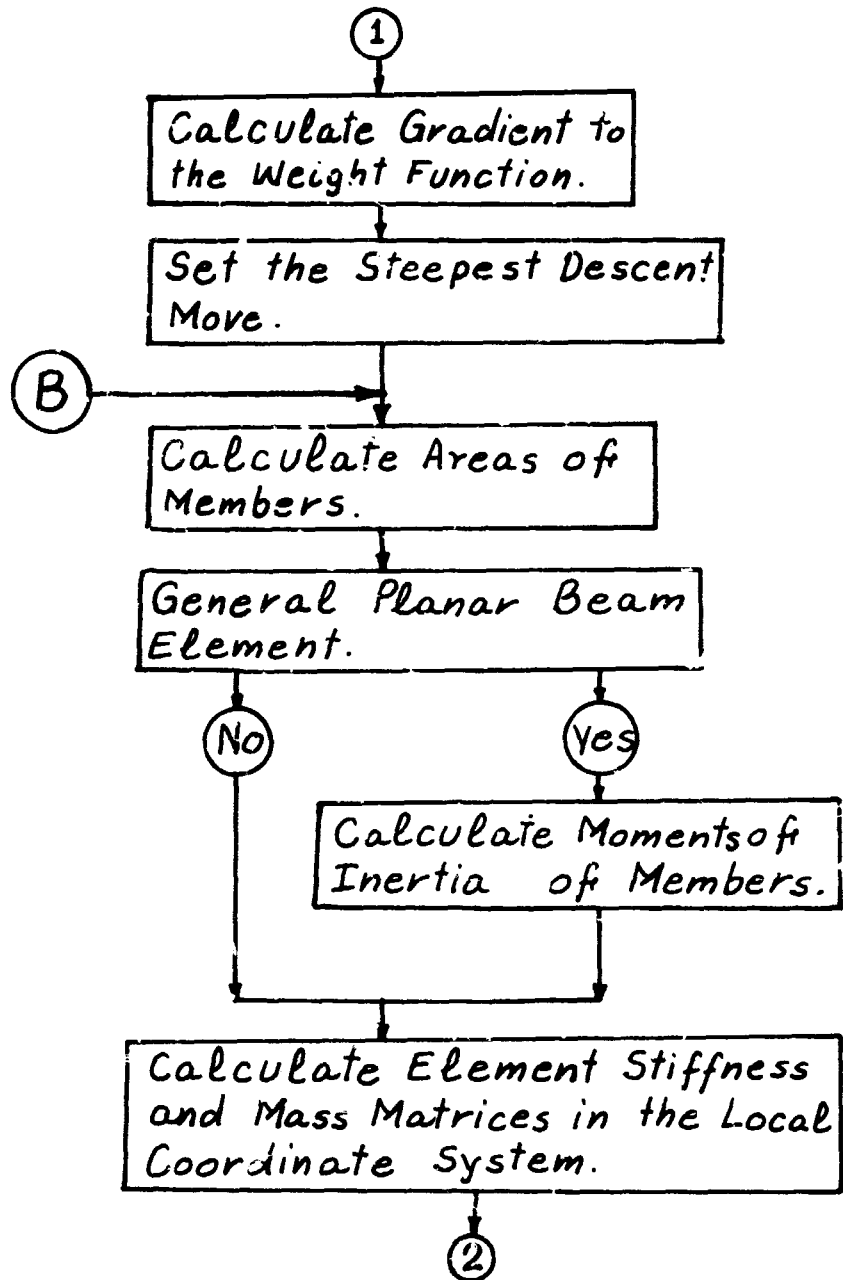
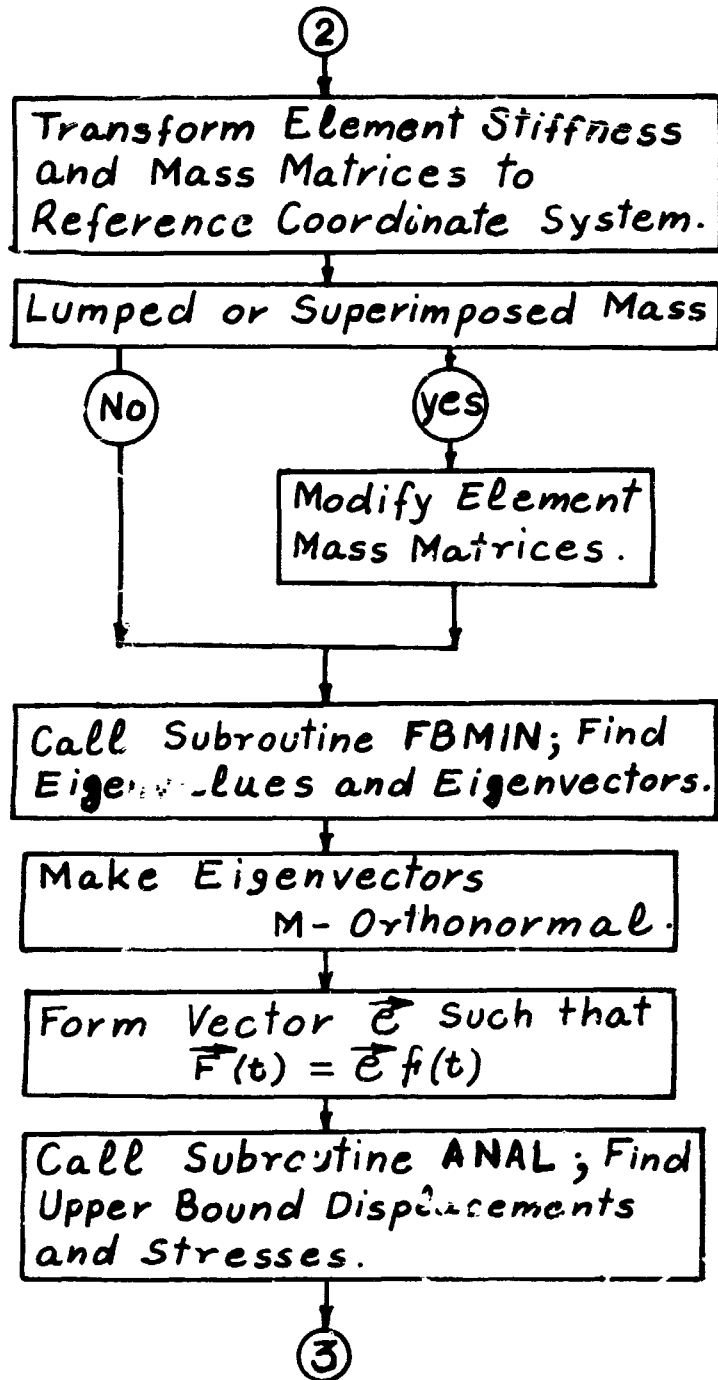


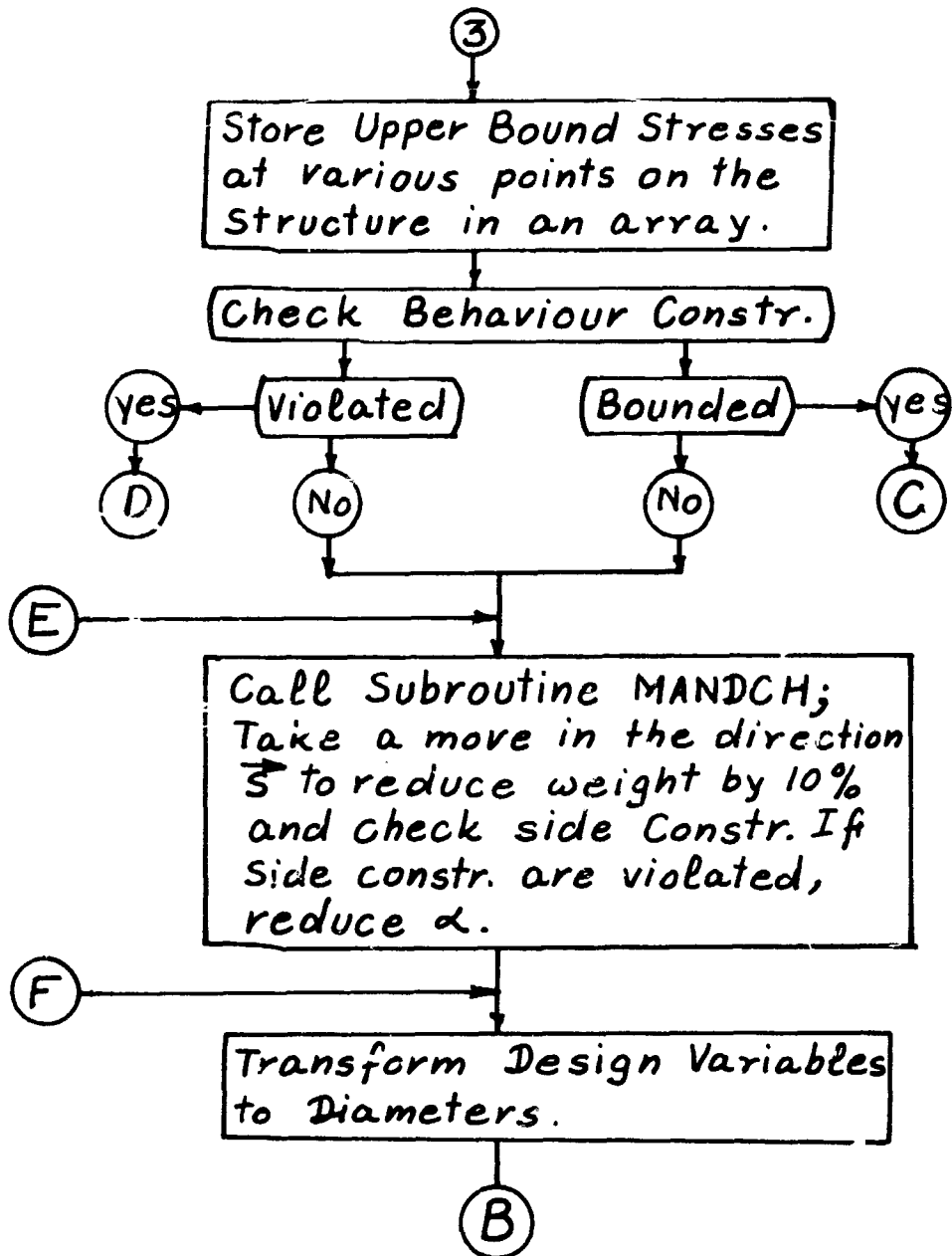
Fig. IV.1 Flow Diagram - MAIN



Flow Diagram - MAIN (Continued)

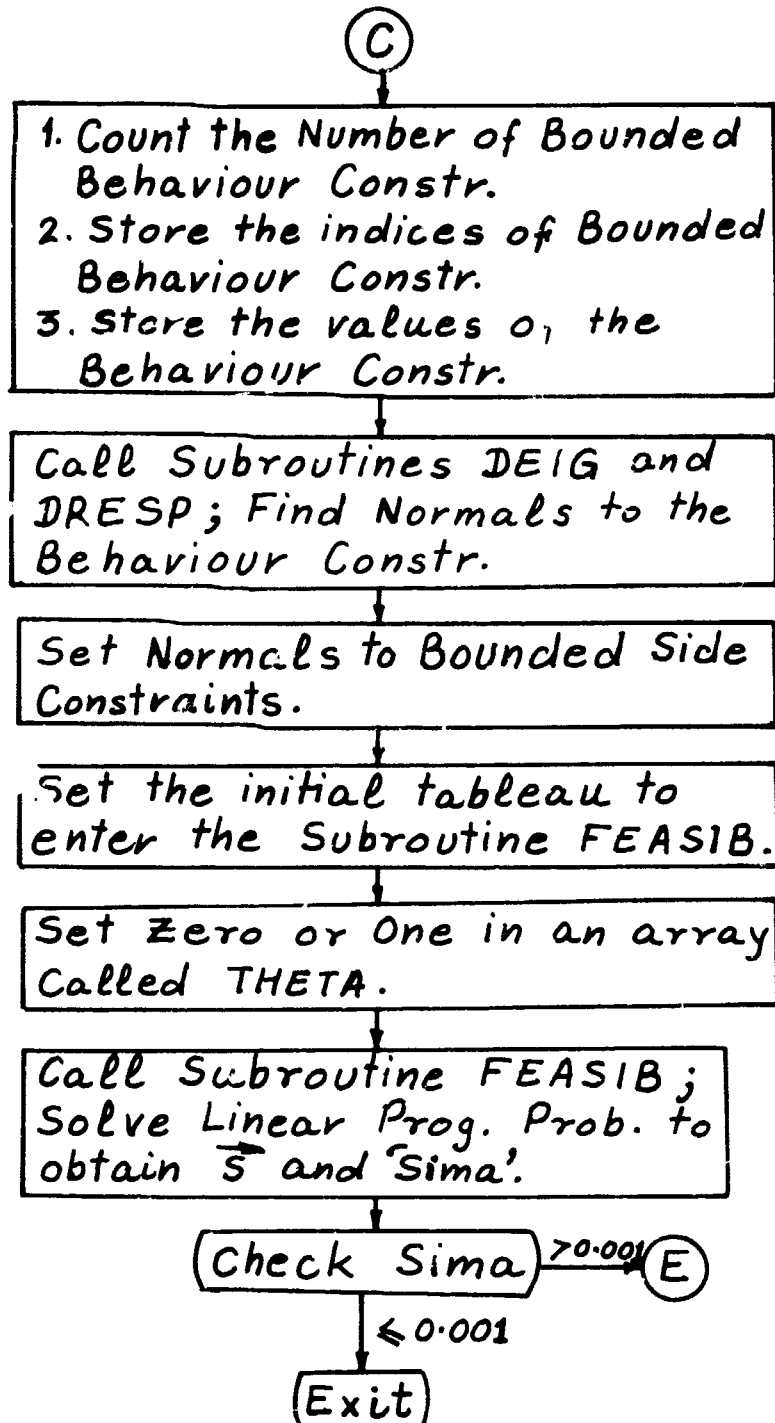


Flow Diagram - MAIN (Continued)

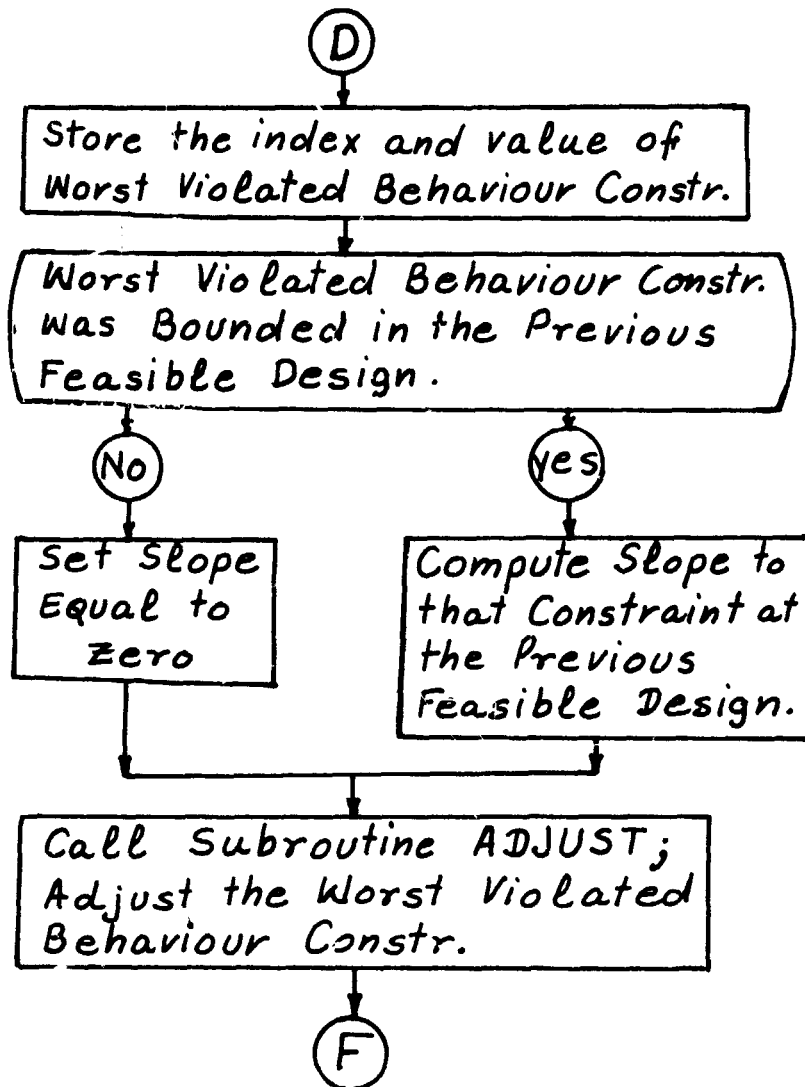


Flow Diagram - MAIN (Continued)





Flow Diagram - MAIN (Continued)



Flow Diagram - MAIN (Continued)

A listing of each of the eleven Fortran Subprograms which make up the complete program for the automated minimum weight design of Planar truss-frame system written in Fortran V for the Univac 1108 digital computer follows in the subsequent pages.

2 FOR FBWIN  
 COMPILE BY CHI UNIVAC 1108 FORTRAN V DATED: OCTOBER 29, 1968  
 THIS COMPILE WAS DONE ON 21 NOV 68 AT 14:25:33

SUBROUTINE FBWIN ENTRY POINT 001664

STORAGE USED (BLOCK, NAME, LENGTH)

|      |        |        |
|------|--------|--------|
| 0001 | CODE   | 003075 |
| 0002 | DATA   | 003247 |
| 0003 | BLANK  | 000000 |
| 0004 | STOP11 | 000001 |
| 0005 | EBLOCK | 000122 |
| 0006 | CBLOCK | 000002 |
| 0007 | DBLOCK | 000001 |
| 0008 | GBLOCK | 000702 |
| 0009 | JBLOCK | 000003 |
| 0010 | KBLOCK | 000013 |
| 0011 | LBLOCK | 000013 |
| 0012 | GMATN  | 000703 |
| 0013 | YBLOCK | 000001 |
| 0014 | ASLOCK | 001760 |

EXTERNAL REFERENCES (BLOCK, NAME)

|      |       |
|------|-------|
| 0016 | KMX   |
| 0017 | SRT   |
| 0020 | NRJOT |
| 0021 | NRJFU |
| 0022 | NRJFU |
| 0023 | NRJFU |
| 0024 | NRJFU |

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

|      |        |       |      |        |       |      |        |       |      |        |       |      |        |       |
|------|--------|-------|------|--------|-------|------|--------|-------|------|--------|-------|------|--------|-------|
| 0000 | 003002 | IF    | 0000 | 003004 | 10F   | 0001 | 001642 | 1000L | 0001 | 001617 | 10036 | 0001 | 001636 | 10136 |
| 0001 | 001722 | 10426 | 0001 | 001746 | 10536 | 0001 | 001771 | 10656 | 0001 | 002252 | 11L   | 0001 | 002043 | 11076 |
| 0002 | 002044 | 11156 | 0001 | 002054 | 11236 | 0001 | 002055 | 11236 | 0001 | 002067 | 11326 | 0001 | 002125 | 11506 |
| 0003 | 002144 | 11536 | 0001 | 002174 | 11646 | 0001 | 002221 | 11766 | 0001 | 002205 | 12L   | 0001 | 002247 | 12046 |
| 0004 | 002272 | 12126 | 0001 | 002423 | 12336 | 0001 | 002427 | 12346 | 0001 | 002445 | 12446 | 0001 | 002451 | 12506 |
| 0005 | 002472 | 12636 | 0001 | 002476 | 12646 | 0001 | 002603 | 13066 | 0001 | 002612 | 13146 | 0001 | 002636 | 13236 |
| 0006 | 002646 | 13266 | 0001 | 002655 | 13356 | 0001 | 002656 | 13406 | 0001 | 002674 | 13476 | 0001 | 002704 | 13546 |
| 0007 | 002705 | 13576 | 0001 | 000007 | 13666 | 0001 | 002741 | 13666 | 0001 | 000010 | 1416  | 0001 | 000020 | 1476  |
| 0008 | 000021 | 15256 | 0001 | 000037 | 1646  | 0001 | 000052 | 1676  | 0001 | 003055 | 19F   | 0001 | 000257 | 200L  |
| 0009 | 000114 | 2-13  | 0001 | 000071 | 21F   | 0001 | 000122 | 2106  | 0001 | 000143 | 2216  | 0001 | 001176 | 229L  |
| 0010 | 000156 | 2316  | 0001 | 002302 | 24L   | 0001 | 000175 | 2436  | 0001 | 000242 | 2616  | 0001 | 000252 | 2676  |
| 0011 | 002305 | 27F   | 0001 | 003011 | 28F   | 0001 | 003013 | 29F   | 0001 | 003025 | 30F   | 0001 | 000771 | 300L  |
| 0012 | 000701 | 3026  | 0001 | 003032 | 31F   | 0001 | 003047 | 32F   | 0001 | 000440 | 3336  | 0001 | 000461 | 3415  |

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0001 000203 350L      0001 000515 3546      0001 000524 3616      0001 000541 3716      0001 000541 38L      0001 000515 42N6
0001 000203 40L      0001 000515 400L      0001 000556 4016      0001 000596 4166      0001 000515 42N6
0001 000203 45L      0001 000570 451L      0001 000570 4546      0001 000625 454L      0001 000752 4726
0001 000203 49L      0001 001022 513L      0001 001060 5256      0001 001100 5346      0001 001113 5446
0001 001122 5516      0001 001151 5663      0001 001132 57L      0001 000425 60L      0001 001217 6046
0001 001251 6146      0001 001304 6306      0001 001333 6416      0001 001352 6536      0001 001372 6656
0001 001422 6776      0001 001400 69L      0001 001471 7076      0001 000533 71L      0001 001460 7176
0001 001423 72L      0001 001467 7246      0001 001476 7316      0001 001515 7416      0001 001527 7456
0001 000434 76L      0001 001562 7623      0001 000506 77L      0001 001257 777L      0001 000510 78L
0001 000133 80L      0001 00107 802L      0001 000146 805L      0001 001067 806L      0001 001307 807L
0001 000246 85L      0001 001317 892L      0001 000171 901L      0001 001366 902L      0001 001571 903L
0001 001343 92L      0004 R 000000 AA      0004 R 000000 AA      0004 R 000000 AA      0004 R 000000 AA      0000 R 001007 ALPHA
0000 R 001005 ALPHA1      0000 R 001006 ALPHA2      0000 R 001006 ALPHA2      0000 R 001016 AG      0000 R 001007 ALPHA
0004 R 000051 BB      0004 R 000051 BB      0000 R 001017 BE      0000 R 001020 BETA      0000 R 002765 A2
0007 R 000000 BX      0000 R 001024 DEB      0000 R 001023 DEL      0000 R 000776 DEN      0007 R 000400 BX
0005 R 000001 DENOM      0000 R 000774 DEJW1      0000 R 001025 DENOM2      0000 R 001027 DEN1      0005 R 000001 DENOM
0000 R 000766 F      0000 R 000765 FR      0000 R 001014 FXEM      0000 R 000773 F1      0000 R 001010 F2
0000 R 000170 G      0000 R 001015 GS      0000 R 000363 GEXM      0000 R 000763 I      0014 I 000000 ID
0000 I 001026 IJ      0000 I 000002 ITER      0000 I 001013 IMAX      0000 I 000762 ITNO      0000 I 001032 IPI
0000 I 000772 IT      0000 I 001025 JJ      0000 I 000002 K      0000 I 003001 KKK      0000 I 000167 IS
0000 I 002331 LL      0000 I 000000 MIV      0000 I 000000 MIN      0000 I 000001 M'NO      0015 R 000000 KR
0015 R 000170 MR      0000 R 001035 N      0000 I 001031 NE      0000 R 002131 NG      0010 I 000001 MNO
0000 R 001740 N1      0000 R 002246 NNN6      0000 R 000032 NNNP      0000 R 002144 NNNG      0000 R 002233 NG
0000 R 000045 NNNP      0000 R 000032 NNNP      0000 R 000017 NPMR      0000 R 000016 NUM      0000 R 002157 NNNNG
0005 R 000000 NUMER      0000 R 000014 NUMER1      0000 R 000017 NPMR      0000 R 000016 NUM      0005 R 000000 NUMER
0000 R 002332 P1      0000 R 001030 RAS      0000 L 0000 TEORL      0000 R 000016 OMEGA      0000 R 00025 P
0003 R 000000 SUIFF      0000 R 002232 SUM      0000 R 000000 SUM      0000 R 000011 TA      0000 R 000676 TEMP
0000 R 001033 TEMP1      0000 R 001034 TEMP2      0000 R 001014 T-ST      0000 R 002403 TM      0000 R 001002 TS1
0000 R 001003 TS2      0000 R 001004 TS3      0000 R 000777 U      0000 R 001000 V      0012 R 000000 VALU
0006 I 000000 VEC      0006 I 000000 VEC      0006 I 000000 VEC      0006 I 000000 VEC      0013 R 000000 VECO
0000 R 001001 W      0000 R 000116      0000 R 000312 XEM      0000 R 000434 XNN      0000 R 000771 XX
0000 R 002230 XXX

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00101 1. C      SUBROUTINE FSW1 (M,ROF,NDFS,SR,KR,MRI)
00101 2. C      THIS SUBROUTINE FINDS THE EIGENVECTORS AND CORRESPONDING EIGENVALU
00101 3. C      ES BY THE MINIMIZATION OF THE RAYLEIGH QUOTIENT.THE CONJUGATE GRAD
00101 4. C      INT METHOD IS USED FOR MINIMIZATION.
00103 5. C      PARAMETER MDEMI,NVEC=11
00103 6. C      ND IS NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM
00103 7. C      NVEC IS NUMBER OF EIGENVALUES-EIGENVECTORS REQUIRED
00104 8. C      INTEGER RE,LSR,VEC
00105 9. C      LOGICAL REARL
00105 10. C      REAL NVEC,NUMBER1,NUMBER2,NUM,NP,NNP,NP,NP,NP,KR,MRI
00107 11. C      COMMON/STG1/SCIFF
00107 12. C      COMMON/BLCK/AR498
00107 13. C      COMMON/BLCK/NUMBER1
00107 14. C      COMMON/BLCK/VEC
00107 15. C      COMMON/BLCK/XX
00107 16. C      COMMON/BLCK/NTIMINGINTER
00107 17. C      COMMON/BLCK/NTIMING

```



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00236 72. WIN=1
00237 73. MINO=1
00240 74. 201 CONTINUE
00242 75. 407 X(I)=X(I)/XX
00243 76. IT=0
00247 77. 49 ITER=0
00251 78. 35J CALL FANGG(NDFS,SR,X,.TRUE.,F1,G)
00252 79. NUMER1=NUMER
00253 80. DENOM1=DENOM
00254 81. 11 (VEC .NE. NDFS) GO TO 85
00255 82. VAL1(VEC)=F1
00256 83. OM*GAIVEC)=SORT(F1)
00257 84. DO 67 I=1,NDFS
00260 85. 67 VEC(VEC,I)=X(I)
00263 86. GO TO 69
00265 87. 85 DO 50 I=1,NDFS
00266 88. 50 S(I)=G(I)
00271 89. ITER=1
00273 90. 20J CALL FANGG(NDFS,SR,S,.FALSE.,F,G)
00274 91. NUMER2=NUMER
00275 92. DENOM2=DENOM
00276 93. NUM=0.0
00277 94. CEN=0.0
00301 95. DO 51 I=1,NDFS
00301 96. NUM=NUM+X(I)*AA(I)
00304 97. LEN=DEN+X(I)*BB(I)
00305 98. 51 U=(NUMER2*DEN)-(NUM*DENOM2)
00307 99. V=(NUMER2*DENOM1)-(NUMER1*DENOM2)
00310 100. W=(NUM*DENOM1)-(NUMER1*DEN)
00311 101. V=W/V
00312 102. W=W/W
00313 103. U1=0.0
00314 104. TS1=U/(2.0+U)
00315 105. TS2=(V**2-4.0*U*W)**0.5/(2.0+U)
00316 106. TS3=-TS2
00317 107. ALPHA1=TS1+TS2
00320 108. ALPHA2=TS1+TS3
00321 109. IF ((ALPHA1 .LT. 0.0) .AND. (ALPHA2 .LT. 0.0)) GO TO 76
00322 110. IF ((ALPHA1 .GT. 0.0) .AND. (ALPHA2 .GT. 0.0)) GO TO 60
00324 111. ALPHA=MAX1(ALPHA1,ALPHA2)
00325 112. GO TO 71
00327 113. 60 ALPHA=AMT.1*(ALPHA1+ALPHA2)
00330 114. GO TO 71
00331 115. 76 GO TO 71
00332 116. 79 XEN(I)=X(I)+ALPHA1*S(I)
00333 117. CALL FANGG(NDFS,SR,XEN,.FALSE.,F1,G)
00337 118. DO 80 I=1,NDFS
00340 119. 80 XEN(I)=X(I)+ALPHA2*S(I)
00343 120. CALL FANGG(NDFS,SR,XEN,.FALSE.,F2,G)
00345 121. IF (F1 .GT. F2) GO TO 77
00346 122. ALPHA=ALPHA*2
00349 123. GO TO 76
00351 124. 77 ALPHA=ALPHA*1
00352 125.

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00353 126.
00356 127.
00360 128.
00363 129.
00365 130.
00366 131.
00367 132.
00370 133.
00373 134.
00374 135.
00376 136.
00377 137.
00400 138.
00403 139.
00405 140.
00406 141.
00407 142.
00411 143.
00413 144.
00414 145.
00415 146.
00420 147.
00422 148.
00423 149.
00426 150.
00430 151.
00431 152.
00432 153.
00433 154.
00434 155.
00435 156.
00436 157.
00437 158.
00440 159.
00444 161.
00445 162.
00447 163.
00450 164.
00451 165.
00452 166.
00453 167.
00456 168.
00460 169.
00461 170.
00463 171.
00464 172.
00466 173.
00467 174.
00471 175.
00474 176.
00475 177.
00477 178.
00500 179.

79 JO 81 I=1,NDFS
81 XEM(I)=X(I)*ALPHA*S(I)
DO 82 I=1,NDFS
82 X(I)=XEM(I)
IT=IT+ITER
GO TO 350
7' TA=0.0
DO 52 I=1,NDFS
TACTAG(I)=6(I)
52 XEM(I)=X(I)*ALPHA*S(I)
RMAX=0.0
IMAX=0
DO 451 I=1,NDFS
IF (ABS(XEM(I))) .LE. RMAX) GO TO 451
RMAX=ABS(XEM(I))
IMAX=I
451 CONTINUE
IF (RMAX .LE. 5.0) GO TO 454
MINOR=MIN
MIN=IMAX
DO 452 I=1,NDFS
452 X(I)=XEM(I)
XEX(MIN)
DO 450 I=1,NDFS
450 X(I)=X(I)/X
IT=IT+ITER
GO TO 49
454 CALL FANDG(NDFS,TR,XEM,.TRUE.,FXEM,GXEM)
NUMER=NUMER
DENOM=DENOM
86 GS=0.0
AG=0.0
BE=0.0
DO 53 I=1,NDFS
GS=GS+S(I)*GXEM(I)
AG=AG+S(I)*S(I)
53 BE=BE+GXEM(I)*GXEM(I)
BETA=BE/TA
AG=AG/TA
BE=BE/TA
TEST=GS/(AG*BE)
DO 54 I=1,NDFS
54 S(I)=GXEM(I)+BETA*S(I)
DIF=FI-FXEM
IF (DIF .LT. 0.0) GO TO 460
56 CEL=2.0*SCIFF
IF (CEL .LT. FI) GO TO 460
DEBDE/10.0
IF (DEB .LT. FI) .AND. (ABS(TEST) .GT. 0.1) GO TO 460
99 DO 55 I=1,NDFS
55 S(I)=GXEM(I)
FI=FI*CY
ITER=ITER+1

```



```

00501      IF (ITER .GT. ITAD) GO TO 300
00502      GO TO 200
00503      300 IT=IT+ITER
00504      GO TO 350
00505      400 IT=IT+ITER
00506      INT(VEC)=IT
00507      INT(VEC)=IT
00508      VALU(VEC)=FNEW
00509      OMEGA(VEC)=SORT(FNEW)
00510      DO 401 I=1, NDFS
00511      401 VEC(VEC,I)=XNEW(I)
00512      IF (VEC .EQ. NVEC) GO TO 69
00513      VEC=VEC+1
00514      CALL XMX(NDFS,SR,XNEW,RR,MR,.TRUE.)
00515      JJ=VEC-1
00516      DO 58 I=1,NDFS
00517      58 BX(I,JJ)=BB(I)
00518      FR=1.0E+30
00519      LU=VEC
00520      DO 40 I=1,NDFS
00521      40 P(I)=VEC(I,JJ)
00522      IF (VEC .GT. 2) GO TO 57
00523      DENI=0.0
00524      DO 64 I=1,NDFS
00525      64 DENI=DENI+P(I)*BX(I,1)
00526      DO 65 I=1,NDFS
00527      65 X(I)=P(I)-DENI*BX(I,1)
00528      IF (ID .EQ. 0) GO TO 92
00529      GO TO 77
00530      57 IF (VEC .GT. 3) GO TO 229
00531      IF (IJ .GT. VEC) GO TO 72
00532      DENI=0.0
00533      DO 59 I=1,NDFS
00534      59 DENI=DENI+X(I,1)*BX(I,2)
00535      RAS=(1.0-(DENI)*P2)
00536      XNN(1,1)=1.0/RAS
00537      XNN(1,2)=DENI/RAS
00538      XNN(2,1)=DENI/RAS
00539      GO TO 72
00540      229 IF (IJ .GT. VEC) GO TO 72
00541      VEC=VEC-1
00542      DO 558 I=1,NDFS
00543      558 BB(I)=X(I,JJ)
00544      CALL XGEV(NDFS,BY,XNN,RB)
00545      VEC=VEC+1
00546      74 CALL XGQV(NDFS,NVEC,BY,XNN,B,XNNP,XNNP)
00547      DO 61 I=1,NDFS
00548      61 X(I)=B(I)*XNNP(I)
00549      IF (ID .EQ. 0) GO TO 92
00550      777 CALL XNOS(NDFS,SR,XNEW,RR,MR,.FALSE.,IG)
00551      IF (ITER .GT. ITAD) GO TO 300
00552      FREE
00553      K=1
00554      DO 300 I=1,NDFS
00555      300
00556      161.
00557      162.
00558      163.
00559      164.
00560      165.
00561      166.
00562      167.
00563      168.
00564      169.
00565      170.
00566      171.
00567      172.
00568      173.
00569      174.
00570      175.
00571      176.
00572      177.
00573      178.
00574      179.
00575      180.
00576      181.
00577      182.
00578      183.
00579      184.
00580      185.
00581      186.
00582      187.
00583      188.
00584      189.
00585      190.
00586      191.
00587      192.
00588      193.
00589      194.
00590      195.
00591      196.
00592      197.
00593      198.
00594      199.
00595      200.
00596      201.
00597      202.
00598      203.
00599      204.
00600      205.
00601      206.
00602      207.
00603      208.
00604      209.
00605      210.
00606      211.
00607      212.
00608      213.
00609      214.
00610      215.
00611      216.
00612      217.
00613      218.
00614      219.
00615      220.
00616      221.
00617      222.
00618      223.
00619      224.
00620      225.
00621      226.
00622      227.
00623      228.
00624      229.
00625      230.
00626      231.
00627      232.
00628      233.

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00632 234. 877 SX(IJ)=X(IJ)
00634 235. 807 IJ=IJ+1
00635 236. IF (IJ.GT. NVEC) GO TO 892
00637 237. GO TO 806
00640 238. 892 DO 809 J=1,NDFS
00643 239. X(J)=VECO(K,J)
00644 240. VECO(K,J)=VECO(VEC,J)
00645 241. VECO(VEC,J)=X(J)
00646 242. 809 X(IJ)=SX(IJ)
00650 243. 92 XX=0.0
00651 244. KIN=0
00652 245. DO 902 I=1,NDFS
00655 246. IF ((ABS(X(I))) .LE. XX) 60 TO 902
00657 247. XX=X(I)
00660 248. KINO=MIN
00661 249. M1=N1
00662 250. 902 CONTINUE
00664 251. DO 62 I=1,NDFS
00667 252. 62 X(I)=X(I)/XX
00671 253. IT=0
00672 254. 60 TO 49
00673 255. 69 WRITE (6,32) (INT(I),I=1,NVEC)
00675 256. WRITE (6,27)
00703 257. WRITE (6,28) (VALU(I),I=1,NVEC)
00705 258. WRITE (6,30)
00713 259. WRITE (6,28) (OMEGA(I),I=1,NVEC)
00715 260. DO 8008 J=1,NVEC
00723 261. 8008 WRITE (6,29) (VALU(J),VECO(J,I),I=1,NDFS)
00726 262. C
00728 263. C *ARRANGE THE EIGENVALUES IN ASCENDING ORDER AND ACCORDINGLY REARRAN
00729 264. C *GE THE NUMBER OF THE EIGENVECTORS
00736 265. REORL=.FALSE.
00737 266. NE=NVEC-1
00740 267. DO 903 I=1,NE
00743 268. IPI=I+1
00744 269. DO 903 J=IPI,NVEC
00747 270. IF (VALU(I)-VALU(J)) 903,903,904
00752 271. 904 REORL=.TRUE.
00753 272. TEMP1=VALU(I)
00754 273. TEMP2=OMEGA(I)
00755 274. VALU(I)=VALU(J)
00756 275. OMEGA(I)=OMEGA(J)
00757 276. VALU(J)=TEMP1
00760 277. OMEGA(J)=TEMP2
00761 278. DO 905 K=1,NDFS
00764 279. TEMP(K)=VECO(I,K)
00765 280. VECO(I,K)=VECO(J,K)
00766 281. VECO(J,K)=TEMP(K)
00770 282. 905 CONTINUE
00773 283. IF (.NOT. REORL) GO TO 1000
00775 284. *RITE (6,31)
00777 285. *RITE (6,27)
00801 286. *RITE (6,26) (VALU(I),I=1,NVEC)
01007 287. *RITE (6,30)

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01011 288. *RITE (6*2R) (OMEGA(I),I=1,NVEC)
01017 289. 1000 RETURN
01017 290.
01020 291. SUBROUTINE FANDG(NDFS,SR,X,LEG,F,G)
01020 292. C THIS INTERNAL SUBROUTINE EVALUATES THE VALUE OF THE HAYLEIGH QUOTI
01020 293. C ENT WHICH IS STORED AS 'F', EVALUATES THE GRADIENT TO THE RAYLEIG
01020 294. C H QUOTIENT, AND PROJECTS THE GRADIENT VECTOR ON TO THE RIGHT SPACE
01020 295. C OF SEARCH BY THE USE OF THE PROJECTION MATRIX.
01023 296. C PARAMETER M=14,PROF=6,NDFS=41,NVEC=11
01023 297. C M IS NUMBER OF ELEMENTS
01023 298. C NDFS IS NUMBER OF DEGREES OF FREEDOM IN REFERENCE COORDINATE SYSTEM.
01023 299. C ND IS NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM
01023 300. C NVEC IS NUMBER OF EIGENVALUES=EIGENVECTORS REQUIRED
01024 301. C INTEGER SR,VEC
01024 302. C REAL NUMER,N,NV,NNG,NNGG,KR,MR
01026 303. C LOGICAL LEG
01026 304. C LEG=FALSE MEANS COMPUTE ONLY FUNCTION VALUE,OTHERWISE BOTH THE
01026 305. C FUNCTION VALUE AND THE GRADIENT.
01027 306. C COMMON/ABLOCK/KR,MR
01030 307. C COMMON/BBLOCK/AA,BB
01031 308. C COMMON/CBLOCK/NUMER,DENOM
01032 309. C COMMON/DBLOCK/VEC
01033 310. C COMMON/GBLOCK/BX
01034 311. C COMMON/JBLOCK/MIN,MIND,ITER
01035 312. C DIMENSION X(NDFS),G(NDFS),AA(ND),BB(ND),N(N,NVEC),N(N,NVEC,NVEC),
01035 313. C 1 SR(M,PROF),BX(ND,NVEC),NNGG(ND),NNG(NVEC),NNG(NVEC)
01035 314. C 2 KR(M,PROF),PROF,MR,MR,PROF
01036 315. C CALL KXX(NDFS,SR,X,KR,MR,FALSE,.)
01036 316. C CALCULATE NUMERATOR AND DENOMINATOR
01037 317. C NUMER=0.0
01040 318. C DENOM=0.0
01041 319. C DO 22 I=1,NDFS
01044 320. C NUMER=NUMER+X(I)*AA(I)
01045 321. C DENOM=DENOM+X(I)*BB(I)
01045 322. C CALCULATE FUNCTION VALUE
01047 323. C F=NUMER/DENOM
01047 324. C IF (.NOT. LEG) GO TO 24
01050 325. C CALCULATE GRADIENT
01052 326. C DO 23 I=1,NDFS
01055 327. C G(I)=(2.0/DENOM)*(AA(I)-F*BB(I))
01057 328. C IF (VEC .EQ. 1) GO TO 34
01061 329. C IF (MIN .NE. MIND) GO TO 40
01063 330. C V(MIN)=1.0
01064 331. C DO 25 I=1,NDFS
01067 332. C V(I)=X(I)
01071 333. C IF (ITER .SE. 1) GO TO 11
01073 334. C IF (VEC .GT. 2) GO TO 12
01075 335. C X=X+V*(MIND)
01076 336. C RAS=(1.0-(XXX)*#2)
01077 337. C V(1)=1.0/RAS
01080 338. C V(1/2)=X/RAS
01101 339. C V(1/2)=X/RAS
01102 340. C V(1/2)=1.0/RAS
01103 341. C GO TO 11

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01240 396. N6(I)=SUM
01241 397. 31 CONTINUE
01242 398. DO 33 I=1,VEC
01243 399. SUM=0.0
01244 400. DO 36 J=1,VEC
01245 401. SUM=SUM+N(I,J)*NG(J)
01246 402. NNG(I)=SUM
01247 403. 33 CONTINUE
01248 404. DO 35 I=1,NDFS
01249 405. SUM=0.0
01250 406. DO 36 J=1,VEC
01251 407. SUM=SUM+N(I,J)*NNG(J)
01252 408. NNG(I)=SUM
01253 409. 35 CONTINUE
01254 410. RETURN
01255 411.
01256 412. SUBROUTINE INGEN(NDFS,N,NNG,X)
01257 413. C THIS INTERNAL SUBROUTINE GENERATES THE INVERSE OF A Q*Q MATRIX.GIV
01258 414. C EN THE INVERSE OF A (Q-1)*(Q-1) MATRIX,AND THE VECTOR X.
01259 415. C
01260 416. C NO 15 NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM
01261 417. C NVEC 15 NUMBER OF EIGENVALUES-EIGENVECTORS REQUIRED
01262 418. C REAL NN,NNG,NNG,NNG
01263 419. C INTEGER VEC
01264 420. C COMMON/DLOCK/VEC
01265 421. C DIMENSION PN(ND),TM(NVEC,NVEC),A1(NVEC,NVEC),A2(NVEC,NVEC),X(NDFS)
01266 422. C ,NN(NVEC,NVEC),N(ND,NVEC),NG(NVEC,NNG(NVEC),NNG(ND)
01267 423. 1. CALL PROD(NDFS,NVEC,X,N,NNG,NNG,NNG)
01268 424. DO 10 I=1,NDFS
01269 425. 10 PN(I)=X(I)-NNG(I)
01270 426. A2=0.0
01271 427. DO 11 I=1,NDFS
01272 428. 11 A2=A2+PN(I)*PN(I)
01273 429. A2=1.0/A2
01274 430. KKK=VEC+1
01275 431. DO 12 I=1,KKK
01276 432. DO 13 J=1,KKK
01277 433. 13 TM(I,J)=NNG(I)*NNG(J)
01278 434. 12 CONTINUE
01279 435. DO 14 I=1,KKK
01280 436. DO 15 J=1,KKK
01281 437. 15 A1(I,J)=TM(I,J)+A2*TM(I,J)
01282 438. 14 CONTINUE
01283 439. DO 16 I=1,KKK
01284 440. 16 A2(I)=A2*NNG(I)
01285 441. DO 17 I=1,KKK
01286 442. DO 17 J=1,KKK
01287 443. 17 NN(I,J)=A1(I,J)
01288 444. J=VEC
01289 445. DO 18 I=1,KKK
01290 446. NN(I,J)=NN(I)
01291 447. DO 18 J=1,KKK
01292 448. NN(I,J)=NN(I)
01293 449. RETURN
01294 450.

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01375 450. END  
01375 451.  
END OF COMPILATION: NO DIAGNOSTICS.  
TOTAL COMPILATION TIME: 5.3086 SECONDS)

A FOR KMX  
 COMPILE BY CHI UNIVAC 1108 FORTRAN V DATED: OCTOBER 29, 1968  
 THIS COMPILE WAS DONE ON 21 NOV 68 AT 14:25:39

SUBROUTINE KMX ENTRY POINT 000215

STORAGE USED (BLOCK, NAME, LENGTH)

0001 \*CODE 000235  
 0002 \*DATA 000306  
 0003 \*BLANK 000000  
 0004 BLOCK 000122  
 0005 ARESP 000124

EXTERNAL REFERENCES (BLOCK, NAME)

0001 NSRTOX  
 0002 NERR35

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001 000022 1L 0001 000011 1136 0001 000012 1166 0001 000035 1336 0001 000036 1366  
 0001 000040 1426 0001 000072 1536 0001 000075 1626 0001 000124 1736  
 0001 000145 20L 0001 000132 2016 0001 000133 2046 0001 000200 22L 0001 000160 2246  
 0001 000174 2316 0003 R 000000 AA 0003 R 000051 88 0000 I 000251 I 0000 I 000000 18  
 0000 I 000252 J 0000 I 000253 JJ 0000 I 000255 K 0000 I 000256 KK 0000 R 000257 CASI  
 0000 R 000254 SJW 0000 R 000001 TSI 0000 R 000125 T52 0004 R 000000 Y

00101 1. SUBROUTINE KMX(INDEX,SR,KR,M,R,BB)  
 00101 2. THIS SUBROUTINE POSTMULTIPLIES THE ELEMENT STIFFNESS AND MASS MAT  
 00101 3. RICES BY THE ELEMENT DISPLACEMENT VECTOR,Y, AND THEN ASSEMBLES THE  
 00101 4. (KIX AND M)X VECTORS.  
 00103 5. PARAMETER M=1,K=ROF,INDEX=4  
 00103 6. M IS NUMBER OF ELEMENTS  
 00103 7. ROF IS NUMBER OF DEGREES OF FREEDOM IN REFERENCE COORDINATE SYSTEM.  
 00103 8. INDEX IS NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM  
 00104 9. LOGICAL SUB  
 00105 10. INTEGER SR  
 00106 11. REAL KR,M,K  
 00107 12. COMMON/COMMON/AA=BB  
 00110 13. COMMON/ARESP/Y  
 00111 14. DIMENSION AR(4,ROF,R),B(4,ROF,R),SR(4,ROF),X(INDEX),Y(4,ROF),  
 00111 15. TSI(4,ROF),T52(4,ROF),AA(INDEX,INDEX)  
 00111 16. 1. ASSIGN SYSTEM DEGREES OF FREEDOM TO ELEMENT DISPLACEMENT VECTOR  
 00112 17. DO 10,INDEX

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00115      DO 11 J=1,RDF
00120      Y(I,J)=0.0
00121      J=SR(I,J)
00122      IF (JJ) 110,11,110
00123      110 Y(I,J)=X(JJ)
00124      11 CONTINUE
00125      10 CONTINUE
00126      C
00127      CALCULATE (K)X AND (J)X
00128      DO 12 J=1,M
00129      DO 13 J=1,RDF
00130      SUM=0.0
00131      DO 14 K=1,RDF
00132      14 SUM=SUM+R(I,J,K)*Y(I,K)
00133      13 TS1(I,J)=SUM
00134      12 CONTINUE
00135      DO 15 I=1,M
00136      DO 16 J=1,RDF
00137      SUM=0.0
00138      DO 17 K=1,RDF
00139      17 SUM=SUM+R(I,J,K)*Y(I,K)
00140      16 TS2(I,J)=SUM
00141      15 CONTINUE
00142      DO 18 J=1,NDFS
00143      AA(I)=0.0
00144      DO 19 K=1,M
00145      DO 20 K=1,RDF
00146      TS3(K,KK)
00147      IF (I) 21,20,21
00148      21 AA(I)=AA(I)+TS1(K,KK)
00149      BB(I)=BB(I)+TS2(K,KK)
00150      20 CONTINUE
00151      19 CONTINUE
00152      IF (.NOT. LBB) 60 TO 22
00153      RAS1=0.0
00154      DO 23 J=1,NDFS
00155      23 RAS1=RAS1+BB(I)*BB(I)
00156      DO 24 J=1,NDFS
00157      24 BB(I)=BB(I)/((RAS1)*0.5)
00158      22 RETURN
00159      END

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END OF COMPILATION: MC DIAGNOSTICS.  
TOTAL COMPILATION TIME: 1.046- SECS(S)



A FOR SHOCK  
 COMPILATION BY CHI UHIVAC 1108 FORTRAN V DATED: OCTOBER 29, 1968  
 THIS COMPILATION WAS DONE ON 21 NOV 68 AT 14:25:40

SUBROUTINE SHOCK ENTRY POINT 000130

STORAGE USED (BLOCK, NAME, LENGTH)

0001 \*CODE 000144  
 0000 \*DATA 000037  
 0002 \*BLANK 000000  
 0003 ASHOCK 000002

EXTERNAL REFERENCES (BLOCK, NAME)

0004 NSRTOR  
 0305 NERR35

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001 000017 1066 0001 000037 11L 0001 000061 12L  
 0000 R 000005 DY 0000 I 000000 I 0003 R 000000 PF  
 0003 R 000001 XG 0000 R 000001 XX 0008 R 000004 YY  
 0001 000102 13L 0001 000105 13L  
 0000 R 000002 TS 0000 R 000003 TTY

00101 1. SUBROUTINE SHOCK(INVEC,X,Y,DY)  
 00103 2. COMMON/ASHOCK/PE,XG  
 00104 3. DIMENSION X(INVEC),Y(INVEC),DY(INVEC)  
 00105 4. DO 10 I=1,NVEC  
 00110 5. XX=X(I)  
 00111 6. IF (XX .GE. 0.275 .AND. XX .LE. 0.62) GO TO 11  
 00113 7. GO TO 12  
 00114 8. 11 TS=-0.785\*XX  
 00115 9. YY=5.5\*(2.7163)\*TS  
 00116 10. YV=YY\*XX  
 00117 11. DYV=YY/(2.\*PF)\*(1.0+TS)  
 00120 12. GO TO 15  
 00121 13. 12 IF (XX .LT. 0.275) GO TO 13  
 00123 14. TS=-0.737\*XX  
 00124 15. YY=2.72\*(2.7163)\*TS  
 00125 16. DYV=YY\*(-C.737/(2.\*PF))  
 00126 17. GO TO 15  
 00127 18. 13 YY=1.0  
 00130 19. DYV=0.0  
 00131 20. 15 Y(I)=YV\*G  
 00132 21. 10 DY(I)=YY\*G

00130 22. RETURN.  
00135 23. END  
00135 24.  
END OF COMPILATION: NO DIAGNOSTICS.  
TOTAL COMPILATION TIME: 1.2516 SECONDS

A FOR POLYN  
 COMPILATION BY CH1 UNIVAC 1108 FORTRAN V DATED: OCTOBER 29, 1968  
 THIS COMPILATION WAS DONE 04 21 NOV 68 AT 14:25:42

SUBROUTINE POLYN ENTRY POINT 000246

STORAGE USED (BLOCK, NAME, LENGTH)

0001 \*CODE 000305  
 0002 \*DATA 000100  
 0002 \*BLANK 000000

EXTERNAL REFERENCES (BLOCK, NAME)

0003 MERR33

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001 000077 1266 0001 000122 1276 0001 000126 1346 0001 000176 1526  
 0001 000202 1576 0001 000153 50L 0001 000217 51L 0000 R 000001 DY  
 0000 I 000016 I 0000 I 000007 IS 0000 I 000017 J 0000 I 000020 K 0000 L 000000 TB

00101 1. SUBROUTINE POLYN (RDF,TRR,TL,IP,PI,MPY,MPH)  
 00102 2. PARAMETER R024  
 00103 3. INTEGER RDF  
 00104 4. LOGICAL TR  
 00105 5. TRR,TRR, MEANS TRUSS STRUCTURE.  
 00106 6. RDEF=0  
 00107 7. DIMENSION TRR(RDF,RDF),MPY(MP,RDF),MPH(MP,RDF),PI(MP,DY(RD),  
 00108 8. )  
 00109 9. TRR=FALSE.  
 00110 10. PI(1)=0.  
 00111 11. PI(2)=TL  
 00112 12. DY(1)=1./TL  
 00113 13. DY(2)=DY(1)  
 00114 14. IF (TR) GO TO 50  
 00115 15. DO 11 151,MP  
 00116 16. C=13\*(1./TL+03)\*(12.\*PI(1))-6.\*TL  
 00117 17. C=16\*(1.-C/13)  
 00118 18. C=15\*(1./TL+02)\*(6.\*PI(1))-6.\*TL\*(1.-1.0)  
 00119 19. C=16\*(1./TL+02)\*(6.\*PI(1))-2.\*TL\*(1.-1.0)  
 00120 20. DO 12 151,MP  
 00121 21. MPY(1,1)=C  
 00122 22. MPH(1,1)=0  
 00123 23. DO 13 151,MP  
 00124 24. C=13\*(1.-C/13)

```

00136      24.      HPY(I,J)=HPY(I,J)+DY(KI)*TRR(K,J)
00137      25.      13 HPY(I,J)=HPY(I,J)+DW(KI)*TRR(K,J)
00141      26.      12 CONTINUE
00143      27.      11 CONTINUE
00145      28.      10 GO TO 51
00146      29.      50 DO 52 I=1,NP
00151      30.      40 DO 53 J=1,NPF
00151      31.      30 HPY(I,J)=0.0
00155      32.      HPY(I,J)=0.0
00156      33.      DO 54 K=1,NRF
00161      34.      54 HPY(I,J)=HPY(I,J)+DY(KI)*TRR(K,J)
00163      35.      53 CONTINUE
00165      36.      52 CONTINUE
00167      37.      51 RETURN
00170      38.      END
00170      39.

```

END OF COMPILATION: NO DIAGNOSTICS.  
TOTAL COMPILATION TIME: 0.8652 SECOND(S)

SUBROUTINE ANAL      ENTRY POINT: 000537  
STORAGE USED (BLOCK, NAME, LENGTH)

| *CODE  | 000577 |
|--------|--------|
| *DATA  | 000406 |
| *BLANK | 000000 |
| ABJCK  | 001760 |
| EMAIN  | 000770 |
| BNJCK  | 000122 |
| KALASH | 000034 |
| PYARI  | 000016 |
| MANOJ  | 000054 |
| JAGAT  | 000520 |
| GMAIN  | 000703 |
| UBLOCK | 000013 |
| RAJA   | 000024 |
| RANI   | 030024 |

**EXTERNAL REFERENCES (BLOCK, NAME)**

|      |         |
|------|---------|
| 0016 | SHOCK   |
| 0017 | KMX     |
| 0020 | POLYN   |
| 0021 | MOSBU   |
| 0022 | NI02S   |
| 0023 | NI01S   |
| 0024 | NIERR3S |

 STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME) |

|      |        |       |        |        |       |      |        |        |      |        |        |      |        |      |
|------|--------|-------|--------|--------|-------|------|--------|--------|------|--------|--------|------|--------|------|
| 0000 | 000255 | 1F    | 000001 | 000024 | 134G  | 0001 | 000043 | 144G   | 0001 | 000070 | 155G   | 0001 | 000107 | 163G |
| 0001 | 000012 | 167G  | 0001   | 000126 | 176G  | 0000 | 000257 | 2F     | 0000 | 000145 | 206G   | 0000 | 000303 | 21F  |
| 0001 | 000015 | 213G  | 0001   | 000001 | 217G  | 0000 | 000022 | 232G   | 0001 | 000242 | 231G   | 0000 | 000243 | 234G |
| 0001 | 000267 | 245G  | 0001   | 000372 | 251G  | 0000 | 000310 | 261G   | 0001 | 000320 | 267G   | 0000 | 000331 | 27G  |
| 0001 | 000351 | 272G  | 0000   | 000322 | 26F   | 0000 | 000263 | 3F     | 0001 | 000353 | 306G   | 0001 | 000354 | 303G |
| 0001 | 000401 | 317G  | 0001   | 000406 | 324G  | 0000 | 000411 | 331G   | 0001 | 000475 | 356G   | 0001 | 000505 | 362G |
| 0001 | 000274 | 4F    | 0001   | 000366 | 43L   | 0000 | 000472 | 88L    | 0001 | 000327 | 93L    | 0000 | 000000 | AA   |
| 0005 | 000051 | B5    | 0001   | 000026 | C     | 0000 | 000472 | CS1G4S | 0000 | 000327 | CS1G8S | 0006 | 000000 | D    |
| 0010 | 000013 | D1C   | 0006   | 000016 | E     | 0000 | 000063 | EVEL   | 0000 | 000323 | H      | 0000 | 000247 | HPY  |
| 0000 | 000027 | I     | 0000   | 000344 | IK    | 0000 | 000001 | I5     | 0000 | 000340 | JW     | 0000 | 000245 | JJ   |
| 0030 | 000041 | K     | 0003   | 000700 | KR    | 0007 | 000000 | L      | 0003 | 000770 | MP     | 0000 | 000000 | N4M  |
| 0013 | 000030 | O4E3A | 0010   | 000341 | PR2C  | 0011 | 000250 | F4     | 0000 | 000002 | PX     | 0011 | 000000 | PD   |
| 0000 | 000250 | S14AS | 0000   | 000251 | S1C3S | 0000 | 000254 | S1GM   | 0015 | 000000 | S1GMA  | 0000 | 000247 | TD   |

```

0000 R 000246 TE      0000 R 000242 TL      0004 R 000000 TH      0000 R 000004 TRR      0012 R 000000 VECO
0000 R 000050 XC      0010 R 000000 YC      0014 R 000000 YN      0000 R 000243 YMA      0000 R 000000 VECO

SUBROUTINE ANAL (N,RDF,SR,EG,XG,PF)
PARAMETER N=14,RD=6,LDF=6,NDFS=41,NP=2,NDFI=20,NVEC=11
INTEGER RDF,SR
REAL L,KR,MR,IUM
COMMON/ABLOCK/KR,MR
COMMON/EMAIN/TR
COMMON/BBLOCK/AA,BB
COMMON/KALASH/D,E
COMMON/PYARI/L
COMMON/MANOJ/TC,DYC,C,PROD
COMMON/JAGAT/PT,PW
COMMON/SHAIN/VECO
COMMON/UBLOCK/OMEGA
COMMON/RAJA/TM
COMMON/RANI/SIGMA
1 FORMAT (10E11.6)
2 FORMAT (2X,13H XC=OMEGA/24P)
3 FORMAT (2X,42H Y COORDINATE READ FROM THE SHOCK SPECTRUM)
4 FORMAT (2X,33H MODAL PARTICIPATION COEFFICIENTS)
21 FORMAT (29H0 RESULTS OF DYNAMIC ANALYSIS)
27 FORMAT (1H0-28H MAXIMUM DISPLACEMENT VECTOR, (2X,10E11.6))
28 FORMAT (1H0-23H MAXIMUM STRESS-MEMBER, (2X,3E12.6))
DIMENSION D(1N),E(1N),L(1N),SR(N,RDF),OMEGA(NVEC),PX(NP),
* KR(NM,RD,RD),WR(NM,RD,RD),TR(NM,LDF,LDF),TRR(RD,RD),
* VECO(NVEC,NDFS),XC(NVEC),YC(NVEC),DYC(NVEC),EG(NDFS),
* C(NVEC),PROD(NVEC),AA(NDFS),BB(NDFS),EVEL(NM,RD),YM(NDFI),
* PT(NM,NP,RD),PW(NM,NP,RD),HPY(NP,RD),HPR(NP,RD),
* SIGMA(1N,NP)
C DYNAMIC ANALYSIS
WRITE (6,21)
DO 77 I=1,NVEC
77 XC(I)=(OMEGA(I))/(2.*PF)
WRITE (6,2)
WRITE (6,1) (XC(J),J=1,NVEC)
CALL SHOCK (NVEC,XC,YC,DYC)
WRITE (6,3)
WRITE (6,1) (YC(I),I=1,NVEC)
EVALUATE R,H,S, FACTOR OF DIFFERENTIAL EGN. OF MOTION
CALL KMX (NDFS,SR,EG,KR,MR,FALSE.)
DO 81 I=1,NVEC
NUN=0.0
DO 82 K=1,NDFS
82 NUN=NUN+VECO(I,K)*BR(K)
81 C(I)=NUN
C EVALUATE MODAL PARTICIPATION COEFFICIENTS
DO 85 I=1,NVEC
85 PROD(I)=C(I)*YC(I)
WRITE (6,4)

```

```

00204      WRITE (B,I) (PROD(J),J=1,NVEC)
00205      C
00206      ASSEMBLE POLYNOMIAL AND TRANSFORM TO REFERENCE COORDINATE SYSTEM
00207      DO 440 I=1,M
00208      TL=I
00209      DO 441 J=1,NDF
00210      CO 441 K=1,NDF
00211      TRR(J,K)=TR(I,J,K)
00212      CALL POLYN (RDF,TRR,TL,NP,PR,HPY,HPW)
00213      DO 101 J=1,NP
00214      DO 101 K=1,NP
00215      PY(I,J,K)=HPY(J,K)
00216      C
00217      EVALUATE MAXIMUM DISPLACEMENT
00218      DO 86 J=1,NDF
00219      YMA=0.0
00220      DO 87 I=1,NVEC
00221      87 YMA=YMA+ABS(PROD(I)*VECO(I,J))
00222      86 YM(J)=YMA
00223      WRITE (6,27) (YM(K),K=1,NDF)
00224      IK=1
00225      DO 48 I=1,M
00226      DO 48 J=1,NP
00227      48 SIGMA(I,J)=0.0
00228      C
00229      TRANSFORM EIGENVECTORS TO ELEMENT DISPLACEMENTS
00230      93 DO 43 I=1,M
00231      DO 43 J=1,NDF
00232      EVEL(I,J)=0.0
00233      JJSR(I,J)
00234      IF (JJ) 44,43,44
00235      44 EVEL(I,J)=VECO(IK,JJ)
00236      43 CONTINUE
00237      DO 47 I=1,M
00238      TEE(I)
00239      TD=I
00240      DO 45 J=1,NP
00241      SIGAS=0.0
00242      SIGBS=0.0
00243      DO 46 K=1,NDF
00244      SIGAS=SIGAS+PY(I,J,K)*EVEL(I,K)
00245      SIGBS=SIGBS+PW(I,J,K)*EVEL(I,K)
00246      46 SIGBS=SIGBS*TE/J2
00247      C
00248      STRESSES IN PROPORTION TO PARTICIPATION COEFF.
00249      CSIGAS=PROD(IK)*SIGAS
00250      CSIGBS=PROD(IK)*SIGBS
00251      SIGMA=ABS(CSIGAS)+ABS(CSIGBS)
00252      SIGMA(I,J)=SIGMA(I,J)+SIGM
00253      45 CONTINUE
00254      47 CONTINUE
00255      IF (IA,E,NVEC) GO TO 64
00256      IK=IK+1
00257      GO TO 93
00258      44 DO 44 K=1,NP
00259      102.

```

00357 103. 808 WRITE (6,2P) (I,(SIGMA(I,J),J=1,N))  
00367 104. RETURN  
00370 105. END  
00370 106.  
END OF COMPILATION: NO DIAGNOSTICS.  
TOTAL COMPILATION TIME: 1.8616 SECONDS



A FOR MANOCH  
 COMPILE BY CHI UNIVAC 1106 FORTRAN V DATE: OCTOBER 29, 1968  
 THIS COMPILE WAS DONE ON 21 NOV 68 AT 14:25:45

SUBROUTINE MANOCH ENTRY POINT 000303  
 STORAGE USED (BLOCK, NAME, LENGTH)

0001 \*CODE 000346  
 0000 \*DATA 000125  
 0002 \*BLANK 000000  
 0003 RAM 000007  
 0004 DAM 000001  
 0005 OLD 000007

EXTERNAL REFERENCES (BLOCK, NAME)

0006 NOSBEU  
 0007 NIOZS  
 0010 NIOIS  
 0011 NERRS3

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

|        |               |        |             |        |             |        |             |        |              |
|--------|---------------|--------|-------------|--------|-------------|--------|-------------|--------|--------------|
| 0000   | 000010 1F     | 0000   | 000065 10F  | 0001   | 000015 125G | 0001   | 000033 135G | 0001   | 000045 143G  |
| 0001   | 000074 154G   | 0001   | 000114 164G | 0001   | 000125 173G | 0000   | 000017 2F   | 0001   | 000254 237G  |
| 0000   | 000024 3F     | 0000   | 000033 4F   | 0000   | 000043 5F   | 0001   | 000234 53L  | 0001   | 000182 54L   |
| 0001   | 000155 55L    | 0001   | 000167 56L  | 0001   | 000212 57L  | 0001   | 000065 59L  | 0000   | 000055 6F    |
| 0001   | 000152 61L    | 0001   | 000267 62L  | 0000   | 000063 9F   | 0000   | 000006 AA   | 0004   | 000000 ALPHA |
| 0000 R | 000007 ALPHA1 | 0000 R | 000005 B    | 0000 R | 000004 DIFF | 0003 R | 000000 DV   | 0000 I | 000001 I     |
| 0000 I | 000003 IC     | 0000 I | 000000 IS   | 0005 R | 000000 OS   | 0000 R | 000002 TS   |        |              |

00101 1.  
 00101 2. C  
 00101 3. C  
 00101 4. C  
 00101 5. C  
 00101 6. C  
 00103 7. C  
 00104 8. C  
 00105 9. C  
 00106 10. C  
 00107 11. C  
 00110 12. C  
 00111 13. C

SUBROUTINE MANOCH (S, P, C, T, R, W, T, G, W, NDV, SCON, BSC, NBSC, BSCL, DELTA)  
 THIS SUBROUTINE MAKES A MOVE IN THE NEGATIVE DIRECTION OF THE  
 GRADIENT TO THE WEIGHT FUNCTION CHECKS THE SIDE CONSTRAINTS OF THE  
 NEW DESIGN AND ADJUSTS BY INTERPOLATION ANY OF THE DESIGN VAR.  
 VIOLATED  
 PARAMETER NDVP=7, NQV=14  
 INTEGER BSC  
 LOGICAL BSCL  
 COMMON/RAV/DV  
 COMMON/CAV/ALPHA  
 COMMON/CLT/OC  
 1 FORVAT (1=0, 2=1, 2dH \*\*\*\*\* PREVIOUS DESIGN \*\*\*\*\*//)  
 2 FORVAT (1=0, 2=1, 2dH ALPHA 15, 2X, E11.6)

```

00113 14. 3 FORMAT (1H0,2X,27H ***** CURRENT DESIGN *****/)
00114 15. 4 FORMAT (1H0,2X,29H BOUNDED SIDE CONSTRAINTS ARE,/,36I3)
00115 16. 5 FORMAT (1H0,2X,39H NUMBER OF BOUNDED SIDE CONSTRAINTS ARE,2X,I2)
00116 17. 6 FORMAT (1H0,2X,18H ADJUSTED ALPHA IS,E11.6)
00117 18. 9 FORMAT (10E11.6)
00120 19. 10 FORMAT (1H0,2X,26H RESULTS FROM MOVEANDCHECK,/)
00121 20. DIMENSION S(NDV),DV(NDV),GM(NDV),SCON(NNDV),BSC(NNDV),OD(NNDV)
00122 21. WRITE (6,10)
00124 22. DO 50 I=1,NDV
00127 23.   OD(I)=CV(I)
00131 24.   WRITE (6,11)
00133 25.   WRITE (6,9) (OD(I),I=1,NDV)
00141 26.   TS=0.0
00142 27.   DO 51 I=1,NDV
00145 28.     TS=TS+S(I)*GM(I)
00147 29.     ALPHA=-(PCT*TS)/(TS)
00150 30.     WRITE (6,2) ALPHA
00153 31.   DO 52 I=1,NDV
00156 32.     DV(I)=OD(I)+ALPHA*S(I)
00160 33.     WRITE (6,3)
00162 34.     WRITE (6,9) (DV(I),I=1,NDV)
00170 35.     IC=0
00171 36.     BSCLE=FALSE.
00172 37.     DO 53 I=1,NNDV
00175 38.       IF (I .GT. NDV) GO TO 54
00177 39.       DIFF=DV(I)-SCON(I)
00200 40.       BOD(I)=SCON(I)
00201 41.       GO TO 61
00202 42. 54 DIFF=SCON(I)-DV(I,NDV)
00203 43.       B=SCON(I)-OD(I,NDV)
00204 44. 55 DIFF=DIFF-B
00206 45. 61 IF (SCON(I) .EQ. 0.0) GO TO 55
00207 46.       AA=DIFF/ABS(SCON(I))
00210 47.       B=DIFF/ABS(SCON(I))
00211 48.       GO TO 56
00212 49.       AA=DIFF
00214 50.       IF (AA .GT. 0.0) GO TO 57
00216 51.       IF (AA .LT. DELTA) GO TO 53
00217 52.       STORE THE INDEX AND NUMBER OF BOUNDED SIDE CONSTRAINTS
00218 53.       IC=IC+1
00219 54.       BSCLE=TRUE.
00220 55.       BSC(IC)=I
00221 56.       GO TO 53
00222 57.       ALPHA=ALPHA
00223 58.       ALPHA=(B-DELTA/2.)*ALPHA/(AA-B)
00224 59.       WRITE (6,6) ALPHA
00227 60.       GO TO 59
00230 61. 53 CONTINUE
00232 62.       IF (.NOT. BSCLE) GO TO 62
00234 63.       NBSC=IC
00235 64.       WRITE (6,4) (BSC(I),I=1,NBSC)
00243 65.       WRITE (6,5) NBSC
00246 66. 62 RETURN
00247 66. END

```

\*DIAGNOSTIC\* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.

00247 67.  
END OF COMPILATION: 1 DIAGNOSTIC.  
TOTAL COMPILATION TIME: 0.9408 SECONDS

A FOR ADJUST  
 COMPILED BY CHI JUIYAC 1108 FORTRAN V DATED: OCTOBER 29, 1968  
 THIS COMPIATION WAS DONE ON: 21 NOV 68 AT 14:25:48

SUBROUTINE ADJUST ENTRY POINT 000413

STORAGE USED (BLOCK, NAME, LENGTH)

0001 \*COJE 000474  
 0002 \*DATA 000134  
 0003 \*BLANK 000000  
 0004 RAM 000007  
 0005 HARI 000001  
 0006 OLD 000007

EXTERNAL REFERENCES (BLOCK, NAME)

0006 NO8FU  
 0007 NI01S  
 0010 NI02S  
 0011 N8RTOR  
 0012 NERR3S

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0000 000013 IF 0001 000013 1243 0001 000155 1709  
 0001 000024 2136 0001 000235 2223 0001 000363 2646  
 0000 000041 5F 0001 000141 51L 0001 000343 53L  
 0001 000335 557L 0001 000302 56L 0001 000325 57L  
 0001 000376 62L 0000 000053 7F 0000 000065 8F  
 0001 000102 85L 0001 000114 86L  
 0000 R 000010 DIFF 0003 R 000000 3V 0004 L 000000 EXIT  
 0000 I 000000 1S 0005 R 000030 OD 0000 R 000003 9A  
 0000 R 000006 6D  
 0001 000205 2036  
 0000 000032 4F  
 0001 000300 55L  
 0001 000265 61L  
 0001 000125 81L  
 0000 R 000011 B  
 0000 I 000007 IC  
 0000 R 000005 GC

00101 1. SUBROUTINE ADJUST (S\*MCV\*MCV\*ALPHA\*BSCL\*ASC\*NSC\*DELTA\*SCON\*NDV,  
 00101 2. LG\*SLDPE)  
 00103 3. PARAMETER SUBROUTINE ADJUST  
 00103 4. C THIS SUBROUTINE ADJUSTS THE WORST VIOLATED BEHAVIOUR CONSTRAINT  
 00103 5. C (BY INTERPOLATION) AND CHECKS THE SIZE CONSTRAINTS  
 00104 6. INTEGER ASC  
 00105 7. LOGICAL BSCL\*EXIT\*LG  
 00106 8. COMMON/PAV/DA  
 00107 9. COMMON/HARI/EXIT  
 00110 10. COMMON/OLD/CO

```

00111 11. DIMENSION S(NDV),DV(NDV),OD(NDV),BSC(NDV),SCON(NDV)
00112 12. 1 FORMAT (1H0,2X,27M ADJUSTED ALPHA IS NEGATIVE,2X,E11.6)
00113 13. 2 FORMAT (1H0,2X,5M DVAR,/)
00114 14. 3 FORMAT (1H0,2X,9M S VECTOR,/)
00115 15. 4 FORMAT (1H0,2X,18M ADJUSTED ALPHA IS,2X,E11.6)
00116 16. 5 FORMAT (2X,10E11.6)
00117 17. 6 FORMAT (1H0,2X,29M BOUNDED SIDE CONSTRAINTS ARE,/,36I3)
00120 18. 7 FORMAT (1H0,2X,39M NUMBER OF BOUNDED SIDE CONSTRAINTS ARE,2X,12)
00121 19. 8 FORMAT (1H0,2X,36M TIME TO LEAVE THE MACHINE AND THINK)
00122 20. WRITE (6,5) (OD(I),I=1,NDV)
00123 21. ALPHA=ALPHA
00130 22. IF (.NOT. LB) GO TO 80
00131 23. QA=ODCV-(DELTA/2.)
00133 24. QB=SLOPE
00134 25. OC=WCV-OCV-SLOPE*ALPHA)/((ALPHA)**2)
00135 26. WRITE (6,5) QA
00136 27. WRITE (6,5) QB
00141 28. GO=(QB)**2-4.*OC*QA
00147 29. IF (GO.GT. 0.0) GO TO 85
00150 30. SET DELTA EQUAL TO TWICE OMCV, THEREFORE QA=0.0
00151 31. ALPHA=-QB/OC
00152 32. GO TO 85
00153 33. 85 ALPHA=(-QB*OD**0.5)/(2.*OC)
00154 34. GO TO 81
00155 35. 80 ALPHA=-OMCV-DELTA/2.*ALPHA/(WCV-OCV)
00156 36. 81 IF (ALPHA.GT. 0.0) GO TO 51
00157 37. WRITE (6,1) ALPHA
00161 38. GO TO 57
00164 39. 51 IF (ALPHA.LT. 1.0E-04) GO TO 557
00165 40. DO 52 I=1,NDV
00167 41. 52 DV(I)=OD(I)*ALPHA*(1)
00172 42. WRITE (6,4) ALPHA
00174 43. WRITE (6,3)
00177 44. WRITE (6,5) (S(I),I=1,NDV)
00201 45. WRITE (6,2)
00207 46. WRITE (6,5) (DV(I),I=1,NDV)
00211 47. IC=0
00217 48. BSC=.FALSE.
00220 49. DO 53 I=1,NDV
00221 50. IF (I.GT. NDV) GO TO 54
00224 51. DIF=DV(I)-SCON(I)
00226 52. B=OD(I)-SCON(I)
00227 53. GO TO 61
00230 54. 54 DIFF=SCON(I)-DV(I-NDV)
00231 55. B=SCON(I)-OD(I-NDV)
00232 56. *DIAGNOSTIC* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00233 57. 61 IF (SCON(I).EQ. 0.0) GO TO 55
00235 58. AA=DIFF/ABS(SCON(I))
00236 59. B=B/ABS(SCON(I))
00237 60. GO TO 56
00240 61. 55 AA=DIFF
00241 62. 56 IF (AA.GT. 0.0) GO TO 57
00243 63. IF (AA.LT. DELTA) GO TO 53

```

```

00243      C      STORE THE INDEX AND NUMBER OF BOUNDED CONSTRAINTS
00244      64.      IC=IC+1
00245      65.      BSCLE=TRUE.
00246      66.      BSC(1C)=1
00247      67.      GO TO 53
00250      68.      57 WRITE (6,2)
00251      69.      557 EXIT=.TRUE.
00253      70.      GO TO 62
00254      71.      53 CONTINUE
00255      72.      IF (.NOT. BSCLE) GO TO 62
00257      73.      BSC=IC
00261      74.      WRITE (6,6) (BSC(I),I=1,NBSC)
00262      75.      WRITE (6,7) NBSC
00270      76.      62 RETURN
00273      77.      END
00274      78.
00274      79.
      END OF COMPILATION:      1 DIAGNOSTIC.
TOTAL COMPILATION TIME: 1.1340 SECOND(S)

```

\* FOR DEIG  
 COMPILATION BY CUI JIYAC 1108 FORTRAN V DATED: OCTOBER 29, 1968  
 THIS COMPILATION WAS DONE ON 21 NOV 68 AT 14:25:49

## SUBROUTINE DEIG ENTRY POINT 001054

STORAGE USED (BLOCK, NAME, LENGTH)

| *CODE | US1104 |
|-------|--------|
| 0001  | 010506 |
| 0002  | 000000 |
| 0003  | 000000 |
| 0004  | 000033 |
| 0005  | 000016 |
| 0006  | 000034 |
| 0007  | 000001 |
| 0008  | 000770 |
| 0009  | 000013 |
| 0010  | 000013 |
| 0011  | 000703 |
| 0012  | 001634 |
| 0013  | 000115 |
| 0014  | 006125 |
| 0015  | 006125 |

EXTERNAL REFERENCES (BLOCK, NAME)

0015 VER438

## STORAGE ASSIGNED (BLOCK, TYPE, RELATIVE LOCATION, NAME)

[illegible]

```

1. SUBROUTINE DEIG (NDFS,SF,NDV,MWAX,LDVCR)
2. PARAMETER SF=1,N=12,LOF=6,ROF=6,NDF=41,NV=7,NVEC=11
3. INTEGER SR
4. REAL L
5. LOGICAL B1
6. COMMON/KALASH/D,E
7. COMMON/PYARI/L
8. COMMON/DMAIN/T,RO
9. COMMON/LMAIN/B1
10. COMMON/EMAIN/TR
11. COMMON/CHAIN/VALU
12. COMMON/GRAIN/VECO
13. COMMON/EDEIG/OTS
14. COMMON/TEMP/CLAMB
15. COMMON/TOT/CVEC
16. DIMENSION D(M,T),E(M,RO),W(M,L),SR(M,ROF),DA(M),ZMI(M),
17. DKL(M,LOF,LOF),DML(M,LOF,LOF),TR(M,LOF,LOF),
18. TS(M,LOF,LOF),DKR(M,ROF,ROF),DMR(M,ROF,ROF),VALU(NVEC),
19. VECO(NVEC,ND),X(ND),Y(NVEC,M,ROF),TS1(ROF),TS2(ROF),
20. CL(NVEC),CLAMB(NVEC,M),DVEC(M,ND),OTS(NVEC,M,ROF),
21. LDVCR(NDV,MWAX),CLAMB(NVEC,NV),CVEC(NVEC,NV,ND)
22.
23. 1 FORMAT (10E11.6)
24. 29 FORMAT (55H1 DERIVATIVE OF EIGENVALUES AND EIGENVECTORS *****
25. 1,/,/)
26. 11 FORMAT (26H0 DERIVATIVE OF EIGENVALUE,12,5X,
27. 23H W,R,T, MEMBER VARIABLE,12,E15.7)
28. 15 FORMAT (1H0,26H DERIVATIVE OF EIGENVALUE,12,5X,
29. 23H R,T, MEMBER VARIABLE,12/(12X,10E11.6))
30. 30 FORMAT (26H0 DERIVATIVE OF EIGENVALUE,12,5X,
31. 23H W,R,T, DESIGN VARIABLE,12,E15.7)
32. 31 FORMAT (1H0,26H DERIVATIVE OF EIGENVALUE,12,5X,
33. 23H W,R,T, DESIGN VARIABLE,12/(12X,10E11.6))
34. DO 80 I=1,N
35. 80 DA(I)=3.1415926*(I)
36. IF (B1) GO TO 82
37. CGN=3.1415926/9.
38. DO 81 I=1,M
39. 81 DW1(I)=CO*(T(I))*((D(I))*e2)+((T(I))*e2))+((D(I))*e2))+((2.0*D(I)
40. 1 *(T(I))*e2))))
41. DO 83 I=1,V
42. 83 J0 84 J=1,LOF
43. DO 84 K=1,LOF
44. 84 DWL(I,J,K)=0.0
45. DWJ1=DA(I)*E(I)/(L(I))
46. DWJ2=RO(I)*E(DA(I))*CL(I)/(L(I))*u20.
47. DWL(I,1)=DWJ1
48. DWL(I,2)=DWJ1
49. DWL(I,1,1)=DWJ1
50. DWL(I,1,1)=140.*JWJ2

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00170 51. DML(I,2,1)=70.*DMD2
00171 52. DML(I,2,2)=140.*DMD2
00172 53. IF (.NOT. B1) GO TO 85
00174 54. DML(I,3,3)=140.*DMD2
00175 55. DML(I,4,3)=70.*DMD2
00176 56. DML(I,4,4)=140.*DMD2
00177 57. GO TO 86
00200 58. DMD3=2.*(E(I))*DML(I,1)/(L(I))
00201 59. DMD4=2.*DMD3
00202 60. DMD5=3.*(DMD3)/(L(I))
00203 61. DMD6=6.*(DMD3)/(L(I))*2)
00204 62. DML(I,3,3)=DMD6
00205 63. DML(I,4,3)=DMD6
00206 64. DML(I,4,4)=DMD6
00207 65. DML(I,5,3)=DMD5
00210 66. DML(I,5,4)=DMD5
00211 67. DML(I,5,5)=DMD4
00212 68. DML(I,6,3)=DMD5
00213 69. DML(I,6,4)=DMD5
00214 70. DML(I,6,5)=DMD3
00215 71. DML(I,6,6)=DMD4
00216 72. DML(I,3,3)=156.*DMD2
00217 73. DML(I,5,3)=22.*(L(I))*DMD2
00220 74. DML(I,4,3)=54.*DMD2
00221 75. DML(I,4,4)=156.*DMD2
00222 76. DML(I,5,4)=13.*(L(I))*DMD2
00223 77. DML(I,5,5)=4.*(L(I))*2)*DMD2
00224 78. DML(I,6,3)=13.*(L(I))*DMD2
00225 79. DML(I,6,4)=22.*(L(I))*DMD2
00226 80. DML(I,6,5)=3.*(L(I))*2)*DMD2
00227 81. DML(I,6,6)=4.*(L(I))*2)*DMD2
00230 82. DO 87 J=1,LDF
00233 83. DO 87 K=1,LDF
00236 84. DML(I,J,K)=DML(I,K,J)
00237 85. DML(I,J,K)=DML(I,K,J)
00242 86. C
00242 87. TRANSFORM DML TO REFERENCE COORDINATE SYSTEM
00244 88. DO 101 I=1,M
00247 89. DO 102 J=1,LDF
00252 90. DO 102 K=1,RDF
00255 91. S'=0.0
00256 92. DO 103 KK=1,LDF
00261 93. 103 SUMESUM=DML(I,J,KK)*TR(I,KK,K)
00263 94. 102 TS(I,J,K)=SUM
00266 95. 101 CONTINUE
00270 96. DO 104 I=1,M
00273 97. DO 88 J=1,RDF
00276 98. DO 86 K=1,RDF
00301 99. SUM=0.0
00302 100. DO 89 KK=1,LDF
00305 101. 89 SUMESUM=TR(I,KK,J)*TS(I,KK,K)
00307 102. 88 DKR(I,J,K)=SUM
00312 103. 104 CONTINUE
00312 104. C
00312 104. TRANSFORM DML TO REFERENCE COORDINATE SYSTEM

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00314      DO 90 I=1,N
105.    00315        DO 91 J=1,NDF
106.    00316          DO 91 K=1,NDF
107.    00317            SUM=0.0
108.    00318            DO 92 K=1,NDF
109.    00319              SUM=SUM+OMU(I,J,KK)*TR(I,KK,K)
110.    00320            TS1(J,K)=SUM
111.    00321          CONTINUE
112.    00322        DO 93 I=1,N
113.    00323          DO 94 J=1,NDF
114.    00324            DO 94 K=1,NDF
115.    00325              SUM=0.0
116.    00326              DO 95 K=1,NDF
117.    00327                SUM=SUM+TR(I,KK,J)*TS1(I,KK,K)
118.    00328              TS2(I,J)=SUM
119.    00329            CONTINUE
120.    00330          DO 60 I=1,NVEC
121.    00331            DO 61 K=1,NDFS
122.    00332              XIA=VECO(I,K)
123.    00333            DO 62 I=1,N
124.    00334              DO 63 J=1,NDF
125.    00335                Y(I,I,JJ)=0.0
126.    00336              JESR(I,JJ)
127.    00337              IF (J) 64,63,64
128.    00338              Y(I,I,JJ)=X(J)
129.    00339            CONTINUE
130.    00340          CONTINUE
131.    00341        DO 105 I=1,NVEC
132.    00342          DO 69 J=1,N
133.    00343            DO 65 K=1,NDF
134.    00344              SUM=0.0
135.    00345              SUM=0.0
136.    00346              DO 66 K=1,NDF
137.    00347                SUM=SUM+OKR(J,K,KK)*Y(I,J,KK)
138.    00348              SUM=SUM+OKR(J,K,KK)*Y(I,J,KK)
139.    00349              TS1(K)=SUM3
140.    00350              TS2(K)=SUM4
141.    00351            CONTINUE
142.    00352          DO 67 K=1,NVEC
143.    00353            SUM=0.0
144.    00354            SUM=0.0
145.    00355            DO 68 K=1,NDF
146.    00356              SUM=SUM+Y(K,J,KK)*TS1(KK)
147.    00357              SUM=SUM+Y(K,J,KK)*TS2(KK)
148.    00358            CALCULATE DERIVATIVE OF EIGENVALUES
149.    00359            IF (K.EQ.1) GO TO 70
150.    00360            CLAME(I,J)=SUM1-VALU(I)*SUM2
151.    00361            CALCULATE COEFFICIENTS
152.    00362            CLUE=SUM2/2.
153.    00363            GO TO 67
154.    00364          CLUE=SUM1-VALU(I)*SUM2/(VALU(I)-VALU(K))
155.    00365        CONTINUE
156.    00366      CALCULATE DERIVATIVE OF EIGENVECTORS
157.    00367
158.    00368
159.    00369

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00460 159. DO 78 K=1,NDFS
00471 160. SUM=0.0
00472 161. DO 79 AL=1,NVEC
00475 162. 79 SUM=SUM+CL(KL)*VECO(KL,K)
00477 163. 78 DVEC(J,K)=SUM
00501 164. 69 CONTINUE
00501 165. C FIND DERIVATIVE OF EIGENVALUES AND EIGENVECTORS *.R.T. DESIGN VAR.
00503 166. DO 106 KK=1,NDV
00506 167. CLAMB(I,KK)=0.0
00507 168. DO 107 JJ=1,NDFS
00512 169. 107 CVEC(I,KK,JJ)=0.0
00514 170. DO 108 KJ=1,NMAX
00517 171. 11=IDVCR(KK,KJ)
00520 172. IF (11) 109,106,109
00523 173. 109 CLAMB(I,KK)=CLAMB(I,KK)+LAMB(I,11)
00524 174. DO 110 JJ=1,NDFS
00527 175. 110 CVEC(I,KK,JJ)=CVEC(I,KK,JJ)+DVEC(I1,JJ)
00531 176. 108 CONTINUE
00533 177. 106 CONTINUE
00535 178. 105 CONTINUE
00537 179. 52 RETURN
00540 180. END
00540 181.
END OF COMPILATION: NO DIAGNOSTICS.
TOTAL COMPILATION TIME: 2.4276 SECOND(S)

```

A FOR DRESP  
 COMPILE BY CHI JIVAC 1108 FONTRAY V DATED: OCTOBER 29, 1968  
 THIS COMPILE WAS DONE ON 21 NOV 68 AT 14:25:52

SUBROUTINE DRESP ENTRY POINT 000762  
 STORAGE USED (BLOCK, NAME, LENGTH)

|      |        |        |
|------|--------|--------|
| 0001 | *CODE  | 001017 |
| 0002 | *DATA  | 002503 |
| 0003 | *BLANK | 000000 |
| 0004 | ABLOCK | 001760 |
| 0005 | GNAIN  | 000703 |
| 0006 | UBLOCK | 000013 |
| 0007 | KALASH | 000034 |
| 0008 | BLOCK  | 000122 |
| 0009 | WANOJ  | 000054 |
| 0010 | EDE16  | 001634 |
| 0011 | TEMP   | 000115 |
| 0012 | TOT    | 006125 |
| 0013 | JAGAT  | 000520 |
| 0014 | RAVI   | 000016 |
| 0015 | ARESP  | 000124 |
| 0016 | KISHAN | 000214 |
| 0017 | AMESH  | 000304 |

EXTERNAL REFERENCES (BLOCK, NAME)

|      |        |
|------|--------|
| 0021 | KMX    |
| 0022 | NERR35 |

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

|      |        |      |      |        |      |      |        |        |      |        |      |        |        |        |
|------|--------|------|------|--------|------|------|--------|--------|------|--------|------|--------|--------|--------|
| 0001 | 000164 | 109L | 0001 | 000074 | 131G | 0001 | 000076 | 134G   | 0001 | 000100 | 140G | 0001   | 000126 | 150G   |
| 0002 | 000136 | 153G | 0001 | 000146 | 157G | 0001 | 000175 | 175G   | 0001 | 000176 | 200G | 0001   | 000201 | 204G   |
| 0003 | 000224 | 214G | 0001 | 000225 | 217G | 0001 | 000240 | 225G   | 0001 | 000241 | 230G | 0001   | 000256 | 236G   |
| 0004 | 000357 | 241G | 0001 | 000273 | 247G | 0001 | 000274 | 252G   | 0001 | 000327 | 274G | 0001   | 000331 | 277G   |
| 0005 | 000336 | 305G | 0001 | 000245 | 32F  | 0001 | 000405 | 325G   | 0001 | 000466 | 33F  | 0001   | 000406 | 330G   |
| 0006 | 000407 | 333G | 0001 | 000430 | 342G | 0001 | 000437 | 345G   | 0001 | 000440 | 350G | 0001   | 000503 | 370G   |
| 0007 | 000512 | 375G | 0001 | 000515 | 402G | 0001 | 000461 | 43L    | 0001 | 000606 | 441G | 0001   | 000611 | 446G   |
| 0008 | 000554 | 447L | 0001 | 000557 | 445L | 0001 | 000562 | 449L   | 0001 | 000563 | 450L | 0001   | 000574 | 451L   |
| 0009 | 000577 | 458L | 0001 | 000602 | 453L | 0001 | 000603 | 454L   | 0001 | 000732 | 88L  | 0001   | 000316 | 886L   |
| 0010 | 000577 | 458L | 0001 | 000312 | 890L | 0001 | 000315 | 891L   | 0001 | 000421 | 93L  | 0007   | R      | 000000 |
| 0011 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0012 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0013 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0014 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0015 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0016 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0017 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0018 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0019 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0020 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0021 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0022 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0023 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0024 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0025 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0026 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0027 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0028 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0029 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0030 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0031 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0032 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0033 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0034 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0035 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0036 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0037 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0038 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0039 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0040 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0041 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0042 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0043 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0044 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0045 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0046 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0047 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0048 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0049 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0050 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0051 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0052 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0053 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0054 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0055 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0056 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0057 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0058 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0059 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0060 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0061 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0062 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0063 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0064 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0065 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0066 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0067 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0068 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0069 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0070 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0071 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0072 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0073 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0074 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0075 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0076 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0077 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0078 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0079 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0080 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0081 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0082 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0083 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0084 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0085 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0086 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0087 | 000551 | B5   | 0001 | 000026 | C    | 0000 | R      | 000574 | COC  | 0012   | R    | 000000 | CLAMB  | 0000   |
| 0088 | 000551 | B5   |      |        |      |      |        |        |      |        |      |        |        |        |

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0000 I 002427 1K
0000 I 002442 AK
0010 R 002441 PKCJ
0000 R 002433 SIGAS
0000 R 002434 SU
0000 R 002432 TJ
0010 R 000000 YC

0000 I 000335 IS
0003 R 000000 KR
0014 R 000250 PM
0000 R 002434 SIGES
0000 R 002420 SUM1
0000 R 002431 TE
0000 R 002417 YMS

0000 I 002413 J
0003 R 000770 WM
0014 R 000000 PT
0000 R 002435 S05
0000 R 002421 SUM2
0000 R 000705 VALU

0000 I 002430 JJ
0000 R 000334 NUM
0000 R 002425 SC
0000 R 002440 SSIGN1
0000 R 002422 SUM3
0004 R 000000 VECC

0000 I 002415 K
0005 R 000000 OMEGA
0000 R 002426 SCYC
0000 R 002441 SSIGN2
0000 R 002424 SYC
0016 R 000000 Y

SUBROUTINE DRESP (M,RDF,NDFS,SR,EG)
PARAMETER NM=14,RD=6,ND=41,NDV=7,NMAX=2,NDFT=20,NP=2,NVEC=11
INTEGER RDF,SR
REAL KR,MR,NUM
COMMON/ABLOCK/KR,MR
COMMON/GMAIN/VECO
COMMON/UBLOCK/OMEGA
COMMON/KALASH/D.E
COMMON/BLOCK/AA,BB
COMMON/MANOJ/YC,DYC,C,PROD
COMMON/DEDE16/DTS
COMMON/TEMP/CLAMB
COMMON/TOT/CVEC
COMMON/JAGAT/PT,PW
COMMON/RAVI/IDVCR
COMMON/ARESP/Y
COMMON/KISHAN/DY
COMMON/RAVESH/DZSIGMA
DIMENSION SR(1:RDF),KR(NM,RD,RD),MR(NM,RD,RD),AA(ND),BB(ND),
* Y(NM,RD),DC(NVEC,NM),CDC(NVEC,NDV),VALU(NVEC),
* VECO(NVEC,ND),DYC(NVEC),DOMEGA(NVEC,NDV),PROD(NVEC),
* DY(NDFT,NDV),D(NM),E(NM),
* DSIGN(NVEC,NDFT),YC(NVEC),C(NVEC),DB(NVEC,NDV),
* CLAMB(NVEC,NDV),CVEC(NVEC,NDV),IDVCR(NDV,ND),DTS(NVEC,NM,RD),
* CSIGMAIN(NP,NDV),REVEL(NM,ND),PT(NM,NP,RD),PW(NM,NP,RD),
* CVECCL(NM,RD,NDV),IDVCR(NDV,NMAX),OMEGA(NVEC),EG(NDFS)
32 FORMAT (1H0,42H DERIVATIVE OF MAXIMUM DISPLACEMENT VECTOR,5X,
2 23H W-R-T, DESIGN VARIABLE,12/(2X,10E11.6))
33 FORMAT (1H0,36H DERIVATIVE OF MAXIMUM STRESS-MEMBER,12,2X,
1 23H W-R-T, DESIGN VARIABLE,12,5X,2(10E11.6,2X))
1 CALL KMX(NDFS,SR,EG,KR,MR,FALSE)
C
EVALUATE SECOND TERM OF CC
DO 106 I=1,NVEC
DO 106 J=1,N
SUM=0.0
DO 107 K=1,RDF
107 SUM=SUM+DTS(I,J,K)*Y(J,K)
106 CC(I,J)=SUM
DO 108 I=1,NVEC
DO 109 J=1,NDV
CC(I,J)=0.0
DO 110 K=1,NMAX
110 IDVCR(J,K)

```

```

00162      IF (I1) 111,109,111
00165      111 CDC(I,J)=CDC(I,J)+DC(I,I1)
00166      110 CONTINUE
00167      109 CONTINUE
00170      108 CONTINUE
00172      C
00174      EVALUATE FIRST TERM OF DC
00177      DO 83 I=1,NVEC
00179      DO 83 J=1,NV
00181      NUM=0.0
00183      DO 84 K=1,NDES
00185      84 NUM=NUM+VEC(I,J,K)*BB(K)
00187      83 DC(I,J)=NUM
00189      C
00191      ADD THE TWO TERMS OF DC
00193      DO 500 I=1,NVEC
00195      DO 500 J=1,NV
00197      500 CDC(I,J)=CDC(I,J)+DC(I,J)
00199      C
00201      EVALUATE DERIVATIVE OF FREQUENCIES W.R.T. DESIGN VAR.
00203      DO 103 I=1,NVEC
00205      DO 103 J=1,NV
00207      103 DOMEGA(I,J)=10.5*(CLAMB(I,J)/OMEGA(I))
00209      C
00211      EVALUATE DG W.R.T. DESIGN VAR.
00213      DO 112 I=1,NVEC
00215      DO 112 J=1,NV
00217      112 DG(I,J)=DYC(I)*DOMEGA(I,J)
00219      C
00221      STORE SIGN VECTOR TO FIND DER. OF MAX. DISPLACEMENT
00223      DO 887 I=1,NVEC
00225      DO 886 J=1,NV
00227      YMS=PROD(I)*VECO(I,J)
00229      IF (YMS .LT. 0.0) GO TO 889
00231      IF (YMS .GT. 0.0) GO TO 890
00233      GO TO 891
00235      889 DSIGN(I,J)=-1.0
00237      GO TO 886
00239      890 DSIGN(I,J)=1.0
00241      GO TO 886
00243      891 DSIGN(I,J)=0.0
00245      886 CONTINUE
00247      887 CONTINUE
00249      C
00251      CALCULATE DERIVATIVE OF MAXIMUM DISPLACEMENT VECTOR W.R.T. DESIGN
00253      C
00255      VARIABLES
00257      DO 115 K=1,NV
00259      DO 113 J=1,NV
00261      SUM1=0.0
00263      SUM2=0.0
00265      SUM3=0.0
00267      DO 114 I=1,NVEC
00269      DSIGN(I,J)=DCSIGN(I,J)
00271      DSIGN(I,J)=DCSIGN(I,J)
00273      DSIGN(I,J)=DCSIGN(I,J)
00275      DSIGN(I,J)=DCSIGN(I,J)
00277      SUM1=SUM1+DCSIGN(I,K)*VECO(I,J)
00279      SUM2=SUM2+DCSIGN(I,K)*VECO(I,J)
00281      SUM3=SUM3+DCSIGN(I,K)*VECO(I,J)
00283      113 D(J,K)=SUM1+SUM2+SUM3
00285      114
00287      115
00289      116
00291      117
00293      118
00295      119
00297      120
00299      121
00301      122
00303      123
00305      124
00307      125
00309      126
00311      127
00313      128
00315      129
00317      130

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00321 98.
00321 99.
00323 100.
00324 101.
00327 102.
00332 103.
00335 104.
00335 105.
00341 106.
00344 107.
00347 108.
00352 109.
00353 110.
00354 111.
00355 112.
00360 113.
00361 114.
00362 115.
00365 116.
00365 117.
00367 118.
00372 119.
00373 120.
00374 121.
00377 122.
00400 123.
00401 124.
00404 125.
00405 126.
00407 127.
00410 128.
00411 129.
00411 130.
00412 131.
00413 132.
00414 133.
00416 134.
00420 135.
00421 136.
00422 137.
00423 138.
00424 139.
00425 140.
00426 141.
00430 142.
00432 143.
00433 144.
00434 145.
00435 146.
00436 147.
00437 148.
00437 149.
00440 150.
00443 151.

115 CONTINUE
C STORE SIG: VECTORS TO F1:0 DER. OF STRESSES
IK=1
DO 48 I=1,N
DO 46 J=1,NP
DO 46 K=1,NV
48 SIGMA(I,J,K)=0.0
C TRANSFORM DERIVATIVE OF EIGENVECTORS
93 DO 42 K=1,NV
DO 43 I=1,N
DO 43 J=1,NDF
CVEL(I,J,K)=0.0
CVEL(I,J,K)=0.0
JJ=SR(I,J)
IF (JJ) 44,43,44
44 CVEL(I,J,K)=CVEL(I,K,JJ)
EVEL(I,J)=VECO(I,K,JJ)
43 CONTINUE
42 CALCULATE STRESSES AT VARIOUS POINTS ON MEMBERS
DO 47 I=1,N
TE=E(I)
TDED(I)
DO 45 J=1,NP
SIGAS=0.0
SIGBS=0.0
DO 46 K=1,NDF
SIGAS=SIGAS+PY(I,J,K)*EVEL(I,K)
SIGBS=SIGBS+PW(I,J,K)*EVEL(I,K)
50=SIGBS*TE/2.
SIGAS=SIGAS+TE
SIGBS=SIGBS+TE*0.2
C STRESSES IN PROPORTION TO PARTICIPATION COEFF.
CSIGAS=PROD(IK)*SIGAS
CSIGBS=PROD(IK)*SIGBS
IF (CSIGAS .LT. 0.0) GO TO 447
IF (CSIGAS .GT. 0.0) GO TO 449
GO TO 449
447 SSIGAS=-1.0
GO TO 450
448 SSIGAS=1.0
GO TO 450
449 SSIGAS=0.0
450 IF (CSIGBS .LT. 0.0) GO TO 451
IF (CSIGBS .GT. 0.0) GO TO 452
GO TO 453
451 SSIGAS=-1.0
GO TO 454
452 SSIGAS=1.0
GO TO 454
453 SSIGAS=0.0
454 EVALUATE DER. OF STRESSES AT VARIOUS POINTS ON MEMBERS
DO 45 I=1,N
CSIGAS=0.0

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00449 152. DSIGBS=C*E
00445 153. DO 117 A=1,2*E
00450 154. DSIGAS=DSIGAS+Y(I,J,K)*C*VECEL(I,J,K,K)
00451 155. 117 DSIGBS=DSIGBS+P(I,J,K)*C*VECEL(I,J,K,K)
00453 156. DSIGAS=IE*DSIGAS
00454 157. DSIGBS=IE*DSIGBS/2.
00455 158. DSIGBS=DSIGBS+SOS
00456 159. DSIGAS=SSIGN1*((CDC(I,K,K)*YC(I,K)*C(I,K)+DQ(I,K,K))*SIGAS+
00457 160. 1 PROD(I,K)*DSIGAS)
00457 161. DSIGBS=SSIGN2*((CDC(I,K,K)*YC(I,K)*C(I,K)+DQ(I,K,K))*SIGBS+
00457 162. 1 PROD(I,K)*DSIGBS)
00460 163. 116 DSIGAS(I,J,K,K)=DSIGAS(I,J,K,K)+DSIGAS*DSIGBS
00462 164. 45 CONTINUE
00464 165. 47 CONTINUE
00466 166. IF (IK.EQ. NVEC) GO TO 88
00470 167. IK=IK+1
00471 168. GO TO 93
00472 169. 86 CONTINUE
00473 170. RETURN
00474 171. END
00474 172.

```

END OF COMPILATION: NO DIAGNOSTICS.  
TOTAL COMPILATION TIME: 2.9736 SECOND(S)



A FOR FLESI:  
 COMPILED BY CH1 ON IAC 1105 FOR TPA V. DATED: OCTOBER 20, 1968  
 THIS COMPILED WAS DONE ON 21 NOV 68 AT 10:25:55

SUBROUTINE FLESI: ENTRY POINT 00075  
 STORAGE USED (BLOCK, NAME, LENGTH)

0001 \*CODE 001336  
 0000 \*DATA 003352  
 0002 \*BLANK 000000

EXTERNAL REFERENCES (BLOCK, NAME)

0003 SORT  
 0004 NOSFJ  
 0005 NIORS  
 0006 NIOLS  
 0007 \*ERRRS

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

|      |        |       |      |        |       |      |        |      |      |        |      |      |        |      |
|------|--------|-------|------|--------|-------|------|--------|------|------|--------|------|------|--------|------|
| 0000 | 003225 | IF    | 0000 | 003227 | 10F   | 0000 | 003175 | 10F  | 0000 | 003200 | 11F  | 0000 | 003205 | 12F  |
| 0001 | 000057 | 1223  | 0001 | 000061 | 1266  | 0001 | 000077 | 124G | 0001 | 000147 | 14F  | 0001 | 000102 | 14G  |
| 0000 | 003215 | 15F   | 0001 | 000123 | 1823  | 0000 | 003216 | 15F  | 0001 | 000147 | 14G  | 0001 | 000174 | 174G |
| 0000 | 003222 | 16F   | 0001 | 000205 | 2013  | 0001 | 000214 | 20F  | 0001 | 000261 | 22G  | 0001 | 000270 | 23G  |
| 0001 | 000301 | 236G  | 0001 | 000324 | 2469  | 0001 | 000407 | 23G  | 0001 | 000433 | 305G | 0001 | 000474 | 323G |
| 0001 | 000475 | 325G  | 0001 | 000520 | 3263  | 0001 | 000537 | 346G | 0001 | 000557 | 354G | 0001 | 000577 | 370G |
| 0001 | 000630 | 402G  | 0001 | 000700 | 4215  | 0001 | 000712 | 42G  | 0001 | 001031 | 453G | 0001 | 001077 | 471G |
| 0001 | 001046 | 50L   | 0001 | 001122 | 51L   | 0001 | 001155 | 514G | 0001 | 001176 | 531G | 0001 | 001176 | 531G |
| 0001 | 001233 | 54L   | 0001 | 001226 | 5413  | 0001 | 001010 | 56L  | 0001 | 001236 | 57L  | 0001 | 001243 | 58L  |
| 0001 | 000354 | 60L   | 0001 | 000370 | 62L   | 0001 | 000374 | 62L  | 0001 | 000426 | 65L  | 0001 | 000544 | 66L  |
| 0001 | 000570 | 67L   | 0001 | 000566 | 68L   | 0001 | 000610 | 71L  | 0001 | 000642 | 72L  | 0001 | 000726 | 75L  |
| 0001 | 000731 | 76L   | 0001 | 000637 | 78L   | 0001 | 000634 | 79L  | 0000 | 003173 | 9F   | 0000 | 003170 | AA   |
| 0000 | 000300 | ARTS  | 0000 | 000901 | 431V  | 0000 | 003123 | 2    | 0000 | 003100 | I    | 0000 | 003117 | 181  |
| 0000 | 000214 | 1551V | 0000 | 000762 | 1651V | 0000 | 003113 | 113  | 0000 | 003116 | I    | 0000 | 003117 | 181  |
| 0000 | 000303 | 15V   | 0000 | 000245 | 15    | 0000 | 003101 | 1    | 0000 | 003115 | J    | 0000 | 003115 | K    |
| 0000 | 000372 | 1CC   | 0000 | 000311 | 1     | 0000 | 003112 | W4V  | 0000 | 003127 | N4A  | 0000 | 003106 | K3   |
| 0000 | 000310 | 1CC   | 0000 | 000311 | 1     | 0000 | 000376 | W4V  | 0000 | 003127 | N4V  | 0000 | 002646 | R45  |
| 0000 | 000371 | R45   | 0000 | 000316 | 15    | 0000 | 003112 | 151  | 0000 | 003103 | 152  | 0000 | 003104 | 151  |

0001 1: SUBROUTINE FLESI: ENTRY POINT 00075  
 0002 2: PARAMETER CODES  
 0003 3: INTERIOR AXIS  
 0004 4: 184.0

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00106 5. 9 FORMAT (6E15.6//)
00107 9. 10 FORMAT (9H1 PHASE I)
00110 7. 11 FORMAT (21H NO FEASIBLE SOLUTION)
00111 6. 12 FORMAT (15H ASIN = PHASE I)
00112 5. 13 FORMAT (16H IBSIN = PHASE I)
00113 4. 14 FORMAT (30H)
00114 3. 15 FORMAT (30H)
00115 2. 16 FORMAT (16H ICSIN = PHASE I)
00116 1. 17 FORMAT (16H ICSIN = PHASE I)
00117 18. DIMENSION ASIN(NCD),RHS(NCD),IBSIN(NCD),ICSIN(NCD),ISV(NCD),
00118 * NC(25,NDV),GW(NDV),HETA(25),SINOV)
00119 19. VACENAC=1
00120 16. VMOV=NDV+1
00121 17. ZERO ASIN
00122 13. DO 50 I=1,NCD
00123 18. RMS(I)=0.0
00124 19. DO 50 J=1,NCD
00125 20. ASIN(I)=0.0
00126 21. 50 NORMALIZE CONSTRAINT VECTORS AND CALCULATE SUM OF COMPONENTS
00127 22. DO 51 I=1,NAC
00128 23. TS1=0.0
00129 24. TS2=0.0
00130 25. DO 52 J=1,NDV
00131 26. TTENC(I,J)
00132 27. TS1=TS1+TTT
00133 28. TS2=TS2+TTT*TTT
00134 29. 52 TS2=SGRT(TS2)
00135 30. RMS(I)=TS1/TS2
00136 31. DO 53 J=1,NDV
00137 32. ASIN(I)=J*NC(I,J)/TS2
00138 33. 53 CONTINUE
00139 34. 51 NORMALIZE HEIGHT GRADIENT VECTOR AND CALCULATE SUM OF COMPONENTS
00140 35. TS1=0.0
00141 36. TS2=0.0
00142 37. DO 54 I=1,NDV
00143 38. TTENC(I)
00144 39. TS1=TS1+TTT
00145 40. TS2=TS2+TTT*TTT
00146 41. 54 TS2=SGRT(TS2)
00147 42. RMS(NAC)=TS1/TS2
00148 43. DO 55 I=1,NDV
00149 44. ASIN(NAC)=I*NC(I)/TS2
00150 45. 55 INSERT SIGNA COEFFICIENTS
00151 46. DO 56 I=1,NAC
00152 47. ASIN(I)=I*HETA(I)
00153 48. 56 ASIN(NAC)=NDV+1.0
00154 49. FILL OUT UPPER BOUNDS
00155 50. DO 57 I=1,NDV
00156 51. KENNAC(I)
00157 52. ASIN(NAC)=I*1.0
00158 53. 57 CONTINUE
00159 54. 54 INSERT OBJECTIVE
00160 55. VACENAC=V+1
00161 56. 56
00162 57. 57
00163 58. 58
00164 59. 59

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00220 .CC=CB+1
00221 ASIM(NCC,NDOV)=1.0
00222 SET OF TRIAL BASIS
00223 DO 58 I=1,NDOV
00224 58 ICSIM(I)=1
00225
00226 C THESE ARE SLACK VARIABLES
00227 CC 59 I=1,NCA
00228 59 IBSIM(I)=NDOV+I
00229
00230 C ADD ARTIFICIAL VARIABLES AND CLEAR ARTIFICIAL OBJECTIVE
00231 ARTS=0
00232 DO 60 I=1,NMAC
00233 IF (RHS(I) .GE. -1.0E-07) GO TO 60
00234 NDOV=NDOV+1
00235 ASIM(I,NDOV)=1.0
00236 ARTS=ARTS+1
00237 DO 61 J=1,NDOV
00238 TS1=ASIM(I,J)
00239 ASIM(I,J)=TS1
00240 61 ASIM(NCC,J)=ASIM(NCC,J)-TS1
00241 RMS(I)=RHS(I)
00242 RMS(NCC)=RHS(NCC)-TS1
00243 ICSIM(NDOV)=IBSIM(I)
00244 IBSIM(I)=2*NDOV+NAC+2*ARTS
00245 60 CONTINUE
00246
00247 NDOV=NDOV+1
00248 IF (ARTS .EQ. 0) GO TO 62
00249 NENAC=NDOV+2
00250 GO TO 63
00251 62 NENAC=NDOV+1
00252 63 NEM=1
00253 DO 64 I=1,N
00254 64 ASIM(I,NDOV)=RHS(I)
00255
00256 IF (ARTS .EQ. 0) GO TO 65
00257 PHASE = I
00258
00259 C WDOV=NDOV-1
00260 CALL SIMPLE (WDOV,N,ASIM,IBSIM,ICSIM)
00261
00262 65 I=C
00263
00264 DO 77 J=1,NCA
00265 IIB=IBSIM(J)
00266 77 I=MAX(0,IIB)
00267 K=2*DO+NAC+2
00268 IF (I .LE. K) GO TO 66
00269 WRITE (6,11)
00270 WRITE (6,12)
00271 11. ((ASIM(I,J),J=1,CC),I=1,NCC)
00272 12. WRITE (5,16)
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00339 113. WRITE (6,15) ICSIM
00340 114. WRITE (6,16)
00341 115. WRITE (6,15) IUS14
00342 116. GO TO 76
00343 117. SET UP ISV FOR SQUEEZE
00344 118. DO 67 I=1,NDV
00345 119. IF (ICSIM(I) .LE. K) GO TO 66
00346 120. ISV(I)=0
00347 121. GO TO 67
00348 122. 60 ISV(I)=1
00349 123. 61 CONTINUE
00350 124. ISV(NDV)=1
00351 125. SQUEEZE MATRIX
00352 126. K=0
00353 127. DO 70 I=1,NDV
00354 128. IF (ISV(I)) .GT. 0) GO TO 71
00355 129. K=K+1
00356 130. GO TO 72
00357 131. 71 IF (K .EQ. 0) GO TO 79
00358 132. I=I-K
00359 133. DO 73 J=1,N
00360 134. 73 ASIM(J,K)=ASIM(J,I)
00361 135. GO TO 74
00362 136. 79 I=I-K
00363 137. 76 ICSIM(I)=ICSIM(I)
00364 138. 72 J=I-K
00365 139. 70 CONTINUE
00366 140. C PHASE II
00367 141. NEW=1
00368 142. I=J=1
00369 143. CALL SIMPLE (IJ,ASIM,ISV,ICSIM)
00370 144. C
00371 145. *S* VECTOR
00372 146. SIGN=ASIM(I,J)
00373 147. DO 75 I=1,NDV
00374 148. S(I)=1.0
00375 149. NEW=1
00376 150. DO 75 I=1,N
00377 151. IF (SIGN(I) .GT. NDV) GO TO 75
00378 152. I=I+SIGN(I)
00379 153. S(I)=ASIM(I,J)-1.0
00380 154. 75 CONTINUE
00381 155. 70 RETURN
00382 156. SUBROUTINE SIMPLE (N,M,A,IS,IC)
00383 157. PARAMETER N=25
00384 158. I=FORMAT (10H,10)
00385 159. 10 FORMAT (10H SOLUTION UNROUTED)
00386 160. DIMENSION A(M,N),IS(N),IC(MN),D(MN)
00387 161. N=25
00388 162. C FIND NEW BASIC VARIABLE
00389 163.

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00450 167. 50 J=0
00451 168. TS=0.0
00452 169. DO 50 I=1,N
00453 170. AA=AI(I)
00454 171. TS=AVINI(TS,AA)
00455 172. *DIAGNOSTIC* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00456 173. IF (TS .NE. AA) GO TO 50
00457 174. J=1
00458 175. 50 CONTINUE
00459 176. IF (TS .GT. -1.0E-05) GO TO 50
00460 177. C DETERMINE THE BASIC TO REPLACE
00461 178. K=0
00462 179. TS=1.0E+30
00463 180. DO 51 I=1,N
00464 181. AA=AI(I)
00465 182. RM=SZAI(I,N)
00466 183. IF (AA .LE. 1.0E-07) GO TO 51
00467 184. TS=RRMS/AA
00468 185. IF (TS2 .GE. TS) GO TO 51
00469 186. TS=TS2
00470 187. K=I
00471 188. 51 CONTINUE
00472 189. IF (K .EQ. 0) GO TO 57
00473 190. C CHANGE BASIC INDEX VECTORS
00474 191. I=IB(K)
00475 192. IB(K)=IC(I)
00476 193. IC(I)=I
00477 194. C SET UP CUMVY VECTOR FOR PIVOT OPERATIONS
00478 195. DO 52 I=1,N
00479 196. V(I)=AI(I)
00480 197. 52 A(I)=V(I)
00481 198. C DIVIDE PIVOT ROW BY PIVOT TERM
00482 199. A(I)=A(I)/V(I)
00483 200. A(I)=A(I)
00484 201. DO 53 I=1,N
00485 202. A(I)=A(I)/A(I)
00486 203. C ELIMINATE PIVOT COLUMN
00487 204. DO 54 I=1,N
00488 205. IF (I .EQ. K) GO TO 54
00489 206. A(I)=A(I)
00490 207. IF (A(I) .LE. 1.0E-07) GO TO 54
00491 208. DO 55 I=1,N
00492 209. A(I)=A(I)-A(K)*A(I,K)
00493 210. 55 A(I)=A(I)-A(K)*A(I,K)
00494 211. 54 CONTINUE
00495 212. A(K)=A(K)
00496 213. 57 WRITE (6,17)
00497 214. 58 STOP
00498 215.

```

END OF SUBROUTINE 1 DIAGNOSTIC  
 SUBROUTINE 1 2000-2 SEC 1(1)



[illegible]

|        |       |      |        |       |      |        |       |      |        |       |      |        |       |
|--------|-------|------|--------|-------|------|--------|-------|------|--------|-------|------|--------|-------|
| 005250 | 1F    | 0000 | 005337 | 10F   | 0001 | 003594 | 1000L | 0001 | 001232 | 1001G | 0001 | 001135 | 1001L |
| 001233 | 1206G | 0001 | 001447 | 1061G | 0001 | 001477 | 1064F | 0001 | 001437 | 1075G | 0000 | 005341 | 11F   |
| 001340 | 1206G | 0001 | 001541 | 1153G | 0001 | 001592 | 1156G | 0001 | 001500 | 1173G | 0000 | 001501 | 1155G |
| 001603 | 1177G | 0001 | 001651 | 1153G | 0001 | 001656 | 1154L | 0000 | 001543 | 1161G | 0001 | 001545 | 1156G |
| 001640 | 1202G | 0001 | 001646 | 1202G | 0001 | 001662 | 121L  | 0000 | 001593 | 12F   | 0001 | 001651 | 1156G |
| 001645 | 1223G | 0001 | 001646 | 1226G | 0001 | 001674 | 123L  | 0001 | 001607 | 1211G | 0001 | 001607 | 1221G |
| 001737 | 1255G | 0001 | 001737 | 1255G | 0001 | 001742 | 123L  | 0001 | 001647 | 1231G | 0001 | 001651 | 1225G |
| 001815 | 129L  | 0000 | 001841 | 1265G | 0001 | 001857 | 1267G | 0001 | 001773 | 1275G | 0001 | 001845 | 126L  |
| 002015 | 1311G | 0001 | 002047 | 1330G | 0001 | 002067 | 1346G | 0001 | 002012 | 1304G | 0001 | 002045 | 131L  |
| 002132 | 1361G | 0001 | 002172 | 137L  | 0001 | 002235 | 138L  | 0001 | 002070 | 1342G | 0001 | 002123 | 1356G |
| 002303 | 142L  | 0001 | 002320 | 143L  | 0001 | 002335 | 145L  | 0001 | 002153 | 146L  | 0001 | 002161 | 147L  |
| 002317 | 146L  | 0001 | 002303 | 146L  | 0001 | 002342 | 15F   | 0001 | 002140 | 15L   | 0001 | 002147 | 157L  |
| 002327 | 151L  | 0001 | 002435 | 1521G | 0001 | 002482 | 1526G | 0001 | 002240 | 153L  | 0001 | 002456 | 157G  |
| 002462 | 154L  | 0001 | 002510 | 155L  | 0001 | 002541 | 156L  | 0001 | 002240 | 153L  | 0001 | 002456 | 157G  |
| 002516 | 162G  | 0001 | 002510 | 161L  | 0001 | 002531 | 1617G | 0001 | 002121 | 162L  | 0001 | 002641 | 1624F |
| 002714 | 1646G | 0001 | 002716 | 1646G | 0001 | 002550 | 165L  | 0001 | 002121 | 162L  | 0001 | 002641 | 1624F |
| 002776 | 1676G | 0001 | 002776 | 1676G | 0001 | 002587 | 168L  | 0001 | 002724 | 1653G | 0001 | 002641 | 1624F |
| 003031 | 170L  | 0001 | 003021 | 171L  | 0001 | 003032 | 171G  | 0001 | 003043 | 1717G | 0001 | 003037 | 1754G |
| 003131 | 174L  | 0001 | 003121 | 1742G | 0001 | 003126 | 1747G | 0001 | 003343 | 175L  | 0001 | 003133 | 1754G |
| 003147 | 1763G | 0000 | 003460 | 18F   | 0001 | 003434 | 180L  | 0001 | 003362 | 184L  | 0001 | 003472 | 185L  |
| 003651 | 156L  | 0001 | 003743 | 188L  | 0001 | 003671 | 189L  | 0001 | 003571 | 19F   | 0001 | 003673 | 185L  |
| 003815 | 192L  | 0000 | 003827 | 19F   | 0001 | 003552 | 20F   | 0001 | 003321 | 2005G | 0001 | 003322 | 2103G |
| 003830 | 210G  | 0001 | 003273 | 202G  | 0001 | 003315 | 2041G | 0001 | 003353 | 2041G | 0001 | 003310 | 2172G |
| 003950 | 215L  | 0001 | 003430 | 2101G | 0001 | 003343 | 2111G | 0001 | 003344 | 2113G | 0001 | 003356 | 2121G |
| 003962 | 2121L | 0001 | 003430 | 2127G | 0001 | 003316 | 2143G | 0001 | 003010 | 215G  | 0001 | 003356 | 2121G |
| 003982 | 223G  | 0000 | 003529 | 23F   | 0001 | 000034 | 231G  | 0000 | 003551 | 24F   | 0001 | 003512 | 24F   |
| 003982 | 236G  | 0000 | 003573 | 25F   | 0001 | 000074 | 254G  | 0000 | 003517 | 26F   | 0001 | 003510 | 246G  |
| 003983 | 24F   | 0000 | 003572 | 271G  | 0001 | 000134 | 277G  | 0000 | 003584 | 26F   | 0001 | 003567 | 25F   |
| 004023 | 31F   | 0000 | 003573 | 30F   | 0001 | 000150 | 306G  | 0000 | 0035   |       |      |        |       |

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0000 006053 802F      0000 005321 9F      0001 003537 999L      0000 R 002637 A      0005 R 000000 AA
0000 005223 AAA      0000 R 005233 AAA1      0023 R 000000 ALPHA      0005 R 000051 BB      0000 I 002515 BBC
0000 L 002603 BACL      0000 R 004745 BOFR      0000 I 002477 BSC      0000 L 002602 BSCL      0011 L 000000 B1
0025 R 000000 CLAMB      0000 R 005230 CON      0000 R 002607 CX      0000 R 002623 CY      0006 R 000000 D
0000 R 005213 DELTA      0000 R 005222 DIFF      0000 R 003645 DIV      0027 R 000000 DSIGMA      0024 L 000000 DV
0026 R 000000 DY      0000 R 003700 DYC      0006 F 000016 E      0000 R 004560 EG      0024 L 000000 EXIT
0000 R 004727 GA      0000 I 005210 I      0000 I 005072 IAR      0030 I 000000 ID      0024 L 000000 EXIT
0017 I 000000 IDVCR      0000 I 005246 I1111      0000 I 005244 IM      0000 I 005243 INP      0030 I 000000 ID
0000 I 002606 IS      0000 I 005211 J      0000 I 005224 JJ      0000 I 005242 JK      0000 I 005234 IAC      0000 I 005226 JT
0000 I 005212 K      0000 I 005220 KK      0000 R 000060 KL      0004 R 000000 KR      0000 I 005227 KT      0000 I 005227 KT
0007 R 000000 L      0000 R 002605 LQ      0000 R 000016 LI      0000 R 000052 MD1      0000 R 000053 MD2      0000 R 000053 MD2
0000 R 000054 MC3      0000 R 000770 MR      0000 R 000034 M1      0000 I 000057 MQ6      0000 R 000000 M1      0000 R 000000 M1
0000 I 005225 NBSC      0000 R 002040 NC      0000 I 005214 N1N11      0000 I 005245 NAC      0000 I 005216 NBSC      0000 I 005237 DJIFF
0013 R 000000 OMEGA      0000 R 005032 OSX      0000 R 005066 OVALU      0000 I 005215 N1N12      0000 R 005006 OYV      0000 R 005006 OYV
0000 I 002317 PE      0016 R 000000 PF      0000 R 004237 P4      0000 R 003765 PX      0000 R 003767 PY      0000 R 004631 SCOV
0000 I 002335 OE      0010 R 000016 QD      0000 R 005216 RS      0000 R 004736 S      0000 R 004631 SCOV
0003 R 000000 S1IFF      0021 R 000000 SIGMA      0000 R 005207 SIMA      0000 R 005241 SLOPE      0000 R 004752 SV
0000 R 002353 SR      0000 R 005232 SU4      0010 R 000000 T      0000 R 005157 THETA      0012 R 000000 TR      0000 R 004647 UBD
0000 R 003721 TRR      0000 R 002655 TS      0000 R 005217 TFS      0000 R 004647 UBD      0000 R 004573 UBS      0000 R 005231 WT
0014 R 000000 VALU      0000 L 002604 VSCL      0015 R 000000 VECO      0000 R 005235 WCV      0000 R 005231 WT
0000 R 004507 X      0000 R 003660 XC      0016 R 000001 XG      0000 R 003673 YC      0020 R 000000 YW

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00134 29. A(LM),MI(M),L(M),L1(M),M1(M),KL(M,LDF,LDF),ML(M,LDF,LDF),
00134 30. TR(M,LDF,LDF),TS(M,LDF,LDF),KR(M,RDF,RDF),MR(M,RDF,RDF),
00134 31. VALU(INVEC),VECO(INVEC,NDFS)
00134 32. , OMEGA(INVEC),DIV(INVEC),XC(INVEC),YC(INVEC),OYC(INVEC),
00134 33. , TR(RDF,RDF),PX(INP),PY(INP),NP,RDF),P2(M,NP,RDF)
00134 34. , EDUCR(NOV),MMAXI,X(NDFS),AA(NDFS),BB(NDFS),ES(NDFS),
00134 35. , SCON(NOV),UBD(NOV),JUBS(MNP),OV(NOV),88(NOV),STNOV),
00134 36. , BOFRITCR,BSC(NOV),S(MNP),BBC(BCNO),SIGMA(M,NP),
00134 37. , Y(NOV),OY(MNP),OS(MNP),OVALUTCFR),IAR(BCNO)
00134 38. , , OY(NOV),OV),DEGMA(M,NP,NOV),CLAMB(INVEC,NOV)
00134 39. , , NC(25,NOV),THETA(25)
00134 40. 1 FORMAT (2X,10E11.6)
00134 41. 315 FORMAT (10(1X,F7.5))
00134 42. 316 FORMAT (1X(1X,F5.1))
00134 43. 317 FORMAT (1X,10F7.3)
00134 44. 318 FORMAT (8(1X,F8.5))
00134 45. 320 FORMAT (11(1X,F6.1))
00134 46. 2 FORMAT (20(1X,F5.1))
00134 47. 3 FORMAT (8(1X,E9.1))
00134 48. 39 FORMAT (16(1X,F8.2))
00134 49. 4 FORMAT (16(1X,F4.1))
00134 50. 5 FORMAT (1X,25I3)
00134 51. 6 FORMAT (16(F4.1,1X))
00134 52. 7 FORMAT (5(1X,E15.8))
00134 53. 9 FORMAT (1H0,2X,27H ***** INITIAL DESIGN *****
00134 54. 9 FORMAT (2X,1H0,67H INITIAL DESIGN VIOLATES SIDE CONSTRAINTS IN DES
00134 55. ,IGN VARIABLE NUMBER,14)
00134 56. 10 FORMAT (10E8.1)
00134 57. 11 FORMAT (1H0,2X,33H UPPER BOUNDS ON DESIGN VARIABLES,/(4X,10E11.4))
00134 58. 12 FORMAT (3X,35H LOWER BOUNDS ON DESIGN VARIABLES,/(4X,10E11.4))
00134 59. 13 FORMAT (1H0,2X,30H UPPER BOUNDS ON DISPLACEMENTS,/(4X,10E11.4))
00134 60. 14 FORMAT (1H0,2X,27H ASSEMBLY CONTROL MATRIX SR,/(2X,11H MEMBER NO.,
00134 61. ,5X,19H DEGREES OF FREEDOM,/(20(6X,12,10X,6(12,2X),/))
00134 62. 15 FORMAT (1H0,2X,33H DESIGN VARIABLE CORRELATION TABLE,/(2X,4H NOV,
00134 63. , 8X,18H NUMBER OF MEMBERS,/(10(3X,12,10X,2(12,2X),/))
00134 64. 15 FORMAT (1H0,2X,25H UPPER BOUNDS ON STRESSES,/(4X,10E11.4))
00134 65. 17 FORMAT (1X,10F6.1)
00134 66. 18 FORMAT (1H0,2X,28H UPPER BOUNDS ON FREQUENCIES,/(4X,10E11.4))
00134 67. 19 FORMAT (3X,28H LOWER BOUNDS ON FREQUENCIES,/(4X,10E11.4))
00134 68. 20 FORMAT (1X,E9.2)
00134 69. 21 FORMAT (1H0,2X,10H WEIGHT IS,5X,E11.6)
00134 70. 22 FORMAT (1H0,2X,45H STARTING BOUND ON CONSTRAINTS ***** DELTA IS,
00134 71. ,/(4X,E11.4))
00134 72. 23 FORMAT (1H0,2X,56H UNACCEPTABLE DESIGN SINCE MAXIMUM DISPLACEMENT
00134 73. ,VARIABLE,2X,14,2X,30H OF INITIAL DESIGN IS VIOLATED)
00134 74. 24 FORMAT (1H0,2X,50H UNACCEPTABLE DESIGN SINCE MAXIMUM STRESS VARIAB
00134 75. ,LE,2X,14,2X,30H OF INITIAL DESIGN IS VIOLATED)
00134 76. 25 FORMAT (1H0,2X,62H UNACCEPTABLE DESIGN SINCE LOWER BOUND OF FREQUE
00134 77. ,NCY CONSTRAINT,2X,14,2X,30H OF INITIAL DESIGN IS VIOLATED)
00134 78. 26 FORMAT (1H0,2X,52H UNACCEPTABLE DESIGN SINCE UPPER BOUND OF FREQUE
00134 79. ,NCY CONSTRAINT,2X,14,2X,30H OF INITIAL DESIGN IS VIOLATED)
00134 80. 27 FORMAT (1H0,2X,34H BOUNDED BEHAVIOUR CONSTRAINTS ARE,/(36I3)
00134 81. , 2X,12)
00134 82. 28 FORMAT (1H0,2X,44H NUMBER OF BOUNDED BEHAVIOUR CONSTRAINTS ARE,

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00177 23 FORMAT (1H0,2X,48H INDEX OF WORST VIOLATED BEHAVIOUR CONSTRAINT IS
00178 *2X,12,2X,17H AND ITS VALUE IS,2X,E11.6,2X,52H VALUE OF SAME CONS
00179 *TRAIT AT THE OLD DESIGN POINT IS,2X,E11.6)
00200 30 FORMAT (13)
00201 31 FORMAT (1H0,2X,69H *** FREQUENCY CONSTRAINTS CONVERTED INTO EIGENV
00202 *VALUES CONSTRAINTS ***
00203 32 FORMAT (1H0,2X,28H UPPER BOUNDS ON EIGENVALUES,/(4X,10E11.4))
00204 33 FORMAT (1H0,2X,28H LOWER BOUNDS ON EIGENVALUES,/(4X,10E11.4))
00205 34 FORMAT (1H0,2X,35H GRADIENT TO THE HEIGHT FUNCTION IS,/(10E11.6)
00206 35 FORMAT (1H0,2X,44H SINA IS,5X,E11.6)
00207 36 FORMAT (1H0,2X,32H FEASIBLE DIRECTION VECTOR,S, IS,/)
00208 37 FORMAT (1H0,2X,79H NUMBER OF ACTIVE CONSTRAINTS ARE MORE THAN TEN,
00209 *CHANGE THE DIMENSION STATEMENTS)
00210 38 FORMAT (1H0,2X,34H SUPPORT POINT DISPLACEMENT VECTOR)
00211 801 FORMAT (1H0,2X,29H BOUNDED SIDE CONSTRAINTS ARE,/,36I3)
00212 802 FORMAT (1H0,2X,39H NUMBER OF BOUNDED SIDE CONSTRAINTS ARE,2X,12)
00213 C READ SECTION CHARACTERISTICS
00221 READ (5,2) (O(1),I=1,M)
00222 READ (5,2) (T(1),I=1,M)
00223 WRITE (6,1) ((O(1),T(1)),I=1,M)
00227 C READ MATERIAL PROPERTIES
00236 READ (5,3) (E(1),I=1,M)
00244 READ (5,39) (RO(1),I=1,M)
00252 WRITE (6,1) ((E(1),RO(1)),I=1,M)
00253 C READ NODAL COORDINATES
00261 READ (5,4) (CX(1),I=1,N)
00267 READ (5,4) (CY(1),I=1,N)
00275 WRITE (6,1) ((CX(1),CY(1)),I=1,N)
00304 C READ COINCIDENCE MATRIX
00312 READ (5,5) (PE(1),I=1,M)
00320 READ (6,5) ((PE(1),GE(1)),I=1,M)
00327 *WRITE (6,5) ((PE(1),GE(1)),I=1,M)
00340 READ (5,5) ((SR(1,J),J=1,NDF),I=1,M)
00352 WRITE (6,14) ((1*(SR(1,J),J=1,NDF)),I=1,M)
00355 DO 40 K=1,NVEC
00363 READ (5,7) (VECO(K),I=1,NDFS)
00376 40 CONTINUE
00386 READ (5,5) ((IDVCR(1,J),J=1,MNAX),I=1,NDV)
00390 WRITE (6,15) ((1*(IDVCR(1,J),J=1,MNAX)),I=1,NDV)
00410 C FIRST NDV CONSTRAINTS ARE UPPER BOUNDS ON THE DESIGN VARIABLES AND
00411 THE NEXT NDV CONSTRAINTS ARE LOWER BOUNDS ON THE DESIGN VARIABLES.
00416 *WRITE (6,11) (SCON(1),I=1,NDV)
00424 J=NDV+1
00425 *WRITE (6,12) (SCON(1),I=J,MNDV)
00433 READ (5,2) (UBD(1),I=1,NDF1)
00441 *WRITE (6,13) (UBD(1),I=1,NDF1)
00447 C *READ (5,3) (UBS(1),I=1,NUP)
00455 *WRITE (6,16) (UBS(1),I=1,NUP)
00463 READ (5,17) (BOFR(1),I=1,TCFR)
00465 C FIRST CFR CONSTRAINTS ARE LOWER BOUNDS ON THE FIRST CFR FREQUENCY
00466 S AND FROM (CFR+1) TO TCFR ARE THE UPPER BOUNDS ON THE FIRST
00467 (TCFR-CFR) FREQUENCIES.
00468 C

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00471 137.      WRITE (6,19) (80FR(I),I=1,CFR)
00477 138.      J=CFR+1
00500 139.      WRITE (6,18) (80FR(I),I=J,TCFR)
00506 140.      READ (5,20) (DELTA)
00511 141.      WRITE (6,22) (DELTA)
00511 142.      CALCULATE MASS DENSITY
00514 143.      DO 55 I=1,N
00517 144.      55 RO(I)=RO(1)/386.
00521 145.      SDIFF=1.0E+06
00522 146.      X6=0.05
00523 147.      PF=250.0
00524 148.      NINT1=NDOFT+MNP
00525 149.      NINT2=NINT1+CFR
00526 150.      EXIT=.FALSE.
00526 151.      C      CONVERT FREQUENCY CONSTRAINTS INTO EIGENVALUE CONSTRAINTS
00527 152.      RS=2.43,1415926
00530 153.      DO 500 I=1,TCFR
00533 154.      TTS=RTS*80FR(I)
00535 155.      500 80FR(I)=TTS
00537 157.      WRITE (6,31)
00541 158.      WRITE (6,33) (80FR(I),I=1,CFR)
00547 159.      J=CFR+1
00550 160.      WRITE (6,32) (80FR(I),I=J,TCFR)
00556 161.      CALL CLOCK
00556 162.      C      TRANSFORM DIAMETER VECTOR TO DESIGN VARIABLE VECTOR
00557 163.      DO 501 I=1,NDV
00562 164.      KK=IDVCR(I,1)
00563 165.      501 DV(I)=D(KK)
00565 166.      ID=0
00565 167.      C      CHECK SIDE CONSTRAINTS FOR INITIAL DESIGN
00565 168.      C      INITIAL DESIGN IS TAKEN TO BE A FREE OR BOUNDED POINT
00566 169.      IC=0
00567 170.      BSCL=.FALSE.
00570 171.      DO 118 J=1,NNDV
00573 172.      IF (J.LE. NDV) GO TO 119
00575 173.      DIFF=SCON(J)-DV(J-NDV)
00576 174.      GO TO 120
00577 175.      119 DIFF=V(J)-SCON(J)
00600 176.      *DIAGNOSTIC* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.
00600 176.      120 IF (SCON(J).EQ. 0.0) GO TO 121
00602 177.      AAA=DIFF/ABS(SCON(J))
00603 178.      GO TO 2121
00604 179.      121 AAA=DIFF
00605 180.      2121 IF (AAA.GT. 0.0) GO TO 122
00607 181.      IF (AAA.LT. DELTA) GO TO 118
00607 181.      C      STORE THE INDEX AND NUMBER OF BOUNDED SIDE CONSTRAINTS
00611 183.      IC=IC+1
00612 184.      BSCL=.TRUE.
00613 185.      BSCIC=J
00614 186.      GO TO 118
00615 187.      122 IF (J.LE. NDV) GO TO 123
00617 188.      J=J-NDV
00620 189.      WRITE (6,9) JJ

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00623 190.      GC TO 1000
00624 191.      123 WRITE (6,9) J
00627 192.      50 TO 1000
00630 193.      110 CONTINUE
00632 194.      IF (.NOT. ESCL) GO TO 870
00634 195.      NBSC=IC
00635 196.      WRITE (6,801) (3SC(J),J=1,NBSC)
00643 197.      WRITE (6,802) NBSC
00643 198.      C      CALCULATE LENGTH OF MEMBERS AND DIRECTION COSINES
00646 199.      800 DO 54 I=1,N
00651 200.      J=RE(I)
00652 201.      K=SE(I)
00653 202.      L(I)=(CX(KT)-CX(JT))*2*(CY(KT)-CY(JT))*2**0.5
00654 203.      L(I)=(CX(KT)-CX(JT))/(L(I))
00655 204.      54 M(I)=(CY(KT)-CY(JT))/(L(I))
00657 205.      WRITE (6,316) (L(I),I=1,N)
00657 206.      C      EVALUATE GRADIENT VECTOR OF WEIGHT FUNCTION. NOTE IT IS CONSTANT
00665 207.      DO 61 I=1,NDV
00670 208.      GW(I)=0.0
00671 209.      DO 62 J=1,NMAX
00674 210.      J=IDVCR(I,J)
00675 211.      IF (JJ) 50,61,50
00675 212.      50 GW(I)=GW(I)+T(JJ)*L(JJ)*RO(JJ)
00701 213.      62 CONTINUE
00703 214.      61 CONTINUE
00705 215.      DO 661 I=1,NDV
00710 216.      GW(I)=GW(I)*3.1415926*386.
00711 217.      661 S(I)=-GW(I)
00713 218.      WRITE (6,34) (GW(I),I=1,NDV)
00721 219.      WRITE (6,8)
00723 220.      WRITE (6,1) (DV(I),I=1,NDV)
00731 221.      DO 183 I=1,BCNO
00734 222.      183 IAR(I)=0
00736 223.      1001 CALL MARK
00736 224.      C      CALCULATE AREA OF MEMBERS
00737 225.      DO 51 I=1,M
00742 226.      51 A(I)=3.1415926*D(I)*T(I)
00744 227.      B1=.FALSE.
00745 228.      C      B1=TRUE MEANS THAT STRUCTURE CONSISTS OF TRUSS ELEMENTS
00745 229.      IF (B1) GO TO 52
00745 230.      C      CALCULATE MOMENT OF INERTIA OF MEMBERS
00747 231.      CON=3.1415926/8.
00750 232.      DO 53 I=1,N
00753 233.      53 MI(I)=CON*D(I)*T(I)*(D(I))*2*(T(I))*2
00755 234.      WRITE (6,7) (MI(I),I=1,N)
00755 235.      C      EVALUATE HEIGHT OF STRUCTURE
00763 236.      52 H=0.0
00764 237.      DO 49 I=1,N
00767 238.      49 H=H+A(I)*L(I)*RO(I)
00771 239.      H=H*386.
00772 240.      WRITE (6,21) H
00772 241.      C      CALCULATE ELEMENT STIFFNESS AND MASS MATRICES
00775 242.      DO 56 I=1,N
00775 243.      DO 57 J=1,LDI

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01003 284.
01006 285.
01007 286.
01012 287.
01013 288.
01014 289.
01015 290.
01016 291.
01017 292.
01018 293.
01021 294.
01022 295.
01024 296.
01025 297.
01026 298.
01027 299.
01030 300.
01031 301.
01032 302.
01033 303.
01034 304.
01035 305.
01036 306.
01037 307.
01040 308.
01041 309.
01042 310.
01043 311.
01044 312.
01045 313.
01046 314.
01047 315.
01050 316.
01051 317.
01052 318.
01053 319.
01054 320.
01055 321.
01056 322.
01057 323.
01060 324.
01063 325.
01066 326.
01067 327.
01072 328.
01073 329.
01074 330.
01077 331.
01102 332.
01105 333.
01110 334.
01111 335.
01112 336.
01113 337.

      JO 57 K=1,LDF
      KL(I,J,K)=0.0
57      MD1=(A(I))*E(I)/(L(I))
      MD2=(RO(I))*A(I)/(L(I))/420.
      KL(I,1,1)=MD1
      KL(I,2,1)=-MD1
      KL(I,1,1)=180.*MD2
      KL(I,2,1)=70.*MD2
      KL(I,2,2)=180.*MD2
      IF (.NOT.B1) GO TO 58
      KL(I,3,3)=180.*MD2
      KL(I,4,3)=70.*MD2
      KL(I,4,4)=180.*MD2
      GO TO 59
58      MD3=2.*E(I)*E(I)/(L(I))
      MD4=1.*MD3
      MD5=3.*MD3/(L(I))
      MD6=6.*MD3/(L(I)*E2)
      KL(I,3,3)=MD6
      KL(I,4,4)=MD6
      KL(I,3,3)=-MD5
      KL(I,5,7)=MD5
      KL(I,5,1)=MD4
      KL(I,6,3)=-MD5
      KL(I,6,4)=MD5
      KL(I,6,5)=MD3
      KL(I,6,6)=MD4
      KL(I,3,3)=156.*MD2
      KL(I,4,4)=156.*MD2
      KL(I,5,3)=-22.*E(I)/(L(I))*MD2
      KL(I,5,4)=-13.*E(I)/(L(I))*MD2
      KL(I,5,5)=4.*E(I)/(L(I))*E2)*MD2
      KL(I,6,3)=13.*E(I)/(L(I))*MD2
      KL(I,6,4)=22.*E(I)/(L(I))*MD2
      KL(I,6,5)=-3.*E(I)/(L(I))*E2)*MD2
      KL(I,6,6)=24.*E(I)/(L(I))*E2)*MD2
59      DO 60 J=1,LDF
      DO 60 K=1,LDF
      KL(I,J,K)=KL(I,K,J)
60      KL(I,J,K)=KL(I,K,J)
56      CONTINUE
      C
      CALCULATE TRANSFORMATION MATRIX
      DO 75 I=1,M
      DO 76 J=1,LDF
      DO 76 K=1,RDF
76      TR(I,J,K)=0.0
      TR(I,1,1)=L(I)
      TR(I,1,3)=E(I)
      TR(I,2,1)=L(I)
      TR(I,2,4)=E(I)

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01114 298. TR(I,3,1)=MI(I)
01115 299. TR(I,3,3)=LI(I)
01116 300. TR(I,4,2)=MI(I)
01117 301. TR(I,4,4)=LI(I)
01118 302. IF (B1) GO TO 75
01119 303. TR(I,5,5)=1.0
01120 304. TR(I,6,6)=1.0
01121 305. 75 CONTINUE
01122 306. C TRANSFORM STIFFNESS MATRIX TO REFERENCE COORDINATE SYSTEM
01123 307. DO 63 I=1,M
01124 308. DO 64 J=1,LDF
01125 309. DO 64 K=1,RDF
01126 310. SUMED=0
01127 311. DO 65 KK=1,LDF
01128 312. 65 SUMESUM=KL(I,J,KK)*TR(I,KK,K)
01129 313. 64 TS(I,J,K)=SUM
01130 314. 63 CONTINUE
01131 315. DO 66 I=1,M
01132 316. DO 67 J=1,RDF
01133 317. DO 67 K=1,RDF
01134 318. SUMED=0
01135 319. DO 68 KK=1,LDF
01136 320. 68 SUMESUM=TR(I,KK,J)*TS(I,KK,K)
01137 321. 67 KK(I,J,K)=SUM
01138 322. 66 CONTINUE
01139 323. C TRANSFORM MASS MATRIX TO REFERENCE COORDINATE SYSTEM
01140 324. DO 69 I=1,M
01141 325. DO 70 J=1,LDF
01142 326. DO 70 K=1,RDF
01143 327. SUMED=0
01144 328. DO 71 KK=1,LDF
01145 329. 71 SUMESUM=ML(I,J,KK)*TR(I,KK,K)
01146 330. 70 TS(I,J,K)=SUM
01147 331. 69 CONTINUE
01148 332. DO 72 I=1,M
01149 333. DO 73 J=1,RDF
01150 334. DO 73 K=1,RDF
01151 335. SUMED=0
01152 336. DO 74 KK=1,LDF
01153 337. 74 SUMESUM=TR(I,KK,J)*TS(I,KK,K)
01154 338. 73 MR(I,J,K)=SUM
01155 339. 72 CONTINUE
01156 340. C ADD SUPERIMPOSED MASSES
01157 341. MR(4,2)=MR(4,2)+1.0
01158 342. MR(4,4)=MR(4,4)+1.0
01159 343. MR(5,2)=MR(5,2)+1.0
01160 344. MR(6,4)=MR(6,4)+1.0
01161 345. CALL FSWIN (M,RDF,NDFS,SR,MR,MR)
01162 346. CALL XMARK(1000)
01163 347. C NORMALISE EIGENVECTORS M.R.T. M-MATRIX
01164 348. DO 39 I=1,NEC
01165 349. DO 90 J=1,NDFS
01166 350. 90 X(IJ)=VECO(IJ)
01167 351. CALL KRYN(NDFS,SR,X,MR,MP,.FALSE.)
01168 352.
01169 353.
01170 354.
01171 355.
01172 356.
01173 357.
01174 358.
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01814 998.
01815 999.
01816 1000.

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01265 352. DIV(I)=0.0
01266 353. DO 91 J=1,NDFS
01271 354. 91 DIV(I)=DIV(I)+(A)*B(I,K)
01273 355. DIV(I)=SQR(DIV(I))
01274 356. DO 92 J=1,NDFS
01277 357. 92 VEC(I,J)=VEC(I,J)/DI/(I)
01301 359. 89 CONTINUE VECTOR
01303 360. C FORM EG VECTOR
01306 361. DO 78 I=1,NDFS
01310 362. 78 EG(I)=0.0
01313 363. DO 79 I=1,M
01316 364. IC=SR(I,4)
01317 365. IF (IC .NE. 0) GO TO 80
01320 366. 80 ES(IC)=1.0
01322 368. 79 CONTINUE
01326 369. IF (ID .GT. 0) GO TO 779
01327 370. WRITE (6,38)
01330 371. 779 CALL ANAL (NORDF,SR,EG,XG,PP)
01334 372. C
01336 373. STORE SIGMA MATRIX AS A VECTOR
01337 374. K=1
01338 375. DO 133 I=1,M
01341 376. DO 132 J=1,NP
01344 377. ITESIGMA(I,J)
01345 378. SM(I,ETS)
01346 379. K=K+1
01347 380. 132 CONTINUE
01351 381. 133 CONTINUE
01353 382. GO TO 146
01354 383. C 161 CALL MANDCH (S=0.10,MT,GM,NOV,SCO,NBSC,NBSC,BSCL,DELTA)
01354 384. TRANSFORM DESIGN VARIABLE VECTOR TO DIAMETER VECTOR
01355 385. 162 DO 145 I=1,NOV
01360 386. DO 144 J=1,MHAX
01363 387. K=IDVCR(I,J)
01364 388. IF (K .EQ. 0) GO TO 145
01366 389. 144 DIKRI=DI(I)
01370 390. 145 CONTINUE
01372 391. ID=ID+1
01373 392. GO TO 1001
01373 393. C CHECK BEHAVIOUR CONSTRAINTS OF CURRENT DESIGN
01374 394. 146 I=0
01375 395. AAA=0.0
01376 396. QBCL=FALSE.
01377 397. 147 CONTINUE
01400 398. 147 J=1,BCNO
01403 399. IF (J .GT. NOFT) GO TO 148
01406 400. DIFF=V(J)-UBD(J)
01406 401. AAA=DIFF/UBD(J)
01407 402. GO TO 153
01410 403. 148 IF (J .GT. MHNT1) GO TO 150
01412 404. DIFF=SM(I,J)-UBS(I,J)-NOFT)
01413 405. AAA=DIFF/UBS(I,J)-NOFT)

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01414 406.      GO TO 153
01415 407.      150 IF (J .GT. .NINT2) GO TO 151
01416 408.      JIFF=FOR(IJ-NINT1)-VALU(J-NINT1)
01417 409.      AA=DIFF/ROFR(J-NINT1)
01418 410.      GO TO 153
01419 411.      151 JIFF=VALU(J-NINT2)-ROFR(J-NINT1)
01420 412.      AA=DIFF/ROFR(J-NINT2)
01421 413.      152 IF (AAA .LT. 0.0) GO TO 154
01422 414.      IF (AAA .LT. DELTA) GO TO 147
01423 415.      C      STORE THE INDEX AND THE NUMBER OF BOUNDED BEHAVIOUR CONSTRAINTS
01424 416.      I=I+1
01425 417.      BBCL=TRUE.
01426 418.      BBCL=J
01427 419.      GO TO 147
01428 420.      154 IF (ID .GT. 0) GO TO 128
01429 421.      IF (J .GT. NDFT) GO TO 142
01430 422.      WRITE (6,23) J
01431 423.      GO TO 1000
01432 424.      142 IF (J .GT. NINT1) GO TO 143
01433 425.      WRITE (6,24) J
01434 426.      GO TO 1000
01435 427.      143 IF (J .GT. NINT2) GO TO 138
01436 428.      WRITE (6,25) J
01437 429.      GO TO 1000
01438 430.      138 WRITE (6,26) J
01439 431.      GO TO 1000
01440 432.      128 BBCL=TRUE.
01441 433.      WRITE (6,30) J
01442 434.      AA=MAX1(AA1,AAA)
01443 435.      IF (AA1 .NE. AAA) GO TO 147
01444 436.      I=J
01445 437.      GOV=AA1
01446 438.      IF (IAR(J) .NE. 1) GO TO 147
01447 439.      LG=TRUE.
01448 440.      147 CONTINUE
01449 441.      NBBC=I
01450 442.      IF (.NOT. BBCL) GO TO 131
01451 443.      WRITE (6,27) (BBCL(I),I=1,NBBC)
01452 444.      WRITE (6,28) NBBC
01453 445.      IF (VBCL) GO TO 727
01454 446.      C      ZERO OUT THE INTEGER ARRAY
01455 447.      DO 717 I=1,BCNO
01456 448.      717 IAR(I)=0
01457 449.      GO 182 J=1,NBBC
01458 450.      K=BBCL(J)
01459 451.      182 IAR(K)=1
01460 452.      727 WRITE (6,5) (IAR(K),K=1,BCNO)
01461 453.      131 IF (ID .GT. 0) GO TO 137
01462 454.      IF (.NOT. BBCL) .AND. (.NOT. BBCL) GO TO 161
01463 455.      GO TO 164
01464 456.      137 IF (.NOT. VBCL) GO TO 164
01465 457.      .ASC=C
01466 458.      IF (I=1) GO TO 155

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\*DIAGNOSTIC\* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.





|      |        |      |        |
|------|--------|------|--------|
| 513. | 0.1730 | 513. | 0.1730 |
| 514. | 0.1731 | 514. | 0.1731 |
| 515. | 0.1732 | 515. | 0.1732 |
| 516. | 0.1733 | 516. | 0.1733 |
| 517. | 0.1735 | 517. | 0.1735 |
| 518. | 0.1736 | 518. | 0.1736 |
| 519. | 0.1740 | 519. | 0.1740 |
| 520. | 0.1740 | 520. | 0.1740 |
| 521. | 0.1741 | 521. | 0.1741 |
| 522. | 0.1746 | 522. | 0.1746 |
| 523. | 0.1746 | 523. | 0.1746 |
| 524. | 0.1751 | 524. | 0.1751 |
| 525. | 0.1753 | 525. | 0.1753 |
| 526. | 0.1756 | 526. | 0.1756 |
| 527. | 0.1760 | 527. | 0.1760 |
| 528. | 0.1762 | 528. | 0.1762 |
| 529. | 0.1765 | 529. | 0.1765 |
| 530. | 0.1766 | 530. | 0.1766 |
| 531. | 0.1770 | 531. | 0.1770 |
| 532. | 0.1771 | 532. | 0.1771 |
| 533. | 0.1773 | 533. | 0.1773 |
| 534. | 0.1777 | 534. | 0.1777 |
| 535. | 0.1775 | 535. | 0.1775 |
| 536. | 0.1776 | 536. | 0.1776 |
| 537. | 0.1777 | 537. | 0.1777 |
| 538. | 0.1777 | 538. | 0.1777 |
| 539. | 0.1780 | 539. | 0.1780 |
| 540. | 0.1782 | 540. | 0.1782 |
| 541. | 0.1787 | 541. | 0.1787 |
| 542. | 0.1792 | 542. | 0.1792 |
| 543. | 0.1805 | 543. | 0.1805 |
| 544. | 0.1820 | 544. | 0.1820 |
| 545. | 0.1821 | 545. | 0.1821 |
| 546. | 0.1823 | 546. | 0.1823 |
| 547. | 0.1824 | 547. | 0.1824 |
| 548. | 0.1825 | 548. | 0.1825 |
| 549. | 0.1826 | 549. | 0.1826 |
| 550. | 0.1827 | 550. | 0.1827 |
| 551. | 0.1831 | 551. | 0.1831 |
| 552. | 0.1834 | 552. | 0.1834 |
| 553. | 0.1835 | 553. | 0.1835 |
| 554. | 0.1836 | 554. | 0.1836 |
| 555. | 0.1840 | 555. | 0.1840 |
| 556. | 0.1843 | 556. | 0.1843 |
| 557. | 0.1845 | 557. | 0.1845 |
| 558. | 0.1846 | 558. | 0.1846 |
| 559. | 0.1848 | 559. | 0.1848 |
| 560. | 0.1851 | 560. | 0.1851 |
| 561. | 0.1853 | 561. | 0.1853 |
| 562. | 0.1854 | 562. | 0.1854 |
| 563. | 0.1855 | 563. | 0.1855 |
| 564. | 0.1856 | 564. | 0.1856 |
| 565. | 0.1857 | 565. | 0.1857 |
| 566. | 0.1860 | 566. | 0.1860 |

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02063 567. 175 NC(U,KK)=ESTSWA(IV,IMP,KK)
02064 568. GO TO 170
02065 569. 173 IF (A .GT. NINT2) GO TO 180
02070 570. K2A=I*Y1
02071 571. GO 177 KK=I*RV
02074 572. 177 NC(U,KK)=CLAMB(K,KK)
02076 573. GO TO 170
02077 574. 180 K2K=N*Y2
02100 575. DO 178 KK=1,NDV
02103 576. 178 NC(U,KK)=CLAMB(K,KK)
02105 577. 170 CONTINUE
02107 578. WRITE (6,1) ((NC(I,J),J=1,NDV),I=1,NAC)
02120 579. DO 181 I=1,NBSC
02123 580. 181 THETA(I)=0.0
02125 581. I1111=NBSC+1
02126 582. DO 179 I=1111,NAC
02131 583. 179 THETA(I)=1.0
02133 584. CALL FEASIS (NAC,NDV,NC,3,THETA,SIVA,S)
02134 585. WRITE (6,36)
02136 586. WRITE (6,1) (S(I),I=1,NDV)
02144 567. WRITE (6,35) SIVA
02147 588. IF (SIVA .LE. 1.0E-02) GO TO 1000
02151 589. GO TO 181
02152 590. 999 WRITE (6,37)
02154 591. 1000 END

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END OF COMPILE: 2 DIAGNOSTICS.  
TOTAL COMPILE TIME: 6.9216 SECOND(S)

### IV.3 Program Input

The input to the computer program consists of two parts:

- (i) The input program deck
- (ii) The input data deck

#### Input Program Deck

The complete program of the synthesis of Planar Truss-Frame Structure is on the deck of cards and is read into the computer memory at the start of each run. The MAIN program and various subroutines contain a parameter card which governs the dimensions of matrices and arrays used in the program. These are detailed below:

#### 1. MAIN PROGRAM

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                                       |
|------------|-------------------|--------------------------------------------------------------------------|
| 1          | M                 | Total number of elements                                                 |
| 2          | N                 | Number of nodes                                                          |
| 3          | LDF               | Number of degrees-of-freedom<br>of an element in local coordinates       |
| 4          | RDF               | Number of degrees-of-freedom<br>of an element in system co-<br>ordinates |
| 5          | NDFS              | Number of degrees-of-freedom of<br>the system                            |
| 6          | NP                | Number of points at which upper                                          |

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                                                                                           |
|------------|-------------------|------------------------------------------------------------------------------------------------------------------------------|
|            | NP(con't)         | bound stress is computed                                                                                                     |
| 7          | NDV               | Number of design variables                                                                                                   |
| 8          | MMA               | Maximum number of elements<br>governed by a design variable                                                                  |
| 9          | NVEC              | Number of eigenvectors used for<br>modal transformations                                                                     |
| 10         | NNDV              | Total number of constraints on<br>design variables (lower and<br>upper limits)                                               |
| 11         | NDFT              | Number of degrees-of-freedom of<br>system in the coordinate direc-<br>tions (i.e. ignoring rotational<br>degrees-of-freedom) |
| 12         | MNP               | $M \times NP$                                                                                                                |
| 13         | CFR               | Number of frequencies on which<br>constraints are imposed                                                                    |
| 14         | TCFR              | Total number of constraints on<br>frequencies (lower and upper<br>limits)                                                    |
| 15         | BCNO              | Total number of constraints on<br>the system                                                                                 |

2. Subroutine FBMIN

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                     |
|------------|-------------------|----------------------------------------|
| 1          | ND                | Number of degrees-of-freedom of system |
| 2          | N                 | Number of eigensolutions desired       |

2a. FANDG

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                               |
|------------|-------------------|------------------------------------------------------------------|
| 1          | M                 | Total number of elements                                         |
| 2          | RDF               | Number of degrees-of-freedom of an element in system coordinates |
| 3          | ND                | Number of degrees-of-freedom of system                           |
| 4          | NVEC              | Number of eigensolutions desired                                 |

2b. INGEN

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                     |
|------------|-------------------|----------------------------------------|
| 1          | ND                | Number of degrees-of-freedom of system |
| 2          | NVEC              | Number of eigensolutions desired       |

3. Subroutine KXMX

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>       |
|------------|-------------------|--------------------------|
| 1          | M                 | Total number of elements |

3. Subroutine KXXM (con't)

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                               |
|------------|-------------------|------------------------------------------------------------------|
| 2          | RDF               | Number of degrees-of-freedom of an element in system coordinates |
| 3          | ND                | Number of degrees-of-freedom of the system                       |

4. Subroutine POLYN

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                               |
|------------|-------------------|------------------------------------------------------------------|
| 1          | RD                | Number of degrees-of-freedom of an element in system coordinates |

5. Subroutine ANAL

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                               |
|------------|-------------------|------------------------------------------------------------------|
| 1          | NM                | Total number of elements                                         |
| 2          | RD                | Number of degrees-of-freedom of an element in system coordinates |
| 3          | LDF               | Number of degrees-of-freedom of an element in local coordinates  |
| 4          | NDFS              | Number of degrees-of-freedom of the system                       |
| 5          | NP                | Number of points at which upper bound stresses are computed      |

5. Subroutine ANAL (con't)

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                                                                                    |
|------------|-------------------|-----------------------------------------------------------------------------------------------------------------------|
| 6          | NDFT              | Number of degrees-of-freedom of the system in the coordinate directions (i.e. ignoring rotational degrees of freedom) |
| 7          | NVEC              | Number of eigenvectors used for modal transformation                                                                  |

6. Subroutine MANDCH and ADJUST

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>         |
|------------|-------------------|----------------------------|
| 1          | NDVP              | Number of design variables |
| 2          | NNDV              | 2*NDVP                     |

7. Subroutine DEIG

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                               |
|------------|-------------------|------------------------------------------------------------------|
| 1          | M                 | Total number of elements                                         |
| 2          | N                 | Number of nodes                                                  |
| 3          | LDF               | Number of degrees-of-freedom of an element in local coordinates  |
| 4          | RDF               | Number of degrees-of-freedom of an element in system coordinates |
| 5          | ND                | Number of degrees-of-freedom of the system                       |
| 6          | NV                | Number of design variables                                       |



7. Subroutine DEIG (con't)

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                      |
|------------|-------------------|---------------------------------------------------------|
| 7          | NVEC              | Number of eigenvectors used for<br>modal transformation |

8. Subroutine DRESP

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                                                                                           |
|------------|-------------------|------------------------------------------------------------------------------------------------------------------------------|
| 1          | NM                | Total number of elements                                                                                                     |
| 2          | RD                | Number of degrees-of-freedom of<br>an element in system coordinates                                                          |
| 3          | ND                | Number of degrees-of-freedom of<br>system                                                                                    |
| 4          | NDV               | Number of design variables                                                                                                   |
| 5          | MMAX              | Maximum number of elements governed<br>by a design variable                                                                  |
| 6          | NDFT              | Number of degrees-of-freedom of<br>system in the coordinate<br>directions (i.e. ignoring rota-<br>tional degrees of freedom) |
| 7          | NP                | Number of points at which upper<br>bound stress is computed                                                                  |
| 8          | NVEC              | Number of eigenvectors used for<br>modal transformation                                                                      |

9. Subroutine FEASIB

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>                                                                              |
|------------|-------------------|-------------------------------------------------------------------------------------------------|
| 1          | NCD               | NAC + NDV + some arbitrary number<br>(taken to be 25) in the attached<br>listing of the program |

9a. SIMPLE

| <u>No.</u> | <u>Identifier</u> | <u>Explanation</u>               |
|------------|-------------------|----------------------------------|
| 1          | MN                | Same as NCD in Subroutine FEASIB |

Input Data Deck

The required input data is described below in the order in which it is read.

1. Mean diameter of members in inches (D)
2. Wall thickness of members in inches (T)
3. Young's Modulus of Elasticity of the material in  $\text{lbs/in}^2$  (E)
4. Weight density in  $\text{lbs./in}^3$  (RO)
5. X-coordinates of the nodes in inches (CX)
6. Y-coordinates of the nodes in inches (CY)
7. P-end of members (PE - Integers)
8. Q-end of members (QE - Integers)
9. Assembly control table (SR - Integers)

10. Starting points for eigensolution (VECO)
11. Design variable correlation table (IDVCR - Integers)
12. Upper and lower bounds on the design variables (SCON)
13. Upper limits on upper bound displacements (UBD)
14. Upper limits on upper bound stresses (UBS)
15. Lower and upper bounds on frequencies (BOFR)
16. Constraint tolerance (DELTA - Negative Quantity)

NOTE: Over and above the above information read through the data deck, the program requires

- 1) The convergence criterion for the termination of eigenvalue routine. This is handled through card number 145 of MAIN by the identifier SDIFF.
- 2) The shock pulse information. This is provided to the computer through card numbers 146 and 147 by the identifiers XG representing maximum ground displacement in inches and PF representing the frequency of the forcing function in radians/sec.

#### IV.4 Program Output

The information which is written out as the output for the automated optimum design of truss-frame structures is described below:

1. The mean diameter and wall thickness of the elements used to model the structure
2. The Young's modulus and weight density of the material of elements
3. 'X' and 'Y' coordinates of the nodes (common joints between the elements)
4. Coincidence table describing the two nodes, a particular member  $i$  connects
5. Assembly control table - SR
6. Design variable correction table
7. Upper bounds on design variables
8. Lower bounds on design variables
9. Upper bounds on displacements
10. Upper bounds on stresses
11. Lower bounds on frequencies
12. Upper bounds on frequencies
13. Constraint thickness - DELTA
14. Lower bounds on eigenvalues
15. Upper bounds on eigenvalues
16. Lengths of elements
17. Gradient to the weight function
18. Initial design
19. Indices of bounded side constraints, if any
20. Number of bounded side constraints, if any
21. Moment of inertia of elements

22. Weight of the structure
23. Number of iterations required for the convergence of Rayleigh quotient to obtain the eigensolution
24. Eigenvalues
25. Frequencies
26. Eigenvectors
27. Reordered eigenvalues
28. Reordered frequencies
29. Time taken to obtain the eigensolution
30. Vector,  $\vec{e}$
31.  $X_i = \omega_i/2p$  where  $\omega_i$ ,  $i = 1, \dots, NVEC$  are the frequencies of the structure and  $p$  is the frequency of the forcing function
32.  $Y_i = \xi_p(\omega_i)^L$ , read from shock spectrum
33. Peak modal participation coefficients
34. Upper bound displacement vector
35. Upper bound stress in the elements at the two ends
36. Violated behavior constraints, if any
37. Bounded behavior constraints, if any
38. A vector whose components are zeros or ones. The location of ones denotes the bounded behavior constraints
39. Index of worst violated behavior constraint and its value at the new infeasible design point and old bounded design point, if there are violated

39(cont) constraints.

- 40. The slope at the old design point. It is set to zero if linear interpolation is executed
- 41. The constraint normals
- 42. The feasible direction vector,  $\vec{S}$
- 43. Value of extra variable  $s_{ima}$
- 44. Previous design
- 45. Step length in the direction of  $\vec{S}$
- 46. New design

The information given in (19) through (46) is written out for all the designs. When the procedure converges to the final optimum design, the program stops.

#### IV.5 Sample Problem - Input/Output Listing

The following pages contain a computer listing of the input deck and two representative sections of program output for the automated optimum design of truss-frame structures subject to dynamic response restrictions.



[illegible]

|                  |                |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |        |
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 -5.4931532,-01  
 8.4996566,-02  
 -1.7147101,-01  
 -7.0325986,-02  
 -3.5184931,-01  
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 -6.2620318,-01  
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 1.6469821,-01  
 -9.7579774,-01  
 -6.8125405,-01  
 -4.0924133,-01  
 2.7815793,-01  
 4.7593890,-01  
 3.5827950,-01  
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 5.7627007,-02  
 1.3781159,-01  
 1.5839691,-01





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UPPER BOUNDS ON DISPLACEMENTS
.2000-00 .2000-00 .2000-00 .2000-00 .2000-00 .2000-00 .2000-00 .2000-00
.2000-00 .2000-00 .2000-00 .2000-00 .2000-00 .2000-00 .2000-00 .2000-00

UPPER BOUNDS ON STRESSES
.5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05
.5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05
.5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05 .5000+05

LOWER BOUNDS ON FREQUENCIES
.2500+02 .2500+02 .2500+02 .2500+02 .2500+02 .2500+02 .2500+02 .2500+02

UPPER BOUNDS ON FREQUENCIES
.9000+02

STARTING BOUND ON CONSTRAINTS ***** DELTA IS --.1000+00

*** FREQUENCY CONSTRAINTS CONVERTED INTO EIGENVALUES CONSTRAINTS ***
LOWER BOUNDS ON EIGENVALUES
.2467+05 .2467+05 .2467+05 .2467+05 .2467+05 .2467+05 .2467+05 .2467+05

UPPER BOUNDS ON EIGENVALUES
.3198+06
CLOCK INTERROGATED AT 14:26:09
7.5 7.5 7.5 7.5 7.5 6.0 9.0 9.6 9.6 9.6 9.6 9.6 9.6 9.6 9.6 9.6 9.6 9.6
9.6

GRADIENT TO THE WEIGHT FUNCTION IS
.131947+01 .131947+01 .131947+01 .105558+01 .168974+01 .168974+01 .168974+01 .168974+01

***** INITIAL DESIGN *****
.250000+01 .250000+01 .250000+01 .250000+01 .250000+01 .250000+01 .250000+01 .250000+01
.61457404-00 .61457404-00 .61457404-00 .61457404-00 .61457404-00 .61457404-00 .61457404-00 .61457404-00
.61457404-00 .61457404-00 .61457404-00 .61457404-00 .61457404-00 .61457404-00 .61457404-00 .61457404-00
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WEIGHT IS .252080+02

NUMBER OF ITERATIONS
110 53 47 29 95 59 102 113 85 70 55

EIGENVALUES ARE
.9687689+05 .9344875+06 .1780836+07 .1780836+07 .1780836+07 .1780836+07 .1780836+07 .1780836+07
.6283413+08 .6284662+08 .1412217+09 .1412217+09 .1412217+09 .1412217+09 .1412217+09 .1412217+09

FREQUENCIES ARE
.3112505+03 .9666889+03 .1334440+04 .1334440+04 .1334440+04 .1334440+04 .1334440+04 .1334440+04
.7926798+04 .7927599+04 .1188367+05 .1188367+05 .1188367+05 .1188367+05 .1188367+05 .1188367+05
EIGENVECTOR CORRESPONDING TO
-.69655567-01 -.1392950-06 .9687689+05 .9687689+05 .9687689+05 .9687689+05 .9687689+05 .9687689+05
.1432414-00 .7162418-01 .1432414-00 .1432414-00 .1432414-00 .1432414-00 .1432414-00 .1432414-00
.7545755+04 .7545755+04 .7545755+04 .7545755+04 .7545755+04 .7545755+04 .7545755+04 .7545755+04
.1577970-02 .1577970-02 .1577970-02 .1577970-02 .1577970-02 .1577970-02 .1577970-02 .1577970-02

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Lengths of Elements.

starting Design Bounded.  
Moment of Inertia of Elements.

|                              |              |              |              |              |              |              |              |
|------------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| -.4503789-01                 | .7213745-01  | .1491526-00  | .2977786-00  | .6496325-00  | .1000000+01  | .1638120-00  | .3277314-00  |
| -.1435952-00                 | .3127691-00  | .6859438-00  | .1648217-00  | -.1990092-01 | -.1985558-01 | -.1978985-01 | -.4697077-01 |
| -.4482122-01                 | -.4464655-01 | -.3904843-01 | -.3952940-01 | -.3992401-01 | -.2185207-01 | -.2184927-01 | -.2184927-01 |
| -.1673419-01                 | -.1664835-01 | -.1651864-01 | -.1511239-01 | -.1798823-01 | -.1782734-01 | -.2343126-01 | -.2355080-01 |
| -.2365674-01                 |              |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
| -.1007798-02                 | .9344875+04  | .6463320-00  | .1000000+01  | .1362109-00  | .3122040-00  | .1427419-00  | .3045524-00  |
|                              | .07-00       | .3740380-00  | .7332520-00  | .2988036-00  | -.1473085-00 | .5073511-00  | .9962656-00  |
|                              | .6615841-00  | .1713556-00  | .7332520-00  | .5035902-01  | -.4886635-01 | -.4726752-01 | .5739186-01  |
|                              | .3711126-00  | .8450708-00  | .5174415-00  | .6977845-01  | -.6426981-01 | -.6655719-01 | -.6824658-01 |
|                              | .5882222-01  | .5882222-01  | .6923254-01  | -.5092651-01 | -.4622638-01 | -.2360955-02 | -.1637936-02 |
|                              | -.2118285-01 | -.2139572-01 | -.5449411-01 |              |              |              |              |
|                              | -.1007798-02 |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
|                              | .1780836+07  | .9427336-00  | .1193967+01  | .5005461-00  | .1000000+01  | .3846797-00  | .6579933-00  |
|                              | .3447612-00  | .6889624-00  | -.1314598+01 | -.5691836-00 | .6087021-00  | -.7345543-00 | -.1410453+01 |
|                              | .1159962+01  | .4894439-00  | -.7212208-00 | .935471-01   | .8797422-01  | .8104718-01  | -.1243236+00 |
|                              | -.7081395-00 | .1363567+01  | -.7650760-01 | .5840250-01  | .8743694-01  | .9430482-01  | .9991652-01  |
|                              | -.1286571-00 | -.1311040-00 | -.7650760-01 | .2503651-01  | .2137757-01  | -.2941204-01 | -.2573382-01 |
|                              | .8594845-01  | .765313-01   | .2773091-01  |              |              |              |              |
|                              | -.2225703-01 |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
|                              | .3590898+07  | .3727229-00  | -.5082392-00 | .5012745-00  | .1000000+01  | -.1378146-00 | .3957971-00  |
|                              | -.1179895+00 | -.2352146-00 | .1786645-00  | -.7162455-01 | .3105783-00  | .7909327-01  | .1463184-00  |
|                              | .2780670-00  | .5348975-00  | .1215167+00  | -.1333935-01 | -.1190775-01 | -.1027045-01 | .3361781-01  |
|                              | .1140555+00  | .1628762-00  | .3825259-01  | -.2134536-01 | -.8376829-02 | -.9850046-02 | -.1098457-01 |
|                              | .3275531-01  | .3109011-01  | -.3113635-01 | -.3932956-01 | -.2325013-01 | -.1061499+00 | .1032053+00  |
|                              | -.1994549-01 | -.1519875-01 | -.5198046-01 |              |              |              |              |
|                              | -.9908867-01 |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
|                              | .2231323+08  | .1844292-02  | .1023540-03  | .1023540-03  | .2270712-03  | -.7958179-00 | -.1381202-01 |
|                              | -.1159156-01 | -.2811781-01 | .3917407-01  | .3394465-01  | .3116870-04  | -.2784024-02 | -.3518523-02 |
|                              | .4025733-02  | .7800278-01  | -.9300549-01 | -.5566448-02 | -.2716374-02 | .6220740-03  | -.5661265-03 |
|                              | .1000000+01  | .1800915-01  | -.5506119-02 | .6614038-03  | .2504393-03  | .2606903-03  | .4421046-03  |
|                              | .2725842-02  | .5509426-02  | -.8927533-03 | .7049505-03  | .1146810-04  | .2606903-03  | .4421046-03  |
|                              | -.2073884-03 | -.2635725-02 | .1922533-01  | .1293547-03  | -.1894635-01 | -.1229365-02 | -.1961491-02 |
|                              | -.2559238-02 |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
|                              | .2344099+08  | .2793846-02  | .3540392-02  | .1653364-02  | .3250408-02  | -.1130141-02 | .1203488-02  |
|                              | .2283266-02  | -.3540392-02 | -.3982708-01 | -.8407389-02 | -.3440711-02 | -.2086787-02 | -.3366951-02 |
|                              | -.7996274-00 | .2882521-01  | -.5982708-01 | .2504393-03  | .2504393-03  | -.2031660-03 | .6252463-03  |
|                              | .7760344-03  | .3485941-02  | -.2637818-05 | -.2100659-00 | .1238529-03  | .2321352-03  | .3644672-03  |
|                              | -.1561072-05 | -.6262222-03 | .8014397-02  | .3392272-04  | -.7954568-02 | .1367002-04  | -.0876553-03 |
|                              | -.3318581-03 | .2286651-03  |              |              |              |              |              |
|                              | -.2699330-03 |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
|                              | .2339316+08  | .4504560-03  | -.1549111-02 | -.1549111-02 | -.3045605-02 | -.7283569-01 | -.2509933-02 |
|                              | -.7216564-03 | -.4861066-03 | .1753981-03  | .1422573-02  | .3746112-04  | -.3541953-02 | -.4046256-02 |
|                              | -.3223096-01 | .8008600-00  | -.1753981-03 | .1422573-02  | .3746112-04  | -.3541953-02 | -.4046256-02 |
|                              | .9024893-01  | -.1963578-02 | .3655677-01  | .2893104-03  | -.1639447-04 | .2747184-03  | -.2738168-03 |
|                              | .1071894-04  | .2867596-03  | .0033891-02  | -.8123514-02 | -.6070061-04 | .2763367-03  | .5834285-03  |
|                              | -.1892111-01 | -.4491544-04 | .1883723-01  | .3012013-03  | .2100172-00  | .2006392-03  | .1405689-03  |
|                              | .5484946-04  |              |              |              |              |              |              |

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EIGENVECTOR CORRESPONDING TO
-.1193993-01 -.230207-01 .5693842+09 .7482968-03 .583030-04 .832151-04 .9236494-01 -.1984954-01
-.4868656-02 .1903199-02 .1000000+01 .1540325-00 .1066276+01 -.2842256-02 -.3497845-01 -.5651065-02
-.3626302-01 .7603203-01 .2146259-02 .1250358-03 .2025948-00 -.1142484-01 .1847220-00 -.1986642-00
.1164222-01 .5164744-00 .1264488-02 .1323336-03 .1394701-02 .6420201-02 .4257492-03 .7082634-02
.1903698-01 .51485616-02 .2774444-01 .2892905-03 .3155233-03 .5897214-03 .4495986-03 .1597739-02
-.3998869-02

EIGENVECTOR CORRESPONDING TO
.6283412+04 .7341056-04 .8931998-05 .2610906-04 .2140200-02 .8066814-04
-.1175305-03 -.3946691-03 .1370032-03 .3537022-02 .1000000+01 .2951958-02
-.1866738-03 .1505140-02 .6916334-00 .2909368-03 .1865928-00 .1700724-00
.1094314-02 .2581638-03 .1306358-03 .2873254-03 .1756856-03 .2250002-03
.9158640-04 .1699543-00 .6253373-04 .2873254-03 .1665592-03 .4304401-03
.4258349-03 .1534516-03 .7311968-03 .3095740-03 .1604452-03 .3951266-03
.6222731-04

EIGENVECTOR CORRESPONDING TO
.6284682+08 .3314210-05 .3621155-05 .1503084-04 .1141058-02 .2068365-02
.2978260-03 .5755621-03 .5018090-03 .2816383-02 .5854734-00 .2684322-02
-.1752065-02 .1390950-02 .5907939-00 .319223-03 .1235908+00 .1231275+00
.7370751-03 .2838092-02 .6276533-04 .102372-03 .1395512-03 .1982928-03
-.9728234-04 .1229852+00 .4102372-03 .7387726-04 .4395512-03 .2098215-00
-.3566362-03 .1768565-03 .5195190-03 .2988970-03 .1684452-03 .3951266-03
.4802279-03

EIGENVECTOR CORRESPONDING TO
.14121217+09 .16108075-02 .1243739-02 .2178926-02 .2002435-01 .1000000+01
.7003275-01 .1269075-00 .6911577-01 .2782782-01 .1797369-01 .4121917-04
-.1074870-02 .8662877-03 .1863928-01 .9176573-03 .5451994-02 .2827384-02
.5219921-01 .1453539-01 .1050876-01 .9176573-03 .5451994-02 .2827384-02
-.2820325-02 .5303082-02 .1963936-03 .1084277-03 .2089113-03 .3505778-03
-.1254463-01 .1094984-01 .1060090-01 .1081862-03 .8164427-04 .6564248-04
.2551145-00

REORDERED THE EIGENVALUES AND CORRESPONDINGLY THE EIGENVECTORS

EIGENVALUES ARE
.9687689+05 .9344875+06 .1780036+07 .3590998+07 .2281423+08 .2339316+06 .2344099+08 .5693842+08
.6283412+08 .6284682+08 .1412217+09 .1412217+09

FREQUENCIES ARE
.3112505+03 .9668689+03 .1334480+04 .1894966+04 .4776320+04 .4636647+04 .4841590+04 .7545755+04
1000 INTERVAL TIME 26.77 SECOND(S)

SUPPORT POINT DISPLACEMENT VECTOR
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FEASIBLE 31

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|------------------------------|-------------|-------------|-------------|-------------|--------------|--------------|-------------|
| •2966940-00                  | •5925954-00 | •7970363-00 | •1007000+01 | •2606919-00 | •5606835-00  | •7431810-00  | •5039830-00 |
| •8498171-00                  | •2061936-00 | •4231357-00 | •8131684-01 | •1573638-00 | •4480588-00  | •4223815-00  | •8225099-00 |
| •4654555-00                  | •2725983-00 | •2725983-00 | •3306693-00 | •5757145-01 | •5446358-01  | •5028565-01  | •8287037-01 |
| •8434201-01                  | •8434201-01 | •5495105-01 | •3793192-01 | •1847208-01 | •5226322-01  | •5483327-01  | •5702672-01 |
| •8407177-01                  | •5295253-01 | •5908523-01 | •1483715-01 | •1546381-01 | •1083774-01  | •7056401-03  | •2555301-02 |
| •4531696-02                  |             |             |             |             |              |              |             |
| EIGENVECTOR CORRESPONDING TO |             |             |             |             |              |              |             |
| •1021857+00                  | •2037908-00 | •3392906+07 | •3880025-00 | •5011018-00 | •1000000+01  | •1298031-00  | •4243288-00 |
| •3486954-00                  | •5720941-00 | •6235461-01 | •1195323+00 | •6080105-01 | •2343674-00  | •4910211-01  | •8476343-01 |
| •9428964-01                  | •1021113+00 | •1270304-00 | •1302658-00 | •8493730-02 | •7777855-02  | •7502897-02  | •2435749-01 |
| •2361199-01                  | •2266669-01 | •4271433-01 | •3288309-01 | •2012876-01 | •4004854-02  | •5597208-02  | •7021337-02 |
| •1676225-01                  | •1152922-01 | •4155727-02 | •5374403-01 | •3696424-01 | •1587992-01  | •1067519+00  | •1006926+00 |
| •9293419-01                  |             |             |             |             |              |              |             |
| EIGENVECTOR CORRESPONDING TO |             |             |             |             |              |              |             |
| •6175679-02                  | •1222358-01 | •1521341+08 | •1774643-02 | •7381522-04 | •1406332-03  | •7963697-00  | •6591228-02 |
| •5869743-02                  | •7292454-01 | •2126023-01 | •2422137-01 | •1614108-01 | •1017967-03  | •1997023-02  | •2719468-02 |
| •1000000+01                  | •1082732-01 | •7470718-02 | •9226532-01 | •3467485-02 | •1684857-02  | •3945713-03  | •7118147-03 |
| •1653959-02                  | •2402576-02 | •1357920-02 | •6993078-04 | •1254074-02 | •3960635-04  | •1907905-03  | •3141326-03 |
| •2080895-00                  | •1552280-02 | •2056942-00 | •1920608-01 | •1201815-03 | •1902677-01  | •6756810-03  | •1046499-02 |
| •1142120-02                  |             |             |             |             |              |              |             |
| EIGENVECTOR CORRESPONDING TO |             |             |             |             |              |              |             |
| •3389935-03                  | •5842943-03 | •1545097+08 | •4498340-03 | •1299319-02 | •2632159-02  | •7326445-01  | •2151702-02 |
| •3181518-01                  | •800431-00  | •4436668-04 | •6386811-03 | •5340927-04 | •712739-04   | •2099420-02  | •3324485-02 |
| •9278718-01                  | •201278-02  | •3668492-01 | •1000000+01 | •2311399-03 | •284927-04   | •3131420-03  | •1113097-03 |
| •3546129-04                  | •6597153-03 | •7985568-02 | •4485935-04 | •8055881-02 | •1217381-03  | •2255588-03  | •3182490-03 |
| •1909997-01                  | •6632623-05 | •1913277-01 | •2095357-00 | •2537527-03 | •2099318-00  | •3287433-03  | •1900296-03 |
| •1090597-04                  |             |             |             |             |              |              |             |
| EIGENVECTOR CORRESPONDING TO |             |             |             |             |              |              |             |
| •7563737-03                  | •1525802-02 | •1547633+09 | •2387486-02 | •1417579-02 | •2789809-02  | •2644834-02  | •2529389-02 |
| •7995951-00                  | •2905613-01 | •3372369-02 | •2738344-02 | •3914428-02 | •2754479-02  | •1730597-02  | •2827445-02 |
| •2947964-02                  | •2765344-02 | •1000000+01 | •3961055-01 | •5997432-03 | •1278130-03  | •2703175-03  | •2617340-03 |
| •1549119-04                  | •2417000-03 | •2099589-00 | •2115112-04 | •2099783-00 | •1199125-03  | •1933976-03  | •2500282-03 |
| •7551734-03                  | •1982917-03 | •9985883-03 | •8007075-02 | •2934761-04 | •17972262-02 | •12083988-03 | •1062259-03 |
| •9891745-05                  |             |             |             |             |              |              |             |
| EIGENVECTOR CORRESPONDING TO |             |             |             |             |              |              |             |
| •3580574-02                  | •7008260-02 | •4003536+08 | •3459144-03 | •114591-04  | •1941235-04  | •2765389-01  | •6643691-02 |
| •1598624-02                  | •6610313-03 | •1000000+01 | •5416790-01 | •6399195-01 | •1793935-03  | •1909153-02  | •2338292-02 |
| •1086651-01                  | •2635982-01 | •9183716-03 | •4176195-03 | •2069574-00 | •4067545-02  | •2007733-00  | •4430986-02 |
| •3631269-02                  | •1120478-03 | •4034758-03 | •105725-03  | •4946976-03 | •1879026-04  | •1557310-03  | •3040758-03 |
| •6324737-02                  | •3300901-02 | •9273735-02 | •9643819-04 | •1284597-03 | •2095601-03  | •2734302-03  | •6002604-03 |
| •1390768-02                  |             |             |             |             |              |              |             |
| EIGENVECTOR CORRESPONDING TO |             |             |             |             |              |              |             |
| •3619631-02                  | •1856387-01 | •7078187+08 | •5101526-03 | •1110874-04 | •9043105-04  | •5029075-01  | •3658984-01 |
| •2471598-02                  | •1151133-02 | •9508702-02 | •1271216-00 | •1000000+01 | •22980690-02 | •9412085-02  | •3010813-02 |
| •6464159-02                  | •6408145-01 | •2487476-03 | •2451579-03 | •3412950-01 | •1041491-01  | •4675968-01  | •1891406-00 |
| •959885-02                   | •2042479-00 | •6515597-03 | •2245368-04 | •6510902-03 | •1494847-02  | •2032778-03  | •1839267-02 |
| •1294748-01                  | •1005608-01 | •1497798-01 | •2302227-03 | •2078433-03 | •2728328-03  | •5794265-02  | •1704314-02 |
| •8606625-02                  |             |             |             |             |              |              |             |



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EIGENVECTOR CORRESPONDING TO
.5673453-03 .1100118-02 .8843343+04 .7202234+05 -.2134705-04 -.4115523-04 -.A309183-03 .1314528-01
-.1207588-02 .1281668-02 .8470800-03 -.2201081-02 .9458862-02 -.4665012-05 .1000000+01 -.3228960-02
.4947227-03 -.2809984-02 -.1356303-03 -.1550342-03 -.3077773-03 .2010488-03 .4603734-03 -.2320007-02
-.1680671-03 .2088377-02 .2355554+03 .2379340-03 -.2720151-03 .2899102-00 .2431997-03 -.2095409-00
-.3391300-03 .2450896-03 .3240128-03 -.3010434-03 .2651842-03 .2789514-03 -.3215509-02 .0208999-03
.3398788-02

EIGENVECTOR CORRESPONDING TO
.3334267-01 .6203668-01 .9732656+03 -.1219313-02 -.9629996-03 -.1802179-02 .1047680-01 .1000000+01
-.7063200-03 -.4457807-03 .6909249-02 -.1172992-01 .2785682-01 .7037678-04 -.7600504-02 .2184182-03
.2446557-01 -.5476081-02 -.1143340-02 .806907-03 .806907-03 -.2051543-02 .9871946-03 .2883612-02 -.7497899-02
-.8210714-03 .6324188-02 -.1785315-03 .3469560-04 .2444894-03 .1644730-02 -.1823727-04 .1628583-02
-.5485674-02 .5288699-02 .4712761-02 -.1582681-03 .1244056-03 .8568568-04 -.2493722-00 .024352-02
-.2586091-00

1000 INTERVAL TIME 14.17 SECOND(S)

RESULTS OF DYNAMIC ANALYSIS
XCOMEGA/2*P
.599167-00 .186148+01 .251441+01 .368397+01 .780087+01 .786154+01 .786799+01 .126547+02 .168264+02 .186501+02
.197308+02
Y COORDINATE READ FROM THE SHOCK SPECTRUM
.842287-01 .344923-01 .213173-01 .900295-02 .433169-03 .414227-03 .412263-03 .121076-04 .559494-06 .127775-06
.657879-07
MODAL PARTICIPATION COEFFICIENTS
.104753+00 .220378-01 .566690-02 .120614-02 -.969087-05 -.976962-05 .117642-05 -.378929-06 -.300008-08 -.381913-08
.389216-09

MAXIMUM DISPLACEMENT VECTOR
.188869-01 .217630-01 .306563-01 .395249-01 .873158-02 .174531-01 .111220-01 .831847-02 .174437-01 .921077-02
.230795-01 .451758-01 .683913-01 .100998+00 .263295-01 .523504-01 .230845-01 .487705-01 .723993-01 .269251-01

MAXIMUM STRESS-MEMBER 1 .436000+05 .442761+05
MAXIMUM STRESS-MEMBER 2 .442691+05 .436653+05
MAXIMUM STRESS-MEMBER 3 .356499+05 .359499+05
MAXIMUM STRESS-MEMBER 4 .358657+05 .355129+05
MAXIMUM STRESS-MEMBER 5 .349632+05 .353610+05
MAXIMUM STRESS-MEMBER 6 .353628+05 .349611+05
MAXIMUM STRESS-MEMBER 7 .182372+05 .184046+05
MAXIMUM STRESS-MEMBER 8 .183488+05 .191377+05
MAXIMUM STRESS-MEMBER 9 .296797+05 .301833+05
MAXIMUM STRESS-MEMBER 10 .302029+05 .297297+05
MAXIMUM STRESS-MEMBER 11 .374044+05 .379927+05

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MAXIMUM STRESS-MEMBER 12 .351219+05 .374233+05  
 MAXIMUM STRESS-MEMBER 13 .375545+05 .333213+05  
 MAXIMUM STRESS-MEMBER 14 .363673+05 .376410+05

RESULTS FROM NOVEANDCHECK

\*\*\*\*\* PREVIOUS DESIGN \*\*\*\*\*

.203128+01 .276526+01 .296872+01 .203128+01 .203128+01 .203128+01 .203128+01

ALPHA IS .421946-00

\*\*\*\*\* CURRENT DESIGN \*\*\*\*\*

.160944+01 .300399+01 .339056+01 .160944+01 .160944+01 .160944+01 .160944+01  
 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00  
 .153197+01 .153197+01 .153197+01 .153197+01 .153197+01 .153197+01 .153197+01  
 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00

WEIGHT IS .204185+02

NUMBER OF ITERATIONS

46 46 43 42 30 6 8 20 30 17 6

EIGENVALUES ARE

.790918+05 .755727+06 .136553+07 .326033+07 .962145+07 .971098+07 .972355+07 .255455+08  
 .624587+08 .784186+08 .115626+09 .115626+09 .115626+09 .115626+09 .115626+09 .115626+09

FREQUENCIES ARE

.281232+03 .869325+03 .116856+04 .180564+04 .310184+04 .311624+04 .311626+04 .505426+04  
 .790308+04 .865542+04 .107529+05 .107529+05 .107529+05 .107529+05 .107529+05 .107529+05

EIGENVECTOR CORRESPONDING TO

.790918+05 .755727+06 .136553+07 .326033+07 .962145+07 .971098+07 .972355+07 .255455+08  
 .624587+08 .784186+08 .115626+09 .115626+09 .115626+09 .115626+09 .115626+09 .115626+09  
 .160944+01 .300399+01 .339056+01 .160944+01 .160944+01 .160944+01 .160944+01  
 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00  
 .153197+01 .153197+01 .153197+01 .153197+01 .153197+01 .153197+01 .153197+01 .153197+01  
 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00 .164344+00

EIGENVECTOR CORRESPONDING TO

.139471+06 .278799+06 .464015+06 .464015+06 .464015+06 .464015+06 .464015+06 .464015+06  
 .390351+06 .808126+01 .383180+06 .383180+06 .383180+06 .383180+06 .383180+06 .383180+06  
 .378640+06 .696995+06 .371376+06 .371376+06 .371376+06 .371376+06 .371376+06 .371376+06  
 .637880+01 .639746+01 .623821+01 .623821+01 .623821+01 .623821+01 .623821+01 .623821+01  
 .227017+01 .221240+05 .221240+05 .221240+05 .221240+05 .221240+05 .221240+05 .221240+05  
 .140701+01 .140701+01 .140701+01 .140701+01 .140701+01 .140701+01 .140701+01 .140701+01

Step Length in the same feasible direction, 5.

New Design Bounded.



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--.394007-02      .327545-02      .2+14543-02      -.1334124-03      .7321542-04      .1267734-03      -.2556361-00      .3016150-02
.2603306-00

EIGENVECTOR CORRESPONDING TO
--.121367-01      -.229974-01      .7+41860+04      .612+844-03      --.5156683-04      --.1006749-03      .7934820-01      .6751460-01
--.321287-02      .1100602-02      -.1168918-01      .2191054-00      .1000000+01      -.2077204-01      -.7736818-02      -.3718351-02
.4531852-02      .111372+00      -.1016247+03      -.5382441-03      .3478928-01      -.2077204-01      -.5009969-01      -.1741844-00
.1840208-01      .2002775-00      .7734157-03      .1556873-04      -.7907522-03      -.1014445-02      .2771720-03      .1475311-02
.2724034-01      -.2572807-01      -.1171671-01      -.3634288-03      .4093477-03      .1012415-03      -.2212376-01      -.2131062-02
.1487702-01

EIGENVECTOR CORRESPONDING TO
--.2625831-03      -.156261+09      .1156261+09      .2203002-04      .9939044-06      .2250089-05      -.133417-02      -.5697433-03
--.1582959-02      .1630026-02      .5926515-03      -.2372161-02      .7441591-02      -.1481761-04      .1000000+01      -.3660999-02
--.1233632-03      -.3168859-02      -.1013140-03      -.5572415-04      -.3088828-03      .2594242-03      .3777082-03      -.1958991-02
--.1883442-03      .1709956-02      .4644728-03      -.5345024-03      -.1937335-03      .2099159-00      .2753302-03      -.2094949-00
--.2398660-04      .1801944-03      .2251370-03      -.6744608-03      .4339427-03      .2119885-03      .2709160-04      -.2360487-04
--.1560561-04

1000 INTERVAL TIME 10.70 SECONDS

RESULTS OF DYNAMIC ANALYSIS
XC=OMEGA/2*P
.562466-00 .173665+01 .233712+01 .361128+01 .620369+01 .623249+01 .623652+01 .101085+02 .158062+02 .177109+02
.215059+02
Y COORDINATE READ FROM THE SHOCK SPECTRUM
.814806-01 .377604-01 .242929-01 .949840-02 .140565-02 .137613-02 .137205-02 .790734-04 .118671-05 .291540-06
.177822-07
MODAL PARTICIPATION COEFFICIENTS
.101846+00 .248562-01 .389683-02 .112787-02 -.286471-04 -.286679-04 .315134-05 -.219851-05 -.450183-08 -.261395-08
.569198-09

MAXIMUM DISPLACEMENT VECTOR
.114040-01 .227977-01 .300793-01 .373398-01 .639886-02 .127914-01 .117717-01 .911226-02 .170899-01 .759463-02
.238802-01 .467733-01 .673832-01 .991804-01 .273866-01 .545654-01 .242148-01 .506775-01 .715251-01 .289177-01

MAXIMUM STRESS-MEMBER 1 6456450+05 662057+05
MAXIMUM STRESS-MEMBER 2 662137+05 656780+05
MAXIMUM STRESS-MEMBER 3 .292021+05 .293806+05
MAXIMUM STRESS-MEMBER 4 .292939+05 .290878+05
MAXIMUM STRESS-MEMBER 5 .256451+05 .258098+05
MAXIMUM STRESS-MEMBER 6 .258757+05 .256128+05
MAXIMUM STRESS-MEMBER 7 .204541+05 .208905+05
MAXIMUM STRESS-MEMBER 8 .207849+05 .204103+05
MAXIMUM STRESS-MEMBER 9 .328943+05 .336055+05
MAXIMUM STRESS-MEMBER 10 .337281+05 .329872+05

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*Output picked up at a later stage of the run.*

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.1616425-J2
EIGENVECTOR CORRESPONDING TO
.2683307-02 .4100614-02 .1709718+05 .6704511-03 .1000000+01
.3887671-04 .1938363-03 .2844463-02 .2663887-05 .5056468-04
.2757204-02 .6294601-03 .7350450-03 .1701099-03 .1925028-03
.9594950-04 .1354948-03 .1466384-03 .1816507-04 .1584525-03
.5336796-03 .2508199-03 .8152501-03 .3082412-04 .6534716-04
.2616876-00
EIGENVECTOR CORRESPONDING TO
.2018431-01 .3821978-01 .1146287-02 .2106278-05 .1199320+00
.2432677-02 .1367701-02 .3438818-00 .3123622-02 .5497816-01
.9634759-02 .1753131-00 .2577606-03 .5038473-01 .6115058-01
.2536664-01 .1949112-00 .5541850-03 .1339638-03 .2594788-03
.5334324-01 .5217185-01 .4521817-02 .4981616-03 .1096131-01
.6950416-02
EIGENVECTOR CORRESPONDING TO
.6979437-03 .1290419-02 .6862129+04 .6310966-06 .4099174-02
.1228275-02 .1220899-02 .3868401-02 .1170634-03 .1060000+01
.1501688-03 .4243685-02 .1044243-03 .1145520-01 .1230077-02
.8434405-03 .7981854-02 .5398317-03 .8992126-04 .2668992-04
.1839122-02 .1977017-02 .2714493-03 .4383574-03 .3908850-03
.2264453-03
1000 INTERVAL TIME 4.23 SECOND(S)
RESULTS OF DYNAMIC ANALYSIS
X=0.00000000
Y=0.00000000
Z=0.00000000
V COORDINATE READ FROM THE SHOCK SPECTRUM
.760806-01 .427146-01 .290541-01 .128591-01 .379682-02 .294614-02 .269323-02 .480417-03 .306607-03 .831928-06
.677065-06
MODAL PARTICIPATION COEFFICIENTS
.951313-01 .278593-01 .459912-02 .169490-02 .15812-04 .663635-04 .461626-04 .125418-04 .547091-06 .111453-07
.193647-07
MAXIMUM DISPLACEMENT VECTOR
.121638-01 .283648-01 .308147-01 .372473-01 .693066-02 .138593-01 .130556-01 .105496-01 .177862-01 .865218-02
.229169-01 .433328-01 .621456-01 .942512-01 .278799-01 .533913-01 .230319-01 .493680-01 .697533-01 .299563-01
MAXIMUM STRESS-MEMBER 1 487846+03 498757+05 488491+05
MAXIMUM STRESS-MEMBER 2 499283+05 259463+05
MAXIMUM STRESS-MEMBER 3 250881+05 259463+05
MAXIMUM STRESS-MEMBER 4 259344+05 257781+05
MAXIMUM STRESS-MEMBER 5 277334+05 260852+05
MAXIMUM STRESS-MEMBER 6 260733+05 277965+05

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**BOUNDED BEHAVIOUR CONSTRAINTS ARE**

[illegible]

### FEASIBLE DIRECTION: VECTOR, S. IS

RESULTS FROM MOVEA+CHECK

$$-121062+01 \cdot 208022+01 \cdot 261178+01 \cdot 830655-00 \cdot 134761+01 \cdot 125183+01 \cdot 137342+01$$

\*\*\*\*\* CURRENT DESIGN \*\*\*\*\*

ADJUSTED AND SI 7402520

10783301 .45531-00 .159399-00 .22060101 .38000901 .15110201 .539091-00

*An intermediate infeasible design.*

ADJUSTED ALPHA IS .225822+01

\*\*\*\*\* CURRENT DESIGN \*\*\*\*\*

.182578+01 .630000-00 .353559-00 .209712+01 .360583+01 .149049+01 .605887-00

ADJUSTED ALPHA IS .198178+01

\*\*\*\*\* CURRENT DESIGN \*\*\*\*\*

.175097+01 .906441-00 .630000-00 .194208+01 .332939+01 .146128+01 .699443-00

\*\*\*\*\* SOUNDED SIDE CONSTRAINTS ARE \*\*\*\*\*

10

NUMBER OF SOUNDED SIDE CONSTRAINTS ARE 1

.21149951-00 .21149951-00 .29602814-01 .29602814-01 .10066722-01  
 .10066722-01 .28841124-00 .28841124-00 .14505920+01 .14505920+01  
 .12310819-00 .12310819+00 .13735483-01 .13735483-01

WEIGHT IS .156652+02

NUMBER OF ITERATIONS

61 112 219 90 54 12 251 136 102 51 83

EIGENVALUES ARE

.4700520+05 .4912130+06 .6540416+06 .1706476+07 .4081486+07 .8018117+07 .1869965+07 .8325527+07  
 .3009304+08 .3827698+08 .3643894+08 .1306321+04 .2020269+04 .2831628+04 .1367467+04 .2685399+04

FREQUENCIES ARE

.2168063+03 .7008659+03 .8087284+03 .1306321+04 .2020269+04 .2831628+04 .1367467+04 .2685399+04  
 .5485712+04 .6186439+04 .6036467+04 .4700520+05 .1888414-00 .1267410-00 .1267410-00 .1267410-00  
 .3144355-01 .1267410-00 .1267410-00 .1267410-00 .1267410-00 .1267410-00 .1267410-00 .1267410-00  
 .1232206+00 .2623168+00 .6414980+00 .1430523-00 .1430523-00 .1430523-00 .1430523-00 .1430523-00  
 .5045551-01 .4777102-01 .4326635-01 .4361495-01 .4361495-01 .4361495-01 .4361495-01 .4361495-01  
 .1435863-01 .1408085-01 .1408085-01 .1408085-01 .1408085-01 .1408085-01 .1408085-01 .1408085-01  
 .2644333-01 .2644333-01 .2644333-01 .2644333-01 .2644333-01 .2644333-01 .2644333-01 .2644333-01

EIGENVECTOR CORRESPONDING TO

.1705550-00 .3412445-00 .1373147+01 .2404615+01 .7971661-00 .1594569+01 .1719296-00 .9749355-00  
 .2074122+01 .6852855-00 .2908444-00 .5801482-00 .3209324-00 .1646026-01 .1646026-01 .1646026-01  
 .2349151-00 .7900558-00 .4104675-00 .3411557-00 .3870688-01 .3870688-01 .3870688-01 .3870688-01  
 .3128431-01 .4194765-01 .4474263-01 .6671725-01 .6671725-01 .6671725-01 .6671725-01 .6671725-01  
 .1232612-01 .1232612-01 .1232612-01 .1232612-01 .1232612-01 .1232612-01 .1232612-01 .1232612-01  
 .1025151+00 .1025151+00 .1025151+00 .1025151+00 .1025151+00 .1025151+00 .1025151+00 .1025151+00

EIGENVECTOR CORRESPONDING TO

.5540416+06



|                              |              |               |              |              |              |              |              |
|------------------------------|--------------|---------------|--------------|--------------|--------------|--------------|--------------|
| -.3312934-00                 | -.662743-00  | -.3349832-00  | -.5967704-02 | -.8645619-00 | -.1726110+01 | -.3425262-00 | -.1201663+01 |
| -.7468041-00                 | -.5672981-00 | .1132611+01   | .8203702+01  | .1077918+01  | -.4624935-00 | .1532039+01  | .2767240+01  |
| .1116762+01                  | .2486480+01  | .1000000+01   | .1757489+01  | -.1529585-00 | -.1874403-00 | .1398940-00  | .1363776-00  |
| .1773761-00                  | .2229313-00  | .1566957-00   | .1888654-00  | .2142342-00  | -.1504588-00 | -.375601-00  | -.2178279-00 |
| -.1140637+00                 | -.1112480+00 | -.1075067+00  | -.1368888-00 | -.5901701-01 | .1713209-01  | .9064094-01  | .8876102-01  |
| .8647830-01                  |              |               |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |               |              |              |              |              |              |
| -.3003383-01                 | -.5996736-01 | .1706476+07   | -.1563296-00 | .8874582-01  | .1772412-00  | -.3211296-01 | .6011688-01  |
| .1136732-01                  | .8519717-00  | .4813264-01   | .8764984-01  | -.5181575-02 | -.9856553-01 | .1769133-01  | .9191411-01  |
| .9649904-01                  | .8985172-01  | -.4945548-02  | .1000000+01  | -.6711423-02 | -.5881510-02 | .4893800-02  | .1199311-01  |
| .1280038-01                  | .1286128-01  | -.3228589-02  | -.3142149-02 | -.2718059-02 | -.1126291-01 | -.8478097-02 | -.1228542-03 |
| -.6059722-02                 | -.5550875-02 | -.4921657-02  | -.2083697-00 | -.1066997-01 | .1929845-00  | -.2011531-01 | -.1979539-01 |
| -.1930967-01                 |              |               |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |               |              |              |              |              |              |
| .733837-04                   | .1450306-03  | .4081486+07   | -.7314100-04 | .4189991-04  | .8193805-04  | .4601606-04  | .1531991-03  |
| .8833449-03                  | .1253618-03  | -.2622130-03  | -.5373438-03 | -.6028773-03 | -.4971150-04 | .1000000+01  | -.6821312-03 |
| -.2429263-03                 | -.6099088-03 | -.1874413-02  | -.2267829-03 | .2738713-04  | .3943610-04  | .3836636-04  | .3118480-04  |
| -.6158163-04                 | -.8518477-04 | -.2308127-03  | -.5278935-04 | .2187450-03  | .2096951-00  | .9773950-04  | -.2095096-00 |
| .1769200-04                  | .2487208-04  | .3105946-04   | -.1660170-03 | .2105354-03  | -.7622800-04 | -.8261922-05 | .5802775-05  |
| .1692485-04                  |              |               |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |               |              |              |              |              |              |
| .3948212-03                  | .8530388-03  | .8018117+07   | .1740290-02  | .8160388-03  | .1562072-02  | .6774777-03  | .1431384-02  |
| -.7998591-00                 | .3784563-02  | -.1344142-02  | -.1626036-02 | -.2389001-02 | -.2045919-02 | .5633003-02  | -.1750020-02 |
| -.1114500-02                 | -.1705648-02 | .1000000+01   | .2843536-02  | .2195371-03  | .1160217-03  | -.5314575-05 | .2264705-03  |
| .5333553-04                  | -.1326978-03 | .2097998-00   | .1498242-04  | -.2087749-00 | .1650780-02  | .1376089-03  | -.1458245-02 |
| .1622647-03                  | .8719716-04  | .1505162-04   | -.8237224-03 | .5589556-04  | .8370413-03  | -.1215720-03 | -.5840654-04 |
| .7921462-06                  |              |               |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |               |              |              |              |              |              |
| .1099919-02                  | .2182705-02  | .1869965+07   | .5013839-02  | -.3091569-02 | -.6173660-02 | .1122477-02  | -.2030726-02 |
| .7587038-03                  | .7985581-00  | -.2082014-02  | -.3697121-02 | .1670868-02  | .3174555-02  | -.8407079-03 | -.4082696-02 |
| -.1874211-02                 | -.3887978-02 | -.2137274-02  | .1000000+01  | .2921190-03  | .2525345-03  | .1894835-03  | -.8381891-03 |
| .4610756-03                  | -.2258907-04 | -.2943709-03  | .8058676-04  | .4084758-03  | .5288464-03  | .3690133-03  | .6043872-04  |
| .2336903-03                  | .2201681-03  | .2106823-03   | -.2096325-00 | .4710760-03  | .2102480-00  | .7012909-03  | .6969439-03  |
| .6872174-03                  |              |               |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |               |              |              |              |              |              |
| -.5948881-03                 | -.1070903-02 | .8325527+07   | .8858410-04  | .2352983-04  | .4641198-04  | -.8053386-03 | -.3047402-03 |
| -.5045894-03                 | .1819260-02  | .4957460-02   | .3916836-02  | .1000000+01  | -.9781649-03 | .4994498-03  | -.9001989-03 |
| .2183396-02                  | .1421047-02  | -.1019326-03  | -.1237084-02 | .3635905-03  | .3890979-03  | .3589151-03  | -.2093375-00 |
| .7428810-03                  | .2051791-00  | .3961851-03   | -.1455799-04 | -.3394810-03 | .2049360-04  | -.4426756-04 | -.3953767-03 |
| .2892039-04                  | -.3105839-03 | -.21309238-03 | -.1045988-03 | .7950460-04  | -.4175321-03 | -.1427902-03 | .7969731-04  |
| .3525418-04                  |              |               |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |               |              |              |              |              |              |
| -.7729158-02                 | -.1512060-01 | .3009304+04   | .3443367-03  | .5192729-04  | .2311049-05  | -.4909036-01 | -.1332147-01 |
| -.8766099-03                 | .7687716-03  | .1000000+01   | .3777886-01  | -.1858246-01 | .4540731-04  | -.2878454-01 | -.2080827-02 |
| .7108424-01                  | .7105792-01  | .712578-03    | -.1315755-03 | -.2078633-00 | -.2850987-02 | .2033966-00  | .9422880-02  |
| .3249900-02                  | -.6338565-02 | .2514820-03   | -.1418823-03 | -.2605894-03 | .1126603-03  | .3483017-03  | -.2431415-03 |
| -.1278627-01                 | -.2212285-02 | .9133110-02   | -.1971821-02 | .1900279-02  | -.1717223-02 | .3267987-03  | -.1337062-02 |
| -.2701384-02                 |              |               |              |              |              |              |              |

EIGENVECTOR CORRESPONDING TO

|              |              |              |              |              |               |
|--------------|--------------|--------------|--------------|--------------|---------------|
| .3627698+08  | .1623077-02  | .7524789-04  | .1193052-03  | -.8033447-00 | -.9342832-01  |
| -.505429-01  | -.5005254-01 | -.3518244-01 | -.3518244-01 | .3725868-03  | -.4932330-02  |
| -.1708295-02 | -.3216950-00 | .7275616-01  | -.8800313-02 | -.4540358-01 | .1952505-01   |
| .1000000+01  | .5615625-01  | .7560857-03  | -.6291397-03 | .1315382-02  | -.1928718-02  |
| .1292991-01  | -.1552707-01 | .5823125-03  | -.1057045-02 | .3782685-02  | -.86355215-02 |
| -.2044886-00 | -.8650060-02 | .1907939-00  |              |              |               |
| -.1945686-01 |              |              |              |              |               |

EIGENVECTOR CORRESPONDING TO

|              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|
| .3643894+08  | -.2691491-04 | .4115882-03  | .7962669-03  | .1147433-02  | -.1504707-02 |
| -.151710-03  | -.3268015-04 | -.5742230-02 | -.1358241-04 | -.1621003-02 | .9821522-03  |
| .6131233-03  | -.2503786-03 | .3604290-03  | -.1296954-02 | -.1318883-02 | -.3925993-03 |
| -.1180803-02 | .9144078-03  | .1003685-04  | -.1166677-03 | .2306546-02  | -.2422219-02 |
| .6053466-04  | -.3245525-04 | -.3951293-04 | -.1000000+01 | .3374516-03  | -.3118200-04 |
| .2991839-03  | -.3951293-04 |              |              |              |              |
| -.5703753-03 |              |              |              |              |              |

# REORDERED THE EIGENVALUES AND CORRESPONDINGLY THE EIGENVECTORS

## EIGENVALUES ARE

|             |             |             |             |             |             |             |             |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| .4700520+05 | .4912130+06 | .0540416+06 | .1706476+07 | .1869965+07 | .4081486+07 | .8018117+07 | .8325527+07 |
| .3009304+03 | .3643894+08 | .3827698+08 |             |             |             |             |             |

## FREQUENCIES ARE

|             |               |             |             |             |             |             |              |
|-------------|---------------|-------------|-------------|-------------|-------------|-------------|--------------|
| .21e8656+03 | .700659+03    | .8057284+03 | .1306321+04 | .1367467+04 | .2020269+04 | .2931523+04 | .21.95390+04 |
| .54e5712+14 | .603467+04    | .618633+04  |             |             |             |             |              |
| 1000        | INTERVAL TIME | 37.50       |             |             |             |             |              |

## RESULTS OF DYNAMIC ANALYSIS

XCRO-EGA/2AP

.433614+00 .140173+01 .151746+01 .261264+01 .273493+01 .404054+01 .566326+01 .577080+01 .109714+02 .120729+02

.123737+02

Y COORDINATE READ FROM THE SHOCK SPECTRUM

.654135-01 .454035-01 .412884-01 .192896-01 .181196-01 .592237-02 .209344+02 .193392-02 .418636-04 .165895-04

.145938-04

VCF - PARTICIPATION COEFFICIENTS

.023745-01 .163141-01 .293701-01 .329979-03 .226854-03 .812675-04 .608674-05 .639319-05 .142902-05 .857527-02

.340332-06

## MAXIMUM DISPLACEMENT VECTOR

.963060-02 .173701-01 .247298-01 .320737-01 .192247-01 .384236-01 .880047-02 .193191-01 .201613-01 .205403-01

.206736-01 .412720-01 .594566-01 .800448-01 .313270-01 .512271-01 .207704-01 .462596-01 .597332-01 .340132-01

MAXIMUM STRESS MEMBER 1 .347917+05 .351550+05

MAXIMUM STRESS MEMBER 2 .352054+05 .348493+05

MAXIMUM STRESS MEMBER 3 .528604+05 .546902+05

MAXIMUM STRESS MEMBER 4 .548152+05 .330773+05

|                          |                   |                   |
|--------------------------|-------------------|-------------------|
| MAXIMUM STRESS-MEMBER 5  | <u>.773983+05</u> | <u>.794223+05</u> |
| MAXIMUM STRESS-MEMBER 6  | <u>.797509+05</u> | <u>.771414+05</u> |
| MAXIMUM STRESS-MEMBER 7  | <u>.250429+05</u> | <u>.251699+05</u> |
| MAXIMUM STRESS-MEMBER 8  | <u>.250935+05</u> | <u>.249677+05</u> |
| MAXIMUM STRESS-MEMBER 9  | <u>.235605+05</u> | <u>.236083+05</u> |
| MAXIMUM STRESS-MEMBER 10 | <u>.238276+05</u> | <u>.236630+05</u> |
| MAXIMUM STRESS-MEMBER 11 | <u>.230372+05</u> | <u>.246261+05</u> |
| MAXIMUM STRESS-MEMBER 12 | <u>.246027+05</u> | <u>.228301+05</u> |
| MAXIMUM STRESS-MEMBER 13 | <u>.517792+05</u> | <u>.551337+05</u> |
| MAXIMUM STRESS-MEMBER 14 | <u>.554101+05</u> | <u>.514406+05</u> |

25 }  
26 }  
27 }  
28 }  
29 }  
30 }  
31 }  
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41 }  
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43 }  
44 }  
45 }  
46 }  
47 }  
48 }  
49 }

Violated constraints.

INDEX OF WORST VIOLATED BEHAVIOUR CONSTRAINT IS 31 AND ITS VALUE IS .595018-00  
VALUE OF SAVE CONSTRAINT AT THE OLD DESIGN POINT IS -.438524-00  
C00000 — slope set to zero; linear interpolation executed. Previous Bounded Design.  
.121462+01 .253922+01 .251175+01 .330655-00 .134761+01 .125183+01 .137342+01

ADJUSTED ALPHA IS .744981-00

S VECTOR

.270640-00-.100000+01-.100000+01 .560822-00 .100000+01 .105688+00-.339886-00

DUAL

.141524+01 .214324+01 .126686+01 .124846+01 .209259+01 .133056+01 .112022+01 — New Design Bounded.  
.1121674+00 .11210574+00 .38745173-00 .38745173-00 .85621092-00  
.25621092-00 .75905525-01 .75905525-01 .36066385-00 .36066385-00  
.93027616-01 .93027616-01 .55643281-01 .55643281-01

WEIGHT IS .101548+02

NUMBER OF ITERATIONS

|            |            |            |            |            |            |            |            |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 6053207005 | 6697030006 | 0015631007 | 2159148007 | 6653811007 | 1597189008 | 1991458008 | 3508515008 |
| 4721610007 | 9222990008 | 3801598009 |            |            |            |            |            |

|            |            |            |            |            |            |            |            |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 2460327+03 | 213538+03  | 1007785+04 | 1469404+04 | 2579498+04 | 3996485+04 | 4462576+04 | 5923272+04 |
| 2172927+34 | 9673640+04 | 5195710+04 |            |            |            |            |            |

|             |             |             |              |             |             |              |             |
|-------------|-------------|-------------|--------------|-------------|-------------|--------------|-------------|
| -7857323-01 | -1571942-00 | -1903915-00 | -2236030-00  | -5754228-01 | -1151807+00 | -8037835-01  | -2315532-01 |
| -4853115-01 | -5962905-01 | -1485794-00 | -2966671-00  | -6479811-00 | -1000000+01 | -1697857-00  | -3374110-00 |
| -1506282-00 | -3171101-00 | -6615689-00 | -31712284-00 | -1379014-00 | -1975568-01 | -1972901-00  | -4683103-00 |
| -4889162-01 | -4695033-01 | -3929470-01 | -3920732-01  | -3638394-01 | -2227151-01 | -2249600-01  | -2275738-01 |
| -1763825-01 | -1717036-01 | -1670599-01 | -1914731-01  | -1738281-01 | -1681194-01 | -82223614-01 | -2263638-01 |
| -2331582-01 |             |             |              |             |             |              |             |

|            |           |           |           |           |           |           |            |
|------------|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| 1469120000 | 293731600 | 647165300 | 100000000 | 866021600 | 173137600 | 136925900 | 240317300  |
| 559755900  | 143166200 | 526413600 | 101343900 | 347387000 | 333422800 | 406570600 | 157269300  |
| 520956500  | 12939700  | 657802900 | 857813300 | 716946800 | 761724400 | 882402000 | 882402000  |
| 931836500  | 911624000 | 110808600 | 105555000 | 954940400 | 100294600 | 954980300 | 103955300  |
| 341230000  | 318213400 | 286947300 | 842263400 | 710358300 | 519658200 | 861592600 | 9790942100 |

|           |           |           |           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 13836700  | 123550701 | 516937501 | 210103201 | 681892000 | 130283701 | 669885100 | 131009201 |
| 13836700  | 137922400 | 72394500  | 138550301 | 319541600 | 771985900 | 682629100 | 130360601 |
| 13836700  | 134742600 | 134742600 | 100000001 | 935345801 | 932866901 | 35141001  | 143511500 |
| 143979100 | 146696000 | 100081100 | 600767301 | 953662802 | 809862801 | 875773201 | 925442801 |
| 109395700 | 157334701 | 320371101 | 799831501 | 133557301 | 13557301  | 173784201 | 565057302 |
| 550291102 |           |           |           |           |           |           |           |

[illegible][illegible]

6046E16E1 0 00000000 00000000



```

.3508515+08      .3801598+08      .9222990+08
FREQUENCIES ARE

.2460327+03      .8183538+03      .1007785+04      .2579498+04      .3996485+04      .4462576+04
.5923272+04      .6165710+04      .9603640+04      .2172927+04
1000  INTERVAL TIME      18.53      SECOND(S)

RESULTS OF DYNAMIC ANALYSIS
XC=OMEGA/2*P
.492065-00 .163671+01 .201557+01 .293861+01 .434585+01 .515900+01 .799297+01 .892515+01 .118465+02 .123314+02
.192073+02
Y COORDINATE READ FROM THE SHOCK SPECTRUM
.752400-01 .407067-01 .307894-01 .155917-01 .552752-02 .303572-02 .375986-03 .189148-03 .219650-04 .153650-04
.967670-07
MODAL PARTICIPATION COEFFICIENTS
.933653-01 -.269639-01 -.595282-02 .185332-02 -.905844-04 .670074+05 -.108206-04 -.501705-05 .512443-06 -.684590-07
.248312-11

MAXIMUM DISPLACEMENT VECTOR
.102762-01 .205522-01 .294417-01 .383188-01 .830599-02 .166144-01 .105227-01 .872475-02 .165264-01 .104995-01
.219558-01 .428893-01 .612096-01 .924712-01 .274683-01 .580135-01 .221116-01 .484550-01 .674105-01 .311567-01
MAXIMUM STRESS-MEMBER 1      .411996+05      .416491+05
MAXIMUM STRESS-MEMBER 2      .416579+05      .412373+05
MAXIMUM STRESS-MEMBER 3      .356389+05      .358979+05
MAXIMUM STRESS-MEMBER 4      .357238+05      .355623+05
MAXIMUM STRESS-MEMBER 5      .332958+05      .338469+05
MAXIMUM STRESS-MEMBER 6      .339005+05      .333477+05
MAXIMUM STRESS-MEMBER 7      .292031+05      .296848+05
MAXIMUM STRESS-MEMBER 8      .297496+05      .292106+05
MAXIMUM STRESS-MEMBER 9      .280134+05      .287767+05
MAXIMUM STRESS-MEMBER 10     .287585+05      .280574+05
MAXIMUM STRESS-MEMBER 11     .418736+05      .432176+05
MAXIMUM STRESS-MEMBER 12     .430939+05      .418234+05
MAXIMUM STRESS-MEMBER 13     .428821+05      .451824+05
MAXIMUM STRESS-MEMBER 14     .452130+05      .430234+05

BOUNDED BEHAVIOUR CONSTRAINTS ARE
46 47

```

NUMBER OF BOUNDED BEHAVIOUR CONSTRAINTS ARE 2  
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
 .260175+04 .251514+04 .547468+04 .104995+04 .288576+03 .286336+04 .164129+05 .268749+04 .260024+04 .554865+04  
 .112728+04 .398333+03 .289822+04 .164764+05

FEASIBLE DIRECTION VECTOR'S IS

-.100000+01-.100000+01-.100000+01-.100000+01-.100000+01-.100000+01 .997791-00

SIMA IS .173749+01

RESULTS FROM MOVEANDCHECK

\*\*\*\*\* PREVIOUS DESIGN \*\*\*\*\*

.141624+01 .214324+01 .186680+01 .124046+01 .209259+01 .133056+01 .112022+01

ALPHA IS .240848-00

\*\*\*\*\* CURRENT DESIGN \*\*\*\*\*

.117539+01 .190239+01 .162595+01 .100761+01 .185174+01 .108971+01 .136053+01  
 .64230593-01 .64230593-01 .27111855-00 .27111855-00 .16944261-00  
 .16944261-00 .40568779-01 .40568779-01 .25007231-00 .25007231-00  
 .51243641-01 .51243641-01 .99432064-01 .99432064-01

WEIGHT IS .145393+02

NUMBER OF ITERATIONS

31 42 34 33 5 7 34 14 16 3 1

EIGENVALUES ARE

.5329327+05 .6091048+06 .9350585+06 .1918367+07 .4473944+07 .6947193+07 .1255957+08 .1382656+08  
 .2495575+08 .2663599+08 .1357068+09 .1357068+09

FREQUENCIES ARE

.2308533+03 .7804517+03 .9669842+03 .1385051+04 .2115170+04 .2635753+04 .3543949+04 .3719411+04  
 .4995573+04 .5161006+04 .1164933+05 .1164933+05

EIGENVECTOR CORRESPONDING TO

-.7792070-01 -.1558450-00 -.1887980-00 -.2217363-00 .6137781-01 .1227288+00 -.7796841-01 -.1698936-01  
 -.5301826-01 .6390828-01 .1404008-00 .2765270-00 .6399916-00 .1000000+01 .1559514-00 .3128932-00  
 .1383528-00 .2947241-00 .6608829-00 .1596173-00 -.1840522-01 -.1802600-01 -.4859779-01 -.4859779-01  
 -.4924918-01 -.4785396-01 -.3803135-01 -.3916972-01 -.4008057-01 -.2095895-01 -.2084185-01 -.2075274-01  
 -.1632129-01 -.1630938-01 -.1628250-01 -.1735627-01 -.1671200-01 -.1605500-01 -.2308157-01 -.2324622-01  
 -.2326751-01

EIGENVECTOR CORRESPONDING TO

.6091048+06

New Design Bounded.

1332550-00 .2665442-00 .5194819-00 .7722139-00 .1189447+00 .2378502-00 .1243657+00 .2676003-00  
 4883102-00 .1553440-00 .3101763-00 .5901554-00 .2132151+00 .1741118-00 .5142456-00 .1000000+01  
 3065031-00 .7953332-00 .3065031-00 .5458186-00 .4218560-01 .3572339-01 .4971136-01 .1000000+01  
 5110743-01 .5195482-01 .6796070-01 .6600375-01 .5982886-01 .6340306-01 .6680274-01 .6948952-01  
 1762727-01 .1546913-01 .1289324-01 .5760700-01 .4882394-01 .3818970-01 .1829195-02 .2482774-02  
 6411062-02

## EIGENVECTOR CORRESPONDING TO

3673870-00 .9350585+06 .1006288+01 .1236638+01 .2923707-00 .5843765-00 .4297453-00 .7076017-00  
 1057653-01 .10589598-00 .5441265-00 .9960249-00 .2458253-00 .5216301-00 .5205787-00 .1000000+01  
 5511296-00 .9983962-00 .4221519-00 .5823532-00 .7581101-01 .6696437-01 .5597598-01 .9808982-01  
 1015688+00 .1028894+00 .7757572-01 .4229008-01 .2742823-03 .6173433-01 .711162-01 .7073195-01  
 7630914-01 .46626507-01 .5373777-01 .3029395-01 .2213608-01 .1228770-01 .7637724-02 .1603316-01  
 2325353-01

## EIGENVECTOR CORRESPONDING TO

9601625-01 .1918367+07 .2852913-00 .3740676-00 .506289-00 .1000000+01 .1073933+00 .4564574-00  
 2533955-01 .5965503-00 .4075643-01 .8519529-01 .6866041-01 .2101628-00 .1369592-01 .1330168-01  
 5420299-01 .4916807-01 .2383624-00 .1262050-00 .5263426-02 .5990312-02 .5905588-02 .2084392-01  
 1973687-01 .1814716-01 .6396211-01 .3691146-01 .4216975-02 .7357455-03 .1039665-02 .2274052-02  
 1220305-01 .9920566-02 .6940395-02 .5723572-01 .3414078-01 .6343191-02 .1137233+00 .9993411-01  
 3516741-01

## EIGENVECTOR CORRESPONDING TO

486111-03 .9105086-03 .4473944+07 .1263895-02 .8924803-03 .1759771-02 .9364430-03 .1742665-02  
 7997818-00 .3964093-02 .1428673-02 .1655970-02 .2128465-02 .1483098-02 .1273611-02 .1758710-02  
 1424079-02 .1705631-02 .1000000+01 .2975081-02 .2286847-03 .1108713-03 .5103295-05 .1103777-03  
 1211746-04 .3299880-00 .2096980-00 .2599987-04 .2097502-00 .800013-04 .117117-03 .1920441-04  
 2228628-03 .1024870-03 .3415081-04 .7848348-03 .1585669-04 .8102867-03 .1656424-03 .16967047-04  
 3955301-04

## EIGENVECTOR CORRESPONDING TO

1578863-03 .2901283-03 .6947193+07 .1618017-03 .9871293-03 .1941956-02 .2572969-02 .223502-02  
 2747829-02 .7997590-00 .1893493-02 .1464987-02 .7342373-03 .1546277-03 .7961108-03 .2063806-02  
 2638337-02 .1795018-02 .5488082-02 .1000000+01 .3307377-03 .1079701-03 .1536904-03 .9144766-04  
 1092251-03 .1218889-03 .8773690-03 .2107377-04 .8432057-03 .1785643-03 .1419864-03 .8712852-04  
 8291351-03 .8834169-04 .7668254-03 .2094867-00 .1677244-03 .2097363-00 .5277747-03 .1981775-03  
 1743811-03

## EIGENVECTOR CORRESPONDING TO

343822F-02 .1669399-01 .7483127-02 .1874991-02 .3027679-05 .2209581-04 .7928795-00 .1839089-01  
 1250191-02 .2403711-00 .2403711-00 .2403711-00 .3237626-01 .1558470-04 .3706308-02 .2749683-02  
 1000000+01 .1794381-01 .1462404-02 .4960322-02 .4651224-01 .2889574-02 .4420664-01 .3547382-03  
 2639777-02 .5227674-02 .8807164-03 .1102327-03 .2849673-03 .3038490-03 .1914797-03 .6449851-03  
 2072401-03 .2374536-02 .2036267-00 .8295255-03 .1404634-03 .6606800-03 .1244796-02 .1354484-02  
 4156412-02

## EIGENVECTOR CORRESPONDING TO

5161630-03 .1581625-02 .1382658+08 .1706127-03 .1132730-04 .2142232-04 .6435270-01 .6138557-02  
 277329-03 .9195214-03 .1000000+01 .1400525-01 .1237568-01 .1677425-04 .1011646-02 .8958715-03  
 7435347-03 .6592678-02 .193275-03 .6862070-03 .2086748-00 .1090863-02 .2071925-00 .2461452-03  
 9754913-03 .2024037-02 .1163026-03 .4402369-04 .1467787-03 .7123040-04 .4715625-04 .1694859-03  
 1446963-01 .7039469-03 .1756097-01 .2104775-03 .4566509-04 .2573346-03 .1351093-02 .1792019-03  
 1558951-02



MAXIMAX DISPLACEMENT VECTOR

BOUNDED BEHAVIOUR CONSTRAINTS ARE

NUMBER OF BOUNDED BEHAVIOUR CONSTRAINTS ARE 2

3566979+04 - 8959913+04 - 2681733+04 - 770753+03 - 222217+03 - 147706+05 - 319375+04 - 1089603+05 - 848473+04 - 186166+04  
3955849+04 - 146811+04 - 901467+05 - 904059+04 - 108666+03 - 837327+04 - 188133+04 - 578439+04 - 130955+04 - 136768+04  
996564+04 - 847171+04 - 901467+04 - 762565+04 - 141051+03 - 334723+02 - 147626+03 - 825915+04 - 955297+04 - 336255+04  
177311+04 - 883043+03 - 963733+03 - 358437+05 - 731388+04 - 110476+05 - 515751+04 - 525553+03 - 215395+04 - 23522+04  
367966+04 - 944934+04 - 108832+05 - 4999945+04 - 667365+05 - 205187+04 - 220352+04 - 389603+05 - 938158+04 - 93621+04  
367914+04 - 174728+04 - 901773+03 - 997033+03 - 356886+05 - 745186+04

..516144-00 .100000+01-.100000+01 .236824-00-.100000+01

RESULTS FROM MOVED CHECK

..117533+71 .130239+01 .162595+01 .130761+01 .185174+01 .108971+71 .136053+01

\*\*\*\*\*  
CURRENT DESIGN \*\*\*\*\*

914054-00 .250241+01 .232597+01 .307590-00 .115172+01 .125550+01 .000514-00

\*\*\*\*\*5753C 13400\*\*\*\*\*

950293-00 .22600+01 .200356+01 .530000-00 .147413+01 .117914+01 .962924-00

## 11 BOUNDED SIDE CONSTRAINTS ARE

NUMBER OF BOUNDED SIDE CONSTRAINTS ARE 1

.37401402-01 .46633726-00 .31662683-00  
 .31662683-00 .10066722-01 .12637576-00  
 .64844003-01 .64844003-01 .37678336-01

WEIGHT IS .137550+02

NUMBER OF ITERATIONS

50 46 53 58 6 6 42 15 19 4 1

EIGENVALUES ARE

.5119546+05 .5304178+06 .9154616+06 .2094511+07 .3643936+07 .5232377+07 .8035089+07 .9950465+07  
 .9654664+07 .4040242+08 .7117373+08 .1909910+04 .2257439+04 .2834623+04 .3154436+04

FREQUENCIES ARE

.2262641+03 .7289797+03 .9567976+03 .1443784+04 .1909910+04 .2257439+04 .2834623+04 .3154436+04  
 .3107196+04 .6356290+04 .8436452+04 .5119546+05 .2022382+06 .1756720+06 .4888072+06 .655214-01  
 .655214-01 .4888072+06 .1552786+06 .1078015-00 .1398770-00 .4975719-01 .9953975-01 .9128199-01  
 .1608310-00 .3617262-01 .1594848+00 .6833192+03 .1797814+00 .3716558-01 .3716558-01 .2413931-01  
 .2413931-01 .4600137-01 .4577769-01 .3669978-01 .1770235-01 .1802162-01 .1812454-01 .2481362-01  
 .1920627-01 .1833664-01 .1747775-01 .1770235-01 .1802162-01 .1812454-01 .2481362-01 .2300855-01  
 .2104610-01 .2104610-01

EIGENVECTOR CORRESPONDING TO

.1497589+00 .2996738+00 .4717009+00 .6432715-00 .4266510-01 .8532701-01 .1495785+00 .2333873+00  
 .364431+00 .1056566+00 .2732608+00 .5024752+00 .1398770-00 .3716558-01 .5090128+00 .1000000+01  
 .2511555+00 .7513129+00 .3840415+00 .5787156+00 .3716558-01 .3716558-01 .2826619-01 .4815958-01  
 .453709-01 .457715-01 .6761337-01 .6894307-01 .6651444-01 .6678351-01 .6678351-01 .6847740-01  
 .1056566+00 .1073624-01 .1073624-01 .1025303+01 .15864501-01 .4420928-01 .2556208-01 .7589805-02  
 .2613688-01

EIGENVECTOR CORRESPONDING TO

.9154616+06 .1301026+01 .1078445+01 .1551859+01 .3896181+03 .7783985+00 .6250366+00 .9240911+00  
 .1301026+01 .3333029+00 .6080317+00 .1073494+01 .2322987+00 .6190851+00 .4672285+00 .9240911+00  
 .5624469+00 .1000000+01 .3222148+03 .5326013+00 .8533524+01 .7255795+01 .5490484+01 .1116336+00  
 .1131456+00 .1138714+00 .7319064+01 .3834429+01 .933165+03 .6031481+01 .5177369+01 .6280157+01  
 .1025097+00 .7377914+01 .4936012-01 .2571257-01 .1286389-01 .3052103+02 .1949916-01 .2315534-01  
 .270454-01

EIGENVECTOR CORRESPONDING TO

.2084511+07 .2953799+00 .231547+00 .1684242+00 .1499419+00 .5097560+01 .5359325+00 .5359325+00  
 .1551859+01 .8555472+00 .1499419+00 .1684242+00 .231547+00 .2156707+00 .5097560+01 .5359325+00  
 .1242251+00 .1316911+00 .2377570+00 .4903355+00 .2457457+01 .2774621+02 .2611816+01 .2611816+01  
 .2563371+01 .2513947+02 .6525641+01 .3303325+01 .5760166+02 .5610553+02 .5370382+02 .7047747+02  
 .1227768+01 .1466364+01 .5489859+02 .1272369+00 .11272369+00 .3903461-01 .6025150+01 .1397466+00  
 .1050826+00



-.1033160-03    .2833550-03    -.1209104-04    -.1073763-04    -.7241622-04    .3140768-03    .8296309-04    -.2527905-03  
 -.1662018-03    .1659381-03    .1084211-03    .9905514-00    -.9995170-00    .1000000+01    -.6352907-05    .1688239-03  
 .2050924-03

# REORDERED THE EIGENVALUES AND CORRESPONDINGLY THE EIGENVECTORS

## EIGENVALUES ARE

.5119546+05    .5304178+06    .9154616+06    .2084511+07    .3643936+07    .5232377+07    .8035089+07    .9654664+07  
 .9950465+07    .4040242+08    .7117373+08

## FREQUENCIES ARE

.2262641+03    .7282979+03    .9567976+03    .1443784+04    .1908910+04    .2287439+04    .2834623+04    .3107196+04  
 .3154436+04    .6356290+04    .84436452+04  
 1000 INTERVAL TIME    10.02    SECONDS

## RESULTS OF DYNAMIC ANALYSIS

XCOORDINATE READ FROM THE SHOCK SPECTRUM  
 YCOORDINATE READ FROM THE SHOCK SPECTRUM  
 .452528-00    .145660+01    .191360+01    .288757+01    .381782+01    .485748+01    .566925+01    .621439+01    .630887+01    .127126+02  
 .168729+02  
 .713758-01    .464853-01    .331926-01    .161918-01    .815722-02    .466901-02    .208422-02    .139461-02    .130081-02    .116020-04  
 .540639-06  
 MODAL PARTICIPATION COEFFICIENTS  
 .895073-01    .300182-01    .518018-02    .177778-02    .124667-03    .102439-04    .553663-04    .340146-04    .247421-05    .283455-06  
 .276175-10

## MAXIMUM DISPLACEMENT VECTOR

.123619-01    .247215-01    .318578-01    .389784-01    .690003-02    .137969-01    .135708-01    .115348-01    .185124-01    .105091-01  
 .220770-01    .407101-01    .576904-01    .891318-01    .283677-01    .566506-01    .218768-01    .486618-01    .667190-01    .331886-01

MAXIMUM STRESS-MEMBER 1    .495695+05    .506809+05

MAXIMUM STRESS-MEMBER 2    .507384+05    .497460+05

MAXIMUM STRESS-MEMBER 3    .286578+05    .287837+05

MAXIMUM STRESS-MEMBER 4    .287898+05    .285762+05

MAXIMUM STRESS-MEMBER 5    .276451+05    .279368+05

MAXIMUM STRESS-MEMBER 6    .279791+05    .276916+05

MAXIMUM STRESS-MEMBER 7    .425123+05    .439640+05

MAXIMUM STRESS-MEMBER 8    .441695+05    .428005+05

MAXIMUM STRESS-MEMBER 9    .292636+05    .304701+05

MAXIMUM STRESS-MEMBER 10    .304842+05    .293046+05



|                              |              |              |              |              |              |              |              |
|------------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| .2288440+03                  | .7501988+03  | .9602235+03  | .1424637+04  | .2143658+04  | .2231720+04  | .3065309+04  | .3304517+04  |
| .3750096+04                  | .5969551+04  | .9474489+04  |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
| -.8467819-01                 | -.192717-00  | .5236956+05  | -.2244703-00 | .5367220-01  | .1073872+00  | -.8722250-01 | -.3048555-01 |
| -.6657877-01                 | .1529849-00  | -.1978641-00 | .2989635-00  | .6501079-00  | .1000000+01  | .1712789-00  | .3438181-00  |
| .1527594-00                  | .3218049-00  | .6807299-00  | .1742407-00  | -.2073421-01 | -.1979400-01 | -.1921561-01 | -.4690255-01 |
| -.4675230-01                 | -.3567719-01 | -.5867719-01 | -.3754296-01 | -.3530240-01 | -.2304334-01 | -.2291163-01 | -.2278294-01 |
| -.1834009-01                 | -.1768145-01 | -.1695806-01 | -.1799398-01 | -.1752445-01 | -.1698772-01 | -.2315592-01 | -.2295251-01 |
| -.2291225-01                 |              |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
| -.1405719-00                 | .2812151-00  | .5627983+06  | .6672779-00  | .6279704-01  | .1254728-00  | .1466264-00  | .2295443-00  |
| -.3873773-00                 | .1157354+00  | -.2765649-00 | .5412937-00  | .1703269-00  | -.2090791-00 | .5098795-00  | .1000000+01  |
| .2633402-00                  | .7708944-00  | -.4063609-00 | .5667160-01  | -.3746815-01 | -.3609997-01 | -.3479909-01 | .4898868-01  |
| .5011715-01                  | .5079287-01  | .6781389-01  | .6759007-01  | .6345502-01  | -.6447467-01 | -.6684687-01 | -.6865322-01 |
| -.1148273-01                 | -.1293005-01 | -.1402771-01 | -.5827893-01 | -.4535023-01 | -.2996976-01 | .5899591-02  | .1356712-01  |
| .1931083-01                  |              |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
| -.4620878-00                 | -.9237468-00 | .9220292+06  | -.1408625+01 | -.3507695-00 | -.7011984-00 | -.5222968-00 | -.8716805-00 |
| -.1188325-01                 | -.3176768-00 | -.1166892+01 | .1000000+01  | .2594082-00  | -.5627603-00 | .4687883-00  | .9091185-00  |
| .5765847-00                  | .9547595-00  | .3389055-00  | .4968639-00  | -.7898929-01 | -.6734040-01 | -.5280211-01 | .1015256+00  |
| .1042993+00                  | .1061699+00  | .7163527-01  | .3757123-01  | .9584987-04  | -.5722672-01 | -.6089422-01 | -.6344359-01 |
| -.8596775-01                 | -.7144000-01 | -.5354833-01 | -.2304759-01 | -.1503700-01 | -.4303224-02 | -.3672468-02 | -.1742697-01 |
| -.3333352-01                 |              |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
| -.1114906+00                 | -.2245324-00 | .2029733+07  | -.3654166-00 | .5008385-00  | .1000000+01  | -.1255487-00 | .4636297-00  |
| .4559850-00                  | .7837672-00  | -.6883730-01 | .1440021-00  | -.3518469-01 | -.2155395-00 | .3143400-01  | .7212132-01  |
| .6935978-01                  | .1081046+00  | -.2448210-00 | .3648500-00  | -.7824097-02 | -.1053786-01 | -.9315259-02 | .2367554-01  |
| .2403331-01                  | .2601923-01  | -.6767019-01 | -.3432252-01 | .9658003-02  | -.5579366-02 | -.4938602-02 | -.3900789-02 |
| -.1660613-01                 | -.1339480-01 | -.8704021-02 | -.1025203+00 | -.3717982-01 | .3598072-01  | -.1188382+00 | -.1039842+00 |
| -.1606724-01                 |              |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
| .1782258-03                  | .3487086-03  | .4595268+07  | .3424941-03  | -.1157935-02 | -.2291713-02 | -.2210530-02 | -.2218281-02 |
| .2655528-02                  | .7996003-00  | -.1839012-02 | -.1309321-02 | -.5816612-03 | .2964531-03  | -.9047263-03 | -.1802128-02 |
| .3124704-02                  | -.1567326-02 | -.5336765-02 | .1000000+01  | .3362468-03  | .9091697-04  | -.1740906-03 | -.8367018-04 |
| -.1081370-03                 | -.1278073-03 | -.9517077-03 | .6764901-06  | .9561095-03  | .1190897-03  | .1284144-03  | .1262534-03  |
| -.5513894-03                 | .6154357-04  | .6659022-03  | -.2094393-00 | .1677931-03  | .2096935-00  | .5402061-03  | .2343659-03  |
| -.1233092-03                 |              |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
| .4617327-03                  | .9369367-03  | .4980574+07  | .1290663-02  | .9730250-03  | .1965731-02  | .9697737-03  | .1787469-02  |
| -.7997510-02                 | .5149300-02  | -.1845972-02 | -.1669686-02 | -.1993800-02 | -.1626239-02 | -.1171334-02 | -.1740616-02 |
| -.1464928-02                 | -.1710836-02 | .1000000+01  | .4319231-02  | .2335789-03  | .1174159-03  | -.2535035-04 | .6957097-04  |
| -.1859674-05                 | -.7335693-04 | .2097066-01  | -.3296287-04 | -.2097330-00 | .5632888-04  | .1192298-03  | .178218-03   |
| .2263421-03                  | .1019147-03  | -.3905210-04 | -.1023725-02 | .1261284-04  | .1039004-02  | -.1582938-03 | -.8266513-04 |
| .1458925-04                  |              |              |              |              |              |              |              |
| EIGENVECTOR CORRESPONDING TO |              |              |              |              |              |              |              |
| -.5142430-02                 | -.1323577-01 | .9396120+07  | .1913227-02  | .1649572-04  | .2864429-04  | -.7935639-00 | -.2297295-01 |

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-8615649-03      -283374-02      .9091193-01      .2880499-01      .1915676-01      .2094412-04      -1396447-02      -.2033835-02
.100000+01      .1343289-01      .1300853-02      -.4575226-02      -.1775847-01      .2090149-02      .1434660-01      .8642437-03
.1967341-02      .2878743-02      .4608840-03      -.3055031-04      -.4915747-03      .5131351-04      .1541994-03      .2135648-03
-.2076933-0C      -.1695781-02      .2051350-00      .8007975-03      .1068979-03      -.6923242-03      .3799975-02      -.8293897-03
-.5328630-02

EIGENVECTOR CORRESPONDING TO
.1092050+08      .1092050+08      .2913781-03      .8279806-05      .3038589-04      .2532992-01      -.1075821-01
-.9430571-03      -.1809334-02      -.7614898-03      -.7614898-03      .1000000+01      .1000000+01      .8343132-03
-.377190-03      .7964458-03      .1592448-01      .8940543-02      -.9899311-05      -.6275770-03      -.8343132-03
-.2953278-01      .6583116-02      .2479117-03      .5376563-03      -.2087719-00      .207159-00      .4650078-03
.9524337-03      .1339407-02      .1204560-03      -.3507295-04      -.1560283-03      .4361582-05      .9470745-04
.7102124-02      -.7203645-03      -.8138558-02      -.1777158-03      .4323544-04      .2194911-03      -.1711326-03
-.2724898-02

EIGENVECTOR CORRESPONDING TO
.1406324+08      .1406324+08      -.3969659-03      -.3969659-03      -.3594972-03      -.7228909-03      -.2977203-02      .1000000+01
.1860607-02      .3732093-02      .1693160-02      .7140098-02      -.6316293-03      .9826276-05      .6006144-04      .5474262-04
-.2296994-03      .1654085-04      .7140098-02      .1354542-02      -.1547837-02      .1115932-03      .1682957-02      -.9124621-04
.5411108-02      -.6491038-03      -.3855257-03      .5040541-03      -.1747075-04      .8913532-04      .3779259-05      .19551951-05
-.9175613-04      -.7963536-04      -.7599464-04      .1747075-04      .3035406-04      .1066513-03      -.2611132-00      .4668252-03
-.1140329-02      .2123424-03      .1424075-02      -.8464860-04

EIGENVECTOR CORRESPONDING TO
.3563673+08      .3563673+08      -.1838144-05      .7886652-05      .1895833-04      -.6661512-03      -.1565065-03
.1364679-03      .240157-03      .1296534-03      -.1838144-05      .1296533-04      .1000000+01      .2044753-02
-.8851145-03      .6758201-03      .7484713-03      -.1690426-02      .9298393-03      .3452143-03      .7526233-04
.1254754-03      .1945303-02      -.1432210-04      -.2761997-03      -.2150033-03      .1555711-03      .2094917-00
-.1238571-03      -.1259211-03      .1863294-03      .1846963-03      .1594908-03      .2097126-00      .1285954-03
.1379274-03      .1229414-03      .2331565-03      -.2391999-03      .2173848-03      .1276382-03      .7852526-04      .2190020-04
-.48633055-04

EIGENVECTOR CORRESPONDING TO
.8976594+08      .8976594+08      .8685860-05      .8685860-05      -.6403917-03      -.1226143-02      -.5757582-03      .3058115-03
.1690362-03      .2209997-03      .1157652-03      .1157652-03      .9162441-03      .6851260-05      .8502356-03      .1513407-02
-.580998-03      .2233500-03      .4691109-03      -.1610139-02      .162441-03      .1298509-03      .1298509-03      -.4367861-03
.403091-04      -.1668297-02      -.6082229-03      .2900355-03      -.1204284-03      .2042612-05      .1298509-03      .2772790-03
.1355333-03      .2666055-03      -.4418919-06      -.1279820-04      -.7851917-04      .382201-03      .1378466-03      .1693046-03
-.1779316-03      .1721955-03      .1114904-03      .9996022-00      -.9995063-00      .1000000+01      -.2851518-04      .1693046-03
.2225426-03

1000 INTERVAL TIME 5.00 SECOND(S)

RESULTS OF DYNAMIC ANALYSIS
XMOVES/2P
.457685-00      .130040+01      .192045+01      .284937+01      .428732+01      .446344+01      .613062+01      .566923+01      .750020+01      .119393+02
.199490+02
Y CORRELATIVE READ FROM THE SHOCK SPECTRUM
.718975-01      .450086-01      .330254-01      .166540-01      .577121-02      .508867-02      .148343-02      .104249-02      .540627-03      .205136-04
.117059-06
LOCAL PARTICIPATION COEFFICIENTS
.684277-01      .233818-01      .624563-02      .217037-02      .928150-04      .174278-04      .399508-04      .253487-04      .151298-05      .483766-06
.303135-11
MAXIMUM DISPLACEMENT VECTOR
.121355-01      .256333-01      .319494-01      .398304-01      .804413-02      .160844-01      .130513-01      .118169-01      .197281-01      .110635-01
.217740-01      .413242-01      .582228-01      .892886-01      .278280-01      .551844-01      .214276-01      .481066-01      .674134-01      .315529-01

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|                          |            |            |
|--------------------------|------------|------------|
| MAXIMUM STRESS-MEMBER 1  | .482778+05 | .492053+05 |
| MAXIMUM STRESS-MEMBER 2  | .490398+05 | .482806+05 |
| MAXIMUM STRESS-MEMBER 3  | .317844+05 | .318496+05 |
| MAXIMUM STRESS-MEMBER 4  | .318868+05 | .315967+05 |
| MAXIMUM STRESS-MEMBER 5  | .322426+05 | .326079+05 |
| MAXIMUM STRESS-MEMBER 6  | .327327+05 | .322422+05 |
| MAXIMUM STRESS-MEMBER 7  | .378725+05 | .368399+05 |
| MAXIMUM STRESS-MEMBER 8  | .386445+05 | .377962+05 |
| MAXIMUM STRESS-MEMBER 9  | .287013+05 | .297814+05 |
| MAXIMUM STRESS-MEMBER 10 | .297868+05 | .287369+05 |
| MAXIMUM STRESS-MEMBER 11 | .431536+05 | .444150+05 |
| MAXIMUM STRESS-MEMBER 12 | .442911+05 | .431394+05 |
| MAXIMUM STRESS-MEMBER 13 | .447028+05 | .469404+05 |
| MAXIMUM STRESS-MEMBER 14 | .470972+05 | .447257+05 |

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FEASIBLE DIRECTION VECTOR, S, IS
.100000+01 .414353-00-.100000+01-.100000+01-.100000+01-.100000+01 .592299-00

S1M4 IS .747974-00

RESULTS FROM MOVEANDCHECK

***** PREVIOUS DESIGN *****
.104352+01 .215789+01 .188145+01 .752109-00 .159624+01 .115022+01 .110503+01

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