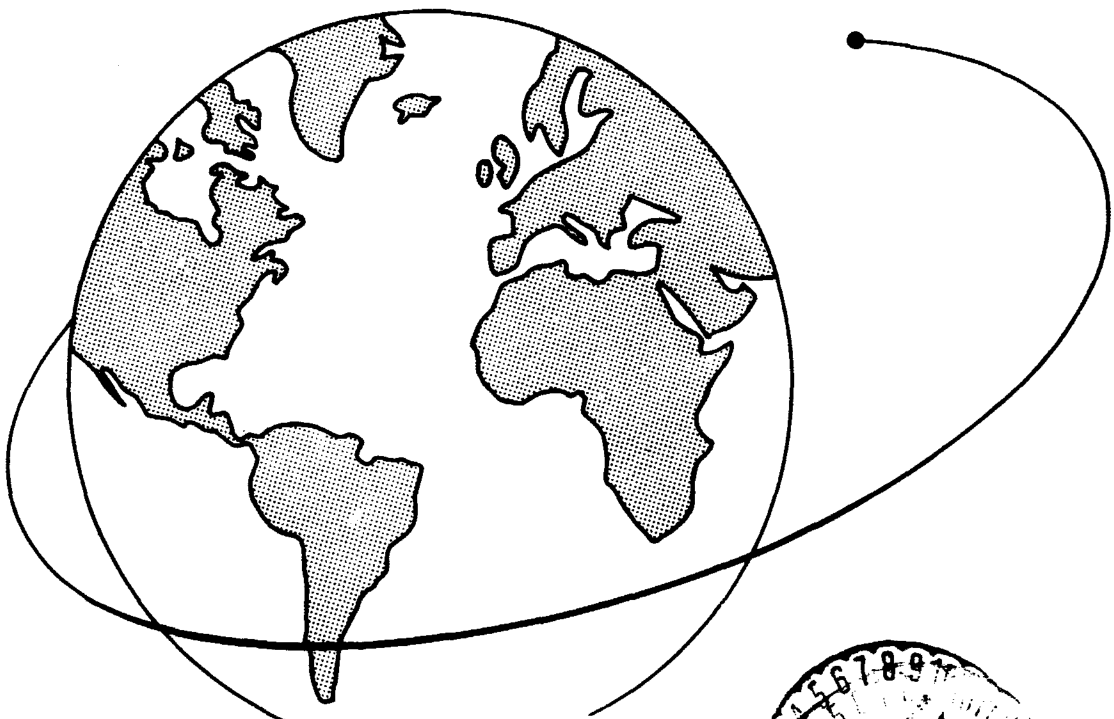
REVISED ZONAL HARMONICS IN THE GEOPOTENTIAL

Y. KOZAI



Smithsonian Astrophysical Observatory
SPECIAL REPORT 295

Research in Space Science
SAO Special Report No. 295

REVISED VALUES FOR COEFFICIENTS OF ZONAL SPHERICAL HARMONICS IN THE GEOPOTENTIAL

Yoshihide Kozai

February 28, 1969

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

809-81

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

ABSTRACT

From precisely reduced Baker-Nunn observations for 12 artificial satellites with inclinations between 28° and 96°, coefficients of zonal spherical harmonics up to the 21st order in the expression of the gravitational potential of the earth are derived.

RÉSUMÉ

Nous avons étudié d'une façon précise les observations photographiques Baker-Nunn pour 12 satellites artificiels ayant une inclinaison entre 28° et 96°, et à partir de ces observations nous avons déduit les coefficients des harmoniques sphériques zonales jusqu'au 21ème ordre dans l'expression du potentiel de gravitation de la terre.

КОНСПЕКТ

Выведены коэффициенты зональных сферических гармоник до 21го порядка в выражении гравитационного потенциала земли исходя из точно обработанных Бэкер-Нунн наблюдений для 12 искуственных спутников с наклонами между 28° и 96°.

REVISED VALUES FOR COEFFICIENTS OF ZONAL SPHERICAL HARMONICS IN THE GEOPOTENTIAL

Yoshihide Kozai

1. INTRODUCTION

Since Kozai's (1968) determination of the coefficients of zonal spherical harmonics for the geopotential, E. M. Gaposchkin and his colleagues at SAO have obtained, from precisely reduced Baker-Nunn observations, orbital elements of very high accuracy for several satellites. Consequently, complete analyses have been made for eight satellites in order to determine (O-C) for secular motions and amplitides of long-periodic terms, where the computed values are based on Kozai's coefficients determined in 1964. These values (Kozai, 1964) are the following:

$$
\begin{aligned}
J_2 &= 1082.639, & J_3 &= -\ 2.546, \\
J_4 &= -\ 1.649, & J_5 &= -\ 0.210, \\
J_6 &= \ \ 0.646, & J_7 &= -\ 0.333, \\
J_8 &= -\ 0.270, & J_9 &= -\ 0.053, \\
J_{10} &= -\ 0.054, & J_{11} &= \ \ 0.302, \\
J_{12} &= -\ 0.357, & J_{13} &= -\ 0.114, \\
J_{14} &= \ \ 0.179, & & \qquad (1)
\end{aligned}
$$

where the unit is 10^{-6} and the following values are used for the geocentric gravitation constant GM and the equatorial radius of the earth a_e:

$$
\begin{aligned}
GM &= 3.98601 \times 10^{20}\ \mathrm{cm}^3\ \mathrm{sec}^{-2}, \\
a_e &= 6.37816 \times 10^8\ \mathrm{cm}. \qquad (2)
\end{aligned}
$$

This work was supported in part by grant NGR 09-015-002 from the National Aeronautics and Space Administration.

In the present determination, 12 satellites are chosen. Their names, as well as their anomalistic mean motions in revolutions per day, inclinations, eccentricities, and periods of observations used, are given in Table 1, which also presents approximate values of the secular motions and amplitudes of long-periodic terms due to spherical harmonics of odd orders.

For the four satellites 1960 ν1, 1959 α1, 1960 ι2, and 1962 $\beta\mu$1, the same observational data as those employed in the previous determination (Kozai, 1968) are used; the satellite 1962 $\beta\tau$2, which was also used in the previous determination, has been dropped here because of the poor accuracy of its orbital elements. For the other eight satellites, additional data are used.

Of these eight satellites, 1963 26A, 1964 01A, and 1965 81A were not included in the previous determination. Furthermore, for the other eight satellites, the accuracy of the data used here is much higher than that in the previous paper.

Table 1. Satellites chosen with their mean orbital elements, secular motions, and amplitudes of $\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}\omega$ terms

Satellite	n (rev/day)	i	e	Periods (MJD)	$\dot{\omega}$ (deg/day)	$\dot{\Omega}$ (deg/day)	A_ω	$A_\Omega \times 10^2$	$A_i \times 10^3$	$A_e \times 10^4$
60 ν1	13.454	28°.33	0.0166	38924 - 39062	8°.265	-5°.065	1°.656	0°.10	-0°.85	4.80
59 α1	11.442	32.88	0.1660	36620 - 38530	5.258	-3.498	0.156	0.75	-6.98	4.62
	11.480	32.88	0.1642		5.293	-3.521	0.158	0.73	-6.92	4.62
62 αε1	9.126	44.80	0.2428	37870 - 38606	1.987	-1.860	0.112	1.64	-7.79	5.23
60 ι2	12.197	47.23	0.0114	37192 - 38576	2.978	-3.101	3.330	1.17	-0.40	6.63
63 26A	14.099	49.74	0.0614	38222 - 38988	3.505	-4.168	0.735	1.18	-2.38	7.94
	14.116	49.74	0.0607		3.514	-4.179	0.744	1.17	-2.35	7.95
62 βμ1	13.345	50.14	0.0070	37974 - 38574	2.963	-3.609	6.363	1.15	-0.26	7.84
65 89A	11.968	59.38	0.0717	39074 - 39574	0.653	-2.247	1.008	3.76	-3.05	12.50
61 15A	13.870	66.82	0.0080	37548 - 38390	-0.695	-2.425	1.886	0.78	-0.05	2.64
64 01A	13.920	69.91	0.0015	38550 - 38866	-1.276	-2.133	29.000	0.04	-0.02	7.31
64 64A	13.746	79.70	0.0129	38698 - 39132	-2.535	-1.078	4.850	0.07	-0.15	10.99
65 81A	13.797	87.37	0.0747	39090 - 39472	-3.042	-0.282	0.910	0.08	-0.23	11.75
	13.814	87.37	0.0739		-3.050	-0.283	0.920	0.08	-0.23	11.76
61 αδ1	8.677	95.85	0.0121	38428 - 38972	-0.978	0.210	3.760	-0.02	0.06	7.91

2. EVEN-ORDER HARMONICS

Table 2 gives equations of condition to improve coefficients of zonal harmonics of even orders. In Table 2a the upper line indicates the secular motion in degrees per day of the argument of perigee; and the lower line, that of the longitude of the ascending node for each satellite identified in Table 1. The values of (O-C) are based on my previous coefficients given in equation (1), and the standard deviations mentioned there are from the analyses of observations.

In the computation of the weights of the equations of condition, however, the standard deviations derived from the observations are not used, since the equations of condition include coefficients only up to 20th order. Neglect of higher order terms causes some errors in the computed values. Therefore, the standard deviations assigned are increased for some data and are given as those of the residuals v. The weight assigned to each equation is inversely proportional to the square of the increased standard deviation.

Table 2b gives equations of condition for amplitudes of long-periodic terms with argument 2ω, ω being the argument of perigee. The first column identifies the satellite. The orbital elements are the argument of perigee, the longitude of the ascending node, the inclination, and the eccentricity, respectively.

The coefficient of J_2 is always zero in Table 2b, as J_2 does not produce long-periodic terms with argument 2ω, although J_2^2 terms in the disturbing function do produce them. And, since the value of J_2 is known to at least three figures, the terms from J_2^2 can be evaluated with sufficient accuracy.

Table 2. Equations of condition, residuals, and weights for even-order harmonics
a) Secular motions
(deg./day)

Satellite	Orbital elements	J_2	J_4	J_6	J_8	J_{10}	J_{12}	J_{14}	J_{16}	J_{18}	J_{20}	$(O-C) \times 10^6$	$v \times 10^6$
60 ν1	$\dot{\omega}$	7623	-5477	-2224	6040	-3259	-1674	3727	-2042	- 743	1977	170° ± 100°	40° ± 100°
	$\dot{\Omega}$	-4670	5167	-2136	- 945	1922	-1070	- 157	702	- 486	17	-125 ± 5	- 1 ± 5
59 α1	$\dot{\omega}$	4880	-1565	-2722	2484	412	-1313	931	670	-1052	245	32 ± 1	1 ± 10
	$\dot{\Omega}$	-3244	2548	- 201	-1100	790	127	- 524	276	142	- 266	- 9 ± 1	0 ± 5
62 αε1	$\dot{\omega}$	1835	1039	- 821	- 643	398	340	- 202	- 177	105	94	40 ± 6	2 ± 6
	$\dot{\Omega}$	-1716	300	511	- 126	- 207	60	96	- 31	- 47	16	7 ± 3	2 ± 3
60 ι2	$\dot{\omega}$	2753	2686	-1224	-2302	316	1425	40	- 763	- 118	368	220 ± 50	47 ± 50
	$\dot{\Omega}$	-2864	261	1168	- 16	- 480	- 37	104	34	- 76	- 21	- 11 ± 1	4 ± 10
63 26A	$\dot{\omega}$	3248	5112	- 768	-6159	-1787	5016	3291	-3153	-3709	1390	920 ± 10	- 52 ± 80
	$\dot{\Omega}$	-3858	- 145	2338	648	-1338	- 766	682	692	- 276	- 547	1 ± 1	19 ± 40
62 βμ1	$\dot{\omega}$	2740	4130	- 333	-4065	-1361	2596	1847	-1187	-1601	304	600 ± 60	60 + 100
	$\dot{\Omega}$	-3334	- 188	1667	489	- 747	- 441	278	301	- 69	- 175	- 42 ± 1	8 ± 15
65 89A	$\dot{\omega}$	605	2453	2144	40	-1392	-1096	- 11	604	438	- 12	-110 ± 20	- 26 ± 20
	$\dot{\Omega}$	-2075	- 976	260	562	240	- 92	- 163	- 64	32	50	- 70 ± 10	- 7 ± 10
61 15A	$\dot{\omega}$	- 640	1895	4421	4326	1625	-1623	-3302	-2742	- 813	1020	-300 ± 80	65 ± 80
	$\dot{\Omega}$	-2240	-2037	- 809	331	811	657	219	- 150	- 284	- 211	22 ± 1	- 3 ± 10
64 01A	$\dot{\omega}$	-1190	789	3596	4891	3802	1122	-1567	-2986	-2771	-1428	600 ± 800	620 ± 800
	$\dot{\Omega}$	-1994	-7082	-1236	- 216	475	684	521	207	- 64	- 198	56 ± 8	9 ± 8
64 64A	$\dot{\omega}$	-2341	-2482	-1456	15	1379	2313	2710	2622	2189	1574	-400 ± 100	-110 ± 100
	$\dot{\Omega}$	- 997	-1299	-1253	-1026	- 735	- 454	- 224	- 58	45	96	90 ± 10	15 ± 10
65 81A	$\dot{\omega}$	-2813	-3980	-4365	-4292	-3961	-3500	-2991	-2484	-2010	-1582	620 ± 30	- 8 ± 80
	$\dot{\Omega}$	- 261	- 375	- 422	- 430	- 417	- 391	- 360	- 326	- 293	- 260	50 ± 1	- 27 ± 18
61 αδ1	$\dot{\omega}$	- 903	- 637	- 331	- 144	- 53	- 15	- 2	2	2	1	- 35 ± 50	- 47 ± 50
	$\dot{\Omega}$	194	145	83	42	20	9	4	2	1	0	-2.9 ± 0.5	0.6 ± 0.5

Table 2. Equations of condition, residuals, weights, scale factors, and computed perturbations for even-order harmonics b) $\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}$ 2ω terms (degrees except for eccentricity)

Satellite	Orbital element	J_2	J_4	J_6	J_8	J_{10}	J_{12}	J_{14}	J_{16}	J_{18}	J_{20}	F_1	(O - C)	v	C	F_2
59 α1	ω	0	-45	86	-38	-54	82	-23	-47	53	- 5	2	3 ± 5	-3	64	-4
	Ω	0	- 4	- 1	14	-13	- 5	17	- 9	- 7	13	2	-2 ± 2	-2	5	-4
	i	0	-20	33	- 9	-21	24	- 2	-14	11	1	1	-3 ± 6	-5	37	-5
	e	0	13	-21	6	15	-17	2	12	-10	- 1	0	0 ± 1	1	-25	-6
62 α1	ω	0	-64	22	70	-29	-47	22	29	-14	-17	2	-1 ± 3	-2	85	-4
	Ω	0	- 7	-17	11	17	- 8	-12	5	7	- 3	2	-1 ± 1	1	- 7	-4
	i	0	-40	5	34	- 6	-18	4	8	- 2	- 4	1	4 ± 4	4	43	-5
	e	0	27	- 4	-23	4	12	- 3	- 6	1	3	0	0 ± 1	0	-29	-6
60 ι2	ω	0	-10	- 2	13	2	- 9	- 2	5	2	- 3	3	-3 ± 4	-2	8	-3
	e	0	2	0	- 3	0	2	0	- 1	0	1	0	0 ± 1	0	- 2	-6
63 26A	ω	0	-13	- 9	26	19	-24	-26	16	27	- 6	3	-6 ± 2	0	4	-3
	Ω	0	- 1	- 4	0	8	4	- 9	- 9	7	11	2	2 ± 2	3	- 5	-4
	i	0	- 4	- 3	8	6	- 7	- 8	4	8	- 1	1	-1 ± 3	1	1	-5
	e	0	14	10	-27	-20	24	26	-15	-26	5	0	3 ± 2	-3	- 4	-6
62 βμ1	ω	0	-13	- 9	20	15	-14	-16	6	13	0	3	3 ± 6	6	3	-3
	e	0	2	1	- 3	- 2	2	2	- 1	- 2	0	0	1 ± 1	0	0	-6
65 89A	ω	0	-21	-52	-15	34	37	5	-20	-17	- 1	2	6 ± 2	2	-26	-4
	Ω	0	- 5	-24	-19	4	20	15	- 1	-10	- 8	2	4 ± 2	0	-12	-4
	i	0	- 6	-16	- 5	10	11	2	- 5	- 5	0	1	5 ± 5	4	- 8	-5
	e	0	26	65	19	-41	-44	- 7	22	19	1	0	-4 ± 1	1	32	-6
61 15A	ω	0	0	8	11	6	- 3	-10	- 9	- 3	3	4	-1 ± 2	0	2	-2
	e	0	- 1	-11	-16	- 9	5	14	13	5	- 4	0	1 ± 2	-1	- 3	-6
64 64A	ω	0	- 1	- 1	0	1	2	2	2	2	2	4	0 ± 2	0	1	-2
	e	0	3	2	1	- 2	- 4	- 5	- 5	- 5	- 3	0	4 ± 4	3	- 2	-6
65 81A	ω	0	-13	-21	-25	-26	-25	-22	-20	-17	-14	3	7 ± 3	3	17	-3
	Ω	0	0	- 1	- 2	- 3	- 5	- 6	- 7	- 8	- 9	2	1 ± 1	0	1	-4
	i	0	0	- 1	- 1	- 1	- 1	- 1	0	0	0	1	-2 ± 8	-2	0	-5
	e	0	16	27	31	32	30	26	22	18	14	0	-6 ± 2	-1	-22	-6

Table 3. Normal equations for even-order harmonics and solutions

J_2	J_4	J_6	J_8	J_{10}	J_{12}	J_{14}	J_{16}	J_{18}	J_{20}	$(O - C) \times 10^6$
2423892	-1115747	219058	453420	-372931	88518	96540	-146869	45329	61548	24440
	1645749	-287517	-345531	455945	-190250	-115665	156890	-60303	-12813	-30401
		435391	72410	-221373	115324	- 2248	- 70616	71054	- 3678	3527
			205370	- 86346	- 23218	54573	- 13535	-10516	15391	896
				220668	- 60669	- 24932	63869	-35976	- 7157	-10217
					117932	- 368	- 46066	36686	- 6278	6638
						38110	83	-13204	7171	1819
							37724	-13985	- 2606	- 4832
								28673	- 3143	757
									7268	9
										1299
-0.009 ±5	0.037 ±14	-0.108 ±25	0.082 ±37	-0.179 ±40	0.152 ±37	-0.056 ±30				216
-0.008 ±6	0.033 ±17	-0.101 ±33	0.072 ±49	-0.166 ±56	0.139 ±54	-0.046 ±43	-0.011 ±31			216
-0.011 ±2	0.056 ±7	-0.144 ±14	0.152 ±20	-0.301 ±24	0.318 ±25	-0.255 ±22	0.192 ±18	-0.234 ±15		33
-0.011 ±2	0.056 ±7	-0.144 ±14	0.152 ±20	-0.300 ±25	0.315 ±27	-0.252 ±28	0.187 ±26	-0.231 ±22	-0.005 ±22	33

3. ODD-ORDER HARMONICS

In order to determine corrections to coefficients of odd-order harmonics, the 46 equations of condition given in Table 4 for the amplitudes of the long-periodic terms with argument ω are used. The same system of numbering the equations is employed as in Table 2b. All the coefficients must be multiplied by the F_1th power of 10; and (O - C) and v, by the F_2th power of 10.

The weight for each equation is computed from the standard deviation assigned to (O - C); when the standard deviations derived from the observations are different from these values, they are given in the last column.

Table 5 gives the normal equations and solutions. The equations are solved with 7, 8, 9, and 10 unknowns, and the equations with Σv^2 are given. After the inclusion of J_{21}, the value of Σv^2 is reduced to 22. However, the standard deviations computed for the solution are not small enough, because of correlations among the coefficients.

Table 4. Equations of condition, residuals, weights, and scale factors for odd-order harmonics (degrees except for eccentricity)

Satellite	Orbital element	J_3	J_5	J_7	J_9	J_{11}	J_{13}	J_{15}	J_{17}	J_{19}	J_{21}	F_1	(O - C)	v	F_2	Alternate standard deviation
60 ν1	ω	-654	575	-68	-293	289	-76	-98	123	-47	-29	3	40 ± 10	-1	-4	
	Ω	-7	-8	21	-15	-4	16	-12	0	8	-7	2	0 ± 3	0	-4	
	i	3	-3	0	1	-1	0	0	-1	0	0	2	0 ± 3	0	-4	
	e	-187	164	-19	-84	82	-21	-28	35	-13	-8	0	16 ± 10	6	-7	
59 α1	ω	-646	473	114	-421	225	132	-246	78	116	-132	2	-17 ± 6	0	-4	±3
	Ω	-53	-97	137	-12	-96	73	17	-61	29	21	2	-2 ± 3	2	-4	±2
	i	288	-143	-71	114	-33	-38	39	-4	-19	14	1	1 ± 5	-4	-5	
	e	-193	95	48	-76	22	25	-26	3	13	-9	0	-31 ± 5	-1	-7	
62 αε1	ω	-507	-79	364	37	-213	-19	119	10	-66	-5	2	-1 ± 2	-1	-4	
	Ω	-45	-195	-9	110	12	-57	-8	30	4	-15	2	2 ± 3	3	-4	
	i	319	98	-123	-31	45	11	-18	-4	8	2	1	-2 ± 3	-4	-5	
	e	-215	-66	83	21	-30	-7	12	3	-5	-1	0	-15 ± 8	2	-7	
60 ι2	ω	-136	-83	70	45	-27	-22	9	10	-3	-4	4	-24 ± 10	-10	-3	±3
	Ω	-2	-15	-4	10	5	-5	-4	2	2	-1	2	0 ± 10	3	-5	
	i	16	10	-8	-5	3	3	-1	-1	0	1	1	-6 ± 6	-6	-5	
	e	-271	-165	138	90	-53	-44	18	20	-6	-9	0	-26 ± 12	3	-7	±6
63 26A	ω	-293	-312	182	283	-60	-220	-16	155	54	-99	3	-17 ± 4	-1	-3	±2
	Ω	-12	-127	-74	99	118	-43	-119	-6	96	36	2	-6 ± 4	1	-4	±1
	i	9	10	-5	-8	2	6	1	-4	-1	2	2	14 ± 15	10	-5	
	e	-311	-332	179	283	-49	-201	-20	127	47	-72	0	-12 ± 3	2	-6	±1
62 βμ1	ω	-245	-253	110	179	-17	-102	-17	50	23	-18	4	-59 ± 15	0	-3	±4
	Ω	-1	-13	-8	8	10	-2	-8	-1	5	3	2	-2 ± 2	-2	-4	
	i	1	1	-1	-1	0	0	0	0	0	0	2	0 ± 1	0	-4	
	e	-301	-311	136	220	-20	-126	-21	62	28	-21	0	-8 ± 2	-1	-6	±0.8
65 89A	ω	-252	-809	-392	152	292	120	-63	-101	-37	25	3	3 ± 2	0	-3	±1.5
	Ω	-9	-642	-532	-96	195	200	56	-58	-71	-24	2	10 ± 2	2	-4	
	i	77	249	120	-40	-78	-32	14	23	8	5	1	-8 ± 8	-7	-5	
	e	-314	-1019	-489	164	321	131	-57	-93	-34	-19	0	-4 ± 2	-2	-6	±1.0

Table 4 (Cont.)

Satellite	Orbital element	J_3	J_5	J_7	J_9	J_{11}	J_{13}	J_{15}	J_{17}	J_{19}	J_{21}	F_1	(O - C)	v	F_2	Alternate standard deviation
61 15A	ω	-265	746	1049	654	48	-346	-397	-216	7	134	4	-19 ± 8	- 8	-2	±2
	Ω	- 1	-123	-126	- 39	47	75	46	- 2	- 32	- 33	2	- 3 ± 4	0	-4	
	i	7	- 20	- 29	- 18	- 1	9	11	6	0	- 4	1	0 ± 5	0	-5	
	e	-370	1039	1461	910	67	-481	-552	-299	9	185	0	-11 ± 5	4	-6	±1
64 01A	ω	-156	131	291	268	137	- 2	- 86	-101	- 68	- 19	5	-200 ± 10	1	-2	
	e	-380	320	710	654	335	- 5	-211	-247	-166	- 47	0	-58 ± 9	- 9	-6	±1
64 64A	ω	-174	-111	- 37	17	47	58	55	45	32	20	4	-11 ± 2	3	-2	
	Ω	- 1	- 11	- 19	- 21	- 19	- 14	- 8	- 2	3	6	2	6 ± 3	1	-4	
	i	5	3	1	- 1	- 2	- 2	- 2	- 1	- 1	- 1	1	0 ± 8	0	-5	
	e	-394	-252	- 85	38	107	131	124	101	72	44	0	-34 ± 5	- 2	-6	
65 81A	ω	-310	-300	-255	-210	-169	-135	-107	- 83	- 64	- 49	3	60 ± 5	3	-3	±2
	Ω	- 1	- 15	- 29	- 41	- 49	- 54	- 57	- 57	- 56	- 53	2	20 ± 2	2	-4	±1
	i	1	1	1	1	0	0	0	0	0	0	2	- 1 ± 1	- 1	-4	
	e	-401	-376	-308	-241	-185	-139	-103	- 76	- 55	- 39	0	60 ± 3	- 2	-6	
61 αδ1	ω	-139	- 64	- 24	- 8	- 2	- 1	0	0	0	0	4	- 3 ± 5	- 4	-2	
	Ω	0	2	2	2	1	1	0	0	0	0	2	- 2 ± 2	- 2	-4	
	i	- 29	- 14	- 5	- 2	- 1	0	0	0	0	0	1	- 6 ± 7	- 6	-5	
	e	-293	-134	- 51	- 17	- 5	- 1	0	0	0	0	0	30 ± 15	0	-7	

Table 5. Normal equations for odd-order harmonics and solutions

J_3	J_5	J_7	J_9	J_{11}	J_{13}	J_{15}	J_{17}	J_{19}	J_{21}	$(O - C) \times 10^6$
1056315	- 203228	-121575	118951	-227689	73704	108788	-78523	42882	41860	4038
	1145266	331106	-252115	71494	-103646	- 94328	83108	- 8421	- 1992	- 3584
		475160	184061	- 91128	- 95821	- 41380	-16921	1724	6304	-13588
			313258	- 43573	- 26609	- 9473	-69117	- 2513	11732	-11222
				195767	18057	- 59236	6503	-25212	- 9945	- 3515
					63109	22273	-18615	- 722	2583	- 641
						49541	8139	8041	3144	916
							46461	7827	- 6118	871
								14785	3287	- 336
									7083	- 126
										1367
-0.021 ±12	0.026 ±19	-0.097 ±32	0.030 ±36	-0.120 ±32	-0.007 ±26	-0.102 ±20				223
0.010 ±6	-0.027 ±9	0.002 ±15	-0.113 ±18	0.026 ±17	-0.197 ±18	0.073 ±16	-0.179 ±13			38
0.010 ±6	-0.029 ±9	0.007 ±17	-0.122 ±22	0.040 ±25	-0.217 ±31	0.097 ±34	-0.203 ±34	0.021 ±26		38
0.008 ±4	-0.020 ±7	-0.028 ±15	-0.047 ±23	-0.100 ±35	-0.009 ±49	-0.174 ±61	0.085 ±65	-0.216 ±52	0.145 ±29	22

4. DISCUSSION

The coefficients determined in the present analyses are as follows:

$$
\begin{aligned}
J_2 &= 1082.628 \pm 2, & J_3 &= -2.538 \pm 4, \\
J_4 &= -1.593 \pm 7, & J_5 &= -0.230 \pm 7, \\
J_6 &= 0.502 \pm 14, & J_7 &= -0.361 \pm 15, \\
J_8 &= -0.118 \pm 20, & J_9 &= -0.100 \pm 23, \\
J_{10} &= -0.354 \pm 25, & J_{11} &= 0.202 \pm 35, \\
J_{12} &= -0.042 \pm 27, & J_{13} &= -0.123 \pm 49, \\
J_{14} &= -0.073 \pm 28, & J_{15} &= -0.174 \pm 61, \\
J_{16} &= 0.187 \pm 26, & J_{17} &= 0.085 \pm 65, \\
J_{18} &= -0.231 \pm 22, & J_{19} &= -0.216 \pm 53, \\
J_{20} &= -0.005 \pm 22, & J_{21} &= 0.145 \pm 29,
\end{aligned}
\tag{3}
$$

where the unit is in 10^{-6}.

The geoid height computed by the coefficients equation (3) with respect to the reference ellipsoid with the flattening 1/298.25 is shown as a function of geocentric latitude in Figure 1.

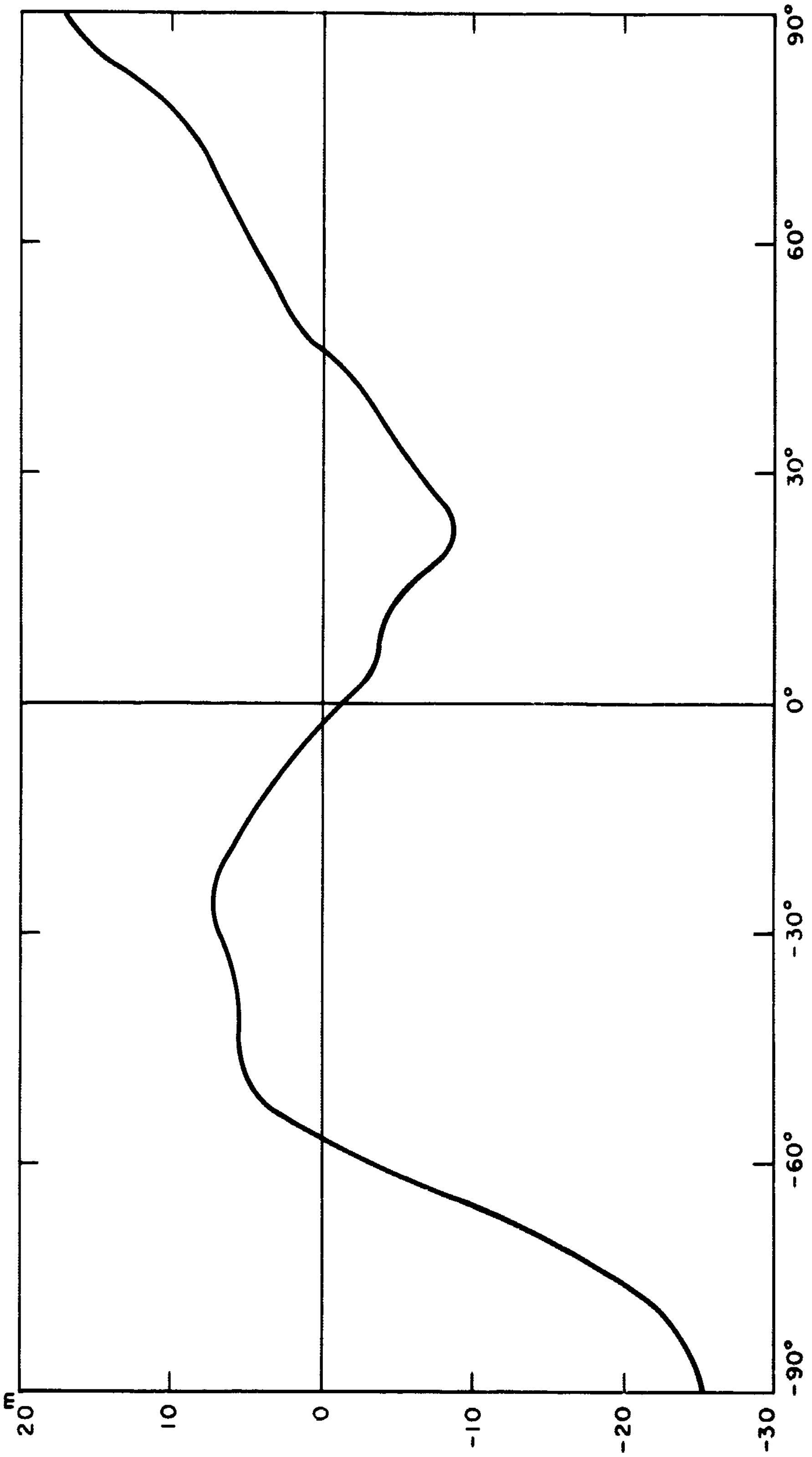


Figure 1. Geoid height with flattening 1/298.25.

Although 12 satellites are chosen in the present determination, it can be said that essentially 10 satellites are used, since the inclinations of three satellites, 1960 ι2, 1963 26A, and 1962 $\beta\mu$1, are near 50°. Two high satellites, 1962 $\alpha\epsilon$ 1 and 1961 $\alpha\delta$1, cannot contribute to the determination of higher order coefficients.

In the determination of even-order coefficients, the equations of condition for the secular motions for the argument of perigee and for the longitude of the ascending node are independent of each other; therefore, the number of independent equations is twice as large as the number of satellites.

Of the equations of condition for determining odd-order coefficients, the equations for the inclination and for the eccentricity are not independent of each other, since the amplitudes of $\sin \omega$ for the two elements are proportional. If the eccentricity is small, the equation for the argument of perigee is not independent of that for the eccentricity, and the amplitude for the longitude of the node is small and can contribute little to the determination. In reality, of the 12 satellites chosen, 10 have small eccentricities.

Therefore, although nearly 50 equations of condition are used in each determination, the number of equations that can really contribute is not large enough to permit the solution for more than 10 unknowns, especially for odd-order harmonics.

To reduce standard deviations for the solutions, more data and the inclusion of much higher order harmonics are necessary. There are gaps in the inclination around 40° and below 25°. Particularly, satellites with inclinations less than 25° are needed to reduce correlation among coefficients in the equations of condition.

In effect, when the number of changes of sign for the coefficients in Tables 2a and 4 is counted for each satellite, it can be noticed that the lower the inclination is, the larger the number becomes. For example, the sign changes six times and seven times, respectively, in the equations for 1960 ν1 in Table 2a and it changes three times for 1965 89A, although the signs are always negative for 1965 81A. This means that the correlations, especially among higher order coefficients, are quite strong without low-inclination satellites.

To reduce correlations in the present analyses, weights to the data for 1960 $\nu 1$ and 1959 $\alpha 1$ are increased artificially. But the correlations are still quite strong.

However, it is certain that the coefficients given in equation (3) are quite reliable up to 12th-order harmonics.

REFERENCES

KOZAI, Y.

1964. New determination of zonal harmonics coefficients of the earth's gravitational potential. Smithsonian Astrophys. Obs. Spec. Rep. No. 165, 38 pp.

1968. Improved values for coefficients of zonal spherical harmonics in the geopotential. In Geodetic satellite results during 1967, ed. by C. A. Lundquist, Smithsonian Astrophys. Obs. Spec. Rep. No. 264, pp. 43-56.

BIOGRAPHICAL NOTE

YOSHIHIDE KOZAI received his doctorate from Tokyo University in 1958. He has been associated with the Tokyo Astronomical Observatory since 1952, and has held concurrent positions as staff astronomer with that observatory and consultant to SAO since 1958.

Dr. Kozai specializes in celestial mechanics, his research at SAO being primarily in the determination of zonal coefficients in the earth's gravitational potential by use of precisely reduced Baker-Nunn observations. He is also interested in the seasonal variations of the earth's potential.

DATE

FILMED

JUL 15 1969