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MUTUAL PERTURBATIONS OF MARS' SATELLITES

by

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SUMMARY

Formulas are obtained, which yield the secular and long-period perturbations of Mars' satellite Deimos under the influence of Phobos. The evaluation of the perturbations is made.

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We constructed in our own work [1] the intermediate orbit of Mars' satellites, based upon the solution of the generalized problem of two fixed centers. Such an intermediate orbit takes into account the perturbations linked with the existence of second and third zonal harminics of Mars' potential. In it a, e, i, l, g, h are the elements of the intermediate orbit, whereupon the elements h, l, g vary with time according to formulas:

$$l = n_0 (1 + \lambda)(t - t_0) + l_0, \quad g = \nu l + g_0, \qquad (1)$$
$$h = \mu l + h_0,$$

where n_0, λ, μ, ν are constants depending upon the elements a, e, i. The coordinates of intermediate motion are represented in the form of series by powers of small magnitude

$$\varepsilon = \frac{c}{a(1 - e^2)}$$

and by powers of eccentricity, which we assume to be small. Here c is a constant linked with harmonic I_2, for Mars with c = 150.304 km. The influence of the Sun on the motion of Mars' satellites is studied in [2]. Utilizing the system of canonic variables L, G, H, l, g, h, linked with the intermediate elements a, e, i, l, g, h, one succeeds in untegrating the equations of the perturbed

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motion and in obtaining the expressions for the perturbations from the Sun.

Basing ourselves upon the intermediate orbit [1], we integrate the canonic system of equations in elements L, G, H, l g, h, just as in [2], and we obtain the secular and long-period perturbations caused by mutual influence of Mars' satellites Deimos and Phobos. The main part of the perturbing function, leading to secular and long-period variations in elements will be taken in the form of a series by powers of small quantities e, e', sin i, sin i', where e and sin i are respectively the eccentricity and the sine of Deimos orbit inclination to the equatorial plane of Mars, while the respective quantities with primes are related to Phobos, which figures in the given problem as the perturbing body, since, according to estimates in [3], the mass of Phobos is 7 times greater than that of Deimos.

When averaging the perturbation function by variables l and l', we disregard by the same token the terms with argument $4l-l'$, which in case of Phobos and Deimos may result in long-period perturbations, since

$$n_\Phi = 1128°\cdot 84396 \text{ deg/day}, \quad n_p = 285°\cdot 16196 \text{ deg/day}.$$

But, the amplitudes of such terms are proportional to e^3, as is shown in [4], so that they will not result in substantial variations of orbit elements.

Taking into account the terms proportional to second powers with respect to e, e', sin i and sin i' and independent of the mean satellite anomalies, we may assume, according to [5], the perturbing function in the form

$$R=\frac{1}{2}fm'A_0+\frac{1}{8}fm'B_1(e^2+e'^2)-\frac{fm'}{8}B_1(s^2+s'^2)+$$
$$+\frac{1}{4}fm'B_1ss'\cos(h-h')-\frac{1}{4}fm'B_2ee'\cos(g-g'+h-h'), \tag{2}$$
$$s=\sin i,\ s'=\sin i',$$

where f is the gravitational constant $aA_0=L_0^{(1/2)}(\alpha)$, $aB_i=\alpha L_i^{(3/2)}(\alpha)$, $\alpha=\frac{a'}{a}<1$, and $L_i^{(s)}$ are the Laplace coefficients. The calculation of the coefficients A_0, B_1, B_2 presents no difficulties, since they are linked by close relations with the total elliptical integrals of 1-st and 2-nd kind, for which computations are available in the table published in [6].

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SECULAR PERTURBATIONS

The part of the perturbing function, leading to secular perturbations of elements l, g, h , looks as follows

$$R_v = \frac{1}{2} fm' A_0 + \frac{1}{8} fm' B_1 (e^2 + e'^2) - \frac{fm'}{8} B_i (s^2 + s'^2). \tag{3}$$

Denoting the coefficients at secular perturbations of elements l, g, h respectively by Δn, Δn_1, Δn_2, we obtain for them the expressions:

$$\begin{aligned} \Delta n &= \frac{m'}{m_0} \{\tilde{A} + \tilde{B} e^2 - 2\bar{B} (e'^2 - s^2 - s'^2), \\ \Delta n_1 &= \frac{1}{4} \frac{m'}{m_0} a B_1 (2 - s^2), \\ \Delta n_2 &= -\frac{1}{4} \frac{m'}{m_0} \frac{\cos i}{\sqrt{1-e^2}} B_1 a, \end{aligned} \tag{4}$$

where m_0 is the mass of Mars,

$$\begin{aligned} \tilde{A} &= \frac{1}{4\alpha} (4B_0 a - 5B_1 a \cdot \alpha), \\ \tilde{B} &= \frac{1}{4(1-\alpha^2)} [3\alpha B_0 a + (2-\alpha^2) B_1 a], \\ \bar{B} &= -\frac{1}{8(1-\alpha^2)} [3\alpha B_0 a + B_1 a]. \end{aligned}$$

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LONG-PERIOD PERTURBATIONS

The elements of Deimos are subject to two forms of long-period pe turbations induced by Phobos: long-period perturbations of 1-st class with period of 2.30527 years and perturbations of 2-nd class with period of 26.8939 years. For long-period perturbations in intermediate elements a, e, i, l, g, h, the formulas have the following form:

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\begin{aligned}
&\delta a = 0,\\
&\delta l = \nu_2 a_2 \cos(g_\Delta + h_\Delta),\\
&\delta i = \nu_1 b_1 \cos h_\Delta + \nu_2 b_2 \cos(g_\Delta + h_\Delta),\\
&\delta l = \nu_1 c_1 \sin h_\Delta + \nu_2 c_2 \sin(g_\Delta + h_\Delta),\\
&\delta g = \nu_1 d_1 \sin h_\Delta + \nu_2 d_2 \sin(g_\Delta + h_\Delta),\\
&\delta h = \nu_1 f_1 \sin h_\Delta + \nu_2 f_2 \sin(g_\Delta + h_\Delta),
\end{aligned}
\tag{5}
$$

where

$$\nu_1 = \frac{m'}{m_0}\frac{1}{\mu_\Delta},\quad \nu_2 = \frac{m'}{m_0}\frac{1}{\nu_\Delta + \mu_\Delta},\quad \mu_\Delta = \mu - \frac{n'}{n}\mu',\quad \nu_\Delta = \nu - \frac{n'}{n}\nu',$$

$$g_\Delta = g - g', \qquad h_\Delta = h - h',$$

$$a_2 = -\frac{1}{2}e'Da,$$

$$b_1 = -\frac{1}{4}s'B_1 a, \qquad b_2 = -\frac{1}{8}ee'\sin i\cdot Da,$$

$$c_1 = -4ss'\overline{B}, \qquad c_2 = -\frac{1}{2}\frac{e'}{e}[Da + e^2\tilde{D}a],$$

$$d_1 = -\frac{1}{8}\frac{s'}{s}(2 + e^2 - 2s^2)B_1 a, \qquad d_2 = \frac{1}{2}\frac{e'}{e}\sqrt{1 - e^2}\,Da,$$

$$f_1 = \frac{1}{8}\frac{s'}{s}\cos i\,(2 + e^2)B_1 a \qquad f_2 = 0$$

$$Da = \frac{3}{2}B_0 a + \frac{1 + \alpha^2}{\alpha}B_1 a, \qquad \tilde{D}a = -\frac{3}{2}\frac{1 - 3\alpha^2}{1 - \alpha^2}B_0 a + \frac{1 + 3\alpha^2 - 3\alpha^4}{\alpha(1 - \alpha^2)}B_1 a.$$

Formulas (4) – (5) may be applied for the study of the influence of Deimos on Phobos. In this case $a < a'$, and we must substitute in formulas (4) and (5) α by $1/\alpha$.

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ESTIMATES OF PHOBOS' AND DEIMOS' MASSES AND OF THEIR MUTUAL PERTURBATIONS

The most probable values of Mars' mass and density, determined by the motion of satellites, and also with the utilization of Mariner-4 data of 1965, are given in the work [7]:

$$m_0 = (0.6428 \pm 0.0013)\cdot 10^{27}\ \text{g},$$

$$\frac{1}{m_0} = 3{,}096{,}000 \pm 6000,$$

$$\rho = (3.89 \pm 0.05)\ \text{g/cm}^3.$$

Assuming Phobos and Deimos as being spheres with uniform density, equal to that of Mars and with respective diameters of 15 and 8 km, it is possible to obtain the estimates for the masses of satellites:

$$m_D = 0\cdot 10{,}428\cdot 10^{19}\ \text{g}, \quad m_\Phi = 0\cdot 68{,}742\cdot 10^{19}\text{g}.$$

We may assume for Phobos and Deimos the following systems of elements:

PHOBOS ELEMENTS	DEIMOS ELEMENTS
$a' = 9378.954$ km,	$a = 23477.636$ km,
$e' = 0.0219160$,	$e = 0.00304127$,
$\sin i' = 0.0155449$.	$\sin i = 0.0306596$.

We finally obtain the following result: the mean motion of Deimos on account of Phobos' effect increases by 0.00221 degree per year, and for Δn_1 and Δn_2 we shall obtain the values $\Delta n\cdot n_0 = 0.000347$ deg/year, $n_0\cdot\Delta n_2 = -0.000173$ deg/year. The corresponding values for Phobos are:

$$\Delta n = -0.000241\ \text{deg/year}, \quad \Delta n_1 = -0.000222\ \text{deg/year},$$
$$\Delta n_2 = -0.000111\ \text{deg/year}.$$

The amplitudes of long-period terms are very small. Varying most are the mean anomaly and the longitude of the pericenter. These terms have for amplitudes respectively + 1"034 and − 1".033. The remaining terms lead to long-period variations with amplitudes of less than one second of arc.

*** T H E   E N D ***

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