N69-28088
NASA CR-101381

USS-T
Gagarin

National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No.NAS-5-12487

ST-PS-CM-10844

INTERMEDIATE ORBITS OF MARS' SATELLITES

by

N. S. Vashkov'yak

(USSR)

T-22915

4 JUNE 1969

ST — PS — CM — 10844

INTERMEDIATE ORBITS OF MARS' SATELLITES

Vestnik Moskovskogo Universiteta
seriya III, Fizika i Astronomiya
No.3, 35 - 45,
Izdatel'stvo MGU, MOSKVA, 1968

by S. N. Vashkov'yak
Chair of
Celestial Mechanics and Gravimetry
Moscow State University

SUMMARY

Studied in this work are the intermediate orbits of Mars' satellites, based upon the solution of the generalized problem of two fixed centers. The obtained formulas describe the motion of satellites with any orbit inclinations to the equatorial plane and with eccentricities not exceeding 0.1. The application is considered of the formulas thus derived to the natural satellites of Mars — Phobos and Deimos.

*
* *

INTRODUCTION

The object of this work is the study of intermediate orbits of the satellites of Mars, based upon the solution of the generalized problem of two fixed centers. The advantage of such intermediate orbits by comparison with Kepler orbits is the fact that they are rigorously taking into account both the second and the third zonal harmonics of Mars' gravitational potential. At the same time, we shall assume for initial formulas those of work [1], in which E. P. Aksenov obtained the solution of the problem of two fixed centers in the case of asymmetrical disposition of fixed centers relative to planet's equatorial plane.

There are works in the press, in which the motion of natural satellites of Mars is examined. At the beginning of the Twentieth Century G. Struve [2] was preoccupied with the construction of the analytical theory of motion of Phobos and Deimos, making use of observations conducted from 1877 to 1910 at different observatories. Several works by M. P. Kosachevskiy were released during the past 10 years, in which he investigated the influence of the Sun on the motion of Phobos and Deimos, and also the mutual influence of these two satellites [3] - [5].

During the construction of the analytical theory of motion of Mars' natural satellites, the fact was utilized that both Phobos and Deimos move over nearly circular orbits near planet's equator. Currently, and in connection with the fact that the appearance of AS of Mars may be anticipated in the near future, a more general problem must be stated, namely, to study the motion of Mars' satellites, whose orbits have different inclinations to planet's equatorial plane, but low values of eccentricities.

In the following we shall assume that the eccentricity does not exceed 0.1. With such a problem setup the obtained formulas will be, on the one hand, describing the motion of Phobos and Deimos, and, on the other hand, they may be used for the study of orbits of Mars' artificial satellites with small eccentricities.

STATEMENT OF THE PROBLEM

Let us introduce a rectangular system of coordinates Oxyz with origin at the mass center of Mars, whose plane Oxy coincides with the equatorial plane, and the axis Ox is directed to the point of intersection of Mars' and Earth's equators.

For the time being we shall postulate that Mars is an asymmetrical body. Then the gravitational potential V of Mars will be given by the formula

$$V = \frac{fm}{r}\left\{1 + \sum_{k=2}^{\infty} I_k \left(\frac{R_0}{r}\right)^k P_k\left(\frac{z}{r}\right)\right\}, \qquad (1)$$

in which f is a gravity constant, m is the mass of Mars, I_h are certain dimensionless constants, R_0 is the equatorial radius of Mars, r is the radius-vector of the satellite, $P_k\left(\frac{z}{r}\right)$ are Legendre polynomials of k-th order.

In order to obtain the intermediate orbit, potential V is approximated by potential U of the generalized problem of two fixed centers:

$$U = \frac{fm}{2}\left\{\frac{1+i\sigma}{\sqrt{x^2+y^2+[z-c(\sigma+i)]^2}} + \frac{1-i\sigma}{\sqrt{x^2+y^2+[z-c(\sigma-i)]^2}}\right\}, \qquad (2)$$

where c and σ are constants, $i = \sqrt{-1}$.

If we expand U in series by Legendre polynomials, we shall have

$$U = \frac{fm}{r}\left\{1 + \sum_{k=2}^{\infty} \frac{\gamma_k}{r^k} P_k\left(\frac{z}{r}\right)\right\}, \tag{3}$$

where

$$\gamma_k = \frac{c^k}{2}\{(1+i\sigma)(\sigma+i)^k + (1-i\sigma)(\sigma-i)^k\}. \tag{4}$$

Choosing c and σ from the conditions

$$\gamma_2 = I_2 R_0^2, \quad \gamma_3 = I_3 R_0^3,$$

we shall obtain

$$c = R_0\left\{-I_2 - \left(\frac{I_3}{2I_2}\right)^2\right\}^{1/2},$$

$$\sigma = \frac{I_3}{2I_2} \cdot \frac{R_0}{c}.$$

The coefficient $I_2 = -0.0020$, determined by the motion of Phobos, is sufficiently reliably known [6]. If we postulate $I_2 = -0.0020$, $I_3 = 0$, $R_0 = 3360$ km, we shall have c = 150.2638, $\sigma = 0$. If we assume that for Mars, I_3 is the same as for the Earth, that is, $I_3 = 2.3 \cdot 10^{-6}$, we shall have c = 150.2625 km and $\sigma = -0.012858$.

In the chosen system of coordinates the satellite's equations of motion for zero mass may be written as follows:

$$\frac{d^2x}{dt_2} - \frac{\partial U}{\partial x} = \frac{\partial R}{\partial x}; \quad \frac{d^2y}{dt_2} - \frac{\partial U}{\partial y} = \frac{\partial R}{\partial y}; \quad \frac{d^2z}{dt^2} - \frac{\partial U}{\partial z} = \frac{\partial R}{\partial z}. \tag{5}$$

Equations (5) will be describing the perturbed motion of the satellite, provided we include in function R the difference U—V between the potential of real Mars and the force function of the problem of two fixed centers, and also the tesserial harmonics of Mars' potential, the terms conditioned by the attraction of the Sun, of natural satellites of Mars and so forth.

****

FORMULAS FOR THE INTERMEDIATE MOTION

According to work [1], the general solution of equations

$$\frac{d^2x}{dt_2} - \frac{\partial U}{\partial x} = 0; \quad \frac{d^2y}{dt^2} - \frac{\partial U}{\partial y} = 0; \quad \frac{d^2z}{dt^2} - \frac{\partial U}{\partial z} = 0$$

has the form

$$x = \rho'(\cos\varphi\cos\bar{\Omega} - \alpha\sin\varphi\sin\bar{\Omega} - \beta\sin\bar{\Omega}),$$
$$y = \rho'(\cos\varphi\sin\bar{\Omega} + \alpha\sin\varphi\cos\bar{\Omega} + \beta\cos\bar{\Omega}), \qquad (6)$$
$$z = c\sigma + \Delta(s\cdot\sin\varphi + \gamma),$$

$$\rho' = \frac{\sqrt{(\xi^2 + c^2)(1 - \varepsilon^2\sigma^2)}}{1 + d\sin\varphi},$$
$$\Delta = \frac{\xi}{1 + d\sin\varphi}, \qquad (7)$$
$$\xi = \frac{a[(1 - e\bar{e}) + (\bar{e} - e)\cos\psi]}{1 + \bar{e}\cos\psi},$$

$$\varphi = \bar{\varphi} + \frac{k_1^2}{8}\sin 2\bar{\varphi} - \frac{k_2^2}{8}\sin 2\psi, \qquad (8)$$

$$\bar{\varphi} = (1 + \nu)\psi + \omega_0. \qquad (9)$$

The variable ψ is linked with time t by the equations:

$$\operatorname{tg}\frac{\psi}{2} = \sqrt{\frac{1 + \bar{e}}{1 - \bar{e}}}\operatorname{tg}\frac{E}{2}, \qquad (10)$$

$$E = M + e^{*}\sin E + \lambda\psi - \lambda_1\sin\psi - \bar{\lambda}_1\cos\bar{\varphi} - \bar{\lambda}_3\cos 3\bar{\varphi} - \bar{\lambda}_2\sin 2\bar{\varphi}, \qquad (11)$$
$$M = n_0(t - t_0) + M_0, \qquad (12)$$

and $\bar{\Omega}$ is given by the formula

$$\bar{\Omega} = \mu\psi + \Omega_0 + \mu_1\sin\psi + \mu_2\sin 2\psi + \bar{\mu}_1\cos\bar{\varphi} + \bar{\mu}_2\sin 2\bar{\varphi}. \qquad (13)$$

Here a, e, i, ω_0, Ω_0, M_0 are the elements of the intermediate orbit, which, for $c = \sigma = 0$ convert respectively into the major semiaxis, the eccentricity, the inclination to equatorial plane, the longitude of the ascending node, the distance from node to pericenter, the mean anomaly in the epoch. In formulas (6) — (13), the coefficients are series by powers of the small parameter

$$\varepsilon = \frac{c}{a(1 - e^2)}. \qquad (14)$$

Inasmuch as $a(1 - e) < R_0$, it results that, taking into account (14) we find $\varepsilon < \frac{1}{20}$.

Considering ε as a term of first order of smallness and preserving the terms to fifth order inclusive, we shall commit an error of the order 10^{-7}. At such a precision for the constants entering into formulas (6) — (13), we shall have

$$s = \sin i, \quad \alpha = \sqrt{1 - s^2},$$

$$\bar{e} = e\{1 + \varepsilon^2(1-e^2)(1-2s^2) + \varepsilon^4(3-16s^2+14s^4)\}.$$

$$e^* = e\{1 - \varepsilon^2(1-e^2)(1-s^2) + 3\varepsilon^4 s^2(1-s^2)\},$$

$$k_1^2 = \varepsilon^2 s^2\{1 - e^2 + \sigma^2 - 4\varepsilon^2(1-s^2)\},$$

$$k_2^2 = \varepsilon^2 e^2\{s^2 - \varepsilon^2(1-10s^2+11s^4)\}. \tag{15}$$

$$\beta = 2\varepsilon\sigma\alpha\{s - \varepsilon^2 s(4-5s^2)\},$$

$$\gamma = -\varepsilon\sigma\{1 - 2s^2 - \varepsilon^2(3-12s^2+10s^4)\},$$

$$\nu = \frac{\varepsilon^2}{4}(1+\sigma^2)(12-15s^2) + \frac{\varepsilon^4}{64}(288-1296s^2+1035s^4). \tag{16}$$

$$\mu = -\frac{3}{2}\varepsilon^2\alpha\left\{1 + \sigma^2 + \frac{\varepsilon^2}{8}(6-17s^2)\right\},$$

$$\mu_1 = -2\varepsilon^2\alpha e\left\{1 + \frac{\varepsilon^2}{8}(4-28s^2)\right\},$$

$$\mu_2 = -\frac{\varepsilon^2}{4}e^2\alpha\left\{1 - \frac{\varepsilon^2}{4}(22+s^2)\right\},$$

$$\bar{\mu}_1 = \varepsilon^3\sigma\alpha s; \quad \bar{\mu}_2 = \frac{\varepsilon^4}{32}\alpha s^3. \tag{17}$$

$$\lambda = -\frac{\varepsilon^4}{16}(24-96s^2+75s^4),$$

$$\lambda_1 = -\frac{1}{4}\varepsilon^4 s^2 e(4-5s^2),$$

$$\bar{\lambda}_1 = \frac{1}{2}\varepsilon^3\sigma s(4-5s^2), \quad d = \varepsilon\sigma s[1-\varepsilon^2(5-6s^2)]. \tag{18}$$

$$\bar{\lambda}_2 = -\frac{1}{4}\varepsilon^2 s^2(1-e^2)^{3/2}, \quad \bar{\lambda}_3 = -\frac{1}{6}\varepsilon^3\sigma s^3.$$

The rectangular equatorial coordinates of the satellite, x, y, z depend on angular variables ψ, ϕ, $\bar{\Omega}$, which in their turn are linked with the temporal equations (10 — (13). The calculation of coordinates x, y, z will be significantly simplified if we obtain formulas expressing the variables ψ, ϕ, $\bar{\Omega}$ as explicit functions of time. Deriving such formulas will be the task of the next section.

* * *

DEPENDENCE OF VARIABLE ψ ON TIME

The relation (10) between ψ and E is analogous to the equation linking the true and eccentric anomalies in Kepler motion. This is why, according to (7), we have

$$E = \psi + 2\sum_{k=1}^{\infty}(-1)^k \frac{\bar{\beta}^k}{k} \sin k\psi, \tag{19}$$

where

$$\bar{\beta} = \frac{1}{\bar{e}}(1 - \sqrt{1-\bar{e}^2}) = \frac{\bar{e}}{2} + \frac{\bar{e}^3}{8} + \frac{\bar{e}^5}{16} + \frac{5}{128}\bar{e}^7. \tag{20}$$

In the series by powers e we shall take into account the terms to e^7 inclusive. Expressing with the aid of (5) sin E as a function of the angle ψ, Eq.(11) will be rewritten in the form

$$\psi + 2\sum_{k=1}^{7}(-1)^k \frac{\bar{\beta}^k}{k} \sin k\psi = M + e^* \sum_{k=1}^{6} \bar{C}_k \sin k\psi + \lambda\psi -$$
$$- \lambda_1 \sin\psi - \bar{\lambda}_1 \cos\bar{\varphi} - \bar{\lambda}_3 \cos 3\bar{\varphi} - \bar{\lambda}_2 \sin 2\bar{\varphi}, \tag{21}$$

where the coefficients have the form

$$\bar{C}_1 = 1 - \bar{\beta}^2 + \frac{29}{360}\bar{\beta}^6,$$
$$\bar{C}_2 = -\bar{\beta} + \bar{\beta}^3, \quad \bar{C}_3 = \bar{\beta}^2 - \bar{\beta}^4 - \frac{1}{120}\bar{\beta}^6,$$
$$\bar{C}_4 = -\bar{\beta}^3 + \bar{\beta}^5, \quad \bar{C}_5 = \bar{\beta}^4 - \bar{\beta}^6; \quad \bar{C}_6 = -\bar{\beta}^5.$$

The quantity $\bar{\beta}$ may be represented in the form of series by powers e, if formulas (20) and (15) are utilized. In Eq.(21) the coefficients of variables ψ and $\bar{\phi}$ depend on e, s, ε^2 and σ, whereupon one may separate the terms depending only on e, s, and the terms with multiplier ε^2. After such a separation, Eq.(21) will be written in the form

$$\psi = M(1+\lambda) + e\sum_k f_k \sin k\psi + \varepsilon^2\Phi(\psi), \tag{22}$$

where

$$\Phi(\psi) = a_1 \cos\bar{\varphi} + a_2 \cos 3\bar{\varphi} + a_3 \sin 2\bar{\varphi} + \sum_{k=1}^{4} f_k^* \sin k\psi; \tag{23}$$

../..

$$a_1 = -\frac{1}{2}\varepsilon\sigma s(4-5s^2),$$

$$a_2 = \frac{1}{6}\varepsilon\sigma s^3; \quad a_3 = \frac{1}{4}s^2(1-e^2)^{1/2},$$

$$f_1^* = -es^2 + \frac{e^2}{4}(2+s^2) + \frac{3}{8}e\varepsilon^2 s^4.$$

$$f_2^* = -\frac{e^2}{2}(1-3s^2)(1-e^2),$$

$$f_3^* = \frac{e^3}{4}(2-5s^2); \quad f_4^* = -\frac{e^4}{8}(3-7s^2).$$

At $\varepsilon = 0$, variable ψ becomes the true anomaly, and Eq.(22) passes to the equation of the center:

$$\psi_0 = M(1+\lambda) + \sum_{k=1}^{7} f_k^0 \sin kM, \tag{24}$$

where the coefficients f_k are well known series by powers of eccentricity. Inasmuch as at $\varepsilon = 0$, $\psi = \psi_0$, Eq.(22) will be written as follows:

$$\psi - \Psi_0 - \varepsilon^2\Phi(\psi) = 0. \tag{25}$$

Considering Eq.(25) as a Lagrange equation, and, according to [8], we may seek the root of this equation in the form of series by powers of the small parameter ε^2

$$\psi = \sum_{k=0}^{\infty} \frac{(\varepsilon^2)^k}{k!}\frac{d^{k-1}}{d\Psi_0^{k-1}}[\Phi^{(k)}(\Psi_0)]. \tag{26}$$

Relation (24), substituted into Eq.(26) provides the possibility of obtaining the variable ψ as a function of M.

Let us introduce instead of elements N, ω_0, Ω_0, figuring in Eqs.(8)-(13), new elements l, h, g or $\bar{g}$, linked with the former as follows:

$$\begin{aligned} l &= M(1+\lambda) = n_0(t-t_0) + l_0, \\ g &= \nu l + \omega_0 = n_1(t-t_0) + g_0, \\ h &= \mu l + \Omega_0 = n_2(t-t_0) + h_0, \\ \bar{g} &= g - \frac{\pi}{2} = n_1(t-t_0) + \bar{g}_0, \end{aligned} \tag{27}$$

where

$$l_0 = M_0(1+\lambda), \quad g_0 = \nu l_0 + \omega_0, \quad h_0 = \mu l_0 + h_0,$$

$$\bar{g}_0 = g_0 - \frac{\pi}{2}; \quad n_1 = \nu n_0; \quad n_2 = \mu n_0,$$

$$n_0 = \sqrt{\frac{fm}{a^3}}\left\{1 - \frac{3}{2}\varepsilon^2(1-e^2)(1-s^2) + \frac{3}{8}\varepsilon^4(1-e^2)(1-s^2)(1-11s^2)\right\},$$

a, μ, ν, λ being given by formulas (16)-(18).

If we conduct such a substitution in Eq.(26), we shall finally obtain:

$$\psi = l + \sum K_{kj}\sin(kl + j\bar{g}). \tag{28}$$

$$K_{10} = e\left[2 - \varepsilon^2 s^2 + \frac{3}{8}\varepsilon^4 s^4\right] - \frac{e^3}{8}[2 - 3\varepsilon^2(4-5s^2)] + \frac{5}{96}e^5 + \frac{107}{4608}e^7,$$

$$K_{20} = \frac{e^2}{4}[5 - 2\varepsilon^2(1-s^2)] - \frac{e^4}{24}[11 - 4\varepsilon^2(9-14s^2)] + \frac{17}{192}e^6,$$

$$K_{30} = \frac{e^3}{24}[26 - 3\varepsilon^2(4-5s^2)] - \frac{43}{64}e^5 + \frac{95}{512}e^7,$$

$$K_{40} = \frac{e^4}{96}[103 - 16\varepsilon^2(3-4s^2)] - \frac{451}{480}e^6,$$

$$K_{50} = \frac{1097}{960}e^5 - \frac{5957}{4608}e^7,$$

$$K_{60} = \frac{1223}{960}e^6, \qquad K_{70} = \frac{47273}{32256}e^7.$$

The remaining coefficients K_h for $j \neq 0$ are represented in the form of Table 1. [Table 1 is reproduced on the next page].

*

* *

DERIVING FORMULAS FOR ϕ AND $\bar{\Omega}$

Substituting into Eqs.(8), (9) and (13) instead of μ the already found expression (28), we finally arrive at the following formulas:

$$\varphi = l + \bar{g} + \frac{\pi}{2} + \sum L_{kj}\sin(kl + j\bar{g}), \tag{29}$$

$$\bar{\Omega} = h + \sum M_{kj}\sin(kl + j\bar{g}), \tag{30}$$

../..

TABLE 1

COEFFICIENTS K_{kj}

k \ j	1	2	−2	3	4
0	$-\frac{1}{2}\varepsilon^3\sigma se(4-5s^2)$	$-\frac{1}{16}\varepsilon^2 s^2 c^2(3-4c^2)$	—	—	—
1	$\frac{1}{2}\varepsilon^3\sigma s(4-5s^2)$	$\frac{1}{16}\varepsilon^2 s^2 c(8-19e^2)$	$\frac{1}{48}\varepsilon^2 s^2 c^3$	—	—
2	$-\frac{1}{2}\varepsilon^3\sigma sc(4-5s^2)$	$-\frac{1}{64}\varepsilon^2 s^2(16-88e^2+157c^4)$	$\frac{1}{96}\varepsilon^2 s^2 e^4$	$-\frac{1}{2}\varepsilon^3\sigma s^3 e$	—
3	—	$-\frac{1}{16}\varepsilon^2 s^2 e(8-39e^2)$	—	$\frac{1}{6}\varepsilon^3\sigma s^3$	—
4	—	$-\frac{1}{48}\varepsilon^2 s^2 c(39-188c^2)$	—	$\frac{1}{2}\varepsilon^3\sigma s^3 e$	$-\frac{1}{16}\varepsilon^4 s^4$
5	—	$-\frac{59}{48}\varepsilon^2 s^2 e^3$	—	—	—
6	—	$-\frac{115}{64}\varepsilon^2 s^2 c^4$	—	—	—

where

$$L_{10} = \frac{e}{32}\,[64 + 16\varepsilon^2(12 - 17s^2) + 3\varepsilon^4(96 - 464s^2 + 385\,s^4) +$$

$$+ 48\varepsilon^2\sigma^2(4 - 5s^2)] - \frac{e^3}{16}\,[4 - \varepsilon^2(12 - 11s^2)] + \frac{5}{96}\,e^5 + \frac{407}{4608}\,e^7,$$

$$L_{20} = \frac{e^2}{16}\,[20 + \varepsilon^2(52 - 69s^2)] - \frac{e^4}{96}\,[44 - \varepsilon^2(12 - 11s^2)] + \frac{17}{192}\,e^6,$$

$$L_{30} = \frac{e^3}{48}\,[52 + 3\varepsilon^2(44 - 59s^2)] - \frac{43}{64}\,e^5 + \frac{95}{512}\,e^7,$$

$$L_{40} = \frac{e^4}{384}\,[412 + \varepsilon^2(1044 - 1445s^2)] - \frac{151}{480}\,e^6,$$

$$L_{50} = \frac{1097}{960}\,e^5 - \frac{5957}{4608}\,e^7,$$

$$L_{60} = \frac{1223}{960}\,e^6, \qquad L_{70} = \frac{47273}{32256}\,e^7,$$

and the coefficients L_{kj} for $j \neq 0$ and coefficients M_{kj} are compiled in the form of Tables 2 and 3 (see page 11).

*

* *

APPLICATION TO PHOBOS AND DEIMOS

Since the quantity ε^2 is small, the initial values of Phobos and Deimos elements may be borrowed from work [3] for computing the coefficients in formulas (28)-(30). The elements are brought out for the initial epoch 1880.0.

PHOBOS	DEIMOS
a = 9383.69 km	a = 23479.57 km
e = 0.0217	e = 0.0
i = 0°53'27".0	i = 1°45'25".0

If for Mars we assume $m = \frac{1}{3093500}\,m_\odot$, $I_2 = -0.0020$, $R_0 = 3360.0$ km, we shall have $\varepsilon_\phi = 0.016022$, $\varepsilon_D = 0.00639978$, where ε_ϕ and ε_D are the values of parameter ε respectively for Phobos and Deimos, while the mass of Mars is expressed in masses of Sun. Upon computations we arrive at the following formulas:

..to page 12..

T A B L E 2

COEFFICIENTS L_{kj}

k \ j	1	2	−2	3	4
0	$-\frac{1}{2}\varepsilon^3\sigma e s(4-5s^2)$	$-\frac{3}{64}\varepsilon^2 s^2 e^2(6-7e^2)$	—	—	—
1	$\frac{1}{2}\varepsilon^3\sigma s(4-5s^2)$	$\frac{1}{32}\varepsilon^2 s^2 e[24-53e^2+4\varepsilon^2(4-7s^2)+8\sigma^2]$	$\frac{1}{32}\varepsilon^2 s^2 e^3$	—	—
2	$\frac{1}{2}\varepsilon^3\sigma s e(4-5s^2)$	$-\frac{1}{128}\varepsilon^2 s^2[48-256e^2+433e^4+8\varepsilon^2(4-7s^2)+16\sigma^2]$	$\frac{1}{64}\varepsilon^2 s^2 e^4$	$-\frac{1}{2}\varepsilon^3\sigma e s^3$	—
3	—	$-\frac{1}{32}\varepsilon^2 s^2 e[24-113e^2+4\varepsilon^2(4-7s^2)+8\sigma^2]$	—	$\frac{1}{6}\varepsilon^3\sigma s^3$	—
4	—	$-\frac{3}{64}\varepsilon^2 s^2 e^2(26-121e^2)$	—	$\frac{1}{2}\varepsilon^3\sigma s^3 e$	$-\frac{1}{16}\varepsilon^4 s^4$
5	—	$-\frac{59}{32}\varepsilon^2 s^2 e^3$	—	—	—
6	—	$-\frac{345}{128}\varepsilon^2 s^2 e^4$	—	—	—

COEFFICIENTS M_{kj}

T A B L E 3

k \ j	0	1	2
0	—	$\varepsilon^3\sigma s e\cos i$	—
1	$-\frac{1}{8}\varepsilon^2 e\cos i[40-21e^2+\varepsilon^2(26-119s^2)+24\sigma^2]$	$-\varepsilon^3\sigma s\cos i$	$\frac{1}{16}\varepsilon^4 s^2 e\cos i$
2	$-\frac{1}{48}\varepsilon^2 e^2\cos i(198-193e^2)$	$-\varepsilon^3\sigma s e\cos i$	$-\frac{1}{16}\varepsilon^4 s^2\cos i$
3	$-\frac{35}{8}\varepsilon^2 e^3\cos i$	—	$-\frac{1}{16}\varepsilon^4 s^2 e\cos i$
4	$-\frac{977}{192}\varepsilon^2 e^4\cos i$	—	—

FOR PHOBOS

$$\psi = 862272'',08 + 2812'',34\,t + 8951'',36 \sin(2812'',34\,t + 862272'') +$$
$$+ 121'',38 \sin 2(2812'',34\,t + 862272'') + 2'',28 \sin 3(2812'',34t + 862272),$$
$$\varphi = 717272'',72 + 2814'',51\,t + 8958'',27 \sin(2812'',34\,t + 862272'') +$$
$$+ 121,''49 \sin 2(2812'',34t + 862282'') + 2'',28 \sin 3(2812'',34t + 862227''),$$
$$\bar{\Omega} = 638859'',75 - 1'',08t - 5'',74 \sin(2812'',34\,t + 862272'') -$$
$$- 0'',10 \sin 2(2812'',34\,t - 862272'');$$

FOR DEIMOS

$$\varphi = 94581'',08 + 711'',64\,t + 1279'',00 \sin(710'',55\,t + 8856'',0) +$$
$$+ 2'',48 \sin 2(710'',55\,t + 8856'',0),$$
$$\bar{\Omega} = 559774'',42 - 0'',043\,t - 0'',127 \sin(710'',55\,t + 8856'',0).$$
$$\psi = 8856'',0 + 710'',55t + 1279'',0 \sin(710'',55\,t + 8856'') +$$
$$+ 2'',48 \sin 2(710'',55\,t + 8856'',0),$$

Here the time must be taken in min. (t).

The intermediate orbit, proposed in this work, for Mars' satellites, accounts for the basic perturbations linked with the second and third zonal harmonic of Mars' potential. Elements a, e, i will be constant, while elements l, $\bar{g}$, h are linked with time by relations (27).

On the example of Phobos and Deimos one may get convinced to what extent the intermediate orbit chosen by us describes the secular perturbations in the element h, which are conditioned by Mars' oblation. Annual variations of element h, determined by Struve [3]from observations of the satellites of Mars.

Observations yield

$$\delta h_{\Phi}^{(н)} = - 158°,4841,$$
$$\delta h_{Д}^{(н)} = - \quad 6°,2795. \qquad (31) \qquad (31)$$

The very same variations of element h, computed by formulas (17) and (27), are

$$\delta h_{\Phi}^{(в)} = - 158°,5122,$$
$$\delta h_{Д}^{(в)} = - \quad 6°,3871. \qquad (32)$$

From the comparison of (31) and (322 it is possible to derive the conclusion that the intermediate orbit, based upon the solution of the generalized problem of two fixed centers, provides a good representation on the real motion of satellites in the gravitational field of Mars. On the other hand, the closeness of (31) and (32) points to the fact that the variation of element h is mainly conditioned by the existence of planet oblation.

The author expresses his gratitude to E. P. Aksenov for useful advises and indications during the preparation of the present work.

*** T H E E N D ***

Received on 28 June 1967

Chair of Celestial Mechanics and Gravimetry
of Moscow State University (MGU)

REFERENCES

1. E. P. AKSENOV., Vestnik MGU, ser, fiz. i astronom. No.6, 1967.
2. H. STRUVE. Beobachtungen der Marstrabanten in Washington, Pilkovo und Lickobobservatory, S. Pb. 1898.
3. M. P. KOSACHEVSKIY. Opredelniye promezhutochnykh orbit sputnikov Marsa (Determination of intermediate orbit of Mars'satellites). Trudy "FAISH", 28, 1960.
4. M. P. KOSACHEVSKIY. Vozmushchaushchiye deystviye Solntsa na dvizheniye sputnikov Marsa (Perturbing action of the Sun on the satellites of Mars). Trudy "GAISCH", 28, 1960.
5. M. P. KOSACHEVSKIY, Vestnik MGU, ser. matem. i mekh. astronom i fizika, 4,1958.
6. "PLANETY I SPUTNIKI". Sb.statey pod redaktsiyey Kuypera.IL (For.Lit,) M, 1957.
7. D. BRAUER, J. Klemens. Met. nebesnoy mekhaniki (Metods of Cel.Mec. M.IL, 1964.
8. G. DUBOSHIN. Nebesnaya Mekhanika (Celestial Mechanics). Osnovnyye zadachi i metody (Basic Problems and Methods)., M., Fizmatgiz, 1963.

CONTRACT No.NAS-5-12487
VOLT INFORMATION SCIENCES, INC.
1145 - 19th St.NW
WASHINGTON D.C. 20036
Tel: (202)- 223-6700 (X-36)

Translated by ANDRE L, BRICHANT
on 3 - 4 June 1969