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FLUIDIC PROPORTIONAL THRUSTER SYSTEM

By T. S. Honda, W. L. Keltz and R. K. Rose

May, 1969

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Prepared under Contract No. NAS2-4490 by
Specialty Fluidics Operation
GENERAL ELECTRIC COMPANY
Schenectady, New York



for

AMES RESEARCH CENTER

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SCHENECTADY, NEW YORK

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FOREWORD

This final report was prepared by the General Electric Company for the System Engineering Facility of the Ames Research Center National Aeronautics and Space Administration, Moffett Field, California, under Contract No. NAS 2-4490. The Ames Project Engineer was Mr. D. M. Chisel.

The authors wish to acknowledge the contributions of Mr. J. Murphy, NASA/ARC, and Mr. M. Stewart of Electronic Associates for their assistance in providing hardware and conducting various computer simulations during the design and evaluation portions of the program.

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TABLE OF CONTENTS

<u>Section</u>	<u>Page</u>
Foreword	i
List of Figures	iv
Abstract	viii
1 <u>INTRODUCTION</u>	1
2 <u>SYNOPSIS</u>	2
Control Functions	2
System Comparisons	2
Thruster Control System Implementation	4
System Performance	6
Conclusion and Recommendations	8
3 <u>TECHNICAL DISCUSSION</u>	9
System Performance	9
Hybrid Simulation Results	9
Air Bearing Results	9
Control Mode Design	19
Coarse Control	19
Reduction of Initial position Error	19
The Selected Coarse Control Mode	30
Fine Control Position Loop Response	35
Pitch and Yaw Axes	35
Roll Axis	39
Performance of Simulator and Hardware	41
Pitch Axis	41
Roll Axis	41
Component Design and Development	52
Vortex Valve Steady State Design and Evaluation	52
Fabrication Technique	52
Valve Evaluation	52
Thruster Servo	56
Torque Motor Spring Rates, K_e , K_s , K_m	61
Torque Motor Radii, r_{al} , r_a , r_f	61
Inverted Flapper Gain K_f	61
Vortex Valve Gain K_v	63
Thruster Servo Evaluation	63
Pneumatic Component Dynamics	63
Gain/Phase Tests (Closed Loop)	63
High Pressure Tests	73

Table of Contents (continued)

<u>Section</u>	<u>Page</u>
Electronic Design	75
Torque Motor Driver Amplifier	75
Error Sources	80
Noise	80
Hysteresis	83
4 <u>APPENDICES</u>	
Nomenclature (1)	85
Noise Analysis Method (2)	88
Hysteresis (3)	95
Backlash and Hysteresis Limit Cycling	93
Computer Programs (4)	101
HYS	101
RMS	101
FREQ	107
TIME	114

LIST OF FIGURES

<u>Figure No.</u>	<u>Description</u>	<u>Page</u>
1	Thrust/Time Profile	3
2	Weight Trade-off Study Results	4
3	SPARCS Attitude Control Package Circuit Diagram	5
4	Fluidic Proportional Thruster	7
5	NASA/Ames Hybrid Simulation	10
6	Hybrid Simulation Yaw Channel Results	11
7	Hybrid Simulation Pitch Channel Results	12
8	Air Bearing Test System	13
9	Air Bearing Supported Vehicle Pitch (or Yaw) Position Loop	14
10	Air Bearing Vehicle Pointing Accuracy vs Electronic Gain	15
11	Air Bearing Results in Yaw with Output Filtered	16
12	Air Bearing Results in Yaw with Output Unfiltered	17
13	Pitch or Yaw Coarse Position Loop	20
14	Coarse Control Switching Lines	22
15	Torque to Inertia Ratio, Rate, and Angle for Optimum Positioning	24
16	Example of Optimum and Non-Optimum Phase Plane Trajectories	25
17	Optimum Lead Break Frequency vs Initial Angle	27
18	Initial Angle vs Time to Optimally Position	28
19	Value of Optimum Coarse Control Lead Break Frequency vs Rate at Angle of 50 deg.	29
20	Pitch or Yaw Coarse Position Loop with Two Switching Lines	31
21	Phase Plane Trajectory with Two Switching Lines	32
22	Coarse Acquisition Time using One Lead Break Frequency	33

List of Figures (continued)

<u>Figure No.</u>	<u>Description</u>	<u>Page</u>
23	Coarse Acquisition Time using Two Lead Break Frequencies	34
24	Fine Pitch Control Position Loop	36
25	Pitch Closed Loop Root Locus Diagram	37
26	Simplified Pitch Closed Loop Root Locus Diagram	38
27	Fine Control Roll Position Loop	40
28	Breadboard FPT Output Differential Noise Pressure	42
29	Hybrid Pitch Position Loop using Breadboard FPT	43
30	Pitch Uncertainty with Breadboard FPT	44
31	Fine Pitch Position Loop Small Step Response	45
32	Fine Pitch Position Loop Large Step Response	46
33	Hybrid Roll Position Loop using Breadboard FPT	47
34	Roll Uncertainty with Breadboard FPT	48
35	Roll Position Loop Small Step Response	50
36	Roll Position Loop Large Step Response	51
37	Lamination Vortex Valve	53
38	Radial Vortex Valve Characteristic	54
39	Button Vortex Valve Characteristic	55
40	Rough Thruster Loop Schematic	57
41	Detailed FPT Block Diagram	58
42	Simplifier FPT Block Diagram	59
43	Torque Motor Scale Factor Test Results	60
44	Bellows, Compensation, Torque Motor and Inverted Flapper Gain	62
45	Vortex Valve Gain	64
46	Vortex Valve Frequency Response Configuration	65
47	Vortex Valve Assembly Block Diagram and Circuit Parameters	66
48	Measured Gain and Phase Vortex Valve Controls to Bellows	67

List of Figures (continued)

<u>Figure No.</u>	<u>Description</u>	<u>Page</u>
49	Measured Phase Vortex Valve Controls to Bellows for Various Thruster Channels	68
50	High Altitude Open Thruster Loop Bode Plot	69
51	Sea Level Open Thruster Loop Bode Plot	70
52	Sea Level Closed Thruster Loop Bode Plot	71
53	High Altitude Closed Thruster Loop Bode Plot	72
54	High Pressure Test Result	74
55	Torque Motor Amplifier	76
56	Torque Motor Amplifier w/o Compensation	77
57	Torque Motor Amplifier with Compensation	77
58	General Form of the Torque Motor Amplifier	78
59	Torque Motor Amplifier Block Diagram	78
60	Block Diagram of Uncompensated Torque Motor Amplifier	79
61	Block Diagram of Compensated Torque Motor Amplifier	79
62	Pitch or Yaw Uncertainty due to Closed Loop Thruster Noise	81
63	Roll Uncertainty due to Closed Loop Thruster Noise	82
64	Pitch (or Yaw) Hysteresis Limit Cycle vs Electronic Gain	84
65	Fine Position Loops	91
66	Back Lash	94
67	Back Lash Describing Function	94
68	Back Lash Describing Function Plot	95
69	Reduction of a General Back Lash Element	96
70	Pitch (or Yaw) Position Loop Examined for Hysteresis Limit Cycling	97
71	Pitch (or Yaw) Position Loop Rearranged for Hysteresis Limit Cycle Analysis	98
72	Graphical Solution for Pitch/Yaw Hysteresis Limit Cycle	99
73	HYS Listing	102
74	Back Lash Describing Function Values	103

List of Figures (continued)

<u>Figure No.</u>	<u>Description</u>	<u>Page</u>
75	RMS Listing	108
76	RMS Sample Problem Solution	111
77	FREQ Listing	112
78	FREQ Sample Problem Solution	113
79	TIME Listing	115
80	TIME Sample Problem Solution	117

ABSTRACT

This report summarizes the work performed for the National Aeronautics and Space Administration, Ames Research Center, Moffett Field, California, under Contract No. NAS 2-4490. A proportional thrust control concept, based on the use of a pair of vortex valves, is described. A description of the three axis thruster control package developed for a solar pointing Aerobee rocket control system is also presented. The report contains the results of hardware evaluation with analog simulation of the vehicle and on an air bearing simulator. The vortex valve development and attitude control system study and synthesis included in the program work are discussed.

INTRODUCTION

This report summarizes work conducted for the NASA Ames Research Center by General Electric's Specialty Fluidics Operation, Schenectady, New York. The program concerned development of a control approach and hardware to implement a proportional attitude control system.

The proportional concept was conceived by NASA/ARC as a means of achieving an order of magnitude improvement in pointing accuracy of sounding rocket payloads. The proportional concept uses a pair of vortex valves controlled by an electromagnetic torque motor to provide differential flow modulation to thrust control nozzles. This type of proportional control approach is unique. It offers significant control performance improvement by providing reduced limit cycle rates and better pointing stability. The application of fluidic technology provides hardware implementation with no sliding parts. As a result, sensitivity to gas contamination and temperature effects is decreased. Threshold and hysteresis effects are also minimized. The inherent insensitivity to gas contamination makes possible the use of high temperature gas generator gases as the reaction control media.

The fluidic proportional thruster (FPT) is a closed loop thrust control servo consisting of a pair of vortex valves controlled by an electromagnetic torque-motor to produce differential flow modulation from the vortex valves. The pressure output of the vortex valves is fed back through fluidic shaping networks and bellows to the torque motor so that the steady state thrust output is proportional to the current input to the torque motor.

The specific program objective was to develop an integrated control package utilizing fluidic proportional thrusters in the roll, pitch and yaw channels. The control package was also designed to perform despin, coarse acquisition and fine control functions. Orderly accomplishment of the program goals was achieved by performing three major tasks as follows:

- Task I - Steady state system design and initial control synthesis.
- Task II - Vortex valve development.
- Task III - Attitude control system analysis and fluidic proportional thruster development.

It is emphasized that the control system finally delivered to NASA resulted from an integrated engineering approach which included detailed systems engineering studies to select a hardware approach, develop control laws, determine system parameters such as configuration and weight, and assess the installed accuracy and performance of the developed hardware. The true performance of the developed control system was continually evaluated in various computer simulations and air bearing tests during the course of the program.

SYNOPSIS

CONTROL FUNCTIONS

The development program was structured around a specific sounding rocket vehicle selected by NASA/Ames as the Aerobee 350. Pertinent characteristics of that vehicle are as follows:

Pitch and yaw inertia	280 slug ft ²
Roll inertia	9.35 slug ft ²
Pitch and yaw distance of C _G to nozzles	56 inches
Roll distance of C _G to nozzles	12 inches

The functional requirements of the attitude control system are based on a possible SPARCS* 350 mission. The attitude control system is activated upon booster and payload separation and the system aligns the vehicle in pitch, yaw and roll. Sun sensors provide the pitch and yaw attitude reference and the direction of the Earth's magnetic field is sensed by a magnetometer to establish the roll reference.

The acquisition functions of the system are:

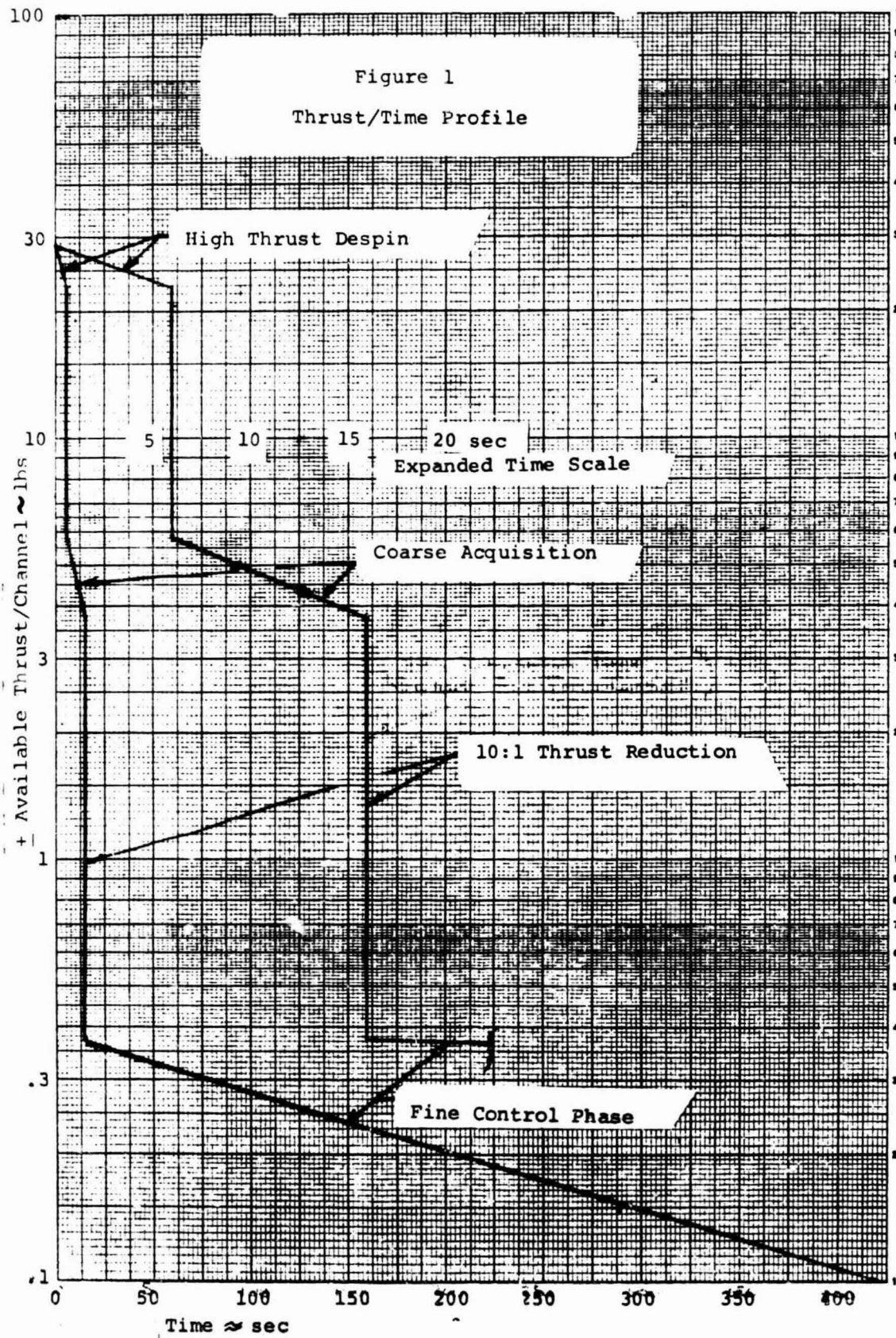
1. Despin of the payload by reaction jets from a spin rate of about 2.5 rps. During this function, all of the available gas flow is applied to the roll channel. The despin maneuver is accomplished in 5 to 6 seconds.
2. Three axis coarse acquisition is made in the pitch, yaw and roll axis. Coarse acquisition is performed in 10 to 20 seconds for maximum initial errors in all control channels.
3. When the coarse control attitude errors are reduced to a low level, the available thrust is reduced by an order of magnitude and fine acquisition is completed in the next 5 to 10 seconds.

The mission is completed in 400 seconds during which the attitude control system is to maintain less than 1 arcsecond rms pointing accuracy to the sun. The control characteristic (available thrust vs. time) is depicted in Figure 1.

SYSTEM COMPARISONS

A comparative weight study of several alternative control implementations was conducted for impulse requirements in the range of 425 to 450 lbsec. It included a comparison of a conventional cold gas bang-bang system with vortex amplifier systems using solid

*Solar Pointing Aerobee Rocket Control System.



propellants, hydrazine, and cold gas. This comparison is summarized in Figure 2.

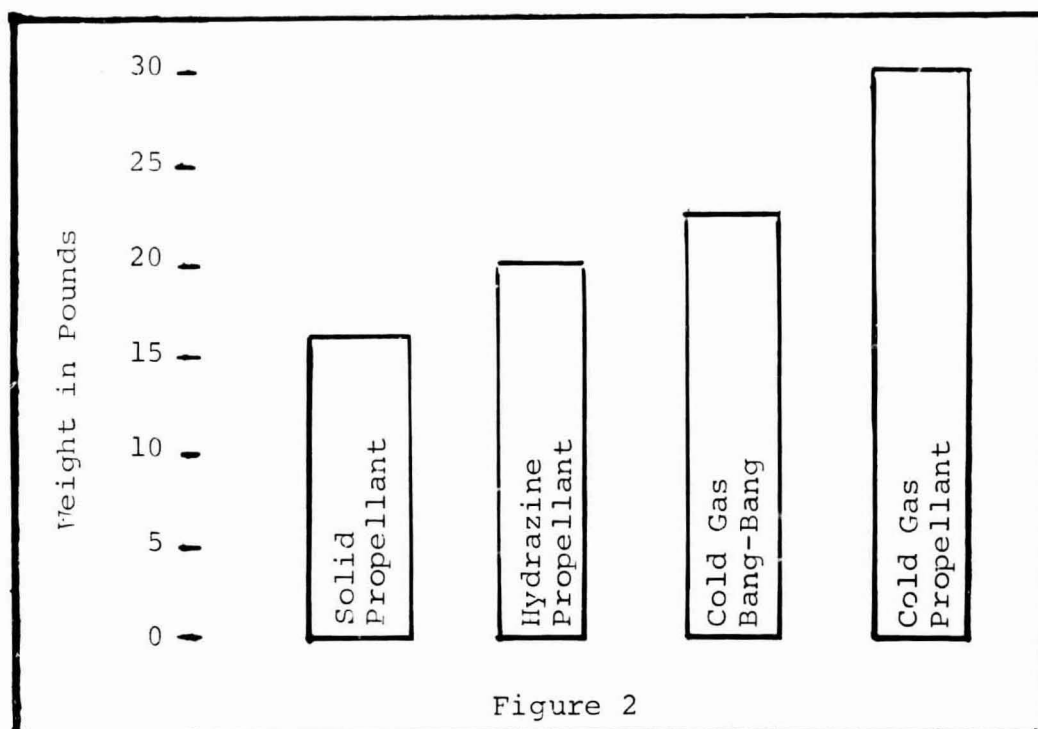


Figure 2

The fluidic proportional and bang-bang systems using cold gas are approximately the same in weight. Although the proportional system requires approximately 3.8 lb more gas than the bang-bang system, the additional gas weight is offset by lower hardware weight. (This results from eliminating the gas regulator and requiring fewer solenoids). The electrical power input to the vortex thruster system is also lower; hence, savings in storage battery weight is also possible, although it is not included in the above results.

The gas consumption of the proportional system can be further decreased if the vortex valves are nulled near their minimum saturation limit. This, however, requires two regulated pressures in the system. This also introduces the possibility of developing a deadband limit cycle, if the null shifts too far into the low gain region. Therefore, when weight, system simplicity and competitive performance are considered for low impulse systems, the fluidic proportional thruster (FPT) system appears most attractive.

The developed FPT system was based on the use of cold gas but has definite growth aspects to hot gas with the attendant weight advantage shown above. As payload size and total impulse requirements increase the use of hot gas becomes most attractive.

THRUSTER CONTROL SYSTEM IMPLEMENTATION

The thrust control implementation which evolved as a result of this study is shown in Figure 3. It includes the gas supply, pressure dropping orifices, shutoff valves, thrust level selector valves,

Orifice Parameters

$$R_C = .052"$$

$$R_f = .018"$$

$$R_v = .078" \text{ Channels 1 \& 3,}$$

$$R_i = .014" \text{ .081" Channel 2}$$

$$R_o = .021"$$

5

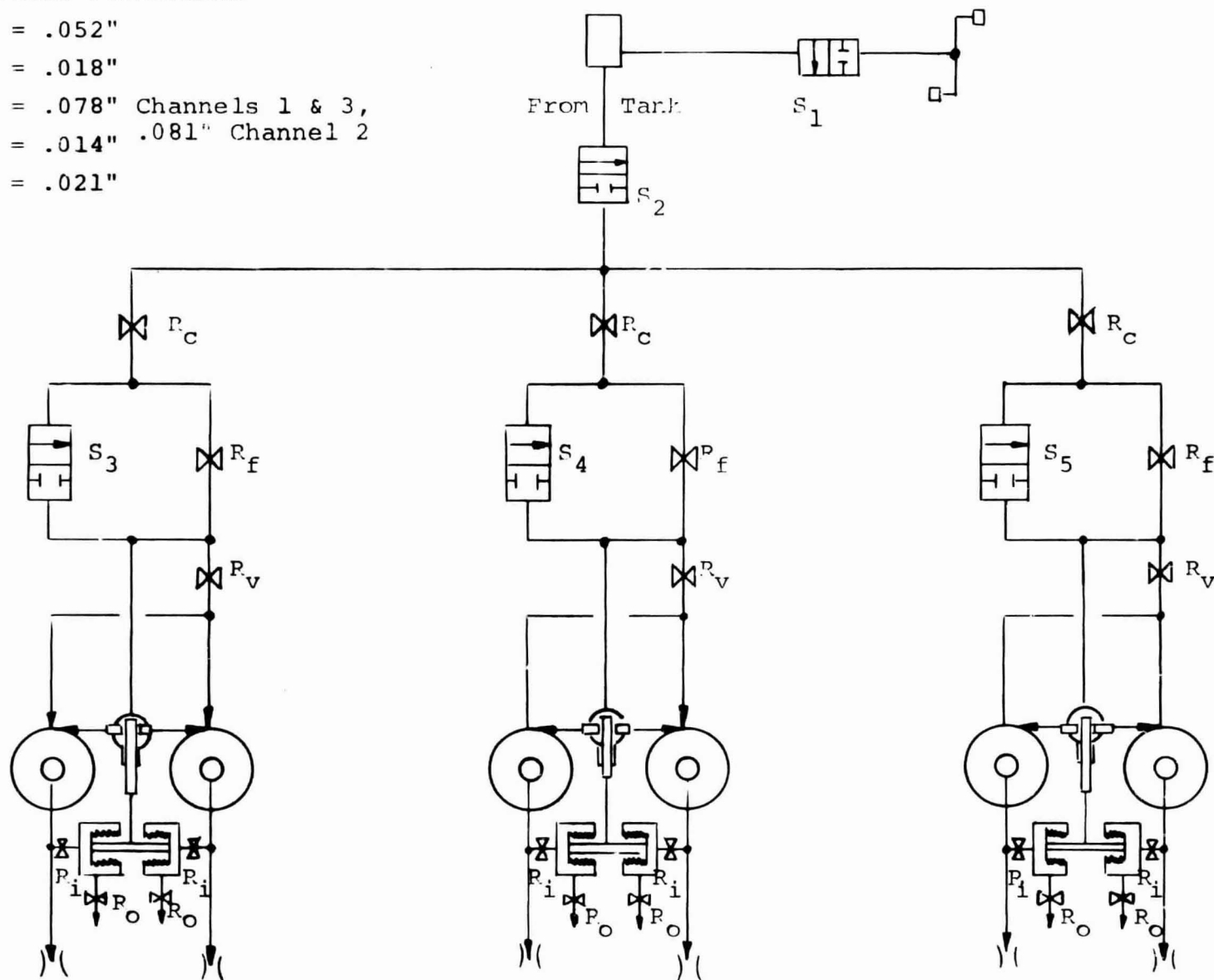


Figure 3

SPARCS Attitude Control Package Circuit Diagram

vortex valve thrusters, and despin switching valve.

Since the vehicle is spun up in a specific orientation, only one valve and a pair of reaction nozzles is required for despin. In Figure 3, the normally closed solenoid valve S_1 is energized to despin the vehicle. When despin is completed, valve S_1 is de-energized and solenoid valve S_2 is turned on to port supply pressure to the three thruster channels. At this time, the coarse control solenoid valves S_3 , S_4 and S_5 are also energized to bypass flow in each channel around its fine control dropping orifice. Coarse control valves are used in each axis to provide the required capability of independent coarse mode operation of each channel. When coarse acquisition is achieved the electrical command signals to solenoid valves S_3 , S_4 and S_5 are removed and the valves are closed. This action reduces the dropping orifice area by 10:1 thus reducing the available thrust and flow consumption for fine control.

An unregulated gas supply from the tank supplies the control package. The fluidic proportional thrusters consist of a pair of vortex valves controlled on push-pull by an electromagnetic torque-motor driven flapper-nozzle and produce a differential thrust proportional to the electrical input signal.

The effects of null drift, hysteresis and nonlinear characteristics of the torque-motor, flapper-nozzle and vortex valves were minimized by applying thrust feedback from the vortex valve output to the torque motor. This feedback was lagged for stability and a bellows actuator provided the torque motor force feedback.

Figure 4 is a photograph of the developed hardware. The envelop of the control package is equivalent to an 8 inch diameter by 8 inch long cylinder; the complete control weight is 10 pounds 2 ounces. A subsequent study has shown that a somewhat simplified control scheme for a similar vehicle could be packaged in an envelope 5 x 5 x 3 inches at a weight of six pounds.

SYSTEM PERFORMANCE

The single most compelling reason for utilizing proportional reaction control system is the ultimate accuracy achievable as opposed to a bang-bang system. This accuracy is further achievable at force levels sufficient to overcome external disturbance torques. Gain distributions, dynamics, and control schemes adopted were designed specifically to take maximum advantage of the proportional system's accuracy potential (minimization of noise, hysteresis, etc.).

The accuracy performance of the system was demonstrated at NASA/ARC on both a three degree of freedom air bearing supported vehicle and on an analog computer. The analog computer simulated only the vehicle dynamics and sensor and the physical hardware was transduced into and out of the computer. The best demonstrated air bearing accuracy was .5 arcseconds rms and the best demonstrated analog accuracy was .05 arcseconds rms. It is suspected that the limit of accuracy of the air bearing was reached; disturbance torques due to

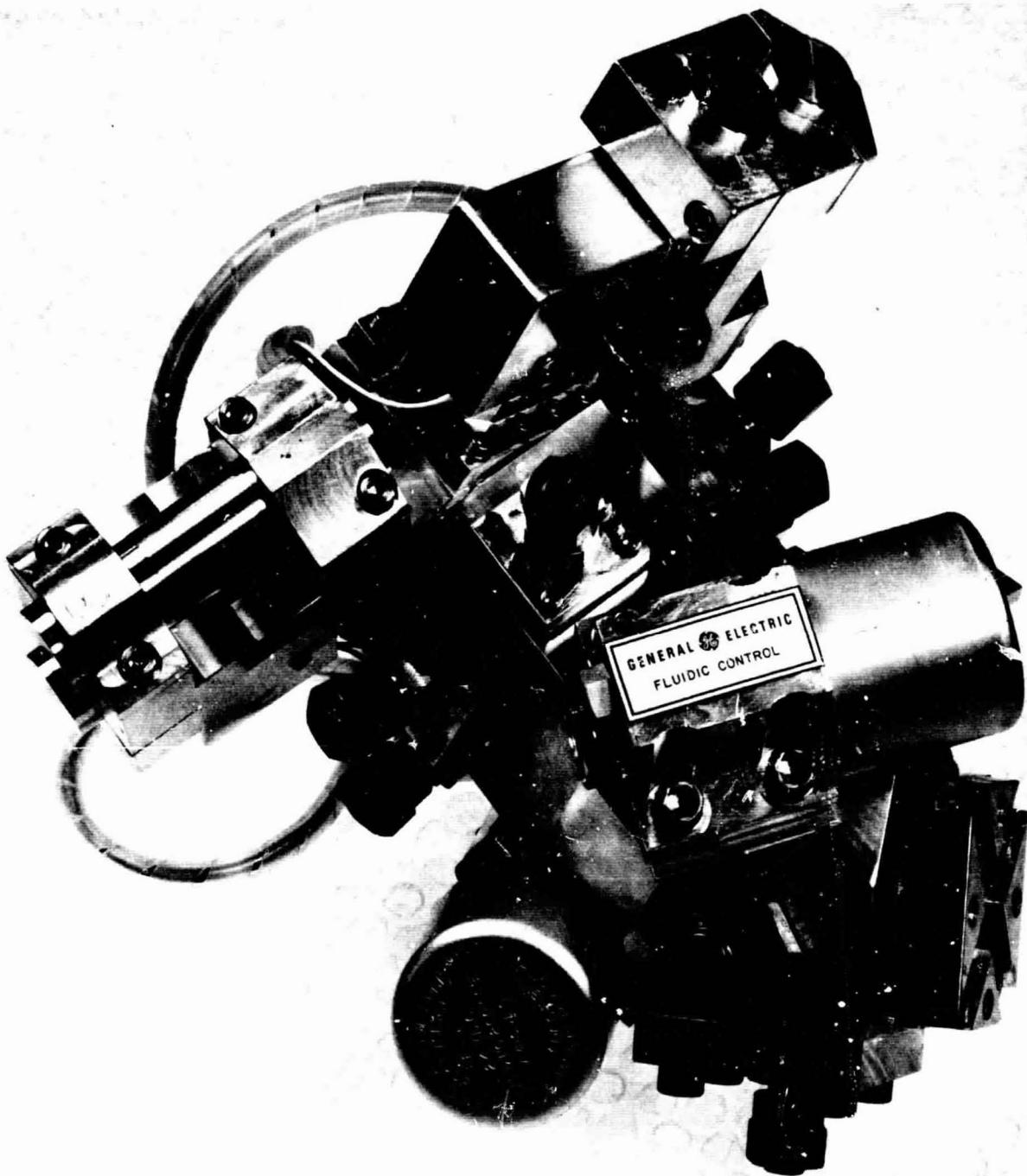


Figure 4
Fluidic Proportional Thruster

windage as well as electronic noise may be a future limit on the air bearing tests.

CONCLUSIONS AND RECOMMENDATIONS

The results of the program described in this report demonstrate the feasibility of a fluidic proportional control concept to provide the improved pointing accuracy required by the SPARCS missions. Specific conclusions resulting from these investigations and development work are as follows.

1. Vortex valves are ideally suited for proportional thrust control. They promise excellent potential application in hot gas reaction control systems where available quiescent flow is high and where high temperature and contamination must be tolerated.
2. Fluidic proportional thrust control with unregulated gas supply is feasible even for applications requiring very precise vehicle positioning.

In view of the conclusions reached, the following recommendations are made:

1. Development should be undertaken to adapt the fluidic proportional thruster concept for hot gas reaction control applications for larger payload vehicles.
2. Redesign of the fluidic proportional thruster should be made to obtain the potentials available for reduction in weight, size and gas utilization.
3. Application studies should be made to apply the fluidic proportional thruster concept to other attitude and vehicle stabilization systems. These may include V/STOL, hovercraft (air cushion vehicles) and high lift re-entry bodies. The concept is also applicable to other precise flow control requirements.

The Technical Discussion to follow will present detailed analyses and data concerning system performance, control mode design and implementation as well as the design of the FPT hardware.

TECHNICAL DISCUSSION

SYSTEM PERFORMANCE

Following delivery of the FPT hardware to NASA/Ames, both single axis hybrid (combination hardware and analog computer) as well as three axis air bearing simulations were conducted. The performance discussion which follows is a summary of the most pertinent test information obtained.

Hybrid Simulation Results

The pitch and yaw simulations were conducted prior to the air bearing tests. Referring to Figure 5, the differential pressure out of the thruster package was measured and converted to thrust through appropriate scale factors and applied as an input to the analog computer simulating the vehicle dynamics (two integrations) and the sun-sensor gain. The air bearing test electronic package was interfaced with the computer and drove the FPT. The gains from the thruster differential pressure, P_O , to the angular acceleration, $\ddot{\theta}$, were adjusted so that for $P_O = 20$ psid, then $\ddot{\theta} = .214 \text{ deg/sec}^2$.

Figure 6 shows the hybrid simulation results using thruster channel three with the yaw channel electronics. Figure 7 presents the results using thruster channel one with the pitch channel electronics. The hybrid peak to peak pointing accuracy was between .09 and .18 arcsec.

Air Bearing Vehicle Test Results

Figure 8 is a photograph of the air bearing test system and Figure 9 is a block diagram of the pitch (or yaw) control loop. The initial electronic gain, K , of Figure 9 was 6.5 v/v making the total open loop gain 8.74 per sec^2 . The primary test variable during the air bearing series of runs was the electronic gain K . The effect of variations in K on system accuracy are shown in Figure 10.

Figures 11 and 12 (Runs II-8 and II-12) show data slices taken during fine acquisition and holding. Run II-8 was taken with the telemetered sun sensor output coupled through a high pass filter designed to cut-off frequencies below 1/4 HZ. Run II-12 is essentially a duplicate run without the telemetry filtering. Addition of the high pass filter aided in identifying high frequency vehicle perturbations and separated the effects of low frequency external disturbance forces (cg. offsets, etc). Run II-12 is characteristic of all the unfiltered data which was taken. Very low frequency disturbances with a 10 to 15 second period would be superimposed on the high frequency changes. The amplitude of the low frequency disturbance varied inversely with the electronic gain.

For electronic gains exceeding 45 v/v the loop was unstable. This gain agrees with linear analysis results presented in a following section. The locus of roots contains unstable roots for electronic

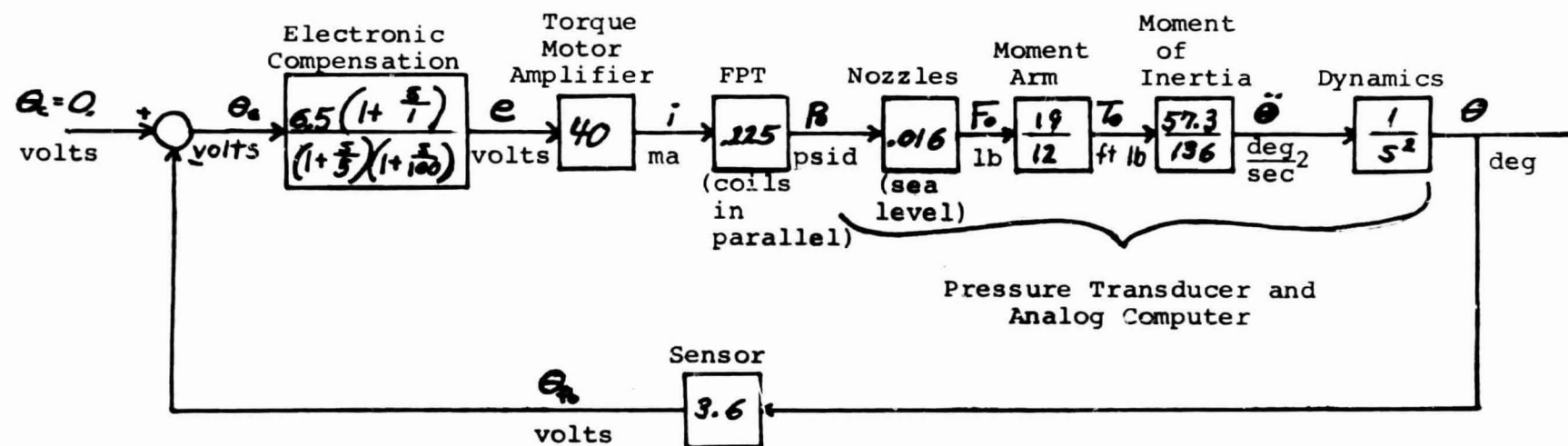


Figure 5
NASA/Ames Hybrid Simulation

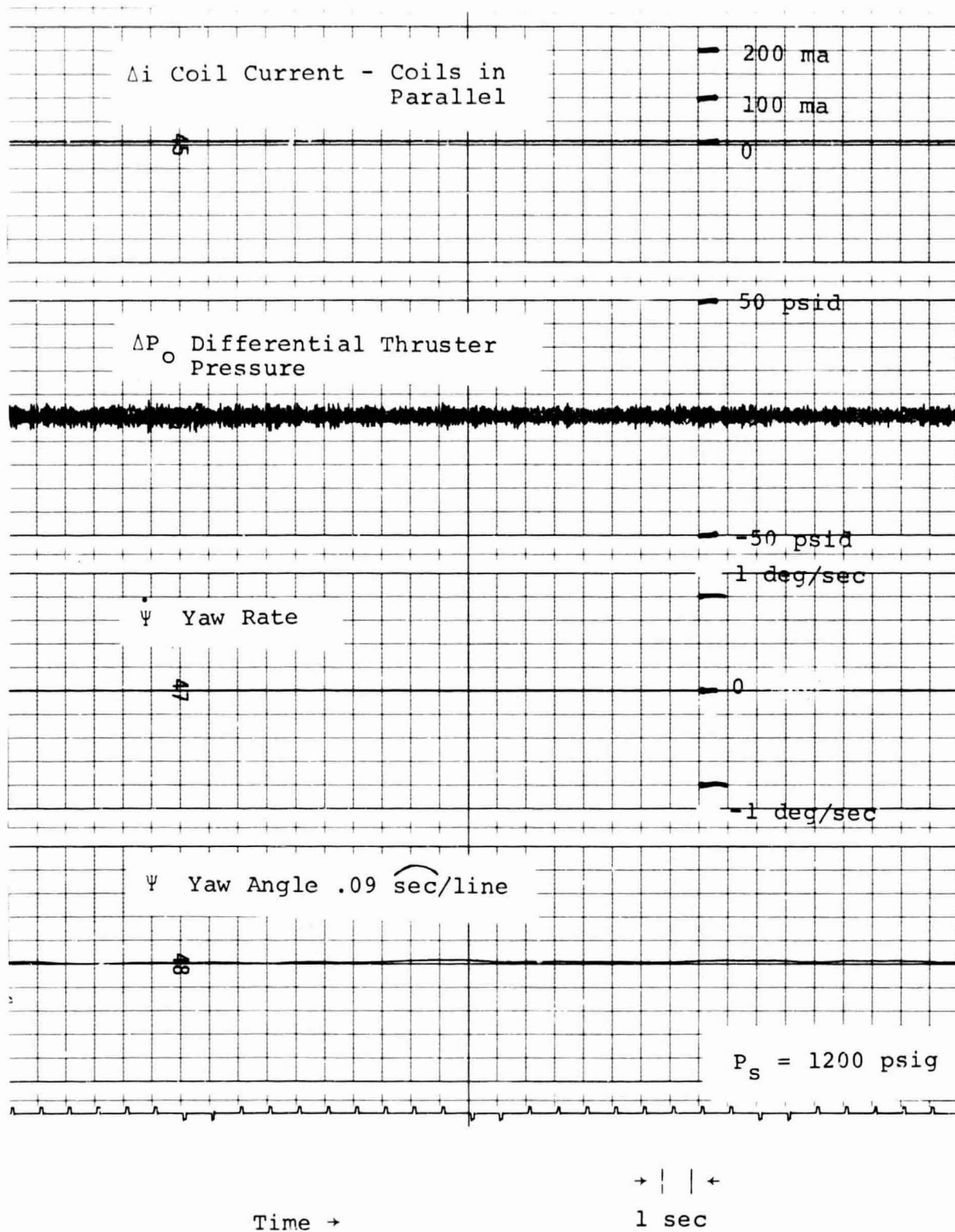


Figure 6
Hybrid Simulation Yaw Channel Results

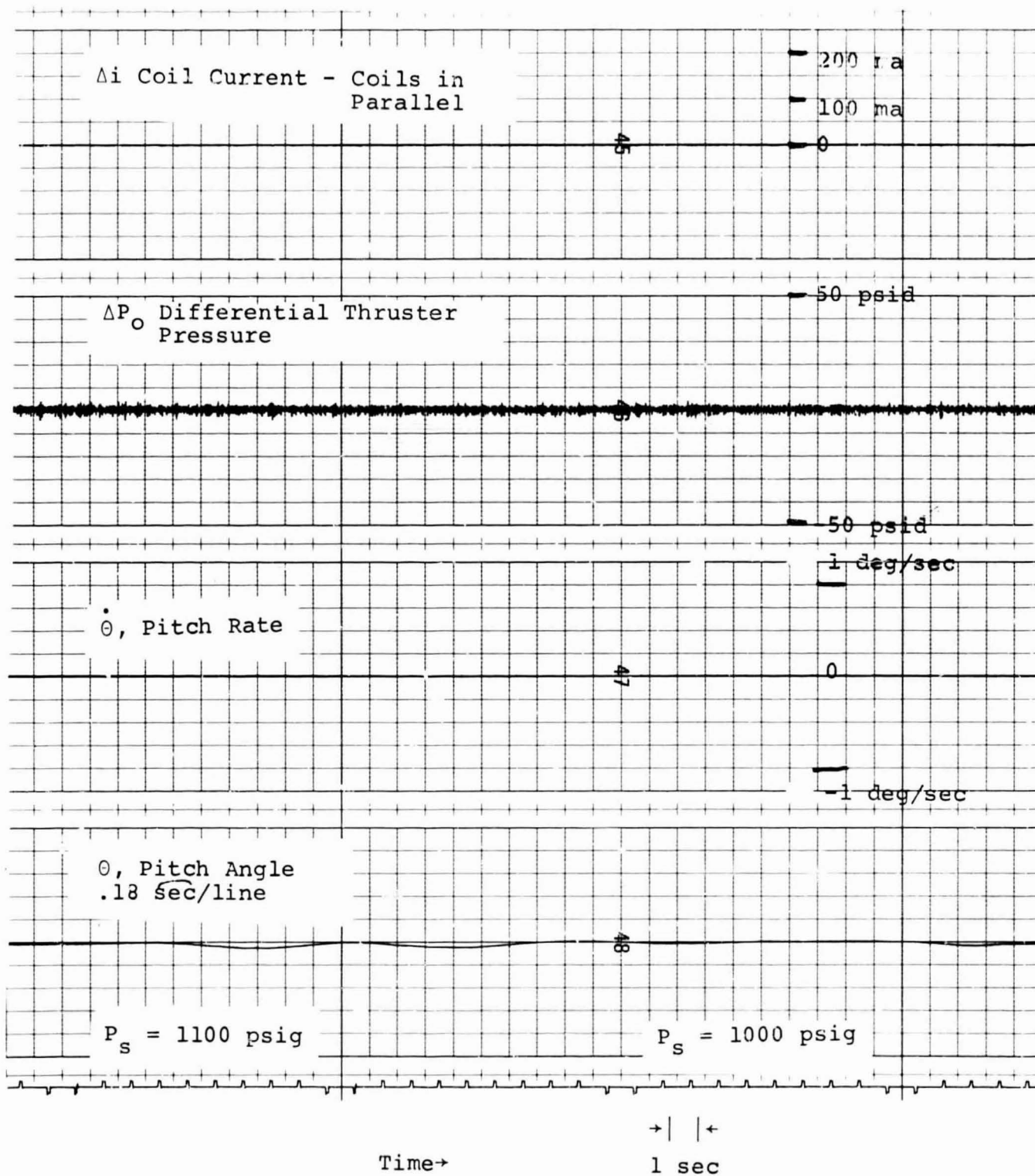


Figure 7
Hybrid Simulation Pitch Channel Results

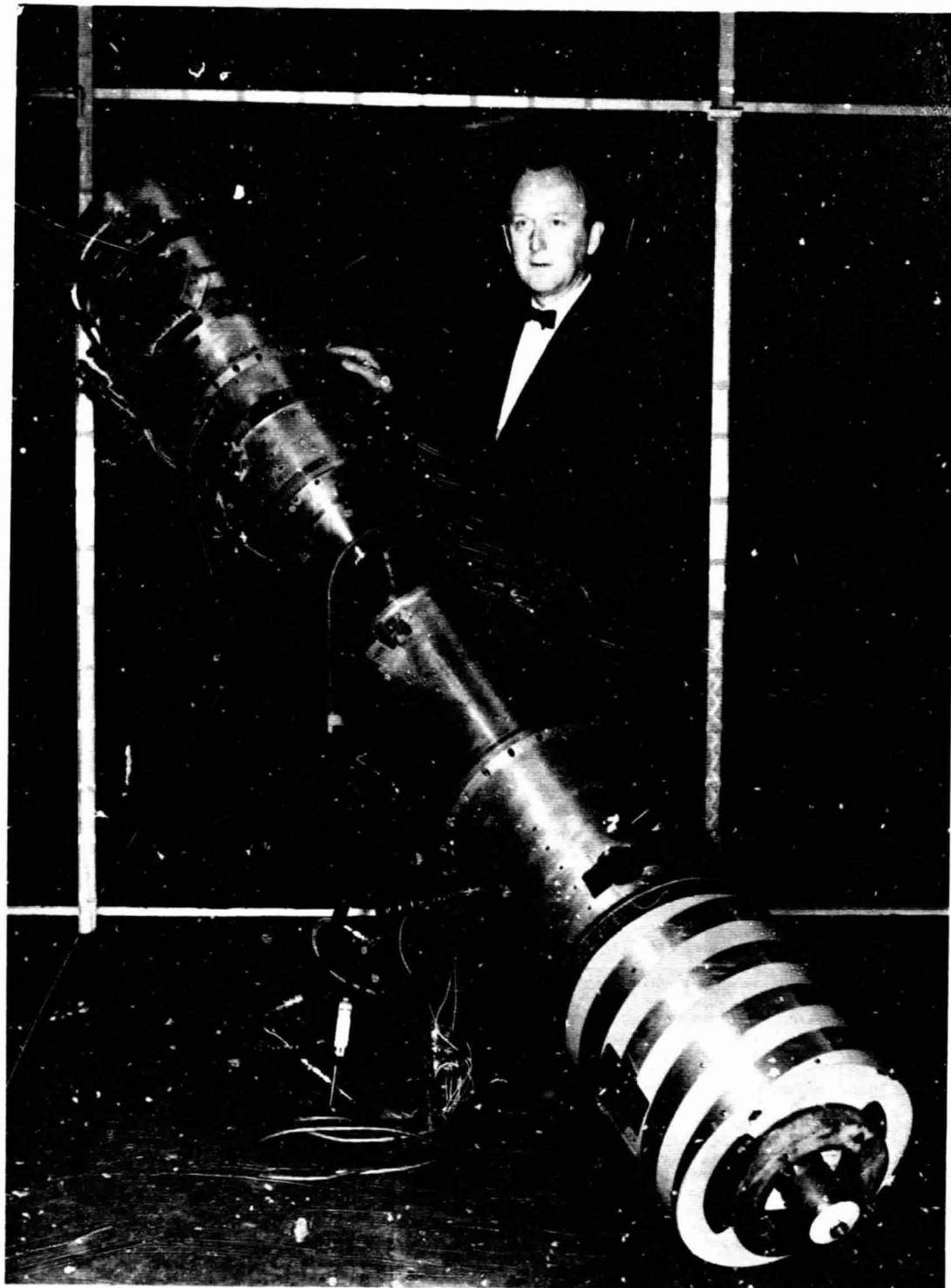


Figure 8
Air Bearing Test System

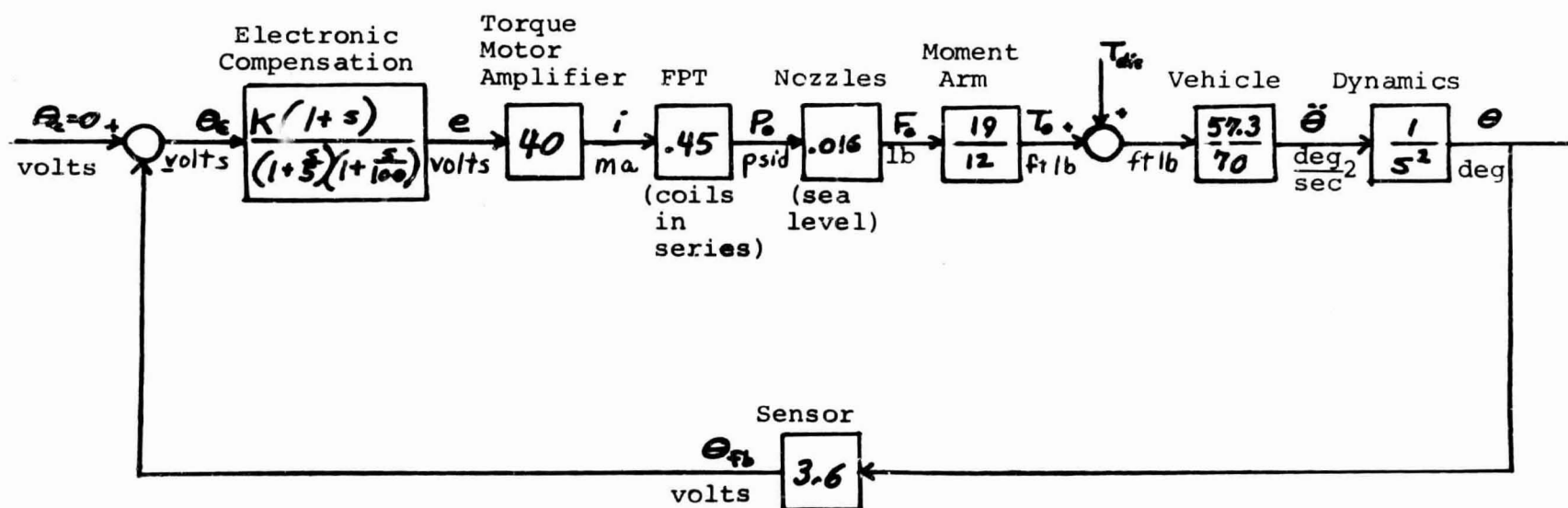
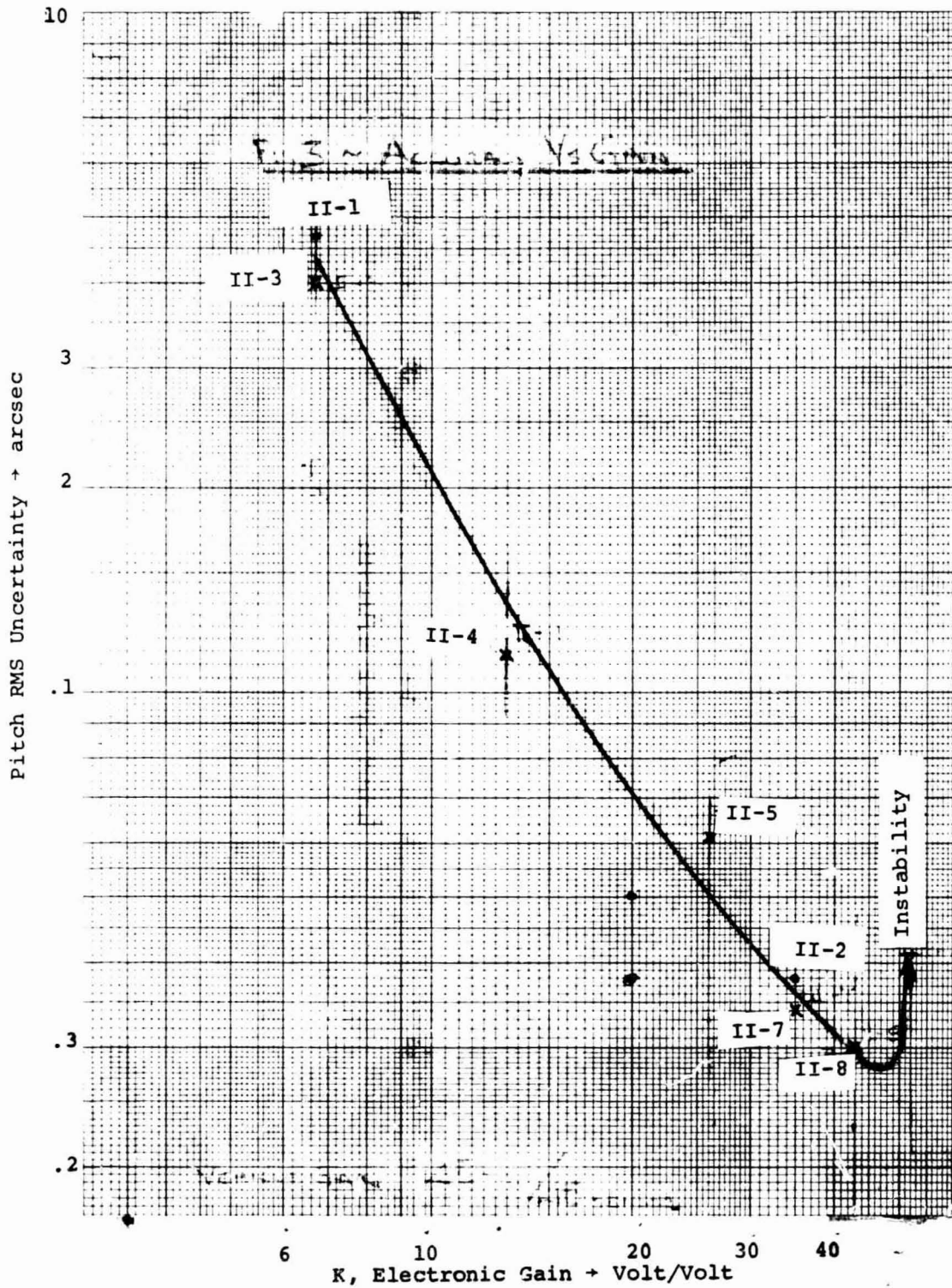


Figure 9
Air Bearing Supported Vehicle Pitch (or Yaw) Position Loop

Figure 10
Air Bearing Vehicle Accuracy vs. Electronic Gain



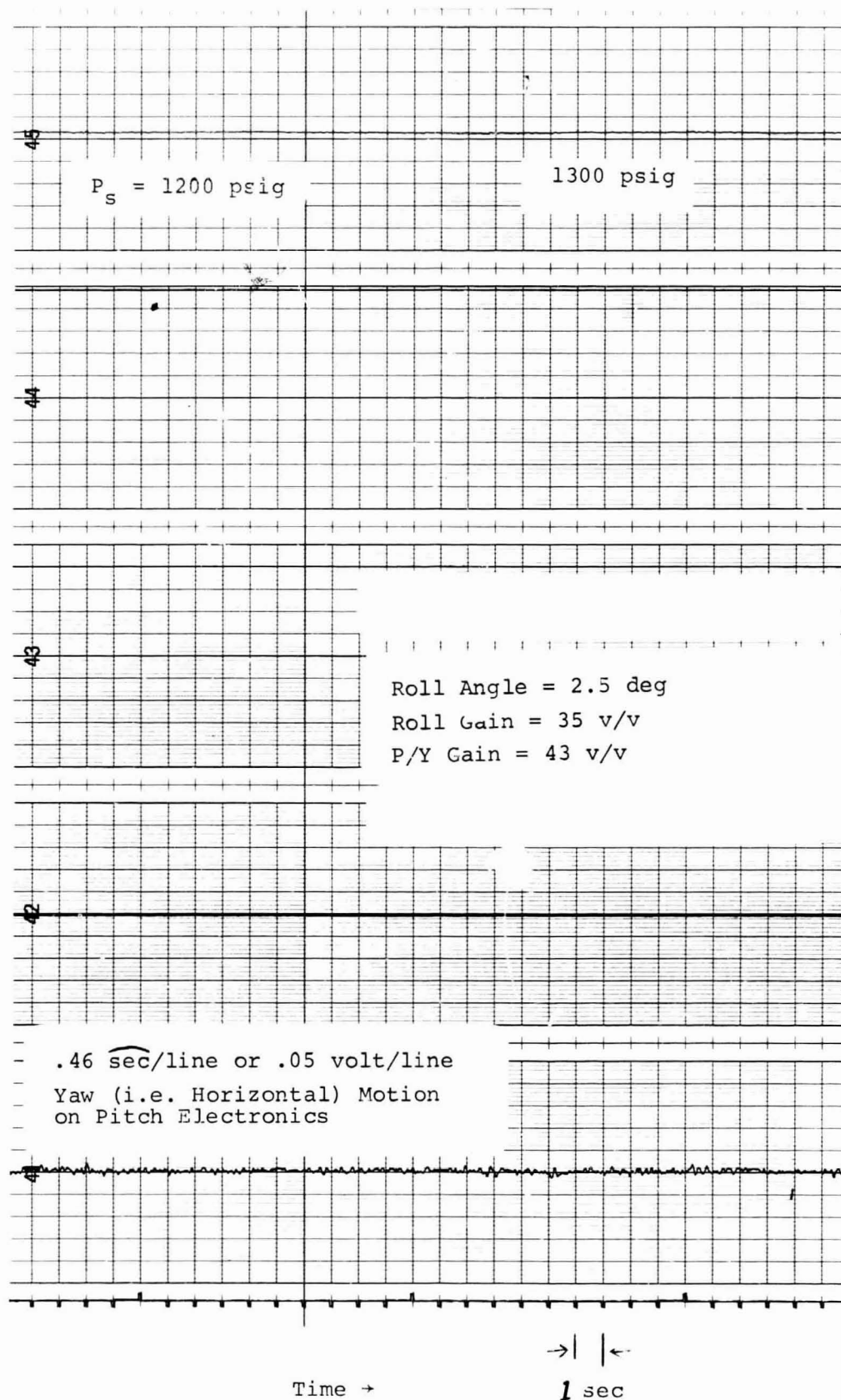


Figure 11
Air Bearing Results in Yaw with Output Filtered

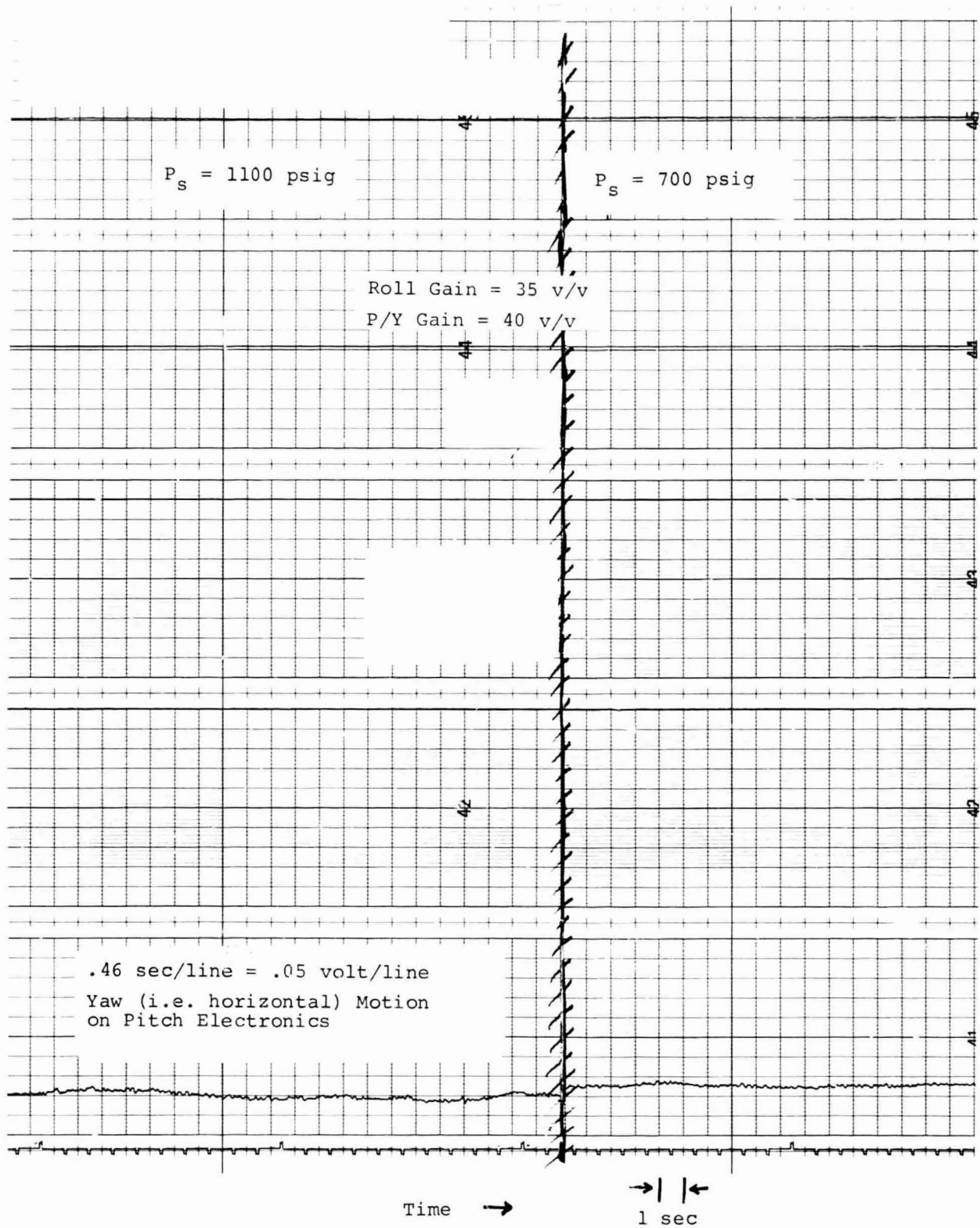


Figure 12
Air Bearing Results in Yaw with Output Unfiltered

gains exceeding 50 v/v.

The low frequency disturbances of Figure 11 are believed not to be system hysteresis since:

1. The disturbances did not appear on the analog.
2. Frequencies associated with hysteresis limit cycle are about four times those observed.
- and 3. Component calibration data (presented in a later section) shows that the hysteresis approaches zero for small signal operation about null.

A simple analysis of the effect of disturbance torques was made to determine the magnitude of possible air flow disturbances around the vehicle while on the air bearing. The tested system would be especially susceptible to aero-torques because of the large vehicle cross sectional area and moment arm to the center of pressure. In addition, the proportional system with its greater quiescent gas flow tends to agitate the surrounding air. After completion of the air bearing test, hot-wire anemometer surveys around the vehicle showed ambient air velocities as high as 9 inches/sec. Air velocities of the order of 4 inches/sec cause aero-torques of sufficient magnitude to perturbate the vehicle up to 1 arcsec.

Additional testing by NASA/Ames also showed that a significant amount of simulator, sensor and electronic noise at a frequency of 3 to 5 cps was present which could possibly explain the high frequency perturbations of Figures 11 and 12.

CONTROL MODE DESIGN

Coarse Control

This report section concerns the synthesis and analysis of the vehicle control laws. The control problem can be broken into two distinct parts: A coarse control phase wherein the vehicle acquires a pointing attitude from angles as large as 90 degrees in pitch and yaw and a fine control phase wherein acquisition to sub-arcsecond errors from initial angles of .1 to 1 degree. The transition from coarse to fine may further be regarded as another control phase. Most efficient use of control gas is achieved if the large angle acquisition is accomplished bang-bang and the small angle acquisition proportional. During all control phases, rate information is generated from lead networks and rate gyros are not employed.

As discussed later, the control law selected uses two separate lead break frequencies; this approach reduced the time in coarse control to a maximum of 14 seconds.

Reduction of Initial Position Error: The coarse control problem is difficult because

1. the control torque to inertia ratio decreases as the supply pressure decreases (the FPT utilizes an unregulated supply).
- and 2. angular rate information is not directly available but must be generated electronically.

Concerning 1. above, most bang-bang control experience has been obtained with systems employing a fixed amplitude torque producer. With the FPT blowdown system, the angular acceleration could vary over a range of 7.6 to 4.2 deg/sec² which leads to significantly different phase plane trajectories*. The second reason simply reduces the number of possible control configurations which can be used.

First consider the two signals available for control:

1. Coarse sun sensor output, θ_{ss}
2. Lead network output, θ_{fb}

The location of these signals in the loop is shown in Figure 13. In Figure 13 the thruster loop, the electronic compensation, and the vehicle have been lumped together as a bang-bang switch since this is the way they should function during coarse control.

The output of the coarse sun sensor is:

$$1) \quad \theta_{ss} = 10 \sin\left(\frac{\theta\pi}{180}\right)$$

* The use of a regulated gas supply, of course, minimizes the control problems; however, the reliability increase obtained in any pneumatic system by elimination of the pressure regulator justified the search for a control approach which allowed blowdown operation.

Electronics, Thruster, and Vehicle
Equivalence

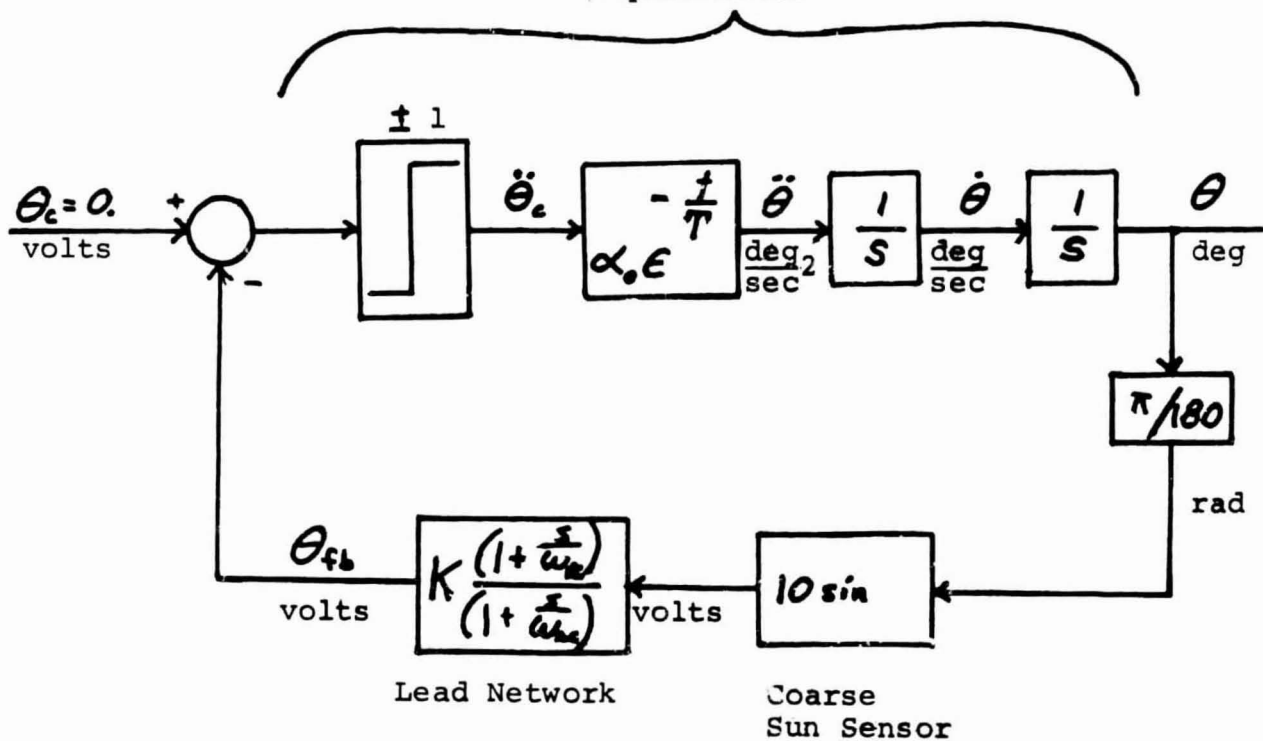


Figure 13
Pitch or Yaw Coarse Position Loop

As long as $\omega_2 \gg \omega_1$ the output of the lead network is approximately:

$$2) \quad \Theta_{fb} = K\dot{\Theta}_{ss} + \frac{K}{\omega_1} \ddot{\Theta}_{ss} = 10K \left[\sin\left(\frac{\Theta\pi}{180}\right) + \frac{\dot{\Theta}\pi}{180\omega_1} \cos\left(\frac{\Theta\pi}{180}\right) \right]$$

where the above variables are presented in Figure 13. It is the sign of Θ_{fb} which determines the direction of thrust in coarse control. The sign of Θ_{fb} is dependent on:

$$3) \quad \text{sgn}(\Theta_{fb}) = \text{sgn} \left(\sin\left(\frac{\pi\Theta}{180}\right) + \frac{\pi\dot{\Theta}}{180\omega_1} \cos\left(\frac{\pi\Theta}{180}\right) \right)$$

A useful way to determine the sign of Θ_{fb} is to plot equation 4.

$$4) \quad \sin\left(\frac{\pi\Theta}{180}\right) + \frac{\pi\dot{\Theta}}{180\omega_1} \cos\left(\frac{\pi\Theta}{180}\right) = 0.$$

or

$$4) \quad \dot{\Theta} = -\frac{180\omega_1}{\pi} \tan\left(\frac{\pi\Theta}{180}\right)$$

Equation 4 is plotted in the phase plane in Figure 14. Equation 4 represents a switching line. Note that for $|\Theta|$ less than 30 degrees the switching line is nearly straight. For larger values of $|\Theta|$ the switching line curves move more and more to infinity. This characteristic is undesirable because it makes control system performance (from large angle) very sensitive to the lead break frequency, ω_1 , and the torque to inertia ratio.

From Equations 1 and 2 the angular rate can be solved in terms of the lead network output and the coarse sun sensor output. The result is:

$$5) \quad \dot{\Theta} = \left[\frac{1}{1 - \left(\frac{\Theta_{ss}}{10}\right)^2} \right] \left[\frac{180\omega_1}{10K\pi} (\Theta_{fb} - K\Theta_{ss}) \right] \text{ deg/sec}$$

The right hand side of 5 is bracketed into two factors. The right factor contains the expression which is desirable to use in extracting rate information. The left factor is a correction term which must be applied to get true vehicle rate. It is equal to the secant of Θ so that Equation 5 can, also, be expressed as:

$$5) \quad \dot{\Theta} = [\sec \Theta] \left[\frac{180\omega_1}{10K\pi} (\Theta_{fb} - K\Theta_{ss}) \right] \text{ deg/sec}$$

Both forms of Equation 5 show that the vehicle rate cannot be extracted from only a linear combination of the outputs of the coarse sun sensor and the lead network. A correction factor which is a function of vehicle angle must be applied to get the rate whenever the angular error is large. This correction factor probably

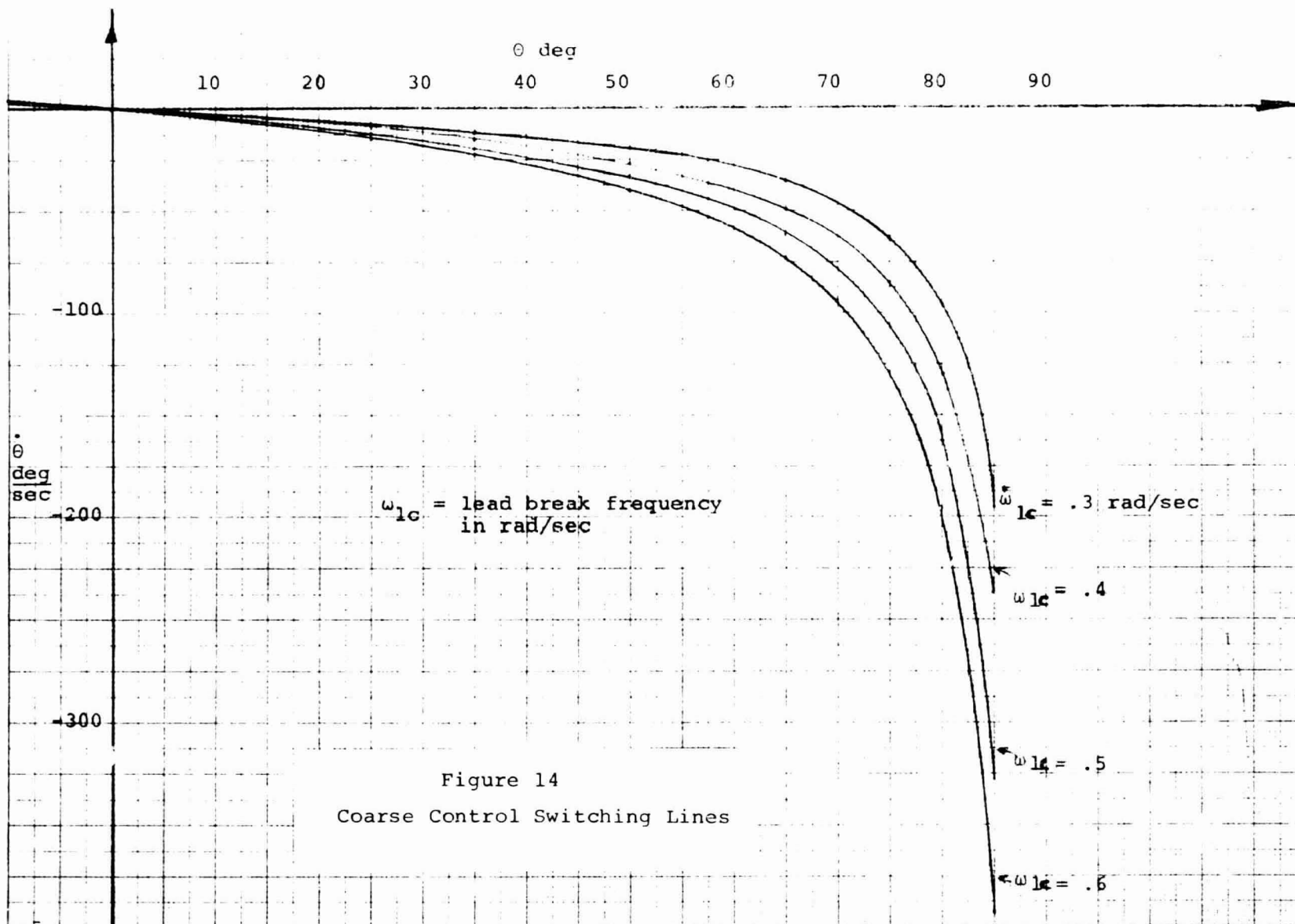


Figure 14
Coarse Control Switching Lines

would be provided by a function generator. The amount of complexity is undesirable, so a control approach which does not require rate at large angular errors is sought.

The first control approach was based on the position loop shown in Figure 13. A lead break frequency, ω_{1c} , was determined to optimize positioning from an initial position error of 90 degrees and zero tip-off rate. By optimum it is meant that the thruster was commanded to deliver two impulses equal in magnitude and opposite in sign. At the end of the second impulse the vehicle angle and rate were approximately zero, and the fine control could complete the positioning. Figure 15 shows a representative time history for torque to inertia ratio, rate, and angle for optimum positioning. The torque to inertia ratio in this case was assumed to follow the tank blow down as a function of time. Because the blow down time constant during coarse control can be as low as 22 seconds and coarse control can last over 10 seconds, the resultant change in thrust must be considered.

Figure 16 shows the trajectory of Figure 13 plotted in the phase plane. Note that for an initial angle of 90 degrees the vehicle acquires a near zero angle in an optimum way. For an initial angle of 60 degrees the phase plane trajectory reaches the switching line sooner than it should for optimum control. As a result, the phase plane trajectory is not optimum, and it creeps up the switching line. This can cause a longer acquisition time than resulted from the 90 degree initial angle.

The expressions for the switching line and phase plane trajectories in Figure 6 are:

$$6) \quad \dot{\theta} = -\frac{180\omega_{1c}}{\pi} \tan\left(\frac{\pi\theta}{180}\right) \quad \text{Switching line}$$

$$7) \quad \theta = T(\dot{\theta}_0 - \dot{\theta}) + T(\dot{\theta}_0 - \alpha_0 T) \ln\left(\frac{\alpha_0 T}{\dot{\theta} - \dot{\theta}_0 + \alpha_0 T}\right) + \theta_0 \quad \text{phase plane trajectory}$$

where: ω_{1c} = coarse lead break frequency in rad/sec

$\dot{\theta}$ = rate in deg/sec

θ = angle in degrees

T = blow down time constant in seconds

$\dot{\theta}_0$ = value of rate at last switch time

θ_0 = value of angle at last switch time

α_0 = value of torque to inertia ratio at last switch time

$$8) \quad \alpha_0 = \alpha_{00} e^{-t/T} \quad \text{deg/sec}^2$$

where t is the value of time at the last switch.

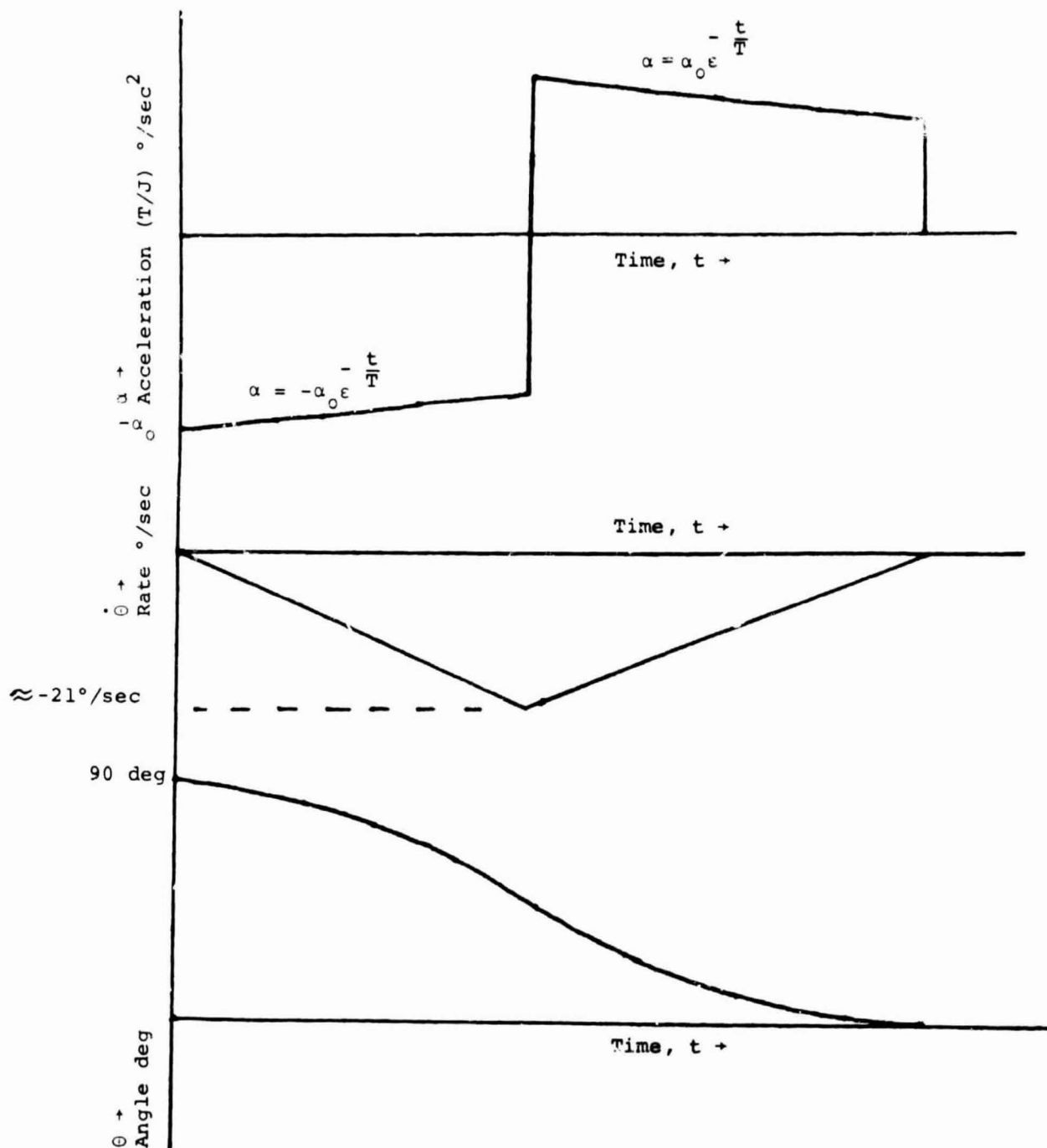
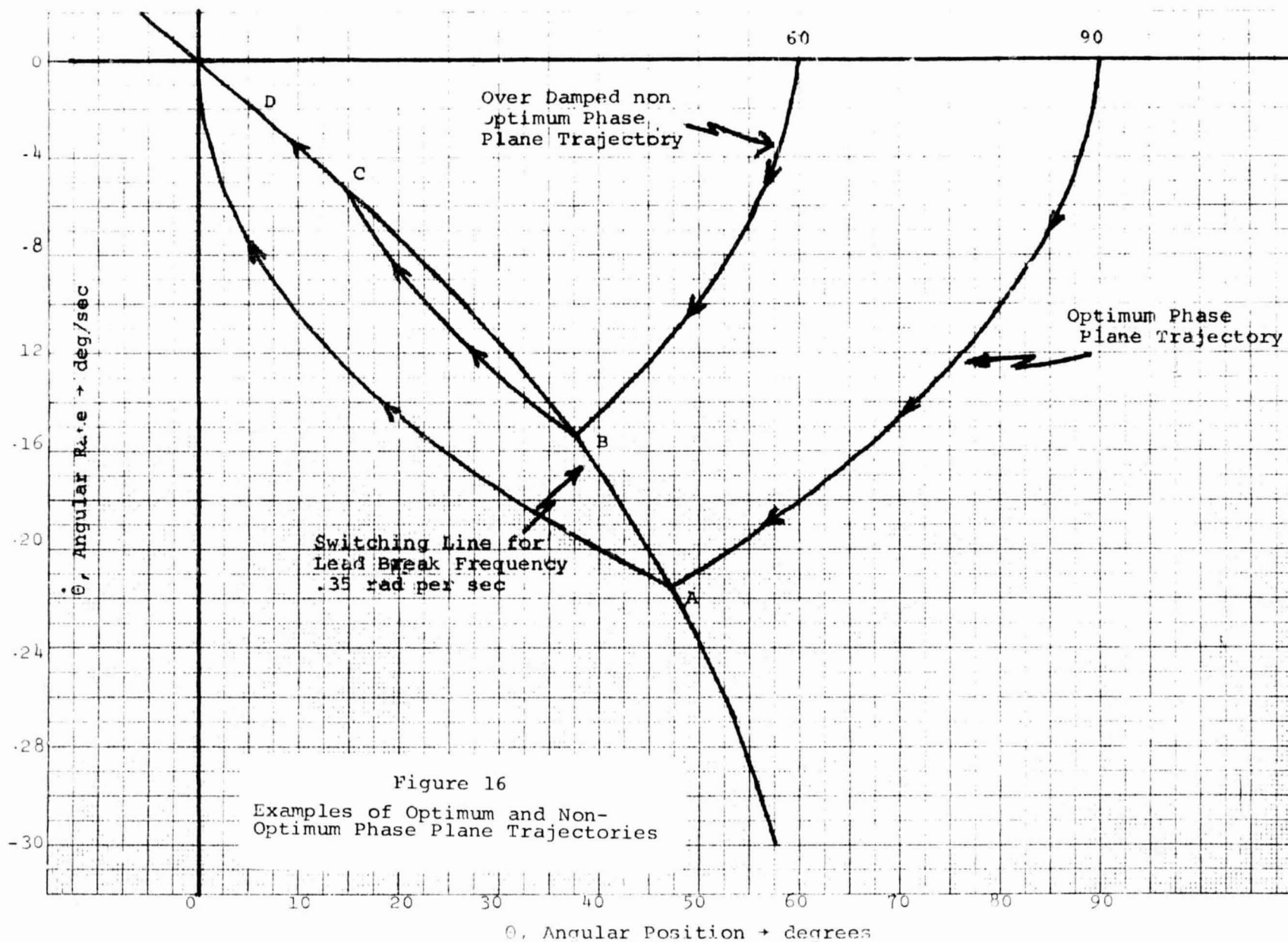


Figure 15
Torque to Inertia Ratio, Rate, and Angle
For Optimum Positioning



The time to go from one switch to the next switch is:

$$9) \quad t = T \ln \left(\frac{\alpha_0 T}{\dot{\theta} - \dot{\theta}_0 + \alpha_0 T} \right)$$

The length of time to creep up the switching line from a point C to a point D is given by:

$$10) \quad \Delta t \approx \frac{1}{\omega_{1C}} \ln \left(\frac{\theta_C}{\theta_D} \right)$$

Because vehicle performance during coarse control depends on the initial angle, the value of torque to inertia ratio when acquisition begins, and the switching line, a digital computer program was written to solve for the optimum switching line. The program computes the optimum coarse control lead break frequency for any particular pair of initial angle and torque to inertia ratio. That value of lead break frequency causes the vehicle to follow a trajectory in the phase plane which brings it close to zero angle and rate at the end of the second control thrust. The results are plotted in Figures 17 and 18.

The first control approach was based on selecting a coarse control lead break frequency which optimized positioning from an initial angle of 90 degrees. That approach led to poor performance when the initial angle was less than 90 degrees. The second control approach was based on a two step procedure. These steps were:

1. For any angle greater than 50 degrees the thruster would be full on in a direction which would reduce the angle.
2. When the angle reached 50 degrees, the angular rate would be estimated and an appropriate coarse control lead break frequency switched into the lead network which would then take over the control function.

A digital computer program provided the optimum coarse control lead break frequency along with angular rate when the angle is 50 degrees. The results are shown in Figure 19. Figure 19 suggests that an optimum lead break frequency may be chosen only if the torque to inertia ratio is known. If this were implemented, then three values of lead break frequency could be selected. For example, suppose the torque to inertia ratio were held at 6.28 deg/sec². Then the following selection might be made.

$$\omega_{1C} = .35 \text{ rad/sec for } |\theta| > 18.5 \text{ deg/sec}$$

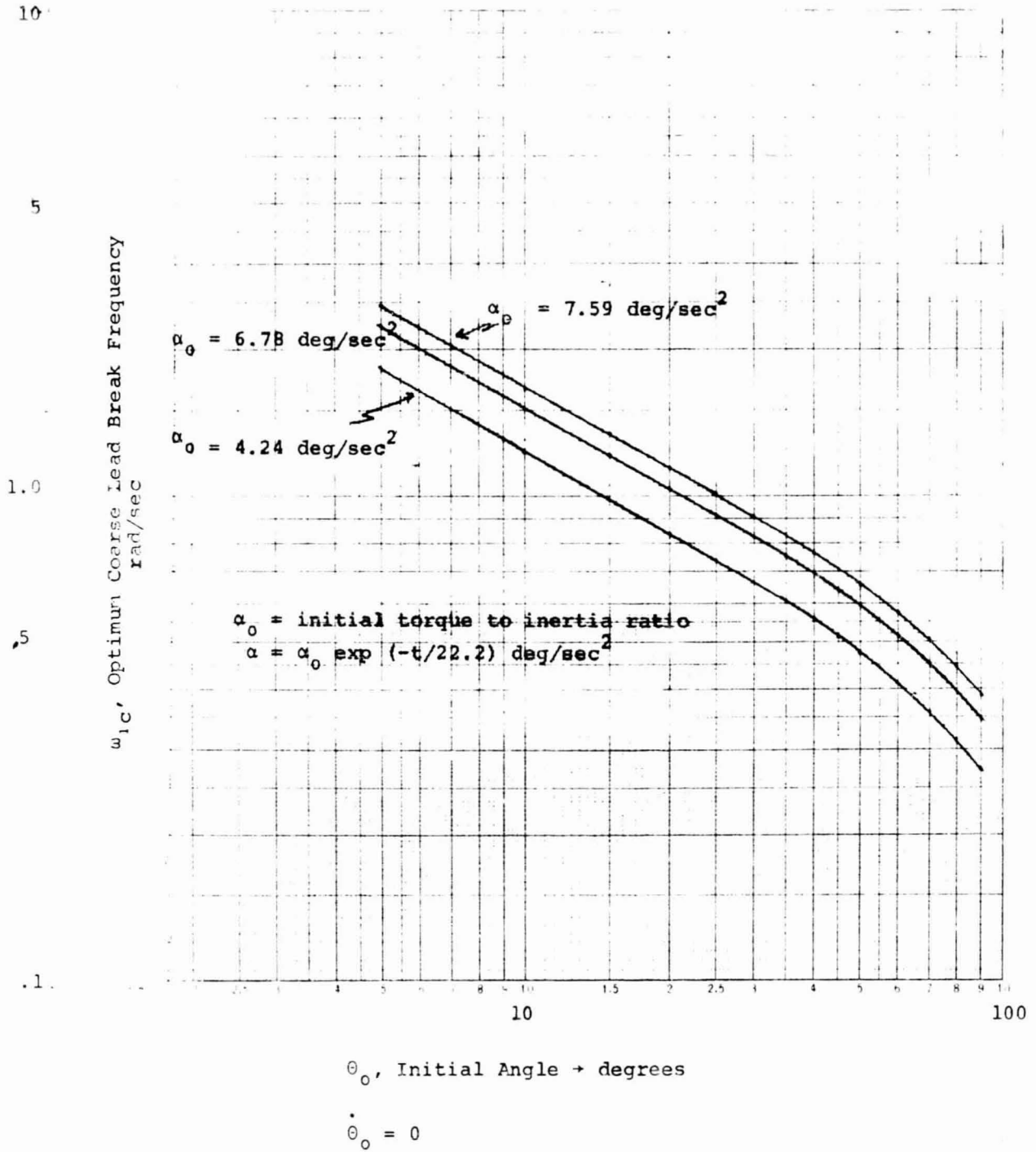
$$|\theta| = 50 \text{ deg}$$

$$\omega_{1C} = .40 \text{ rad/sec for } 12.5 \leq |\theta| \leq 18.5 \text{ deg/sec}$$

$$|\theta| = 50 \text{ deg}$$

Figure 17

Optimum Coarse Lead Break Frequency
vs. Initial Angle



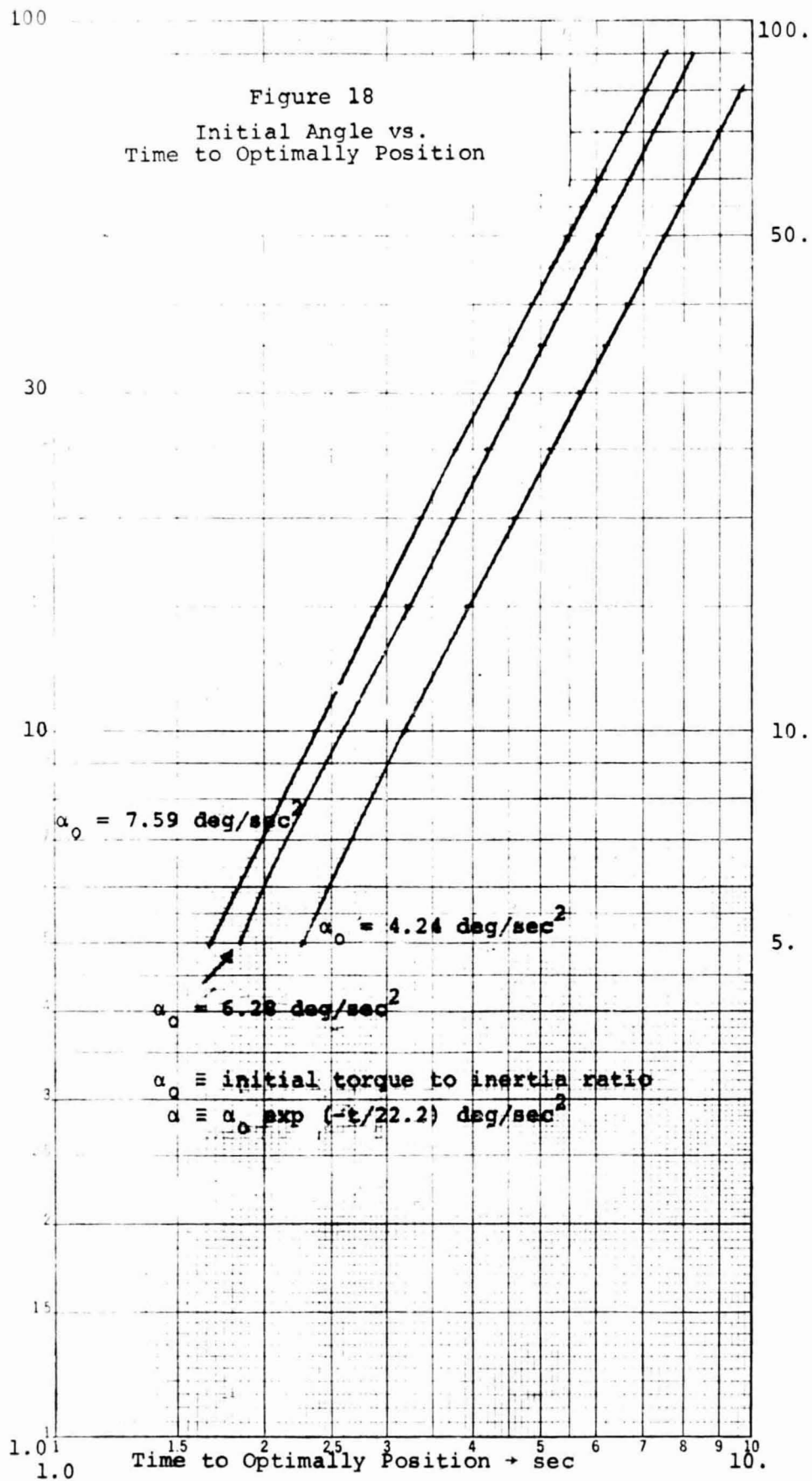
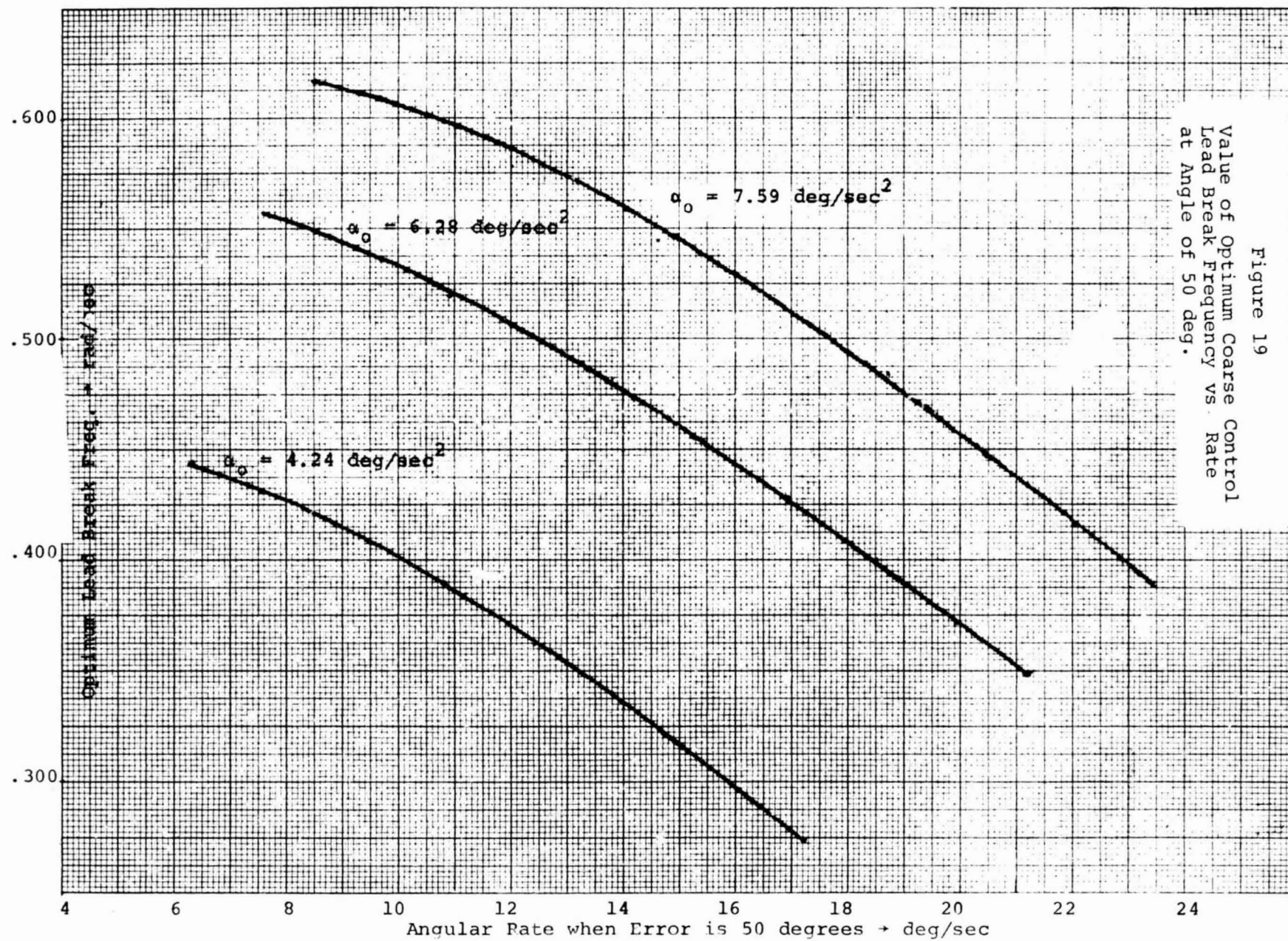


Figure 19
Value of Optimum Coarse Control
Lead Break Frequency vs. Rate
at Angle of 50 deg.



$$\omega_{1c} = .50 \text{ rad/sec for } |\dot{\theta}| < 12.5 \text{ deg/sec}$$

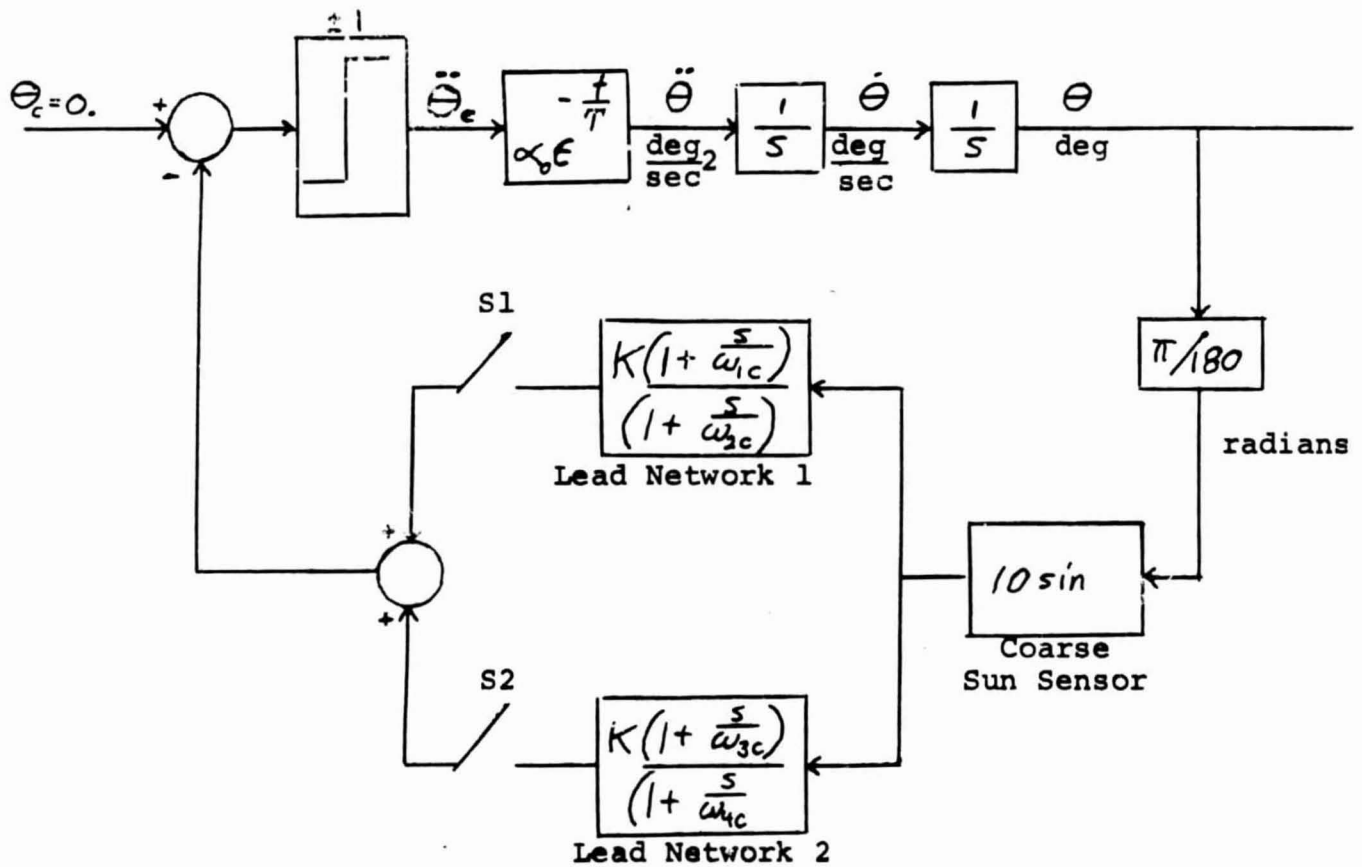
$$|\theta| = 50 \text{ deg.}$$

The Selected Coarse Control Mode: The control approach finally selected uses two distinct values of ω_{1c} during coarse control. The value of ω_{1c} used depended on the value of the pitch angle. Figure 20 shows a schematic representation. For example, if the pitch angle exceeded 9° , ω_{1c} might be set at .24 rad/sec. Such a value introduces enough rate damping to cause the phase plane trajectory to exhibit creeping for small angles, i.e. angles of 9° or less. Such a trajectory appears in Figure 21 where the switching line used for $\theta > \theta_s$ corresponds to the switching line that would result for $\omega_{1c} = .24$ rad/sec. Although creeping along a switching line is wasteful of gas, it is not altogether bad. It allows an accurate estimate of angular rate at any angle where creeping occurs. The next step is to switch in a new switching line which enables the vehicle to be near optimally positioned from the set of known initial conditions made possible by the creeping situation. This is shown in Figure 21 as the steep switching line along with the affect it has on the phase plane trajectory. For example, at an angle θ_s of 9° , ω_{1c} might be increased to 1.1 rad/sec. The larger value of ω_{1c} reduces the amount of rate feedback and allows the phase plane trajectory to stop creeping and approach the desired angle of zero degrees quickly.

To evaluate the effectiveness of using two values of coarse lead break frequency during coarse control a computer program was developed which computed the time to acquire to small angles by solving the phase plane trajectory equations. Figure 22 presents the time to acquire from initial angles between 10° and 90° using a control based on Figure 13. Coarse lead break frequencies of .24, .50, and .70 rad/sec were considered with initial torque to inertia ratios of 4.9 and 8.76 deg/sec². The torque to inertia ratio was assumed to exponentially decay with a 22.2 second blow down time constant. Figure 22 shows that if initial torque to inertia ratios are between 4.9 and 8.76 deg/sec², then acquisition from some initial angles will take at least 23 sec regardless of which coarse lead break frequency is selected. Figure 23 shows that the acquisition time does not exceed 14 seconds if two coarse lead break frequencies are used. Figure 23 is based on the control scheme presented in Figure 20. The data points for Figure 23 were computed by a digital computer program and verified by analog computer runs conducted by NASA/ARC.

The use of two coarse lead break frequencies during coarse control reduces the maximum acquisition time from perhaps 23 seconds down to 14 seconds. This reduction in time is important because it results in a large saving in total impulse which can be translated into reduced bottled gas weight and volume.

Electronics, Thruster, and Vehicle
Equivalence



S1 Open S2 Closed if $\theta < 9$ deg.
S1 Closed S2 Open if $\theta > 9$ deg.

Figure 20
Pitch or Yaw Coarse Position
Loop with Two Switching
Lines

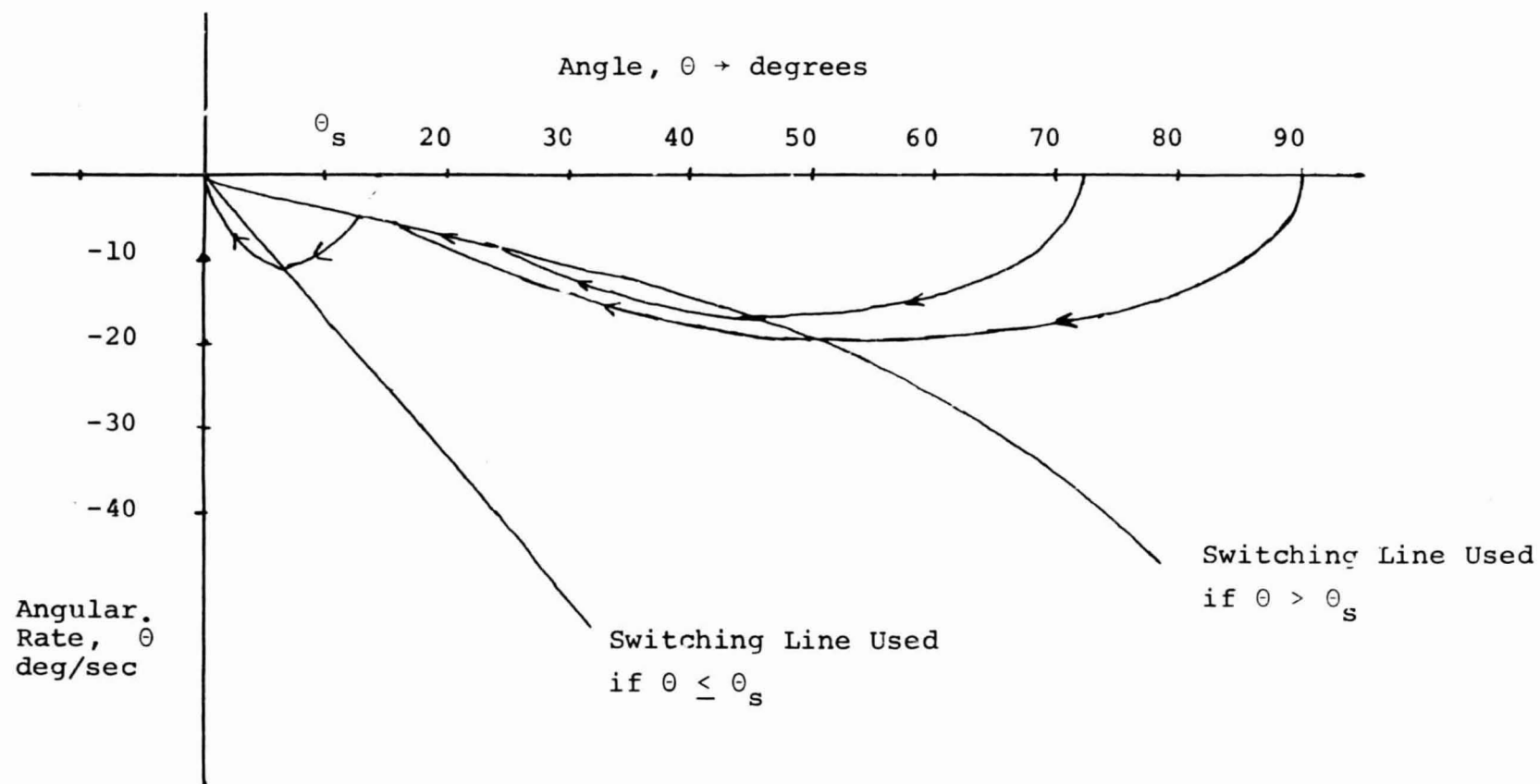


Figure 21
Phase Plane Trajectories
with Two Switching Lines

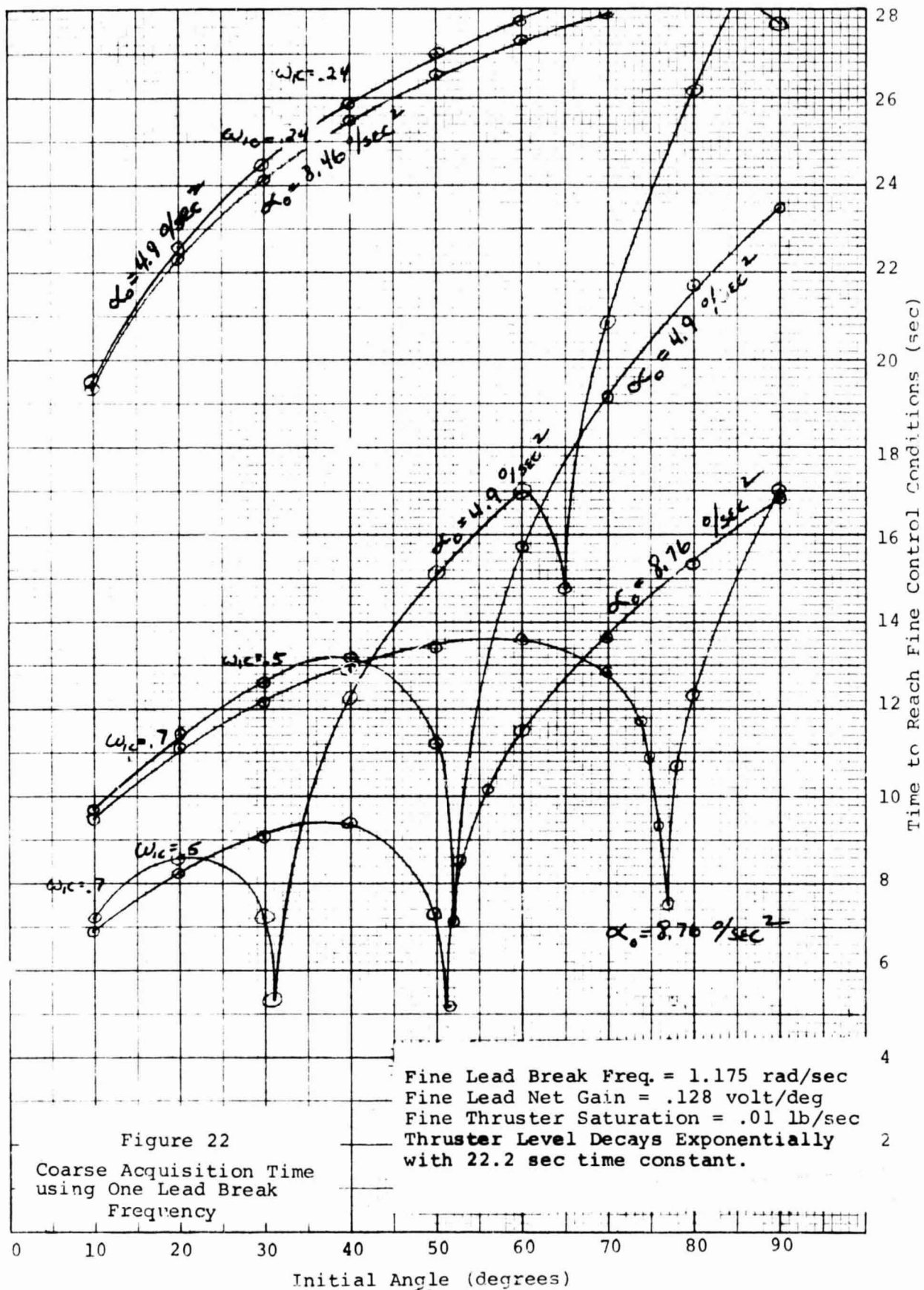


Figure 22
Coarse Acquisition Time
using One Lead Break
Frequency

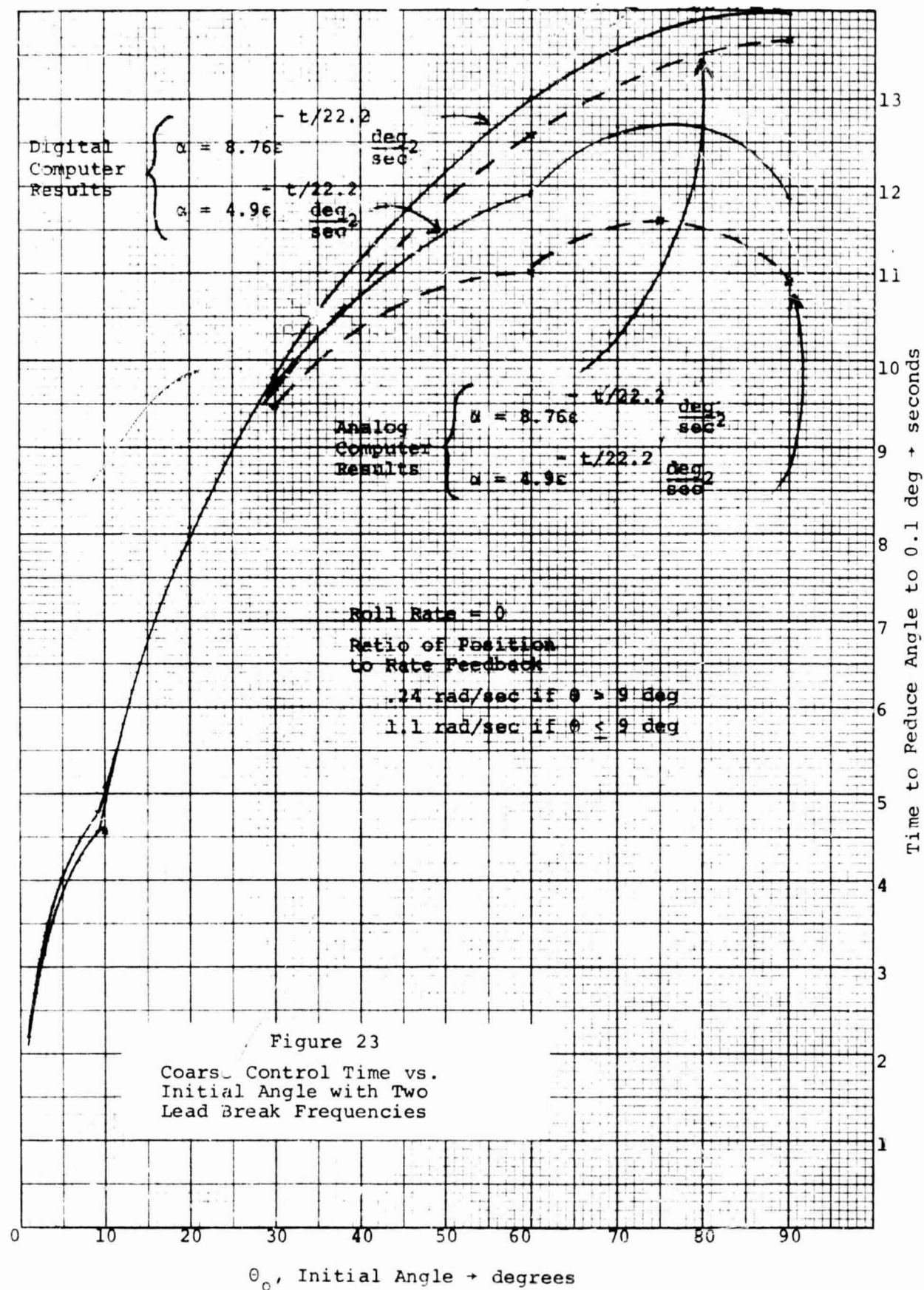


Figure 23
Coars. Control Time vs.
Initial Angle with Two
Lead Break Frequencies

Fine Control Position Loop Response

Pitch and Yaw Axes: Figure 24 presents the key parameters which determine the pitch and yaw position loop linear response. The lower loop in Figure 24 is a simplification of the upper loop which is valid as long as the loop gain is low enough to keep the closed loop bandwidth much less than the discarded dynamic terms. The lead compensation break frequency, ω_1 , is equivalent to the ratio of position to rate feedback which would exist in a position loop which has an inner rate loop. The lead term is essential to insure the stability of the position loop.

The reciprocal of the lead compensation lead break frequency imposes a limit on the closed loop response. For example, a lead break frequency of 0.5 rad/sec would limit the closed loop response to no less than a $1/0.5 = 2$ second time constant. Therefore, fast response requires that ω_1 be as large as possible. The value of ω_1 for fine control was always in the vicinity of 1 rad/sec, so the closed loop time constant was in the vicinity of one second.

The electronic compensation included a low frequency lag, ω_B , which was intended to cancel the lead term in the FPT. This lag would be unnecessary if the FPT lead occurred at the value desired to stabilize the position loop.

Figures 25 and 26 present the locus of the roots of the closed loop characteristic equation of the top and lower loops of Figure 24. For low gains the two figures are nearly the same, but for higher gains Figure 25 shows that the upper loop in Figure 24 becomes unstable. Since the upper loop more closely represents the actual position control, it should represent cases involving high gains.

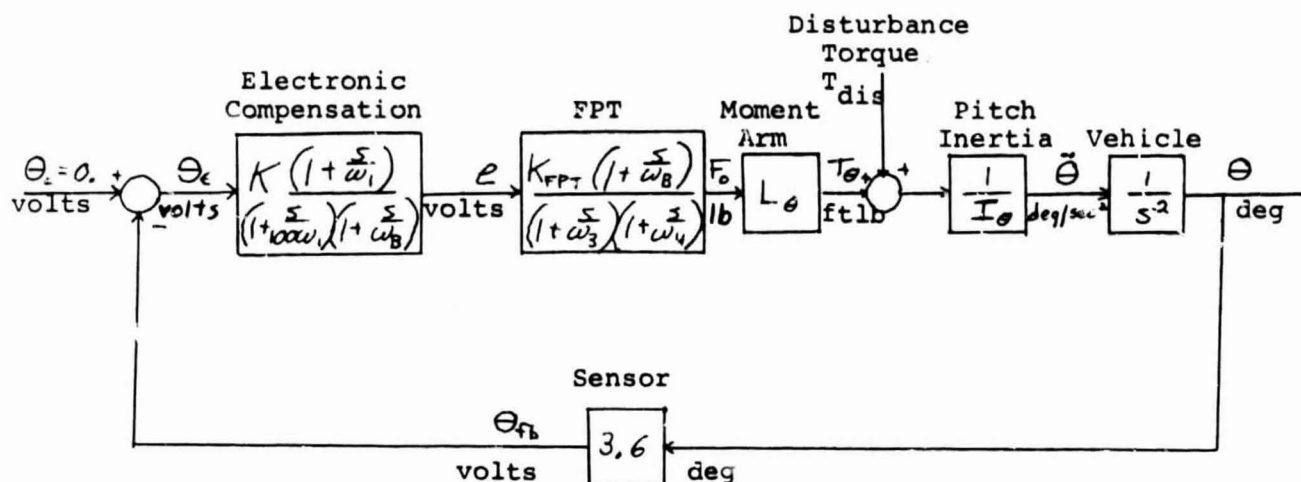
Using the air bearing test values listed previously and letting the electronic gain $K = 6.5$ v/v results in an open loop gain of:

$$\text{Loop Gain} = K K_{\text{FPT}} L_{\Theta} \frac{(3.6)}{I_{\Theta}} = 6.5 \times .288 \times 1.58 \times \frac{3.6}{1.22} = 8.74 \frac{\text{volt}}{\text{volt sec}^2}$$

The closed position loop transfer function of the upper loop of Figure 9 is:

$$\frac{\Theta}{\Theta_c} = \frac{.278 (1+s)}{\left(1 + \frac{s}{1.139}\right) \left(1 + \frac{2(.74)s}{14.7} + \frac{s^2}{14.7^2}\right) \left(1 + \frac{2(.99)s}{91.1} + \frac{s^2}{91.1^2}\right)} \quad \frac{\text{deg}}{\text{volt}}$$

The latter transfer function is misleading because the lead term implies the possibility of cancelling the effect of the low frequency lag. While this is possible, the response to a disturbance torque is more meaningful. Figure 24 shows where a disturbance torque would be applied. The transfer function from disturbance torque to pitch angle for an electronic gain of 6.5 volts per volt is given by:



Fine Control Pitch Position Loop

Air Bearing Values

$$K = 6.5 \text{ to } 35 \text{ volt/volt}$$

$$K_{FPT} = .288 \text{ lb/volt}$$

$$L_{\theta} = 1.58 \text{ ft}$$

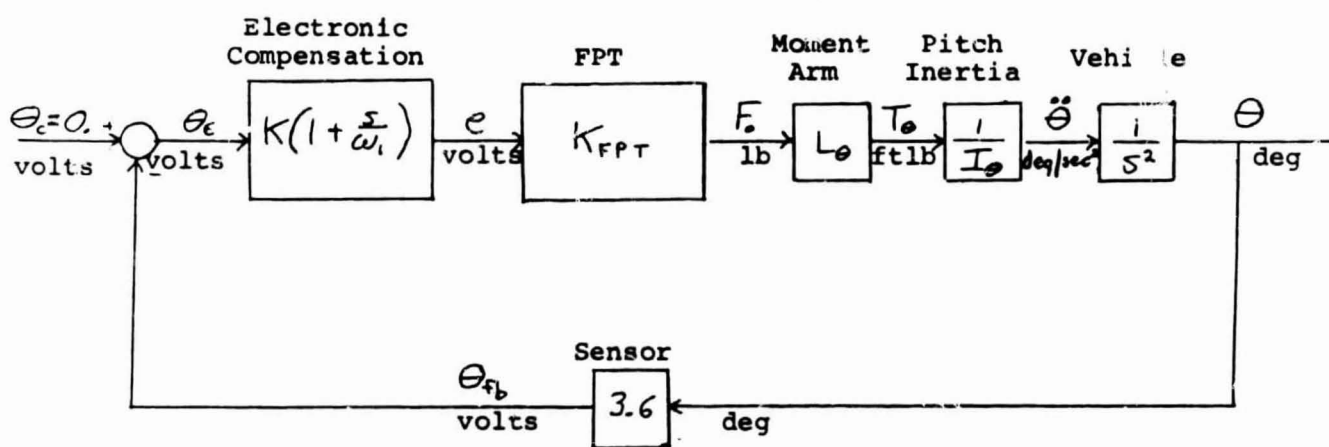
$$I_{\theta} = 70 \text{ slugft}^2 = 1.22 \text{ ftlb} \frac{\text{sec}^2}{\text{deg}}$$

$$\omega_1 = 1 \text{ rad/sec}$$

$$\omega_3 = 33.2 \text{ rad/sec}$$

$$\omega_4 = 70 \text{ rad/sec}$$

$$\omega_B = 5 \text{ rad/sec}$$



Simplified Fine Control Pitch Position Loop

Figure 24
Fine Pitch Control Position Loop

Figure 25
Pitch Closed Loop Root Locus
Applies to Top Loop of Figure 9

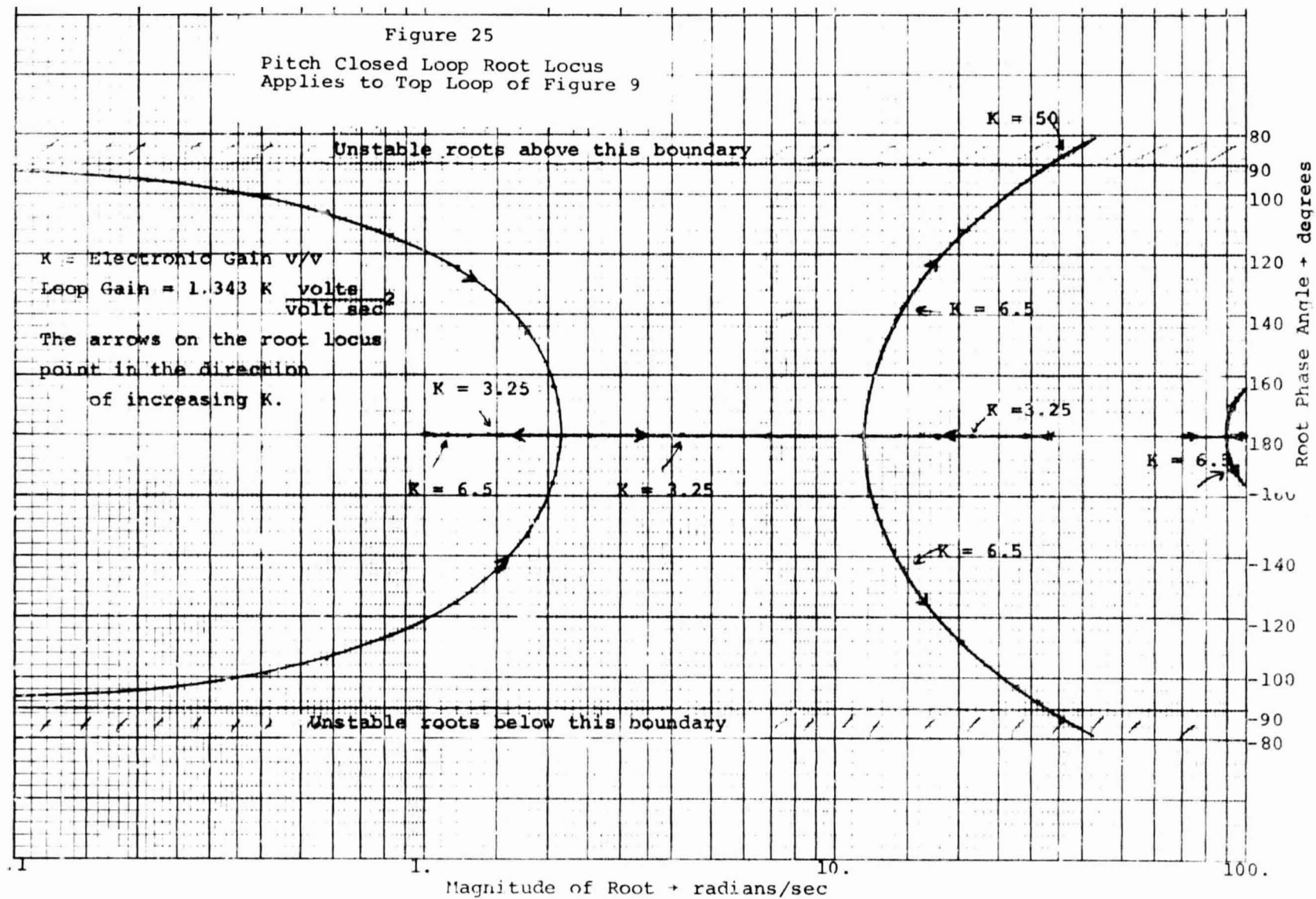
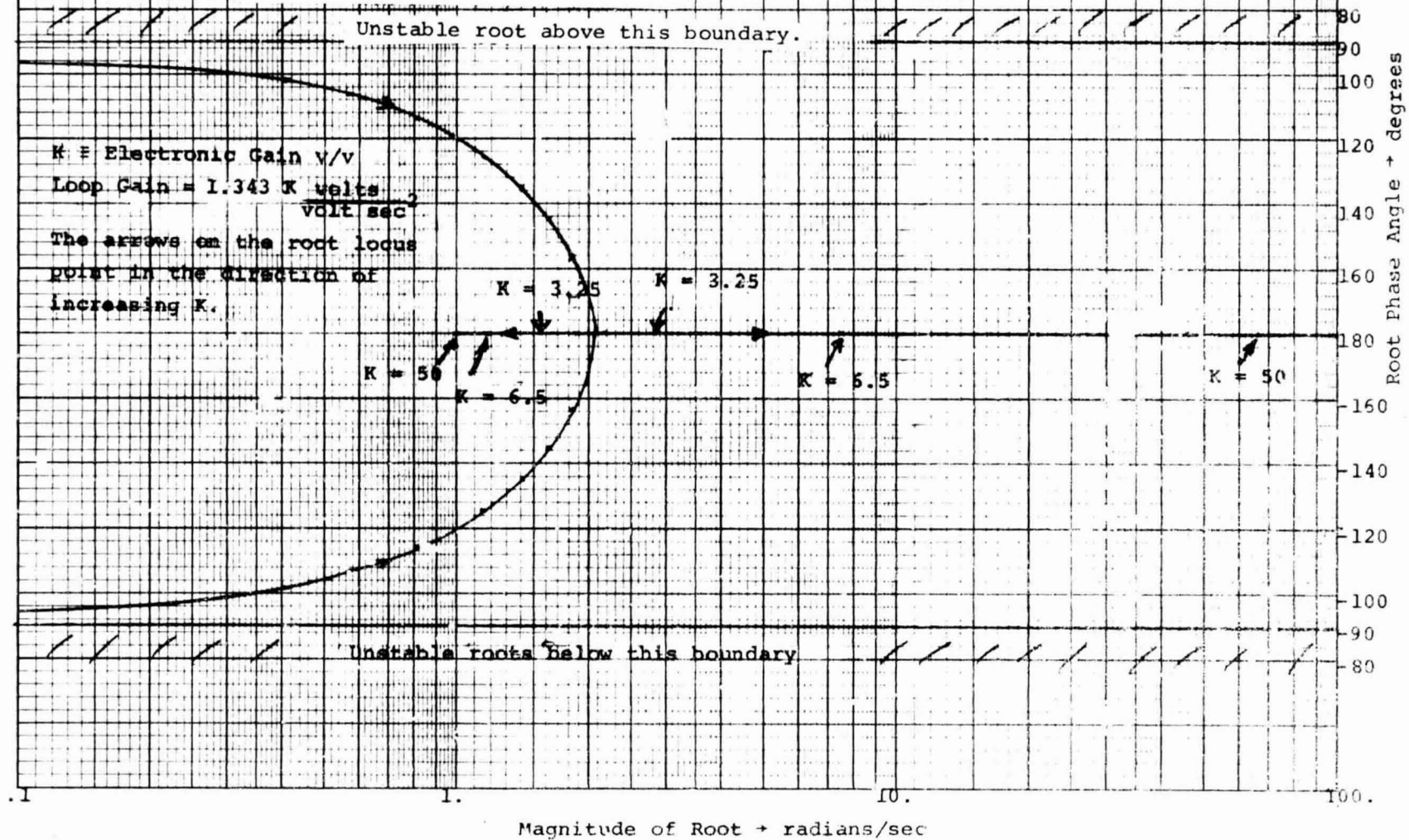


Figure 26
Pitch Closed Loop Root Locus
Applies to Bottom Loop of
Figure 9



$$\frac{\Theta}{T_{dis}} = \frac{1.76 \left(1 + \frac{s}{33.2}\right) \left(1 + \frac{s}{70}\right) \left(1 + \frac{s}{100}\right)}{\left(1 + \frac{s}{1.139}\right) \left(1 + \frac{2(.74)s}{14.7} + \frac{s^2}{14.7^2}\right) \left(1 + \frac{2(.99)s}{91.1} + \frac{s^2}{91.1^2}\right)} \frac{\text{arcsec}}{\text{in-oz}}$$

for $K = 6.5 \text{ v/v}$

The above equation demonstrates two facts.

1. Response to a disturbance torque is primarily that of a first order filter with a 1.139 rad/sec break frequency.
2. Small disturbance torques cause significant output perturbations. This undesirable effect can be reduced by increasing the electronic gain and/or adding lag-lead compensation to remove low frequency disturbances and/or adding electronic compensation to provide essentially accelerometer type feedback to remove higher frequency disturbances.

As will be shown later, the most significant parameters concerning the fine control loop performance is the response of the control to noise inputs and possible hysteresis limit cycling.

Roll Axis: Figure 27 presents the key parameters which determine the roll position loop linear response. The form of the loop is the same as Figure 24, but the gain distribution is different.

Using the air bearing values listed in Figure 27 and letting the electronic gain be 6.5 v/v results in an open loop gain of:

$$\text{Loop Gain} = K K_{FPT} L \phi \frac{(.1747)}{I \phi} = (6.5) (.288) (.667) \frac{(.1747)}{(.0437)} = 4.99 \frac{\text{volt}}{\text{volt sec}^2}$$

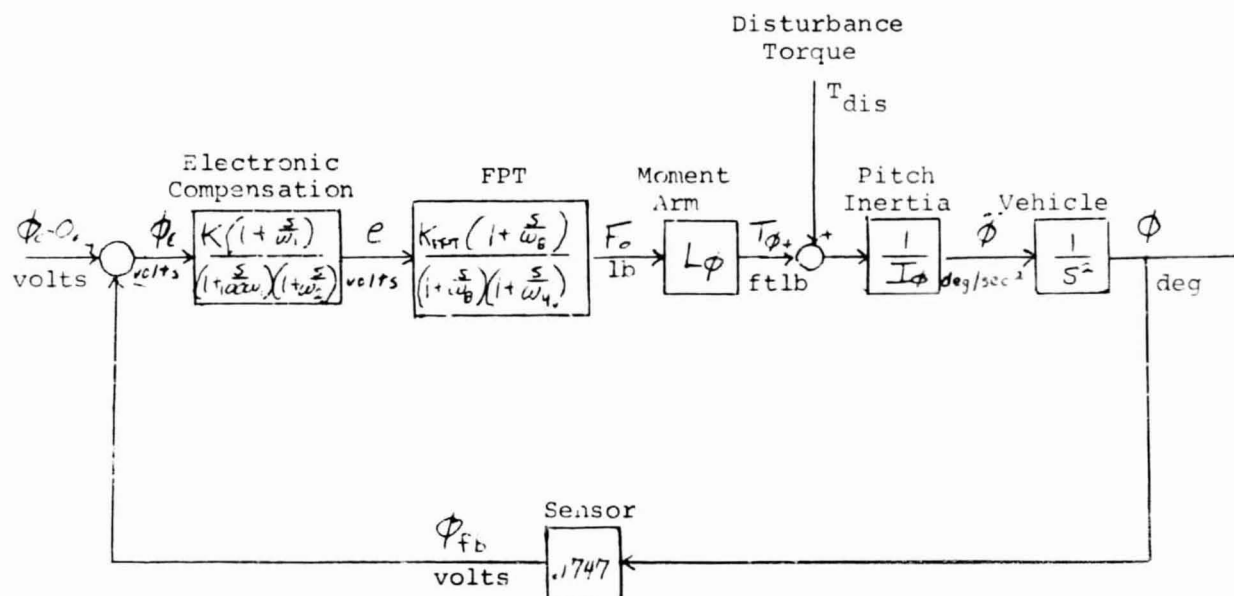
and the closed position loop transfer function of the roll loop is:

$$\frac{\phi}{\phi_c} = \frac{5.72 (1+s)}{\left(1 + \frac{s}{1.34}\right) \left(1 + \frac{s}{5.8}\right) \left(1 + \frac{s}{19.3}\right) \left(1 + \frac{2(.99)s}{88.5} + \frac{s^2}{88.5^2}\right)} \frac{\text{deg}}{\text{volt}}$$

The transfer function from disturbance torque to roll angle is:

$$\frac{\phi}{T_{dis}} = \frac{86.1 \left(1 + \frac{s}{33.2}\right) \left(1 + \frac{s}{70}\right) \left(1 + \frac{s}{100}\right)}{\left(1 + \frac{s}{1.34}\right) \left(1 + \frac{s}{5.8}\right) \left(1 + \frac{s}{19.3}\right) \left(1 + \frac{2(.99)s}{88.5} + \frac{s^2}{88.5^2}\right)} \frac{\text{arcsec}}{\text{in-oz}}$$

Examination of the scale factor shows that the roll position loop is much more sensitive to disturbance torques than the pitch and yaw loops for identical electronic gains. As in the case of the pitch and yaw loops, this sensitivity can be reduced by increasing the electronic gain and/or adding electronic compensation.



Air Bearing Values

$K = 6.5$ to 35 volt/volt

$K_{FPT} = .283$ lb/volt

$L_\phi = .667$ ft

$I_\phi = 2.5$ slug ft² = $.0437$ ft-lb $\frac{\text{sec}^2}{\text{deg}}$

$\omega_1 = 1$ rad/sec

$\omega_3 = 33.2$ rad/sec

$\omega_4 = 70$ rad/sec

$\omega_B = 5$ rad/sec

Figure 27
Fine Control Roll Position Loop

Performance of Simulator and Hardware

Pitch Axis: A closed loop simulation was conducted to verify the control design parameters. The differential pressure out of the thruster package was measured and applied as an input to an analog computer which simulated the vehicle dynamics, the position sensor, and the lead stabilizing network.

Figure 28 shows the noise at the breadboard FPT output. The noise was recorded on a Hewlett Packard model 7030Z X-Y recorder. This recorder has a transfer function with two lags at 50 radians per sec.

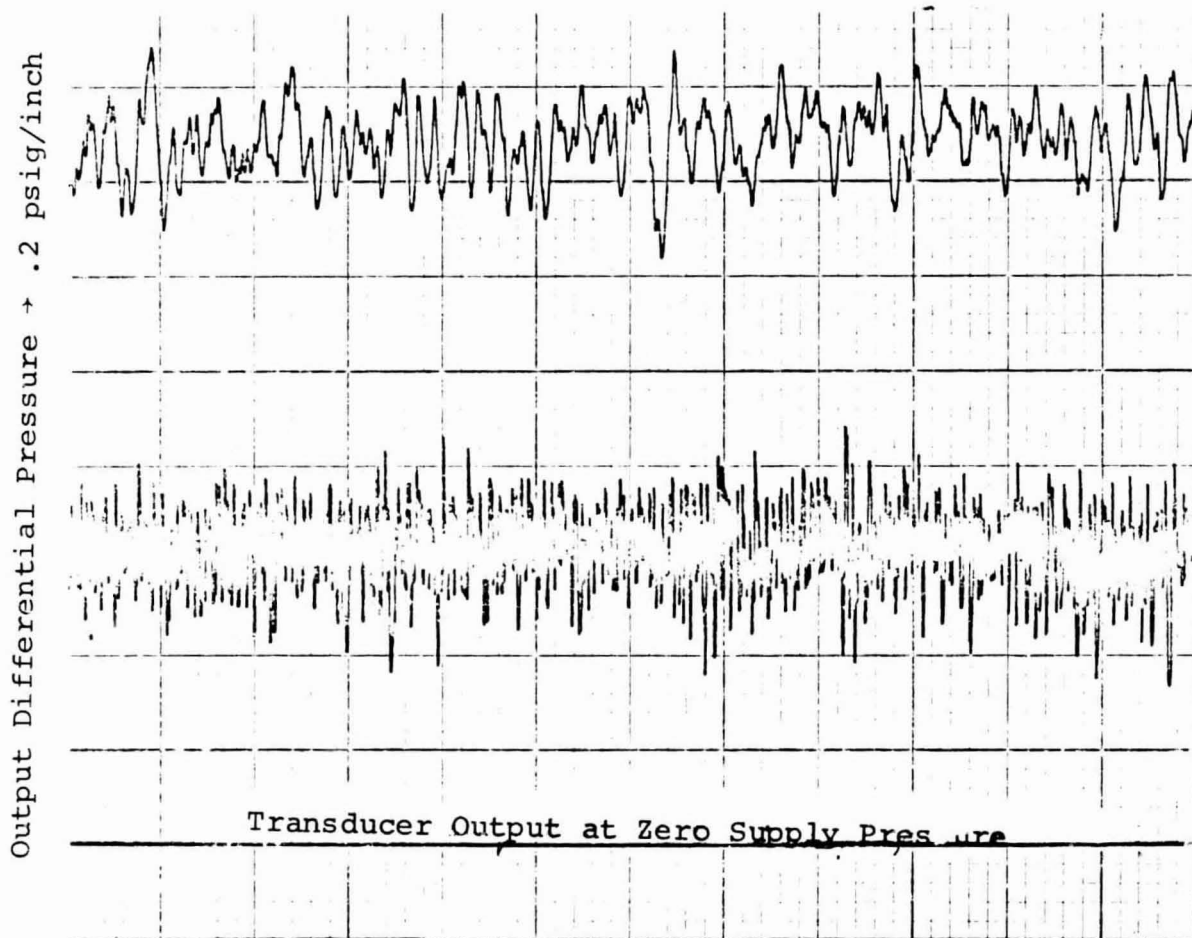
Figure 29 is a block diagram of the breadboard hybrid simulated pitch control loop. Figure 30 is a trace of the analog simulated vehicle steady state pitch position as a function of time. The lower trace, which was swept at 10 sec per inch shows an rms pitch variation of .061 arcsec. The loop gain distribution was similar to that used to generate Figure 62, and the FPT signal to noise ratio was around 1300. The thruster supply pressure was set at its fine control level of 90 psia. For these conditions the noise analysis results presented in Figure 62 predict a rms pitch error of .14 arcsec. This exceeds the simulated measurement by two to one, but it indicates that both values are in the same order of magnitude. The difference probably arises from the assumptions made in computing the rms values for noise and angle and the noise power spectral density. The results indicate hysteresis was no problem. A more complete treatment of noise and hysteresis is presented later under Error Sources.

The response to small angle step commands appears in Figure 31. Good tolerance to the electronic gain value is exhibited.

The response to large angle step commands appears in Figure 32. It also indicates tolerance to the electronic gain value. Figure 32 indicates the fine thrust level should be adequate for positioning the vehicle from angles less than six degrees as long as the angular rate at a position of six degrees is small. Any attempt to reduce the FPT noise level by reducing the maximum thrust must be accompanied by a similar reduction in the maximum angle which the fine thrust level attempts to control. For example, a redesign of the FPT which limits the thrust to half the present value should reduce the noise level by one half. If a redesigned FPT were used in the vehicle of this simulation, the fine thrust level should be adequate for acquiring angles up to three degrees instead of six degrees with the present FPT.

Roll Axis: Figure 33 shows the roll position loop with the FPT included as part of the loop.

Figure 34 is a trace of the analog simulated steady state roll position as a function of time. The lower trace, which was swept at 10 sec per inch, shows an rms roll variation of 1.37 arcsec. The loop gain distribution was similar to that used to generate Figure 63, the FPT signal to noise ratio was around 1300, and the thruster supply pressure was around 95 psia. For those conditions the noise analysis results presented in Figure 63 predict a rms roll error



Time $\rightarrow$

2 sec/inch for top trace

10 sec/inch for lower two traces

$$\text{Recorder transfer functions} = \frac{K}{\left(1 + \frac{S}{50}\right)^2}$$

Figure 28

Breadboard FPT Output Differential Noise Pressure
 $(P_s = 80 \text{ psig and volume dumps to ambient})$

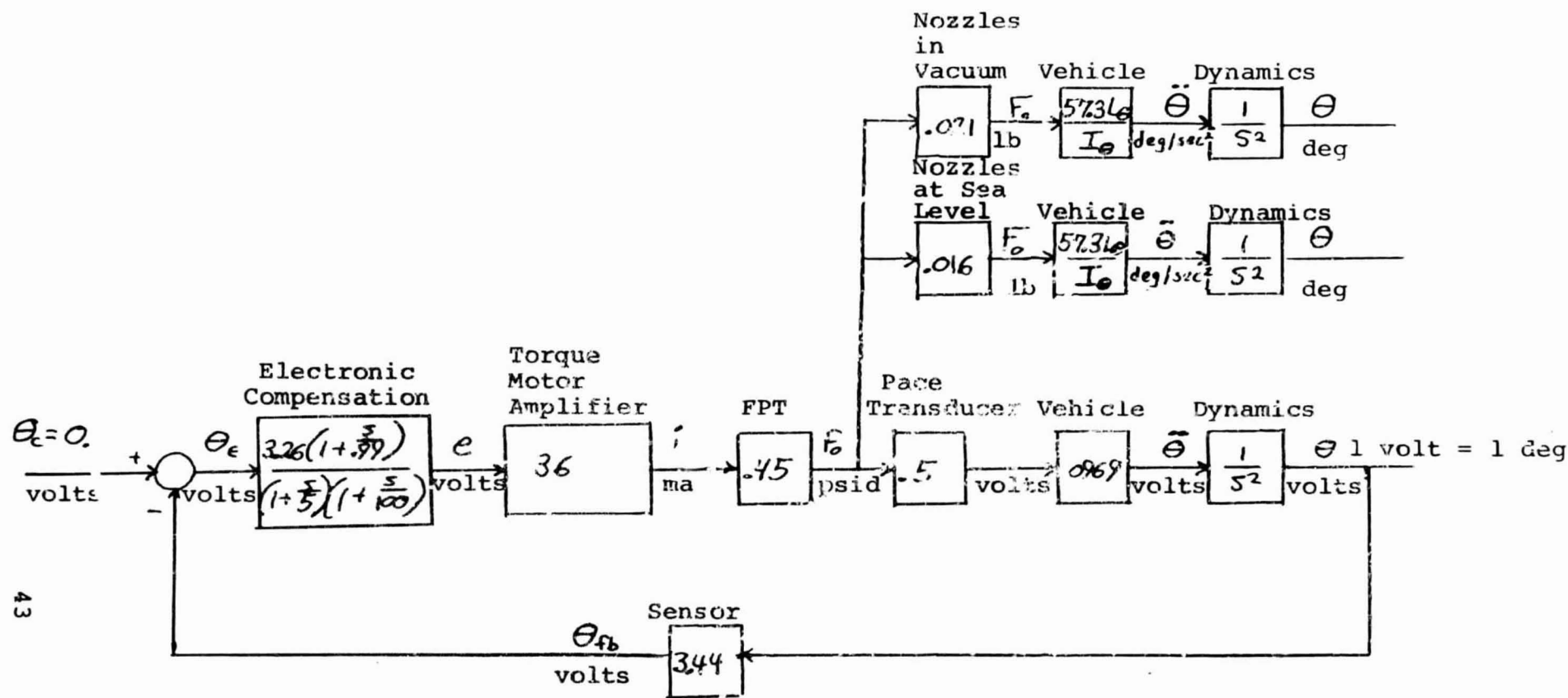


Figure 29
Hybrid Pitch Position Loop Simulation Using Breadboard FPT

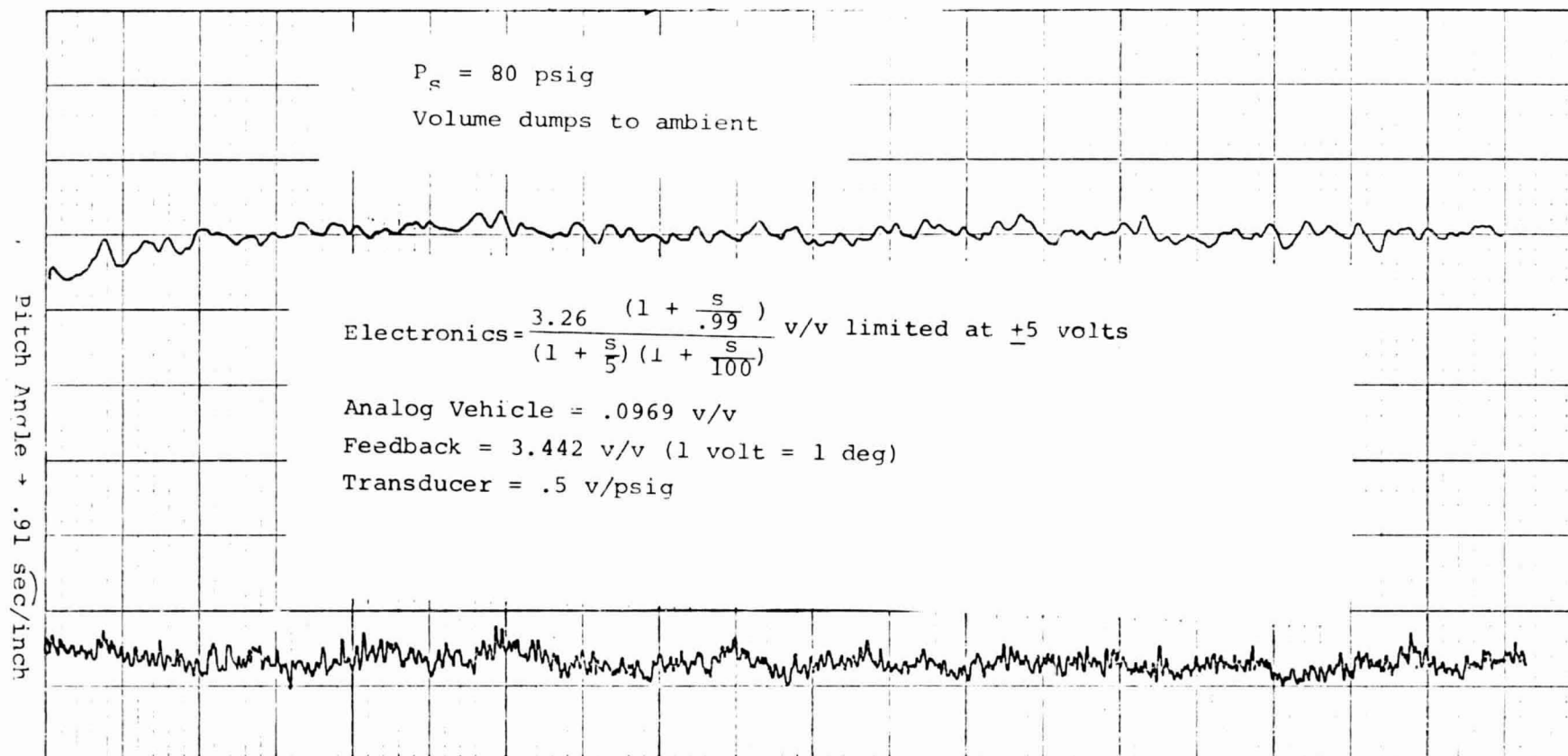
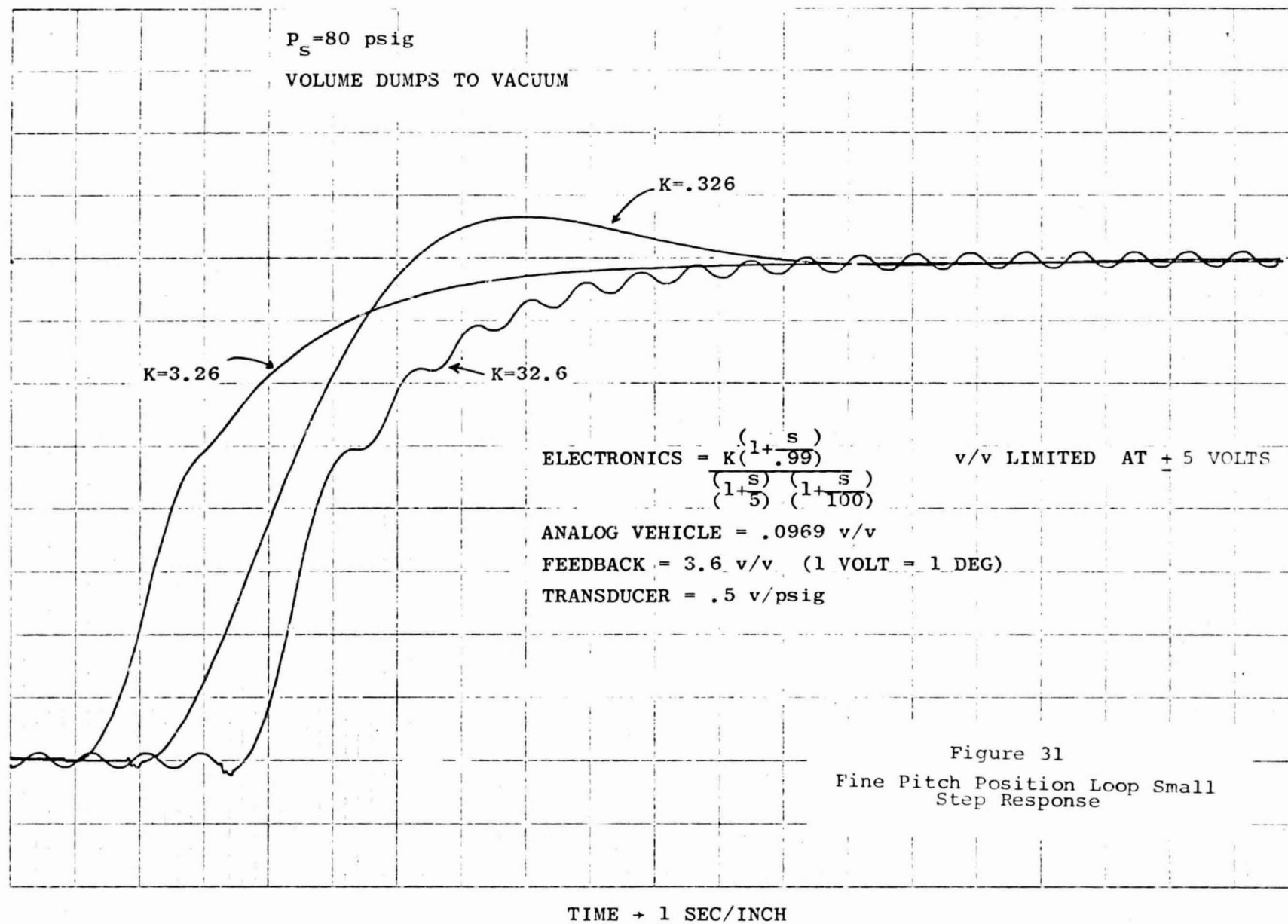


Figure 30

Pitch Uncertainty with
Breadboard FPT

Time + 2 sec/in for top trace; 10 sec/in for bottom trace



$P_s = 80$ psig

Volume dumps to vacuum

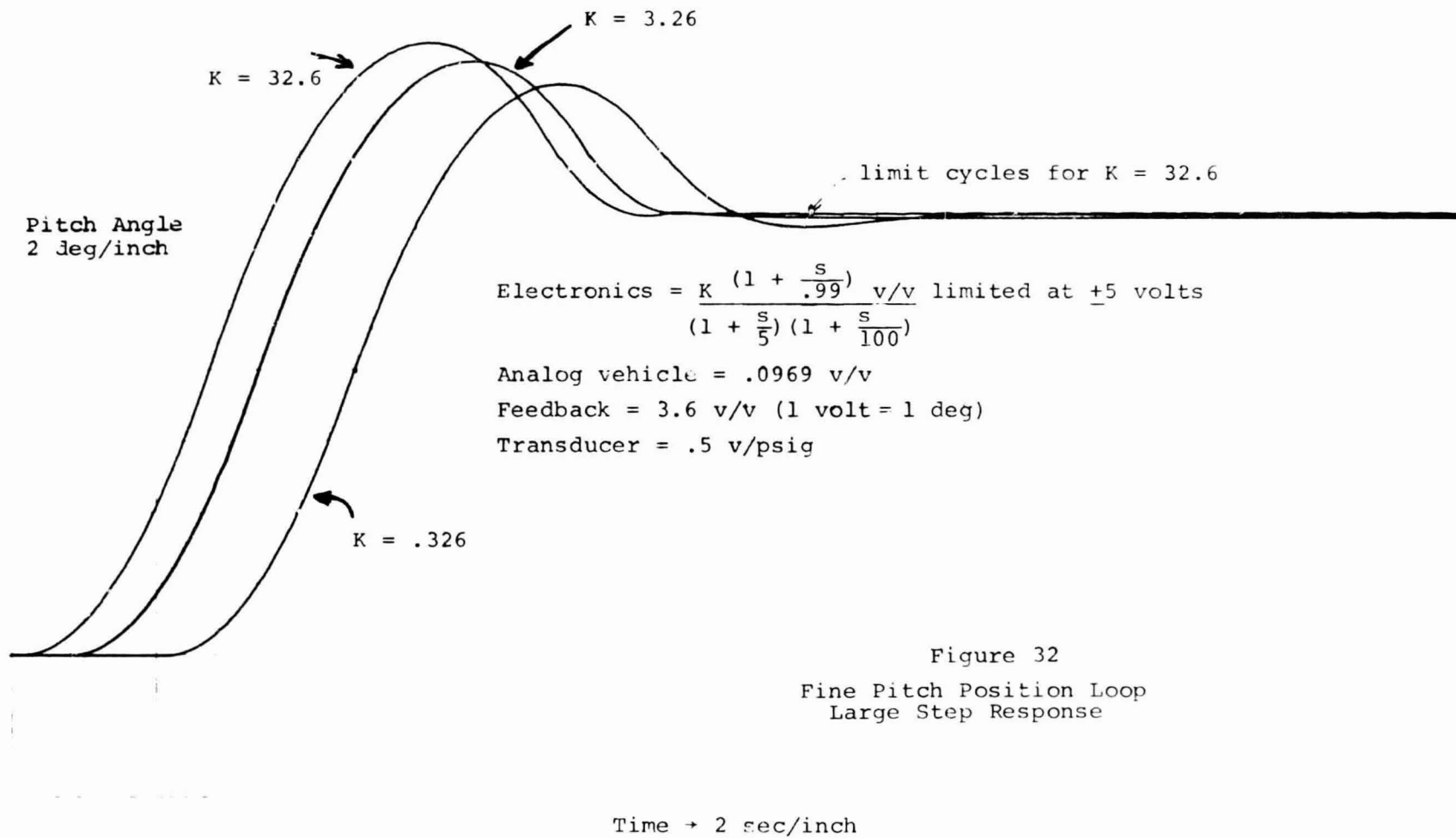


Figure 32
Fine Pitch Position Loop
Large Step Response

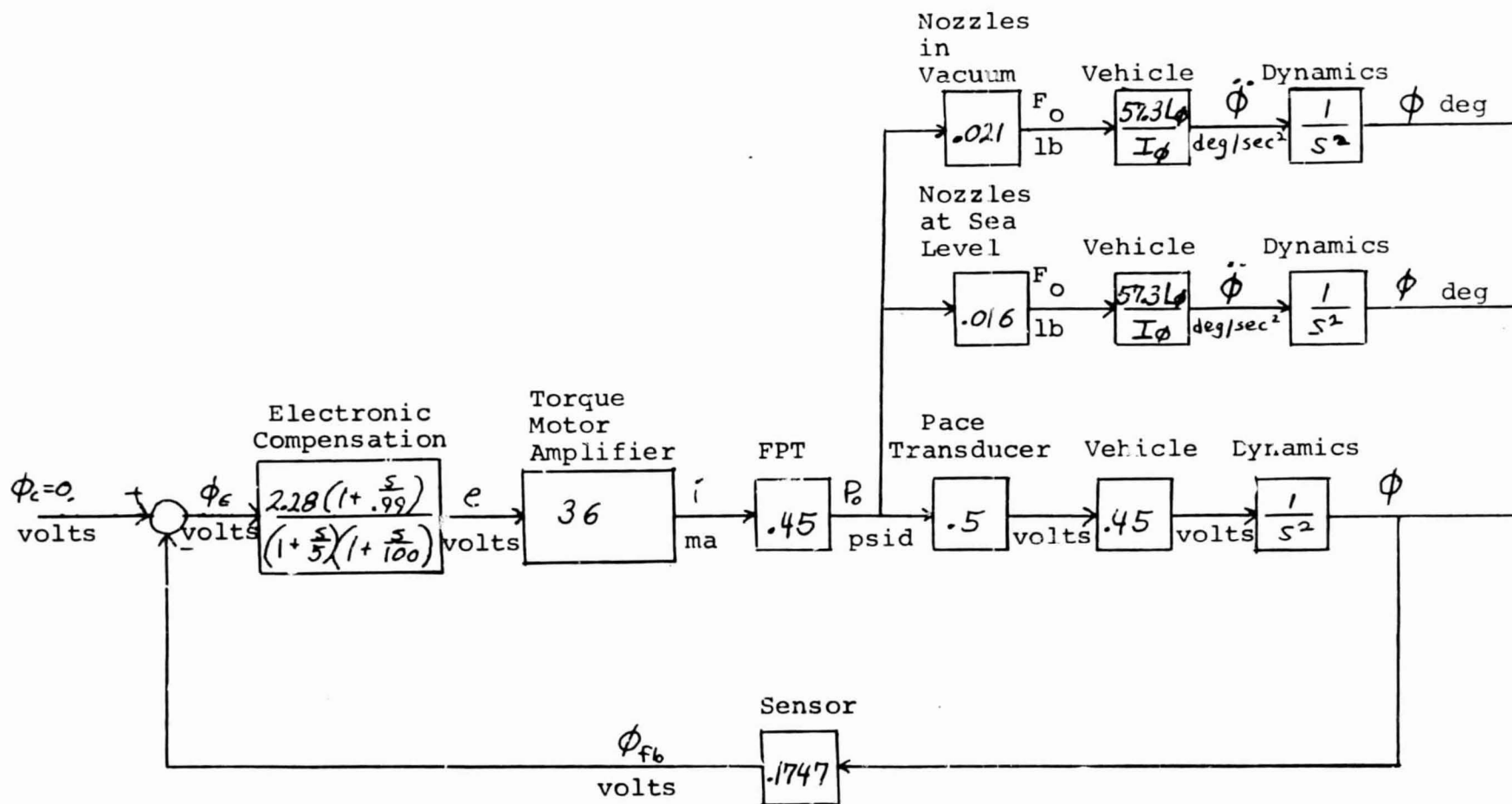
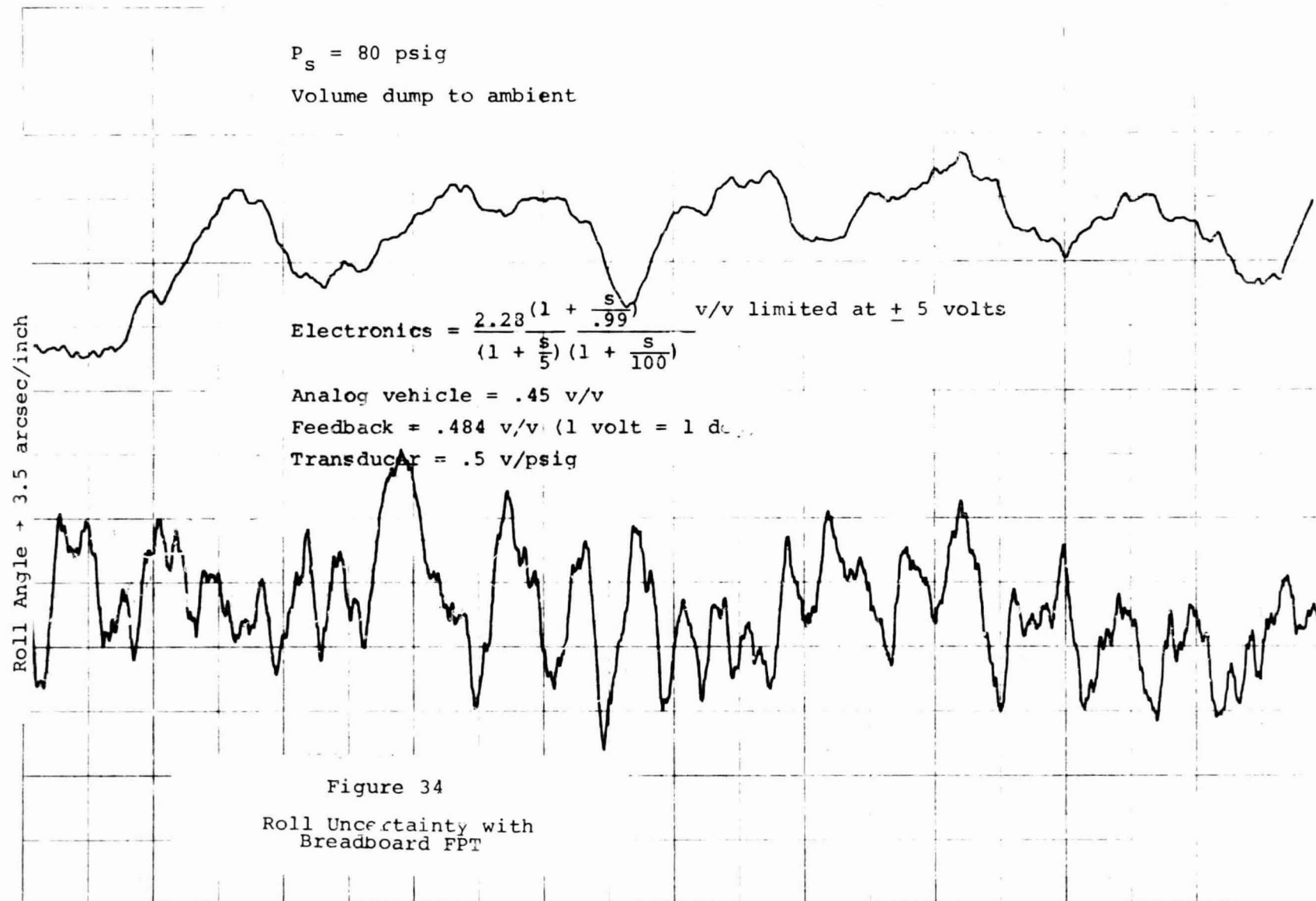


Figure 33
Hybrid Roll Position Loop Simulation Using Breadboard FPT



Time $\rightarrow 2 \text{ sec/in}$ for top trace; 10 sec/in for bottom trace

of 3.5 arcsec. As in the case of the pitch simulation, the predicted uncertainty exceeds the measured value by about two to one. The difference probably arises from the assumptions made in computing the rms values for noise and angle and the noise power spectral density. The roll results also indicated that hysteresis was no problem.

It may be pointed out that the roll accuracy requirements for a typical sounding rocket vehicle are typically in the 100 to 1000 arcsec range. The FPT performance is thus considerably better than needed.

The response to small roll commands is presented in Figure 35. The nominal electronic gain was 2.28 v/v. The response to large roll commands appears in Figure 36. Figure 36 demonstrates the ability of the FPT fine force level to acquire roll from a twenty degree error.

$P_s = 80$ psig

Volume dumps to vacuum

Roll Angle
.5 deg/inch

$K = .228$

$K = 2.28$

$K = 22.8$

$$\text{Electronics} = \frac{K \left(1 + \frac{s}{.99}\right)}{\left(1 + \frac{s}{5}\right) \left(1 + \frac{s}{100}\right)} \quad \text{v/v limited at } \pm 5 \text{ volts}$$

Analog vehicle = .45 v/v

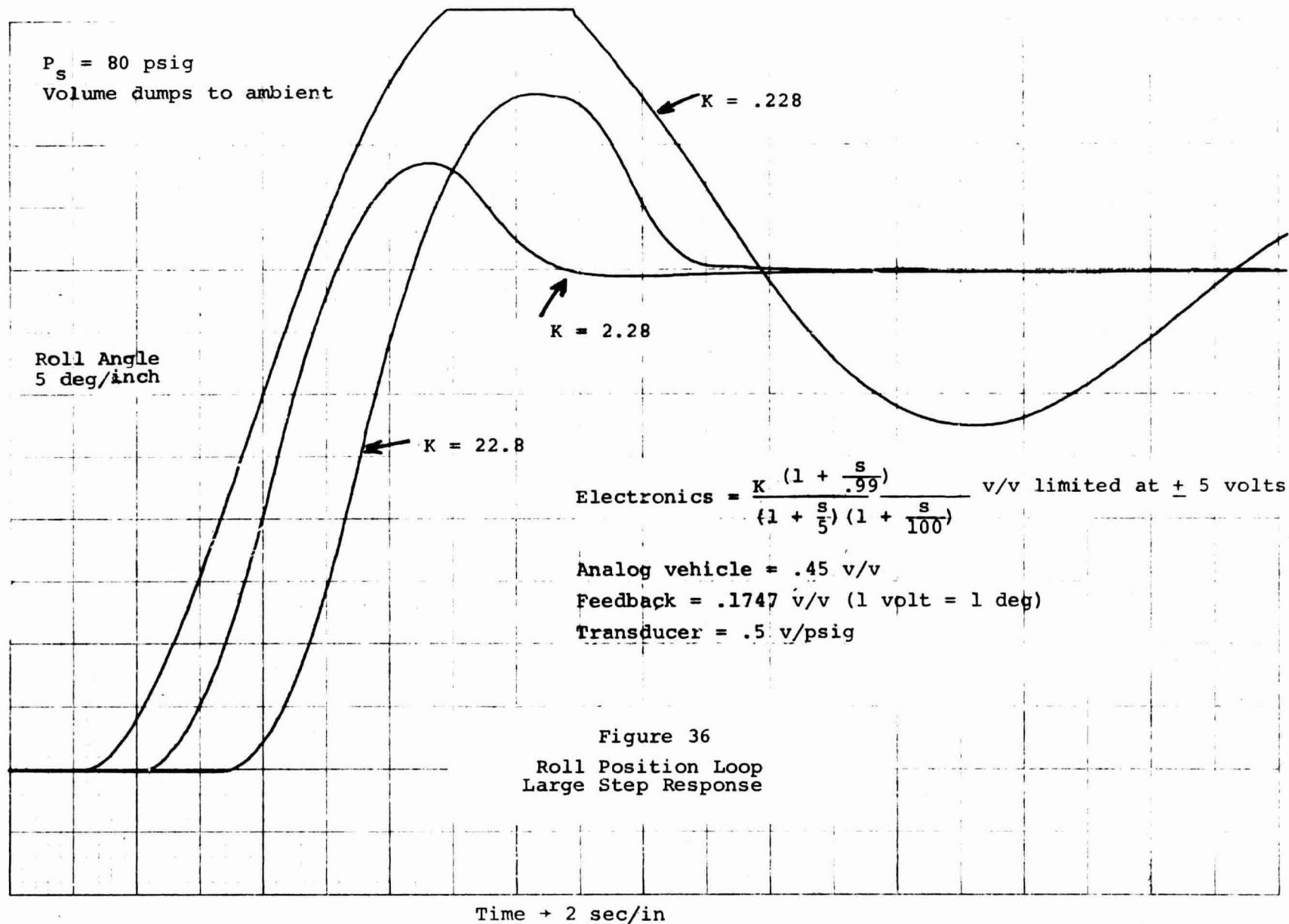
Feedback = .1747 v/v (1 volt = 1 deg)

Transducer = .5 v/psig

Figure 35

Roll Position Loop Small
Step Response

Time → 2 sec/in



COMPONENT DESIGN AND DEVELOPMENT

This report section presents detailed information concerning the various components used in the FPT assembly. Where pertinent test and performance data are included to substantiate the design.

Vortex Valve Steady State Design and Evaluation

Fabrication Technique: Several vortex valve configurations designed to meet the SPARCS 350 system flow requirements, i.e. to provide .129 lb/sec full flow at 1000 psig supply pressure were designed and evaluated. A radial-inlet, double-outlet configuration which provided full turndown ratio of 7.5 with a control to supply pressure ratio of 1.2 to 1.3 was selected for use in the thruster. Selection was made on the basis of good turndown, good linearity, minimum hysteresis and low noise. In general, all of the above design factors cannot be optimized independently. For example, a valve with high turndown ratio usually has poor linearity and may exhibit some hysteresis.

The metal-etch manufacturing process that is a standard on all of the General Electric fluidic amplifier assemblies lends itself well to this experimental optimization of the valve parameters. This fabrication technique affords flexibility with which valve parameters can be quickly and easily changed. A stack-up of chemically-etched metal laminations make up the vortex spin chamber, supply inlets, control nozzles and outlet interconnection. The laminations were etched from .004 inch thick stainless steel laminations. The optimized configuration consist of a stack of metal laminations with the profile shown in Figure 37. The laminations are sandwiched between an end cover plate and manifold block which contain the outlet orifices, and control and supply ports to comprise a vortex valve.

Valve Evaluation: Several different lamination arrangements were evaluated to establish an optimum valve configuration. Test data were compiled to evaluate the effects of chamber width, control nozzle area, control nozzle distribution, outlet configuration and operating pressures.

Typical output flow vs. control pressure characteristics obtained for a radial valve is shown in Figure 38 and that for a button configuration is shown in Figure 39. Comparison of the two curves clearly indicates that the radial valve exhibits better characteristics for a proportional control application. The radial valve has good linear range, and no apparent hysteresis. The same turndown ratio is achieved as with the button configuration at a greater linear range. In addition, the button valve exhibits a tendency toward latching and hysteresis in the high gain region. The radial valve was thus selected for the thruster application.

The valve design parameters were fixed as follows:

Type: radial inlet, double-outlet
Spin chamber diameter: 0.5 inch

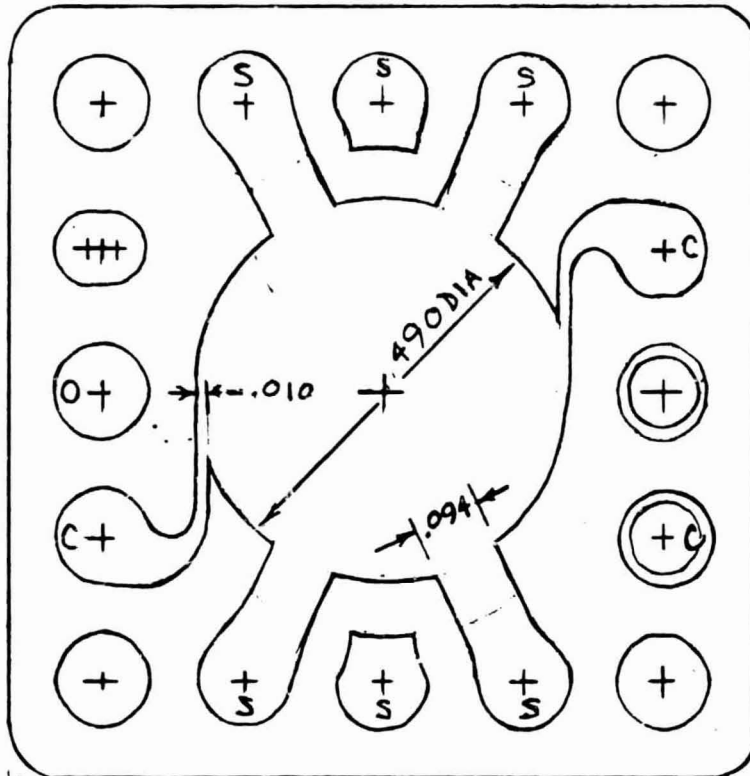


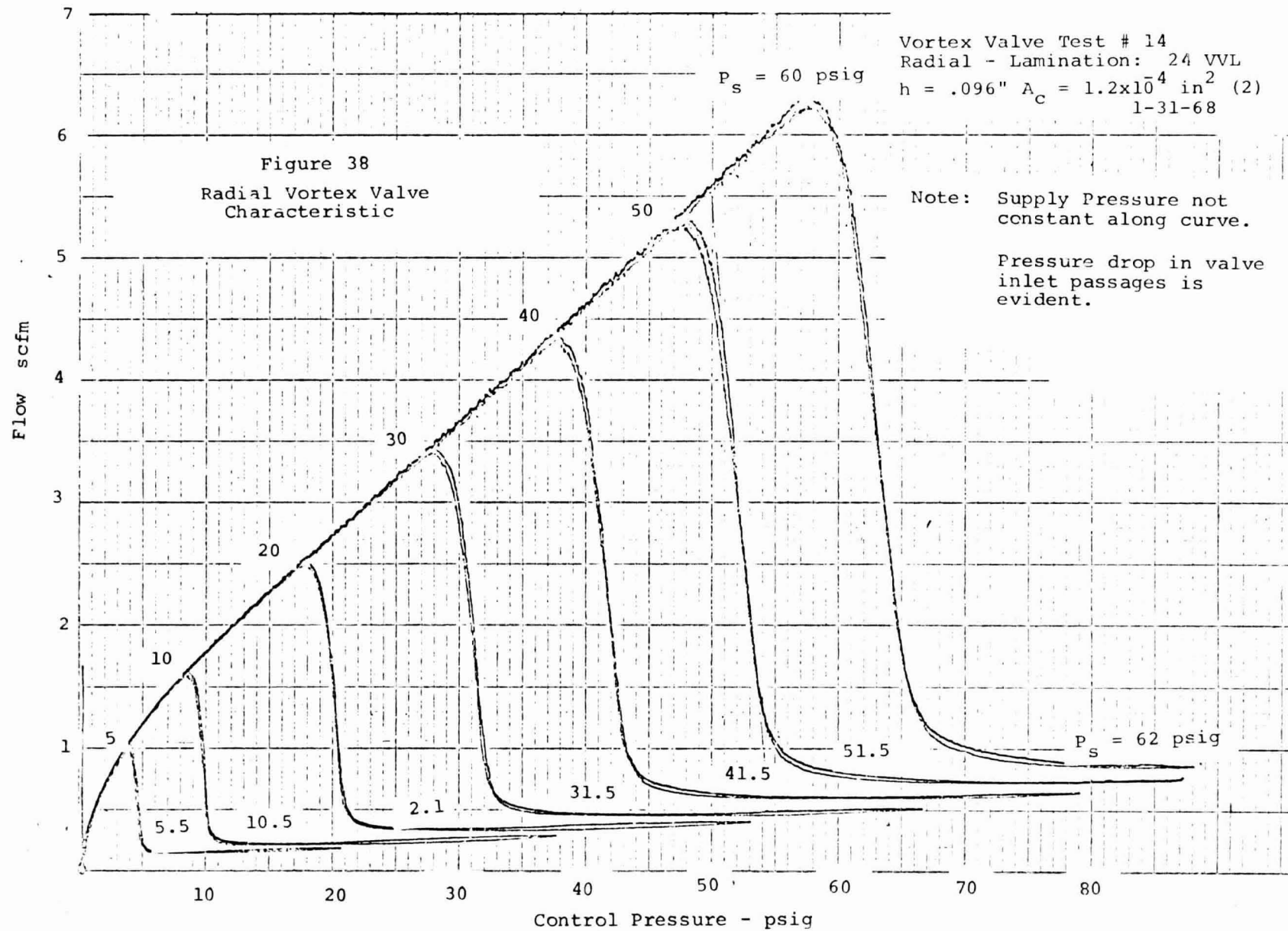
Figure 37
Lamination Vortex Valve

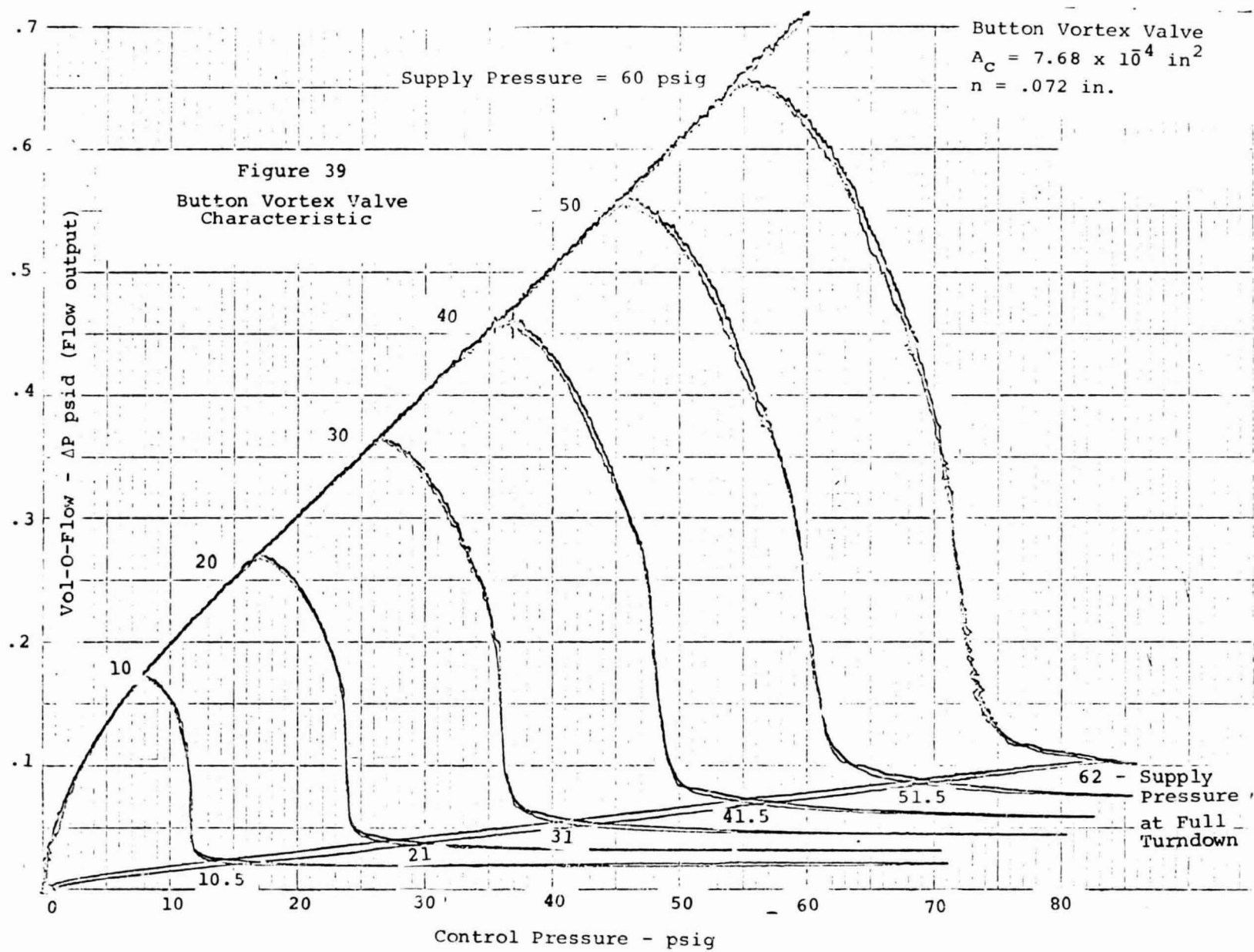
Vortex Valve Test # 14
 Radial - Lamination: 24 VVL
 $h = .096''$ $A_C = 1.2 \times 10^{-4} \text{ in}^2$ (2)
 1-31-68

Figure 38
 Radial Vortex Valve
 Characteristic

Note: Supply Pressure not
 constant along curve.

Pressure drop in valve
 inlet passages is
 evident.





Ratio of supply to outlet area: 4.5
Ratio of control to outlet area: .225

Other conclusions obtained during the test evaluation series are:

- 1) A smaller ratio of control nozzle to outlet area produces larger turndown ratios but valve gain decreases since higher control pressures are required. At the same time, linearity is also degraded.
- 2) For a given configuration, valve gain remains constant regardless of supply pressure variations. For the valve selected, the gain is in the order of .001 lb/sec per psi.
- 3) A comparison was made of sharp-edged orifice, short tube and converging nozzle outlet configurations. No significant difference was observed.

Noise and hysteresis data was obtained for a push-pull pair of valves, as used in the final thruster assembly. Analysis of this data is presented in a later section.

Thruster Servo

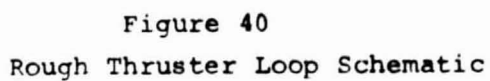
Figure 40 shows the thruster loop with its essential components and Figure 41 is a detailed block diagram showing the significant system dynamics and feedback loops. The parameter values of Figure 41 were either directly or indirectly evaluated from laboratory tests. NASA/ARC simulated Figure 41 on an analog computer. Their evaluation resulted in a simpler representation of the entire loop as shown in Figure 42.

The major simplifying assumption of Figure 42 is that mechanical resonances associated with the torque motor and bellows could be neglected. Lab tests at GE and NASA/ARC validated this assumption.

Note that a mechanical position limit is shown in the lower section of Figure 41 in series with K_{2ra1} . The function of this position limit is to reduce the thruster feedback for large signal (coarse control) operation. Limiting of the feedback forces the thruster to operate bang-bang for large position and rate errors.

A brief summary of the physical hardware characteristics making up the most significant gains shown in the thruster block diagram follows.

Torque Motor Gain, K_t : Figure 43 presents the test results which applied to the FPT torque motor. For that torque motor the gain was .00875 in.lb per ma with the coils connected in series. The gain per coil was one half the latter value or .004375 in.lb per ma. This gain depends on the torque motor pole piece magnetization. The magnetization varied to the extent that the gain per coil would vary from .0035 to .005 in.lb per ma from torque motor to torque motor.



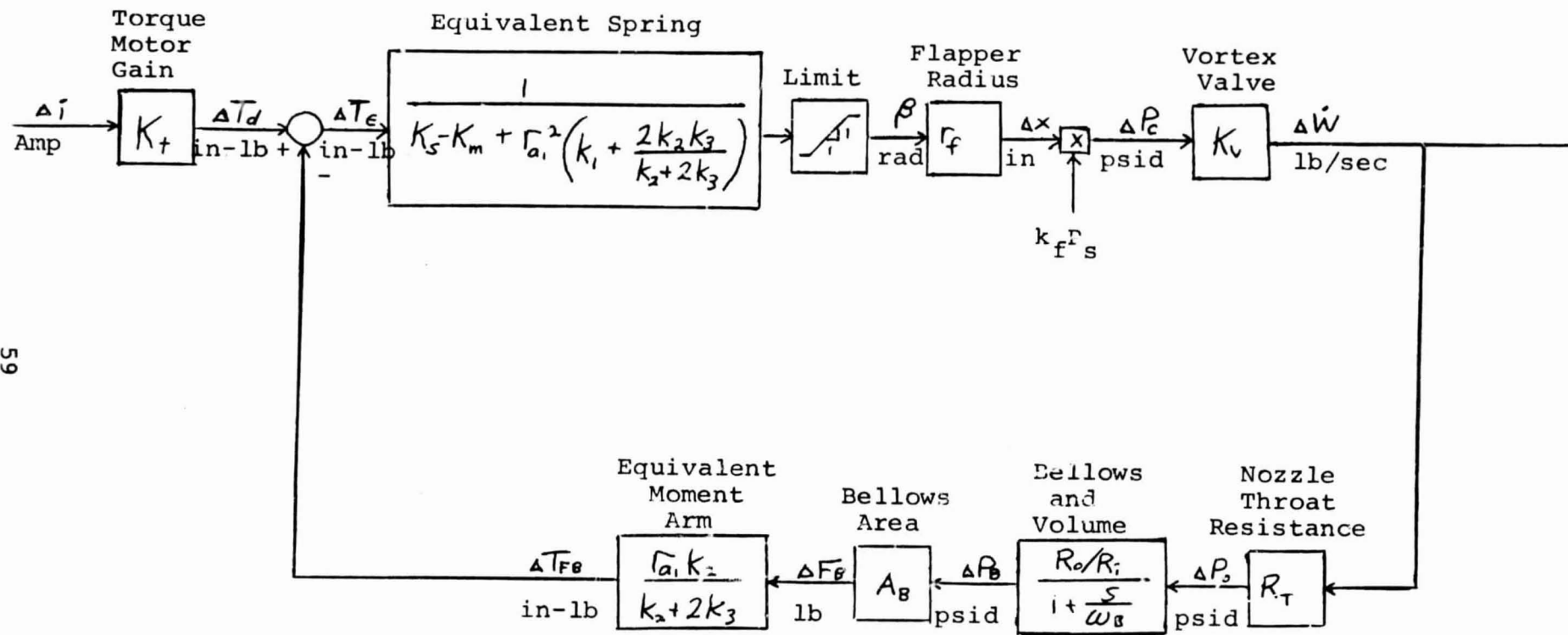


Figure 42
Simplified FPT Block Diagram

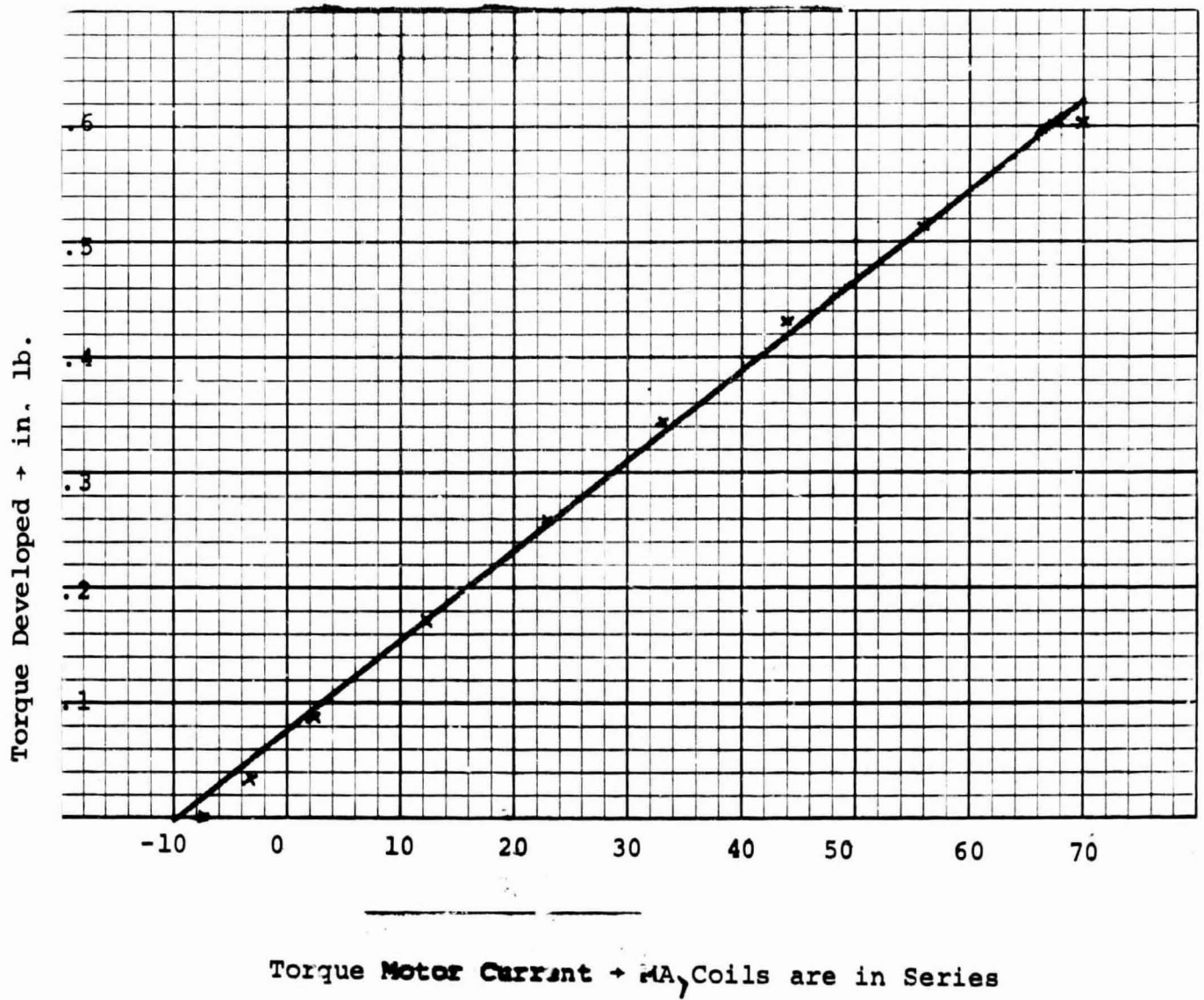


Figure 43
Torque Motor Scale Factor Test Results

Torque Motor Spring Rates, K_e , K_s , K_m : The equivalent torque motor spring, K_e , is defined as:

$$K_e \equiv K_s - K_m$$

The value of K_e was measured for one torque motor after it was magnetized but before it was installed in the FPT. The torque motor was driven by a sinusoidal current input and the output displacement measured. Ninety degrees of phase lag occurred at 315 HZ. From geometric considerations, the torque motor moment of inertia was computed to be 5.99×10^6 in.lb.sec². The value of K_e was then computed from the expression for natural frequency in a second order system.

$$f_o = 315 \text{ HZ} = \frac{1}{2\pi} \sqrt{\frac{K_e}{J}} = \frac{1}{2\pi} \sqrt{\frac{K_e}{5.99 \times 10^6}}$$

From the above:

$$K_e \equiv K_s - K_m = 23.4 \text{ in.lb per radian}$$

The torque motor spring constant is about 65 in.lb/per radian. Thus:

$$K_m = K_s - K_e = 65 - 23.4 = 41.6 \text{ in.lb per radian}$$

K_m is the magnetic spring rate which tends to decrease the equivalent torque motor spring constant, K_e .

Torque Motor Radii, r_{al} , r_a , r_f : Figure 40 provides an explanation of the three radii. Two of the radii were fixed by the torque motor design. These are:

$$r_a = .563 \text{ inch}$$

$$r_{al} = .78 \text{ inch}$$

The flapper radius, r_f , was made as long as possible to obtain the highest possible FPT gain. The value selected ($r_f = .795$ inches) was about as large as packaging limitations would allow.

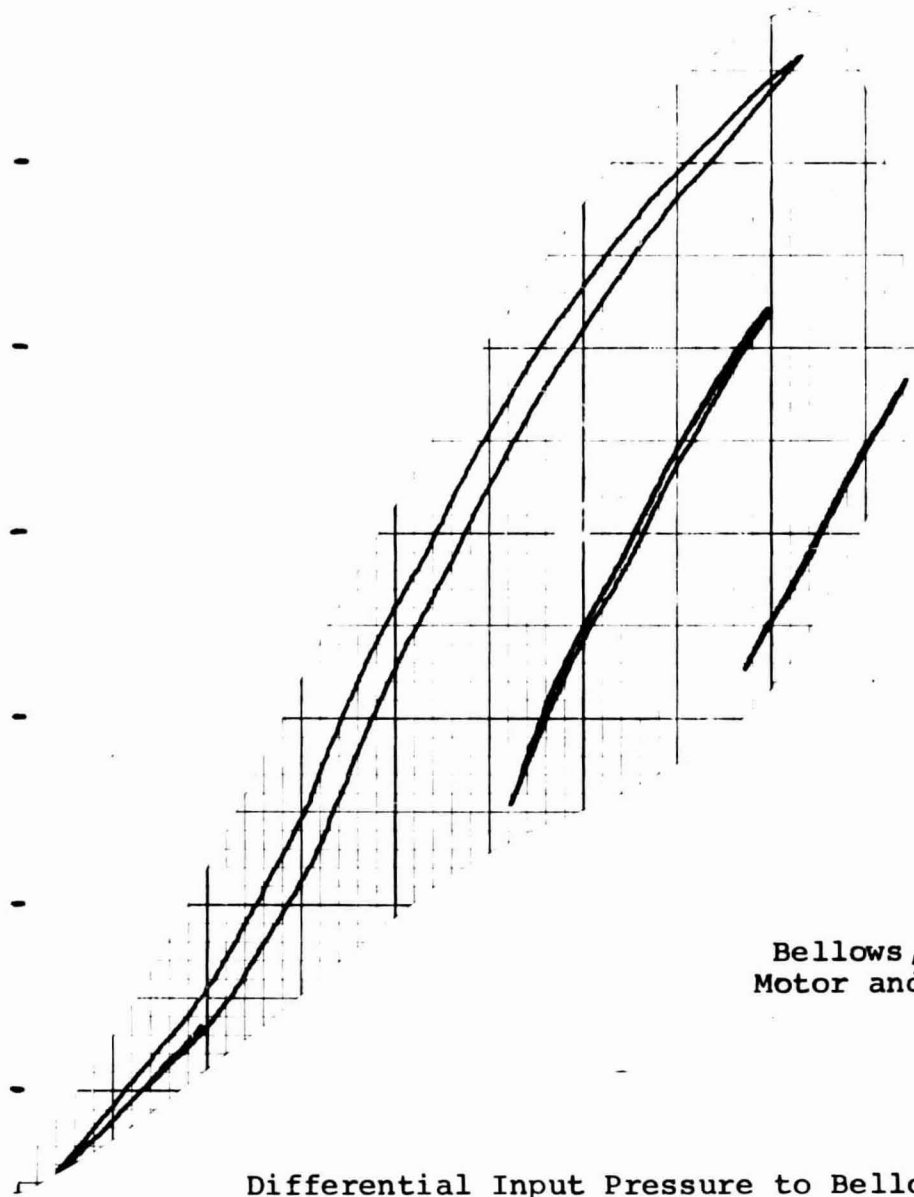
Inverted Flapper Gain K_f : Figure 44 shows the FPT open loop characteristic from the bellows input to the vortex valve control pressure input; $\Delta P_c / \Delta P_{ol} = 1.81$ psig/psig at $P_s = 95$ psia. From Figure 42 it can be seen that

$$\frac{\Delta P_c}{\Delta P_{ol}} = (R_c/R_i) A_B \frac{r_{al} k_2}{(k_2 + 2k_3)} \frac{1}{\left(K_e + r_{al}^2 \left(k_1 + \frac{2k_2 k_3}{k_2 + 2k_3} \right) \right)} r_f k_f P_s$$

Using the previously mentioned constants and further setting

$$\frac{R_o}{R_i} = .444 \text{ and } A_B = .226 \text{ in}^2,$$

Differential Input Pressure to Vortex Valve
1 inch = 2 psid



Differential Input Pressure to Bellows
1 inch = 2 psid

Figure 44
Bellows, Compensation, Torque
Motor and Inverted Flapper Gain

leads to a gain, K_f , of 38.2 per inch. As evident from the block diagram in Figure 42, the thruster loop gain varies with supply pressure P_s .

Vortex Valve Gain K_v : Figure 45 is an X-Y recording of the vortex valve characteristic; the input/output slope is 4.65 psid/psid. The resistance of the thruster nozzles loading the valve was 3210 sec per in². Thus:

$$K_v = \frac{\text{slope}}{\text{resistance}} = \frac{4.65}{3210} = 1.45 \times 10^{-3} \text{ in}^2 \text{ per sec.}$$

Thruster Servo Evaluation

Pneumatic Component Dynamics: The preceding section has presented the essential steady state design constants used in the FPT loop. The dynamic response between control pressure applied to the vortex amplifiers and the thruster line pressure fed back to the torque motor assembly were first estimated using lumped parameter modeling techniques and then verified by measurement. In summary, the combination of vortex amplifier and line dynamics can be represented by a transport delay between 4 and 8 milliseconds for line lengths 10 to 51 inches.

Figure 46 shows the configuration which was subjected to the frequency response test. The frequency response from the vortex valve input, ΔP_C , to the simulated bellows output, ΔP_B , was measured. The frequency response was determined for channel lengths of 10, 15, 24 and 51 inches. The channel length simulated the distance from the vortex valve output to the thruster nozzle.

Figure 47 shows the lumped parameter equivalent of the portion of Figure 46 which lies between ΔP_C and ΔP_B . Figure 48 is a Bode plot which compares the lumped parameter estimates with the test results; the agreement between theoretical and experimental results is excellent. Figure 49 is a Bode phase plot of tested phase lag between ΔP_C and ΔP_B with the thruster channel length a parameter. The dynamic testing verified that good response should be obtained when the complete loop is closed; line lengths up to 51 inches did not materially affect the system stability.

Frequency response data were taken (Figures 48, 49 and 50) for altitude as well as sea level operation. Figure 51 presents the open loop gain and phase test results for the sea level performance case. In this test the orifice which vents the feedback bellows was dumping to the local ambient pressure, so the flow was not choked. The effect of operating in ambient condition is to reduce the open loop gain and increase the phase lag as a comparison of Figures 50 and 51 reveals.

Gain/Phase Tests (Closed Loop): The FPT closed loop frequency response was measured for sea level (Figure 52) and altitude conditions (Figure 24). The results were consistent with the open loop measurement. The small signal closed loop FPT transfer function is:

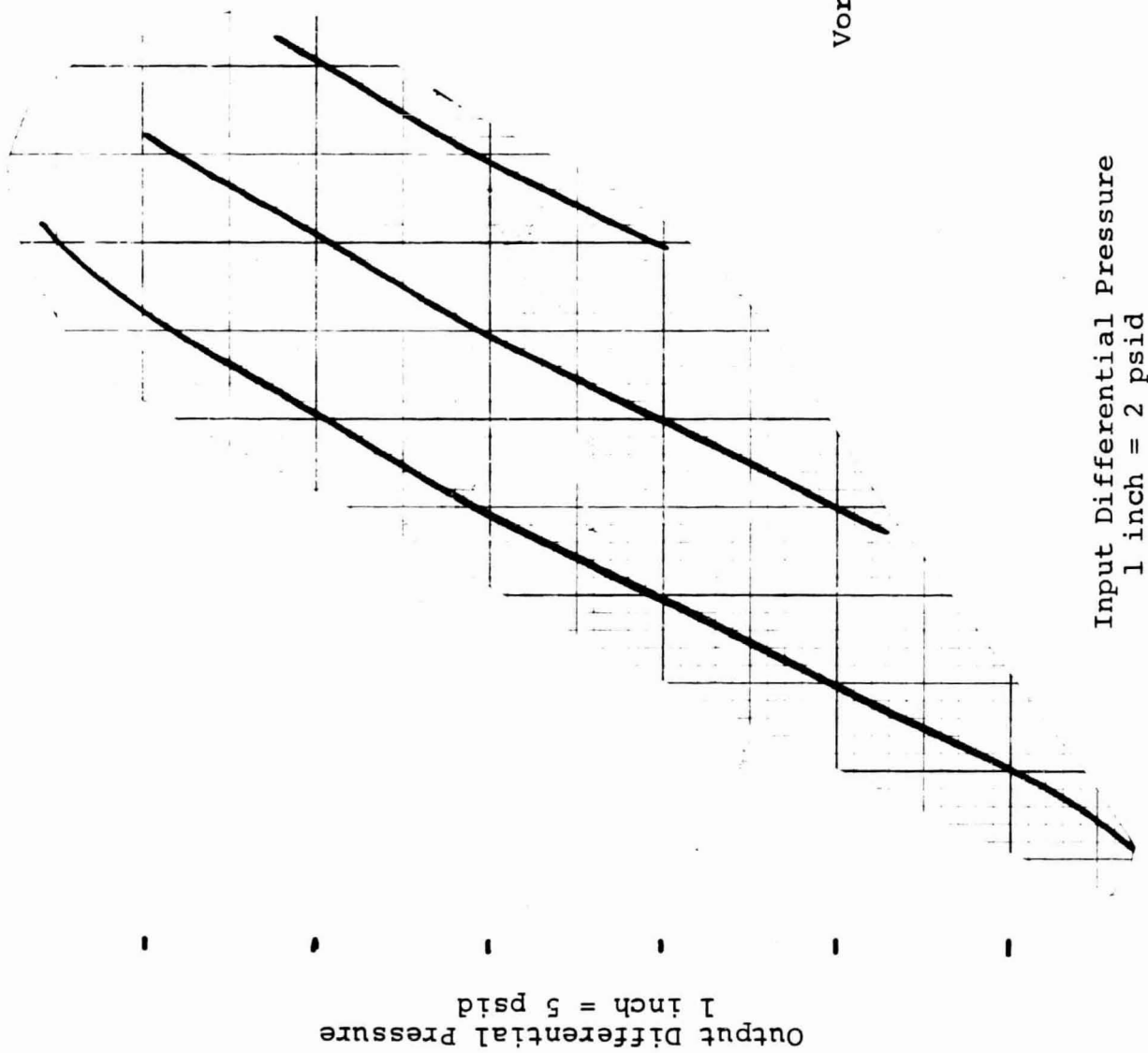


Figure 45
Vortex Valve Gain

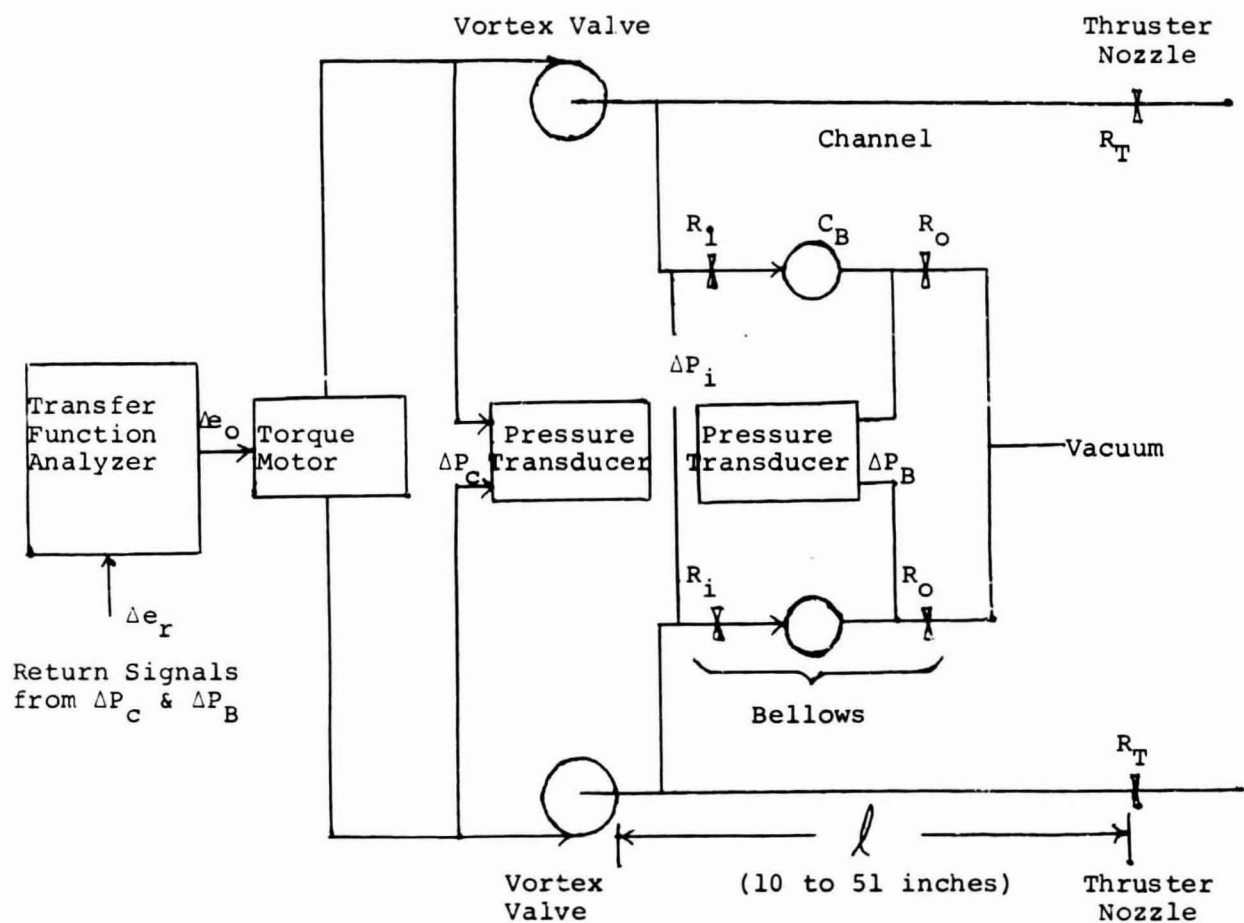
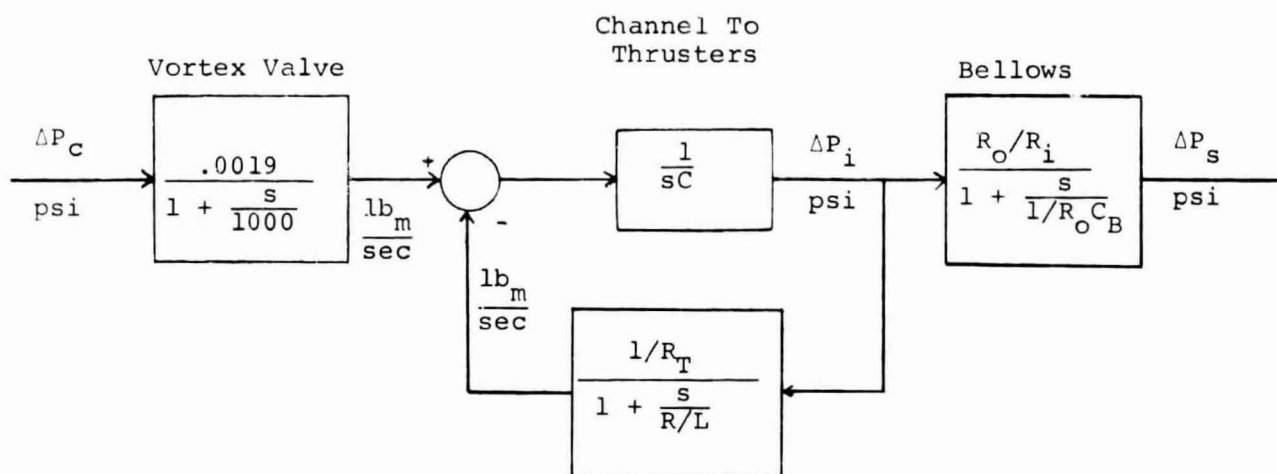
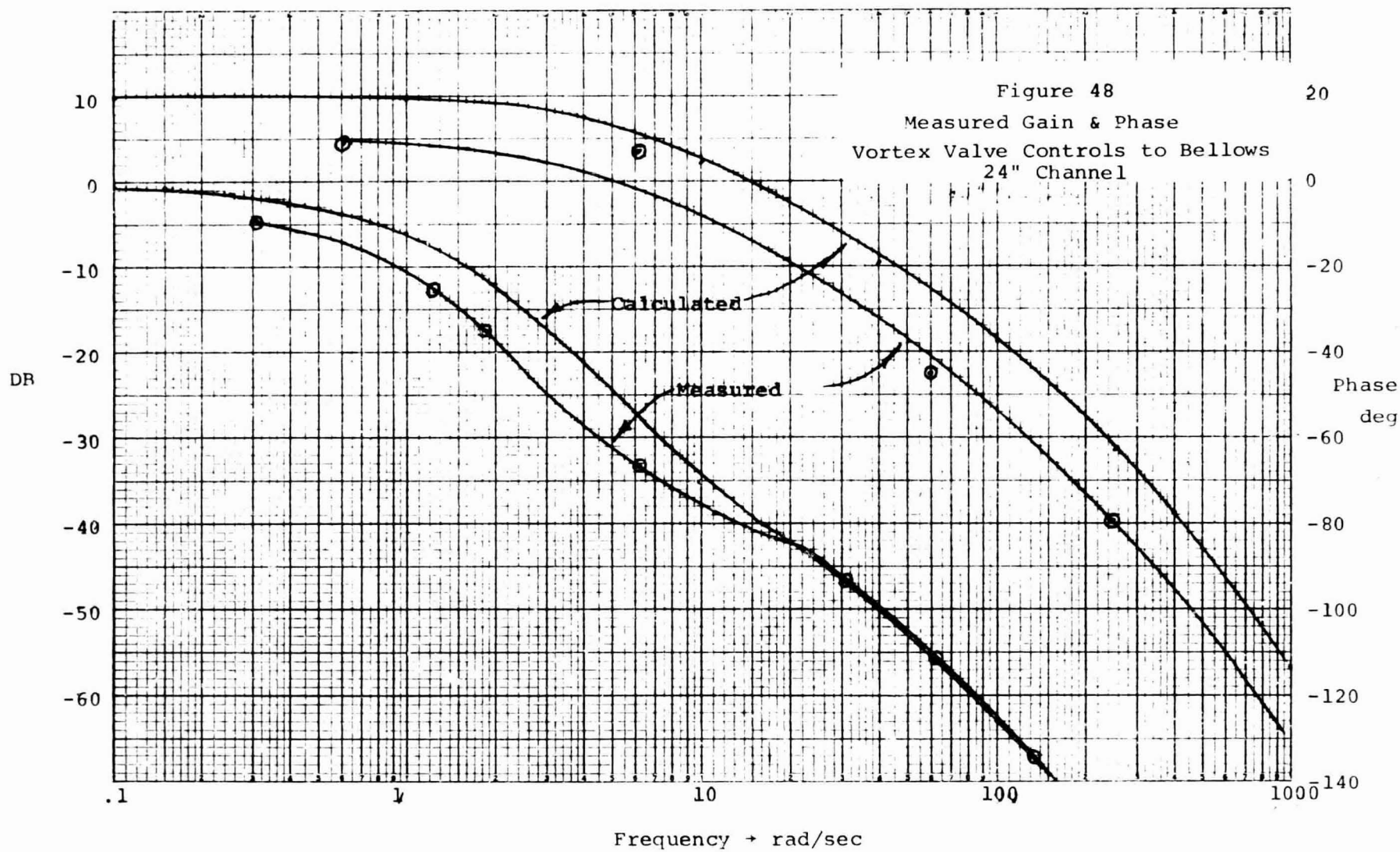


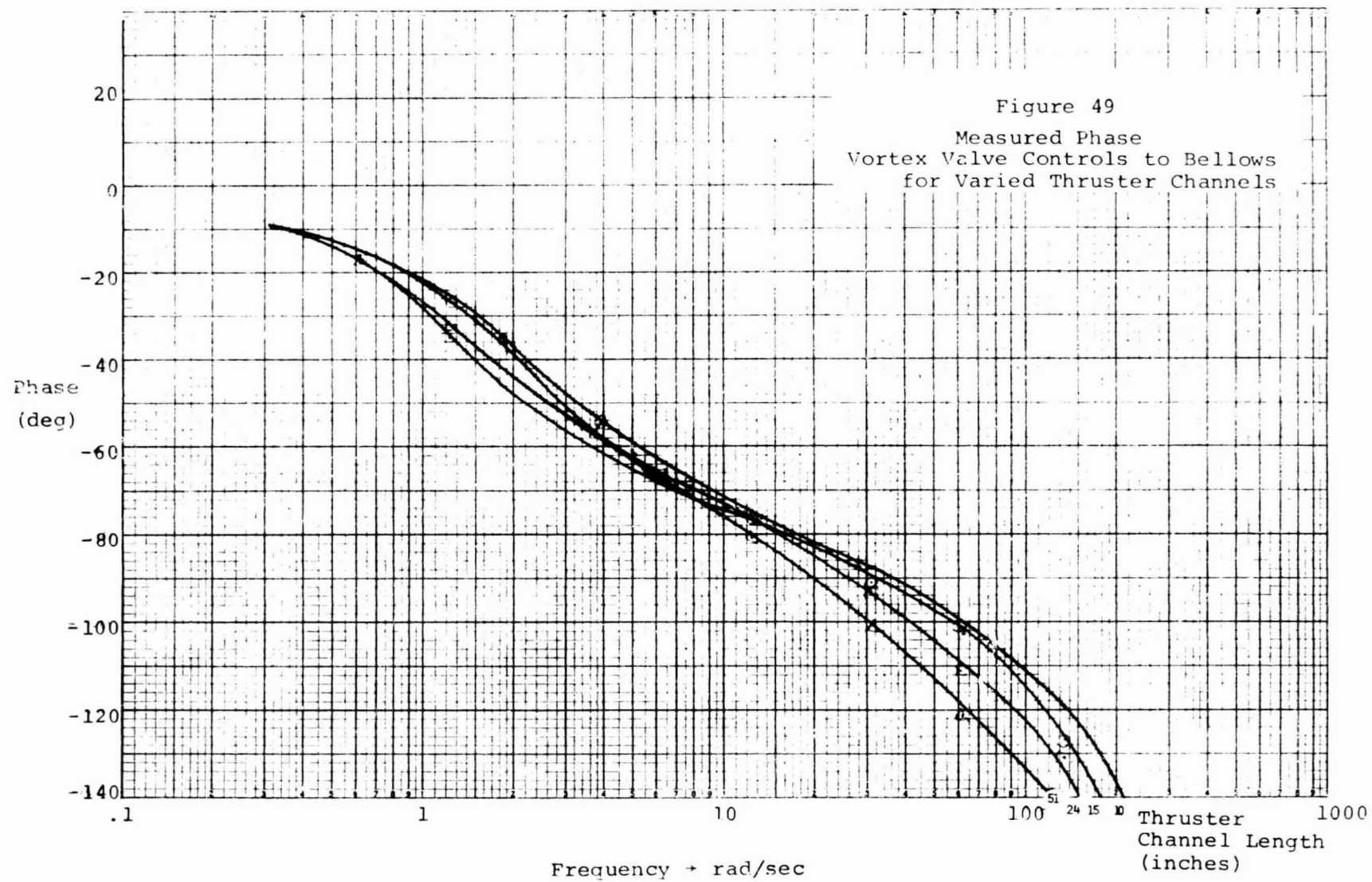
Figure 46
Vortex Valve Frequency Response Configuration

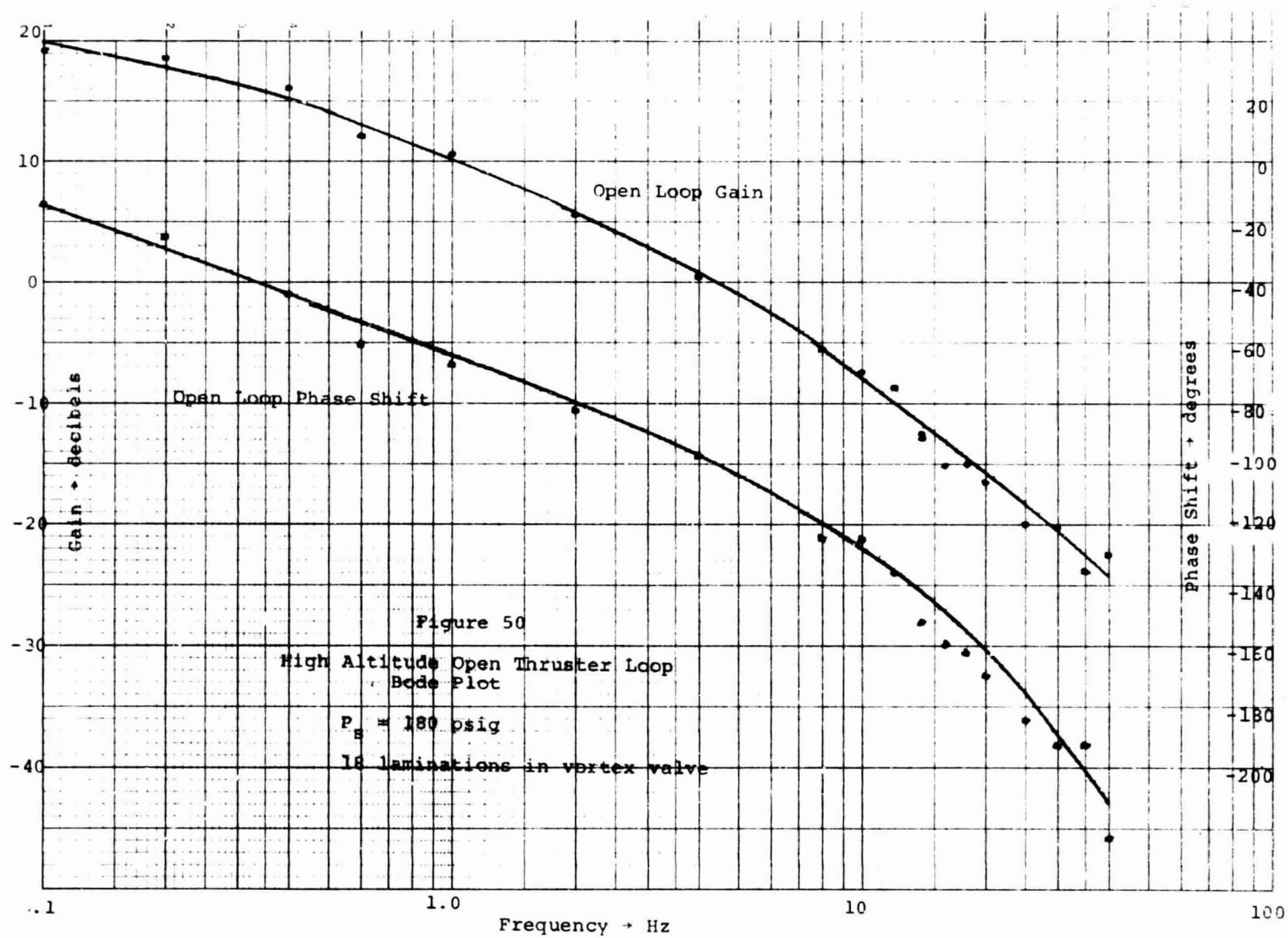


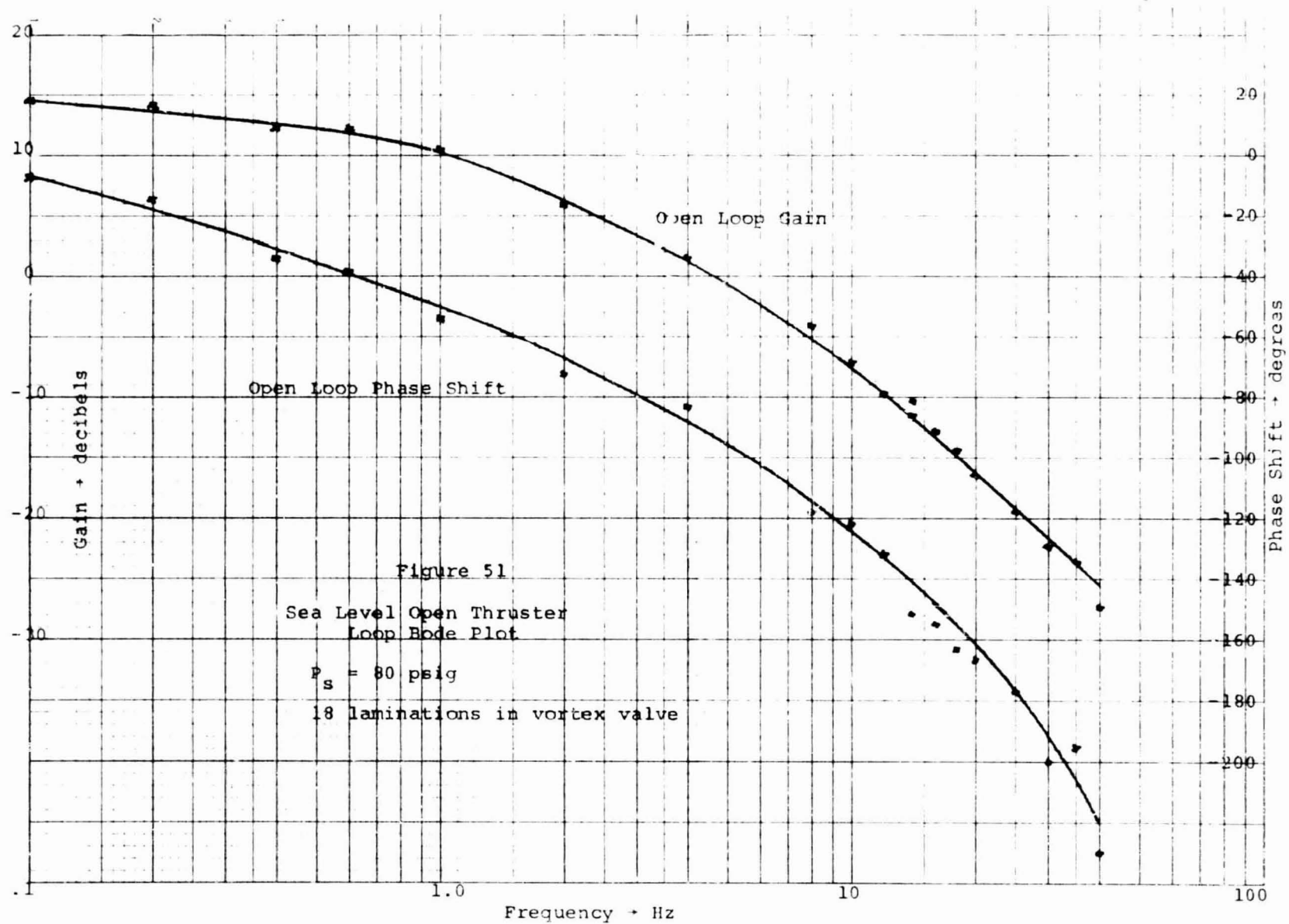
		Channel Length			
		10"	15"	24"	51"
1/C	in ⁻²	1.25x10 ⁶	.833x10 ⁶	.521x10 ⁶	.245x10 ⁶
1/R	$\frac{\text{lb}_m/\text{sec}}{\text{psi}}$	2.67x10 ⁴	2.67x10 ⁴	2.67x10 ⁴	2.67x10 ⁴
R/L	radians/sec	5560	3710	2280	1080
R _O /R _i	$\frac{\text{psi}}{\text{psi}}$	.444	.444	.444	.444
1/R _O C _B	radians/sec	4.43	4.43	4.43	4.43

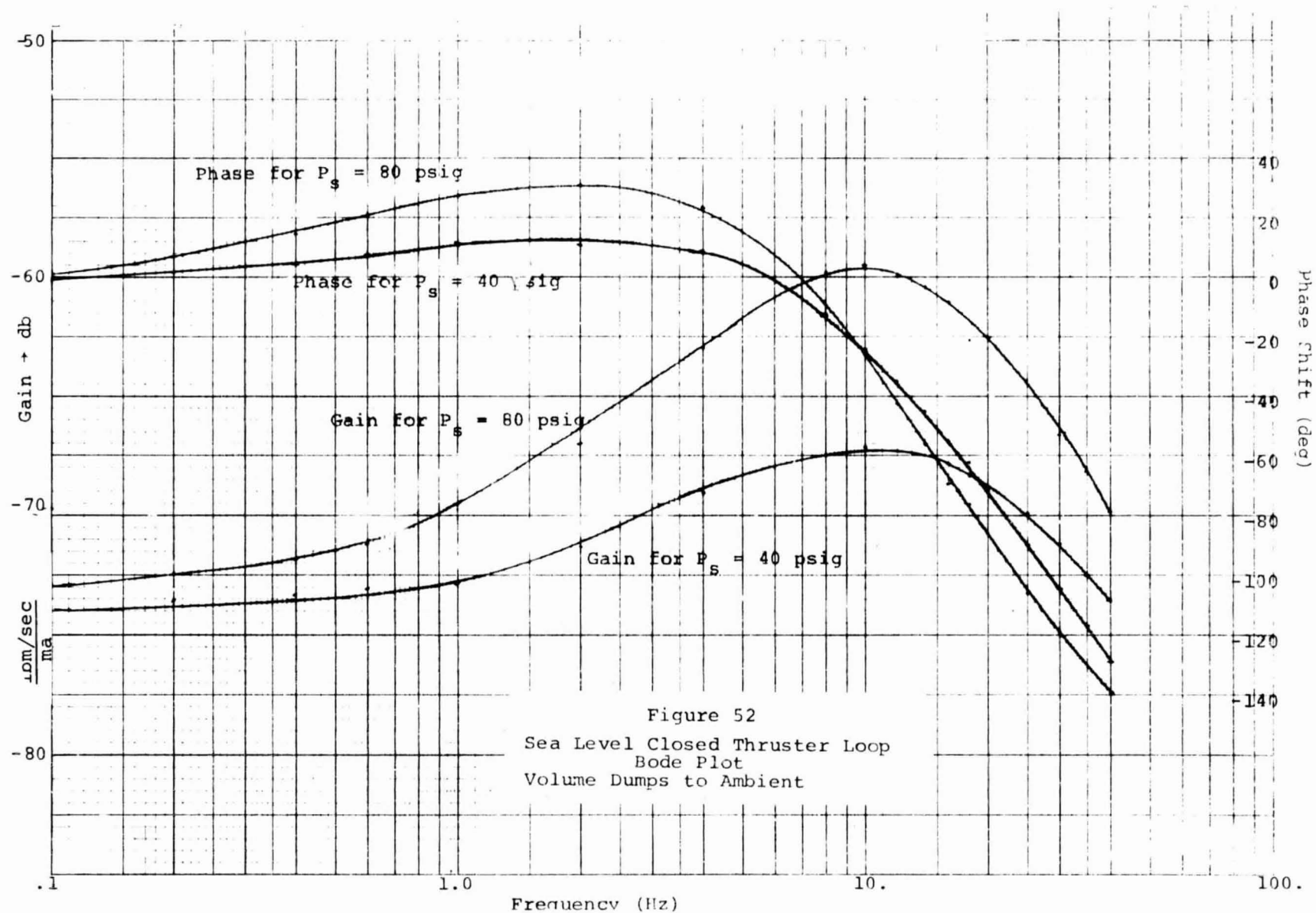
Figure 47
Vortex Valve Assembly
Block Diagram and Circuit Parameters

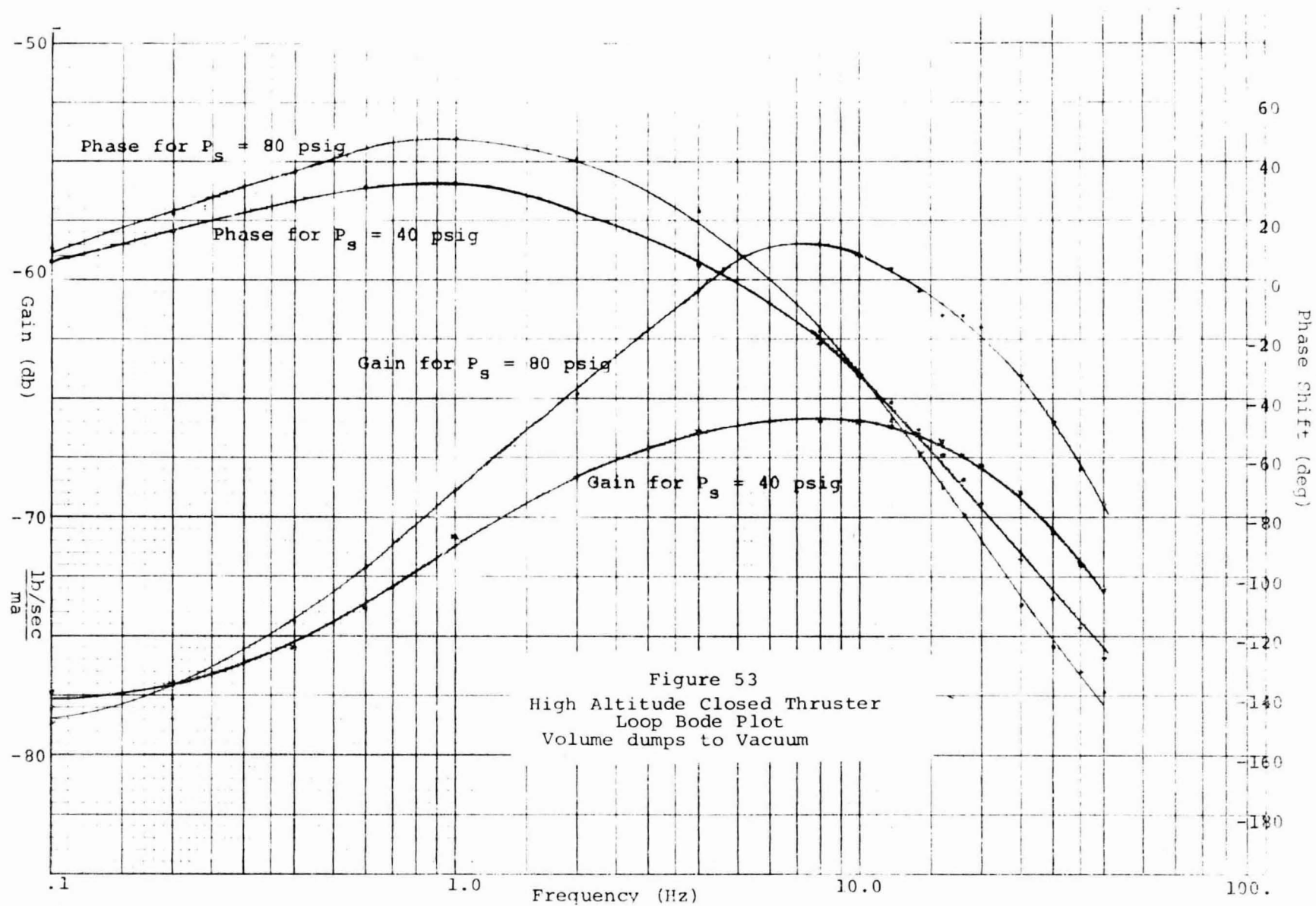












$$\frac{\Delta \dot{\omega}_Q}{\Delta i} = \frac{k \left(1 + \frac{s}{\omega_B}\right)}{\left(1 + \frac{s}{33.2}\right) \left(1 + \frac{s}{70}\right)} \quad \frac{\text{lb/sec}}{\text{amp}}$$

where: K depends on the torque motor coil connection and the supply pressure to the thruster.

$$3.5 \leq \omega_B \leq 5 \text{ rad/sec}$$

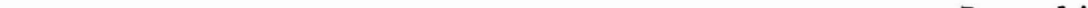
The section entitled "Performance of Simulator and Hardware" presented the system performance as measured on a breadboard development for fine control operation. The final hardware performance for all three axes of control was essentially identical and will not be repeated here.

High Pressure Tests: As indicated earlier, the FPT thrust feedback is limited so that bang-bang action is achieved for large position errors.

Figure 54 is a Sanborn trace showing the functioning of the FPT both at high pressures and for a transition to low pressure. This test verified the ability of the FPT to follow a step input command and to operate bang-bang when on coarse control. During the test the thruster supply pressure was reduced to simulate fine control conditions. The FPT continued to follow the input command. Tank pressures up to 2000 psia were run during the high pressure test series.

$\Delta P_O = 230 \text{ psig}$

$\Delta P_O = 18 \text{ psig}$

$P_T = 1350$ psig  $P_T = 1450$ psig

74

Electronic Design

Torque Motor Driver Amplifier: Figure 55 shows a circuit diagram for the torque motor driver. The circuit consists of an operational amplifier in series with a Darlington power stage to provide driving current capability. Figures 56, 57, 58, and 59 show uncompensated and compensated versions of the amplifier. The amplifier configuration shown in Figure 57 had a series resistance-capacitance feedback (R_C and C_C) connected directly around the operational amplifier. The purpose of the R_C C_C feedback compensation was to provide high loop gain at low frequencies and low loop gain at higher frequencies - in particular, where the phase shift exceeds 180 degrees.

Figure 60 indicates the closed loop will be stable, but could be lightly damped if the crossover frequency were larger than both ω_1 and $(R_m + R_L)/L_m$.

Figure 59 reduces to Figure 61 if $Z_C = R_C + \frac{1}{sC_C}$. The lag due to the torque motor inductance can be cancelled by the lead due to the compensation, R_C and C_C by setting:

$$R_C C_C = \frac{L_m}{R_m + R_L}$$

Typical parameter values are:

$L_m = .06$ henry torque motor self and mutual inductance
 $R_m = 80$ ohms torque motor coils in series
 $R_L = 10$ ohms resistance to ground

Then:

$$R_C C_C = \frac{L_m}{R_m + R_L} = \frac{.06}{80+10} = .667 \text{ msec}$$

One method employed in determining optimum circuit parameters was to subject the uncompensated torque motor amplifier to a step response test. The output current should not ring about the desired step command. If it does, then R_C and C_C should be selected to reduce the ringing. An initial R_C C_C product in the neighborhood of one millisecond should be selected.

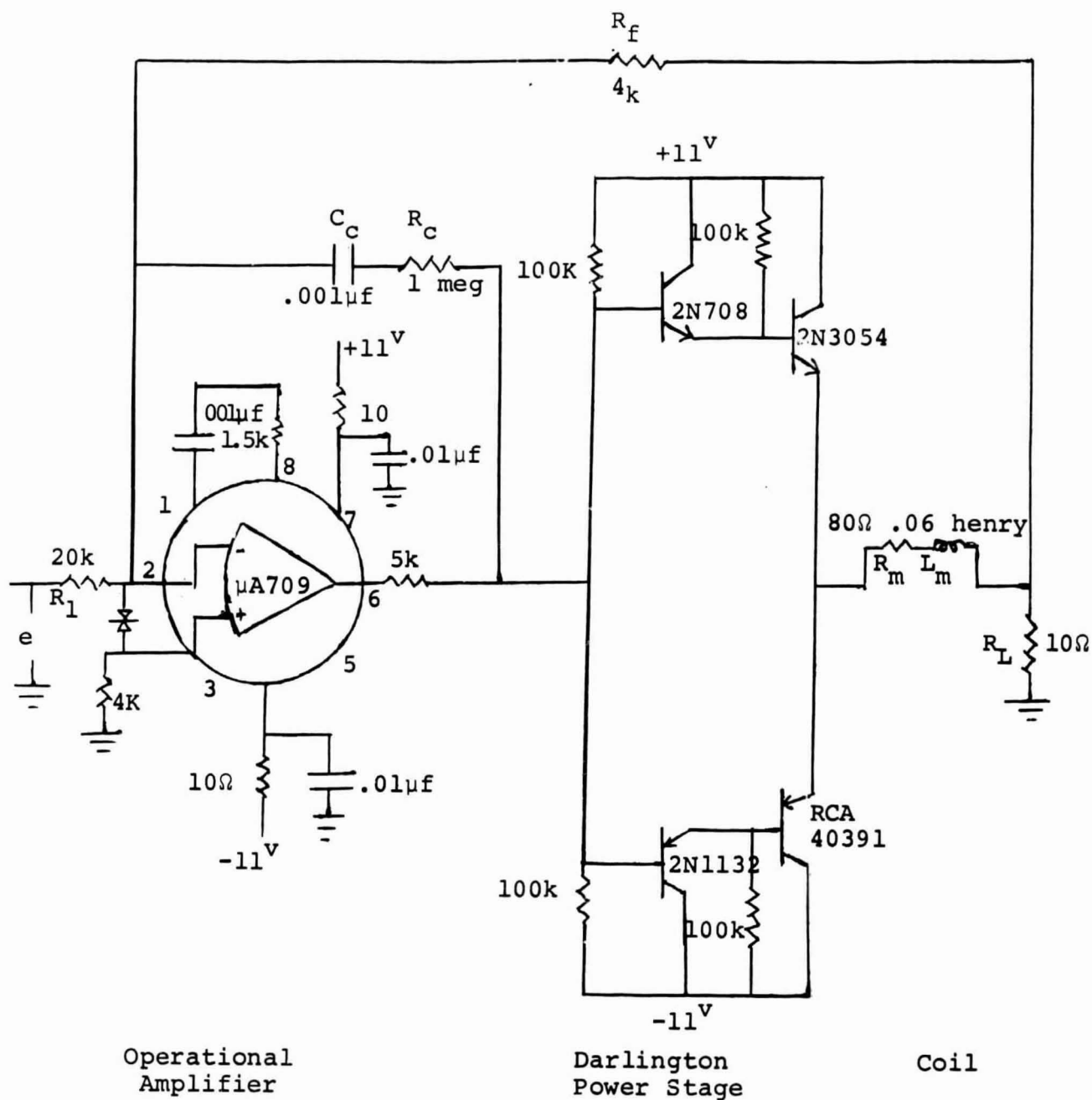


Figure 55
Torque Motor Amplifier

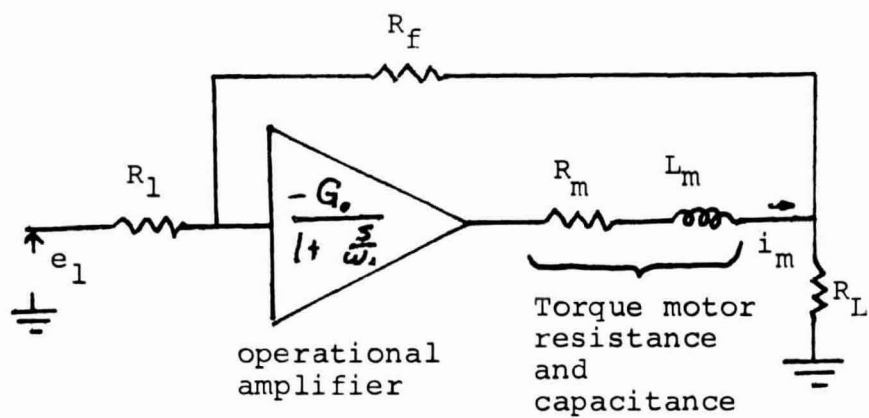


Figure 56
Torque Motor Amplifier w/o Compensation

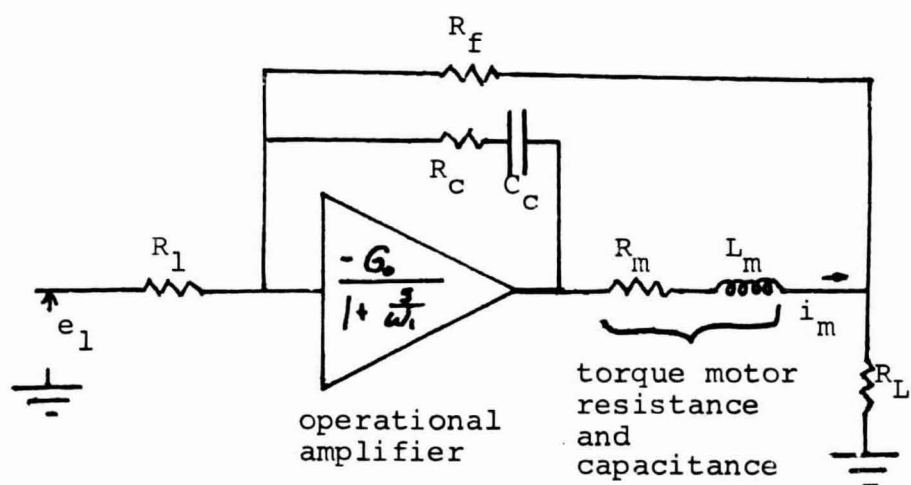


Figure 57
Torque Motor Amplifier with Compensation

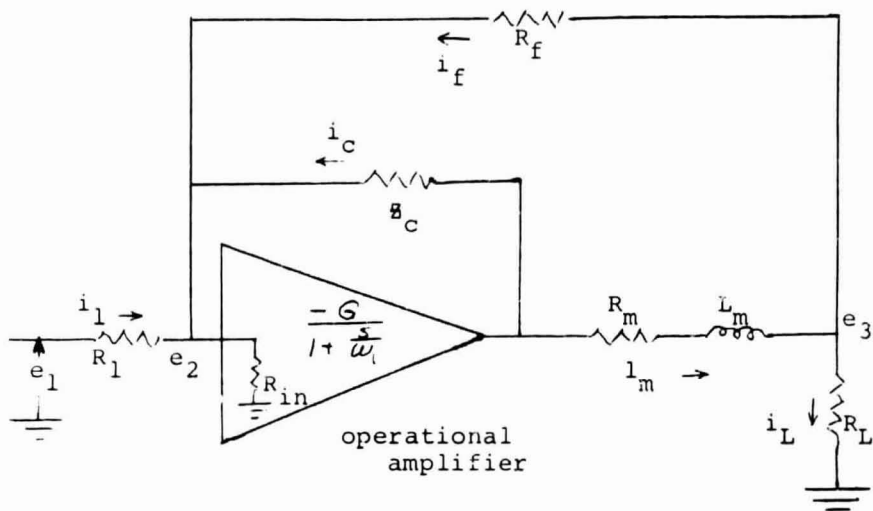


Figure 58
General Form of the Torque Motor Amplifier

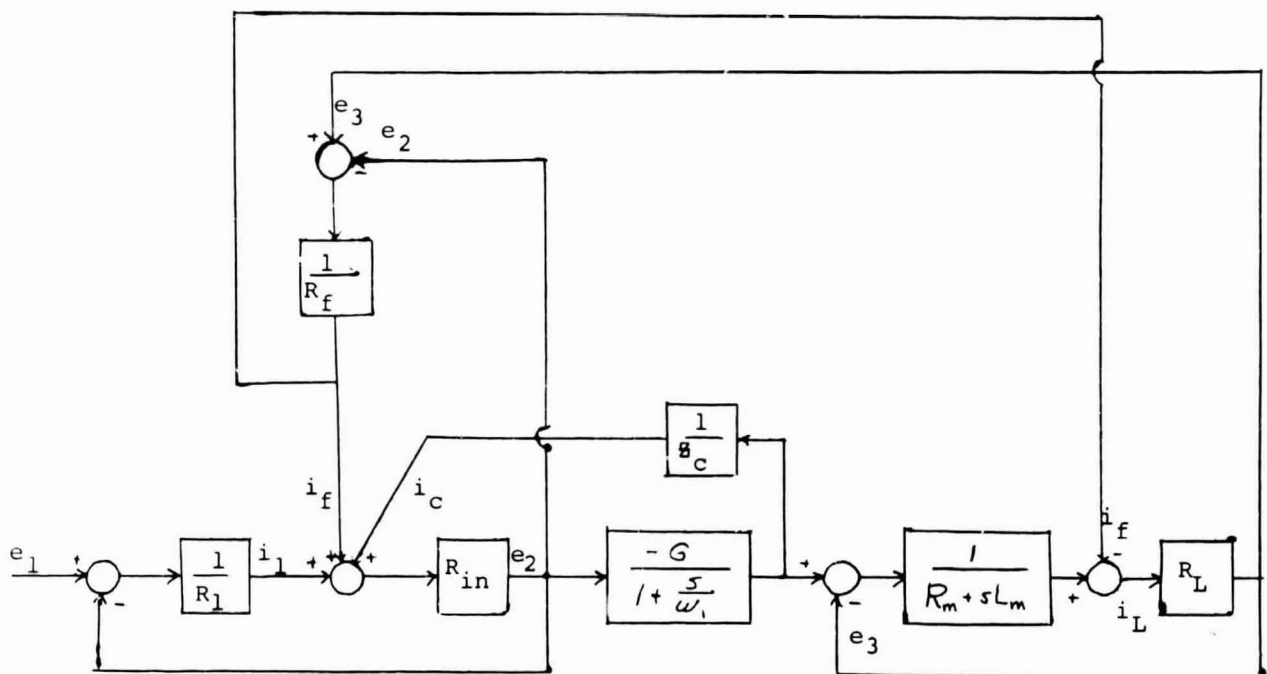


Figure 59
Torque Motor Amplifier Block Diagram

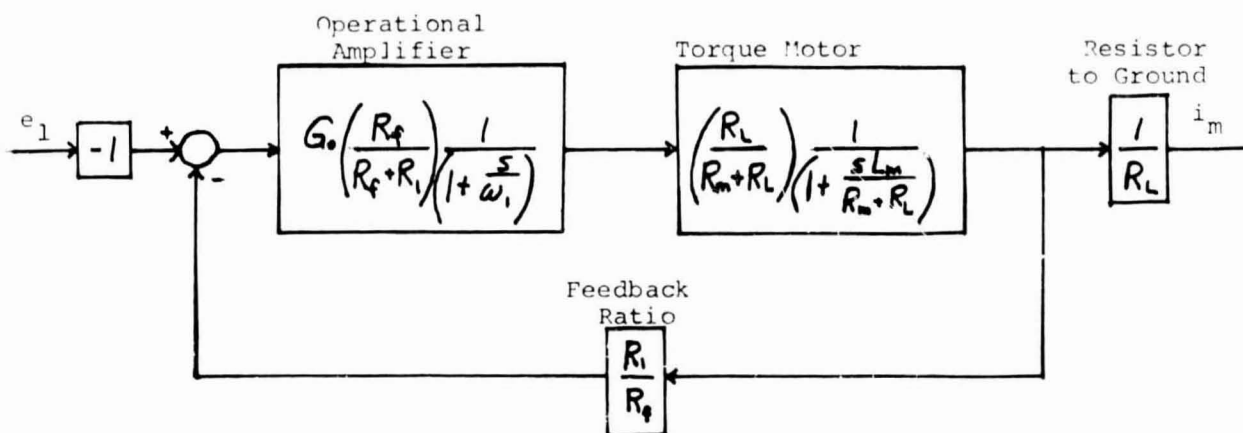


Figure 60
Block Diagram of Uncompensated Torque Motor Amplifier

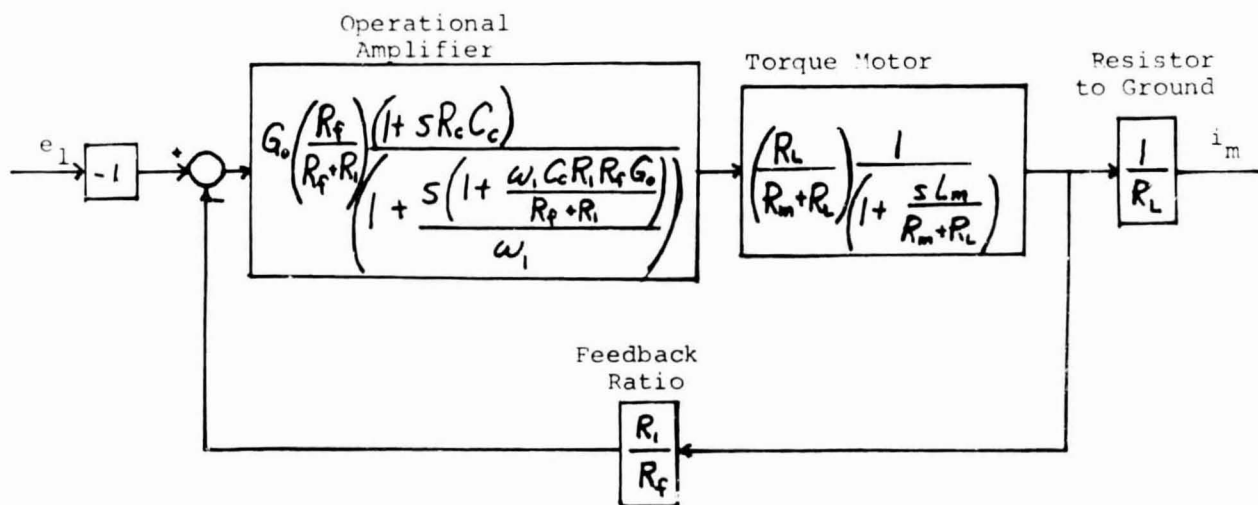


Figure 61
Block Diagram of Compensated Torque Motor Amplifier

ERROR SOURCES

There are two major sources of error associated with the FPT. These error sources, noise and hysteresis, are treated analytically in the following section (as well as in Appendix 2 and 3). In addition, a discussion of air bearing test error sources is also included.

NOISE

The outputs of the vortex valves contain random pressure (or flow) fluctuations which can be described in terms of a power spectral density. Appendix 2 shows the method used to predict the pointing errors due to output noise in the vortex valve push-pull pairs. The power spectral density was determined from laboratory tests and position loop gains and compensation were selected to provide about a one second response. The evaluation of noise effects was conducted first by considering only the valve noise and theoretically closing the FPT loop around the valve and later, as the FPT hardware became available, the output noise of the assembly was evaluated. The noise at the FPT output was reduced due to the action of the feedback loop around the valve. Since the loop noise more accurately describes the actual situation when the vehicle and control are coupled, only that analysis is included here.

Figures 62 and 63 show predicted pitch/yaw and roll pointing errors due to the noise. The assumptions made for the pitch/yaw predictions of Figure 62 are:

1. The gain from vortex valve flow to angular acceleration is 155.4 deg/sec^2 per lb/sec.
2. The open position loop gain is $14.8/\text{s}^2$.
3. The lead compensation break frequency is $.99 \text{ rad/sec}$.
4. For $P_s = 100 \text{ psia}$ the maximum differential flow out of a vortex valve push-pull pair is 0.01 lb/sec .

The assumptions made for the roll predictions of Figure 63 are:

1. The gain from vortex valve flow to angular acceleration is 721 deg/sec^2 per lb/sec.
2. The open position loop gain is $2.3/\text{s}^2$.
3. The lead compensation break frequency is $.99 \text{ rad/sec}$.
4. For $P_s = 100 \text{ psia}$ the maximum differential flow out of a vortex valve push-pull pair is 0.01 lb/sec .

The measured FPT signal to noise ratio was in the neighborhood of 1300:1. Figure 62 shows the rms pitch-yaw pointing error to be approximately $.16 \text{ arcsec}$ for $S/N = 1300$ and $P_s = 100 \text{ psia}$. Figure 63 shows that under the same circumstances the roll rms pointing error to be around 3.8 arcsec . Early in the development program, noise predictions based on only the vortex amplifier performance in

Figure 62

Pitch or Yaw Uncertainty due to Closed
Loop Thruster Noise
S/N = Closed Thruster Loop Signal to Noise Ratio

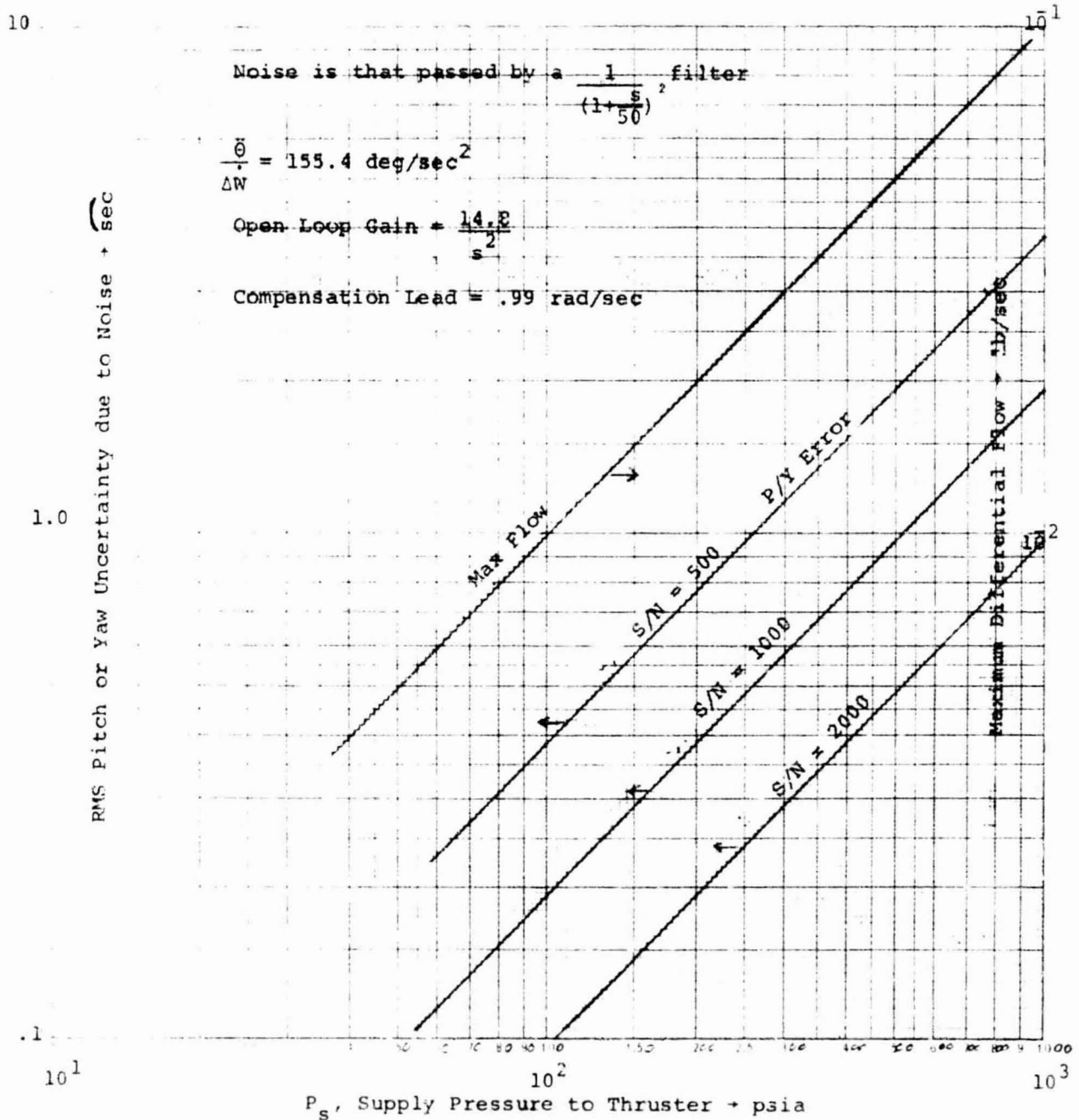
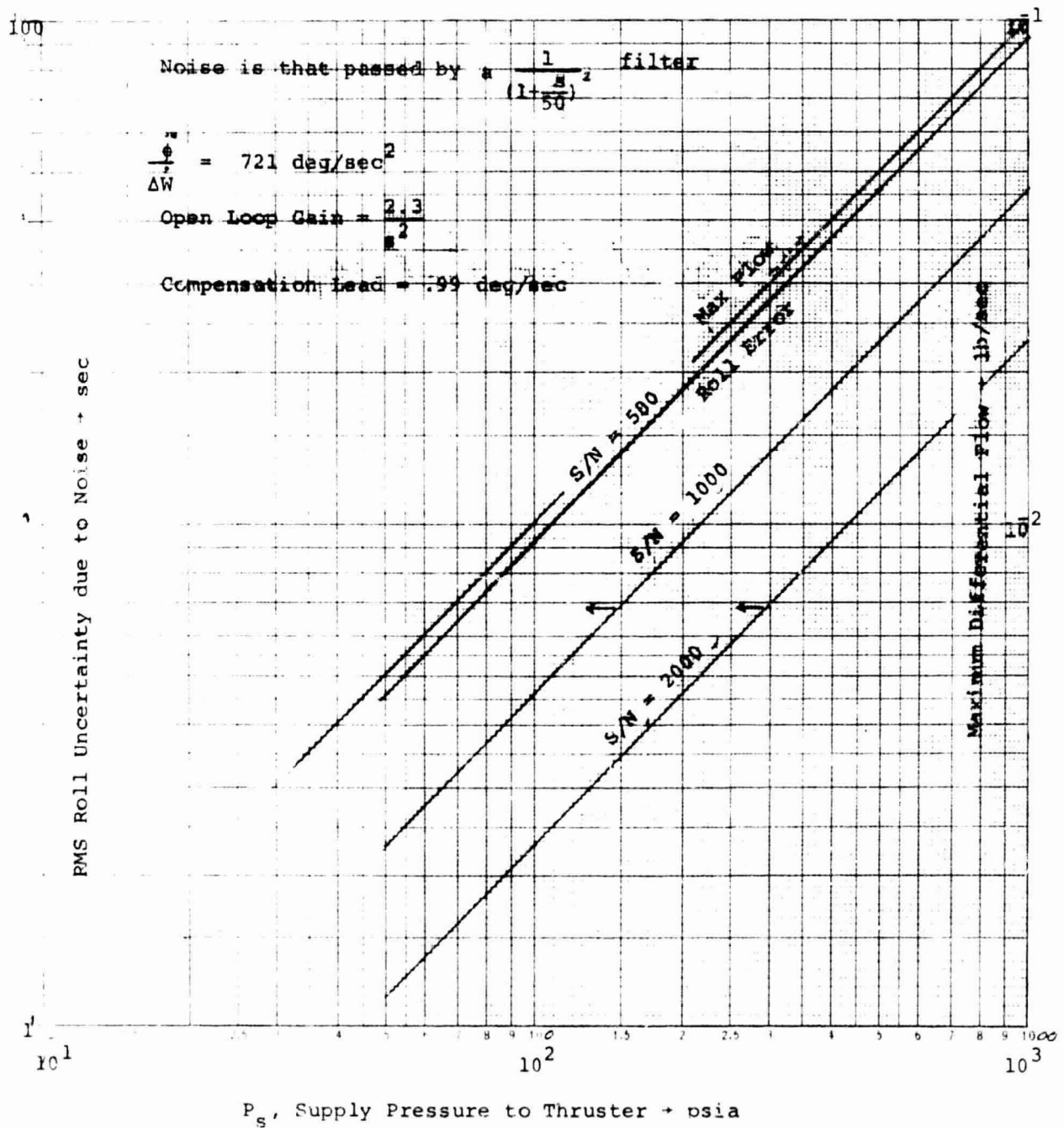


Figure 63

Roll Uncertainty Due to Closed Loop Thruster Noise

S/N = Closed Thruster Loop Signal to Noise Ratio



conjunction with theoretical estimates of the remaining control loop and system dynamics agreed very closely with the above value for pitch of .16 arcseconds; .1 arcseconds was predicted at that time.

HYSTERESIS

Hysteresis within the mechanical and fluidic portions of the FPT can cause a low frequency limit cycle and attendant error in pointing accuracy. Two types of hysteresis can exist in physical systems. The hysteresis width may be proportional to signal amplitude or the width may be constant (backlash). For the FPT dynamics only backlash is significant. That is, a limit cycle exists only if backlash is present. Backlash of the order of 1% is significant whereas hysteresis (as a percent of amplitude) may be as high as 15 to 20% and not affect system accuracy.

The backlash limit cycle amplitude and frequency was predicted using describing function techniques.

The result of the limit cycle analysis is presented in Figure 64. Appendix 3 explains the derivation of Figure 64. Appendix 3 also defines the various system gains in the pitch and yaw position loops.

Actual test results indicate the FPT hysteresis decreases with input signal amplitude to the point that it is obscured by the resolution of the measuring instruments. Conservatively assuming 0.1 percent hysteresis and an electronic gain of 20 volts per volt, then Figure 64 predicts a limit cycle amplitude $0.1 \times .25 = .025$ arcsec at a limit cycle frequency of 3.62 HZ. Closed loop simulations have also shown little tendency toward backlash limit cycling.

Figure 64

Pitch (or Yaw) Hysteresis Limit Cycle
vs.
Electronic Gain

10.

$$I = 70 \text{ slug ft}^2$$

$$L = 1.25 \text{ ft}$$

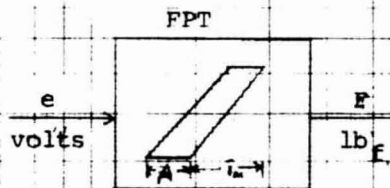
Thruster gain = .144 lbf/volt (sea level performance with torque motor coils in parallel)

Sensor = 3.6 volt/deg

10.

Limit Cycle Amplitude per 1% Hysteresis

1.0



$$\% \text{ Hysteresis} = 100 \frac{A}{E}$$

Limit Cycle Frequency, Hz

1.0

Frequency

Amplitude

1

10

100

K, Electronic Gain + Volt/Volt

6.6K
1/15/67

APPENDIX 1

NOMENCLATURE

A_B	= bellows area
A_T	= nozzle throat area
A_O	= bellows exit orifice area
C_D	= discharge coefficient
F_B	= bellows force
F_O	= FPT output force
FPT	= abbreviation for Fluidic Proportional Thruster
I_S	= specific impulse
I_θ	= pitch moment of inertia
I_ϕ	= roll moment of inertia
K	= electronic gain
K_e	= equivalent torque motor spring constant
K_f	= inverted flapper gain coefficient
K_{FPT}	= FPT gain
K_m	= magnetic spring constant
K_t	= torque motor gain
K_s	= demagnetized torque motor spring constant
K_v	= vortex valve gain
L_θ	= pitch moment arm
L_ϕ	= roll moment arm
M_O	= torque motor magnetization
N_C	= number of turns per coil in torque motor
P_B	= bellows output differential pressure
P_C	= vortex valve control pressure
P_O	= vortex valve output pressure
P_S	= supply pressure to thruster
R_i	= fluidic resistance into bellows
R_O	= fluidic resistance out of bellows
R_T	= thruster nozzle fluidic resistance
R_v	= fluidic resistance providing control pressure bias
R	= gas constant
S/N	= signal to noise ratio
T	= temperature

Nomenclature (continued)

T_d	=	torque motor developed torque
T_{dis}	=	disturbance torque acting on vehicle
T_e	=	error torque which positions FPT flapper
T_{fb}	=	torque fed back to torque motor
T_θ	=	pitch torque
T_ϕ	=	roll torque
V_{BV}	=	volume of bellows and added volume
$\dot{W}$	=	flow
$\Delta \dot{W}_{max}$	p to p	= maximum peak to peak thruster flow
$\Delta \dot{W}_{rms}$		= rms noise flow
$\dot{W}_O$		= thruster output flow
d_i	=	bellows entrance orifice diameter
d_o	=	bellows exit orifice diameter
e	=	torque motor amplifier input
g	=	acceleration due to gravity
g_l	=	torque motor armature to pole air gap
i	=	current in torque motor coil
k_1	=	bias spring fastened to case
k_2	=	bias spring fastened to bellows
k_3	=	bellows spring constant
k_p	=	constant relating maximum flow to P_s
m_B	=	bellows mass
r_a	=	torque motor radius on which magnetization acts
r_{al}	=	torque motor armature radius on which bias springs act
r_f	=	flapper radius
rms	=	abbreviation for root mean square
x	=	flapper translational displacement
α	=	torque to inertia ratio
α_θ	=	pitch torque to inertia ratio
β	=	flapper angular displacement
γ	=	ratio of specific heats
θ	=	pitch angle
θ_c	=	pitch command

Nomenclature (continued)

Θ_e	=	pitch error
Θ_{fb}	=	pitch feedback angle
τ	=	transport delay
ϕ	=	roll angle
ϕ_c	=	roll command
ϕ_e	=	roll error
ϕ_{fb}	=	roll feedback angle
ψ	=	yaw angle
ω_1	=	lead compensation break frequency
ω_{1c}	=	coarse control lead break frequency
ω_3	=	FPT lag break frequency
ω_4	=	FPT lag break frequency
ω_B	=	FPT feedback break frequency
ω_{nf}	=	noise filter break frequency

APPENDIX 2

NOISE ANALYSIS METHOD

This appendix summarizes the approach followed in measuring and evaluating the effect of system noise. Generally, the output of the vortex valve thruster consists of a desired signal plus noise. The noise is measured by setting the desired signal to zero so that whatever output remains is noise. The noise has a power distribution which is a function of frequency. The effect of frequency sensitive elements between the noise source and the pitch or yaw position is estimated as follows:

1. Determine $G_{N\theta}$, the transfer function from the noise input to the pitch (or yaw) angle.
2. Determine $N(\omega)$, the noise spectral density of the output of the vortex valve.
3. Compute the rms pointing error from the relation:

$$\theta_{\text{rms}} = \sqrt{\int_0^{\infty} |G_{N\theta}|^2 N(\omega) d\omega}$$

Noise is random and can be characterized by its spectral density (ω). In the case of the vortex valve the spectral density of the output noise has the dimensions of $(\text{lb}_m/\text{sec})^2$ per (rad/sec) .

An ideal way to measure the spectral density would be to filter the vortex valve noise through a one radian per second band pass filter, measure the rms output, and finally, square the rms value. The result would be the spectral density at the center frequency of the band pass filter. This technique is difficult to apply because filters with an ideal band pass of one radian per second are not available.

A more common (but more time consuming and expensive in required computer time) way to measure the spectral density is to:

1. obtain a time history of the noise signal.
2. determine the autocorrelation function of the noise signal.
3. compute the Fourier transform of the autocorrelation function (which is the power spectral density).

Past experience based on obtaining the spectral density by the correlation and transform approach has shown that this approach can be simplified so that commonly available lab equipment combined with straightforward computation techniques can be used.

Generally:

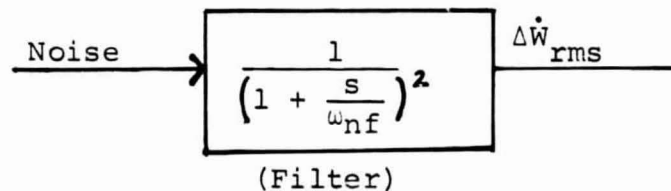
1. The noise spectral density associated with the tested elements can be approximated by white noise at low frequencies.
2. The rms value of the noise can be assumed to be 1/6 the peak to peak value of noise.

For the subject application, the relationship between power spectral density, supply pressure to the thruster, and signal to noise ratio is derived first. The signal to noise ratio is defined as:

$$11) \quad \frac{S}{N} = \frac{\Delta \dot{W}_{\max \text{ p to p}}}{\Delta \dot{W}_{\text{rms}}}$$

where: $\Delta \dot{W}_{\max \text{ p to p}}$ is the maximum peak to peak FPT differential output flow. e.g. for $P_s = 100$ psia the maximum FPT differential flow is $+0.01$ lb/sec so $\Delta \dot{W}_{\max \text{ p to p}} = 2 \times 0.01 = 0.02$ lb/sec.

$\Delta \dot{W}_{\text{rms}}$ is the root mean square noise measured at a filter output.



For white noise the power spectral density is given by

$$12) \quad N(\omega) = \frac{4}{\pi \omega_{nf}} (\Delta \dot{W}_{\text{rms}})^2$$

Also, the maximum differential thruster flow is proportional to the thruster supply pressure. If the constant of proportionality is K_p , then equation 13 follows.

$$13) \quad \Delta \dot{W}_{\max \text{ p to p}} = K_p P_s$$

Combining the preceding equations

$$14) \quad N(\omega) = \frac{4}{\pi \omega_{nf}} \left(\frac{\Delta \dot{W}_{\max \text{ p to p}}}{S/N} \right)^2 = \frac{4}{\pi \omega_{nf}} \left(\frac{K_p P_s}{S/N} \right)^2$$

The constant K_p was evaluated from experimental data. The rms noise passed by a filter with $\omega_{nf} = 50$ rad/sec was 1.46×10^5 lb/sec and the supply pressure to the thruster for that measurement was 90 psia. The maximum differential FPT flow was ± 0.0095 lb/sec. Then:

$$P_s = 95 \text{ psia}$$

$$\Delta \dot{W}_{\text{rms}} = 1.46 \times 10^5 \text{ lb/sec}$$

$$\frac{S}{N} = \frac{2 \times .0095}{1.46 \times 10^{-5}} = 1305$$

From equations 11 and 13

$$14) \quad \Delta \dot{W}_{\text{rms}} = \frac{k_p P_s}{S/N}$$

Using equation 14 to solve for k_p , the result is:

$$k_p = \frac{\Delta \dot{W}_{\text{rms}} (S/N)}{P_s} = \frac{1.46 \times 10^5 \times 1.305 \times 10^3}{.95 \times 10^2} = 2.01 \times 10^4 \frac{\text{lb/sec}}{\text{psia}}$$

The latter constants can be combined so the power spectral density becomes:

$$N(\omega) = \frac{4}{\pi \omega_{\text{nf}}} \left(\frac{K_p P_s}{S/N} \right)^2 = \frac{4}{\pi (50)} (2.01 \times 10^4)^2 \left(\frac{P_s}{S/N} \right)^2 = 1.025 \times 10^9 \left(\frac{P_s}{S/N} \right)^2 \frac{(\text{lb/sec})^2}{\text{rad/sec}}$$

The above equation is valuable because it expresses the power spectral density in terms of easily measured quantities, i.e. the FPT supply pressure and the FPT output signal to noise ratio.

The next step is to evaluate the transfer function from the noise input to the vehicle pitch, yaw and roll positions.

Figure 65 shows the pitch/yaw and roll position loops. For this analysis the sensor gain is unimportant. The pointing error depends on the loop gain, which can be adjusted electronically, and the gain from flow to angular acceleration.

The noise transfer functions of Figure 65 are: For pitch and yaw:

$$\frac{\theta}{N} = \frac{37750 \left(1 + \frac{s}{33.2}\right) \left(1 + \frac{s}{70}\right) \left(1 + \frac{s}{100}\right)}{\left(1 + \frac{s}{1.06}\right) \left(1 + \frac{2(.444)s}{19.1} + \frac{s^2}{19.1^2}\right) \left(1 + \frac{2(.978)s}{94.5} + \frac{s^2}{94.5^2}\right)} \frac{\text{arcsec}}{\text{lb/sec}}$$

and for roll:

$$\frac{\phi}{N} = \frac{1.122 \times 10^6 \left(1 + \frac{s}{33.2}\right) \left(1 + \frac{s}{70}\right) \left(1 + \frac{s}{100}\right)}{\left(1 + \frac{2(.777)s}{1.634} + \frac{s^2}{1.634^2}\right) \left(1 + \frac{s}{26.9}\right) \left(1 + \frac{s}{78.6}\right) \left(1 + \frac{s}{94.7}\right)} \frac{\text{arcsec}}{\text{lb/sec}}$$

The rms pointing error is evaluated from Parseval's theorem. For pitch or yaw:

$$15) \quad \theta_{\text{rms}} = \sqrt{\int_0^\infty N(\omega) \left| \frac{\theta}{N} \right|^2 d\omega} = 6.02 \times 10^4 \sqrt{N(\omega)}$$

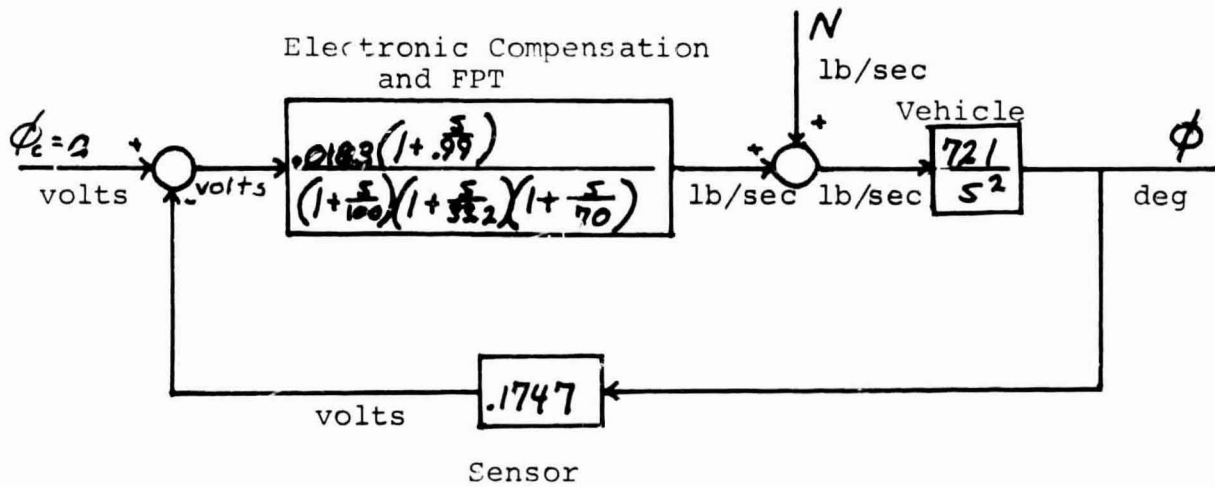
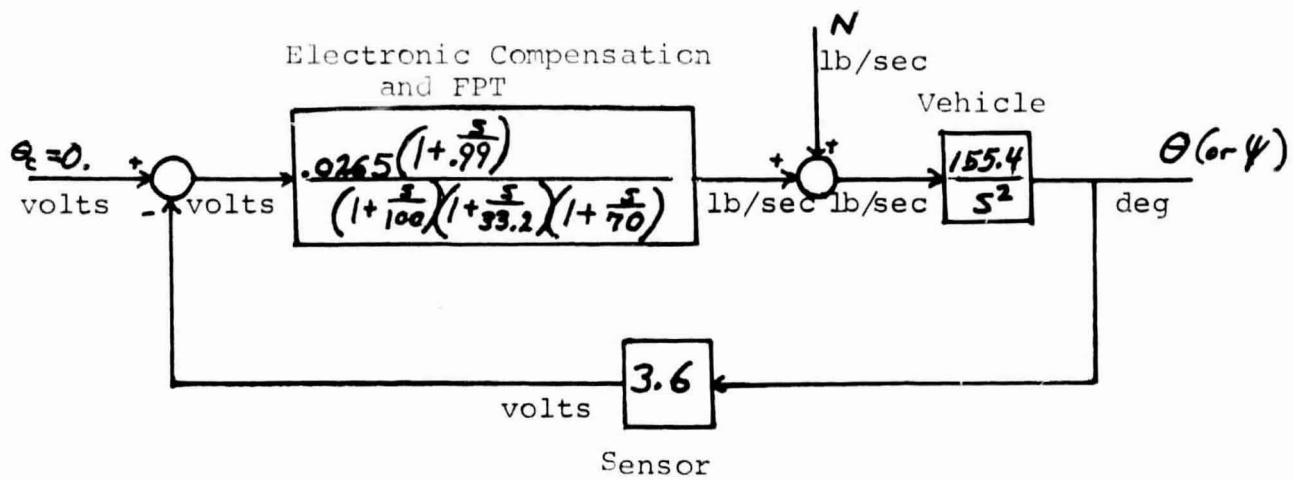


Figure 65
Fine Position Loops

For roll:

$$16) \quad \phi_{\text{rms}} = \sqrt{\int_0^{\omega_c} N(\omega) \left| \frac{\phi}{N} \right|^2 d\omega} = 1.442 \times 10^6 \sqrt{N(\omega)}$$

In the latter equations the integrals were numerically evaluated by a time sharing computer program. Substituting the previously derived value of $N(\omega)$ leads to the final desired result.

$$15) \quad \theta_{\text{rms}} = 6.02 \times 10^4 \sqrt{N(\omega)} = 6.02 \times 10^4 \sqrt{1.025 \times 10^{-9} \left(\frac{P_s}{S/N} \right)^2} = 1.92 \frac{P_s}{S/N}$$

$$16) \quad \phi_{\text{rms}} = 1.442 \times 10^6 \sqrt{N(\omega)} = 1.442 \times 10^6 \sqrt{1.025 \times 10^{-9} \left(\frac{P_s}{S/N} \right)^2} = 46 \frac{P_s}{S/N}$$

These equations are plotted in Figures 62 and 63.

APPENDIX 3

HYSTERESIS

Backlash and Hysteresis Limit Cycling

Backlash has an input-output characteristic shown in Figure 66. It is more easily visualized in terms of gears whose teeth must move through some small distance before the direction of torque transmitted by the gears can be reversed.

Figure 67 shows how backlash is represented by a transfer function. The backlash is sometimes referred to as hysteresis. The transfer function is the describing function. It has gain and phase characteristics which are independent of frequency but depend on input amplitude. The gain and phase are given by:

$$17) \quad G_H = (a^2 + b^2)^{1/2}$$

$$18) \quad \phi_H = \tan^{-1} (a/b)$$

$$\text{where: } a \equiv \frac{1}{\pi} (r) (r-2)$$

$$b \equiv \frac{1}{\pi} \left\{ \frac{\pi}{2} + \sin^{-1} (1-r) + (1-r) [2r-r^2]^{1/2} \right\}$$

$$r \equiv h/M \text{ refer to Figures 66 and 67}$$

The gain and phase are plotted in Figure 68.

The backlash of Figures 66, 67, and 68 is for a device with unity gain in the linear region. Usually the gain in the linear region is unequal to unity. In this case, the element with back lash is split into two parts as shown in Figure 69.

The hysteresis limit cycle amplitude was computed for the position loop in Figure 70. To make use of the backlash describing function the loop was rearranged to the form in Figure 71. The loop in Figure 71 oscillates when the loop characteristic equation satisfies the relation:

$$19) \quad 1 + G_H = 0$$

$$\text{or } 19) \quad H = -1/G_H$$

The two transfer functions H and G depend on frequency and the ratio of $.144A$ to ΔF^1 . The two transfer functions are plotted together in Figure 72 for several different values of electronic gain. The intersection of the curves is the desired solution ($H = -1/G$).

For example, an electronic gain of 3.25 volts per volt causes the H curve in Figure 72 to intersect the $-1/G$ curve at a magnitude of 2.34 and an angle of -137 degrees. The limit cycle frequency is 1.05 rad/sec and the ratio of $.144A$ to ΔF^1 is 1.30. Therefore ΔF^1 is:

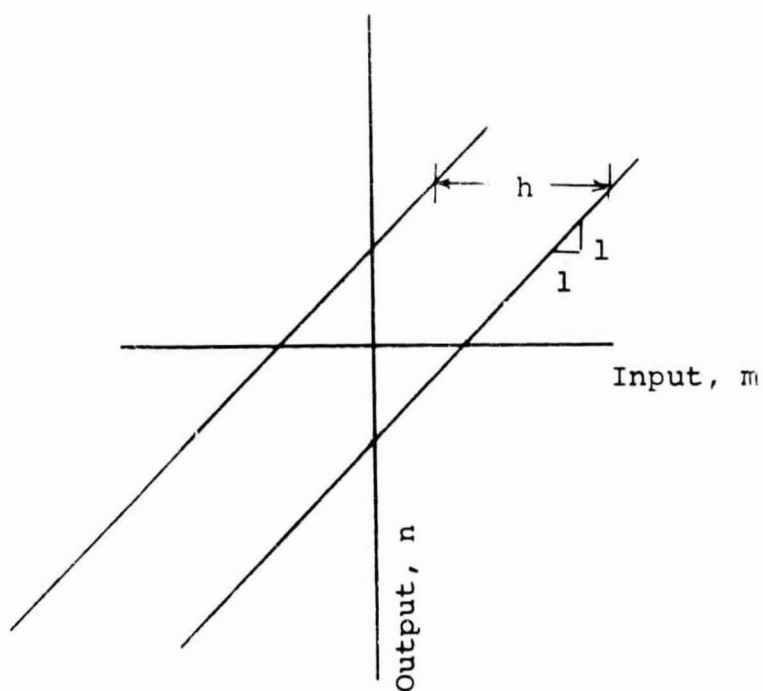


Figure 66
Backlash

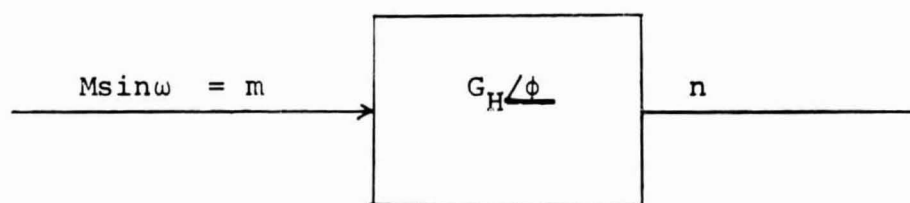
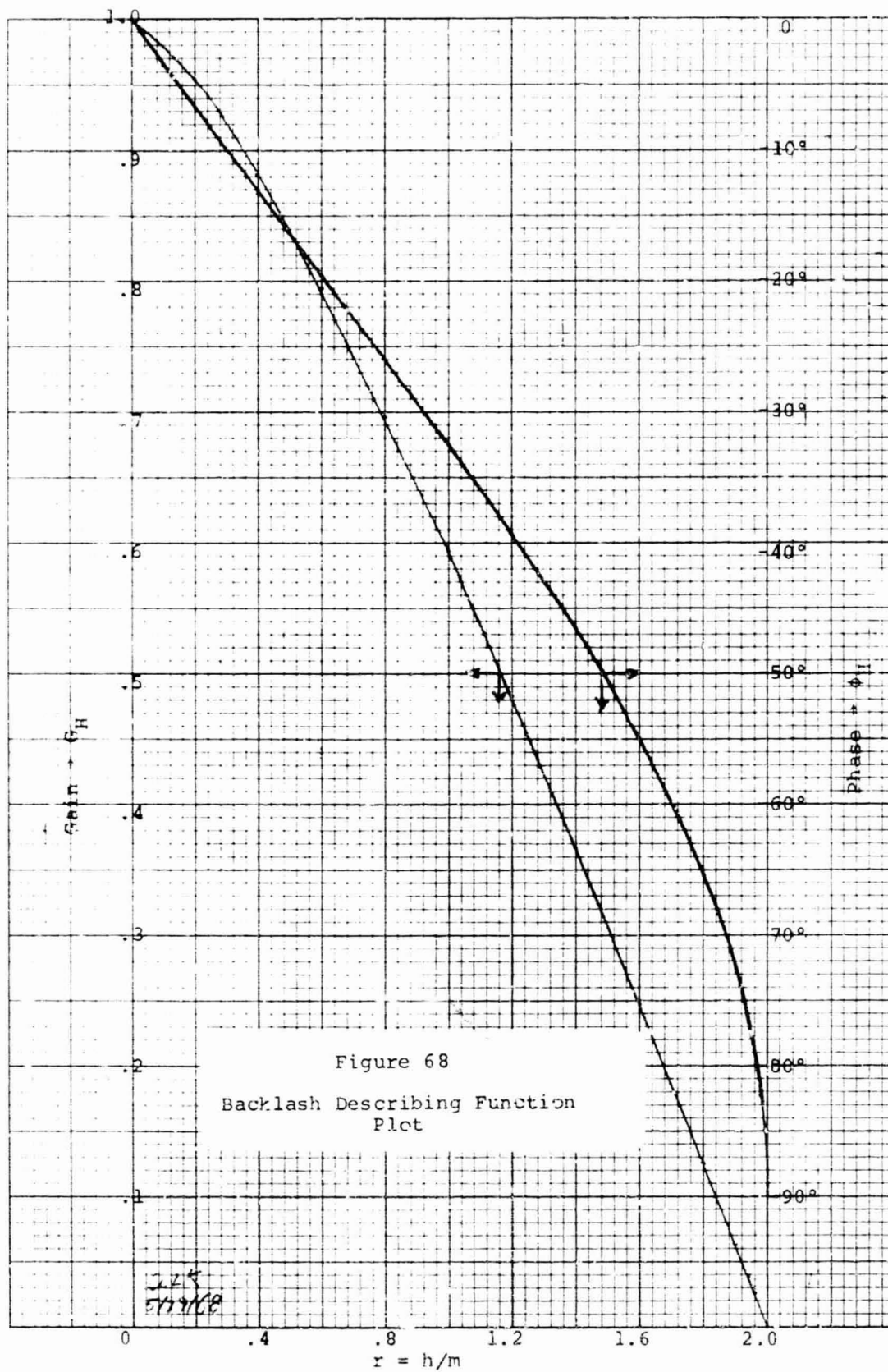
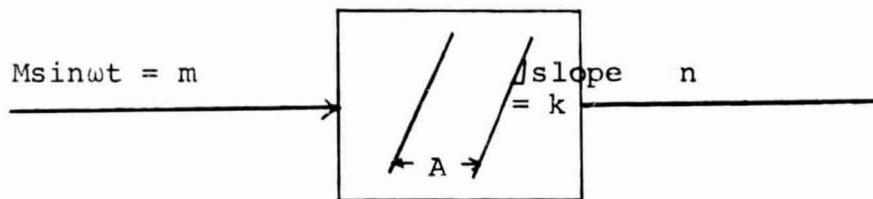


Figure 67
Backlash Describing Function Definitions

$$\begin{aligned} G &= G(h/M) = G(r) \\ \phi &= \phi(h/M) = \phi(r) \\ r &\equiv h/M \end{aligned}$$





The above element with backlash, H , and linear gain of k is equivalent to the two blocks below.

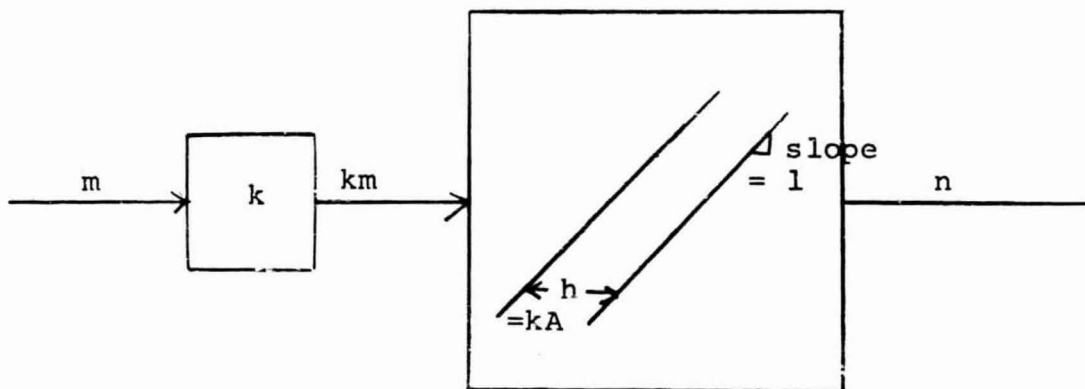
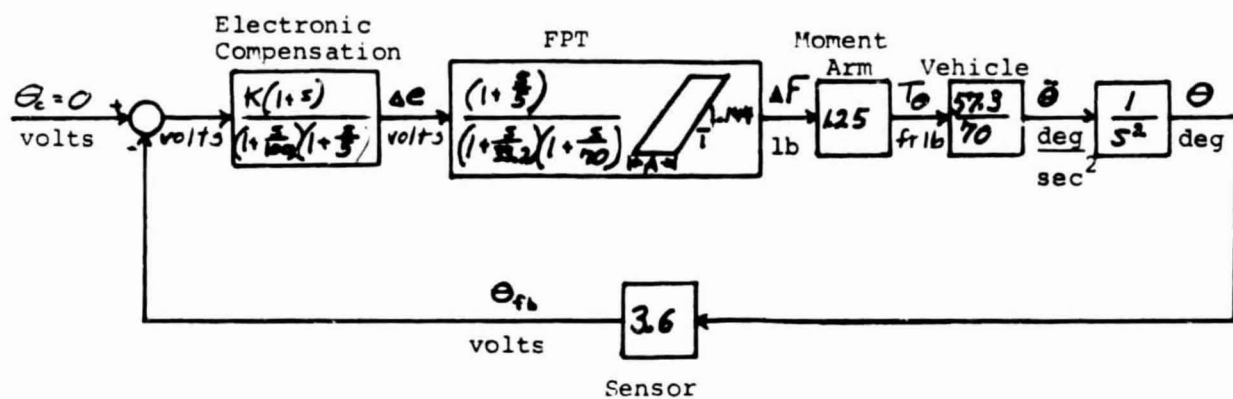


Figure 69
Reduction of a General Backlash Element



$K \equiv$ Electronic Gain

Figure 70
Pitch (or Yaw) Position Loop Examined
for Hysteresis Limit Cycling

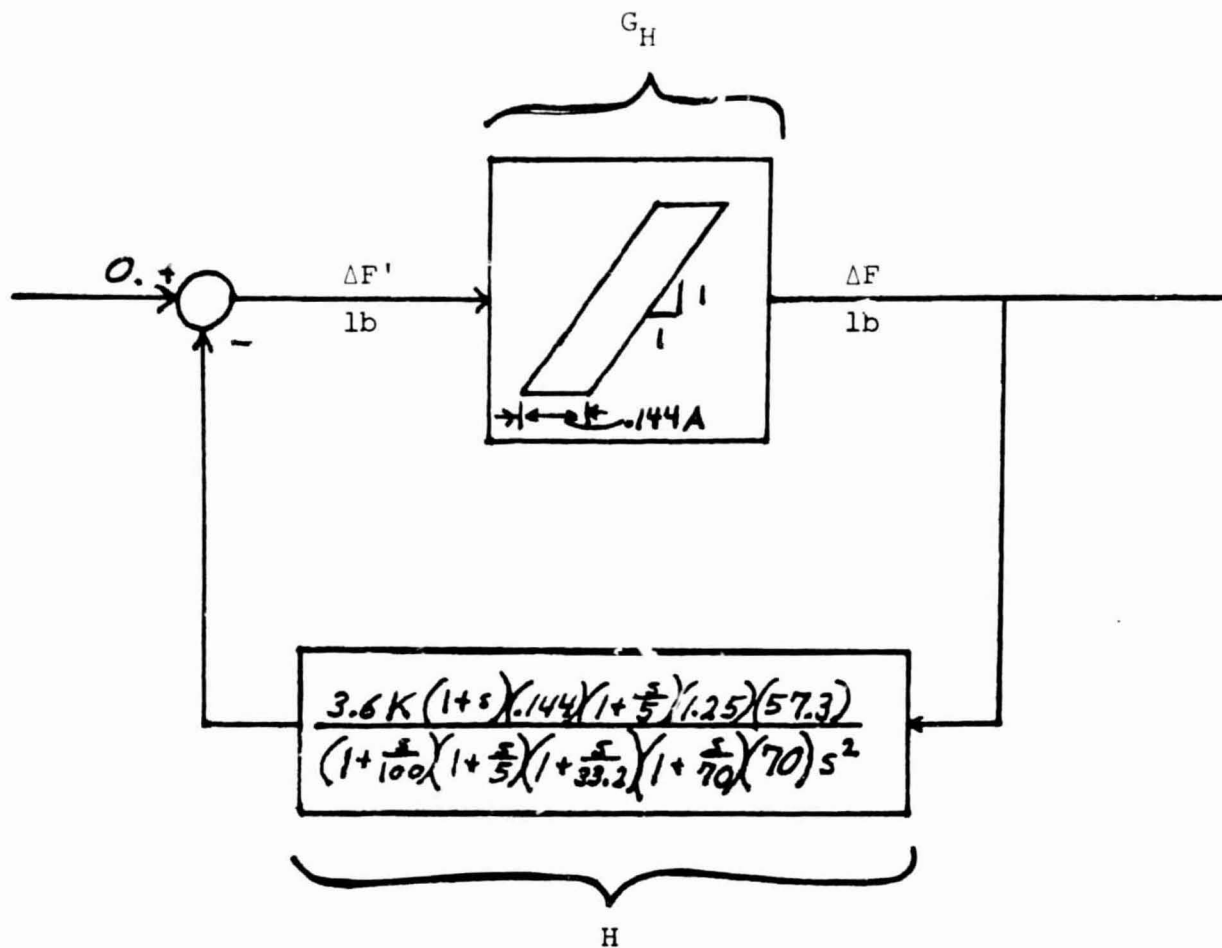


Figure 71

Pitch (or Yaw) Position Loop Rearranged
for Hysteresis Limit Cycle Analysis

100

90

80

70

60

50

40

30

20

10

Magnitude

-90

-110

-130

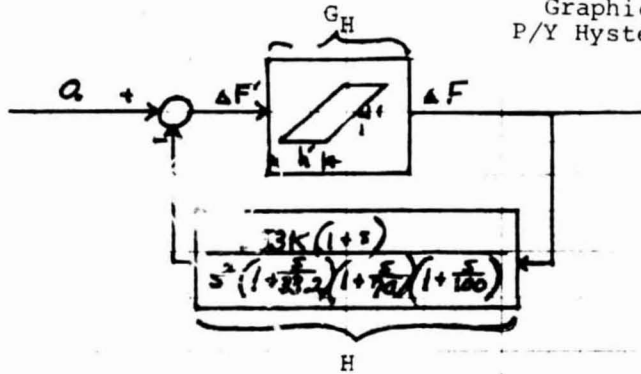
-150

-170

-190

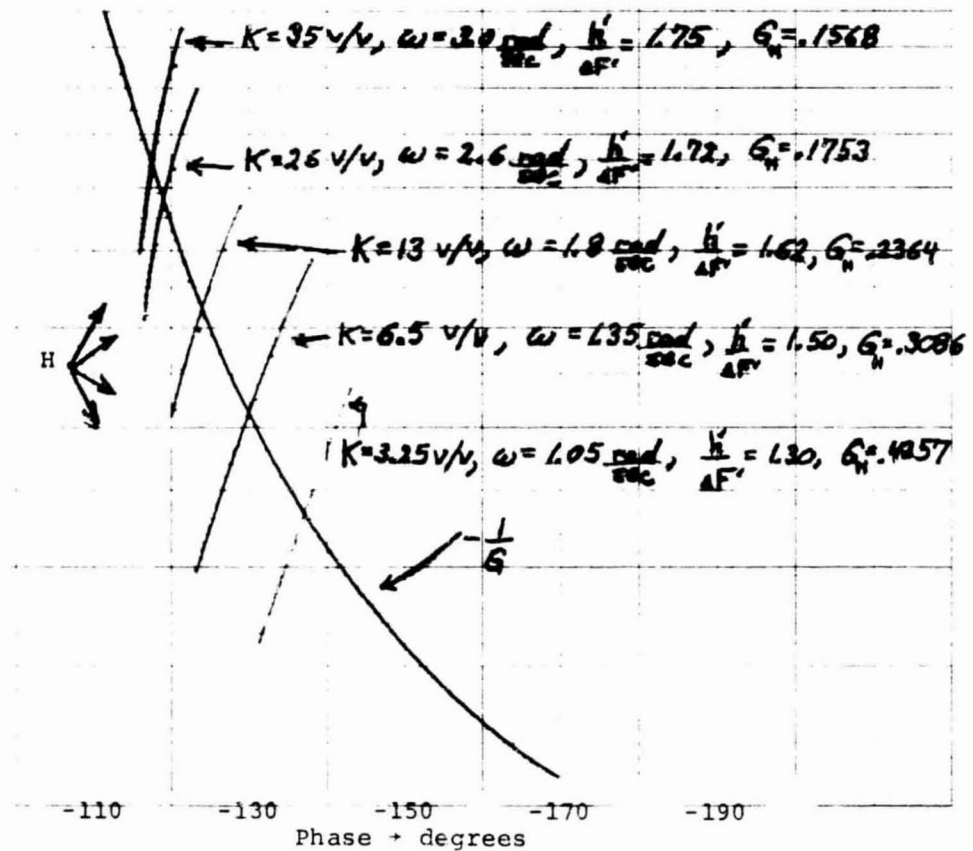
Phase + degrees

Figure 72

Graphical Solution for
P/Y Hysteresis Limit Cycle

$h' = .144$ A where A is hysteresis in volts at the input of the torque motor amplifier.

$K =$ electronic gain



$$\Delta F^1 = \frac{.144A}{1.3} \quad 1b$$

Next we want to determine the value of A which should be called one percent hysteresis. Test results with the torque motor coils in parallel indicate the FPT output saturates for an input of +1 volt. Then according to the convention shown in Figure 64 the percent hysteresis is:

$$\% \text{ hysteresis} = \frac{100A}{2 \times 1} = \frac{100A}{2}$$

Then for 1% hysteresis, $A = .02$ volts.

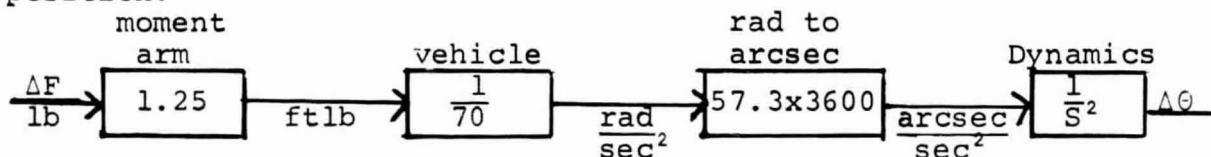
Using this value of A in the expression for ΔF^1 gives

$$\Delta F^1 = \frac{.144A}{1.3} = \frac{.144 \times .02}{1.3} = .00222 \quad 1b.$$

The gain, G_H , in Figure 71 is taken from Figure 72 as the reciprocal of the intersection magnitude. G_H equals .4257. Then

$$\Delta F = G_H \Delta F^1 = .4257 \times .00222 = .000945 \quad 1b.$$

ΔF is the amplitude of the first harmonic of the FPT output as predicted by the describing function analysis. Going from ΔF to $\Delta \theta$, the pitch limit cycle amplitude simply requires reflecting the FPT output through the appropriate transfer function to get output position.



From Figure 72 the limit cycle frequency should be 1.05 rad/sec for $K = 3.25$ volt per volt. Then:

$$\Delta \theta = \frac{1.25 \times 57.3 \times 3600 \Delta F}{70 S^2} = \frac{1.25 \times 57.3 \times 3600 \times .000945}{70 \times 1.05^2} = 3.16 \text{ arcsec}$$

Therefore for an electronic gain of 3.25 volt/volt a one percent FPT hysteresis should cause a limit cycle amplitude of about 3.16 arcsec at a frequency of 1.05 rad/sec (i.e. .167 HZ). This is plotted in Figure 64.

The other points in Figure 64 were similarly determined.

APPENDIX 4

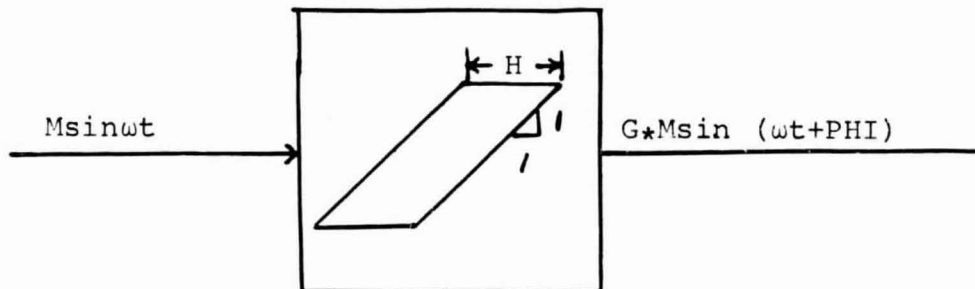
COMPUTER PROGRAMS

This contains the listings of the various time sharing computer programs used. All of these programs were entered and run from remote terminals. The programs presented are:

1. HYS - This program calculates the backlash describing function.
2. RMS - This program computes the rms output of a transfer element. The transfer function of the element and the power spectral density input to the element must be read in at running time.
3. FREQ - This program computes the optimum lead break frequency (i.e. the optimum position to rate feedback) for a bang-bang position control with exponentially decaying thrust.
4. TIME - This program computes the time required for an exponentially decaying bang-bang control system to position the vehicle.

HYS

This program (Figure 73) calculates the backlash describing function whose formula is given in Appendix 3. The program is short, but the results are not readily available in control system literature. The results are presented in Figure 74. The heading meanings in Figure 74 are based on the figure below.



The last two columns of Figure 74 are useful for determining the limit cycle frequency using the graphical approach of Figure 72.

RMS

This program computes the following integral:

$$\text{Integral} = \int_{\text{Lower Limit}}^{\text{Upper Limit}} \Phi(\omega) * |G(s)|^2 * d\omega$$

HYS

```
010 PI=3.1415927; P=1./PI; Q=.5*PI
020 RAD=57.29578
030 X=.1
040 PRINT 100
041 100 FORMAT(5H H/M,8X,1HG,7X,3HPHI,7X,3H1/G,3X,11H(-180 -PHI))
050 DO 20 I=1,190
060 A=X*(X-2.)*P
070 XNUM=1.-X
080 XDEN=SQRT(2.*X-X*X)
090 W1=ATAN(XNUM/XDEN)
100 B=P*(Q+W1+(1.-X)*XDEN)
110 G=SQRT(A*A+B*B)
120 PHI=ATAN(A/B)*RAD
130 G1=1./G
140 PHI1=-180.-PHI
150 PRINT 10,X,G,PHI,G1,PHI1
160 10 FORMAT(F5.2,F10.4,F10.2,F10.3,F10.2)
170 20 X=X+.01
180 STOP
190 END
```

Figure 73
HYS Listing

H/M	G	PHI	1/G	(-180 -PHI)
.10	.9832	-3.53	1.017	-176.47
.11	.9807	-3.87	1.020	-176.13
.12	.9781	-4.21	1.022	-175.79
.13	.9755	-4.55	1.025	-175.45
.14	.9728	-4.89	1.028	-175.11
.15	.9700	-5.22	1.031	-174.78
.16	.9671	-5.56	1.034	-174.44
.17	.9641	-5.90	1.037	-174.10
.18	.9611	-6.23	1.040	-173.77
.19	.9580	-6.56	1.044	-173.44
.20	.9549	-6.89	1.047	-173.11
.21	.9516	-7.22	1.051	-172.78
.22	.9484	-7.55	1.054	-172.45
.23	.9451	-7.88	1.058	-172.12
.24	.9417	-8.21	1.062	-171.79
.25	.9382	-8.54	1.066	-171.46
.26	.9348	-8.86	1.070	-171.14
.27	.9312	-9.19	1.074	-170.81
.28	.9277	-9.51	1.078	-170.49
.29	.9240	-9.84	1.082	-170.16
.30	.9204	-10.16	1.087	-169.84
.31	.9167	-10.48	1.091	-169.52
.32	.9129	-10.80	1.095	-169.20
.33	.9091	-11.13	1.100	-168.87
.34	.9053	-11.45	1.105	-168.55
.35	.9014	-11.77	1.109	-168.23
.36	.8975	-12.09	1.114	-167.91
.37	.8935	-12.41	1.119	-167.59
.38	.8896	-12.73	1.124	-167.27
.39	.8855	-13.04	1.129	-166.96
.40	.8815	-13.36	1.134	-166.64
.41	.8774	-13.68	1.140	-166.32
.42	.8733	-14.00	1.145	-166.00
.43	.8691	-14.31	1.151	-165.69
.44	.8649	-14.63	1.156	-165.37
.45	.8607	-14.95	1.162	-165.05
.46	.8565	-15.26	1.168	-164.74
.47	.8522	-15.58	1.173	-164.42
.48	.8479	-15.90	1.179	-164.10
.49	.8435	-16.21	1.185	-163.79
.50	.8392	-16.53	1.192	-163.47
.51	.8348	-16.84	1.198	-163.16
.52	.8304	-17.16	1.204	-162.84
.53	.8259	-17.47	1.211	-162.53
.54	.8214	-17.79	1.217	-162.21
.55	.8169	-18.10	1.224	-161.90
.56	.8124	-18.42	1.231	-161.58
.57	.8079	-18.73	1.238	-161.27
.58	.8033	-19.05	1.245	-160.95
.59	.7987	-19.36	1.252	-160.64
.60	.7941	-19.68	1.259	-160.32
.61	.7894	-19.99	1.267	-160.01
.62	.7847	-20.31	1.274	-159.69
.63	.7800	-20.62	1.282	-159.38
.64	.7753	-20.94	1.290	-159.06
.65	.7706	-21.25	1.298	-158.75
.66	.7658	-21.57	1.306	-158.43

Figure 74
Backlash Describing
Function Values

H/V	G	PHI	1/G	(-180-PHI)
.67	.7610	-21.88	1.314	-158.12
.68	.7562	-22.20	1.322	-157.80
.69	.7514	-22.51	1.331	-157.49
.70	.7466	-22.83	1.339	-157.17
.71	.7417	-23.15	1.348	-156.85
.72	.7368	-23.46	1.357	-156.54
.73	.7319	-23.78	1.366	-156.22
.74	.7270	-24.10	1.376	-155.90
.75	.7220	-24.41	1.385	-155.59
.76	.7171	-24.73	1.395	-155.27
.77	.7121	-25.05	1.404	-154.95
.78	.7071	-25.37	1.414	-154.63
.79	.7021	-25.68	1.424	-154.32
.80	.6970	-26.00	1.435	-154.00
.81	.6920	-26.32	1.445	-153.68
.82	.6869	-26.64	1.456	-153.36
.83	.6818	-26.96	1.467	-153.04
.84	.6767	-27.28	1.478	-152.72
.85	.6716	-27.60	1.489	-152.40
.86	.6664	-27.92	1.501	-152.08
.87	.6613	-28.24	1.512	-151.76
.88	.6561	-28.57	1.524	-151.43
.89	.6509	-28.89	1.536	-151.11
.90	.6457	-29.21	1.549	-150.79
.91	.6405	-29.54	1.561	-150.46
.92	.6352	-29.86	1.574	-150.14
.93	.6300	-30.19	1.587	-149.81
.94	.6247	-30.51	1.601	-149.49
.95	.6194	-30.84	1.614	-149.16
.96	.6141	-31.17	1.628	-148.83
.97	.6088	-31.49	1.643	-148.51
.98	.6034	-31.82	1.657	-148.18
.99	.5981	-32.15	1.672	-147.85
1.00	.5927	-32.48	1.687	-147.52
1.01	.5873	-32.81	1.703	-147.19
1.02	.5820	-33.14	1.718	-146.86
1.03	.5765	-33.48	1.734	-146.52
1.04	.5711	-33.81	1.751	-146.19
1.05	.5657	-34.14	1.768	-145.86
1.06	.5602	-34.48	1.785	-145.52
1.07	.5548	-34.82	1.803	-145.18
1.08	.5493	-35.15	1.820	-144.85
1.09	.5438	-35.49	1.839	-144.51
1.10	.5383	-35.83	1.858	-144.17
1.11	.5328	-36.17	1.877	-143.83
1.12	.5273	-36.51	1.897	-143.49
1.13	.5217	-36.85	1.917	-143.15
1.14	.5162	-37.20	1.937	-142.80
1.15	.5106	-37.54	1.958	-142.46
1.16	.5050	-37.89	1.980	-142.11
1.17	.4994	-38.24	2.002	-141.76
1.18	.4938	-38.58	2.025	-141.42
1.19	.4882	-38.93	2.048	-141.07
1.20	.4826	-39.29	2.072	-140.71
1.21	.4770	-39.64	2.097	-140.36
1.22	.4713	-39.99	2.122	-140.01
1.23	.4656	-40.35	2.148	-139.65

Figure 74 (continued)

H/M	G	PHI	1/G	(-180-PHI)
1.24	.4600	-40.71	2.174	-139.29
1.25	.4543	-41.06	2.201	-138.94
1.26	.4486	-41.42	2.229	-138.58
1.27	.4429	-41.79	2.258	-138.21
1.28	.4371	-42.15	2.288	-137.85
1.29	.4314	-42.51	2.318	-137.49
1.30	.4257	-42.88	2.349	-137.12
1.31	.4199	-43.25	2.381	-136.75
1.32	.4142	-43.62	2.415	-136.38
1.33	.4084	-43.99	2.449	-136.01
1.34	.4026	-44.37	2.484	-135.63
1.35	.3968	-44.74	2.520	-135.26
1.36	.3910	-45.12	2.558	-134.88
1.37	.3852	-45.50	2.596	-134.50
1.38	.3793	-45.89	2.636	-134.11
1.39	.3735	-46.27	2.677	-133.73
1.40	.3676	-46.66	2.720	-133.34
1.41	.3618	-47.05	2.764	-132.95
1.42	.3559	-47.44	2.810	-132.56
1.43	.3500	-47.84	2.857	-132.16
1.44	.3441	-48.24	2.906	-131.76
1.45	.3382	-48.64	2.957	-131.36
1.46	.3323	-49.04	3.009	-130.96
1.47	.3264	-49.45	3.064	-130.55
1.48	.3205	-49.86	3.121	-130.14
1.49	.3145	-50.27	3.179	-129.73
1.50	.3086	-50.69	3.241	-129.31
1.51	.3026	-51.10	3.305	-128.90
1.52	.2966	-51.53	3.371	-128.47
1.53	.2907	-51.95	3.440	-128.05
1.54	.2847	-52.38	3.513	-127.62
1.55	.2787	-52.82	3.588	-127.18
1.56	.2727	-53.26	3.668	-126.74
1.57	.2666	-53.70	3.750	-126.30
1.58	.2606	-54.14	3.837	-125.86
1.59	.2546	-54.59	3.928	-125.41
1.60	.2485	-55.05	4.023	-124.95
1.61	.2425	-55.51	4.124	-124.49
1.62	.2364	-55.98	4.230	-124.02
1.63	.2304	-56.45	4.341	-123.55
1.64	.2243	-56.92	4.459	-123.08
1.65	.2182	-57.41	4.583	-122.59
1.66	.2121	-57.89	4.715	-122.11
1.67	.2060	-58.39	4.855	-121.61
1.68	.1999	-58.89	5.003	-121.11
1.69	.1938	-59.40	5.161	-120.60
1.70	.1876	-59.91	5.330	-120.09
1.71	.1815	-60.43	5.510	-119.57
1.72	.1753	-60.97	5.703	-119.03
1.73	.1692	-61.51	5.911	-118.49
1.74	.1630	-62.05	6.135	-117.95
1.75	.1568	-62.61	6.376	-117.39
1.76	.1507	-63.18	6.638	-116.82

Figure 74 (continued)

H/M	G	PHI	1/G	(-180-PHI)
1.77	.1445	-63.76	6.922	-116.24
1.78	.1383	-64.35	7.232	-115.65
1.79	.1321	-64.96	7.572	-115.04
1.80	.1259	-65.57	7.946	-114.43
1.81	.1196	-66.21	8.359	-113.79
1.82	.1134	-66.85	8.818	-113.15
1.83	.1072	-67.52	9.331	-112.48
1.84	.1009	-68.20	9.908	-111.80
1.85	.0947	-68.91	10.562	-111.09
1.86	.0884	-69.63	11.310	-110.37
1.87	.0821	-70.38	12.173	-109.62
1.88	.0759	-71.16	13.180	-108.84
1.89	.0696	-71.98	14.370	-108.02
1.90	.0633	-72.82	15.797	-107.18
1.91	.0570	-73.71	17.542	-106.29
1.92	.0507	-74.65	19.724	-105.35
1.93	.0444	-75.65	22.529	-104.35
1.94	.0381	-76.73	26.268	-103.27
1.95	.0317	-77.89	31.504	-102.11
1.96	.0254	-79.17	39.358	-100.83
1.97	.0191	-80.63	52.448	-99.37
1.98	.0127	-82.35	78.627	-97.65
1.99	.0064	-84.60	157.166	-95.40

Figure 74 (concluded)

where: ϕ $\equiv$ Noise power spectral density expressed in even powers of frequency.
 $G(s)$ $\equiv$ transfer function of the transfer element under consideration

Figure 75 is the listing of RMS. Figure 76 presents the sample problem.

$$\text{Integral} = \int_{.01}^{100} 1 * \left| \frac{1}{1+s} \right|^2 d\omega = 1.551$$

FREQ

This program (Figure 77) computes the optimum lead break frequency (i.e. the optimum position rate feedback) for an exponentially decaying bang-bang position control. This program generated the points for Figures 17 & 18. This program selects a ratio of position to rate feedback which makes the vehicle acquire to within 0.1 degrees from an arbitrary initial angle and rate in the sense presented in Figure 15.

Figure 78 is a sample run of FREQ. The inputs indicate the thrust level is given by:

$$\text{Thrust} = 6.28 \exp(-t/22.2) \text{ deg/sec}^2$$

where t is in seconds. The initial angle is 90 degrees. The initial rate is 0 deg/sec. For the second case the initial angle will be $90-20 = 70$ deg. The cases continue down to an initial angle of 10 degrees.

The print out terms have the following meanings for the first case:

TIME0 = initial time = 0 seconds
 THETA0 = initial angle = 90 degrees
 W1 = optimum lead break frequency = .345 rad/sec
 TIME2 = 3.772 sec = time thrust switches direction
 THETA2 = 47.755 deg = angle at TIME2
 RATE2 = -21.784 deg/sec = rate at TIME2
 TIME3 = 8.323 sec = final time
 THETA3 = -.076 deg = final angle
 RATE3 = .020 deg/sec = final rate

Ideally THETA3 = RATE3 = 0, but this condition can only be approached using the digital computer.

RMS

```

10000 COMMON A(10),B(10),C(10),D(10),E(10),F(10),FSN(10),FSD(10),
10010 +AP(10),BP(10),CP(10),DP(10),EP(10),FP(10),PN(10),PD(10)
10021 PRINT,"THIS PROGRAM COMPUTES THE RMS NOISE OUTPUT OF A",
10022 PRINT," TRANSFER"
10023 PRINT,"ELEMENT WHICH IS SUBJECTED TO A NOISE INPUT."
10030 1 DO 2 I=1,10
10040 A(I)=1.;B(I)=0.;C(I)=0.;AP(I)=1.;BP(I)=0.;CP(I)=0.
10045 CP(I)=0.
10046 D(I)=1.
10047 E(I)=0.
10048 F(I)=0.
10049 DP(I)=1.
10050 D(I)=1.;E(I)=0.;F(I)=0.;DP(I)=1.;EP(I)=0.
10060 2 FP(I)=0.
10070 PNW=1.;PDW=1.;XN=1.;XD=1.;S=0.
10079 PRINT," "
10080 PRINT,"TRANSFER FUNCTION INPUTS"
10090 PRINT,"TYPE NUMBER OF NUMERATOR SECOND ORDER FACTORS",
10100 INPUT,NNUM
10110 PRINT,"TYPE NUMBER OF DENOMINATOR SECOND ORDER FACTORS",
10120 INPUT,NDEN
10130 PRINT,"TYPE NUMERATOR COEFFICIENTS E.G. A +B*S +C*S**2"
10140 INPUT,(A(I),B(I),C(I),I=1,NNUM)
10150 PRINT,"TYPE DENOMINATOR COEFFICIENTS E.G. D +E*S +F*S**2"
10160 INPUT,(D(I),E(I),F(I),I=1,NDEN)
10170 PRINT," "
10180 PRINT," "
10190 PRINT,"POWER SPECTRAL DENSITY FUNCTION INPUTS"
10200 PRINT,"TYPE NUMBER OF NUMERATOR FOURTH ORDER FACTORS",
10210 INPUT,NPN
10220 PRINT,"TYPE NUMBER OF DENOMINATOR FOURTH ORDER FACTORS",
10230 INPUT,NPD
10240 PRINT,"TYPE NUMERATOR COEFFICIENTS E.G. A +B*W**2 +C*W**4"
10250 INPUT,(AP(I),BP(I),CP(I),I=1,NPN)
10260 PRINT,"TYPE DENOMINATOR COEFFICIENTS E.G. D +E*W**2 +F*W**4"
10270 INPUT,(DP(I),EP(I),FP(I),I=1,NPD)
10280 PRINT," "
10290 PRINT,"TYPE LOWER LIMIT, UPPER LIMIT, AND INCREMENT RATIO"
10292 INPUT,W,WF,R
10300 PRINT," "
10301 PRINT,"TYPE 0 FOR COMPLETE PRINT OUT. TYPE 1 FOR ANSWER",
10302 PRINT," ONLY."
10303 INPUT,NPRINT
10304 IF(NPRINT) 71,70,71
10309 70 CONTINUE
10310 PRINT," "
10320 PRINT," "
10330 PRINT,"          W          P(W)          G(W)**2    P(W)*G(W)**2"
10340 PRINT," "
10350

```

Figure 75
RMS Listing

RMS CONTINUED

```
10360' ROUTING FOR RUNGE-KUTTA INTEGRATION
10370'
10371 71 CONTINUE
10380 KPATH=1
10390 39 IPATH=1
10400 40 GO TO(41,42),KPATH
10410 41 0=S
10420 NPATH=1
10430 KPATH=2
10440 IPATH=2
10450 GO TO 47
10460'
10470' RUNGE-KUTTA INTEGRATION
10480'
10490 42 GO TO(43,44,45,46),NPATH
10500'
10510 43 S=0
10520 01=DW*PL
10530 0=S+.5*01
10540 NPATH=2
10550 W=W-.5*DW
10560 GO TO 47
10570'
10580 44 02=DW*PL
10590 0=S+.5*02
10600 NPATH=3
10610 GO TO 47
10620'
10630 45 03=DW*PL
10640 0=S+03
10650 NPATH=4
10660 W=W+.5*DW
10670 GO TO 47
10680'
10690 46 04=DW*PL
10700 0=S+(01+2.*02+2.*03+04)/6.
10710 NPATH=1
10720 IPATH=2
10730'
10740' COMPUTATION OF INTEGRAND, PL
10750'
10760 47 DO 10 I=1,10
10770 FSN(I)=(A(I)-C(I)*(W**2))**2+(B(I)*W)**2
10780 FSD(I)=(D(I)-F(I)*(W**2))**2+(E(I)*W)**2
10790 PN(I)=AP(I)+BP(I)*W*W+CP(I)*W*W*W*W
10800 10 PD(I)=DP(I)+EP(I)*W*W+FP(I)*W*W*W*W
10810 DO 15 I=1,10
10820 XN=XN+FSN(I)
10830 XD=XD+FSD(I)
10840 PNW=PNW+PN(I)
```

Figure 75 (continued)

RMS CONTINUED

```
10850 15 PDW=PDW*PD(I)
10860 GW=XN/XD
10870 PW=PNW/PDW
10880 PL=GW*PW
10890 GO TO (21,22,22,22),NPATH
10891 21 IF(NPRINT) 22,72,22
10900 72 PRINT 23,W,PW,GW,PL
10910 23 FORMAT(1PE12.3,3E12.3)
10920 22 XN=1.;XD=1.;PNW=1.;PDW=1.
10930'
10940' END OF COMPUTATION OF INTEGRAND
10950'
10960 GO TO(40,54),IPATH
10970'
10980' TEST FOR UPPER INTEGRATION LIMIT
10990'
11000 54 IF(WF-W) 56,56,55
11010 55 DW=W*(R-1.)
11020 W=W*R
11030 GO TO 39
11040'
11050' INTEGRATION COMPLETE
11060' COMPUTE RMS
11070'
11080 56 RMS=SQRT(0)
11090 PRINT," "
11100 PRINT,"THE INTEGRAL IS",
11110 PRINT 18,0
11120 PRINT,"THE RMS VALUE IS",
11130 PRINT 18,RMS
11140 18 FORMAT(1PE11.3)
11150 DO 20,I=1,5
11160 20 PRINT," ";PRINT," "
11170 GO TO 1
11180 END
```

Figure 75 (concluded)

THIS PROGRAM COMPUTES THE RMS NOISE OUTPUT OF A TRANSFER
ELEMENT WHICH IS SUBJECTED TO A NOISE INPUT.

TRANSFER FUNCTION INPUTS

TYPE NUMBER OF NUMERATOR SECOND ORDER FACTORS? 1
TYPE NUMBER OF DENOMINATOR SECOND ORDER FACTORS? 1
TYPE NUMERATOR COEFFICIENTS E.G. $A + B*S + C*S**2$
? 1. 0. 0.
TYPE DENOMINATOR COEFFICIENTS E.G. $D + E*S + F*S**2$
? 1. 1. 0.

POWER SPECTRAL DENSITY FUNCTION INPUTS

TYPE NUMBER OF NUMERATOR FOURTH ORDER FACTORS? 1
TYPE NUMBER OF DENOMINATOR FOURTH ORDER FACTORS? 1
TYPE NUMERATOR COEFFICIENTS E.G. $A + B*W**2 + C*W**4$
? 1. 0. 0.
TYPE DENOMINATOR COEFFICIENTS E.G. $D + E*W**2 + F*W**4$
? 1. 0. 0.

TYPE LOWER LIMIT, UPPER LIMIT, AND INCREMENT RATIO

? .01 100. 1.1

TYPE 0 FOR COMPLETE PRINT OUT. TYPE 1 FOR ANSWER ONLY.

? 1

THE INTEGRAL IS 1.551E+00

THE RMS VALUE IS 1.245E+00

Figure 76
RMS Sample Problem Solution

FREQ

```

10000 TH0(A0,B0,T,TH0)=A0*B0*(EXP(-T/B0)-1.0)+TH0
10010 THETA(A0,B0,T,TH,TH0)=A0*B0*B0*(1.-EXP(-T/B0))+(TH0-A0*B0)*T+TH
10020 ERROR(A,W,R)=-1.72*(SIN(A)+(R/W)*COS(A))
10030 RAD=3.141593/180.
10040 3 CONTINUE
10050 PRINT,"TORQUE TO INERTIA RATIO,BLOW DOWN TIME CONSTANT,"
10060 PRINT,"INITIAL ANGLE, INITIAL RATE, AND ANGULAR INCREMENT"
10070 INPUT,A0,B0,TH0,TH00,DTHO
10080 PRINT," ";PRINT," "
10090 PRINT,"      TIME0  THETA0      W1"
10100 PRINT,"      TIME2  THETA2    RATE2"
10110 PRINT,"      TIME3  THETA3    RATE3"
10120 5 CONTINUE
10130 20 RATE1=TH0(A0,B0,T1,TH00)
10140 W1=SQRT(4.*A0/TH0)
10150 40 T2=SQRT(TH0/A0)
10160 50 TH2=THETA(A0,B0,T2,TH0,TH00)
10170 RATE2=TH0(A0,B0,T2,TH00)
10180 RR2=RAD*RATE2;RTH2=RAD*TH2
10190 E2=ERROR(RTH2,W1,RR2)
10200 E=ABS(E2)
10210 IF(.001-E) 45,60,60
10220 45 T2=T2*SQRT((1.72-E2)/1.72)
10230 60 T0 50
10240 60 A02=-A0*EXP(-T2/B0)
10250 T3=T2*EXP(T2/B0)
10260 65 CONTINUE
10270 TH3=THETA(A02,B0,T3,TH2,RATE2)
10280 RATE3=TH0(A02,B0,T3,RATE2)
10290 RR3=RAD*RATE3;RTH3=RAD*TH3
10300 E3=ERROR(RTH3,W1,RR3)
10310 E=ABS(E3)
10320 IF(.001-E) 70,75,75
10330 70 T3=T3*SQRT((1.72+E3)/1.72)
10340 75 T0 65
10350 75 TH3A=ABS(TH3)
10360 IF(.1-TH3A) 80,85,85
10370 80 W1=W1*SQRT((.5*TH0+T3)/(.5*TH0))
10380 85 T0 50
10390 85 T3=T2+T3
10400 PRINT," "
10410 PRINT 100,T0,TH0,W1,T2,TH2,RATE2,T3,TH3,RATE3
10420 100 FORMAT(3F8.3)
10430 TH0=TH0-DTH0
10440 IF(5-TH0) 5,5,2
10450 2 PRINT," ";PRINT," ";PRINT," ";PRINT," "
10460 60 T0 3
10470 STOP
10480 END

```

Figure 77

FREQ Listing

TORQUE TO INERTIA RATIO, BLOW DOWN TIME CONSTANT,
 INITIAL ANGLE, INITIAL RATE, AND ANGULAR INCREMENT
 ? 6.23 22.2 90. 0. 90.

TIME0	THETA0	NI
TIME2	THETA2	RATE2
TIME3	THETA3	RATE3
.000	90.000	.346
3.772	47.755	-21.784
8.323	-.076	.320
.000	70.000	.452
3.330	35.347	-19.417
7.251	-.058	.012
.000	50.000	.591
2.817	25.110	-16.612
6.045	-.043	.009
.000	30.000	.823
2.136	15.477	-13.073
4.620	-.035	.045
.000	10.000	1.510
1.265	5.069	-7.722
2.614	-.059	.039

Figure 78
 FREQ Sample Problem Solution

TIME

This program (Figure 79) computes the time required for an exponentially decaying bang-bang control system to position the vehicle. It helped generate the data points for Figures 22 and 23.

Figure 80 is a sample running of TIME.

The output of Figure 80 is read as follows. For the first case

TO = 10 degrees initial angle

AO = 4.9 deg/sec² initial torque to inertia ratio

Not printed out, but contained in the program is:

W1 = .224 rad/sec lead break frequency

T = 22.2 sec thruster decay time constant.

The printout shows that for path number 1 (refer to Figure 16. This is the phase plane path from the initial angle to B):

TIME = .440589 sec in path number 1

ANGLE = 9.52754 deg when thrust switches direction

RATE = -2.1376 deg/sec when thrust switches direction

FOURTH QUADRANT = The thrust switches direction when the phase plane trajectory is in the fourth quadrant.

For path number 2 the printout headings have the same meaning as for path number 1. Path number 2 corresponds to the trajectory from B to C in Figure 16.

The phase plane trajectory from C through D towards the origin (again, referring to Figure 16) creeps along the switching line. The printout states this and prints the time (.100499 sec) necessary to creep near the origin.

TIME

```

10000 PRINT
10010 LET A0=4.9
10020 LET A9= A0
10030 LET G= .128
10040 LET W2= 1.175
10050 LET W1= .224
10060 LET A1= A0
10070 LET T= 22.2
10080 LET H3= 9.313
10090
10100 FOR T0= 10 TO 90 STEP 80
10110 LET A0= A9
10120 LET T9= T0
10130 PRINT "T0=";T0;"A0=";A0
10140 LET T9= T0
10150 LET T3= 0
10160 PRINT
10170 PRINT
10180 LET Z=0
10190 LET Z1= 0
10200 LET Z5= 0
10210 LET H=1
10220 LET S= (ABS(T0/A0))*.5
10230 LET T1= T*T*A0*(1-EXP(-S/T))+(T3-A0*T)*S+T0
10240 LET T5= A0*T*(EXP(-S/T)-1)+T3
10250 LET T7= (-180*W1/3.1416)*TAN(3.1416*T1/180)
10260 IF ABS ((T5-T7)/T7)< .01 THEN 10330
10270 LET R= SIN(3.1416*T1/180)+(T5*3.1416/(180*W1))*COS(3.1416*T1/180)
10280 IF H= 4 THEN 10310
10290 LET R= -R
10300 LET M= 1/(10*ABS(R))
10310 LET S= S*(ABS(1-R/1.54))*M
10320 GO TO 10230
10330 LET Z5= Z5+1
10340 PRINT
10350 PRINT "FOR PATH NUMBER";Z5
10360 PRINT "TIME=";S;"SECONDS"
10370 PRINT "ANGLE =" ;T1;"DEGREES"
10380 PRINT "RATE=";T5;"DEGREES/SEC"
10390 LET T0= T1
10400 LET T3= T5
10410 LET A0= A0*(EXP(-S/T))
10420 IF T1< H2 THEN 10500
10430 GO TO 10680
10440 PRINT
10450 PRINT
10460 PRINT "THIS IS A CREEPER"
10470 LET S1= (LOG(T1/H3))/W1
10480 PRINT "CREEP TIME IS";S1;"SECONDS"
10490 GO TO 10510

```

Figure 79
TIME Listing

TIME CONTINUED

```
10500 PRINT "LIMIT HAS BEEN REACHED"
10510 FOR X= 1 TO 4
10520 PRINT
10530 NEXT X
10540 PRINT "-----"
10550 PRINT
10560 LET T0= T9
10570 LET A0= A1
10580 NEXT T0
10590 FOR Z5= 1 TO 8
10600 PRINT
10610 NEXT Z5
10620 READ A0
10630 LET A9= A0
10640 DATA 6.28, 7.59, 0
10650 IF A0= 0 THEN 10880
10660 GO TO 10030
10670 GO TO 10880
10680 IF T1<0 THEN 10780
10690 IF T5<0 THEN 10720
10700 PRINT "FIRST QUADRANT. WHAT DO I DO NOW?"
10710 GO TO 10880
10720 LET Z= 4
10730 PRINT "FOURTH QUADRANT"
10740 IF Z= H THEN 10440
10750 LET A0=-A0
10760 LET H= Z
10770 GO TO 10220
10780 IF T5> 0 THEN 10810
10790 PRINT "THIRD QUADRANT. WHAT DO I DO NOW?"
10800 GO TO 10880
10810 LET Z= 2
10820 PRINT "SECOND QUADRANT. YOU DON'T WANT THIS"
10830 GO TO 10510
10840 IF Z= H THEN 10440
10850 LET A0= -A0
10860 LET H=Z
10870 GO TO 10220
10880 END
```

Figure 79 (concluded)

TO= 10 AO= 4.9

FOR PATH NUMBER 1
TIME= .440589 SECONDS
ANGLE = 9.52754 DEGREES
RATE=-2.1376 DEGREES/SEC
FOURTH QUADRANT

FOR PATH NUMBER 2
TIME= 1.17636E-03 SECONDS
ANGLE = 9.52573 DEGREES
RATE=-2.13195 DEGREES/SEC
FOURTH QUADRANT

THIS IS A CREEPER
CREEP TIME IS .100477 SECONDS

TO= 90 AO= 4.9

FOR PATH NUMBER 1
TIME= 3.95010 SECONDS
ANGLE = 53.9420 DEGREES
RATE=-17.7312 DEGREES/SEC
FOURTH QUADRANT

FOR PATH NUMBER 2
TIME= 3.81755 SECONDS
ANGLE = 14.4958 DEGREES
RATE=-3.34658 DEGREES/SEC
FOURTH QUADRANT

THIS IS A CREEPER
CREEP TIME IS 1.97520 SECONDS

Figure 80
TIME Sample Problem Solution