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CARL A. WAGNER
BRUCE C. DOUGLAS
ELISE R. FISHER

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GSFC
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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by

Carl A. Wagner
Mission Trajectory Determination Branch
Mission and Trajectory Analysis Division
Tracking and Data Systems Directorate
Goddard Space Flight Center

Bruce C. Douglas
Wolf Research and Development Corporation
Riverdale, Maryland

Elise R. Fisher
Mathematics and Computing Branch
Laboratory for Theoretical Studies
Space Sciences Directorate
Goddard Space Flight Center

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ABSTRACT

A general resonant orbit and gravity constant determining program has been developed accepting short arc mean Kepler elements as data. The evolution of these elements is calculated by numerical integration of their long period and secular variations.

With only slowly-changing mean element coordinates being integrated, a step size of the order of an orbit revolution or more is achieved. Satellite ephemerides over 5000 revolutions are calculated in about a minute on an IBM/360 computer. Partial derivatives of the evolved mean elements with respect to initial values and gravity constants are readily evaluated from numerically generated variant trajectories.

To test the method, simulated and actual orbit data from long resonant twelve-hour trajectories have been processed for gravity information by a least-squares technique. The gravity recovery is in excellent agreement with the model

for the simulated data, and with previous analytic results from the actual data on Cosmos 41.

Extensive data from many resonant 12 and 24 hour satellites is currently being processed by the mean element program. Preliminary determinations from this data are given. The final result should be definitive information on more than a quarter of the longitude harmonics through 8, 8.

RESONANT SATELLITE GEODESY BY HIGH SPEED

ANALYSIS OF MEAN KEPLER ELEMENTS

INTRODUCTION

Resonant satellite geodesy has undergone considerable elaboration since 1961 when A. H. Cook of England wrote (to our knowledge) the first theoretical treatise on the subject (1). At present we think the theoretical aspects are understood for most orbit classes, although a general analytic theory of satellite orbital resonance is still an unsolved problem of celestial mechanics. However, we have developed a general semi-analytic theory that should be adequate for the purposes of resonant satellite geodesy. This theory and it's application to the solution for the geopotential is the main subject of our paper.

Since 1965, there has been a startling increase of the data base for resonant satellite geodesy. What used to be regarded as a rather esoteric theoretical subject with only a few isolated examples has multiplied to such an extent that we should now speak of the nonresonant orbit as the isolated exception rather than the rule. In April 1968, 600 objects were being tracked in the United States. Ninety seven of these objects had primary resonant beat periods greater than 10 days (see fig. 1). The primary resonant orbits are those with nearly repeating ground tracks in one day. Eighteen of these had beat periods greater than one-hundred days.

Most remarkably, six objects were found with nearly commensurate orbits in two days (2). These also had greater than 100 day beat periods. When analyzed, they should provide the first really good information from satellites on many constants of odd order between m = 17 and 29.

Many orbits in shallow resonance (i.e., with less than about 10 day beat period) have already been used by the Smithsonian Astrophysical Observatory (SAO) and others for geodetic solutions (3,4,5,6). In addition to these, many deeply resonant orbits (7) should provide definitive solutions for a large number of longitude harmonics by analyses of mean elements similar to those of Kozai (8) and King-Hele's group (9, 10) on the zonals.

Referring to fig. 1 there appears to be a sufficient number of existing orbits resonant with terms of order 12 to 15 with a good distribution of inclinations and eccentricities to allow the definitive determination of many of these constants. For 8th to 12th order terms, there are fewer resonant orbits with a more restricted distribution of elements, but possibly still enough to give definitive determinations of a few constants of these orders. At the low frequency end of the spectrum the situation is very favorable for definitive geodetic solutions because of the large number of deliberately commensurate communications satellites.

INTEGRATION OF RESONANT ORBITS

Previous analytic solutions for the evolution of resonant orbits have been restricted in one or more ways which preclude universal application. This general problem includes not only all relevant resonance effects from rapidly circulating to librating orbits, but also should not be restricted by orbit inclination or eccentricity. We achieve a uniform solution by numerically integrating the coupled Lagrange planetary equations for the Keplerian elements (see ref. 11, p. 23). The Earth's disturbing potential defined in terms of these elements is a fourfold Fourier series due to W. M. Kaula (11).

$$R = \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \sum_{p=0}^{\ell} \sum_{q=-\infty}^{\infty} V_{\ell mpq},$$

where:

$$V_{\ell mpq} = \frac{\mu}{a}\left(\frac{a_e}{a}\right)^{\ell} F_{\ell mp}(i)\, G_{\ell pq}(e)\, J_{\ell m} \begin{bmatrix} \cos\Psi \\ \sin\Psi \end{bmatrix}^{\ell-m \text{ even}}_{\ell-m \text{ odd}},$$

and:

$$\Psi = [(\ell - 2p)\,\omega + (\ell - 2p + q)\,M + m\,(\Omega - \theta_g - \lambda_{\ell m})]\,.$$

The $C_{\ell m}$, $S_{\ell m}$ and $J_{\ell m}$, $\lambda_{\ell m}$ sets of gravity coefficients are related by:

$$C_{\ell m} = J_{\ell m} \operatorname{Cos} m\,\lambda_{\ell m}$$

$$S_{\ell m} = J_{\ell m} \operatorname{Sin} m\,\lambda_{\ell m}$$

In this potential representation; a, e, i, m, Ω, and ω are the conventional Kepler elements, semi-major axis, eccentricity, inclination, mean anomaly, right ascension of the ascending node and argument of perigee. The constants μ, a_e and θ_g are the Earth's Gaussian gravity constant, mean equatorial radius, and Greenwich hour angle.

This integration technique would offer no advantages over the straight numerical integration of the equations of motion in cartesian coordinates except that the Fourier analysis of the disturbing potential enables us to ignore the great majority of terms which have only short period effects. Only the resonant tesseral and secular and long period zonal (oblateness) terms are calculated for the geopotential effects. These terms are those for which:

$$\ell - 2p + q = 0,\ m = 0\ \text{(zonals)}$$

and

$$\ell = 2p + q = m/s,\ m \neq 0\ \text{(longitude terms)},$$

where s is the resonant orbit's commensurability ratio expressed as it's nearest rational mean motion in revolutions per day.

It can be shown that integrating the long period and secular variations of the elements is equivalent to a 1st order integration of canonical variables without short period terms. These variables are exactly those which would be obtained by a Von Zeipel transformation (following Brouwer (12)) or through the method of Brown and Shook (13).

The long period sun and moon effects of W. M. Kaula (14) are also included in the integration. Drag is incorporated by inclusion of the long period variations of the elements given by formulae similar to those of King-Hele (15).

Integration of the slowly evolving mean element coordinates permits time steps of about one day, in contrast to about 1/100 revolution when short period effects are included.

DIFFERENTIAL CORRECTION PROGRAM

A differential correction program using independently determined mean Kepler elements as the data type has been built around the numerical integrator described in the previous section. The great efficiency which can be achieved by integrating mean elements permits us to solve numerically for the partials of the Kepler element updates with respect to initial mean elements and gravity constants in reasonable computing time from variant trajectory differencing. Using these partials and the mean element residuals from a reference trajectory we form and solve the normal equations for a least squares estimation (of the corrections to the initial elements and gravity constants) in the usual way (11). The differential correction process is iterated until the sum of the squared residuals reaches a minimum.

GRAVITY RECOVERY WITH COSMOS 41 DATA

Figure 2 shows the variation in a set of North American Air Defense Command (NORAD) mean semi-major axes for Cosmos 41 (1964-49D) in 1965. Actually this data represents a conversion from the NORAD mean-mean motion to a semi-major axis ($\bar{a}$) averaged with respect to mean anomaly. This gives a semi-major axis compatible with the integration described previously. The conversion is:

$$\bar{a} = \left[\frac{\mu^{1/2}}{\bar{n}} + \frac{3}{2} J_2 \left(\frac{a_e}{a}\right)^2 \left(1 - \frac{3}{2} \sin^2 i\right) (1 - e^2)^{-3/2}\right]^{2/3},$$

where $J_2 \doteq 1083 \times 10^{-6}$, and $\bar{n}$ is the mean-mean motion. This conversion was necessary because the NORAD semi-major axes were defined from $\bar{n}$ according to Kozai (1959) (16) and do not represent mean elements averaged over mean anomaly.

The variation in fig. 2 is almost entirely due to resonance with 2nd and 4th order terms in the geopotential since other effects have been found to be very small or negligible.

In the numerical analysis with the high speed program, we chose to fix the (2, 2) harmonic and solve for (3, 2) and (4, 4) to try to duplicate the analytic solution over the same data obtained by Wagner (1968) (17). Starting with (3, 2) and (4, 4) values quite far from the final ones, the differential correction program using this data converged to an improved initial semi-major axis, (3,2) and (4,4) constants in less than a minute of running time on an International Business Machines (IBM) 360/91 electronic computer. The trajectories were computed in this run at about 5000 revolutions per minute. The final residuals after convergence are shown in fig. 3 and appear to be quite random. The scatter is typical of that seen for NORAD elements. The root mean square residual is slightly lower than in the analytic solution.

Figure 4 shows a comparison of the corrected gravity constants with the previous analytic results, and is also encouraging. However, the residuals in mean anomaly from the corrected solution show small periodicities consistent with 6th order resonant effects superimposed on a secular trend which is most probably due to a slight incompatibility remaining in the semi-major axis definition.

Figure 5 shows the results of differential corrections of this orbit using mean anomaly data as the observation type. The solution for (3, 2) and (4, 4) is compatible with the previous results on the semi-major axis according to the random variances allowed by the data for the constants. The solutions for (6, 6) are intriguing when compared to the SAO-66 M1 (3) nonresonant solution. But they (as well as (3, 2) and (4, 4)) are not to be considered definitive because, as the last corrected solution indicates, there is still a considerable influence on this data from other resonant harmonics. Definitive determinations will require the use of a number of resonant orbits.

CONCLUSION

A high speed differential correction program accepting mean elements as observation data has been developed and checked out on both simulated (not shown here) and actual data from 12 hour satellites. It appears possible that with this program, uniform, efficient analysis of the more than a score of deeply resonant satellites of 12 and 24 hour period will ultimately produce definitive constants for more than a quarter of the longitude geopotential terms of degree and order less than 8.

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LIST OF CAPTIONS

CHARACTERISTICS OF 97 EXISTING RESONANT ORBITS WITH BEAT PERIODS GREATER THAN 10 DAYS

INCLINATION - DEGREES	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
TOTALS →	23	11					1	4	3	2	2	14	6	26	5
160–140														2	
140–120															
120–100															
100–80							1	4	1			4	1	11	
80–60		9										2		5	2
60–40	4								2	2		4	1	6	1
40–20	1										2	4	4	2	2
20–0	18	2													

FREQUENCY, S, REVOLUTIONS/DAY

FIGURE 1

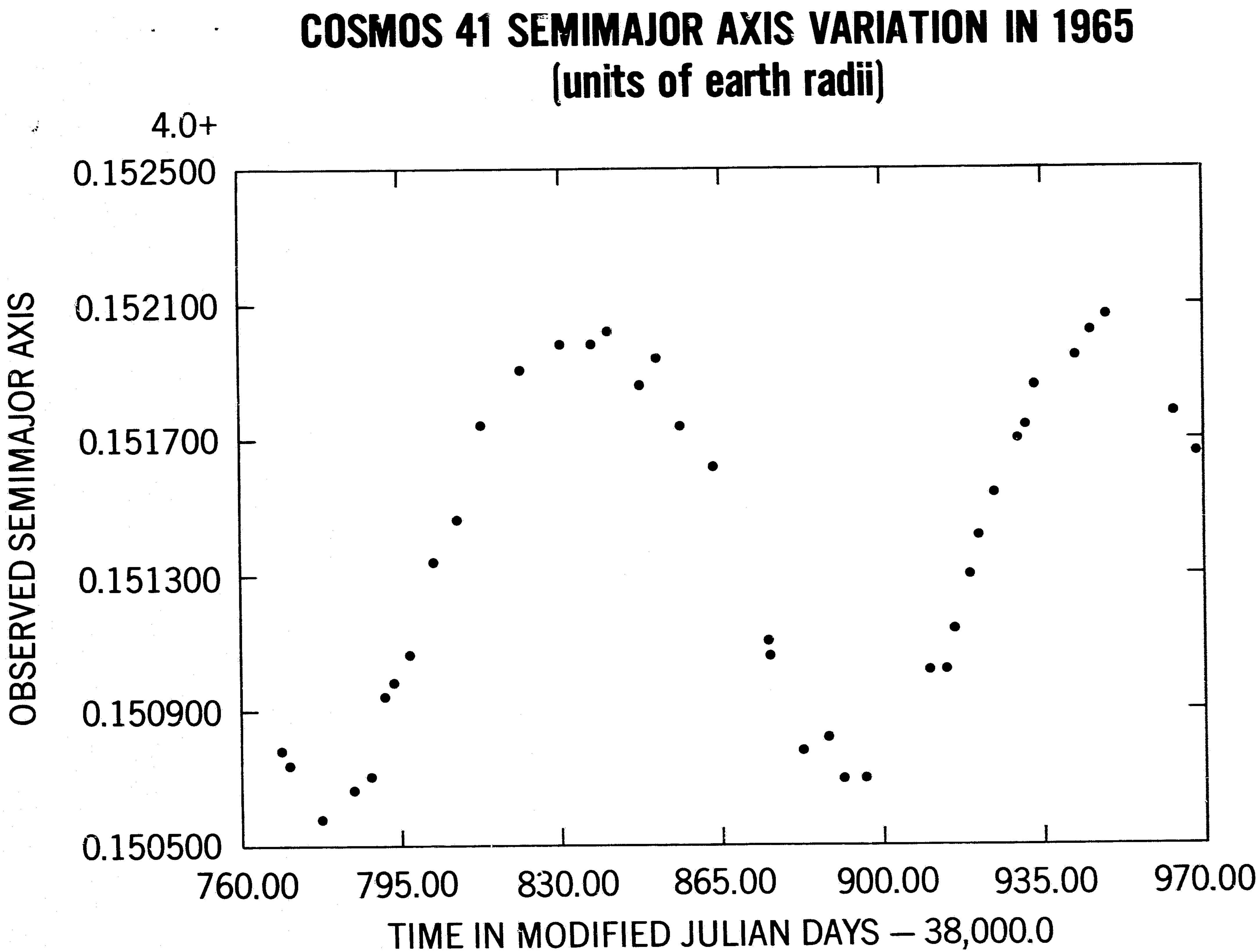
COSMOS 41 SEMIMAJOR AXIS VARIATION IN 1965
(units of earth radii)
4.0+
0.152500
0.152100
0.151700
0.151300
0.150900
0.150500
OBSERVED SEMIMAJOR AXIS
760.00
795.00
830.00
865.00
900.00
935.00
970.00
TIME IN MODIFIED JULIAN DAYS — 38,000.0

FIGURE 2

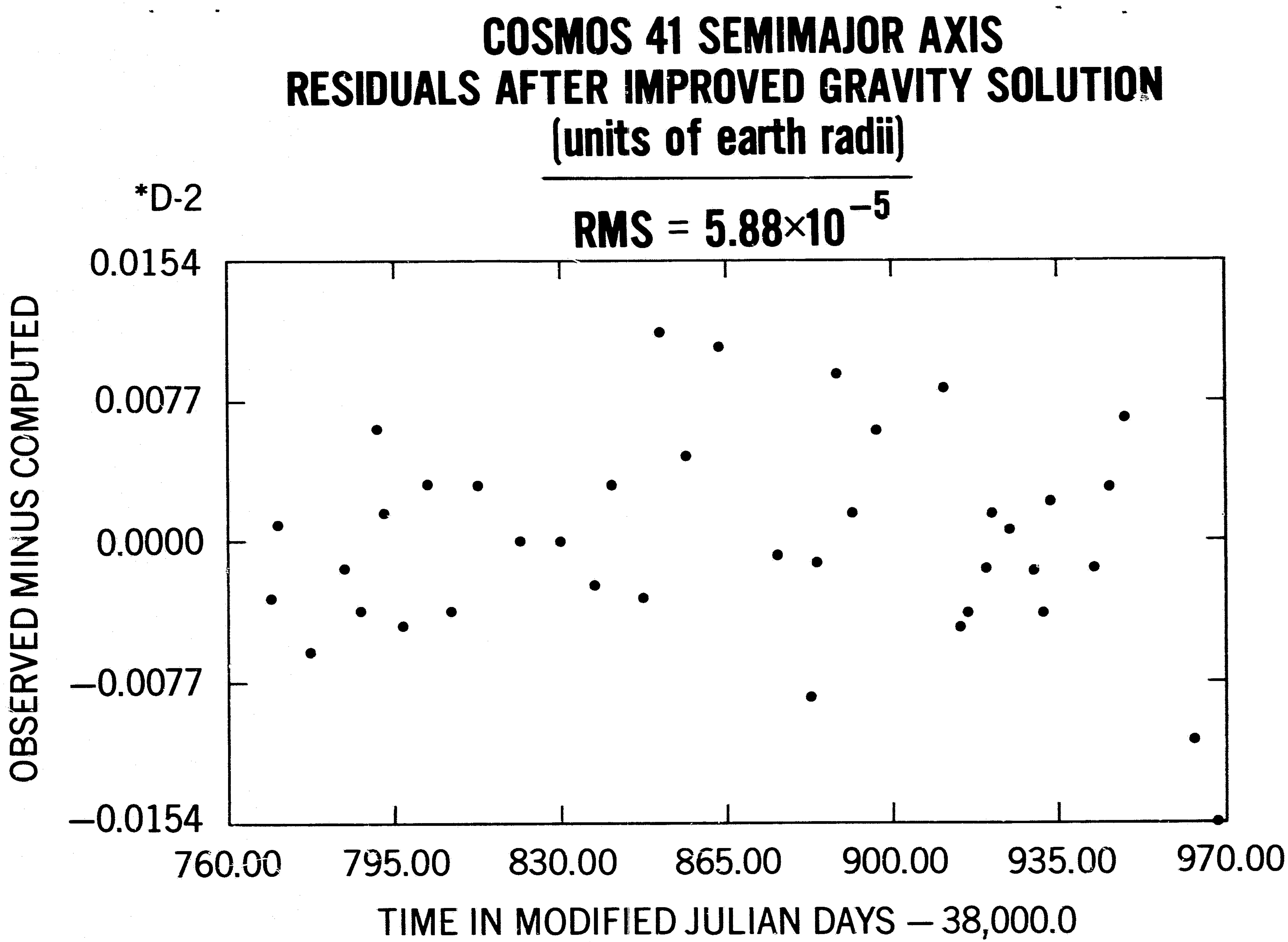
COSMOS 41 SEMIMAJOR AXIS
RESIDUALS AFTER IMPROVED GRAVITY SOLUTION
(units of earth radii)
RMS = 5.88×10^-5
*D-2
0.0154
0.0077
0.0000
−0.0077
−0.0154
OBSERVED MINUS COMPUTED
760.00
795.00
830.00
865.00
900.00
935.00
970.00
TIME IN MODIFIED JULIAN DAYS — 38,000.0

FIGURE 3

COSMOS 41 MEAN ANOMALY RISIDUALS AFTER IMPROVED GRAVITY SOLUTION (units of degrees)

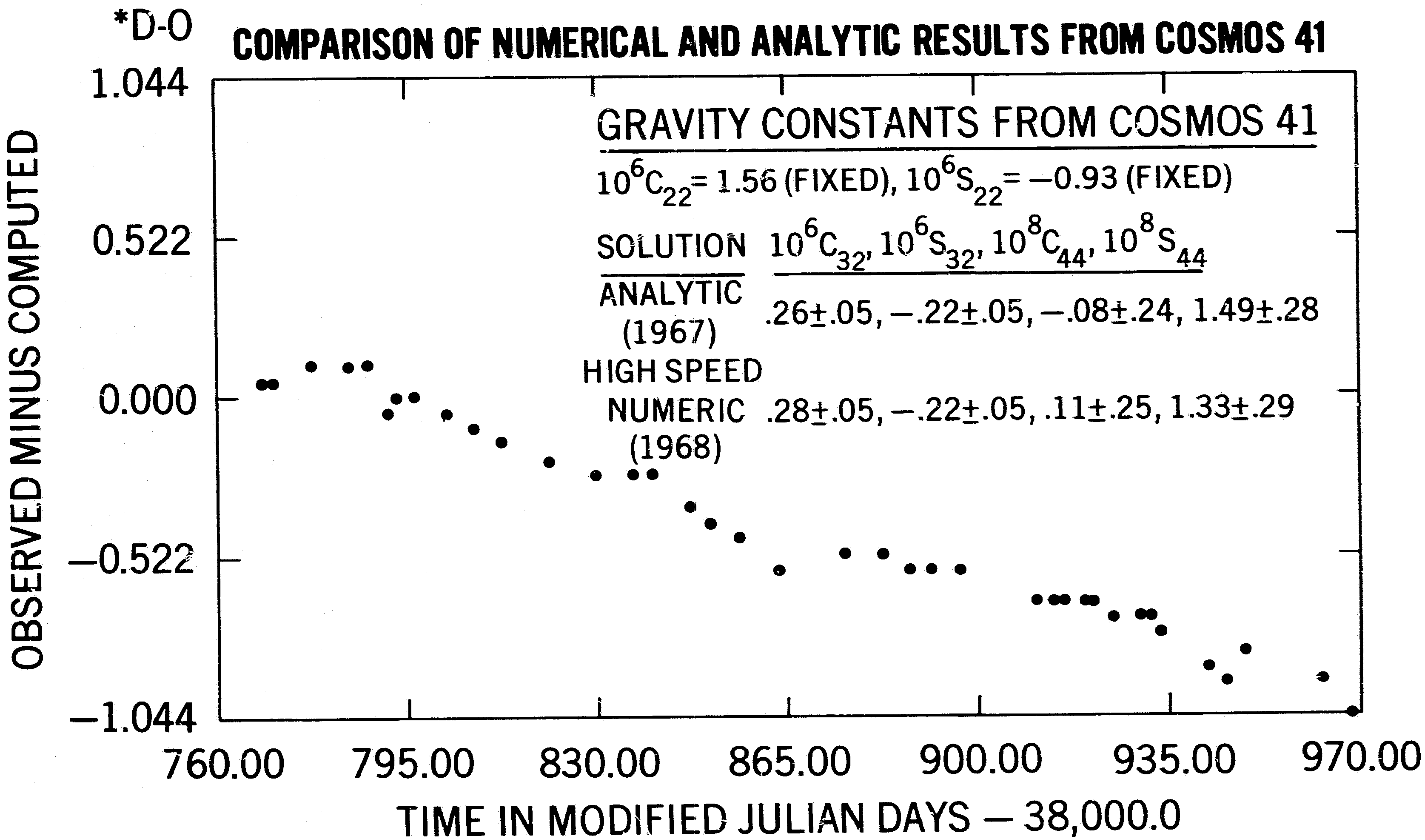

FIGURE 4

GRAVITY CONSTANTS FROM COSMOS 41

MEAN ANOMALY DATA

$10^6 C_{22} = 1.56$ (Fixed), $10^6 S_{22} = -0.93$ (Fixed)						
SOLUTION σ (°)	$10^6 C_{32}$	$10^6 S_{32}$	$10^8 C_{44}$	$10^8 S_{44}$	$10^{11} C_{66}$	$10^{11} S_{66}$
0.038	0.32	-0.21	0.28	1.27		
0.036	0.33	-0.20	0.35	1.26	4.0	-1.2
*0.036	0.33	-0.20	0.35	1.26	3.6	-2.8
SAO 1966, M1 Solution	0.26	-0.22	-0.08	1.49	-0.9	-3.6

*THIS SOLUTION INCLUDES (7, 6) AND (8, 6) EFFECTS WITH CONSTANTS FROM SAO 1966 STANDARD EARTH (M1 SOLUTION)

FIGURE 5