MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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ACKNOWLEDGEMENTS

This report was prepared under National Aeronautics and Space Administration Electronics Research Center Contract No. NAS 12-2081. NGR-22-009-229

Publication of this report does not constitute approval of the findings or conclusions by the National Aeronautics and Space Administration. Distribution is provided in the interest of information exchange.

Grateful acknowledgement is made for use of the M.I.T. Man Vehicle Control Laboratory computing facility. The authors are indebted to Noel Van Houtte who assisted in programming the hybrid computer.

ABSTRACT

A VTOL aircraft is modeled as a point mass moving in a vertical plane acted upon by thrust, gravity, lift, and drag. Approach and departure paths are studied under various constraints using a hybrid computer simulation. The pilot has control over angle of attack, thrust magnitude, and thrust direction. Fuel consumption is taken to be proportional to the time integral of the thrust. Constraints are placed on the approach path plus the maximum value of velocity, acceleration, and angle of attack. Data are presented to show how the fuel cost varies as a function of the constraints imposed. The most significant consideration for fuel economy is minimization of the time during which the aircraft flies below conventional stall speed. In general, the steeper the approach or departure path, the greater the fuel cost. Fuel-optimum approaches call for high descent rates at low altitude. When the rate of descent is constrained, the fuel cost increases but loses its sensitivity to approach angle. Automatic velocity control is necessary to maintain the glide path during steep approaches.

NOTATION

AR	aspect ratio
C_{D_o}	profile drag coefficient
C_{D_I}	induced drag coefficient
C_{L_α}	lift curve slope
D	drag, lb
e	wing efficiency factor
g	gravitational acceleration, ft/sec^2
h	vertical coordinate or altitude, ft
i	thrust direction relative to flight reference line
L	lift, lb
m	aircraft mass, sec^2/lb-ft
$\dot{m}_a$	air mass flow rate, sec/lb-ft
S	wing reference area, ft^2
T	thrust, lb
t	time, sec
t_f	flight time, sec
V	velocity, ft/sec
V_e	engine exhaust velocity, ft/sec
V_{eq}	equilibrium velocity, ft/sec
W	aircraft weight, lb
x	horizontal coordinate or range, ft
α	angle of attack
γ	flight path angle

γ_{eq} equilibrium flight path angle

δ thrust direction relative to horizontal

θ pitch angle

κ ratio of cruise miles to lb-sec of thrust, mi/lb-sec

ρ air density, $sec^2/lb\text{-}ft^4$

Introduction

The objective of this work has been to establish optimum approach and departure paths for VTOL terminal operations considering the interrelated aspects of economy, safety, traffic, and noise. In order to formulate the problem analytically, it was assumed that the considerations of safety, traffic, and noise would establish constraints on the flight path which could otherwise be chosen to minimize total fuel expended. The VTOL vehicle was modeled as a point mass moving in a vertical plane acted upon by thrust, gravity, lift, and drag. The pilot has control over angle of attack, thrust magnitude, and thrust direction. Fuel consumption was taken to be proportional to the time integral of the thrust. The results are representative of, but not necessarily restricted to, the case of a vectored-thrust jet-lifter, an example of a VTOL aircraft which was considered most likely to benefit from an optimized flight path. Two previous studies[(1),(2)] using the same analytic model have shown that the unconstrained optimum calls for the aircraft to dive underground during both the initial acceleration after take-off and the final deceleration prior to landing. In this work the aircraft has been constrained not to descend below the ground. The angle of attack has been constrained not to exceed a critical value for stall. The maximum acceleration has been limited for structural safety and passenger comfort. The effect of constraining the angle of approach and departure has also been investigated. Another facet of the problem has been to consider how the pilot might actually reproduce a computed, optimum flight path. In a conventional aircraft it is possible to fly optimally by reproducing an optimum state history (position and velocity of the vehicle). The two controls (angle of attack and thrust magnitude) are determined by the state history. This is not possible for the VTOL aircraft because three controls cannot be uniquely determined from the state history.

Three different techniques were used to study the problem. The first extracted physical insight from classical solutions to the equations of motion in steady-state conditions. The second simulated the response of the vehicle on a GPS 290T hybrid computer. Control inputs were established by a human "pilot" who could observe the output indications of vehicle position and velocity in real time. A third approach using optimal control theory to determine accurate, constrained-optimum flight paths on the digital computer is the topic of a subsequent report. The computational search for a constrained optimum is generally more difficult and time-consuming than the search for an unconstrained optimum. However, the constrained optimum often calls for the control to lie on the boundary of the constraint. Physical insight makes it possible to predict the optimum control in this case on the basis of logic. The fuel cost under that control can then be found using the hybrid computer simulation. The flight path is relatively easy for the pilot to follow since the control lies on the boundary. Data has been taken to show how the fuel cost changes as a function of constraints imposed. A subjective evaluation has been made of the difficulty involved in following optimum flight paths.

The particular VTOL aircraft simulated is designed to transport 80 passengers for a range of 200 miles at a cruise altitude of 20,000 feet.[3] This particular vehicle was chosen in order to be able to compare the simulator results with those obtained by Mehra and Bryson.[1] Agreement was taken as a test of the validity of the simulation. Constrained optima were constructed by trial and error on the simulator guided by the conclusions drawn from steady-state models. The results have not been shown to satisfy any mathematical criteria of optimality.

Hybrid Computer Simulation

For the purpose of this study, the aircraft is assumed to be a mass point acted upon by lift, drag, and thrust. The rotational dynamics are assumed to have no direct effect on the overall performance. Only motion in a vertical plane is considered. The effects of wind gusts and shears are neglected.

The diagram in Fig. 1 shows how the various quantities are defined. The angles are positive in the direction of the arrows. The equations of motion resolved in the x-h coordinate frame are

$$m\ddot{x} = T\cos\delta - D\cos\gamma - L\sin\gamma + \dot{m}_a V[\cos\delta - \cos\gamma] \qquad (1)$$

$$m\ddot{h} = T\sin\delta - D\sin\gamma + L\cos\gamma - mg + \dot{m}_a V[\sin\delta - \sin\gamma] \qquad (2)$$

where

$$L = 1/2\ \rho S C_{L_\alpha}\ \alpha V^2 \qquad (3)$$

$$D = 1/2\ \rho S V^2 (C_{D_o} + C_{D_I}\ \alpha^2) \qquad (4)$$

$$V = (\dot{x}^2 + \dot{h}^2)^{1/2} \qquad (5)$$

$$\gamma = \tan^{-1}(\dot{h}/\dot{x}) \qquad (6)$$

$$\alpha = \theta - \gamma \qquad (7)$$

$$\delta = i + \theta \qquad (8)$$

$$C_{D_I} = C_{L_\alpha}{}^2/\pi e AR \qquad (9)$$

$$\dot{m}_a = T/[(65)(32.2)] \quad \text{sec/lb-ft if T is in lb.} \qquad (10)$$

The $\dot{m}_a V$ terms result from the directional change in the momentum of the air as it passes through the engine. These same equations, resolved in velocity-flight path angle coordinates, were used in Ref. 1. The advantage of the equations in the form given here is that there is no singularity at zero velocity.

The aircraft mass is taken to be constant during the takeoff and landing. The air density varies with altitude according to

$$\rho = \rho_{SL}(1 - .6875\text{x}10^{-5}h)^{4.2561} \tag{11}$$

where h is in feet.

The optimum takeoff and landing is the one that minimizes fuel consumption. Since the fuel mass flow rate is generally proportional to the thrust, the quantity to be minimized is the integral $\int_0^{t_f} T dt$. The takeoff and landing consist of the complete ascent and descent, respectively, between the ground and cruise altitude.

It is assumed that there is closed loop control over the pitch angle so that making either θ or α the control variable is equivalent. The other controls are thrust magnitude and thrust direction. The wings are rigidly attached to the aircraft, but the engine thrust direction can be rotated relative to the wings through the full 360 degrees. It is assumed that minimum thrust is zero, and maximum is 25 percent greater than the aircraft weight. The cases for maximum thrust 10 and 5 percent greater than the aircraft weight were also considered. The thrust varies with altitude according to

$$T = T_{SL}(1 - .55h/30{,}000) \quad (12)$$

where T_{SL} is the sea-level thrust, and h is in feet.

Table 1 gives the values for the aircraft parameters that were used (Ref. 1).

Table 1. Aircraft Parameters

Parameter	Value
C_{D_o}	.027
C_{L_α}	5.73
e	.9
AR	6.0
S	421 ft^2
W	56,900 lb
m	1765 $sec^2/lb\text{-}ft$
C_{D_I}	1.93

The angle of attack that provides the maximum lift to drag ratio is found as follows:

$$L/D = \frac{C_{L_\alpha}\alpha}{(C_{D_o} + C_{D_I}\alpha^2)} \quad (13)$$

$$\frac{d(L/D)}{d\alpha} = \frac{C_{L_\alpha}}{C_{D_o} + C_{D_I}\alpha^2} - \frac{2C_{L_\alpha}C_{D_I}\alpha^2}{(C_{D_o} + C_{D_I}\alpha^2)^2} = 0 \qquad (14)$$

$$\alpha_{(L/D)_{max}} = (C_{D_o}/C_{D_I})^{1/2} = 6.8 \text{ deg} \qquad (15)$$

The maximum angle of attack is taken to be 15 deg, which is slightly more than twice $\alpha_{(L/D)_{max}}$.

The velocities for stall and maximum lift to drag ratio are found by assuming the aircraft is in unaccelerated level flight with zero thrust angle. From the equations of motion, it is seen that the lift is equal to the weight for this condition. Thus, equating lift with weight gives

$$V_{(L/D)_{max}} = [2W/(\rho S C_{L_\alpha}\alpha_{(L/D)_{max}})]^{1/2} \qquad (16)$$

$$V_{stall} = [2W/(\rho S C_{L_\alpha}\alpha_{max})]^{1/2} \qquad (17)$$

Consider (17) to be the definition of conventional <u>stall</u> <u>speed</u>. The sea-level stall speed is 280 feet per second. The sea-level velocity for maximum lift to drag ratio is 420 feet per second. It was assumed that the aircraft cruises at the speed for maximum lift to drag ratio which is the speed for maximum endurance. The thrust required for cruise, T_{CR}, is found from Eq. (1) with $\ddot{x} = \gamma = i = 0$. This gives T_{CR} = 4580 lbs. The number of cruise miles that can be travelled per lb-sec of thrust, κ, is given by

$$\kappa = V_{(L/D)_{max}} \Big/ T_{CR} \tag{18}$$

$$= 2.32 \times 10^{-5} \text{ mi/lb-sec}$$

Thus, each second of thrusting at the maximum thrust of 1.25W is equivalent to 1.65 miles of range. Thrusting at one weight for one minute is equivalent to 79 cruise miles. A larger value for κ results if the velocity for maximum range is used instead of the velocity for maximum endurance. Consequently, the fuel costs presented here are conservative.

Figure 2 shows a flow diagram of the system used to simulate the VTOL aircraft. The analog computer is the heart of the simulator. The other main components are the digital computer, control box, displays, and recorders.

The analog computer used was a GPS 290T. It received values for $\ddot{x}$ and $\ddot{h}$ from the digital computer and integrated them to give $\dot{x}$, $\dot{h}$, x, and h.

The analog computer received signals from the controls to provide values for θ, T, and i. It integrated the thrust to give the fuel consumption and also provided the necessary information for the displays and recorders.

The digital computer used was a PDP-8. It received values for $\dot{x}$, $\dot{h}$, h, T, i, and θ from the analog computer. It then computed the right-hand side of the equations of motion (Eqs. 1 and 2) which yielded values for $\ddot{x}$ and $\ddot{h}$. The cycle was completed every 3.9 millisec.

The three controls for the VTOL aircraft came from a three-degree-of-freedom control stick. The diagram in Fig. 3 illustrates how the controls were actuated by the stick.

The quantities that were displayed to aid in controlling the aircraft were $\dot{h}$, $\dot{x}$, h, x, α, and δ. The altitude and range were shown by an x-y plotter. The other variables were displayed by a Brush recorder. These displays, plus knowledge of the thrust magnitude by stick position, were ample to control the aircraft.

The only recordings that were made, in addition to the quantities displayed, were T, i, and fuel consumption. The time histories of T and i were provided by a Brush recorder. The fuel consumption was read by a digital voltmeter after each run. Additional details of the simulation are available in Ref. 4.

Steady-State Thrust Requirement

Neglecting the small forces due to the directional change in the momentum of the air passing through the engine, the equations of motion for the vehicle give the thrust in vector form as

$$\underline{T} = \underline{\dot{V}} - \underline{L} - \underline{D} - \underline{W} \qquad (19)$$

The fuel cost is the time integral of the magnitude of the thrust.

$$|\underline{T}| \leq |\dot{\underline{V}}| + |\underline{L}| + |\underline{D}| + |\underline{W}| \tag{20}$$

The maximum contribution to the cost due to each term in (20) can be estimated in cruise miles. If $\dot{V}$ is in a fixed direction and V changes monotonically, then the cost of the acceleration term will be less than the total velocity change expressed in cruise miles. The cost due to the acceleration term alone in going from rest to stall speed is less than 10 cruise miles. The cost of the lift term is negligible since thrust will almost never be used to counter lift. The lift is used to counter the weight, and thrust is used to counter the drag. The maximum value of the drag (neglecting the intentional use of high drag devices wherein the drag is not cancelled by thrust) is at stall speed where the cost is about 9 cruise miles each minute. At velocities below stall speed, thrust must be used to counter both weight and drag. The cost of countering the weight is 79 cruise miles per minute and is clearly the most significant cost. The comparatively low integrated cost of the acceleration term seems to justify the use of steady-state models which neglect acceleration and equate the thrust to the sum of lift, drag, and weight. These models are classical to aircraft performance theory and can be found in any standard text such as Ref. 5. Using such models the thrust required for level flight can be expressed as a function of velocity as shown in Fig. 4. The graph has been extended to velocities below conventional stall speed by assuming that the angle of attack is held at its maximum value and the thrust direction adjusted to achieve equilibrium. A glance at Fig. 4 shows that the most important consideration for fuel economy is the minimization of the time spent at velocities below conventional stall speed. If considerable time must be spent in this speed range, then it is important to have the angle of attack at its maximum value because it minimizes the thrust required at these speeds under steady-state conditions. The same conclusion has been reached by Gallant et al[6] for the tilt-wing VTOL aircraft where they suggested independent control over wing position and thrust direction in order to maintain the angle of attack near maximum. With angle of attack fixed, the specification of the flight path uniquely determines the remaining two controls (thrust magnitude and thrust direction).

Optimum Takeoff

To duplicate Mehra and Bryson's results, the aircraft was controlled manually to match the time histories for α and i as closely as possible. Mehra and Bryson used Eqs. (1) and (2) resolved in velocity and flight path angle coordinates. To avoid the singularity at $V = 0$, a small initial velocity of 50 ft/sec with an initial flight path angle of 7 degrees had to be assumed. These initial conditions were used for the duplication. The time to climb to 20,000 feet was minimized without specification of the range or final horizontal velocity component. Figures 5 - 11 show the results. Although the simulator trajectory climbed slightly steeper with a slight loss in range, the total time was the same. The theoretical optimum without constraint calls for a downward acceleration of about 10 g's to bring the flight path angle to zero at the end of the climb. The simulation could obtain only about 2 g's of downward acceleration causing an overshoot of the cruise altitude. The optimum violates several physical constraints. Not only does the aircraft go through the ground and sustain 10 g's of acceleration, but the pitch angle becomes too large for passenger comfort and the velocity becomes large enough for compressibility effects to occur.

However, this method does get the aircraft to an altitude of 20,000 feet in only 53 seconds. The resulting fuel consumption is equivalent to 87 miles of range. No other method was discovered with the simulator that required less fuel than this method. In addition, if the method of control deviated very much from the one presented here, the time and fuel to reach cruise altitude increased. The broken lines in Figs. 5 - 11 show a case where the aircraft did not go below the ground. This non-optimum example took 96 cruise miles of fuel.

Mehra and Bryson considered a vertical takeoff with a pitch angle of 90 degrees. This allows the angle of attack and, in turn, the lift and induced drag to remain zero. Thus, the aircraft rises to 1000 feet with an essentially constant acceleration. After 16 seconds the altitude of 1000 feet is reached with a vertical velocity of 125 ft/sec. The optimum control theory solution started with the conditions at the end of 16 seconds. These initial conditions were also used for the simulator duplication. Again the simulator results were very close to the optimum control theory results as seen in Fig. 12. The same deviations as for the unconstrained takeoff were again present. The angle of attack was slightly large so some range was lost with the simulator. The large downward acceleration at the end of the climb could not be obtained.

Although the aircraft does not enter the ground during this takeoff, it still violates some physical constraints. The 90 degree takeoff followed by a rapid decrease in pitch angle exceeds the level for passenger comfort. Later in the climb, the pitch angle again exceeds the comfort level. The accelerations incurred are also too large for passenger comfort and aircraft structure. The velocity reaches the onset of compressibility effects.

The aircraft climbs to the cruise altitude of 20,000 feet in 68 seconds with this method, using 112 cruise miles of fuel. With these initial conditions, no better method of climbing was found with the simulator. Again, if the method of control varied very much from the optimum, additional fuel was required to reach the cruise altitude.

Further constraints were imposed on the takeoff to make it more physically realizable. The pitch angle was not allowed to exceed a comfortable level of ±30 degrees. The angle of attack was not allowed to exceed the maximum value of 15 degrees. The velocity was kept below 600 ft/sec. The aircraft was required to rise vertically to an altitude of 300 feet and then execute a 20 degree angle of climb to an altitude of 3000 feet. Horizontal flight had to be obtained at the end of the climb with a downward acceleration of 1 g or less.

This constrained takeoff was flown first with a constant thrust of 1.25W. The resulting flight path and time histories of the flight variables are shown in Figs. 13 - 20. These show that the thrust direction had to be raised to be almost straight up at 5000 feet in order to keep the velocity from exceeding 600 ft/sec. It took 91 seconds or 151 cruise miles of fuel to reach 20,000 feet. This is 64 miles more than that required with the unconstrained optimum. It is 51 miles more than that required with the vertical optimum, if the initial vertical rise is to 300 feet instead of 1000 feet for the vertical optimum.

The case when the maximum thrust is 1.10W was also considered. The flight was very similar to the one with a thrust of 1.25W available. It took 114 seconds or 165 cruise miles of fuel to reach 20,000 feet. Most of the additional 14 miles of fuel were spent in the initial vertical climb to 300 feet. After this initial climb the lower thrust level is almost as efficient as the higher level because of the velocity constraint.

Climb angles other than 20 degrees were also tried. The results are nearly the same for all climb angles less than 30 degrees because of the velocity constraint, pitch angle constraint, and the acceleration constraint at the end of the climb. An 8 degree takeoff uses 5 cruise miles of fuel less than the 20 degree takeoff. Thus, a low climb angle requires somewhat less fuel. However, the velocity constraint reduces the benefits of a low climb angle. The small amount of time saved in obtaining the maximum velocity is mostly cancelled by the altitude that is lost in the process.

Variable thrust was used next to try to reduce the fuel consumption. The results are shown by the dashed lines in Figs. 13 - 20. The thrust is kept at its maximum level of 1.25W at first. Then, it is beneficial to reduce the thrust when the maximum velocity is reached. The thrust is kept just large enough to maintain the velocity at this maximum value. In this manner 115 cruise miles of fuel are required, a saving of 36 miles over the similar constant thrust method. This is 15 miles more than that for the less constrained constant thrust vertical optimum, which initially climbs vertically to 300 feet.

By reducing the thrust earlier, a somewhat lower maximum velocity can be imposed without a large cost in fuel. However, if the maximum velocity is much below 500 ft/sec, the fuel requirement increases markedly. For maximum velocities of 450 and 400 ft/sec, the fuel required is approximately 20 and 50 cruise miles more, respectively, than that for 600 ft/sec.

The results from the various control methods used point out several guidelines for minimizing fuel consumption while climbing to the cruise altitude. The aircraft should be provided with a large maximum thrust and the maximum should be used initially. Then, the thrust should be kept just large enough to maintain the maximum allowable velocity. The initial vertical climb should be continued to as low an altitude as possible. The climb angle should be small at first to increase the velocity to its maximum rapidly. This is accomplished by keeping the thrust angle small after the initial vertical climb. At the end of the climb, the aircraft should be leveled to horizontal flight at the last possible moment, not exceeding the allowable acceleration. This leveling of the flight path should be started at an altitude between 18,000 and 19,000 feet.

Optimum Landing

The final touchdown of a VTOL vehicle is expected to be under visual conditions from a hover 20 to 40 feet above the touchdown point. The optimization problem is to get from cruise at 20,000 feet to hover at around 20 feet with minimum fuel. The unconstrained optimum would put the vehicle in a power-off glide pulling up vertically from below the final touchdown point so as to reach zero velocity at the final time without the expenditure of fuel. With the constraint that the vehicle remain above ground level, the power-off glide needs to be terminated with thrust. The angle of attack should be maximum at the end of the power-off glide, and the altitude should be as low as possible. This results in a minimum velocity to be nulled by thrusting. The maximum-range, power-off glide is with an equilibrium flight path angle of -4.8 degrees for $\alpha_{(L/D)_{max}}$. At an altitude of 4000 feet the glide is transitioned to -7.2 degrees with the angle of attack at its maximum value. The vertical component of velocity is -35 ft/sec at low altitude for both angles of attack.

The search for the optimum thrust maneuver was begun by keeping the thrust at its maximum level once it was applied. Various methods of employing the other two controls were used to determine the method that required the least fuel. The solid lines in Figs. 21 and 22 show the flight path and vertical velocity for the optimum landing method that was found. These curves begin at the end of the power-off equilibrium glide. First, an oscillation is produced which allows the aircraft to level off. Twenty-two seconds later the thrust is applied at its maximum level of 1.25W to bring the aircraft to a stop. The angle of attack is kept at its maximum value of 15 degrees throughout, except when it is decreased to produce the oscillation. To produce the oscillation, the angle of attack is reduced to 12 degrees and then brought back to 15 degrees over a 10 second interval. This disturbance occurs at an altitude of 1050 feet. The flight path angle decreases to a minimum value of -14 degrees. Then, when it has increased nearly to zero, thrust is applied. The thrust is applied at a low altitude of less than 20 feet and a range of 1500 feet from the landing point. In this manner the thrust has to be applied for only 7.9 seconds. The fuel that is consumed is equivalent to 13 cruise miles.

The optimum method of control is the same if the maximum available thrust is less than 1.25W. The dashed lines in Figs. 21 and 22 show the case when the maximum thrust is only 1.05W. The thrust has to be applied for 15 seconds. An additional 625 feet of range are required. The fuel needed for this case is equivalent to 17 cruise miles.

The broken lines in Figs. 21 and 22 show a landing in which the equilibrium glide is not disturbed before thrusting. The thrust is applied at the maximum level of 1.25W, the angle of attack is kept at its maximum value, and the flight path angle is kept constant at the equilibrium value.

Thrusting is begun at an altitude of 335 feet and a range of 2700 feet from the landing point. Eleven seconds of thrust or 17 cruise miles of fuel are required.

The optimum method is the easiest to perform. The difficulty in control increases with methods that require increased fuel consumption. In all cases it was difficult to hit the precise landing point. However, the final range could be held within ±100 feet of the landing point with reasonable consistency. For the optimum method the final altitude was fairly consistently between the ground and 40 feet above the ground. For the optimum method, in which the equilibrium glide is not disturbed, the final altitude was within 20 feet of the ground with the same consistency. The distance by which the landing point was missed increased with methods that require increasing fuel consumption.

Variable thrust was used to try to find a method of landing that requires less than 13 cruise miles of fuel. No such variable thrust method was found. It is reasonable that the maximum thrust should be used in order to decrease the time that gravity has to be opposed.

The acceleration in the negative x direction produced by using constant maximum thrust may be somewhat large for passenger comfort. An acceleration greater than .7 g is produced with most of the maximum thrust methods. The optimum method produces an acceleration of 1.20 g in the negative x direction.

The thrust was varied to keep this acceleration below .5 g. It was brought rapidly to 25,000 pounds. Then it was increased to the maximum of 1.25W, keeping the acceleration below .5 g. The flight path angle was held constant and the angle of attack was kept at its maximum value. Both disturbed and undisturbed equilibrium glide landings were performed. In each case the fuel requirement was just slightly greater than that for constant maximum thrust. Thus, if the acceleration is constrained to be less than .5 g, more time, altitude, and range are required; but the fuel penalty is not large.

Figures 23 - 25 compare landings using variable thrust with those in which the thrust is kept at its maximum value once thrusting is begun. Figure 24 shows that the thrust is increased rapidly to 25,000 pounds and then increased at a constant rate to the maximum. The variable thrust landings require only one cruise mile more fuel than the corresponding constant maximum thrust landings.

It was considerably more difficult to control the aircraft while performing the variable thrust landings. However, the final range could be held within ±200 feet of the landing point with reasonable consistency. The final altitude was fairly consistently between the ground and 40 and 60 feet above the ground for the undisturbed and disturbed glide landings respectively.

Steep Approaches

The aircraft was constrained to maintain constant flight path angles of -7, -15, and -25 degrees below an altitude of 2000 feet. Each of these angles was flown with the pitch angle set to provide the maximum angle of attack of 15 degrees. It was very difficult to keep the aircraft on the -15 and -25 degree flight paths by controlling the thrust magnitude and direction directly. These flight paths could be maintained only for several hundred feet of altitude before large oscillations were encountered. Other investigators[(7),(8)] have reported the same difficulty in maintaining path control during steep approaches with several types of VTOL aircraft. A simple velocity control system was added in order to maintain the desired constant flight path angles. The velocity control system eliminated the difficulty in maintaining path control during steep approaches. Similar results have been reported by Rhodes and Tymczyszyn[(8)] using a helicopter simulation. The diagram for the velocity control system is shown in Fig. 26. The error signals are obtained by comparing the desired velocity with the actual velocity. The vertical velocity component error signal produces a thrust magnitude input, while the horizontal velocity component error signal produces a thrust direction input. The limiter keeps the thrust between zero and 1.25W. The angle of attack was maintained at its maximum value. This particular velocity control system will give economic operation at velocities below stall speed. A different control system would be called for at higher speeds. The -7 degree approach required 19 cruise miles of fuel which was one cruise mile less efficient than direct control. The fuel cost increased with flight path angle at an almost constant rate of .7 cruise mile per degree. Figure 27 shows the vertical velocity profiles. The steeper approaches call for higher vertical velocities.

If the vertical velocity is constrained to be less than some small value below an initial altitude, then the aircraft will take the same length of time in performing the descent and the fuel cost will not be dependent upon approach angle. The fuel cost, however, is very large. A 20 ft/sec descent from 1200 feet, for example, requires over 150 cruise miles of fuel. The steep approach angle and slow rate of descent force the aircraft velocity to be well below stall speed. Thrust must be approximately equal to weight, and the fuel cost goes to 79 cruise miles for each minute the aircraft spends in the descent. It can be seen that the fuel cost for the optimum landing is swamped by any constraint that requires flight below stall speed for longer than a single minute.

Conclusions

To minimize fuel it is most important to minimize the time spent at airspeeds below conventional stall speed. At very slow airspeeds the thrust is approximately equal to the weight. At cruise speed the thrust is the weight divided by the lift to drag ratio, less than one-twelfth the thrust required at very slow speeds. With a cruise speed of over six miles per minute, the cost of only one minute of very slow flight is 79 miles range. Optimum flight below conventional stall speed requires that the angle of attack be held at its maximum limiting value. This is the most important consideration for optimization of the transition or for sustained flight at speeds just below conventional stall speed. Optimum methods of control during the takeoff and landing were found by trial and error. While an optimum found in this manner is not guaranteed to be the absolute mathematical optimum, the fuel costs under the various constraints are representative. The optimum takeoff is obtained if the initial vertical rise is to a minimum altitude. Rapid acceleration to the maximum allowable velocity should take place with as small a climb angle as permissable. The maximum thrust is used initially and then is kept just large enough to maintain the maximum velocity. After reaching the maximum velocity, the climb angle is increased until the pitch angle reaches its maximum value. The flight path is leveled as late as possible using the maximum comfortable acceleration. In this manner 115 cruise miles of fuel are required to obtain the cruise altitude of 20,000 feet.

For the optimum landing the engines are off for most of the descent. At an altitude of 4000 feet, an equilibrium power-off glide with the maximum angle of attack is entered. Then just before thrust is applied, an oscillation is produced to bring the flight path angle to zero when the aircraft is a few feet above the ground. The thrust is applied at as large a value as possible, keeping the acceleration just below the comfort level. The angle of attack is kept at its maximum value, and the flight path angle is held near zero while thrusting. Consequently, the pitch angle is held constant. The thrust direction is horizontal initially. It is then raised toward the vertical, keeping the flight path angle constant. In this manner 13 cruise miles of fuel are required for the landing.

The cost in fuel for various non-optimum methods of control was determined. During takeoff it is important to use the maximum available thrust initially and then reduce it when the maximum velocity is reached. Other methods of applying the thrust can cost 30 to 50 cruise miles of fuel. The maximum allowable velocity should be greater than 500 ft/sec. If the maximum velocity is only 400 ft/sec, the cost in fuel is approximately 50 cruise miles. The flight path angle that is used in the early part of the climb is not critical. The cost of a large climb angle early in the flight is between 5 and 10 cruise miles.

During the landing the engines should be off until the end of the descent when the thrust is applied strongly. Any preliminary thrusting is wasted since the aircraft returns to equilibrium conditions when the thrust is discontinued. Once thrust is applied, the maximum value should be used, not exceeding the maximum allowable acceleration. If the maximum allowable thrust is not used, allowing the acceleration to vary, a cost of 30 cruise miles of fuel can easily occur. If the angle of attack that provides the maximum lift to drag ratio is used instead of the maximum, 5 to 10 additional miles of fuel may be required. The cost in fuel is large if the approach velocity is reduced below the stall velocity before the optimum altitude for thrusting is reached. A saving of 3 to 10 cruise miles of fuel can be obtained by producing an oscillation to bring the flight path angle nearly to zero just before thrusting. If the acceleration is constrained to be less than .5 g, the penalty in fuel is only one cruise mile.

Steep approaches increase the fuel cost about .7 cruise mile per degree for approaches greater than 7 degrees. The vertical velocity also increases with steeper approaches. If the vertical velocity is constrained, the fuel cost increases greatly but loses its sensitivity to approach angle. Steep approaches require an automatic velocity control system to keep the aircraft on the prescribed flight path. The velocity control system makes flight path control relatively easy when used with a position display showing altitude vs. range. The use of a velocity control system adds a negligible fuel cost.

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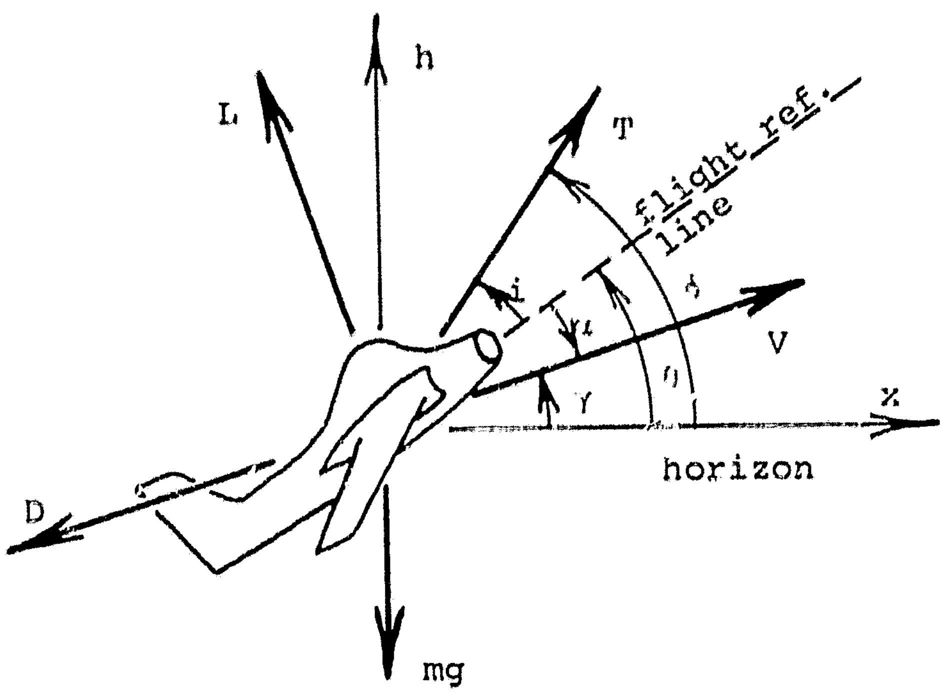

Fig. 1 Force Diagram for Aircraft

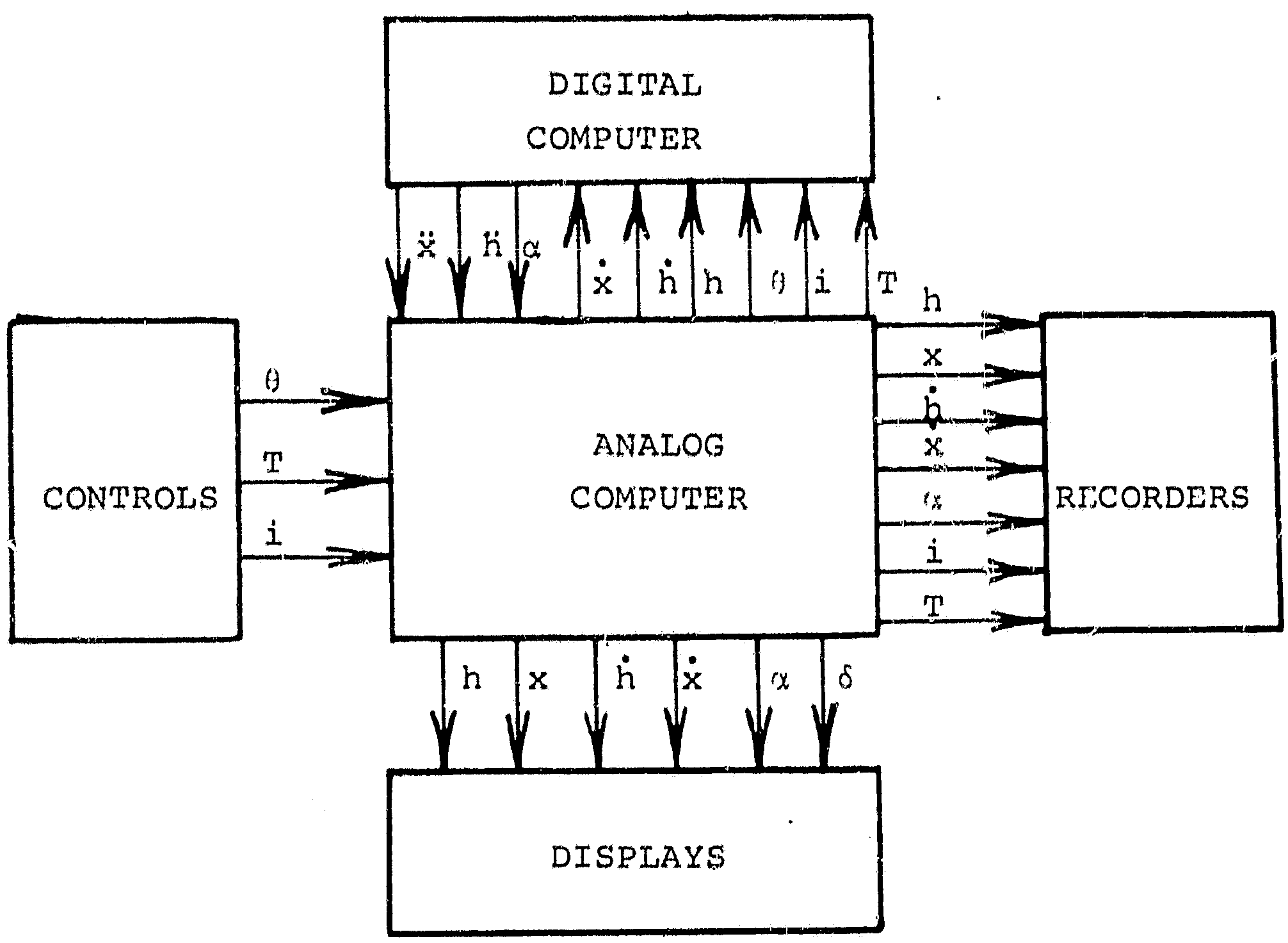

Fig. 2 Simulator Flow Diagram

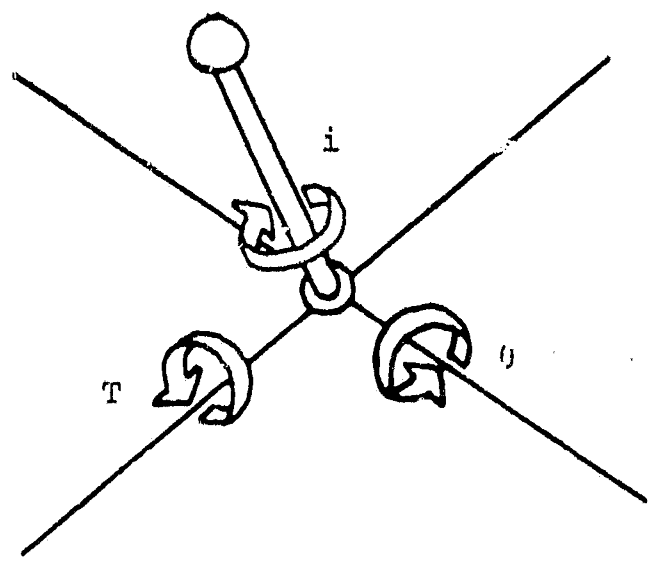

Fig. 3 Control Stick

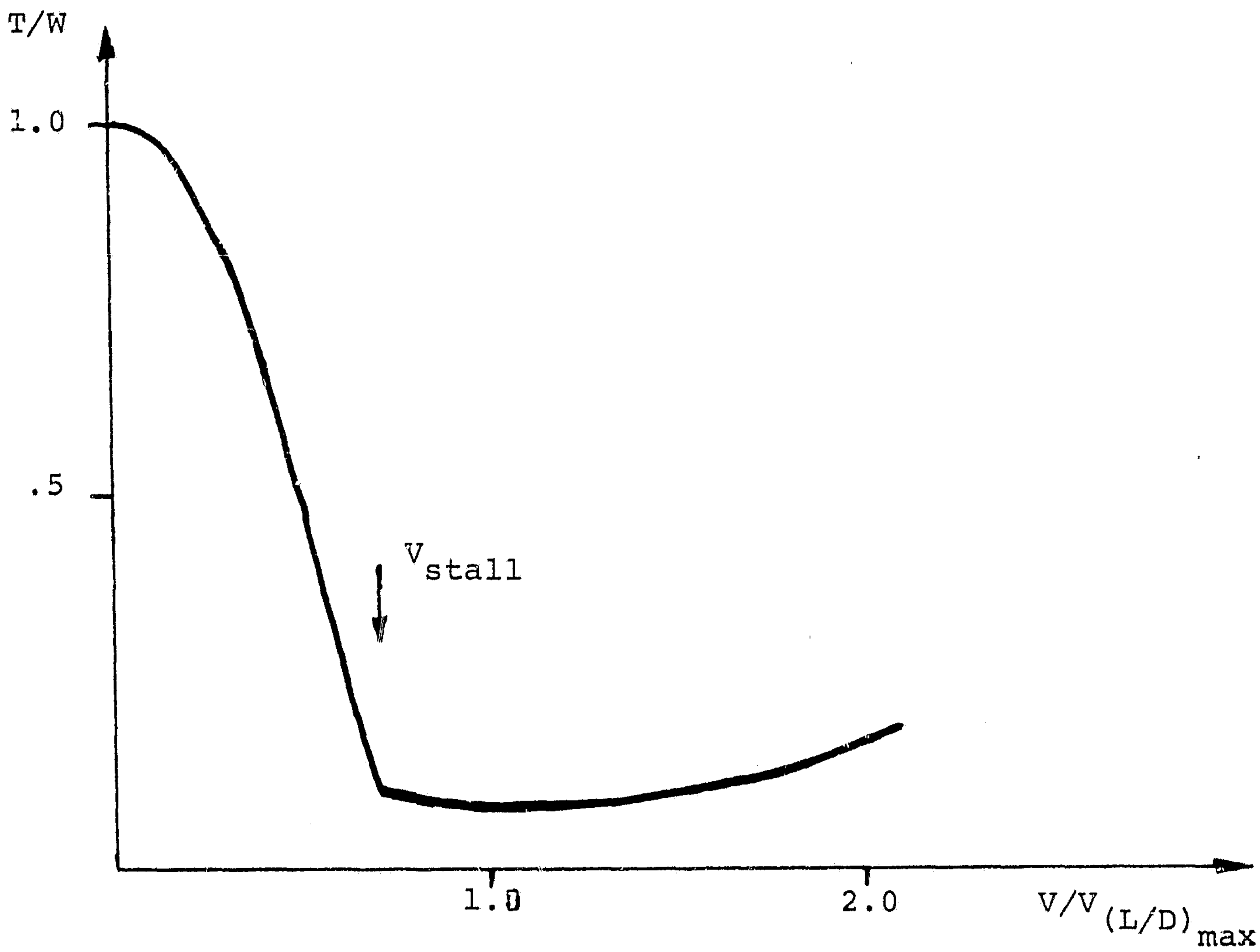

Fig 4. Thrust Required for Level Flight

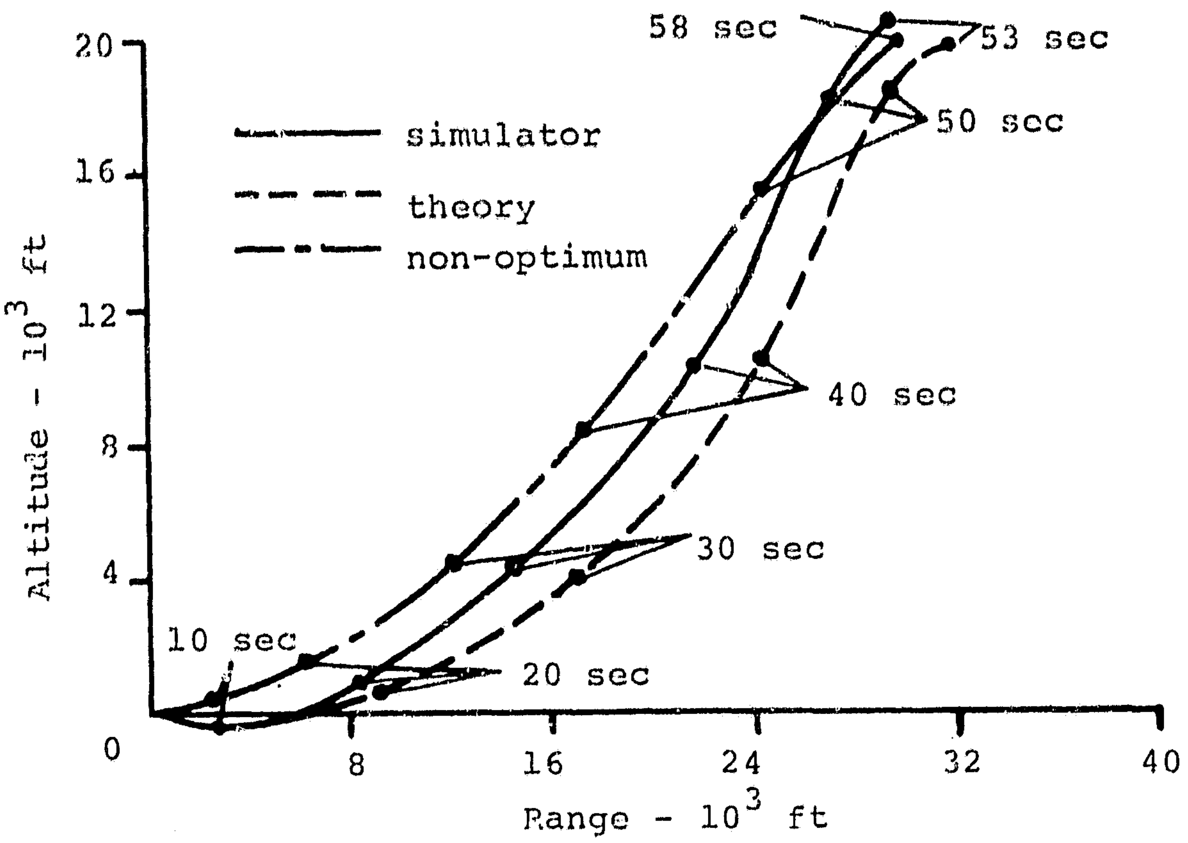

Fig. 5 Flight Path for Unconstrained Takeoff

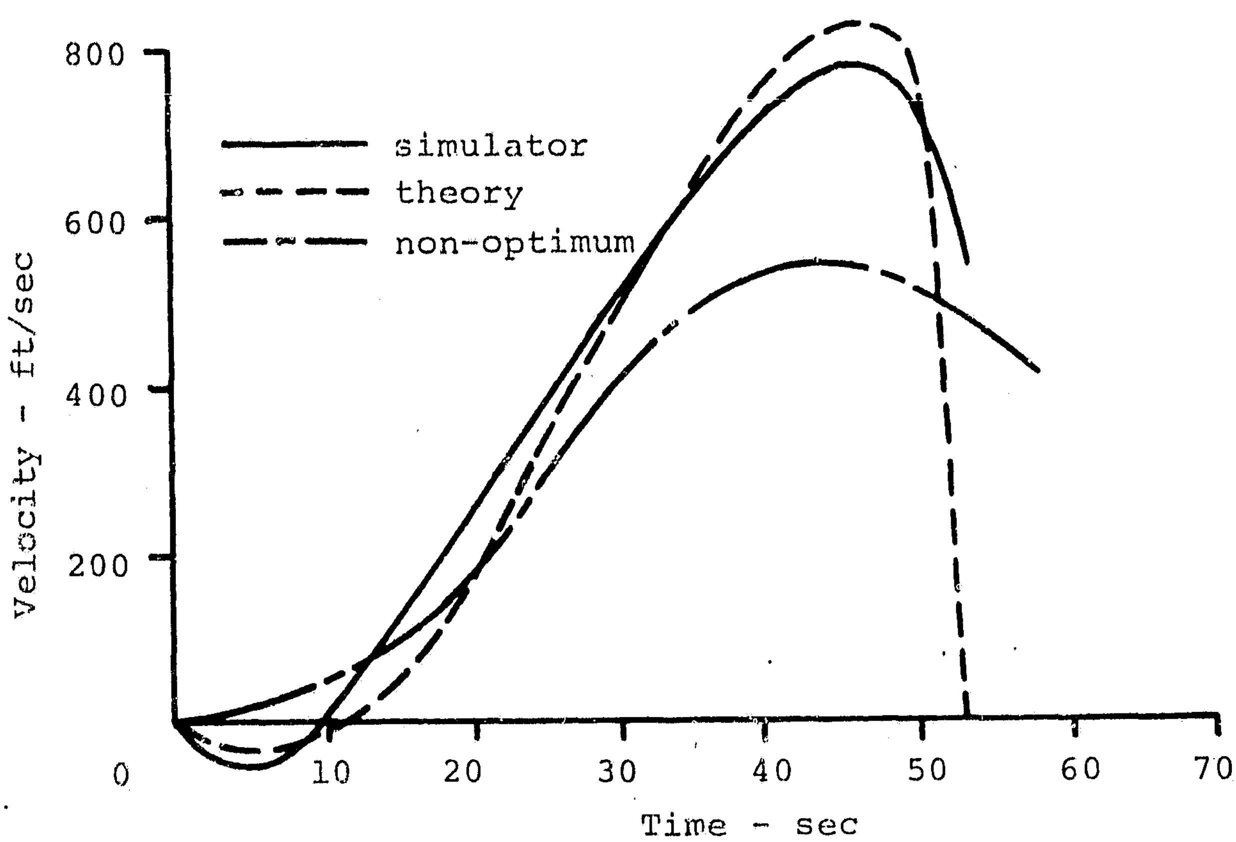

Fig. 6 Vertical Velocity for Unconstrained Takeoff

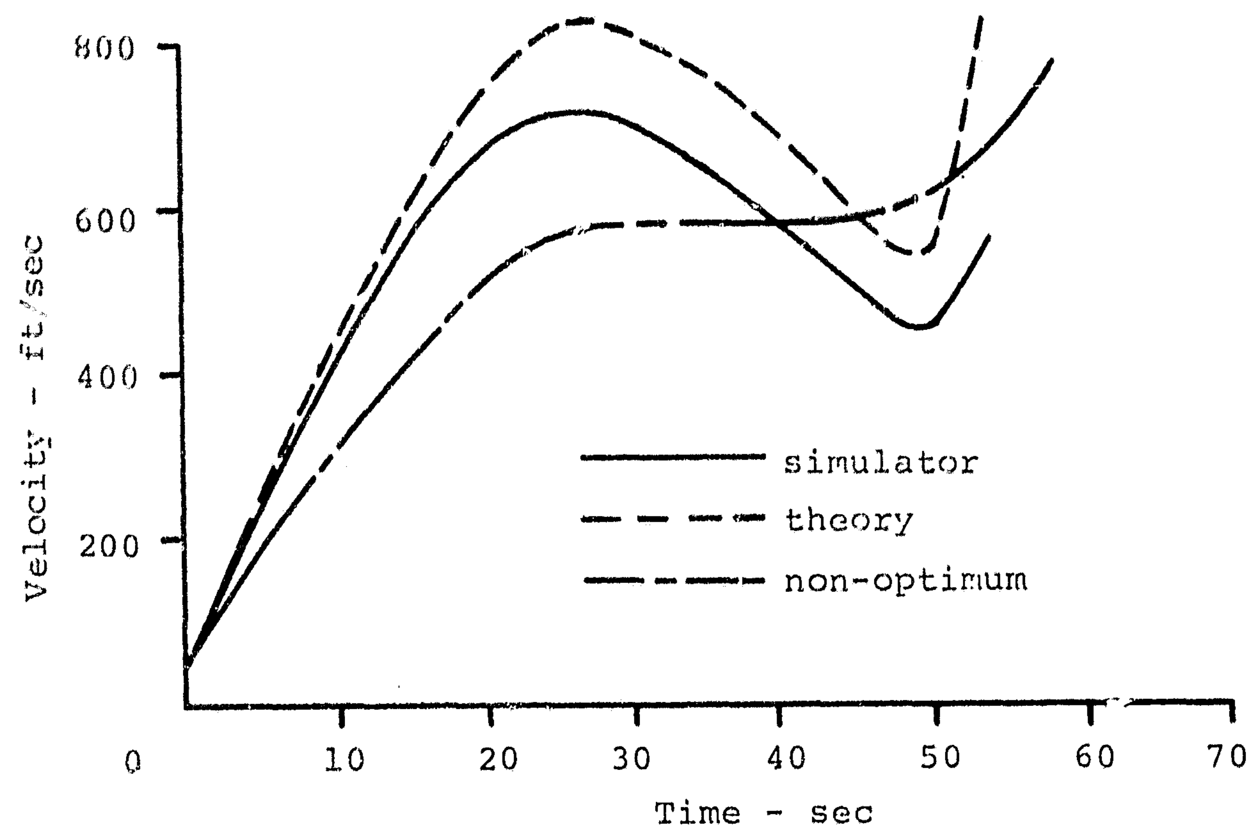

Fig. 7 Horizontal Velocity for Unconstrained Takeoff

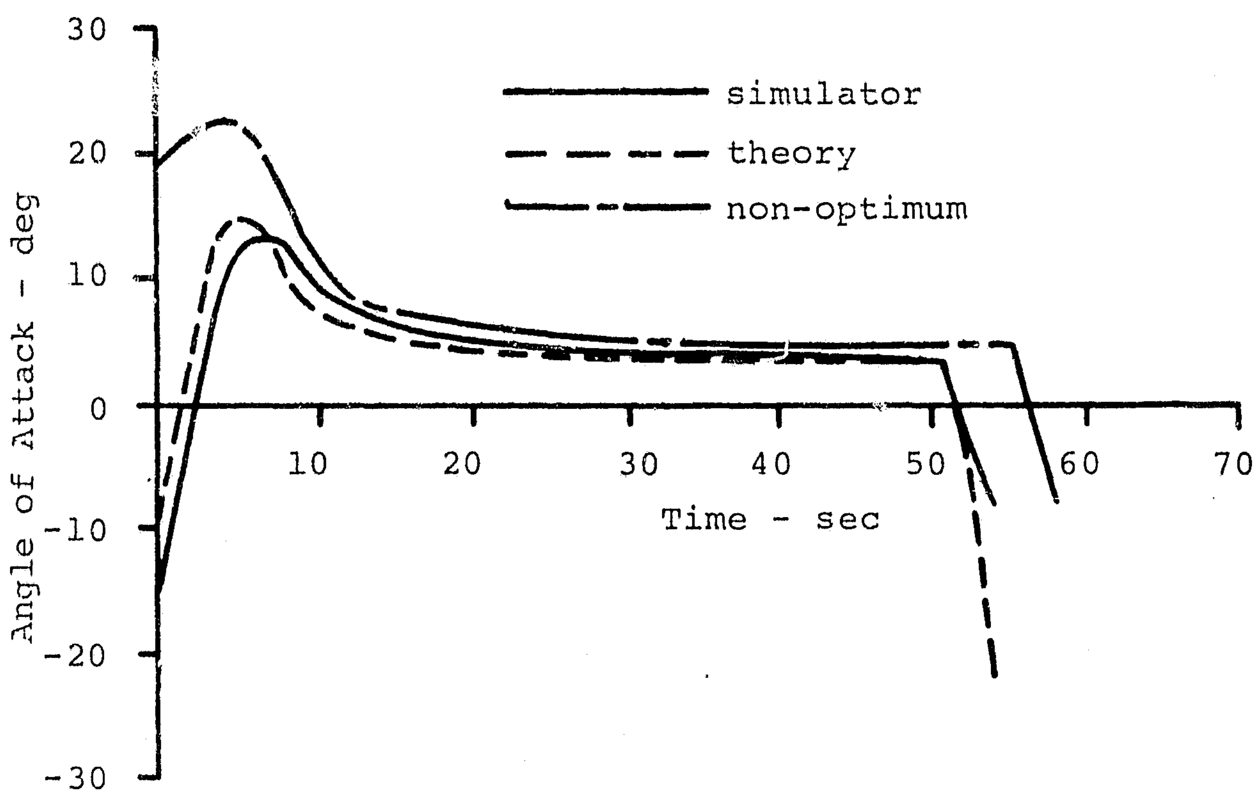

Fig. 8 Angle of Attack for Unconstrained Takeoff

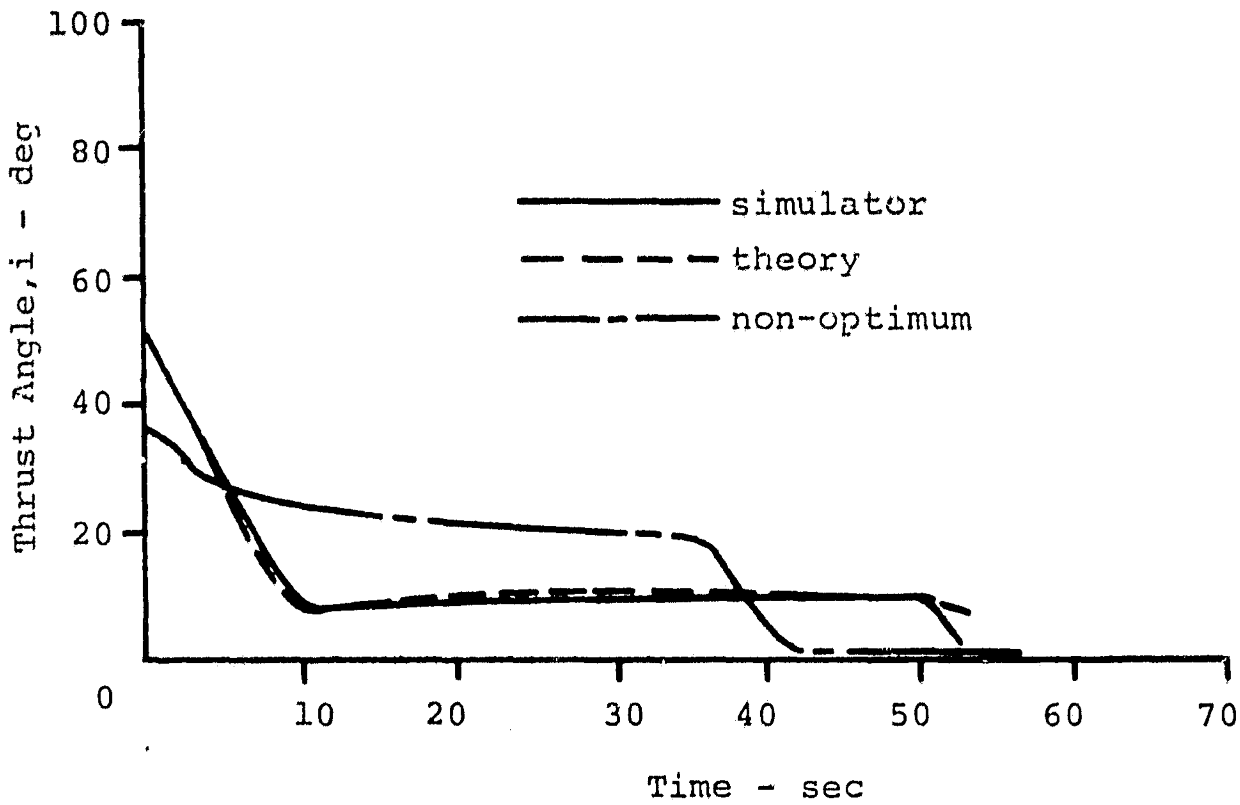

Fig. 9 Thrust Angle for Unconstrained Takeoff

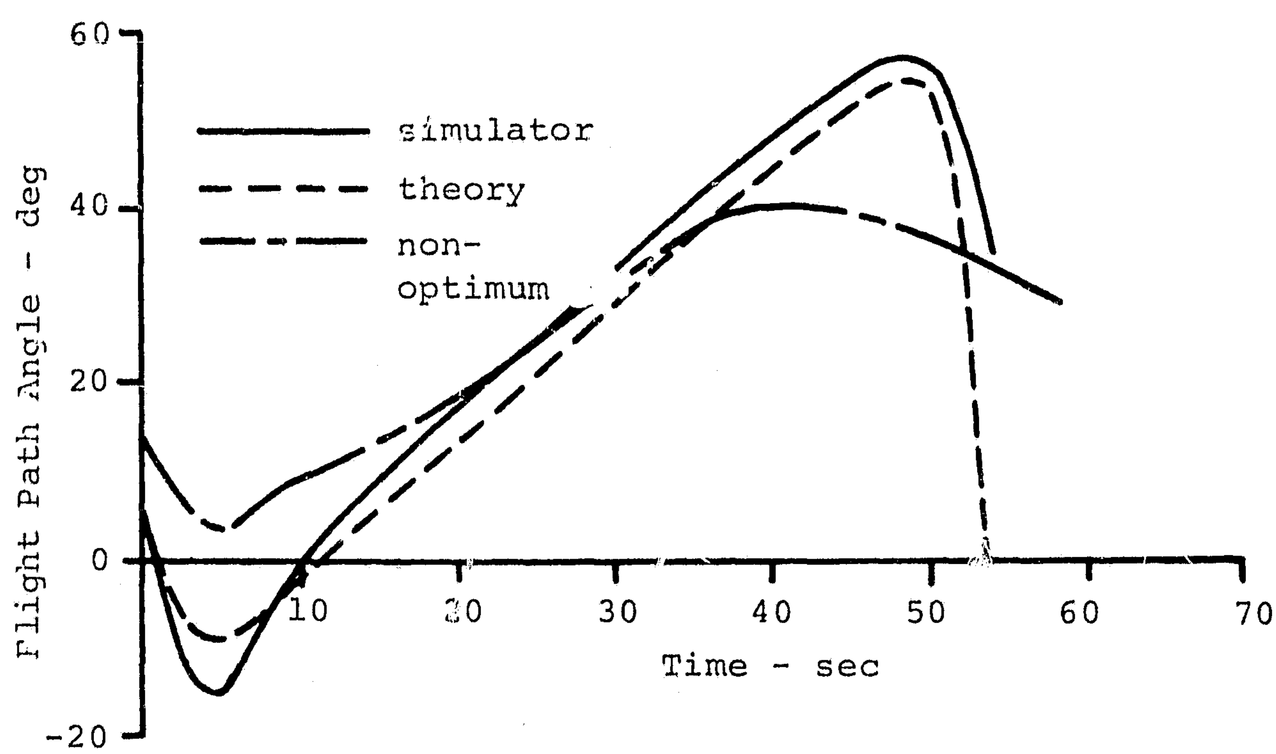

Fig. 10 Flight Path Angle for Unconstrained Takeoff

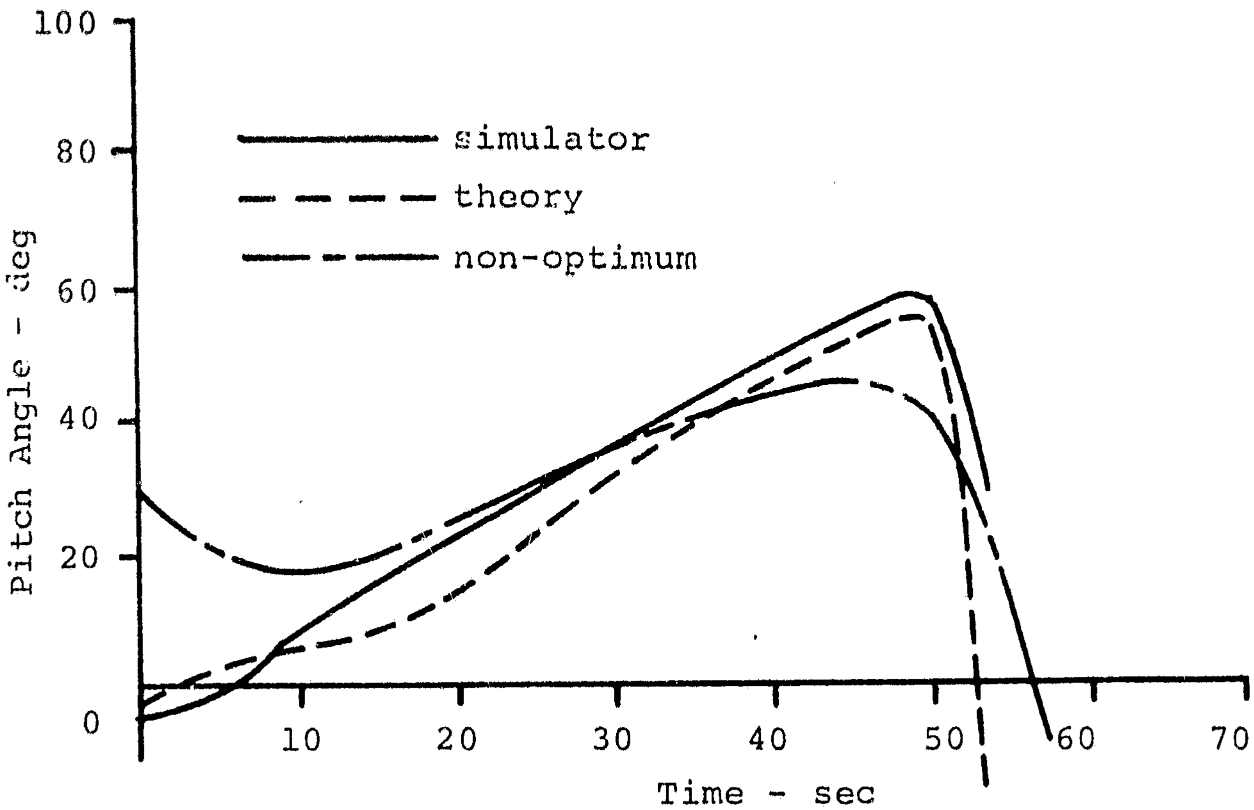

Fig. 11 Pitch Angle for Unconstrained Takeoff

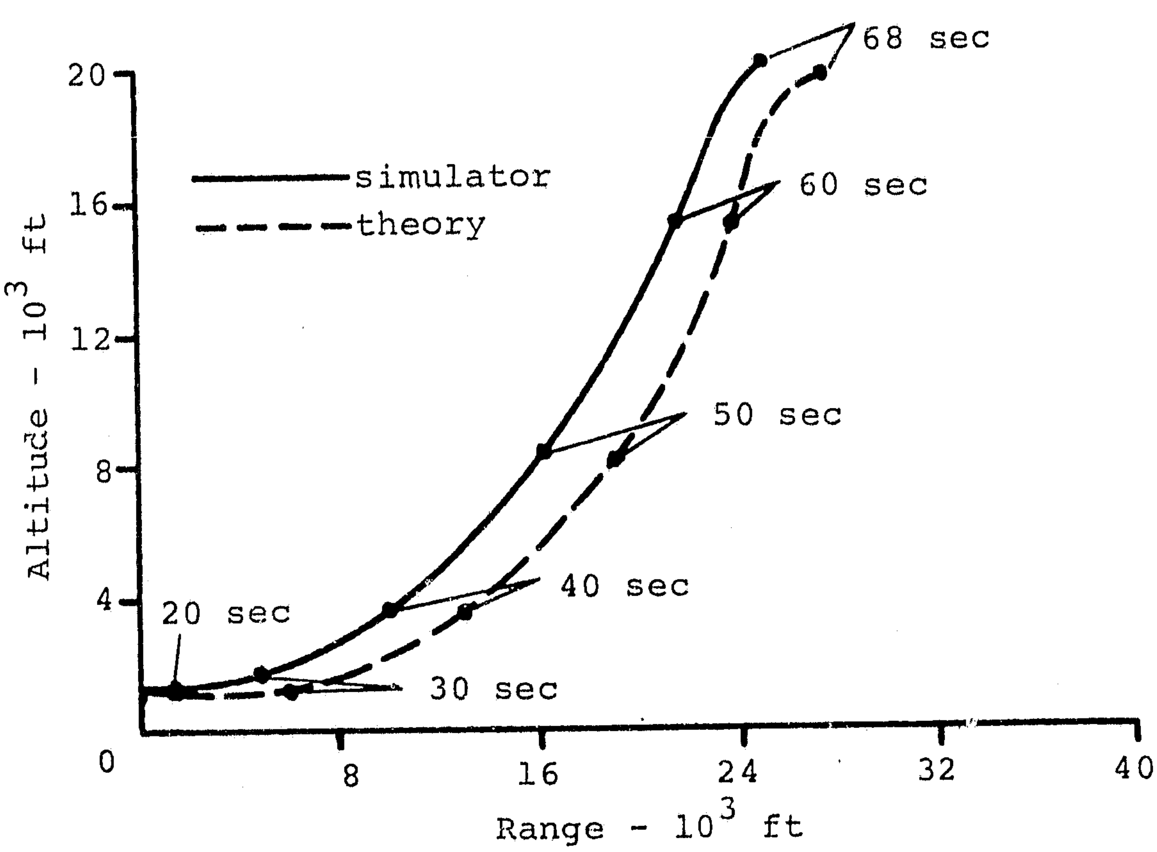

Fig. 12 Flight Path for Vertical Takeoff

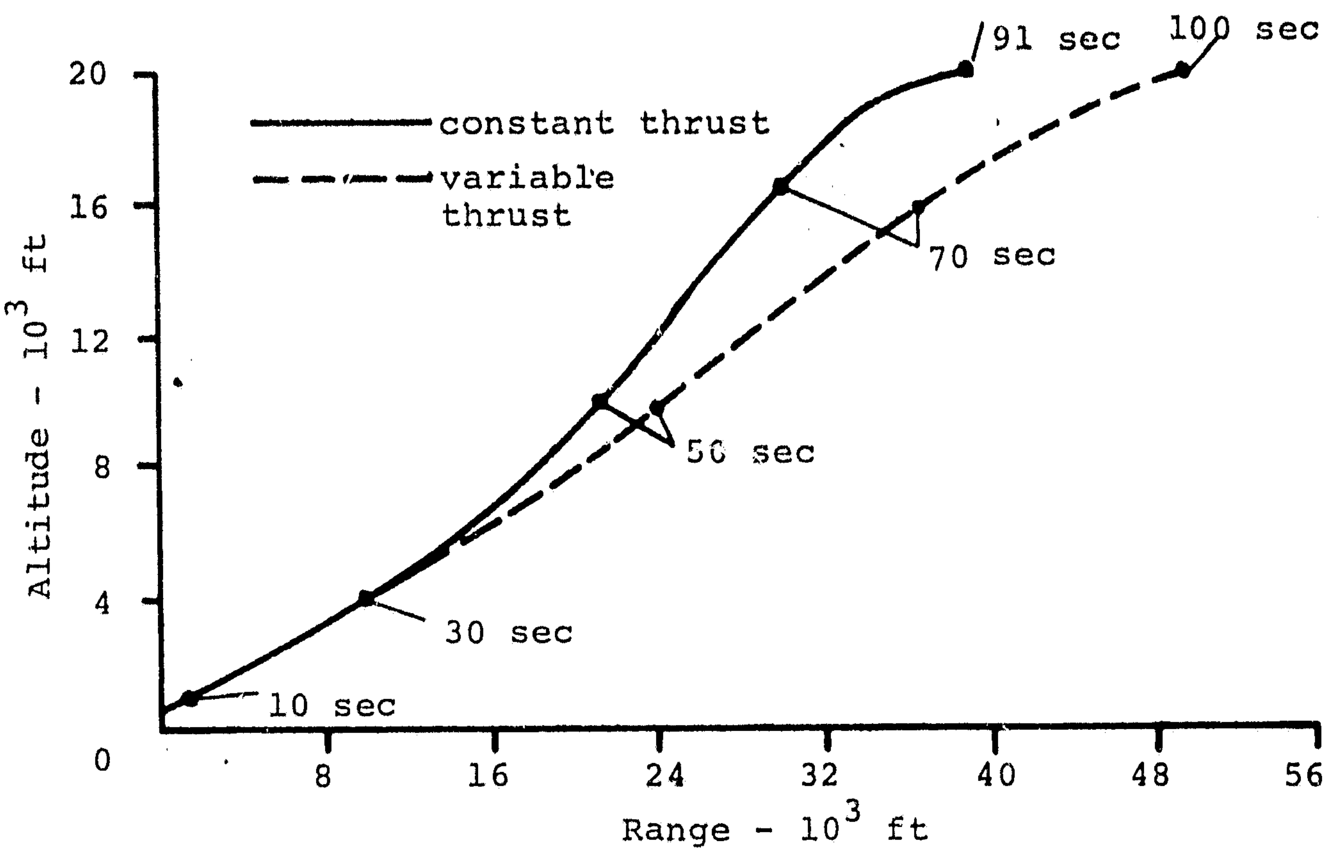

Fig. 13 Flight Path for Constrained Takeoff

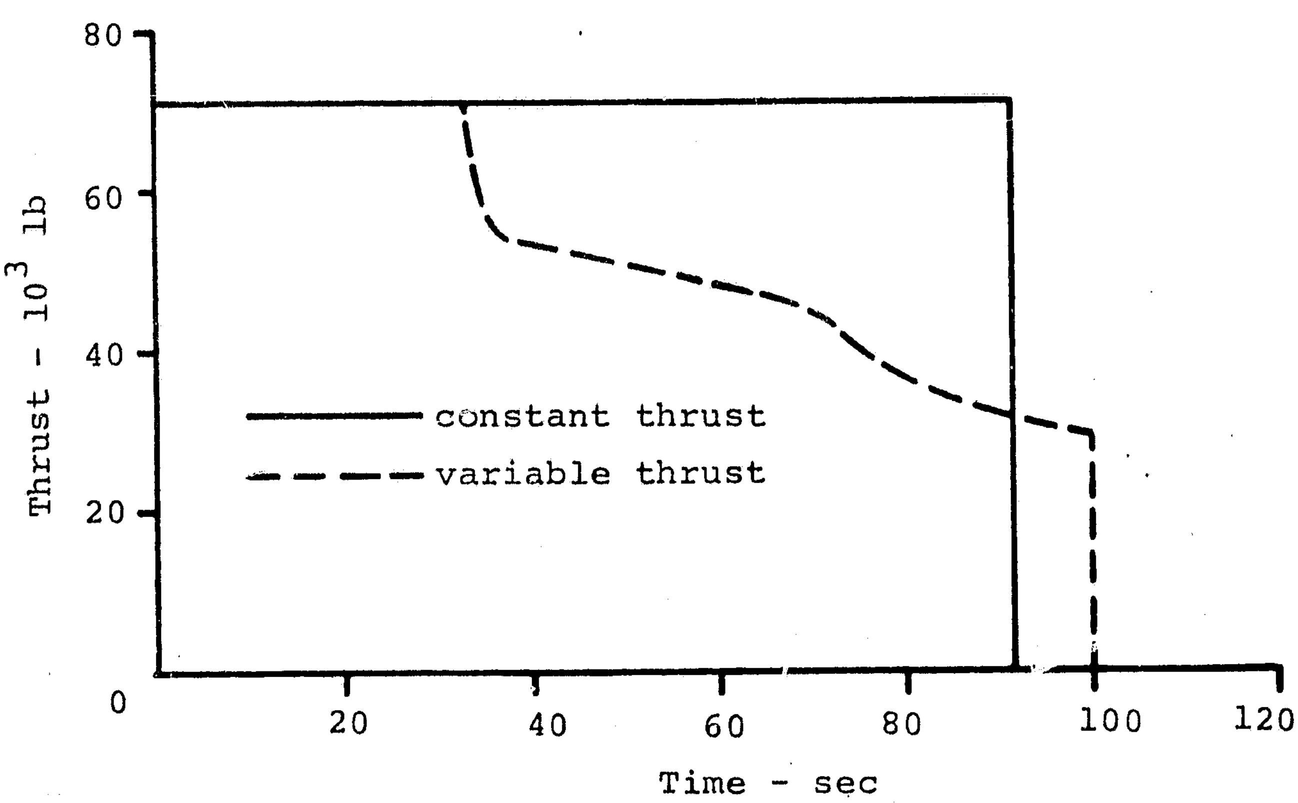

Fig. 14 Thrust for Constrained Takeoff

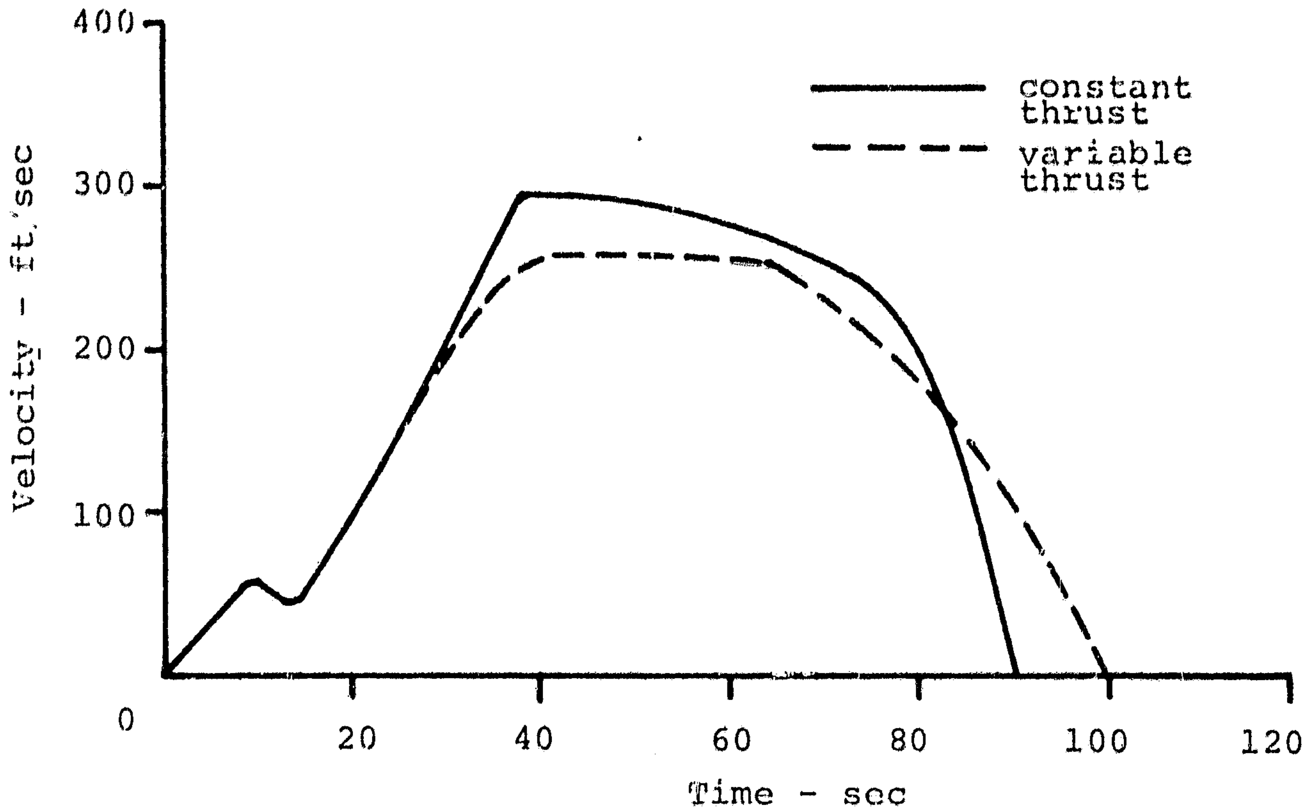

Fig. 15 Vertical Velocity for Constrained Takeoff

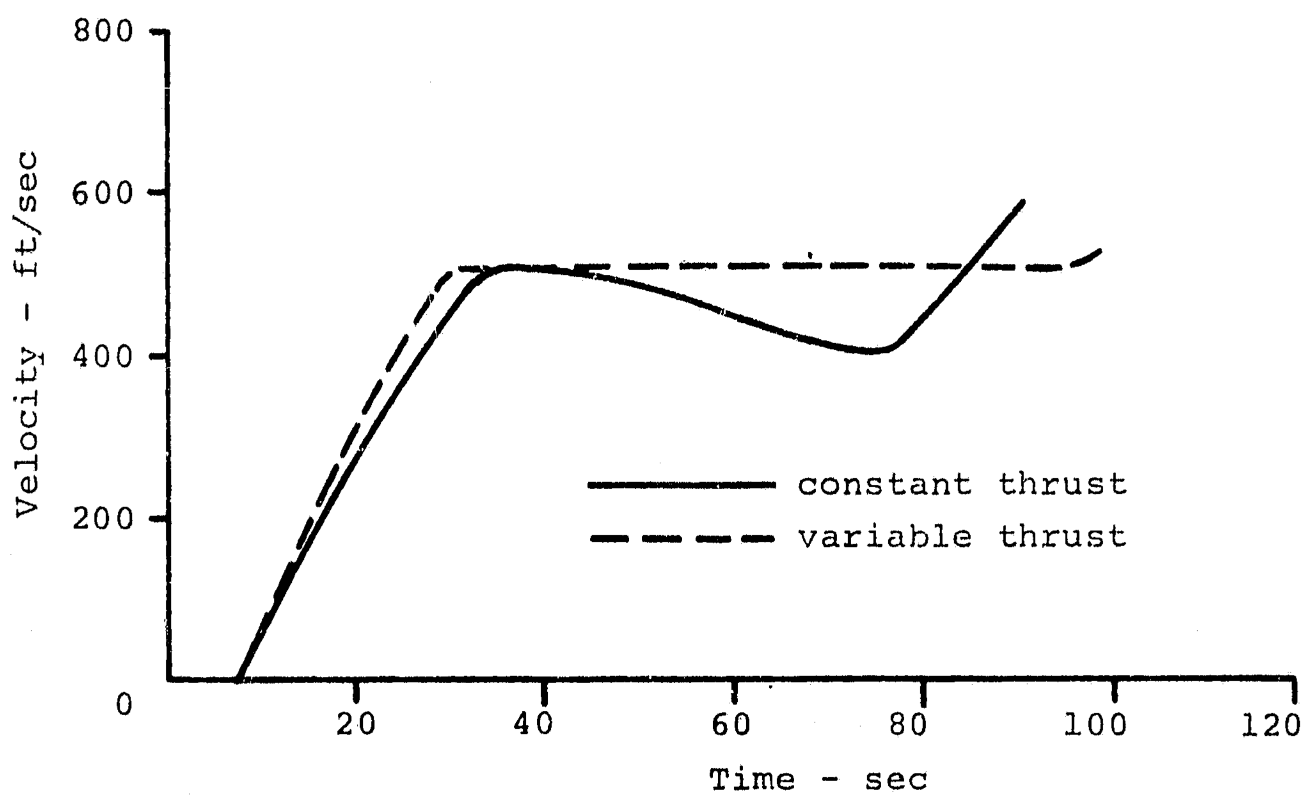

Fig. 16 Horizontal Velocity for Constrained Takeoff

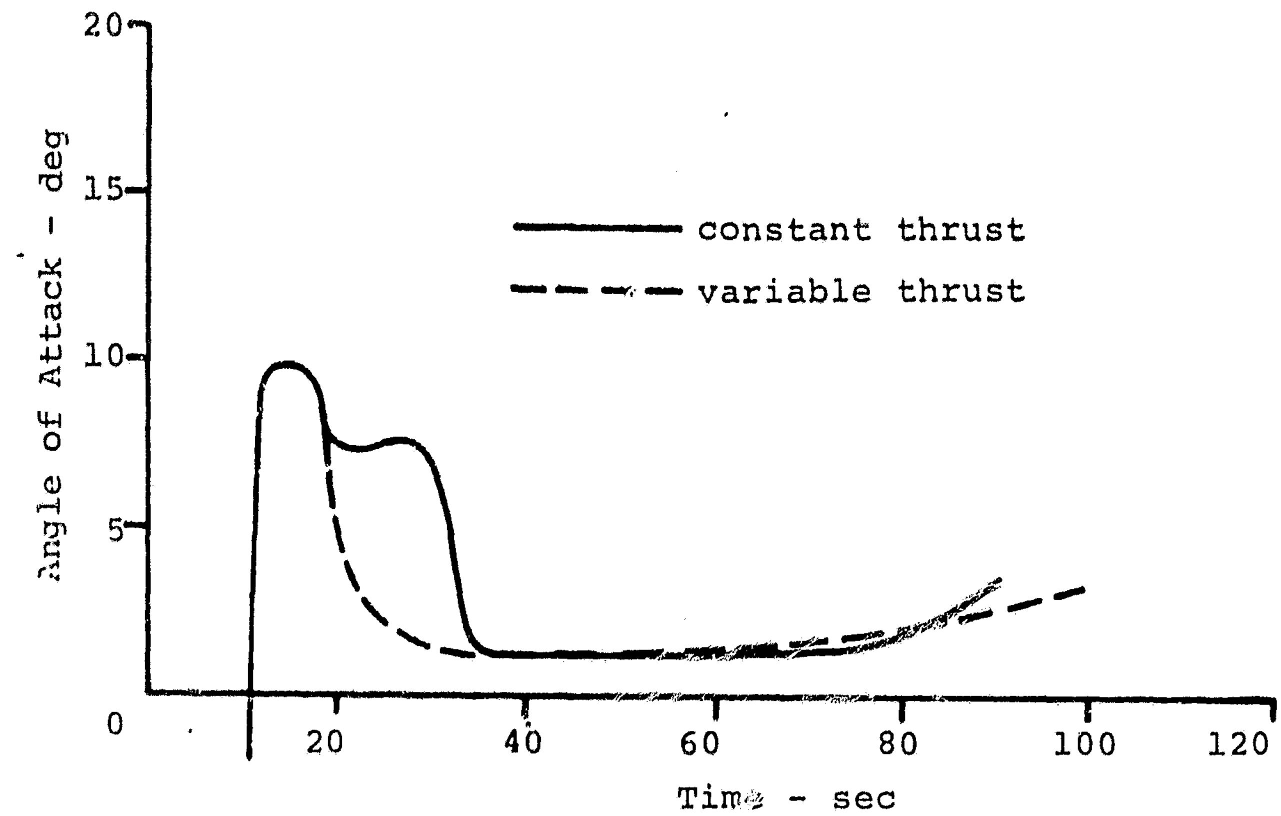

Fig. 17 Angle of Attack for Constrained Takeoff

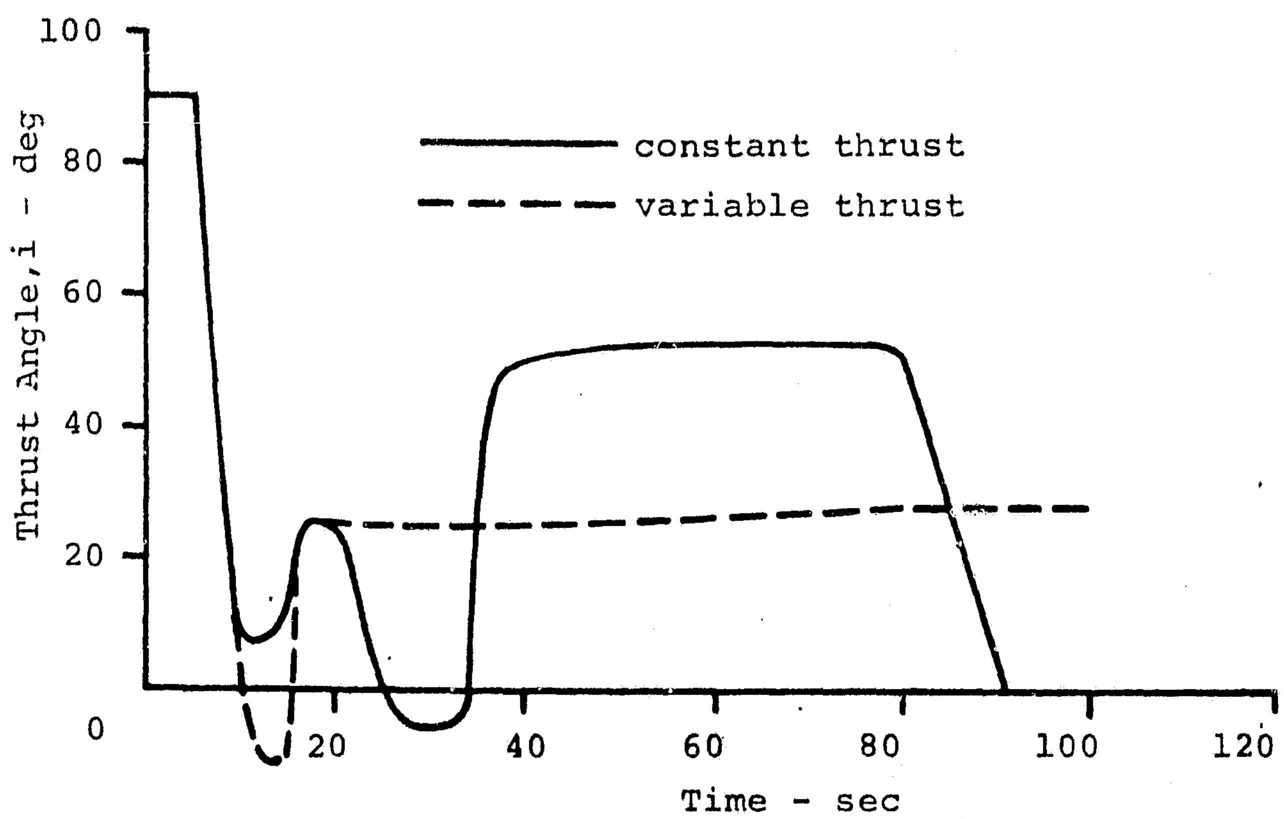

Fig. 18 Thrust Angle for Constrained Takeoff

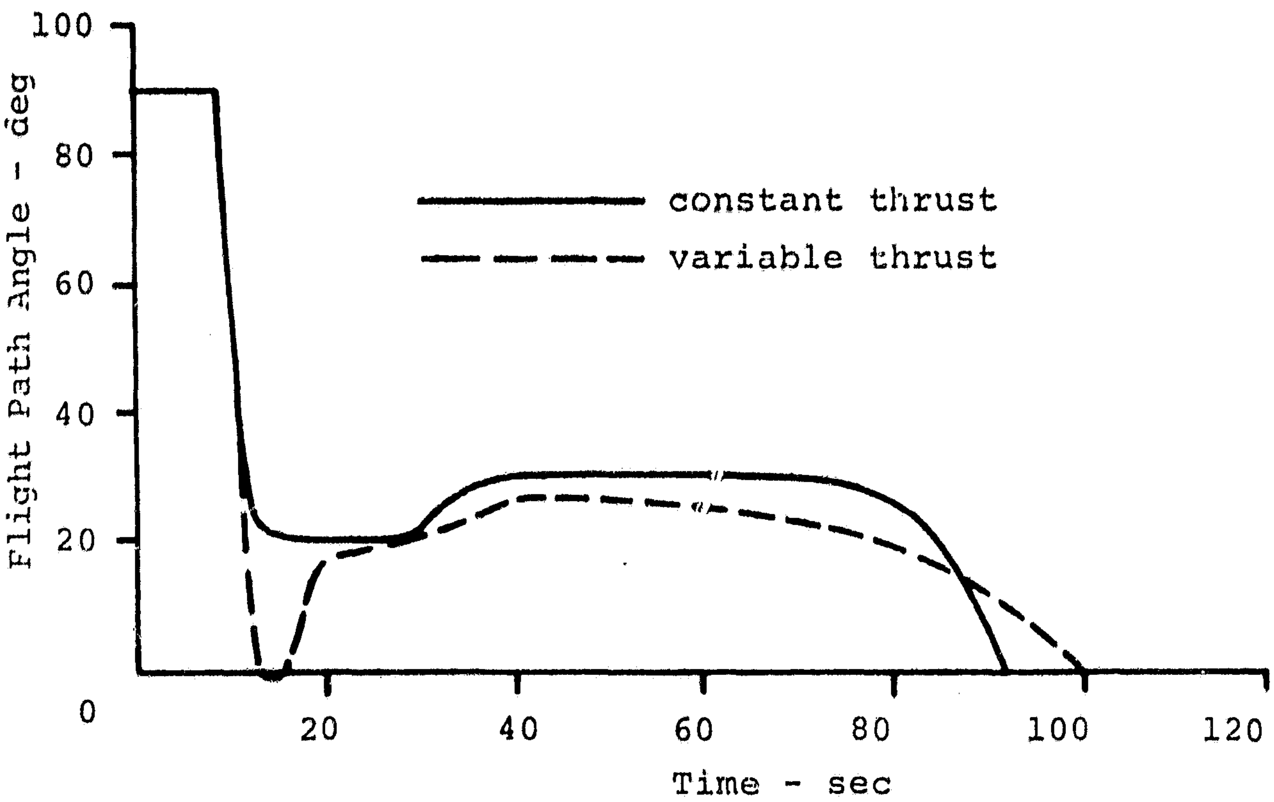

Fig . 19 Flight Path Angle for Constrained Takeoff

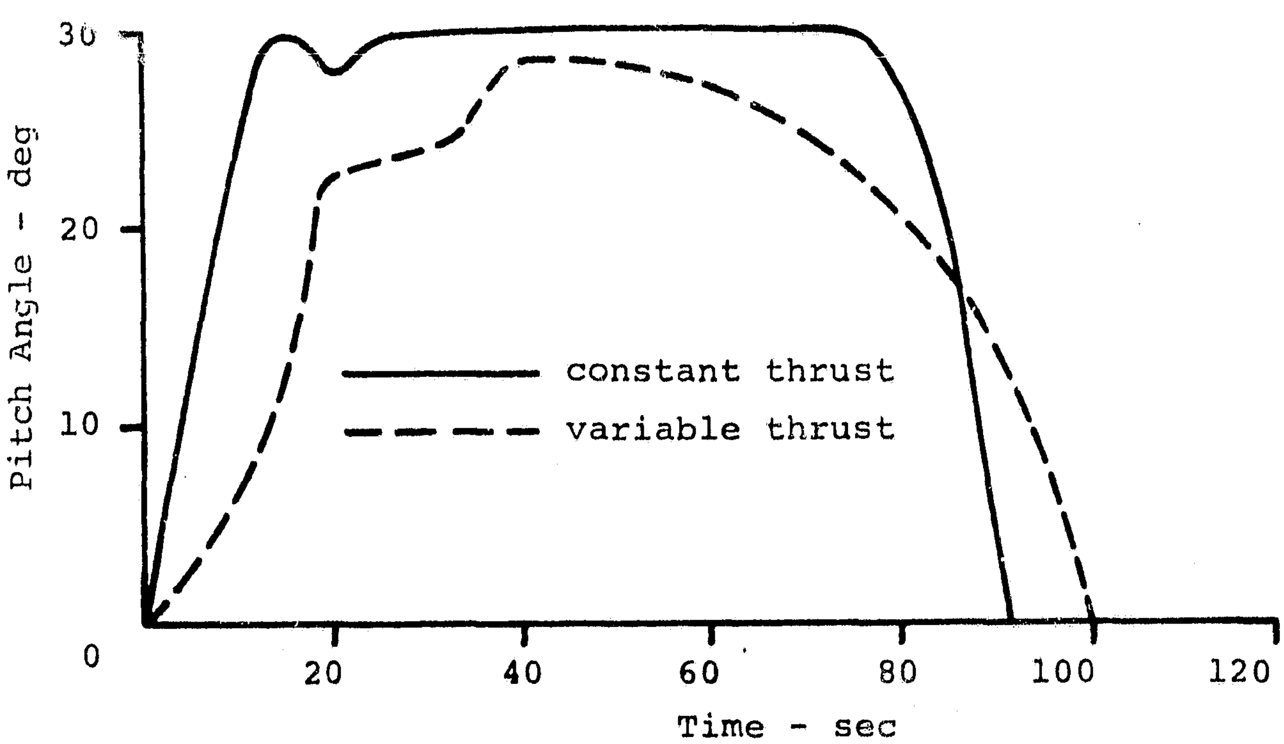

Fig. 20 Pitch Angle for Constrained Takeoff

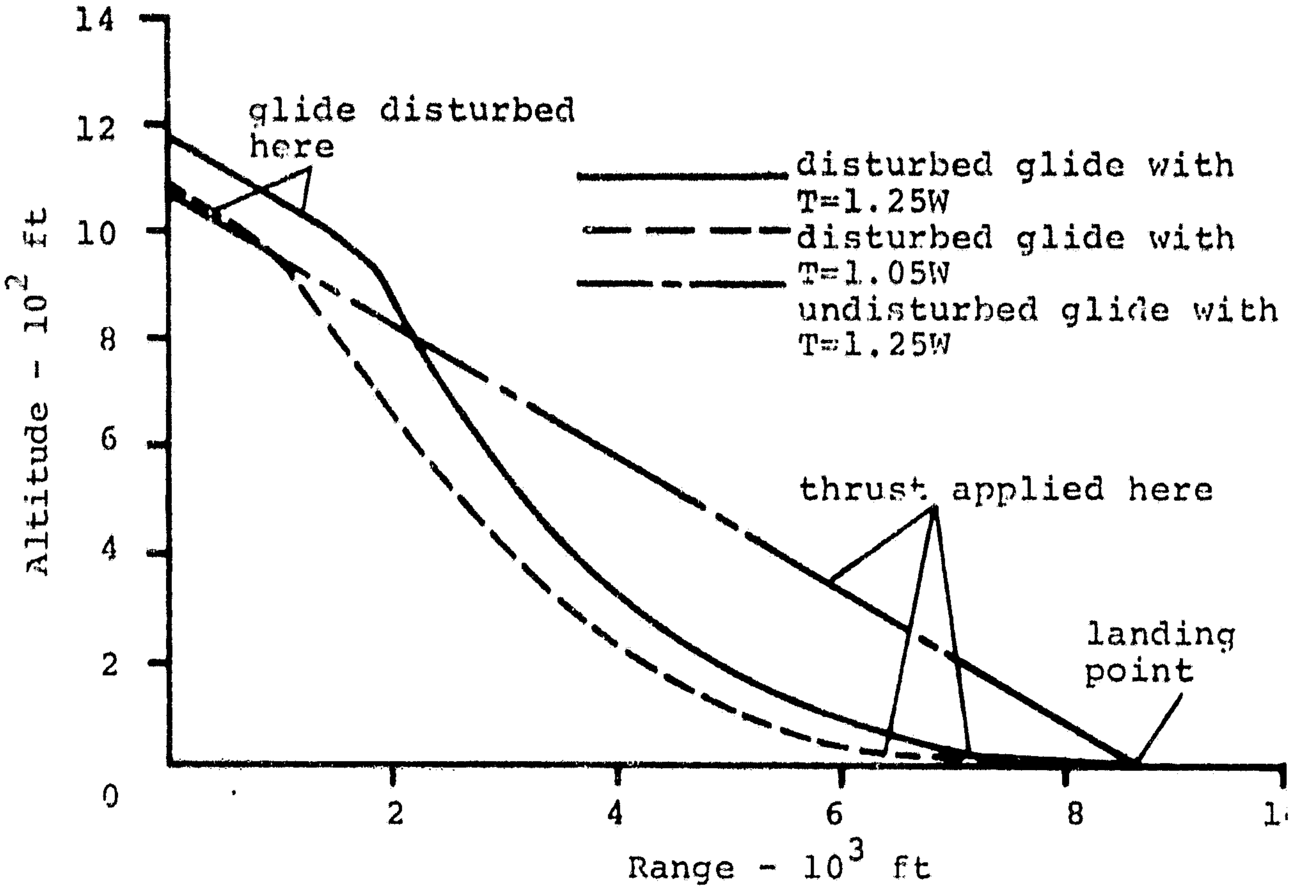

Fig. 21 Flight Path for Constant Thrust Landing

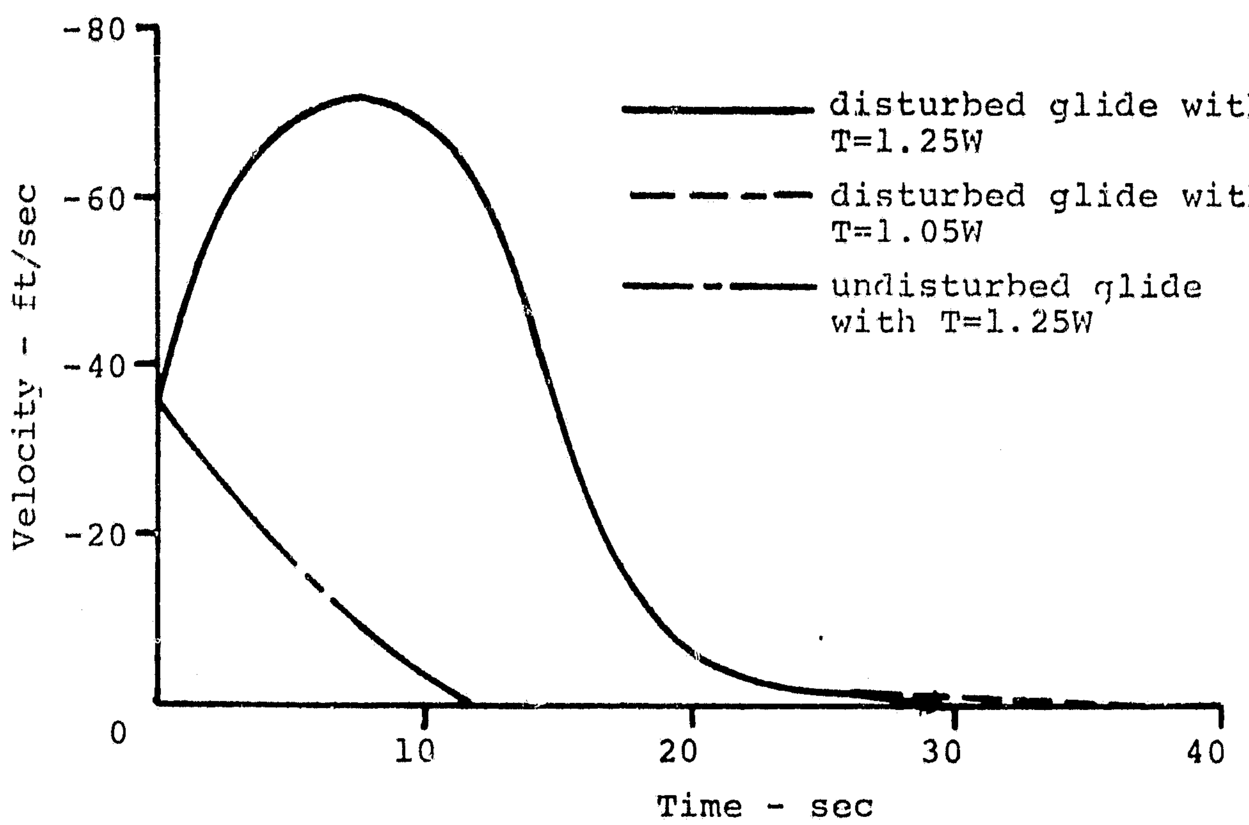

Fig. 22 Vertical Velocity for Constant Thrust Landing

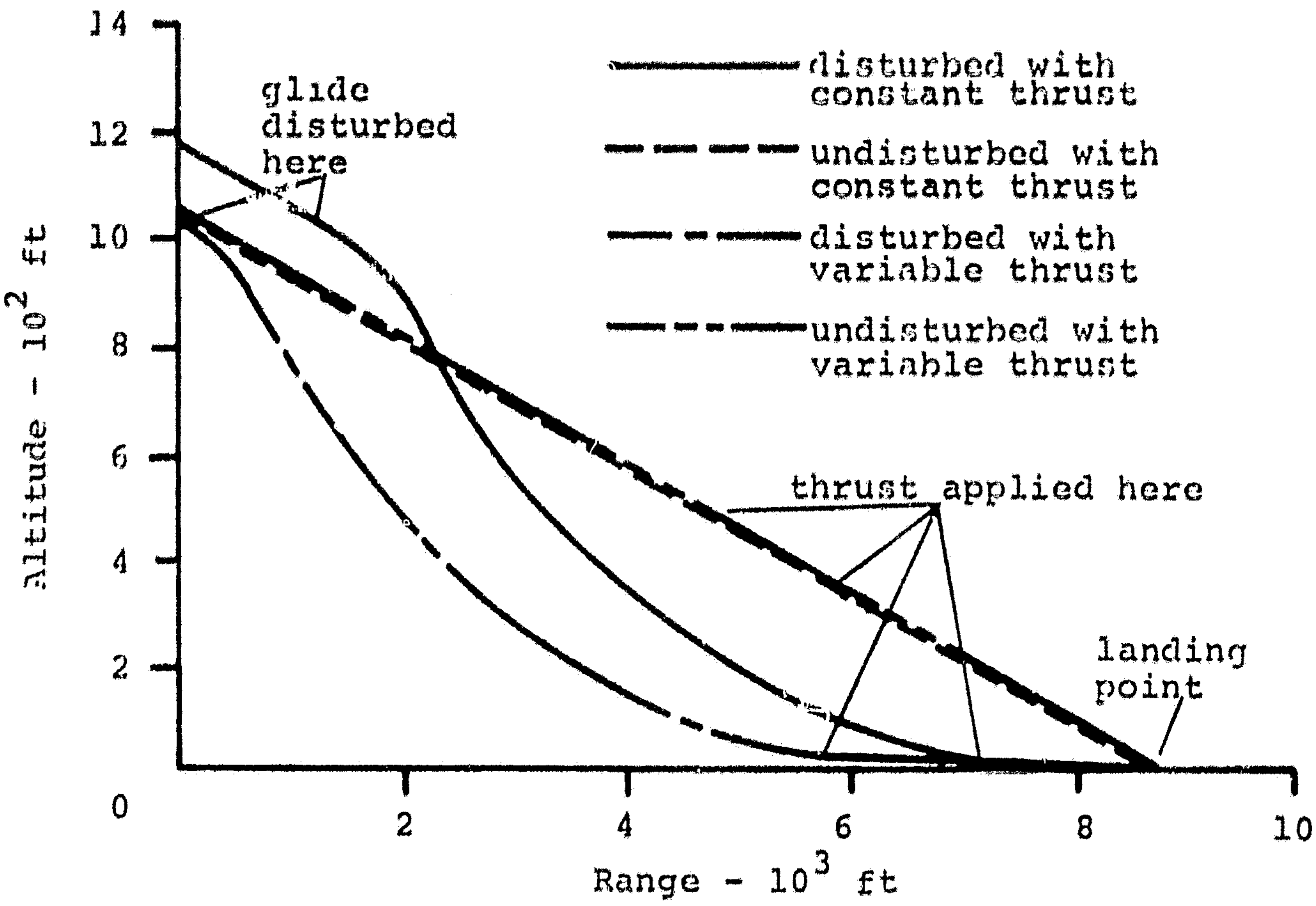

Fig. 23 Flight Path for Variable Thrust Landing

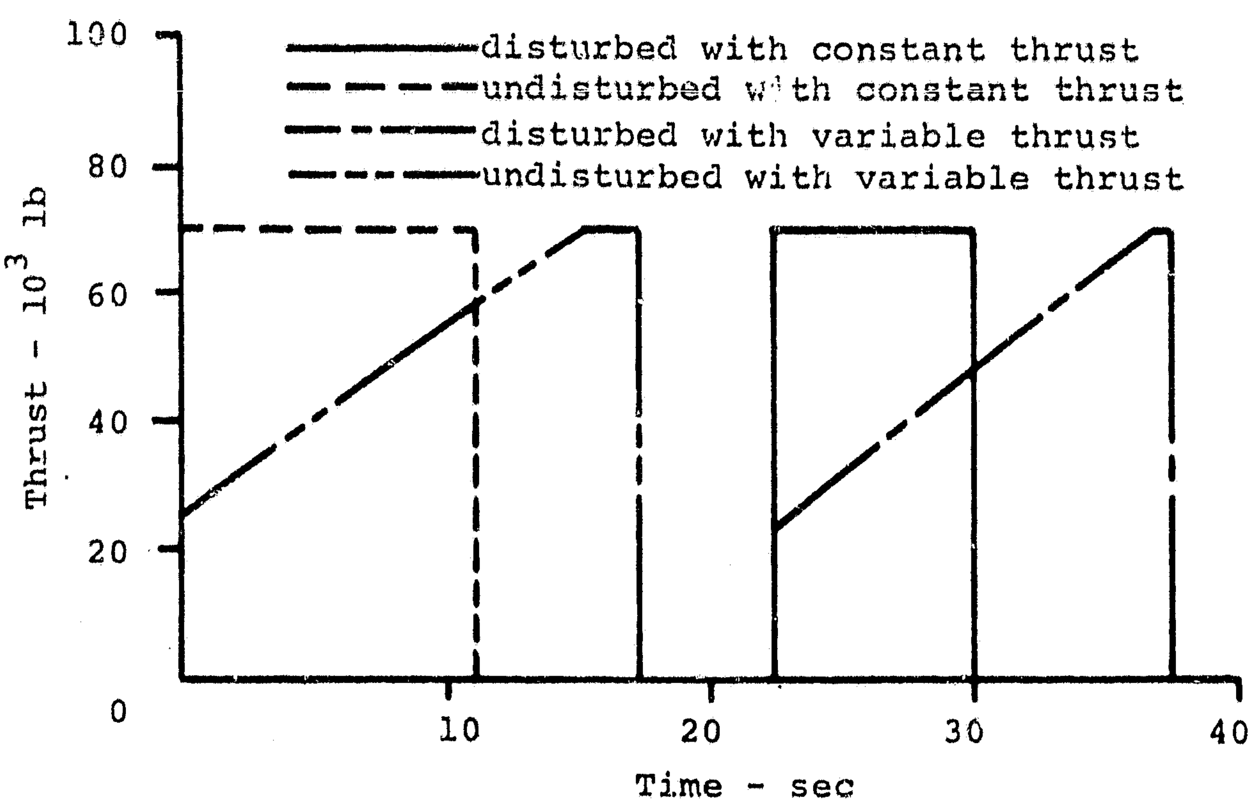

Fig. 24 Thrust for Variable Thrust Landing

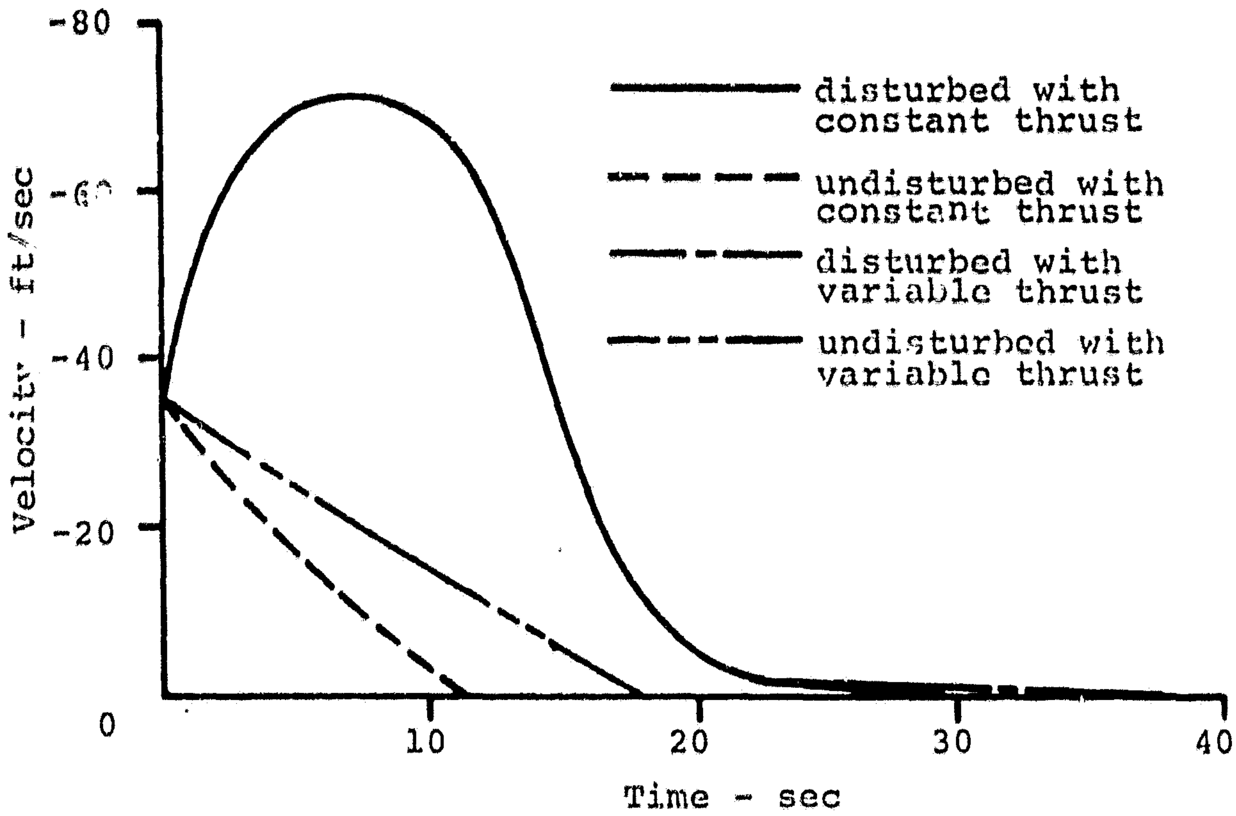

Fig. 25 Vertical Velocity for Variable Thrust Landing

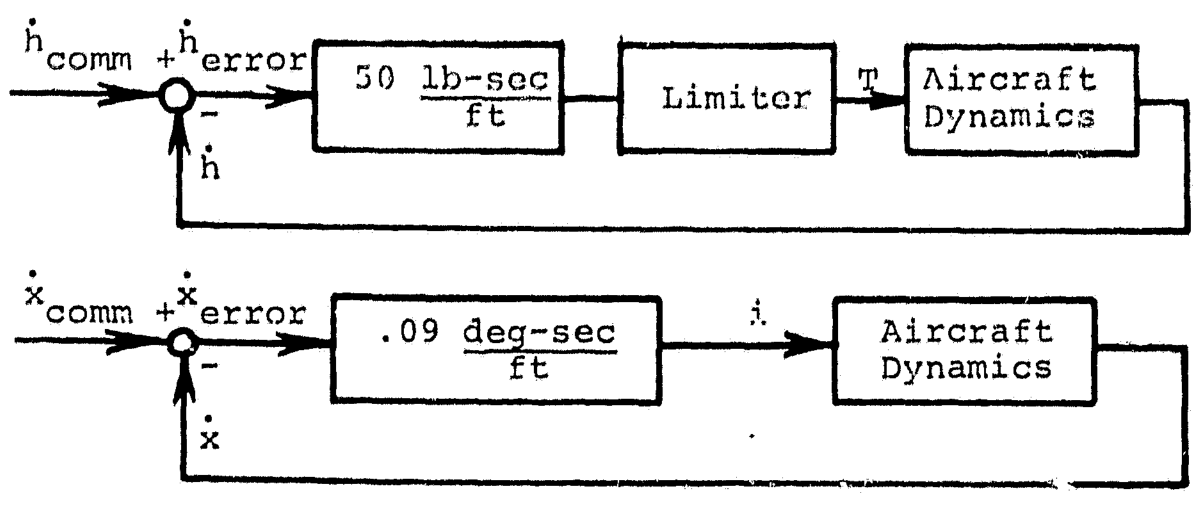

Fig. 26 Velocity Control System

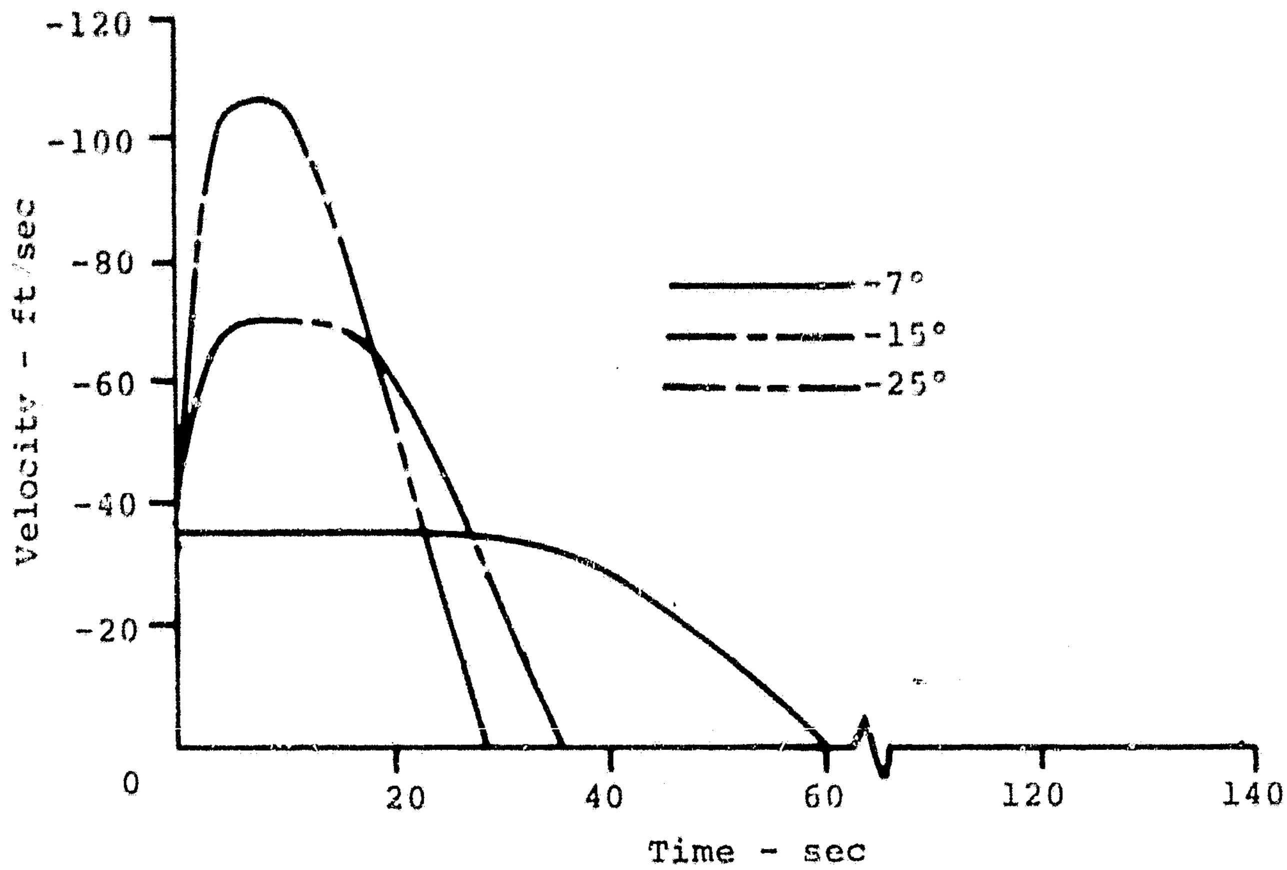

Fig. 27 Vertical Velocity for Constant Flight Pat. Angle Landing