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ANALYTICAL INVESTIGATION OF TWO-DIMENSIONAL LOSS CHARACTERISTICS OF SUPERSONIC TURBINE STATOR BLADES

by Louis J. Goldman and Michael R. Vanco

Lewis Research Center
Cleveland, Ohio

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ABSTRACT

An analysis was conducted to determine the loss characteristics of two-dimensional, minimum length, supersonic nozzles with sharp-edged throats. Boundary layer characteristics were calculated and used to obtain the conditions downstream of the nozzles after the flow had mixed to a uniform state. Subsonic, sonic, and supersonic aftermixing axial Mach number solutions were obtained for this model. The loss characteristics were investigated for nozzles designed over an exit Mach number range of 1.5 to 5.0. The effect of nozzle flow angle, throat Reynolds number, and specific heat ratio on losses were studied.

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Page 7, line 3: Equation (1) should read

$$\eta = \left(\frac{V_2}{V_{2,id}}\right)^2$$

Page 7, line 4: The term $V_{2,1D}$ should be $V_{2,id}$.

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ANALYTICAL INVESTIGATION OF TWO-DIMENSIONAL LOSS CHARACTERISTICS OF SUPERSONIC TURBINE STATOR BLADES

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SUMMARY

An analysis was conducted to determine the loss characteristics of two-dimensional, minimum length, supersonic nozzles with sharp-edged throats. Boundary layer characteristics were calculated and used to obtain the conditions downstream of the nozzle after the flow had mixed to a uniform state. Subsonic, sonic, and supersonic aftermixing axial Mach number solutions were obtained when the free-stream axial Mach number at the nozzle exit (before mixing) was supersonic. The subsonic solution corresponds to mixing plus oblique shock losses, whereas the supersonic solution corresponds to shockless mixing. The sonic solution corresponds to the limiting conditions possible if the nozzles exhaust into a constant area passage.

The nozzle loss characteristics were studied as a function of nozzle flow angle, free-stream Mach number level, and throat Reynolds number. The effect of specific heat ratio was also investigated.

The results of the analysis indicated that higher nozzle efficiencies occurred for the supersonic solution than for the subsonic solution, because of the absence of shock losses in the supersonic solution. Intermediate efficiencies were obtained for the sonic solution. For the supersonic solution, it was found that the aftermixing flow angle deflects towards the axial direction resulting in further flow expansion on mixing. This results in higher aftermixing Mach numbers and total-to-static pressure ratios for the supersonic solution than for the subsonic solution. The nozzle Mach number level has little effect on the nozzle efficiency for the supersonic solution. However, for the subsonic solution, higher Mach numbers result in higher oblique shock losses, and therefore lower efficiencies. Increasing either the throat Reynolds number or specific heat ratio results in an increase in nozzle efficiency.

INTRODUCTION

Supersonic turbines have potential application in turbopump and open-cycle auxiliary power systems (ref. 1) where high-energy fluids are used and high pressure ratios are available. This has resulted in a need for design procedures applicable to this type of turbine. Some experimental data on the overall performance of supersonic turbines has been reported in references 2 to 4. Unfortunately this data does not include sufficient information to assess the individual performance of the stator and rotor.

Supersonic stators and rotors can both be designed by the method of characteristics as applied to the isentropic flow of a perfect gas. Computer programs for the design of two-dimensional supersonic nozzles and rotor blades have been reported by Vanco and Goldman (ref. 5) and Goldman and Scullin (ref. 6), respectively. The design of blading by these procedures must then be supplemented by a knowledge of the loss characteristics of the nozzle and rotor.

The purpose of this report is to study analytically the loss characteristics of minimum length supersonic nozzles with sharp-edged throats under conditions applicable to auxiliary space power systems. This type of nozzle produces uniform parallel flow at the exit. To obtain a theoretical estimate of the losses, the following calculations are required: (1) isentropic design of the minimum length supersonic nozzle, (2) calculation of the boundary layer characteristics (momentum and displacement thicknesses) for the nozzle, and (3) determination of the losses due to mixing downstream of the nozzle. The isentropic nozzle design is obtained from the computer program of reference 5. The boundary layer parameters are calculated by the method developed by Cohen and Reshotko (ref. 7). Finally, the aftermixing losses are found by the procedure given by Stewart (ref. 8) for turbomachine blades.

The loss characteristics of two-dimensional minimum length supersonic nozzles with sharp-edged throats were investigated over an ideal free-stream Mach number range of 1.5 to 5.0. The effects of nozzle flow angle and throat Reynolds number are studied over this Mach number range. The effect of specific heat ratio is also included.

SYMBOLS

M Mach number

p pressure, psia (N/m^2)

Re Reynolds number, $Re = yV/\nu$

u tangential direction

V velocity, ft/sec (m/sec)

x axial direction

y width, ft (m)

α nozzle flow angle measured from axial direction, deg

γ ratio of specific heats

η nozzle efficiency

ν kinematic viscosity, ft^2/sec (m^2/sec)

Subscripts:

fs free-stream

id ideal

t throat

0 station upstream of nozzle (stagnation conditions)

1 station at nozzle exit

2 station downstream of nozzle

METHOD OF ANALYSIS

The calculation of the loss characteristics of two-dimensional supersonic nozzles that produce uniform parallel flow in the minimum distance is described herein. This type of nozzle has a sharp-edged throat. The nozzle loss characteristics were obtained by first designing a series of loss-free nozzles for given exit Mach numbers, nozzle flow angles, throat Reynolds numbers, and specific heat ratios. The boundary layer characteristics (momentum and displacement thicknesses) for the ideal nozzles were then obtained and the nozzle profile corrected to include the effect of the displacement thickness. Finally, the Mach number, flow angle, pressure ratio, and kinetic energy loss were calculated assuming the flow mixes to uniform conditions downstream of the nozzles.

The losses obtained in this analysis are for two-dimensional blade rows. In an actual design, three-dimensional effects would have to be included. A method of estimating three-dimensional losses from two-dimensional losses has been described by Stewart, et al. (ref. 9).

Nozzle Description and Design

As seen in figure 1, a typical nozzle consists of three sections: (1) a converging section, (2) a diverging section, and (3) a straight section on the suction surface. The

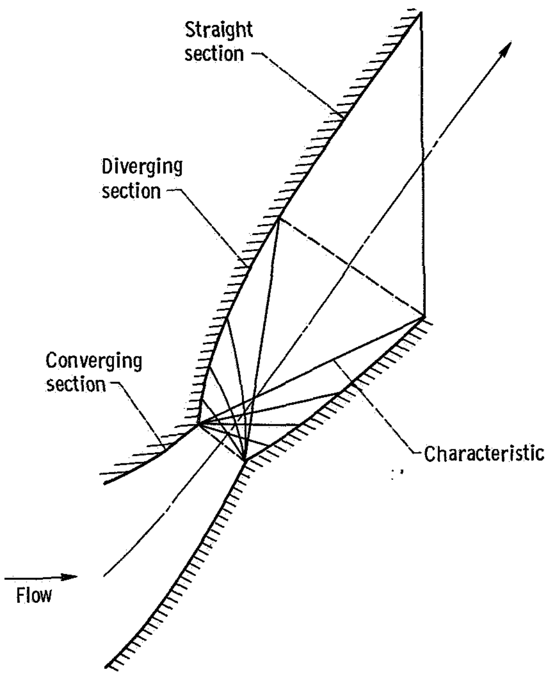


Figure 1. - Supersonic nozzle with sharp-edged throat.

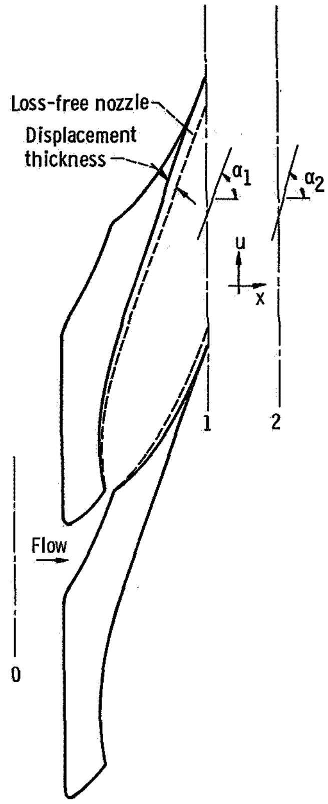


Figure 2. - Supersonic nozzle design.

converging section produces the flow turning with little losses. The diverging section accelerates the flow to the desired free-stream Mach number at the exit. As shown in figure 1, this section is designed by the method of characteristics. The computer program described in reference 5 was used for this purpose. The straight section on the suction surface completes the nozzle profile and its length is determined by the required nozzle exit flow angle.

For the low flow rate open-cycle auxiliary power system of current interest, laminar flow would occur. The boundary layer characteristics for the ideal nozzles were obtained by use of the compressible laminar-boundary layer theory of Cohen and Reshotko (ref. 7). The final nozzle profile was obtained by adding the displacement thicknesses to the loss-free nozzle coordinates. Figure 2 shows a nozzle designed in this manner. The dashed line represents the loss-free nozzle profile. The displacement and momentum thicknesses at the nozzle exit (station 1, fig. 2) were used to calculate the conditions downstream of the nozzles.

Loss Characteristics

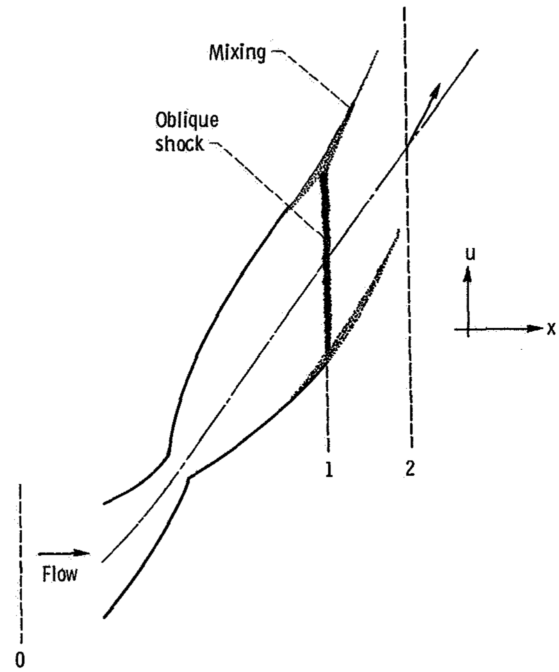
The calculation of the losses due to mixing downstream of turbomachine blade rows has been described by Stewart (ref. 8) in terms of boundary layer characteristics. In this loss model, the flow sufficiently downstream of the blade row is assumed to be uniform and parallel. Application of the continuity, momentum, and energy equations between stations 1 and 2 (fig. 2) results in the determination of the aftermixing velocity, flow angle, pressure ratio, and kinetic energy loss. For supersonic free-stream velocities two cases have to be considered: (1) supersonic free-stream axial Mach numbers and (2) subsonic free-stream axial Mach numbers.

Supersonic free-stream axial Mach numbers. - For supersonic free-stream axial Mach numbers at the nozzle exit (station 1, fig. 2), two aftermixing solutions are possible for this model. One solution results in supersonic aftermixing axial Mach numbers and is hereafter referred to as the supersonic solution. The mixing losses are the result of the nonuniformity of the flow at the blade exit, and for zero boundary layer this solution corresponds to straight through flow. No shock losses occur for this solution. The second solution results in subsonic aftermixing axial Mach numbers and will be referred to as the subsonic solution. For zero boundary layer, this solution corresponds to an oblique shock wave occurring at the nozzle exit plane (station 1). Therefore, with a boundary layer, the losses are a combination of mixing and shock losses. A schematic representation of these solutions is shown in figure 3.

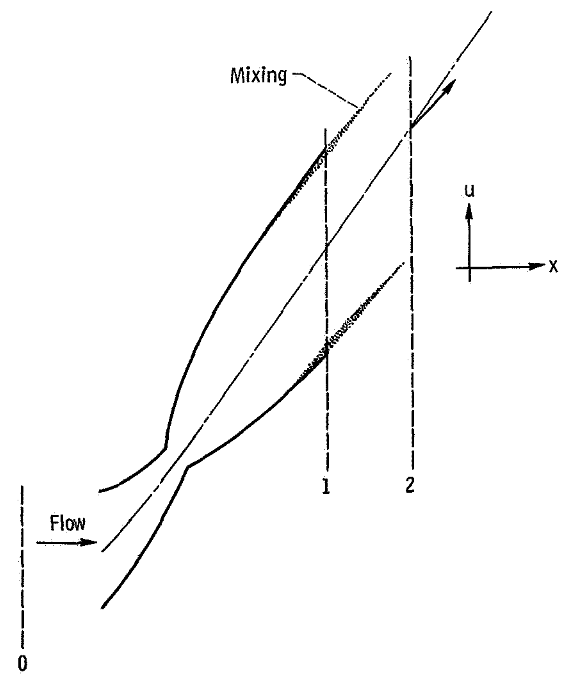
The supersonic solution might not be physically possible in certain situations. For example, if the nozzles exhausts into a constant area annulus, the annulus could become choked as the exhaust pressure is decreased. This is a consequence of the system of shock waves passing through the nozzles during start-up. For these conditions the maximum aftermixing axial Mach number would be one. Supersonic axial Mach numbers could be obtained if the annulus area increases. The solution where the aftermixing axial Mach number is sonic is also presented.

An approximate method was used to obtain the sonic solution. It was first observed that the subsonic solution can also be obtained by first mixing the nonuniform flow to uniform conditions (supersonic solution) and then having the uniform flow undergo an oblique shock of equal strength to a shock occurring across the exit plane. If the strength of oblique shock is decreased it is possible to estimate flow conditions when the aftermixing axial Mach number is one. A schematic of the sonic solution is shown in figure 4. In this analysis all three solutions (subsonic, sonic, and supersonic aftermixing axial Mach numbers) are presented.

Subsonic free-stream axial Mach numbers. - For subsonic axial Mach numbers at the nozzle exit, only one aftermixing solution is possible for this loss model. The aftermixing axial Mach number is subsonic for this solution and is again referred to as the subsonic solution. Only mixing losses occur for this situation.



Subsonic solution



Supersonic solution

Figure 3. - Subsonic and supersonic solutions.

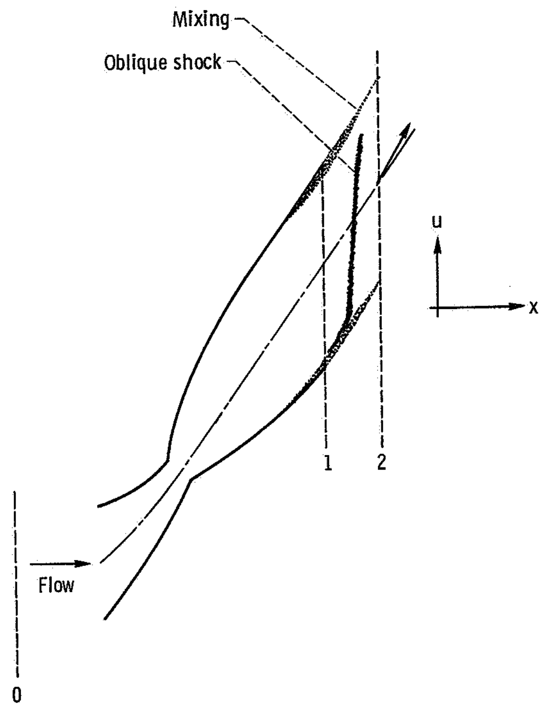


Figure 4. - Sonic solution.

The loss characteristics for the nozzles are indicated by the aftermixing Mach number, flow angle, pressure ratio, and efficiency. The nozzle efficiency η is defined as

$$\eta = \frac{V_2}{V_{2,1D}} \tag{1}$$

where V_2 is the aftermixing velocity, and $V_{2,1D}$ is the ideal velocity at station 2 obtained by isentropic expansion to the aftermixing static pressure. The effect of working fluid on losses was obtained by varying the specific heat ratio and the throat Reynolds number. The throat Reynolds number Re_t is defined as

$$Re_t = \frac{y_t V_t}{\nu_o} \tag{2}$$

where y_t is the throat width, V_t is the throat velocity, and ν_o is stagnation kinematic viscosity. The effect of nozzle flow angle on losses was also obtained. The nozzle flow angle was varied over a range that includes both subsonic and supersonic free-stream axial Mach numbers at the nozzle exit plane (station 1).

RESULTS AND DISCUSSION

The results of the study of the loss characteristics of supersonic nozzles are presented in this section. Although, the loss characteristics were investigated for nozzles designed over a free-stream Mach number range of 1.5 to 5.0, similar trends were found for the different Mach number levels. Therefore, only the primary results for a single Mach number level of 2.5 are discussed. First, the nozzle efficiency and nozzle aftermixing conditions are discussed. Then, the effects of Reynolds number and specific heat ratio on nozzle efficiency are presented. Finally, the effect of Mach number level on nozzle efficiency is discussed. The results for all the Mach number levels are included in the appendix since they may be used for preliminary design purposes.

Nozzle Efficiency and Aftermixing Conditions at Mach Number of 2.5

Nozzle efficiency. - The nozzle efficiency η is shown in figure 5. The throat Reynolds number Re_t and specific ratio γ are kept constant at values of 10 000 and 1.4, respectively. As discussed previously, three solutions corresponding to subsonic, sonic, and supersonic aftermixing axial Mach numbers are shown in the figure.

The efficiency for the supersonic solution is higher than for the subsonic solution because of the absence of shock losses in the supersonic solution. Intermediate efficiencies occur for the sonic solution. It is interesting to note that for the subsonic solution the efficiency exhibits a maximum.

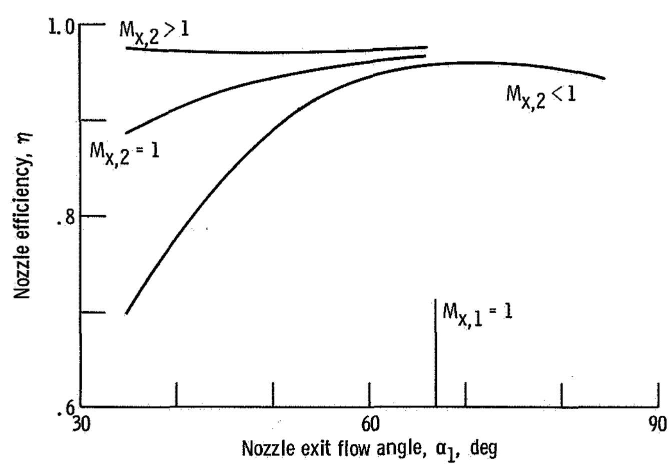
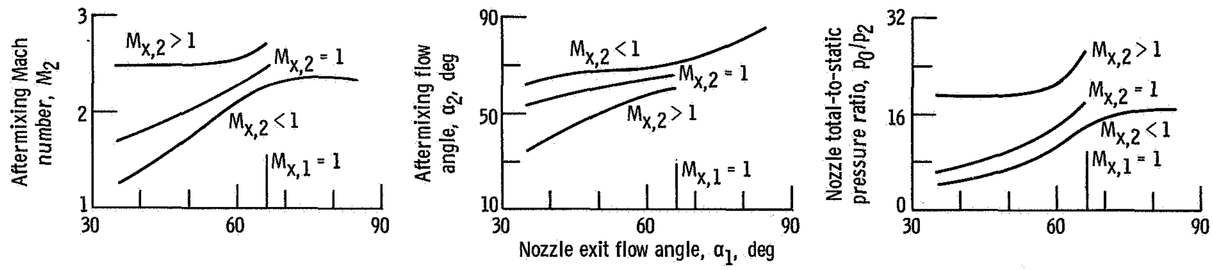


Figure 5. - Effect of nozzle exit flow angle on nozzle efficiency for nozzles designed for free-stream Mach number of 2.5, throat Reynolds number of 10 000, and specific heat reatio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} < 1$.)



(a) Aftermixing Mach number. (b) Aftermixing flow angle. (c) Nozzle total-to-static pressure ratio.

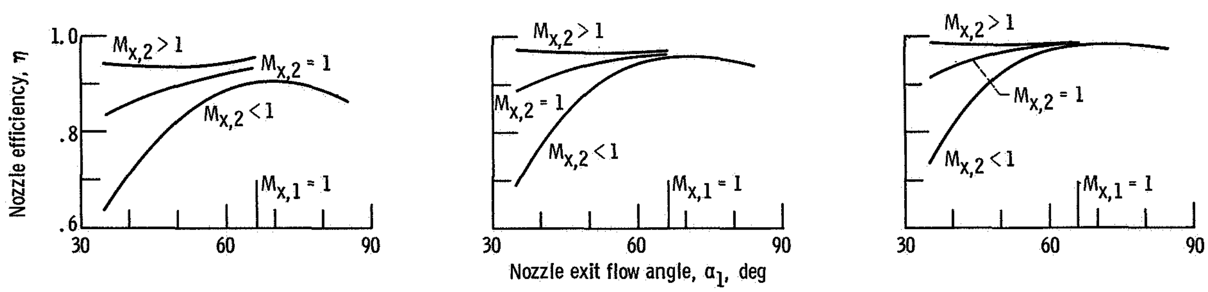
Figure 6. - Aftermixing conditions for nozzles designed for free-stream Mach number of 2.5, throat Reynolds number of 10 000, and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

Nozzle aftermixing conditions. - The aftermixing conditions are shown in figure 6. The aftermixing Mach number M_2 is shown in figure 6(a). The effect of nozzle flow angle on aftermixing Mach number is similar to that exhibited by the nozzle efficiency. For the supersonic solution, aftermixing Mach numbers equal to and greater than the free-stream Mach number, $M_{fs,1}$, occur. As will be seen subsequently, this is the result of further flow expansion occurring on mixing.

The aftermixing flow angle α_2 is shown in figure 6(b). For the subsonic solution the flow deflects away from the axial direction. The deflection becomes larger as the nozzle flow angle decreases because of the increased strength of the oblique shock that occurs for this solution. For the supersonic solution, the flow deflects toward the axial direction resulting in further flow expansion.

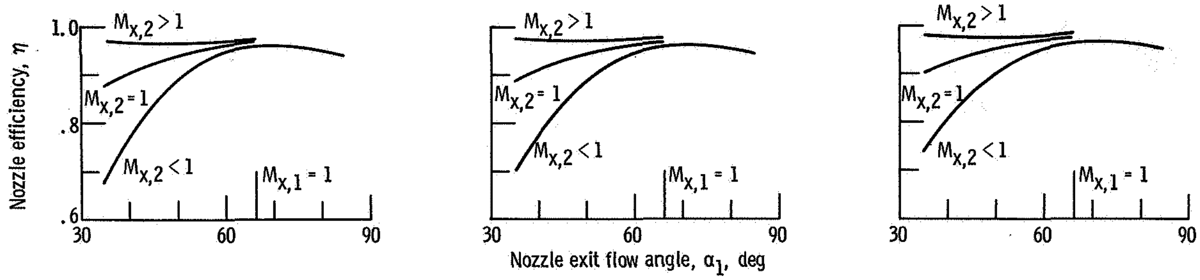
The nozzle total-to-static pressure ratio p_0/p_2 is shown in figure 6(c). The flow expansion occurring for the supersonic solution results in lower aftermixing static pressures p_2 than occur for the subsonic solution. Therefore, the total-to-static pressure ratio is higher for the supersonic solution.

Effect of Reynolds number on efficiency. - The effect of throat Reynolds number on nozzle efficiency is shown in figure 7. The free-stream Mach number and specific heat



(a) Throat Reynolds number, 2000. (b) Throat Reynolds number, 10 000. (c) Throat Reynolds number, 50 000.

Figure 7. - Effect of throat Reynolds number on nozzle efficiency of nozzles designed for free-stream Mach number of 2.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)



(a) Specific heat ratio, 1.3. (b) Specific heat ratio, 1.4. (c) Specific heat ratio, 1.66.

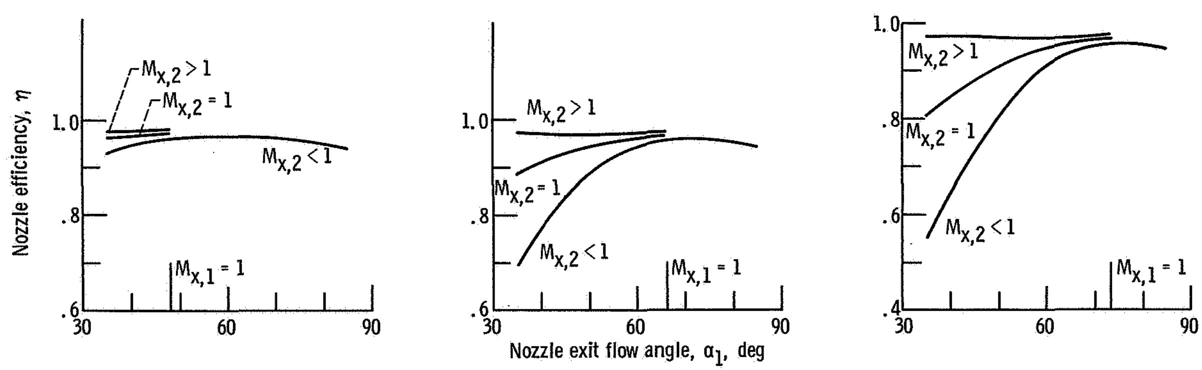
Figure 8. - Effect of specific heat ratio on nozzle efficiency for nozzles designed for free-stream Mach number of 2.5 and throat Reynolds number of 10 000. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

ratio are kept constant at values of 2.5 and 1.4, respectively. As expected, the nozzle efficiency increases as the Reynolds number increases. For laminar flow, the skin friction or viscous losses are known to vary inversely with the square root of Reynolds number. Therefore as the throat Reynolds number is increased, the viscous losses in the subsonic, sonic, and supersonic solutions decrease, resulting in higher nozzle efficiencies.

Effect of specific heat ratio on efficiency. - The effect of specific heat ratio γ on nozzle efficiency is shown in figure 8. The free-stream Mach number and Reynolds number are kept constant at values of 2.5 and 10 000, respectively. It is seen that an increase in specific heat ratio results in an increase in nozzle efficiency.

Effect of Nozzle Mach Number Level on Nozzle Efficiency

The efficiency of nozzles designed for free-stream Mach numbers of 1.5, 2.5, and



(a) Free-stream Mach number, 1.5. (b) Free-stream Mach number, 2.5. (c) Free-stream Mach number, 3.5.

Figure 9. - Effect of free-stream Mach number on nozzle efficiency for throat Reynolds number of 10 000 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

3.5 are shown in figure 9. The throat Reynolds number and specific heat ratio are kept constant at values of 10 000 and 1.4, respectively. For the supersonic solution, the nozzle Mach number level has little effect on the efficiency. For the subsonic solution, higher Mach number levels result in lower efficiencies. This is to be expected, since the strength of the oblique shock occurring in the subsonic solution increases as the Mach increases. Therefore, at the higher Mach number levels, larger shock losses occur for the subsonic solution.

SUMMARY OF RESULTS

An analysis was conducted to determine the loss characteristics of two-dimensional, minimum length, supersonic nozzles with sharp-edged throats. Boundary layer characteristics were calculated and used to obtain the conditions downstream of the nozzles after the flow had mixed to a uniform state. Subsonic, sonic, and supersonic aftermixing axial Mach number solutions were obtained when the free-stream axial Mach number at the nozzle exit (before mixing) was supersonic. The subsonic solution corresponds to mixing plus oblique shock losses, whereas the supersonic solution corresponds to shockless mixing. The sonic solution corresponds to the limiting conditions possible if the nozzles exhaust into a constant area passage.

The nozzle loss characteristics were studied as a function of nozzle flow angle α_1, free-stream Mach number level $M_{fs,1}$, and throat Reynolds number Re_t. The effect of specific heat ratio was also investigated. The following results were obtained:

1. The nozzle efficiency for the supersonic solution is higher than for the subsonic solution, because of the absence of shock losses in the supersonic solution. Intermediate efficiencies occur for the sonic solution.

2. The aftermixing flow angle deflects away from the axial direction for the subsonic solution, but deflects towards the axial direction for the supersonic solution. The supersonic solution, therefore, represents further flow expansion on mixing which results in higher aftermixing Mach numbers and higher total-to-static pressure ratios than occur for the subsonic solution.

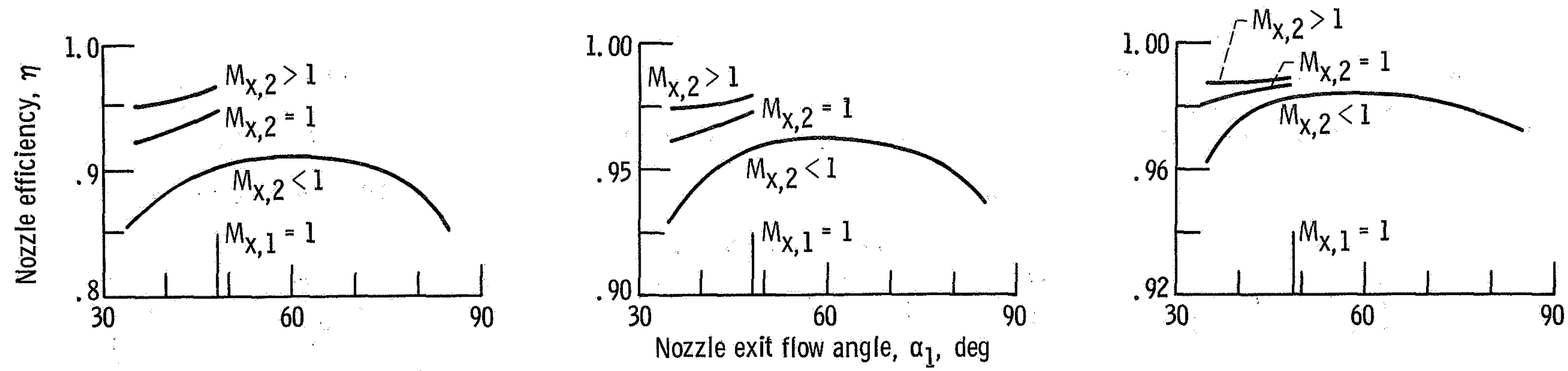
3. Increasing either the throat Reynolds number or specific heat ratio results in an increase in nozzle efficiency.

4. The nozzle Mach number level has little effect on the nozzle efficiency for the supersonic solution. However, for the subsonic solution, higher Mach numbers result in higher oblique shock losses, and therefore lower efficiencies.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, April 8, 1969,
128-31-32-07-22.

APPENDIX - LOSS CHARACTERISTICS OF TWO-DIMENSIONAL SUPERSONIC NOZZLES WITH SHARP-EDGED THROATS

The nozzle loss characteristics for nozzles designed for free-stream Mach numbers of 1.5, 2.5, 3.5, and 5.0 are presented herein. These curves may be used for preliminary design purposes. For each Mach number level, curves of nozzle efficiency, after-mixing Mach number, flow angle, and total-to-static pressure ratio are presented. The nozzle flow angle has been varied so as to include both subsonic and supersonic free-stream axial Mach numbers. Results for three different throat Reynolds numbers are given. The results for free-stream Mach numbers of 1.5, 2.5, 3.5, and 5.0 are shown in figures 10 to 13, 14 to 17, 18 to 21, and 22 to 25, respectively.

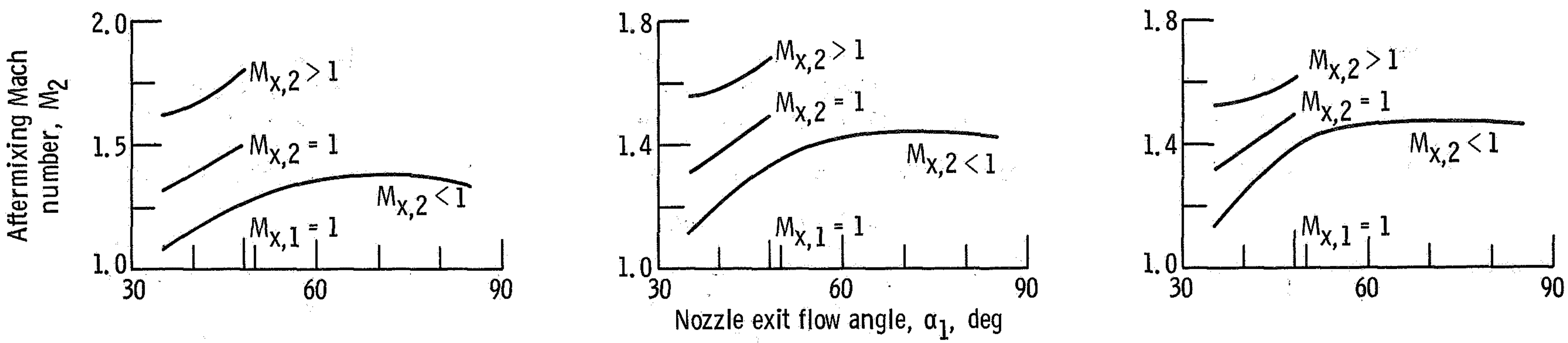


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 10. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on nozzle efficiency for free-stream Mach number of 1.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

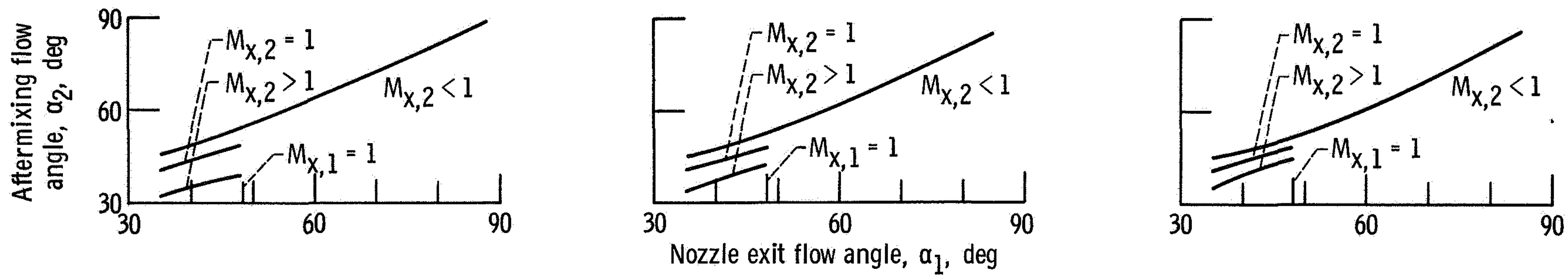


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 11. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on aftermixing Mach number for free-stream Mach number of 1.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

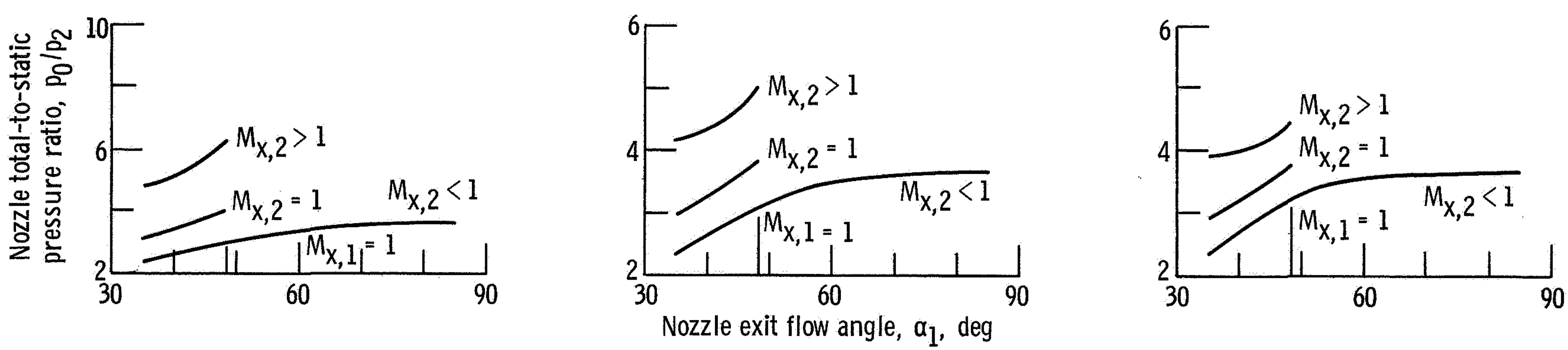


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 12. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on aftermixing flow angle for free-stream Mach number of 1.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

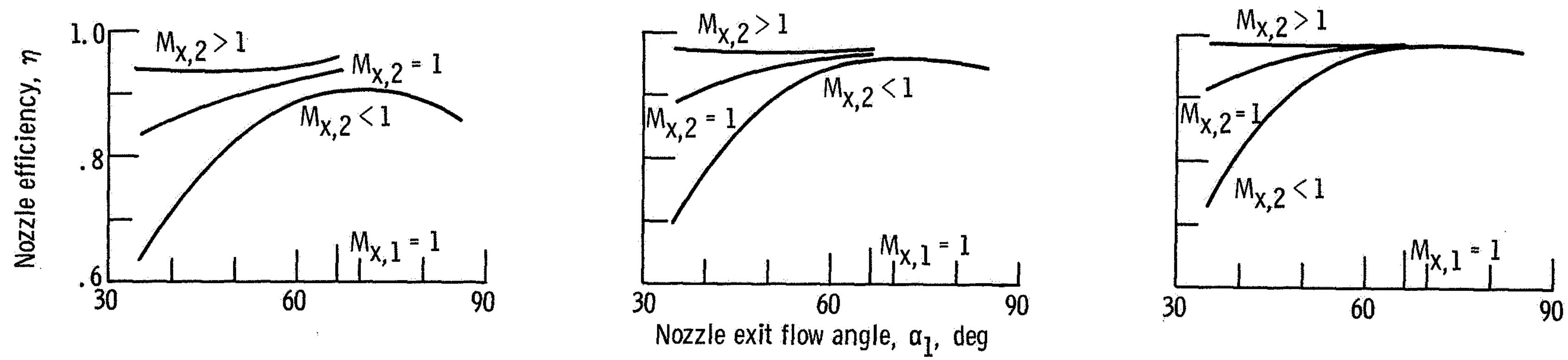


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 13. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on total-to-static pressure ratio for free-stream Mach number of 1.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

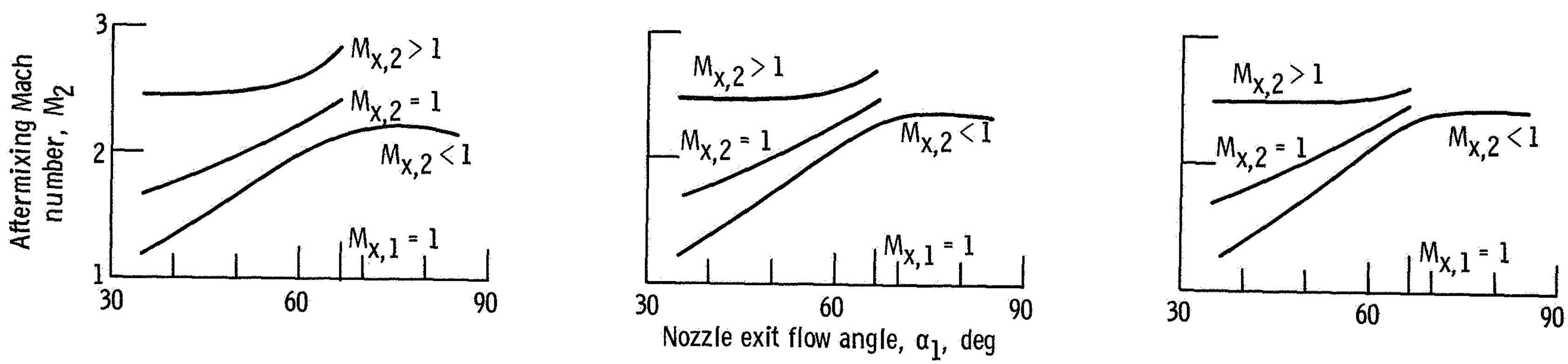


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 14. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on nozzle efficiency for free-stream Mach number of 2.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

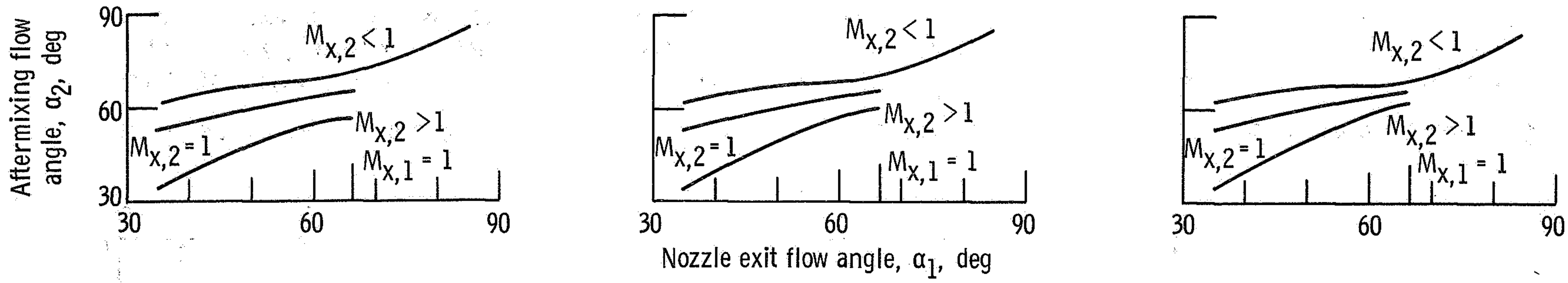


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 15. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on aftermixing Mach number for free-stream Mach number of 2.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

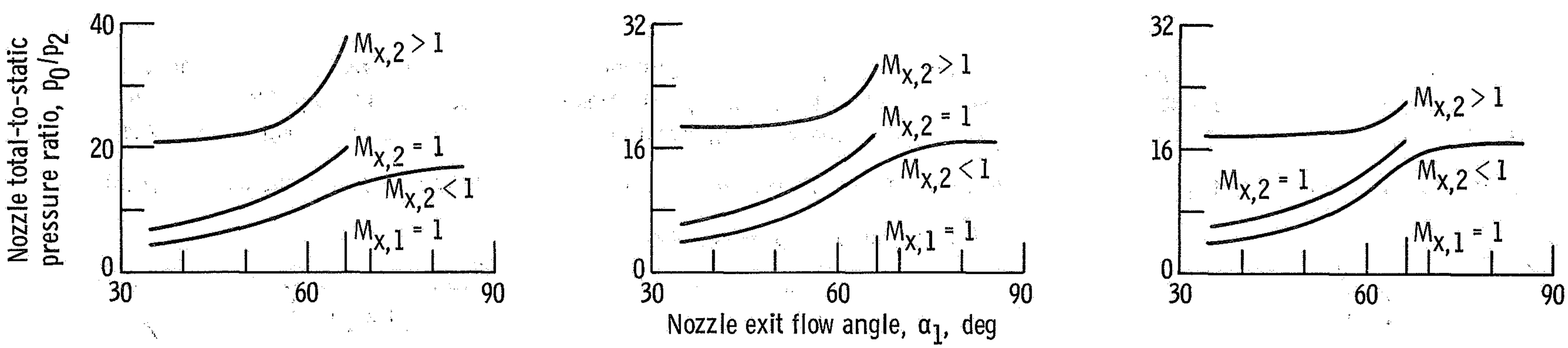


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 16. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on aftermixing flow angle for free-stream Mach number of 2.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

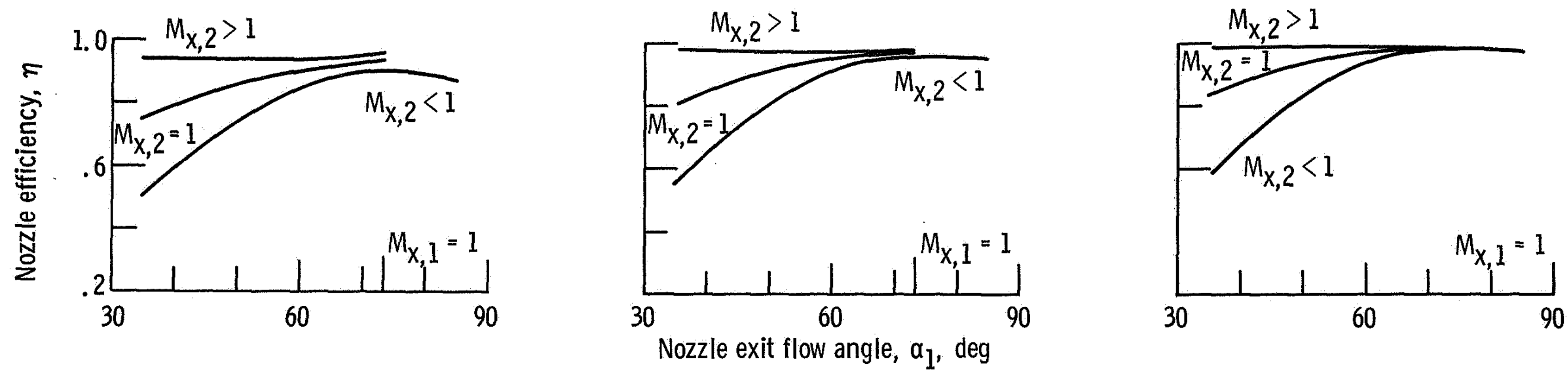


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 17. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on total-to-static pressure ratio for free-stream Mach number of 2.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

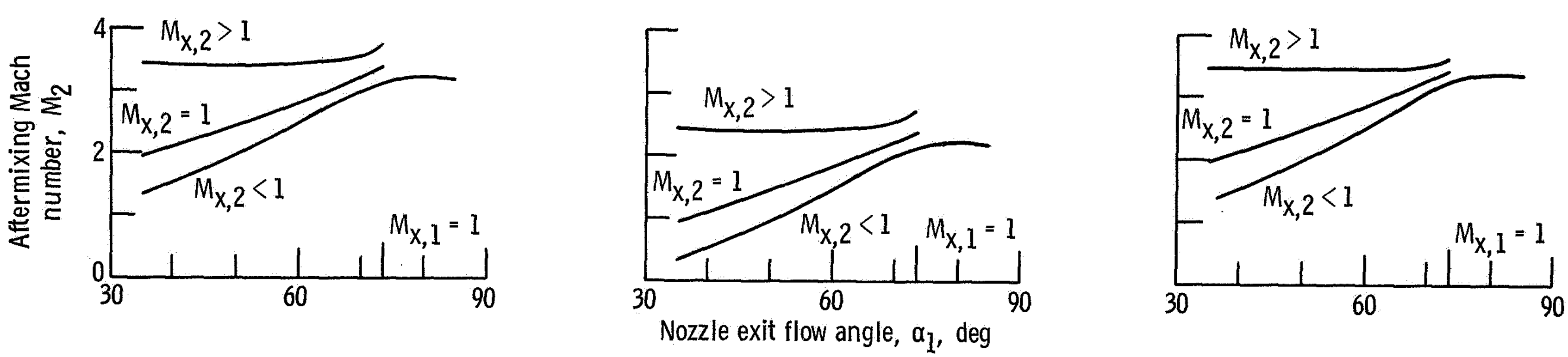


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 18. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on nozzle efficiency for free-stream Mach number of 3.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.

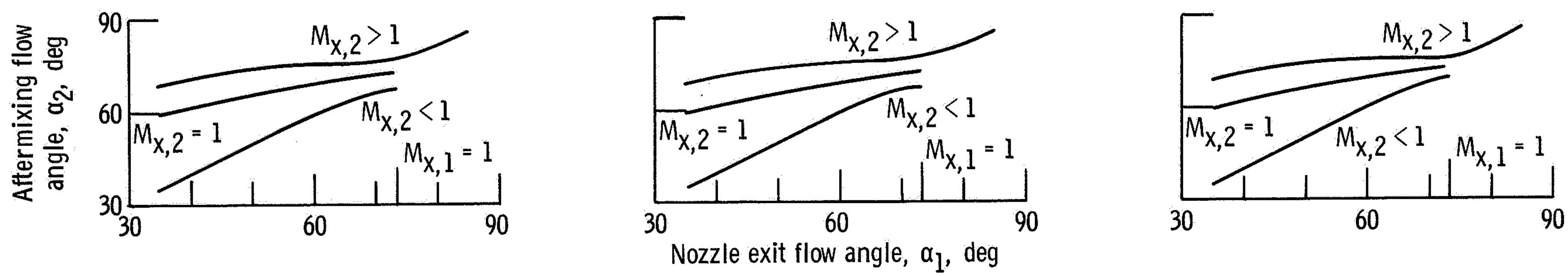


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 19. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on aftermixing Mach number for free-stream Mach number on 3.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

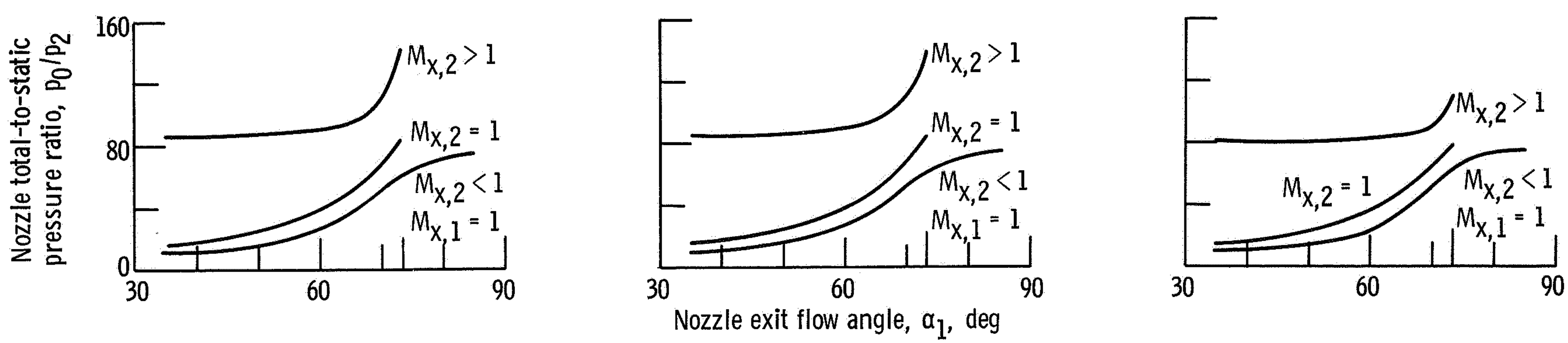


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 20. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on aftermixing flow angle for free-stream Mach number of 3.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

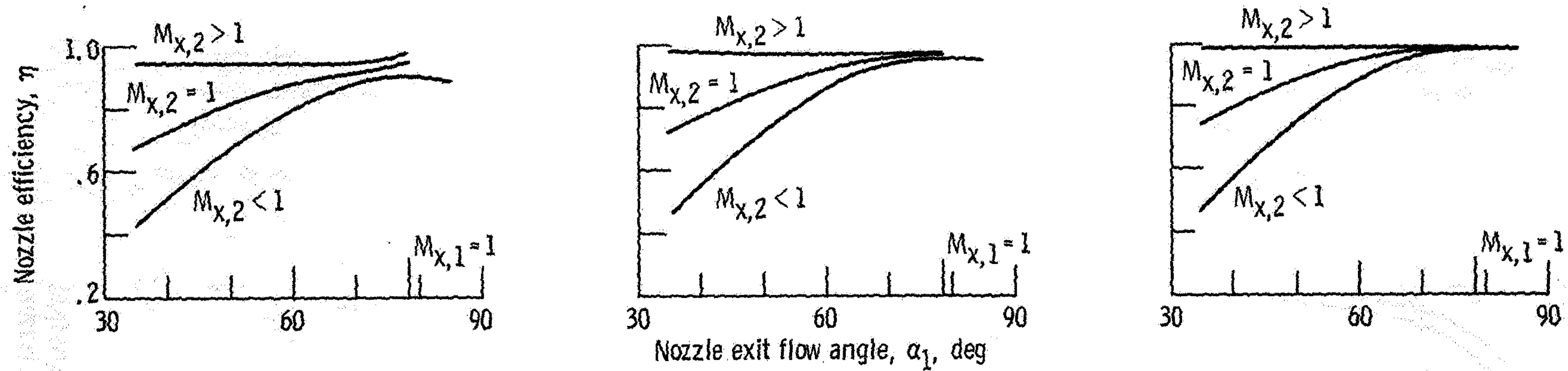


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

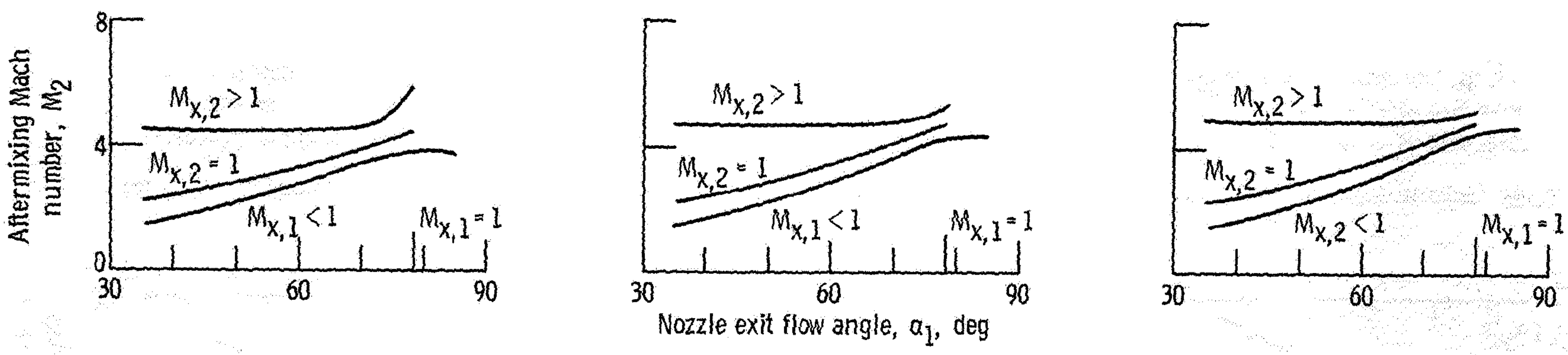
(c) Throat Reynolds number, 50 000.

Figure 21. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on total-to-static pressure ratio for free-stream axial Mach number of 3.5 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream Mach number, $M_{x,1} = 1$.)



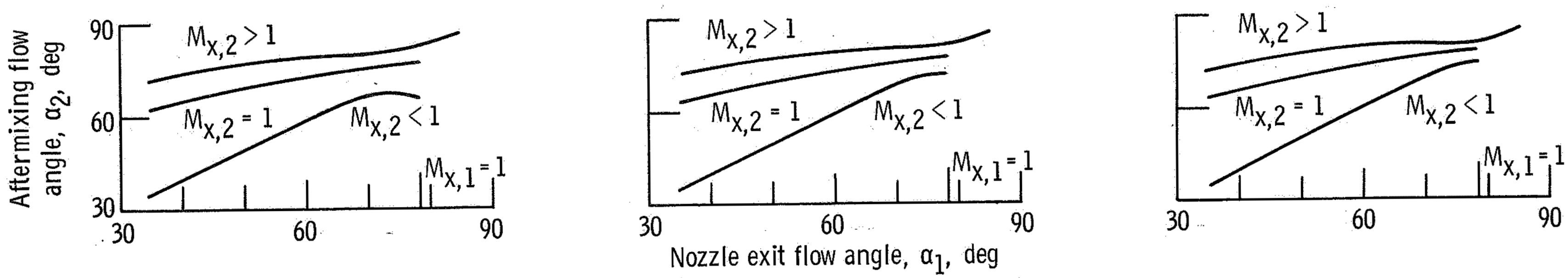
(a) Throat Reynolds number, 2000. (b) Throat Reynolds number, 10 000. (c) Throat Reynolds number, 50 000.

Figure 22. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on nozzle efficiency for free-stream Mach number of 5.0 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)



(a) Throat Reynolds number, 2000. (b) Throat Reynolds number, 10 000. (c) Throat Reynolds number, 50 000.

Figure 23. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on aftermixing Mach number for free-stream Mach number of 5.0 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)

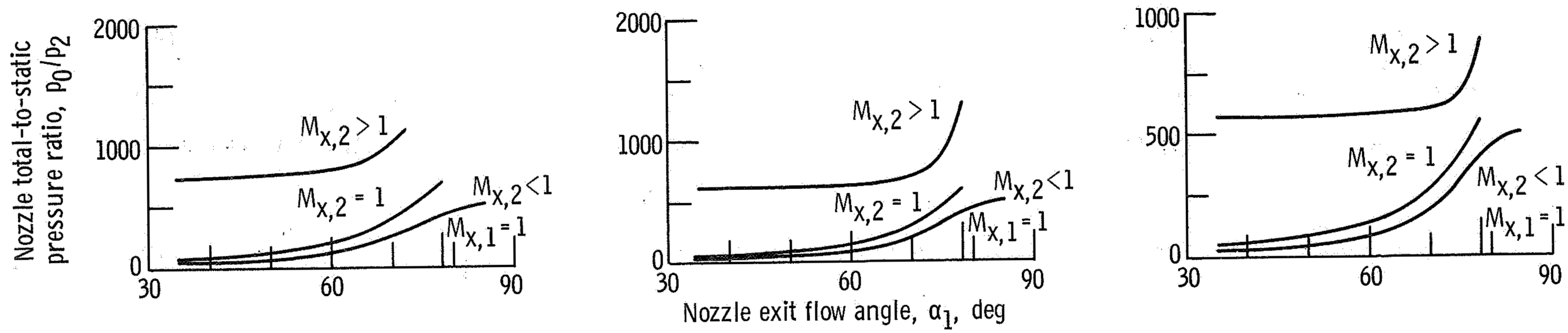


(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 24. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on aftermixing flow angle for free-stream Mach number of 5.0 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number, $M_{x,1} = 1$.)



(a) Throat Reynolds number, 2000.

(b) Throat Reynolds number, 10 000.

(c) Throat Reynolds number, 50 000.

Figure 25. - Effect of nozzle exit flow angle and nozzle throat Reynolds number on total-to-static pressure ratio for free-stream Mach number of 5.0 and specific heat ratio of 1.4. (Supersonic aftermixing axial Mach number, $M_{x,2} > 1$; sonic aftermixing axial Mach number, $M_{x,2} = 1$; subsonic aftermixing axial Mach number, $M_{x,2} < 1$; sonic free-stream axial Mach number,

REFERENCES

1. Vanco, Michael R.: Thermodynamic and Turbine Characteristics of Hydrogen-Fueled Open-Cycle Auxiliary Space Power System. NASA TM X-1337, 1967.
2. Moffitt, Thomas P.: Design and Experimental Investigation of a Single-Stage Turbine with a Rotor Entering Relative Mach Number of 2. NACA RM E58F20a, 1958.
3. Moffitt, Thomas P.; and Klag, Frederick W., Jr.: Experimental Investigation of the Partial-Admission Performance Characteristics of a Single-Stage Mach 2 Supersonic Turbine. NASA TM X-80, 1959.
4. Moffitt, Thomas P.; and Klag, Frederick W., Jr.: Experimental Investigation of Partial - And Full - Admission Characteristics of a Two-Stage Velocity-Compounded Turbine. NASA TM X-410, 1960.
5. Vanco, Michael R.; and Goldman, Louis J.: Computer Program for Design of Two-Dimensional Supersonic Nozzle with Sharp-Edged Throat. NASA TM X-1502, 1968.
6. Goldman, Louis J.; and Scullin, Vincent J.: Analytical Investigation of Supersonic Turbomachinery Blading. I - Computer Program for Blading Design. NASA TN D-4421, 1968.
7. Cohen, Clarence B.; and Reshotko, Eli: The Compressible Laminar Boundary Layer with Heat Transfer and Arbitrary Pressure Gradient. NACA Rep. 1294, 1956.
8. Stewart, Warner L.: Analysis of Two-Dimensional Compressible-Flow Loss Characteristics Downstream of Turbomachine Blade Rows in Terms of Basic Boundary-Layer Characteristics. NACA TN 3515, 1955.
9. Stewart, Warner L.; Whitney, Warren J.; and Wong, Robert Y.: Use of Mean-Section Boundary-Layer Parameters in Predicting Three-Dimensional Turbine Stator Losses. NACA RM E55L12a, 1956.

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