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PRELIMINARY RESULTS FROM MEAN ELEMENT ANALYSES OF 12-HOUR RESONANT ORBITS

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ABSTRACT

A general resonant orbit and gravity constant determining program has been developed accepting short arc mean Kepler elements as the data type. The long term evolution of these elements is calculated by numerical integration of coordinates without short-period variation.

With only slowly-changing mean element coordinates being integrated, a single step size of the order of an orbit revolution or more is achieved. Satellite ephemerides over 5000 revolutions are calculated in about a minute on an IBM/360 computer. Partial derivatives of the evolved mean elements with respect to initial values and gravity constants are readily evaluated from numerically generated variant trajectories.

Extensive data from many resonant 12- and 24-hour satellites is currently being processed by the mean element program. Preliminary determinations from the 12-hour data are given.

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OF 12-HOUR RESONANT ORBITS

Over the past year there has been developed at the Goddard Space Flight Center a high speed trajectory determination program specifically designed to recover gravity information from the long term changes of the Kepler elements of resonant orbits.[1]

In this program, the equations of satellite motion, integrated numerically, are the classic Lagrange Planetary equations with coordinates of Kepler elements without short period variation, so called mean elements. The geopotential in terms of these elements derives from the work of Bill Kaula[2] and the original straight trajectory generator for resonant orbits was designed by Bruce Douglas. The program now includes atmospheric drag as well as long term sun and moon perturbations. The observation data used so far with this program on 12-hour satellites are mean Kepler element updates reduced from independent Baker-Nunn camera, radar skin track and Minitrack data spans. The updated elements are generally spaced a week or two apart. For the Molniya and Cosmos satellites the reduction has been from Baker-Nunn camera and radar skin track data by an orbit determination program based on a satellite theory due to Kozai.[3] For the Intelsat II F1 satellite (what I call Looney Bird because it was the first Comsat Lani Bird gone awry) the raw data are Minitrack observations reduced by Goddard Space Flight Center to mean elements by a Brouwer differential correction program. It has been found that the Brouwer element data is completely compatible with the high speed resonant gravity recovery program. The Kozai element data has been made reasonably compatible by converting the mean-motion in the element updates from the North American Air Defense Command (NORAD), to a Brouwer equivalent semi-major axis. I hope in the future to determine my own short arc mean elements from the raw NORAD data, when I receive it.

For geodetic solutions, only satellite arcs that are above about 750 km altitude are generally used, to avoid the uncertainties of the drag calculation.

But I did have a pleasant surprise in analyzing one long arc for the satellite Molniya 4 which decayed in September last year. This was not a true atmospheric decay but rather a relatively catastrophic depression of perigee under the influence of the sun and moon, a common characteristic of high eccentricity orbits.[4] However, during the last two months of decay when perigee was less that 500 km, the mean motion rose very rapidly due to heavy drag.

A very sensitive test of this part of the program was provided by the mean element updates during this period. I found that the best fit to the data then was with a satellite area to mass ratio of 0.12 centimeters squared per gram. This happened to be exactly the average ratio estimated for Molniya 1 during its early life as reported to me by a Russian investigator.[5] I have what appears to be an adequate single arc reduction of this drag influenced data but have not yet incorporated it into the multiple satellite solutions.

Now I want to summarize the satellite arcs and gravity harmonics I have examined on both 12- and 24-hour orbits (see Figure 1). The 2, 2 harmonic is actually dominant on both kinds of satellites and is most strongly determined from the 24 hour data. The quality of the 12-hour solutions are thus best represented by fixing 2, 2 and solving for less well know effects. 2, 2 is also quite well known now from non resonant satellites.

On the right of Figure 1 you see the quality of the harmonics I hope to determine from the mean element data already available on the 10 high-altitude satellites. The solutions I have made so far make me confident that this promise will be fulfilled in large part. But on the other hand, it is doubtful that really definitive constants will be forthcoming from this data alone for all but the lowest order and degree effects. This judgment is based only on the analysis of the data available now, principally its lack of a good variety of orbital elements. Also there are problems with the compatibility and data quality of the short arc NORAD elements and more fundamentally with the possibility of unaccounted for small orbit maneuvers in the Russian communication satellites.

Figure 2 shows six individual arc solutions with 12-hour satellites using mean anomaly as the observation data type, solving for initial semi-major axis and mean anomaly in the arcs as well as a minimum number of sensitive harmonics. These arcs are from 200 to 500 days long and the mean anomaly oscillations without considering resonant effects have amplitudes of from 2 to 20 degrees. In two Cosmos 41 and the two Molniya arcs, a pair of 2nd- and 4th-order constants were the minimum sensitive set because these were non stationary satellites with reasonable longitude sampling. One Cosmos 41 arc and the Looney Bird arc were stationary within 15 degrees. For these, only a pair of 2nd-order constants were solved for though the SAO-66-M1 4th-degree and order constants[6] were included in the trajectory. But even with only two free gravity constants, the stationary arc solutions still showed a high correlation between these constants. However, taking this high correlation into account, the error ellipse in the (3,2) plane for the stationary solutions appears to be compatible with those for the circulation arcs.

The major problem in this analysis is to insure that the force model is adequate to explain the element data over the long arc, to the short arc accuracy level. The large range of sigmas in these long arcs is an indication that this is not the case for some of them. Examination of residuals in the arcs with mean anomaly sigmas over a tenth of a degree appears to show the effects of radiation

SATELLITE	ORBIT FREQUENCY (REVS./DAY)	INCLINATION (DEGREES)	NUMBER OF ARCS BEING USED (100-500 DAYS PER ARC)	QUALITY OF DETERMINATION OF H_{lm} WITH COMBINED SOLUTION (S=STRONG, W=WEAK)
SYNCOM 2	1	29-33	6	
SYNCOM 3	1	0-3	4	
EARLY BIRD	1	0-3	2	
INTELSAT II-F4	1	1	1	
INTELSAT II-F1 (LOONEY BIRD)	2	18	1	
COSMOS 41	2	65-66	2	
COSMOS 41 ROCKET	2	67-69	2	
MOLNIYA 1	2	64-65	4	
MOLNIYA 4	2	65	1	
MOLNIYA 7	2	65	1	

l \ m	1	2	3	4	5	6	7	8
2		S						
3	S	S	S					
4		S		S				
5	W	W	W	S	W			
6		W		W		S		
7				W		W		
8						W		W

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Figure 1. Resonant Satellites and Harmonics in Current Low Order Gravity Solutions

12 HOUR SATELLITE	ARC	I	e	ω	$10^6 H_{32}$		$10^8 H_{44}$		σ (STANDARD DEVIATION OF FIT)	COMMENT
					C	S	C	S		
COSMOS 41	23-65	65°	.74	323°	.32	−.21	.28	1.27	.04°	
COSMOS 41R	29-55	68°	.72	312°	.08	−.92	M1*	M1*	.07°	STATIONARY, HIGH CORRELATIONS
COSMOS 41R	74-108	69°	.71	282°	.21	−.27	.10	1.12	.17°	NEARLY STATIONARY, HIGH CORRELATIONS
MOLNIYA 1	35-118	65°	.73	321°	.33	−.23	.30	1.16	.08°	
MOLNIYA 1	131-162	65°	.69	306°	.22	−.15	−.37	.65	.15°	
LOONEY BIRD	1-21	18°	.64	39°-107°	.57	−.18	M1*	M1*	.02°	STATIONARY, HIGH CORRELATIONS

*SAO 1966 (M1) SOLUTION FOR H_{44} USED IN THE TRAJECTORY

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Figure 2. Single Arc Solutions Data Type: Mean Anomaly. H_{22} Fixed ($10^6 C_{22} = 1.56$, $10^6 S_{22} = -0.93$)

pressure which has not yet been included in the trajectories force model. Since the periodicity of this effect is comparable to the long resonant beat periods, the gravity solutions in these arcs are undoubtedly absorbing some of the force due to radiation pressure. Residual drag at perigee has been mentioned in these high eccentricity orbits. But only for Molniya 4, not yet included in the combined solutions, does this appear to be a significant problem.

There remains the nagging doubts about orbit maneuvers in the Russian satellite arcs. We have already isolated many of these from obviously large changes of mean motion between orbit updates. Others have been suspected from the discontinuous way the mean anomaly residuals behave in gravity solutions with semi-major axis data. But the doubts still remain even in the presumably free drift arcs, such as those here.

I will not be able to resolve these doubts, particularly for the large sigma arcs, until I look at the raw observation data. However, there is a valuable test that can be made of the compatibility of these presumably free drift gravity solutions, short of this raw data analysis. There is available, analytic formulas for the longitude acceleration due to the resonant gravity harmonics.[7] Thus one can readily calculate standard accelerations and variances for a typical orbit from the statistics of each arc solution, taking into account the likely effect of neglected harmonics in the individual solutions. If these standard accelerations agree within a reasonable variation allowed by the individual arc statistics, the free drift assumption is a good one.

Already, the combined solutions that have been made (Figure 3) indicate there is such reasonable compatibility among the six arcs here. In the first line of Figure 3 is the result of a combined solution with these arcs including the effects of (6, 6), weighted according to the individual arc sigmas for mean anomaly. The sigma shown in this figure is a non dimensional, or normalized measure of the combined data standard deviation. A sigma of zero for the combined solution would have variances for each arc as low as in the best individual arc solutions. Combined arc sigmas less than 1.0 would appear to be a reasonably compatible solution if the variances in the arcs of the solution are also reasonably close to their best individual arc fits.

The first combined arc solution here has a sigma just less than 1. Yet examination of the individual arc residuals of this simplest solution clearly shows the effects of other harmonics, especially those of 2nd-order, which are not being completely absorbed by the solved-for constants. The sigma on the second line is the result of a solution for all 2nd- and 4th-order harmonics through (5, 4) and also (6, 6). The actual solution is not shown because high correlation and perhaps neglect of radiation pressure appear to have made some of the constants quite unrealistic. But the distance between the sigmas in lines 1 and 2 is a good measure of the information in these arcs on higher degree effects.

	12 HOUR SATELLITE SOLUTIONS, 6 ARCS	$10^6 \bar{H}_{22}$		$10^6 \bar{H}_{32}$		$10^6 \bar{H}_{44}$		$10^6 \bar{H}_{66}$		σ (NORMALIZED, COMBINED ARCS STANDARD DEVIATION)	COMMENTS
		C	S	C	S	C	S	C	S		
1.	H_{22} FIXED	2.42	−1.44	.94	−.53	.27	.57	.03	−.12	0.94	
2.	H_{22} FIXED	2.42	−1.44	NOT REALISTIC BECAUSE OF HIGH CORRELATIONS						0.22	SOLVES FOR ALL EFFECTS THROUGH $H_{5,4}$ WITH $H_{6,6}$
3.	SOLVING FOR INITIAL SEMIMAJOR AXIS ONLY WITH: SAO-1966(M1) GRAVITY CONSTANTS	2.38	−1.35	.73	−.54	−.05	.23	−.05	−.16	8.2	INCLUDES ALL EFFECTS THROUGH $\bar{H}_{7,6}$
4.	SOLVING FOR INITIAL SEMIMAJOR AXIS ONLY WITH: NWL- 8D (1967) GRAVITY CONSTANTS	CLASSIFIED								6.8	INCLUDES ALL EFFECTS THROUGH $\bar{H}_{7,6}$

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Figure 3. Combined Arc Solutions 12 Hour Satellites Data Type: Mean Anomaly

An important question about this low-order resonant data is how adequate are recent gravity fields in explaining the oscillations. I have made two tests of this data so far, with perhaps the best camera-satellite and doppler-satellite fields. The result of using the SAO-66-M1 field through (7, 6) in the trajectory model and solving only for initial semi-major axis in the six arcs gave a combined sigma of 8.2 as shown in line 3. A similar test with a classified 1967 Naval Weapons Laboratory gravity field[8] gave a combined sigmas of 6.8, as shown in line 4.

Clearly the information in these 12-hour arcs should sharpen considerably knowledge of many low-order geopotential terms. However, with the limited analysis I have made to date on this data, I cannot yet report a definitive set of these terms. It may be that the best that can be done with the 12- and 24-hour data in a combined solution will be a good solution for only the lowest degree and order resonant terms (2, 2), (3, 3), (4, 4), and (6, 6). But the set of excellent resonant accelerations now being determined from these orbits should, when combined with the best non-resonant satellite results, give definitive solutions for at least 1/2 of the longitude geopotential terms through (8, 8).

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