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THE EFFECT OF DAILY (24 HOUR) PRECESSION OF THE GEOMAGNETIC DIPOLE ON THE CREATION OF Sq-VARIATIONS

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1. It is a known fact that regular quiet sun-daily variations of the magnetic field of the Earth (Sq) are effected by the systems of electrical currents in the near-Earth space. It is a generally accepted concept that these currents can be found in the thin spherical E-layer of the ionosphere and are created due to the atmospheric dynamo effect. According to the dynamo theory the currents originate as a consequence of the motion of electrically conductive air masses in the Earth magnetic field.

The dynamo theory is developed at present in many theoretical and experimental studies [1,2]. The value of the current originated is usually evaluated with the aid of ionosphere parameters and the Earth's magnetic field. The direct magnetic dipole field serves as a magnetic field; the axis of the former must be running along the axis of the Earth's rotation. In using such interpretation of the dynamo theory the magnetic disturbances on the Earth's surface should not be dependent on the UT (universal time) in view of the homogeneity of the direct dipole magnetic field. The non-coincidence of the magnetic and geographic axes of the Earth which actually takes place, may lead to daily variations of the Sq-field, controlled by the universal time. The effect of the Sq dependence upon the universal time has been revealed in articles [3], [4], [5].

It is necessary to remember that the daily procession of an inclined dipole around the geographic axis of the Earth results not only in the dependence of the Sq-field upon UT but also in other phenomena of an electro-dynamic character in the near-Earth space. Thus, for instance, Ya. P. Terletskiy [6] has pointed out the contribution created by the variations of the Earth's magnetic field in the process of particle acceleration in the near Earth space. The present article aims to develop the dynamo theory of Sq-variations, taking into account the non-coincidence of magnetic and geographic axes of the Earth.

2. According to [1] we can utilize a generalized Ohm's law in order to calculate currents in the dynamo region.

$$j_i = \sigma_{ij} E_j , \quad i,j = 1,2,3, \qquad (1)$$

where J_i, E_j are the components of the current density and the aggregate electric field (including the dynamo field), σ_{ij} is the conductivity tensor. The distribution of electric conductivity in the ionosphere according to altitude is such that the dynamo currents may be considered approximately as surface currents.

This fact permits the introduction of a current function I determined in the following manner:

$$\vec{j} = [\vec{n}\,\nabla J]. \qquad (2)$$

where $\vec{n}$ is the unit vector of the normal to the current-carrying surface.

By introducing the dynamo field $\vec{E}_g = [\vec{v}\vec{H}]$ (3) (in the system c = 1), the polarization field

$$E_n = -\operatorname{grad} S \qquad (4)$$

and the induction electric field $\vec{E}_u$ which is established by the Maxwell equation

$$\frac{\partial \vec{H}}{\partial T} = -\operatorname{rot} \vec{E}_u, \qquad (5)$$

and combining (1 + 5), we obtain

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial J}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 J}{\partial t^2} = -\sigma a^2\frac{\partial H_z}{\partial T} + \frac{a\sigma}{\sin\theta}\left[\frac{\partial}{\partial\theta}(v_x H_z \sin\theta) + \frac{\partial}{\partial t}(v_y H_z)\right], \qquad (6)$$

where a is the radius of the dynamo region of the atmosphere, the effective conductivity, and v the wind velocity.

Equation (6) differs from the corresponding dynamo equation as given in /1, 2/ by the first member, which appears in the right part of the equation. It is easy to find that in the case of an inclined dipole the vertical component of the magnetic field H_z is written out as:

$$H_z = C\left|\cos\theta + \gamma \sin\theta \cos(t - T - \lambda_0)\right| , \qquad (7)$$

where $C \simeq 0.5$ gauss, $\gamma = \mathrm{tg}\,\theta_0$, $\theta_0 = 11{,}5°$, $\lambda = 291°$ are the coordinates of the northern geomagnetic pole.

The existence of the part of the Sq currents dependent upon the universal time is caused by the second member in (7). It should be noted at this point that if no motions of ionized gas existed in the ionosphere the rotation of the inclined magnetic dipole would still result in the creation of a certain non-potential current system and, therefore, to daily regular variations of the magnetic field at the Earth's surface. The computations lead us to believe that a part of the non-potential current system corresponding to the turbulent induction field makes up to 10 to 15% of the dynamo currents.

Inasmuch as experimental data on the velocity of ionospheric winds are insufficient, we introduce into the study, for illustrative purposes, the potential winds; in other words, we assume:

$$V = -\mathrm{grad}\,\psi , \quad \psi = a V_0 P_1^1(\cos\theta)\sin(t + \alpha) , \qquad (8)$$

where $V_0 = 80$ m/sec is the amplitude of the daily (24 hour) winds wave, and P_1^1 (cos θ) is the Legendre polynom.

By substituting (8) and (7) in (6) we obtain:

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial J}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 J}{\partial t^2}=\frac{\gamma G C a^2}{T_0}\sin(t-T-\lambda_0)\sin\theta+$$
$$+GCaV_0\left[-\frac{3}{2}\sin 2\theta\sin(t+\alpha)+\gamma(\cos 2\theta+\cos^2\theta)\times\right.$$
$$\left.\times\sin(t+\alpha)\cdot\cos(t-T-\lambda_0)-\gamma\sin(2t-T-\lambda_0+\alpha)\right]. \qquad (9)$$

Here T_o is the period of the Earth's rotation.

3. Let us now take up the finding of the solution to equation (9). The solution to the homogeneous equation is

$$J(\theta,t)=\sum_{n=1}^{\infty}(A_n tg^n\,\theta/2+B_n ctg^n\,\theta/2)\begin{matrix}\sin(nt+\alpha)\\ \cos(nt+\alpha)\end{matrix}. \qquad (10)$$

We shall limit ourselves to the value $I(\Theta,t)$ at $n = 1$.

$$J(\theta,t)=(A_1 tg\,\theta/2+B_1 ctg\,\theta/2)\sin(t+\alpha). \qquad (10a)$$

The general solution will be a sum of solutions corresponding to each member in the right part. For purposes of illustration we shall find the solution for the equation

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial J}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2 J}{\partial t^2}=f(\theta)\sin(nt+\alpha). \qquad (11)$$

We seek the solution to this equation as

$$J(\theta,t)=J_1(\theta)\sin(nt+\alpha)$$

Substituting in (11), we obtain

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dJ}{d\theta}\right)\frac{n^2 J_1}{\sin^2\theta}=f(\theta) \qquad (11a)$$

By replacing shx = ctgθ the homogeneous equation corresponding to (11a) will become

$$\frac{d^2 J_1}{dx^2}-n^2 J_1=0.$$

Whencefrom we obtain

$$J_1(\theta)=A_n e^{nx}+B_n e^{-nx},$$

or, using the previous variable

$$J_1(\theta)=A_n tg^n\,\theta/2+B_n ctg^n\,\theta/2.$$

The general solution (11a) is found by the method of variation of constants A_n, B_n. In this manner we can find the solution to the illustrative equation (11). The solution found for the dynamo equation (9) by this method, using (10a) has the following aspect:

$$J(\theta,t,T)=-M(\sin\theta+A_0 tg\,\theta/2+B_0 ctg\,\theta/2)\sin(t-T-\lambda_0)\cdot$$
$$-\frac{N}{4}\left(\frac{\sin 2\theta}{2}+A_1 tg\,\theta/2+B_1 ctg\,\theta/2\right)\sin(t+\alpha)+$$
$$+\frac{\gamma}{4}N\left(\sin^2\theta+A_2 tg^2\theta/2+B_2 ctg^2\theta/2\right)\sin(2t+\alpha-T-\lambda_0), \qquad (12)$$

where $M = \gamma C \sigma a^2$, $N = \sigma C a v_0$. T enters the expression for the current function as a parameter.

4. Inasmuch as data on conductivity of the ionosphere are not complete, by following the mathod indicated in /2/ we may divide the dynamo region of the ionosphere into five zones with sharply differing values of conductivity. The borders of individual zones and the corresponding values of conductivity are presented below:

Zones	Borders	Conductivity value in the CGSE system
I	$0 < \theta < 25°$	$0{,}6 \cdot 10^{-8}$
II	$25° < \theta < 86°$	$3 \cdot 10^{-8}$
III	$86° < \theta < 94°$	$30 \cdot 10^{-8}$
IV	$94° < \theta < 155°$	$3 \cdot 10^{-8}$
V	$155° < \theta < 180°$	$0{,}6 \cdot 10^{-8}$

We have considered at this point that the conductivity in the zones located symmetrically in relation to the equator are equal. As we can see from the conductivity values presented, we have a co-relation $\sigma_{II} : \sigma_{III} = 1:5:50$ which corresponds to the observed values of variations of the magnetic field in the indicated zones. The third zone, with a sharply increased conductivity corresponds to the location of the equatorial electric flux.

At the border of the division the normal component of the current density

$$j_x = \frac{1}{a} \frac{\partial J}{\partial t}$$

and the tangential component of the electric vector

$$E_t = \frac{1}{a \delta \sin \theta} \frac{\partial J}{\partial \theta}$$

are uninterrupted. Starting with this statement we shall build a system of equations to determine the constsnts of integration, which enter (12). In order to obtain an analytical solution for the entire region, including the particular regular points we should select B_o, B_1, B_2 = 0 for the first zone, and A_o, A_1, A_2 = 0 for the fifth zone. The values found for the remaining constants are presented in the table below:

A_0^{I}	A_0^{II}	A_0^{III}	A_0^{IV}	A_1^{I}	A_1^{II}	A_1^{III}	A_1^{IV}	A_2^{I}	A_2^{II}	A_2^{III}	A_2^{IV}
1,26	-0,004	-0,45	-0,62	1,33	0,41	0,23	0,07	0,064	0,15	-0,006	-0,062

B_0^{II}	B_0^{III}	B_0^{IV}	B_0^{V}	B_1^{II}	B_1^{III}	B_1^{IV}	B_1^{V}	B_2^{II}	B_2^{III}	B_2^{IV}	B_2^{V}
-0,45	-0,004	1,26	-0,07	-0,23	-0,41	-1,83	-0,006	-0,44	0,12	-0,43	-2,4

where the symbols over the constsnts relate to corresponding zones.

5. Upon determining the analytical expression for the current function we built flow-charts for current systems flowing in the dynamo region, these were thereafter compared with corresponding curves obtained from experimental data for various instants of the universal time.

Fig. 1 shows the trajectory of the motion of focuses with the universal time for both the northern and the southern hemisphere.

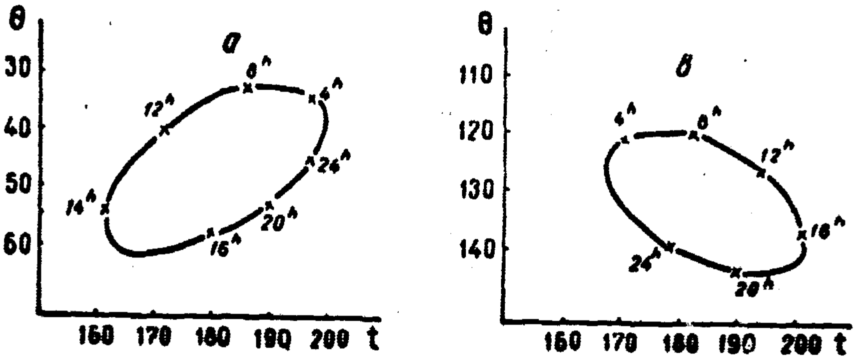

Fig. 1. Trajectory of the focuses movement
a) northern hemisphere; b) southern hemisphere
x = points where focuses are located at a given
universal time

Fig. 2 shows the variations of the magnitude of the currents functions in the focuses of the two hemispheres, depending upon the universal time. As can be determined from the drawing, at given assumptions, the universal time variations in intensities of the Sq-turbulences in the two hemispheres of the ionosphere occur in the anti-phase.

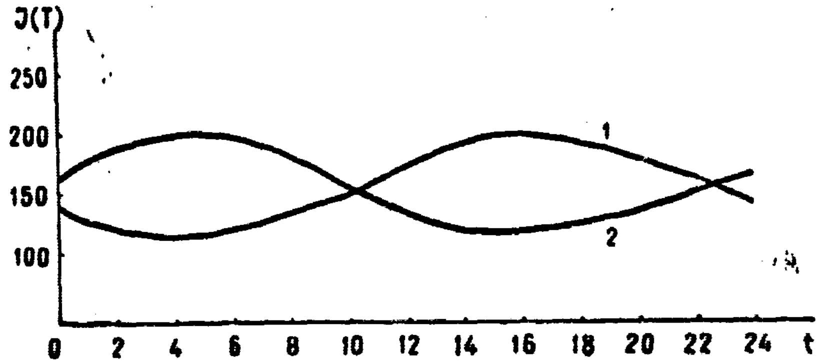

Fig. 2. Chart of function I(T) in focuses:
I, Northern hemisphere; II, southern hemisphere
I is measured in units of 1000 amp.

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