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ABSTRACT

A theory is presented to explain the observed variations of the magnetic field and plasma in the vicinity of the moon. Under the guiding-center approximation, solutions for the plasma flow near the moon are obtained from the kinetic equation. The creation of a plasma cavity in the core region of the lunar optical shadow disturbs the interplanetary magnetic field. Maxwell's equations are used to study perturbations of the magnetic field in the lunar wake. The acceleration drift current, which was omitted from the earlier work, is included in the present theory in the calculation of the total electric current in the lunar wake. Numerical solutions of Maxwell's equations are obtained. When the interplanetary magnetic field lines penetrate into the lunar body, due to sudden change of magnetic permeability the magnetic field is disturbed at the lunar limbs. Propagations of this disturbance with magnetoacoustic speed form a Mach cone downstream, which is sometimes observed as the exterior increase of field magnitude in the lunar penumbra. Perturbations of the magnetic field are restricted to the region inside the Mach cone; the region outside remains undisturbed. The numerical results agree extremely well with experimental data from the Explorer 35 spacecraft.

I. INTRODUCTION

Measurements[1-6] of the interplanetary magnetic field and plasma in the vicinity of the moon have been made from lunar orbit on the Explorer 35 spacecraft. The purpose of this paper is to present a theory which can explain the observed variations of the field and plasma in the lunar wake.

When the solar wind interacts with the moon, no shocks are observed in the vicinity of the moon. Figure 1 shows a simultaneous measurement of the interplanetary field and plasma on Explorer 35 when the moon is outside the earth's bow shock. The major effect of the moon on the solar wind plasma is the creation of a plasma cavity in the umbral region of the lunar shadow. In this cavity the magnitude of the magnetic field increases, i.e., the field is observed to be stronger than the undisturbed interplanetary field. On either side of the umbral increase, the field decreases, i.e., the field becomes weaker than the undisturbed condition. These penumbral decreases occur at the location where the plasma density is about half of the undisturbed plasma density, and they are often bounded on the exterior by additional small increases in the field magnitude. A positive correlation between the increase of the plasma flux, the plasma density and the penumbral increase of field magnitude has also been observed.

From the kinetic theory point of view, the plasma flow in the vicinity of the moon can be analyzed under a guiding-center approximation. The zeroth-order solution for the plasma flow obtained by Whang[7] is in good agreement with the measured plasma density, plasma flux, and plasma flow direction.

The first order guiding-center theory is used to describe the plasma and the electric current in the lunar wake. Whang[8] has previously studied the perturbation of the magnetic field in the vicinity of the moon by assuming that the total electric current in the lunar wake is composed of the gradient drift current, the curvature drift current and the magnetization current. This present theory produces more realistic results than the earlier theory in three aspects: (1) The present theory infers that perturbations of the magnetic field in the lunar wake are confined to a region bounded by a Mach cone (a standing magnetoacoustic wave), outside of which the interplanetary field is undisturbed. (2) The penumbral decreases of the field magnitude are of the same amplitude as the observed decreases. (3) The umbral increase of the field magnitude is of the same spatial extent as the observed increase. The basic difference of the present theory from Whang's previous one[8] is the inclusion of the acceleration drift current in the calculation of the total electric current in the lunar wake. This current turns out to be at least as large as the other terms.

The solar wind flows approximately radially outward from the sun at 1 AU at a supersonic velocity. The solar magnetic lines of force are carried outward by the solar wind and are twisted into the form of Archimedean spirals, thus the interplanetary magnetic field is in general not aligned with the direction of the solar wind velocity. Thermal motion of the solar wind ions is anisotropic; the ratio of $T_{\parallel}$ and $T_{\perp}$ varies between 1 and 4. In the umbral region immediately behind the moon, only those ion particles with high parallel thermal energy are present, while the perpendicular thermal energy is directly proportional to field intensity. Thus, the plasma thermal anisotropy, $T_{\parallel}/T_{\perp}$, increases in the core of the lunar wake. The propagation speed of magnetoacoustic waves in an anisotropic plasma is again highly anisotropic when the thermal energy of the anisotropic solar wind plasma is of the same order of magnitude as the energy of the interplanetary magnetic field. All these real physical conditions of the solar wind are taken into account in the present model.

The solar wind interaction with the moon has also been studied by other authors[6,9-11] using the continuum theory approach. In these models, the thermal anisotropy of the solar wind plasma and the anisotropic acoustic waves are not considered, and a simple polytropic relation for pressure and density is used. Some have only considered the case where the magnetic field upstream of the moon is aligned with the direction of the solar wind velocity.

Two dimensional steady-state solutions of Maxwell's equations are studied. Thus the present results actually correspond to the interaction of the solar wind with a cylindrical moon. Under the two-dimensional cylindrical-moon approximation, the solution will become less accurate at large distances behind the moon.

The first order solution of the kinetic equation is also studied in the last part (section VII) of this paper.

II. KINETIC EQUATION

The Larmor radius of the ion particles in the solar wind is only a small fraction of the radius of the moon, therefore, the lowest-order solution for the ion flow in the vicinity of the moon can be calculated[7] by approximating each ion particle by a guiding-center particle. It is convenient to use the guiding-center distribution function[12] which is defined as a function of $\underline{x}$, t, v and μ

$$F = F(\underline{x}, t, v, \mu) \tag{1}$$

where v is the component of the particles velocity parallel to $\underline{B}$, and μ the magnetic moment. From the first order theory of guiding-center motion[13],

$$\frac{dv}{dt} = \underline{e}_1 \cdot \left(\frac{e}{m_i} \underline{E} - \frac{\mu}{m_i} \nabla B + \nabla \frac{U_\perp^2}{2} \right) + v \, \underline{\kappa} \cdot \underline{U}_\perp \, , \tag{2}$$

and

$$\frac{d\mu}{dt} = 0 \, , \tag{3}$$

where $d/dt = \partial/\partial t + (\underline{U}_\perp + \underline{e}_1 v) \cdot \nabla$, $\underline{e}_1 = \underline{B}/B$, $\underline{U}_\perp = c\, \underline{E} \times \underline{B} / B^2$ and $\underline{\kappa} = \underline{e}_1 \cdot \nabla \underline{e}_1$.

The collisionless Boltzmann equation can be expanded following the Chew-Goldberger-Low theory[14,15]. The lowest-order equation can be reduced to

$$\mathcal{D}(F/B) = 0 \, , \tag{4}$$

where the operator $\mathcal{D}$ denotes the time derivative following the trajectory of a guiding-center in phase space,

$$\mathcal{D} = \frac{\partial}{\partial t} + (\underline{U}_\perp + \underline{e}_1 v)\cdot\nabla + \frac{dv}{dt}\frac{\partial}{\partial v} \; . \tag{5}$$

Equation (4) states that F/B is conserved following the trajectory of a guiding-center in phase space.

Solutions of the kinetic equation (4) are studied subjected to the boundary conditions: (i) the moon absorbs all particles that hit its surface, and (ii), the distribution function is Maxwellian in the undisturbed flow upstream. Once F is known, the ion density, parallel velocity, and parallel pressure can be calculated from

$$n = \iint F \, d\mu \, dv \, , \tag{6}$$

$$U_\parallel = \frac{1}{n}\iint v F \, d\mu \, dv \, , \tag{7}$$

and

$$P_\parallel = \iint m_i (v - U_\parallel)^2 F \, d\mu \, dv \, . \tag{8}$$

The average magnetic moment is related to the perpendicular pressure by

$$\begin{aligned} \langle \mu \rangle &= \frac{1}{n}\iint \mu F \, d\mu \, dv \\ &= P_\perp / nB . \end{aligned} \tag{9}$$

Consider that in the undisturbed upstream flow, the solar wind moves with a constant velocity $\underline{U}_o$ with respect to the center of the moon, and carries with it a steady uniform magnetic field $\underline{B}_o$. We choose a coordinate system (Fig. 2) with its origin located at the center of the moon, the x axis parallel to $\underline{U}_o$, and the z axis along the direction of the electric field in the upstream $\underline{E}_o = -\underline{U}_o \times \underline{B}_o / c$.

The scale of the coordinate system is normalized by using the radius of the moon as a unit length.

The zeroth order equations are those in which the guiding-center velocity of each particle is assumed to be constant in both magnitude and direction. In this order the equation may be solved analytically. The earlier theory[7] presented the solution for the special case in which the magnetic field was constant. Here an analytical solution is presented in which the magnitude of the field is allowed to vary.

As in the earlier work the space near the moon is divided into four regions as shown in Fig. 2. Let ϕ_o denote the direction angle (the angle between $-\underline{U}_o$ and $\underline{B}_o$) in the upstream region. The analytical zeroth-order solution gives

$$n/n_o = P_{\parallel}/P_{\parallel o} = B/B_o ,$$

and

$$U_{\parallel} = -U_o \cos \phi_o$$

in region 0. In the other regions the solutions are functions of B/B_o, the two solar wind parameters ϕ_o and S $[S = U_o/(2kT_{\parallel o}/m_i)^{1/2}]$, and the two position parameters

$$\chi = \arcsin\left[(1-z^2)^{1/2}(x^2+y^2)^{-1/2}\right],$$

and

$$\lambda = \arctan(y/x).$$

The solutions are:

$$\frac{n}{n_o} = \frac{1}{2}\frac{B}{B_o}\left[\operatorname{erfc}(Y_1 S) + \operatorname{erfc}(Y_2 S)\right], \qquad (10)$$

$$\frac{U_{\parallel}}{U_0} = -\cos\phi_0 + \frac{\exp(-Y_2^2 S^2) - \exp(-Y_1^2 S^2)}{2\pi^{1/2} S \; nB_0/n_0 B} \tag{11}$$

and

$$\frac{P_{\parallel}}{P_{\parallel 0}} = (n/n_0)\left[1 - S^2(\cos\phi_0 + U_{\parallel}/U_0)^2\right] + $$
$$+ (\pi)^{-1/2}(B/B_0)\, S\left[Y_1 \exp(-Y_1^2 S^2) + Y_2 \exp(-Y_2^2 S^2)\right], \tag{12}$$

where

Y_1	$= \sin(\chi + \lambda)/\sin(\phi_0 - \chi - \lambda)$	in regions 1,3
	$= \infty$	in region 2
Y_2	$= \sin(\chi - \lambda)/\sin(\phi_0 + \chi - \lambda)$	in regions 2,3
	$= \infty$	in region 1 .

Equations (10) - (12) are slightly different from the solution given in reference 7, because the frozen-in effects of the variation of field magnitude are included in the present form. With the inclusion of the factor B/B_0, the present solution is capable of explaining the positive correlation between plasma density and magnetic field strength in the exterior penumbra.

A solution to the first order equations in which the deflection of the guiding-center trajectory in the non-uniform magnetic and electric fields is taken into account is obtained numerically in section VII.

III. MAXWELL'S EQUATIONS

Maxwell's equations are used to describe the variation of the magnetic field. The total electric current is composed of the magnetization current, the gradient drift current, the curvature drift current and the acceleration drift current. Let D/Dt denote the derivative $\partial/\partial t + \underline{U}\cdot\nabla$, $\underline{U}$ the fluid velocity and ρ the fluid density. The acceleration drift current density is

$$\underline{J}_A = \frac{c}{B}\, \underline{e}_1 \times \rho \frac{D\underline{U}}{Dt}.$$

The acceleration drift current is sometimes known as the polarization current[16], and was not included in the early theoretical treatment[3,8] of the solar wind interaction with the moon. This term is at least as important as the other terms of the current density. Maxwell's equations can now be written as

$$\nabla\cdot\underline{B} = 0 \tag{13}$$

and

$$\left(1 + \frac{4\pi P_\perp}{B^2}\right)\nabla\times\underline{B} = \frac{4\pi}{B}\,\underline{e}_1\times\left(P_\perp \nabla \ln n + P_\parallel \underline{\kappa} + \rho\frac{D\underline{U}}{Dt}\right). \tag{14}$$

Here n is the number density of the plasma, $P_\perp$ and $P_\parallel$ respectively the perpendicular and the parallel plasma pressure.

In the wake region, the electric field is only slightly disturbed from the undisturbed electric field $\underline{E}_0$. The perpendicular component of the fluid velocity can be calculated from

$$\underline{U}_{\perp} = c\, \underline{E}_o \times \underline{B} / B^2 . \tag{15}$$

The perpendicular pressure is directly proportional to the product nB. Once n, $U_{\parallel}$ and $P_{\parallel}$ are known functions of $\underline{x}$ and B, Equations (13) and (14) can be solved for the variation of $\underline{B}$ in the lunar wake.

Two dimensional steady-state solutions of Maxwell's equations are studied in this paper.

IV. THE EQUATIONS OF CHARACTERISTICS

In a collisionless plasma, the magnetoacoustic wave[17] propagates along the field direction with speed $a=[(P_\perp - P_\parallel + B^2/4\pi)/\rho]^{1/2}$, and perpendicular to the field direction with speed $b=[(2P_\perp + B^2/4\pi)/\rho]^{1/2}$. The solar plasma flow is supersonic in the vicinity of the moon except in the umbral region of the lunar wake, where the magnetoacoustic speed becomes very large because the plasma density and pressure decreases. In the supersonic region outside the lunar umbra the method of characteristics is used to calculate the variation of the magnetic field.

Let B and ϕ denote respectively the magnitude and the direction angle of the vector $\underline{B}$, and $\underline{e}_2 = \underline{e}_z \times \underline{e}_1$, then one can write

$$\nabla \times \underline{B} = (-\underline{e}_2 \cdot \nabla B + B\, \underline{e}_1 \cdot \nabla \phi)\, \underline{e}_z ,$$

$$\underline{e}_1 \times \nabla \ln n = (\underline{e}_2 \cdot \nabla \ln n)\, \underline{e}_z ,$$

$$\underline{e}_1 \times \underline{\kappa} = \nabla \times \underline{e}_1 = (\underline{e}_1 \cdot \nabla \phi)\, \underline{e}_z$$

and

$$\underline{e}_1 \times (\underline{U} \cdot \nabla \underline{U}) = - \underline{U} \cdot (\nabla U_\perp \underline{e}_2 + U_\parallel \nabla \underline{e}_1) \times \underline{e}_1 = \underline{U} \cdot (- U_\perp \cdot \nabla \ln B + U_\parallel \nabla \phi)\, \underline{e}_z .$$

Making use of these relations, one can write Maxwell's equations [(13), (14)] as

$$\underline{e}_1 \cdot \nabla \ln B + \underline{e}_2 \cdot \nabla \phi = 0 \qquad (16)$$

and

$$\underline{A}_1 \cdot \nabla \ln B + \underline{A}_2 \cdot \nabla \phi = H , \qquad (17)$$

where

$$\underline{A}_1 = U_\perp \underline{U} - b^2 \underline{e}_2 ,$$

$$\underset{\sim}{A}_2 = a^2 \underset{\sim}{e}_1 - U_{\parallel} \underset{\sim}{U},$$

and

$$H = (P_{\perp}/\rho)\, \underset{\sim}{e}_2 \cdot \nabla \ln (n/B).$$

We now consider the linear combination of the two equations (16) and (17) obtained by multiplying equation (16) by a factor τ and add the two equations,

$$\underset{\sim}{L} \cdot \nabla \ln B + \underset{\sim}{M} \cdot \nabla \phi = H \tag{18}$$

where

$$\underset{\sim}{L} = (U_{\perp} U_{\parallel} + \tau)\, \underset{\sim}{e}_1 + (U_{\perp}^2 - b^2)\, \underset{\sim}{e}_2$$

and

$$\underset{\sim}{M} = (a^2 - U_{\parallel}^2)\, \underset{\sim}{e}_1 + (\tau - U_{\perp} U_{\parallel})\, \underset{\sim}{e}_2 .$$

The magnitude and the direction of vectors $\underset{\sim}{L}$ and $\underset{\sim}{M}$ may be adjusted by the variation of the factor τ. Now, suppose for some choice of τ, the vectors $\underset{\sim}{L}$ and $\underset{\sim}{M}$ are parallel and tangent to a curve C; then only the derivatives along the C-direction are contained in equation (18). In this case the curve C is known as a characteristic curve.

$\underset{\sim}{L}$ and $\underset{\sim}{M}$ are parallel vectors when

$$\tau^2 = a^2 U_{\perp}^2 + b^2 U_{\parallel}^2 - a^2 b^2$$

or

$$\tau_+ = -\tau_- = (a^2 U_{\perp}^2 + b^2 U_{\parallel}^2 - a^2 b^2)^{1/2} .$$

Substituting these two roots into (18), one can replace the Maxwell's equations by a pair of characteristic equations

$$\begin{aligned} \underset{\sim}{L}_+ \cdot \nabla \ln B + \underset{\sim}{M}_+ \cdot \nabla \phi &= H \\ \underset{\sim}{L}_- \cdot \nabla \ln B + \underset{\sim}{M}_- \cdot \nabla \phi &= H, \end{aligned} \tag{19}$$

where

$$\underset{\sim}{L}_{\pm} = \tau_{\pm}\, \underset{\sim}{e}_1 + \underset{\sim}{A}_1$$

$$\underset{\sim}{M}_{\pm} = \tau_{\pm}\, \underset{\sim}{e}_2 + \underset{\sim}{A}_2 .$$

$\underset{\sim}{L}$ and $\underset{\sim}{M}$ are parallel vectors, thus each of the above equations contains the derivative of B and ϕ along the direction of a characteristic curve.

The characteristic equations (19) are used to calculate the variation of magnetic field in the supersonic region (where $\tau^2 > 0$) of the lunar wake. In the subsonic and transonic region a different method, which is developed in the next section, is used in calculations.

In equation (19), if all three terms in one of the two equations are zero, its solution may be called the simple wave solution in frozen-in plasmas.

V. THE UMBRAL ANALYSIS

The method of characteristics derived in section IV can be used to calculate the variations of magnetic field in the supersonic region of the lunar wake. In the subsonic and transonic regions of the lunar wake, approximate solutions of Maxwell's equations are calculated under the assumptions (i) the magnetic field, $\underline{B}$, changes rapidly in the y-direction, but very slowly along the x-direction, and (ii) B_x and B_y are of the same order of magnitude.

The divergence of $\underline{B}$ equation requires that $\partial B_x/\partial x = -\partial B_y/\partial y$. Thus the order of magnitude for the derivatives of $\underline{B}$ can be measured by

$$\frac{\partial B_x}{\partial y} : \frac{\partial B_x}{\partial x} : \frac{\partial B_y}{\partial y} : \frac{\partial B_y}{\partial x} \sim 1 : \epsilon : \epsilon : \epsilon^2 .$$

Dropping out the second order terms, Maxwell's equations (13), (14) may be written as

$$Z_1 \frac{\partial B_x}{\partial y} = Z_2 + Z_3 \frac{\partial B_y}{\partial y} , \tag{20}$$

where

$$Z_1 = a^2 B_y^2 + b^2 B_x^2 - U_y^2 B^2 ,$$

$$Z_2 = -(B^3 P_\perp / \rho)\, \underline{e}_1 \times \nabla \ln (n/B) ,$$

and

$$Z_3 = 2(a^2 - b^2) B_x B_y - 2 B^2 U_x U_y .$$

For given x, the method of characteristics is used to determine the solution for $\underline{B}$ in the supersonic region outside the lunar umbra. Thus, conditions at the lower and the upper boundary of the umbra are determined from the characteristics solution,

$$B_x = B_{x1} \quad \text{at} \quad y = y_1 \tag{21}$$

$$B_x = B_{x2} \quad \text{at} \quad y = y_2 \tag{22}$$

$$B_y = B_{y1} \quad \text{at} \quad y = y_1 \tag{23}$$

$$B_y = B_{y2} \quad \text{at} \quad y = y_2 . \tag{24}$$

The divergence of $\underline{B}$ equation also requires that, when x is fixed

$$\left(\int_{-\infty}^{y_1} + \int_{y_1}^{y_2} + \int_{y_2}^{\infty} \right)(B_x - B_o \cos \phi_o) dy = 0 . \tag{25}$$

Subjected to the above five conditions, Eq. (20) can be integrated numerically for the solution of $\underline{B}$ in the interval $y_1 \leq y \leq y_2$.

In Eq. (20), the density gradient in Z_2 is the main driving force for the variation of B_x along the y-direction in the umbral region. Integration of Eq. (20) is carried out by assuming a series form for the first order term $\partial B_y / \partial y$. Let

$$B_y = c_o + c_1 y + c_2 y^2 + c_3 y^3 , \tag{26}$$

where the four constants c_o, c_1, c_2, c_3 are adjusted to satisfy the conditions (21)-(25). Making use of the condition (23) and (24), c_2 and c_3 can be determined as functions of c_o, c_1. As the first order term $\partial B_y / \partial y$ is a known function of c_o, c_1 and y, numerical integration of Eq. (20) can be carried out starting at $y = y_1$ with $B_x = B_{x1}$. The solution for B_x will be in the form of

$$B_x = B_x(c_o, c_1, y) . \tag{27}$$

Here we still have two adjustable constants c_o and c_1. Their values can be determined when both Eq. (22) and Eq. (25) are satisfied. The method described above indicates that by assuming a cubic polynomial solution for the first order term, one can calculate the solution for B_x satisfying all required conditions.

VI. NUMERICAL SOLUTION

Given the speed ratio S and direction angle ϕ_o, the zeroth order solutions of n/n_o, $U_{\parallel}/U_o$ and $P_{\parallel}/P_{\parallel o}$ for ion particles can be calculated from Equations (10) - (12). These analytical solutions are used to approximate the plasma conditions in the lunar wake. Then based on the mathematical method developed in the two preceding sections, solutions for the perturbation of magnetic field in the lunar wake can be carried out numerically. The results quantitatively reproduce the major features of the observed magnetic field perturbations in the lunar wake.

The perturbations of the magnetic field in the lunar wake depend upon four solar wind parameters: the plasma anisotropy $T_{\parallel o}/T_{\perp o}$, the speed ratio S, the direction angle ϕ_o, and the parameter $\beta = 8\pi P_{\perp o}/B^2$. β is defined as the ratio of the perpendicular thermal pressure to the field pressure (the plasma thermal pressure includes both the ion and the electron thermal pressure). For typical interplanetary conditions at the orbit of the earth, the ratio of $T_{\parallel}$ and $T_{\perp}$ varies between 1 and 4, the speed ratio S is of the order of $7 \sim 10$, the magnetic field has a direction angle $\phi \sim 135^\circ$ or 315° and the β value is of the order of unity. Experimental evidence has shown that the field anomalies are strongly affected by the β value[18].

A numerical solution for some typical solar wind parameters is plotted in Figures 3 and 4.

Figure 3 shows the variation of the field magnitude in the lunar wake region. The exterior increase of the field magnitude actually represents a standing magneto-acoustic wave attached to the lunar limbs; it resembles the Mach cone in aerodynamics.

Perturbations of the magnetic field are restricted to the region bounded by the Mach cone; the region outside remains unperturbed.

Figure 4 shows that the width of the umbral increase is always slightly less than one lunar diameter. This feature agrees with observed data, and was not predicted by previous theories. The penumbral decrease of field magnitude obtained here again agrees with measurements far better than Whang's early result[8].

The numerical solution is carried out by assuming that the field is slightly disturbed at the initial line $x = 0$ near the lunar limbs. These disturbances propagate downstream to form the standing magnetoacoustic wave, which is observed as penumbral increases. If the field is completely undisturbed at the initial line, then numerical solutions cannot produce the observed penumbral increases. This suggests that when the magnetic field lines penetrate into the lunar body, due to sudden change of magnetic permeability across the sunny side of the lunar surface, the magnetic field is disturbed near the lunar limbs. These disturbances propagate in the supersonic flow to the downstream. If the field magnitude increases at the limbs, the propagation of this disturbance would be observed as a penumbral increase of the field magnitude in the lunar wake.

VII. FIRST ORDER SOLUTION OF KINETIC EQUATION

The zeroth order solutions of the kinetic equation discussed in section II are obtained under the assumption that the guiding-center particles are not accelerated; their trajectories are approximated by straight lines. However since the electric and the magnetic fields are perturbed in the lunar wake, the velocity and the trajectory of a guiding-center will also be perturbed. The effect of the induced electric field on the guiding-center trajectories has been discussed by Alfonso-faus and Kellogg[19]. In this section the combined effect of the induced electric field and the perturbed magnetic field are included in calculating the first order solution of the guiding-center kinetic equation.

We consider that in the perturbed region the electron density is equal to the ion density and the electrons always assume an equilibrium distribution in the potential field. The potential of induced electric field depends on the ratio n/n_o according to the logarithmic law

$$\Phi = (kT_e/e)\ \ln(n/n_o) \tag{28}$$

From this equation, one can calculate that the magnitude of the induced electric field is very small compared with the upstream electric field E_o.

The guiding-center kinetic equation states that F/B is constant on the path of a guiding-center in phase space

$$\mathcal{D}(F/B) = 0$$

One may write the operator $\mathcal{D}$ in the following form

$$\mathcal{D} = R_1 \frac{\partial}{\partial x} + R_2 \frac{\partial}{\partial y} + R_3 \frac{\partial}{\partial v}, \tag{29}$$

which indicates that the guiding-center trajectory is described by

$$\frac{dx}{R_1} = \frac{dy}{R_2} = \frac{dv}{R_3}. \tag{30}$$

The expressions for R_i's are of the following forms:

$$R_1 = U_\perp \sin\phi - v\cos\phi,$$

$$R_2 = -U_\perp \cos\phi - v\sin\phi,$$

and

$$R_3 = \frac{dv}{dt} = \underset{\sim}{e}_1 \cdot \left(\nabla \frac{U_\perp^2}{2} - \frac{\mu}{m_i}\nabla B - \frac{e}{m_i}\nabla\Phi\right) + v\,\underset{\sim}{\kappa}\cdot\underset{\sim}{U}.$$

The procedure of calculating the first order solution for the distribution function $F(v,\mu)$ at any point in the perturbed region is the following. If $\underset{\sim}{B}$ and Φ are known, then one can integrate Eq. (30) to obtain the trajectory of a guiding center and the variation of v along its trajectory. The magnetic moment μ is an adiabatic invariant as the guiding center moves. In the undisturbed region upstream, the guiding-center trajectories are straight lines, and the distribution function is Maxwellian, F_o. If the trajectory intercepts the lunar surface, the distribution function $F(v,\mu)$ is zero. Otherwise, the trajectory will end at the undisturbed region with its parallel velocity changed to v_o, the distribution function can now be calculated from

$$F(v,\mu) = (B/B_o)\,F_o(v_o,\mu). \tag{31}$$

Following this procedure, one can numerically calculate the guiding-center distribution function at any point in the perturbed region as functions of v and μ. Taking moments of this distribution function one can calculate the first order solution for n, $U_{\parallel}$ and $P_{\parallel}$ at the point of interest.

One further simplifying assumption is used to carry out some first order solutions in this paper, that is, all particles are assumed to possess the same magnetic moment

$$\mu = P_{\perp o} / n_o B_o .$$

Under this assumption, the guiding-center plasma has thermal motion only along the field line; its distribution function depends on one velocity variable v . The plasma behaves as a one-dimensional gas.

Consider that (i) the electron temperature $T_e = 2T_{\parallel o}$, (ii) the potential is calculated from the zeroth-order n, and (iii) the magnetic field $\underline{B}$ is calculated from the method outlined in the previous sections. One can calculate the first-order solution for F. Some first order solutions are plotted in Figure 5. The first order solution shows a slightly decrease of the plasma density gradient across the boundary. The analytical solution for the zeroth order density is practically good enough for the purposes of understanding the plasma flow in the lunar wake.

Figure 5 also shows that (i) the penumbral decreases of field magnitude occur at the location where the plasma density is about half of the upstream density, and (ii) a positive correlation exists between the increases of plasma density and magnetic field strength in the penumbra.

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FIGURE CAPTIONS

Figure 1 The simultaneous measurements of field and plasma obtained on August 5, 1967 from lunar orbit on the Explorer 35 spacecraft. The trajectory of the spacecraft is shown projected on the ecliptic plane and positionally correlated with the data through UT annotation. The x axis is parallel to the sun-moon line.

Figure 2 Four characteristic regions of perturbed plasma flow in the vicinity of the moon.

Figure 3 Distributions of the magnitude of magnetic field in the lunar wake. Perturbations of the field are restricted to the region bounded by the Mach cone. The dashed lines indicate the direction of plasma velocity.

Figure 4 Perturbations of the field magnitude in the lunar wake calculated for $T_{\parallel o}/T_{\perp o} = 1.5$, $S = 10$, $\Phi_o = 135°$, and $\beta = 1$.

Figure 5 The zeroth order and the first order solutions of the ion density. The first order solutions are calculated for $T_e = 2T_{\parallel o}$.

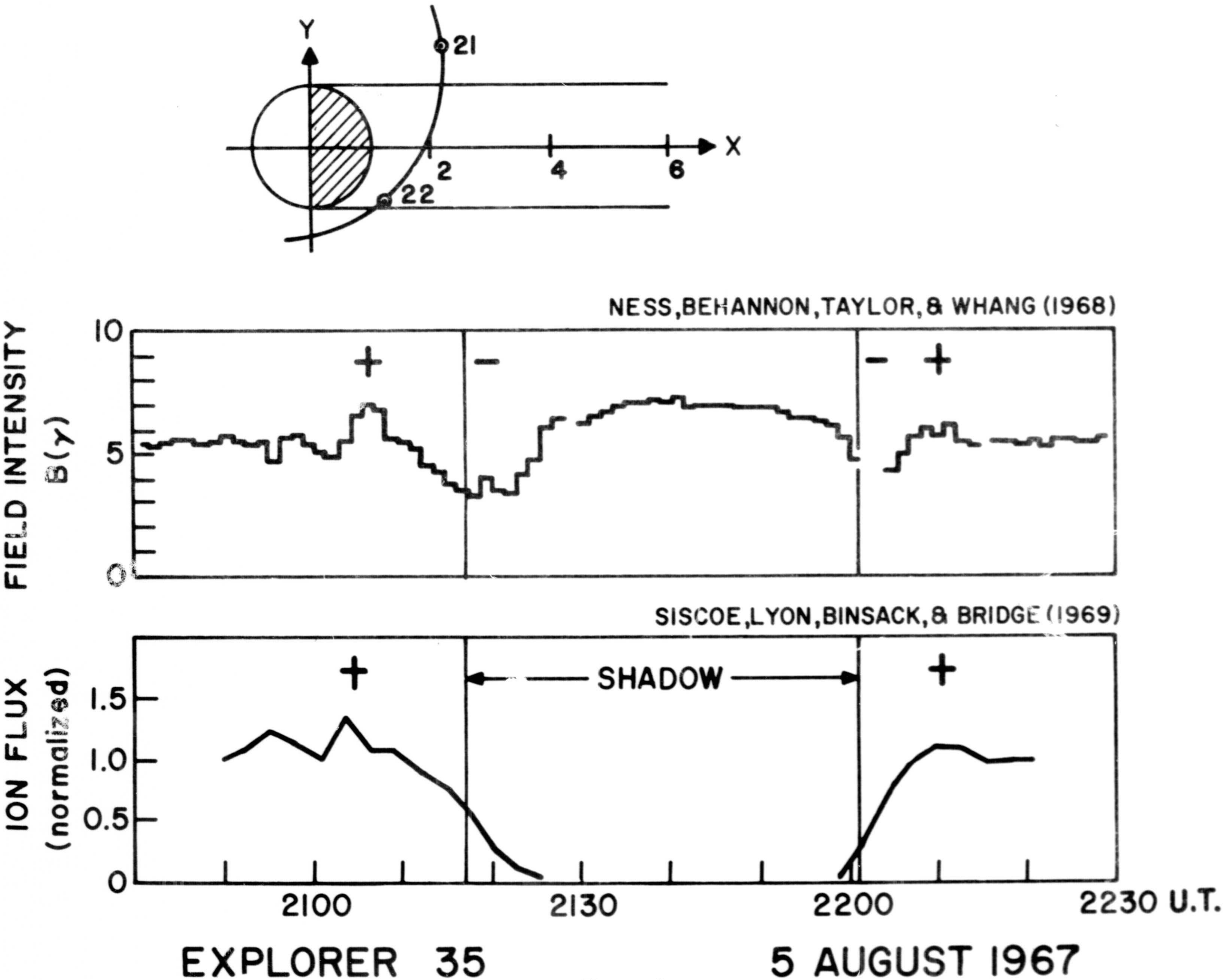

Y
X
21
22
2
4
6
NESS, BEHANNON, TAYLOR, & WHANG (1968)
FIELD INTENSITY
B(γ)
10
5
0
+
-
- +
SISCOE, LYON, BINSACK, & BRIDGE (1969)
ION FLUX
(normalized)
1.5
1.0
0.5
0
+
SHADOW
+
2100
2130
2200
2230 U.T.
EXPLORER 35
5 AUGUST 1967

Figure 1

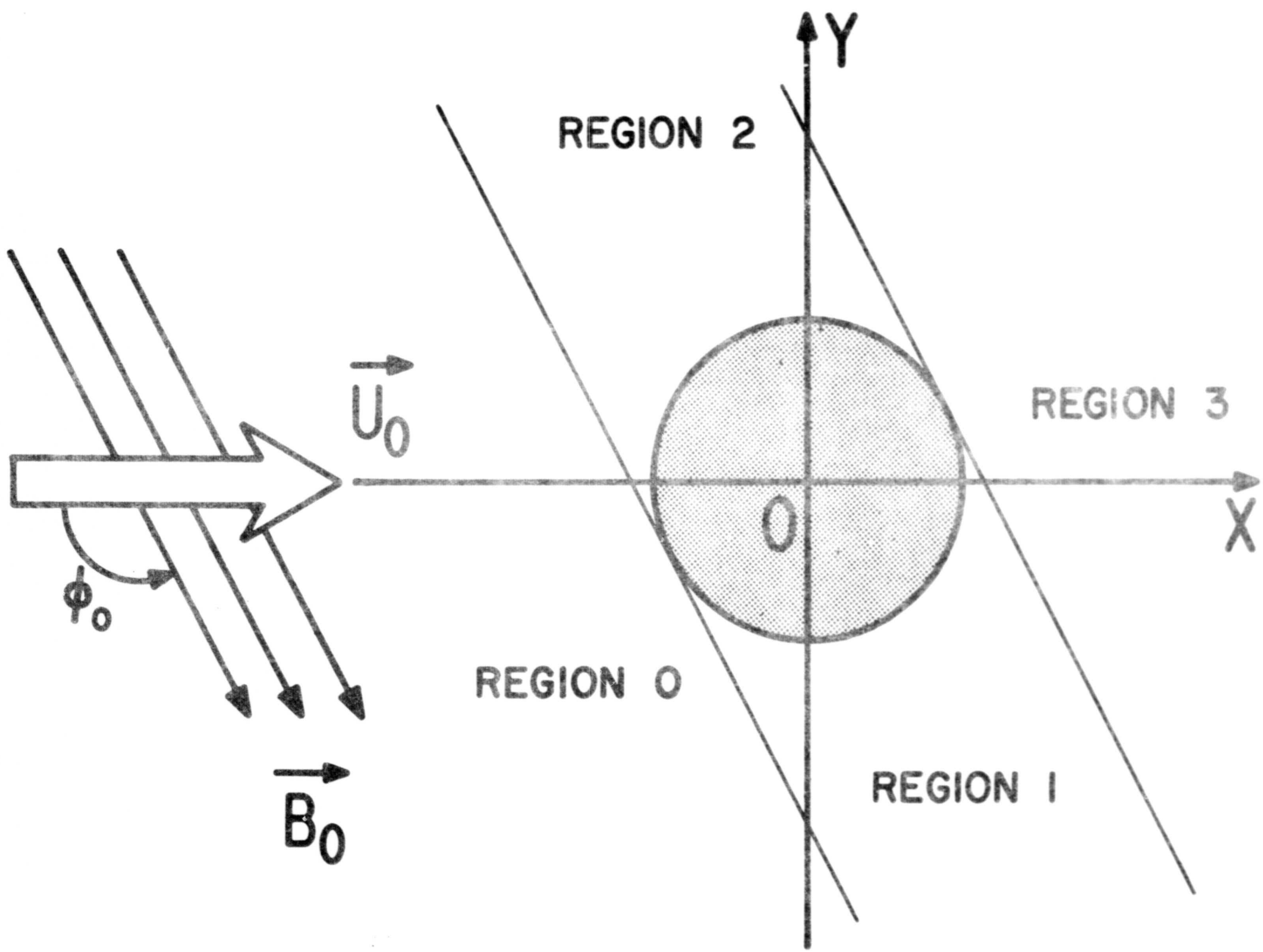


Figure 2

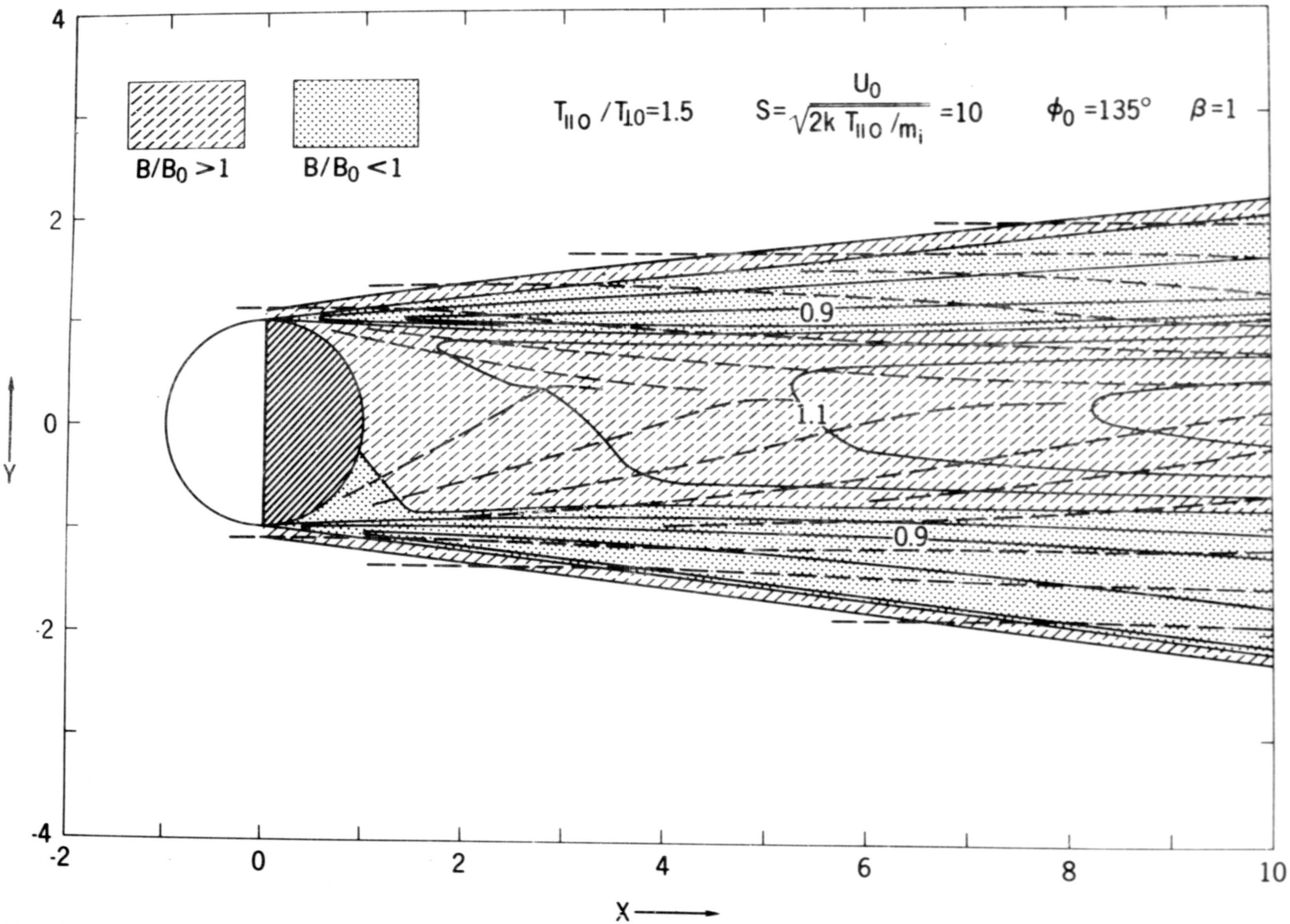

B/B0 >1
B/B0 <1
T∥0 / T⊥0=1.5
S= U0 / √(2k T∥0 /mi) =10
φ0 =135°
β=1
0.9
1.1
0.9
4
2
0
-2
-4
-2
0
2
4
6
8
10
X
Y

FIGURE 3

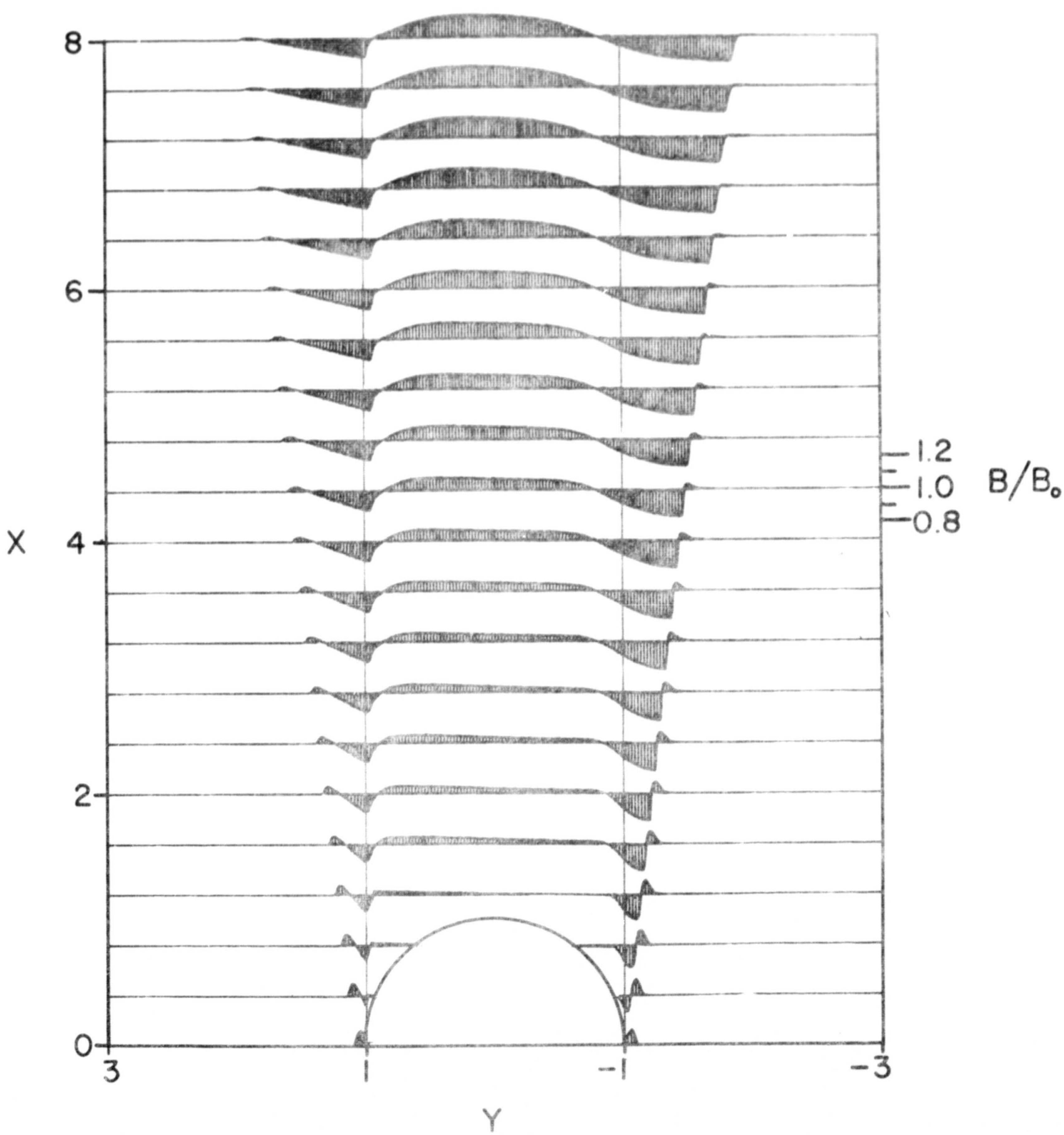
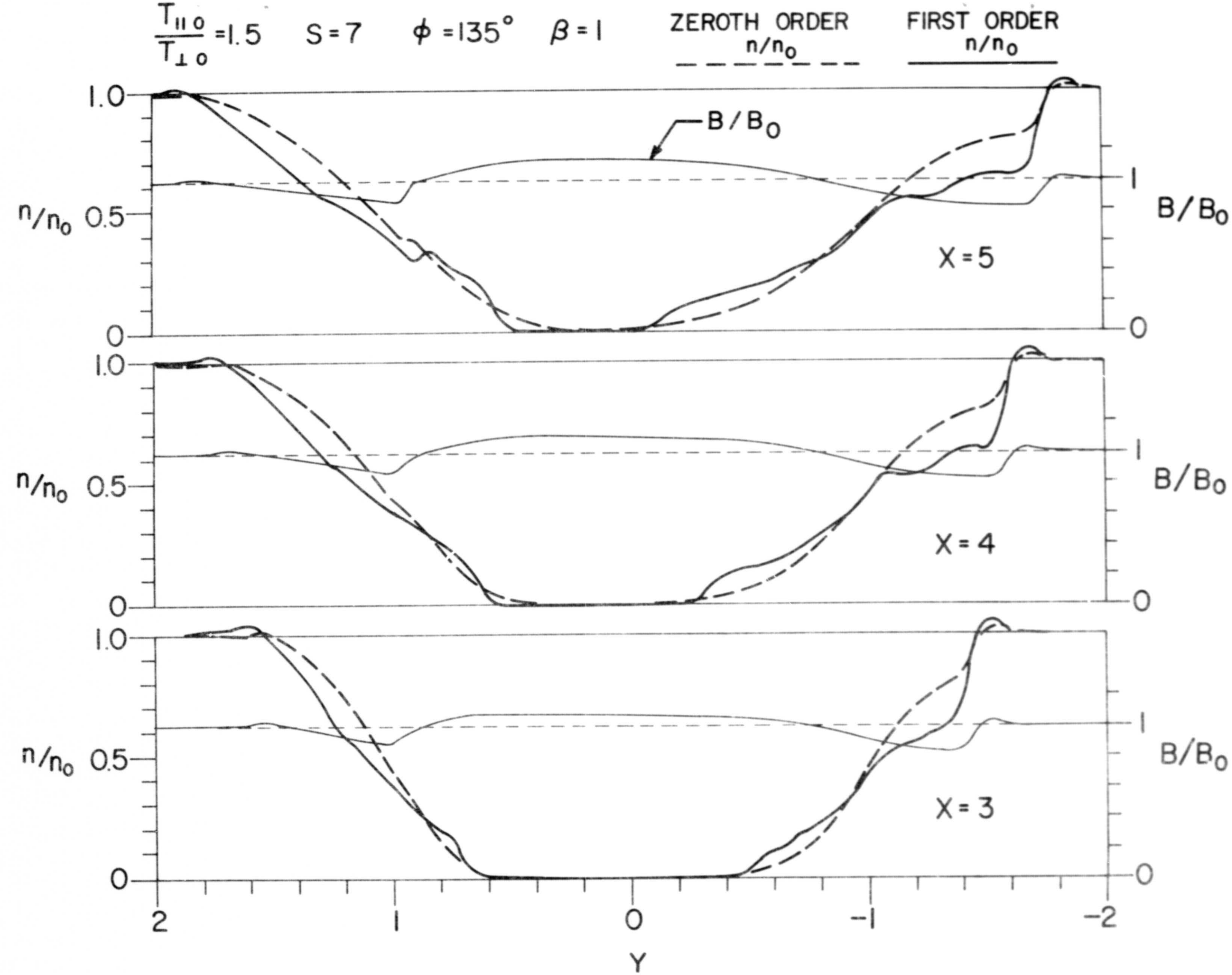


Figure 4



$\frac{T_{\parallel 0}}{T_{\perp 0}} = 1.5$ S = 7 $\phi = 135°$ $\beta = 1$
ZEROTH ORDER n/n_0
FIRST ORDER n/n_0
B/B_0
n/n_0
X = 5
X = 4
X = 3
Y

Figure 5