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VARIATION OF THE ION TEMPERATURE GRADIENT ALONG FIELD LINES IN THE OUTER PLASMASPHERE

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J. M. Grebowsky and N. K. Rahman*

ABSTRACT

The equations of thermal diffusion are used to explore the variation of the ion temperature in the outer plasmasphere, where H^+ is the dominant positive ion. It is shown that the ion temperature gradient component in the direction of the magnetic field is a function of only the ratio of the H^+ scale height to the He^+ scale height along the field direction when either the ion and electron temperatures are equal or the scale height ratio is unity. The ion temperature gradients are determined throughout the plasmasphere where the H^+, He^+ concentration ratio is assumed to be spatially invariant. At an altitude of 1000 kilometers in the protonosphere, the magnitude of the computed upward directed ion temperature gradient component along the field line increases from 0.9°K/Km near L = 1.3 to 1.5°K/Km at L = 5. The computed ion temperature variations are in general agreement with electron temperature observations in the topside ionosphere and the outer plasmasphere.

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INTRODUCTION

Ion composition measurements made by the ion mass spectrometers on the eccentric orbiting satellites OGO 1 and OGO 3 (Taylor et al., 1965; Brinton et al., 1968) have indicated that the H^+, He^+ concentration ratio is relatively constant (of the order of 100/1) from perigee to the highest altitudes at which ions were detected near 30,000 kilometers. It has been noted by Walker (1967) and Colin et al., (1969) that this behavior of the light ions may be attributed to the existence of upward directed plasma temperature gradients along the field lines.

In this paper the equations of thermal diffusion (Walker, 1967) are used to determine the relationship between the plasma scale heights and the temperature gradients along the field lines in the outer plasmasphere (i.e., in the protonospheric region bounded by the plasmapause) assuming a state of stationary diffusive equilibrium prevails. The magnitudes of the ion temperature gradients are explicitly determined for a model plasmasphere in which the H^+, He^+ concentration ratio is constant.

THEORY

Assuming the ambient H^+, He^+, e^- plasma existing at altitudes within the outer plasmasphere is in a state of diffusive equilibrium, the total ion-electron pressure variation along a magnetic field line is governed by the hydrostatic balance equation:

$$\frac{dP}{ds} = -\rho g_{11} \tag{1}$$

where

$$P = [n(H^+) + n(He^+)]\, k\, [T_e + T_i] \tag{2}$$

and where ρ is the total mass density, n is the number density, k is the Boltzmann constant, T_i is the ion temperature, T_e is equal to the electron temperature, s is the spatial coordinate along a field line measured from the surface of the earth, and g_{11} is the magnitude of the downward gravitational acceleration component along the field line minus the centrifugal acceleration due to corotation with the earth.

Since the H^+ density in the upper plasmasphere is generally more than 100 times larger than the He^+ density (Taylor et al., 1965), the total mass density is to a good approximation equal to the H^+ mass density. Another assumption to be made is that throughout the plasmasphere

$$\left| \frac{dn(He^+)}{ds} \right| << \left| \frac{dn(H^+)}{ds} \right| . \tag{3}$$

Using the relationship

$$\frac{dn}{ds} = -\frac{n}{h} \tag{4}$$

where h is the scale height distance along a field line, it is readily seen that (3) is satisfied in the outer plasmasphere where $n(H^+) \approx 100\ n(He^+)$ and where the ratio $h(H^+)/h(He^+)$ is of the order of unity.

Under the above assumptions, the hydrostatic equation (1) can be written

$$\frac{1}{h(H^+)} = \frac{m(H^+)\, g_{11}}{k(T_e + T_i)} + \frac{1}{(T_e + T_i)} \frac{d}{ds} (T_e + T_i) \tag{5}$$

and from the theory of thermal diffusion (see Walker, 1967 and Burgers, 1960 for a complete discussion of the thermal diffusion equations) the equation which governs the distribution of the minor ion He^+ along a field line is

$$\frac{1}{h(He^+)} = \frac{m(He^+)\, g_{11}}{kT_i} - \frac{T_e}{T_i} \frac{1}{h(H^+)} + \frac{(1 - \alpha)}{T_i} \frac{dT_i}{ds} + \frac{1}{T_i} \frac{dT_e}{ds} \tag{6}$$

where the thermal coefficient α equals 1.7 for the plasma model under consideration.

The relationship between the ion temperature gradient and the scale height ratio $h(H^+)/h(He^+)$ is determined by dividing equation (6) by (5):

$$\frac{dT_i}{ds} = \frac{\left[\frac{T_e}{T_i} + \frac{h(H^+)}{h(He^+)}\right]\left[\frac{m(H^+)\, g_{11}}{k} + \frac{dT_e}{ds}\right] - \left[\frac{T_e}{T_i} + 1\right]\left[\frac{4m(H^+)\, g_{11}}{k} + \frac{dT_e}{ds}\right]}{1 - \frac{h(H^+)}{h(He^+)} - 1.7\left[\frac{T_e}{T_i} + 1\right]} \tag{7}$$

When either the ion and electron temperatures are equal everywhere or the ratio $h(H^+)/(h(He^+)$ is unity, then it follows from equation (7) that

$$\frac{dT_i}{ds} = \frac{\left(7 - \frac{h(H^+)}{h(He^+)}\right) \frac{m(H^+) g_{11}}{2k}}{\left(\frac{h(H^+)}{h(He^+)} + 0.7\right)} . \qquad (8)$$

Hence, under the above conditions, if the local ratio of the H^+ to He^+ scale heights along the field line is known at any point in the outer plasmasphere, the ion temperature gradient component along the field direction at that point can be determined without specification of any other plasma parameter.

In Figure 1, it is seen that one branch of the hyperbolic relation (8) for dT_i/ds (in units of $m(H^+)\ g_{11}/2k$) corresponds to the scale height ratios $h(H^+)/h(He^+) > -0.7$ and the other branch to the ratios $h(H^+)/h(He^+) < -0.7$. Setting $T_e = T_i$ in equation (5) it is readily seen that when g_{11} is positive, the former branch corresponds to a diffusive equilibrium state in which the H^+ density locally decreases (i.e., $h(H^+) > 0$) with altitude along the field lines, whereas the latter branch describes local plasma states in which the H^+ density increases with altitude locally (i.e., $h(H^+) < 0$). As a function of the scale heights ratio the He^+ density can only increase with altitude when

$$-0.7 < \frac{h(H^+)}{h(He^+)} < 0$$

which corresponds to physical states in which very large temperature gradients may exist since the magnitude of the temperature gradient diverges as $h(H^+)/h(He^+)$ approaches -0.7.

MODEL TEMPERATURE GRADIENTS

Since the ratio of the He^+ density to the H^+ density is observed to be approximately constant throughout the plasmasphere regions traversed by the eccentric orbiting satellites OGO 1 and OGO 3 (Taylor et al., 1965; Brinton et al., 1968), the magnitudes of the ion temperature gradients along the magnetic field lines in the outer plasmasphere can be determined by setting the ratio $h(H^+)$ / $h(He^+)$ equal to unity in equation (7), or equivalently in equation (8). Ion temperature gradients (Figure 2) were computed in this manner from the equator to an altitude of 1000 kilometers on dipole field lines characterized by L coordinates between 1.3 and 5. Since the present analysis is only valid in the protonosphere, the computations at the low altitudes can only be valid on the night side where the O^+ - H^+ transition level is below 1000 kilometers in contrast to the dayside where the O^+ - H^+ transition may occur above 1000 kilometers (Mayr et al., 1967).

At the 1000 kilometer altitude, the computed temperature gradients increase with geomagnetic latitude (i.e., with the L coordinate) from approximately 0.9°K/Km near 19° latitude (L = 1.3) to 1.5°K/Km near 61° latitude (L = 5). The magnitudes of these gradients are generally consistent with the results of Mahajan and Brace (1968) who have detected, using Alouette II and Explorer XXII measurements, electron temperature gradients at 1000 kilometers on the nightside which increase in magnitude from 0.6°K/Km at L = 1.4 to values as high as 2° K/Km at an L coordinate of 5. Colin et al., (1969) from a diffusion study which compared high and low altitude ion composition measurements have also arrived at the conclusion that plasma temperature gradients of the order of 1°/Km exist at this altitude on the night side of the earth.

At the equator the ion temperature gradient component in the field direction vanishes as it must for any physical model which requires symmetry about the dipole equatorial plane. On the L = 5 field line, near the equator the ion temperature gradient becomes slightly negative (i.e., the temperature decreases with increasing altitude along the field line) due to the dominance of the centrifugal acceleration over the gravitational acceleration component.

Under the assumption that the scale heights $h(H^+)$ and $h(He^+)$ are equal throughout the outer plasmasphere, the ion temperature variations along the field lines were computed from the previously derived ion temperature gradients. The resulting ion temperature profiles are plotted in Figure 3. Brace et al. (1967) have shown that the electron temperature at 1000 kilometers altitude on the night side of the earth increases from approximately 1000°K near the equator to a value as high as 3000°K at L = 5. Hence if the ion and electron temperatures are approximately equal at this altitude, then the computed ion temperature variations (Figure 3) require that the ion temperature in the equatorial plane increase with altitude to a value as high as 11,000°K at L = 5. This is in general agreement with the electron temperatures measured near the equatorial plane by IMP 2 (Serbu and Maier, 1966). Hence if the ratio $h(H^+)/h(He^+)$ is to a good approximation equal to unity throughout the outer plasmasphere, then the ion temperatures are everywhere comparable to the electron temperatures.

SUMMARY

If diffusive equilibrium exists within the plasmasphere and nonvanishing plasma temperature gradients exist along the field lines, then the equations of thermal diffusion govern the relationship between the ion composition and the temperature gradient. In the outer plasmasphere, where H^+ is the major ion and He^+ is present in small quantities, the ion temperature gradient component in the direction of the magnetic field is a function of only the H^+, He^+ scale height ratio when either (a) the electron and ion temperatures are equal everywhere, or (b) the H^+, He^+ scale heights are identical.

Since Taylor et al. (1965) and Brinton et al. (1968) have observed that the H^+, He^+ concentration ratio appears constant along the OGO 1 and OGO 3 trajectories, the above condition (b) was applied in order to determine the ion temperature gradients in the outer plasmasphere. The model ion temperature variations computed from these gradients are in general agreement with measured electron temperature variations in the outer plasmasphere.

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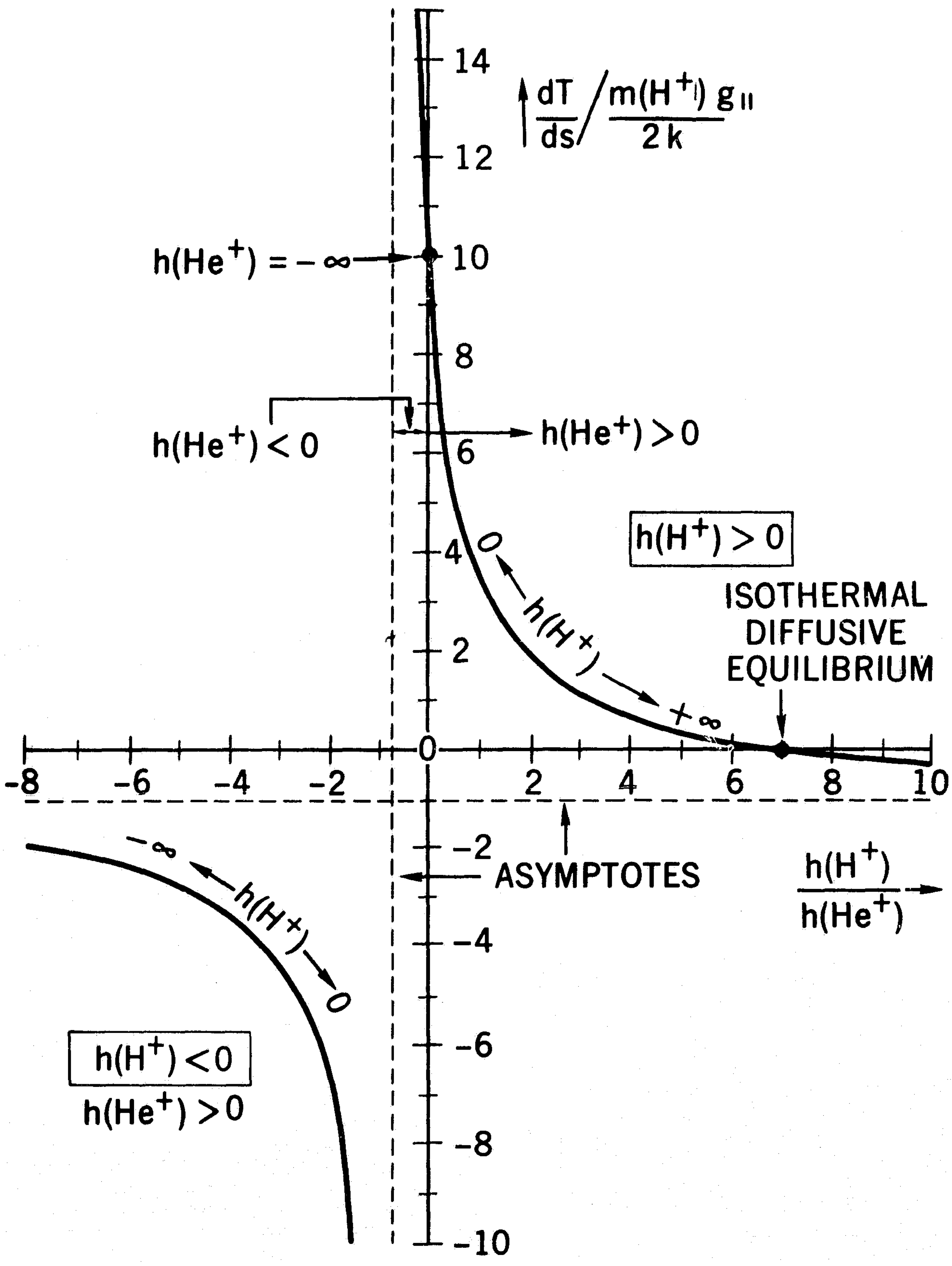

Figure 1—The ion temperature gradient component along a field line is shown as a function of the ratio of the H^+, He^+ scale heights along the field line. The different equilibrium regimes are explicitly labeled. It is assumed that the resultant external force is directed along the field line towards the earth (i.e., $g_{11} > 0$) and that $T = T_e = T_i$.

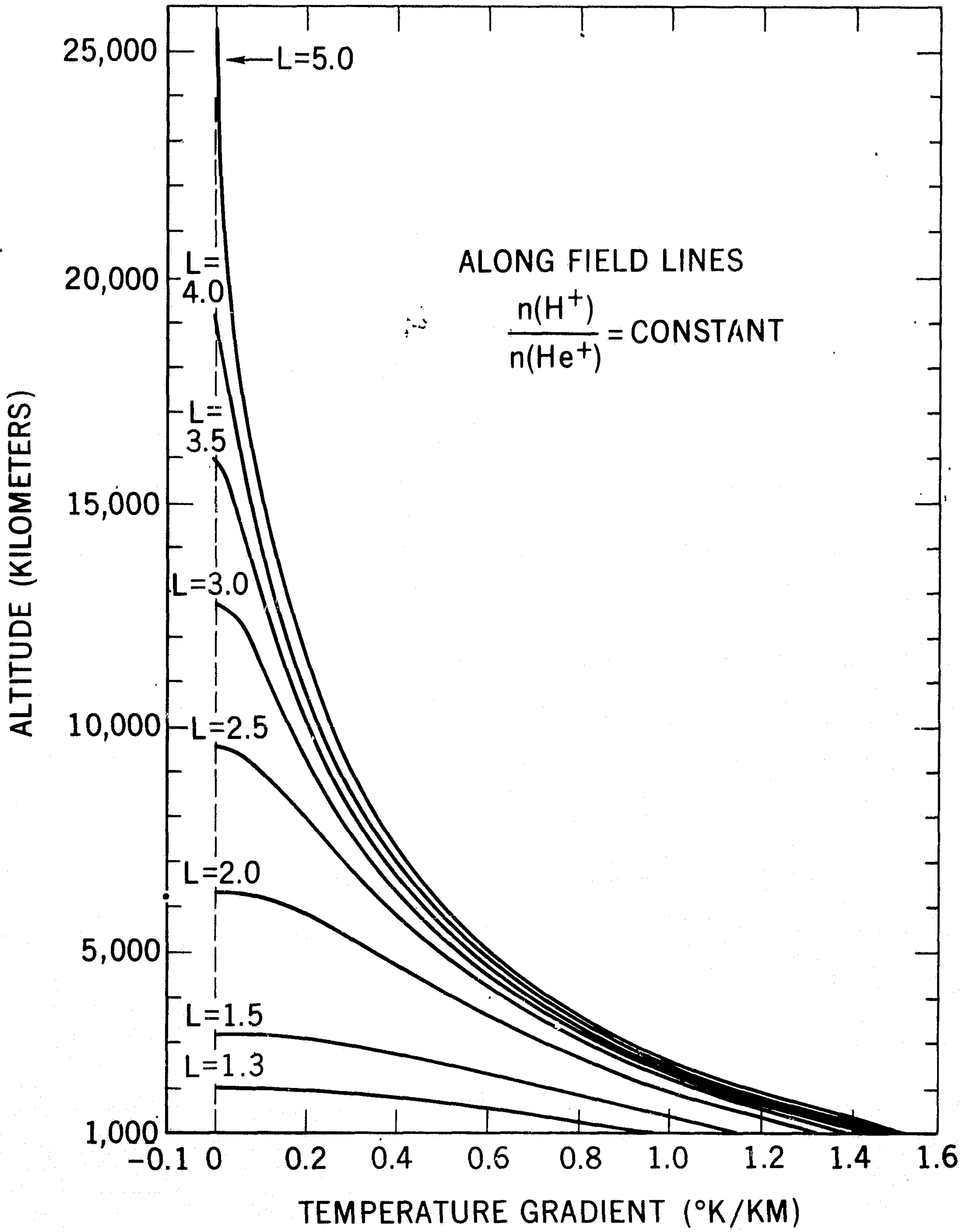

Figure 2–The magnitude of the upward directed ion temperature gradient component along the field line is plotted as a function of altitude assuming the H^+, He^+ scale height ratio is unity at each point.

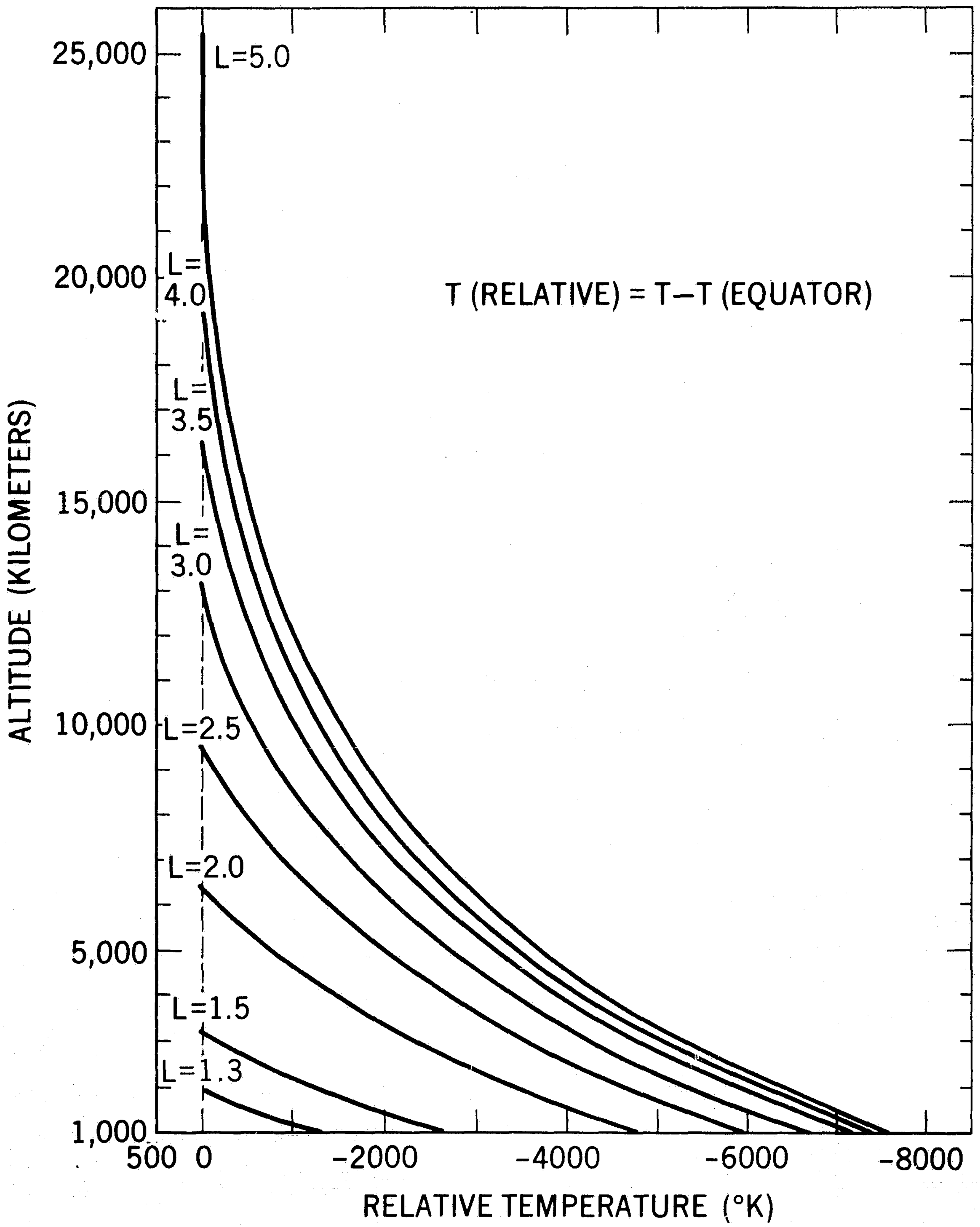

Figure 3—The ion temperature variation along the dipole field lines is shown relative to the equatorial temperature. It is assumed that the H^+, He^+ scale height ratio is unity everywhere.