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PARTICLES OF LUNAR ORIGIN

1. DETERMINATION OF ELEMENTS OF GEOCENTRIC ORBITS OF LUNAR PARTICLES IN THE SPATIAL CASE

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I. DETERMINATION OF ELEMENTS OF GEOCENTRIC ORBITS OF LUNAR PARTICLES IN THE SPATIAL CASE

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ABSTRACT

Elements are derived of geocentric orbits of particles ejected from the Moon by shock explosions of meteorites as a function of primary conditions of ejection: selenocentric longitude λ_0 and latitude ϕ_0, initial velocity V_0, azimuth A_0 and the zenithal distance δ_0 of the direction of ejection.

It is shown that the lines of nodes of geocentric orbits of particles of lunar origin are concentrated near the position of the Moon at time of particle ejection.

The region of the lunar surface is determined, from which vertical motion of particles toward the Earth is possible.

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Part of the lunar matter ejected by the impact of a meteoritic body returns to the Moon, while the other drifts away in outer space [1, 2].

A certain fraction of lunar particles probably hits the meteoric ring around the Earth which is disposed along the entire lunar orbit [3].

We shall examine the spatial motion of lunar particles of dimension greater than 10^{-4} cm with the help of the approximate method of spheres of influence. In this case the trajectory is found by conjugation of the Kepler

(*) [Translated by request through GSFC Procurement].

selenocentric orbit in the sphere of influence of the Moon with the Kepler geocentric orbit in the sphere of influence of the Earth [4, 5].

The Moon (L_0), with the part of the sphere of influence of radius r_{inf} = 102,000 km surrounding it, is shown in Figure 1 hereafter. From a certain part of the lunar surface $M_0(\lambda_0, \phi_0)$ the particle escapes in the direction M_0M_0'. The vertical (radial) direction of motion is preserved through the particle's flying out of the sphere of influence of the Moon. It is evident that in this case the selenocentric coordinates of the point of emergence on the sphere of influence of the Moon will be as follows:

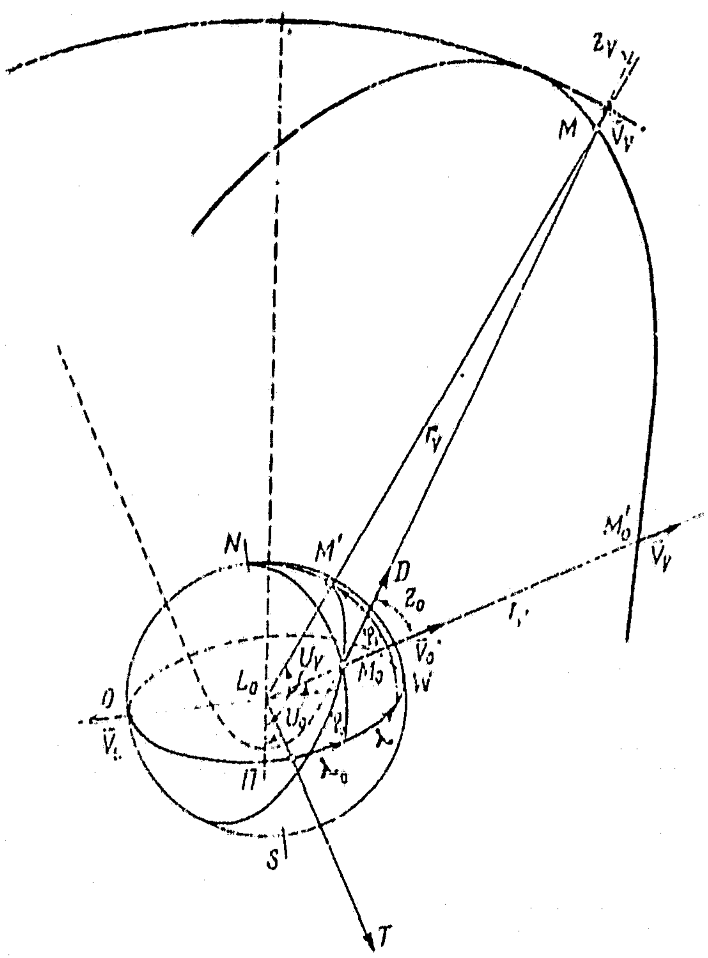

Fig.1. Trajectory of the lunar particle M_0 in the sphere of influence of the Moon

$$\lambda_V = \lambda_0 - \Delta_t, \tag{1}$$

$$\phi_V = \phi_0. \tag{2}$$

The longitude λ_0 has changed because during the flight time t_V of the particle in the sphere of influence of the Moon, the latter moved away along the orbit by an angle Δ_t, which is equal to

$$\Delta_t = \omega_L t_V, \tag{3}$$

where ω_L is the angular velocity of the Moon (Fig.2).

If the particle leaves the Moon in any direction, for example M_0D for an azimuth A_0 and a zenithal distance z_0, the point M of escape from the sphere of influence of the Moon will be displaced by a substantial distance from the corresponding point M_0' of the radial trajectory.

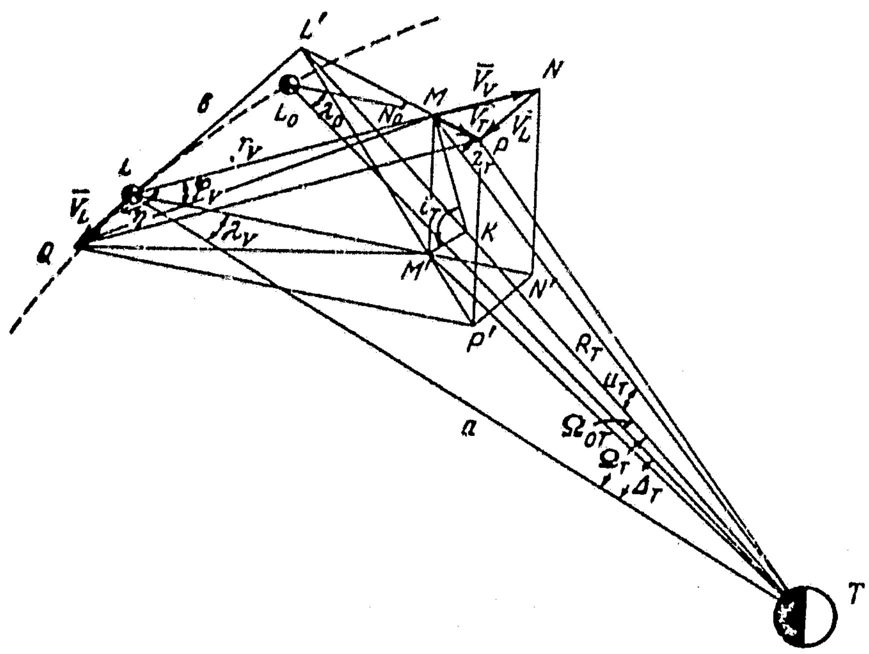

Fig.2. Scheme of transformation of selenocentric outgoing data (V_V, λ_V, ϕ_V, r_V) on the boundary of Moon's sphere of influence into the geocentric entrance data (V_T, δ_T, R_T, U_T, i_T, Ω_{0T})

The elements of the selenocentric orbit are determined by the formulas of the problem of two bodies: the constant C_L of the area law, the orbit parameter P_L, the major semiaxis a_L, the eccentricity e_L, the true anomaly of the particle in the positions M_0 and M correspondingly to v_0 and v_V.

From the polar spherical triangle M_0NM' we have

$$\lambda = \lambda_0 + \arcsin\left[-\frac{\sin A_0}{\cos \varphi_v}\sin(v_v - v_0)\right]; \tag{4}$$

then

$$\lambda_v = \lambda - \Delta_l, \tag{5}$$

$$\varphi_v = \arcsin\left[\sin\varphi_0 \cos(v_v - v_0) - \cos\varphi_0 \sin(v_v - v_0)\cos A_0\right]. \tag{6}$$

The escape velocity (or rate of discharge) V_V of the particle from the sphere of influence of the Moon is determined from the energy integral

$$V_v = \sqrt{V_0^2 - 5.545}, \tag{7}$$

where V_V and V_0 are expressed in km/sec.

The angle of the rate of discharge (or of the escape velocity) relative to radius L_0M is found with the help of the constant of area law:

$$z_v = \arcsin \frac{C_L}{r_L V_v}. \tag{8}$$

Computation shows that the outgoing angle is quite small; for example, even at horizontal flying out ($z_0 = 90°$) with initial velocity $V_0 = 2.5$ km/sec, the angle z_V is equal to about 3° and at $z_0 = 45°$ the angle z_V constitutes about 2° [1]. Such an insignificant difference in the direction of emergence from the sphere of influence of the Moon from the radial allows us to admit particles with radial direction of escape for the basic ones, and the more so since, as follows from the theory of explosive processes, only particles having escaped vertically during the explosion and upward, are endowed with maximum velocity [2].

Consequently, the entire combination of particle trajectories, when these escape from any point of the surface of the Moon in any direction and with any velocity, may be represented at the boundary of the sphere of influence of the Moon in the form of a "spherical hedgehog", in which every radial "needle" is the corresponding selenocentric outgoing or escape velocity V_V (Fig.3). It represents schematically the system Earth- Moon. The sphere of influence around the Moon is described by the radius $LM_1 = r_V$. On its surface velocity vectors $\bar{V}_V$ are plotted along the radii.

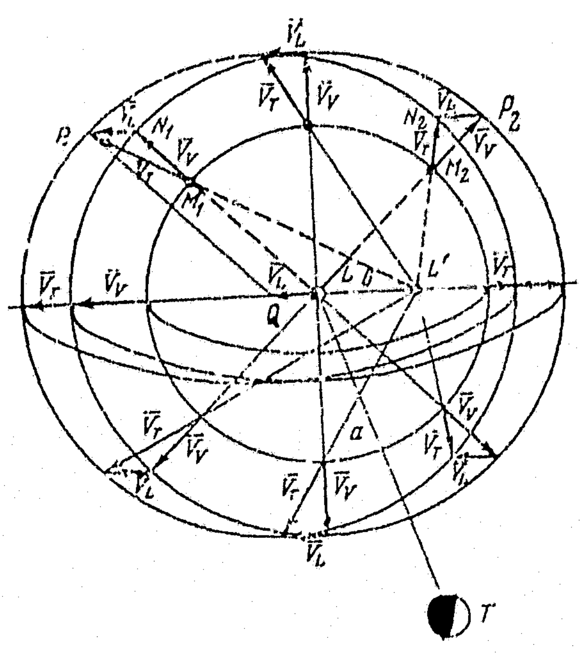

Fig.3. Transformation of the sphere of outgoing selenocentric velocities V_V into the sphere of entrance geocentric velocities V_T

If we sort the particles which leave the Moon with identical velocity V_0 (see (7)), they also intersect the sphere of influence with identical velocity V_V. The ends of the "needles" of the outgoing velocities from the surface of a sphere of radius LN_1. Let us now separate from the sphere of velocities the velocity cone LN_1P_2 on the selenocentric latitude ϕ. When passing from the selenocentric to geocentric system of coordinates it is imperative to take into account the relative velocity of Moon displacement along the orbit $V_L = 1.02$ km/sec, i. e. to sum up geometrically the selenocentric escape velocity $\overline{V}_V$ with the lunar $\overline{V}_L$, which would yield the entrance geocentric velocity

$$\overline{V}_T = \overline{V}_V + \overline{V}_L. \tag{9}$$

We shall then obtain the cone of entrance geocentric velocities $L'P_1N_2$, of which the base will be distant from that of the selenocentric cone by a quantity V_L in the direction of motion of the Moon, while the summit L' is shifted in the opposite direction. The shift $LL' = b$ is easy to find from the similitude of the triangles $LL'M_2$ and $M_2N_2P_2$:

$$b = r_v \frac{V_L}{V_v}. \tag{10}$$

Evidently, the same can be referred to the entire sphere of entrance geocentric velocities, the center Q being displaced relative to the center L of the sphere of selenocentric escape velocities by the quantity V_L and the point of "emergence" L' of all velocities V_T being distant from the center of the Moon by the quantity $\underline{b}$.

Let us consider the particle M which leaves the sphere of influence of the Moon on the selenocentric longitude λ_V and latitude ϕ_V (Fig.2) along the selenocentric radius $LM = r_V$. The magnitude of the geocentric radius R_T is easily found from the triangle TLM:

$$R_T = \sqrt{a^2 + r_v^2 - 2ar_v \cos\varphi_v \cos\lambda_v}, \tag{11}$$

where $\underline{a}$ is the mean distance TL between the Earth and the Moon.

The geometrical summing up of velocities $\overline{V}_V$ and $\overline{V}_L$ yields the magnitude of the entrance geocentric velocity V_T:

$$V_T = \sqrt{V_V^2 + V_L^2 - 2V_VV_L \cos\phi_V \sin\lambda_V} \tag{12}$$

The longitude of the ascending node Ω_T of the geocentric orbit relative to the direction toward the Moon at time of particle emergence from the sphere of action of the Moon is

$$\Omega_T = \text{arc tg } b/a, \tag{13}$$

or, taking into account (10),

$$\Omega_T = \text{arc tg}\left[\frac{1}{V_0}\left(\frac{V_L r_v}{a}\right)\right]. \tag{14}$$

Since $V_L r_V / a$ is constant for all the considered orbits, Ω_T depends only on the magnitude of the inital velocity V_0.

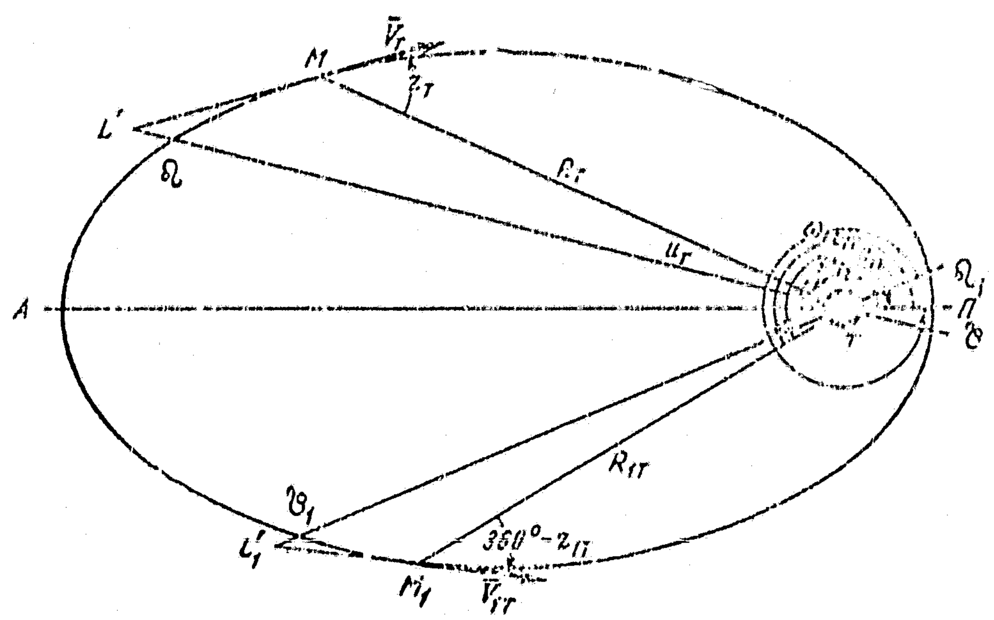

Fig.4. Determination of the argument of the perigee ω_T of the geocentric orbit of particles M and M_1

In order to determine the longitude of the asecnding node Ω_{0T} relative to the straight line uniting the Earth and the Moon, one must subtract at time of particle breakaway from the surface of the Moon Ω_T from the angle Λ_t determined by formula (3):

$$\Omega_{0T} = \Omega_T - \Delta_t. \tag{15}$$

As the initial escape velocity V_0 increases, the longitude Ω_{0T} decreases and coincides at node line limit with the direction Earth - Moon. For example, when the initial radial velocity increases from 2.4 to 11 km/sec, the longitude decreases from 4.64 to 0.02°. Consequently, the node lines of geocentric orbits of particles of lunar origin concentrate near the direction Earth-Moon at time of particle escape from the surface of the Moon.

After a series of elementary transformations, from the triangle TL'P we find the angle z_T between the entry geocentric velocity and the geocentric radius R_T:

$$z_T = \arccos\left[\frac{(aV_v \cos\lambda_v + r_v V_L \sin\lambda_v)\cos\varphi_v - r_v V_v}{R_T V_T}\right], \tag{16}$$

The area law constant is

$$C_T = R_T V_T \sin z_T. \tag{17}$$

From the triangle MKM' we obtain the inclination of the geocentric orbit to the orbit plane of the Moon:

$$i_T = \arcsin\left(\frac{\sqrt{a^2 V_v^2 + r_v^2 V_L^2}}{C_T} \cdot \sin\varphi_v\right). \tag{18}$$

The orbit parameter is

(19)

wher $\mu_T = k^2 m$ the gravitational parameter of the Earth.

The orbit's major semiaxis is

$$a_T = \frac{\mu_T R_T}{V_T^2 R_T - 2\mu_T}. \tag{20}$$

The orbit's eccentricity is

$$e_T = \sqrt{1 - \frac{p_T}{a_T}}, \tag{21}$$

The position of the particle on the orbit is determined by the latitude argument u_T, which is the angle measuring the distance of the particle from the ascending node ☊. The angle u_T varies from 0 to 360° in the direction of particle motion.

The argument of the latitude u_T of the particle at the boundary of the sphere of action of the Moon at the point M is obtained from the triangles TMK, MKM', LMM':

$$u_T = \arcsin\left(\frac{r_v}{R_T} \cdot \frac{\sin\varphi_v}{\sin i_T}\right). \tag{22}$$

The true anomaly v_T, which is measured by the angle between the orbit perigee and the radius-vector of the particle counterclockwise from 0 to 360° is found with the aid of the well known formula

$$v_T = \arccos\left[\frac{1}{e_T}\left(\frac{p_T}{R_T} - 1\right)\right]. \tag{23}$$

The last element, that is, the argument of the perigee ω_T, is measured by the angle between the ascending node ☊ and the perigee in the direction of motion of the particle.

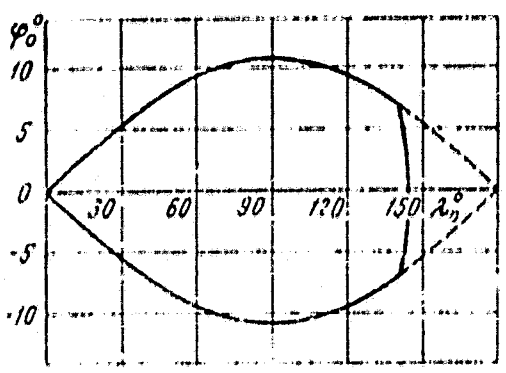

Fig.5. Region of the lunar surface from which the vertically escaping particles hit the Earth.

From Fig.4 it is obvious that

$$\omega_T = u_T \pm v_T \tag{24}$$

whereupon «+» is for $\phi_V > 0$ (position of M),

and «—» is for $\phi_V < 0$ (position of M_1).

In the case when the latitude argument u_T is smaller than the true anomaly v_T, we have

$$\omega_T = 2\pi + u_T - v_T \tag{25}$$

where $\phi_0 < 0$.

On the basis of the derived elements of the geocentric orbits we could determine the region of the lunar surface from which a vertical escape of particles hitting the Earth is possible (on the condition that the perigee distance $R_p \leqslant 6{,}500$ km). The region of vertical rise is represented in selenocentric coordinates λ_0, ϕ_0 in Fig.5. This is the part of the lunar surface in the western hemisphere, cut out by spherical sector with aperture angle of 21.4°, symmetrical relative to lunar equator. In this case the northern and southern boundary curves are described by the formula

$$\operatorname{tg} \phi_0 = \pm \operatorname{tg} 10.7° \cdot \sin \lambda_0 \tag{26}$$

To the East of the longitude $\lambda_0 = 140 - 144°$ part of the sector does not satisfy the hitting the Earth, for in these conditions partciles drift away all the time from Earth along hyperbolic trajectories, though formally their perigee $R_p \leqslant$ 6,500 km.

The maximum width of the sector is attained at longitude $\lambda_0 = 90°$

*** T H E   E N D ***

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