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I. INTRODUCTION

In a gaseous squeeze-film bearing, load capacity is generated by oscillating one of the bearing surfaces at a high frequency. An asymptotic analysis for large squeeze number was formulated in [1] and the performance for various squeeze-film bearing geometry was given in Refs. 2 to 5. It was shown that in order for the squeeze action to be effective the squeeze number must be large and the excursion amplitude be comparable to the mean film thickness. In the literature cited above, the supported mass was assumed to be stationary disregarding the oscillatory forces imposed by the squeeze motion. In reality the supported mass may respond dynamically with the squeeze action. This was investigated analytically in [6] by a simplified approach with the response assumed to be synchronous. The results indicate that the load capacity may be considerably affected by the synchronous response depending on the squeeze frequency and the mass of the float.

The supported mass motion was also studied in [7]. For moderately high squeeze number, numerical solution for the response was obtained using small perturbation analysis. The supported mass respond was found to be synchronous with the squeeze motion; this was also verified by experimental observation. The dynamic response of a spherical squeeze-film bearing with a time scale much larger than that of the squeeze motion was investigated in [8] using the results of [9].

The purpose of this report is to investigate the synchronous response of squeeze-film bearings with very high squeeze number. The geometry of a double squeeze-film circular disk has been chosen for simplicity. Three methods of solution are presented and the results compared.

II. ANALYSIS

Consider a double squeeze-film thrust plate as shown in Fig. 1. When the supported mass is at its central position, the mean film thickness on either side is C. Let ℓ be the thickness of the thrust plate, then the mean distance between the two squeeze surfaces is $2C + \ell$. Denote the instantaneous position of the lower squeeze surface, the thrust plate and the upper squeeze surface by y_1, y_2, and y_3 respectively. We have, then,

$$y_1 = -C \epsilon \cos \omega t \tag{1}$$

$$y_2 = C \delta \tag{2}$$

$$y_3 = 2C + \ell + C \epsilon \cos \omega t \tag{3}$$

where ϵ is the dimensionless excursion ratio and ω, the squeeze frequency. Normally, if one neglects the dynamic response of the plate due to squeeze-film actions, the dimensionless displacement δ would be time-independent. However, the dynamic response is, in general, not negligible. Therefore, δ is a periodic function of time. The time-average of δ should be related to the steady-state displacement as follows:

$$\frac{1}{2\pi} \int_0^{2\pi} C \delta \, d(\omega t) = C - C\eta_o \tag{4}$$

where η_o is the dimensionless steady-state displacement from the central position; η_o is defined to be positive for a downward displacement.

The instantaneous film thicknesses of A and B are, from (1), (2) and (3),

$$h_A = y_2 - y_1 = C \delta + C \epsilon \cos \omega t \tag{5}$$

$$h_B = y_3 - y_2 - \ell = 2C + C \epsilon \cos \omega t - C \delta \tag{6}$$

Assume that the squeeze number σ is very high so that we can apply the results of an asymptotic approximation. Let T_A and T_B represent the asymptotic approximation of (ph) in film A and B respectively. Using the results of [1], we obtain

$$T_A = p_a C \left\{ \frac{\int_0^{2\pi} (\delta + \varepsilon \cos \tau)^3 \, d\tau}{\int_0^{2\pi} (\delta + \varepsilon \cos \tau) \, d\tau} \right\}^{1/2} \tag{7}$$

$$T_B = p_a C \left\{ \frac{\int_0^{2\pi} (2 + \varepsilon \cos \tau - \delta)^3 \, d\tau}{\int_0^{2\pi} (2 + \varepsilon \cos \tau - \delta) \, d\tau} \right\}^{1/2} \tag{8}$$

The instantaneous pressures in A and B can be easily obtained by

$$p_A = \frac{T_A}{h_A} = \frac{T_A}{C (\delta + \varepsilon \cos \tau)} \tag{9}$$

$$p_B = \frac{T_B}{h_B} = \frac{T_B}{C (2 + \varepsilon \cos \tau - \delta)} \tag{10}$$

Note that although T_A and T_B are time-independent, p_A and p_B are functions of time. Furthermore, for a given excursion ratio ε, the pressures p_A and p_B are not known, because δ is an unknown quantity. Therefore, one more equation is needed for δ, which is the equation of motion of the supported mass,

$$M C \omega^2 \frac{d^2 \delta}{d \tau^2} = F_y + p_A A - p_B A$$

using Eqs. (7) to (10). Here, M is the mass, A is the bearing area, and F_y is the time-independent external force (positive upward).

Define

$$\bar{F}_y = \frac{F_y}{p_a\, A}$$

$$B = \frac{M\, C\, \omega^2}{p_a\, A}$$

$$I_A = \left\{ \frac{\int_0^{2\pi} (\delta + \varepsilon \cos \tau)^3 \, d\tau}{\int_0^{2\pi} \delta \, d\tau} \right\}^{1/2} \tag{11}$$

$$I_B = \left\{ \frac{\int_0^{2\pi} (2 + \varepsilon \cos \tau - \delta)^3 \, d\tau}{4\pi - \int_0^{2\pi} \delta \, d\tau} \right\}^{1/2}$$

then, the equation of motion becomes

$$B \frac{d^2 \delta}{d\tau^2} = \bar{F}_y + \frac{I_A}{\delta + \varepsilon \cos \tau} \quad \frac{I_B}{2 + \varepsilon \cos \tau - \delta} \tag{12}$$

Integrating (12) over a complete squeeze cycle we obtain a relationship between $\bar{F}_y$ and ε and δ.

$$\bar{F}_y = - \frac{I_A}{2\pi} \int_0^{2\pi} \frac{d\tau}{\delta + \varepsilon \cos \tau} + \frac{I_B}{2\pi} \int_0^{2\pi} \frac{d\tau}{2 + \varepsilon \cos \tau - \delta} \tag{13}$$

With $\bar{F}_y$ given by (13), Eq. (12) can be considered as the differential equation for δ. If we are interested in the steady-state dynamic response rather than the transient, the boundary conditions to be satisfied are the periodicity conditions:

$$\delta\ (\tau) = \delta\ (\tau + 2\ \pi) \qquad (14)$$

$$\frac{d\ \delta\ (\tau)}{d\ \tau} = \frac{d\ \delta\ (\tau + 2\pi)}{d\ \tau} \qquad (15)$$

It should be pointed out here that Eq. (12) is non-linear in δ.

III. METHODS OF SOLUTION

A. Harmonic Solution

Assume that δ can be expressed by

$$\delta = 1 - \eta_o - \lambda \cos \tau - \nu \sin \tau \tag{16}$$

where η_o is the steady-state downward displacement from the central position; λ and ν are the in-phase and out-of-phase components of the response. The higher harmonics have been truncated in (16). Substituting (16) into Eqs. (11), (12) and (13), we obtain

$$\left.\begin{aligned} I_A &= \left\{(1 - \eta_o)^2 + \frac{3}{2}\left[(\varepsilon - \lambda)^2 + \nu^2\right]\right\}^{1/2} \\ I_B &= \left\{(1 + \eta_o)^2 + \frac{3}{2}\left[(\varepsilon + \lambda)^2 + \nu^2\right]\right\}^{1/2} \\ \bar{F}_y &= - \frac{I_A}{\sqrt{(1 - \eta_o)^2 - (\varepsilon - \lambda)^2 - \nu^2}} + \frac{I_B}{\sqrt{(1 + \eta_o)^2 - (\varepsilon + \lambda)^2 - \nu^2}} \end{aligned}\right] \tag{17}$$

The equation of motion (12) takes the form

$$B(\lambda \cos \tau + \nu \sin \tau) = \bar{F}_y + \frac{I_A}{(1 - \eta_o) + (\varepsilon - \lambda) \cos \tau + \nu \sin \tau} - \frac{I_B}{(1 + \eta_o) + (\varepsilon + \lambda) \cos \tau + \nu \sin \tau} \tag{18}$$

Multiply both sides of (18) by $\sin \tau$ and integrate from 0 to 2π,

$$\pi B \nu = I_A \int_0^{2\pi} \frac{\sin \tau \, d\tau}{(1 - \eta_o) + (\varepsilon - \lambda) \cos \tau - \nu \sin \tau} - I_B \int_0^{2\pi} \frac{\sin \tau \, d\tau}{(1 + \eta_o) + (\varepsilon + \lambda) \cos \tau + \nu \sin \tau}$$

Expanding the intergrands in power series of $\frac{\epsilon-\lambda}{1-\eta_o}$ cos τ and $\frac{\nu}{1-\eta_o}$ sin τ, we obtain

$$\pi B \nu = \frac{I_A}{1-\eta_o}\pi \left[\frac{\nu}{1-\eta_o} + \frac{3}{4}\frac{\nu}{1-\eta_o}\left(\frac{\epsilon-\lambda}{1-\eta_o}\right)^2 + \frac{3}{4}\left(\frac{\nu}{1-\eta_o}\right)^3 + \ldots\right]$$

$$- \frac{I_B}{1+\eta_o}\pi\left[-\frac{\nu}{1+\eta_o} - \frac{3}{4}\frac{\nu}{1+\eta_o}\left(\frac{\epsilon+\lambda}{1+\eta_o}\right)^2 - \frac{3}{4}\left(\frac{\nu}{1+\eta_o}\right)^3 \ldots\right] \quad (19)$$

Take the first power of ν in the right hand side of (19), then

$$\nu\left\{B - \frac{I_A}{(1-\eta_o)^2}\left[1 + \frac{3}{4}\left(\frac{\epsilon-\lambda}{1-\eta_o}\right)^2 + \ldots\right] - \frac{I_B}{1+\eta_o{}^2}\left[1+\frac{3}{4}\left(\frac{\epsilon+\lambda}{1+\eta_o}\right)^2 + \ldots\right]\right\} = 0 \quad (19a)$$

Since the quantity in the bracket is, in general, not equal to zero, the only way to satisfy (19a) is $\nu = 0$. Furthermore, because the right hand side of (19) is a converging power series in ν, we conclude that

$$\nu \equiv 0 \quad (20)$$

Using (20) and multiplying both sides of (18) by cos τ and integrate from 0 to 2π, we obtain

$$\pi B \lambda = I_A\left[\frac{2\pi}{\epsilon-\lambda} - 2\pi\frac{1-\eta_o}{\epsilon-\lambda}\frac{1}{\sqrt{(1-\eta_o)^2-(\epsilon-\lambda)^2}}\right]$$

$$- I_B\left[\frac{2\pi}{\epsilon+\lambda} - 2\pi\frac{1+\eta_o}{\epsilon+\lambda}\frac{1}{\sqrt{(1+\eta_o)^2-(\epsilon+\lambda)^2}}\right] \quad (21)$$

which is the equation to be solved for λ. Before we solve Eq. (21) which is obviously non-linear, let us first obtain a linearized solution.

For the case that

$$\left.\begin{array}{l} \dfrac{\varepsilon - \lambda}{1 - \eta_o} << 1 \\ \\ \text{and} \\ \\ \dfrac{\varepsilon + \lambda}{1 + \eta_o} << 1 \end{array}\right] \tag{22}$$

we can linearize Eq. (21) and obtain

$$\lambda = \frac{\dfrac{2\eta_o}{1-\eta_o^2}}{\dfrac{2}{1-\eta_o^2} - B} \varepsilon \tag{23}$$

It is seen that when $B = \frac{2}{1-\eta_o^2}$, λ becomes infinitely large; the response of the mass is unbounded according to the linear approximation of a truncated harmonic analysis. Suppose that η_o is positive, i.e., the mass has a steady-state displacement downward. If $B < \frac{2}{1-\eta_o^2}$, λ has the same sign as ε and the response is in-phase with the squeeze motion of the lower bearing surface. If $B > \frac{2}{1-\eta_o^2}$, the response is 180^o out-of-phase with the squeeze motion and the squeeze action is enhanced there.

If condition (22) is not valid, we have to solve the nonlinear Equation (21). An iterative method may be used taking the linearized results as the initial guess for the iterative process. The numerical computation has been programmed on a computer.

B. <u>Exact Numerical Solution</u>

Rewrite Eq. (12) in the form,

$$\frac{d^2 \delta}{d \tau^2} = G(\delta) \tag{24}$$

where $$G(\delta) = B^{-1} \left(\bar{F}_y + \frac{I_A}{\delta + \varepsilon \cos \tau} - \frac{I_B}{2 + \varepsilon \cos \tau - \delta} \right) \tag{25}$$

$\bar{F}_y$, I_A and I_B are given by (11) and (13).
An iteration method will be used to solve Eq. (24). Let δ_i be the i-th approximation for the solution of (24). Let u_i be a correction function to δ_i, so that the sum,

$$\delta_{i+1} = \delta_i + u_i \tag{26}$$

will be an improved approximation. Thus,

$$\frac{d^2 \delta_i}{d \tau^2} + \frac{d^2 u_i}{d \tau^2} = G (\delta_i + u_i) \tag{26a}$$

Taking the first two terms of Taylor's Series expansion of the right hand side, we have

$$\frac{d^2 \delta_i}{d \tau^2} + \frac{d^2 u_i}{d \tau^2} = G (\delta_i) + u_i G' (\delta_i) \tag{27}$$

or $$\frac{d^2 u_i}{d \tau^2} - u_i G' (\delta_i) = G (\delta_i) - \frac{d^2 \delta_i}{d \tau^2} \tag{27a}$$

Since δ_i is a periodic function and so is δ_{i+1}, Eq. (26) then indicates that u_i must also be periodic. Thus,

$$\left. \begin{aligned} u_i (\tau) &= u_i (\tau + 2\pi) \\ \frac{d\, u_i (\tau)}{d \tau} &= \frac{d\, u_i (\tau + 2\pi)}{d \tau} \end{aligned} \right] \tag{28}$$

Note that $G (\delta_i)$ is a function of δ_i as well as a functional of δ_i because δ_i is implicitly involved in the definite integrals I_A and I_B. For the iterative process, an approximate expression for $G'(\delta_i)$ can be obtained by treating I_A, I_B and $\bar{F}_y$ as independent of δ_i. Thus,

$$G'(\delta_i) = B^{-1} \left[- \frac{I_A}{(\varepsilon \cos \tau + \delta_i)^2} - \frac{I_B}{(2 + \varepsilon \cos \tau + \delta_i)^2} \right] \tag{29}$$

Since the approximate expression (29) is used to determine u_i for the next approximation in δ_i, it would affect the speed of convergence of the iterative process; however, it would not affect the accuracy of the resulting final solution.

Let there be Q points between 0 and 2π, then the interval is

$$\Delta = \frac{2\pi}{Q - 1} \tag{30}$$

In finite-difference form,

$$\frac{d^2 u_i^q}{d\tau^2} = \frac{u_i^{q+1} - 2u_i^q + u_i^{q-1}}{\Delta^2}$$

$$\frac{d^2 \delta_i^q}{d\tau^2} = \frac{\delta_i^{q+1} - 2\delta_i^q + \delta_i^{q-1}}{\Delta^2}$$

The superscripts q, q + 1 and q - 1 indicate stations. Substitute into Eq. (27),

$$u_i^{q+1} - 2u_i^q + u_i^{q-1} - \Delta^2 u_i^q G'(\delta_i^q)$$

$$= \Delta^2 G(\delta_i^q) - [\delta_i^{q+1} - 2\delta_i^q + \delta_i^{q-1}];\ q = 1,2,..Q-1$$

Or, in matrix form

$$\left[\quad M \quad \right] \begin{bmatrix} u_i^1 \\ u_i^2 \\ \cdot \\ \cdot \\ \cdot \\ u_i^{Q-2} \\ u_i^{Q-1} \end{bmatrix} = \left[\quad v \quad \right] \tag{31}$$

where

$$M = \begin{bmatrix} -2-\Delta^2 G'(\delta_i^1) & 1 & 0 & --- & 0 & 1 \\ 1 & -2-\Delta^2 G'(\delta_i^2) & 1 & & 0 & 0 \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ 1 & & & & \cdot & \cdot \\ 0 & 0 & 1 & & -2-\Delta^2 G(\delta_i^{q-2}) & 1 \\ 1 & 0 & 0 & & 1 & -2-\Delta^2 G(\delta_i^{q-1}) \end{bmatrix} \tag{32}$$

$$v = \begin{bmatrix} \Delta^2 G(\delta_i^{1}) - [\delta_i^{2} - 2\delta_i^{1} + \delta_i^{0}] \\ \Delta^2 G(\delta_i^{2}) - [\delta_i^{3} - 2\delta_i^{2} + \delta_i^{1}] \\ \vdots \\ \Delta^2 G(\delta_i^{q-2}) - [\delta_i^{q-1} - 2\delta_i^{q-2} + \delta_i^{q-3}] \\ \Delta^2 G(\delta_i^{q-1}) - [\delta_i^{q} - 2\delta_i^{q-1} + \delta_i^{q-2}] \end{bmatrix} \tag{33}$$

We have used the peroidicity conditions

$$\delta_i^{q} = \delta_i^{1}; \quad \delta_i^{0} = \delta_i^{q-1} \tag{34}$$

$$u_i^{q} = u_i^{1}; \quad u_i^{0} = u_i^{q-1} \tag{35}$$

To start the iteration, the result of the harmonic solution, (16) can be used as an initial guess. The u_i's can be easily calculated from (31) by inverting the matrix M. Repeat the process as indicated in (26) until the elements of u_i's are less than a specified value.

Typical results are obtained for a squeeze frequency of 18000 rad/sec, $\varepsilon=0.3$, $\eta_o = 0.2$. $C = 0.001$ in, $A = 1$ in^2, $p_a = 14.7$ psi and $M = 0.1$ lb. The 2π interval is equally divided into 25 parts ($Q = 26$). The results of the numerical solution are tabulated as follows:

t	δ (t)
0.0000	0.88907
0.2513	0.88694
0.5027	0.88058
0.7540	0.87012
1.0053	0.85578
1.2566	0.83793
1.5079	0.81711
1.7592	0.79412
2.0106	0.77010
2.2619	0.74662
2.5132	0.72571
2.7646	0.70975
3.0159	0.70104
3.2672	0.70104
3.5185	0.70975
3.7699	0.72571
4.0212	0.74662
4.2725	0.77010
4.5238	0.79412
4.7752	0.81711
5.0265	0.83793
5.2778	0.85578
5.5291	0.87012
5.7805	0.88058
6.0318	0.88694
6.2831	0.88907

Thus, from Eq. (16)

$$\delta(t) = 1 - \eta_o - \lambda \cos \tau$$
$$= 0.8 + 0.0884 \cos \tau$$

On the other hand, the non-linear harmonic solution yields $\lambda = -0.0884$.

The linearized harmonic solution yields an λ of -0.035. It can be easily seen that only the non-linear harmonic solution agrees well with the tabulated numerical solution whereas the linear harmonic solution does not.

LOAD CAPACITY

We can identify $(-\bar{F}_y)$ as the dimensionless load capacity $\bar{W}$ of the double squeeze-film thrust plate. Several expressions for $\bar{W}$ can be obtained depending on how the synchronous response is calculated; they are enumerated as follows:

1) Load capacity without taking the synchronous response into account can be obtained from Eq. (17) by setting $\lambda = 0$.

$$\bar{W}_o = -\bar{F}_{yo} = \sqrt{\frac{(1 - \eta_o)^2 + \frac{3}{2}\ \epsilon^2}{(1 - \eta_o)^2 - \epsilon^2}} - \sqrt{\frac{(1 + \eta_o)^2 + \frac{3}{2}\ \epsilon^2}{(1 + \eta_o)^2 - \epsilon^2}} \tag{36}$$

2) Load capacity using the harmonic solution for the synchronous response is directly obtainable from Eq. (17).

$$\bar{W} = \bar{F}_y = \sqrt{\frac{(1 - \eta_o)^2 + \frac{3}{2}(\epsilon - \lambda)^2}{(1 - \eta_o)^2 - (\epsilon - \lambda)^2}} - \sqrt{\frac{(1 + \eta_o)^2 + \frac{3}{2}(\epsilon + \lambda)^2}{(1 + \eta_o)^2 - (\epsilon + \lambda)^2}} \tag{37}$$

The load capacity is designated by $\bar{W}_L$ and $\bar{W}_N$ for respectively the linear and non-linear results of λ.

3) Load capacity $\bar{W}_E$ can be calculated from Eq. (13) using the exact numerical solution for δ.

$$\bar{W}_E = \frac{I_A}{2\pi} \int_0^{2\pi} \frac{d\tau}{\delta + \epsilon \cos \tau} - \frac{I_B}{2\pi} \int_0^{2\pi} \frac{d\tau}{2 + \epsilon \cos \tau - \delta} \tag{38}$$

In order to visualize the effect of synchronous response of the supported mass to the load capacity of a squeeze-film bearing, it is illustrative to normalize the load capacity with respect to $\bar{W}_o$. In Figure 2, $\bar{W}_E/\bar{W}_o$, $\bar{W}_N/\bar{W}_o$ and $\bar{W}_L/\bar{W}_o$ are plotted against B for $\epsilon = 0.3$ and $\eta_o = 0.2$. It is seen that when B is large, the three normalized load capacities approach to unity asymptotically. This indicates that for large B, the amplitude of the synchronous response is small so that its effect to load capacity is negligibly small. It is interesting to note that in the region where $B > 6$, the normalized load capacities are positive and greater than unity; this is because the response is 180^o out of phase with the motion of the closer squeeze surface (the lower surface if η_o is positive). This out-of-phase response enhances the squeeze action of the lower film A, while decreases the squeeze action of the upper film B. Because the lower film is dominating for positive η_o, the net effect is an increase in load capacity. However, if B is small ($B < 2$, say), the response is in phase with the lower surface which results in a decrease in load capacity. As seen in Fig. 2, the normalized load capacity becomes negative which is clearly undesirable.

The response of the supported mass is seen to be similar to the response of the forced motion of an undamped spring-mass system. When the frequency is lower than the critical frequency the response is in-phase with the forcing function. But, when the frequency is higher than the critical frequency (super critical), the response is 180^{o} out-of-phase with the forcing function. It is in this supercritical region we should design and operate a squeeze-film bearing.

In Fig. 2, it is clear that $\bar{W}_N$ is very close to $\bar{W}_E$, especially in the super-critical region. In other words, the non-linear harmonic solution agrees well with the exact numerical solution. This feature will be utilized in future squeeze-film bearing analysis (Synchronous Response of Conical and Spherical Squeeze-film Bearings).

In Figures 3 and 4, $\bar{W}_E/\bar{W}_o$ is plotted for ϵ = 0.1 and 0.3 respectively with η_o as a parameter. Note that in the supercritical region the normalized load capacity increases with η_o. This indicates that the synchronous response increases not only the load capacity but also the stiffness.

IV. DISCUSSIONS

In the exact numerical solution of the dynamic response of a double-film squeeze bearing, periodic boundary conditions for the function and its first derivative have been assumed at $\tau = 0$. In so doing, we have included the synchronous response and its higher harmonic responses, whereas the sub-harmonic responses are automatically discarded.

In order to investigate the importance of the sub-harmonic responses it is only necessary to impose the periodic boundary conditions at 0 and $n2\pi$. Clearly, for $n = 2$, for example, the solution would include half-harmonic, synchronous and higher harmonic responses.

An actual plot is shown in Fig. 5 for ε=0.3 and $\eta_o = 2$; the solid curve is for n = 1 and the dotted one for n = 2. Note that although the periodic boundary conditions are imposed at 0 and 4π for the n = 2 case, the actual numerical solution shows that it repeats itself from 2π on. Also, the fact that the two curves being so close together indicates that the response is mainly synchronous. The computed load capacity, $\bar{W}_E/\bar{W}_0$, is 2.831 for n = 1 and 2.906 for n = 2. Therefore, it is not necessary to take the sub-harmonic responses into consideration.

Another point of significance is that the pressure forces in a squeeze film were obtained by an asymptotic approximation. In so doing, the damping part of the squeeze-film force becomes zero. The damping can be realized by considering the edge correction of the asymptotic approximation as will be seen in the following. Starting with the Reynolds' equation of a squeeze-film thrust plate,

$$\frac{H^3}{R} \frac{\partial}{\partial R} \left(PR \frac{\partial P}{\partial R} \right) = \sigma \frac{\partial (PH)}{\partial \tau} \qquad (39)$$

where P, R and H are normalized quantities.

$$\text{Let } \psi \equiv PH \qquad (40)$$

An asymptotic analysis shows that we can write

$$\psi\ (R,\ \tau) = \psi_{\infty}\ (R) + \psi_e\ (R,\ \tau) \qquad (41)$$

Here, ψ_{∞} (R) is the asymptotic approximation for large σ, and ψ_e (R, τ) is the edge correction which is important only in a region near the edge; the extent of which is of the order of $1/\sqrt{\sigma}$.

The governing equation for ψ_e is

$$\frac{H}{2} \frac{\partial^2 \psi_e{}^2}{\partial \xi^2} = \frac{-\psi_e}{\partial \tau} \qquad (42)$$

with boundary conditions

$$\psi_e\ (\xi = o,\ \tau) = H$$

$$\psi_e\ (\xi \rightarrow \infty,\ \tau) = \psi_{\infty}\ (R = 1) \qquad (43)$$

and the periodicity condition

$$\psi_e\ (\tau) = \psi_e\ (\tau + 2\pi) \qquad (44)$$

where $\xi = \sqrt{\sigma}\ (1-R)$ (45)

From the structure of (44), it is not difficult to see that if H involves a cos τ variation in time, ψ_e would have both sin τ and cos τ terms. Therefore, when we obtain the instantaneous film pressure from

$$p\ (R,\ \tau) = \frac{\Psi_{\infty}(R) + \psi_e\ (R,\ \tau)}{H} \qquad (46)$$

it is clear that ψ_e (R, τ) would contribute to a damping. Thus, the damping occurs only in the edge region and is of the order of $1/\sqrt{\sigma}$.

The stability of this double squeeze-film thrust plate has been considered using the same approach as [10], i.e. the small parameter stability technique. Starting from Eq. (12), it is found that (See Appendix I) for $\varepsilon = 0.3$ and $\eta_o = 0.2$, the bearing would be stable if $B \geq 12$. As shown in Fig. 2, $B = 12$ is already in the supercritical region. Actually, this stability result is conservative because damping has been neglected.

V. CONCLUSIONS

The dynamic response of a double squeeze-film thrust plate has been analyzed theoretically. Based on the results obtained the following conclusions can be drawn:

1. The dynamic response is mainly synchronous; sub-harmonic responses are negligibly small in magnitude.

2. Damping of a squeeze-film occurs only in the edge region and is of the order of $1/\sqrt{\sigma}$. For large squeeze number, the damping can be neglected.

3. When the squeeze frequency is high (large B), the response is 180° out-of-phase with the squeeze action. This enhances the squeeze action, and thus increases the load capacity and stiffness. The converse is true for low squeeze frequency (small B).

4. The harmonic solution for the dynamic response without linearization agrees well with the exact numerical solution. The linearized harmonic solution has only qualitative agreement with the exact numerical solution.

APPENDIX I STABILITY CONSIDERATION

The stability of the double squeeze-film thrust plate can be formulated using Eq. (12) and setting

$$\delta = \delta_o + \delta' \qquad (I.1)$$

where $\delta_o = 1 - \eta_o$ and δ' is a small perturbation from the equilibrium position δ_o.

Substituting (I.1) into Eq. (12) we have

$$B \frac{d^2 \delta'}{d \tau^2} + \left[\frac{I_A}{\delta_o} \left(\frac{1}{\delta_o} - 2 \frac{\varepsilon \cos \tau}{\delta_o^2}\right) + \frac{I_B}{2 - \delta_o} \left(\frac{1}{2 - \delta_o} - 2 \frac{\varepsilon \cos \tau}{(2 - \delta_o)^2}\right)\right] \delta' = 0 \qquad (I.2)$$

Note that in the above equation, we have neglected non-homogeneous terms such as $\frac{I_A}{\delta_o}\left(1 - \frac{\varepsilon}{\delta_o} \cos \tau\right)$ and non-linear terms involving δ'^2, etc. This is justified for a small parameter stability analysis.

Rearranging Eq. (I.2) Eq. (I.2) we can write

$$\frac{d^2 \delta'}{d \tau^2} + (a_1 + a_2 \cos \tau)\, \delta' = 0 \qquad (I.3)$$

where

$$a_1 = \frac{1}{B} \left[\frac{I_A}{\delta_o^2} + \frac{I_B}{(2 - \delta_o)^2}\right]$$

$$a_2 = - \frac{2}{B} \left[\frac{I_A}{\delta_o^3} + \frac{I_B}{(2 - \delta_o)^3}\right] \varepsilon \qquad (I.4)$$

Clearly, the sign of a_2 is irrelevant because $a_2 \cos \tau$ is only a simusoidally fluctuating term. Neglecting the squares of δ', I_A and I_B can be written as

$$I_A = \delta_o^{\,2} + \frac{3}{2}\ \epsilon^2$$

$$I_B = (2 - \delta_o)^2 + \frac{3}{2}\ \epsilon^2$$

Thus,

$$a_1 = \frac{1}{B}\left[1 \div \frac{3}{2}\left(\frac{\epsilon}{1\ \eta_o}\right)^2 + 1 + \frac{3}{2}\left(\frac{\epsilon}{1 + \eta_o}\right)^2\right] \qquad \text{(I.5)}$$

$$a_2 = \frac{2}{B}\left[\frac{\epsilon}{1 - \eta_o}\left\{1 + \frac{3}{2}\left(\frac{\epsilon}{1 - \eta_o}\right)^2\right\} + \frac{\epsilon}{1 + \eta_o}\left\{1 + \frac{3}{2}\left(\frac{\epsilon}{1 + \eta_o}\right)^2\right\}\right] \qquad \text{(I.6)}$$

Note that we have used the asymptotic approximation for large σ. In so doing, the damping in the squeeze-film is neglected. The stability result is therefore conservative.

Results of [10] and [11] indicated that for marginal stability

$$a_1 = 0.25 - 0.5\ a_2 \qquad \text{(I.7)}$$

If $\epsilon = 0.3$ and $\eta_o = 0.2$, Eq. (I.7) yields B = 12. Therefore, for stable operation B should be greater than or at least equal to 12.

NOMENCLATURE

A	area
B	$MC\omega^2/p_aA$
C	mean film thickness when bearing is unloaded
e	amplitude of excursion
F_y	external force
$\bar{F}_y$	F_y/p_aA
G	defined in (25)
h_A, h_B	film thicknesses given in (5) and (6)
I_A, I_B	defined in (11)
M	mass of float; matrix defined in (32)
p	pressure
p_a	ambient pressure
P	p/p_a
r	radial coordinate
r_o	radius of plate
R	r/r_o
T_A, T_B	asymptotic approximations of ph
v	defined in (33)
$\bar{W}$	dimensionless load capacity
$\bar{W}_o$	" " " without supported mass motion
$\bar{W}_L$, $\bar{W}_N$, $\bar{W}_E$	" " capacities with supported mass motion calculated by linear and non-linear harmonies solution and by exact numerical solution
y_1, y_2, y_3	displacements defined in Fig. 1 and Eqs. (1), (2) and (3).
δ	dimensionless displacement of supported mass
Δ	defined in (30)
ε	e/C
η_o	dimensionless steady-state displacement of supported mass from central position
λ, ν	amplitudes of mass motion
μ	viscosity

σ	squeeze number = $\frac{12\ u\ \omega}{p_a}\left(\frac{R}{C}\right)^2$
τ	ωt
ψ	PH
ω	squeeze frequency

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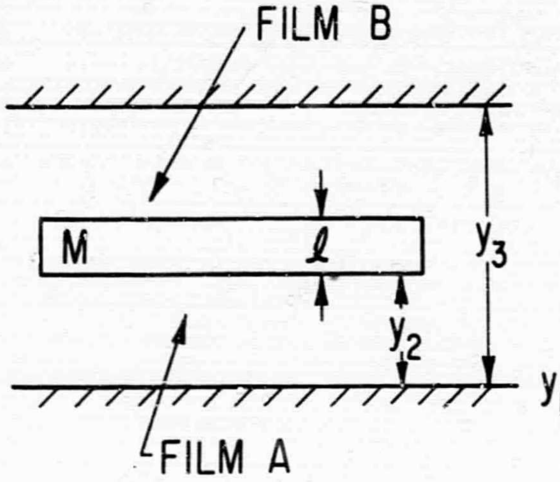

Fig. 1 Double Squeeze-Film Thrust Plate

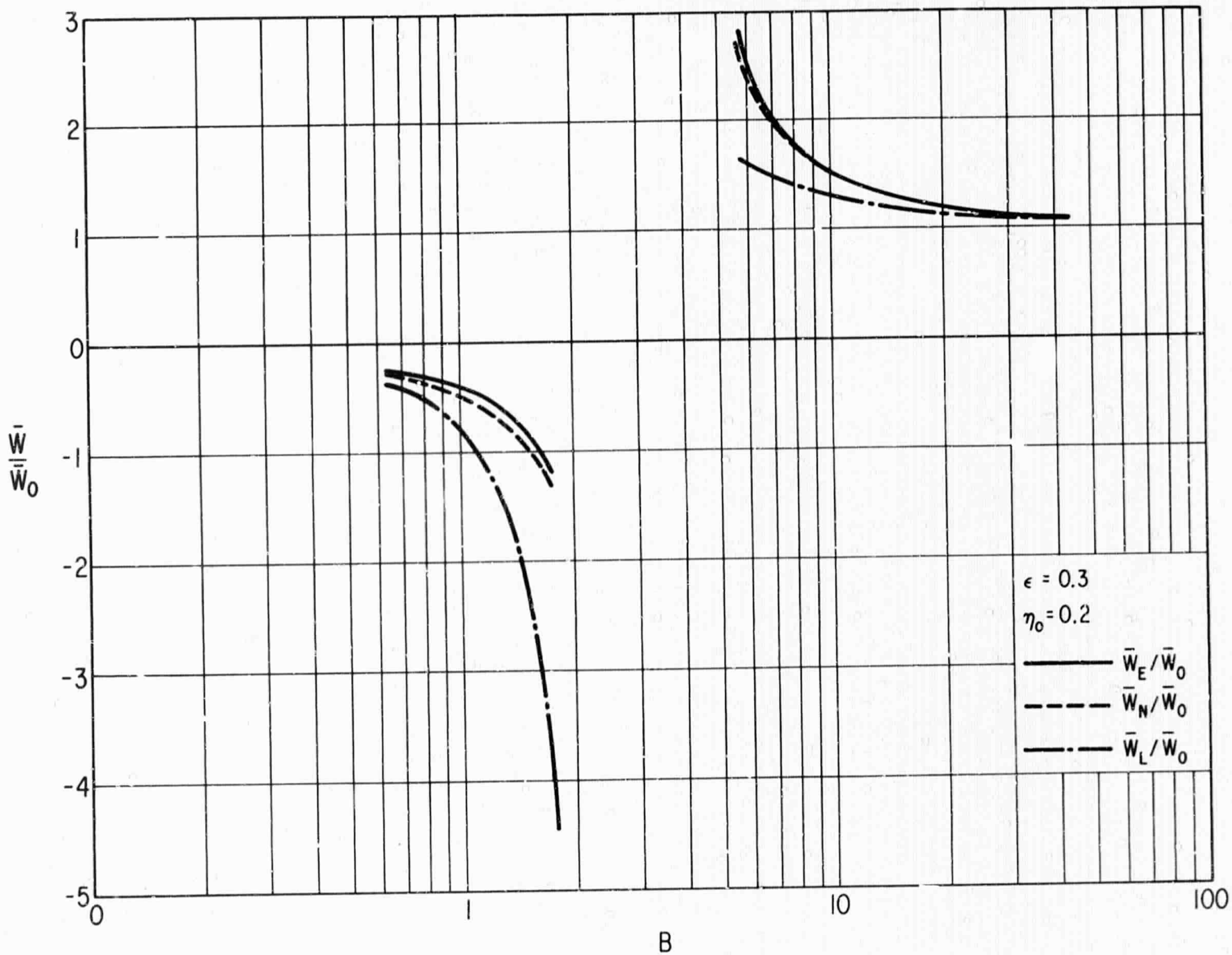

Fig. 2 Normalized Load Capacity Versus B

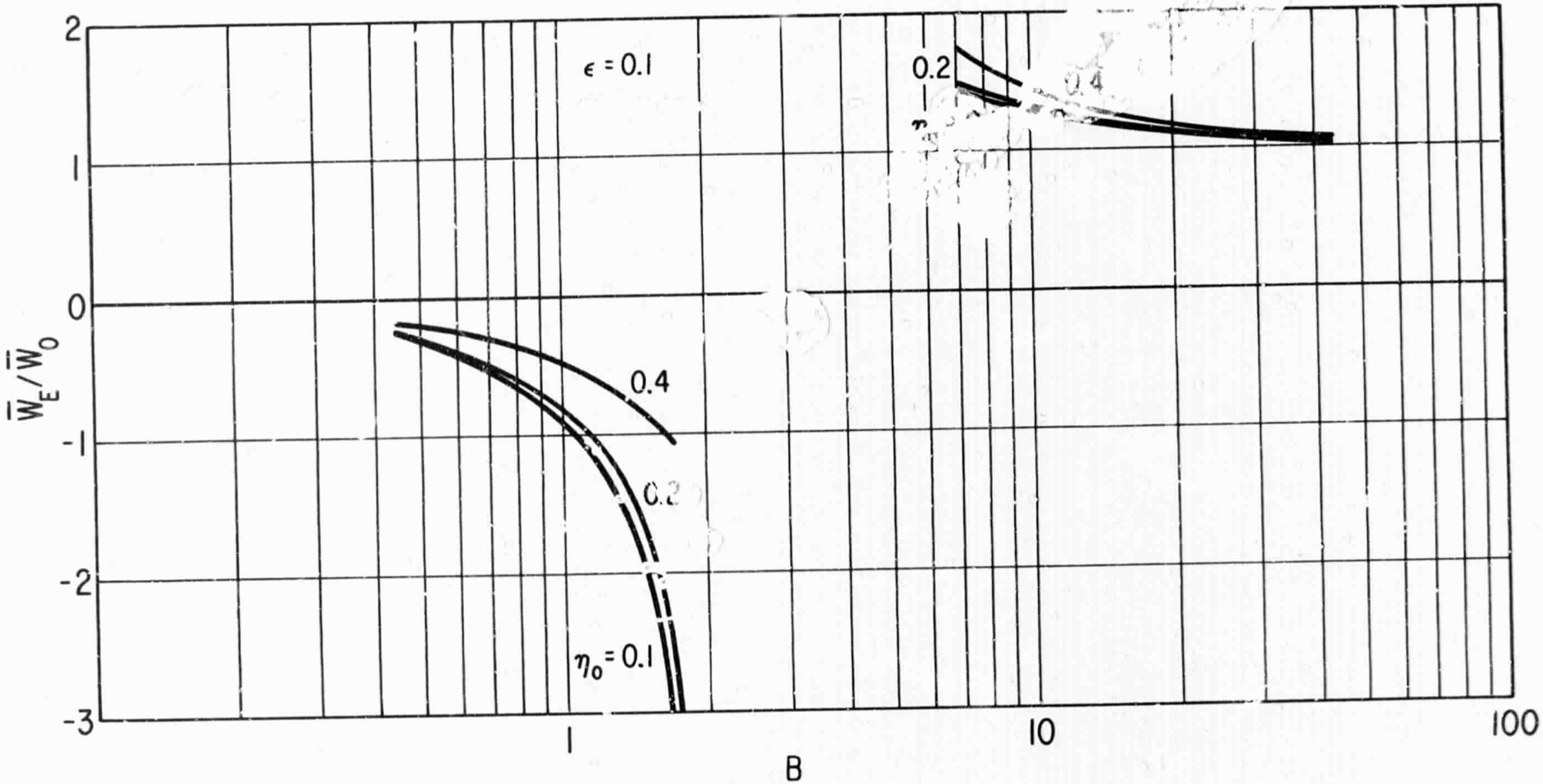

Fig. 3 Normalized Load Capacity Versus B for ε = 0.1.

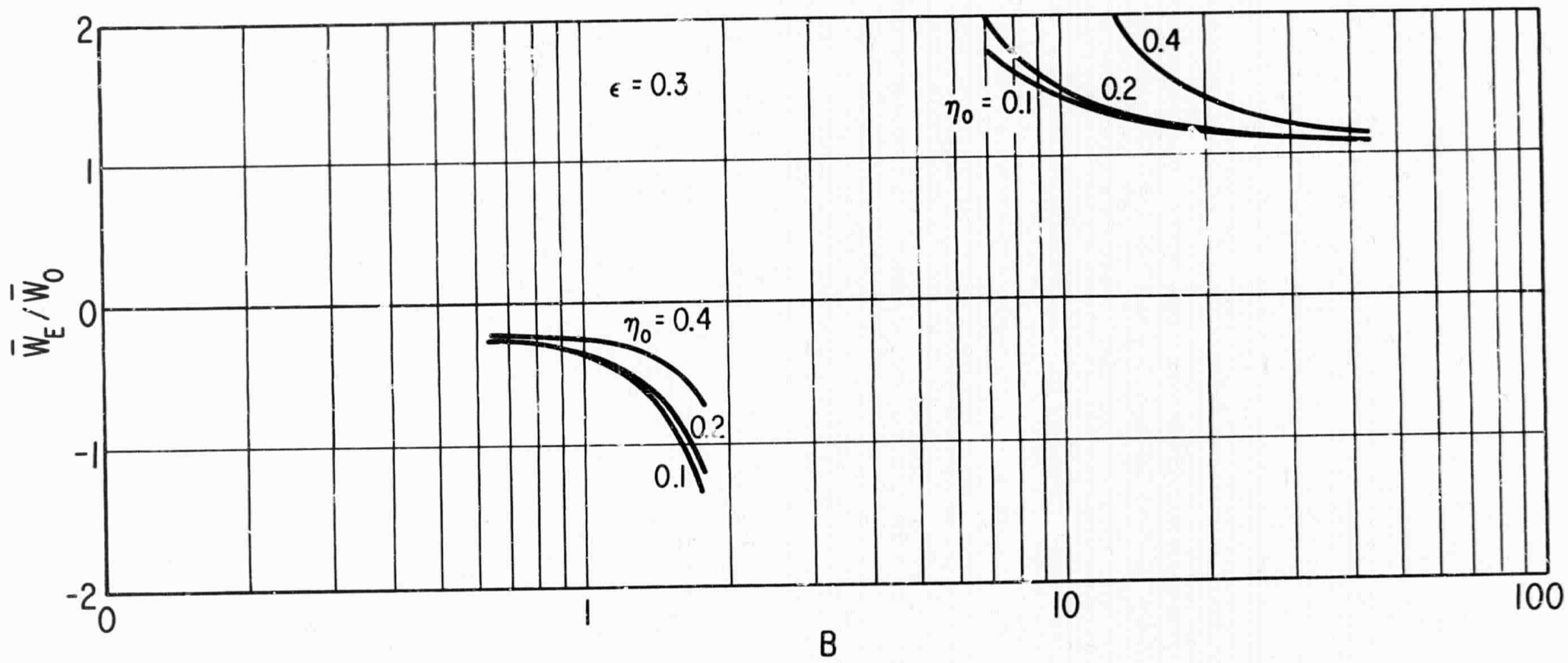

Fig. 4 Normalized Load Capacity Versus B for $\epsilon = 0.3$.

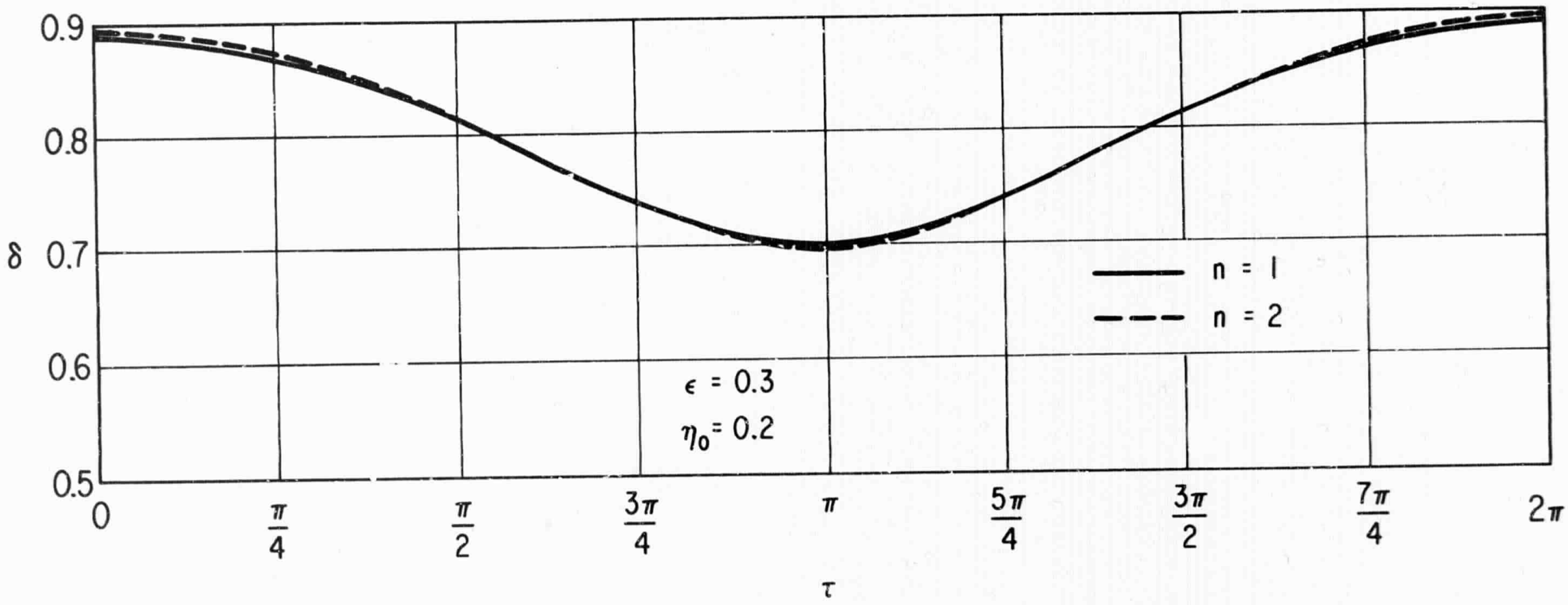

Fig. 5 Subharmonic (n = 2) and Harmonic (n = 1) Responses