NASA CR-86201

INTERIM REPORT

A STUDY OF GUIDANCE MODES FOR A UNIFIED GUIDANCE SYSTEM

31 MARCH 1969

NAS 12-593

TRW SYSTEMS GROUP

ONE SPACE PARK • REDONDO BEACH, CALIFORNIA

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By

C.G. Pfeiffer

W.H. Berry

31 MARCH 1969

Prepared under Contract No. NAS-12-593

By

TRW Systems Group
Redondo Beach, California

for

ELECTRONICS RESEARCH CENTER
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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LIST OF SYMBOLS

SYMBOL	MEANING
a, a_T	Thrust acceleration magnitude
$\underline{a}$	Thrust acceleration vector
α	Parameter vector
A,B,C,D	TRW Hybrid guidance parameters
b	Massless closest approach distance
β	Mass flow rate
c	Correction capability
C_1	Specific angular momentum
C_3	Vis-viva energy
$\underline{\delta r}$, $\underline{\delta v}$	Deviations from nominal $\underline{r}$ and $\underline{v}$
E	Energy
$E[\ ]$	Expected value operator
e	Eccentricity
$\underline{g}$	Gravitational acceleration
γ	Flight path angle
g	Final conditions
H	Hamiltonian
h	Angular momentum
J	Performance index
L	Likelihood function
$\underline{\lambda}$	Lagrange multiplier vector
Λ	Covariance matrix
m	Vehicle mass

LIST OF SYMBOLS (Cont'd)

SYMBOL	MEANING
μ	Gravitational constant
N	Normality matrix
ν	Lagrange multiplier vector
p	Semi-latus rectum
p	Power
Ψ	Terminal conditions
Ψ_p	Pitch angle
$\underline{r}$	Radius vector
r	Radial distance
ρ	Radius of curvature of reachable set of points
T	Thrust termination time
θ	Down range angle and true anomaly
$\underline{u}$	Thrust acceleration direction
u_c, u_r	Circumferential and radial direction cosines of $\underline{u}$
$\underline{v}$	Velocity vector
v	Velocity magnitude
v_∞	Asymptotic velocity
v_e	Effective exhaust velocity
$\underline{x}$,$\underline{y}$,$\underline{z}$	State vectors
χ	Steering angle

SUMMARY

This Report discusses space guidance theory and practice from a general point of view. Different types of powered flight guidance algorithms (modes) are classified and performance characteristics of the classes are discussed. Illustrative simulation results are presented to compare the performance of the Iterative Guidance Mode (IGM) and the TRW Hybrid Guidance Mode with the Calculus of Variations solution. Certain problems which must be solved before a truly unified guidance system can be developed are identified and discussed in detail.

1.0 INTRODUCTION AND SUMMARY

1.1 UNIFIED GUIDANCE AND NAVIGATION

Guidance and navigation systems for present day missions are developed on an ad hoc basis, in the sense that guidance and navigation software and hardware is designed so that specific mission objectives can be met for a specific launch vehicle. Little thought is usually given to adaptability to other applications. One would like to take a more general point of view and construct what might be called a "unified" guidance and navigation system. By a unified system we mean a set of software and hardware modules which can be used to control all segments of the trajectory for a wide class of missions and launch vehicles. This is a desirable objective, for savings in cost, reliability, and development t[illegible] could be achieved. Furthermore, the cost in time and money of mission design, preflight preparation, and flight readiness verification could be reduced by using the unified guidance capability to develop preflight analysis software (the 'quick reaction" problem). The task of designing and building a unified guidance and navigation system is not an easy one, for different types of difficult mathematical problems must be solved to accomplish a space mission. Furthermore, an on-board (self-contained) system must solve these problems with a relatively rudementary computer.

Assuming that guidance and navigation analysis can be treated separately, it is apparent that the development of a unified guidance and navigation system will only be feasible if efficient algorithms (modes) for solving the guidance problem can be devised. With this consideration in mind, the purpose of the study described here is to examine the state-of-the-art in guidance mode development; classify existing and proposed modes; define measures of their performance; compare the modes with respect to these measures; describe some of the problem areas that may be encountered, and recommend directions for further research and development. Anticipating the forthcoming discussion, it can be said that the analytical and numerical studies reported here indicate that it is feasible to design a unified guidance mode capable of on-board implementation if one is willing to exploit the numerical integration and iteration capability of present day computers. To accomplish this end, certain problem areas requiring further research have been identified. These are briefly discussed in Sections 1.4 and 1.8; more detailed analyses are presented in Sections 2,6,7,8, 9 and 10.

The studies reported here cover only some aspects of the unified guidance problem. Further information and supporting analysis is contained in Reference [1], where mission requirements are analyzed, design concepts for a unified, modularized guidance hardware/software systems are developed and an error analysis of a preliminary modular design is performed. The unified guidance problem is discussed from the "quick reaction" point of view in References [2] and [3]. The theory of guidance and navigation is discussed in References [4] and [5]-[7].

1.2 DEFINITIONS

Guidance and navigation analysis is concerned with the solution of the two-point boundary value problem which arises when one attempts to estimate the state (e.g., position and velocity) of a space vehicle and use this information to control* the translational motion so as to attain desired end conditions at mission completion. That is, a mathematical description of the given vehicle dynamics and the navigation information to be obtained, and a specification of the desired end conditions, such as the orbital elements of the final satellite orbit about the moon or planet, the guidance analyst must design an algorithm for calculating the translational accelerations to be applied to satisfy mission objectives in the presence of navigation errors and trajectory disturbances. When:

<u>Definition 1:</u> Given a mathematical model of the motion of a space vehicle, a description of the translation acceleration which can be commanded by the guidance system, and an estimate of the state of the overall dynamic system, guidance is the task of calculating and executing a realizable acceleration profile which will cause the trajectory of the space vehicle to attain desired end conditions, where

<u>Definition 2:</u> The state of the space vehicle system consists of the position and velocity of the vehicle, the parameters determining the vehicle performance capability, and the parameters determining the gravitational and atmospheric accelerations.

* Guidance theory is a special case of final value control theory.

The estimate of the state is obtained from the navigation system, where

Definition 3: Navigation is the task of estimating the state of the space vehicle system from sensed data, such as the first and second integrals of on-board accelerometer data, and/or earth-based tracking data and/or on-board observation of a celestial reference.

Definition 4: If the state estimate is available only at some initial time, the guidance system is said to be operating open-loop. If continually updated state information is available from the navigation system, the guidance system is said to be operating closed-loop.

Definition #1 states that guidance encompasses guidance theory (software) as well as the mechanization of the theory (hardware). Guidance mechanization usually concerns the guidance theoretician only to the extent that it affects his analytical treatment of the problem. For example, the analytical treatment will certainly be dependent upon the functional form of the guidance accelerations, which might be applied in the form of impulsive changes of velocity, realized by thrusting with a relatively high acceleration level for a relatively short time; by starting, throttling, steering and/or shutting off a rocket engine which thrusts for a relatively long period of time, and/or by applying lift and/or drag accelerations during motion in the atmosphere, realized by commanding motion of aerodynamic surfaces. On the other hand, the guidance analyst usually assumed that the attitude control problem can be ignored, where

Definition 5: Attitude Control is the task of attaining and stabilizing the vehicle in the attitude configuration called for by the guidance system. Separate treatment of guidance and attitude control is reasonable for most applications, because their response times are usually so different that there is negligible interaction. The stability problem must be considered, however, where

Definition 6: A guidance or attitude control system is said to be unstable if arbitrarily small errors can result in arbitrarily large commands; if this phenomenon cannot occur the system is said to be stable. Note that stability is usually not the most important consideration in the guidance problem, for the duration of guidance is finite and short compared to the mission duration. This is not true for the attitude control problem, indeed, stability is usually the primary design goal.

The primary purpose of the study reported here is the analysis of guidance modes, where

Definition 7: A guidance mode is an algorithm for calculating the parameters and functions which will accomplish the guidance task.

Since navigation information will be gathered during the mission in order to update the estimate of the state of the system, a guidance mode must be capable of acting as a real-time, feedback, final-value control law.

In general, there exists an infinite variety of guidance modes which will accomplish mission objectives. Thus one seeks an optimal guidance policy which satisfies the end conditions while minimizing some performance index, such as engine propellant expenditure, or else one prespecifies a functional form which is near-optimal. Present practice is to simplify the overall optimization problem by treating it as a sequence of two-point boundary value problems. That is, the overall mission is thought of as a sequence of "phases", usually characterized by the means available for applying the guidance accelerations.

Definition 8: A guidance phase is a segment of a trajectory, usually characterized by the means available for applying the guidance accelerations, having a distinct guidance objectives, i.e., specified end conditions.

The objective of the guidance system for any given phase is to attain the intermediate set of pre-specified end conditions. For example, a guidance phase might consist of transfer by means of relatively high rocket thrust acceleration from a near-earth circular parking orbit to a specified earth-escape hyperbola. (A more detailed description of guidance phases is given in Section 1.3). Such intermediate end conditions must be obtained by a "targeting" method, where

Definition 9: Targeting a given guidance phase is the task of analytically and/or numerically specifying the objectives of that phase.

Thus targeting is concerned with the practical task of piecing together the solutions of sequence of two-point boundary value problems so as to devise an overall solution of the complete problem. The targeting problem is almost synonomous with the guidance theory problem to analysts primarily concerned with guidance maneuvers which take the form of velocity impulses, while analysts concerned with continuous thrusting think of targeting in terms of specifying end conditions. In the terminology of optimization theory, targeting may be thought of as the task of specifying the transversality conditions for any guidance phase, given that the trajectory has been segmented into phases.

The imposition of intermediate "boundary" conditions by the targeting process leads to a sub-optimal overall guidance law, but, since each phase can be treated individually, the design of appropriate guidance modes is much simplified. In practice, guidance modes for the individual phases are usually quite different in form. Considering also the diverse forms of guidance mechanization employed for the various phases, it is true that essentially different guidance systems are presently used on a given mission.

It seems clear that it would be desirable to design a unified guidance system for future applications, where

Definition 10: A unified guidance system is one capable of guiding all phases of a given mission.

Lastly, for the purpose of classification and analysis of the contribution of software errors, to overall system performance, it is necessary to define what is meant by precision.

Definition 11: A software algorithm is said to be precise if the system error introduced by mathematical approximations is small compared to the total root-sum-square system error; otherwise the algorithm is said to be approximate.

1.3 DESCRIPTION AND EXAMPLES OF GUIDANCE PHASES

Present practice in guidance and navigation system design is to segment the overall trajectory into a sequence of phases (see Definition 8). The guidance mode in each phase attempts to null errors resulting from the previous phase, plus errors due to any current disturbances, by attaining the end conditions specified for the phase. Thus a guidance system might be called upon to solve in real time many different types of two-point boundary value problems for a wide range of initial conditions. Possible types of guidance phases for advanced missions are:

- <u>High Thrust Continuous Guidance</u>

  Launch vehicle guidance

  a. Initial ascent to altitude

  b. Booster stage

  c. Ascent to orbit

  d. Injection from parking orbit

  Terminal guidance

  a. Retro into lunar or planetary orbit

  b. Injection into earth satellite orbit

  c. Descent from orbit

  d. Soft landing and hovering

- <u>Low Thrust Continuous Guidance</u>

  Spiral escape from earth

  Earth to target transfer

  a. Lunar

  b. Interplanetary

Spiral capture by target body

Continuous orbit adjustment

a. Earth satellite

b. Lunar satellite

c. Planetary satellite

d. Earth-target transfer orbit

- Impulsive Guidance

  a. Midcourse

  b. Approach

  c. Terminal

  d. Satellite orbit trim

  e. Descent from orbit

  f. Soft landing retro

- Aerodynamic Guidance

  a. Control of lifting reentry

  b. Control of ballistic reentry

  c. Drag brake control

  d. Parachute control

Since there are many types of guidance phases, it is usually the case that more than one guidance mode will be employed during a mission, and more than one command mechanization subsystem will be used. Indeed, the guidance techniques for the various phases are so different that essentially different guidance systems are used during a mission.

EXAMPLES:† Guidance Phases for a Typical Lunar Mission

Ascent Phase. The ascent phase begins at launch and extends to injection into a nearearth circular parking orbit, which might have a nominal altitude of 100 n.mi. above the earth's surface. A typical ascent phase might last 8 to 10 minutes. The objectives of the guidance system is to attain circularity (eccentricity equal to zero) at an altitude close to the standard value. Guidance corrections are applied by steering the vehicle with the gimbaled rocket nozzle and by making small changes in the thrust termination time of each rocket stage. The distrubances to the flight path consist of imperfectly applied thrust acceleration and external forces, such as wind and air density variations. The position and velocity of the vehicle are measured by integrating the outputs of accelerometers mounted on an inertially fixed platform within the vehicle, or from ground-based tracking radars, or from both these sources.

Parking Orbit Phase. The parking orbit phase begins at parking orbit injection and extends to the restart of the launch vehicle for the injection phase. Typical parking orbit durations are 1 to 20 minutes (4 to 80^0 of coast arc) but they can be indefinitely long. There are usually no guidance corrections required during this phase, but some vernier adjustment of the errors remaining from the ascent phase might be made. The disturbances to the flight path are negligibly small for short coast arcs, but otherwise atmospheric drag becomes important. The position and velocity of the vehicle are determined as in the ascent phase, but celestial observations can be incorporated if the parking orbit

† This material is from Reference [5].

is sufficiently long. A rendezvous and docking of two or more vehicles may occur in this phase, the purpose being to assemble a spacecraft capable of completing the remainder of the mission. The rendezvous does not alter the essential here.

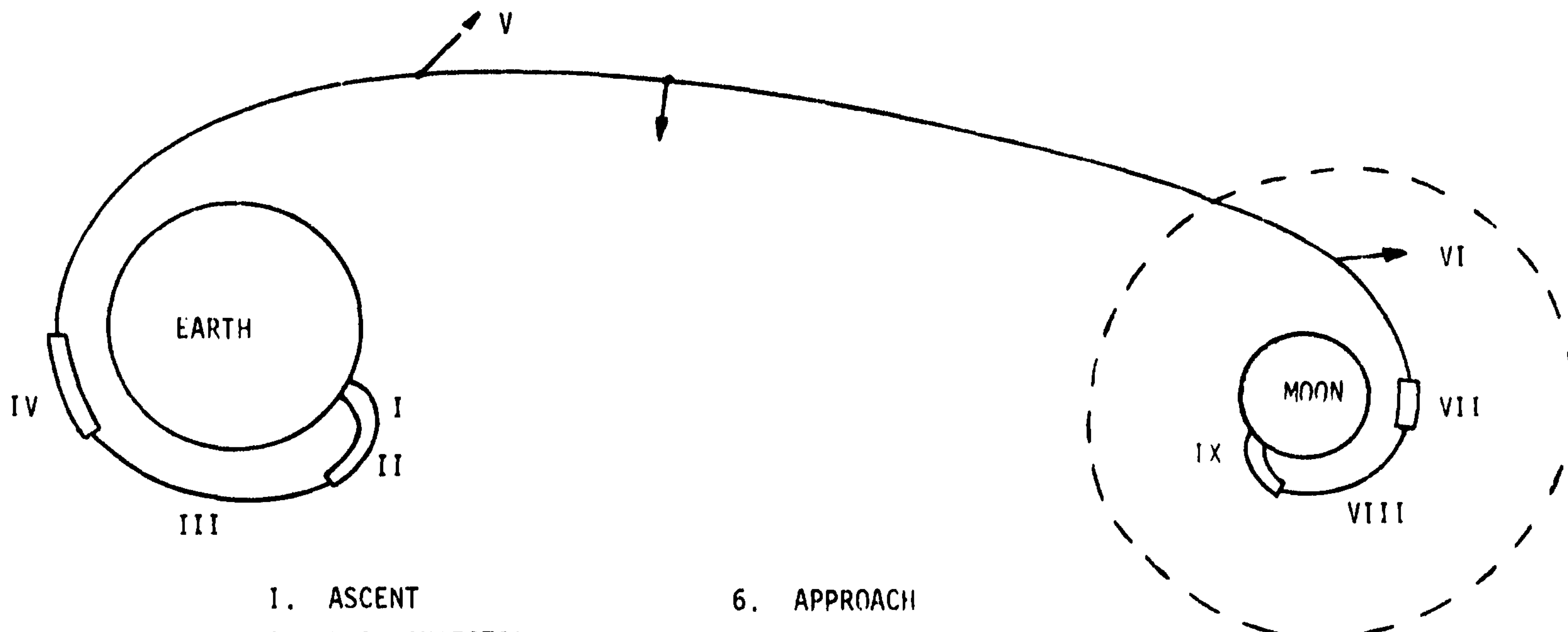
Injection Phase. The injection phase begins at restart of the launch vehicle and extends to injection into earth-moon transfer orbit (which is an ellipse relative to the earth with eccentricity of 0.987 for a 66 hour transfer). The duration of the injection phase is typically 2 to 3 minutes. The objective of the guidance system is to attain a transfer orbit which will cause the spacecraft to impact the desired target point on the moon. The corrections are made as in the ascent phase. The disturbances to the flight path are primarily due to imperfectly applied thrust acceleration. The position and velocity of the vehicle are determined as in the ascent phase.

Midcourse Phase. The midcourse phase begins at injection and extends until the spacecraft enters the "sphere of influence" of the moon, a point which is not precisely defined but is approximately 60,000 km from the moon. The duration of a typical midcourse phase is roughly 50 hours. The spacecraft is separated from the launch vehicle during this period. The primary objective of the guidance system is to correct for errors in the injection phase, thus providing a vernier adjustment. There are in addition some small distrubances to the flight path to consider, such as solar winds, leaking gas jets in the attitude control system, and errors in the assumed values of the physical constants which define the mathematical model used to construct the standard trajectory. The orbit is determined from celestial sightings and/or earth-based radar data. The guidance corrections are performed by applying short-duration impulses of acceleration (on the order of a minute long) with a small rocket engine so as to achieve essentially instantaneous changes of velocity. The magnitude of the correction is determined by the duration of the thrusting, and the desired direction is attained by properly pointing the spacecraft. One or more corrections might be made, the first no sooner than 5 hours after injection so as to allow time to determine the orbit, and others (usually not more than two) as required to null errors in the previous correction. The total velocity correction applied in the midcourse phase depends primarily on the injection error, but is typically less than 100 m/sec.

Approach Phase. The approach phase begins when the spacecraft enters the sphere of influence of the moon and extends until just prior to beginning of the terminal phase, a period of typically 15 hours. The objective of the guidance system, the disturbances to the flight path, and the techniques for determining the orbit and applying the guidance corrections are the same as in the midcourse phase. The trajectory is a moon-centered hyperbola with a hyperbolic excess velocity of typically 1.0 to 1.2 km/sec. Two or more corrections will probably be made, based on orbit determination measurements which sense the position and/or velocity of the spacecraft relative to the moon. Examples of such observations would be on-board sightings of the angles between the target center and certain stars and/or measurement of the change in spacecraft speed as it is acted upon by the moon's gravitational attraction. It is the gathering of this target-relative type of orbit information which distinguishes the approach phase. Since the ultimate mission accuracy very likely will depend on this information, the approach guidance phase is one of the most important of all. It supplies the final vernier corrections to the orbit.

Terminal Phase. The terminal phase begins at the completion of the last approach correction and extends through the final thrusting required to complete the mission, which might be a retro-braking into satellite orbit, a direct descent to the lunar surface, or a combination of these two maneuvers in order to descend to the surface from parking orbit. Integrated accelerometer data would be used during the thrusting periods, the initial conditions being obtained from the orbit parameters estimated during the approach phase. Celestial measurements and/or earth-based tracking data would be employed, if possible, during the coast periods. The only small impulsive corrections made would be during the parking orbit, if there is one. Thus the terminal phase is similar to the ascent-to-injection phases, with appropriately modified guidance objectives.

FIGURE 1.1 LUNAR MISSION PHASES



I. ASCENT
2. P.O. INJECTION
3. P.O.
4. TRANSLUNAR INJECTION
5. MIDCOURSE
6. APPROACH
7. DEBOOST
8. LUNAR P.O.
9. DESCENT

FIGURE 1.2: JUPITER INJECTION GUIDANCE TEST CASE

TRAJECTORY

VEHICLE

OBJECTIVES: CONTROL ENERGY, ECCENTRICITY, ARGUMENT OF PERIGEE

1.4 THE SEPARATION OF GUIDANCE, NAVIGATION AND ERROR ANALYSIS

It will usually be assumed throughout this report that guidance, navigation, and error analysis can be treated as separate problems. This assumption is consistent with present practice in guidance analysis, but the theoretical justification is not at all obvious. Indeed, simple examples can be constructed to show that the separate treatment of the guidance and navigation problems (control and estimation problems) can lead to incorrect results. Separability conditions can be established, however, and these are usually tacitly assumed for guidance work. For example, if system and navigation errors are small, so that a linearized analysis is legitimate, then separability follows. To properly account for random errors in such an analysis, one sometimes applies the method of aposteriori constraints. That is, a guidance law is derived, an error analysis is performed, and revised end conditions and constraints on end conditions are aposteriori specified in order to yield an acceptable performance in the presence of the stochastic effects. Such an approach is usually justified heuristically or empirically, and works well for many applications. Those cases where it does not need to be analyzed further.

The separability question is discussed in more detail in Section 2.0. Some new results are presented to indicate that a reformulation of the guidance and navigation problem in terms of maximum likelihood end conditions can lead to a rationale for separability, or a relatively simple way of deriving stochastic corrections. In particular, it is shown that separability applies for the case of hamiltonian dynamic systems, which is a useful result for space guidance work. The main difficulty in such an approach is justification of the maximum likelihood formulation. Cases where it is not applicable arise in post-injection impulsive guidance, and discussion of this problem is presented in Section 10

1.5 CLASSIFICATION AND PERFORMANCE OF GUIDANCE MODES

Assuming that the separate treatment of guidance,navigation and error analysis is indeed legitimate, a classification of guidance modes was devised and measures of performance were defined. In general, the modes were classified according to the assumptions introduced in their derivation. That is, the designer of a guidance mode can base his derivation upon either an approximate or a precise model of the dynamic system. He can employ either an arbitrary steering function or a special parameterized form. He can develop the guidance law in terms of a series expansion with coefficients precalculated and stored before the flight, where the expansion can be linear or non-linear, or he can call upon the computer to solve the guidance problem in real time, where the end conditions to be met are either explicitly or numerically specified. Thus a certain logic tree is followed to completion by the guidance analyst, terminating in a special class of guidance modes. This classification is discussed in detail in Section 3. Some examples of presently employed guidance mode are described and classified according to this scheme.

Measures of performance of a guidance mode are also described in Section 3, and the various classes of guidance modes are qualitatively discussed from the point of view of these measures of performance.

1.6 SIMULATION OF JUPITER INJECTION GUIDANCE

A limited amount of computer simulation was carried out in order to compare typical examples of guidance modes. The test case chosen was the injection guidance phase of a Jupiter mission. The guidance modes chosen for numerical simulation were the precise calculus of variations algorithm, the Iterative Guidance Mode, (IGM) and the TRW hybrid guidance mode. The trajectory (shown in Figure 1.2) was an unusual one, chosen not as an example of optimal performance but instead as a severe test case for guidance. The vehicle had an ascent phase consisting of preprogrammed ascent through the atmosphere plus guided ascent to a circular 96 nautical mile parking orbit, and an injection phase, consisting of two burns to a velocity suitable for satisfying the Jupiter mission. The arc turned over the earth's surface by the last two stages was 96°, the burnout altitude was 2647 nautical miles, and the path angle at burnout was 49.4°. The reason for this long burning arc was that energy, eccentricity and argument of perigee were specified, thereby causing the vehicle to inject onto a precisely fixed hyperbla. The long burning arc was required by the particular vehicle. The hypothesized vehicle consisted of a 260 inch solid rocket first stage, ar SIVB second stage, an improved Centaur third stage, and a fourth "Kick" stage.

The results were interesting. The TRW hybrid and IGM guidance techniques failed because the approximations introduced in their deviation were not applicable to a trajectory with a long arc length traversed over the earth's surface and a very large altitude change between ignition and burnout. The calculus of variations approach of course succeeded, but some difficulty was obtained in generating the trajectory.

After some initial attempts to improve the approximations in the TRW hybrid law proved unfruitful, an iterative technique was devised for improving the performance. The motivation for such an approach followed from the realization that the approximations of the equations of motion were, in effect a crude one-step integration. It therefore makes sense to replace the approximations with more accurate numerical integration. This approach led to excellent performance, resulting in only a 36 pound payload loss from the calculus of variation solution, and exact satisfaction of end conditions. Complete details are given in Section 4.

1.7 SIMULATION OF ATLAS-CENTAUR GUIDANCE

In order to make the numerical evaluation of guidance modes more complete, studies of Atlas-Centaur guidance carried out by TRW Systems Group under NASA Contract NAS-3-3231 (Reference 8) were summarized in Section 5. These studies evaluated the operation of the same two finite parameter guidance modes discussed in Section 4: the Iterative Guidance Mode (IGM) and the TRW Hybrid Guidance Mode. The eight missions simulated for the study are: one-burn lunar (direct ascent), two-burn lunar (parking orbit), earth orbital, polar earth orbital, one-burn planetary (Mars, direct ascent), two-burn planetary (Mars, parking orbit), synchronous satellite, and solar probe. While these eight missions represent a spectrum of situations, performance of the two guidance modes was satisfactory in each case. This conclusion is to be contrasted with the results of Section 4, where neither mode was adequate for guidance of the unusual Jupiter injection phase trajectory.

1.8 PARAMETERIZED GUIDANCE

The injection guidance simulation results studied the feasibility of what might be called approximate parameterized guidance. These guidance modes, which will later be classified as finite dimensional (for example, the Iterative Guidance Mode, IGM, and the TRW Hybrid Guidance Mode), are constructed by imposing a functional form on either the steering angle (χ) or a portion of the vehicle state vector (e.g., radial distance r). This leads to the formulas $\tan \chi = A + Bt$ for the IGM mode and $\ddot{r}/a_T = A + Bt$ for the TRW Hybrid mode, where a_T is the thrust acceleration. The equations of motion are then integrated in approximate form and parameter values are selected so that the desired end conditions are satisfied. Approximations to the integrated equations of motion are in form of closed analytic expressions which give, in general, crude estimates of the actual integrals. Time-to-go to burnout and the down range transfer angle are often so approximated. These estimates improve as cut-off time is approached so that an acceptable trajectory results when the system is operated closed-loop. The approximations are good for the early portions of flight if the guidance phase involves a relatively short burn time and down range transfer angle, but otherwise the approximate values of the acceleration integrals are no longer adequate. This was seen on the Jupiter mission Injection Phase, and an iterative "precise" parameterized guidance mode was required.

Precise parameterized guidance retains the parameterization of the steering law or a portion of the state vector, but replaces the crude approximations of the acceleration integrals by more exact numerical integration of the equations of motion. The accuracy of this numerical integration need be limited only by the computing capabilities of the system. The final values of the state vector components become functions of the parameter values, linked by means of the numerical integration scheme, and desired end conditions may be achieved through application of the Newton-Raphson algorithm. Extra degrees of freedom, which occur when there are more guidance parameters than desired end conditions, may be utilized for optimization of final payload or for the treatment of state and/or control variable constraints along the trajectory. This general method is discussed in more detail in Section 6.

1.9 PROBLEM AREAS

Certain theoretical problem areas requiring further research and development for unified guidance were identified and analyzed as a part of the study reported here.

- Separability of Guidance, Navigation, and Error Analysis

  As discussed in Section 1.4, the question of separability is a potentially troublesome one. The conditions for separability are discussed in Section 2, with primary emphasis upon understanding and justifying present practice in guidance analysis. Some interesting new results on maximum likelihood guidance techniques are briefly described, and it is concluded that further work is required to exploit this approach.

- <u>Targeting</u>

  Targeting, which consists of specifying parameters in the explicit end conditions functions for guidance phases, is an important consideration in the development of a unified guidance system. In Section 7 existing techniques for solving the targeting problem are described from an intuitive point of view, and some new results with an improved numerical technique are briefly described. Based upon these results, it appears that more efficient targeting techniques can be developed.

- <u>Constraints</u>

  The anlaytic treatment of mission and launch vehicle constraints is another important consideration in guidance mode development. From a control theoretic point of view, these can be either state variable or control variable inequality or equality constraints. It is well known that such control problems can be difficult to formulate and solve. In practical guidance work solutions are sometimes obtained in an empirical manner, using using a trialand error procedure to iterate on the form of the guidance law and the numerical values of the parameters in law. The constraint problem has not been treated in detail in this report, but a new approach is suggested in Section 8. It is concluded that further work should be spent on developing efficient analytical techniques for treating state variable and control variable constraints.

- <u>The Switching Time Problem</u>

  It is well known that not all end conditions can be controlled near the end of a guidance phase, for the guidance capability near thrust termination reduces to a velocity impulse. In Section 9 the shut-off time phenomenon is discussed from a theoretical point of view, showing that the problem can be understood in terms of the classical abnormality condition. Based upon this consideration, a shut-off region is defined and a steering and shut-off mode of operation is suggested.

  The classical conjugate point phenomenon is discussed in relation to optimizing the start time of a guidance phase. It is shown that optimization of start time can cause a conjugate point to exist on a trajectory when it otherwise might not. That is, the simultaneous optimization of start time and steering angle function leads to a different conjugate point analysis than the optimization of steering angle alone. These interesting questions might well be further pursued.

* Stochastic Guidance

  The separate treatment of navigation, guidance, and error analysis may not be justified in certain cases. For example, the impulsive guidance corrections applied during the midcourse, approach, and orbital phases of flight may require a stochastic guidance algorithm. Stochastic control problems of this type are very difficult to solve, and at this time only relatively primitive results have been obtained. In Section 10 this class of guidance problems is briefly discussed, and some recently completed work toward practical solutions is briefly described.

1.10 CONCLUSIONS AND RECOMMENDATIONS

The results of this study have led to the following conclusions and recommendations:

1. The capability of on-board computers to do real time numerical integration and iteration makes unified self contained guidance possible.

2. Certain problem areas exist, but relatively straightforward extensions of existing theoretical results could lead to development of algorithms suitable for real-time, on-board, unified guidance.

3. It is recommended that further analysis be carried out in the following areas:

a. Separability of guidance, navigation, and error analysis

b. Targeting

c. Analytic treatment of constraints

d. Switching time analysis

e. Impulsive stochastic guidance

4. A parameterized form of guidance law is tentatively recommended for unified guidance, both for use as an on-board algorithm and for generation of preflight reference trajectories. The parameterized form may also be used to generate reference trajectories for rudimentry guidance systems which cannot duplicate the complete parameterized form in real time.

5. More work on parameterized guidance is called for. In particular:

   a. It is necessary to devise more universal functional forms which are nearly optimal for a wider class of guidance phases

   b. The use of free parameters for the purpose of optimality and/or satisfaction of constraints must be examined

   c. An efficient analytic technique for treatment of constraints should be developed.

These conclusions and recommendations must be considered tentative until further analysis can be carried out, and a wider class of guidance problems can be studied and simulated.

2. THE SEPARATION OF GUIDANCE, NAVIGATION, AND ERROR ANALYSIS

2.1 INTRODUCTION

Implicit in the definitions of guidance and navigation are the assumptions that guidance and navigation can be treated as separate problems, and that statistical error considerations do not affect the design of a guidance mode. That is, in present practice the guidance theorist designs a guidance mode by assuming that the state is known perfectly, and for real-time applications employs the estimated state in place of its true value. This assumption, which is essential to a meaningful discussion of existing guidance modes, requires further clarification.

Strictly speaking, the deterministic derivation of a guidance mode is not correct, for the predicted end conditions which determine the guidance functions become random variables if there are random estimation errors and random systematic disturbances to the trajectory. In effect, the state of the system can no longer be defined simply by position, velocity, and system parameter vectors. Instead, the state must be thought of as the expected value of these quantities plus all the statistical moments of their distribution. In other words, the state can only be described by the conditional probability density function of the state, given the navigation data. From the point of view of guidance optimization theory, the random behavior of the dynamic system implies that there no longer exists a field of solutions which are the characteristics of the deterministic Hamilton-Jacobi partial differential equation. Thus, at least theoretically, the notion of a predictable reference trajectory has to be abandoned. In theoretical treatments of the stochastic control problem, one usually defines the control (guidance) so as to cause the first moment (expected value) of the deterministic state, conditioned on the navigation data, to achieve the desired value. Such an approach is elegant, but, in general, impossibly difficult to implement.

Simple examples of stochastic control problems (See Section 2.2) would seem to indicate that the deterministic guidance analysis is not at all valid for realistic problems. As a practical matter, however, stochastic guidance analysis is not required for those applications where (1) an apriori reference trajectory is available, and (2) the random navigation errors and random systematic errors are small. That is, the deterministic analysis applies when the first variation (or perhaps the first and second variations) about some reference trajectory is the dominant consideration. In a first variation analysis the random errors enter linearly and the deterministic approach can be theoretically justified (See Reference [9]). Consideration of the second variation does not change this conclusion, for it then follows that approximate results similar to the linear case are obtained with additional terms in the estimation and control equations to account for non-linear bias. Such assumptions can be thought of as devices to simplify the difficult computational problem of computing expectations.

A guidance law resulting from determiniatic analysis is sometimes modified to account for neglected stochastic effects by adding aposteriori "constraints". That is, a linearized or Monte Carlo closed-loop analysis of random navigation and systematic errors is performed to test system performance, appropriate state variable or control variable constraints are then empirically defined to cause the mission objectives to be met in the presence of these errors, and the guidance law is appropriately adjusted. The procedure can be iterated as many times as necessary.

Still another, and perhaps most important, way of justifying a separate, deterministic analysis of guidance is to introduce the notion of maximum likelihood guidance. In this case one seeks to control the "most likely" value of the deterministic state rather than its expected value. This most likely value is the root of the differentiated likelihood function, and is the value of the state which maximizes its (conditional) probability density function. Not all guidance problems can be meaningfully formulated in this way (e.g., impulsive midcourse guidance with execution errors proportional to the applied correction), but the technique does apply in important cases (e.g., injection guidance). In these cases it can be shown that the deterministic guidance law is also the stochastic guidance law, and navigation and error anlaysis can indeed be treated separately, if the equations of motion describe a hamiltonian system. For non-hamiltonian systems the stochastic correction to the deterministic law is relatively easy to compute.

The assumptions required to justify deterministic guidance analysis usually apply to those phases of a space mission where continuous guidance accelerations are applied, but stochastic considerations become important when the guidance is applied in the form of a sequence of small velocity impulses at unspecified times. The difficulty in this case is finding the times of application. Present practice for the Ranger-Mariner-Surveyor type of mission (Reference 7) is to prespecify these times and apply the method of constraints. That is, the application times are chosen by heuristic or empirical rules, and the real time corrections are calculated with a linearized deterministic rule. The non-linear effects of the corrections are treated by an iterative technique. Stochastic considerations are introduced by employing a non-linear maximum likelihood estimator in the navigation equations, and approximately modifying the targeted end conditions so as to take into account the statistics of the estimation and systematic errors. Small real time (adaptive) variations in the time of application of the corrections are sometimes allowed. This approach demonstrably works well for many applications.

In summary, the separate treatment of guidance, navigation and error analysis can be justified by

- the assumption of linearity
- the imposition of aposteriori constraints
- the maximum likelihood formulation.

Those cases where such separation is not justified are very difficult to treat, and further research is called for in this area. Further discussion and some recently obtained results are presented in Section 10.

2.2 AN EXAMPLE OF OPTIMAL STOCHASTIC GUIDANCE

The stochastic control problem can be illustrated by a simple example. Suppose that at the initial time t_0 the final state $x_1(t_f)$ is known to be of the form

$$x_1(t_f) = g\ (u,\alpha) \tag{2.1}$$

where u is some scalar control parameter, and α is some scalar parameter characterizing the motion between t_0 and t_f. For example, α might be the initial condition $x_1(t_0)$, or the magnitude of a perturbing acceleration acting between t_0 and t_f, and u might be the magnitude of the guidance correction applied at t_0. Suppose the navigation system has provided an estimate of α, denoted by α^*, but there is an error in this estimate, denoted by $\varepsilon = (\alpha^* - \alpha)$. Suppose this unknown error is a zero mean, Gaussian random variable with variance σ^2 over the ensemble of all similar experiments. The problem is to choose the u which in some sense minimizes x_1. If the estimates were perfect ($\varepsilon = 0$), one would seek a u^0 such that

$$\frac{\partial g}{\partial u}\ (u^0,\ \alpha^*) = 0 \tag{2.2}$$

In the stochastic case, however, the error in the estimate can be arbitrarily large, so one must deal with the statistical expectation ($E[\cdot]$) of the derivative. This might be expressed in the form of a Taylor series as

$$
\begin{aligned}
0 = E\left[\frac{\partial g}{\partial u}(u^{o},\alpha)\right] &= E\left[\left(\frac{\partial g}{\partial u}\right) + \left(\frac{\partial^2 g}{\partial u\,\partial\alpha}\right)\varepsilon + \frac{1}{2}\left(\frac{\partial^3 g}{\partial u\,\partial\alpha^2}\right)\varepsilon^2\right. \\
&\quad + \frac{1}{3!}\left(\frac{\partial^4 g}{\partial u\,\partial\alpha^3}\right)\varepsilon^3 + \frac{1}{4!}\left(\frac{\partial^5 g}{\partial u\,\partial\alpha^4}\right)\varepsilon^4 \\
&\quad \left. + \text{ higher order terms in } \varepsilon^n\right] \\
&= \left(\frac{\partial g}{\partial u}\right) + \frac{1}{2}\left(\frac{\partial^3 g}{\partial u\,\partial\alpha^2}\right)\sigma^2 + \frac{3}{4!}\left(\frac{\partial^5 g}{\partial u\,\partial\alpha^4}\right)\sigma^4 \\
&\quad + [\text{higher order terms in } \sigma^2]
\end{aligned}
\tag{2.3}
$$

where the coefficients of the Taylor series are evaluated as functions of u^0 and α^*, and properties of a Gaussian distribution have been used in computing the expectation (i.e., the expected value of the odd moments are zero, and the expected value of the even moments are expressible in terms of σ^2). Thus it can be seen that the statistics of the errors in the estimate become inseparably mixed into the optimization problem, and it could be difficult to find u^0. The Taylor series method might not be the best approach, especially for states of higher dimension. In any case, one is conceptually faced with solving an infinite number of optimization problems corresponding to every possible value of ε, and choosing a statistically averaged solution, weighted according to the probability of occurrence of the values of ε.

A still more subtle problem arises if it is desired to choose

$$x_1(t_f) = a_1(u_1,\ u_2,\ \alpha) \tag{2.4}$$

subject to

$$x_2(t_f) = g_2(u_1, u_2, \alpha) = \text{given value} \tag{2.5}$$

Analogous to the deterministic case and the previous example, one is tempted to minimize

$$E[g_1(u_1, u_2, \alpha) + \nu\, g_2\,(u_1, u_2, \alpha)] \tag{2.6}$$

where ν is a Lagrange multiplier. Conceptually, this corresponds to solving an infinite number of optimization problems where the end condition is satisfied each time, and the Lagrange multiplier has to be treated as a random variable. There is no reason to expect that such solutions exist, however. Alternatively, one seeks to set

$$E[g_1(u_1,u_2,\alpha)] + \nu\, E[g_2(u_1,u_2,\alpha)] = \text{minimum} \tag{2.7}$$

where ν is a fixed constant. In this case the end conditions are satisfied "on the average", that is, the expected value of the end conditions satisfy the constraint but individual members of the ensemble generally do not.

Stochastic control considerations analogous to those discussed in this simple example will arise in the case of impulsively applied guidance corrections.

2.3 LINEARIZED STOCHASTIC GUIDANCE WITH CONSTRAINTS

A common approach to stochastic guidance problems is to linearize the system of equations with respect to random error sources. All statistical moments then drop out of the optimization equation and one obtains the deterministic guidance law. For example, in the simple problem discussed above, setting $\left(\frac{\partial^{n+1} g}{\partial u \partial \alpha^n}\right) = 0$ for $n > 1$ reduces equation (2.3) to $\frac{\partial g}{\partial u}(u^o, \alpha^*) = 0$. Similarly, equation (2.7) reduces to the deterministic form

$$\left[\frac{\partial g_1}{\partial u_i}(u_1, u_2, \alpha^*)\right] + \nu \left[\frac{\partial g_2}{\partial u_i}(u_1, u_2, \alpha)\right] = 0 \qquad (2.8)$$

$$i = 1, 2$$

It is clear that such an approach can at best yield an approximately correct answer, for, in the case of equation (2.3), terms of the form $\left(\frac{\partial^{n+1} g}{\partial u \partial \alpha^n}\right) \sigma^n$ could be large.

The effect of neglected random errors is sometimes taken into account by what can be called the "method of constraints". Suppose that the solution of equation (2.8) is (u_1^o, u_2^o), so that the expected value of the desired end condition is (since the odd moments are zero)

$$E(x_2) = \left[g_2(u_1^o, u_2^o, \alpha^*)\right] + \left[\frac{\partial^2 g^2}{\partial \alpha^2}(u_1^o, u_2^o, \alpha^*)\right] \frac{\sigma^2}{2}$$
$$+ \frac{3}{4!}\left[\frac{\partial^4 g^2}{\partial \alpha^2}\; u_1^o, u_2^o, \alpha^*)\right] \sigma^4 + \ldots \qquad (2.9)$$

Thus there is a stochastic bias in the desired end condition due to the higher order terms, which can be accounted for by introducing an offset aiming point. More precisely, one specifies that the probability of achieving the end condition x_2 within a given region R should be a certain number (see Figure 2.1), and adjusts the aim point to satisfy this probabilistic constraint. The calculation of this probability is a difficult task, and cannot be directly inferred from the statistical moments. It can be numerically determined by the Monte-Carlo method, or, as a reasonable approximation, it can be assumed that the probability density is Gaussian with bias nearly equal to the value given by equation (2.9) and variance calculated by linearized analysis. In either case, iteration is required to calculate the guidance law. That is, equation (2.8) is solved for u_1^o, u_2^o, the probability constraint is tested, new end conditions on x_2 are defined, and the procedure is iterated until the probability constraint is satisfied. This procedure will only be workable if the linearized analysis is approximately correct, however.

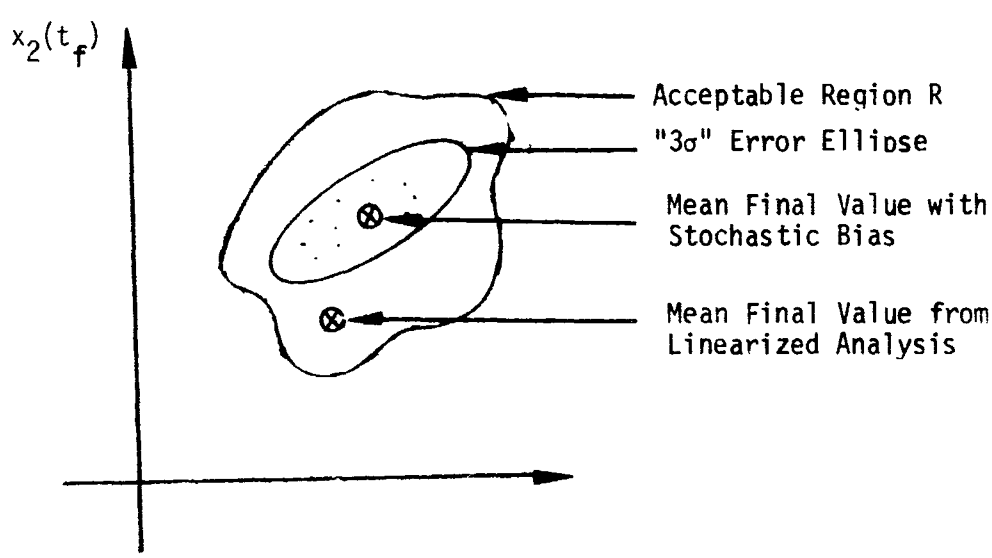


FIGURE 2.1: STOCHASTIC BIAS OF DESIRED END CONDITIONS

2.4 THE METHOD OF MAXIMUM LIKELIHOOD

A different and more workable approach to the stochastic guidance problem is to use the concept of maximum likelihood end conditions, rather than expected values. That is, suppose the task of the guidance system is to cause the "most likely" predicted final state to be a desired value, where the maximum likelihood end condition is that value which maximizes its probability density function.

Consider first the simple example of Section 2.2, where x_1 is to be maximized and the problem is to choose the proper value of α to solve $\frac{\partial q}{\partial u}(u^o, \alpha) = 0$. Suppose that, for a fixed value of the parameter u, there is a one-to-one mapping between x_1 and α, so that the following inverse relationship exists:

$$\alpha = g^{-1}(u, x_1) = f(u, x_1) \tag{2.10}$$

Then the probability density function for x is

$$\begin{aligned} p(u,x_1) &= \left[\text{prob}\ (\alpha\ (x_1, u))\right] \left|\frac{d\alpha}{dx_1}\right| \\ &= (2\pi\sigma^2)^{-\frac{1}{2}} \left|\frac{\partial f}{\partial x_1}(u,x_1)\right| \exp\left\{\frac{-f^2(u,x_1)}{2\sigma^2}\right\} \end{aligned} \tag{2.11}$$

The maximum likelihood value of x_1, denoted by $\hat{x}_1$, is found as the root of

$$\frac{\partial p}{\partial x_1} = 0 = \left\{ (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left(- \frac{f^2}{2\sigma^2} \right) \right\} \left\{ \left[\frac{\partial^2 f}{\partial x_1^2} (u, \hat{x}_1) \right] \right. \tag{2.12}$$

$$\left. - \frac{1}{\sigma^2} \left[f(u, \hat{x}_1) \right] \left[\frac{\partial f}{\partial x_1} (u, \hat{x}_1) \right]^2 \right\}$$

From equation (2.12) we can obtain a relationship between the parameter u and the most likely value of x_1, given by

$$\hat{x}_1 = h(u) \tag{2.13}$$

Then the value of u which maximizes $\hat{x}_1$ is found as the root of

$$\frac{\partial h}{\partial u} = 0 \tag{2.14}$$

Maximum likelihood control of end conditions for space guidance applications can be described by a relatively simple but typical example, where the control correction is based upon only apriori information rather than data gathered during the mission. Suppose that at time t_0 a single correction impulse vector u is to be applied so as to cause the "most likely" state at the final time T to be a desired value. Let the equations of motion be

$$\frac{dx}{dt} = f(x,t) \qquad t_0 < t \leq T \tag{2.15}$$

where x is the state vector. The initial condition at $t_0^{(+)}$ is x_0, where x_0 is a Gaussian vector with covariance matrix Λ_0 and mean equal to [m + Ku]. The K is supposed to be a given matrix. Thus the control u impulsively changes the state at t_o according to

$$x_o = x(t_o^{(+)}) = x(t_o^{(-)}) + Ku \tag{2.16}$$

where $x(t_o^{(-)})$ is a Gaussian vector with mean equal to m and covariance Λ_o. Assuming a one-to-one mapping of x_o to the final state $x_T = x(T)$ of the form $x_T = g(x_o)$, the probability density function of x_T is

$$p_T(x_T) = c \exp\left\{-\frac{1}{2}[q(x_T)]^T \Lambda_o^{-1} [q(x_T)]\right\} \left|\frac{\partial x_o}{\partial x_T}\right| \tag{2.17}$$

where c is the coefficient of the Gaussian density function of x_o, and

$$q(x_T) = g^{-1}(x_T) - (Ku + m) \tag{2.18}$$

$$\left|\frac{\partial x_o}{\partial x_T}\right| = \text{determinant} \left|\frac{\partial g^{-1}(x_T)}{\partial x_T}\right| \tag{2.19}$$

Define the likelihood function

$$L(x_T) = -\ln p_T(x_T) = \frac{1}{2} q(x_T)^T \Lambda_o^{-1} q(x_T) - \ln \left|\frac{\partial x_o}{\partial x_T}\right| - \ln c \tag{2.20}$$

But $\left|\frac{\partial x_o}{\partial x_T}\right|$ is the inverse determinant of the state transition matrix $\left|\frac{\partial x_T}{\partial x_o}\right|$, and it can be shown that (see [10], page 28).

$$\left|\frac{\partial x_o}{\partial x_T}\right| = \exp\left\{-\int_{t_o}^{T} \text{trace}\left[\frac{\partial f}{\partial x}(t;\, x_o(x_T))\right]dt\right\}. \tag{2.21}$$

Then the most likely value of x_T is that value $\hat{x}_T$ which satisfies

$$\frac{\partial L(x_T)}{\partial x_T} = 0 = \Lambda_o^{-1} \, q(x_T) + \frac{\partial}{\partial x_T} \int_{t_o}^{T} \text{trace} \, [\frac{\partial f}{\partial x}] \, dt \qquad (2.22)$$

and the control u should be chosen to set $\hat{x}_T$ equal to the desired value.

2.5 SEPARABILITY FOR HAMILTONIAN SYSTEMS

For hamiltonian systems, where the state transition matrix is symplectic (see [6], pp. 306) it can be shown that $|\frac{\partial x_T}{\partial x_o}| = 1$ and hence trace $[\frac{\partial f}{\partial x}] = 0$. In the example discussed above we have $q(\hat{x}_T) = 0$, and

$$Ku = q^{-1} \, (\hat{x}_T) - m \qquad (2.23)$$

In other words, for any desired final state $\hat{x}_T$ we can find the control u from (2.23), just as though the initial conditions were not random. That is, hamiltonian systems can be treated as though they were deterministic. This is an important conclusion. For example, the deterministic analysis of injection guidance can be justified in this way. For the case of non-hamiltonian systems, relatively simple stochastic corrections can be obtained from equation (2.22).

The example considered in Section 2.4 treated random initial condition disturbances only. Thus it is not at all clear that the separability condition applies to hamiltonian systems where estimation errors are present as well as systematic disturbances to the equations of motion. This extension of the problem is analyzed in Reference [11], from which it can be inferred that guidance, navigation, and error analysis can be separated if:

(1) the maximum likelihood formulation is appropriate

(2) the dynamic system is hamiltonian

(3) the probability density functions of disturbances and tracking data noise are Gaussian

(4) the tracking data noise is statistically uncorrelated.

Justifying the maximum likelihood formulation is a central difficulty for space guidance applications, for it is necessary to have a one-to-one relationship between random disturbances and end conditions in order to define the likelihood function. For example, this condition does not apply if the squared value of the end condition is to be controlled. This is the case in some problem formulations (Reference 12), which can arise in post-injection impulsive guidance (see Section 10). For the purpose of guidance mode analysis, however, it appears to be the case that separability of guidance, navigation, and error analysis is a reasonable hypothesis.

3. GENERAL STRUCTURE OF CONTINUOUS GUIDANCE MODES

3.1 INTRODUCTION

The purpose of guidance is to control the magnitude and direction of a rocket thrust vector to meet the targeting requirements of a mission phase. This section examines the basic operation of the continuous (as opposed to impulsive) high thrust guidance modes and describes a method of classifying modes according to basic differences in the underlying algorithms.

Open-loop guidance (definition 3) is useful for the lift-off phase of a mission during which the guidance commands are pre-programmed and independent of the actual state. If the propulsion hardware could perform the guidance commands perfectly, and if the mathematical model used to describe the mission were in complete accord with the real world, then open-loop guidance would be sufficient for the entire mission. Since this ideal is far from realization, it is necessary to occasionally measure the true state of the vehicle and revise the guidance commands in accordance with the new information. This is closed-loop guidance. The present section will be concerned exclusively with closed-loop guidance, which will now be called simply guidance.

Figure 3.1 illustrates the flow of signals in the total control system, where

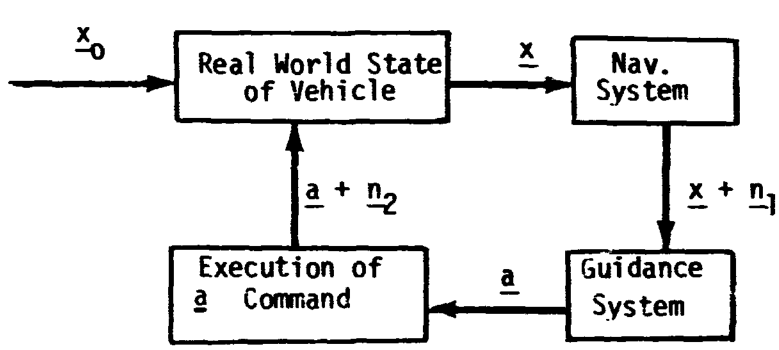


FIGURE 3.1: GENERAL CONTROL SYSTEM

$\underline{x}$ is the state of the vehicle; $\underline{a}$ is the commanded thrust acceleration from the guidance system; $\underline{n}_1$ is the error between the actual vehicle state and the estimated state received from the navigation system, and $\underline{n}_2$ is the error between the acceleration command and actual acceleration resulting from execution of the command. Consistent with the discussion of Section 2, it will be assumed that $\underline{n}_1$ and $\underline{n}_2$ are zero. The space vehicle system shown in Figure 3.1 may now be reduced to the system depicted in Figure 3.2.

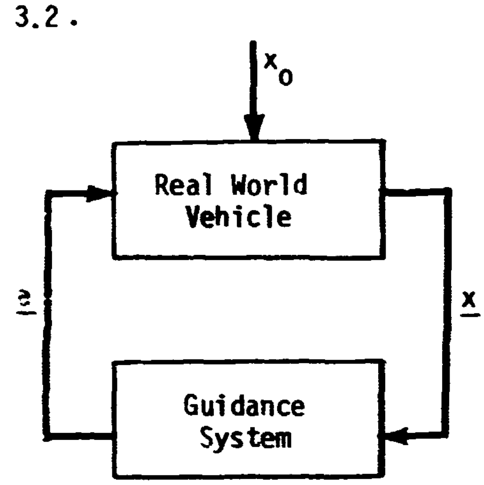


FIGURE 3.2: SIMPLIFIED CONTROL SYSTEM

The function of the guidance system is seen to be the computation of an acceleration vector $\underline{a}$ for each value of t and $\underline{x}$, in some region pertinent to the mission. Since the guidance system is usually incapable of providing a continuous output of acceleration vectors corresponding to a continuous input of the vehicle state, it is common practice to require guidance to output an acceleration vector function $\underline{a}(t) = \underline{a}(t, t_0, \underline{x}_0)$ for $t > t_0$, corresponding to the state vector $\underline{x}_0$ at time t_0.

The function $\underline{a}(t)$ is used to command the vehicle propulsion system until a new state vector $\underline{x}_1$ at time $t_1 > t_0$ is received. Note that a single computation at t_0, (open-loop operation) would be sufficient if there were no navigation or system errors and if there were no approximations in the guidance calculation. Neither of these conditions usually apply, however. Because of the closed-loop operation, it is difficult to assess the effect of errors and approximations except by computer simulation. It has been demonstrated in practice that a guidance mode which apparently is crude in the approximations made in the algorithm is quite effective in steering to the desired end conditions in a near optimal fashion.

3.2 THE MATHEMATICAL PROBLEM

The vehicle state will consist of the position $\underline{r}$, the velocity $\underline{v}$, and the mass m of the vehicle. Thus, $\underline{x} = (\underline{r}, \underline{v}, m)$. For a given guidance phase these variables are related by the equations:

$$\ddot{\underline{r}} = \dot{\underline{v}} = \underline{g} + a\,\underline{u} \tag{3.1}$$

$$\dot{m} = -\beta \tag{3.2}$$

where
- $\underline{g}$ = gravitation acceleration (or gravitational force per unit mass)
- a = thrust acceleration magnitude = $\frac{V_e \beta}{m}$
- β = mass flow rate
- V_e = effective exhaust velocity
- $\underline{u}$ = thrust acceleration direction, $|\underline{u}| = 1$.

and a dot over a vector denotes differentiation by time. The objectives of the guidance phase may be expressed by the equations

$$g_i\ (T,\ \underline{x}(T)) = 0, \quad i = 1,2,\ \ldots,\ m \tag{3.3}$$

or simply

$$g(T,\ \underline{x}(T)) = 0 \tag{3.4}$$

where $g = (g_1, g_2, \ldots, g_m)$ and T = time of termination of the phase (thrust cut-off). The time T may be either fixed or free. In addition to the dynamic operations (3.1) and (3.2) and the desired final conditions (3.4), a performance index J is specified:

$$J = J(T,\ x(T)). \tag{3.5}$$

It is common to select J to be the amount of fuel consumed to final time T. It is desired to choose the guidance commands a and $\underline{u}$ so that the resulting trajectory $\underline{x}(t)$, determined by (3.1) and (3.2), satisfies the end conditions (3.4), while minimizing the performance index J.

The selection of the guidance commands is of course not arbitrary. The realizable thrust acceleration is limited by the capabilities of the vehicle. Two common situations are outlined below.

A. Mass Flow Rate Limited (chemical rockets). The effective exhaust velocity (V_e) is constant and the mass flow rate is bounded:

$$0 \le -\dot{m} \le q.$$

Often the mass flow rate can assume only two values, o and q. Then

$$m(t) = m(t_0) - qt, \tag{3.6}$$

hence

$$a(t) = \frac{-\dot{m}\, v_e}{m} = \frac{qv_e}{m(t_0) - qt} = \frac{v_e}{\tau - t}, \tag{3.7}$$

where $\tau = m(t_0)/q$.

B. Power Limited (Plasma or Ionic Rockets). The instantaneous propulsive power is limited, with:

$$p = \frac{1}{2} v_e^2 \beta, \; p_{min} \le p \le p_{max} \tag{3.8}$$

Both v_e and β are treated as control variables.

3.3 CLASSIFICATION OF GUIDANCE MODES

In this section various guidance modes will be classified according to the mathematical approximations and/or assumptions introduced in their derivation. Although any attempt to classify guidance modes is difficult, the present scheme has been found workable and useful as a basis for discussion*.

* The need for a classification of guidance modes was first recognized by Mr. W.E. Miner. The present scheme of classification is based upon his suggestions.

The classification of guidance modes proceeds according to the steps the analyst introduces in their development. One must first define the problem, which requires a mathematical model of the system dynamics, a definition of system performance, a specification of the desired end conditions, both at mission completion and at the end of each individual guidance phase, and finally a description of the constraints imposed on the guidance system. The constraints may be of two types: state variable and control variable constraints.

Given the problem definition, the analyst must first decide whether the derivation of the guidance mode equations is to be based upon a precise model of the system dynamics or an approximate mode. The approximate model would be selected to simplify the derivations or to facilitate real time operation. All modes based on an approximate model of the dynamics are operated closed-loop so that the errors introduced through the approximations are removed through feed-back action.

In either case the analyst must next decide on either an unrestricted infinite dimensional steering law or seek to limit the available degrees of freedom through the introduction of a parameterized steering law. The latter approach will be called finite dimensional steering. In general, the infinite dimensional law will result from a calculus of variations solution; a finite dimensional law is a fixed functional form containing a finite number of parameters. In this case an ordinary optimization problem must be solved if there are more parameters than end conditions.

The distinction between infinite and finite, which is really a distinction between non-parameterized and parameterized, is consistent with theory and practice.

After the guidance law has been derived, it may be implemented in a number of ways. There are basically two approaches. Before the mission takes place one can compute the steering law for an ensemble of state vectors, and store the steering law as a function of time and the vehicle state. This is called the stored expansion method. Such expansions can be further classified according to whether they are linear or non-linear. By linear we mean an expansion which develops the control as a linear function of the state, even though non-linear analysis may be required to do this. For example, the so-called second variation method (Reference 13 and 14) considers first and second order variational terms in order to develop the steering law, which nevertheless turns out to be a linear function of the state. Another approach to the problem is to solve the optimization problem in real-time. This will be called real time iteration, for, in general, no closed form solution to the guidance problem exists and an iterative technique must be employed.

If real time iteration is selected one has a choice of specifying the end conditions either explicitly or numerically. In the explicit approach the end conditions are given in functional form. For example, the end conditions might require attainment of given values of energy and angular momentum at burnout, which can be explicitly described as functions of the vehicle state by the well known conic formulas. The numerical end condition approach specifies some point or some hyper-surface in state space to be achieved. The latter may be described in the form of a Taylor series expansion with numerical coefficients. In either case, one must specify certain numbers. For explicit end conditions one must specify the numerical value of, say, the energy and angular momemtum to be achieved. In the numerical end condition case one must specify the coefficients of the series. Targeting is the task of determining these coefficients. This important part of the guidance pre-flight preparation is further discussed elsewhere.

The scheme of classification is summarized in Tables 3.1 and 3.2, and representative guidance modes are shown as members of the various classes. The examples listed comprise modes which have already been employed for space guidance, as well as schemes which have only been analytically developed. Because of our classification method it is possible for some modes to fit into more than one class.

3.4 THE FORM OF THE STEERING LAW

This subsection discusses in more detail the structure of infinite dimensional vs finite dimensional steering laws as defined in Section 3.3.

<u>Infinite Dimensional Steering</u>: Whether a precise or an approximate model of the system dynamics is employed in the problem formulation, a basic guidance class comprises those modes which employ an infinite

TABLE 3.1: CLASSIFICATION OF GUIDANCE MODES

DEFINITION OF PROBLEM: SYSTEM DYNAMICS, SYSTEM PERFORMANCE, END CONDITIONS, CONSTRAINTS

PRECESE MODEL OF SYSTEM DYNAMICS							
INFINITE DIMENSIONAL STEERING LAW				FINITE DIMENSIONAL STEERING LAW			
STORED EXPANSION		REAL-TIME ITERATION		STORED EXPANSION		REAL-TIME ITERATION	
LINEAR	NON-LINEAR	EXPLICIT END CONDITION	NUMERICAL END CONDITION	LINEAR	NON-LINEAR	EXPLICIT END CONDITION	NUMERICAL END CONDITION
1. Delta - steer to null deviations from nominal trajectory 2: Lambda matrix - Ref. 15 3. Second variation Refs. 13 & 14	1. Polynominal Storage - Ref. 16 3. Dynamic programming - Ref. 17	1. Calculus of variations - Ref. 18 2. Successive approximations - Ref. 19,21 3. Newton - Raphson - Ref. 20	1. Calculus of variations 2. Successive approximations 3. Newton-Raphson	1. C Matrix Ref. 6	1. Polynominal storage	1. TRW Iterative Hybrid - Ref. 8 and Section 4 2. C Matrix Ref. 6	1. Ranger/Mariner/Surveyor Mid-Course - Ref. 7

TABLE 3.2: CLASSIFICATION OF GUIDANCE MODES

DEFINITION OF PROBLEM: SYSTEM DYNAMICS, SYSTEM PERFORMANCE, END CONDITIONS, CONSTRAINTS

APPROXIMATE MODEL OF SYSTEM DYNAMICS							
INFINITE DIMENSIONAL STEERING LAW				FINITE DIMENSIONAL STEERING LAW			
STORED EXPANSION		REAL-TIME ITERATION		STORED EXPANSION		REAL-TIME ITERATION	
LINEAR	NON-LINEAR	EXPLICIT END CONDITION	NUMERICAL END CONDITION	LINEAR	NON-LINEAR	EXPLICIT END CONDITION	NUMERICAL END CONDITION
1. Delta - Steer to null deviations from nominal trajectory 2. Lambda matrix - Ref. 15 3. Second variation Refs. 13 & 14	1 Polynominal storage - Ref. 16 2. Dynamic programming Ref. 17	1. Open - Ref. 3 2. Brown-Johnson - Ref. 22 3. Calculus of variations Ref. 18 4. Successive approximations - Ref. 19,21 5. Newton - Raphson - Ref. 20	1. Calculus of variations 2. Successive approximations 3. Newton - Raphson	1. C Matrix Ref. 16	1. Polynominal storage	1. TRW Hybrid - Ref. 8 2. IGM - Ref. 23 3. Perkins- Ref. 24 4. Teren - Ref. 25 5. Cherry - Ref. 26	1. TRW Hybrid - 2. IGM 3. Perkins 4. Teren 5. Cherry

dimensional steering law. That is, the steering law is restricted only by the vehicle and state variable constraints and not by artificially introduced parameterization of the steering angles. The calculus of variations is the basic analytical tool for determining the time varying steering angles. The procedure is to introduce the adjoint variables and proceed as in the theory of optimal control. The following discussion outlines the major features of this method.

The basic equations for the calculus of variations solution will now be developed under the assumption that burnout mass is to be maximized with a mass-flow-rate-limited propulsion system. The equations of motion are as given by (3.1) and (3.2). The control variables are the mass flow rate ($\beta(t)$, the thrust direction u(t), and the total burning time (T), where

$$o \leq \beta \leq \beta_{max} \tag{3.9}$$

$$\underline{u} \cdot \underline{u} = 1 \tag{3.10}$$

The state of the vehicle is to satisfy the initial conditions

$$\underline{r}(o) = \underline{r}_0,\ \underline{v}(o) = \underline{v}_0,\ m(o) = m_0 \tag{3.11}$$

and the final conditions

$$g_i(\underline{r}(T),\ \underline{v}(T),\ T) = 0 \quad i = 1,2\ldots m \leq 6 \tag{3.12}$$

$$m(T) \geq M_{min}. \tag{3.13}$$

The guidance variables $\beta,\underline{u}$ are to be selected so that the resulting state vectors with initial conditions (3.11) satisfy (3.12) and minimize the quantity $-m(T)$. The adjoint state vector

$$\underline{\lambda} = \begin{pmatrix} \underline{\lambda}_r \\ \underline{\lambda}_v \\ \underline{\lambda}_m \end{pmatrix} \tag{3.14}$$

is introduced along with the Hamiltonian H defined by

$$H(\underline{x},\underline{\lambda}) = \underline{\lambda}^T \underline{v} + \underline{\lambda}^T (\underline{g} + a\ \underline{u}) + \lambda_m(-\beta) \tag{3.15}$$

From the calculus of variations it may be inferred that if optimal guidance $\beta,\underline{u}$ exists, then there exists an adjoint vector $\underline{\lambda}$ which satisfies the equations

$$\dot{\underline{\lambda}}_r = -\left(\frac{\partial \underline{g}}{\partial \underline{r}}\right)^T \underline{\lambda}_v = \frac{\partial H}{\partial \underline{r}} \tag{3.16}$$

$$\dot{\underline{\lambda}}_v = -\underline{\lambda}_r = -\frac{\partial H}{\partial \underline{v}} \tag{3.17}$$

$$\dot{\lambda}_m = \underline{v}^T \underline{u} \ \frac{\beta v_e}{m^2} \ . \tag{3.18}$$

and the final conditions

$$\underline{\lambda}_r^T \, \underline{\delta r} + \underline{\lambda}_v^T \, \underline{\delta v} = 0 \tag{3.19}$$

where $\underline{\delta r}$, $\underline{\delta v}$ are arbitrary vectors which satisfy

$$\frac{\partial g_i}{\partial \underline{r}} \, \underline{\delta r} + \frac{\partial g_i}{\partial \underline{v}} \, \underline{\delta v} = 0. \tag{3.20}$$

The maximum principle implies, furthermore, that

$$\underline{u} = \underline{\lambda}_v / |\underline{\lambda}_v|, \text{ and} \tag{3.21}$$

$$\beta = \begin{cases} \beta_{max} \text{ if} \left(\underline{\lambda}_v^T \, \underline{u} \, \frac{v_e}{m}\right) - \lambda_m > 0 \\ \\ 0 \text{ if} \left(\underline{\lambda}_v^T \, \underline{u} \, \frac{v_e}{m}\right) - \lambda_m < 0. \end{cases} \tag{3.22}$$

If the function $\left(\underline{\lambda}_v^T \, \underline{u} \, v_e/m - \lambda_m\right)$ is identically 0 on an interval, β is not specified. In this case the trajectory is said to have a singular sub-arc. Letting $y = \underline{\lambda}_v^T \, \underline{u} \, v_e/m - \lambda_m$, it is seen from (3.16) that

$$\dot{y} = (\underline{\lambda}_v^T \, \underline{u}) \, \frac{v_e}{m} = |\underline{\lambda}_v| \, \frac{v_e}{m} \tag{3.23}$$

It is thus unnecessary to determine λ_m from (3.16); integration of the above equation will suffice. It is also seen that

$$\ddot{\underline{\lambda}}_v = G^T \underline{\lambda}_v, \qquad (3.24)$$

where $G = \partial \underline{g}/\partial \underline{r}$, so that is is sufficient[*] to introduce only the three dimensional adjoint vector $\underline{\lambda} = \underline{\lambda}_v$. The problem is then reduced to the following set of differential equations and end conditions:

$$\dot{\underline{r}} = \underline{v} \qquad (3.25)$$

$$\dot{\underline{v}} = \underline{g} + \beta \frac{v_e}{m} \frac{\lambda}{|\lambda|} \qquad (3.26)$$

$$\dot{m} = - \beta \qquad (3.27)$$

$$\ddot{\underline{\lambda}} = G^T \underline{\lambda} \qquad (3.28)$$

$$\dot{y} = |\lambda| \, v_e/m, \qquad (3.29)$$

$$\underline{r}(o) = \underline{r}_o, \ \underline{v}(o) = v_o, \ m(o) = m_o \qquad (3.30)$$

$$g_i(\underline{r}(t), \underline{v}(t)) = 0 \quad i = 1,2,\text{---}m \leq 6 \qquad (3.31)$$

and

$$\underline{\lambda}^T \, \underline{\delta v} - \dot{\underline{\lambda}}^T \, \underline{\delta r} = 0 \text{ at } t = T \qquad (3.32)$$

---

[*] This procedure is no longer valid in the presence of atmospheric drag. The dynamic equation has an additive position and velocity dependent acceleration term which makes elimination of $\underline{\lambda}_r$ impossible.

for all $\underline{\delta r}$, $\underline{\delta v}$ satisfying

$$\frac{\partial g_i}{\partial \underline{r}} \underline{\delta r} + \frac{\partial g_i}{\partial \underline{v}} \underline{\delta v} = 0 \quad i = 1,2,\text{---}m. \qquad (3.33)$$

If the m vectors $(\partial g_i/\partial \underline{r}, \partial g_i/\partial \underline{v})$ are linearly indeoendent, then the conditions (3.33) represent m-6 conditions on the variables $\underline{\lambda}$, $\dot{\underline{\lambda}}$ at time t = T. The system of equations

$$\ddot{\underline{r}} = \dot{\underline{v}} = \underline{g} + a\,\underline{u} \qquad (3.34)$$

$$\ddot{\underline{\lambda}} = G^T \underline{\lambda} \qquad (3.35)$$

of order twelve is thus constrained by a 6 initial conditions and 6 final conditions on the variables $\underline{r}$, $\dot{\underline{r}}$, $\underline{\lambda}$,$\dot{\underline{\lambda}}$. This system and the corresponding boundary values will be called the two point boundary value problem.

There are many methods for implementing these equations in an exact or approximate form. For example, there is the gradient method (References 27, 28), the Newton-Raphson method (References 13, 14, 19, 20), various forms of successive approximation techniques (Reference 21), or an approximate solution of the Hamilton-Jacobi equation (Reference 29). A fundamental difference is between real time solution and pre-flight solution giving the required steering as a function of the vehicle state. The latter technique, the so-called stored solution method, may also be accomplished in a number of ways. The guidance commands may be stored in the form of functions of the actual vehicle state; these will, in general, be non-linear functions of the state. Another popular method is to store the guidar commands in the form of functions of deviations of the vehicle state from a nominal state. The nominal state in such a case is generally computed by solution of the two point boundary value problem given by variational theory. The functional form for the guidance/state vector relationship may be either linear or non-linear. These methods generally are called first or second order methods. The major drawbacks to the stored solution method is the enormous amount of pre-flight calculation required to cover all contingencies, e.g., variations in launch date, possible abort modes, et cetera, and the bulk of the data which needs to be stored. The data storage problem is considerably alleviated when the guidance commands are given as linear functions of the state vector deviations from a nominal. The on-board and real time computer capabilities are also at a minimum for this type of operation.

Real time solution of the two point boundary value problem is suggested as a feasible guidance mode in light of recent advances in computer technology. Never the less, the computational demands are considerable. More design work and testing will be required before feasibility can be definitely established. Although this method is capable of yielding the optimal steering law, optimality is but one measure of performance of a guidance mode. It is often desirable to sacrifice optimality in favor of accuracy, ease of mechanization, or a wider region of applicability. These considerations will be discussed further in Section 3.8.

FINITE DIMENSIONAL STEERING: This mode of operation means that either the steering law or a portion of the state vector has been limited to a finite number of degrees of freedom. Again, either an approximate or a precise model of the system dynamics may be used. Some of the equations of motion must be integrated numerically if the precise model is used. This approach has been termed parameterized guidance in this report and is more fully discussed in another section. Most of the operational guidance modes are finite dimensional with an approximate model of the dynamics. The corrdinate systems and the parameterization employed are so selected that the equations of motion may be (approximately) integrated in closed form, thereby expressing the end conditions in terms of the parameters. The parameters are then determined to obtain the desired end conditions, assuming that the dynamic model is correct. These modes must be run closed-loop so that the errors due to the approximate model may be removed by subsequent guidance commands. Since these schemes are relatively simple, there is no need for the stored solution mode of operation. Further discussion and examples of finite dimensional steering are to be found in Sections 4, 5, and 6.

3.5 APPLICATION OF THE CLASSIFICATION METHOD TO ATLAS-AGENA-RANGER GUIDANCE

The modes used for guidance of the Atlas, Agena, and Ranger vehicles for lunar missions can be classified according to the method of Section 3.3. The mission objective in this case was to impact a given point on the lunar surface. Different guidance modes were used for each of the three vehicles. The Atlas system was radio-commanded guided, with the pitch and yaw steering angles choosen at every guidance cycle to be linear functions of time. Thus there were two parameters to choose for each steering angle, i.e., the initial angle and the angle rate for each. The end conditions for Atlas burnout were predicted at each cycle time t by assuming a constant, average gravity vector acting between t and the final time. The desired end conditions were explicitly given as the energy and angular momentum of a prespecified coast ellipse at burnout. The ellipse was chosen such that mission objectives would be met if subsequent Agena vehicle were to perform nominally. The calculations of steering angle and predicted time-to-go were iterated in real time as new tracking data was obtained and processed. Thus the Atlas system is to be considered an example of a guidance mode which employs: (1) approximate model of system dynamics; (2) a finite dimensional steering law; (3) real time iteration, and (4) explicit end conditions. The parking orbit coast time for the Agena vehicle was calculated and stored at Atlas shut-off by using the burnout position and the pre-calculated inertial direction of the "pseudo-asymptote" to determine the coast angle. The pseudo-asymptote concept will be described in Section 7. The numerical values of these conic parameters required for Atlas guidance were determined by a targeting process.

The Agena vehicle was guided in more rudimentary fashion. A prespecified nominal pitch and yaw attitude angle was commanded, where the inertially-sensed pitch angle was corrected in flight by a horizon sensor. Thrust termination was commanded when the integrated sensed velocity reached its nominal value. This rudimentary system can only be justified theoretically by assuming an approximate model of the vehicle motion. Thus, the Agena system is to be considered an example of a guidance mode which employs: (1) an approximate model of system dynamics; (2) a finite dimensional steering law; and (3) a stored expansion with a linear form. Note that the parking orbit duration, that is, the time of starting the translunar injection phase, is an open-loop guidance parameter after Atlas burnout.

Guidance of the Ranger spacecraft was accomplished by tracking from the earth to determine injection conditions, and then iterating in real time to determine the three components of the velocity impulse to be applied at the prespecified midcourse maneuver time in order to impact the desired point on the moon. There was adequate time available to accomplish these calculations, for the correction was not applied until 15 to 20 hours after injection. The iteration process was carried out by a Newton-Raphson procedure. Thus the Ranger system is to be considered an example of a guidance mode which employs: (1) a precise model of system dynamics; (2) a finite dimensional steering law; (3) real time iteration, and (4) numerical end conditions.

Thus it can be seen that the relatively simple Atlas-Agena-Ranger lunar mission contained three distinctly separate guidance modes.

3.6 APPLICATION OF THE CLASSIFICATION METHOD TO ATLAS-AGENA-MARINER GUIDANCE

The Atlas-Agena-Mariner vehicle employed for interplanetary missions was guided in much the same fashion as for Ranger missions, the primary difference being that the direction of the earth-escape hyperbolic asymptote was used for controlling the parking orbit duration rather than the lunar pseudo-asymptote. The specification of the escape asymptote is described in Section 7. The midcourse guidance corrections for the Mariner vehicle were determined as for the Ranger mission, except that the "impact point" was specified by a fly-by distance at the planet.

3.7 APPLICATION OF THE CLASSIFICATION METHOD TO SATURN V APOLLO GUIDANCE

The Saturn V vehicle used for Apollo missions is guided by the so-called IGM guidance mode, which stands for Iterative Guidance Mode. In this method the pitch and yaw steering angles are chosen to be linear functions of time, and end conditions are predicted assuming a constant gravitational acceleration between present and final time. An additional approximation is made in the calculation of time-to-go to burnout, which corresponds to a rough one step integration between present time and predicted final time. End conditions can be specified in various forms, either numerically in terms of a desired state at burnout, or explicitly from the conic formulae. The calculations are iterated at each guidance cycle as new inertial navigation data is obtained. Thus, as in the case of Atlas guidance for Ranger and Mariner missions, we have an example of a guidance mode which employs: (1) an approximate model of system dynamics; (2) a finite dimensional steering law; (3) real time iteration, and (4) either explicit or numerical end conditions. The main difference between previously discussed Ranger-Mariner mission guidance and Apollo mission guidance is that the iterative guidance mode is employed all the way to injection, including a closed-loop computation of parking orbit duration, rather than by using a rudimentary guidance mode of the Agena type for translunar injection.

The mode used for midcourse and approach guidance of the Apollo spacecraft is similar to that used for the Ranger/Mariner vehicles. The modes used for subsequent phases of the Apollo mission are of the same class as the IGM, but will not be discussed here.

3.8 MEASURES OF GUIDANCE MODE PERFORMANCE

The choice of a guidance mode is often ad hoc, and one must consider many factors. Some measures of guidance system performance are:

1. Optimality - given that there is a performance index to be minimized, say propellant expenditure, how does the obtained value of the performance index compare to the theoretical minimum?

2. Accuracy - given that approximations are introduced into the derivation and mechanization of the guidance equations what are the resulting errors in the desired terminal conditions? These errors can be classified according to:

- approximation errors - due to analytic approximations introduced into the derivation of the guidance equation.

- computer errors - due to the inaccuracies of the numerical algorithms used to implement the guidance equations (truncation and roundoff).

- mechanization errors - due to the inability of the vehicle to physically respond to the guidance commands.

There are also navigation errors, but, insofar as the guidance and navigation problems are separable (i.e., assuming superposition of effects), these errors need not be considered in the design of a guidance mode.

3. Stability - does the guidance mode call for high frequency attitude changes, or can small input errors result in large non-standard maneuvers of the vehicle?

4. Constraint Compability - is the guidance mode capable of generating commands and a trajectory which satisfy all imposed control and state variable constraints? If so, how difficult is the task of incorporating the constraints into the analytical formulation?

5. preflight prepartion - what is the cost in time and money of preflight preparation of the guidance equations. In particular, how long does it take to prepare the guidance guidance system to accomplish a given mission? (the "quick reaction" problem).

6. verification requirements - what is the cost in time and money of preflight verification of guidance system performance? (another aspect of the "quick reaction" problem)

7. flexibility - what are the types of missions which the guidance mode can perform, and how well can it adapt to changes in the mission, such as variations of launch azimuth? (another aspect of the "quick reaction" problem).

8. region of applicability - what is the range of perturbations which can be adequately treated by the guidance mode?

9. computer factors - what are the real time on-board and/or earth-based computer requirements, in particular, how much storage space is required, what is the length of the computing cycle for each iteration of the guidance equations, and how complex must the computer be?

10. growth potential - what is potential applicability of the guidance mode to future missions?

These measures of performance are qualitatively discussed in Section 3.9 for various classes of guidance modes.

3.9 DISCUSSION OF GUIDANCE MODE PERFORMANCE

It is difficult to compare the various guidance modes with respect to the many measures of performance, for there is a wide variety of missions, mission phases, and launch vehicles to be considered. This section does not attempt a thorough comparison of specific guidance modes on the basis of the performance indices, but rather contains a more general outline of the properties of the classes of guidance modes. Specific results on optimality, accuracy and region of applicability may be found in the simulation results. The following is organized according to the general classification scheme.

- PRECISE MODEL-INFINITE DIMENSIONAL STEERING

The steering law in this case is determined via the calculus of variations.

Optimality: Both the stored expansion and real time iteration methods are capable of performing arbitrarily close to optimum, the former being limited by any numerical approximentations of the control function which may be introduced and the latter being limited by the number of iterations one is willing to perform.

Accuracy: Accuracy is primarily determined by the form of the steering algorithm near the final time, and the criterion used to terminate thrust. As in the case of all guidance modes, there is a natural uncontrollability near burnout (see Section 9).

Stability: Relatively stable, since the optimal steering law is a smoothly varying function with only moderate changes in slope at switching (staging) times. As in the case of all guidance modes, however, there is a natural potential instability at the final time (see Section 9.)

Constraint Compatibility: Thrust vector (control variable) constraints can be treated relatively easily. State vector constraints pose difficulties for the real time iteration modes, however, for there is no satisfactory theory which allows straight-forward implementation (see Reference 30). When the solutions are stored, state variable constraints may be included at the expense of additional pre-flight computation.

Pre-flight Preparation: Relatively little required for real-time iteration methods. The stored expansion method, however, requires that many steering commands be calculated to cover all feasible contingencies. These include changes in launch data, off-nominal vehicle performance, and abort operations. There is a direct trade-off between the amount of pre-flight preparation and the region of applicability in the sense that the requirement for a larger region (e.g., non-linear rather than linear control law) leads to more pre-flight computation.

Verification Requirements: Involves simulation of all feasible contingencies. The stored expansion method is more difficult to verify than the real-time iteration method because of the many input numbers to be checked.

Flexibility: Potentially very flexible since the form of the guidance law can be quite general and all mission parameters could be included as input variables.

Region of Applicability: The real-time iteration method can be designed to easily accommodate a wide region, being limited only by the convergence region of the iteration algorithm. In some cases this region can be determined analytically. The region of applicability of the stored expansion method depends upon the form of the expansion (e.g., linear vs non-linear) as discussed above in Pre-flight Preparation.

Computer Factors: The stored solution technique requires minimal on-board computer capability but large data storage. Real time iterations require a relatively sophisticated on-board computer capability, but a minimum of data storage.

Growth Potential: Limited for the stored solution method since new forms of the control law may be required for different applications. Growth potential is good for the real-time iteration method, since the form of the solution is a general one.

- PRECISE MODEL-FINITE DIMENSIONAL STEERING

This class is called precise parameterized guidance in this report (see Section 6). The same performance may be expected of this technique as for Infinite Dimensional

Steering-Real-Time Iteration (discussed above), with the exception that optimality may be reuced and instabilities may be induced by the selected form of the control law.

- APPROXIMATE MODEL-INFINITE DIMENSIONAL STEERING

  The guidance modes in this category are essentially the same as those discussed under Precise Model-Infinite Dimensional Steering, except the algorithms are applied to an approximate dynamic model. The purpose of introducing the approximate model is to reduce the number and/or complexity of the calculations required. The corrective effect of closed-loop operation is relied upon to maintain accuracy; the main limitation is some loss of optimality.

- APPROXIMATE MODEL-FINITE DIMENSIONAL STEERING

  These guidance modes are based upon an approximate model of system dynamics and a parameterized form of the guidance law.

  Optimality: The cost in optimality is slight within the region of applicability, but there can be definite degradation in optimality if missions further from the designed region are attempted.

  Accuracy: The final value accuracy can be made arbitrarily good for most missions, subject only to the natural uncontrollability suffered by all guidance modes near burnout.

  Stability: Stable if the form of the steering law is suitably chosen (except at the firal time as noted above).

  Constraint Compatibility: State constraints are more easily handled than in the infinite dimensional case since enforcement of these constraints may normally be accomplished by constraining the guidance parameters.

Pre-flight Preparation: Minimal because of the parameterized form of the guidance law and the simplified system dynamics.

Verification Requirements: Additional pre-launch simulation and verification of the guidance equations may be required in order to check the approximations of the system dynamics.

Flexibility: Can be made very flexible within the region of applicability, depending only on the form of the guidance law.

Region of Applicability: At present limited to the particular set of mission geometries to which the approximations apply. The finite dimensional modes can overcome deviations from nominal propulsion performance with relative case.

Computer Factors: Minimal computer capability is required, both on-board and on the ground (see above Pre-flight Preparation).

Growth Potential: Limited only by the form of the guidance law and the dynamic system approximations; these can be modified relatively easily.

4. SIMULATION OF JUPITER INJECTION GUIDANCE

4.1 INTRODUCTION

Computer simulation of the launch-to-injection phase of a Jupiter mission was performed in order to compare typical examples of guidance modes. The trajectory profile was an unusual one, chosen not as an example of optimal performance but instead as a severe test case for guidance analysis. The guidance modes chosen were the precise calculus of variations algorithm, the Iterative Guidance Mode (IGM) as described in Reference 23, the TRW Hybrid guidance mode, as described in Reference 8, and an improved iterative form of TRW Hybrid guidance. The IGM guidance mode is briefly discussed in Section 3.7, and the TRW Hybrid guidance mode is briefly discussed in Section 6.3. These modes are examples of parameterized guidance laws. In all cases the guidance simulations were carried out open-loop, in the sense that no random navigation or systematic errors were introduced

4.2 TRAJECTORY DESCRIPTION

The launch-to-injection phase of the Jupiter fly-by mission used the STACK launch vehicle defined by NASA Electronics Research Center. STACK vehicle consists of four stages: a 260" solid first stage, an SIVB second stage, an improved Centaur third stage, and an advanced Kick stage. Table I presents the vehicle mass and performance characteristics obtained from a reference trajectory provided by ERC.

The first two stages boost the payload into a circular parking orbit of 96.13 nautical mile altitude, an inclination of 26 degrees, and a descending node of 101.881 degrees (the angle measured from the launch meridian to the descending node). Holding this plane of motion fixed, the last two stages achieve the required velocity for injection into the departure hyperbola of 3.171 eccentricity, an energy at cut-off 7.1138×10^8 ft^2/sec^2, and an inertially fixed position of periapsis. The trajectory is to be designed to maximize payload at final injection, subject to these final value constraints, i.e., orientation of the plane of motion, energy, eccentricity, and position of periapsis.

The optimal launch-to-injection Jupiter fly-by trajectory was obtained by using the Multiple Vehicle N-Stage (MVNS) program with the gravity turn, Iterative Guidance Mode (IGM), program in the role of a preliminary optimization tool and a Calculus of Variations program. Optimization of the trajectory was conducted in two phases: injection into the specified parking orbit and injection in the hyperbolic transfer orbit.

The basic assumptions involved in solving the steering optimization problem relative to a three-dimensional model of rocket motion were as follows:

1. Spherical rotating earth
2. Uniform inverse-square gravitational model
3. U.S. 1962 standard atmosphere (first stage only); no aerodynamic forces for second and upper stages
4. Ambient pressure corrected thrust
5. Minimize time of power flight to achieve specified final conditions.

- ASCENT TO PARKING ORBIT TRAJECTORY OPTIMIZATION

The STACK vehicle is launched from Cape Kennedy at an azimuth of 90 degrees. An instantaneous kick turn is used to tilt the missile after a 10 second vertical rise time. A gravity turn program carries the vehicle to first stage burnout. The second stage, which burns in vacuum, uses the IGM program with both pitch and yaw steering, to

TABLE 4.1

STACK Launch Vehicle

Weight Summary

Payload at hyperbolic injection	7,480	
Kick stage propellant consumed	11,770	
Kick stage ignition weight		19,250
Improved Centaur dry weight	4,568	
Improved Centaur propellant consumed	37,305	
Improved Centaur wet weight		41,873
S-IV B dry weight	38,482	
S-IV B propellant consumed	217,096	
S-IV B interstage	5,600	
S-IV B wet weight		261,178
First stage dry weight	402,735	
First stage propellant consumed	3,298,986	
First Stage wet weight		3,701,721
	Lift-off weight	4,024,022

Performance Summary

Stage	Vehicle	Thrust (Vacuum) lb.	Specific Impulse sec.	Flow Rate lb/sec.
I	260" solid	6,429,857	265	24,300.13
II	S-IV B	205,070	425	482.00
III	Improved Centaur	31,711	465	68.20
IV	Advanced Kick Stage	7,927	465	17.05

steer the vehicle into the circular parking orbit described above. The inertial velocity is the cut-off parameter.

Optimization of the payload into parking orbit is accomplished by varying the kick angle, while constraining the first stage to a gravity turn and constant burn time, and using the near-optimum steering law of the IGM program to minimize the burn time of the second stage for the specified end conditions. An iteration procedure built into the MVNS program generates a set of first and second stage trajectories and provides the kick angle to optimize first stage burnout altitude for a given burnout weight. Figure 4.1 shows the variation of the second stage burnout weight with the first stage burnout altitude. The near maximum weight inserted into the parking orbit is obtained with a first stage burnout altitude of 195,494 feet.

Further improvement in the second stage trajectory is obtained by using the calculus of variations package with the MVNS program. This program option uses initial guesses of pitch and yaw attitude and attitude rates to generate a set of Lagrange multipliers that permit, simultaneously, solution of the equations of motion and the Euler-Lagrange equations for minimum propellant consumption. The calculus of variations solution increased the second stage burnout weight by 380 pounds over the IGM approximation. A payload increase of 6,126 pounds reference value was obtianed by these optimization methods.

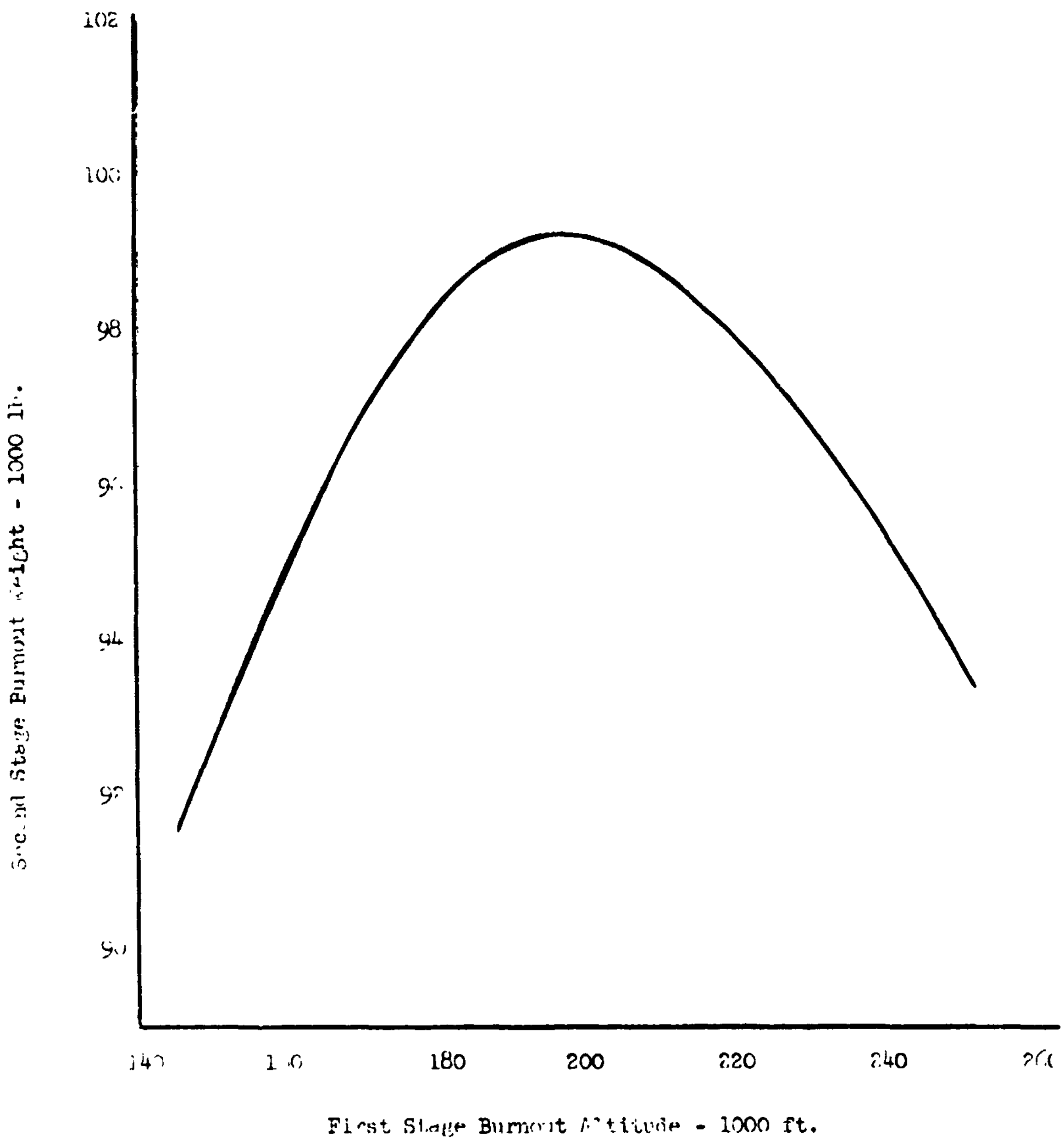


FIGURE 4.1: SECOND STAGE BURNOUT WEIGHT

- <u>HYPERBOLIC ORBIT INJECTION OPTIMIZATION</u>

Transfer from parking orbit to e departure hyperbola requires a long burn time and a large range angle. The linear tangent steering law used in the Iterative Guidance Mode package is no longer suitable to optimize the third and fourth stage trajectories. The calculus of variations option is therefore used for the hyperbolic orbit injection phase. No out-of-plane steering is necessary since the desired plane change is accomplished during the second stage burn into parking orbit.

Since the parking orbit duration can be freely chosen to control the position of periapsis of the departure hyperbola, an arbitrary ignition time is assumed for the third stage and the optimal trajectory is designed with initial conditions corresponding to the position and velocity in the parking orbit at this time. With fixed burning time for the third stage, the fourth stage burnout is defined by the final energy constraint. Iteration is then performed to satisfy the angular momentum constraint and the transversality condition

$$\bar{v}\cdot\bar{\lambda}_r - \frac{\mu}{r^3}\bar{r}\cdot\bar{\lambda}_v = 0$$

where $\bar{r}$ and $\bar{v}$ are the instantaneous radius and inertial velocity vectors $\bar{\lambda}_r$ and $\bar{\lambda}_v$ the set of Lagrange multipliers, and μ the gravitational parameter.

Initial values of Lagrange multipliers are computed from assumed initial values of the pitch attitude, Ψ_p, and pitch attitude rate, $\dot{\Psi}_p$. To determine a proper starting combination of pitch and pitch rate, several nominal trajectories are simulated for ranges of initial values

Ψ_p and $\dot{\Psi}_p$. The errors in angular momentum and transversality constraint for each of these combinations are plotted in Figures 4.2 and 4.3 to show the combinations of Ψ_p $\dot{\Psi}_p$ which allow each of the errors to be nulled. Fig. 4.3 shows that there is, in general, more than one value of Ψ_p corresponding to each value of $\dot{\Psi}_p$ which allows the transversality constraint to be satisfied. This is due to the highly non-linear behavior of the transversality boundary condition and makes the extrapolation shown questionable.

Through a cross plot of the combination of Ψ_p and $\dot{\Psi}_p$ which allows each of the constraints to be zero, it is possible to determine the combination which simultaneously nulls both constraints. This cross plot is shown in Fig. 4.4. The two curves for the zero transversality correspond to the left and right crossings of the curves in Figure 4.3 Iteration for an optimum near the intersection of the right zeros with the nulled angular momentum constraint allows rapid convergence to the only optimal injection trajectory. The extrapolated left zeros and the nulled angular momentum curves will not intersect since the solution is unique.

After optimization, the third stage ignition time in the parking orbit is altered to produce the desired argument of perigee in the final injection orbit to be 159.44 degrees (the angle measured in the orbit plane from the ascending node to a line passing through the perigee of the hyperbolic conic). This is possible because the payload at injection is independent of where third stage ignition occurs in the circular parking orbit.

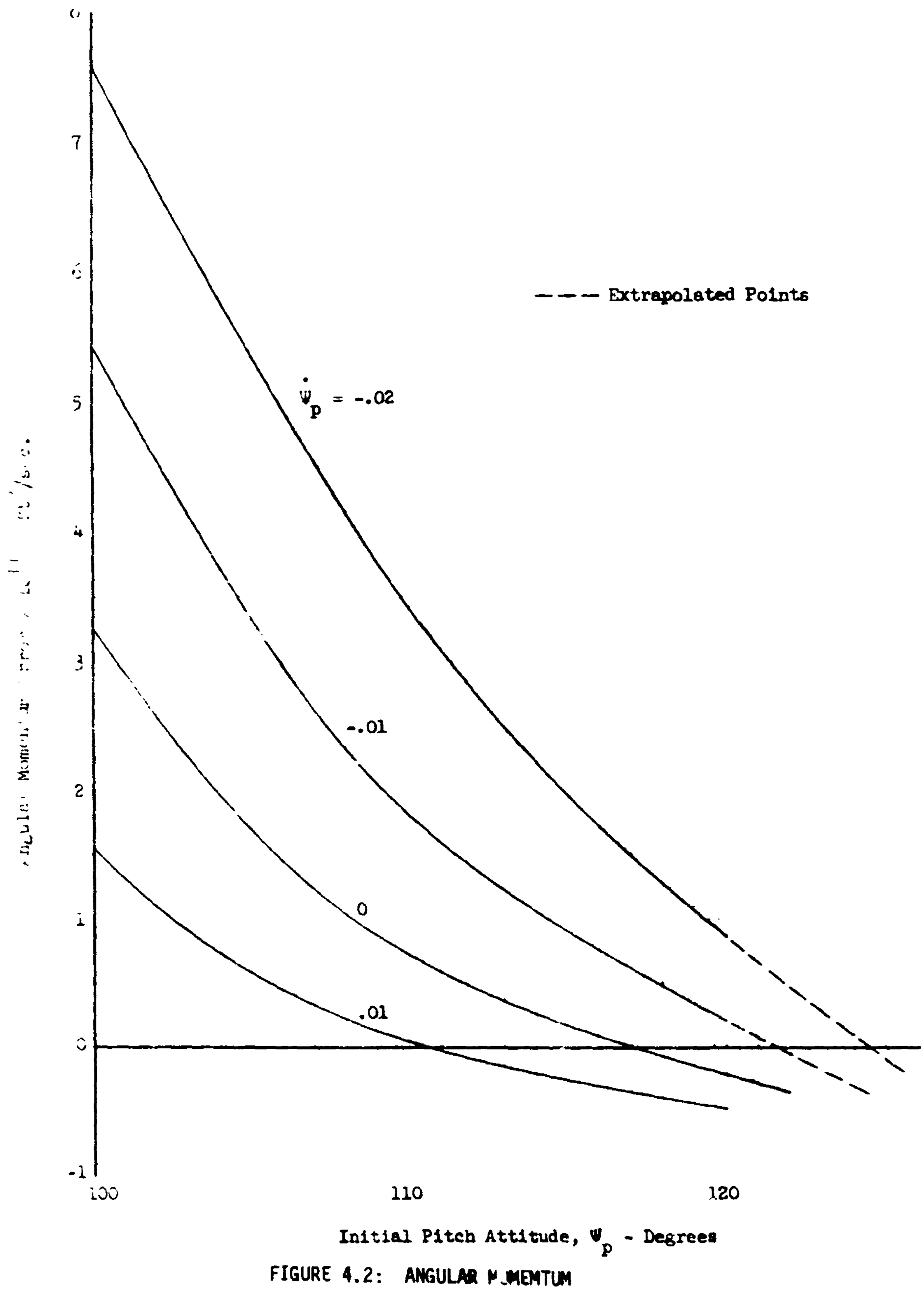


FIGURE 4.2: ANGULAR MOMENTUM

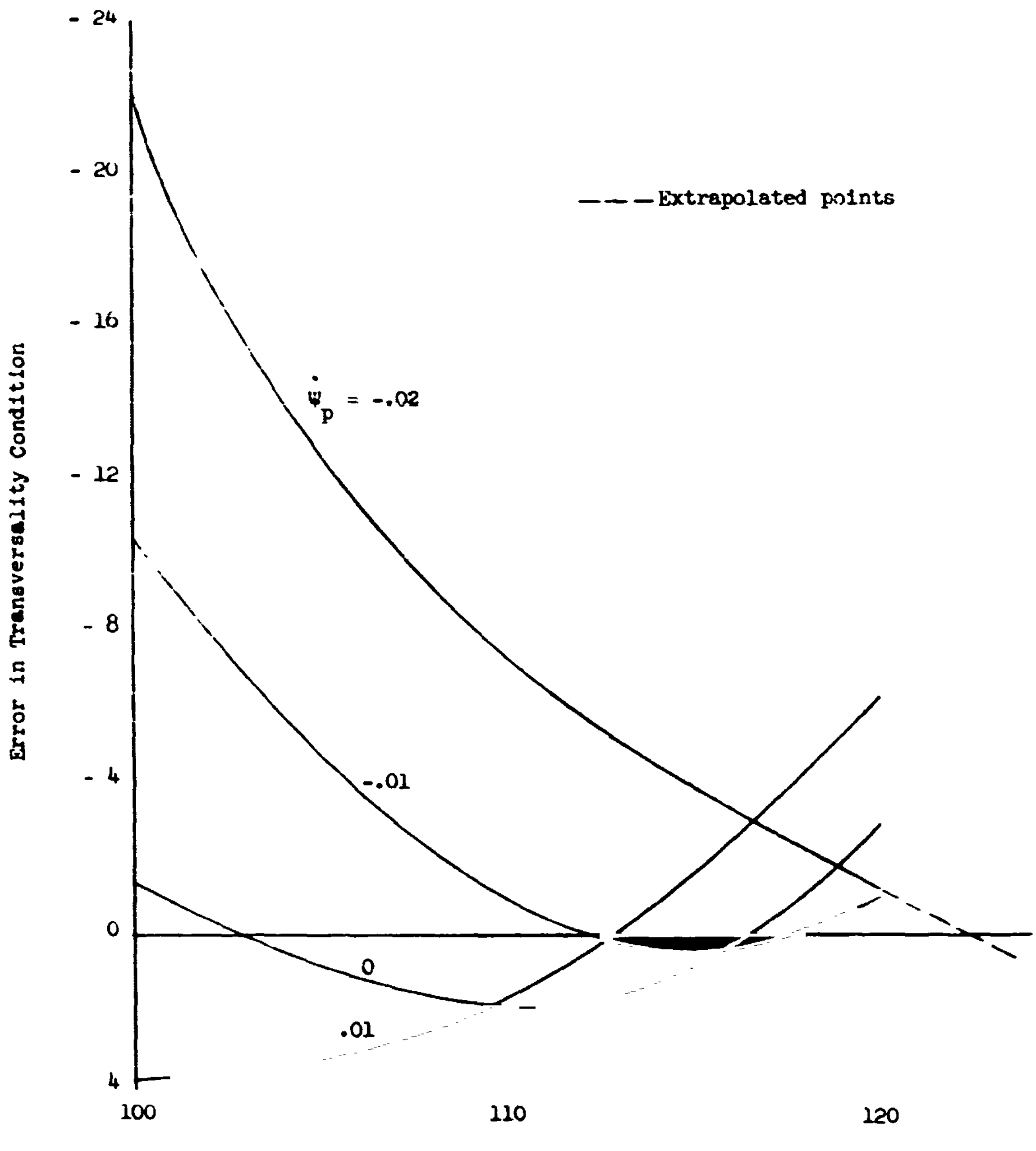


FIGURE 4.3: TRANSVERSALITY CONSTRAINT ERROR

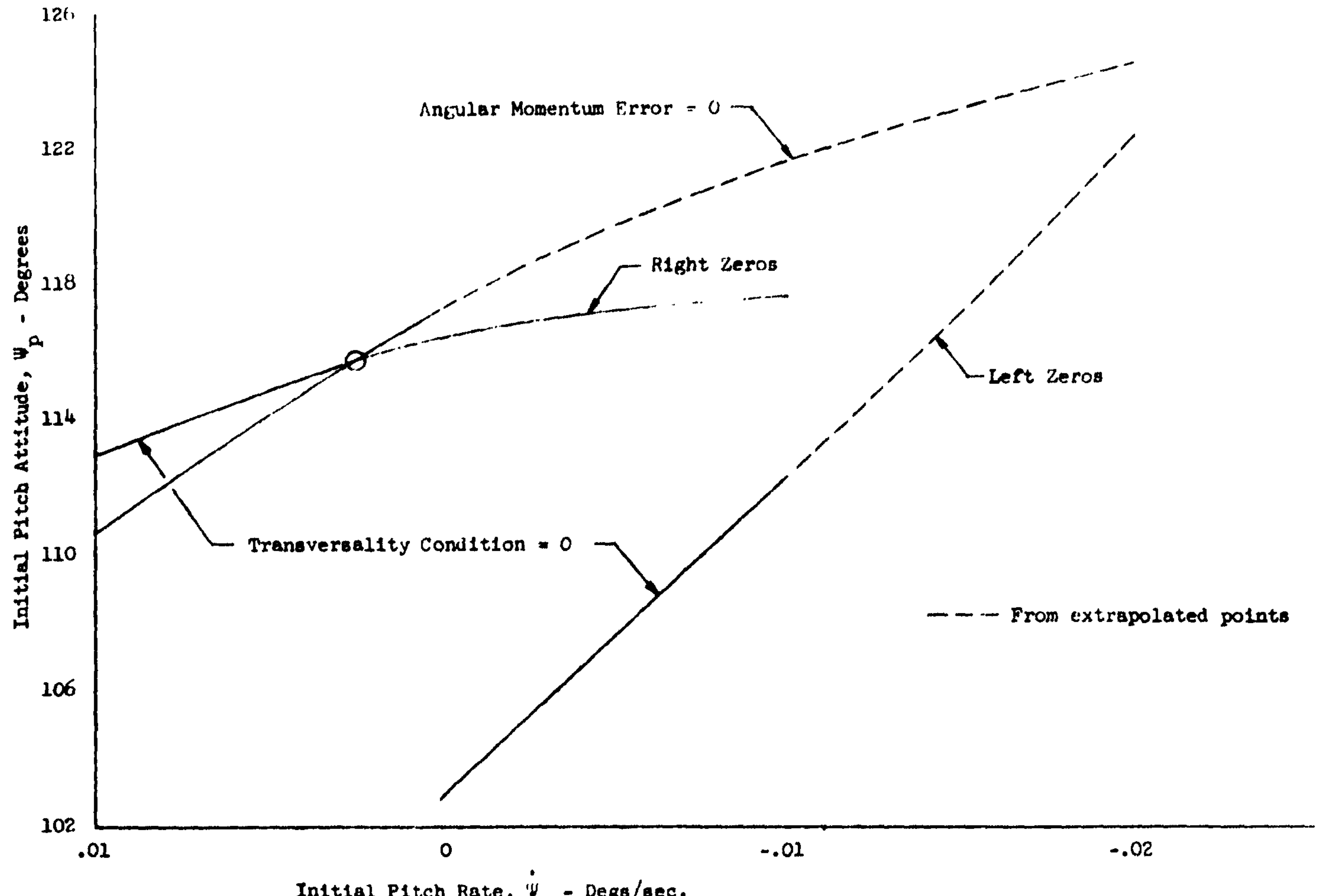


FIGURE 4.4: END CONDITION CROSS PLOT

The maximum payload at injection is 7,480 lb, or 1,024 lb in excess of ERC's payload. This assumes that the 6,126 lb additional weight obtained at second stage burnout is removed from the S-IVB stage as excess fuel and is carried through the third and fourth stages. Due to the increased burning time of the fourth stage (5,102 lb of fuel burned), the final injection conditions are achieved at an altitude that is 1,104 n.mi. above that of the ERC reference trajectory. Table 4.2 shows a comparison of the TRW optimal trajectory with the trajectory provided by ERC.

The trajectory just described provides an optimum solution under the ground rules stipulated by ERC. If the periapsis position of the departure hyperbola is not specified, however, and only the direction of the escape asymptote and the escape energy are specified, better performance may be obtained. Another simulation performed for this project achieved the same asymptotic escape velocity using a (non-optimal) zero angle of attack steering program (that is, the thrust vector paralled to the velocity vector), and achieved a 100 pound increase in payload at injection.

4.3 GUIDANCE SIMULATION RESULTS - ASCENT PHASE

The ascent phase of the mission was simulated using three guidance modes: the IGM, the TRW Hybrid, and a variation of the TRW Hybrid using a numerical iteration technique to compute the parameters of the steering law. The results show a 0.37%, 0.14% and a 0.15% decrease, respectively, from the second stage burnout weight for the optimized trajectory.

TABLE 4.2a

ERC - TRW TRAJECTORY COMPARISON

EVENT	ERC REFERENCE				
	TIME SEC.	ALTITUDE		WEIGHT	
		n.mi.	km.	lb.	kg.
Launch	0	0	0	4,024,022	1,825,256
Burnout Stage I	135.76	44.364	44.364	725,036	329,869
Ignition Stage II	135.76	44.364	44.364	322,302	146,193
Burnout Stage II	599.11	96.138	178.048	93,480	42,402
Initiate Coast	599.11				
Terminate Coast	997.95				
Ignition Stage III	997.95	96.136	178.048	55,000	24,948
Burnout Stage III	1545.95	252.95	466.993	17,695	8,026
Ignition Stage IV	1545.95	252.156	466.993	13,126	5,954
Burnout Stage IV	1937.20	1542.568	2856.835	6,456	2,928

TABLE 4.2b

ERC - TRW TRAJECTORY COMPARISON

EVENT	TRW OPTIMAL				
	TIME SEC.	ALTITUDE		WEIGHT	
		n.mi.	km.	lb.	kg.
Launch	0	0	0	4,024,022	1,825,256
Burnout Stage I	135.76	32.174	59.587	725,036	328,869
Ignition Stage II	135.76	32.174	59.587	322,302	146,193
Burnout Stage II	586.16	96.127	178.028	99,606	45,180
Initiate Coast					
Terminate Coast					
Ignition Stage III	943.90	96.111	177.998	61,124	27,725
Burnout Stage III	1490.90	221.138	409.547	23,819	10,804
Ignition Stage IV	1490.90	221.138	409.547	19,250	8,732
Burnout Stage IV	2181.25	2647.035	4902.308	7,480	3,393

The objectives of the first phase were to fix the inclination, longitude of the ascending node, eccentricity (equal to zero), and the radius of the circular orbit. In order to use the TRW Hybrid equations, a few simple modifications were made to the equations of Reference 8, which do not allow a constraint of the orientation of the final orbit plane. (The guidance mode described in Reference 8 constrains the eccentricity and semi-major axis, with the plane of the final orbit plane constrained only to contain an input target vector). The results of the ascent phase are shown for the TRW Hybrid equations and the IGM equations in Table 4.3. The results of the optimized trajectory are also shown in Table 4.3 for comparison.

The TRW iterative guidance is discussed in Section 6.3 where the guidance parameters A_2, B_2, C_2 and D_2 are defined. This technique requires an initial guess of the guidance law parameters. The trajectory is then integrated from the initiation of guidance (immediately after S-IVB-solid separation) to burnout, defined as the time when the velocity vector reaches the prescribed value. The initial guess of the guidance parameters A_2, B_2, C_2, and D_2 produces a set of terminal errors. Next, four separate integrations are performed perturbing each guidance parameter successively (two for pitch steering and two for yaw steering). This allows a matrix of partial derivatives to be determined. The proper values of the guidance parameters can then be determined to null the terminal errors. It was found that an appropriate set of trajectory variables to iterate upon is (1) position directed out of the desired orbit

TABLE 4.3: **Comparison of Certain Trajectory Variables at Circular Parking Orbit Injection**

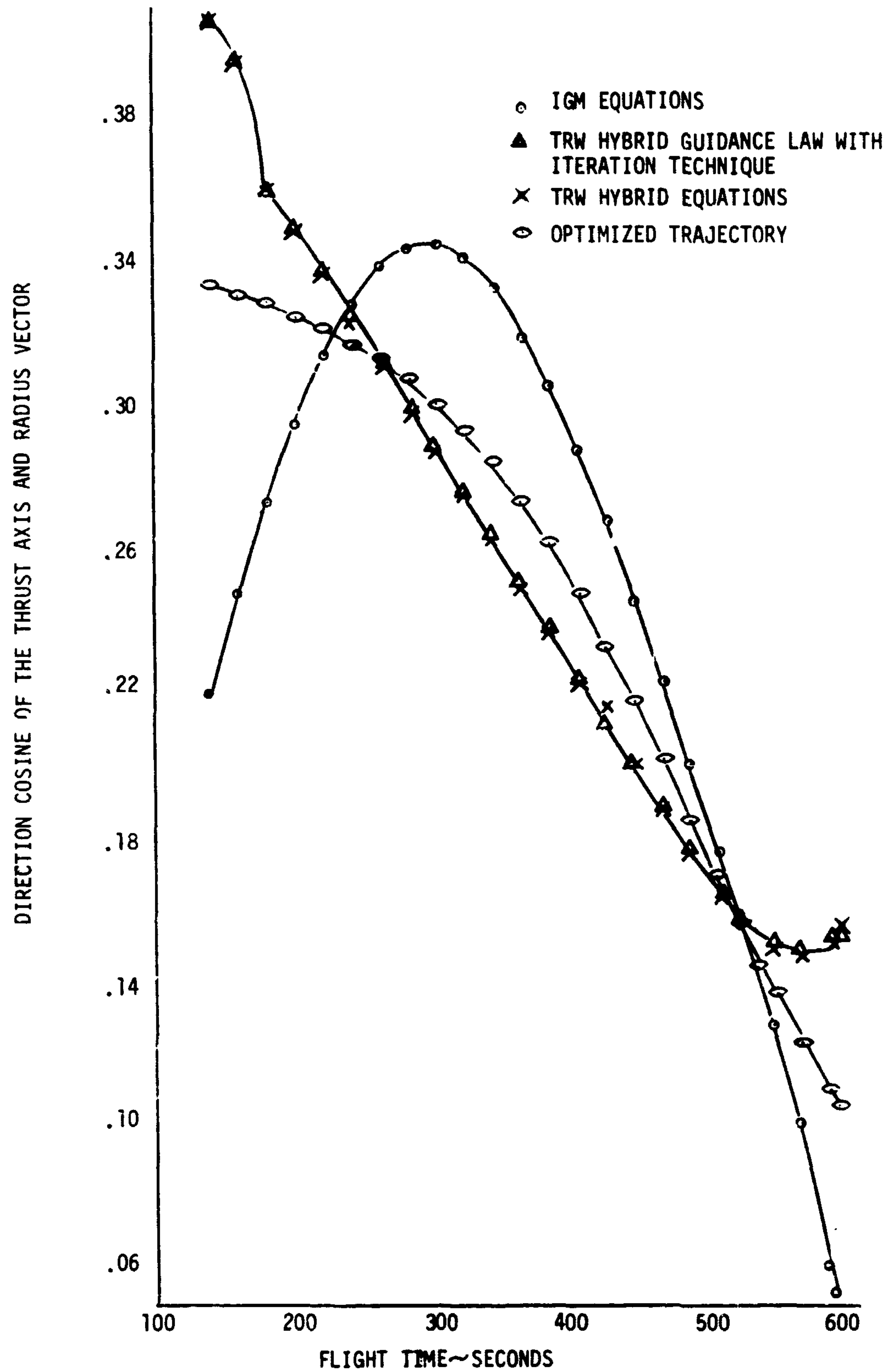
	Optimized Trajectory	TRW Hybrid	TRW Guidance Law Using Numerical Iteration Technique	IGM
Time of Burnout (seconds)	586.164	586.458	586.498	586.933
Burnout Weight (pounds)	99606.5	99464.9	99445.7	99235.8
Magnitude of the Inertial Velocity (feet/second)	25605.9	25606.5	25605.9	25605.9
Eccentricity	$.109329 \times 10^{-4}$	$.430750 \times 10^{-4}$	$.576890 \times 10^{-5}$	$.647625 \times 10^{-4}$
Inclination (degrees)	26.0000	26.0002	25.9998	25.0002
Longitude of the Ascending Node (degrees)	201.277	201.273	201.280	201.274
Apogee (nautical miles)	96.1588	96.4232	96.1473	96.3642
Perigee (nautical miles)	96.0614	96.1181	96.1063	95.9056
Geocentric Radius (feet)	21510186.	21510130.	21510188.	21510188.
Flight Path Angle (degrees)	$-.613213 \times 10^{-3}$	$.155449 \times 10^{-3}$	$-.329018 \times 10^{-3}$	3.70970×10^{-3}

plane, (2) radial distance, and (3) flight path angle. In this simulation convergence was achieved after seven integrations, which included a nominal trajectory, four perturbations to form the matrix of partials and two attempts at nulling the terminal errors. The converged result of these iterations is shown in Table 4.3. Figure 4.5 shows the cosine of the angle between the thrust axis and the radius vector for the optimized trajectory, the trajectory obtained by simulating TRW Hybrid equations, the trajectory obtained by the TRW Iterative equations, and the trajectory obtained using the IGM equations.

4.4 GUIDANCE SIMULATION RESULTS - INJECTION PHASE

The objectives of the second phase were to fix the eccentricity, the argument of perigee, and the energy of the departive hyperbola, holding the plane of motion fixed (no out-of-plane steering). Neither the IGM guidance law nor the TRW Hybrid guidance law of Reference 8 would perform the second phase of the Jupiter fly-by trajectory, since the long arc length and high burnout altitude of the reference trajectory violated the basic assumptions required to derive these guidance modes. Only the TRW iterative equations produced a satisfactory trajectory. A problem was encountered with the continuity conditions at staging (separation of the Centaur and Kick Stage), however. The equations of Reference 8 enforce continuity of the direction cosine (between the thrust axis and the radius vector) and the direction cosine rate at staging. It was found that enforcing continuity of the direction cosine rate did not produce good results. Hence only the continuity of the

FIGURE 4.5: DIRECTION COSINE OF THE THRUST AXIS VS. FLIGHT TIME ASCENT PHASE



direction cosine was enforced at staging. Figure 4.6 shows the variation of the direction cosine with flight time for the second phase of the Jupiter fly-by trajectory, which indicates the discontinuity of the direction cosine rate at staging for the Hybrid guidance law.

The trajectory integrations and iterations were carried out as in the ascent phase, where the constraints for this portion of the trajectory were energy, eccentricity, and argument of perigee. Since, including the total burning time T, there were four guidance parameters, B_3, D_3, D_4, and T (B_4 being constrained by the continuity condition of staging) and only three constraints, the burnout weight can be maximized. Convergence was achieved in thirteen iterations. Given an initial guess of the parameters B_3, D_3, and D_4 (which remain constant throughout the flight) the terminal errors were determined at the time (T) when the velocity magnitude reached the specified value. The parameters B_3, D_3, and D_4 were then successively perturbed in order to form a matrix of partial derivatives. Three integrations were necessary to converge on the parameters D_3 and D_4 which would null the terminal errors for a given B_3, which was then perturbed again and the above process repeated until convergence was achieved. At that time, three pairs of values of burnout weight and B_3 were available. Burnout weight was then fit to a quadratic versus B_3 and the optimum value of B_3 obtained. This burnout weight was only 36 pounds less than the weight achieved for the optimized trajectory of Reference 2. The sensitivity of the burnout weight to variations in the control parameter B_3 was determined. The results are shown in Figure 4.7. The comparison of the resulting trajectory with the optimized trajectory is shown in Table 4.4.

FIGURE 4.6: DIRECTION OF THE THRUST AXIS VS FLIGHT TIME
HYPERBOLIC INJECTION

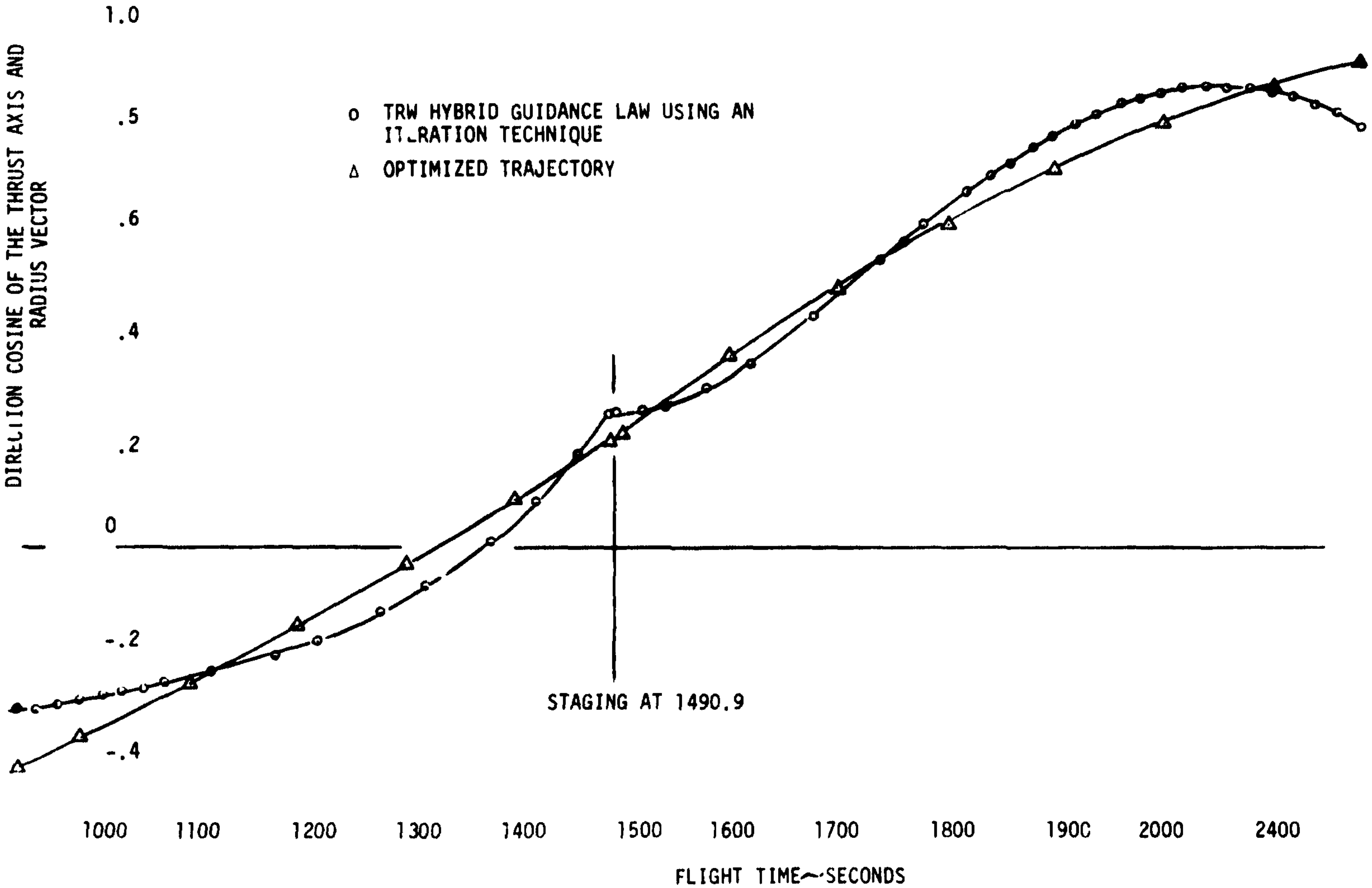


FIGURE 4.7: BURNOUT WEIGHT AT INJECTION VS B_3

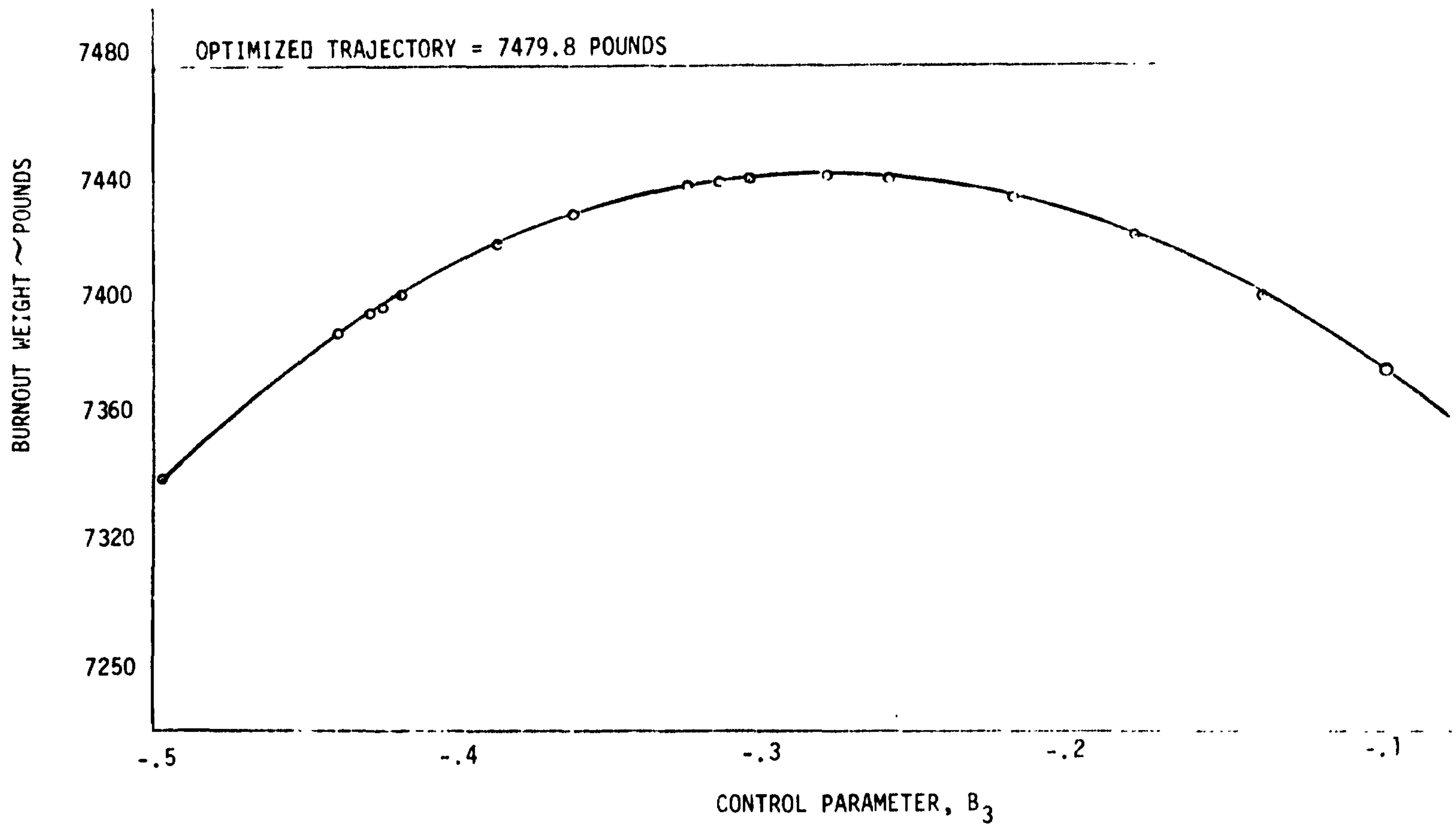


TABLE 4.4: COMPARISON OF CERTAIN TRAJECTORY VARIABLES AT INJECTION INTO A HYPERBOLIC TRAJECTORY

	OPTIMIZED TRAJECTORY	TRW HYBIRD GUIDANCE LAW USING NUMERICAL ITERATION TECHNIQUE
Time of Burnout (seconds)	2181.25	2183.38
Burnout Weight (pounds)	7479.86	7443.59
Magnitude of the Inertial Velocity (feet/second)	46743.1	46732.6
Magnitude of the Radius Vector (feet)	37009800.	37057500.
Total Energy $(\text{feet})^2/(\text{second})^2$	711,385,000.	711,385,000.
Radial Velocity (feet/second)	35500.9	35520.7
Tangential Velocity (feet/second)	30407.3	30368.0
Flight Path Angle (degrees)	49.4192	49.4717
Eccentricity	3.17157	3.17156
Argument of Perigee (degrees)	159.445	159.445
True Anomaly (degrees)	63.2744	63.3380
Inclination (degrees)	26.0000	26.0000
Longitude of the Ascending Node (degrees)	201.277	201.277

This procedure was carried out only for the given nominal state vector at third stage ignition, while for a realistic application it would be repeated as new navigation data as obtained. Such iterations would not be necessary, of course, if no navigation or systematic errors were present. Note, however, that no optimization would be possible for iterations carried out during fourth stage, for then there would be just as many end conditions to satisfy (three) as guidance parameters remaining to be chosen (B_4, D_4 and T). This would not be true if only two end conditions were specified, e.g., energy and direction of the escape asymptote.

5. SIMULATION OF ATLAS-CENTAUR GUIDANCE

5.1 INTRODUCTION

In order to make the numerical evaluation of guidance modes more complete, studies of Atlas-Centaur guidance carried out by TRW Systems Group under NASA Contract NAS-3-3231 (Reference 8) are summarized in this Section. These studies evaluated the operation of the same two finite parameter guidance modes discussed in Section 4: the Iterative Guidance Mode (IGM) and the TRW Hybrid Guidance Mode. The eight missions simulated for the study are described in Section 5.2, and guidance results are given in Section 5.4. While these eight missions represent a spectrum of situations, performance of the two guidance modes was satisfactory in each case. This conclusion is to be contrasted with the results of Section 4, where neither mode was adequate for guidance of the unusual Jupiter injection phase trajectory.

5.2 ATLAS-CENTAUR MISSION DESCRIPTIONS

The simulations results described in this section compare the performance of the IGM and TRW Hybrid guidance modes with the optimal solution. A set of eight Atlas-Centaur missions was selected for simulation. This set of missions may be considered to provide a cross-section of future planning for this vehicle. The missions are not particularly severe from the standpoint of guidance, and these results indicate primarily the adequacy of existing modes. The missions are:

1. One-Burn Lunar (Direct Ascent)
2. Two-Burn Lunar (Parking Orbit)
3. Earth Orbital
4. Earth Orbital, Polar
5. One-Burn Planetary (Mars, Direct Ascent)
6. Two-Burn Planetary (Mars, Parking Orbit)
7. Synchronous Satellite
8. Solar Probe.

General descriptions of these missions are given below.

- <u>One-Burn Lunar (Direct Ascent)</u>

The One-Burn or Direct Ascent Lunar Mission is designed to inject the Atlas/Centaur payload into a lunar transfer orbit with one continuous burn of the Centaur engine.

The Centaur guidance sub-phase for this mission is initiated as described previously and the engine burns for a period of approximately 429 seconds. Termination occurs when the desired cutoff velocity of 36060 feet per second (nominal trajectory) is reached. Payload weight and total injection time (that is, from liftoff to translunar injection) are about 6662 pounds and 675 seconds, respectively

• Two-Burn Lunar Mission (Parking Orbit)

The Two-Burn Lunar Mission has the same objectives as the Direct Ascent Lunar; namely, the injection of payload into a lunar transfer orbit. However, in this case the Centaur guidance phase has two burn periods. The first burn has a duration of about 324 seconds and injects the vehicle into a nominal 100 nautical mile circular orbit. The vehicle coasts through approximately 25-30^{0} in this orbit for phasing, then reignites and burns for approximately another 108 seconds to inject into the desired translunar orbit. As with the direct ascent mission, this desired injection orbit is specified by the apogee, perigee, inclination, ascending node and argument of perigee. The purpose of the two burns, i.e., injecting into the parking orbit, is to obtain a much larger launch window than would be possible with the direct ascent mission.

This mission used the previous ground rules. Total time from liftoff to final injection was about 1170 seconds and Centaur total burn time was approximately 432 seconds. Payload delivered was nearly the same; however, the tradeoff penalty in obtaining the larger launch window was an increased trip time for the two burn mission.

• Low Earth Orbit

The earth orbit mission is an injection into a low 100 nautical mile circular earth orbit (particular earth orbit missions including the polar orbit and the parking orbit phase of several missions are discussed below).

A 90-degree launch azimuth was used and total Centaur burn time was nominally about 325 seconds. This particular mission is nearly identical to the parking orbit phase of other missions.

- Polar Orbit Mission

The polar orbit mission differs from the usual earth orbital mission in that the injection of the Atlas/Centaur payload is into a 90-degree inclination orbit, using a single continuous Centaur burn. The reference orbit was 100 nautical miles, circular. The maximum allowable launch azimuth was assumed to be 147 degrees, which necessitated a sizeable plane change during the overall powered flight phase. Thus, injection into this mission mode provides a thorough test of the yaw steering capability of the guidance scheme.

Most of the qualitative aspects of the launch and guidance phases are the same as described for the Direct Ascent Mission. Overall injection and Centaur burn times were 612 seconds and 365 seconds, respectively. Payload weight was approximately 11050 pounds.

- One-Burn Planetary Mission (Mars, Direct Ascent)

A direct ascent Mariner simulation was chosen to represent the one burn planetary mission. The Centaur objective is to inject the vehicle into a hyperbolic orbit which will intercept Mars. The limitations of the one burn lunar mission apply to this case in that a very small launch window is available with this mode, since a coast phase is not available for phasing improvement. Almost 450 seconds of Centaur burn time were required for this mission.

A feature of the solar probe, and the two-burn planetary mission (below), is that the guidance equations must be able to treat both elliptical and hyperbolic injections.

- Two-Burn Planetary Mission (Mars, Parking Orbit)

The Mariner Mission is designed to inject the vehicle into a hyperbolic earth escape orbit which places it in an interplanetary trajectory intercepting Mars. The profile consists of injection into a nominal 100 nautical mile circular parking orbit, followed by a 240^{0} earth orbit. This mission should reveal any sensitivities in the guidance equations due to the definition of the orbital elements.

The launch and sustainer guidance phases for this mission are the same as for the Two-Burn Lunar Mission. The Centaur guidance phase differs from that mission in that a slightly different parking orbit is used and the parking orbit coast is almost a complete period.

- Synchronous Orbit Mission

A synchronous satellite is an orbiting vehicle which would be permanently stationed (if earth was spherical and homogeneous) over a point of earth. To accomplish this, the orbital period must equal that of the earth, i.e., be a 24 hour satellite, and it must lie in the equatorial plane. The proper altitude for this synchronous orbits is approximately 19326 nautical miles.

The Centaur guidance subphase of the mission accomplishes this mission by initially placing the vehicle into a nominal circular parking orbit. When the phasing is proper, the vehicle is injected into a Hohmann transfer orbit with an apogee at the 24-hour synchronous satellite altitude and at a point above the earth near the final desired longitude. At apogee, the transfer orbit is circularized and the necessary yaw maneuver is made to bring the final orbit onto the equatorial plane.

Guidance requirements imposed by this mission are two coast periods, a large plane change, and achievement of a 0^o inclination for the equatorial orbit. The last requirement can be a targeting problem for some schemes in that the ascending node is undefined at the 0^o inclination.

Centaur burn time total about 451 seconds for this mission and total injection time was about 20740 seconds for the case considered.

- Solar Probe Mission

The solar probe mission is designed to achieve a close perihelion distance using the Atlas/Centaur vehicle. If the apohelion distance is approximately equal to the earth's orbital distance, the sun-centered ellipse described by the probe must have a lower orbital energy than the earth's circular orbit possesses. This necessitates a retrograde launch from the earth with respect to the sun (to lower energy), or adjusting the semi-major axis to the desired value. A small plane change would be necessary for a launch if an ecliptic plane probe orbit were desired. The out-of-plane effects have been minimized, however, by injecting with the velocity vector parallel to the ecliptic and tolerating the small angular difference between the planes. The mission simulation was also designed such that a line-of-sight communications link would be possible at the time of perihelion passage.

5.3 THE ATLAS-CENTAUR LAUNCH VEHICLE

In order to reduce the complexity of the study, portions of each mission were made as nearly similar as possible. The main differences occur in the number and duration of the Centaur burns, and in launch azimuth. The mission profile is separated into a launch phase, which is common to all missions, and a guidance phase which steers the vehicle to applicable end conditions. The common launch phase will be described first.

The Atlas booster has three independent sets of engines as shown in the sketch below:

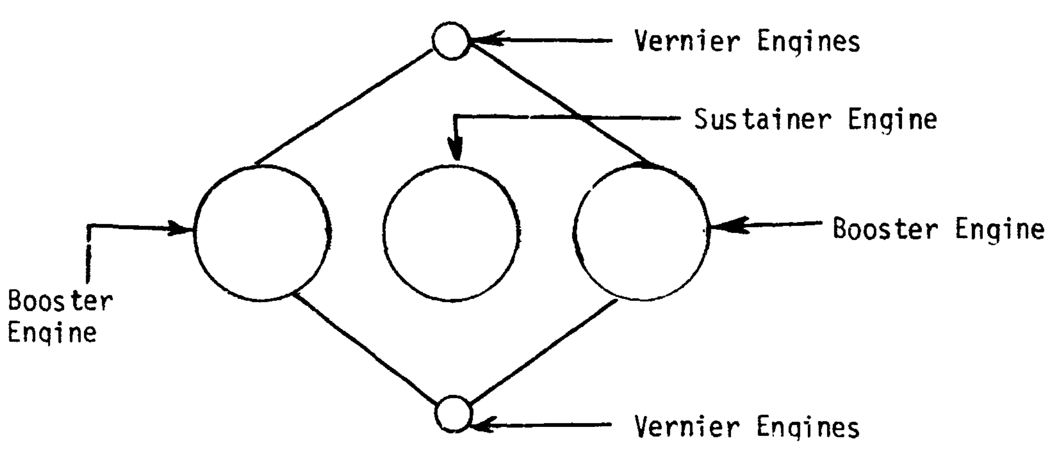


FIGURE 5.1: ATLAS ENGINE CONFIGURATION

The assumed sequence of events is shown in the list below:

TIME (minutes)	DESCRIPTION OF EVENT
0.0	Lift-off, Start booster
0.3	End Vertical Rise. Start Gravity Turn
2.5	BECO (Booster Engine Cut-Off), Begin Sustainer Phase
4.2	SECO (Sustainer Engine Cut-Off) VECO (Vernier Engine Cut-Off) Pre-Centaur Start, Separation
4.4	Centaur Main-Engine Full Thrust.

The launch phase consists of that flight portion from lift-off to 10 seconds after BECO, at which point active guidance is initiated. For the Atlas/Centaur configuration, the overall guidance phase can be considered to consist of the sustainer and Centaur subphases.

The sustainer guidance phase is similar for all missions and is characterized by staging operations such as the jettisoning of various equipment and the Atlas/Centaur separation. The Centaur guidance phase consists of coasting and powered sub-phases as specified for the particular mission. These are discussed in Section 5.2. The initiation of the Centaur phase however, is common to all missions. The phase begins 8.6 seconds after SECO with a low thrust and flow-rate requiring 1.315 seconds until full thrust and flow rate are achieved. The same start-up procedure applies to subsequent reignitions.

TABLE 3.1: NOMINAL VEHICLE--ACCURACY AND PERFORMANCE RESULTS AT INJECTION FOR VARIOUS MISSIONS

Mission	Scheme	* Payload (lb)	INJECTION ERRORS Altitude (ft)	Altitude Rate (ft/sec)	** Velocity Magnitude (ft/sec)	Yaw Velocity (ft/sec)
Direct Ascent	Hybrid	0.6	-3	- .05	.02	-.02
Lunar	IGM	-1.0	0	- .22	0	0
Polar Orbit	Hybrid	-0.3	-1	- .03	.01	0
	IGM	5.5	0	- .10	0	0
Two Burn Lunar	Hybrid	2.2	-8	- .05	-.01	.01
	IGM	-6.9	0	.24	0	0
Two Burn Planetary	Hybrid	-6.9	-8	- .04	0	-.02
	IGM	-4.0	-1	1.86	0	0
Two Burn Solar Probe	Hybrid	-6.2	-13	- .04	.01	.02
	IGM	-2.8	-1	- 3.88	0	0
Synchronous Orbit	Hybrid	-6.9	2	0	-.13	.02
	IGM	-0.5	4	.14	0	0

* Closed Loop minus Calculus of Variations.

** The IGM simulation does not use the time-to-go cutoff routine, but achieves a perfect cutoff on desired velocity. All of the IGM results above would also be slightly degraded if this routine were used. It is estimated that the resulting error in velocity magnitude would be less than 0.1 ft/sec.

TABLE 5.2: VEHICLE DISPERIONS--ACCURACY AND PERFORMANCE RESULTS AT INJECTION FOR THE DIRECT ASCENT LUNAR MISSIONS

Dispersion	Dispersion Magnitude	Scheme	Variations: Perturbed Value-Nominal Value		
			Payload (lb)	Perigee (ft)	Midcourse Corrections* (ft/sec)
Sustainer I_{sp}, Nominal Thrust	+1.5%	Hybrid	53.3	-1	.01
		IGM	51.9	3	.02
	-1.5%	Hybrid	-52.1	7	.24
		IGM	-52.2	3	.15
Centaur I_{sp}, Nominal Thrust	+1.5%	Hybrid	169.9	3	.01
		IGM	168.7	0	.03
	-1.5%	Hybrid	-169.7	1	.24
		IGM	-169.7	1	.23
Thrust and I_{sp} of all stages (Nominal Flow Rate)	+3%	Hybrid	475.4	3	.13
		IGM	693.0	1	.22
	-3%	Hybrid	-702.2	4	.09
		IGM	-699.5	5	.26
Pitch Rate (Booster Pitch Program)	+4.6%	Hybrid	-19.1	11	.12
		IGM	-19.6	-3	.18
	-4.6%	Hybrid	-15.7	10	.04
		IGM	-15.2	0	.15
Launch Azimuth	$+2^0$	Hybrid	-16.9	5	.07
		IGM	-16.8	-1	.16
	-2^0	Hybrid	2.2	9	.10
		IGM	1.8	0	.10

* An approximate figure based on the miss at the nominal impact time predicted from two-body equations, and a midcourse time-to-go of 160,000 sec.

TABLE 5.3

VEHICLE DISPERSIONS--ACCURACY AND PERFORMANCE RESULTS AT INJECTION FOR THE POLAR ORBIT MISSION

Dispersion	Dispersion Magnitude	Scheme	VARIATIONS: PERTURBED VALUE-NOMINAL VALUE				
			Payload (lb)	Altitude (ft)	Altitude Rate (ft/sec)	Velocity Magnitude (ft/sec)	Yaw Velocity (ft/sec)
Sustainer I_{sp} (Nominal Thrust)	+1.5%	Hybrid	75.6	-1	-.03	0	0
		IGM	77.0	0	-.09	0	0
	-1.5%	Hybrid	-77.6	-1	-.03	0	0
		IGM	-77.2	0	.21	0	0
Centaur I_{sp} (Nominal Thrust)	+1.5%	Hybrid	189.8	-1	-.03	0	0
		IGM	190.0	-1	.12	0	0
	-1.5%	Hybrid	-192.7	-1	-.03	.01	0
		IGM	-193.0	1	.13	0	0
Thrust and I_{sp} of all stages (Nominal Flow Rate)	+3%	Hybrid	979.2	-1	-.03	0	0
		IGM	976.0	-1	.43	0	0
	-3%	Hybrid	-1012.3	-1	-.02	0	0
		IGM	-1006.0	1	.05	0	0
Pitch Rate (Booster Pitch Program)	+4.6%	Hybrid	-140.4	-1	-.03	.01	0
		IGM	-122.0	0	.02	0	0
	-4.6%	Hybrid	50.1	-1	-.03	0	0
		IGM	39.0	1	.23	0	0
Launch Azimuth	$+2^{o}$	Hybrid	157.5	-1	-.03	0	0
		IGM	149.0	-1	.04	.0	0
	-2^{o}	Hybrid	-162.6	-1	-.03	0	0
		IGM	-173.0	1	.04	0	0

The vehicle Atlas/Centaur 8 is simulated as specified in the GDC report series "Centaur Monthly Configuration, Performance and Weight Status Report". The thrust and mass flow rates of the sustainer and Centaur phases are approximately as given below:

	Thrust (lbs)	Mass Flow Rate (lbs/sec)
Sustainer	81,000	270
Centaur	30,000	65

5.4 ATLAS/CENTAUR GUIDANCE SIMULATION RESULTS

Table 5.1 shows accuracy and performance results for the IGM and TRW Hybrid guidance equations. These results show the adequacy of either scheme for the missions simulated.

In addition, perturbations in vehicle performance parameters were introduced in the One-Burn Lunar Mission and the Earth Polar Orbit Mission in order to assess the effects on accuracy. The results are presented in Tables 5.2 and 5.3. These results show that both schemes are able to achieve mission end conditions in spite of off-nominal vehicle performance.

6. PRECISE PARAMETERIZED GUIDANCE

6.1 INTRODUCTION

The success of the TRW Iterative guidance mode on the Injection Phase of the ERC trajectory suggests the feasibility of studying other forms of precise parameterized (or finite dimensional) guidance. This section contains a discussion of these schemes and the advantages which may be expected to result.

There are basically two types of parameterized guidance:

- parameterized control - where the form of the steering law is specified as a function of time containing the arbitrary guidance parameters. The IGM mode is an example, where the steering angle (χ) tangent is chosen to be a linear function of time, tan χ = A + Bt. A crude numerical integration then relates the parameters (A, B) to the end conditions.

- parameterized state - where the form of certain state variables is specified as a function of time containg the arbitrary guidance parameters, and the steering law is thereby implicitly determined via the equations of motion. The TRW Hybrid guidance mode is an example, where the radial distance (r) is chosen to be $\ddot{r}$ = $a_T(t)$ [A + Bt], and $a_T(t)$ is the thrust acceleration.

The precise forms of parameterized guidance, which do not approximate the equations of motion, may be regarded as being as being intermediate to the approximate closed form iterative modes, such as IGM and TRW Hybrid, and the complete calculus of variations solution employing the adjoint variables. Recent advances in on-board computing capability have led to the suggestion that integration of the adjoint equations and iterative search for the correct adjoint variable initial values may be possible in real time. It is further suggested that such a scheme would form a reasonable basis for a guidance mode.

Approximate parameterized schemes such as the IGM and TRW Hybrid modes require a minimum of computer capability. These modes (or variations of them) have been tested and have proven quite successful within the spectrum of missions for which they were designed. These finite dimensional schemes achieve near optimality because the functional forms imposed on the steering function or certain dynamic variables are selected to imitate the behavior of these variables on an optimal trajectory. In addition, certain approximations are introduced to obviate the necessity of integrating the equations of motion. Usually the downrange transfer angle and time-to-go to burnout are approximated through the use of closed form expressions.

The performance of these schemes is superior in several respects:

- Verification Requirements. Being relatively simple, there is less opportunity for system error.

- Region of Applicability. Simulations and flight tests have shown that these modes continue to work well even when booster performance differs from nominal.

- Computer Requirements. - Minimal.

- Pre-flight Preparation. Minimal.

The limitations of these modes arise from the introduction of the closed form approximations in place of integration of the equations of motion. The precise parameterized guidance attempts to retain the desirable features mentioned above and to eliminate the disadvantages by replacing the approximate expressions with trajectory integration and Newton-Raphson iteration. This general idea results in a new class of guidance modes which are intermediate in complexity between the existing parameterized schemes and the calculus of variations solution. An example, the TRW Iterative mode discussed in Section 4 , was seen to perform well on the ERC Jupiter Fly-by Injection Phase even though the IGM and TRW Hybrid modes failed.

6.2 GENERAL DISCUSSION OF PRECISE PARAMETERIZED STATE GUIDANCE

Having selected a coordinate system it ·. assumed that the state vector $\underline{x}$ of dimension n may be portioned into vectors $\underline{y}$ and $\underline{z}$ of dimension n_1 and n_2 respectively, $n_1 + n_2 = n$,

$$\underline{x} = \begin{bmatrix} \underline{y} \\ \underline{z} \end{bmatrix} \tag{6.1}$$

such that the dynamic equations may be written

$$\dot{\underline{y}} = f(t, \underline{y}, \underline{z}, \underline{u}) \tag{6.2}$$

$$\dot{\underline{z}} = g(t, \underline{y}, \underline{z}, \underline{u}) \tag{6.3}$$

It is assumed that the following parameterization is imposed:

$$\dot{y} = h(t, \alpha)$$

where $h(t, \underline{\alpha})$ is a specified function and $\underline{\alpha}$ is a k-dimensional constant vector of parameters. Then the vector function $\underline{y}$ may be determined as a function of t and $\underline{\alpha}$:

$$\underline{y} = \underline{y}\,(t,\underline{\alpha}). \tag{6.5}$$

It is further assumed that the remaining two equations

$$h(t,\underline{\alpha}) = f(t, \underline{y}, \underline{z}, \underline{u}) \tag{6.6}$$

$$\dot{z}(t) = g(t, \underline{y}, \underline{z}, \underline{u}) \tag{6.7}$$

determine $\underline{u}$, and hence $\underline{z}$, uniquely as functions of t and $\underline{\alpha}$. This requires that $m = n_1$.

Suppose the final conditions of the problem are of the form

$$\Psi(T, \underline{x}(T)) = 0 \qquad \text{(dimension } q) \tag{6.8}$$

and the performance index is

$$J(T, \underline{x}(T)). \tag{6.9}$$

Having $\underline{y}$ and $\underline{z}$ as functions of $(t,\underline{\alpha})$ means that the problem becomes the finite dimensional problem:

Choose T, $\underline{\alpha}$ to minimize

$$J(T, \underline{x}(T, \underline{\alpha})) \tag{6.10}$$

Subject to the constraints

$$\Psi(T, \underline{x}(T, \alpha)) = 0. \tag{6.11}$$

It is assumed that $q \leq k + 1$. If the equality holds, there are of course only sufficient degrees of freedom available to satisfy the final conditions.

There are several methods available for solving the finite dimensional problem. All of these require integration of equation (6.3) at least once for each iteration, with additional integrations if numerical differentiation of the end conditions with respect to $\underline{\alpha}$ is to be employed. Integrations of linearized versions of the equations would be required if the derivatives are to be determined from the variational equations. There is thus a considerable amount of extra computation over the IGM and TRW Hybrid modes, yet no more than through introduction of the adjoint variables. Further simulation work is needed to determine the feasibility of such a scheme.

Several comments on possible variations of the scheme and applications are in order:

- State Variable Constraints. It is sometimes necessary to impose state constraints which concern only the portion y of the state and may be written

$$F(y) \leq 0. \tag{6.12}$$

When the parameterized $\underline{y}$ is inserted, this becomes

$$F\left(\underline{y}_0 + \int_0^t h(s, \underline{\alpha})ds\right) \leq 0 . \tag{6.13}$$

If A(t) is the set of $\underline{\alpha}$ satisfying this condition at time t, then

$$A = \bigcap_{0 \leq t \leq T} A(t) \tag{6.14}$$

is the set of $\underline{\alpha}$ satisfying the condition for all $0 \leq t \leq T$. There exists the possibility of selecting $\underline{y}$ and $\underline{\alpha}$ such that this constraint is independent of t, and hence becomes an algebraic constraint on $\underline{\alpha}$. The state variable constraint problem is discussed further in Section 8.2.

- <u>Atmospheric flight</u>. The calculus of variations solution to the guidance problem is considerably more complicated in atmospheric flight. Parameterized guidance, on the other hand, would operate in the same manner as in a vacuum with the possible exception of a minor change in the function $h(t, \underline{\alpha})$. This consideration may become important, for example, when shuttle flights to and from an orbiting space station become routine. Since the shuttle vehicles will require some aerodynamic control upon re-entry, a form of parameterized guidance might be appropriate. This is particularly true if the re-entry phase requires a large downrange transfer arc.

6.3 TRW ITERATIVE GUIDANCE

A specific example of precise parameterized state guidance will now be examined; the TRW Iterative mode used to achieve end conditions on the Injection Phase of the Jupiter Fly-by mission. The scheme is a natural extension of the approximate parameterized TRW Hybrid guidance mode. The same type of extension from approximate to precise parameterized guidance could be applied to the IGM equations or any of the so-called "explicit" schemes.

The equations of motion in a rotating coordinate system (planar motion) become

$$\ddot{r} = a_T u_R + \frac{h^2}{r^3} - \frac{\mu}{r^2} \tag{6.15}$$

$$\dot{h} = r a_T u_c \tag{6.16}$$

$$\dot{\Theta} = \frac{h}{r^2} \tag{6.17}$$

where

r = radial distance

h = angular velocity

a_T = thrust magnitude

Θ = down range angle

u_r = direction cosine of the thrust vector (radial)

u_c = direction cosine of the thrust vector (circumferential)

μ = gravitational constant of the earth.

The form of the control selected is based on the observation that $\ddot{r}/a_T$ is approximately a linear function of time on an optimal trajectory during periods of powered flight. Since the Jupiter Fly-by mission Injection Phase consists of two thrust stages (and no intermediate coast) it is natural to constrain $\ddot{r}/a_T$ by

$$\frac{\ddot{r}}{a_T} = \begin{cases} B_3 + D_3 t & 0 \le t \le T_1 \\ B_4 + D_4(t-T_1) & T_1 \le t \le T \end{cases} \tag{6.18}$$

where third stage ignition occurs at t = 0, fourth stage ignition and cut-off are at $t = T_1$ and $t = T$ respectively. It is assumed that T_1 is fixed so that only T may be varied as a control parameter.

The final conditions are three in number: it is desired to reach prescribed values of

$$\text{energy} \quad E = \frac{1}{2}\left(\dot{r}^2 + \frac{h^2}{r^2}\right) - \frac{\mu}{r} \tag{6.19}$$

$$\text{eccentricity} \quad e = \sqrt{1 + \frac{2Eh^2}{\mu^2}} \tag{6.20}$$

$$\text{argument of perigee } \varpi = \theta - \cos^{-1}\left[\frac{1}{e}\left(\frac{h^2}{\mu r} - 1\right)\right] \tag{6.21}$$

Since

$$u_r = \begin{cases} B_3 + D_3 t - \frac{1}{a_T}\left(\frac{h^2}{r^3} - \frac{\mu}{r^2}\right) & 0 \le t \le T_1 \\ B_4 + D_4(t-T_1) - \frac{1}{a_T}\left(\frac{h^2}{r^3} - \frac{\mu}{r^2}\right) & T_1 \le t \le T \end{cases} \tag{6.22}$$

it follows that at staging $(t = T_1)$ the requirement

$$B_3 + D_3 T_1 = B_4 \tag{6.23}$$

enforces the continuity of the direction cosine u_r. B_4 is thus determined leaving free the four parameters B_3, D_3, D_4 and T to satisfy the three end conditions. With the first three parameters constant, the integration of the equations proceeds until the desired energy is achieved; this determines T. As explaned in Section 4.5, the parameters D_3 and D_4 are varied until the other final conditions are satisfied. Thus, final burn out weight becomes a function of the free parameter B_3. The graph of this realtionship is shown in Figure 4.7 .

7. THE TARGETING PROBLEM

7.1 INTRODUCTION

One of the most important concerns in present day guidance system development is the cost in time and money of preflight preparation and verification. For example, even a relatively simple mission such as a Mars fly-by (e.g., Mariner) might require the generation and verification of as many as 1,000 closed-loop, precisely simulated reference trajectories:

10-15 launch azimuths per daily launch opportunity
x 50-90 days of launch period
= 500-1350 trajectories

As a result, a relatively simple guidance law is usually chosen and an explicit (closed) form of the end conditions for each phase is used.

These end condition functions describe the trajectory after guidance termination, and hence define relationships between end conditions of the guidance phase and desired conditions at mission completion. These are always approximate relationships, for precise closed form solutions do not exist. The targeting problem consists of choosing the parameters in these functions (e.g., desired value of injection energy and Euler angles of the earth-escape asymptote) so as to satisfy mission objectives with negligible error. In effect, targeting consists of constructing an acceptably accurate sequence of simplified two-point boundary value problems.

The conic formulae are used extensively in targeting, for motion during a coast period in a drag-free environment can usually be closely approximated, with perhaps some empirical correction terms, by the solutions of "patched" two-body problems. Thus for guidance purposes a closed form solution is nearly valid in these segments of the trajectory, and the objectives of a given guidance phase can be stated as attaining a certain combination of orbital elements. A dynamic programming argument gives a sequence of desired elements for all phases by working backward from mission termination. For example, the objectives of a retro thrust maneuver to obtain injection into a terminal satellite orbit about a planet can be specified in terms of the elements of that orbit. The objectives of the approach phase can be specified in terms of the elements of the approach hyperbola which will be optimal for the retro phase. The objectives of the midcourse phase can be specified in terms of the elements of a heliocentric transfer ellipse which will yield an optimal approach hyperbola. The objectives of the near-earth injection can be satisfied in terms of the elements of the earth-escape hyperbola which will yield an optimal heliocentric transfer ellipse. Lastly, the objectives of the initial ascent from the launch pad can be specified in terms of the elements of the near-earth parking orbit which yields an optimal injection phase.

The notion of a "patched" conic is not a precise one, for the actual trajectory is continuously attracted by many bodies. The guidance analyst does not consider the conics to be joined at fixed points on the trajectory, however, but instead they are "asympototically matched" in order to yield a much better approximation of the true motion. That is, the target planet can be considered massless for the purpose of injection and midcourse guidance analysis, and the position and velocity at closest approach to the massless target can then be used to determine the asymptotic conditions of the approach hyperbola. The magnitude of the position vector at closest approach is the impact parameter and the velocity is the hyperbolic excess velocity.

The difference between the actual and "massless" time of closest approach can be calculated by relatively simple but crude formulae. The errors introduced by such approximations are due primarily to the gravitational attraction of the target body and other non-target masses acting during the entire trajectory, and can be made negligibly small compared to other sources of guidance system error. The determination of simple yet acceptable correction terms is the central issue in the targeting problem. Thus targeting models for explicitly calculating desired end conditions for deep space missions can be ordered according to increasing levels of precision and computational complexity:

a. flat earth, vacuum - where gravitational acceleration is represented by a constant vector, and the trajectory after guidance termination is a parabola.

b. simple conic - where the trajectory after guidance termination is described by the conic formulae.

c. simple patched conic - where the trajectory after guidance termination is described by a sequence of conic segments patched at the spheres of gravitational influence of the bodies which perturb the spacecraft.

d. asymptotically matched conic - where the trajectory after guidance termination is described by a sequence of conic segments as in (c) above, but the patching is accomplished by matching the "asymptotic" values of position and velocity of the individual (hyperbolic) segments.

e. corrected conic - where the trajectory after guidance termination is described by one or more conic segments, and numerically determined corrections to the approximation are applied.

There are, of course, other types of approximations which could be employed. For example, an approximate explicit description of motion of a high speed vehicle in a dense atmosphere might be obtained by assuming that the gravitational acceleration is negligible compared to the drag acceleration. In this Report, however, we shall be parimarily concerned with the conic approximations.

7.2 EXAMPLES OF CONIC TARGETING FORMULAE

In this section we will give examples of conic formulae which can be used as approximations for explicitly relating end conditions at termination of a given guidance phase to desired conditions at mission completion. In general, the conic formulae define explicit relationships between final speed, velocity path angle, altitude, and downrange angle which must be satisfied in order to meet mission requirements, assuming that such requirements can be specified in terms of certain conic parameters such as energy, angular momentum, orientation of the line of apsides, or orientation of the hyperbolic asymptote. Since two body motion is only an approximation of the actual trajectory, and because additional approximations are sometimes introduced in the derivation of the end condition functions, the targeting problem consists of numerically determining the conic parameters (energy, angle of asymptote, etc.) which will yield acceptable approximation errors.

The examples given here are for guidance in the plane of motion only, and are primarily applicable to injection guidance, where future guidance corrections are nominally zero. All equations follow from the conic relationship between radial distance (r) and true anomaly (θ), given by

$$r = \frac{p}{1 + e \cos \theta} \tag{7.1}$$

where p = semi-latus rectum = $\frac{c_1^2}{\mu}$ = constant

μ = gravitational constant

c_1 = specific angular momentum = $r\ v \cos \gamma$ = constant

v = speed

γ = flight path angle

e = eccentricity = $\left[1 + \frac{c_1^2\ c_3}{\mu^2} \right]^{\frac{1}{2}}$ = constant

c_3 = 2 x (specific energy) = vis - viva = $v^2 - \frac{2\mu}{r}$ = constant

a. Energy-Angular Momentum Guidance - This form of end conditions specification arises when controlling the apogee ($\theta = \pi$) and perigee distances of a near earth ellipse, Via Equation (6.1), these quantities are completely specified by a,p and e, and hence specific energy, (c_3), and specific angular momentum, (c_1). Let r_I be the injection altitude Then the required speed (v_r) and velocity path angle (γ_r) at injection are found from

$$c_3 = \text{specified} = v_r^2 - \frac{2\mu}{r_I} \tag{7.2}$$

$$c_1 = \text{specified} = r_I\ v_r \cos \gamma_r \tag{7.3}$$

From these expressions the $v_r(r_I)$ and $\gamma_r(r_I)$ required to obtain the specified c_1 and c_3 can be calculated. Equation (7.3) can be put into different forms better suited for control of low path angles. For example,

$$\sin \gamma_r = \left[1 - \left(\frac{c_1}{r_I v_r}\right)^2\right]^{\frac{1}{2}}$$ is an equivalent form.

b. Apogee/Apsides Guidance - This explicit form arises when it is required to control the apogee of a near earth ellipse as well as the direction of the line of apsides. Similar formulas would apply for control of perigee and line of apsides. If ψ_I is the down range angle at injection, relative to some arbitrary inertial reference, and θ is the true anomaly at injection, then the control of line of apsides is achieved by setting $(\psi - \theta)$ equal to a fixed value (see Figure 7.1). The true anomaly at injection is found from

$$\cos \theta = \left(\frac{p - r_I}{r_I\, e}\right) \tag{7.4}$$

It follows that

$$\sin \theta = \left(\frac{p \tan \gamma}{r_I\, e}\right) \tag{7.5}$$

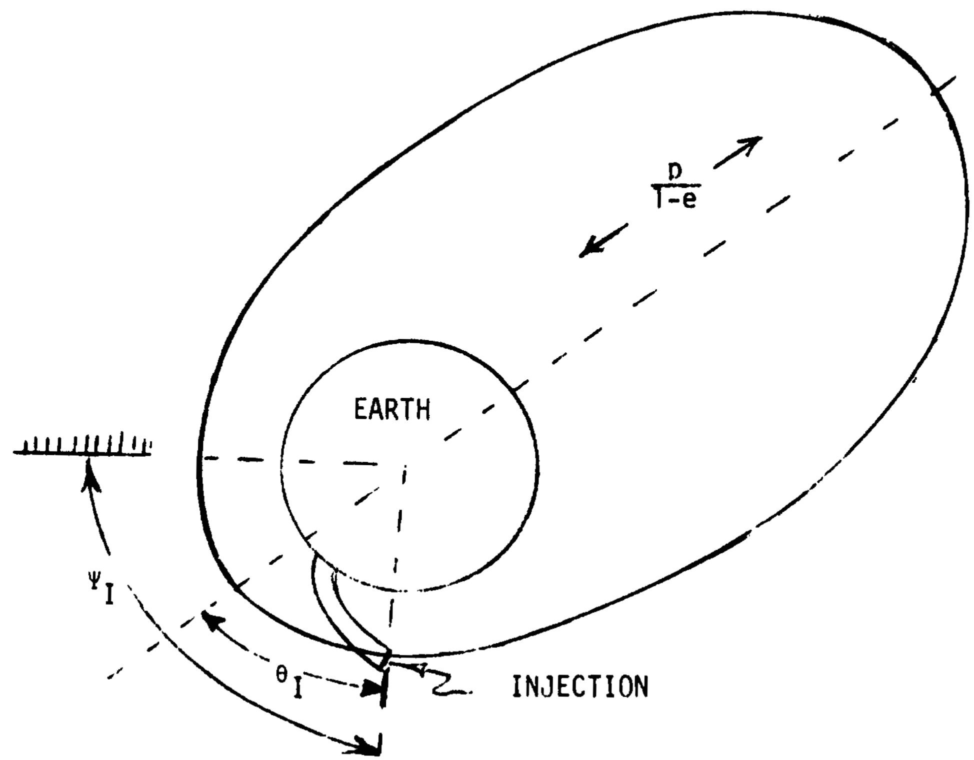
Then

$$\begin{aligned} \cos (\psi - \theta) &= \text{specified} \\ &= \cos \psi \cos \theta + \sin \psi \sin \theta \end{aligned} \tag{7.6}$$

$$\text{apogee distance} = \text{specified} = \left(\frac{p}{1 - e}\right) \tag{7.7}$$

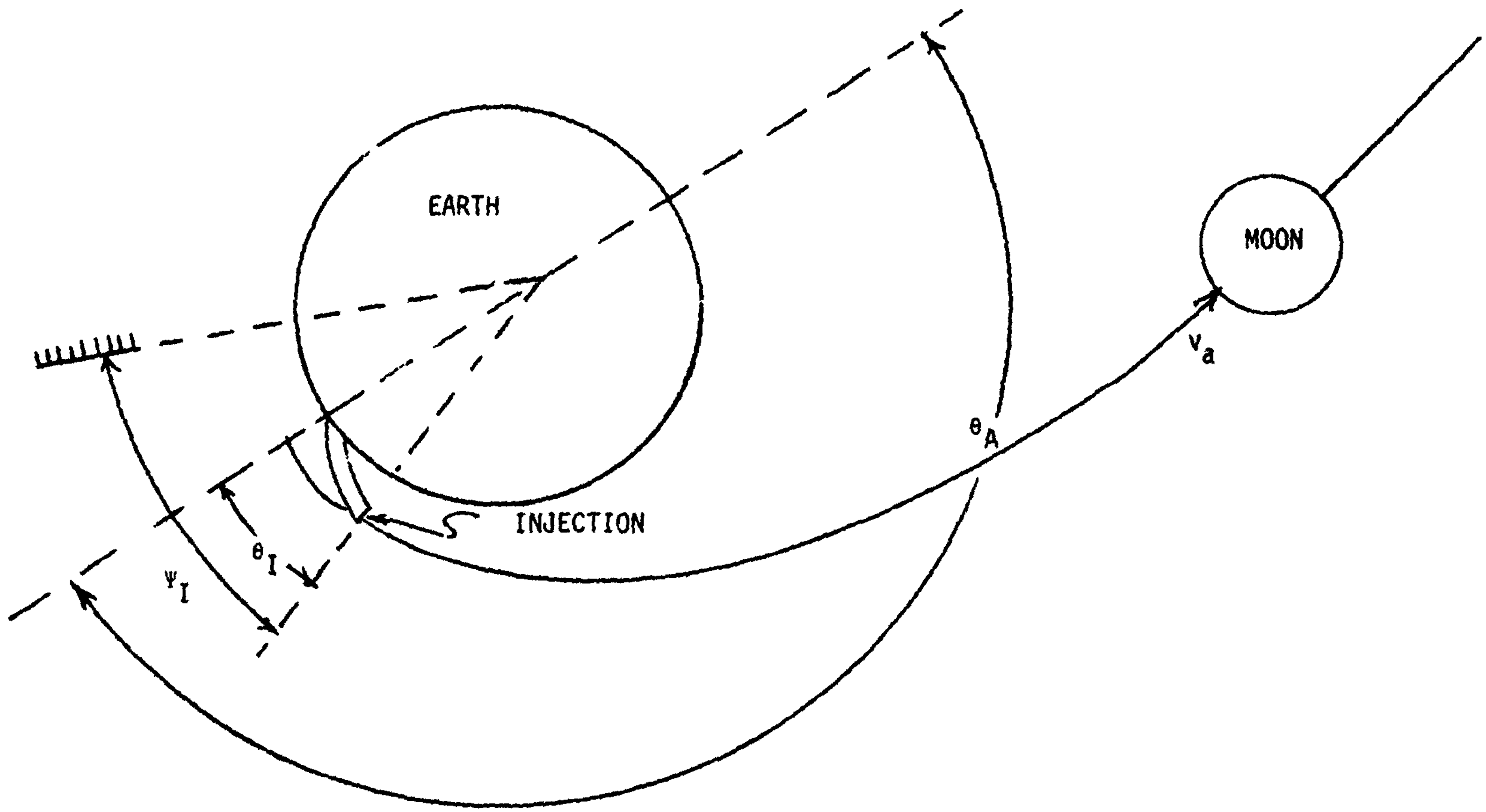
Given ψ_I and r_I, the simultaneous solution of these equations for $v_r(r_I, \psi_I)$ and $\gamma_r(r_I, \psi_I)$ yields the required velocity.

FIGURE 7.1: APOGEE/APSIDES GUIDANCE



c. <u>Lunar Mission Guidance</u> - Suppose the mission objectives is to fly past the moon at a given distance. It will be shown in Section 7.3 that it is legitimate to pretend that the moon has no mass and construct an equivalent "massless" miss to cause the appropriate fly-by distance to be achieved. Then if follows that conditions at injection can be specified by constructing a conic passing throuch a massless point which moves according to the lunar ephemeris. Such an approach requires an application of Lambert's Theorem to find the time-of-flight to the moon's distance as a function of the injection conditions r, v, and γ. The specification of this time, as well as the angular orientation of the earth-centered conic, will satisfy mission requirements, and thus define the required v_r (r_I, Ψ_I) and $\gamma_r(r_I, \Psi_I)$. A simpler (but approximate) approach is to introduce the notion of a "pseudo-asymptote", which is defined by the direction and magnitude of the earth-centered conic velocity at the point of closest approach to the massless moon. Controlling the magnitude of this vector at the (specified) lunar distance fixes energy and (approximately) the time of flight. Controlling the direction of this vector fixes (approximately) the orientation of the earth-centered conic. This law has the practical feature of being similar to the "energy-asymptote" specification used for interplanetary missions.

FIGURE 7.2: LUNAR IMPACT GUIDANCE



Assuming that the distance to the moon is so large that the velocity and position vectors are approximately parallel, the "pseudo-asymptote" is then determined by (see Figure 7.3)

$$c_3 = \text{specified} = v_r^{\ 2} - \frac{2\mu}{r_I} \tag{7.8}$$

$$\begin{aligned} \cos \delta &= \text{specified} = \cos (\psi - \theta_I + \theta_A) \\ &= \cos \psi_I (\cos \theta_A \cos \theta_I + \sin \theta_A \sin \theta_I) \\ &\quad + \sin \psi_I (\sin \theta_A \cos \theta_I - \cos \theta_I \sin \theta_A) \end{aligned} \tag{7.9}$$

where ψ_I is defined as before; θ_I is the injection true anomaly, where $\cos \theta_I$ and $\sin \theta_I$ are determined from equations (7.4) and (7.5), and θ_A is the true anomaly at the moon's distance, given by

$$\theta_A = \cos^{-1} \left(\frac{p}{r_m} - \frac{1}{e}\right) \tag{7.10}$$

where r_m is the moon's distance. Convenient expressions for $\sin \theta_A$ and $\cos \theta_A$ can be obtained from equation (7.10), and thus the required $v_r(r_I, \psi_I)$ and $\gamma_r(r_I, \psi_I)$ can be calculated.

d. Interplanetary Guidance - A more complicated situation arises in the case of interplanetary missions, where there are three conics to be considered i.e., the near earth escape hyperbola, the heliocentric ellipse, and the approach hyperbola near the target. The basic approach is to pretend that the earth and target planet have no mass and design a heliocentric ellipse which will pass between the massless earth and the massless planet with an acceptable earth escape hyperbola, characterized by the energy and direction of the asymptote. The conic injection conditions are then given by (see Figure 7.3)

$$c_3 = |\bar{v}_\infty|^2 = \text{specified} = v_r^{\,2} - \frac{2\mu}{r_I} \tag{7.11}$$

$$\begin{aligned}\cos \beta &= \text{specified} = \cos(\psi_I - \theta_I + \theta_A) \\ &= \cos\psi_I\,[\cos\theta_A \cos\theta_I + \sin\theta_A \sin\theta_I] \\ &\quad + \sin\psi_I\,[\sin\theta_I \cos\theta_A - \cos\theta_I \sin\theta_A]\end{aligned} \tag{7.12}$$

where ψ_I and θ_I are defined as before, and θ_A is the asymptotic true anomaly, defined by

$$\cos\theta_A = -\frac{1}{e} \tag{7.13}$$

Note that the angle of the hyperbolic asymptote is obtained in much the same fashion as for the lunar pseudo-asymptote described in (c). Indeed, success with the energy asymptote concept for interplanetary guidance led to the notion of a pseudo-asymptote for lunar guidance.

7.3 Asymptotically Matched Conics

In the previous section it was assumed that it made sense to pretend that the mass of the target body, be it the moon or a planet, can be set to zero for the purpose of defining injection guidance requirements. It remains to be shown how target errors for a massless moon or planet can be related in a meaningful fashion to the true conic relative to the body. The justification follows from the concept of "asymptotic matching" of the near target and/or near earth hyperbolae to a transfer ellipse (References 31,32,33).

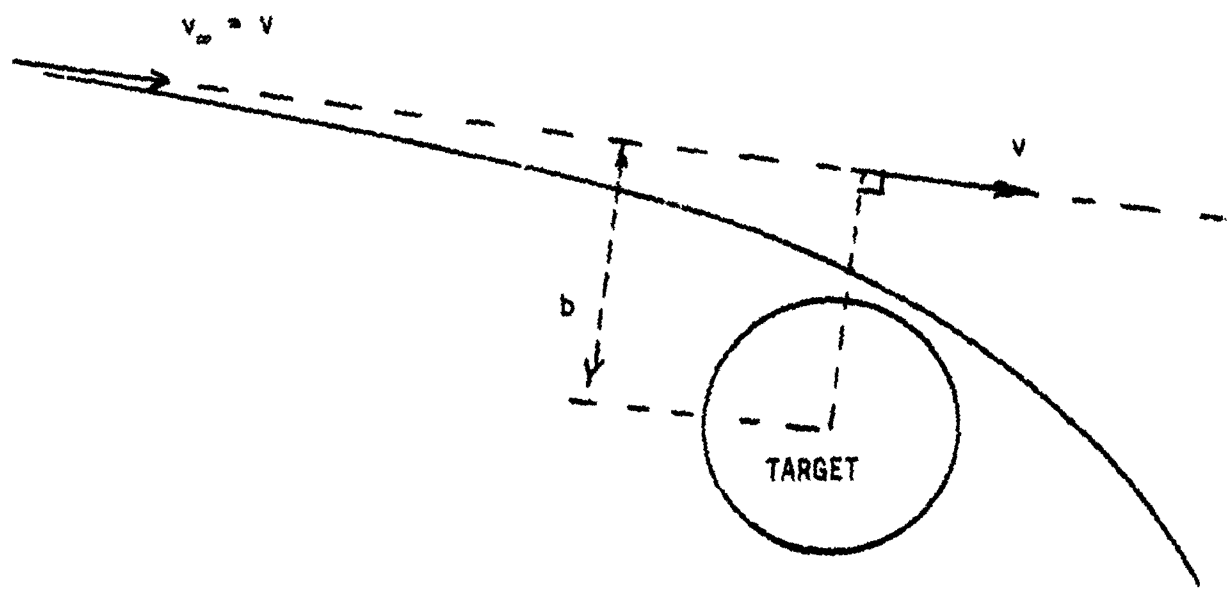


FIGURE 7.3: ASYMPTOTIC PATCHING

Referring to Figure 7.3, let the massless closest approach distance be denoted by b and the massless velocity relative to the target at closest approach by v. Recognizing that the asymptotic speed must be $v_\infty = v$, so that

$$v_\infty{}^2 = v^2 = c_3 \tag{7.14}$$

and that the angular momentum is given by

$$c_1 = bv_\infty \tag{7.15}$$

we note that the energy and angular momentum of the target centered hyperbola are specified by b and v. The orientation of the hyperbola is specified by the inertial direction of $\bar{v}$. It remains to be shown how the time-of-flight to massless closest approach relates to time of flight to actual closest approach. From the conic formula we find that a time from an arbitrary position r to periapsis is

$$t(r) = (\frac{a}{v_\infty}) [e \sinh E - E] \tag{7.16}$$

where

$$\cosh E = (1 + \frac{r}{a}) (\frac{1}{e}) \tag{7.17}$$

and a is the semi-major axis given by $a = (\frac{\mu}{c_3})$. But for large r $(r \to \infty)$, we have (asymptotically)

$$\cosh E \approx (\frac{r}{ae}) \approx (\frac{\exp E}{2}) \approx \sinh E \tag{7.18}$$

$$E \approx \ln (\frac{2r}{a}) - \ln e \tag{7.19}$$

Then

$$t(r) = \left(\frac{r}{v_\infty}\right) - \left(\frac{a}{v_\infty}\right) \ln \left(\frac{2r}{a}\right) + \left(\frac{a}{v_\infty}\right) \ln e \qquad (7.20)$$

The term $\left(\frac{r}{v_\infty}\right)$ is the time to massless closest approach; the term $\left(\frac{a}{v_\infty}\right) \ln \left(\frac{2r}{a}\right)$ is the decrease in that time due to the gravitational acceleration of the target body, and the term $\left(\frac{a}{v_\infty}\right) \ln e$ is the increase in that time due to the bending of the conic trajectory. Similar results can be obtained for the asymptotic patching of the near-earth hyperbola when launching to a planet, or for the near-moon hyperbola when moving from moon to earth.

It should be noted that the asymptotic distance corrections and the time of flight correction $\left(\frac{a}{v_\infty}\right) \ln e$ are commonly used in guidance work. The time of flight correction $\left(\frac{a}{v_\infty}\right) \ln \left(\frac{2r}{a}\right)$ has not been applied, however, because it is very large and probably too approximate. Instead, a numerical integration is usually performed to calculate actual time of flight to closest approach (t_{cA}), and "linearized time-of-flight" (t_L) is used for guidance purposes, defined by

$$t_L = t_{cA} - \left(\frac{a}{v_\infty}\right) \ln e \qquad (7.21)$$

This quantity, called the "local" correction by Breakwell (Reference 33), is linear in the sense that the nonlinear dependence of flight time upon the impact parameter b has been eliminated. As an example of the magnitude of this correction, note that for a lunar mission with $v_\infty = 1.25$ km/sec and a closest approach distance of 100 km, we have $a = 6{,}270$ km, $p = 9{,}190$ km, $e = 2.46$, $\frac{a}{v_\infty} = 5{,}360$ sec, and

$$\frac{a}{V_\infty} \ln e = 4{,}830 \text{ sec} = 1.34 \text{ hr.} \qquad (7.22)$$

The other flight time correction term, called the gross correction by Breakwell (Reference 33), is much larger. If we set r = distance to earth = 384,000 km, then

$$\left(\frac{a}{V_\infty}\right) \ln \left(\frac{2r}{a}\right) = 25{,}800 \text{ sec} = 7.15 \text{ hrs.} \qquad (7.23)$$

Since this correction is adequate only if the motion of the spacecraft relative to the moon is approximately a hyperbola for the entire mission, errors of more than 10% (an hour, say) are to be expected. Thus one can see the motivation for numerically calculating this term.

The asymptote matching method yields very good results if the gross correction term is numerically computed. There nevertheless do remain some errors in the approximation, for the effect of other bodies during the hyperbolic phase has not been precisely treated.

7.4 THE HYBRID CONIC TECHNIQUE

A "hybrid conic" technique has been developed by TRW Systems Group to numerically improve conic approximations of translunar and transearth trajectories.* In this approach, one takes a patched conic as a first approximation, which does not necessarily have to be asymptotically matched, and integrates in a simple way the moon's perturbation on the geocentric conic. The integrations are performed by

* This work was carried out under contract to the Manned Spacecraft Center, Houston under Contract No NAS 9-4810, Phase II, and is reported in TRW Note 69 FMT-728.

FIGURE 7.4: HYBRID AND CONIC POSITION ERRORS

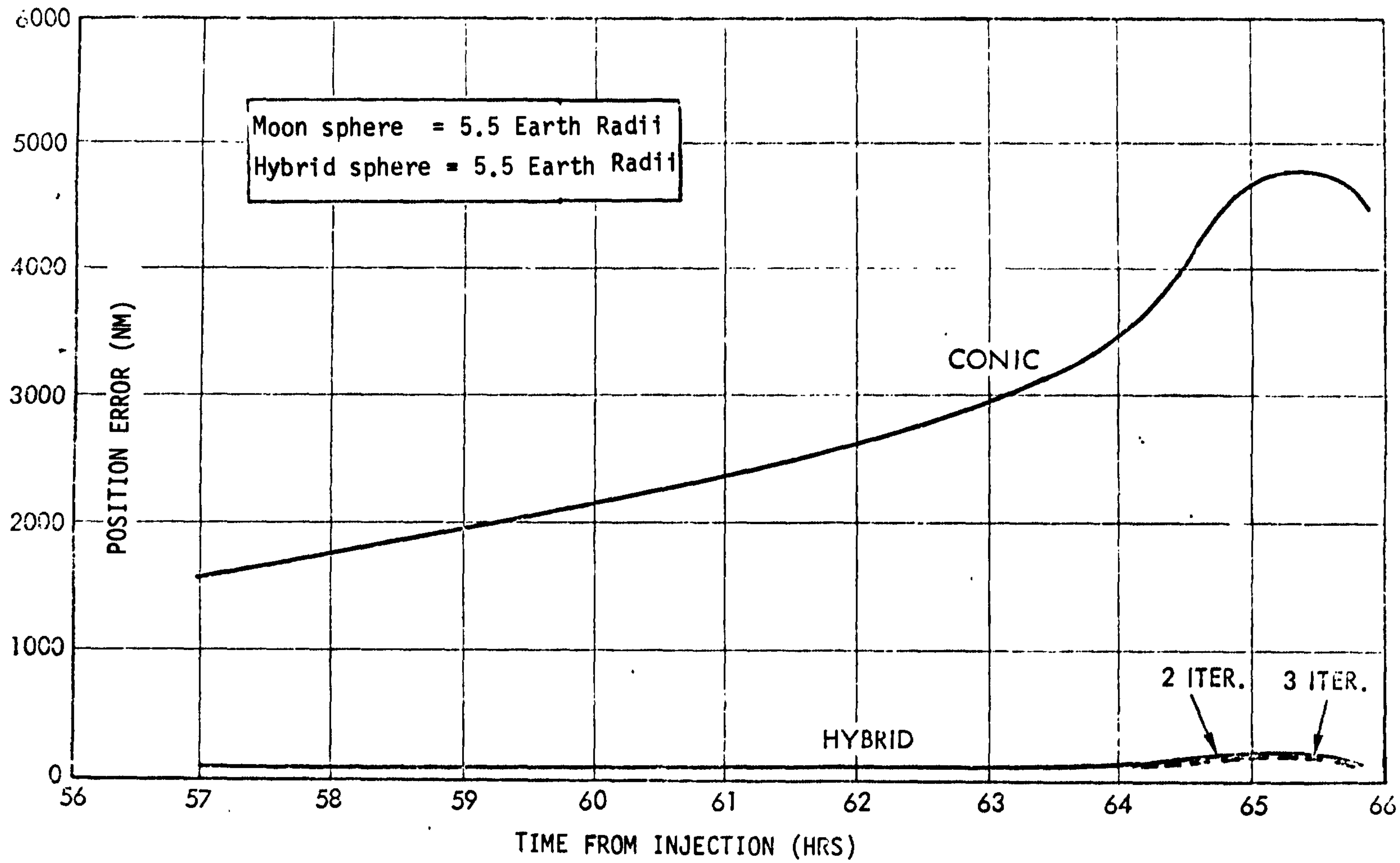
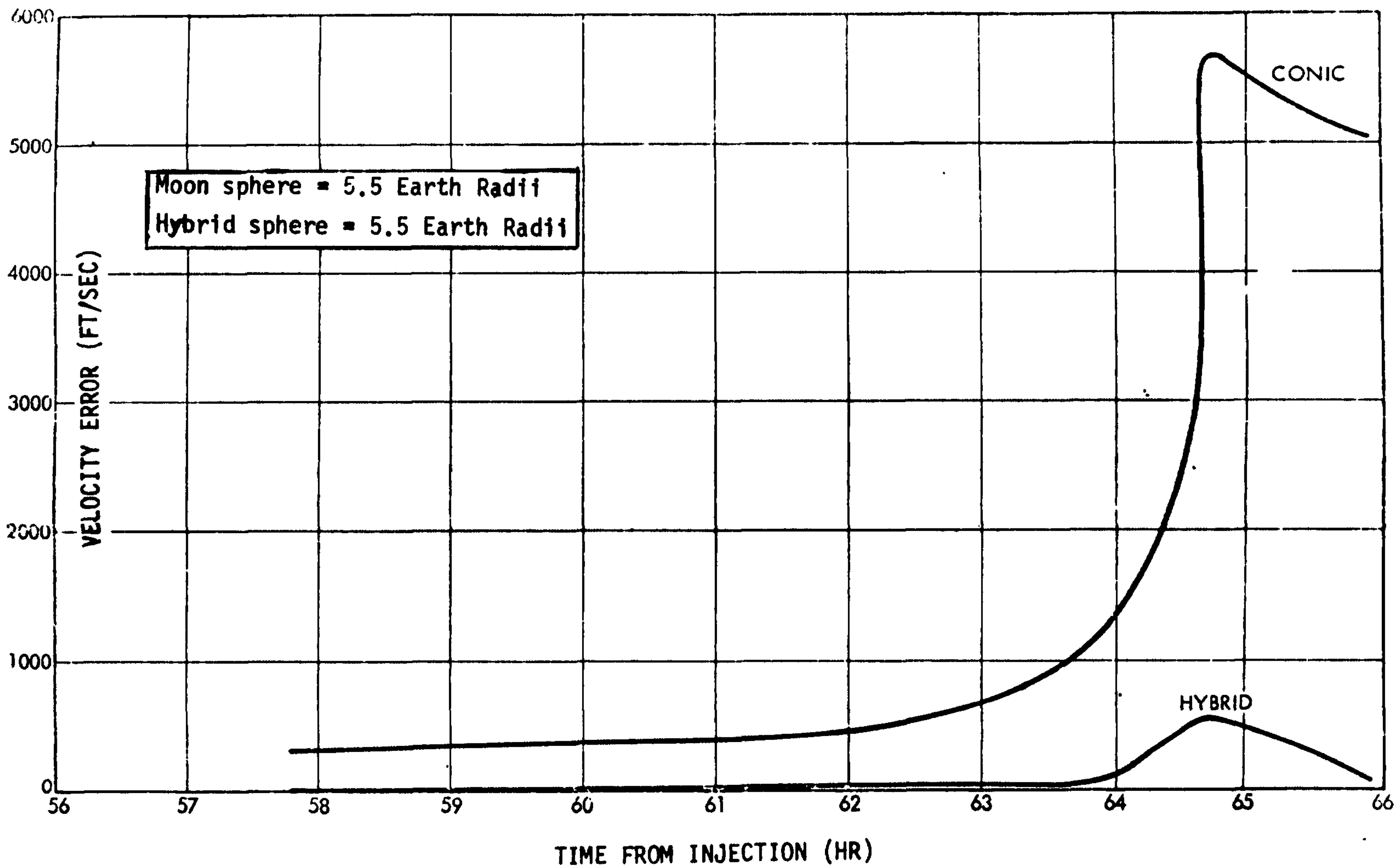


FIGURE 7.5: TRANSLUNAR HYBRID AND CONIC VELOCITY ERRORS



assuming that the perturbing acceleration is of the form

$$\ddot{\varepsilon}(t) = a(t) A + b(t)B + c(t)C \tag{7.24}$$

where a(t), b(t), and c(t) are specified functions which can be integrated in closed form, and A, B, C are constant vectors. The nine constants defining A, B, C are determined by evaluating the three components of $\ddot{\varepsilon}(t)$ at three specified times. The time varying position and velocity corrections to the matched conic, $\varepsilon(t)$ and $\dot{\varepsilon}(t)$, are obtained by quadrature, thereby constructing an improved trajectory approximation. The procedure is repeated by re-evaluating A, B, and C on the improved trajectory, and iterating to convergence. The technique requires very little computing time, and converges rapidly to a trajectory with saml1 errors. Some numerical results are presented in Figures 7.4 and 7.5.

8. THE CONSTRAINT PROBLEM

8.1 INTRODUCTION

Another important consideration in the development of a guidance mode is the analytical treatment of constraints on the mission, the trajectory, and the vehicle. Indeed, it is sometimes the case that these requirements affect the form of a guidance algorithm more than any other factor, for constraints are of prime importance and are difficult to treat analytically. Constraints are basically of two types: mission constraints and vehicle constraints. The mission constraints are those which exist independently of any vehicle, while vehicle constraints are those required by the particular launch vehicle.

Examples of mission constraints would be:

- Total mission time. The time from launch from mission completion may be fixed. This is particularly important in reliability anlaysis, where the probability of failure is a monotonically increasing function of time.
- Tracking. It may be that the trajectory will be required to pass over certain tracking stations at various times during the mission.
- Range Safety. It may be required that the trajectory not pass over certain parts of the earth because of the safety considerations. These constraints can be described in terms of certain impact probabilities, and translate themselves into such considerations as launch azimuth constraints and yaw steering constraints.
- Launch Window Duration. The length of time available to get to launch a vehicle may be restricted by many practical considerations.
- Launch Window Availability. Certain days or months may be restricted for numerous reasons.
- Solar Visibility. Some payloads may require solar power or heating or attitude reference at phases of the mission.

Examples of vehicle dependent constraints which may be specified are as follows:

- The vehicle attitude at various times (e.g., at staging or injection).
- The vehicle attitude rate at various times.
- The velocity correction capability remaining at burnout.
- Maximum dynamic pressure.
- The heating integral, which is the integral of dynamic pressure times velocity.
- Dynamic pressure at first stage separation.
- Angle of attack at first stage separation
- First stage impact distance.
- Radar look angles, for the case of a vehicle which is radio guided.

There are other constraints which might be specified, but, from the point of view of guidance theory, these constraints all translate themselves into one of two types:

1. <u>State variable constraints</u> -- Consisting of certain functional specifications of the trajectory state. For example, the specification that the dynamic pressure must never exceed a certain number is a state variable constraint, as is the specification that the heating integral shall be less than a certain number.

2. Control variable constraints - The specification of vehicle control functions, such as the steering law. For example, if the steering attitude at injection is specified to be a certain value, or the injection attitude rate is specified to be a certain value, these would be control variable constraints.

The control variable or the state variable constraints could be in equality or inequality constraints. For example, specification of a radar look angle as a precise function of time on the nominal trajectory would be a state variable equality constraint. Specification of a radar look angle as lying between two given bounds would be a state variable inequality constraint. Specification of final injection attitude as a fixed number would be a control variable equality constraint, while specification of the final injection attitude as lying within certain bounds would be a control variable inequality constraint. Control variable constraints can usually be treated without too much difficulty, particularly with a parameterized guidance law.

8.2 TREATMENT OF STATE VARIABLE CONSTRAINTS

The analytical treatment of state variable constraints is difficult. The reader is referred to References 17 and 30 for a discussion of this problem from the point of view of optimal control theory. In general, the theory states that one may design a trajectory without regard to constraints if it occurs that the resulting trajectory satisfies all constraints. If this is not the case, a different trajectory must be constructed. For example, in the case of a state variable inequality constraint one must develop a trajectory broken into segments, where on some segments an ordinary unconstrained optimization problem is solved, and on other segments the trajectory moves along the boundary of the constraint inequality surface. The task then is to find the break points, or switching times, where the segments join. With the exception of some special cases, the theory is difficult to implement, and empirical rules are often applied for the design of practical guidance modes. One must be aware of this troublesome problem when evaluating the performance of any given guidance mode.

The state variable constraint problem might be efficiently treated by a parameterized guidance law. (See also the discussion of Section 6.2). Suppose that a steering law similar to the TRW Hybrid guidance mode were used, and, after appropriate approximations of the non-parameterized state variables, the equations of motion could be represented in the form

$$\frac{dx}{dt} = f\ (p,t) \tag{8.1}$$

where p is the guidance parameter vector. Then implicitly we have $x = x(p,t)$. Suppose there is a state variable inequality constraint of the form

$$0 < q(\lambda) = q(x(p,t)) \quad . \tag{8.2}$$

For any given p, the maximum value of q is attained when

$$0 = \frac{dq}{dt} = \left(\frac{\partial q}{\partial x}\right)\left(\frac{dx}{dt}\right) \tag{8.3}$$

Solving for the time (t_m) of maximum q from equation (8.3), we have an implicit constraint on p:

$$q_{max} = q(x(p,t_m)0 \geq 0 \tag{8.3}$$

Thus there is an acceptable region in parameter space, the boundary being determined by setting $q_{max} = 0$ in Equation (8.4). If one can find the intersection of this set of acceptable parameter values with other similarly defined sets corresponding to other state variable constraints, the selection of acceptable p vectors would be possible. If there are more values of p than end conditions and p can be chosen as an interior point, degrees of freedom are available for optimization.

The constraints determine p, however, if it must be a boundary point in order to satisfy the end conditions. If there are just the right number of values of p to satisfy end conditions, the constraint surfaces provide a means for checking whether or not a given p vector is acceptable.

9. THE SWITCHING TIME PROBLEM

9.1 INTRODUCTION

Another important consideration in guidance mode analysis is the specification of engine stop and start times, which are called switching times. Most guidance mode analysis tends to concentrate on the steering laws, but for all schemes some technique for starting and stopping engines must be developed. It is often the case, however, that the largest source of guidance system inaccuracy arises from errors in thrust shutoff time. The selection of the steering law primarily affects the fuel consumption required to get to the end conditions, and has relatively little to do with accuracy. An error and shutoff time, on the other hand, yields an error in state which transforms directly to an error in conditions at mission completion.

There is a fundamental problem in rocket engine shutoff because the guidance system is basically uncontrollable near the final time. For example, it is obviously impossible to correct even a small position error at an arbitrarily short time-to-go from shutoff by steering and shutting off the engine, for in this short time the rocket engine can at best supply an impulse of velocity to the trajectory. According to definition 5 of Section 2, the guidance system can be said to be potentially unstable at the final time. This is a well known phenomenon and has been analyzed from various points of view by a number of people. The usual approach is to relax the guidance requirements in a small region near cutoff. One very useful and elegant technique is the velocity-to-be gained approach, where one defines as a function of given position the velocity which would be required to satisfy mission objectives. The difference between the present velocity and the required velocity is called velocity-to-be gained, and the vehicle is steered to null the velocity to be gained. There are several ways to do this; one method is to thrust in the direction to null the cross product of velocity-to-be gained and the rate of change of velocity-to-be gained. In any case, one must determine the criterion for changing from whatever steering mode has been previously employed to the shutoff mode, be it velocity-to-be gained or any other, and secondly, one must develop a steering and shutoff law which achieves a satisfactory set of end conditions. It should be noted that all three components of velocity-to-be gained should simultaneously go to zero, for otherwise errors will occur. The cross product steering scheme theoretically has this feature.

From the theoretical point of view there are several interesting questions to be asked. First, from the point of view of the classical theory of the calculus of variations, what phenomenon is taking place at the final time? This can be answered in terms of the classical condition of abnormality. Second, given that this phenomenon does indeed occur (i.e., abnormality), what insight does control theory offer as a solution to the problem? Lastly, how do such techniques as velocity-to-be-gained by cross product steering relate to the theoretically optimal law? These questions will be discussed in this section.

9.2 THE FIRST NECESSARY CONDITION FOR OPTIMAL STEERING AND SWITCHING

Let the equations of motion be

$$\dot{x} = f(x, u, t) \qquad t_0 \leq t \leq T \tag{9.1}$$

where $u(t)$ is a continuous scalar, control (steering) variable, and t_0, T are, respectively, the times of starting and stopping times of the guidance phase. In particular, we have in mind the special case

$$\dot{v} = \frac{\mu r}{|r|^3} + a(t) \begin{bmatrix} \cos u \\ \sin u \end{bmatrix} \tag{9.2}$$

$$\dot{r} = v \tag{9.3}$$

where $a(t)$ is the prespecified thrust acceleration, v is the velocity vector, r is the position vector, and μ is the gravitational constant.

We suppose that the problem has been formulated so that, at the final time(T), x_1 is to be minimized and $x_2 \cdots x_r$ are to be set equal to zero. Thus we are considering a special case of the Mayer problem, where the end conditions are linear. There is no loss of generality here, for a guidance problem can be put in such a form by a re-definition of state variables. For example, given the state variables $y_1 \cdots y_m$ and the problem formulation

$$\dot{y} = f(y, u, t) \tag{9.4}$$

$$\int_{t_0}^{T} g_1 \left(y_1, \cdots y_m, t\right) dt = \text{ minimum} \tag{9.5}$$

$$\int_{t_0}^{T} g_i \left(y_1 \cdots y_m, t\right) dt = 0 \qquad i = 2 \cdots r \tag{9.6}$$

then define $x_1 \cdots x_r, x_{r+1} \cdots x_{r+m}$ by

$$\frac{dX_i}{dt} = g_i \qquad i = 1, \cdots r \tag{9.7}$$

and

$$\frac{dX_i}{dt} = f_{i-r} \left(x_{r+1}, \cdots x_{r+m}, u, t\right) \qquad i = r+1, \cdots r+m \tag{9.8}$$

That is, we have set $x_{i+r} = y_i$ and introduced r new state variables to treat end conditions and optimality. The reformulated problem is of the type we have postulated.

To obtain the first necessary conditions for optimality of u(t) and t_0, T, we suppose that an optimal reference trajectory $x_s(t)$ is available, and consider the variations about this path. Let $\delta X(t) = x(t) - x_s(t)$, and $\delta u(t) = u(t) - u_s(t)$ so that the first variation is described by

$$\frac{d}{dt} \delta x = F(t) \delta x + G(t) \delta u \tag{9.9}$$

where

$$F(t) = \left[\frac{\partial f}{\partial x}\left(x_s(t),\ u_s(t),\ t\right)\right] \tag{9.10}$$

$$G(t) = \left[\frac{\partial f}{\partial u}\left(x_s(t),\ u_s(t),\ t\right)\right] \tag{9.11}$$

Holding t_0 and T fixed, the solution of equation (9.9) at the final time T is

$$\delta x(T) = u(T,\ t_0)\quad \delta x(t_0) + \int_{t_0}^{T} \eta(t)\ \delta u(t)\ dt \tag{9.12}$$

where the

$$\eta(t) = U\ (T,\ t)\ G(t) \tag{9.13}$$

and $U(T, t)$ is the state transition matrix, described by

$$\frac{d}{dt}\ U(T,\ t) = -U(T,\ t)\ F(t) \tag{9.14}$$

where $U(T,\ T)$ = the identity. The effect of changing the engine start/stop times t_0 and T by small amounts is to apply impulsive changes in position and velocity at, respectively, the beginning and end of the trajectory. For example, in the special case described by equations (9.2) and (9.3), we have

$$\Delta V(t_0) = a(t_0) \begin{bmatrix} \cos u_s(t_0) \\ \sin u_s(t_0) \end{bmatrix} \tag{9.15}$$

$$\Delta r(t_0) = 0 \tag{9.16}$$

$$\Delta V(T) = a(T) \begin{bmatrix} \cos u_s(T) \\ \sin u_s(T) \end{bmatrix} \tag{9.17}$$

$$\Delta r(T) = 0 \tag{9.18}$$

In general, we have

$$\Delta x\,(t_0) = \alpha(t_0)\, dt_0 \tag{9.19}$$

$$\Delta x\,(T) = \alpha(T)\, dT \tag{9.20}$$

where $\alpha(t_0)$ and $\alpha(T)$ are the accelerations realized by switching the thrust. Then if $dx\,(T) = x\,(T + dt) - x_s(T)$, from equation (9.12), we have

$$dx(T) = \beta(t_0)\, dt_0 + \alpha(T)\, dT + \int_{t_0}^{T} \eta(t)\, \delta u(t)\, dt \tag{9.21}$$

where

$$\beta(t_0) = U(T, t_0)\, \alpha(t_0) \tag{9.22}$$

The limits of the integral can be taken as the nominal times for this first order theory. Then if the trajectory is to be optimal, it is necessary that there be a constant Lagrange multiple vector $\nu^T = [\nu_1 \cdots \nu_r]$ such that

$$\nu \cdot dx(T) = 0 \qquad (9.23)$$

for all $\delta u(t)$, dt_0, and dT. Since these are independent variations, it follows that

$$\nu \cdot \eta(t) = 0 \qquad (9.24)$$

$$\nu \cdot \beta = 0 \qquad (9.25)$$

$$\nu \cdot \alpha = 0 \qquad (9.26)$$

Defining

$$\lambda(t) = \nu^T U(T, t) \qquad (9.27)$$

so that, from the definition of $U(T, t)$,

$$\dot{\lambda}(t) = -\lambda(t) \frac{\partial f}{\partial x}(t) \qquad (9.28)$$

it follows from equation (9.24) and the definition of $\eta(t)$ that

$$\nu \cdot \eta(t) = 0 = \lambda(t) \frac{\partial f}{\partial u}(t) \qquad (9.29)$$

These are the well-known optimality conditions, where the hamiltonian is

$$h(x, u, t) = \lambda(t) \cdot f(x, u, t) \tag{9.30}$$

9.3 ABNORMALITY AND UNCONTROLLABILITY

The first necessary condition for optimality can be derived in an elementary way, which illustrates the relationship between optimality, abnormality, and controllability (Reference 34).

Suppose we define the normality matrix

$$N = N_u + \beta\beta^T + \alpha\,\alpha^T \tag{9.31}$$

where α and β are as given in equations (9.20) and (9.22), and

$$N_u = \int_{t_o}^{T} \eta(t)\, \eta^T(t)\, dt$$

$$= \int_{t_o}^{T} \begin{bmatrix} (\eta_1^2) & (\eta_1\eta_2) & \cdots & (\eta_1\eta_r) \\ (\eta_1\eta_2) & (\eta_2^2) & \cdots & \\ \cdot & \cdot & \cdots & (\eta_r^2) \end{bmatrix} dt \tag{9.32}$$

The N matrix cannot have full rank (cannot be invertable) if the reference trajectory is optimal, for otherwise we could find variations dt_o, dt, and $\delta u(t)$ such that the end conditions are met with the performance variable $x_1(T)$ less than the nominal value. For example, choose the variations

$$dt_0 = \beta^T N^{-1} \varepsilon \tag{9.33}$$

$$dT = \alpha^T N^{-1} \varepsilon \tag{9.34}$$

$$\delta u(t) = \eta^T (t) N^{-1} \varepsilon \tag{9.35}$$

where ε is an arbitrary vector with $|\varepsilon|$ small. Then equation (9.21) becomes

$$dx(T) = [\beta\beta^T + \alpha\alpha^T + N_u] N^{-1} \varepsilon = \varepsilon \tag{9.36}$$

In this case $dx(T)$ can be controlled so as to take on arbitrary values, in particular, $dx_1 = -\varepsilon_1$, $dx_2 = dx_3 = \cdot \cdot \cdot dx_r = 0$. This is a contradiction of the assumption of optimality, and hence N must not have full rank. By setting dt_0 and dT both equal to zero, similar reasoning shows that N_u also must not have full rank. Thus we have established that N (and N_u) has at least one zero eigenvalue, and, from the definition of N, there is a constant vector ν such that

$$0 = \nu^T N\nu = \int_{t_0}^{T} (\nu \cdot \eta(t))^2 \, dt + (\nu \cdot \beta)^2 + (\nu \cdot \alpha)^2 \tag{9.37}$$

From equation (9.31) the optimality conditions given by equations (9.24)-(9.26) immediately follow.

We have shown that the first necessary condition for optimality is that the r by r normality matrix N must have rank not greater than $r - 1$. Abnormality exists in the case where the rank is $r - 2$ or less, for then there is more than one linearly independent vector ν which has the property described by equation (9.37). That is,

<u>Definition</u>: A trajectory is said to be optimal if the r by r normality matrix (N) has rank less than r; it is said to be abnormal of order q if it has rank equal to $(r-1-q)$.

The notions of abnormality and controllability (Reference 35) are closely related.

<u>Definition</u>: The r end conditions $x_1 \cdots x_r$ are said to be first-order completely controllable if the $r \times r$ normality matrix N has full rank; it is said to be first-order uncontrollable of order $q + 1$ if it has rank equal to $(r-1-q)$.

Thus an optimal trajectory is, by defi. :ion, always uncontrollable, and is abnormal of order q if it uncontrollable of order $q + 1$.

Note that this definition of abnormality is slightly different from that of Bliss (Reference 36, page 210), where the abnormality is also said to occur if the vector ν turns out to be $\nu^T = [0, \nu_2, \ldots \nu_r]$. That is, the Lagrange multiplier of the performance variable x_1 cannot be zero. We have also slightly modified Kalman's definition of (first-order) controllability, in the sense that the performance index x_1 is included as a state variable. For example, because of the way state variables have been defined an optimal system of the form given by equations (9.4) - (9.6) would be uncontrollable, even though the original system $y_1 \ldots . y_r$ were completely controllable. One is forced to adopt these modifications if there is to be, in general, a precise relationship between abnormality and controllability. For example, it can be seen that these definitions are meaningful for the case of a minimum time trajectory with all end conditions specified. Essentially, the slightly different definitions arose because abnormality was introduced as a condition required to proceed with aralysis of the second variation (see Reference 34), and controllability was a condition required for the analysis of linear systems with a quadratic performance index (see Reference 35).

9.4 ABNORMALITY AND UNCONTROLLABILITY AT THE FINAL TIME

All trajectories become uncontrollable of order (r-1) at the final time, and all optimal trajectories become abnormal of order (r-2) at the final time. This follows from equations (9.31) and (9.32), for suppose that t_0 is an arbitrary time during the guidance phase (i.e., t_0 is no longer the free start time), and let $t_0 \to T$. Then the normality matrix becomes

$$\lim_{t_0 \to T} N(T_0) = \lim_{t_0 \to T} \left[N_u\,(t_0) + \alpha(T)\ \alpha^T(T) \right] = \left[\alpha(T)\ \alpha^T(T) \right] \tag{9.38}$$

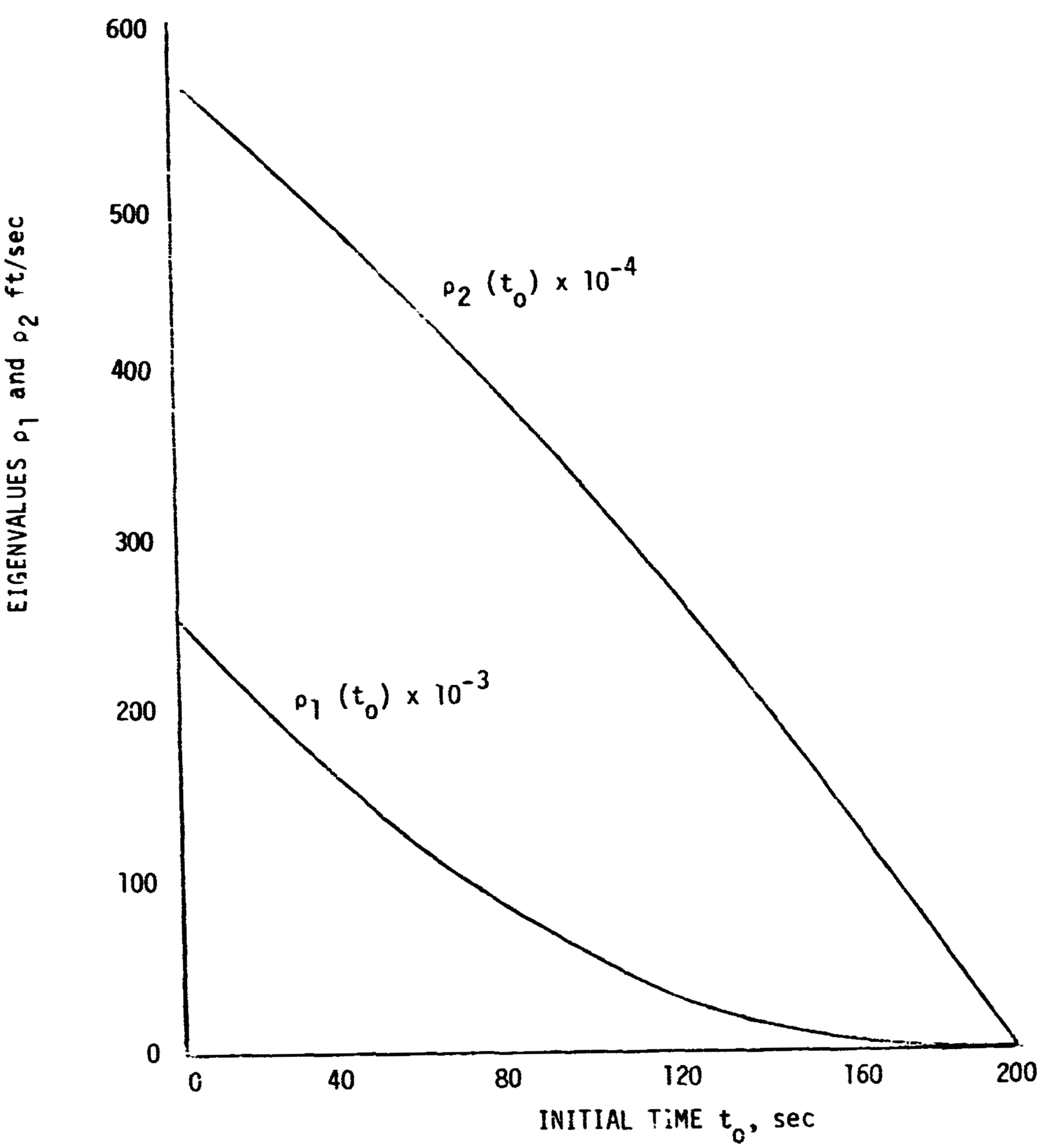
The $\left[\alpha(T)\ \alpha^T(T)\right]$ is obviously a rank one matrix. Furthermore, for times t_0 near T we note that $N_u(t_0)$ is also of rank one, for

$$N_u(t_0) = \int_{t_0}^{T} \eta(t)\ \eta^T(t)\ dt \approx \left[\eta(T)\ \eta^T(T) \right] \left(T - t_0 \right) \tag{9.39}$$

For an optimal trajectory it follows from Equations (9.38) and (9.39) $\eta(T)$ and $\alpha(T)$ are both orthogonal to ν ; for a non-optimal trajectory this is not necessarily the case.

An analysis of the $N_u(t_0)$ matrix is carried out in Reference (37) for the case of optimizing the burnout speed to attain a satellite orbit, controlling path angle and burnout altitude. The eigenvalues of the resulting 3 by 3 normality matrix are shown in Figure 9.1, where, for physical compatibility of units, the position state has been scaled by an energy factor to be expressed in units of ft/sec.

FIGURE 9.1: EIGENVALUES OF THE $N_u\ (t_o)$ MATRIX



9.5 SHUT-OFF AND STEERING NEAR THE FINAL TIME

The degenerary of the normality matrix near thrust termination offers an explanation of the thrust termination phenomenon. Given an optimization problem which is abnormal in the sense defined here, one would be forced to delete from consideration those linear combinations of end conditions corresponding to the zero eigenvalues of the normality matrix. Similarly, in the guidance problem one should attempt to control only one linear combination of end conditions near thrust termination. For the special case described by Equations (9.2)-(9.3) the only control available at the final time is the velocity correction $\Delta v(T)$ given by Equation (9.9). That is, at the final time the vehicle ought to be pointed in the nominal direction, and the shut-off time should be adjusted (by dT) so to null coordinate errors in the direction of Δv. This theoretical result is in agreement with practice, for thrust termination commands are usually generated when the required velocity component in a pre-specified direction reaches the desired value.

It remains to be shown how the region of uncontrollability can be precisely defined, and what the steering law should be in this region. In reference (37) it was suggested that a stable guidance law would be obtained by steering to the boundary of the reachable set of points, which is the the locus of attainable end conditions obtained by an analysis of the second variation. The analysis will not be repeated here, suffice it to say that the resulting steering law tends to null end condition errors sensed early in flight, when the radii of curvature of the boundary (ρ_i) are large, and tends to ignore errors sensed late in flight when the radii of curvature of the boundary (ρ_i) are small. Assuming suitable normalization of the steering angle, the ρ_i near thrust termination are approximately equal to the eigenvalues of the normality matrix (see Figure 9.1). Further work is required to develop this approach into a practical algorithm,but the analysis suggests that a reasonable terminal steering and shut-off criterion might be as follows: (1) enter the shut-off mode of operation when all but one of the eigenvalues of the normality matrix are nearly zero, (2) since the non-zero eigenvalue corresponds to the direction of $\eta(T)$, steer to null velocity errors perpendicular to this direction, but with a limit on the maximum steering angle deviation from nominal, and (3) terminate thrust when the speed error in the direction of $\alpha(T)$ is zero. Note that in effect we are attempting to null velocity errors near shut-off, which can be thought of as components of the velocity-to-be-gained.

9.6 THE CONJUGATE POINT PHENOMENON AT START TIME

The optimal start time can be calculated from the conditions obtained in Section 9.2. Although no difficulty should be expected for most space guidance applications, there is a possibility that the conjugate point phenomenon can occur at t_0 <u>because</u> of the fact that t_0 is a free parameter.

Consider first the case where t_0 and T are fixed, and for simplicity assume there is only one constraint, $x_2(T) = 0$. It is shown in Reference (34) that, by a proper choice of the x_i coordinates, the second order expression for end condition variations can be written in the form

$$\delta x_1(T) = \frac{1}{2}\int_{t_0}^{T}\int \delta u(t)\ Q\ (t,s)\ \delta u(s)\ dt\ ds \tag{9.40}$$

$$\delta x_2(T) = \int_{t_0}^{T} \eta(t)\ \delta u(t)\ dt \tag{9.41}$$

where Q(t,s) is a real symmetric kernel. Equation (9.40) is obtained by expanding the equations of motion (9.1) as a second order Taylor's series, but is, the terms $\delta x^2(t)$ and $\delta u^2(t)$ are retained, and the solution is represented in the form of an integral equation. The x_1 coordinate is chosen in the direction of the Lagrange multiplier ν. (Note that Q(t,s) would be a matrix, and $\delta u(t)$ would be a vector, if the control were to be interpreted as the values of u(t) at a finite number of points). The conjugate point condition is developed by solving the accessory minimum problem to obtain an expression for the boundary of the reachable set of points. It is shown in Reference (34) that the boundary surface is given by

$$\delta x_1 = \frac{\delta x_2^{\ 2}}{2\rho} \tag{9.42}$$

The ρ is the radius of curvature of the reachable set of points given by

$$\rho = \int_{t_o}^{T}\int \eta(t) \ \ Q^{-1} \ (t,s) \ \eta(s) \ dt \ ds \tag{9.43}$$

where Q^{-1} (t,s) is the "inverse Kernel" of Q(t,s). All end conditions within the boundary (9.42) can be reached by applying some control $\delta u(t)$. A conjugate point occurs at a value of t_o where $\rho(t_o) = 0$.

For the case of free initial time t_o, Equation (9.40) becomes

$$\delta x_1(T) = \quad \frac{\gamma dt_o^{\ 2}}{2} + dt_o \int_{t_o}^{T} K(t) \ \delta u(t) \ dt \tag{9.44}$$

$$+ \frac{1}{2}\int_{t_o}^{T}\int \delta u(t) \ Q(t,s) \ \delta u(s) \ dt \ ds \tag{9.45}$$

$$\delta x_2(T) = \ \beta_2 \ dt_o + \int_{t_o}^{T} \eta(t) \ \delta u(t) \ dt \tag{9.46}$$

where β is as defined above, γ is the second partial derivative of x_1 with respect to t_0, and $K(t)$ is the mixed partial derivative. Expressions for γ and K will not be presented here; they may be obtained from the analysis of Reference (37), where the second order effect of initial condition error is calculated. The conjugate point analysis proceeds as before, except that the inverse of a mixed kernel and scalar must be obtained. This can be done (again, think of $\delta u(t)$ as a vector and $Q(t,s)$ as a matrix), and the resulting radius of curvature is

$$\rho = \int_{t_0}^{T}\int \eta(t)\, Q^{-1}\,(t,s)\,\eta(s)\, dt\, ds$$

$$+ \frac{1}{c}\left[\int_{t_0}^{T}\int \eta(t)\, Q^{-1}(t,s)\,\eta(t)\, dt\, ds - \alpha\right]^2 \qquad (9.47)$$

where

$$c = \gamma - \int_{t_0}^{T}\int K(t)\, Q^{-1}\,(t,s)\, K(s)\, dt\, ds \qquad (9.48)$$

As before, the conjugate point phenomenon occurs when $\rho(t_0) = 0$

A comparison of Equations (9.43) and (9.48) shows that the conjugate point condition is indeed modified when t_0 is treated as a free variable. Thus it is possible that optimal switching could cause the conjugate point phenomenon to occur when it otherwise would not, or conversely. These interesting possibilities should be explored further for practical cases.

10. IMPULSIVE STOCHASTIC GUIDANCE

10.1 INTRODUCTION

It was pointed out in section 1.3 that in certain phases of fight the seperability of guidance, navigation and error analysis no longer applies. These cases arise when impulsive velocity corrections are to be applied, and there are execution errors proportioned to the magnitude of the correction and/or a constraint on the total amount of propellant to be utilized. For example, an important objective of some space missions is to achieve a specified eliptical orbit around the target body (e.g. Voyager, Apollo, Lunar orbiter). After the insertion manuver, the spacecraft may reach a dispersed orbit due to orbit determination and manuver execution errors, and a sequence corrections (possibly only two) becomes mandatory in order to accomplish the mission. These corrections are applied in a form of acceleration impulses, causing a step change in velocity, and hence orbital elements, at the correction times. The guidance strategy for such a sequence of corrections is the specification of how many corrections should be applied, when, and what they should be.

The determination of a strategy which minimizes a statistical measure of the final orbit error in the presence of orbit determination (estimation) and guidance execution errors poses an important unsolved problem. The classical optimal orbit transfer analysis which seeks the strategy to minimize propellant (correction capability) expenditure and does not consider random errors, can at best yield that approximately optimal strategy. The theoretically correct approach is **obtained via the** dynamic programming algorithm or its equivalent. This is an impossibly difficult task for realistic guidance systems, so one must take a different approach. Several authors (Refs. 12, 38-41) have dealt with a problem of this type and obtained some interesting results by analysis of simplified cases of by developing a suboptimal guidance strategy. The relation between this work and practical application is somewhat tenuous, for the problems have to be so simplified in order to accomplish the analysis that reality is often lost. In practice one sometimes uses a procedure which might be called the method of constraints. That is, a guidance strategy is chosen, based upon huristically applied deterministic reasoning, and this strategy is modified to take into account random errors. For example, on an interplanetary mission such as Mariner one might arbitrarily decide to make the first mid-course correction at, say, six days after injection, realizing that adequate tracking information can be gathered and processed by that time and that little would be gained by making the corrections slightly sooner or later. The aiming point for the correction, must consider the random correction execution errors, however, for these errors will yield state dispersions at the target which may violate certain mission constraints (e.g., the probability of impacting the planet may be too high). Thus one chooses a biased aim point which yields an acceptable probability of impacting the planet, assuming that there will be a second correction at some later time to adjust the aim point so as to have a high probability of satisfying the mission objectives. (The execution errors for the second correction are considerably smaller because the correction itself is smaller). In this case we are seeking to satisfy certain probabilistic mission objectives while imposing a probabilistic state variable constraint. This type of stochastic guidance problem is, in general, a very difficult one.

10.2 SOME NEW RESULTS IN STOCHASTIC GUIDANCE THEORY

The solution of a rather realistic form of the stochastic guidance problem is described in References (42) and (43). It is assumed that the total guidance correction capability expended during the mission is constrained to be less than a specified number (the resource initially allotted), and one seeks the strategy which minimizes the expected value of a weighted sum of squares of the final orbit errors. These dispersions arise from random estimation and correction execution errors. The analysis employs the dynamic programming technique for determining the optimal stochastic orbital transfer strategy, where it is supposed that the state of the system at any time can be described by the correction capability remaining and the maximum likelihood estimates of the orbit parameters. These estimates do indeed provide sufficient statistics for our problem if we assume that very many tracking data points are processed between corrections, since it can be shown that the probability density function for the estimation errors asymptotically becomes Gaussian with known covariance. A further simplification is introduced by assuming that two modes of operation are possible at each correction opportunity: capability unlimited, where an excess of correction capability is available to complete the mission and the optimal correction can be determined without constraint, and capability limited, where there is not adequate correction capability to complete the mission optimally, and a heuristically justified simple one-correction policy and measure of performance are to be employed. Thus correction capability (c) is a state variable which is constraind to be positive, and the control policy and performance associated with transferring from the state $c>0$ to the state variable boundary $c = 0$ are assumed to be specified. It is reasonable to specify these functions in a heuristic fashion, because for most applications the capability limited case rarely occurs In effect, then, one need only treat the capability unlimited case ($c = \infty$) and tabulate the optimal correction policy, the optimized measure of performance, and the expected correction capability required to complete the mission.

These simplifications are essential to the numerical dynamic programming solution of the problem. Nevertheless, one still encounters the well known numerical difficulty of calculating and storing the many values of the optimized performance index and guidance policy corresponding to all possible values of the state vector for all of the corrections. This problem can be partially overcome by recognizing that the only regions of the performance surface which are of real interest are the neighborhoods of the local minima which result from the guidance corrections, for these determine the "aiming points" in state space for preceeding corrections. (It can be shown that such local minima do indeed exist). Thus for the $i^{\underline{th}}$ guidance correction we need to store,as functions of uncorrected state variables,the coordinates and magnitudes of the local minima of the performance surface resulting from the correction, the expected value of the correction capability required for subsequent corrections, and a quadratic approximation of the local behavior of the performance surface. Given the estimate of orbit parameters prior to the $(i-1)^{\underline{st}}$ correction, these results would be used in a real-time application to determine the coordinates of the "best" reachable local minimum point. This choice specifies the aiming point for the $(i-1)^{\underline{st}}$ correction and (implicity) the correction itself, assuming that the capability required to reach this point plus the expected value to complete the mission is less than the amount presently avilable. This procedure would be repeated for the next correction,after tracking and estimation of the corrected orbit parameters, and all subsequent corrections would be treated similarly. If adequate correction capability is not available at any opportunity it is necessary to either choose a different local minimum which is acceptable, or apply the capability limited mode of operation, or aim for "best" local minimum, assuming it is to be followed with a capability-limited correction. The best of these alternatives should be chosen.

The dynamic programming approach to stochastic orbit transfer described here was applied by Nishimura to a space mission of the Voyager type (Reference 42). The numerical results were obtained with a computer program which determined the optimized performance index resulting from the final correction for a large number of points in state space. The optimal performance index contours were plotted to find the local minima (Figure 10.1), and these coordinates were used to define the intermediate orbits, i.e.. the "aiming points" for the preceeding correction. An example correction sequence is depicted in Figure 10.2, which is interesting to compare to the classical Hahman transfer case. Because of the stochastic dispersion factor, the Hohman transfer will yield poorer performance than the optimized strategy shown in Figure 10.2.

In summary, then, we have described a non-linear stochastic control problem where expenditure of correction capability is introduced as a state variable constraint, where the maximum likelihood estimates of the orbit parameters provide sufficient statistics to define the state of the system, and where the properties of the local minima of performance surface provide adequate information for a dynamic programming solution of the problem. More work is required to develop this approach to the point where it is practical for real-time guidance application. Some possibilities for simplification have recently become apparent and are being investigated.

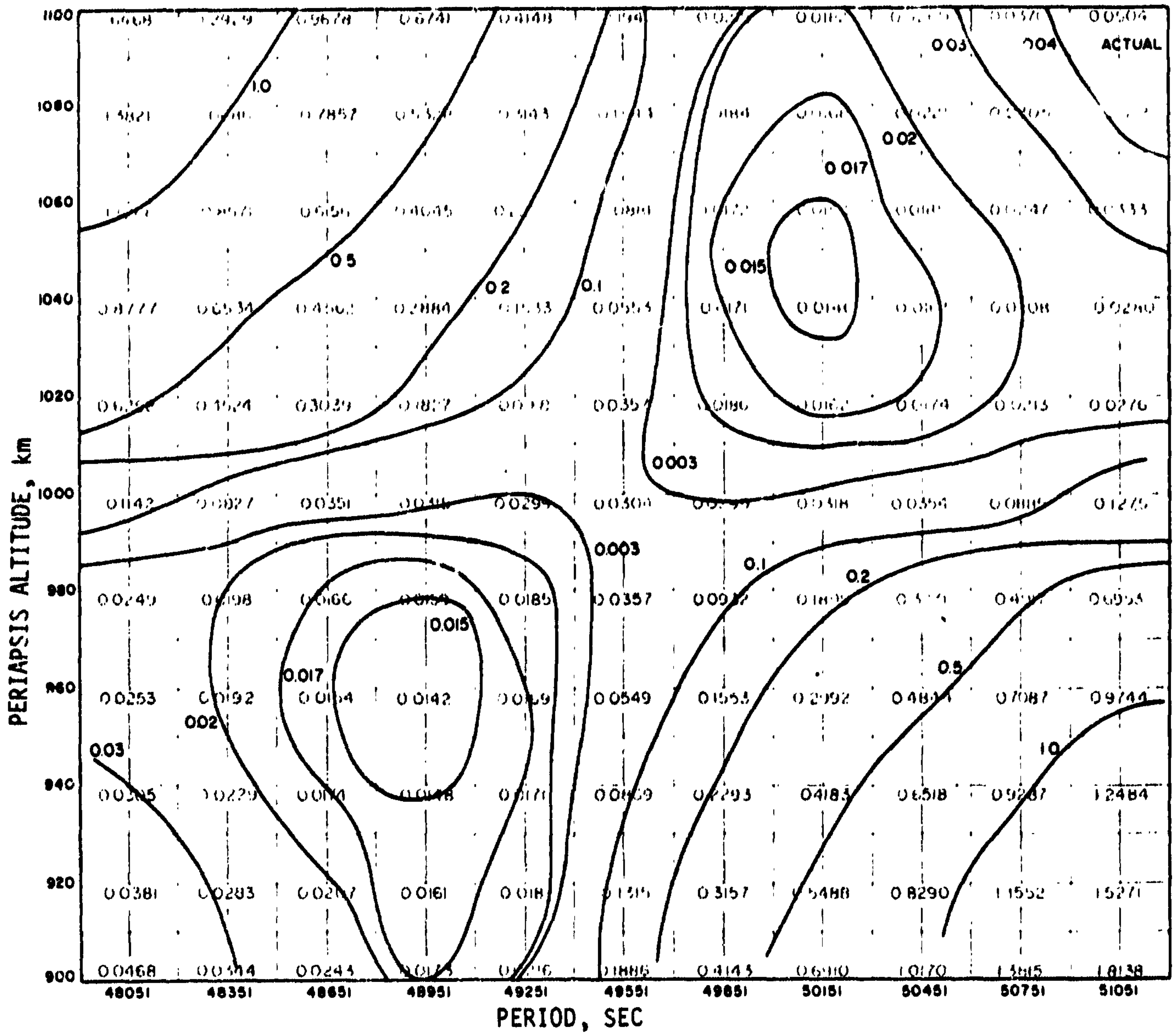


FIGURE 10.1: Performance Index Contour Map

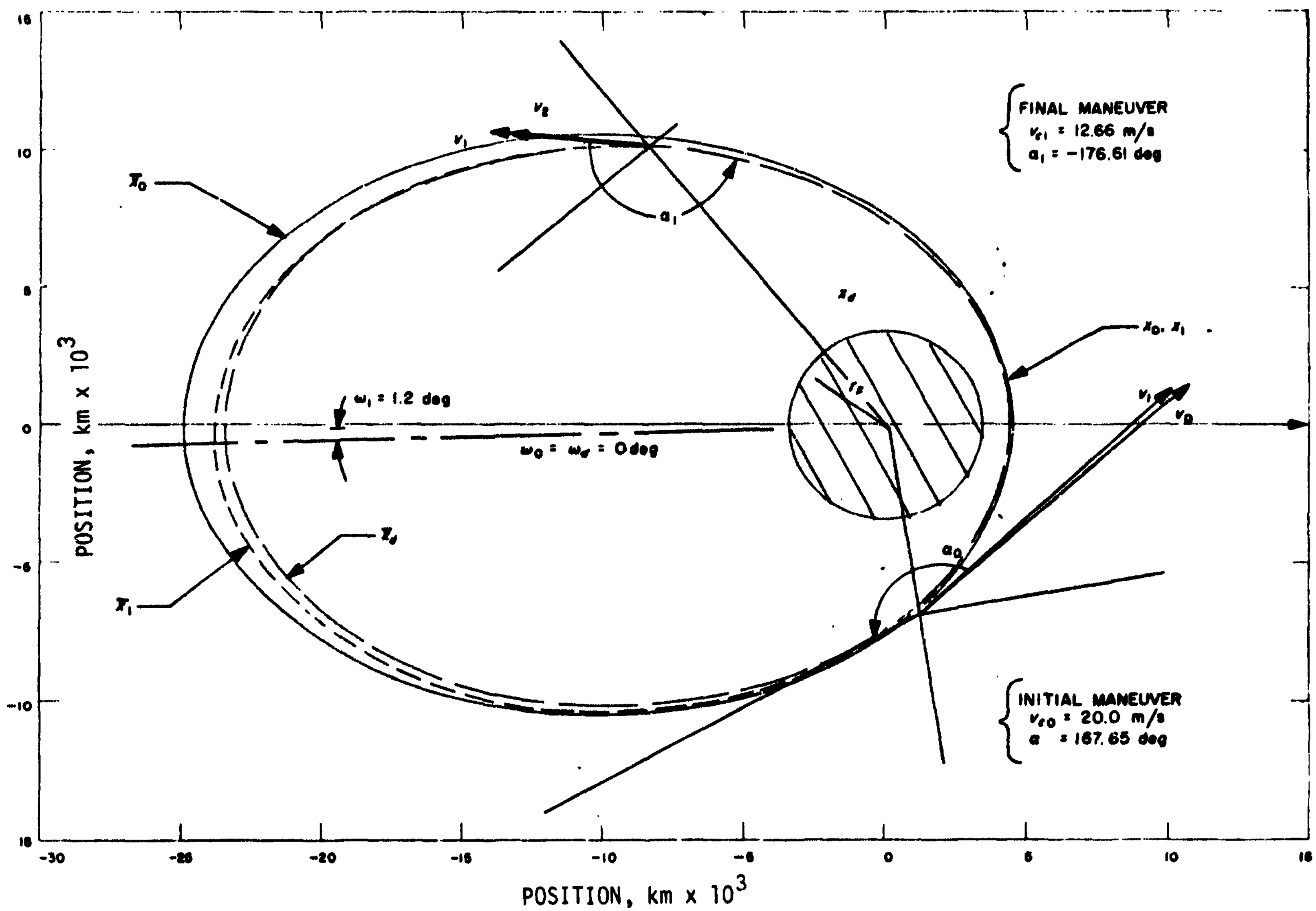


FIGURE 10.2: Example of Two-Stage Optimal Maneuvers

11. CONCLUSION

In this report we have taken a rather general, theoretical view of the "unified" guidance and navigation problem. (By unified system we mean a set of software and hardware modules which can be used to control all segments of the trajectory for a wide class of missions and launch vehicles). Assuming that the guidance and navigation problems can be treated separately, it is apparent that the derivation of a guidance algorithm (mode) is a key element in the development of a unified guidance and navigation system. With this motivation in mind, the purpose of the study described here was to examine the state-of-the-art in guidance mode development; classify existing and proposed modes; define measures of their performance; compare the modes with respect to these measures; describe some of the problem areas that may be encountered, and recommend directions for further research and development.

Specific conclusions and recommendations are presented in Section 1.9 and will not be repeated here. Suffice it to say that the analytical and numerical studies reported here indicate that it is feasible to design a unified guidance mode, capable of on-board implementation, if one is willing to exploit the numerical integration and iteration capability of present day computers. Further analysis and simulation is required in order to fully justify this conclusion. Such effort would be well spent, for savings in cost, reliability, and development time could be achieved by developing a unified guidance system. Furthermore, the cost in time and money of mission design, preflight preparation, and flight readiness verification could be reduced by using the unified guidance capability to develop preflight analysis software (the "quick reaction" problem). It is hoped that this Report will be a worthwhile contribution toward such a development.

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