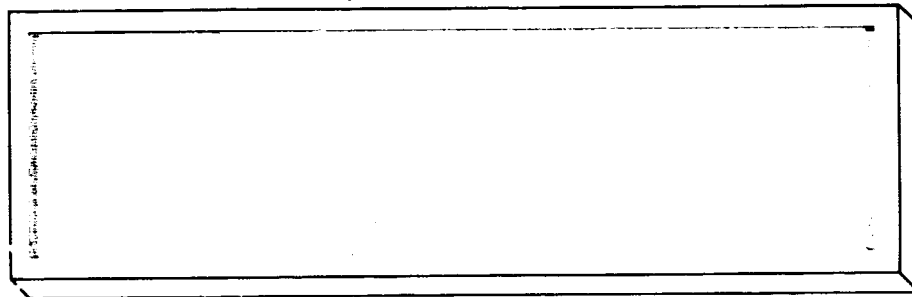
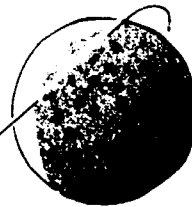


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STABILITY OF A CLASS OF COUPLED SYSTEMS¹

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Stability of a Class of Coupled Systems

Control problems in the area of gyroynamics normally involve a set of coupled second-order differential equations. A special class of such systems is considered and the stability results obtained are shown to have direct application to a current problem involving sub-optimal control of artificial satellites.

The system considered is of the form

$$\ddot{\underline{y}} + [D + S(\underline{y}, \dot{\underline{y}}, t)](a\dot{\underline{y}} + b\underline{y}) = 0 \quad (1)$$

where \underline{y} is an n -vector, D is a constant symmetric $n \times n$ matrix, S is a skew-symmetric $n \times n$ matrix whose elements may be functions of \underline{y} , $\dot{\underline{y}}$, and t , and a and b are constants.¹

Theorem: If $a > 0$, $b > 0$, $S + S' = 0$, and $a^2 D - bI_n > 0$, then the equilibrium at the origin of (1) is globally asymptotically stable.²

¹ It is assumed, as usual, that solutions of (1) do indeed exist.

² I_n represents the $n \times n$ unit matrix.

Proof: Let $\underline{x}_1 = \underline{y}$, $\underline{x}_2 = \dot{\underline{y}}$, and write (1) in the form

$$\dot{\underline{x}}_1 = \underline{x}_2$$

$$\dot{\underline{x}}_2 = - [D + S(\underline{x}_1, \underline{x}_2, t)](a\underline{x}_2 + b\underline{x}_1) \quad (2)$$

Consider the scalar function

$$V = 2ab\underline{x}_1' D \underline{x}_1 + 2b\underline{x}_1' \underline{x}_2 + a\underline{x}_2' \underline{x}_2 \quad (3)$$

having the derivative according to (2)

$$\dot{V} = - 2\underline{x}_2' [a^2 D - bI_n] \underline{x}_2 - 2b^2 \underline{x}_1' D \underline{x}_1 \quad (4)$$

Since (3) may also be written as

$$V = \frac{b}{a} \underline{x}_1' [2a^2 D - bI_n] \underline{x}_1 + a(\underline{x}_2' + \frac{b}{a} \underline{x}_1')(\underline{x}_2 + \frac{b}{a} \underline{x}_1)$$

the theorem follows immediately [1].

The theorem can be applied to a problem in sub-optimal attitude control of an artificial satellite using motor driven inertia wheels as a source of control torque. The angular motion of a satellite whose principle moments of inertia are equal is determined by

$$\ddot{\underline{\theta}} + H\dot{\underline{\theta}} + \underline{\dot{h}} = 0 \quad (5)$$

$$\text{where } H = \begin{vmatrix} 0 & h_3 & h_2 \\ -h_3 & 0 & -h_1 \\ -h_2 & h_1 & 0 \end{vmatrix}, \quad \underline{\theta} = \begin{vmatrix} \theta_1 \\ \theta_2 \\ \theta_2 \end{vmatrix}, \quad \underline{h} = \begin{vmatrix} h_1 \\ h_2 \\ h_3 \end{vmatrix}$$

and h_i = the angular momentum of the i^{th} inertia wheel divided by the principle moment of inertia of the satellite.

$\underline{\theta}$ = the angular displacement (assumed small) of the satellite relative to some non-rotating orthogonal reference frame whose origin is located at the center of mass of the satellite.

The vector $\underline{\dot{h}}$ represents the control effort used to regulate the attitude of the satellite. This vector is to be selected in feedback form as a function of the current values of $\underline{\theta}$, $\dot{\underline{\theta}}$, and \underline{h} , and must drive $\underline{\theta}$ and $\dot{\underline{\theta}}$ to zero while producing an "approximate minimum" of the integral performance index

$$J = \int_0^{\infty} (q\underline{\theta}'\underline{\theta} + p\dot{\underline{\theta}}'\dot{\underline{\theta}} + \underline{h}'\underline{h})dt \quad (6)$$

where p and q are positive constants.

The \underline{h} vector is time-varying and its functional form cannot be specified a priori. In order to determine an adequate control, a method for sub-optimal control proposed in [2] may be applied. This method yields

$$\dot{\underline{h}} = a\dot{\underline{\theta}} + b\underline{\theta} + \frac{b}{a} H\underline{\theta} \quad (7)$$

$$\text{where } a = \sqrt{p+2\sqrt{q}} \quad \text{and } b = \sqrt{q} .$$

This control, however, is acceptable only if (5) is then asymptotically stable. Combining (5) and (7),

$$\ddot{\underline{\theta}} + [I_3 + \frac{1}{a} H](a\dot{\underline{\theta}} + b\underline{\theta}) = 0 \quad (8)$$

and the theorem then assures that (9) is asymptotically stable for all $p > 0$, $q > 0$, regardless of the unknown time-varying matrix H . By (7) it is also clear that $\dot{\underline{h}} \rightarrow 0$ as $t \rightarrow \infty$.

References

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2. W. L. Garrard, N. H. McClamroch, and L. G. Clark, "An Approach to Sub-Optimal Feedback Control of Non-Linear Systems." International Journal of Control (to appear).