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INFLUENCE OF A PLANETARY ATMOSPHERE UPON THE FRESNEL DIFFRACTION OF ULTRASHORT WAVES

by

V. A. Andrianov

(USSR)

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SUMMARY

The asymptotic representation is obtained of the attenuation function in the region of Fresnel's penumbra at ultrashort wave diffraction around a planet surrounded by atmosphere.

The results are presented of numerical calculations for an exponential profile of the atmosphere.

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INTRODUCTION

The diffraction of electromagnetic waves on an ideally conducting sphere situated in a nonuniform medium with dielectric constant profile $\varepsilon(r) = 1 + \gamma^2/r^2$ was examined in the work [1].

Analysis of the field in the region of Fresnel penumbra allows us to obtain a qualitative representation on the influence of a nonuniform medium on the variation of the structure of Fresnel's diffraction field.

In order to discuss the possibility of applying the described effects in case of investigation of planetary atmosphere with emission sources of cosmic sonde types' radioeclipse [2], one must be in possession of a solution of an analogous problem for the profile of dielectric constant, the latter more closely corresponding to a certain average authentic planetary atmosphere profile. For the latter, the following exponential profile is usually applied:

$$\varepsilon(r) = 1 + \Delta\varepsilon_0 e^{-\beta(r-a)} \qquad (1)$$

or

$$n(r) = 1 + \Delta n_0 e^{-\beta(r-a)}; \quad \varepsilon(r) = n^2(r), \qquad (1')$$

where n(r) is the refraction index. We are interested in a wave band, for which the influence of planet's ionosphere may be neglected.

For the exponential profile $\varepsilon(r)$ we shall attempt to construct a short-wave approximation of the stated problem of diffraction on a sphere in the case $ka \gg 1$, where $k = 2\pi/\lambda$ is the wave number, a is the radius of the sphere, and to obtain certain numerical estimates.

1. CONSTRUCTION OF THE SOLUTION FOR THE REGION OF FRESNEL'S PENUMBRA

In order to construct the solution, let us take the integral expression of the radiowave field in a nonuniform atmosphere obtained by Fok [3], and applicable for the very general case of arbitrary course of atmosphere's dielectric constant profile from the height $\varepsilon(h) = \varepsilon(r - a) = \varepsilon(r)$ *.

Then for the Debye potential v of the radial magnetic dipole we shall write the expression by means of the contour integral in the form [1]

$$v = \frac{1}{16}\sqrt{\frac{2}{\pi \sin\vartheta}}\, e^{i(\pi/4)} \int_c \sqrt{\nu}\, F(\nu, r, r_0)\, e^{i\nu\vartheta} d\nu, \tag{2}$$

where r, r_0 are respectively the coordinates of the receiver and of the source; ϑ is the angular distance between them, and function $F(\nu, r, r_0)$ is expressed by the solution of the equation

$$\frac{d^2 f}{dr^2} + \left[k^2 \varepsilon(r) - \frac{\nu^2}{r^2} \right] f = 0 \tag{3}$$

and satisfies the boundary condition $F(\nu, a, r_0) = 0$ and the condition of emission at infinity. The solution of Eq.(3) may, correspondingly with the idea of standard equations method [4], be represented in the form

$$f(\nu, r) = A \sqrt{\frac{\xi(\nu, r)}{\xi'(\nu, r)}}\, H_\nu^{(1,2)}(\xi(\nu, r))[1 + 0(ka)^{-2/3}], \tag{4}$$
$$\xi'(\nu, r) \equiv d\xi/dr,$$

where $\xi(\nu, r)$ is the solution of the equation

$$\int_{kr_1}^{kr} \sqrt{\varepsilon(\rho) - (\nu/\rho)^2}\, d\rho = \nu(\operatorname{tg}\Omega - \Omega), \quad \cos\Omega = \nu/\xi, \tag{5}$$

and the lower limit in the integral kr_1 is determined from the condition

$$\varepsilon(r_1) - (\nu/kr_1)^2 = 0.$$

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* It is assumed that in the interval (a, ∞) $\varepsilon(r)$ is not zero and, as $r \to \infty$, $\varepsilon(r) \to 1$.

Applying for Hankel functions $H_\nu^{(1,2)}[\xi(r)]$ the Debye asymptotic representation, and computing the integral in (2) by the stationary phase method, we shall obtain for the field an expression valid in the illuminated region and corresponding to the geometric optics approximation in the solution of the problem.

For the construction of the solution in the region of Fresnel's penumbra, we shall assume for Hankel functions the asymptotic expression of the form [5]

$$H_\nu^{(1)}(\xi) = -\frac{i}{\sqrt{\pi} M} \exp\left[i\left(\int_{kr_1}^{kr} \sqrt{\varepsilon(\rho) - \left(\frac{\nu}{\rho}\right)^2}\, d\rho - \frac{2}{3} Y^{3/2} \right)\right] w_1(t - Y), \tag{6}$$

where $w_1(x)$ is an Airy function;

$$Y = \frac{M^2}{a^2}(r^2 - a^2);$$

$$M = (ka/2)^{1/3}.$$

The expression for the Debye potential (2) is then transformed as follows:

$$v = \frac{1}{16\pi M}\sqrt{\frac{2ka}{\pi r r_0 \sin\vartheta}} \exp\left\{ i\frac{\pi}{4} - i\frac{2}{3} Y^{3/2} - \right.$$

$$\left. - i\frac{2}{3} Y_0^{3/2} \right\} \int_\Gamma \sqrt{\nu(t)}\, G_r\{\nu(t), r, r_0\} e^{i\varphi(\nu, r, r_0)} w_1(t -$$

$$- Y_0)\left\{ w_2(t - Y) - \frac{w_2(t)}{w_1(t)} w_1(t - Y) \right\} dt,$$

where

$$\varphi(\nu, r, r_0) = \nu\vartheta + \int_{kr_1}^{kr} \frac{\sqrt{q(\nu, \rho)}}{\rho}\, d\rho + \int_{kr_1} \frac{\sqrt{q(\nu, \rho)}}{\rho}\, d\rho \cdot \qquad q(\nu, \rho) = \rho^2 \varepsilon(\rho) - \nu^2.$$

The complex component $\underline{t}$, over which integration now takes place in (7), is linked with ν by the relation [6]

$$\nu = \sqrt{\varepsilon(a)}\, ka + \frac{1}{\sqrt{\varepsilon(a)}} Mt. \tag{9}$$

The latter is important when considering the phase funtion $\phi(\nu, r, r_0)$ in (8). In the remaining cases one may consider $\nu \simeq ka + Mt$. This remark will be of use to us in the following.

For the determination of the form of function $G_r[\nu(t), r, r_0]$, entering into the integral of expression (7), let us turn at the outset to the physical sense of this function.

If in (7) we realize the ultimate transition $\varepsilon(\rho) \rightarrow 1$, we shall obtain the expression for the Debye potential corresponding to the case of absence of atmosphere [7]; postulating $G_r = 1$, i. e. the form of the function is determined by the properties of the nonuniform medium surrounding the sphere, namely: it describes the refraction attenuation of radiowaves. At the same time, it is difficult to determine it from the solution of Eq.(5) in the region of Fresnel penumbra. To that effect, we shall make use of the asymptotic solution of the diffraction problem on a phase screen (8), valid in the caustics' vicinity. As a result, we shall obtain

$$G_r(\nu, r, r_0) = \frac{1}{\sqrt[4]{q(\nu, r)\, q(\nu, r_0)}} \sqrt{\frac{R}{\varphi''(\nu, r, r_0)}}, \qquad (10)$$

where $R = \sqrt{r^2 - 2rr_0 \cos\vartheta + r_0^2}$, and ϕ is differentiated with respect to ν(or $\underline{t}$). It is not difficult to verify that, as $\varepsilon(\rho) \rightarrow 1$, $G_r \rightarrow 1$.

In the particular case of infinitely remote source ($r_0 \rightarrow \infty$), we shall obtain the simpler expressions:

$$G_r(\nu, r, \infty) = 1 / \sqrt[4]{q(\nu, r)}\sqrt{\varphi''(\nu, r, \infty)}. \qquad (11)$$

As a slowly varying one, multiplier $\sqrt{\nu} G_r(\nu, r, r_0)$ may be taken out of the integral sign in (7) at $t = 0$, and we may pass in the expression for the Debye potential to attenuation functions introduced by Fok (7]. This is fone in detail in the work [1].

Let us make use of the results of [1] and pass to the particular case of interest to us, when one of the correspondents is at infinite range ($r_0 \rightarrow \infty$).

Expanding the integrals in (8) in series by powers $(\nu - \sqrt{\varepsilon(a)}ka)$, we shall obtain for $\phi(\nu, r, r_0)$ the approximate expressions:

$$\varphi(\nu, r, r_0) \simeq k\sigma_r + \zeta_r l, \qquad (12)$$

where

$$\sigma_r = \int_{kr_t}^{kr} \sqrt{\varepsilon(\rho) - \varepsilon(a)(ka/\rho)^2}\, d\rho + \int_{kr_t}^{kr_0} \sqrt{\varepsilon(\rho) - \varepsilon(a)(ka/\rho)^2}\, d\rho;$$

$$\zeta_r = \frac{1}{\sqrt{\varepsilon(a)}} M \left[\vartheta - \sqrt{\varepsilon(a)}\, ka \left(\int_{kr_t}^{kr} \frac{d\rho}{\rho^2 \sqrt{\varepsilon(\rho) - \varepsilon(a)(ka/\rho)^2}} + \int_{kr_t}^{kr_0} \frac{d\rho}{\rho^2 \sqrt{\varepsilon(\rho) - \varepsilon(a)(ka/\rho)^2}} \right) \right]$$

It is not difficult to be convinced by means of direct calculation that at $\varepsilon(r) = 1 + (\gamma/r)^2$ we shall obtain the expressions for σ_r and ζ_r from [1], and for $\varepsilon(r) \to 1$, ζ_r from (13) passes into ζ from [8].

Passing to the limit for $r_0 \to \infty$, we shall obtain in the illuminated part of the Fresnel penumbra the asymptotic expression for the attenuation function W [1] in the form

$$|W| \simeq G_r(ka, r, \infty)\left[1 + \frac{1}{\sqrt{\pi}} \frac{\sin(\tau_r^2 - \pi/4)}{|\tau_r|}\right], \tag{14}$$

where $\tau_r = \eta\zeta_r$; $\eta = \sqrt[4]{Y}$.

The expression (14) is fundamental for the discussion of the question of the possibility of applying Fresnel difrraction for the study of planetary atmospheres and conducting numerical estimates. This question is the object of discussion in the next section.

2. INFLUENCE OF PLANETARY ATMOSPHERE ON FRESNEL DIFFRACTION

As already indicated in [1] and as may be seen from (14), the influence of planetary atmosphere is reduced to refraction attenuation of radiowaves, described by the multiplier G_r (ν, r, r_0), and to the variation in the fine structure of the Fresnel field of diffraction described by the quantity τ_r.

In order to conduct numerical estimates, we shall apply the above obtained results to analysis of the diffraction radiowave propagation around planet MARS, basing ourselves, first of all, on the fact that an analogous experiment already has been realized in the course of the flight of space probe MARINER-4 [10]. Moreover, during propagation in denser atmospheres, such as, for example, the Earth's and particularly the Venus' atmospheres, waveguide propagation of radiowaves is possible, at which the diffraction phenomenon is getting more complex. The results of measurements of MARINER-4 have shown that, from the standpoint of its electrodynamic properti s, the atmosphere can be characterized by the following values of parame ers in (1): $\Delta\varepsilon_0 = 7.2 \cdot 10^{-6}$; $\beta = 0.11$. The block diagram of the experiment and the preliminary computations are described in detail in the work [2].

Calculations of the refraction attenuation G_r in the atmosphere of Mars were conducted by us for several values of $\Delta\varepsilon_0 = 2\Delta n_0$ and β. They are shown in Fig.1 by solid lines for certain values of $\Delta\varepsilon_0 = 2\Delta n_0$, so as to enable us to make comparison with the results of analogous calculations in the work [2], indicated in Fig.1 by dashes and drawn on the basis of radial optics considerations When computing G_r by formula (11), one should note that the differentiation with respect to ν under the integral sign in the expression for ϕ should so conducted that no diverging integral be obtained (to that effect one should apply the integration by parts). The calculation of G_r was performed by us with the aid of a BESM-2M computer.

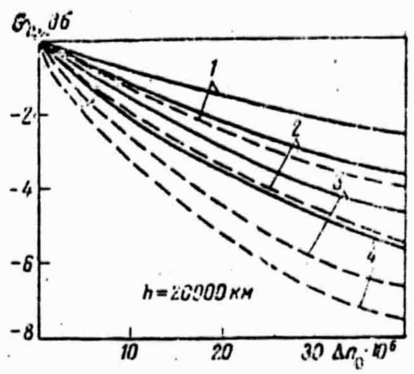

Fig.1. Refraction attenuation in the atmosphere of Mars:

1 — $\beta = 0{,}05$ км$^{-1}$; 2 — $\beta = 0{,}07$ км$^{-1}$; 3 — $\beta = 0{,}09$ км$^{-1}$; 4 — $\beta = 0{,}11$ км$^{-1}$

Comparison with [2] shows that the values of G_r obtained in the latter, are somewhat greater than our own. This may be the consequence of the fact that, gene-

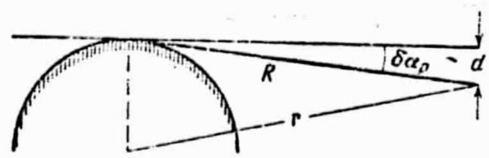

Fig.2

rally speaking, at propagation of radiowaves tangentially to the planet, formulas of radial optics are not valid, and this is why the calculations made on their basis may lead to somewhat overrated values (if one makes in these formulas such approximations that they still allow us to conduct calculations at tangential propagation), which is exactly what we observe. This difference may have a significance when interpreting the results of measurements of the refraction attenuation for the determination of planetary atmosphere's parameters $\Delta\varepsilon_0$ and β[10].

The results of calculations of G_r for Mars' atmosphere parameters, brought out in [10] as a function of the dependence of source (or receiver's) raising above the planet, $h = r - a$ are compiled in Table 1.

TABLE 1

REFRACTION ATTENUATION G_r AS A FUNCTION OF HEIGHT h

G_r, дб	−0,310	−0,515	−0,881	−1,810	−3,026
h, км	5000	10 000	20 000	50 000	100 000

Passing to the study of the influence of the atmosphere on the fine structure of the diffraction curve, we note that here two effects are possible: on account of radiowave refraction there takes place a shadow boundary shift toward the side of shadow's region (and alongside with it of the whole Fresnel curve). A displacement of oscillations is observed at coincidence with the Fresnel curve in case of absence of atmosphere in such a way that their values coincide at the boundary of the "geometric" shadow.

In order to conduct numerical estimates, we shall write

$$\tau_r = \tau + \eta(\zeta_r - \zeta). \tag{15}$$

where $\tau = \eta\zeta = \eta M\psi$, ψ being the angle of diffraction [9]:

$$\psi = \vartheta - \alpha(r) - \alpha(r_0); \quad \alpha = \text{arc cos } (a/r).$$

Substituting ζ_r from (13) into (15), and taking $\sqrt{\varepsilon(a)} \simeq 1 + \frac{1}{2}\Delta\varepsilon_0$, we obtain for the difference $\zeta_r - \zeta$ the expression

$$\zeta_r - \zeta = 2M\delta\alpha_r - \tfrac{1}{2}\Delta\varepsilon_0 M\psi_r, \tag{16}$$

where $\delta\alpha_r = (\alpha_r - \alpha)$ is the angle of refraction;

$$\alpha_r(r) = \sqrt{\varepsilon(a)}\,ka \int\limits_{kr_1}^{kr} \frac{d\rho}{\rho^2\sqrt{\varepsilon(\rho) - \varepsilon(a)\,(ka/\rho)^2}};$$

$$\psi_r = \vartheta - \alpha_r(r) - \alpha_r(r_0),$$

or we have for τ_r the expression

$$\tau_r \simeq \tau + 2\eta M\delta\alpha_r - \tfrac{1}{2}\Delta\varepsilon_0\eta M\psi_r. \tag{17}$$

The position of shadow boundary is determined from the condition $\tau_r = 0$, ψ_r being zero at the same time.

Then, from (17) we obtain the shadow boundary shift in the absence of atmosphere relative to the position of shadow boundary in the presence of atmosphere:

$$\tau = -2\eta M\delta\alpha_r. \tag{18}$$

Formula (18) allows us to determine the angular displacement. The magnitude of the latter may be expressed by the distance from boundary position in the absence of atmosphere (see Fig.2:

$$d \simeq R\delta\alpha_r. \tag{19}$$

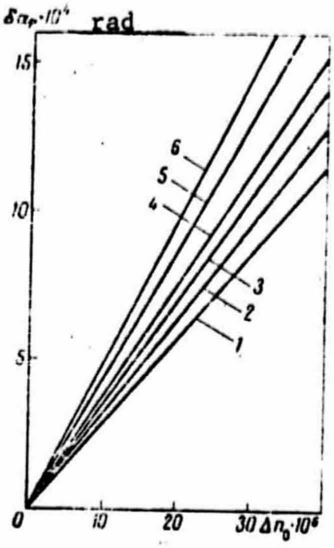

Fig.3. Refraction at tangential relative to the planet of Mars atmosphere radiotranslucence:
1 — β = 0,04 км$^{-1}$; 2 — β = 0,05 км$^{-1}$; 3 — β = 0,06 км$^{-1}$; 4 — β = 0,07 км$^{-1}$; 5 — β = 0,09 км$^{-1}$; 6 — β = 0,11 км$^{-1}$

For the experimental profile of $\varepsilon(r)$, taking into account the parameters characteristics of the atmosphere of Mars, the integral, entering into $\delta\alpha_r$, may be computed approximately. As a result, we obtain

$$\delta\alpha_r \simeq {}^1/_2\Delta\varepsilon_0\sqrt{2\pi a\beta} = \Delta n_0\sqrt{2\pi a\beta}. \tag{20}$$

This coincides with [2]. The result of calculation by formula (20) is plotted in Fig.2.

Besides that, displacement of Fresnel curve's oscillations are expected at coincidence of the two curves: one – in the absence of atmosphere, the other – in the presence of it, so that their values coincide at shadow bpundary. The magnitude of this displacement can be determined with the knowledge of "zero" positions (points of intersection of oscillations with a straight line corresponding to the level $|W| = 1$ [1]:

$$\tau_n' = \tau_{rn} - 2\eta M\delta\alpha_r + {}^1/_2\Delta\varepsilon_0\eta M\psi_r = \tau_n - 2\eta M\delta\alpha_r + {}^1/_2\Delta\varepsilon_0\eta M\psi_r. \tag{21}$$

Since the quantity ψ_r is proportional to the range from shadow boundary, we may obtain for the difference $\tau_n' - \tau_{n+1}' = \delta\tau_n'$, determining the distance between zeros, from Eq.(21):

$$\delta\tau_n' = \delta\tau_n(1 + {}^1/_2\Delta\varepsilon_0). \tag{22}$$

The relative shift is expressed by the formula

$$\delta F = (\delta\tau_n' - \delta\tau_n) / \delta\tau_n = {}^1/_2\Delta\varepsilon_0, \tag{23}$$

i.e., the magnitude of the displacement is practically unnoticeable.

CONCLUSION

A shortwave approximation for the attenuation function of the field of radiowaves emitted by a cosmic sonde at its eclipse by the planet is obtained for the exponential (or little differing from it) profile of the index of refraction of planet's atmosphere, from the point of view of an observer situated on Earth, which allows us to estimate the influence of the planetary atmosphere on the Fresnel diffraction of ultrashort radiowaves.

Estimates were conducted in case of radioeclipses by planet Mars.

The results of computations show that the described effects, that is, the refraction attenuation, the displacement of shadow's geometric boundary and the variation of the oscillation period of the Fresnel curve, are small as far as the atmosphere of Mars is concerned. Besides, a divergence is noted with the results of refraction attenuation obtained by means of formulas of geometric optics. This may have a significance when interpreting the results of measurements, the possibility of conducting same is shown in the experiment with MARINER-4.

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