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OPTIMUM LANDING OF A SPACECRAFT ON THE MOON

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SUMMARY

The optimum landing of a spacecraft at a fixed point on the surface of the Moon from a low circular AMS orbit is investigated. Analyzed are the influence of thrust load, of the height of the initial orbit and of the range of the landing site on the magnitude of spacecraft's final mass. Examples are brought out of optimum trajectories and optimum control programs of thrust magnitude and direction.

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The landing of a spacecraft on the surface of the Moon is one of the most power-requiring among the complexes of maneuvers during the flight to the Moon. Therefore, it is desirable to estimate the minimum need of fuel consumption and to investigate the peculiarities of optimum trajectory landing.

A considerable number of works are dedicated to this problem [1-3]. However, in their investigations all sorts of simplified suggestions are used which narrow the range of application of the obtained results. In the present work the optimum landing is estimated from a circular AMS orbit on the Moon's surface of a spacecraft in which the control of magnitude and direction of thrust and ejection velocity is inertialess and is independent of the magnitude of the thrust. The motion takes place in the central Newtonian gravitational field of the Moon. The assignment of spacecraft's initial position on the circular AMS orbit is connected with the fact that low circular orbits attract attention as possible intermediate portions of the flight trajectory of spacecraft to the Moon with crew on board.

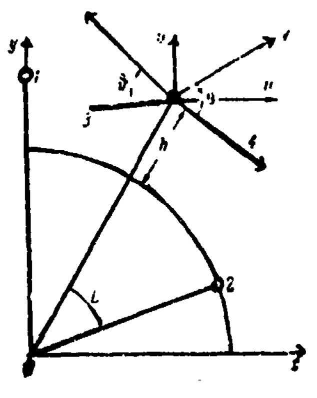

Fig.1.

The denotations used in the work are clear from Fig.1. Employed here is the Descartes' inertial system of coordinates connected with the center of the Moon: 1) is the point of descent from the orbit, 2) is the point of landing, 3) is the horizontal at descent from the orbit, 4) is the local horizontal.

Making use of L.S. Pontryagin's maximum principle, the equations of optimum motions will be written as follows:

$$\dot{u} = \frac{P}{m}\cos\vartheta - \frac{\mu}{R^3}x; \quad \dot{v} = \frac{P}{m}\sin\vartheta - \frac{\mu}{R^3}y;$$
$$\dot{x} = u, \ \dot{y} = v; \quad \dot{m} = -\frac{P}{c};$$
$$\dot{p}_m = -p_x, \ \dot{p}_v = -p_y, \ \dot{p}_x = \frac{\mu}{R^3}\left[p_u - \frac{3x}{R^3}(xp_u + yp_v)\right].$$
$$\dot{p}_y = \frac{\mu}{R^3}\left[p_v - \frac{3y}{R^3}(xp_u + yp_v)\right], \ \dot{p}_m = -\frac{P}{m^2}\rho; \tag{1}$$

where

$$R = (x^2 + y^2)^{1/2}, \quad \rho = (p_u^2 + p_v^2)^{1/2},$$

$m = \dfrac{M(t)}{M(0)}$ is the dimensionless mass, α is the consumption of mass m per second, $\overline{P} = P/g_E$ is the dimensionless reactive thrust, c is the constant outflow velocity of reactive jet, $\mu = g_M R_M^2$, g_M, R_M is the acceleration of the free fall on the surface of the Moon and its radius, $g_E = 9.81$ m/sec^2.

Controlling functions: The magnitude P and the orientation angle ϑ of the reactive force satisfy the required optimum conditions

$$\sin\vartheta = -\frac{p_v}{\rho}; \quad \cos\vartheta = -\frac{p_u}{\rho},$$
$$P = \begin{cases} P_{max}; & \Theta > 0, \\ 0, & \Theta < 0, \end{cases} \tag{2}$$

where $\Theta = \rho + mp_m/c$.

The questions of special control are not investigated in the present work, i.e. according to (2), P assumes only the limiting values P_{max} and 0, and the zeros of function Θ serve as change-over points.

The final value of the mass $m(T) = m^1 = \max$, serves as the maximizing problem of the functional.

The controls (1) have for the first integral

$$\mathscr{H}(t) = -\frac{P}{m}\left(\rho + \frac{m p_m}{c}\right) - \frac{\mu}{R^3}(x p_u + y p_v) + u p_x + v p_y = 0. \qquad (3)$$

The maneuver scheme is presented in Fig.1, where $R = R_M + H$ and H are the radius and the height of the initial orbit, the axis Oy is drawn through the landing point, ψ is the angular range from the assigned point of descent from the orbit (in degrees), $\underline{h}$ and L are the current values of altitude above the surface of the Moon and of the selenocentric range (in km). The boundary conditions have the form

$$t = 0,\ u(0) = u^0 = V_{bound}(H)\cos\psi,\ v(0) = v^0 = V_{bound}(H)\sin\psi, \qquad (4)$$

$$x(0) = x^0 = -R\sin\psi,\ y(0) = y^0 = R\cos\psi,\ m(0) = 1,$$

$$t = T,\ u(T) = u^1 = 0,\ v^1 = 0,\ x^1 = 0,\ y^1 = R_M,\ p^1{}_m = -1. \qquad (5)$$

At the initial moment of time the values of p_u, p_v, p_x, p_y, p_m are unknown. These parameters must be selected in such a manner that the trajectory of system (1) corresponding to them and to condition (4) pass through the point (5). At the same time the values of P, ϑ in system (1) must satisfy the conditons (2). Taking into account (3), the number of selected parameters will be four.

In each trajectory obtained as a result of the solution of the boundary value problem, the maximum principle and the boundary conditions are fulfilled, i.e. all the obtained trajectories satisfy the total combination of required conditions for the local functional's maximum. It is possible to hit the point on the Moon's surface at the range ψ from the fixed point on the orbit by means of flights with angular ranges ψ, $\psi + 360°$, $\psi + 720°$, etc. Only the landing trajectories on the first orbit are investigated in this work.

For the solution of the boundary value problem (1)-(5) the Newtonian method was used with the modification, described in [4]. As a result we obtain the optimum control of thrust and orientation angle magnitudes and their corresponding optimum trajectory.

As an example we shall bring forth the initial data for the descent from the orbit H = 200 km, $\psi = 100°$, $\overline{P}_{max} = 0.4$:

$$p_u^0 = -0.289265358, \qquad p_y^0 = 0.283924199,$$
$$p_v^0 = 0.046420083, \qquad p_m^0 = -0.538128174;$$

p_x^0 being determined from the condition $\mathscr{H} = 0$.

The values of parameters were assumed as follows: $g_M = 1.622$ m/sec^2, $R_M = 1737$ km, $c = 3100$ m/sec. The altitude of the initial orbit varied from 15 to 200 km, while the selenocentric angular range of the landing sector varied from 10° to 180°. The dimensionless spacecraft's mass m^1 rises with the increase of the total range (Fig.2), reaching at ψ 30° - 40° a value close to maximum. This value depends little on the altitude of the initial orbit and on spacecraft's thrust load (see solid lines in Fig.3). Therefore, only the results of computations for the initial relative thrust load $A_E = P_{max}/g_E = \overline{P}_{max} = 0.4$ are presented below.

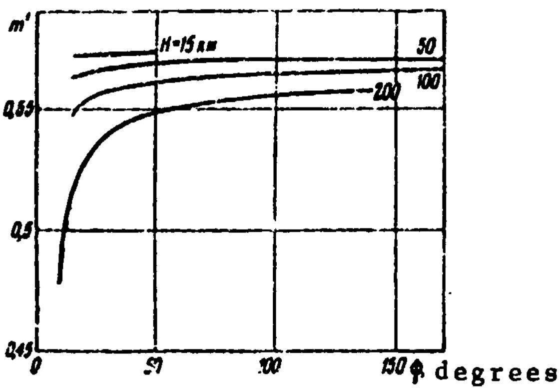

Fig.2

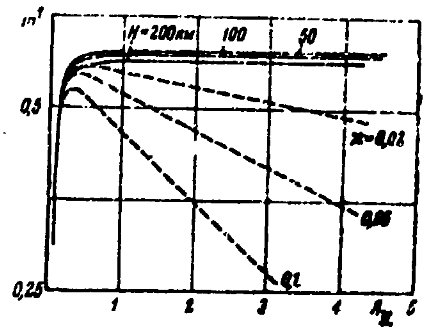

Fig.3

The trajectories shown in Figs.4,5, are typical for smaller and bigger values of ψ. The trajectories are composed of three portions: those of maximum thrust at the beginning and at the end of descent, separated by a passive sector. Such a condition of thrust magnitude control is optimum for the entire investigated re-

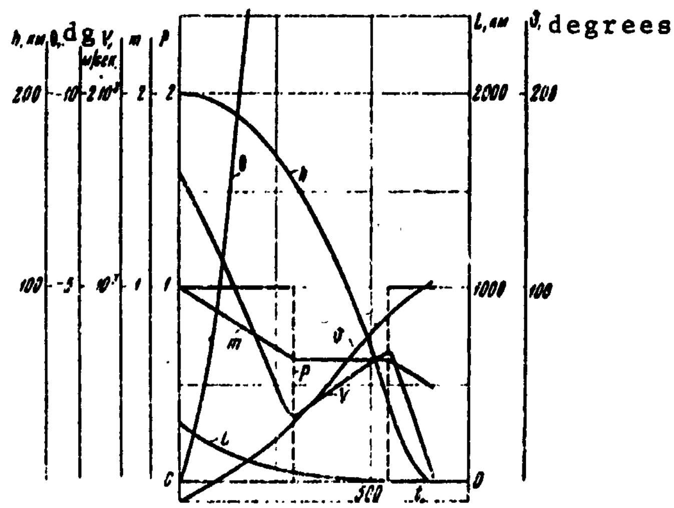

Fig.4

gion. This result agrees well with [5]. For values $\psi > 15°-20°$ the first active sector is small from the standpoint of time (Fig.6) and of the characteristic velocity ΔV_1. In it, the impulse for descent from the orbit is in fact given. The passive flight sector, following it next, compensates the difference in range over trajectories with different ψ. The second active sector, that of deceleration, over which the velocity is damped down to 98%, is fundamental. In the range of small distances there is no such demarcation of active sectors, and they are commensurate with respect to time and characteristic velocity.

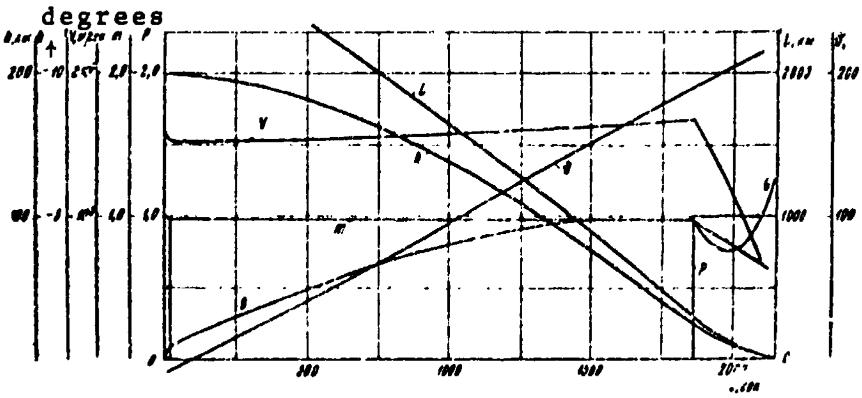

Fig.5

With the increase of ψ (for $\psi > 20°$) the optimum program of thrust ϑ-vector orientation (in the inertial system of coordinates - from the horizon at the point of descent from the orbit), approaches the linear function of t. At the same time, as the total range increases, the value of $\dot{\vartheta}$ decreases and approaches the angular velocity of the AMS at zero height. Shown in Fig.7, is the averaged angular coefficient $\dot{\vartheta}_2$ of the orientation program in the

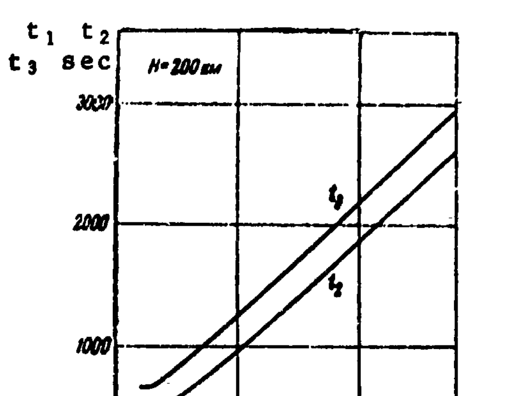

Fig.6

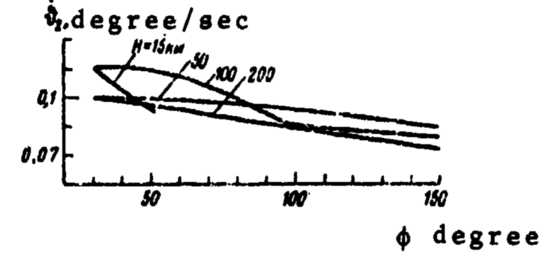

Fig.7

braking sector for those angular distances, where such averaging has sense. Owing to small duration of the descent portion of orbit, its orientation angle varies insignificantly, the thrust being directed almost against the velocity. The averaged orientation angle of the thrust vector is presented in Fig.8. With the decrease of total landing range, the impulse approaches the lateral load. With the increase of initial orbit's height, so does the impulse angle of deflection downward from the transverse direction.

At small values of ψ the orientation program is essentially nonlinear (Fig.4). Such an evolution of the program $\vartheta(t)$ (of orientation) with the variation of total range is explained by the change of type $\vec{p}$-trajectories [5,6]*. For small landing ranges the $\vec{p}$-trajectories are close to elliptical, the angular motion velocity of the imaginary point is not constant. With the increase of ψ the $\vec{p}$-trajectories approach the circular, over which the motion velocity of the imaginary point is constant (Fig.9). Physically the described character of programs' $\vartheta(t)$ variation is explained by the fact that for small angular landing ranges, the gravitational forces for the rotation of the velocity vector are not sufficiently used, and the trajectory distortion is basically achieved with the aid of the thrust.

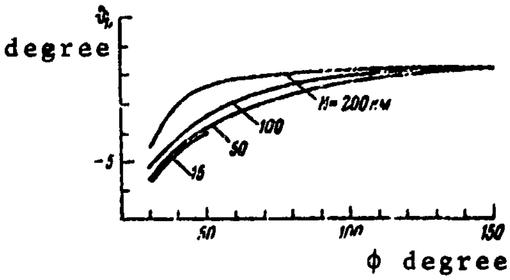

Fig.8

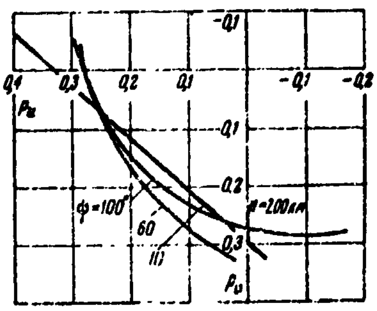

Fig.9

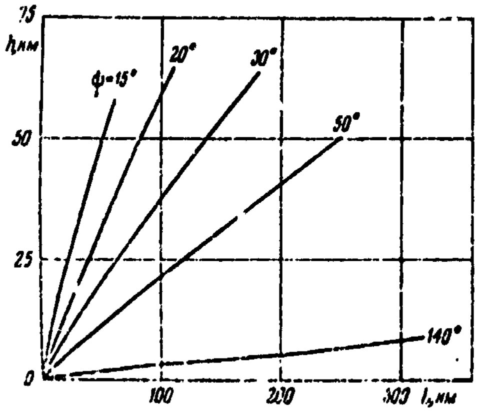

Fig.10

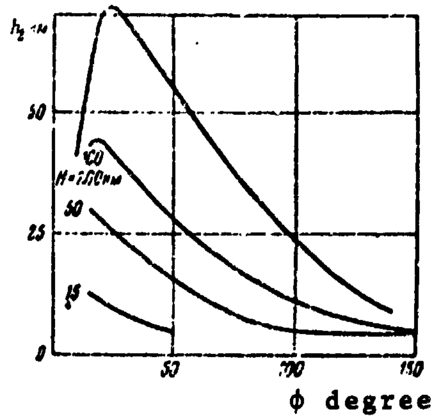

Fig.11

* By $\vec{p}$-trajectories we understand the hodograph of vector $-\vec{p} = -(p_u\vec{i} + p_v\vec{j})$, where $\vec{i},\vec{j}$ are the orts of axes 0_x, 0_y.

Shown in Fig.10 are the trajectories of the braking portion in the plane (h,L). At landing from orbits $H > 100$ km the trajectories are close to spiral (with constant angle ϑ).

In conclusion let us note that the equations of optimum motion (1) are written without taking into account a series of limitations, possible in practice. In a series of cases the obtained values of the functional m^1 cannot be attained in reality. Presented in Fig.11 is the altitude dependence of the beginning of the braking portion on the landing range and orbit height. With great ψ (when $\psi > 100°$) the initial height of the braking sector is of several kilometers.

Trajectories with greater range are inadmissible even from the point of view of safety. The flight's altitude is commensurate with the altitude of lunar mountains over a considerable part of the braking portion. With angular distances of the order of 180° the solution of the variation problem (1)-(5) must be conducted by taking into account the phase limitations, as with the absence of the latter the trajectories pass under the surface of the Moon. However, there is no requirement in the use of trajectories of greater angular range, because the value of the final mass close to maximum is attained starting with $\psi = 40°-50°$.

The dependence $m^1(P_{max})$ obtained in the work (see Fig.3) is universal. As is well known [7,8], it can be used for the selection of motive installation from the condition of payload maximum (in the assumption, that the weight of the engine and the thrust generated by it are linked unambigously. For the linear law $G_{eng} = \kappa P_{max}$ without taking into account the weight of tanks the dependence of relative $m^1_{payload} = m^1 \kappa A_E$ on $A_E = P_{max}/g_E$ is shown in Fig.3 by dashed lines, where it may be seen that it has a well defined extremum.

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