X-641-69-349

PREPRINT

NASA TM X- 63642

MIXING BETWEEN THE STELLAR CORE AND ENVELOPE IN ADVANCED PHASES OF EVOLUTION

DAIICHIRO SUGIMOTO

AUGUST 1969

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

MIXING BETWEEN THE STELLAR CORE AND ENVELOPE IN ADVANCED PHASES OF EVOLUTION

Daiichiro Sugimoto*

Laboratory for Theoretical Studies
NASA-Goddard Space Flight Center
Greenbelt, Maryland

*NRC-NASA Resident Research Associate

ABSTRACT

The evolution of stars of about 30 and 12 solar masses has been studied through the oxygen-burning phase. For the star of 30 solar masses it is found that an extensive surface convection zone invades deep into the stellar core, and the material in the core is mixed into the envelope. When the neutrino loss due to the universal Fermi interaction is not taken into account, the results suggest that the core mass of a pre-supernova will be close to the Chandrasekhar limit. On the other hand, when neutrino loss is included, the mixing after the carbon-burning phase is found to be negligible, as the time scale of evolution is short compared with the time scale of heat transfer even near the bottom of the hydrogen-rich envelope. The evolution of a star of about 6 solar masses has been studied also. If there is neutrino loss, the mixing is found to be strong. Eventually, a pure helium envelope develops because of the co-existence of a deepening surface convection zone and hydrogen-shell burning.

I. INTRODUCTION

The study of stellar evolution in and after the carbon-burning phase has been usually done neglecting the hydrogen-rich envelope. The present status of such studies is summarized in a recent paper by Stothers and Chin (1969). Such studies approximate the evolution of stellar cores with a given core mass. However, Stothers and Chin (1969) have pointed out that extensive mixing between core and envelope occurs in the very advanced phases of evolution because of the extensive surface convection zone. They have studied very general models by assuming constant radiative flux in zones in radiative equilibrium, i.e., mathematically, they have solved the stellar structure only as a boundary-value problem. Such an assumption is valid in cases when the time scale of heat transfer is sufficiently short compared with that of evolution.

When neutrino loss due to the universal Fermi interaction is taken into account, however, the time scale of evolution after carbon burning is shorter than that of heat transfer, even near the bottom of the hydrogen-rich envelope. In order for a mass element in the core to be mixed into the surface convection zone, the value of its entropy should be increased up to the value in the surface convection zone. Thus, it is important to include the equations of energy conservation and of heat transfer. In the present study, we have solved the stellar structure and evolution as a combined initial-boundary value problem by means of a Henyey-type computational method (Sugimoto 1969a).

In the present Note we give only the essential assumptions and results in so far as they are related to the mixing between core and envelope. Other aspects of the evolution, and further discussion will be given in separate papers.

II. HYDROGEN-RICH ENVELOPE AND SURFACE CONVECTION ZONE

In order to simplify the treatment, the hydrogen-rich envelope is replaced by boundary conditions at its innermost layer. In a recent study Arnett (1969) has imposed boundary conditions at a certain point in the carbon-oxygen core to take into account the outer parts of the stars. However, we can formulate the boundary conditions in better approximation at the bottom of the hydrogen-rich envelope, because the core interior to it is of a strongly centrally-condensed type (Hayashi, Hōshi, and Sugimoto 1962; to be referred to as "HHS").

We denote values at the bottom of the hydrogen-rich envelope with the subscript *H*, adding *e* to indicate the exterior side of the chemical composition discontinuity. Other symbols have their usual meanings (see, e.g., HHS) For $U_H \equiv (4\pi r^3\rho/M_r)_H \ll 1$, the following boundary conditions very well take place of an envelope solution (HHS):

$$V_H \equiv \left(\frac{G\, M_r \rho}{r\, P} \right)_{H,e} = (n + 1)_{rad,\, H,\, e}, \tag{1a}$$

$$(n + 1)_{rad,\, H,\, e} \equiv \left(\frac{16\, \pi\, a\, c\, G\, T^4 M_r}{3\, P\, \kappa\, L_r} \right)_{H,\, e}$$

$$= \begin{cases} 4 \text{ (radiative)}, \\ (n + 1)_{ad,\, H,\, e} \text{ (convective)}, \end{cases} \tag{1b}$$

where $(n + 1)_{ad}$ denotes adiabatic polytropic index. In equation (1b), the convective (radiative) boundary condition is used when the surface convection zone does (does not) reach the bottom of the hydrogen-rich envelope.

To find the depth of the surface convection zone, we formulate the so-called "surface condition" of the star in a different way. Following equation (4E26) in HHS, the surface condition can be approximated as

$$L = L_o\, (M/M_\odot)\, (T_{eff}/10^{3.5}\ {}^\circ K)^{-8}\, (E/20)^{-0.1}. \tag{2}$$

HHS gave the values of $\log L_0/L_\odot$ as 3.63 for 15.6 $M_\odot$ (HHS; 7D1), and 3.22 for 4 $M_\odot$ (HHS; 7D5). In an adiabatic envelope the following relation holds between pressure and temperature,

$$P = K\,T^{5/2}\left(1 + \frac{1-\beta}{\beta}\right)\exp\left[\frac{4(1-\beta)}{\beta}\right]. \qquad (3)$$

where β denotes the ratio of gas pressure to total pressure, including non-degenerate gas and radiation. The integration constant $\underline{K}$ is related to the non-dimensional constant $\underline{E}$ by

$$E = 4\pi\, G^{3/2} M^{1/2} R^{3/2}\,(\mu H/k)^{5/2} K\,. \qquad (4)$$

As seen in equation (3), the quantity, $-\ln\,[(\mu H/k)^{5/2}K]$, is entropy per particle in units of the Boltzmann constant $\underline{k}$ (denoted as $\mu\sigma$) plus a constant. For the sake of convenience in computing gravitational energy release (Sugimoto 1969b), the ionic entropy per ion is defined as $k \ln\,(P_i^{\,3/2}/\rho^{5/2})$ + const., where P_i denotes ion pressure and the const. is selected so that the value of the ionic entropy is the same as that of electronic entropy per electron (with quantum statistical zero point) for non-degenerate gas consisting of ^{20}Ne.

Eliminating $\underline{R}$ and T_{eff} from equations (2), and (4), and the relation, $L = \pi a c R^2 T_{eff}^4$, the value of entropy in the convection zone is approximately expressed as

$$\mu\sigma^* = 8.3 + \frac{9}{8} \ln\left(\frac{L}{L_\odot}\right) \quad (5)$$

In equation (5) the additional term involving $\ln(M^{1/8}E^{-77/80})$ + constant has been replaced by a mean value for the relevant range of masses treated in this Note, since we can not solve for the value of E in the framework of the present study. The uncertainty in equation (5) is estimated to be ± 1, which comes from uncertainties in $L_\odot$ and from the terms involving the total mass and E. This uncertainty can be ascribed to an uncertainty in log T_{eff} as small as ∓ 0.2, or to an uncertainty in the value of the core mass of about ± 2 per cent as found after computation of the evolution. Here, it is interesting to note that the nature of the mixing depends only slightly upon the stellar total mass, which may have been reduced as a result of mass loss.

By assuming a value of the core mass and by using boundary conditions (1), we solve for the core structure and obtain the value of the entropy at the bottom of the hydrogen-rich envelope, $(\mu\sigma)_{H,e}$. If this is smaller than $\mu\sigma^*$ in equation (5), the surface convection zone does not reach the bottom of envelope; if larger, a part of the core should have been mixed into the convective envelope, and we then assume a smaller core mass for the next stage of evolution.

III. RESULTS AND DISCUSSIONS

We assume for the initial stage a pure helium core in the helium-burning phase. The products of helium burning were assumed to be half carbon and half oxygen in mass. For the initial core mass we selected 10 $M_\odot$, 3 $M_\odot$, and 1.5 $M_\odot$; these are discussed in the following subsections.

a) Core of 10 $M_\odot$

This core is formed as a result of hydrogen burning of a star of about 30 $M_\odot$ (Stothers 1963). In the phase of contracting carbon-oxygen core, which follows the helium burning phase, the surface convection zone becomes deeper and deeper. Then, the mixing between the core and envelope commences at a stage of the phase of contracting carbon-oxygen core. Core masses found just after each burning phase are as follow: when there is no neutrino loss, 7.4 $M_\odot$ after carbon burning, 6.9 $M_\odot$ after neon burning, and 6.2 $M_\odot$ after oxygen burning. On the other hand, when there is neutrino loss, the core mass found just after the corbon-burning phase is 8.8 $M_\odot$. After that only 0.01 $M_\odot$ is mixed into the lower envelope until the end of the oxygen-burning phase.

In order for a mass element in the core to be mixed into the convective envelope, its entropy must be increased by an amount $\Delta\sigma$. An order of magnitude upper limit estimate of the mass, δM, which can be mixed during a time $\Delta\tau$, is expressed as

$$\delta M \lesssim \frac{\delta L_{ph} \Delta\tau}{(k/\mu H) \, T \, \Delta\sigma} \tag{6}$$

In the above relation δL_{ph} denotes the energy absorbed in the outer part of the core which is in radiative equilibrium, and it is related to the time scale of heat transfer. Stothers and Chin (1969) have discussed the mixing time, i.e., the time scale for establishing constancy of entropy in a convective envelope. However, relation (6) gives a more stringent restriction in limiting mixing.

We consider first the case with neutrino loss. The lifetime from the onset of mixing to the onset of carbon burning is 3.1×10^3 yr, and the lifetime of carbon burning is 3.8×10^2 yr. Thus the core mass is essentially determined at the onset of carbon burning. The lifetime from the exhaustion of carbon through the exhaustion of oxygen is 3×10^1 yr. If we adopt $\Delta\sigma \sim 10$, $T \sim 2 \times 10^7$ $^{\circ}$K, and $\delta L_{ph} \sim 10^5$ $L_{\odot}$, relation (6) gives $\delta M \sim 0.01$ $M_{\odot}$, in agreement with the result of numerical computation. Mixing after the oxygen-burning phase will be even less. The core mass at the pre-supernova stage will be about 8.8 $M_{\odot}$, and the supernova explosion will be triggered by photodisintegration of iron nuclei; this large core mass is one of the main differences between the conclusions of the present Note and those of Stothers and Chin (1969).

On the other hand, when there is no neutrino loss, the lifetime of the carbon-burning phase is as long as 9.7×10^4 yr. and the lifetimes of later phases are of the same order of magnitude. Thus, condition (6) provides no restriction at all, and mixing will proceed as discussed by Stothers and Chin (1969), i.e., the core mass of a pre-supernova will be close to the Chandrasekhar limit, and the products of nuclear burnings will be conveyed outwards to the stellar surface.

b) Core of 3 $M_\odot$

The mass fraction of the helium core left after the hydrogen-burning is about a quarter for a star of mass between 15 $M_\odot$ and 4 $M_\odot$ (see, e.g., HHS). Thus the stellar mass corresponding a core of 3 $M_\odot$ is about 12 $M_\odot$. There is no mixing through the oxygen-burning phase; this result is in essential agreement with that of Stothers and Chin (1969). After the oxygen-burning phase, the same discussion will be applicable as that concerning the core of 10 $M_\odot$.

c) Core of 1.5 $M_\odot$

The evolution of a helium <u>star</u> of 1.5 $M_\odot$ has been computed without neutrino loss. It is found that corbon-burning begins in a non-degenerate carbon oxygen core. Thus, when there is no neutrino loss, the evolution of a core of 1.5 $M_\odot$ will be similar to that of the core of 3 $M_\odot$, described above. No attempt to fit hydrogen-rich envelope by boundary conditions (1) was made for the case without neutrinos.

For the case with neutrino loss we imposed boundary conditions (1). The stellar mass corresponding to a core of 1.5 $M_\odot$ is about 6 $M_\odot$. After the helium has been exhausted in the central region, electrons become degenerate because of cooling by neutrinos (see e.g., Weigert 1966). The central density increases as the core contracts. After the central density has become greater than 4.5×10^6 g cm^{-3}, the surface convection zone invades deeper and deeper into the core. In 1.2×10^4 yr , the core mass has been reduced to 1.07 $M_\odot$. Then two carbon-shell flashes occur successively; similar flashes have been found by Murai, Sugimoto, Hōshi, and Hayashi (1968), and Kutter and Savedoff (1969). Convection develops in the carbon-burning shell, which extends from M_r = 0.55 $M_\odot$ to 1.0 $M_\odot$. The carbon in this convective shell is almost consumed after the two flashes.

The entropy distribution through the core becomes greatly changed by the flashes, and a hydrogen-shell burning becomes active. Simultaneously the surface convection zone becomes deeper and deeper. Thus, helium produced by the hydrogen-shell burning is mixed throughout the envelope, and subsequently the hydrogen in the envelope is uniformly consumed. The core is cooled by neutrino loss and the core becomes smaller and smaller in mass. In 7×10^6 yr hydrogen in the envelope is exhausted, i.e., the stellar envelope is composed of helium. It is to be noticed that the above lifetime of hydrogen-burning in the envelope is comparable to the lifetime of helium burning viz., 3.7×10^6 yr. The core mass at this stage is somewhat uncertain because of uncertainties in equation (5) for such a star; however, its upper limit is estimated to be 0.8 $M_\odot$. Numerical computation has been quitted at this stage of evolution.

Subsequent evolution of the core will be similar to that of Rose (1969), since the difference in mass of the helium envelope is not important for the evolution of such a core. Eventually carbon will begin to burn at the center when the mass of the carbon-oxygen core grows close to the Chandrasekhar limit by the helium-shell burning as studied by Rose (1969) and Paczynski (1969), and carbon detonation will disrupt the star as studied by Arnett (1968, 1969).

Even if an insufficient amount of carbon has been produced as a result of helium burning to result in a carbon-shell flash, hydrogen burning will eventually begin when the core mass is reduced by mixing and the helium envelope will develop. Products of the preceding helium burning as well as products of carbon-shell flashes are processed in the hydrogen-burning shell at temperature of about 1.0×10^8 °K and are conveyed outwards into the envelope. In this connection it is interesting to point out that hydrogen-deficient carbon star R CrB lies near the Cepheid strip in the H-R diagram (Searle 1961), where a star with a helium surface convection zone is expected to lie (Nariai 1969).

The author wishes to thank Drs. R. Stothers, B. Paczynski, and G. S. Kutter for discussion and communications. He is also grateful to Drs. R. C. Cameron, T. Kelsall, and K. Nariai for continuous discussion and encouragement, and to Dr. T. G. Northrop for his stay at the Goddard Space Flight Center as a Resident Research Associate of the National Research Council-National Academy of Sciences. Computation was made with an IBM 360/91 in the Goddard Space Flight Center.

REFERENCES

Arnett, W. D. 1969, Nature, 219, 1344.

Arnett, W. D. 1969 (preprint).

Hayashi, C., Hōshi, R., and Sugimoto, D. 1962, Prog. Theoret. Phys. Suppl. (Kyoto), No. 22, 1 (HHS).

Kutter, G. S., and Savedoff, M. P. 1969, Ap. J., 156, 1021.

Murai, T., Sugimoto, D., Hōshi, R., and Hayashi, C. 1968, Prog. Theoret. Phys. (Kyoto), 39, 619.

Nariai, K. 1969 (private communication).

Paczynski, B. 1969 (private communication).

Rose, W. K. 1969, Ap. J., 155, 491.

Searle, L. 1961, Ap. J., 133, 531.

Stothers, R. 1963, Ap. J., 138, 1074.

Stothers, R., and Chin, C.-w. 1969 (to be published in Ap. J.).

Sugimoto, D. 1969a (submitted to Ap. J.).

Sugimoto, D. 1969b (submitted to Ap. J.).

Weigert, A. 1966, Zs. f. Ap., 64, 395.