CR 73363

AVAILABLE TO THE PUBLIC

A Theory for the Interpretation of Lunar Surface Magnetometer Data

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NAS-2-5153

ABSTRACT

The solution to the problem of the electromagnetic interaction of the moon with spatial irregularities in the interplanetary magnetic field is found. The lunar electrical conductivity is modeled by a two-layer conductivity profile. For the interaction of the moon with the co-rotating sector structure of the interplanetary magnetic field it is found that the magnetic field in the lunar shell is the superposition of an oscillatory uniform field, an oscillatory dipole field and an oscillatory field that is toroidal about the axis of the motional electric field. With various lunar conductivity models and the theory of this paper, lunar surface magnetometer data can be quantitatively interpreted to yield information on the conductivity and consequently the temperature of the lunar core.

INTRODUCTION

The electromagnetic interaction of the moon and the solar wind plasma has been considered by several authors. SONETT and COLBURN (1967, 1968) described the steady state electromagnetic interaction of a moon with electrical conductivity σ moving with velocity $\underline{v}$ relative to a constant magnetic field $\underline{B}$ in terms of an induction generator. For an infinitely conducting plasma, an observer moving with the moon would measure a magnetic field $\underline{B}$ and an electric field $\underline{v}\times\underline{B}$ in the solar wind. The electric field within the homogeneous moon is everywhere the motional field $\underline{v}\times\underline{B}$. Thus currents, whose density is $\sigma\underline{v}\times\underline{B}$, flow in the lunar interior and produce an induced magnetic field which is toroidal with respect to the direction of the motional electric field. Currents at the moon-plasma boundary provide for the completion of closed current paths. HOLLWEG (1968) has applied this steady state solution to a moon with a two-layer electrical conductivity model; i.e., a core of constant electrical conductivity σ_c surrounded by a shell of constant electrical conductivity σ_s. The two-layer conductivity model provides a simple method of accounting for the possibility that the lunar electrical conductivity may change by several orders of magnitude between the moon's surface and its interior (ENGLAND et al, 1968).

The quasi-periodic sector structure of the interplanetary magnetic field necessitates consideration of the time-dependent electromagnetic interaction problem. The relative motion of the moon with respect to spatial irregularities in the incident interplanetary magnetic field produces time-dependent electric and magnetic field fluctuations in the lunar interior. SCHWARTZ and SCHUBERT (1969) have modeled this time-dependent interaction by the relative motion of the moon and a spatially periodic magnetic field $\underline{a}_y\, H_0 \exp(i2\pi z/\lambda)$, where H_0 is the amplitude and λ is the wavelength of the magnetic field oscillation and $\underline{a}_y$ is a unit vector in the y-direction. The moon moves with speed v in the negative z-direction and a coordinate system fixed in the moon is employed. The solar wind plasma is characterized by a magnetic permeability μ and an infinite electrical conductivity, while the moon is assumed to be homogeneous with constant conductivity σ and magnetic permeability μ. In the moving coordinate system, the magnetic and electric fields in the plasma are given by the real parts of

$$\underline{a}_y\, H_0 \exp\left[i\, \frac{2\pi}{\lambda}\, (z-vt)\right] \qquad \text{and} \qquad \underline{a}_x\, \mu v H_0 \exp\left[i\, \frac{2\pi}{\lambda}\, (z-vt)\right]$$

respectively, where $\underline{a}_x$ is a unit vector in the x-direction and z is now a coordinate in the moving reference frame.

The formulation of the electromagnetic boundary value problem requires consideration of both the oscillatory magnetic field and the oscillatory motional electric field. The lunar electromagnetic fields are determined by solving Maxwell's equations subject to the conditions that at the surface of the moon the normal component of the magnetic field and the tangential component of the electric field are continuous.

The time-dependent lunar fields are forced by both the oscillatory magnetic field and the oscillatory motional electric field in the solar wind plasma. In the low frequency limit -- i.e., for length scales of the interplanetary magnetic field irregularities much larger than the moon's radius and for the time scale of the sector structure reversal large compared with the Cowling diffusion time -- the magnetic field fluctuations in the lunar interior are the sum of the variations in the driving interplanetary magnetic field plus induced magnetic field fluctuations whose magnitude is proportional to the product of the lunar magnetic Reynolds number and the magnitude of the interplanetary magnetic field fluctuations. This induced field is toroidal about an axis in the direction of the forcing motional electric field. The low frequency limit of the time-dependent solution is in agreement with the steady state induction generator solution.

In this paper, we present the results of the important extension of the analysis of SCHWARTZ and SCHUBERT (1969) to a radially inhomogeneous moon. We will adopt the two-layer electrical conductivity model as being representative of a lunar conductivity profile that may vary from high values of conductivity in the interior to relatively low values of conductivity near the lunar surface. BLANK and SILL (1969) have described the interaction of a two-layered moon and the rotating sector structure of the interplanetary magnetic field by a model that does not include the time-dependent motional electric field in the solar wind. Instead they have determined the lunar magnetic field by demanding, *a priori*, that the lunar magnetic field be confined to meridional planes passing through an axis parallel to the external magnetic field. This is an incorrect statement of the boundary value problem. The lunar magnetic field is not confined to these meridional planes because of the magnetic field fluctuations forced by the motional electric field. We will show that the induced lunar magnetic field has a toroidal component (about the direction of the motional electric field) that is precisely the induced magnetic field of the steady state induction generator solution (aside from $e^{-i\omega t}$). FULLER and WARD (1969) have considered a multi-layered lunar conductivity profile and have determined the induced lunar dipolar magnetic field and several higher modes of the "magnetic type." In their solution, they also fail to consider the significant induced magnetic field forced by the oscillatory motional electric field.

The major results of the present investigation can be summarized as follows. For the two-layer lunar conductivity model and an infinitely conducting plasma we determine the general solution of the time-dependent electromagnetic boundary value problem. The general solution can be simplified considerably by noting that (1) the wavelength of the quasi-periodic sector structure is much larger than the radius of the moon and (2) the time scale for the field reversals of the corotating sector structure is much larger than the Cowling diffusion time through a moon whose conductivity is everywhere given by the shell conductivity. With these approximations, the magnetic field in the shell is the sum of an oscillating uniform field, parallel to the direction of the interplanetary magnetic field, an oscillating dipole field whose axis is parallel to the interplanetary magnetic field, and an oscillating field which is toroidal about the axis of the motional electric field. In general, all of these contributions are of equal importance. For sufficiently low frequencies, only the uniform field and the toroidal field contribute to the magnetic field in the shell. The DC limit of the solution is in agreement with the induction generator solution for the steady state interaction of a moon moving with constant speed through a uniform magnetic field.

The solution provides us with sufficient physical insight to begin an interpretation of lunar surface magnetometer data. To lowest order in the approximations considered here, we may also construct lunar conductivity models for quantitative comparison with the measured power spectra. Such conductivity models will provide information on the temperature of the lunar interior.

THEORETICAL DESCRIPTION

We generalize the model of SCHWARTZ and SCHUBERT (1969), discussed in the introduction, to account for a radially inhomogeneous lunar conductivity profile. The magnetic and electric fields in the plasma are

$$\underline{a}_y \, H_o \exp\left[i\left(\frac{2\pi z}{\lambda} - \omega t\right)\right] \qquad \text{and} \qquad \underline{a}_x \, \mu v H_o \exp\left[i\left(\frac{2\pi z}{\lambda} - \omega t\right)\right],$$

respectively, where $\mu v H_o = E_m$ is the amplitude of the motional electric field oscillation and $\omega = 2\pi v/\lambda$. The two-layer lunar electrical conductivity profile consists of a core of radius R_c with constant conductivity σ_c surrounded by a shell of thickness $a-R_c$ with constant conductivity σ_s (Figure 1). The electromagnetic field in the plasma forces the response $\underline{E}_s \, e^{-i\omega t}$, $\underline{H}_s \, e^{-i\omega t}$ in the shell and $\underline{E}_c \, e^{-i\omega t}$, $\underline{H}_c \, e^{-i\omega t}$ in the core. The magnetic field in the lunar interior satisfies the vector Helmholtz equation

$$\nabla^2 \underline{H}_{s,c} + k^2_{s,c} \underline{H}_{s,c} = 0 \qquad (1)$$

where ∇^2 is the Laplacian operator,

$$k^2_{s,c} = \mu\epsilon \, \omega^2 + i\sigma_{s,c}\mu\omega \qquad (2)$$

and ϵ is the lunar permittivity. The electric fields $\underline{E}_{s,c}$ are determined from Ampere's law. The boundary conditions are the continuity of the tangential component of the electric field at $r = a$ and $r = R_c$ (r, θ, ϕ are spherical polar coordinates) and the continuity of the tangential component of the magnetic field at $r = R_c$. This model approximates the current layer in the plasma by a sheet of surface current whose strength is determined by the magnitude of the discontinuity in the tangential component of the magnetic field at $r = a$. Discussions of the current sheet approximation have been given by JOHNSON and MIDGLEY (1968) and BLANK and SILL (1969).

The solution to this problem is easily obtained with the aid of STRATTON (1941). The spherical harmonic expansions of the forcing fields are

$$\underline{H} = - H_o e^{-i\omega t} \sum_{n=1}^{\infty} \beta_n (\underline{m}_{e1n}^{(1)} + i\ \underline{n}_{o1n}^{(1)}) \tag{3}$$

$$\underline{E} = E_m e^{-i\omega t} \sum_{n=1}^{\infty} \beta_n (\underline{m}_{o1n}^{(1)} - i\ \underline{n}_{e1n}^{(1)}) \tag{4}$$

where

$$\beta_n = \frac{i^n(2n+1)}{n(n+1)} \tag{5}$$

$$\underline{m}^{(1)}_{\substack{o\\e}1_n} = j_n(kr)\left\{\pm \frac{P^1_n(\cos\theta)}{\sin\theta} \begin{matrix}\cos\\ \sin\end{matrix}\,\varphi\,\hat{\underline{\theta}} - \frac{dP^1_n(\cos\theta)}{d\theta} \begin{matrix}\sin\\ \cos\end{matrix}\,\varphi\,\hat{\underline{\varphi}}\right\} \qquad (6)$$

$$\underline{n}^{(1)}_{\substack{o\\e}1_n} = \frac{n(n+1)}{kr} j_n(kr) P^1_n(\cos\theta) \begin{matrix}\sin\\ \cos\end{matrix}\,\varphi\,\hat{\underline{r}} +$$

$$\frac{1}{kr}\frac{d}{dr}\left\{r j_n\left(\frac{2\pi r}{\lambda}\right)\right\}\left\{\frac{d}{d\theta} P^1_n(\cos\theta) \begin{matrix}\sin\\ \cos\end{matrix}\,\varphi\,\hat{\underline{\theta}} \pm \frac{P^1_n(\cos\theta)}{\sin\theta} \begin{matrix}\cos\\ \sin\end{matrix}\,\varphi\,\hat{\underline{\varphi}}\right\} \qquad (7)$$

where $k = 2\pi/\lambda = \omega/v$. The spherical Bessel functions j_n and the associated Legendre functions $P^1_n(\cos\theta)$ are defined as in STRATTON (1941). The vectors $\hat{r}, \hat{\theta}, \hat{\varphi}$ are the orthogonal unit vectors of the spherical polar coordinate system.

The electric and magnetic fields inside the core are

$$\underline{E}_c = E_m \sum_{n=1}^{\infty} \beta_n \left(a^c_n\, \underline{m}^{(1)c}_{o1n} - i\, b^c_n\, \underline{n}^{(1)c}_{e1n}\right) \qquad (8)$$

$$\underline{H}_c = \frac{v k_c H_O}{\omega} \sum_{n=1}^{\infty} \beta_n \left(b^c_n\, \underline{m}^{(1)c}_{e1n} + i\, a^c_n\, \underline{n}^{(1)c}_{o1n}\right) \qquad (9)$$

where $\underline{m}^{(1)c}_{\substack{o\\e}1_n}$ and $\underline{n}^{(1)c}_{\substack{o\\e}1n}$ are obtained from Eqs. (6) and

(7) by replacing k with k_c. Similarly, within the shell the fields are

$$\underline{E}_s = E_m \sum_{n=1}^{\infty} \beta_n \left(a_n^{(1)s} \underline{m}_{01n}^{(1)s} - i\, b_n^{(1)s} \underline{n}_{e1n}^{(1)s} + a_n^{(3)s} \underline{m}_{01n}^{(3)s} - i\, b_n^{(3)s} \underline{n}_{e1n}^{(3)s} \right) \tag{10}$$

$$\underline{H}_s = - \frac{v k_s H_0}{\omega} \sum_{n=1}^{\infty} \beta_n \left(b_n^{(1)s} \underline{m}_{e1n}^{(1)s} + i\, a_n^{(1)s} \underline{n}_{01n}^{(1)s} + b_n^{(3)s} \underline{m}_{e1n}^{(3)s} + i a_n^{(3)s} \underline{n}_{01n}^{(3)s} \right) \tag{11}$$

where $\underline{m}_{\substack{0\\e}1n}^{(1)s}$ and $\underline{n}_{\substack{0\\e}1n}^{(1)s}$ are obtained from equations (6) and (7) by replacing k with k_s and $\underline{m}_{\substack{0\\e}1n}^{(3)s}$ and $\underline{n}_{01n}^{(3)s}$ are obtained from equations (6) and (7) by substituting k_s for k and also using the spherical Bessel function $h_n^{(1)}$ in place of j_n. The values of the coefficients a_n^c, b_n^c, $a_n^{(1)s}$, $a_n^{(3)s}$ and $b_n^{(3)s}$ are given in Appendix A.

DISCUSSION OF THE SOLUTION

To interpret the data from lunar surface magnetometer experiments we must compute the magnetic field at $r = a$ as a function of ω for particular lunar conductivity models and compare these computations with the measured power spectra. The essential dimensionless parameters on which the solution depends are $ka = 2\pi a/\lambda$, $k_s a$ and $k_c R_c$. For the interaction of the moon with the rotating sector structure of the interplanetary magnetic field ka is $0(10^{-4})$. Thus we will consider the simplification $ka << 1$. For the physical pheonomenon of interest it is appropriate to approximate $k_{s,c}^2$ with $i\,\sigma_{s,c}\mu\omega$. This is consistent with the low frequency approximation $ka << 1$. The absence of a lunar bow shock places an upper limit of 10^{-5} mhos/m on the electrical conductivity of the shell. Thus $\left| k_s^2\, a^2 \right| << 1$ for $\omega << .025\ \text{sec}^{-1}$. Since ω is $0(10^{-5}\text{sec}^{-1})$ for the problem of the interaction of the moon and the rotating sector structure of the interplanetary magnetic field and since the power density spectrum of the interplanetary magnetic field (COLEMAN, 1968) contains significant power at the frequencies associated with field reversals caused by the corotation of the field's sector structure, the frequency range $\omega << .025\ \text{sec}^{-1}$ represents an appropriate one for investigation. We proceed to determine the form of the general solution under the conditions $2\pi a/\lambda \ll 1$ and $|k_s a| \ll 1$. Since $R_c \leq a$, with unity $|k_s R_c|$ is also small compared with unity.

Since we are mainly interested in the magnetic field at r=a, we compute the coefficients $a_n^{(1)s}$, $a_n^{(3)s}$, $b_n^{(1)s}$ and $b_n^{(3)s}$ under the approximations $2\pi a/\lambda \ll 1$ and $|k_s a| \ll 1$ and find

$$a_n^{(1)s} \sim \frac{(ka)^n (k_s a)^{-n}}{\left\{1 - \left(\frac{R_c}{a}\right)^{2n+1} B_n(k_c R_c)\right\}} , \tag{12}$$

$$a_n^{(3)s} \sim \frac{-i}{(2n+1)} \frac{1}{\left\{(2n-1)!!\right\}^2} \frac{(ka)^n (k_s a)^{n+1} \left(\frac{R_c}{a}\right)^{2n+1} B_n(k_c R_c)}{\left\{1 - \left(\frac{R_c}{a}\right)^{2n+1} B_n(k_c R_c)\right\}} , \tag{13}$$

$$b_n^{(1)s} \frac{(ka)^{n-1} (k_s a)^{-n+1}}{\left\{1 - \left(\frac{R_c}{a}\right)^{2n+1}\right\}} , \tag{14}$$

$$b_n^{(3)s} \sim \frac{i(n+1)}{n(2n+1)} \frac{(ka)^{n-1}(k_s a)^{n+2}}{\left\{(2n-1)!!\right\}^2} \frac{\left(\frac{R_c}{a}\right)^{2n+1}}{\left\{1 - \left(\frac{R_c}{a}\right)^{2n+1}\right\}} , \quad (15)$$

where

$$B_n(k_c R_c) = \frac{p_n(k_c R_c) - (n+1) j_n(k_c R_c)}{p_n(k_c R_c) + n\, j_n(k_c R_c)} , \quad (16)$$

and

$$(2n-1)!! = (2n-1)(2n-3)(2n-5).....1 , \quad (17)$$

We must also determine the functions $\underline{m}_{\substack{0\\e}1n}^{(1)s}$, $\underline{m}_{\substack{0\\e}1n}^{(3)s}$, $\underline{n}_{\substack{0\\e}1n}^{(1)s}$ and $\underline{n}_{\substack{0\\e}1n}^{(3)s}$ for $\frac{2\pi a}{\lambda} << 1$ and $|k_s a| << 1$.

It will be sufficient for our purpose to note that

$$\underline{m}_{\substack{0\\e}1n}^{(1)s} = O\left((k_s a)^n\right) , \quad (18)$$

$$\underline{m}_{\substack{0\\e}1n}^{(3)s} = O\left((k_s a)^{-n-1}\right) , \quad (19)$$

$$\underline{n}_{\substack{0 \\ e}\,1n}^{(1)s} = 0(\quad (k_s a)^{n-1}) \quad , \quad (20)$$

$$\underline{n}_{\substack{0 \\ e}\,1n}^{(3)s} = 0(\quad (k_s a)^{-n-2}) \quad , \quad (21)$$

Consider the order of magnitude of each of the terms in the expressions for $\underline{H}_s$

$$\frac{k_s}{k} b_n^{(1)s} \underline{m}_{e1n}^{(1)s} = 0\left\{ (ka)^{n-2} \ (k_s a)^2 \right\} \quad , \quad (22)$$

$$\frac{k_s}{k} a_n^{(1)s} \underline{n}_{01n}^{(1)s} = 0\left\{ (ka)^{n-1} \right\} \quad , \quad (23$$

$$\frac{k_s}{k} b_n^{(3)s} \underline{m}_{e1n}^{(3)s} = 0 \left\{ (ka)^{n-2} \ (k_s a)^2 \right\} \quad , \quad (24)$$

$$\frac{k_s}{k} a_n^{(3)s} \underline{n}_{01n}^{(3)s} = 0 \left\{ (ks)^{n-1} \right\} \quad , \quad (25)$$

To the lowest order of approximation only the n=1 terms need be considered. Two of these terms are

$O(1)$, while the remaining two are $O\left\{(k_s a)^2 (\frac{2\pi a}{\lambda})^{-1}\right\}$. The quantity $(k_s a)^2 (\frac{2\pi a}{\lambda})^{-1} = i(\sigma_s \mu v a)$, is (aside from i) the lunar magnetic Reynolds number based on the conductivity of the shell and the moon's radius. Alternatively, one may consider $\sigma_s \mu v a$ as the ratio of the Cowling diffusion time for a body of dimension a and conductivity σ_s to the time required to move the distance a at speed v. Since the absence of a lunar bow shock requires only that $\sigma_s < 10^{-5}$ mhos/m, the lunar magnetic Reynolds number is an $O(1)$ quantity, i.e. $(k_s a)^2 (\frac{2\pi a}{\lambda})^{-1}$ is $O(1)$ and consequently all the n=1 terms in the expression for $\underline{H}_s$ are of the same order. Thus we find

$$\frac{\underline{H}_s}{H_0} = \underline{a}_y \left\{1-\left(\frac{R_c}{a}\right)^3 B_1(k_c R_c)\right\}^{-1} \left\{1-\left(\frac{R_c}{r}\right)^3 B_1(k_c R_c)\right\}$$
$$+ \frac{3}{2}(\cos\theta \sin\varphi\, \hat{\underline{\theta}} + \cos\varphi\, \hat{\underline{\varphi}}) \left(\frac{R_c}{r}\right)^3 B_1(k_c R_c) \left\{1-\left(\frac{R_c}{a}\right)^3 B_1(k_c R_c)\right\}^{-1}$$
$$- (\sigma_s \mu v R_c)(\sin\varphi\, \hat{\underline{\theta}} + \cos\theta \cos\varphi\, \hat{\underline{\varphi}}) \left\{1-\left(\frac{R_c}{a}\right)^3\right\}^{-1} \left\{\frac{r}{2R_c} + \left(\frac{R_c}{r}\right)^2\right\} \quad (26)$$

where

$$B_1(k_c R_c) = 1 - \frac{3}{k_c^2 R_c^2} + \frac{3\cot k_c R_c}{k_c R_c} \quad (27)$$

Equation (26) is the major result of this paper and its physical content is readily obtained. First we note that if r, a, ψ are spherical polar co-ordinates with the y-axis the polar axis, then

$$\cos\theta \sin\varphi \, \hat{\underline{\theta}} + \cos\varphi \, \hat{\underline{\varphi}} = - \sin\alpha \, \hat{\underline{\alpha}} \tag{28}$$

If r, γ, η are spherical polar coordinates with the x-axis the polar axis then

$$\sin\varphi \, \hat{\underline{\theta}} + \cos\theta\cos\varphi \, \hat{\underline{\varphi}} = - \sin\gamma \, \hat{\underline{\eta}} \tag{29}$$

The expression for the magnetic field in the lunar shell is

$$\frac{\underline{H}_s}{H_0} = \underline{a}_y \frac{\left\{1-(\frac{R_c}{r})^3 B_1(k_c R_c)\right\}}{\left\{1-(\frac{R_c}{a})^3 B_1(k_c R_c)\right\}} - \frac{3}{2}\sin\alpha \, \hat{\underline{\alpha}} \frac{(\frac{R_c}{r})^3 B_1(k_c R_c)}{\left\{1-(\frac{R_c}{a})^3 B_1(k_c R_c)\right\}} + (\sigma_s \mu v R_c)\sin\gamma \, \hat{\underline{\eta}} \frac{\left\{\frac{r}{2R_c} + (\frac{R_c}{r})^2\right\}}{\left\{1-(\frac{R_c}{a})^3\right\}} \tag{30}$$

The term proportional to the magnetic Reynolds number $\sigma_s \mu v R_c$ represents a magnetic field which is toroidal about an axis in the direction of the motional electric field. This toroidal magnetic field is identical (aside from the factor $e^{-i\omega t}$) to the field of the steady state induction generator in the approximation $\sigma_s/\sigma_c \ll 1$ (see Appendix B). The contribution of the toroidal field to the total magnetic field in the shell is, to lowest order, independent of the core conductivity and the frequency (aside from $e^{-i\omega t}$). The remaining terms in the expression for $\underline{H}_s/H_0$ represent the superposition of a uniform field, in the direction of the interplanetary field, and a dipole field whose axis is parallel to the interplanetary field direction. These terms agree with the result of BLANK and SILL (1969), who pointed out that the factor $\left\{1-(\frac{R_c}{a})^3 B_1(k_c R_c)\right\}^{-1}$ represents a volume compression of the field which cannot leak into the solar wind plasma. These authors, however, failed to consider the toroidal magnetic field contribution to the total field in the lunar shell. We note that the uniform and dipolar fields are, to lowest order, independent of the shell conductivity. The real and imaginary parts of the function $B_1(k_c R_c)$ are shown in Fig. (2).

At the lunar surface the magnetic field is

$$\frac{\underline{H}_s}{H_0}(r=a) = \underline{a}_y + (\sigma_s \mu v a) \sin\gamma \, \hat{\underline{\eta}} \, \frac{(\frac{1}{2} + \frac{R_c^3}{a^c})}{(1 - \frac{R_c^3}{a^3})}$$

$$- \frac{3}{2} \sin\alpha \, \hat{\underline{\alpha}} \frac{\left(\frac{R_c}{a}\right)^3 B_1(k_c R_c)}{\left\{1 - \left(\frac{R_c}{a}\right)^3 B_1(k_c R_c)\right\}} \tag{31}$$

If the core is sufficiently conducting that $|k_c R_c| \gg 1$ for $\omega = 0(10^{-5}\text{sec}^{-1})$ and $R_c = 0(10^6\text{m})$, $\sigma_c \gg 10^{-1}\text{mhos/m})$, $B_1(k_c R_c) \approx 1$ and the surface magnetic field becomes

$$\frac{\underline{H}_s}{H_0}(r=a) \xrightarrow[|k_c R_c| \to \infty]{} \underline{a}_y + (\sigma_s \mu v a) \sin\gamma \, \hat{\underline{\eta}} \frac{\left(\frac{1}{2} + \frac{R_c^3}{a^3}\right)}{\left(1 - \frac{R_c^3}{a^3}\right)} \tag{32}$$

$$- \frac{3}{2} \sin\alpha \, \hat{\underline{\alpha}} \frac{(R_c/a)^3}{\left\{1 - (R_c/a)^3\right\}}$$

a result dependent only on the magnetic Reynolds number based on the shell conductivity and the fraction of the lunar radius occupied by core material. For frequencies sufficiently small that $k_c R_c \ll 1$, or for a core conductivity sufficiently small that $k_c R_c \ll 1$ (for $\omega = 0(10^{-5}\text{sec}^{-1})$ and $R_c = 0(10^6\text{m})$, $\sigma_c \ll 10^{-1}\text{mhos/m})$, $B_1(k_c R_c) \approx 0$ and the surface magnetic field becomes

$$\frac{\underline{H}_s}{H_0}(r=a) \xrightarrow[|k_c R_c| \to 0]{} \underline{a}_y + (\sigma_s \mu v a) \sin\gamma \, \hat{\underline{\eta}} \frac{\left(\frac{1}{2} + \frac{R_c^3}{a^3}\right)}{\left(1 - \frac{R_c^3}{a^3}\right)} \tag{33}$$

CONCLUDING REMARKS

The power spectra from lunar surface magnetometer experiments can be quantitatively evaluated using equation (31) in conjunction with various models of the lunar conductivity. The parameters required by the theory are, in addition to knowledge of the interplanetary magnetic field and the solar wind velocity, the magnetic Reynolds number $\sigma_s \mu va$, R_c/a and $\sigma_c \mu R_c^2$. With enough data, the possible values of these parameters should be sufficiently constrained to yield information on σ_c and consequently the temperature of the lunar interior. For the frequencies of interest and a low conductivity core $\sigma_c << 10^{-1}$ mhos/m we note that there is no induced dipolar magnetic field (equation (33)). The existence of a measurable induced dipolar field at the lunar surface, for the frequencies of interest, would rule out the possibility of a cold moon.

ACKNOWLEDGEMENT

The authors are grateful to Dr. C. P. Sonett for stimulating this investigation. We also thank Drs. C. P. Sonett, D. S. Colburn and E. J. Putzer for helpful discussions.

Support for this work was provided under Contract No. NAS2-5153. The paper is based on NASA CR7-

prepared for NASA-Ames Research Center.

APPENDIX A

We list here the values of the coefficients a_n^c, b_n^c, $a_n^{(1)s}$, $b_n^{(1)s}$, $a_n^{(3)s}$ and $b_n^{(3)s}$.

$$a_n^c = j_n(ka)\ \Delta_a^{-1} \left\{ j_n(k_sR_c)q_n(k_sR_c) - h_n^{(1)}(k_sR_c)p_n(k_sR_c) \right\}$$

$$b_n^c = \frac{k_s p_n(\frac{2\pi a}{\lambda})}{k\Delta_b} \left\{ p_n(k_sR_c)h_n^{(1)}(k_sR_c) - q_n(k_sR_c)j_n(k_sR_c) \right\}$$

$$a_n^{(1)s} = \frac{j_n(ka)}{j_n(k_sa) + R_1 h_n^{(1)}(k_sa)}$$

$$b_n^{(1)s} = \frac{k_s p_n(ka)}{k\Delta_{b'}} \left\{ k_s^2\ h_n^{(1)}(k_sR_c)p_n(k_cR_c) - k_c^2 j_n(k_cR_c)q_n(k_sR_c) \right\}$$

$$a_n^{(3)s} = \frac{R_1\ j_n(ka)}{R_1\ h_n^{(1)}(k_sa) + j_n(k_sa)}$$

$$b_n^{(3)s} = \frac{k_s p_n(ka)}{k\Delta_{b'}} \left\{ k_c^2 j_n(k_cR_c)p_n(k_sR_c) - k_s^2 j_n(k_sR_c)p_n(k_cR_c) \right\}$$

The following quantities have been used in the above formulas for the coefficients.

APPENDIX A (Con't)

$$p_n(x) = \frac{d}{dx}\left(xj_n(x)\right) \quad , \quad q_n(x) = \frac{d}{dx}\left(xh_n^{(1)}(x)\right)$$

$$\Delta_{b'} = k_c k_s \Delta_b = p_n(k_s a)\left\{k_s^2\, h_n^{(1)}(k_s R_c) p_n(k_c R_c) - k_c^2 j_n(k_c R_c) q_n(k_s R_c)\right\}$$

$$+ q_n(k_s a)\left\{k_c^2 j_n(k_c R_c) p_n(k_s R_c) - k_s^2 j_n(k_s R_c) p_n(k_c R_c)\right\} .$$

$$R_1 = \frac{j_n(k_s R_c) p_n(k_c R_c) - j_n(k_c R_c) p_n(k_s R_c)}{j_n(k_c R_c) q_n(k_s R_c) - h_n^{(1)}(k_s R_c) p_n(k_c R_c)} \quad .$$

$$\Delta_a = j_n(k_s a)\left\{j_n(k_c R_c) q_n(k_s R_c) - h_n^{(1)}(k_s R_c) p_n(k_c R_c)\right\}$$

$$+ h_n^{(1)}(k_s a)\left\{j_n(k_s R_c) p_n(k_c R_c) - j_n(k_c R_c) p_n(k_s R_c)\right\} \quad .$$

APPENDIX B

The DC limit of the general solution, i.e. the limit $\omega \to 0$, is obtained by considering $k_s^2 a^2$, $k_c^2 R_c^2$ to be $0(\frac{2\pi a}{\lambda})$ as $\frac{2\pi a}{\lambda} \to 0$. The result for the magnetic field in the lunar shell is

$$\frac{\underline{H}_s}{H_0} = \underline{a}_y + \frac{(\sigma_s \mu v a) \sin\gamma \; \hat{\underline{\eta}} \left\{ \frac{r}{2R_c} \left(2 + \frac{\sigma_c}{\sigma_s}\right) - \frac{R_c^2}{r^2} \left(1 - \frac{\sigma_c}{\sigma_s}\right) \right\}}{\frac{a}{R_c} \left(2 + \frac{\sigma_c}{\sigma_s}\right) + \frac{R_c^2}{a^2} \left(1 - \frac{\sigma_c}{\sigma_s}\right)}$$

If one considers $\sigma_s/\sigma_c << 1$, the toroidal field is precisely the toroidal field of the time dependent interaction (aside from $e^{-i\omega t}$). The result should be compared with the toroidal term on the right hand side of Equation (30).

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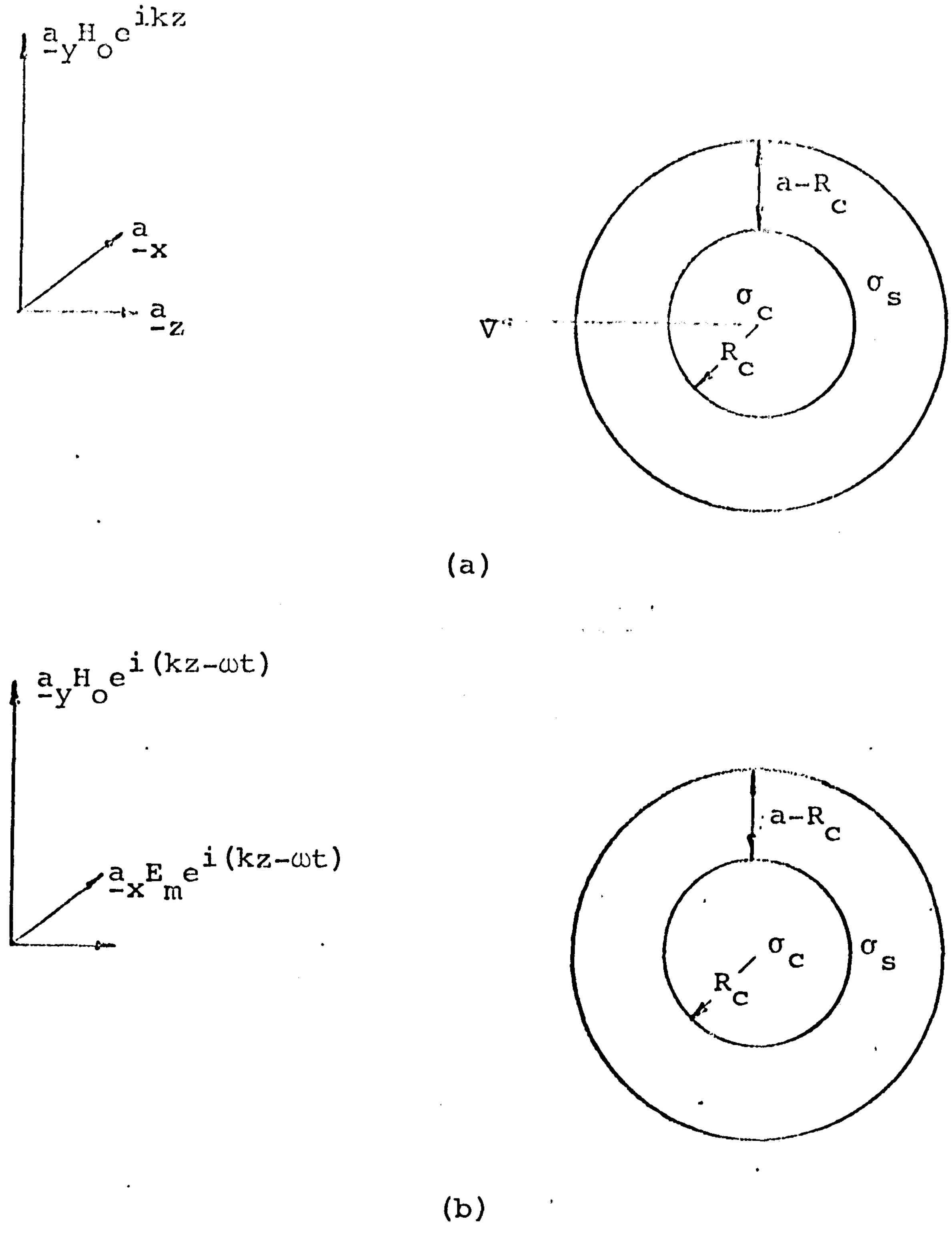
FIGURE CAPTIONS

Fig. 1. (a) Moon with two-layer conductivity profile moving with speed v relative to a spatially periodic magnetic field and (b) The relative motion viewed by an observer co-moving with the moon. A travelling electromagnetic wave appears incident on the moon.

Fig. 2. The real and imaginary parts of B_1 as a function of

$$\left(\frac{\sigma_c \mu \omega R_c^2}{2}\right)^{1/2}$$



$a_y H_o e^{ikz}$
a_x
a_z
$a-R_c$
σ_c
σ_s
R_c
V
(a)
$a_y H_o e^{i(kz-\omega t)}$
$a_x E_m e^{i(kz-\omega t)}$
$a-R_c$
σ_c
σ_s
R_c
(b)

FIGURE 1

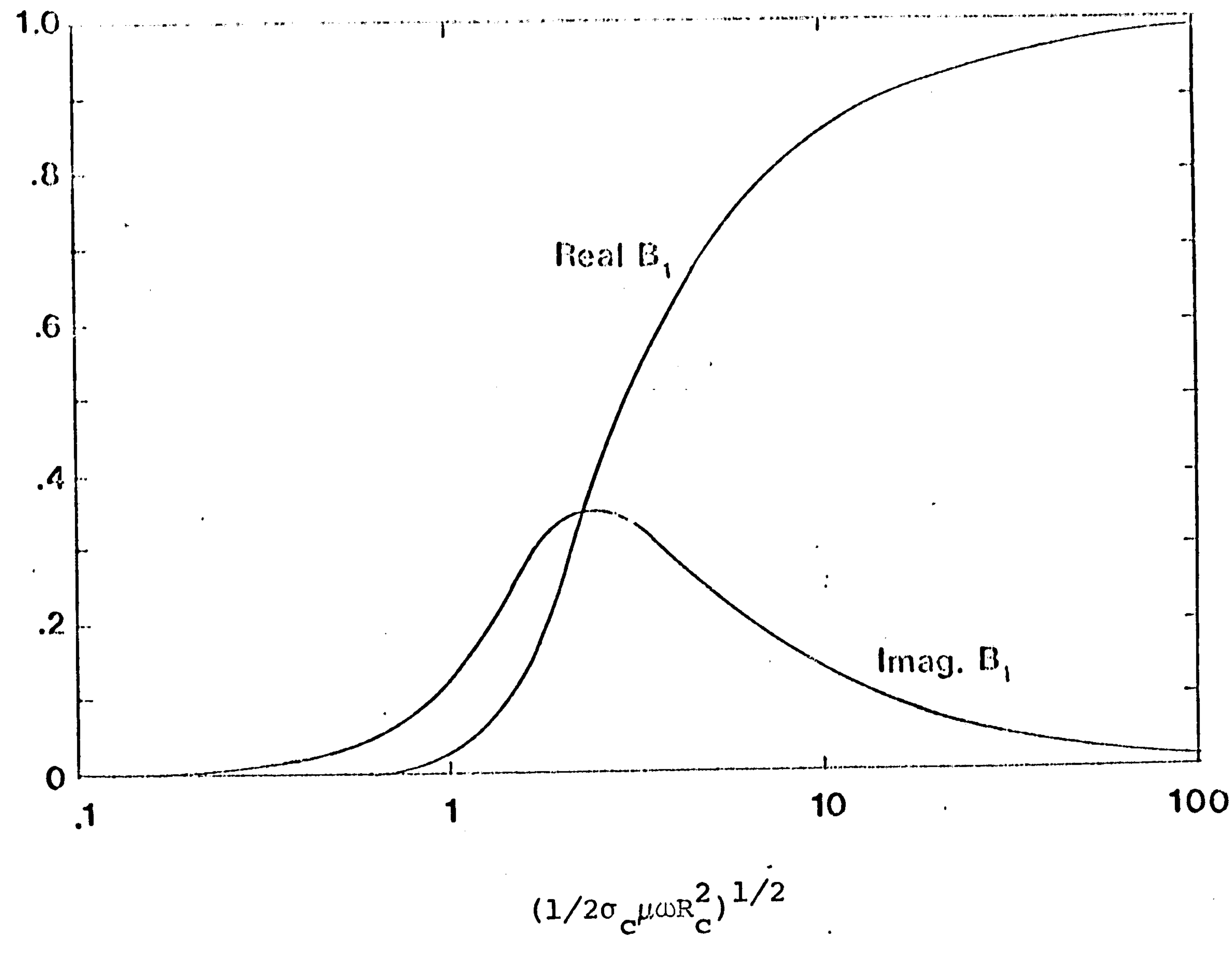


Figure 2