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PREPRINT

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N69 36263

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JULY 1969

GSFC

GODDARD SPACE FLIGHT CENTER

GREENBELT, MARYLAND

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July 1969

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ABSTRACT

The fluctuation in the angular velocity of the present rotation of Mercury is investigated. The instantaneous rotational rate in terms of orbital mean motion at different positions along Mercury's orbit is given. At aphelion the rotational velocity decreases substantially because the solar gravitational torque on the two thermal bulges on Mercury's surface tends to retard the rotation of Mercury before aphelion passage. It is found that the difference between the rotational periods derived from the motions at perihelion and aphelion is 4.68 minutes and that the maximum rate of rotation occurs at $f = \pi/4$ and $f = 7\pi/4$ where f is the true anomaly.

INSTANTANEOUS ROTATIONAL VELOCITY OF MERCURY

I. INTRODUCTION

In the previous paper (Liu, 1969) Liu has suggested that the solar thermal, tidal and gravitational effect could account for Mercury's rotation being locked into a 3:2 resonance with its orbital revolution. Because of this resonance lock the instantaneous rotational period depends strongly on the orientation and position of the planet relative to the Sun. For correct interpretation of the dynamic behavior of such a lock-in motion, a precise knowledge about the velocity of rotation is of special importance. The present paper is devoted to studying the fluctuation of Mercury's rotation in terms of orbital mean motion at different positions along its orbit. The calculation is confined to the planar motion.

II. METHODS OF COMPUTATION

The rotation of Mercury about its polar axis has the form

$$\Phi = f + \phi \tag{1}$$

where f is the true anomaly and ϕ, denoting the angle between the axis of the two thermal bulges with the radius vector, is the solution of the following differential equation (Liu and O'Keefe, 1965)

$$\frac{d^2\phi}{df^2} - \frac{2 e \sin f}{1 + e \cos f}\left(\frac{d\phi}{df} + 1\right) + \frac{3(B - A)}{C(1 + e \cos f)} \cos \phi \sin \phi = 0 \tag{2}$$

where (B-A)/C is the present value of the difference in Mercury's equatorial moments of inertia and e is the orbital eccentricity. At the present time, the influence of the solar thermal and tidal effect on Mercury's rotation is in equilibrium. Therefore, the use of equation (2) for orientation determination is justified.

To investigate the fluctuations in the angular velocity of rotation, it is necessary to transform the variable of function Φ from the true anomaly f to the mean anomaly M. This can be done from Kepler's equation

$$M = n(t - t_0) = E - e \sin E \tag{3}$$

where $t_1 = t_0$ corresponds to the time at perihelion, $n = 2\pi/p$ is the mean motion, p = orbital period and E is the eccentric anomaly which is related to

$$\tan\frac{E}{2} = \left(\frac{1-e}{1+e}\right)^{1/2} \tan\frac{f}{2} \tag{4}$$

We can solve equations (2), (3) and (4) on a computer by applying the Runge-Kutta integration algorithm. The computer program performs the algorithms for successively updating expansions. The over-all precision is consistent with the 10^{-7} truncation level.

III. RESULTS

Results of $d\Phi/dt = \Omega = (3/2 + \delta)\, n$, where δ is the coefficient of fluctuation, are generated for $(B-A)/C = 5 \times 10^{-5}$. The intial conditions are chosen such that the angle between the axis of the two thermal bulges and the radius vector at $f = 0$ is $0.6°$ and the averaged value of $\Omega - 3/2n$ over the period of revolution vanishes. For the orbital eccentricity, the value $e = 0.206$ is adopted. The results of the instantaneous rotational velocity in terms of orbital mean motion n at different positions along Mercury's orbit are given in Table 1. It is found that

$$\Omega - \frac{3}{2} n \geq 0$$

when $|f| \leq 115°$ and

$$\Omega - \frac{3}{2} n \leq 0$$

when $|f| \geq 115°$.

The minimum rate of rotation occurs at aphelion because the solar gravitational torque which exerts a couple on the two thermal bulges on Mercury's surface tends to retard the rotation of Mercury before aphelion passage. During perihelion passage Mercury's rotation increases substantially under the effect of the Sun's attraction. The maximum rate of rotation occurs at $f = 45°$ and $f = 315°$. The dynamic fluctuation in the velocity of rotation is illustrated in Figure 1.

IV. CONCLUSION

The fluctuation in the rotation of Mercury is calculated accurately to 10^{-7} of orbital mean motion at the 3:2 resonance state for which the influence of the solar thermal and tidal effect on Mercury's rotation is in equilibrium.

The difference between the rotational periods derived from the motions at perihelion and aphelion is 4.68 minutes. The maximum rate of rotation occurs at $f = \pi/4$ and $f = 7\pi/4$. In order to test the theory of rotation for Mercury, it seems necessary to improve the accuracy of observations.

ACKNOWLEDGMENT

The numerical analysis and computer programming were performed by W. R. Trebilcock and C. Wade.

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Liu, H. S., 1969, On the Figure of Mercury, (In press)

Liu, H. S. and O'Keefe, J. A., 1965, Theory of Rotation for the Planet Mercury Science 150, 1717.

Table 1. Instantaneous Rotational Velocity at Different Positions Along Mercury's Orbit

f	Ω/n	f	Ω/n	f	Ω/n
0	1.5000243	125	1.4999890	250	1.5000042
5	1.5000243	130	1.4999834	255	1.5000083
10	1.5000244	135	1.4999777	260	1.5000115
15	1.5000245	140	1.4999718	265	1.5000145
20	1.5000246	145	1.4999659	270	1.5000170
25	1.5000248	150	1.4999603	275	1.5000192
30	1.5000249	155	1.4999551	280	1.5000209
35	1.5000251	160	1.4999505	285	1.5000223
40	1.5000254	165	1.4999466	290	1.5000233
45	1.5000254	170	1.4999437	295	1.5000242
50	1.5000254	175	1.4999418	300	1.5000247
55	1.5000251	180	1.4999411	305	1.5000251
60	1.5000247	185	1.4999418	310	1.5000254
65	1.5000242	190	1.4999437	315	1.5000254
70	1.5000233	195	1.4999466	320	1.5000254
75	1.5000209	200	1.4999505	325	1.5000251
80	1.5000209	205	1.4999551	330	1.5000249
85	1.5000192	210	1.4999603	335	1.5000248
90	1.5000170	215	1.4999659	340	1.5000246
95	1.5000145	220	1.4999718	345	1.5000245
100	1.5000115	225	1.4999777	350	1.5000244
105	1.5000083	230	1.4999834	355	1.5000243
110	1.5000042	235	1.4999890	360	1.5000243
115	1.5000000	240	1.4999940		
120	1.4999940	245	1.5000000		

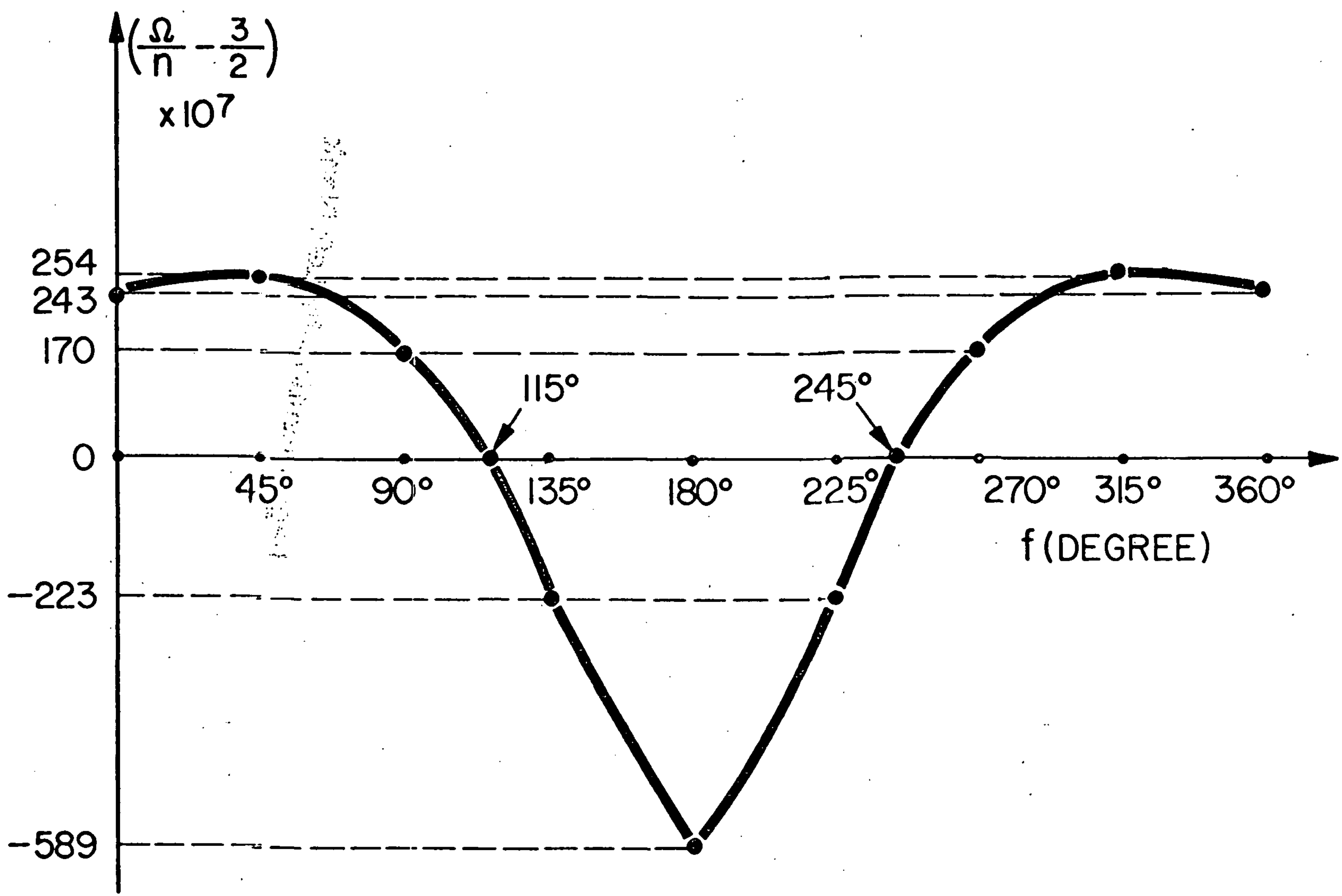

Figure 1—Fluctuation of the Rotational Velocity at Different Positions Along Mercury's Orbit

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