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TIDAL THEORY AND ORBITAL PERTURBATIONS

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1. Ocean Tide Theory

To ensure stability of the solutions for models with highly contoured ocean profiles the time-independent equation was solved for the expansion coefficients $A_{\ell m}$ of the pseudo-potential as given by

$$\psi = U_{\ell^* m^*}\left(1 + k_{\ell^*} - h_{\ell^*}\right) + V(\xi) - g\xi$$

$$= - g(\lambda^2/4\omega^2 - \cos^2\theta)^2 \sum_{\ell,m} A_{\ell m}\, Y_{\ell m}(\theta,\varphi)$$

where $U_{\ell^* m^*}$ is a potential term of the external forces, $V(\xi)$ is the self-gravitation of the tide ξ, λ is the angular frequency of the forcing term, ω is the angular frequency of the earth's rotation, and k_{ℓ^*} and h_{ℓ^*} are Love numbers.

Solved for the tide-height, the equation becomes

$$\xi = \sum_{\ell,m} \sum_{s} g^{-1} D_{\ell s}(|m|) A_{sm} N_{sm}^{-1} \left[1 - \frac{3\rho_w/\rho_e}{2\ell+1}\right.$$

$$\left. \cdot \left(1 + k_\ell' - h_\ell'\right)\right]^{-1} N_{\ell m} Y_{\ell m}(\theta,\varphi)$$

$$+ U_{\ell * m *}\left(1 + k_{\ell *} - h_{\ell *}\right)$$

where the coefficients $D_{\ell s}$ arise from reducing the factor $(\lambda^2/4\omega^2 - \cos^2\theta)^2$, the N's are normalization constants, and k_ℓ' and h_ℓ' are load numbers

With realistic ocean profiles convergence proved slow. Truncation of the series of $A_{\ell m}$ interacts with higher terms of the ocean depth coefficients causing resonances that affect even the lower terms of the $A_{\ell m}$. The process is somewhat analogous to acoustical resonances in finite rooms.

The semi-monthly tide showed better convergence than the semi-diurnal, probably due mathematically to relatively smaller coefficients of high order m, and physically to smaller interactions of the longer wavelengths of the semi-monthly tide with the depth wavelengths defining the continents.

The Love numbers were found to have on the order of a 20% effect on the semi-monthly tide. They accentuated the main features and introduced little phase shifting. Since lateral load numbers ℓ_ℓ', (Longman, J. Geophys. Res., 67, 845, 1962) are small, their exclusion should not alter this greatly.

There may be somewhat larger phase shifts in the semi-diurnal tide due to greater interaction with the continents. However, the absence of the order m in the load number correction (second equation) argues against a large phase shift.

Computations by Pekeris (preprint received 7 April 1969) show that frictional interactions with continental shelves account for much of the phase lag in amphidromic systems. The presence of non-linear viscous terms in the equations might augment the effect of the elastic response terms.

Further progress will require a method for reducing the errors of series truncation. Some possibilities are: a) a higher-degree interative solution; b) variational use of an additional constraint; c) iteration on a more rapidly converging principal solution; d) obtaining physical conclusions without solving the equations; and e) solutions obtained with a Green's function.

2. Tidal Perturbations of Satellite Orbits

The tidal disturbing function with the amplitude factor k and lag angle ϵ expressed as sums of zonal spherical harmonics was developed in two forms, one appropriate for the evolution of the moon's orbit; the other, for the perturbations of close artificial satellites.

In regard to the current evolution of the moon's orbit, the principal finding was that a second degree zonal harmonic in the lag angle would interact with the corresponding term of the lunar disturbing function to produce a significant energy transfer out of the moon's orbit without angular momentum transfer, dependent on the eccentricity. However, a decreasing eccentricity would be contrary to the circumstances required for spin orbit coupling, and decrease in the semi-major axis on backward extrapolation makes this effect insignificant.

Possible effects of tidal friction on the inclination of the moon's orbit close to the earth were examined, to determine whether the fission hypothesis could be retained entirely. It was concluded that tidal friction could do nothing to wrench the moon out of an equatorial orbit, and that an implausible commensurability between the moon's orbital revolution and the earth's rotation was needed for tidal friction to increase a non-zero inclination. It thus seems necessary that at least part of the moon was captured.

Tidal friction effects on close satellites were derived with a view to reconciling lag angle estimates from the moon's orbital deceleration with close satellite orbit analyses by Kozai and Newton. Assuming no significant difference between lunar and solar tide lag angles, the results of Kozai appear impossible to reconcile. The results of Newton require a variation such that phase leads predominate over phase lags beyond latitude 56°, as may be caused by ocean tides. It seems necessary, however, to have a drag-free satellite to resolve the matter.

A paper "Tidal Friction with Latitude-Dependent Amplitude and Phase Angle" was presented at the AGU & COSPAR meetings, and submitted to the Astronomical Journal. (This work was partially supported by NASA contract NAS 12-695).

3. Representation of Shorter Wavelength Variations in the Gravity Field

Satellite-borne altimeters and satellite-to-satellite tracking will be sensitive to much smaller variations in the gravitational field than have been determined by analyses of tracking from ground stations. Hence the continued use of spherical harmonic coefficients as the mode of representation could become excessively cumbrous.

The power spectra of effects on the differing measuring systems were estimated using sate'lite data up to spherical harmonic degree 15 and autocovariance analysis of terrestrial gravimetry up to degree 200. A rule-of-thumb was devised to approximate the orbital perturbation effects. The results are shown in the attached figure. The main point which this figure emphasizes is the grossly different parts of the spectrum to which the different measuring systems are sensitive. For the present accuracy of knowledge of the long-wavelength part of the gravity field, the short-periodic errors in orbit calculation will fall more than an order-of-magnitude below the corresponding part of the geoid height spectrum. Hence the results of altimetry can be represented in terms of residuals with respect to orbits taking into account the low degree harmonics of the gravity field with very little distortion.

The graph includes spectra for 890-km and 260-km altitude satellites. The much slower drop-off of the velocity variations with increasing wave number of the latter indicates that to obtain the full potential benefit of satellite-to-satellite tracking, one of the satellites should be quite low and hence instrumented to be drag-free.

A paper "The Appropriate Representation of the Gravity Field for Satellite Geodesy" was presented at the 4th IAG Symposium on Mathematical Geodesy and will be published in the proceedings.

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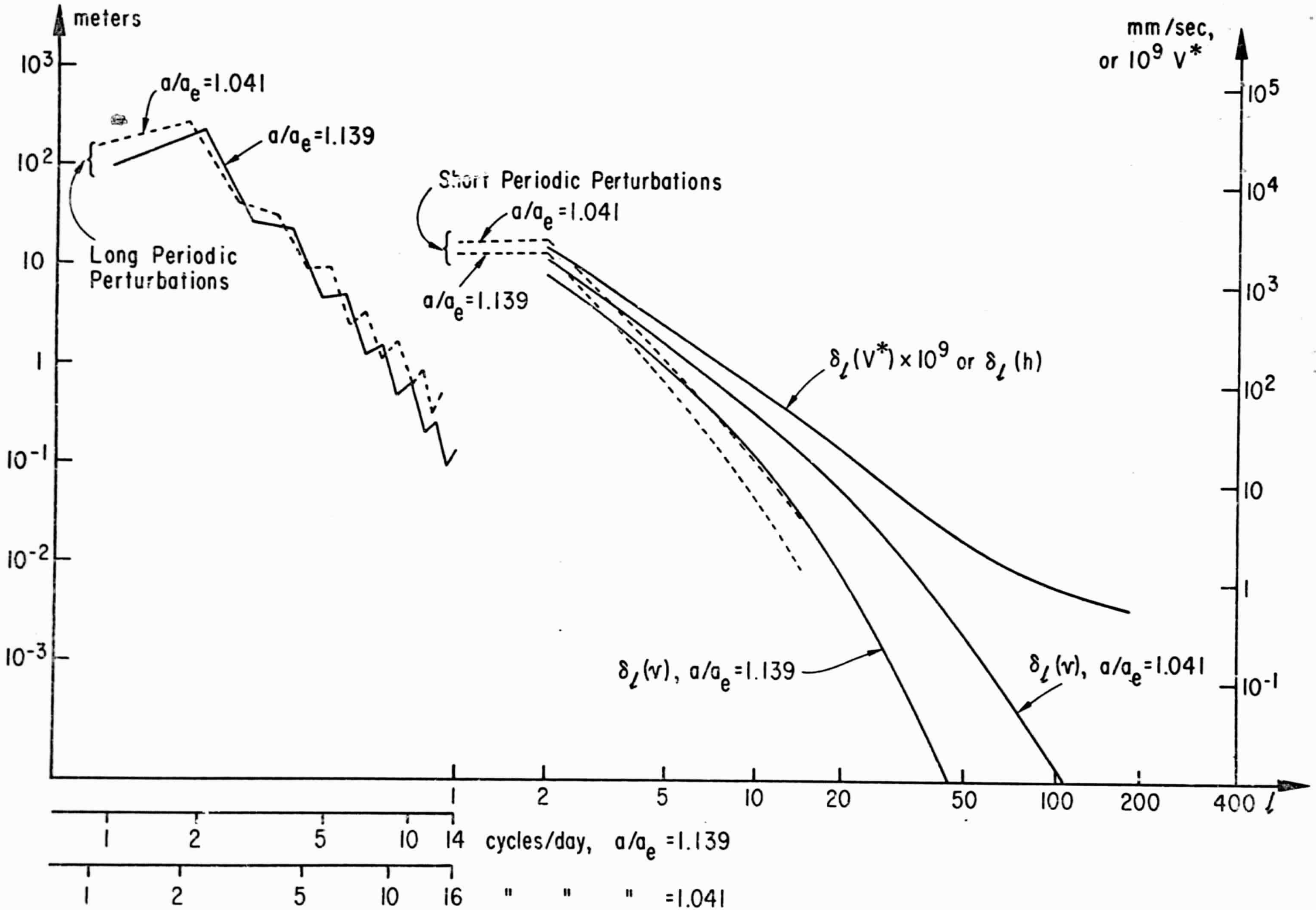

meters
10^3
10^2
10
1
10^-1
10^-2
10^-3
a/a_e=1.041
a/a_e=1.139
Long Periodic Perturbations
Short Periodic Perturbations
a/a_e =1.041
a/a_e=1.139
δ_l(V*) × 10^9 or δ_l (h)
δ_l(v), a/a_e=1.139
δ_l(v), a/a_e=1.041
mm/sec, or 10^9 V*
10^5
10^4
10^3
10^2
10
1
10^-1
1
2
5
10
20
50
100
200
400 l
1
2
5
10
14 cycles/day, a/a_e =1.139
1
2
5
10
16 " " " =1.041