NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1380

Block-Coded Communications

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Preface

The work described in this report was performed by the Telecommunications Division of the Jet Propulsion Laboratory.

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Abstract

A theory for use in the design of block-coded telemetry systems is delineated. This theory is useful in designing and testing the performance of one- and two-way, phase-coherent telemetry systems when a double-conversion, superheterodyne, phase-locked receiver preceded by a bandpass limiter is used to track the carrier. System analysis for either orthogonal or bi-orthogonal codes is given. Design trends, relating the various system parameters, are presented in graphical form for practical code sizes. Emphasis is placed on the case of greatest practical interest: the design situation in which the data rate is large when compared to the design point bandwidth of the carrier tracking loop.

Block-Coded Communications

I. Introduction

Previous work (Refs. 1, 2, 3, and 4) has established performance characteristics and trends required for the design of one- and two-way, phase-coherent, uncoded communications systems. More recently, considerable interest has developed (Ref. 5) in applying known techniques and theories, evolved over the past few years, to the mechanization of block-coded communications systems for deep space applications. Such expressions as "high-rate telemetry" (HRT), implying data rates in excess of a few thousand bits per second, and "system software" are becoming a part of the vocabulary of every communications design engineer faced with advancing the technology of deep space communications. For example, a major objective of the *Mariner* Mars 1969 mission is to obtain television pictures of Mars by applying the theory of block coding to the development of a 16,200-bits/s telemetry system. The HRT system is a modification of the basic digital telemetry system that was used on *Mariners IV* and *V*. The primary difference is that the data detection process is now more efficient.

The purpose of this report is to establish the performance of one- and two-way, phase-coherent communication systems which employ double-conversion, superheterodyne, phase-locked receivers preceded by a bandpass limiter to track the modulation. Such a theory is useful in testing and performance prediction, as well as in evaluation of the design of such systems prior to and after launch.

In this report we shall draw heavily upon the notation and terms established in Refs. 1, 2, 3, and 4, thus shortening the presentation and avoiding duplication of definitions previously established.

II. System Model

Figures 1 and 2 present functional descriptions of the system. Briefly, the data to be transmitted is assumed to be block-encoded into binary symbols. Each code word, say $x_l(t)$, $l = 1, \cdots, N$, to be transmitted is made comma-free (Ref. 6) by adding an appropriate pseudonoise vector plus one bit to facilitate the word synchronization problem at the receiver. The code symbols, appearing at the modulator in the form of a binary waveform, are used to bi-phase-modulate a square-wave data subcarrier (Ref. 3), say $S(t)$. The modulated data subcarrier (Refs. 1

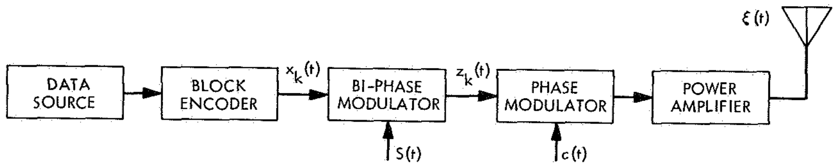

Fig. 1. Transmitter characterization

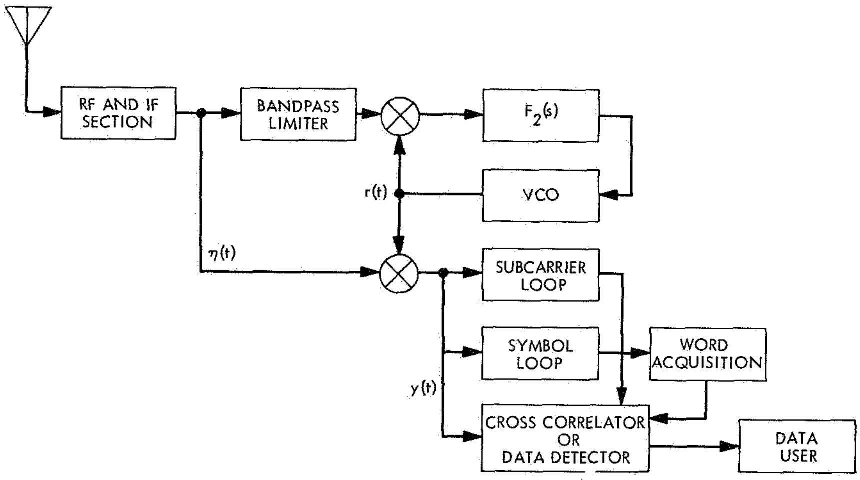

Fig. 2. Receiver characterization

and 2) in turn phase-modulates the RF carrier $c(t)$, which is then amplified and radiated from the spacecraft or vehicle antenna (Ref. 3) as $\xi(t)$. On the ground, a double-conversion, superheterodyne, phase-tracking receiver is used to track the observed RF carrier component, thus providing a coherent reference for synchronously demodulating the subcarrier. The received signal is denoted by $\eta(t)$ (see Fig. 2). Owing to the fact that this reference is derived in the presence of white Gaussian noise of single-sided spectral density N_{02} W/Hz, there will exist phase jitter due to the additive noise on the down-link (Refs. 1, 2, and 3), and if the system happens to be two-way locked (Refs. 1 and 2), the additive white noise on the up-link (assumed to be white Gaussian noise with single-sided spectral density of N_{01} W/Hz) also exerts another component of phase jitter. We shall be concerned here with predicting system performance in both situations. The results are extremely useful in designing systems which must operate with narrow performance margins, i.e., the number of decibels in excess of the sum of the negative tolerances in equipment performance. For deep space telecommunication links, the sum of the negative tolerances is typically 4 to 6 dB. Experience has shown that requiring the design to exceed the sum of the negative tolerances is slightly conservative; hence reducing excess margin results in a much "tighter" or a less conservative design.

At the receiver (Fig. 2), a subcarrier tracking loop (Ref. 3) is assumed to exist for the purposes of providing subcarrier sync. In practice, phase jitter also exists on this reference. However, this phase jitter may usually be made negligibly small by designing a very narrow-band subcarrier tracking loop (Ref. 3). Finally, word sync may be derived at the receiver by making use of the comma-free properties of the transmitted code (Ref. 6). Thus the necessary timing information is provided for triggering the cross-correlation detector in Fig. 2. The output data is the recovered bit stream and may be recorded for the data user.

For convenience we shall assume that the code words $x_l(t)$, $l = 1, 2, \cdots, N$, representing sequences of ± 1s, occur with equal probability, contain equal energies, and exist for $T = kT_b = 2^k T_s$ sec. Here T_b is the time per bit, the reciprocal of the data rate $\mathcal{R}$, T_s is the time per code word symbol, and n is the number of bits per code word. Thus the transmitted waveform may be represented by

$$\xi(t) = (2P)^{1/2} \sin [\omega t + (\cos^{-1} m) z_l(t)] \qquad (1)$$

where P is the total radiated power, and m is the modulation factor which apportions the total power between the carrier component and modulation sidebands. In Eq. (1), the waveform $z_l(t) = x_l(t)S(t)$, $l = 1, 2, \cdots, N$, where $x_l(t)$ is the code word in the form of a sequence of ± 1s to be transmitted, and $S(t)$ is the unmodulated data subcarrier possessing unit power (see Fig. 1). Since $S(t)$ is a sequence of ± 1s, $z_l(t)$ is also a sequence of ± 1s.

Assuming that the channel introduces an arbitrary (but unknown) phase shift θ to $\xi(t)$ and further disturbs $\xi(t)$ by additive white Gaussian noise $n_2(t)$ of single-sided spectral density of N_{02} W/Hz single-sided, one observes at the input to the receiver (Fig. 2)

$$\eta(t) = (2P)^{1/2} \sin [\omega t + (\cos^{-1} m) z_l(t) + \theta] + n_2(t) \tag{2}$$

when the receiver is operating in a one-way locked condition (Ref. 1). If the receiver is operating in a two-way locked condition (Ref. 1), then the input is taken to be

$$\eta(t) = (2P)^{1/2} \sin [\omega t + (\cos^{-1} m) z_l(t) + \hat{\theta}_1 + \theta] + n_2(t) \tag{3}$$

where $\hat{\theta}_1$ represents phase modulation due to the up-link additive noise (Ref. 1), i.e., noise introduced in the spacecraft transponder.

In either case, the output of the receiver's voltage-controlled oscillator (VCO) is denoted by

$$r(t) = (2)^{1/2} \cos (\omega t + \hat{\theta}_2) \tag{4}$$

where $\hat{\theta}_2$ is the estimate of the phase of the observed carrier component. By multiplying $\eta(t)$ by $r(t)$ and neglecting double-frequency terms, it may be shown (Ref. 1) that the output $y(t)$ of the receiver's carrier tracking loop, which is the input to the data detector, is given by

$$y(t) = (S)^{1/2} z_l(t) \cos \phi + n_2'(t) \tag{5}$$

where $S = (1 - m^2)P$, $m^2 = P_c/P$, P_c is the power remaining in the carrier component at frequency $f = \omega/2\pi$, and ϕ is the receiver's phase-error, i.e., $\phi = \theta - \hat{\theta}_2$ if one-way lock is assumed, and $\phi = \theta + \hat{\theta}_1 - \hat{\theta}_2$ if two-way lock is assumed. The probability distribution of the phase-error ϕ is important in determining overall system performance. In Section III we present a model for this distribution when bandpass limiters precede the carrier tracking loop.

III. Probability Distribution for the Phase Error

A. One-Way Link

To characterize the distribution $p_1(\phi)$ requires considerable elaboration (beyond the scope of this report) on the response (signal plus noise) of a phase-locked loop preceded by a bandpass limiter. However, the distribution may be modeled on the basis of experimental and theoretical evidence given in Refs. 7, 8, and 9, in which the distribution for $p_1(\phi)$ is approximated in the region of interest by

$$p_1(\phi) = \frac{\exp(\rho_L \cos \phi)}{2\pi I_0(\rho_L)}, \qquad |\phi| < \pi \tag{6}$$

where

$$\rho_L = \frac{2P_c}{N_0 w_{L0}} \cdot \frac{1}{\Gamma} \left(\frac{1 + r_0}{1 + \dfrac{r_0}{\mu}} \right) \tag{7}$$

and the parameters w_{L0}, r_0, and μ are defined from the closed-loop transfer function $H_2(s)$ of the carrier tracking loop

$$H_2(s) = \frac{1 + \left(\dfrac{r_0 + 1}{2w_{L0}}\right) s}{1 + \left(\dfrac{r_0 + 1}{2w_{L0}}\right) s + \dfrac{\mu}{r_0} \left(\dfrac{r_0 + 1}{2w_{L0}}\right)^2 s^2} \tag{8}$$

Here μ is taken to be the ratio of the limiter suppression factor α_0 at the loop design point (threshold) to the limiter suppression, say α, at any other point; i.e., $\mu = \alpha_0/\alpha$. The filter in the carrier tracking loop is assumed to be of the form (see Fig. 2)

$$F_2(s) = \frac{1 + \tau_1 s}{1 + \tau_2 s} \tag{9}$$

in which case

$$r_0 = \frac{\alpha_0 K \tau_2^2}{\tau_1} \tag{10}$$

and K is the equivalent simple-loop gain (Ref. 7). The zero subscripts refer to the values of the parameters at the loop design point. The parameter w_{L0} is defined by

$$w_{L0} = \frac{1 + r_0}{2\tau_2} \tag{11}$$

The loop bandwidths are conveniently defined by w_L and b_L through the relationship

$$w_L = 2b_L = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} |H_2(s)|^2 \, ds \tag{12}$$

Substitution of Eq. (8) into Eq. (12) yields

$$w_L = w_{L0} \left(\frac{1 + \frac{r_0}{\mu}}{1 + r_0} \right) = 2b_L \tag{13}$$

The relation $w_{L0} = 2b_{L0}$ may be defined in a similar way. Thus Eq. (13) becomes

$$2b_L = (2b_{L0}) \left(\frac{1 + \frac{r_0}{\mu}}{1 + r_0} \right) \tag{14}$$

which is the usual definition of loop bandwidth employed by practicing engineers.

The factor Γ is approximated (Ref. 7) by

$$\Gamma = \frac{1 + 0.345\, \rho_H}{0.862 + 0.690\, \rho_H} \tag{15}$$

where ρ_H is the signal-to-noise ratio at the output of the receiver's IF amplifier; i.e.,

$$\rho_H = \frac{2P_c}{N_0 w_H} \tag{16}$$

The parameter w_H is the two-sided bandwidth of the second IF amplifier in the double-heterodyne receiver. In one-sided bandwidth notation, $w_H = 2b_H$, and

$$\rho_H = \frac{P_c}{N_0 b_H} \tag{17}$$

The parameter ρ_H is also the signal-to-noise ratio at the input to the bandpass limiter.

The remaining parameter to define is the factor $\mu = \alpha_0/\alpha$. It may be shown that limiter suppression α is given by

$$\alpha = \left(\frac{\pi}{2}\right)^{1/2} \left(\frac{\rho_H}{2}\right)^{1/2} \times \exp\left(-\frac{\rho_H}{2}\right) \left[I_0\left(\frac{\rho_H}{2}\right) + I_1\left(\frac{\rho_H}{2}\right) \right] \tag{18}$$

where $I_m(z)$, $m = 1, 2$, is the modified Bessel function of argument z and order. To specify α_0, the parameter ρ_H is rewritten as follows:

$$\rho_H = \frac{P_c}{N_0 b_H} \cdot \frac{b_{L0}}{b_{L0}} = \frac{P_c}{N_0 b_{L0}} \cdot \frac{b_{L0}}{b_H} = zy \tag{19}$$

where

$$z = \frac{P_c}{N_0 b_{L0}}$$

and (20)

$$y = \frac{b_{L0}}{b_H}$$

In practice, the parameters of the carrier tracking loop are specified at the loop design point or threshold. If the design point is defined as $z_0 = \gamma_0 = \text{constant}$, then the parameter α_0 is given by

$$\alpha_0 = \left(\frac{\pi}{2}\right)^{1/2} \left(\frac{\gamma_0 y}{2}\right)^{1/2} \times \exp\left(-\frac{\gamma_0 y}{2}\right) \left[I_0\left(\frac{\gamma_0 y}{2}\right) + I_1\left(\frac{\gamma_0 y}{2}\right) \right] \tag{21}$$

Therefore, it is clear that system performance depends upon the choice of γ_0. In the Deep Space Network (DSN), this choice is usually $\gamma_0 = 2$, so that

$$z_0 = \frac{P_{c0}}{(kT^\circ)(b_{L0})} = 2 \tag{22}$$

or, equivalently,

$$\frac{P_{c0}}{(kT^\circ)(2b_{L0})} = 1$$

at the design point. Here, $N_0 = kT°$, k is Boltzmann's constant, and $T°$ equals the system temperature in degrees Kelvin.

B. Two-Way Link

In order to characterize the probability distribution $p_2(\phi)$ for the phase-error in a two-way link, one must consider the up-link parameters and the mechanization of the transponder in the spacecraft (Ref. 1). As before, the characterization of $p_2(\phi)$ requires considerable elaboration (beyond the scope of this report) on the response (signal plus noise) of phase-locked loops in cascade. Certain theoretical and computer simulation results (Ref. 10) are available to explain the nonlinear behavior of loops in cascade. The characterization here is predicated upon work reported in Refs. 1 and 10. We use the following notation: a subscript 1 refers to up-link parameters and constants associated with the spacecraft transponder mechanization; a subscript 2 refers to down-link parameters and constants associated with the mechanization of the ground receiver.

The generic form discussed in Refs. 1 and 10 for $p_2(\phi)$ is given by

$$p_2(\phi) = \frac{I_0\left[\,|\rho_1 + \rho_2 \exp(j\phi)|\,\right]}{2\pi I_0(\rho_1) I_0(\rho_2)}, \qquad |\phi| \leq \pi \tag{23}$$

The parameter ρ_2, which was equal to ρ_L in Eq. (7), becomes, in the new notation,

$$\rho_2 = \frac{2P_{c2}}{N_{02} w_{20}} \cdot \frac{1}{\Gamma_2} \left(\frac{1 + r_{20}}{\dfrac{1 + r_{20}}{\mu_2}} \right) \tag{24}$$

and

$$r_{20} = \left(\frac{\alpha_{02} K_2 \tau_{22}^2}{\tau_{12}} \right)$$

where the zero subscripts refer to the parameters at the loop design point. The parameter w_{20} replaces the design point loop bandwidth w_{L0} in Eq. (11) and is defined by

$$w_{20} = \frac{1 + r_{20}}{2\tau_{22}} = 2b_{20} \tag{25}$$

when the loop filters are of the form given in Eq. (9), with τ_1 replaced by τ_{12} and τ_2 by τ_{22}. The parameter Γ_2 is defined in Eq. (15) by the addition of the subscript 2 to all symbols. Likewise, in Eqs. (18) and (21) the limiter suppressions α_{22} and α_{02} are defined by the addition of the subscript 2 to all symbols and

$$z_2 = \frac{P_{c2}}{N_{02} b_{20}}, \qquad y_2 = \frac{b_{20}}{b_{H2}} \tag{26}$$

In Eq. (26) we have dropped the L subscript on b_{L0} and replaced it with 2.

The remaining parameter to define is the variable ρ_1, which is given (Ref. 1) by

$$\rho_1 = \frac{2P_{c1}}{N_{01} w_{10}} \cdot \frac{1}{G^2 \Gamma_1 K(k_1, k_2, \beta)} \tag{27}$$

where G is the static phase gain of the spacecraft transponder and is determined by the ratio of the output frequency to the input carrier frequency. The limiter performance factor is defined in Eq. (15). However, the parameter ρ_{H1} is defined by

$$\rho_{H1} = \frac{2P_{c1}}{N_{01} w_{H1}} = \frac{P_{c1}}{N_{01} b_{H1}}$$

where w_{H1} is the two-sided bandwidth of the second IF amplifier in the spacecraft receiver, and b_{H1} is the one-sided bandwidth. The function $K(k_1, k_2, \beta)$ is given by

$$K(k_1, k_2, \beta) = \frac{1}{r_{10} + 1} \left[\frac{k_1(2 + k_1) + 2(k_1 + k_2 + 2)(\beta + \beta^2) + k_2(2 + k_2)\beta^3}{k_1^2 + 2k_1\beta + 2(k_1 + k_2 - k_1 k_2)\beta^2 + 2k_2\beta^3 + k_2^2\beta^4} \right] \tag{28}$$

where

$$k_n = \frac{2\mu_n}{r_{n0}}, \qquad r_{n0} = \frac{\alpha_{n0} K_n \tau_{2n}^2}{\tau_{1n}}$$

$$\mu_n = \frac{\alpha_{nn}}{\alpha_{0n}}$$

$$\beta = \frac{w_{10}}{w_{20}} \left(\frac{r_{20} + 1}{r_{10} + 1} \right)$$

$$w_{n0} = 2b_{n0} = \frac{1 + r_{n0}}{2\tau_{22}\left(\frac{1 + \tau_{22}}{r_{n0}\tau_{12}}\right)}$$

$$\rho_{Hn} = \frac{2P_{cn}}{N_{0n}w_{n0}}$$

$$\alpha_{nn} = \frac{\pi}{2}\left(\frac{\rho_{Hn}}{2}\right)\exp\left(-\frac{\rho_{Hn}}{2}\right)\left[I_0\left(\frac{\rho_{Hn}}{2}\right) + I_1\left(\frac{\rho_{Hn}}{2}\right)\right]$$

with $n = 1,2$. Now α_{0n} is defined by either the design point in the carrier tracking loops of the transponder, $n = 1$, or ground receiver, $n = 2$, through

$$\alpha_{0n} = \left(\frac{\pi}{2}\right)^{1/2}\left(\frac{\gamma_{0n}y_n}{2}\right)^{1/2}$$

$$\times \exp\left(-\frac{\gamma_{0n}y_n}{2}\right)\left[I_0\left(\frac{\gamma_{0n}y_n}{2}\right) + I_1\left(\frac{\gamma_{0n}y_n}{2}\right)\right]$$

IV. System Performance

A. Conditional Word-Error Probability

The problem of evaluating system performance may be described as follows: The output of the carrier tracking loop is given by Eq. (5). For k-bit orthogonal codes, the optimum decoder consists of 2^k cross-correlators whose outputs $C(j)$, $j = 1, 2, \cdots, 2^k$, are

$$C(j) = \int_0^{kT_b} y(t)x_j(t)dt \qquad (29)$$

where T_b is the transmission time per information bit. Once the set $\{C(j)\}$ has been determined, the most-probable transmitted word corresponds to that $x_j(t)$ for which $C(j)$ is greatest. The output of the decoder will be those k bits which, if encoded, would produce this $x_j(t)$.

As 2^k cross-correlators are required to decode a k-bit orthogonal code, the decoder becomes impractical for k of about 8 or greater because of its complexity. Also, the complexity of the decoder and the maximum bit rate at which it will operate are major factors in its design. This report does not outline or investigate techniques for reducing decoder complexity or increasing the maximum bit rate at which it will operate. The interested reader is referred to material contained in Refs. 11 and 12.

The conditional probability of correct word detection $P_c(\phi)$ is shown (Ref. 3) to be given by

$$P_c(k,\phi) = \int_{-\infty}^{\infty} \frac{1}{(2\pi)^{1/2}} \exp\left(\frac{-x^2}{2}\right) dx$$

$$\cdot \left[\int_{x+A_n}^{\infty} \frac{1}{(2\pi)^{1/2}} \exp\left(\frac{-y^2}{2}\right) dy\right]^{2^k-1} \qquad (30)$$

where

$$A_n = (2kR_n)^{1/2}\cos\phi$$

$$R_n = \frac{S_nT_{bn}}{N_{0n}} = \frac{S_n}{N_{0n}\mathcal{R}_n} \qquad (31)$$

where $\mathcal{R}_n = T_{bn}^{-1}$, and k = number of bits per code word. The subscript $n = 1$ is for one-way lock, while $n = 2$ implies two-way lock.

For bi-orthogonal codes of k bits per word, the probability of correct reception of a word, conditioned upon a particular phase-error, is given (Ref. 3) by

$$P_c(k,\phi) = \int_{-\infty}^{\infty} \frac{\exp\left(\frac{-x^2}{2}\right) dx}{(2\pi)^{1/2}} \qquad (32)$$

$$\cdot \left[\int_{x+A_n}^{\infty} \frac{1}{(2\pi)^{1/2}} \exp\left(\frac{-y^2}{2}\right) dy\right]^{2^k-1}$$

where A_n is defined in Eq. (31). The probability of a word error, conditioned upon a fixed value of ϕ, is, of course,

$$P_E(k,\phi) = 1 - P_c(k,\phi) \qquad (33)$$

For convenience, when $n = 1$ we will drop the subscript on A.

B. Average Word- and Bit-Error Probability

To obtain the average word-error probability $P_E(k)$, one averages Eq. (33) over the phase-error distribution.

Thus

$$P_E(k) = 1 - \int_{-\pi}^{\pi} p_n(\phi)P_c(k,\phi)d\phi\,, \qquad n = 1,2 \tag{34}$$

where $p_1(\phi)$ is given in Eq. (6) for one-way lock and $p_2(\phi)$ is defined in Eq. (23) for two-way lock. Substitution of Eq. (6) or (23) and (30) or (32) into Eq. (34) yields integrals which generally cannot be evaluated analytically. However, numerical integration by an IBM 7090 computer is possible.

In certain cases of practical interest, the bit-error probability is of importance. For k-bit orthogonal codes, the bit-error probability is (Ref. 13)

$$P_B(k) = \frac{2^{k-1}}{2^k - 1} P_E(k)$$

while for k-bit bi-orthogonal codes the total bit-error probability is (Ref. 13)

$$P_B(k) = P_1(k) + \frac{(k-1)2^{k-2}}{k(2^{k-1}-1)} P_2(k)$$

where $P_1(k)$ is given in Eq. (34) for orthogonal codes and $P_2(k)$ is given by Eq. (34) for bi-orthogonal codes.

V. Design Results

Since the integrals in Eq. (34) cannot be evaluated numerically, integration by an IBM 7090 computer yielded the results for one-way lock illustrated in Figs. 3, 4, 5, and 6 for code words containing k = 5, 6, 7, and 8 bits of information. These figures depict word-error rates vs the signal-to-noise ratio in the data for various values of the signal-to-noise ratio x in the design point bandwidth of the carrier tracking loop. Clearly, system performance depends upon the choice of a design point γ_0 in the carrier tracking loop. For purposes of presentation, the choice is taken to be that which corresponds to the design point in the Deep Space Network; i.e., $r_0 = 2$, $\gamma_0 = 2$, and $y = 1/400$. Clearly, as x approaches infinity, i.e., the case of perfect RF sync, the deleterious effects of a noisy phase reference disappear and perfect coherent detection is possible.

In the case of two-way lock, system performance for k = 5-, 6-, 7-, and 8-bit orthogonal codes is illustrated in Figs. 7 through 18 for various values of ρ_1 and x_2. The same carrier tracking loop design point is used for this case as was used for the case of one-way lock. Note that in this sequence of figures, as the signal-to-noise ratio in the ground receiver's design point loop bandwidth x increases without limit, the deleterious effects of the up-link noise introduce an irreducible error probability. This irreducible error depends upon the amount of carrier phase jitter introduced by the vehicle's carrier tracking loop. This irreducible error probability may be made arbitrarily small by increasing the up-link transmitter power. In fact, it is easy to show that the irreducible error probability, say $P_{ir}(k)$, is given by

$$P_{ir}(k) = 2\int_{\pi/2}^{\pi} p_n(\phi)d\phi$$

$$P_{ir}(k) = \lim_{R\to\infty} P_E(k) = 2\int_{\pi/2}^{\pi} p_n(\phi)d\phi$$

which is the probability that the phase-error exceeds $\pi/2$; i.e., $Prob(|\phi| > \pi/2)$. It is shown, therefore, that P_{ir} is independent of the code and depends only upon the design of the carrier tracking loops, the available power in the carrier components, and the channel noise. Thus for given channel conditions and fixed loop parameters, large transmitter output power capabilities are certainly desirable.

For $k \geq 5$, the performance of a block-coded, digital communication system using bi-orthogonal codes is essentially the same as one that uses orthogonal codes (Ref. 13). Hence for $k \geq 5$ the results presented may be applied to the design of systems whose code dictionaries are made up of bi-orthogonal codes.

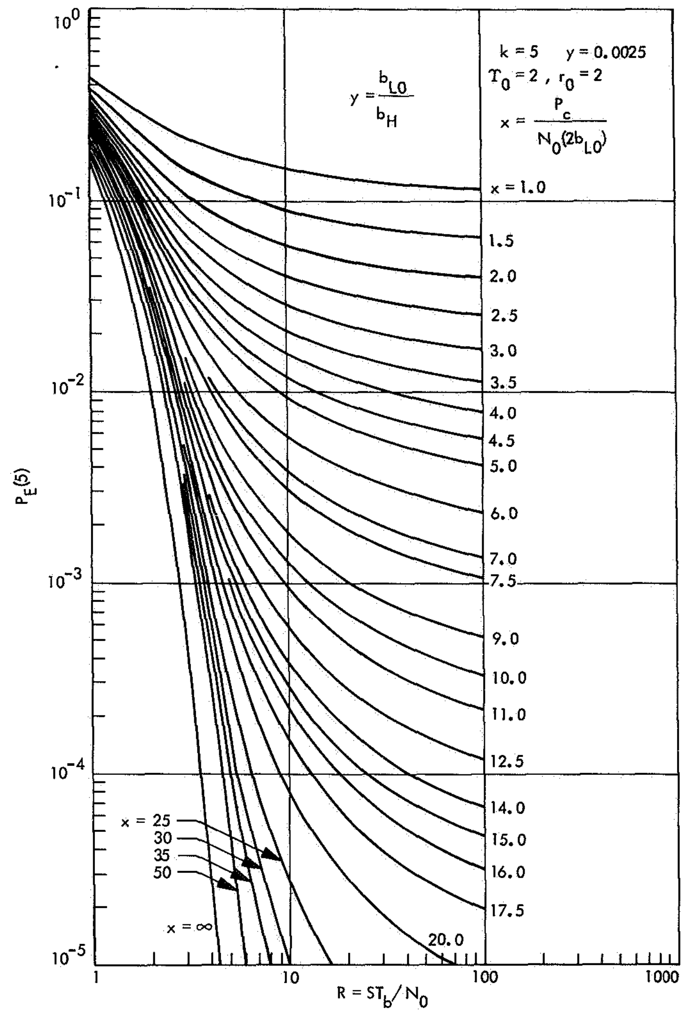

Fig. 3. Word-error probability vs signal-to-noise ratio *R* for various values of the signal-to-noise ratio *x*

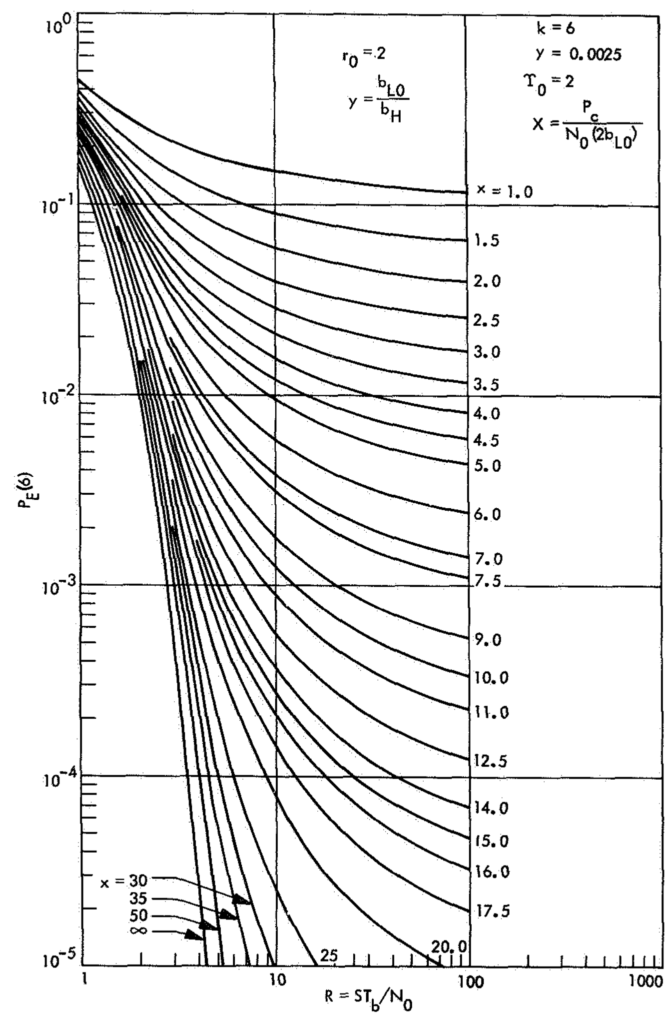

Fig. 4. Word-error probability vs signal-to-noise ratio *R* for various values of the signal-to-noise ratio *x*

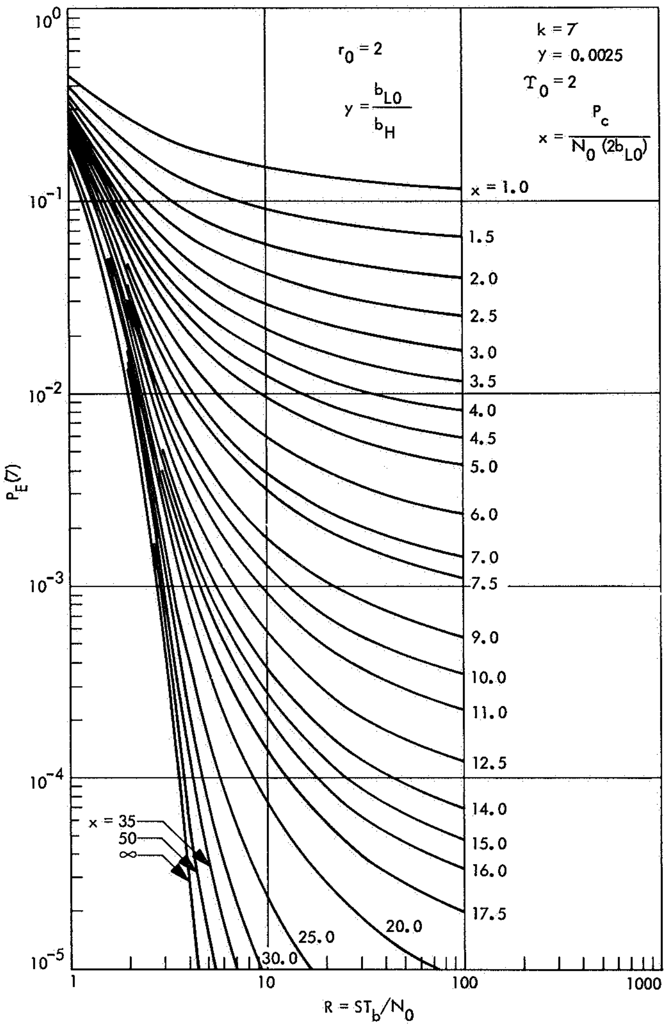

Fig. 5. Word-error probability vs signal-to-noise ratio *R* for various values of the signal-to-noise ratio *x*

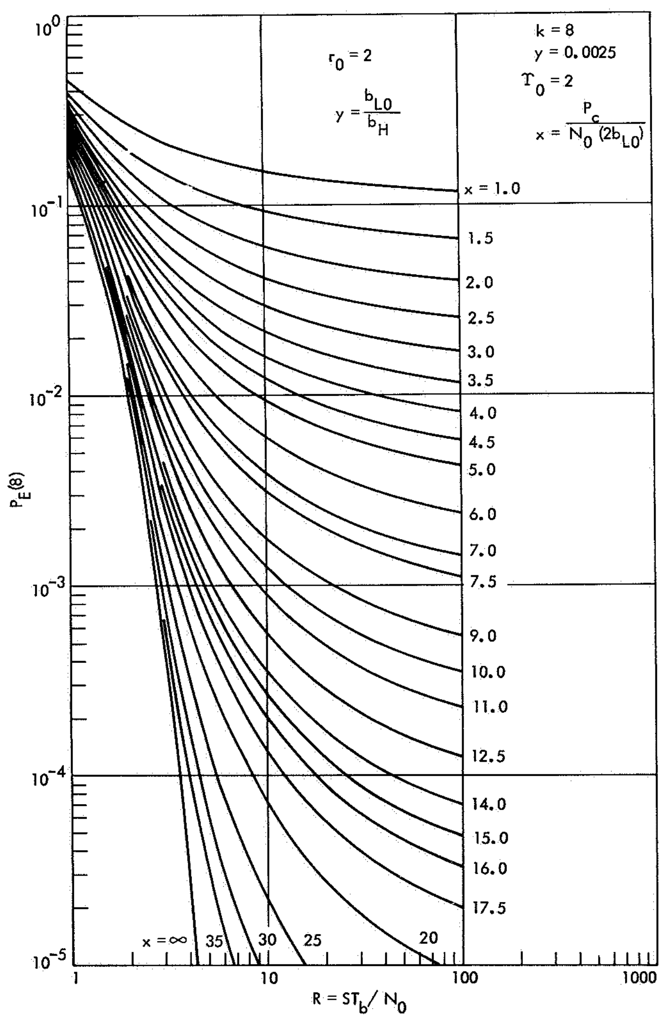

Fig. 6. Word-error probability vs signal-to-noise ratio *R* for various values of the signal-to-noise ratio *x*

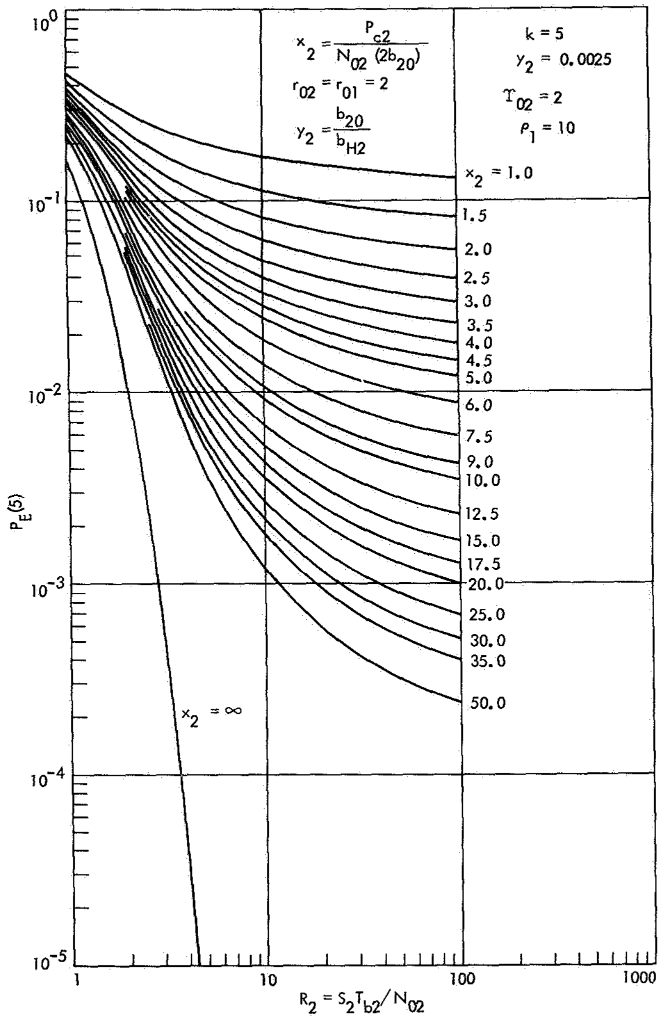

Fig. 7. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

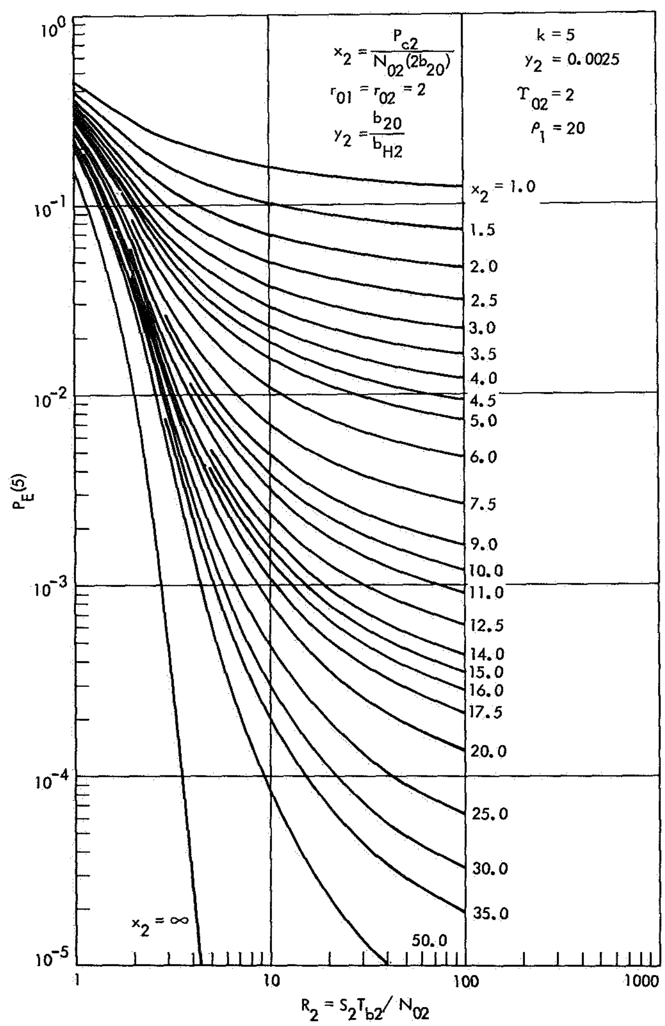

Fig. 8. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

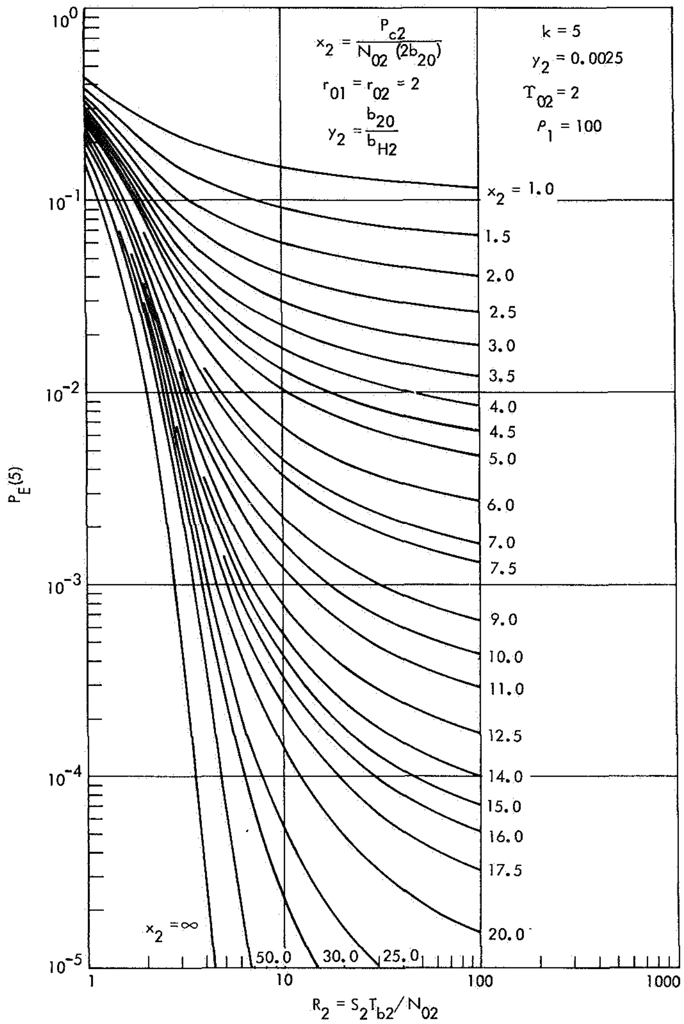

Fig. 9. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

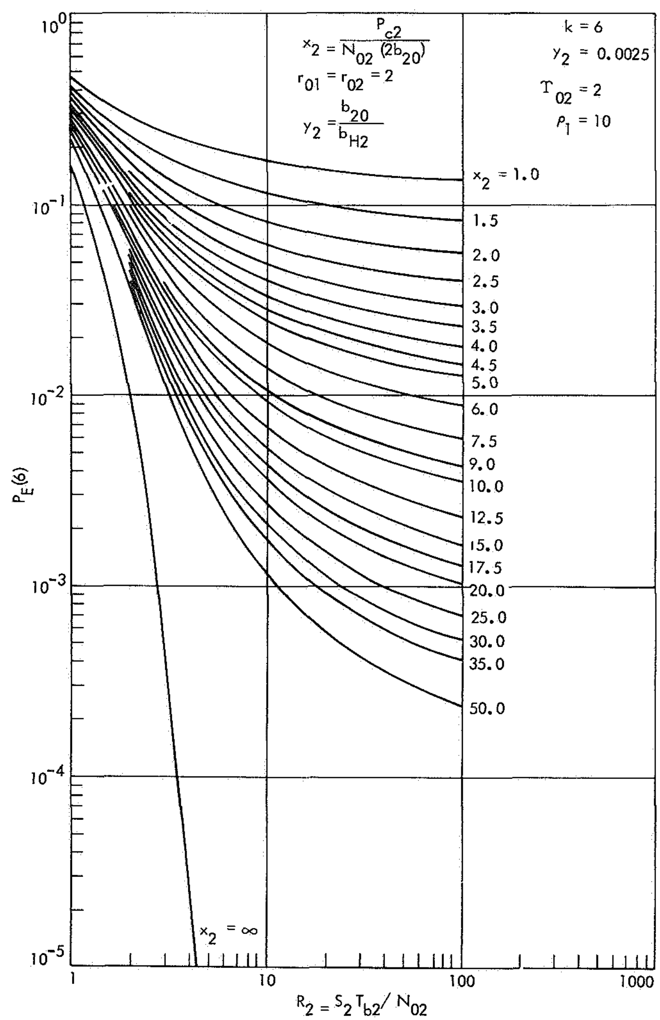

Fig. 10. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

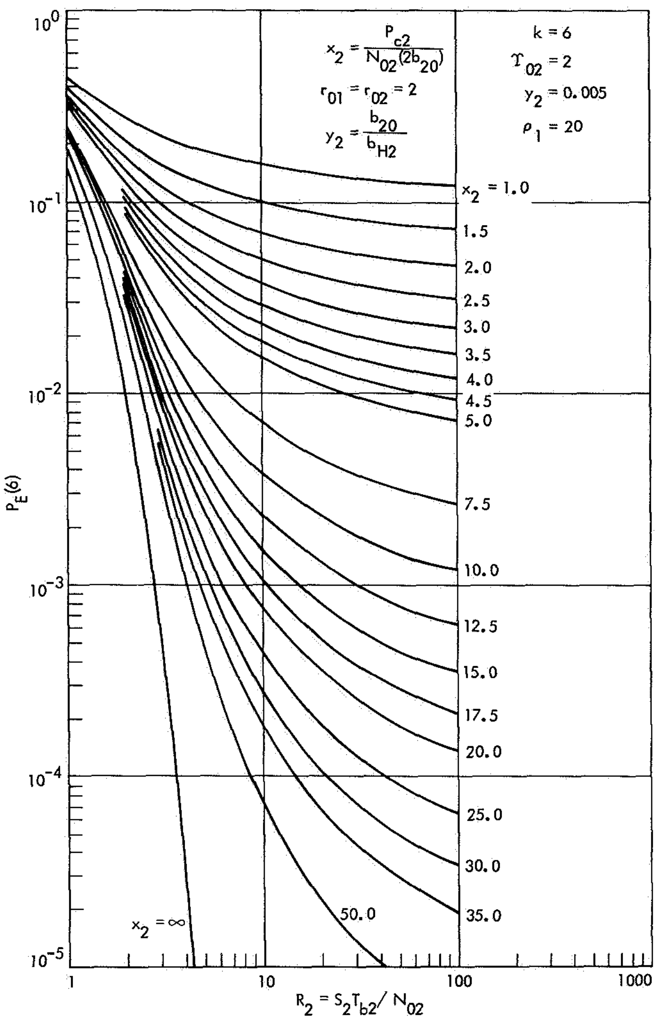

Fig. 11. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

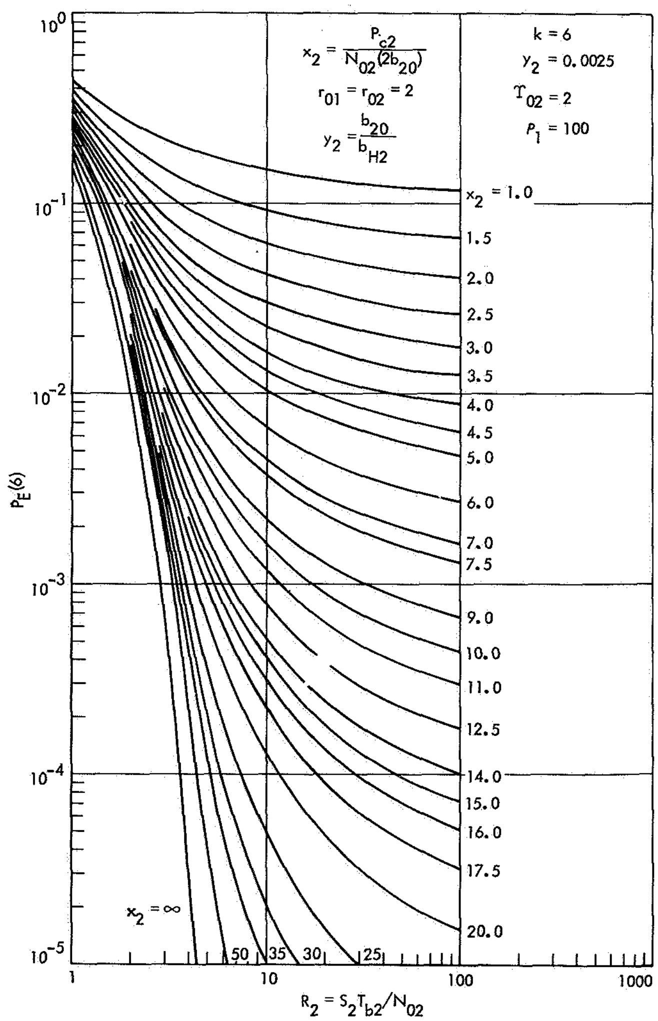

Fig. 12. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

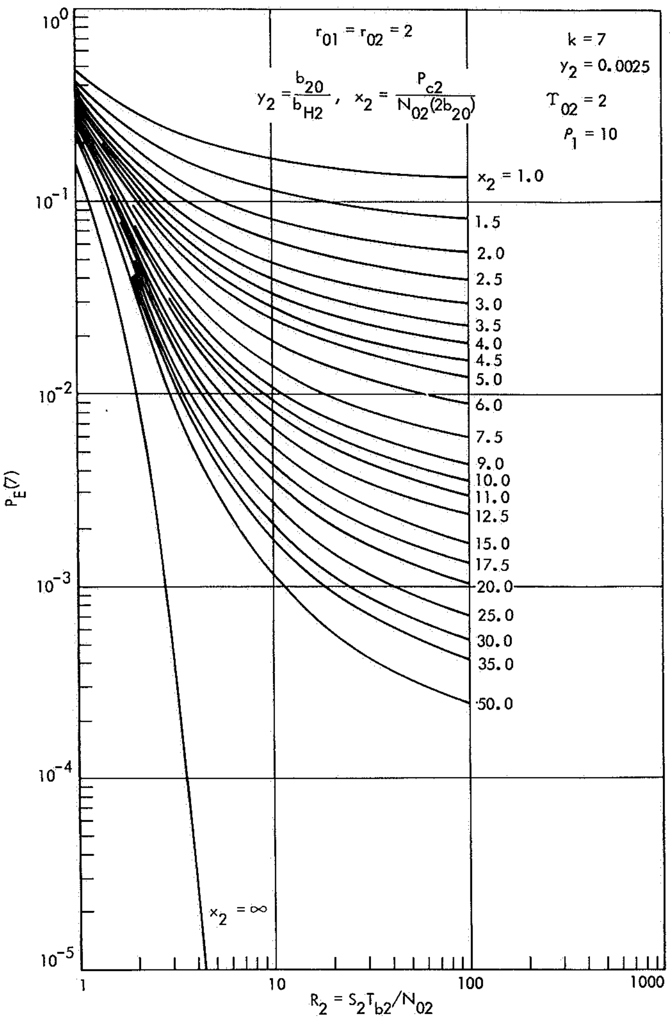

Fig. 13. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

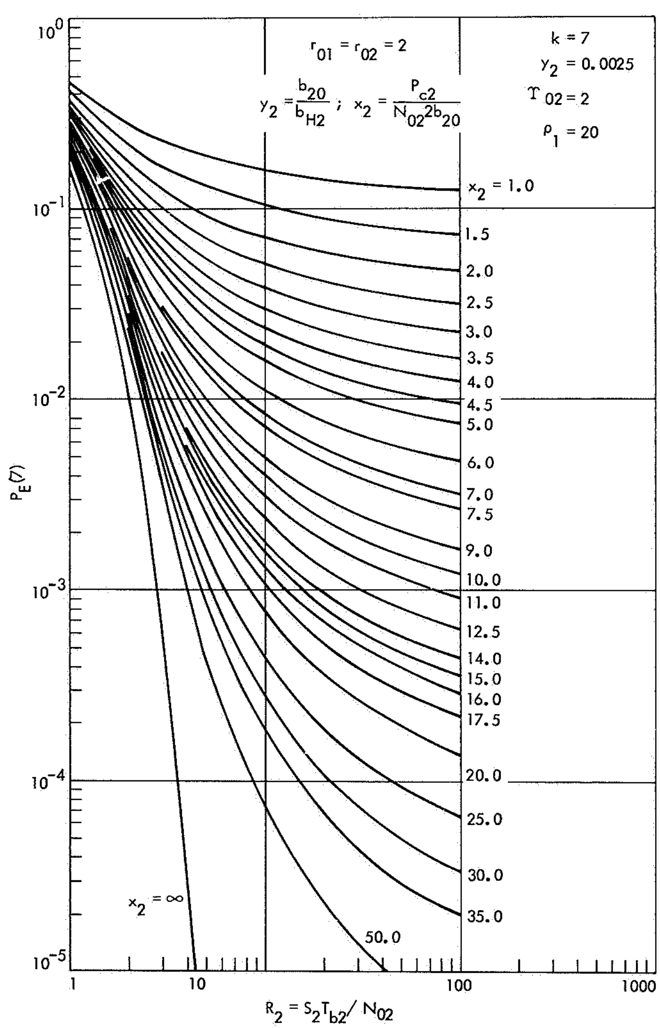

Fig. 14. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

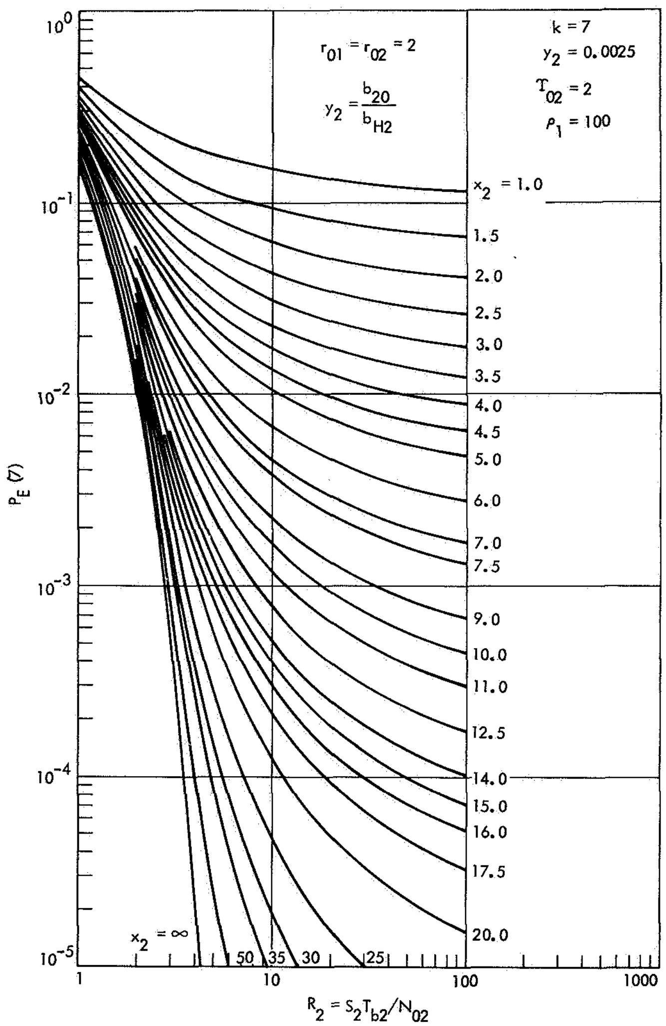

Fig. 15. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

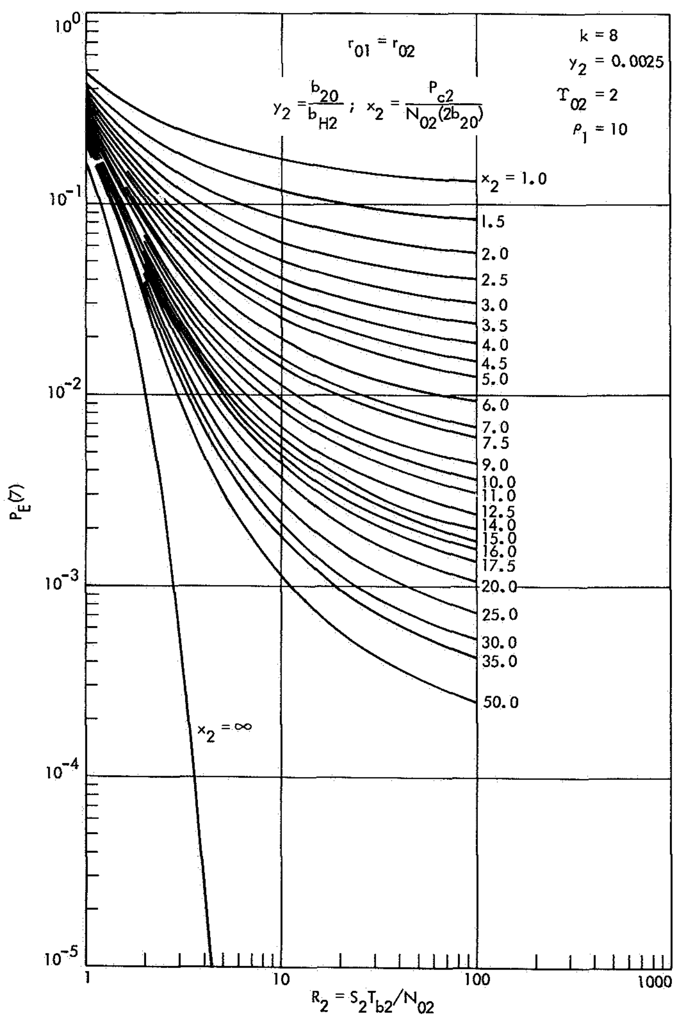

Fig. 16. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

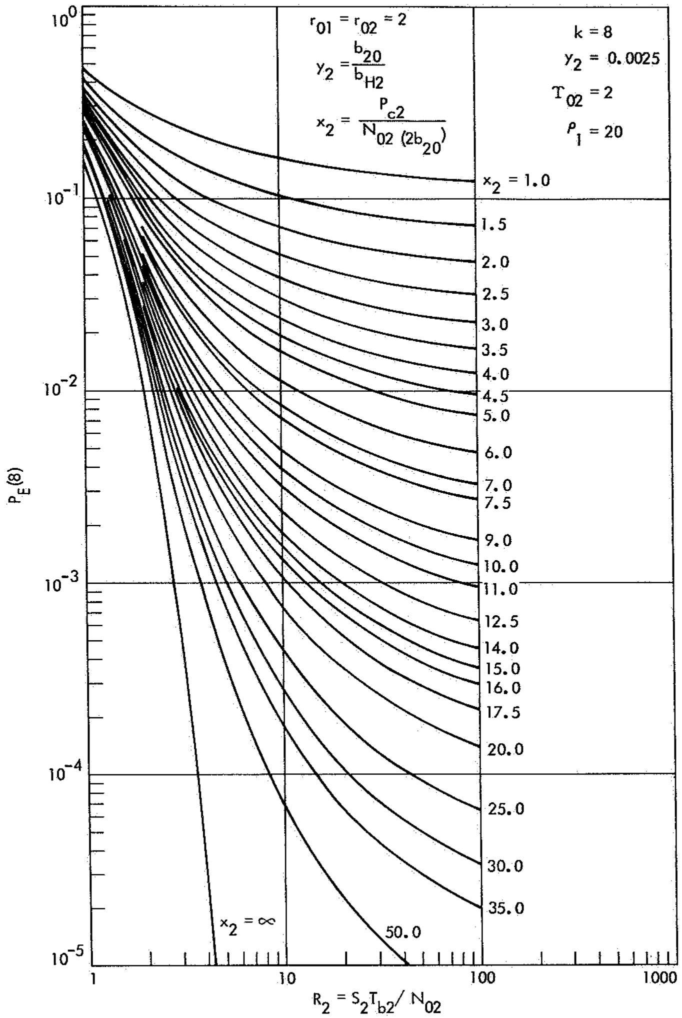

Fig. 17. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

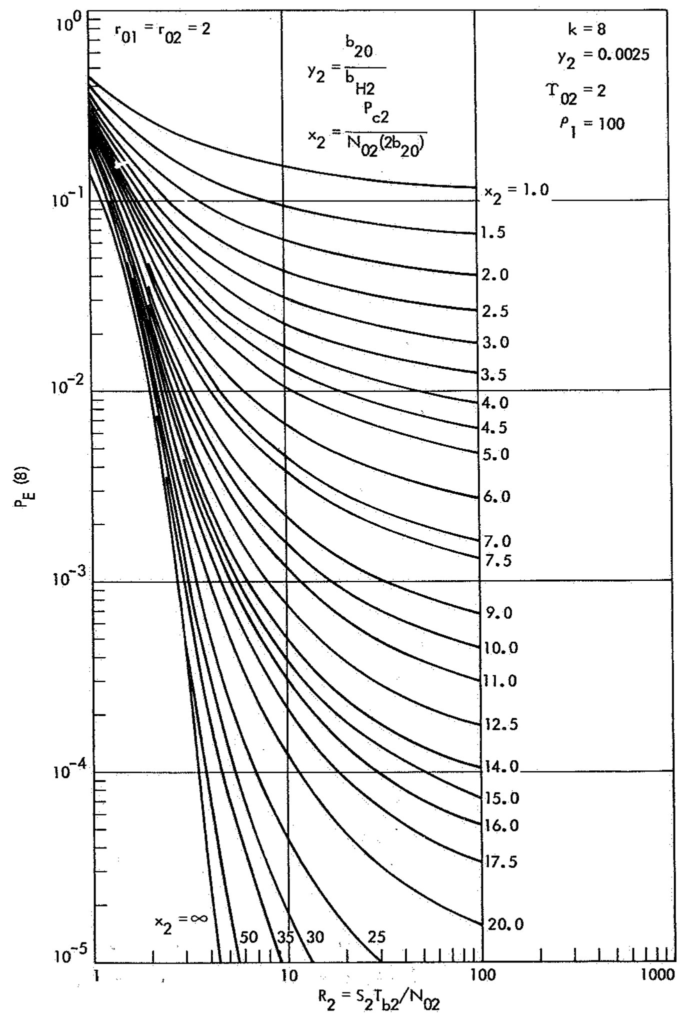

Fig. 18. Word-error probability vs signal-to-noise ratio R_2 for various values of the signal-to-noise ratio x_2

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