2

X-834-69-270

PREPRINT

NASA TM X-63649

RADAR ALTIMETER AS A SPACEBORNE NAVIGATION AID

F. KALIL

JULY 1968

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

X-834-69-270

RADAR ALTIMETER AS A SPACEBORNE NAVIGATION AID

F. Kalil

July 1969

Goddard Space Flight Center
Greenbelt, Maryland

RADAR ALTIMETER AS A SPACEBORNE NAVIGATION AID

by

F. Kalil

ABSTRACT

The purpose of this report is to show the feasibility of using a radar altimeter as a spaceborne navigation aid, particularly on manned spacecraft by showing, at least in a cursory way: (1) the areas or mission phases where it could be advantageous; and (2) that the power, antenna size, and other system requirements are both practical and within the present state-of-the-art. Although the mission analyses contained herein are simplified to permit a cursory evaluation of feasibility, there is sufficient information to permit an engineer, unversed in orbital mechanics to make some "quick-look, desk" computations. Furthermore, the appendices contain some parametric curves for "quick-look" evaluations of atmospheric drag effects on near-earth spacecraft.

PRECEDING PAGE BLANK NOT FILMED.

CONTENTS

PRECEDING PAGE BLANK NOT FILMED.

LIST OF FIGURES

LIST OF TABLES

Table

SUMMARY AND CONCLUSIONS

A radar altimeter could be a valuable navigation aid on-board a manned spacecraft, particular whenever the spacecraft is out-of-view of ground support facilities, such as

1. During certain abort trajectories whenever the spacecraft is within the useful range of the altimeter (~400 km).
2. When the spacecraft is at the backside of the moon or a planet.
3. Possibly, during the non-blackout periods of reentry.
4. While approaching the moon or planet.
5. While in orbit about the Earth, Moon or planet.

For instance, after one orbit about the central body (earth, moon, or planet), the spacecraft can use the altitude measurements in the following stepwise manner:

$$r = h + R$$

$$r_p = h_{min} + R$$

$$r_a = h_{max} + R$$

$$a = (r_a + r_p)/2$$

$$e = (r_a - r_p)/(r_a + r_p)$$

$$v_p = \sqrt{\mu\, r_a / a\, r_p}$$

$$v_a = \sqrt{\mu\, r_p / a\, r_a}$$

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a}\right)}$$

where r is the distance from the central body's center, μ is its gravitational constant, R is its radius, h is the spacecraft's altitude, "a" is the orbit's semi-major axis, e is the orbit's eccentricity, r_p and r_a are the minimum and maximum r (in the case of the earth, they are the perigee and apogee radii) and v is the spacecraft's speed.

Assuming altimeter measurement errors of ±100 m (rms, after smoothing), normally distributed and uncorrelated, and for an orbit about a spherical central body with R = 6367.4 km (the average earth's radius, h_p = 200 km, h_a = 400 km, then the corresponding rms errors in the above determined orbital elements would be:

$$\delta r = \delta r_p = \delta r_a = \delta h = 100 \text{ m}$$

$$\delta a = 141 \text{ m}$$

$$\delta e = 1.05 \times 10^{-5}$$

$$\delta v_a \lesssim \delta v \lesssim \delta v_p \approx 0.12 \text{ m/sec}$$

This leads one to conclude that the spaceborne altimeter could be a powerful navigation tool, which could also be used to determine the shape of the central body if "a priori" information is available from ground tracking data about the orientation of the orbit plane relative to the central body's axis. This latter use would be highly desirable (see ref. 20).

Regarding the altimeter system requirements, for $S/N = 14$ db, and effective antenna area of $1m^2$, a pulse width of 1μ sec, a pulse repetition rate of 500 pps (a duty cycle of 5×10^{-4}), an average radar cross-section per unit area of planet surface of 0.01, and at an altitude of 300 km, the required peak power output of the transmitter is 270 watts, which corresponds to an average power of 135 milliwatts. Off-the-shelf magnetrons are available with such capabilities. Furthermore, by using CW and phase-lock loop techniques to narrow the effective bandwidth, either the average power or antenna diameter may be reduced. Thus, it is concluded that the radar altimeter could be a practical and useful navigation aid, especially for manned spacecraft, and systems trade-off studies are warranted.

RADAR ALTIMETER AS A SPACEBORNE NAVIGATION AID

I. INTRODUCTION

The purpose of this report is to show the feasibility of using a radar altimeter as a useful navigation aid at altitudes of up to about 200 to 400 km.

Present day jet aircraft successfully use pulse type radar altimeters at altitudes of up to 12 km (40,000 ft). It will be shown that by increasing, within practical limits, the transmitter power output and antenna gain over and above what is presently used on jet aircraft, such an altimeter could be feasibly and successfully used at altitudes of up to about 200 to 400 km. Furthermore, the following section on mission analysis will show how such an altimeter could be used as a valuable navigation aid.

II. MISSION ANALYSIS

A radar altimeter could be a valuable navigation aid onboard a manned spacecraft, particularly during those mission phases of trajectories where the spacecraft is out-of-view of ground support facilities (stations, ships, or aircraft). For instance, it could be useful in the following conditions:

1. During contingencies such as abort trajectories especially when the spacecraft altitude is within the useful range of the altimeter, i.e., $\sim$400 km.

2. During lunar or planetary missions when the spacecraft is on the backside of the moon or planet, and the altimeter could be used to give a time history of altitude. If the spacecraft gets too close to the lunar or planetary surface, then corrective actions could be taken such as thrusting the spacecraft to a higher altitude.

3. Perhaps during the non-blackout periods of reentry. Although this area will not be discussed further in this report, it merits mentioning because of the frequent and large reentry footprints expected in the future with those space laboratory type missions which will utilize lifting body spacecraft with high lift-to-drag ratios (L/D) of about 2 or greater as "shuttle buses" to earth and back for logistic support. (See refs. 1, 2, 3, 4.)

4. While in an orbit around the earth, moon, or planets, wherein a time history of altitude could be used to determine the three Keplerian orbital elements which define the shape of the orbital conic but not its orientation, via the following basic equations, wherein first order and higher order effects due to gravitational perturbations, atmospheric drag, etc., are neglected here for the sake of simplicity. The first order gravitational effects and atmospheric effects are discussed in Appendices A, B.

Consider the spacecraft to be in an orbit around the earth, for example. Then the spacecraft distance (r) from the earth's center is related to spacecraft altitude (h) and the Keplerian orbital elements as follows:

$$r(t) = R_e + h(t) = \frac{a(1 - e^2)}{1 + e \cos.\theta(t)} \tag{1}$$

where as shown in Figure 1

R_e = radius of the earth, strictly speaking at the sub-satellite point

$h(t)$ = altitude above the earth's surface at time t

a = semi-major axis of the Keplerian orbit conic

e = eccentricity of the Keplerian orbit

$\theta(t)$ = true anomaly at time t

The angle θ is related to time t in the following, somewhat complicated, way:

$$t - t_p = \sqrt{\frac{a^3}{\mu}}\,(E - e \sin E) \tag{2}$$

where t_p = time at which the spacecraft passes through perigee

$$E = \cos^{-1}\left[\frac{a - r}{ae}\right] = \text{eccentric anomaly} \tag{3}$$

$$\sin E = \frac{r\ .\sin\theta}{a\sqrt{1 - e^2}} \tag{4}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1 + e}{1 - e}}\,\tan\frac{E}{2} \tag{5}$$

This latter equation is perhaps the most useful relation between θ, and E, since $\theta/2$ and $E/2$ are always in the same quadrant (see Figure 2.4, page 39, ref. 5). At minimum altitude (h_{min}), $\theta = 0$ and

$$r(t)_{min} = r_p = R_e + h_{min} = a(1-e) = \text{orbital radius at perigee} \tag{6}$$

At maximum altitude h_{max}, $\theta = \pi$ radians and

$$r(t)_{max} = r_a = R_e + h_{max} = a(1+e) = \text{orbital radius at apogee} \tag{7}$$

The spacecraft speed relative to the earth's center-of-mass can be determined from

$$v(t) = \sqrt{\mu\left(\frac{2}{r(t)} - \frac{1}{a}\right)} \tag{8}$$

$$v_p = \sqrt{\frac{\mu}{a}\left(\frac{1+e}{1-e}\right)}, \quad \text{Spacecraft's orbital speed at perigee} \tag{9}$$

$$v_a = \sqrt{\frac{\mu}{a}\left(\frac{1-e}{1+e}\right)} \quad \text{Spacecraft's orbital speed at apogee} \tag{10}$$

μ = earth's gravitational constant

$= 3.986\,032 \times 10^{14}\ m^3/sec^2$

Since the earth is not spherical in shape, and there are mountainous regions, a good approximation is obtained by replacing R_e with the average radius of the earth $\langle R_e \rangle$ in the equations above. Using the constants given in reference 6, we have

$$\langle R_e \rangle \cong \frac{\int_0^{\pi/2} R_{eq} \, (1 - f \sin^2 \theta) \, d\theta}{\frac{\pi}{2}} \tag{11}$$

where here θ = geocentric latitude

R_{eq} = earth's equatorial radius

= 6,378.165 km

= 3443.934 n. mi.

f = earth's flattening

$$f = \frac{R_{eq} - R_{pole}}{R_{eq}} = \frac{1}{298.30} \tag{12}$$

R_{pole} = earth's polar radius

= 6,356.77 km

= 3432.38 n. mi.

Integrating equation 7,

$$\langle R_e \rangle = R_{eq} \left(1 - \frac{f}{2}\right) = 6367.47 \text{ km} = 3438.15 \text{ n. mi.} \tag{13}$$

Thus, it can be seen from the above discussion that adequate equations exist so that a small onboard computer could be used to determine the three Keplerian elements a, e, and t_p. It should be noted that these three elements define the conic but not the orientation of the conics plane in space, wherein it is assumed that there is no "a-priori" knowledge of the spacecraft's position vector or velocity.

Let us now digress momentarily and examine roughly how the altimeter's accuracy affects its capabilities as a spaceborne navigation aid. Performing a regression of h on t to get a least squares curve fit for the data and to thence get a "band" or "error" for h_{min} is beyond the scope of this report. Furthermore, such a sophisticated approach is not warranted at this time because of the many simplifying assumptions that are used. Hence, the following simplified analyses are only cursory, and their only purpose is to show feasibility. Assuming for simplicity that the spacecraft has completed one orbital revolution about a spherical body of radius R, then

$$r(t) = h(t) + R \tag{14}$$

$$r_p = h_{min} + R \tag{15}$$

$$r_a = h_{max} + R \tag{16}$$

$$a = \frac{r_a + r_p}{2} \tag{17}$$

$$e = \frac{r_a - r_p}{r_a + r_p}. \tag{18}$$

$$v_p = \sqrt{\frac{\mu \, r_a}{a \, r_p}} \tag{19}$$

$$v_a = \sqrt{\frac{\mu \, r_p}{a \, r_a}} \tag{20}$$

$$v(t) = \sqrt{\mu \left[\frac{2}{r(t)} - \frac{1}{a} \right]} \tag{21}$$

Neglecting variations in the surface terrain of the central body for simplicity, then the errors in r_p and r_a are

$$\delta r_p = \delta h_{min} = \delta h; \text{ the altimeter's read-out accuracy after smoothing} \tag{22}$$

$$\delta r_a = \delta h_{max} = \delta h \tag{23}$$

$$\delta r = \delta h \tag{24}$$

Assuming a normal distribution in the altimeter measurement errors and they are uncorrelated

$$\delta a = \sqrt{\left(\frac{\partial a}{\partial r_a} \, \delta r_a\right)^2 + \left(\frac{\partial a}{\partial r_p} \, \delta r_p\right)^2} \tag{25}$$

$$\therefore \delta a = \sqrt{2} \, \delta h \tag{26}$$

In a similar manner, we obtain for the error in e,

$$\delta e = \frac{\delta h}{2a^2} \sqrt{r_a^2 + r_p^2} \tag{27}$$

$$\delta e \approx \frac{\sqrt{2}}{2a} \delta h, \text{ for } 0 \lesssim e \lesssim 0.2 \tag{28}$$

while for the error in v, we get

$$\delta v \doteq \frac{\mu}{v} \delta h \sqrt{\frac{1}{r^4} + \frac{1}{16 a^4}} \tag{29}$$

$$\delta v \cong \frac{\mu}{v} \frac{\delta h}{r^2} \cong v \frac{\delta h}{r}, \text{ for } 0 \lesssim e \lesssim 0.2 \tag{30}$$

These latter equations are applicable for any point in the orbit, i.e., they can be used to determine δv_p or δv_a by merely substituting r_p or r_a for r, respectively.

Consider the case where $\delta h = 100$ m (an assumed rms value after smoothing), $r_p = 200$ km + R, $r_a = 400$ km + R, and assume that R = 6367.47 km, which is the average earth's radius. Then

$$r_p = 6567.47 \text{ km}$$

$$r_a = 6767.47 \text{ km}$$

$$a = 6667.47 \text{ km}$$

$$e = 0.015$$

$$\delta r = \delta r_p = \delta r_a = \delta h = 100 \text{ m (rms)}$$

$$\delta a = 141 \text{ m (rms)}$$

$$\delta e = 1.05 \times 10^{-5} \text{ (rms)}$$

Since δv_p represents the worst case, i.e., δv is maximum at perigee for a given δh, we will look at δv_p:

$$\delta v_p \cong 0.12 \frac{\text{m}}{\text{sec}} \text{ (rms)}$$

Thus, it can be seen from the above example, that the radar altimeter could be a very powerful tool as a navigation aid. Furthermore, if the spacecraft had some "a-priori" knowledge about the orientation of its orbital plane relative to the planet's axis, then the altitude data could be useful in determining the shape of the planet.

The reader is cautioned that the above analyses were simplified to demonstrate feasibility and hence are only cursory. For instance a spherical central body was assumed, whereas the earth is an oblate spheroid to first order and not a smooth spheroid at that because of its mountain ranges, etc. Furthermore, atmospheric drag effects were neglected. Appendices A and B show to first order how the earth's oblate shape (neglecting the effect of mountain ranges) and atmospheric drag, respectively, affect the spacecraft's altitude. Thus, it can readily be seen that further analyses are warranted. However, it is believed that such detailed analyses would be so cumbersome that they would detract from the intended purpose of this report, and hence should be the subject of a separate report.

III. SYSTEM ANALYSIS

Let us now examine the altimeter from the viewpoint of transmitter power and antenna gain requirements.

The two-way range equation for a pulse type radio altimeter is derived as follows:

The incremental signal power (dS_{gnd}) received by the increment of ground area dA_{gnd} (see Figure 2) is

$$dS_{gnd} = \frac{P_T\, G_T\, F_T\,(\theta,\,\phi)\,|d\vec{A}|}{4\pi\, r^2\, L_T} \tag{31}$$

where P_T = transmitter output power

$$G_T = \frac{4\pi\, A_{T,eff}}{\lambda^2} = \eta_R\, \frac{4\pi}{\theta_T^2}. \tag{32}$$

G_T = transmitter antenna power gain

$A_{T,eff}$ = effective transmitter antenna area

$F_T(\theta, \phi)$ = transmitting antenna radiation pattern

λ = carrier wavelength

η_R = antenna radiating efficiency

$\cong$ 0.5 for a circular aperture

θ_T = transmitted beamwidth at the half power points

L_T = transmitter line losses

$|\vec{dA}|$ = component of dA_{gnd} in the direction of incident radiation

$$|\vec{dA}| = dA_{gnd} \cos\theta$$

$$= \frac{h^2 \tan\theta \, d\theta \, d\phi}{\cos\theta} \tag{33}$$

h = altitude of spacecraft altimeter above earth's surface

r = distance of dA_{gnd} from altimeter antenna

$$r = \frac{h}{\cos\theta} \tag{34}$$

Let σ_0 be the radar cross-section (or scatter cross-section) per unit area of illuminated ground. By definition the radar cross-section (or scatter cross-section) of a target is the area intercepting that amount of power which, when scattered equally in all directions, produces an echo at the radar equal to that from the target. In other words

$$\sigma = 4\pi \frac{\text{power scattered back at the source/unit solid angle}}{\text{Power/unit area striking target}} \tag{35}$$

In general, σ_0 is a function of θ, so that henceforth it will be written as $\sigma_0(\theta)$.

The signal power received by the altimeter at the receiver input terminals from the target dA_{gnd} is

$$dS = \frac{dS_{gnd}\, A_R(\theta, \phi)}{4\pi r^2 L_R}\, \sigma_0(\theta) \tag{36}$$

where L_R is the altimeter receiver line loss, and $A_R(\theta, \phi)$ is the effective receiving aperture perpendicular to r and is given by

$$A_R(\theta, \phi) = \frac{G_R F_R(\theta, \phi)\, \lambda^2}{4\pi} \tag{37}$$

where G_R is the receiving antenna power gain, and $F_R(\theta, \phi)$ is its radiation pattern.

Assuming that the altimeter transmitting and receiving apertures are the same, then

$$dS = \frac{P_T G^2 F^2(\theta, \phi)\, \lambda^2 \sigma_0(\theta) \cos\theta\, dA_{gnd}}{64\pi^3 r^4 L_T L_R}\,. \tag{38}$$

and the total signal power is

$$S = \frac{P_T G^2 \lambda^2}{64\pi^3 L_T L_R} \int_{A_{gnd}} \frac{F^2(\theta, \phi)\, \sigma_0(\theta) \cos\theta\, dA_{gnd}}{r^4} \tag{39}$$

but, as can be seen from Figure 2,

$$r = \frac{h}{\cos\theta} \tag{40}$$

$$dA_{gnd} = \frac{h^2 \tan\theta \, d\theta \, d\phi}{\cos^2\theta} \tag{41}$$

so that

$$S = \frac{P_T G^2 \lambda^2}{64 \pi^3 h^2 L_T L_R} \int_\theta \int_\phi F^2(\theta,\phi)\, \sigma_0(\theta) \sin\theta \cos\theta \, d\theta \, d\phi \tag{42}$$

Shown in Figures 3, 4, and 5 (see refs. 7, 8, 9, 10) are some radar cross-sections for roads, forrest and sea at X-band (10 GHz) as a function of γ, the angle of incidence (sometimes called grazing incidence or grazing angle), where γ is the complement of the angle $\theta/2$ used in the above equations and shown in Figure 2, i.e., $\gamma = 90 - \theta/2$. Figure 5 also shows the frequency dependence of σ_0. Thus, it can be seen that the evaluation of the above integral can be difficult except in some simple cases. The above expression is valid for C-W or pulse type altimeters which are beamwidth limited, that is the beamwidth is narrow enough that the difference in the two-way time delay between a ray along the center of the beam and a ray along the edge (or half power point) of the beam is less than the pulsewidth (τ). Thus the radius (ℓ) of the maximum effective ground area illuminated by the beam is from Pythogorean's theorem (see Figure 6)

$$\ell \leq \sqrt{\left(h + \frac{c\tau}{2}\right)^2 - h^2} \quad . \tag{43}$$

$$\leq \sqrt{h c \tau + \left(\frac{c\tau}{2}\right)^2}$$

Since

$$\frac{c\tau}{2} \ll h c \tau \tag{44}$$

$$\ell \lesssim \sqrt{h c \tau}$$

But

$$\ell \approx h \frac{\theta_B}{2} \tag{45}$$

and

$$\theta_B^2 = \frac{4\pi}{G_D} \tag{46}$$

where θ_B is the beamwidth at the half power points of the radiated beam, and G_D is the directional gain and is related to the power gain G by

$$G = \eta_R G_D \tag{47}$$

$$\eta_R = \text{radiating efficiency}$$

$$\therefore h_{max} \cong \frac{\eta_R C\tau G}{\pi} \cong \frac{C\tau G}{4} \tag{48}$$

where here h_{max} is the maximum altitude for beamwidth limited operation for a pulse type altimeter.

Let us now evaluate the equation 21 for a simple case only to get a feeling for the magnitude of the so-called loop loss (S/P_T). Assume the beamwidth is narrow with a constant intensity, i.e., assume beamwidth limited operation, $\sigma_0(\theta)$ is constant over the narrow beamwidth, and $F(\theta;\phi) = 1$. The loop loss becomes

$$\frac{S}{P_T} = \frac{G^2 \lambda^2 \sigma_0^2}{32 \pi^2 h^2} \left[\frac{1 - \cos^2 \frac{\theta_B}{2}}{2} \right] \tag{49}$$

where $\theta_B/2$ is the half beamwidth for the case being considered.

By definition the directive gain is

$$G_D \equiv \frac{4\pi P_{max}}{\int_\Omega P(\theta, \phi)\, d\Omega} \tag{50}$$

or in words

$$G_D = \frac{\text{maximum radiation intensity}}{\text{average radiation intensity}}$$

$$G_D = \frac{4\pi \text{ (Maximum power radiated/unit solid angle)}}{\text{total power radiated}}$$

but

$$P(\theta, \phi) = F(\theta, \phi)\, P_{max} \tag{51}$$

$$d\Omega = \sin\theta\, d\theta\, d\phi \tag{52}$$

and F $(\theta, \phi) = 1$ for the case considered. Integrating equation 50, we have

$$\therefore G_D = \frac{2}{1 - \cos \frac{\theta_B}{2}} \tag{53}$$

and

$$\frac{S}{P_T} = \frac{\eta_R \, G \, \lambda^2 \, \sigma_0}{16 \, \pi^2 \, h^2} \left[\frac{1 + \cos \frac{\theta_B}{2}}{2} \right] \tag{54}$$

For small beamwidths

$$\cos \frac{\theta_B}{2} \cong 1 - \frac{\theta_B^2}{4} \tag{55}$$

$$\left[\frac{1 + \cos \frac{\theta_B}{2}}{2} \right] = 1 - \frac{\theta_B^2}{8} \approx 1$$

Using

$$G = \frac{4 \pi \, A_{T,eff}}{\lambda^2} \tag{56}$$

$$\frac{S}{P_T} \cong \frac{\eta_R \, A_{T,eff} \, \sigma_0}{4 \pi h^2} \tag{57}$$

This formula is accurate enough for the purposes of this paper and agrees with the results obtained in references 10, 11. This equation is valid for the C-W or beamwidth limited pulse type radar altimeters. It is interesting to note that, except for the dependence of σ_0 on frequency, the so-called loop loss is independent of frequency as long as beamwidth limited operation is maintained.

Let us now examine the transmitter power requirements for a particular case. Consider the altimeter to be used on a spacecraft as a navigation aid for determining the energy dependent orbital elements, i.e., the in-plane or Keplerian elements, as discussed earlier in the paper. Assume or consider the following:

$$A_{T,eff} = 1\text{m}^2$$

$$\tau = 1\mu\,\text{sec}$$

$$B \cong = 10^6\ \text{Hz, effective receiver beamwidth} \tag{58}$$

$$\frac{S}{N} = 30 = 14.8\ \text{db, ratio of signal power to noise power}$$

$$f = 10\ \text{GHz, carrier frequency}$$

$$\sigma_0 = 0.01 = -20\ \text{db}$$

$$N_R = 0.5 = -3\ \text{db}$$

$$L_T = L_R = 2 = 3\ \text{db, to be conservative}$$

$$h = 300\ \text{km}$$

Now the noise power N is

$$N = KT_{eff}\,B \tag{59}$$

where

$$K = 1.38 \times 10^{-23} \text{ watt-sec/deg} = \text{Boltzmann's constant},$$

T_{eff} is the effective receiver noise temperature at the receiver input terminals and is given by

$$T_{eff} = T_0 (F_0 - 1) + \frac{T_a}{L_R} + T_L \left(1 - \frac{1}{L_R}\right) \qquad (60)$$

where $T_0 = 290°K$, reference temperature

F_0 = receiver noise figure, and for a receiver with a parametric amplifier, a typical value is about 4 db = 2.5 (see ref. 12).

The T_a is the antenna noise temperature, sometimes called the sky noise temperature, and is due to the background radiation, side lobe noise, spillover, and atmospheric attenuation. For a receiver looking at the earth with a beamwidth equal to or less than the earth's diameter, $T_a \cong 300°K$ (see ref. 13). The T_L is the actual temperature of the receiver input terminals and will be assumed to be 290°K for a spaceborne receiver.

Thus, T_{eff} reduces to

$$T_{eff} \approx T_0 F_0 \qquad (61)$$

$$\approx 725°K$$

and the required peak transmitter power is from equation 37

$$P_T = \frac{(S/N)\ 4\pi h^2\ K\ T_{eff}\ B\ L_T\ L_R}{\eta_R\ A_{T,eff}\ \sigma_0}$$

$$= \frac{30\ (4\pi)\ (300\ \text{km})^2\ \left(1.38 \times 10^{-23}\ \frac{W - sec}{°K}\right)\ (725\ °K)\ (10^6\ Hz)\ (2)\ (2)}{0.5\ (1\ m^2)\ (10^{-6}\ Km^2/m^2)\ (10^{-2})} \tag{62}$$

= 271 watts, peak power

See also Table 1 which gives the above analysis in logarithmic form.

Table 1

Analysis of Altimeter Transmitter Peak Power Requirements for h = 300 km

Signal-to-Noise Power Ratio	S/N = 30	14.8 db
1/Spreading loss	$4\pi h^2 = 1.13 \times 10^6\ (km)^2$	60.5 db $(km)^2$
System Noise Temperature	$KT_{eff} = 10^{-20}$ W	-200.0 dbW/Hz
Noise bandwidth	$B = 10^6$ Hz	60 dbHz
Altimeter transmission loss	$L_T = 2$	3 db
Altimeter reception loss	$L_R = 2$	3 db
1/(radiating efficiency)	$\frac{1}{N_R} = 2$	3 db
1/(effective aperture)	$\frac{1}{A_{eff}} = 10^6\ (km)^{-2}$	60 db/km^2
1/(ground scatter cross-section per unit area)	$\frac{1}{\sigma_0} = 10^2$	20 db
Transmitter Peak Power* required	$P_T = 271$ W	24.3 dbW

*At a duty cycle of 5×10^{-4}, this corresponds to an average power of 135 milliwatts (see following analysis and Table 2).

To check if this example is beamwidth limited, we now compute the maximum altitude for beamwidth limited operation from

$$h_{max} \cong \frac{C \tau G}{4}$$

where

$$G = \frac{4 \pi A_{eff}}{\lambda^2}$$

$$\lambda = \frac{C}{f} = \frac{3 \times 10^8 \text{ m/sec}}{10^{10} \text{ cps}} = 3 \times 10^{-2} \text{ m}$$

$$\therefore G = \frac{4 \pi (1 \text{ m}^2)}{(3 \times 10^{-2} \text{ m})^2} = 1.4 \times 10^4 = 41.4 \text{ db}$$

$$h_{max} = \frac{3 \times 10^5 \text{ km/sec} (10^{-6} \text{ sec}) 1.4 \times 10^4}{4}$$

$$= 1.05 \times 10^3 \text{ km} \quad .$$

Hence for operational altitudes below 1,000 km this system is beamwidth limited.

Let us now examine roughly the pointing requirements, because they would affect the attitude control requirements. Since

$$A_{eff} \cong 0.5 \frac{\pi d^2}{4}$$

$$d \cong \left[\frac{8 A_{eff}}{\pi}\right]^{1/2} \cong 1.6 \text{m}$$

then

$$\theta_B \cong \frac{\lambda}{d} = \frac{3 \times 10^{-2}\ \text{m}}{1.6\ \text{m}} = 1.87 \times 10^{-2}\ \text{rad}$$

$$\theta_B \cong 1.7\ \text{deg}$$

This means that the altimeter must be pointed in the direction of the vertical axis towards the ground (i.e. in the direction of the negative zenith axis) with an accuracy of about $\pm\theta_B/2$ or ± 0.85 deg. which is well within the state-of-the-art even for manned spacecraft. Otherwise the systems gain would be significantly reduced whenever the return signal with the minimum time delay, i.e. the signal along the altitude axis, would be outside the half power beamwidth because of improper antenna orientation.

Perhaps it should also be pointed out here that as the altitude varies, then the Doppler information could also be utilized for navigation purposes, i.e. the two-way Doppler shift $\Delta f = 2\dot{h}/\lambda$, where $\dot{h}$ is the time rate of altitude change, and λ is the wavelength.

Regarding the problems which may be encountered with uneven terrain, particularly on close approaches, both a wide beam and narrow beam could be advantageously used. The wide beam would provide information regarding the average terrain while the narrow beam would correlate the target being approached with the average terrain.

It is interesting to note that the Apollo, lunar module, landing radar uses the principles discussed, i.e. it has one altitude sensor beam used also as a velocity sensor and three velocity sensor beams. Also, the Surveyor landing radar used the above principles.

The maximum pulse repetition rate for resolving altitude (range) ambiguities without having to resort to coding the pulses is derived as follows:

At altitude (h) of 300 km, the two-way propagation time ($2\,t_{prop}$)

$$2\,t_{prop} = \frac{2h}{C} = 2 \times 10^{-3} \text{ sec} \tag{63}$$

To avoid the necessity of coding the pulses in order to resolve altitude ambiguities, this two-way propagation time must be equal to or less than the pulse repetition period (T_r), i.e.

$$2\,t_{prop} = 2 \times 10^{-3} \text{ sec} \leq T_r \tag{64}$$

The pulse repetition rate (f_r) is

$$f_r = \frac{1}{T_r} = 500 \text{ pps.} \tag{65}$$

and the duty cycle is $\tau f_r = 5 \times 10^{-4}$.

Shown in Table 2 are some beacon magnetrons which can more than adequately meet the peak power requirement of 270 W and the duty cycle of 5×10^{-4}, which corresponds to an average power of only 135 milliwatts. Thus, it may be concluded that using an altimeter onboard a spacecraft as a navigation aid is feasible.

Table 2

Beacon Magnetrons

FIXED FREQUENCY
All units positive-pulsed except those indicated by an asterisk.

Model Number (ref. 14)	Freq. GHz	Min. Peak Power w	Duty Cycle	Pulse Width μs	Peak Anode Volt. kV	Peak Anode Cur. A	Approx. Weight oz.
MA-259	7.0-7.5	1kw	0.005	0.5	2.00	2.00	21
MA-221B	8.0-8.8	1.5	CW	–	0.45	0.02	11
MA-231B	8.0-8.8	20	CW	–	0.93	0.06	11
MA-221A	8.0-8.8	10	0.01	1.0	0.50	0.15	11
MA-221C	8.0-8.8	10	0.01	5.0	0.50	0.15	11
MA-221D	8.0-8.8	20	0.005	1.0	0.53	0.30	11
MA-231A	8.0-8.8	200	0.05	1.0	1.00	0.75	11
MA-250	8.5-9.6	4kw	0.005	0.5	3.75	4.00	21
MA-212B	8.8-9.6	1.5	CW	–	0.46	0.02	11
MA-212A	8.8-9.6	10	0.01	1.0	0.52	0.15	11
MA-232B	8.8-9.6	15	CW	–	0.93	0.06	11
MA-212C	8.8-9.6	10	0.01	5.0	0.52	0.15	11
MA-212D	8.8-9.6	20	0.005	1.0	0.56	0.30	11
MA-232A	8.8-9.6	150	0.05	1.0	1.00	0.75	11
MA-252A	8.8-9.6	250	0.005	0.5	1.15	0.65	11
MA-222*	9.3-9.4	7kw	0.002	1.0	5.50	4.50	48
MA-239	13.28-13.37	40	0.25	2.5	0.85	0.24	23
MA-239B	13.0-13.6	10	CW	–	0.95	0.06	23
MA-240	13.7-14.6	500	0.013	1.0	1.70	1.40	16
MA-240A	13.7-14.6	700	0.013	1.0	1.75	1.80	16
MA-245	15.4-15.6	20	0.50	5.0	1.00	0.14	23
MA-257	16.0-16.5	1kw	0.002	0.5	2.50	1.80	16
MA-256*	16.4-16.6	10kw	0.001	0.2	9.00	5.25	22
MA-246A	16.5-16.8	100	0.005	0.5	1.15	0.45	16

*Negative pulsed

Table 2 (Continued)

TUNABLE
All units positive-pulsed except those indicated by an asterisk.

Model Number	Freq. GHz	Min. Peak Power w	Duty Cycle	Pulse Width μs	Peak Anode Volt. kV	Peak Anode Cur. A	Approx. Weight oz.
MA-219F	8.5–9.0	3	CW	–	0.63	0.02	13
MA-219B	8.5–9.6	1	CW	–	0.47	0.02	13
MA-214H	9.0–9.5	3	CW	–	0.65	0.02	13
MA-214B	9.0–10.0	1	CW	–	0.47	0.02	13
MA-249	9.25–9.45	17	CW	–	1.00	0.05	13
MA-219A	8.5–9.6	10	0.005	1.0	0.53	0.15	13
MA-219C	8.5–9.6	20	0.01	1.0	0.56	0.30	13
MA-261	8.5–9.6	50	0.003	1.0	0.82	0.45	13
	8.9–9.4	400	0.0005	0.25	4.00	0.5	24
	8.9–9.4	900	0.003	1.0	4.30	0.8	24
	8.9–9.4	800	0.001	0.2	4.30	0.9	24
MA-214C	9.0–10.0	8	0.005	1.0	0.54	0.15	13
MA-214D	9.0–10.0	16	0.005	1.0	0.57	0.30	13
MA-241	9.1–9.4	150	0.001	1.0	2.75	1.0	24
MA-218*	9.3–10.0	7kw	0.002	1.0	5.90	4.5	48
MA-232T	9.5–9.8	150	0.002	1.0	1.00	0.7	13
MA-260	16.0–16.5	1kw	0.001	0.5	3.00	1.6	18

(per ref. 14)

If C-W operation is desired because a transistorized transmitter can be used without the need for filament power, then the average power output required is

$$P_{ave} = P_T \tau f_r = 270 (10^{-6}) (500) = 135 \text{ mW}$$

for the example being considered.

However, the bandwidth must also be reduced by an equivalent amount in order for the sample system being considered to operate at 300 km altitude and S/N = = 14.8 db. That is

$$B_{CW} = B_{pulse}\ \tau\ f_r = 500\ \text{Hz} \tag{67}$$

Using phase-lock loop techniques, this bandwidth is not only feasible but can be reduced an order of magnitude, in which case the above corresponding average power requirement can also be reduced an order of magnitude. However, it may be more desirable to reduce the antenna aperture instead of the average power. In any event, system trade-offs are possible, and the altimeter is a feasible navigation aid for use on spacecraft. Furthermore, as earlier analyses in this paper indicates, the altimeter's measurements could be useful in determining the shape of the moon or planet, if "a-priori" knowledge is available from ground tracking data about the orientation of the spacecraft's orbital plane relative to the axis of the central body. Determining the shape of the moon and planets is one of the significant goals of the scientific community (see ref. 20).

ACKNOWLEDGMENTS

I would like to thank W. P. Varson and J. P. Shaughnessy for their helpful comments and suggestions.

REFERENCES

1. Baradell, Donald L., and McLellan, Charles H., "Lateral-Range and Hypersonic Lift-Drag-Ratio Requirements for Efficient Ferry Service from a Near-Earth Manned Space Station," NASA Langley Research Center, Langley Station, Hampton, Virginia, Presented at the AIAA Second Manned Space Flight Symposium, Dallas, Texas, April 22-23, 1963.

2. LaFond, Charles D., "Von Braun Urges Rensable Transport," Missiles and Rockets, November 8, 1964, pp. 16.

3. "M2-L2 Lifting Body is Mated with B-52," Aviation Week and Space Technology, October 11, 1965, pp. 83.

4. Thomas, B. K., Jr., "USAF Nears Manned Lifting Body Tests," Aviation Week and Space Technology, July 10, 1967, pp. 99.

5. Battin, R. H., "Astronautical Guidance," McGraw-Hill Book Co., New York, New York, 1964.

6. "Apollo Missions and Navigation Systems Characteristics," by the Apollo Navigation Working Group, GSFC, Greenbelt, Maryland and MSC, Houston, Texas, Report No. AN 1.2, January 17, 1967.

7. Taylor, R. C., "Terrain Return Measurements at X, K_u, K Band," IRE National Convention Record, Vol. 7, pt. 1, pp. 19-26, 1959.

8. Katz, I., and Spetner, L. M., "Polarization and Depression Angle Dependence of Radar Terrain Return," National Bureau of Standards (U.S.) J. Research, Vol. 64D, pp. 483-486, September - October 1960.

9. Grant, C. R., and Yaplee, B. W., "Backscattering from Water and Land at Centimeter and Millimeter Wavelengths," Proc. IRE, Vol. 45, pp. 976-982, July 1957.

10. Skolnik, M. I., "Introduction to Radar Systems," McGraw-Hill Book Company, New York, New York, 1962.

11. "Minimum Performance Standards Airborne Low-Range Radar Altimeters," Radio Technical Commission for Aeronautics, Washington 25, D. C., Paper 96-63/DO-123, August 1963.

12. "Parametric Analysis of Microwave and Laser Systems for Communication and Tracking," by Hughes Aircraft Company, Culver City, California, for GSFC, Greenbelt, Maryland, Contract NAS5-9637, Report No. P65-149, November 30, 1965.

13. Martin, J. A., "Design Approaches for Satellite Antennas," Stanford Research Institute, Menlo Park, California SRI Project No. 3854, Final Report, February 1962.

14. "Master Catalog of Components and Subassemblies," Microwave Associates, Burlington, Massachusetts, 1966.

15. Kalil, F., "Minimum Altitude Variation Orbits about an Oblate Rotating Atmosphere on the Eccentricity, Semi-major Axis, and Period of a Close Earth Satellite," AIAA Journal, Vol. 1, No. 8, August 1963, pp. 1872-1878.

16. Izakov, M. N., "Allowance for a Variable Aerodynamic Drag Coefficient in Deriving the Air Density from Satellite Decelerations," (UDC 551.510.536; 533.6.013.12), translated from Kosmicheskie Issledovaniya, Volume 3, No. 2, March - April, 1965, pp. 297-308.

17. Sterne, T. E., "Effects of the Rotation of a Planetary Atmosphere Upon the Orbit of a Close Satellite," ARS Journal 29, 1959, pp. 777-782.

18. "U. S. Standard Atmosphere, 1962," U. S. Government Printing Office, Washington, D. C.

19. "Manned Space Science Programs Goals and Objectives – Prospectus 1966," OSSA, NASA Headquarters, Washington, D. C., February 1, 1966 (Draft).

BIBLIOGRAPHY

1. "Radar Altimeter and Doppler Velocity Sensor," page 225 in Space Programs Summary 37-41, Vol. IV; Supporting Research and Development, for the period Aug. 1 to Sept. 30, 1966, Jet Propulsion Laboratory, Pasadena, California, Oct. 31, 1966.

2. "Apollo Radar Works," Microwaves, June, 1969, pg. 30.

APPENDIX A

Earth Gravitational Perturbations to First Order

A near-earth satellite experiences altitude variations because of gravitational perturbations due to the non-spherical shape of the earth; i.e., the satellite orbit is not a true ellipse. The satellite orbit to first order is given by (ref. 15)

$$r = \overline{r}_0 \left[1 - e \cos (\beta - \omega) + \frac{J}{6}\left(\frac{R_{eq}}{\overline{r}_0}\right)^2 \sin^2 i_0 \cos 2\beta\right] \tag{A-1}$$

whenever the eccentricity e is of the order of J, where $\overline{r}_0$ is a constant of integration and has the physical significance of being essentially the mean semi-major axis of a perturbed orbit, $J = 1.6235 \times 10^{-3}$ is the coupling coefficient of the second harmonic in the earth's gravitational potential ($J = 3/2\ J_2$), R_{eq} = 6, 378.165 km (3,433.934 n. mi.) is the earth's equatorial radius, i_0 is the mean orbital inclination about which the instantaneous inclination varies, ω is the angle of perigee, and β is the central angle. (See Figure A-1 which depicts the orbital geometry.)

The satellite altitude h is

$$h = r - R_e \tag{A-2}$$

where the earth's local radius R_e is given by, to first order

$$R_e = R_{eq} (1 - f \sin^2 \theta) \tag{A-3}$$

and

$$f = \frac{1}{298.30} = \text{earth's flattening}$$

$$\theta = \text{geocentric latitude}$$

$$\sin \theta = \sin i_0 \sin \beta \tag{A-4}$$

For the case of $e = 0$, $\bar{r}_0 = R_{eq} + 185.2$ km (i.e. $h = 100$ n. mi. at the equator), and $i_0 = 90°$ (a polar orbit where the altitude variation are maximum; see equation A-1), the altitude varies sinusoidally about an average $(\bar{h})$ of ~200 km with an amplitude of 0.8 km as shown in Figure A-2. It is interesting to note that the earth's radius varies by 21.50 km from equator to pole, see Figure A-3. Hence, it may be concluded that in this example, the satellite is closely following the oblate shape of the earth.

APPENDIX B

Atmospheric Drag

A near earth satellite experiences a dissipative drag force as it goes through the earth's atmosphere. The drag force per unit satellite mass is

$$\vec{F} = - B \rho v \vec{v} \tag{B-1}$$

where B is the satellites ballistic coefficient defined by

$$B = \frac{C_D A}{2m} \tag{B-2}$$

C_D is the drag coefficient which depends on the satellite shape, its orientation as it moves through the air, the molecular density of the atmosphere, and the satellite's air speed (v). (See ref. 16). For massive, near-earth satellites such as Apollo, for instance, the $C_D = 2.0 \pm 0.2$ (see ref. 6). The A is the satellite cross-sectional area normal to the satellite air velocity $\vec{v}$, and m is the satellite mass.

This drag force is called a dissipative force because it reduces the total energy of the satellite via the air friction. To illustrate this, consider first for simplicity the case of a near-earth satellite in a circular orbit. Assume for this illustration that the earth's atmosphere is spherical in shape and is not rotating (the more general case is discussed below later on). The satellite's kinetic energy (T) per unit mass is $v^2/2$, and its potential energy (U) per unit mass is $-\mu/r$, where μ is the gravitational constant of the earth ($\mu = 3.986\ 032 \times 10^{14}$ m^3/sec^2; ref. 6). The satellite's total energy (E) per unit mass is

$$E = T + U \tag{B3}$$

In a Keplerian orbit

$$v^2 = -\mu \left(\frac{2}{r} - \frac{1}{a}\right) \tag{B4}$$

where

a = semimajor axis of the elliptic orbit

Substituting from Equation B4 into Equation B3, the satellite's total energy per unit mass becomes

$$E = -\frac{\mu}{2a}, \text{ elliptic orbit} \tag{B5}$$

In the case of a circular orbit ($a = r$),

$$E = -\frac{\mu}{2r}, \text{ circular orbit} \tag{B6}$$

The atmospheric drag changes the satellite's total energy by an amount W_D, the work done by the drag force, i.e.

$$dE = dW_D = \vec{F}_D \cdot \vec{dr} \tag{B7}$$

The amount of change in total energy in one orbital revolution is

$$\frac{\Delta E}{rev} = \frac{\Delta W_D}{rev} = \int_{rev} \vec{F}_D \cdot \vec{dr} \tag{B8}$$

and also, from Equation B6

$$\frac{\Delta E}{rev} = \frac{\mu}{2r^2} \frac{\Delta r}{rev} \tag{B9}$$

Since F_D is antiparallel to $\vec{dr}$, and is essentially constant over one orbital revolution for a circular orbit in a spherical atmosphere

$$\int_{rev} \vec{F}_D \cdot \vec{dr} \doteq F_D \int_{rev} dr = -2\pi r B \rho v_c^2 \tag{B10}$$

where v_c is the circular orbit speed and is given by

$$v_c^2 = \frac{\mu}{r} \tag{B11}$$

so that

$$\frac{\Delta E}{rev} = -2\pi B \rho \mu \tag{B12}$$

The minus sign here shows that the dissipative drag force decreases the total energy of the satellite. Combining Equations B9 and B12, and solving for $\Delta h/rev$ which is equal to $\Delta r/rev$,

$$\frac{\Delta h}{rev} = -4\pi B \rho r^2, \quad \text{for circular orbit, and a circular non-rotating atmosphere} \tag{B13}$$

The minus sign merely indicates that the altitude is decreasing due to atmospheric drag.

The case for an elliptic orbit in an oblate rotating atmosphere has been treated for the cases of $0 \leq e \leq 0.01$ (ref. 17), and $e \geq 0.01$ (ref. 18), and will not be treated here except to make the following observations. It is interesting to note that both the rotation of the atmosphere and its oblate spheroidal shape contribute significantly to the orbital decay. For instance, for a circular orbit in the earth's oblate, rotating atmosphere

$$\frac{\Delta h}{rev} = -4\pi B r^2 (1-d)^2 I_0 (q/2) \rho_\pi \epsilon^{-q/2} \tag{B14}$$

where $I_0 (q/2)$ is the Modified Bessel function of the first kind; $I_0 (q/2) \epsilon^{-q/2}$ accounts for the atmosphere's oblateness; $(1-d)^2$ accounts for the atmosphere's rotation;

ρ_π = atmosphere density at the minimum altitude (h_{min}) of the satellite.

$$q = K R_{eq} f \sin^2 i_0 \tag{B15}$$

$\rho = \rho_\pi \, \epsilon^{-Kz}$, atmosphere density at altitude h

$K = -(d/dz) \ln \rho$, lapse rate of the atmosphere (see ref. 19) (B16)

ϵ = base of the natural logarithms

z = height above the minimum altitude (h_{min})

f = 1/298.30, earth's flattening

R_{eq} = 6,378.165 km, earth's equatorial radius

$d = \Omega_g / n \, (1 - e^2)^{1/2} \cos i_0$ (B17)

e = orbital eccentricity; and is zero in the case for the circular orbit being considered.

$\Omega_e = 4.178\,07416 \times 10^{-3}$ deg/sec, earth's rotational speed about its axis

$n = 2\pi/\tau$, satellite's mean motion (B18)

τ = anomalistic period, i.e. the time in orbit from perigee to perigee

i_0 = orbital inclination

For near-earth orbits and $e \leq 0.01$,

$$d \cong \frac{1}{15} \cos i_0 \tag{B19}$$

From the above equations, it can be seen that: the effect of the atmosphere's rotation on the satellite's altitude decay (Δh/rev), due to the $(1 - d)^2$ term, could be as large as ± 13% per orbital revolution and is greatest when $\cos i_0 = 1$; while the effect of the atmosphere's oblateness on the satellite's altitude decay (Δh/rev), due to the $I_0 \, (q/2) \, \epsilon^{-q/2}$ term, could be as large as -25% at satellite altitudes of 200 km and is greatest when $\sin i_0 = 1$. Hence, both the oblate shape and the rotation of the atmosphere are significant contributors to the satellite's altitude decay.

If we define an effective ballistic coefficient as

$$B_{eff} = B_{actual}\,(1 - d)^2\, I_0\,(q/2)\,\epsilon^{-q/2} \tag{B20}$$

where the actual ballistic coefficient is

$$B_{actual} = \frac{C_D\,A}{2m} \tag{B21}$$

then

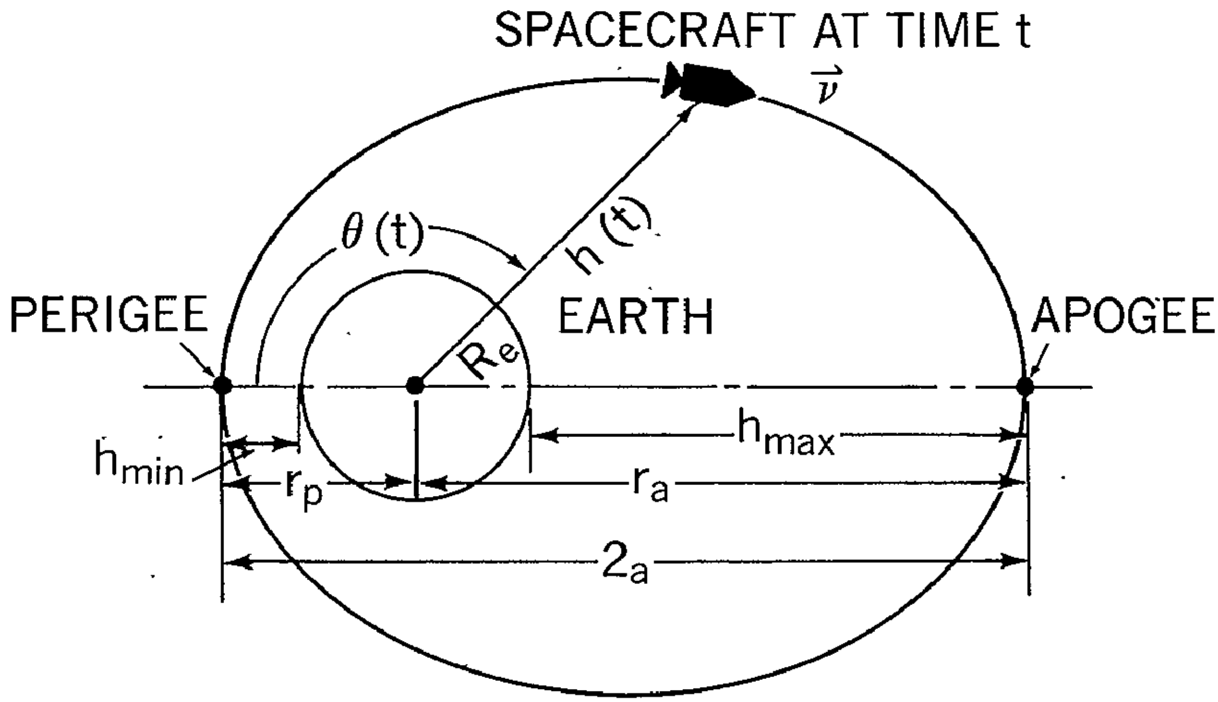
$$\frac{\Delta h}{rev} = -4\pi\, B_{eff}\,\rho\, r^2 \tag{B22}$$

For circular earth orbits and a nominal atmosphere, Figure B1 shows the ratio $\Delta h/rev/B_{eff}$ as a function of satellite altitude, while Figure B2 shows the ratio B_{eff}/B_{actual} as a function of both altitude and inclination angle. To use these figures, one must know the satellites actual ballistic coefficient, i.e. $C_D A/2m$, the circular orbital altitude, and the orbital inclination. Then one may obtain:

(a) the B_{eff} from the B_{eff}/B_{actual} given in Figure B2.

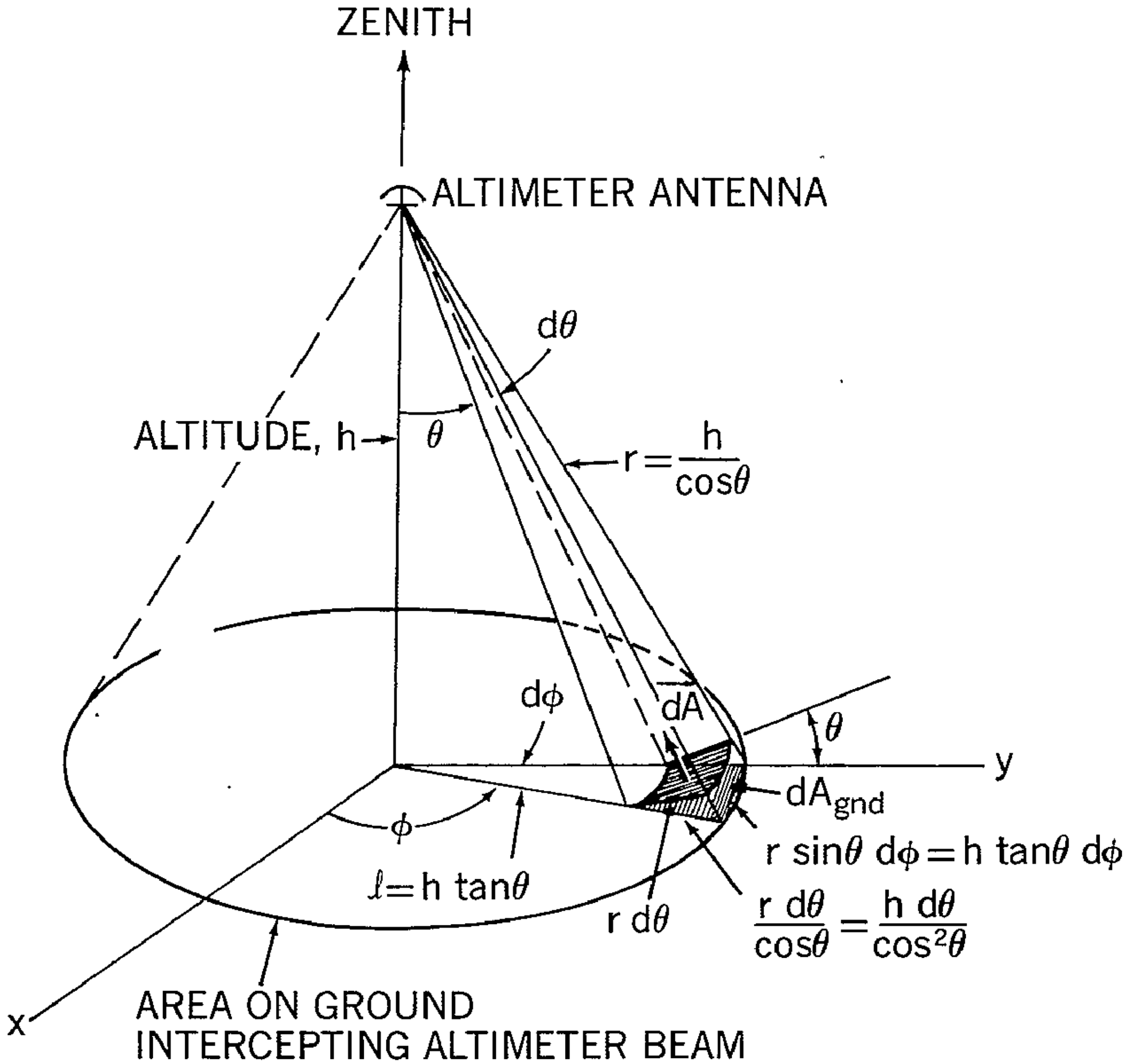
(b) the $\Delta h/rev$ from the $\Delta h/rev/B_{eff}$ given in Figure B1.

It should be pointed out that in the above analysis and considerations, a Keplerian type orbit was assumed. What needs to be considered further is both the gravitational perturbations (at least to first order) and the oblate, rotating atmosphere simultaneously acting on the satellite's motion in order to derive a closed form solution of the type given above.



$$r\ (t) = R_e + h(t) = \frac{a\ (1 - e^2)}{1 + e\ \cos\theta(t)}$$

Figure 1—Orbital Geometry Depicting the Keplerian Orbital Elements.



$$\left|\overrightarrow{dA}\right| = dA_{gnd} \cos\theta = \frac{h^2 \tan\theta \, d\theta \, d\phi}{\cos\theta}$$

Figure 2–Geometry for Computing Effective Ground Reradiating Area.

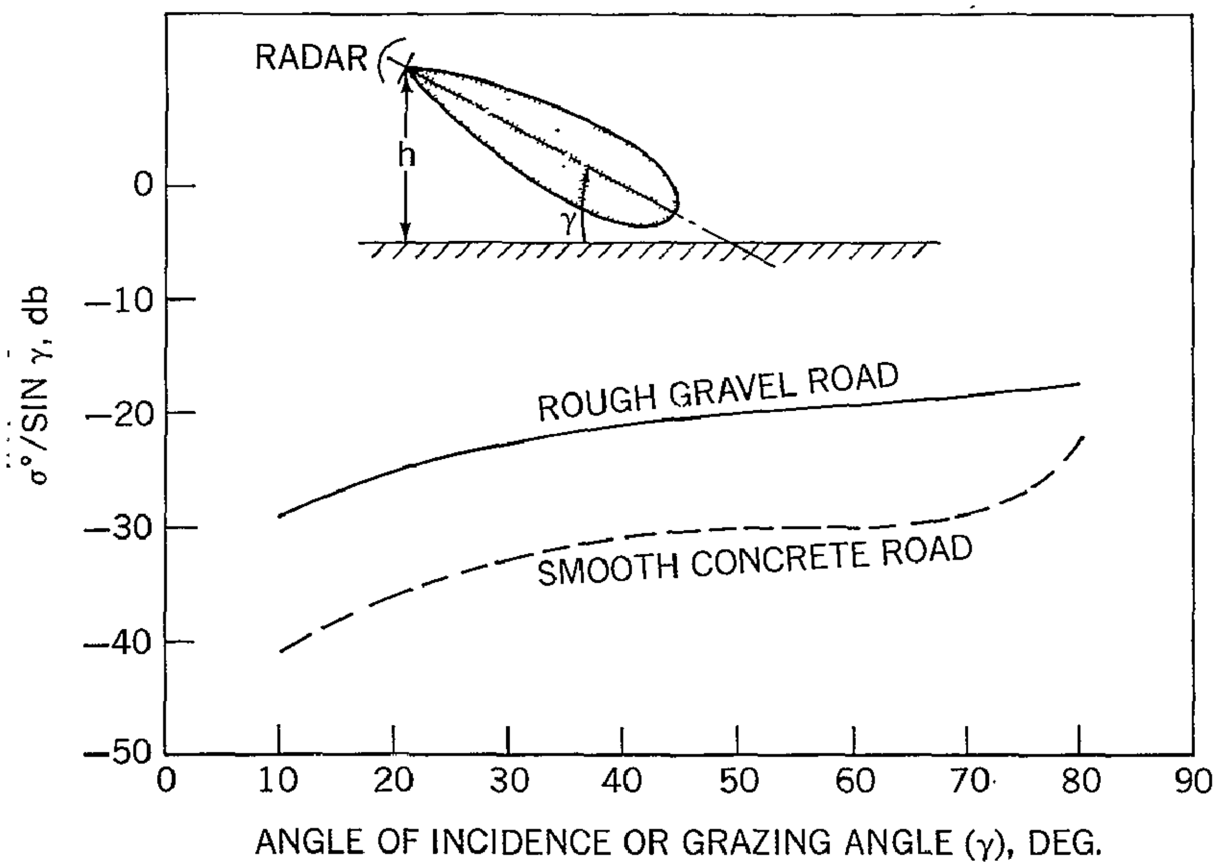


Figure 3—Radar Cross-section (σ_0) of Gravel and Concrete Roads at X-Band (10 GHz) versus Angle of Incidence (γ).

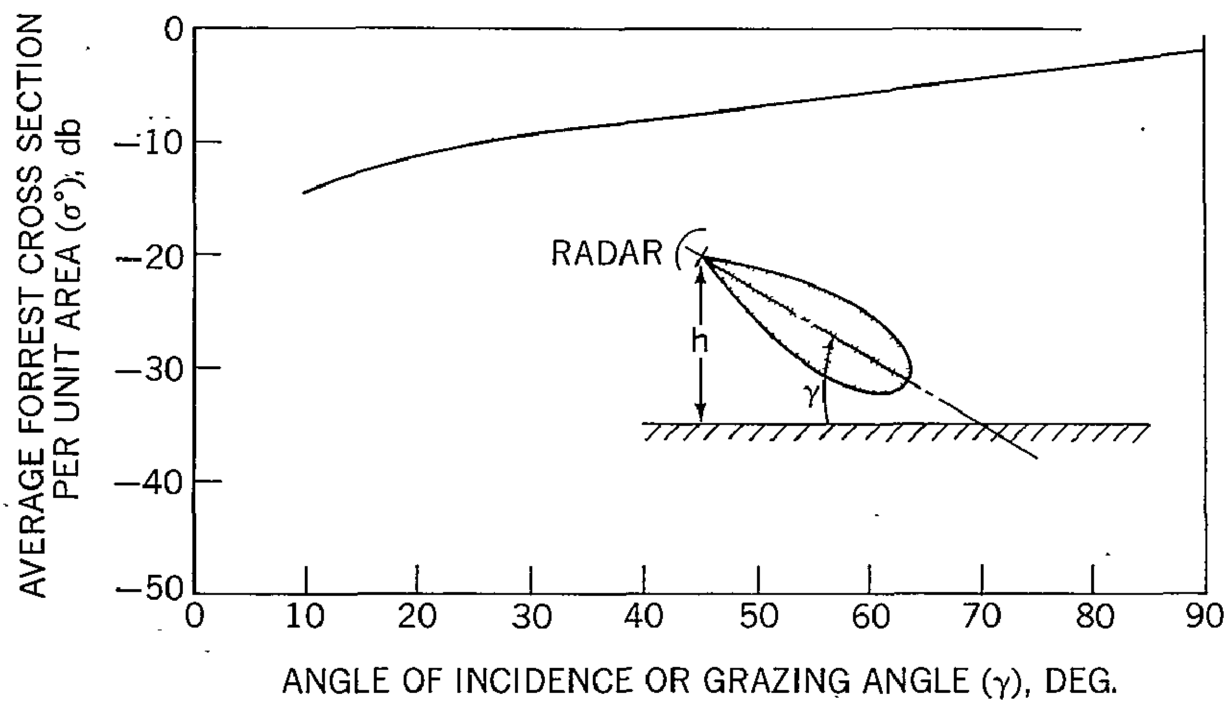


Figure 4–Average Radar Cross-section per Unit Area of Forrest (σ_0) as a Function of Incidence Angle (γ) at X-Band (10 GHz).

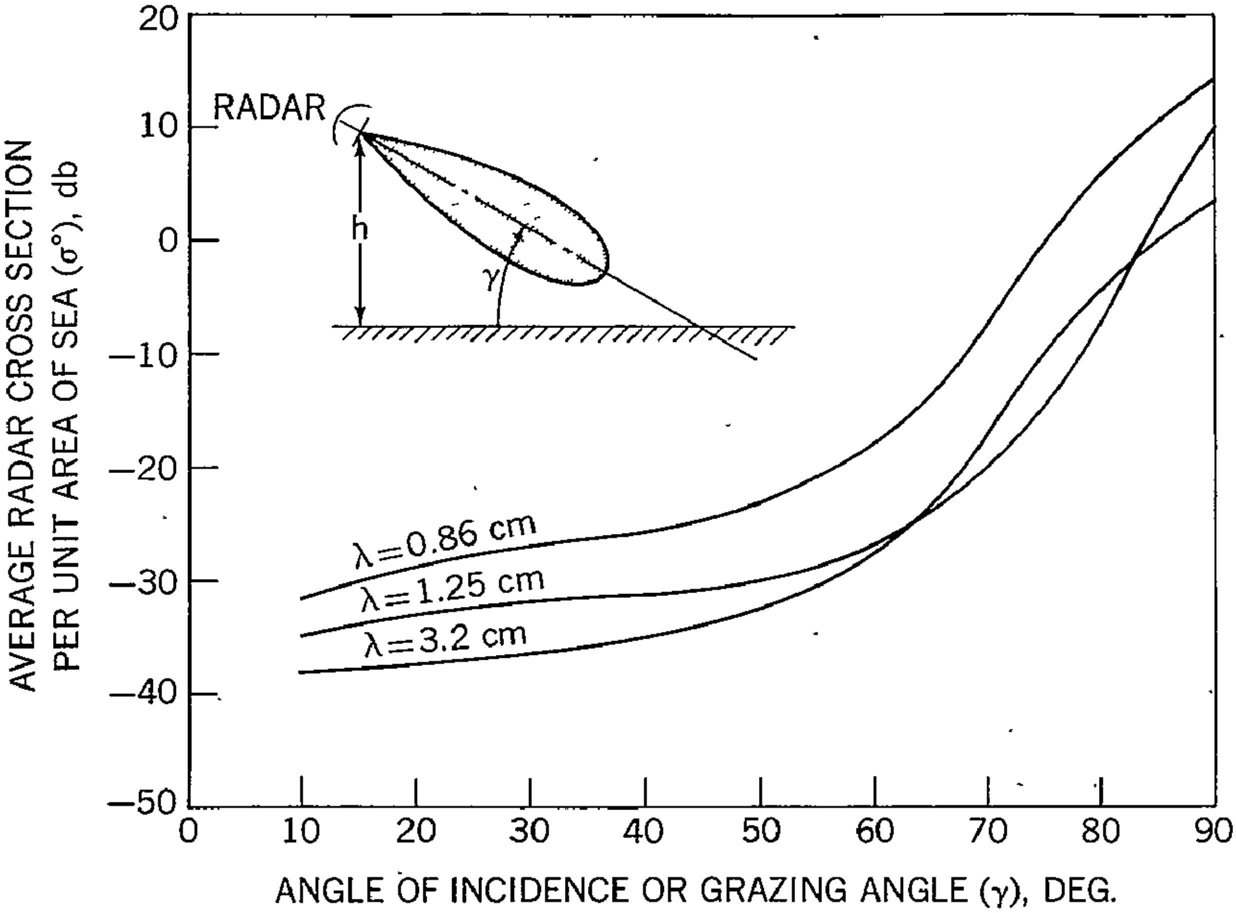


Figure 5–Average Radar Cross-section per Unit Area of Sea as a Function of Wavelength and Incidence Angle (γ).

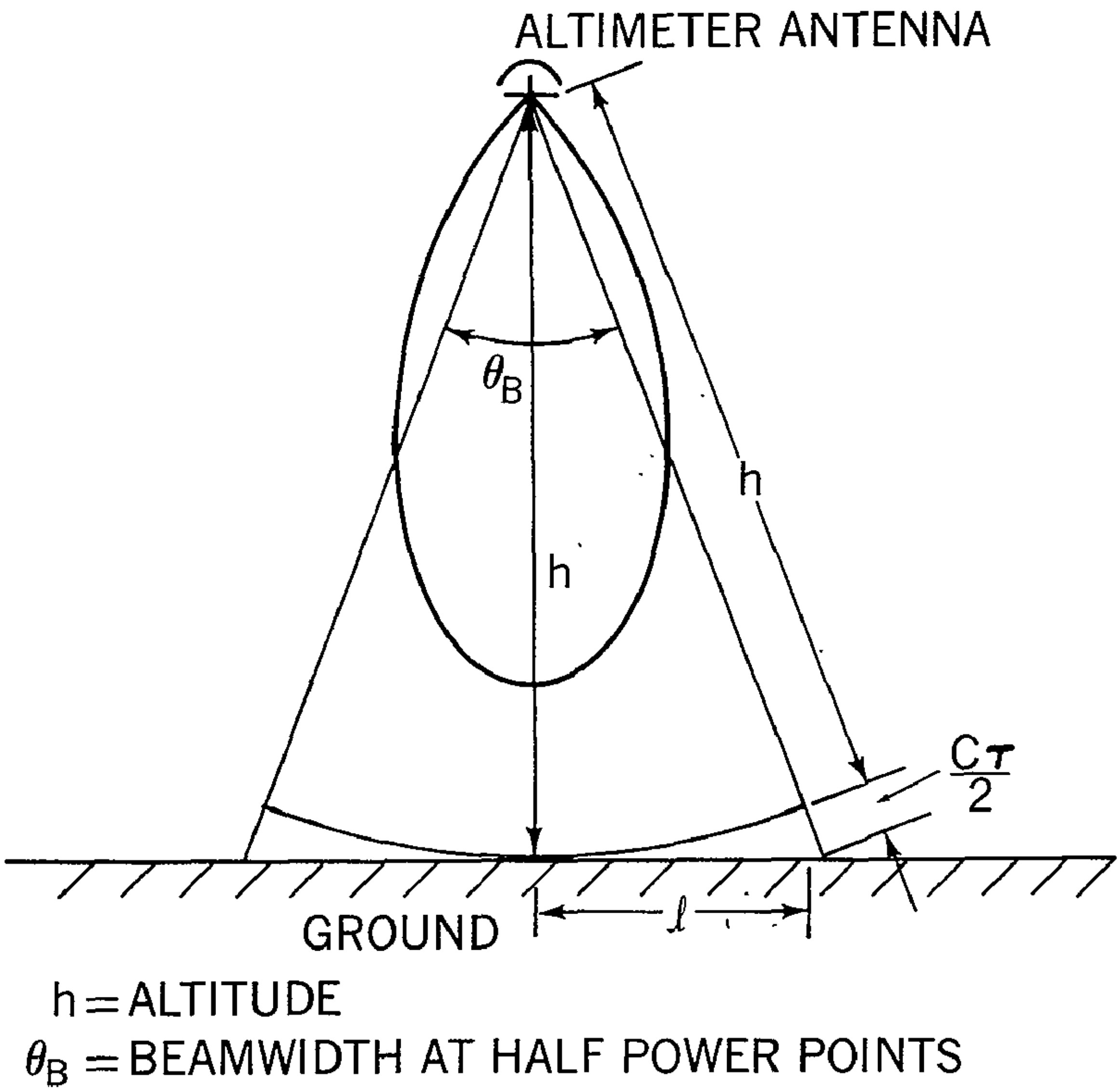
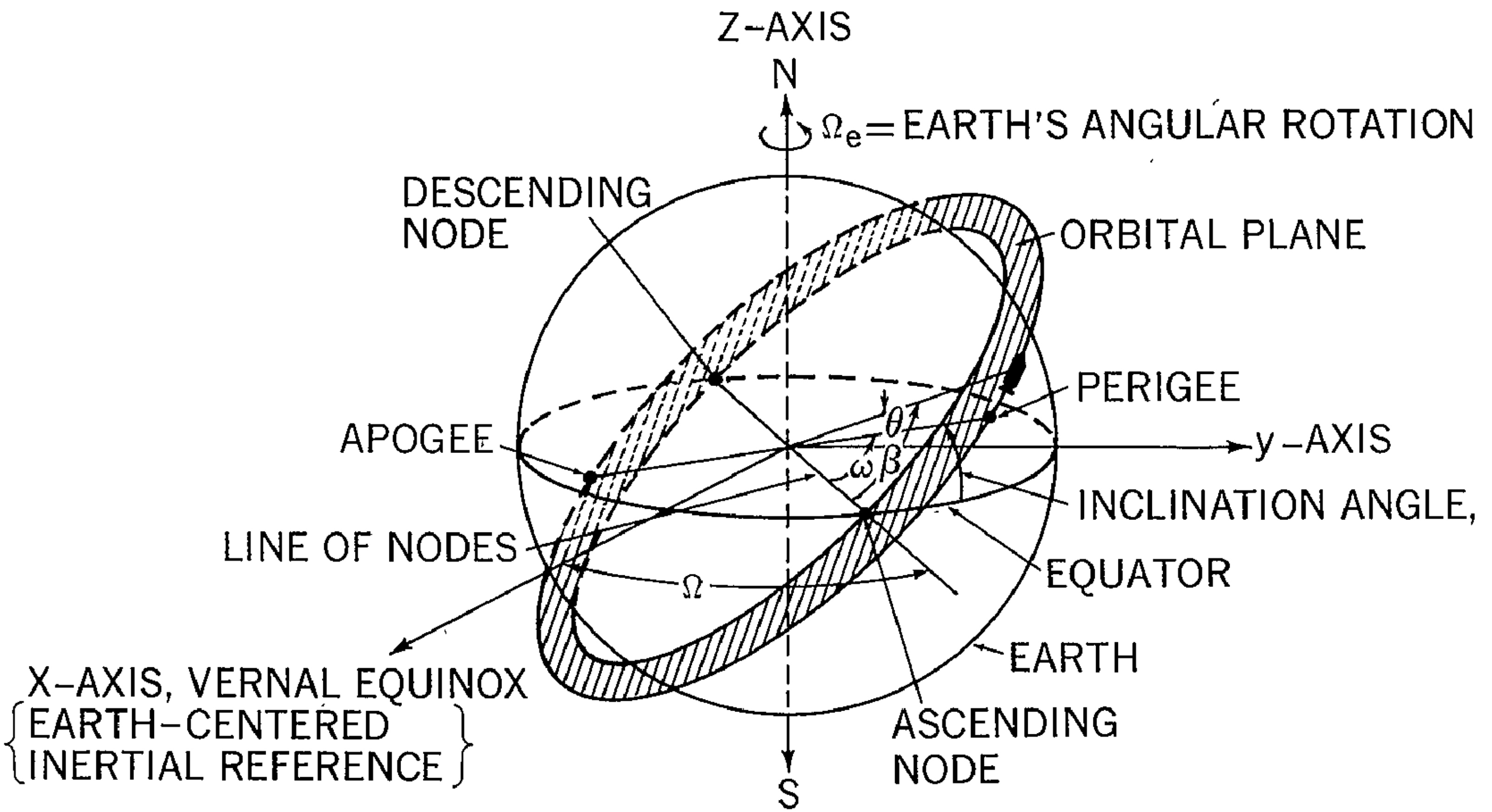


Figure 6–Geometry for Computing the Maximum Altitude for Beamwidth Limited Operation of a Pulsed Type Altimeter.



NOTE: IN THE CASE OF A CIRCULAR ORBIT (e=O), THE PERIGEE IS UNDEFINED.

Figure A1–Orbital Geometry Depicting the Euler Angles.

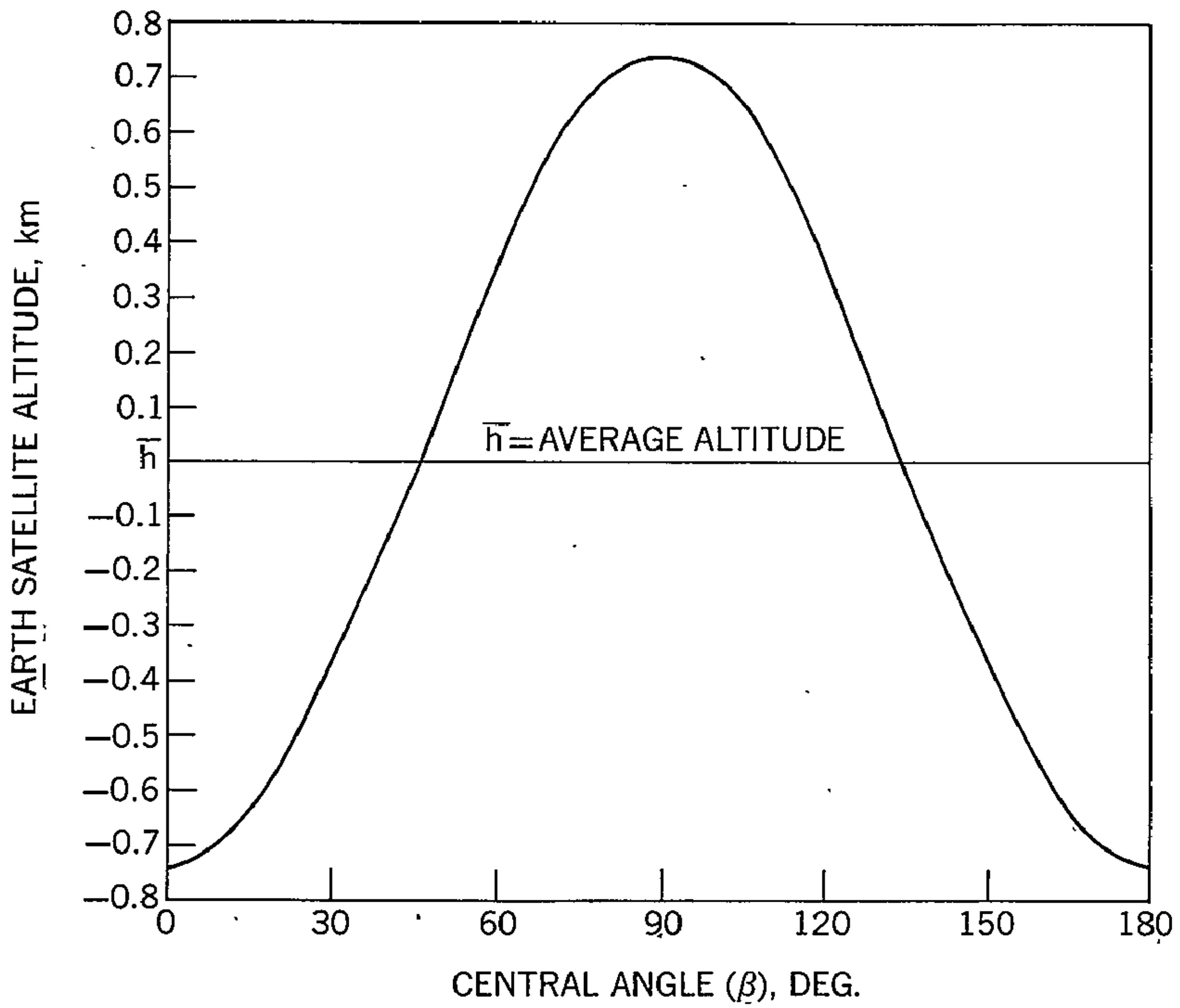


Figure A2–Satellite Altitude Variations About an Oblate Earth to First Order; $e = 0$, $i = 90°$, $\bar{h}' = 200$ km.

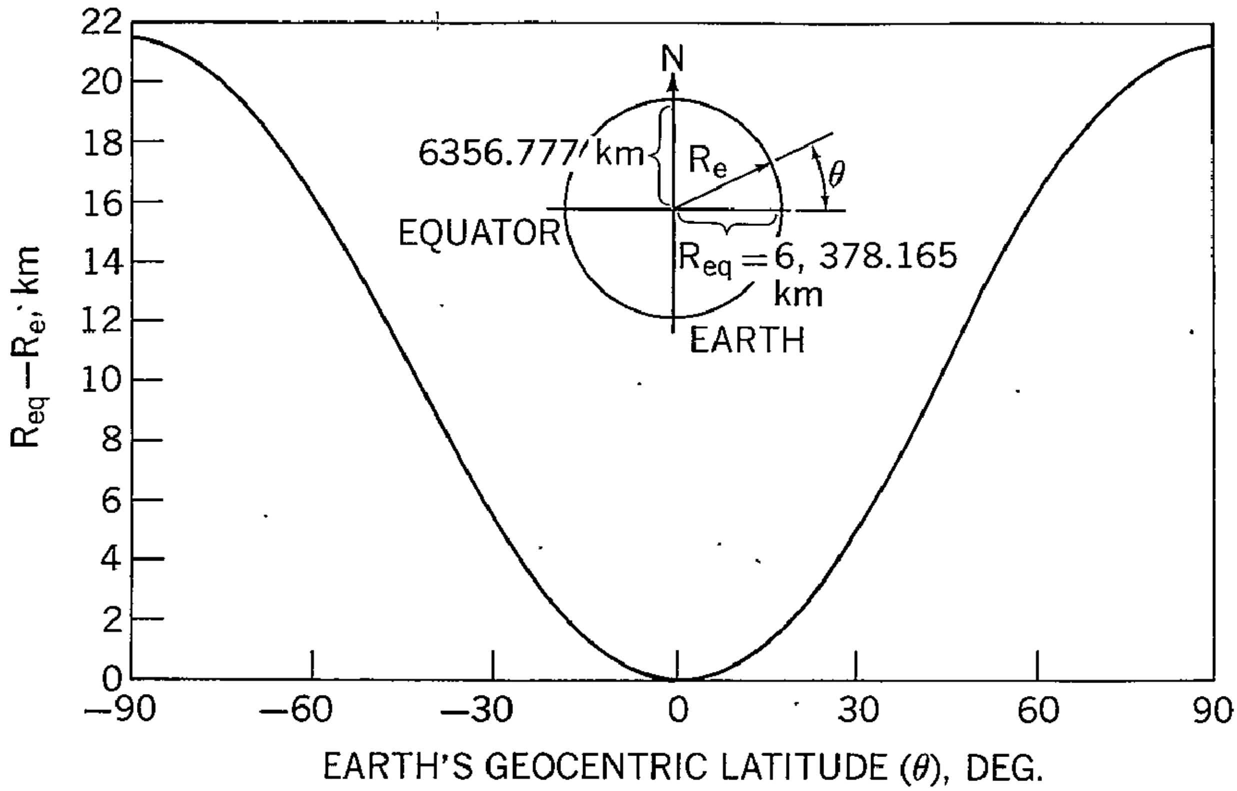


Figure A3–Earth's Radius as a Function of Latitude to First Order.

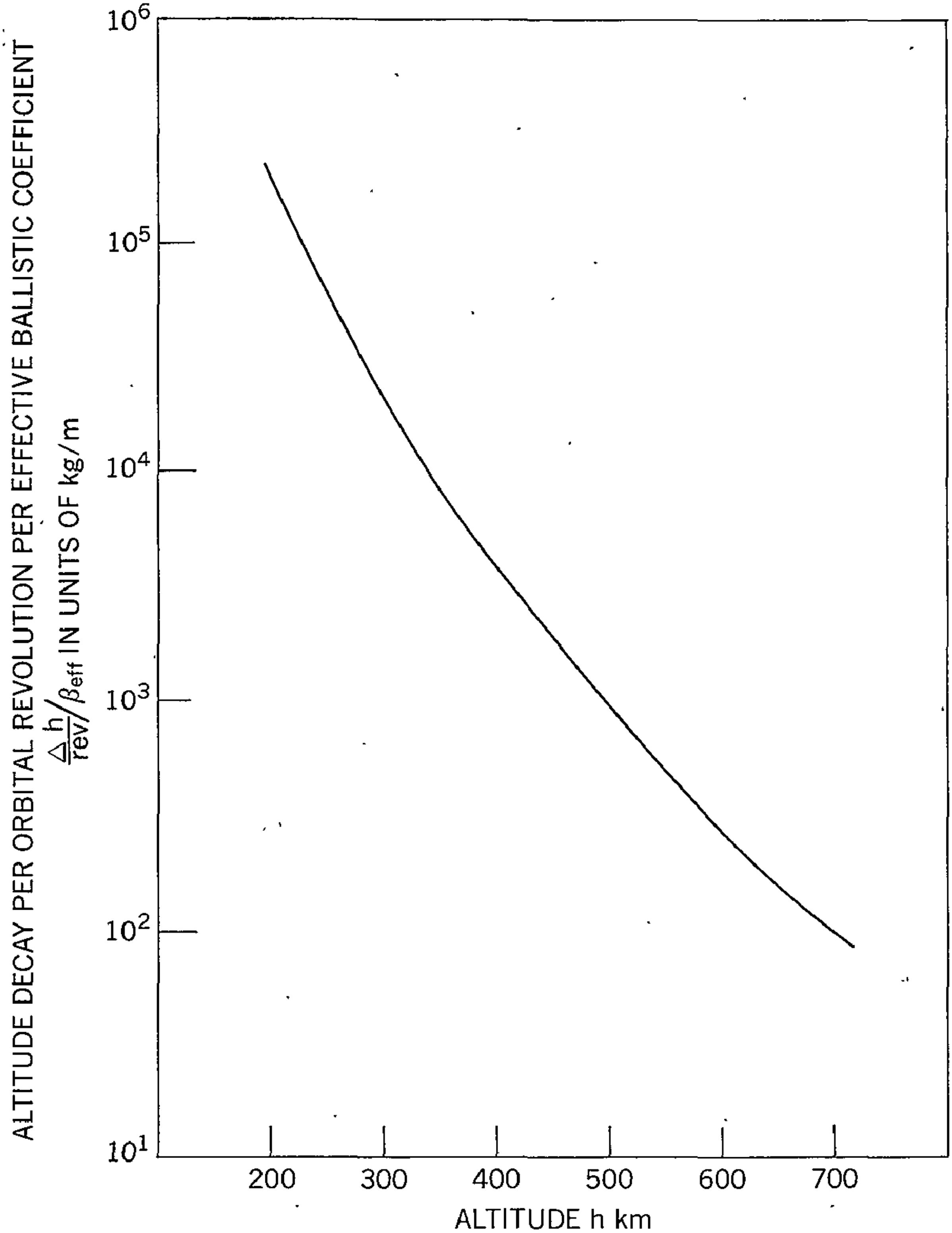


Figure B1–Satellite Altitude Decay in a Circular Earth Orbit; Nominal Atmosphere.

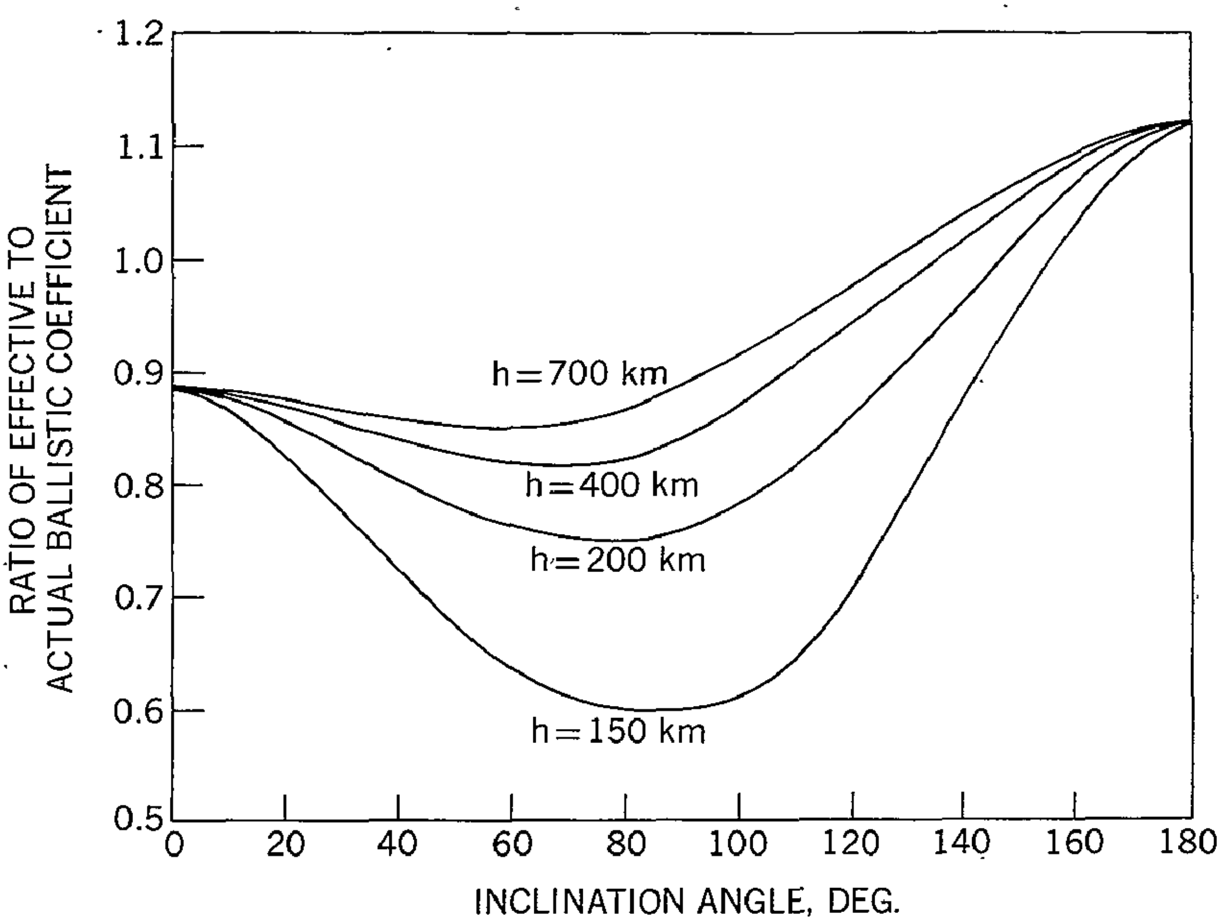


Figure B2–Ratio of Effective to Actual Ballistic Coefficient for Circular Orbits in the Nominal Earth's Oblate Rotating Atmosphere.